# Simulation and Analysis of Complexity of Supply Chain Inventory Systems

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Abstract—An effective supply chain management strategy is crucial for delivering the right goods & services to the right place, in the right quantity at the right time. A supply chain network involves several players and is influenced by their decisionmaking as well as various external factors. This, coupled with the growing uncertainty in consumer demand, makes the supply chain network a complex system. The conventional decentralized inventory management model is susceptible to the problem of demand following the supply chain to get distorted from the end consumers to the manufacturers upstream, i.e., the bullwhip effect. The scope of this article spans the performance analysis of vendor managed inventory (VMI) models or commonly known as the Third-party logistics management inventory (TMI) models in controlling the adverse effects of the bullwhip effect under different demand models using MATLAB and Simulink with the assumption of non-accumulation of the unsatisfied demand. It was found that the Third-party logistics management inventory model performs the best for normally distributed random demand and fixed constant demand. Therefore, it is concluded that the Third-party logistics help in mitigating the effects of the bullwhip effect.

Index Terms—Supply chain networks, Third-party Logistics Management Inventory (TMI), APVIOBPCS production system, Demand Forecasting.

# I. INTRODUCTION

Higher competition in the market has triggered interests towards the supply chain. Each member of each company in the supply chain work is required to maximize his/her efforts in order to cater to the competition between supply chains. Higher the risk, higher the profits, which enables companies in the supply chain to continue the benefits they have been enjoying for a longer time. Even research and development, innovation, and creativity is boosted by this improvement in technology. This, as a cycle, again enables the supply chain to gain more at each level. The American Production and Inventory Control Society (APICS), which is currently known as the Association for Supply Chain Management, defines supply chain as: "The global network used to deliver products and services from raw materials to end customers through engineered flows of information, physical distribution, and cash" [1]. This network involves several organizations where the decisions of each organization drastically affects the behavior of the whole network. This makes supply chain networks very complex and non-linear in nature. This complexity, essentially leads to significant discrepancies between the predicted and actual demand and supply of material caused by uncertainty and a variety of environmental factors interacting with the supply chain [2]. In the traditional methods of modeling supply chains, each entity at each level is only concerned about its own production, inventory control and distribution operations and shares information only with its immediate supplier and customer. This creates a dire lack of visibility of the actual demand which causes a major challenge in supply chains commonly known as the bullwhip effect. The bullwhip effect refers to the increasing fluctuation in inventory due to shifts in consumer demand upstream in the supply chain from the retailer through to the wholesaler and the manufacturer [3]. One of the consequences of the bullwhip effect has been recently witnessed at the early stages of the COVID-19 pandemic, when sudden spikes in demand for medical supplies such as masks or ventilators as well as consumer items such as toilet paper or eggs gave rise to situations of panic buying & hoarding [4]. The evolution of information technology over the recent years facilitated information sharing among all the members in a supply chain. As a result, new models of supply chain such as the Vendor-Managed Inventory (VMI) model and the Third-Party Logistics Management Inventory (TMI) model were formulated. In these new models of supply chain, the information about demand, supply and inventory levels are fully shared among all the entities of the supply chain. Thereby, controlling the problem of gradually increasing demand up the supply chain from the consumer to the manufacturer, i.e. the bullwhip effect [5]. In this article, a TMI model of the supply chain has been constructed with an assumption of non-accumulation of unmet demand and with an APVIOBPCS ordering policy. Behavior of this supply chain has been studied via simulation under a variety of different demand models as an input.

## II. LITERATURE REVIEW

In [2], the authors define supply chains and complex systems and attempt to draw parallels between them on the basis of characteristics found to be common in both e.g. emergence, connectivity, co-evolution, distribution control, non-linearity, state of paradox, far-from-equilibrium, heterogeneity, openness & self-organization. In [3], Lee et al introduce the bullwhip effect as a phenomenon wherein the demand distorts up the supply chain from the retailer all the way to the manufacturer. It was also proposed that, in order to prevent this problem demand and inventory information, which is otherwise proprietary, be shared with the manufacturers. In [5], Disney and Towill simulated and compared the Vendor Managed Inventory model of supply chains with the traditional "serially-linked" supply chain model on the basis of their performance against

the bullwhip effect. This was done on the basis for two measures, i.e., peak order rate to a step input and order rate variance. It was found that the VMI model outperforms the traditional model. In [6], White et al. investigate how the Automatic Pipeline, Variable Inventory and Order Based Production Control System (APVIOBPCS) increases profitability in the electronic RAM production. In another study by Disney et al. [7], the authors have considered the APVIOBPCS within Vendor Managed Inventory (VMI) supply chain model and developed a transfer function model of the system using causal loop diagrams, block diagrams, difference equations and z-transforms. Subsequently, the stability conditions of the same were found. In [8], Zhang et al. use the system dynamics method to establish a nonlinear supply chain system model with supply capacity which assumes non accumulation of unmet demands under third-party logistics management inventory model (TMI). Theory of singular value and Jury Test were used to determine the stable interval of the model.

