

IV SEMESTER

# MULTIPLE CHOICE QUESTIONS

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1. If  $A$  and  $B$  are square matrices of the same order, then  $\text{tr}(AB) =$ 
  - A.  $\text{tr}(A + B)$
  - B.  $\text{tr}(A)\text{tr}(B)$
  - C.  $\text{tr}(BA)$
  - D.  $\text{tr}(A) + \text{tr}(B)$
  
2. If  $A$  and  $B$  are square matrices of the same order, then  $(AB)^T =$ 
  - A.  $A^T B^T$
  - B.  $B^T A^T$
  - C.  $A^T + B^T$
  - D.  $(BA)^T$
  
3. If  $A$  and  $B$  are symmetric matrices of same order, then
  - A.  $AB$  is always symmetric
  - B.  $AB$  is never symmetric
  - C.  $AB$  is skew-symmetric
  - D.  $AB$  is symmetric if and only if  $AB = BA$
  
4. For all square matrices  $A$  and  $B$ , is it true that  $\det(A+B) = \det(A) + \det(B)$ 
  - A. True
  - B. False
  - C. Cannot be determined
  
5. A matrix that is both symmetric and upper triangular must be a
  - A. diagonal matrix
  - B. non-diagonal but symmetric
  - C. both A and B
  - D. none of the above

6. If  $A$  and  $B$  are invertible matrices with the same size, then  $AB$  is invertible and  $(AB)^{-1} =$
- A.  $A^{-1}B^{-1}$
  - B.  $B^{-1}A^{-1}$
  - C. both A and B
  - D. none of the above
7. A matrix  $E$  is called ... if it can be obtained from an identity matrix by performing a single elementary row operation.
- A. equivalent matrix
  - B. echelon matrix
  - C. elementary matrix
  - D. row reduced matrix
8. Let  $A$  be an  $n \times n$  matrix, and  $A$  is invertible. Then which of the following statement is equivalent:
- A.  $Ax = 0$  has only the trivial solution.
  - B.  $A$  cannot be expressed as a product of elementary matrices.
  - C.  $Ax = b$  is inconsistent for every  $n \times 1$  matrix  $b$ .
  - D.  $Ax = b$  has more than one solution for every  $n \times 1$  matrix  $b$
9. A homogeneous linear system in  $n$  unknowns whose corresponding augmented matrix has a reduced row echelon form with  $r$  leading 1's has
- A.  $n$  free variables
  - B.  $n - r$  free variables
  - C.  $r$  free variables
  - D. cannot be determined
10. A linear system is called consistent if it has
- A. infinitely many solutions
  - B. no solution

- C. at least one solution  
 D. none of the above
11. A consistent linear system of two equations in two unknowns has  
 A. exactly one solution  
 B. infinitely many solutions  
 C. exactly two solutions  
 D. either A or B
12. Which of the following matrices is in reduced row echelon form.  
 A.  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$   
 B.  $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$   
 C. both A and B  
 D. none
13. If  $T_A: \mathbb{R}^n \rightarrow \mathbb{R}^m$  and  $T_B: \mathbb{R}^n \rightarrow \mathbb{R}^m$  are matrix transformations, and if  $T_A(\mathbf{x}) = T_B(\mathbf{x})$  for every vector  $\mathbf{x}$  in  $\mathbb{R}^n$ , then  
 A.  $A$  and  $B$  are equivalent but not equal  
 B.  $A$  and  $B$  are equal  
 C.  $A$  and  $B$  cannot be equal  
 D. cannot be determined
14. If  $S = \{v_1, v_2, \dots, v_n\}$  is a set of vectors in a finite-dimensional vector space  $V$ , then  $S$  is called a basis for  $V$  if:  
 A.  $S$  spans  $V$   
 B.  $S$  is linearly independent  
 C. either A or B  
 D. both A and B

