ALDA HW3 Khantil & Aman

Q-5 (b)

(iii)

Data ID	1	2	3	4
(x_1, x_2)	(0,2)	(2,0)	(0,0)	(2,2)
label y	-1	-1	1	1

→ Dot produtcs of data-points:

Data ID	1	2	3	4
1	4	0	0	4
2	0	4	0	4
3	0	0	0	0
4	4	4	0	8

→ Kernel Values $K(x_i, x_j) = (1 + x_i \cdot x_j)^2$

	1	2	3	4
1	25	1	1	25
2	1	25	1	25
3	1	1	1	1
4	25	25	1	81

$$L = \sum \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j x_i x_j y_i y_j$$

$$= \sum \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j \cdot K(x_i, x_j) y_i y_j$$

$$= \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$

$$- \frac{1}{2} \Big[\alpha_1^2 \cdot 25 \cdot 1 + \alpha_1 \cdot \alpha_2 - \alpha_1 \alpha_3 - 25 \alpha_1 \alpha_4$$

$$+ \alpha_1 \cdot \alpha_2 + 25 \alpha_2^2 - \alpha_2 \cdot \alpha_3 - 25 \alpha_2 \alpha_4$$

$$- \alpha_3 \alpha_1 - \alpha_2 \alpha_3 + \alpha_3^2 + \alpha_3 \alpha_4$$

$$- 25 \alpha_1 \alpha_4 - 25 \alpha_2 \alpha_4 + \alpha_3 \alpha_4 + 81 \alpha_4^2 \Big]$$

$$\therefore L = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$

$$- \frac{25}{2} \alpha_1^2 - \frac{25}{2} \alpha_2^2 - \frac{\alpha_3^2}{2} - \frac{81}{2} \alpha_4^2$$

$$- \alpha_1 \alpha_2 + \alpha_1 \alpha_3 + 25 \alpha_1 \alpha_4 + \alpha_2 \alpha_3$$

$$+ 25 \alpha_2 \alpha_4 - \alpha_3 \alpha_4$$

$$\therefore \frac{dL}{d\alpha_1} = 1 - 25\alpha_1 - \alpha_2 + \alpha_3 + 25\alpha_4 = 0 \quad — (1)$$

$$\therefore \frac{dL}{d\alpha_2} = 1 - 25\alpha_2 - \alpha_1 + \alpha_3 + 25\alpha_4 = 0 \quad — (2)$$

$$\therefore \frac{dL}{d\alpha_3} = 1 - \alpha_3 + \alpha_1 + \alpha_2 - \alpha_4 = 0 \quad — (3)$$

$$\therefore \frac{dL}{d\alpha_4} = 1 - 81\alpha_4 + 25\alpha_1 + 25\alpha_2 - \alpha_3 = 0 \quad — (4)$$

(1) − (2)

$$-24\alpha_1 + 24\alpha_2 = 0$$

$$\therefore \boxed{\alpha_1 = \alpha_2} \quad — (5)$$

(2) + (4)

$$2 + 24\alpha_1 - 56\alpha_4 = 0$$

$$1 + 12\alpha_1 - 28\alpha_4 = 0$$

$$\therefore \boxed{12\alpha_1 = 28\alpha_4 - 1} \quad — (6)$$

(2) + (3)

$$2 - 24\alpha_2 + 4\alpha_4 = 0$$

$$1 - 12\alpha_2 + 12\alpha_4 = 0$$

$$\therefore \boxed{12\alpha_4 = 12\alpha_2 - 1} \quad — (7)$$

(6), (7) and (5)

$$12\alpha_1 = 28\alpha_4 - 1 \quad —(6)$$

$$12\alpha_4 = 12\alpha_1 - 1 \quad —(7)$$

Substituting value of (6) into (7)

$$12\alpha_4 = (28\alpha_4 - 1) - 1$$

$$\therefore 16\alpha_4 = 2$$

$$\therefore \boxed{\alpha_4 = \frac{1}{8}} \quad — (8)$$

Substituing value of α_4 in (7).

$$12\left(\frac{1}{8}\right) = 12\alpha_1 - 1$$

$$\therefore \frac{3}{2} = 12\alpha_1 - 1$$

$$\therefore 12\alpha_1 = \frac{3}{2} + 1 = 5/2$$

$$\therefore \boxed{\alpha_1 = \frac{5}{24} = \alpha_2} \quad — (9)$$

Substituiting values $\alpha_1, \alpha_2, \alpha_4$ in (3).

$$1 + \frac{5}{24} + \frac{5}{24} - \frac{1}{8} = \alpha_3$$

$$\therefore \alpha_3 = \frac{24 + 10 - 3}{24} = \frac{31}{24}$$

$$\therefore \boxed{\alpha_3 = \frac{31}{24}}$$

For x' datapoint, (support vector).

$$w \cdot x_1 + w_0 = y_1$$

$$w \cdot \phi(x_1) + w_0 = y_1$$

$$\therefore \sum_{i=1 \text{ to } 4} \alpha_i y_i \, \phi(x_i) \cdot \phi(x_1) + w_0 = y_1$$

$$\therefore \alpha_1 y_1 k(x_1, x_1) + \alpha_2 y_2 k(x_2, x_1) + \alpha_3 y_3 k(x_3, x_1)$$

$$+ \alpha_4 y_4 k(x_4, x_1) + w_0 = y_1$$

$$\therefore \left(\frac{5}{24}\right)(-1)(25) + \frac{5}{24}(-1)(1) + \frac{31}{24}(1)(1) + \frac{1}{8}(1)(25)$$

$$+ w_0 = -1$$

$$\therefore \frac{-125 - 5 + 31 + 75}{24} + w_0 = -1$$

$$\therefore \frac{-24}{24} + w_0 = -1$$

$$\therefore \boxed{w_0 = 0}$$