## III. SYSTEM MODEL

### A. System Description

The TMI supply chain consists of one supplier, one retailer, and one third-party logistics enterprise. The third party logistics have right and are responsible for inventory management and logistics transportation tasks of the entire supply chain. For this, distribution centers are built near retailers. The work of supply chain members is periodic and it follows an event flow given by:

### 1) For retailers (t=0):

- a) Receive replenishment from distribution centers, shipment according to customer demand, inventory, third-party logistics combined with retailer safety stock.
- b) Suggest that retailers replenish.
- c) Retailers issue replenishment notifications to distribution centers.

### 2) For the distribution center (t=0):

- a) Distribution center received replenishment from the warehouse, replenished the goods from the retailer, inventoried the inventory.
- b) Issued a replenishment notice to the warehouse.

# 3) For warehouses (t=0):

- a) Received replenishment from suppliers, replenished them at the distribution center, counted inventory.
- b) Sent replenishment notifications to suppliers.

### 4) For suppliers:

a) Production will be carried out at time t, according to the replenishment notice received at end of period t-1.

# B. System Block Diagrams

Using the discrete system Z transform theory, we have laid out the equations in a block diagram manner using Simulink. Following are the resulting block diagrams.

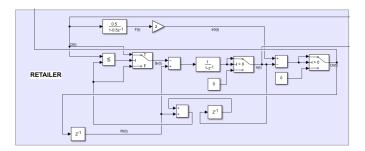


Fig. 1. System Block Diagram of the inventory system of the Retailer.

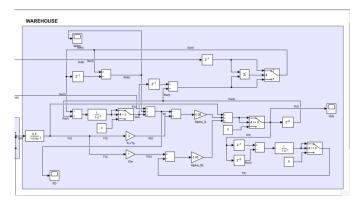


Fig. 2. System Block Diagram of the inventory system of the Warehouse.

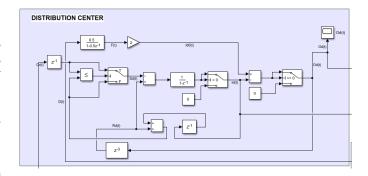


Fig. 3. System Block Diagram of the inventory system of the Distribution Center.

### IV. SYSTEM DIFFERENCE EQUATIONS

### A. Demand Forecasting

This is done by using simple exponential smoothing methods. It is done based on the demands of the supply chain's end customer, and not the retailer. Demand forecasts are more reasonable for the entire supply chain.

$$F(t) = \theta F(t - 1) + (1 - \theta)D(t). \tag{1}$$

# B. Order Policy

APVIOBPCS[4] ordering strategy is used here. The idea here is that ordering has a fixed period, order quantity depending on the demand forecasting, inventory adjustment, and transit adjustment. Suppliers, warehouses, and distribution centers are seen as a system when considering warehouse ordering. The system is called the Warehouse-Distribution Center System. The ordering expression of the system is:

$$O(t) = \max(0, F(t) + \alpha_S \Delta I(t) + \alpha_{SL} \Delta Y(t))$$
 (2)

where,

$$\Delta I(t) = I^0(t) - I(t) \tag{3}$$

$$\Delta Y(t) = Y^{0}(t) - Y(t) \tag{4}$$

The supplier's work-in-process inventory at time t is given by:

$$Y(t) = Y(t-1) + O(t-1) - R_w(t)$$
(5)

The supplier's expected work-in-process inventory at time t is given by:

$$Y^0(t) = G_w F(t) \tag{6}$$

Using the discrete system Z transform theory, we have drawn the block diagram of the inventory system of warehouse as shown in Figure 1.