15. If  $A$  is an  $n \times n$  matrix that is not invertible, then the linear system  $Ax = 0$  has
- A. infinitely many solutions
  - B. exactly one solution
  - C. not possible to find solution
  - D. finitely many solutions
16. Let  $A$  be an  $n \times n$  matrix. The linear system  $Ax = 4x$  has a unique solution if and only if ... is an invertible matrix.
- A.  $A$
  - B.  $A + 4I$
  - C.  $A - 4I$
  - D.  $4A$
17. If  $A$  is an  $m \times n$  matrix, then the codomain of the transformation  $T_A$  is
- A.  $R^n$
  - B.  $R^{m+n}$
  - C.  $R^{mn}$
  - D.  $R^m$
18. If  $T_A : R^n \rightarrow R^n$  and if  $T_A(x) = 0$  for every vector  $x$  in  $R^n$ , then  $A$  is
- A. the  $n \times n$  zero matrix
  - B. the  $n \times n$  identity matrix
  - C. an elementary matrix
  - D. cannot be determined
19. Does the vectors  $v_1 = (-3, 7)$  and  $v_2 = (5, 5)$  form a basis for  $R^2$ ?
- A. Data not complete
  - B. No
  - C. Yes
  - D. Not in  $R^2$

20. Are the vectors  $v_1 = (2, 0, -1)$ ,  $v_2 = (4, 0, 7)$ , and  $v_3 = (-1, 1, 4)$  linearly independent in  $R^3$ ?
- A. linearly dependent
  - B. linearly independent
  - C. Data not complete
  - D. none of the above
21. If  $W$  is a subspace of a finite-dimensional vector space  $V$ , then
- A.  $\dim(W) = \dim(V)$  always
  - B.  $\dim(W) \geq \dim(V)$
  - C.  $\dim(W) \leq \dim(V)$
  - D. none of the above
22. Let  $A$  be an  $n \times n$  matrix. The characteristic polynomial of  $A$  is a polynomial of degree
- A.  $n+1$
  - B. can be greater than  $n$
  - C. can be less than  $n$
  - D.  $n$  always
23. A  $n \times n$  matrix has
- A. at most  $n$  distinct eigenvalues
  - B. at least  $n$  distinct eigenvalues
  - C. exactly  $n$  distinct eigenvalues
  - D. exactly  $n+1$  distinct eigenvalues
24. Let  $\lambda$  is an eigenvalue of a  $n \times n$  matrix  $A$ .
- A. The system of equations  $(\lambda I + A)x = 0$  has only trivial solutions.
  - B. The system of equations  $(\lambda I - A)x = 0$  has only trivial solutions.
  - C. The system of equations  $(\lambda I + A)x = 0$  has nontrivial solutions.

- D. The system of equations  $(\lambda I - A)x = 0$  has nontrivial solutions.
25. If  $A$  is a  $2 \times 3$  matrix, then the domain of the transformation  $T_A$  is
- A.  $R^6$
  - B.  $R^2$
  - C.  $R^3$
  - D.  $R^5$
26. Which of the following is a subspace of  $R^3$ ?
- A. All vectors of the form  $(a, 0, 0)$
  - B. All vectors of the form  $(a, 1, 1)$
  - C. All vectors of the form  $(a, b, c)$  where  $b = a + c + 1$
  - D. None of these
27. Which of the following is a subspace of  $M_{n,n}$ ?
- A. The set of all non-invertible  $n \times n$  matrices
  - B. The set of all  $n \times n$  matrices  $A$  such that  $\det(A) = 0$
  - C. The set of all  $n \times n$  matrices  $A$  such that  $\text{tr}(A) = 0$
  - D. None of these
28. Which of the following is a subspace of  $P_3$ ?
- A. All polynomials  $a_3x^3 + a_2x^2 + a_1x + a_0$  for which  $a_3, a_2, a_1, a_0$  are rational numbers
  - B. All polynomials  $a_3x^3 + a_2x^2 + a_1x + a_0$  for which  $a_3 + a_2 + a_1 + a_0 = 0$
  - C. Both A and B
  - D. Neither A nor B
29. Which of the following are linear combinations of  $u = (0, -2, 2)$  and  $v = (1, 3, -1)$ ?
- A.  $(2, 2, 2)$
  - B.  $(0, 0, 0)$
  - C. Both A and B
  - D. Neither A nor B

30. Which of the following are linear combinations of

$$A = \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}?$$

A.  $\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix}$

B.  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

C. Neither A nor B

D. Both A and B

31. Find the value of  $m$  such that the vector  $(m, 7, -4)$  is a linear combination of vectors  $(-2, 2, 1)$  and  $(2, 1, -2)$ .

