On The Computational Complexity of Syntactic Dependencies

Kenneth Hanson

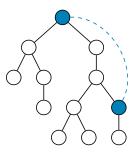
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Language as a computational problem

- 1. What kinds of computations are needed to build linguistic structures?
- 2. What can we gain from this knowledge?



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- Local and long-distance phonological maps (Chandlee and Heinz 2018; Burness et al. 2021)

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- Local and long-distance phonotactics (McMullin 2016; Heinz 2018)
- Local and long-distance phonological maps (Chandlee and Heinz 2018; Burness et al. 2021)
- Most syntactic dependencies

Selection (Graf 2018)

Functional hierarchies (Hanson 2023b)

Adjunction (Hanson under review)

Movement (Graf 2018, 2022b)

Case (Vu et al. 2019; Hanson 2023a)

Agreement (Hanson to appear)

Computational complexity: The old view

Modeled using **surface strings**, syntactic patterns are fairly **complex**

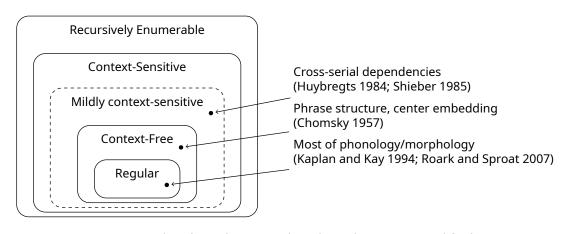


Figure 1: The Chomsky Hierarchy (Chomsky 1959), simplified

Computational complexity: The new view

Modeled using **trees**, syntactic patterns are **subregular**, along with most of phonology and morphology

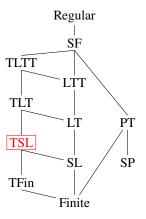


Figure 2: The Subregular Hierarchy (Heinz 2018; Lambert 2023), simplified

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Overall theme:

By focusing on the computational properties of language, we can discover new generalizations and build connections within and beyond linguistic theory.

Roadmap

What is a TSL computation?

A TSL model of agreement

Consequences for typology

Related and ongoing work

Gradience in experimental data

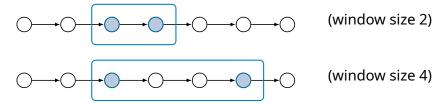
Other syntactic dependencies

Open questions

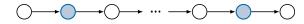
What is a TSL computation?

What does it mean to be local?

Local \rightarrow finitely bounded



Long-distance (non-local) \rightarrow no finite bound



Not just local, strictly local

Strictly local (SL): permitted/forbidden substrings of fixed size

Ex. Local assimilation

Window: 2

Good: nt, nd, mp, mb, ...

Bad: *np, *nb, *mt, *md, ...

Long-distance, but *local over a tier*

Tier-based strictly local (TSL): like SL, but *irrelevant elements are ignored*

Ex. Samala sibilant harmony (Heinz 2018)

Window: 2

Visible elements: s, ∫

Constraints: *s, *s



Also see: Heinz et al. (2011) and Lambert and Rogers (2020)

More about TSL

- SL is the special case of TSL where the tier contains everything
- TSL is distinct from autosegmental phonology (Goldsmith 1976)
 - Autosegmental tier: true multistratal representation
 - TSL: extra arcs in the basic string/tree representation

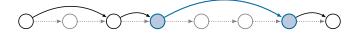


Figure 3: Two adjacent elements on a tier

Invisibility and blocking

TSL with a window size of 2 (TSL-2) can handle both invisibility and blocking

Ex. Slovenian sibilant harmony (McMullin and Hansson 2016)

Window:2Visible elements: s, \int, t Constraints: $*s\int, *\int s$

Also see McMullin (2016), Graf (2022b), and Hanson (to appear).

A TSL model of agreement

Locality of syntactic agreement

- (1) Minimality (Rizzi 1990)
 - a. The cat_{sg} chases_{sg} the rats.
 - b. * The cat **chase_{PL}** the rats_{PL}.

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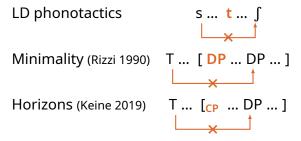
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Hanson (to appear):

The agreeing items must be adjacent on the relevant tier.