### C. For Retailers

The retailer's replenishment point is given by:

$$I_r^0(t) = G_r F(t) \tag{7}$$

The retailer's inventory level at time is given by:

$$I_r(t) = I_r(t-1) + R_r(t) - S_r(t)$$
(8)

And the retailer replenishment is given by:

$$O_r(t) = \max(0, I_r^0(t) - I_r(t))$$
 (9)

Replenishment notification is issued at the end of the period, and the replenishment can be received at the beginning of the next period:

$$R_r(t) = O_r(t-1).$$
 (10)

The retailer's shipments for this period's customer demand is given by:

$$S_r(t) = D(t) \tag{11}$$

Using the discrete system Z-transform theory, we have drawn the block diagram of the inventory system of the retailer as shown in Figure 2.

### D. For Warehouse-Distribution Center System

The distribution center replenishment point is given by:

$$I_d^0(t) = F(t)G_d \tag{12}$$

The distribution center inventory is given by:

$$I_d(t) = I_d(t-1) + R_d(t) - S_d(t).$$
(13)

The distribution center replenishment is given by:

$$O_d(t) = \max(0, I_d^0(t) - I_d(t))$$
 (14)

The distribution center receiving volume is given by:

$$R_d(t) = O_d(t - T_P - 1) (15)$$

The distribution center shipments are given by:

$$S_d(t) = \begin{cases} I_d(t-1) + R_d(t) & I_d(t-1) + R_d(t) \le O_r(t-1), \\ O_r(t-1) & I_d(t-1) + R_d(t) > O_r(t-1). \end{cases}$$
(16)

And the distribution center inventory in transit is given by:

$$W_d(t) = W_d(t-1) + O_d(t-1) - R_d(t)$$
(17)

The block diagram of the inventory system of distribution center is drawn using the Z-transform theory of the discrete system as shown in Figure 3.

The warehouse inventory is given by:

$$I_w(t) = I_w(t-1) + R_w(t) - S_w(t)$$
(18)

The warehouse receipt is given by:

$$R_w(t) = O(t - T_C - 1) (19)$$

The warehouse shipments are given by:

$$S_w(t) = \begin{cases} I_w(t-1) + R_w(t) & I_w(t-1) + R_w(t) \le O_d(t-1), \\ O_d(t-1) & I_w(t-1) + R_w(t) > O_d(t-1). \end{cases}$$
(20)

The expected inventory of the system is given by:

$$I^{0}(t) = F(t) \left( T_{C} + T_{P} \right) \tag{21}$$

And the actual inventory of the system is given by:

$$I(t) = I(t-1) + R_w(t) - S_d(t)$$
(22)

# E. Model Parameters and Variables

Following are the parameters and variables used for the representation of the complex supply chain.

D(t): The actual demand at time t.

 $F_r(t)$ : The expected demand at time t.

 $I_r(t)$ : Retailer's inventory level at time t.

 $R_r(t)$ : Retailer's receipt amount at time t.

 $S_r(t)$ :Retailer's delivery amount at time t.

 $O_r(t)$ : Retailer's order amount at time t.

 $S_d(t)$  : Distribution center's delivery amount to retailers at time t.

 $I_d(t)$ : Distribution center's inventory level at time t.

 $R_d(t)$ : Distribution center's receipt amount at time t.

 $W_d(t)$ : Distribution center's in-transit inventory at time t.

 $O_d(t)$ : Distribution center's order amount at time t.

 $I_r^0(t)$ : Retailer's expected inventory levels at time t.

 $I_{d}^{0}(t)$  : Distribution center's expected inventory levels at time  $t_{\cdot}$ 

 $I^0(t)$ : Warehouse-distribution center system's expected inventory levels at time t.

 $S_{\boldsymbol{w}}(t)$  : Warehouse's delivery amount to distribution center at time t.

 $I_w(t)$ : Warehouse's inventory level at time t.

 $R_w(t)$ : Warehouse's receipt amount at time t.

I(t) : Warehouse-distribution center system's inventory level at time  $t. \ \ \,$ 

O(t): Warehouse-distribution center system's order amount at time t.

Y(t): Supplier's work-in-process inventory at time t.

 $Y^{0}(t)$ : Supplier's expected in-process inventory at time t.

 $\alpha_S$ : The adjustment coefficient of inventory.

 $\alpha_{SL}$  : The adjustment coefficient of work-in-process inventory or stock on the way.

 $\mathcal{T}_{P}$  : Transport lead time.

 $T_C$ : Production lead time.

 $G_r$ : Safety inventory coefficient of retailer.

 $G_d$ : Safety inventory coefficient of distribution center.

 $G_w$ : Safety inventory coefficient of warehouse.