A. 2

B. -2

C. 0

D. -1

32. Which of the following does not span  $R^3$ ?

A.  $x = (2, 2, 2)$ ,  $y = (0, 0, 3)$ ,  $z = (0, 1, 1)$

B.  $x = (2, -1, 3)$ ,  $y = (4, 1, 2)$ ,  $z = (8, -1, 8)$

C. Neither A nor B span  $R^3$

D. Both A and B span  $R^3$

33. Suppose that  $x = (2, 1, 0, 3)$ ,  $y = (3, -1, 5, 2)$ , and  $z = (-1, 0, 2, 1)$ . Which of the following vectors are in  $\text{span}\{x, y, z\}$ ?

A.  $(2, 3, -7, 3)$

B.  $(1, 1, 1, 1)$

C. Both A and B

D. Neither A nor B

34. Let  $f = \cos^2 x$ ,  $g = \sin^2 x$ . Which of the following lie in the space spanned by  $f$  and  $g$ ?

A.  $3 + x^2$

B.  $\sin x$

- C. Both A and B
  - D. Neither A nor B
35. Which of the following is false?
- A. Every subspace of a vector space is itself a vector space.
  - B. Every vector space is a subspace of itself.
  - C. The intersection of any two subspaces of a vector space  $V$  is a subspace of  $V$ .
  - D. The union of any two subspaces of a vector space  $V$  is a subspace of  $V$
36. The polynomials  $x - 1, (x - 1)^2, (x - 1)^3$  span  $P^3$ .
- A. True
  - B. False
  - C. Data not complete
  - D. span  $P^4$
37. Which of the following is true?
- A. Every subset of a vector space  $V$  that contains the zero vector in  $V$  is a subspace of  $V$ .
  - B. Two subsets of a vector space  $V$  that span the same subspace of  $V$  must be equal.
  - C. The set of upper triangular  $n \times n$  matrices is a subspace of the vector space of all  $n \times n$  matrices.
  - D. All are true.
38. The kernel of a matrix transformation  $T_A : R^n \rightarrow R^m$  is a subspace of
- A.  $R^n$
  - B.  $R^m$
  - C.  $R^{n+m}$
  - D.  $R^{nm}$
39. The solution set of a consistent linear system  $Ax = b$  of  $m$



equations in  $n$  unknowns is a subspace of

- A.  $R^m$
- B.  $R^n$
- C.  $R^{n+m}$
- D.  $R^{nm}$

40. Which of the following is true?
- A. A finite set that contains 0 is linearly dependent.
  - B. A set with exactly one vector is linearly independent if and only if that vector is not 0.
  - C. A set with exactly two vectors is linearly independent if and only if neither vector is a scalar multiple of the other.
  - D. All are true.
41. Find the Wronskian of  $1, e^x, e^{2x}$ .
- A.  $2e^{2x}$
  - B.  $3e^{2x}$
  - C.  $2e^{3x}$
  - D.  $3e^{3x}$
42. Which of the following sets of vectors in  $R^3$  are linearly independent.
- A.  $\{(2, 1, 2), (8, 4, 8)\}$
  - B.  $\{(1, 1, 0), (1, 1, 1), (0, 1, -1)\}$
  - C.  $\{(1, 3, 2), (1, -7, -8), (2, 1, -1)\}$
  - D.  $\{(-2, 0, 1), (3, 2, 5), (6, -1, 1), (7, 0, -2)\}$
43. Which of the following is true?
- A. A set containing a single vector is linearly independent.
  - B. The set of vectors  $\{v, kv\}$  is linearly dependent for every scalar  $k$ .
  - C. Every linearly dependent set contains the zero vector.
  - D. None of the above

44. The dimension of zero vector space is
- A. not defined
  - B. 1
  - C. 0
  - D. infinite
45. Which of the following is false?
- A. There is a set of 17 linearly independent vectors in  $R^{17}$ .
  - B. There is a set of 11 vectors that span  $R^{17}$ .
  - C. Both A and B
  - D. Neither A nor B
46. Which of the following is true?
- A. Every linearly independent set of five vectors in  $R^5$  is a basis for  $R^5$ .
  - B. Every set of five vectors that spans  $R^5$  is a basis for  $R^5$ .
  - C. Every set of vectors that spans  $R^5$  contains a basis for  $R^5$ .
  - D. All are true
47. Which of the following is not a vector space.
- A. The set of all  $2 \times 2$  invertible matrices with the standard matrix addition and scalar multiplication.
  - B. The set of all  $2 \times 2$  matrices of the form  $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$  with the standard matrix addition and scalar multiplication.
  - C. The set of all  $2 \times 2$  matrices with real entries with the standard matrix addition and scalar multiplication.
  - D. None of these
48. Which of the following is a vector space.
- A. The set of all pairs of real numbers of the form  $(x, 0)$  with the standard operations on  $R^2$ .
  - B. The set of all pairs of real numbers of the form  $(x, y)$ , where  $x \geq 0$ , with the standard operations on  $R^2$ .

