Week 2
Tutorial - GLMs

Applied Statistical Analysis II

Lecturer: Jeffrey Ziegler, PhD
Teaching Fellow: Shekhar Kedia

ASDS - Trinity College Dublin

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Roadmap for today

- Today:
  - GLMs
    1. Know how to use the ‘glm()‘ function in R
    2. Understand a little about how GLM works, & how it differs from linear regression
    3. Regret not having learned more calculus ☺
- By next week, please...
  - Start problem set #1!

GLMs in R

glms use MLE

ols working by minimizing the residuals

in this case mle and
derivaties with possible set of combinations and try to minimize it

using mle this is the best parameter is.

glm uses mle to predict the best combination of parameters which can help estblaihs relationship between predictor and outcome.

- Today, look at **generalized linear model**
- GLM allows us to extend principle of linear regression to different kinds of outcomes using a *link function*
  - Simply an equation that "links" linear predictors to non-linear outcomes
  - Ex: "yes/no", counts, etc.

GLMs in R

LM function is a special kind of GLM

we should get the same parametrs because weare different estimation approaches we are trying to understand the same best arrival at the pararmeters

Today's goal: More theoretical than usual for our tutorials

- Introduce the 'glm()' function in R
- Begin to see how it works 'under the hood' using what is known as **maximum likelihood estimation**
  - We'll focus on this explicitly next week
- MLE is just a way of estimating parameters of a distribution from the available data
  - For instance, in certain circumstances, OLS method we used last term satisfies objective of MLE

The ‘glm()‘ function

- ‘glm()‘ function is very similar to ‘lm()‘ function we’re already familiar with

- In fact, you can run a linear regression with ‘glm()‘ function, because linear regression is part of the same family as the generalised linear model

```r
lm(Sepal.Length ~ Sepal.Width + Species, data = iris)
```

```r
glm(Sepal.Length ~ Sepal.Width + Species, data = iris,
  family = "gaussian")
```

The ‘glm()‘ function

two different mathods, lle, and mle, the coefficents are the same

if the coefficents are the same what exatclt is different

we rerun with different covariates to determine which gets slcosest to zero

the mroe compelx the model, heavey pentality youll see higher valkules for aic/bic. they are pentalty terms.

	lm()	glm()
(Intercept)	2.25***	2.25***
	(0.37)	(0.37)
Sepal.Width	0.80***	0.80***
	(0.11)	(0.11)
Speciesversicolor	1.46***	1.46***
	(0.11)	(0.11)
Speciesvirginica	1.95***	1.95***
	(0.10)	(0.10)
R^2	0.73	
Adj. R^2	0.72	
Num. obs.	150	150
AIC		183.94
BIC		198.99
Log Likelihood		−86.97
Deviance		28.00

- See how ‘glm()‘ function, when we specify the ‘family = ‘ argument to "gaussian", gives us *almost* same output as ‘lm()‘ function?
- This is because linear regression is based on normal, or Gaussian, distribution
- What do you notice is different though?

These are a bit different, these are all negataive terms. If its clsoe to zero, the BIC, is explanation

dev is how much you deivate from a perfect model

we can try to remodel to try to minimze these values, the closer they all are to zero si whats important

so if you run mutliple different models, you pick the ones that are closest to zero.

The ‘glm()‘ function

logistic = binomial

to run different glm functions you just change the name.

- For example, what if we want to use ‘glm()‘ to perform regression for binary outcome?
- We simply specify distribution for which logistic regression estimates parameters, i.e., binomial distribution

```r
glm(am ~ cyl + disp, data = mtcars, family = "binomial")
```

Logistic (or logit) regression

the predictions that we draw should also be bounded between, for example, zero and one.