# V. SIMULATION RESULTS & ANALYSIS

The stability of the supply chain is greatly dependent upon the demand and its nature. For a fixed demand, the retailer's inventory flattens to a particular level eventually regardless of other parameters and the stability of the supply chain. Whereas, the retailer's inventory becomes extremely volatile under a random uniform distribution despite the stability of the supply chain.

When the demand is random and normally distributed and the supply chain is stable, there is very little fluctuation in the retailer's inventory which is restricted to a limited range. This much uncertainty is expected and very common.

The volatility of the system with fixed constant demand is the lowest, followed by that of the system with normally distributed random demand. While the volatility of the system with random uniformly distributed demand is the highest.

Focusing on the variable of interest here, which is  $I_r(t)$ , Retailer's Inventory level at time t. Other important variables are O(t), the Order Quantity at time t and I(t), the Inventory level of the Warehouse at time t. These variables have to be analysed against D(t), the Demand at time t.

## A. Demand Models

We have modeled the demands in 3 ways:

- Normally distributed random demand Demand Mean is set to 500 and Variance to 100.
- 2) Uniformly distributed random demand Demands ranging from 450 to 550.
- 3) Fixed Constant Demand Fixed Demand of 500.

We have simulated the model for all these demands, and let's gather the observations from them.

# B. Observations & Figures

Following are the results obtained and the observations related with them.

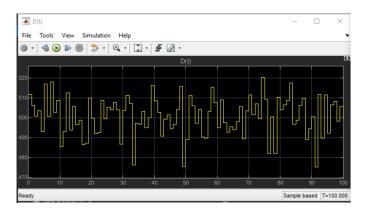


Fig. 4. Demand at time t for Demand following Normal Distribution Random Demand Model.

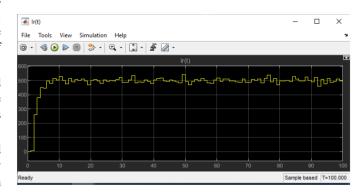


Fig. 5. Retailer's Inventory at time t for Demand following Normal Distribution Random Demand Model.

For Normally distributed random demand, we can see that the Retailer's inventory level oscillates closely around 500. We expect fewer fluctuations in the inventory levels, and this is what can be observed.

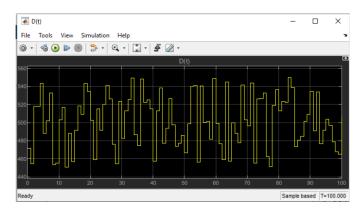


Fig. 6. Demand at time t for Demand following Uniform Distribution Random Demand Model

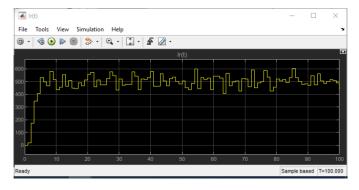


Fig. 7. Retailer's Inventory at time t for Demand following Uniform Distribution Random Demand Model.

For Uniformly distributed random demand, the Retailer's inventory level shows volatility around 500. This is in direct correlation to the demand fluctuations that we see for this demand model.

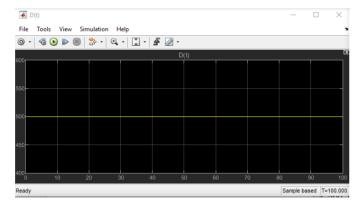


Fig. 8. Demand at time t for Demand following Fixed Constant Demand Model.

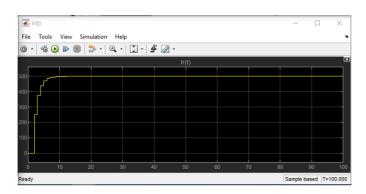


Fig. 9. Retailer's Inventory at time t for Demand following Fixed Constant Demand Model.

For Fixed constant demand, as there are no fluctuations in demand, we see that the Retailer's inventory level settles at 500.

# VI. CONCLUSION

The simulation of the complex Third-Party Logistics model has been approached. We have analysed the complexity of the entities of the supply chain in some detail and drew conclusions from the parameter changes and its effects. We also analysed the Difference Equations involved in the model. We then simulated the model alongside with different Demand Models and recorded the behavior of the system under them. We conclude that this model prevents the bullwhip effect by keeping inventory levels quite stable. The most volatile demand model was found to be the one with demands following the Uniformly distributed random demand model. Also, the Normal distribution random demand model was found to be the most robust to the changes in demands.

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