

EGBE 331 – Control Systems for BME

Project Assignment — 2 people per group

Due: 5 June 2020

1. Create the open-loop transfer function $\frac{K}{s(T_1s+1)(T_2s+1)}$
2. Specify the range of K between 1 to 10 and specify T₁, T₂ in the range 0.1–0.99 sec.
3. Plot the output response of the open-loop system for a unit step input, and measure the following (Maximum overshoot, Settling time, Peak time, Steady-state error).
4. Create the closed-loop system with unity feedback.
5. Plot the output response of the closed-loop system (step input) and measure Maximum overshoot, Settling time, Peak time, Steady-state error.
6. Design a Lead or Lag-Lead compensator using the Root Locus method, according to specifications chosen by the student (maximum overshoot of 25% or less, settling time of 3 sec or less and static velocity error constant K_v is 50 sec⁻¹).
7. Apply the designed compensator to the system, input a unit step, and plot the output. Measure maximum overshoot, settling time, peak time, steady-state error. Compare these results to those from earlier tasks.
8. Draw the circuit diagram of the Lead compensator, including values of R and C from task 6.
9. Plot the Bode Diagram of the system in task 1, and determine Gain margin and Phase margin
10. Design a Lead or Lag compensator using the Bode Diagram method, according to specifications chosen by the student (static velocity error constant K_v is 20 sec⁻¹, the phase margin is at least 50°, and the gain margin is at least 10dB)
11. Apply this compensator to the system, input a unit step, and plot the output. Measure maximum overshoot, settling time, peak time, steady-state error. Compare these results to earlier tasks.
12. Draw the circuit diagram of the Lead compensator from task 10, including R and C values.

13. Design a PD or PI compensator using the Root Locus method, according to chosen specifications (maximum overshoot less than 20% and settling time of 5 sec or less)
14. Apply this compensator to the system, input a unit step, and plot the output. Measure maximum overshoot, settling time, peak time, steady-state error
Compare these results to earlier tasks.
15. Draw the circuit diagram of the PD or PI compensator from task 13, including R and C values

Plot input and output from following open loop transfer function:

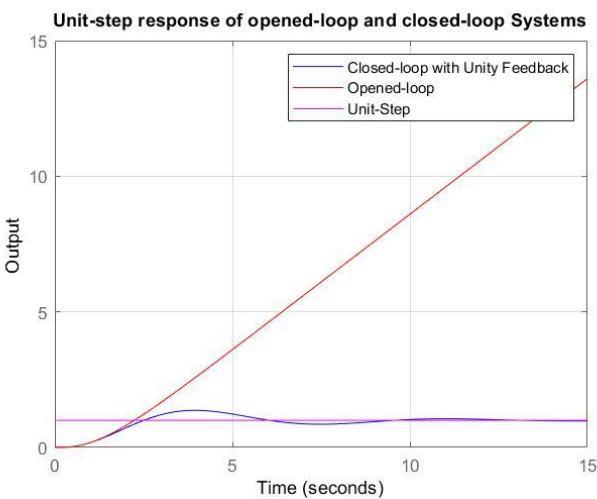
$$\frac{K}{s(T_1s + 1)(T_2s + 1)}$$

with $K = 1$, $T_1 = 0.5$, $T_2 = 0.9$

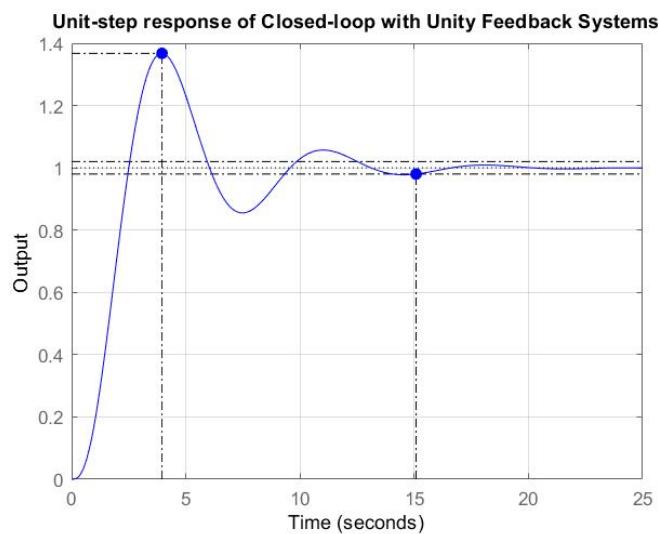
Hence open loop transfer function is

$$\frac{1}{s(0.5s + 1)(0.9s + 1)}$$

Unit-Step response of this transfer function before and after add unity feedback to this system show in Figure 1(a)



(a)



(b)

Figure 1

(a) Unit-Step response curves for the opened-loop and closed-loop systems;

(b) Unit-Step response curves for closed-loop with unity-feedback system.