A unified model of locality

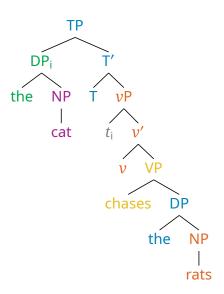
Long-distance dependencies are TSL-2 over their respective structures.

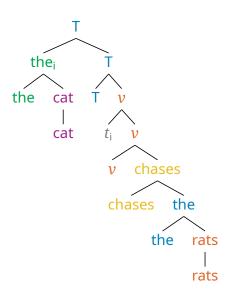


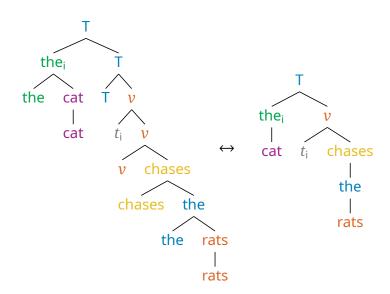
Setup

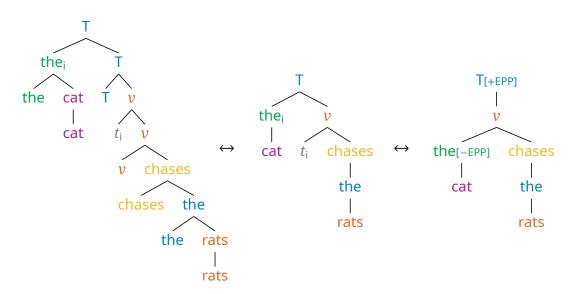
Key ingredients:

- Hierarchical representation
- Way to indicate long-distance dependencies
- Way to pick out the paths (=strings) which matter



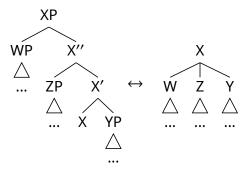






Derivation trees (2)

The rightmost child is the complement; others are specifiers or adjuncts.



See Graf and Kostyszyn (2021). Related: Brody (2000) and Kobele (2002, 2012).

Modeling long-distance dependencies

Diacritics mark items which *actually* move/agree/etc. in the current derivation, as in Minimalist Grammars (MGs, Stabler 1997, 2011).

Operation	Feature Type	Example
Move	Landing site	+EPP
	Mover	–EPP
Agree	Probe (valuee)	$+\phi$
	Goal (valuer)	$-\phi$



Paths, spines

Individual syntactic dependencies don't make use of the entire tree at once.

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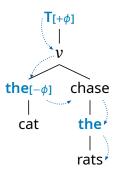
What path (=string) do we need for agreement?

Paths, spines

Individual syntactic dependencies don't make use of the entire tree at once.

What path (=string) do we need for agreement?

- Nodes ordered by derivational prominence
 ≈ reverse order of external Merge
 (Graf and Shafiei 2019)
 - ► Head < (Spec) < Comp
- At each branching point, follow the complement spine (Graf and De Santo 2019)



Also see: Kayne (1984), Uriagereka (1999), Adger (to appear), and Graf and Hanson (under review).

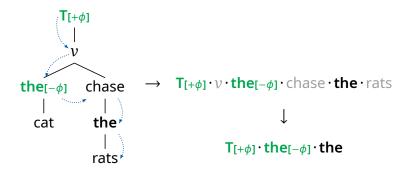
Tiers over spines

TSL grammar for English subject-verb agreement:

- Tier contents:
 all agreement participants (finite T, D) and blockers (C)
- Tier constraints: every ϕ -probe is immediately followed by a ϕ -goal, and vice versa

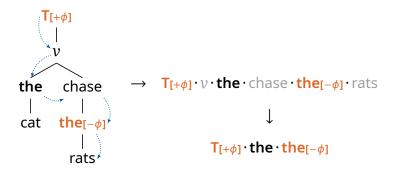
Minimality

✓ The cat chases the rats. (subject agreement)