- C. The set of all pairs of real numbers with the standard vector addition but with scalar multiplication defined by  $k(x, y) = (k^2x, k^2y)$
- D. None of the above
49. Which of the following is a basis for  $M_{22}$  of  $2 \times 2$  matrices.
- A.  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$
- B.  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
- C.  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
- D.  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$
50. Which of the following is not a basis for  $R^3$ .
- A.  $\{(1, 1, 1), (1, 2, 3), (2, -1, -1)\}$
- B.  $\{(2, 0, -1), (4, 0, 7), (-1, 1, 4)\}$
- C.  $\{(1, 1, 2), (1, 2, 5), (5, 3, 4)\}$
- D. All are bases
51. Transition matrices are
- A. not at all invertible.
- B. invertible always.
- C. invertible sometimes.
- D. data not complete.
52. If  $B$  is a basis for a vector space  $R^n$ , then  $P_{B \rightarrow B}$  is
- A. the zero matrix.
- B. a diagonal matrix.
- C. no particular matrix.
- D. the identity matrix.
53. If  $P_{B_1 \rightarrow B_2}$  is a diagonal matrix, then
- A. each vector in  $B_2$  is the same vector as in  $B_1$ .

- B. each vector in  $B_2$  is a scalar multiple of some vector in  $B_1$ .
  - C. each vector in  $B_2$  is one more than the corresponding vector in  $B_1$ .
  - D. no relation between the vectors in  $B_2$  and  $B_1$ .
54. If  $A$  is an  $m \times n$  matrix, then the row space of  $A$
- A. is a subspace of  $R^n$
  - B. is a subspace of  $R^m$
  - C. is a subspace of  $R^{mn}$
  - D. is a subspace of  $R^{\min(m,n)}$
55. Let  $A$  be an  $m \times n$  matrix. The solution space of the homogeneous system of equations  $Ax = 0$  is called the
- A. row space of  $A$ .
  - B. column space of  $A$ .
  - C. null space of  $A$ .
  - D. none of these.
56. A system of linear equations  $Ax = b$  is consistent if and only if
- A.  $b$  is in the row space of  $A$ .
  - B.  $b$  is in the null space of  $A$ .
  - C.  $b$  is in the column space of  $A$ .
  - D. none of these.
57. Which statement is false?
- A. Elementary row operations do not change the null space of a matrix.
  - B. Elementary row operations do not change the row space of a matrix.
  - C. Elementary row operations change the column space of a matrix.
  - D. Elementary row operations do not change the column space of a matrix.

58. If a matrix  $R$  is in row echelon form, then the row vectors with the leading 1's (the nonzero row vectors)
- A. form a basis for the column space of  $R$
  - B. form a basis for the row space of  $R$
  - C. does not form a basis for the column space of  $R$
  - D. does not form a basis for the row space of  $R$
59. Choose the correct statement.
- A. The column space of a matrix  $A$  is the set of solutions of  $Ax = b$ .
  - B. If  $R$  is the reduced row echelon form of  $A$ , then those column vectors of  $R$  that contain the leading 1's form a basis for the column space of  $A$ .
  - C. The system  $Ax = b$  is inconsistent if and only if  $b$  is not in the column space of  $A$ .
  - D. If  $A$  and  $B$  are  $n \times n$  matrices that have the same row space, then  $A$  and  $B$  have the same column space.
60. Choose the wrong statement.
- A. If  $E$  is an  $m \times m$  elementary matrix and  $A$  is an  $m \times n$  matrix, then the column space of  $EA$  is the same as the column space of  $A$ .
  - B. If  $E$  is an  $m \times m$  elementary matrix and  $A$  is an  $m \times n$  matrix, then the row space of  $EA$  is the same as the row space of  $A$ .
  - C. If  $E$  is an  $m \times m$  elementary matrix and  $A$  is an  $m \times n$  matrix, then the null space of  $EA$  is the same as the null space of  $A$ .
  - D. All are true.
61. What do you know about the dimension of the row space and the column space of a matrix?
- A. dimension of the row space is less than that of the column space.