Quickly review what we use logistic regression for, and how it works

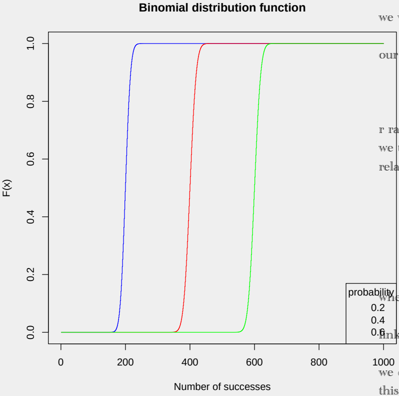
- We have an outcome that can only be true or false, yes or no
- Then, our estimate for this outcome needs to be bounded between zero (for false/no) and one (for true/yes)
- It would make no sense to have an estimate less than zero, or more than 1
- Moreover, if we did get such an estimate, it would suggest that our model was somehow wrong, and that all our other predictions might be off

Logistic (or logit) regression

Basically, we need a distribution that falls between zero and one

- And, we need to be able to link coefficients of our predictors to this distribution

Here's the distribution:



Here's the link function:

we will se a bunomial distribution. How can glm help us to draw a linear conlcusion.

our dist give us a p-value if you plug it in over there

r ration of it happening. or it not happening
we take the log of the odds, not getting y as the function but we are gettings log of r's/ log os r's is
related to beta values in a linear form.

$$\log \frac{p}{1-p} = \beta X$$

when we say the outcome varibale is linearly relatied to the variables this is what we mean.

link function tells us how the distribtion is related to my predictors using a linear relationship.

we can still say the linear equation, but y is the log odd value, and we have the linear equation.
this is incase of logistic function, other family funcitons the link fucntion would change.

Example GLM: Logit regression

using log odds gives us a normal dist.

the way we trasnform y depends on the distribution. logit for binomial dist. values of - infity to infitnity.

if we just take odds and not log odds itll only give us vcalues moving in 1d, take a log distrib, it gives you the full range, needed for normal distribtuion.

- Remember: All link function does is convert from a probability (0 to 1 scale), which has a curve shape, to log odds ($-\infty$ to ∞ scale), which is a straight line
- Typically use log odds for βX (literally, the log of the odds) because these are symmetrical and normally distributed
- Taking log of the odds also converts terms in our model from a multiplicative to an additive relationship (i.e., we add terms rather than multiply them)
  - Which is similar to what we are familiar with from linear regression

Example GLM: Logit regression

- Easy to get confused in logistic regression
- There is a difference between odds and odds ratio, log (odds) and log (odds ratio)
  - Logit link function used to convert between probabilities and log odds
  - Log-likelihood transform used in maximum likelihood estimation to fit predicted line
- Takes time to familiarize yourself with these
- In general, we'll focus on how to interpret our model coefficients and how to make a prediction based on them

Using logistic regression

Let’s compare a logistic regression model with a linear model we’re familiar with, using the ‘iris‘ dataset we’re also familiar with

```r
dat <- iris
dat$set <- ifelse(iris$Species == "setosa", 1, 0)
mod1 <- lm(set ~ Petal.Length + Petal.Width, data = dat))
```

	Dependent variable:
	set
Petal.Length	-0.251^{***}
	(0.032)
Petal.Width	0.010
	(0.073)
Constant	1.266^{***}
	(0.044)
Observations	150
R^2	0.852
Adjusted R^2	0.849
Residual Std. Error	0.183 (df = 147)
F Statistic	421.497^{***} (df = 2; 147)

USING LOGISTIC REGRESSION

- So, we ran a linear model on a binary outcome (is iris of species setosa or not?)
- Let’s make a prediction from our model based upon some imaginary data
  - An iris with petal length of 5.4cm and width of 2.4cm

```r
newdat <- data.frame(Petal.Length = 5.4, Petal.Width = 2.4)
predict(mod1, newdat)
```

we did predict and it gave us a -0,06 value, this is bc lm function can give a while range of functions and is not boudned by zero or one so we run a logit

```
1
-0.0675413
```

Using logistic regression

As we can see, our model gives us a prediction below zero - does this make sense?

Let’s try again with logistic regression:

```r
mod2 <- glm(set ~ Petal.Length + Petal.Width, data = dat, family =
     "binomial")
```

new dataset

	Dependent variable:
	set
Petal.Length	−17.596
	(43,449.070)
Petal.Width	−33.887
	(115,850.800)
Constant	69.447
	(43,042.940)
Observations	150
Log Likelihood	−0.000
Akaike Inf. Crit.	6.000

Using logistic regression

- So, we get a different set of coefficients for our predictor variables, both of which are now negative…
  - What does this mean?
  - How do we make a prediction?
- Let’s try with our same imaginary data

```
predict(mod2, newdat)
```

```
1
-106.8992
```

Using logistic regression

Well, that doesn’t seem too useful - we’re even further below zero than the linear model

Except, if we check the help file for ‘predict.glm()‘...