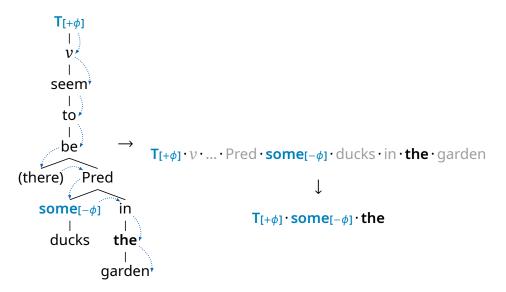
Minimality (2)

X The cat chase the rats. (object agreement)



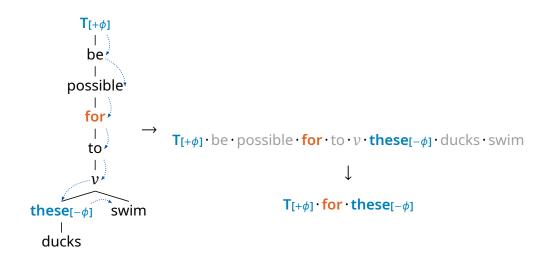
Invisibility

✓ There seem to be some ducks in the garden. (no blockers)



Blocking

X It are possible for these ducks to swim. (C is a blocker)



TSL grammar for subject-verb agreement

Window: 2

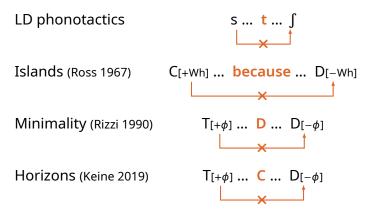
Tier: finite T, all D, all C

 $\text{Constraints:} \left\{ \begin{array}{ll} X_{[+\phi]} \cdot Y_{[+\phi]} & X_{[-\phi]} \cdot Y_{[-\phi]} \\ X_{[+\phi]} \cdot Y & X \cdot Y_{[-\phi]} \\ X_{[+\phi]} \cdot \bowtie & \rtimes \cdot Y_{[-\phi]} \end{array} \right\}$

Consequences for typology

A unified model of locality

TSL-2 captures the pattern of invisibility and blocking which is characteristic of long-distance linguistic dependencies.



Parameters of variation

In general, syntactic dependencies have different visibility conditions, and therefore require distinct tiers.

Dependency	Participants	Blockers
φ-agreement	T _{FIN} , all D	all C
EPP movement	T _{FIN} , all D, <i>there</i>	all C
wh-movement	C _{wh} , all <i>wh</i> -movers	if, because,

Table 1: Visibility conditions for English

Parameters of variation (2)

The contents of a given tier also vary across languages.

Language	Participants	Blockers
English Hindi	T_{FIN} , all D T_{FIN} , $D_{[NOM]}$	all C all C, T _{INF}

Table 2: Verbal agreement in English vs. Hindi

Case-sensitive agreement

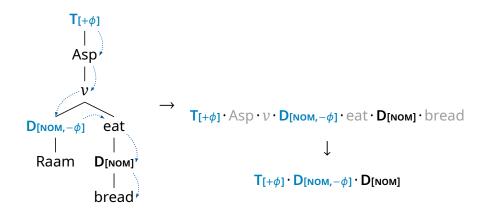
In Hindi, the verb agrees with the closest nominative argument, which may not be the subject.

- (3) Hindi verbal agreement ignores ergatives (Mahajan 1990)
 - a. Raam roţii khaat-aa th-aa.
 Raam.m.nom bread.f.nom eat-IPFV.m be-PST.m
 'Raam ate bread (habitually).'
 - b. Raam-ne <u>roţii</u> khaay-ii. Raam.M-**ERG** bread.F.NOM eat-PFV.F 'Raam ate bread.'

Analysis: Project D only if nominative. Tier constraints are unchanged.

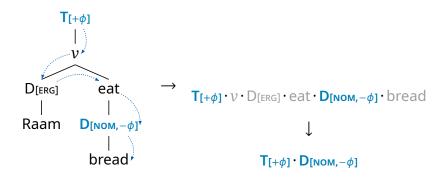
Case-sensitive agreement (2)

'Raam ate bread (habitually).' (nominative subject, subject agrees)



Case-sensitive agreement (3)

'Raam ate bread.' (ergative subject, object agrees)



Long-distance agreement

In Hindi, agreement can extend into vP, but not TP (Keine 2019).