While unit-step response of opened-loop system does not have final value because system is divergence, so maximum overshoot, settling time cannot measure.

The maximum overshoot of unit-step response is 36.8%, the settling time is 15.1 secs, the peak time is 3.96 sec and the steady state error equals to 1 as show in Figure 1(b).

Unit-Ramp response of this transfer function before and after adding unity feedback to this system show in Figure 2

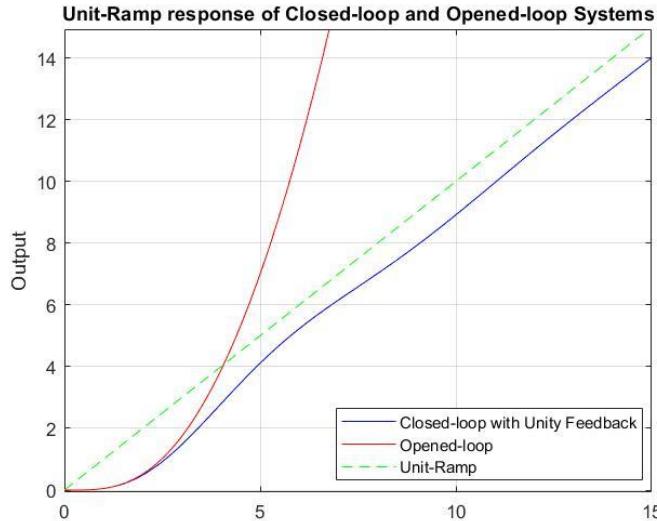


Figure 2

Unit-Ramp response curves for the opened-loop and closed-loop systems.

Design a compensator such that the unit-step response curve will exhibit maximum overshoot of 25% or less, settling time of 3 sec or less and static velocity error constant K_v is 50 sec^{-1}

From $T_s = \frac{4}{\zeta \omega_n}$, $M_p = e^{-\frac{\zeta \pi}{\sqrt{1-\zeta^2}}}$ then we desired $\zeta = 0.4037$ and $\omega_n = 3.3027$ so desired dominant closed-loop poles are located at

$$s = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1} = -1.439 \pm j3.2607$$

Let us assume that a lag-lead compensator has the transfer function

$$G_c(s) = K_c \left(\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} \right) \left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}} \right), (\gamma > 1, \beta > 1)$$

where γ is not equal to β

Since $\left| \frac{1}{s(0.5s+1)(0.9s+1)} \right|_{s=-1.439+j3.2607} = -289.79^\circ$ so the angle of deficiency = 109.79°

The time constant T_1 and the value of γ are determined from

$$\left| \frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} \right| \left| \frac{1}{s(0.5s+1)(0.9s+1)} \right|_{s=-1.439+j3.2607} = 1, \quad \left| \frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} \right|_{s=-1.439+j3.2607} = 109.79^\circ$$

We can locate point A and B such that

$$\angle APB = 109.79^\circ, \quad \frac{PA}{PB} = 0.0575$$

(Use Law of Sine and Law of Cosine) The result is

$$PA = 1.3723, \quad PB = 61.433$$

$$AO = 0.0666, \quad BO = 62.872$$

or

$$T_1 = 15.015, \quad \gamma = 952.57$$

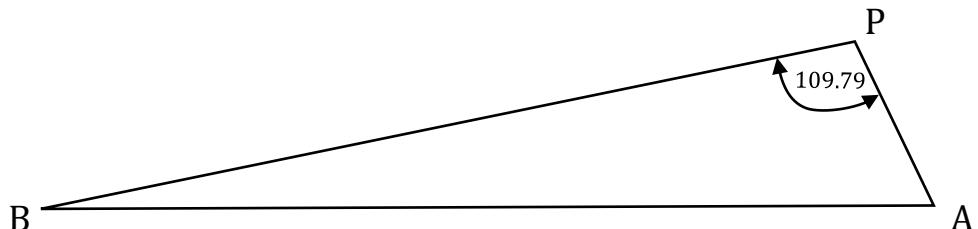


Figure 3

Determination of the desired pole-zero location.