Let’s try again with the argument `type = "response"`

```r
predict(mod2, newdat, type = "response")
```

```
1
2.220446e-16
```

Using logistic regression

- What's the difference?
- By default we get log odds prediction
  - If we want to get the probability from this, we need to reverse the link function, which is...

$$p = \frac{e^{\log(odds)}}{1 + e^{\log(odds)}}$$

Using logistic regression

Let's try this out on a slightly more normal range of data, petal length = 2.3 and petal width = 0.8

```r
newdat2 <- data.frame(Petal.Length = 2.3, Petal.Width = 0.8)
predict(mod2, newdat2, type = "response")
```

```
1
0.8661015
```

```r
log_odds <- predict(mod2, newdat2)
exp(log_odds)/(1+exp(log_odds))
```

```
1
0.8661015
```

```r
1/(1+exp(-log_odds))
```

```
1
0.8661015
```

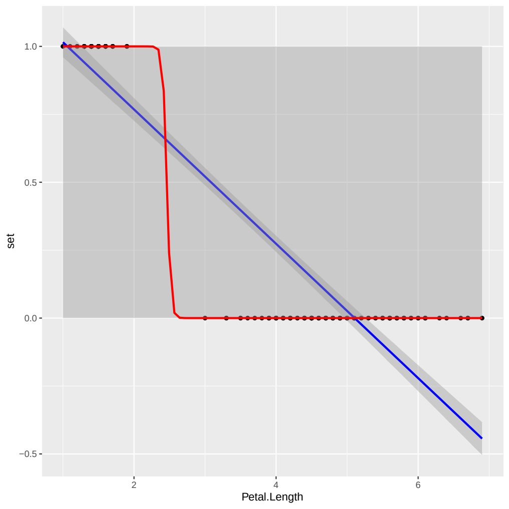
Using logistic regression

- As you can see, we get same prediction!
- Interpreting coefficients and predicted values in logit regression is less straightforward than linear regression
  - We'll look into this in greater detail in the weeks to come
- This site has some helpful examples that explain difference between log odds, odds, and probability, and how we can calculate each with R

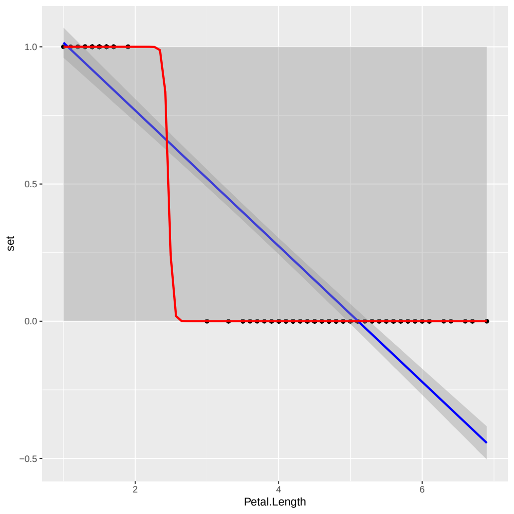
Linear vs Logit regression

Let’s plot probability of an iris being from setosa species according to length of its petals

```
ggplot(data = dat, aes(
    Petal.Length, set)) +
geom_point() +
geom_smooth(method = "lm",
     color = "blue") +
geom_smooth(method = "glm"
    , method.args = list(
    family = "binomial"),
    color = "red")
```



Linear vs Logit regression



- As we can see, the larger the petal, the lower the probability
- However, linear regression line is not bounded by zero on the y axis
  - So, line moves negative above roughly 5cm on x-axis
- Whereas, logit regression is asymptotic to zero and one on y-axis
  - No matter how large the petal, prediction does not go below

Overview

- Right, that's enough calculus for today
- Now, let's practice organizing data, executing a lm() + glm(), and comparing output
- Next week, we'll figure out how to create our own glm with MLE
- Then in week 4, we'll practice logistic regression on some real data