- (4) Hindi LDA and default agreement (Bhatt 2005)
 - a. Ram-ne [vP roții khaa-nii] chaah-ii Ram-ERG bread.F eat-INF.F want-PFV.FSG 'Ram wanted to eat bread.'
 - b. Ram-ne [TP roţii khaa-naa] chaah-aa Ram-ERG bread.F eat-INF.M want-PFV.MSG 'Ram wanted to eat bread.'

Analysis: T_{INF} appears on the tier, but v does not.

Limits of variation

Ergative DPs are not necessarily invisible:

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- Case-visibility hierarchy (Bobaljik 2008):
 Unmarked (Nom/Abs) < Marked (Erg/Acc) < Oblique (Dat, Gen)

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When we consider the full range of variation, TSL-2 gets the right fit.

More parameters

Phenomenon	Change to baseline grammar	
Invisibility	Some D's fail to project	
Blocking	Some non-agreeing items project	
Upward agreement	Swap order of $+\phi/-\phi$	
Chain agreement	Allow sequential $+\phi$	
Multiple agreement	Allow sequential $-\phi$	
Independent subfeatures of ϕ	Each probe gets its own tier/constraints	

Table 3: Parameters for agreement (Hanson to appear)

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Upward complementizer agreement (Lubukusu, Diercks 2013)
 → Only project EPP-movers, −φ precedes +φ

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D[-EPP,-\phi] ... C[+\phi]
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- A'-agreement (Dinka, Van Urk 2015)
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- Parasitic agreement (Hindi, Bhatt 2005)
 - \rightarrow Second "concord tier" allows parasitic elements between $+\phi$ and $-\phi$

$$T[+\phi] \dots v [+\phi] \dots v [+\phi] \dots D[nom, -\phi]$$

Impossible patterns

Logically possible patterns which are not TSL-2 are generally unattested.

Туре	Class	Example	Visible Cs
Unbounded	TSL-2	Aari	Only sibilants
LD w/ blocking	TSL-2	Slovenian	All coronals
Transvocalic	TSL-2	Koyra	All consonants
At most 1 C intervenes	TSL-3	Unattested	_
Exactly 1 C intervenes	TSL-3	Unattested	_
At least 1 C intervenes	TLT/OTSL	Unattested	

Table 4: Typology of consonant harmony (adapted from McMullin and Hansson 2016)

Impossible patterns (2)

Non-TSL-2 and unattested island types (Graf 2022b):

- Gang-up islands: A mover can escape n islands, but not n + 1.
- Cowardly islands: XP is an island iff there are at least *n* XPs in the same clause.
- ..

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Non-TSL-2 and unattested verbal agreement patterns:

- T agrees with the subject unless there is a temporal adjunct, in which case it agrees with the object.
- Only DPs which contain a relative clause which contains two PPs can agree.
- ..

The big picture

- TSL computations are extremely restricted
 - ► No boolean logic
 - No counting of violations
 - Tier contents determined solely by the element labels
- Observed typology emerges from this highly structured logical space
- Syntax is revealed to be much more similar to phonology and morphology than previously thought (Graf 2022a)
 - Different structures, different features, but same computations

$$\mathsf{FIN} \subseteq \mathsf{SL} \subseteq \mathsf{TSL} \subseteq \ldots \subseteq \mathsf{REG} \subseteq \ldots \subseteq \mathsf{CFL} \subseteq \ldots \subseteq \mathsf{CS} \subseteq \ldots \subseteq \mathsf{RE}$$

Related and ongoing work

Islands and gradient acceptability

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Islands and gradient acceptability

- We have been treating islands and other blockers as being categorical
- Experimental studies have found that judgments of island violations are often gradient (Chaves 2022)
- Can this be incorporated into the TSL model?

Probabilistic TSL

Gradient blocking be captured via probabilistic tier projection (Mayer 2021).

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Torres, Hanson, Graf, and Mayer (2023):
Applied pTSL to syntactic islands, modeling data from Sprouse et al. (2016)



Charles Torres
UC Irvine



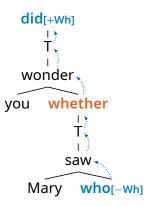
Connor Mayer UC Irvine



Thomas Graf Stony Brook

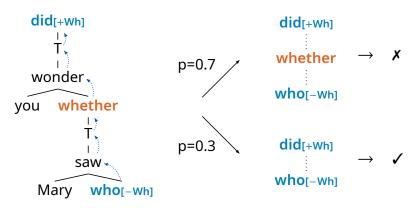
Probabilistic TSL: Example

'Who did you wonder whether Mary saw?'