- B. dimension of the row space is greater than that of the column space.
  - C. dimension of the row space and that of the column space are different.
  - D. dimension of the row space and that of the column space are same always.
62. What is the maximum possible rank of an  $m \times n$  matrix  $A$  that is not square?
- A.  $\text{rank}(A) \geq \min(m, n)$
  - B.  $\text{rank}(A) \leq \min(m, n)$
  - C.  $\text{rank}(A) = \min(m, n)$
  - D.  $\text{rank}(A) = \max(m, n)$
63. If  $A$  is a matrix with  $n$  columns, then
- A.  $\text{rank}(A) + \text{nullity}(A) = n$
  - B.  $\text{rank}(A) - \text{nullity}(A) = n$
  - C.  $\text{rank}(A) + \text{nullity}(A) = 2n$
  - D.  $\text{rank}(A) - \text{nullity}(A) = n/2$
64. Find the number of parameters in the general solution of  $Ax = 0$  if  $A$  is a  $5 \times 7$  matrix of rank 3.
- A. 10
  - B. 5
  - C. 4
  - D. 7
65. Find the rank of a  $5 \times 6$  matrix  $A$  for which  $Ax = 0$  has a two-dimensional solution space.
- A. 8
  - B. 6
  - C. 5
  - D. 4
66. For a matrix  $A$ , the row space of  $A^T$  is same as

- A. row space of  $A$
  - B. column space of  $A$
  - C. column space of  $A^T$
  - D. null space of  $A$
67. Let  $A$  be any matrix. Then
- A.  $\text{rank}(A) = \text{rank}(A^T)$
  - B.  $\text{rank}(A) \neq \text{rank}(A^T)$
  - C.  $\text{rank}(A) < \text{rank}(A^T)$
  - D.  $\text{rank}(A) > \text{rank}(A^T)$
68. If  $W$  is a subspace of  $R^n$ , then which of the following statement is false.
- A.  $W^\perp$  is a subspace of  $R^n$ .
  - B. The only vector common to  $W$  and  $W^\perp$  is 0.
  - C. The orthogonal complement of  $W^\perp$  is  $W$ .
  - D. None of these.
69. If  $A$  is an  $m \times n$  matrix, then
- A. The null space of  $A$  and the row space of  $A$  are orthogonal complements in  $R^n$ .
  - B. The null space of  $A^T$  and the column space of  $A$  are orthogonal complements in  $R^m$ .
  - C. Both A and B are correct.
  - D. Neither A nor B are correct.
70. Let  $A$  be an invertible  $n \times n$  matrix. Pick out the wrong one.
- A. The orthogonal complement of the null space of  $A$  is  $R^n$ .
  - B.  $A$  has nullity  $n$ .
  - C.  $\det(A) \neq 0$ .
  - D. The orthogonal complement of the row space of  $A$  is  $\{0\}$ .

71. Let  $A$  be a  $7 \times 6$  matrix such that  $Ax = 0$  has only the trivial solution. What is the rank of  $A$ ?
- A. 6
  - B. 0
  - C. 7
  - D. 1
72. Let  $A$  be a  $5 \times 7$  matrix with rank 4. What is the dimension of the solution space of  $Ax = 0$ ?
- A. 4
  - B. 3
  - C. 5
  - D. 1
73. Let  $T : R^5 \rightarrow R^3$  be the linear transformation defined by  $T(x_1, x_2, x_3, x_4, x_5) = (x_1 + x_2, x_2 + x_3 + x_4, x_4 + x_5)$ . Find the nullity of the standard matrix for  $T$ .
- A. 5
  - B. 3
  - C. 2
  - D. 1
74. If  $A$  is a  $3 \times 5$  matrix, then the number of leading 1's in the reduced row echelon form of  $A$  is at most ....
- A. 4
  - B. 2
  - C. 5
  - D. 3
75. What is the largest possible value for the rank of  $A$  and the smallest possible value for the nullity of  $A$  if  $A$  is a  $5 \times 3$  matrix.
- A. 3;0
  - B. 5;3