The phase-lead portion of the lag-lead network thus becomes

$$K_c \left(\frac{s + 0.066}{s + 62.87} \right)$$

Next, determine the value of K_c from magnitude condition

$$\left| K_c \left(\frac{s + 0.266}{s + 45.88} \right) \frac{1}{s(0.5s+1)(0.9s+1)} \right| = 1$$

Hence,

$$K_c = 302.4058$$

Since the requirement on the static velocity error constant K_v is 50 sec^{-1} , we have

$$K_v = \lim_{s \rightarrow 0} sG_c G = \lim_{s \rightarrow 0} s K_c \left(\frac{T_1 s + 1}{T_1 s + \gamma} \right) \left(\frac{T_2 s + 1}{T_2 s + \frac{1}{\beta}} \right) \frac{1}{s(0.5s + 1)(0.9s + 1)} = \frac{\beta}{\gamma} K_c$$

$$50 = \frac{\beta}{\gamma} K_c$$

Hence, β is determined as

$$\beta = 156.0918$$

Finally, we choose the value T_2 such that the following conditions are satisfied:

$$\left| \frac{s + \frac{1}{T_2}}{s + \frac{1}{156.09T_2}} \right|_{s=-1.439+j3.2607} \cong 1.00, \quad -5^\circ < \left| \frac{s + \frac{1}{T_2}}{s + \frac{1}{156.09T_2}} \right|_{s=-1.439+j3.2607} < 0^\circ$$

and compare unit-step responses of compensated system for the best performance.

Since $T_2 = 3.03$ satisfies the two conditions, the compensated system will have the open-loop transfer function

$$G_c(s)G(s) = 302.41 \left(\frac{s + 0.066}{s + 62.87} \right) \left(\frac{s + 0.333}{s + 0.0021} \right) \frac{1}{s(0.5s + 1)(0.9s + 1)}$$

The characteristic equation of the compensated system is $s^5 + 65.99 s^4 + 198 s^3 + 812.1 s^2 + 266.8 s + 14.77 = 0$

Hence the new closed-loop poles are located at

$$s = -1.29278 \pm j3.19576$$

The other closed-loop poles are located at

$$s = -0.2814; \quad s = -0.070; \quad s = -63$$

The new maximum overshoot is 17.4% and settling time is 2.74 sec.

The unit-step response curves of the compensated and uncompensated systems are shown in Figure. The unit-ramp response curves for both systems are depicted in Figure 4 (a). And from new static velocity error constant K_v is 50 sec^{-1} , it shows steady-state error of compensated system equal 0.2.

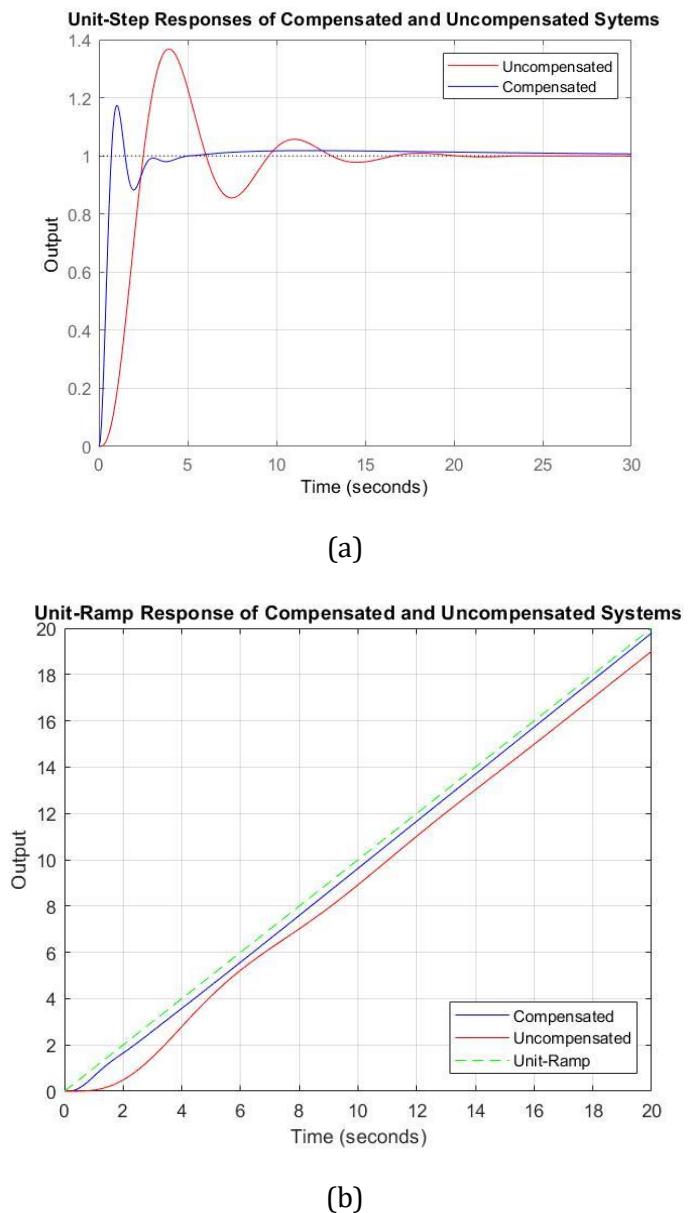


Figure 4

(a) Unit-step response curves for the compensated and uncompensated systems;
 (b) Unit-ramp response curves for the compensated and uncompensated systems.