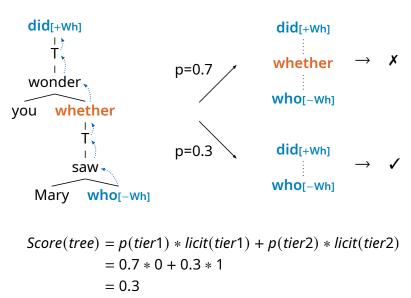
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- Three island types and four conditions

```
\left\{ \begin{array}{l} \textit{whether} \; \text{island} \\ \textit{adjunct} \; \text{island} \\ \textit{complex NP island} \end{array} \right\} \times \left\{ \begin{array}{l} \textit{non-island, matrix clause} \\ \textit{non-island, embedded clause} \\ \textit{island, matrix clause} \\ \textit{island, embedded} \end{array} \right\}
```

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Provided a structural analysis for each sentence

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- Three island types and four conditions

- Provided a structural analysis for each sentence
- Fit projection probability of island nodes to approximate mean Likert rating for each sentence (z-score normalized and transformed to range [0, 1])
 - other nodes fixed to 1 (movers/landing sites) or 0 (others)

Node	Projection probability
that	.46
complex NP whether	.63 .73
adjunct	.89

Table 5: Mean projection probabilities of fitted pTSL island model

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- Replicated superadditivity effect extraction from an island is worse than

 (i) extraction from non-island plus (ii) mere presence of an island
- Model does not capture relative badness of matrix clause movement

Probabilistic TSL: Discussion

- The TSL model is perfectly compatible with gradience in the grammar
 - ▶ categorical tier projection ↔ categorical output
 - ▶ probabilistic tier projection ↔ gradient output
- Further extensions:
 - Can add probabilities to the constraints to produce a stochastic grammar, which can be fit to corpus data in the same manner as a PCFG

Other syntactic dependencies

Local	Selection Functional hierarchies Adjunction	Graf (2018); Hanson (2023b) Hanson (2023b) Hanson (under review)
Long-distance	Movement Case Agreement	Graf (2018, 2022b) Vu et al. (2019); Hanson (2023a) Hanson (to appear)

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Hanson (2023b): modeling local dependencies with spines

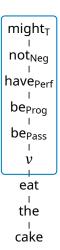
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• English clausal hierarchy (Adger 2003):

$$T < (Neg) < (Perf) < (Prog) < (Pass) < v$$

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- ex. 'The cake might not have been being eaten.'



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- ex. 'The cake might not have been being eaten.'
- TSL grammar

Tier elements: all symbols Window size: 2

Good substrings:

```
T Neg
T Perf Neg Perf
T Prog Neg Prog Perf Prog
T Pass Neg Pass Perf Pass Prog Pass
T v Neg v Perf v Prog v Pass v
```

might_T not_{Nea} have_{Perf} be_{Proq} be_{Pass} eat the cake

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- English clausal hierarchy (Adger 2003):
 T < (Neg) < (Perf) < (Prog) < (Pass) < v
- ex. 'The cake might not have been being eaten.'
- TSL grammar

Tier elements: all symbols Window size: 2
Good substrings:

```
T Neg
T Perf Neg Perf
T Prog Neg Prog Perf Prog
T Pass Neg Pass Perf Pass Prog Pass
T v Neg v Perf v Prog v Pass v
```

```
might<sub>T</sub>
 not_{Nea}
have<sub>Perf</sub>
 be_{Proq}
 be<sub>Pass</sub>
    eat
    the
  cake
```

Case dependencies

- Hanson (2023a): in-depth analysis of case in Japanese*
 - ► Three tiers: verbal domain, nominal domain, lexical case
 - Valency alternations, nominative objects, long-distance case

^{*}See appendix for an example.