- C. 5;2
  - D. 5;0
76. Pick the correct statement.
- A. Either the row vectors or the column vectors of a square matrix are linearly independent.
  - B. The nullity of a nonzero  $m \times n$  matrix is at most  $m$ .
  - C. If  $A$  is square and  $Ax = b$  is inconsistent for some vector  $b$ , then the nullity of  $A$  is zero.
  - D. The nullity of a square matrix with linearly dependent rows is at least one.
77. The rank of a matrix  $A$  is the
- A. dimension of the row space of  $A$ .
  - B. dimension of the column space of  $A$ .
  - C. both A and B
  - D. dimension of the null space of  $A$ .
78. If  $A$  is an  $m \times n$  matrix, then the column space of  $A$
- A. is a subspace of  $R^n$
  - B. is a subspace of  $R^m$
  - C. is a subspace of  $R^{mn}$
  - D. is a subspace of  $R^{\min(m,n)}$
79. For a matrix  $A$ , the column space of  $A^T$  is same as
- A. row space of  $A$
  - B. column space of  $A$
  - C. row space of  $A^T$
  - D. null space of  $A$
80. If  $A$  is an  $m \times n$  matrix, then the null space of  $A$
- A. is a subspace of  $R^n$
  - B. is a subspace of  $R^m$
  - C. is a subspace of  $R^{mn}$
  - D. is a subspace of  $R^{\min(m,n)}$

81. The values of  $r$  and  $s$  for which  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & r-2 & 2 \\ 0 & s-1 & r+2 \\ 0 & 0 & 3 \end{bmatrix}$  has

rank 1?

- A.  $r = 2, s = 2$
  - B.  $r = 2, s = 1$
  - C.  $r = -2, s = 1$
  - D. Cannot have rank 1
82. If  $A$  is a  $3 \times 5$  matrix, then the rank of  $A^T$  is at most
- A. 4
  - B. 2
  - C. 5
  - D. 3
83. If  $A$  is a  $3 \times 5$  matrix, then the nullity of  $A^T$  is at most
- A. 4
  - B. 2
  - C. 5
  - D. 3

84. The values of  $r$  and  $s$  for which  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & r-2 & 2 \\ 0 & s-1 & r+2 \\ 0 & 0 & 3 \end{bmatrix}$  has

rank 2?

- A.  $r = 2, s = 2$
  - B.  $r = 2, s = 1$
  - C.  $r = -2, s = 1$
  - D. Cannot have rank 2
85. The  $6 \times 6$  matrix with all entries 1 have rank
- A. 1
  - B. 2

- C. 4
  - D. 6
86. Pick the odd one out.
- A.  $\lambda$  is an eigenvalue of  $A$ .
  - B.  $\lambda$  is a solution of the characteristic equation  $\det(\lambda I - A) = 0$ .
  - C. The system of equations  $(\lambda I - A)x = 0$  has trivial solutions.
  - D. There is a nonzero vector  $x$  such that  $Ax = \lambda x$ .
87. Let  $A$  is an  $n \times n$  matrix. The eigenspace of  $A$  corresponding to  $\lambda$  is same as:
- A. the null space of the matrix  $\lambda I - A$ .
  - B. the kernel of the matrix operator  $T_{\lambda I - A} : R^n \rightarrow R^n$ .
  - C. the set of vectors for which  $Ax = \lambda x$ .
  - D. All the above.
88. A square matrix  $A$  is invertible if and only if
- A.  $\lambda = 0$  is not an eigenvalue of  $A$ .
  - B.  $\lambda = 0$  is an eigenvalue of  $A$ .
  - C.  $\lambda = 1$  is not an eigenvalue of  $A$ .
  - D.  $\lambda = 1$  is an eigenvalue of  $A$ .
89. Let  $A$  is an  $n \times n$  matrix and suppose  $A$  has rank  $n$ . Then
- A.  $T_A$  is not one-to-one.
  - B.  $\lambda = 0$  is not an eigenvalue of  $A$ .
  - C. The range of  $T_A$  is  $\{0\}$ .
  - D. The kernel of  $T_A$  is  $R^n$ .
90. Let  $T : V \rightarrow V$  be a linear operator and  $T(x) = \lambda x$  for some scalar  $\lambda$ . Then  $x$  is called
- A. an eigenvector of  $T$ .
  - B. an eigenvalue of  $T$ .