From lag-lead compensated systems can make by following electrical circuits as shown values in Figure 5

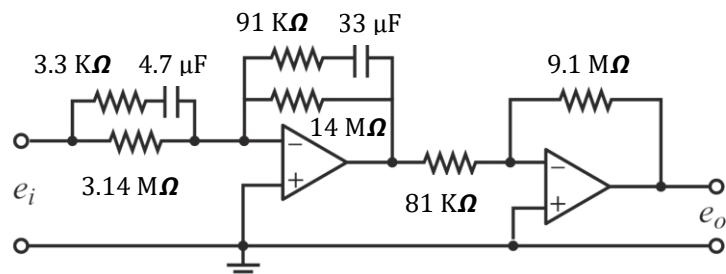


Figure 5

Lag-lead system by operational-amplifier circuits

Design a compensator using Bode diagram for the system so that static velocity error constant K_v is 20 sec^{-1} , the phase margin is at least 50° , and the gain margin is at least 10dB.

Let us assume that a lag-lead compensator has the transfer function

$$G_c(s) = K_c \left(\frac{s + \frac{1}{T_1}}{s + \frac{\beta}{T_1}} \right) \left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}} \right)$$

where $\beta > 1$

It is desired that the static velocity error constant K_v be 20 sec^{-1} , the phase margin is at least 50° , and the gain margin is at least 10 dB.

As we assume a lag-lead compensator given by previous equation, the static velocity error constant K_v can calculate

$$K_v = \lim_{s \rightarrow 0} s G_c G = \lim_{s \rightarrow 0} s G_c(s) \frac{K}{s(0.5s + 1)(0.9s + 1)} = K$$

Hence,

$$K = 20$$

Next, we shall draw the Bode diagram of the uncompensated system with $K = 20$, as shown in Figure 6. The phase margin of the gain-adjusted but uncompensated system is -40.2° , which state that this system is unstable. We choose a new gain crossover frequency where phase = -180° at $\omega = 1.49 \text{ rad/s}$. After selecting new gain crossover frequency to be 1.49 rad/s , the corner frequency $\omega = 1/T_2$ to be 1 decade below the new gain crossover frequency at $\omega = 0.149 \text{ rad/s}$

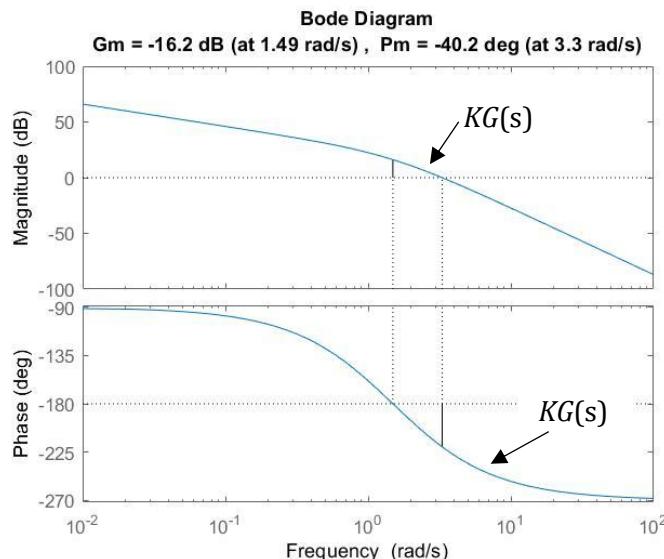


Figure 6

Bode diagram for G (gain-adjusted but uncompensated open-loop transfer function).

And from

$$\sin \phi_m = \frac{1 - \frac{1}{\beta}}{1 + \frac{1}{\beta}} = \frac{\beta - 1}{\beta + 1}$$

we selected $\phi_m = 50^\circ + 11.9^\circ = 61.9^\circ$, we may choose $\beta = 16$

Then the corner frequency $\omega = \frac{1}{\beta T_2} = 0.024845$ rad/s. The transfer function of the phase-lag portion of the lag-lead compensator then becomes

$$\frac{s + 0.14907}{s + 0.02485} = 16 \left(\frac{6.70826s + 1}{40.2414s + 1} \right)$$

At the new gain crossover frequency at $\omega = 1.49$ rad/s, $|G(j1.49)|$ is 16.2dB and the corner for the lead portion can be determine from intersections of straight line of slope 20dB/decade through point (1.49 rad/s, -16.2dB) and $\omega = 0.95831$ rad/s and $\omega = 15.333$ rad/s. Thus, the transfer function of the phase-lead portion of the lag-lead compensator then becomes

$$\frac{s + 0.95831}{s + 15.333} = \frac{1}{16} \left(\frac{1.0435s + 1}{0.06522s + 1} \right)$$

Combining the transfer function of the lag and lead portion of the compensator, we obtain the transfer function $G_c(s)$ of the lag-lead compensator:

$$G_c(s) = \left(\frac{1.0435s + 1}{0.06522s + 1} \right) \left(\frac{6.70826s + 1}{40.2414s + 1} \right)$$

The Bode diagram of the lag-lead compensator $G_c(s)$ and open-loop transfer function $G_c(s)G(s)$ can view from Figure 7. The new phase margin is 52.6° , gain margin is 21.6dB, phase crossover frequency is 1.18 rad/s and gain crossover frequency is 5.55 rad/s.