Case dependencies

- Hanson (2023a): in-depth analysis of case in Japanese*
 - ► Three tiers: verbal domain, nominal domain, lexical case
 - Valency alternations, nominative objects, long-distance case
- Current research: to what extent do the parameters of a TSL-2 model account for cross-linguistic variation in case assignment?
 - Case spreading / multiple assignment
 - Dependent case
 - Accusative/ergative/split alignment
 - Differential argument marking

^{*}See appendix for an example.

• To what extent do the same patterns occur in different dependencies?

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- To what extent do the same patterns occur in different dependencies?
- Are constraint interactions across tiers always limited to intersection?
- How does the computational system interact with other factors, such as the feature system?
- Can we view the mapping from syntax to morphology as a TSL function?

Wrapping up

 To a large extent, linguistic structures can be built with TSL computations: Local or long-distance, trees or strings, harmony or agreement or movement.

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- Syntax and phonology are similar by virtue of their shared computational basis
- Theory can be connected to experiment via the underlying mathematics of TSL
- Many new questions

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Appendices

More agreement case studies

Upward agreement

A' agreement

Parasitic agreement

Learning tiers from positive data

Avoiding overgeneration in the category system

Miscellaneous

More agreement case studies

Upward agreement

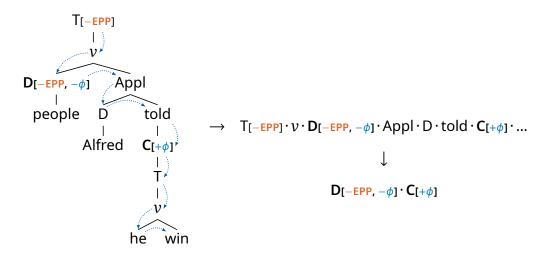
- (5) Complementizer Agreement in Lubukusu (Diercks 2013)
 - a. Ba-ba-ndu ba-bolela Alfredi ba-li a-kha-khile. c2-c2-people c2-said c1.Alfred c2-that c1-FUT-conquer 'The people told Alfred that he will win.'
 - b. Alfredi ka-bolela ba-ba-ndu a-li ba-kha-khile. c1.Alfred c1-said c2-c2-people c1-that c2-FUT-conquer 'Alfred told the people that they will win.'

Analysis:

- Agreement is upward \rightarrow allow $C_{[+\phi]}$ to follow $D_{[-\phi]}$.
- Agreement on C is subject oriented → project only DPs bearing –EPP.

Upward agreement (2)

'The people told Alfred that he will win.'



Analysis: Project DPs only if [-EPP]. Allow $C[+\phi]$ follow $D[-\phi]$.

Syntactic counterfeeding

Agreeing C is impossible in hyperraising structures.

(6) Agreeing complementizer incompatible with hyperraising

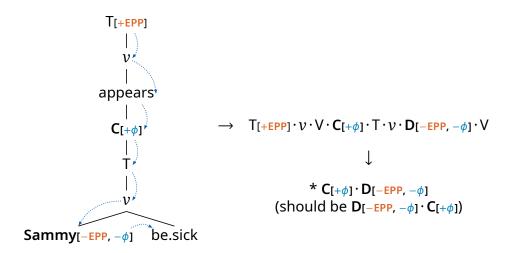
Sammy a-lolekhana mbo (*a-li) a-likho a-lwala. c1.Sammy c1-appears that (*c1-that) c1.PROG c1-be.sick

'Sammy appears to be sick.' (lit. 'Sammy seems that is sick.')

This follows immediately from the TSL analysis!

Syntactic counterfeeding (2)

The hyperraised subject is below C in the derivation tree and cannot agree.



A' agreement

- (7) Dinka verbal agreement with Spec-CP
 - a. Mòc à-cé yîin tîiŋ.

 man 3sg-PRF.SV you see.NF

 'The man has seen you.'