- C. an eigenspace of  $T$ .
  - D. None of these.
91. Suppose that the characteristic polynomial of some matrix  $A$  is found to be  $p(\lambda) = (\lambda - 1)(\lambda - 3)^2(\lambda - 4)^3$ . What is the size of  $A$ ?
- A.  $5 \times 5$
  - B.  $6 \times 6$
  - C.  $5 \times 6$
  - D.  $6 \times 5$
92. If 0 is an eigenvalue of a matrix  $A$ , then the set of columns of  $A$  is
- A. linearly independent or linearly dependent..
  - B. linearly dependent always.
  - C. linearly independent always.
  - D. Cannot be determined.
93. Which of the following is not a similarity invariant?
- A.  $A$  and  $P^{-1}AP$  have the same rank.
  - B.  $A$  and  $P^{-1}AP$  have the same nullity.
  - C.  $A$  and  $P^{-1}AP$  have the same trace.
  - D. None of these.
94. Let an  $n \times n$  matrix  $A$  be diagonalizable. Then
- A.  $A$  has  $n$  linearly independent eigenvectors.
  - B.  $A$  has  $n$  linearly dependent eigenvectors.
  - C.  $A$  has  $n+1$  linearly independent eigenvectors.
  - D.  $A$  has  $n-1$  linearly dependent eigenvectors.
95. If  $A$  is a square matrix, then for every eigenvalue of  $A$ ,
- A. the geometric multiplicity is equal to the algebraic multiplicity.
  - B. the geometric multiplicity is less than or equal to the

algebraic multiplicity.

C. the geometric multiplicity is greater than or equal to the algebraic multiplicity.

D. the geometric multiplicity is strictly less than the algebraic multiplicity.

96. Which of the following is true?

A.  $\langle u, v + w \rangle = \langle u, v \rangle + \langle u, w \rangle$

B.  $\langle u, v + w \rangle = \langle v, u \rangle + \langle w, u \rangle$

C. Both A and B are true.

D. Neither A nor B is true.

97. If  $u$  and  $v$  are vectors in a real inner product space  $V$ , then

A.  $|\langle u, v \rangle| \geq \|u\| + \|v\|$

B.  $|\langle u, v \rangle| \leq \|u\| + \|v\|$

C.  $|\langle u, v \rangle| \geq \|u\|\|v\|$

D.  $|\langle u, v \rangle| \leq \|u\|\|v\|$

98. Find the correct one from the given statements.

A. If  $u$  is orthogonal to every vector of a subspace  $W$ , then  $u = 0$ .

B. If  $u$  and  $v$  are orthogonal, then  $|\langle u, v \rangle| = \|u\|\|v\|$ .

C. If  $u$  and  $v$  are orthogonal, then  $\|u + v\| = \|u\|\|v\|$ .

D. None of these.

99. If  $u$  and  $v$  are orthogonal vectors in a real inner product space  $V$ , then

A.  $\|u + v\|^2 = \|u\|^2 + \|v\|^2$ .

B.  $\|u + v\|^2 < \|u\|^2 + \|v\|^2$ .

C.  $\|u + v\|^2 > \|u\|^2 + \|v\|^2$ .

D.  $\|u + v\|^2 = \|u\|^2\|v\|^2$ .

100. Find the wrong one from the given statements.

- A. If  $A$  is diagonalizable and invertible, then  $A^{-1}$  is diagonalizable.
- B. If  $A$  is diagonalizable, then  $A^T$  is diagonalizable.
- C. If every eigenvalue of a matrix  $A$  has algebraic multiplicity 1, then  $A$  is diagonalizable.
- D. An  $n \times n$  matrix with fewer than  $n$  distinct eigenvalues is not diagonalizable.

## ANSWER KEY

1. C
2. B
3. D
4. B
5. A
6. B
7. C
8. A
9. B
10. C
11. D
12. A
13. B
14. D
15. A
16. C
17. D
18. A
19. C

20. B

21. C

22. D

23. A

24. D

25. C

26. A

27. C

28. B

29. C

30. D

31. A

32. B

33. C

34. D

35. D

36. B

37. C

38. A

39. A



40. D

41. C

42. B

43. B

44. C

45. B

46. D

47. A

48. A

49. C

50. C

51. B

52. D

53. B

54. A

55. C

56. C

57. D

58. B

59. C

60. A

61. D

62. B

63. A

64. C

65. D

66. B

67. A

68. D

69. C

70. B

71. A

72. B

73. C

74. D

75. A

76. D

77. C

78. B

79. A

- 80. B
- 81. D
- 82. D
- 83. D
- 84. B
- 85. A
- 86. C
- 87. D
- 88. A
- 89. B
- 90. A
- 91. B
- 92. C
- 93. D
- 94. A
- 95. B
- 96. C
- 97. D
- 98. D
- 99. A
- 100. D

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