After combine unit-step response of this compensated system, characteristic equation is $(0.06522s + 1)(40.2414s + 1)s(0.5s+1)(0.9s+1)+20(1.0435s + 1)(6.70826s + 1) = 1.18104 s^5 + 21.8123 s^4 + 59.5038 s^3 + 181.708 s^2 + 156.035s + 20$. The performance is shown in Figure 8.

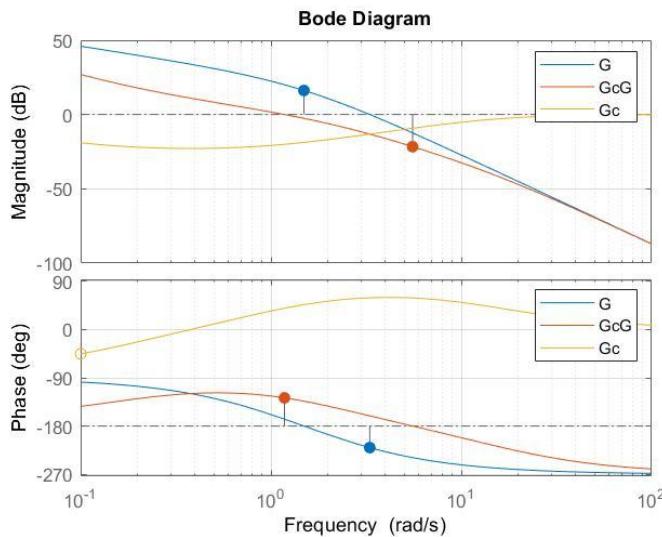


Figure 7

(a) Bode diagram for G (gain-adjusted but uncompensated open-loop transfer function), G_c (lag-lead compensator), and G_cG (open-loop transfer function).

(a)

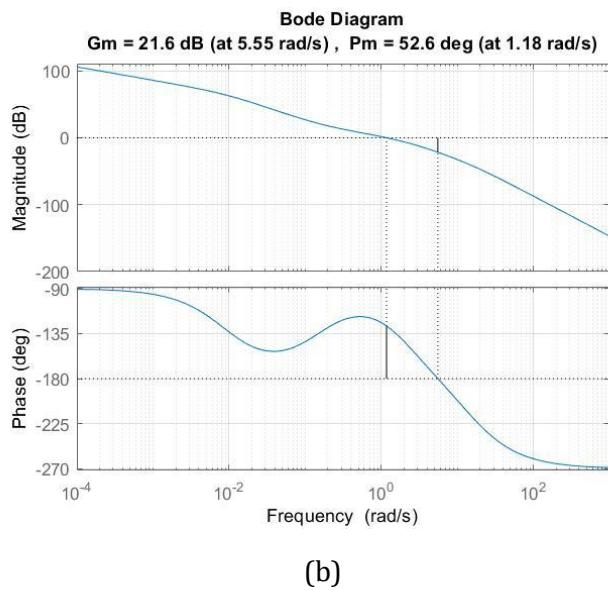


Figure 7

(b) Bode diagram for G_cG (open-loop transfer function).