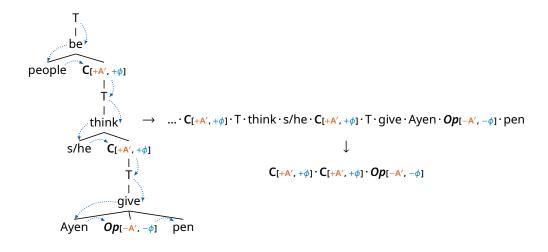
 (Van Urk 2015, ch. 4, 19b)
 - b. Yîin Ø-cíi môc tîiŋ.
 you 2-PRF.OV man.GEN see.NF
 'You, the man has seen.'
 (Van Urk 2015, ch. 4, 20a)
- (8) Agreement in both matrix and embedded clause

```
Yè kôɔc-kó [CP Op é-kè-yá ké tàak [CP è ____
be people.CS-which PST-PL-HAB.2SG 3PL think.NF C
é-kè-cíi Áyèn ké gâam gàlàm]]?
PST-PL-PRF.OV Ayen.GEN 3PL give.NF pen
```

'Which people did (s)he think that Ayen had given a pen to?'
(Van Urk 2015, ch. 5, 14a)

A' agreement (2)

'Which people did (s)he think that Ayen had given a pen to?'



Analysis: Project all D bearing [-A'], allow $C_{[+\phi]}$ to iterate.

Parasitic agreement

- (9) Hindi participles and infinitives agree iff the main verb does
 - a. Shahrukh-ne [vP tehnii kaaṭ-nii] chaah-ii thii Shahrukh-ERG branch.F cut-INF.F want-PFV.F be.PST.FSG 'Shahrukh had wanted to cut the branch.'
 - b. Shahrukh-ne [TP tehnii kaat-naa] chaah-aa thaa Shahrukh-ERG branch.F cut-INF.M want-PFV.MSG be.PST.MSG 'Shahrukh wanted to cut a/the branch.'

Analysis: Verbs agree iff they occur along a chain from $T_{[+\phi]}$ to $D_{[-\phi]}$.

Complication:

T-agreement and V-agreement must be regulated on separate tiers.

Parasitic agreement (2)

If T agrees with DP, then all verbs along the path also agree.

```
T-agreement tier: T_{\text{FIN}[+\phi]} \text{Aux}_{[+\phi]} v_{[+\phi]} D_{[\text{ERG}]} V v_{[+\phi]} V D_{[\text{NOM},-\phi]} V-agreement tier: T_{\text{FIN}[+\phi]} \text{Aux}_{[+\phi]} v_{[+\phi]} D_{[\text{ERG}]} V v_{[+\phi]} V D_{[\text{NOM},-\phi]}
```

Why isn't one tier enough?

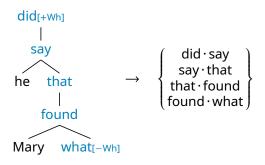
- Agreement can fail, so non-agreeing pairs $T \cdot Aux$, $Aux \cdot v$, etc., must be allowed.
- Once you do this, agreement is incorrectly predicted to always be optional.

Learning tiers from positive data

Learning tiers from paths

Heuser et al. (2024): track *licit bigrams along a movement path*, and generalize to categories when allowed by the Tolerance Principle (Yang 2016)

Hanson (2024): this algorithm already tracks the information needed by a formal TSL learner \rightarrow we can adapt it to produce a TSL-2 grammar!



See appendix for details.

Learnability considerations

TSL constraints are easy to learn if the tier is already known (Lambert et al. 2021)...

But identifying the tier itself is not trivial

• If there are n symbols, then there are 2^n possible tiers

Current formal learners can do efficiently this for pure TSL-2 languages (Jardine and McMullin 2017; Lambert 2021)...

But natural language always involves intersecting tiers

Starting point

Let's assume that:

- The learner already knows the basics of constituency and selection
- The learner can identify the dependent elements, e.g. mover and landing site, probe and goal

The dependent elements must appear on the tier, and must be adjacent

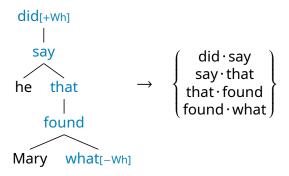
Next step: identify the blockers

Blockers for wh-movement

- ✓ What did Mary find ——?
- ✓ What did he say that Mary found ——?
- X What did he wonder whether Mary found ___?
- **X** What did he mutter that Mary found ___?

Learning blockers from paths

Heuser et al. (2024): learn movement blockers by tracking licit bigrams along a movement path (plus generalization to categories as in Yang 2016)



Key point: Bigrams like {wonder·whether, whether·found, ...} will not be attested along any *wh*-movement path

Learning blockers from paths (2)

Hanson (2024): the path grammar contains everything we need to construct a TSL-2 grammar

- Extract the elements from the set of path bigrams for movement type X
- The elements which are conspicuously missing are just the tier blockers

$$\left\{ \begin{array}{c} \text{that} \\ \text{whether} \\ \text{say} \\ \text{mutter} \\ \dots \end{array} \right\} - \left\{ \begin{array}{c} \text{that} \\ \text{say} \\ \dots \end{array} \right\} = \left\{ \begin{array}{c} \text{whether} \\ \text{mutter} \\ \dots \end{array} \right\}$$

Learning blockers from paths (3)

Advantages to the combined system:

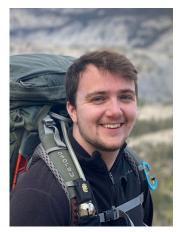
- Combines the typological merits of TSL-2 with an empirically-motivated acquisition model
- Produces a version of the Height-Locality Connection (Keine 2019), which is not inherent to the TSL-2 model
- Could be adapted for learning constraints on agreement

Summary

- The space of possible tiers is too large to search exhaustively, but we don't actually need to do this
- When learning non-local constraints, the local constraints should be factored out
- Some empirical generalizations may derive from the details learning process rather than the grammar formalism itself

Avoiding overgeneration in the category system

Collaborators



Logan Swanson Stony Brook



Thomas Graf Stony Brook

The problem with subcategorization

- There is a LI of category V that selects a CP
- …only if that CP contains an AP
- ...and also contains three VPs, none of which contain one another
- ...but only if the LI does not also select a PP
- ...in which case the three VPs must instead be self-embedded
- ...and CP must also contain a non-finite TP
- ...

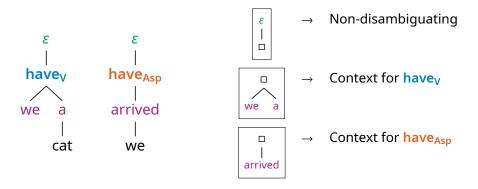
Without further restrictions, any system of category selection can do this

Reining in category features

- Any regular tree constraint can be indirectly implemented via selection
- Selection is SL-2 over trees
 - → Massive overgeneration
 - → All subregular distinctions are lost
- Conjecture: category and subcategorization features must be inferable from the local tree context → ISL-recoverable (Graf 2020)
- This project: test this conjecture against MGBank (Torr 2017, 2018)

ISL-Recoverability

It should be possible to infer the category of every lexical item based only on the phonological features of its **selector** or its **selectees** (or maybe both).



Results

Forms with ambiguous category:	8369
ISL-2 recoverable in all contexts:	79-86%
ISL-2 recoverable in at least one context:	95–97%

- Most errors are due to empty heads or misparsed sentences (good!)
- Subcategorization features are recoverable at a similar rate (good!)
- Movement features are recoverable at a similar rate (not good!)

Discussion

- It should not be possible to accurately identify all movement features from the local context
- Simulations suggest that a Zipfian distribution of feature specifications over phonetic exponents produces similar quantitative results
 → ISL inferrability might be epiphenomenal
- In any case, local inferrability makes a good heuristic for learning: identical items in the same context must have the same category

Miscellaneous

What makes a possible tier?

We know that the tier elements can be fairly arbitrary, but they can't be completely arbitrary.

- T = {the[Nom], all[Acc], for_C, smile_V, already_{Adv}, ...}
- $\bar{T} = \{ \text{the}_{[Acc]}, \text{ all}_{[Nom]}, \text{ that}_{C}, \text{ wash}_{V}, ... \}$

Usually, we want entire classes of items:

all D, all D[NOM], all D[-ЕРР], ...

A couple of possibilities:

- a tier must be defined in terms of natural classes (insert your favorite theory of syntactic features here)
- unnatural tiers are never posited by the learner under realistic conditions

Case in Japanese

Hanson (2023a): in-depth analysis of case in Japanese, relativizing both dominance and precedence to a tier \rightarrow **tree tier**

(10) Mearii no imooto ga Jon ni ringo o ageta. Mary GEN sister NOM John DAT apple ACC gave 'Mary's sister gave John an apple.'