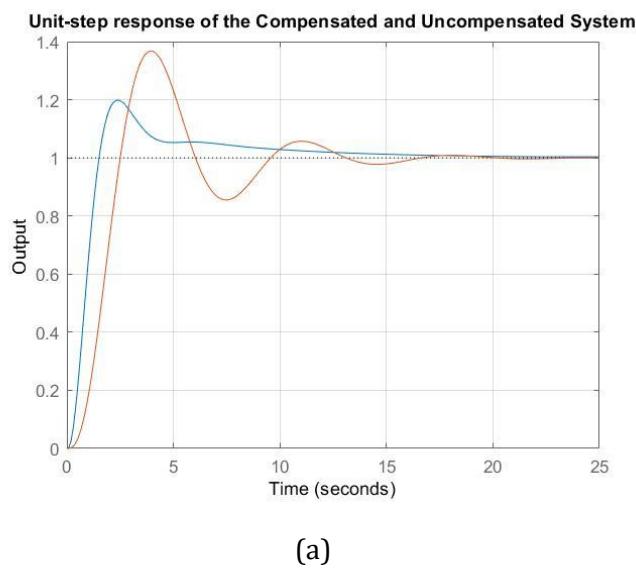
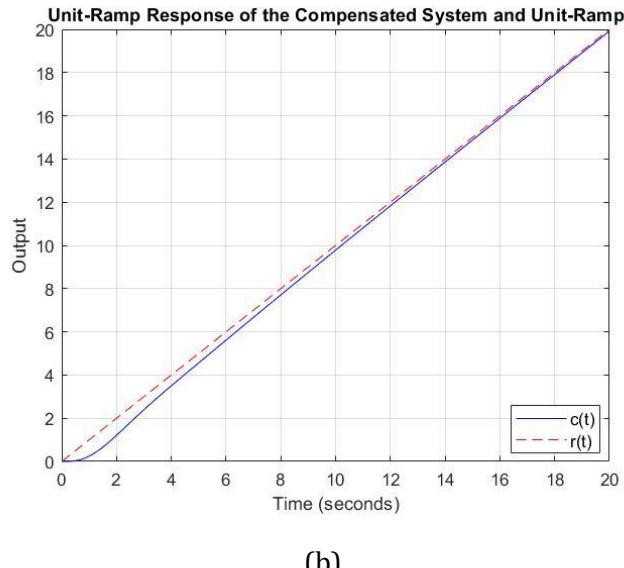


Figure 8

(a) Unit-Step response of the compensated and uncompensated systems;

(b) Unit-Ramp response of the compensated system and unit-ramp.



From lag-lead compensated systems can make by following electrical circuits as shown values in Figure 9

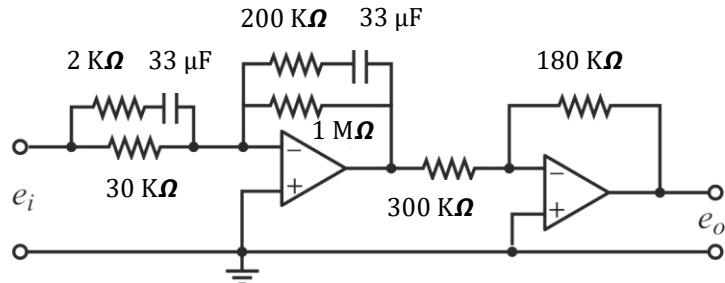


Figure 9

Lag-lead system
by operational-
amplifier circuits.

Design a PID controller for the system so that maximum overshoot less than 20% and settling time of 5 sec or less.

The PID controller has the transfer function

$$G_C(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

Apply a Ziegler-Nichols tuning rule for determine the values of parameters Kp, Ti, and Td.

From following the second method of Ziegler-Nichols tuning rules. We obtain the closed-loop transfer function:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(0.5s + 1)(0.9s + 1) + K_p}$$

The value of Kp that makes the system marginally stable can be calculate by using Routh's stability criterion. Since the characteristic of closed-loop system is

$$0.45s^3 + 1.4s^2 + s + K_p = 0$$

We find that sustained oscillation will occur if $K_p = 28/9$. Thus, the critical gain K_{cr} is

$$K_{cr} = 28/9$$

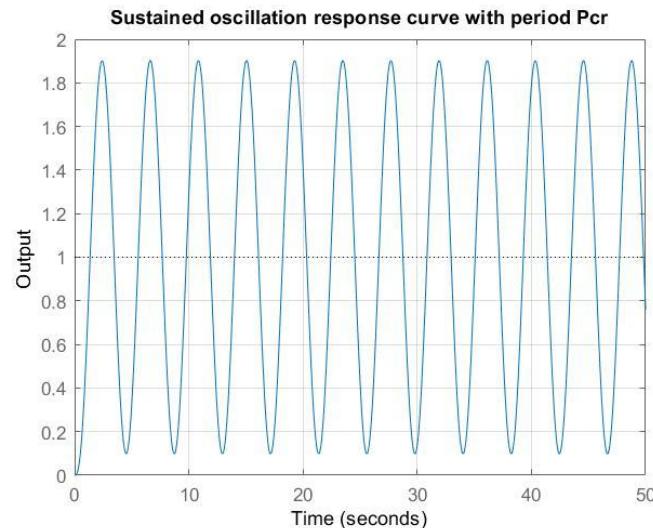


Figure 10

Sustained oscillation response curve with period P_{cr} .

We can determine P_{cr} from Figure 10. Hence P_{cr} is

$$P_{cr} = 4.214$$

Referring to Ziegler-Nichols methods, K_p , T_i and T_d are determined as follows:

$$K_p = 0.6K_{cr} = 1.867$$

$$T_i = 0.5P_{cr} = 2.107, K_i = 0.886$$

$$T_d = 0.125P_{cr} = 0.527, K_d = 0.984$$

The transfer function of the PID controller is thus

$$\begin{aligned} G_C(s) &= K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \\ &= 1.867 \left(1 + \frac{1}{2.107 s} + 0.527 s \right) \\ &= 0.98373 \frac{(s + 0.9984)^2}{s} \end{aligned}$$

The PID controller has a pole at the origin and double zero at $s = -0.9984$. The resulting unit-step response curve is shown in Figure 11. The maximum overshoot in the unit-step response is 62.9%

And from Figure 12, Root Locus show that although we keep same zero of PID transfer function and adjust gain does not improve performance of system. Hence, we change zeros of PID controller.

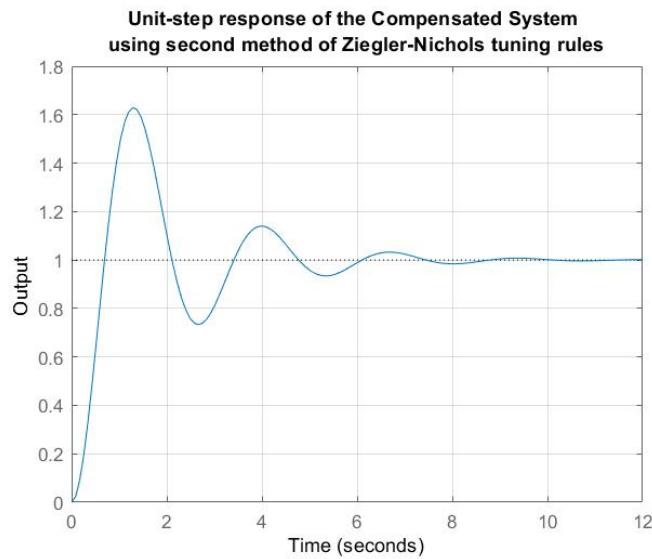


Figure 11

Unit-step response of the compensated System using second method of Ziegler-Nichols tuning rules.

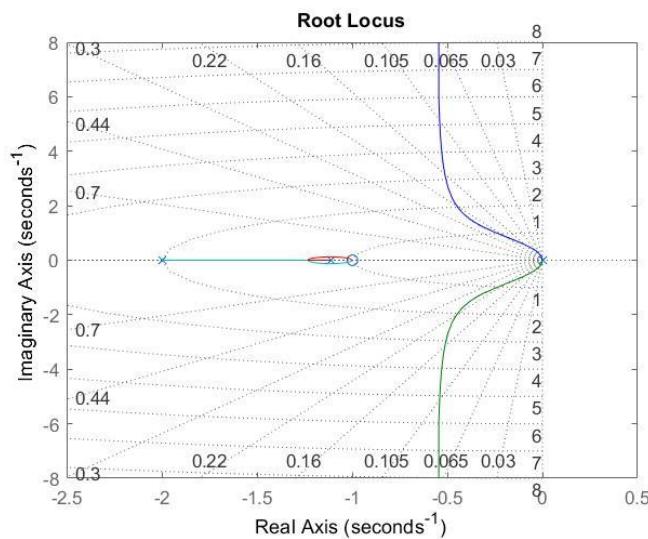


Figure 12

Root Locus of transfer function of the PID controller.

From

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right) = K \frac{(s+a)(s+b)}{s}, \quad K_p = (a+b)K, \quad T_i = \frac{a+b}{ab}, \quad T_d = \frac{1}{a+b}$$

The pole of PID controller is located at $s = 0$ and zeros at $s = -a, s = -b$

We selected

$$K = 4.2$$

$$a = 0.28$$

$$b = 0.05$$

so, we get transfer function of PID is

$$G_C(s) = 4.2 \frac{(s + 0.28)(s + 0.05)}{s} = 1.386 \left(1 + \frac{1}{23.5714s} + 3.0303s \right)$$

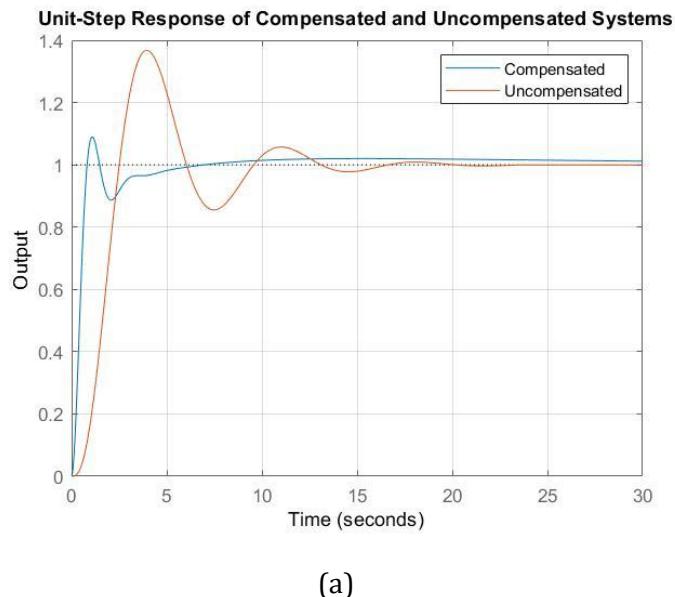
or

$$K_p = 1.386$$

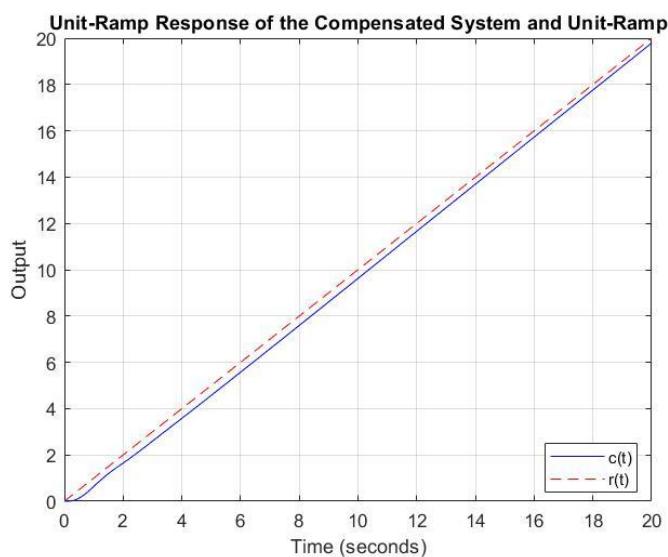
$$T_i = 23.5714, K_i = 0.0588$$

$$T_d = 3.0303, K_d = 4.2$$

The unit-step response of compensated system is on Figure 13(a), the maximum overshoot is 9.03%, settling time is 4.89 sec and new static velocity error constant K_v is 50 sec^{-1} , it shows steady-state error of compensated system equal 0.2.



(a)



(b)

Figure 13

(a) Unit-Step response of the compensated and uncompensated systems;

(b) Unit-Ramp response of the compensated system and unit-ramp.

From figure 14, it states that the new transfer function of the PID controller is more stable than the old one.

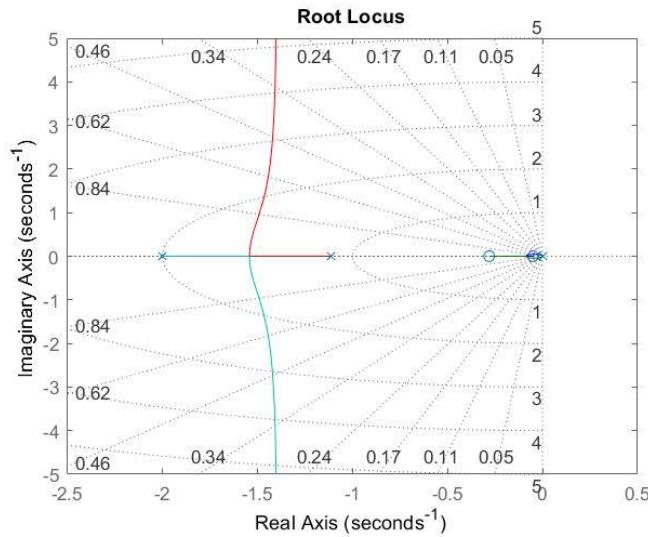


Figure 14

Root Locus of new transfer function of the PID controller.

From PID compensated systems can make by following electrical circuits as shown values in Figure 15.

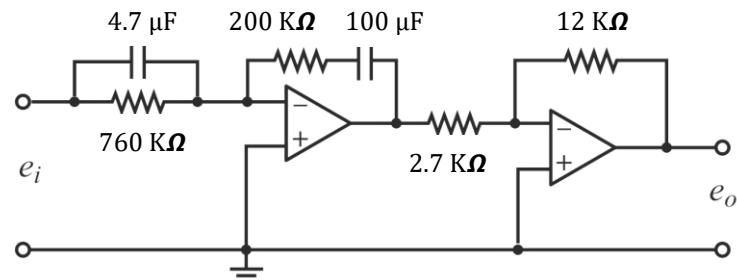


Figure 15

PID system by operational-amplifier circuits.

Closed-loop frequency-response

For following transfer function:

$$\frac{K}{s(0.5s + 1)(0.9s + 1)}$$

The Nichols Chart is shown in Figure 16 which we can obtain Gain margin, Phase margin, Gain crossover frequency and Phase crossover frequency.

Figure 16

Nichols Chart of
Open-loop transfer
function.

