Numerical Method

~~Questionxx~~ 01:-

$$v(t) = \frac{gm}{c} \cdot \left[1 - e^{-ct/m}\right] \quad \text{--- } 1$$

$g = 9.8 \ m/s^2$

$c = 15 \ kg/s$

$v = 35 \ m/s$

$t = 9s$

$\varepsilon_s = 0.1\%$

Use Newton-Raphson.?

Answer:-

Substituting given values, in eq 1

$$35 = \frac{9.81(m)}{15}\left[1 - e^{(-15/m)9}\right]$$

$$9.81m(1 - e^{-\frac{135}{m}}) - 525 = 0$$

So,

$$f(m) = 9.81^m(1 - e^{\frac{-135}{m}}) - 525 \quad \text{--- } 3$$

the interval within which the roots lies is $X_l = 55$ $X_u = 60$

Formula for Newton-Raphson's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad — (2)$$

and $f(X_l) = -31.7985$

$f(x_u) = 1.562$

Now let upper limit only as, $x_0 = 60$

$f(x_0) = 1.562$.

Taking derivative of $f(m)$ we get

$$f'(m) = 9.81\left[1 - e^{-135/m}\left[1 + \frac{135}{m}\right]\right] \quad — (4)$$

using eq (2) with $n=0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = 60 - \frac{1.562}{6.4496}$$

Hence $x_1 = 59.7578$

and $f(x_1) = -0.002561$

$f'(x_1) = 6.4707$ ∴ (by putting value in eq (4))

Again using eq (2)

$$x_2 = 59.7578 - \frac{(-0.002561)}{6.4707}$$

$x_2 = 59.7582.$

$f(x_2) = -0.000005$

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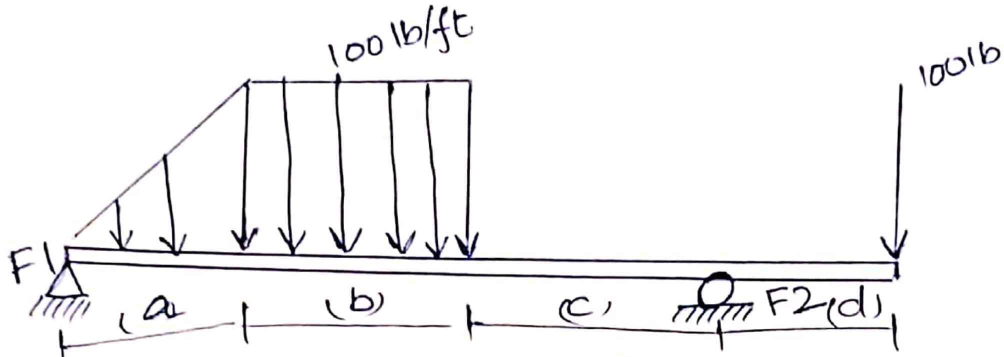
③ Hence the root required is

$$\boxed{m = 59.7582}$$

Question no. 02.



use bisection Method. to find position of no moment

Answer:-

Let 2 supporting forces be F1 and F2

Sum of torque and forces be equal to zero.

$F_a = 3 \times \frac{100}{2}$ $\qquad$ $F_b = 3 \times 100$

$F_a = 150\,lb$ $\qquad$ $F_b = 300\,lb$

$F_c = 0$ $\qquad$ $F_d = 100\,lb$

Hence forces w.r.t sections are

$F_a = 150\,lb$

$F_b = 300\,lb$

$F_c = 0$

$F_d = 100\,lb$

Sum of all forces

$= 550\,lb$

④

We have forces supporting:-

as $F_1 = 265 lb$

$F_2 = 285 lb$

NOW Applying bisection method,

Limits

$x_l = 6$ $\quad$ $M = -185x + 1650$

$x_u = 10$

$M(x_l) = -185(x_l) + 1650$

$M(x_l) = 540$

$M(x_u) = -200$

1st

$x_r = 8$ as $\left(\frac{x_l + x_u}{2}\right)$

$M(x_r) = -185(8) + 1650$

$= 170$

Hence $x_l = x_r$

So $x_r = \frac{9+10}{2} = 9$

2nd

$M(x_r) = -15$

$M(x_l) = \cancel{-200}\ 170$

$M(x_u) = -200$

3rd

So, $x_r = \frac{8+9}{2} = 8.5$

$M(x_r) = -185(8.5) + 1650 = 77.5$

$M(x_l) = 77.5$

$M(x_u) = -15$

4th

4.5

Again

$$\left.\begin{aligned} M(x_u) &= -15 \\ M(x_r) &= 31.25 \\ M(x_l) &= 31.25 \end{aligned}\right\} 5^{th}$$

Hence $x_r = 8.873$

$M(x_u) = -15$ $\underbrace{M(x_r) = 8.125 \quad M(x_l) = 8.125}_{6^{th}}$

$x_r = 8.9375$

$$\left.\begin{aligned} M(x_r) &= -15 \\ M(x_u) &= -3.4375 \\ M(x_l) &= 8.125 \end{aligned}\right\} 7^{th}$$

$$\left.\begin{aligned} x_r &= 8.90625 \\ M(x_r) &= -3.4375 \end{aligned}\right\} 8^{th}$$

$M(x_u) = 2.3475$

$M(x_l) = 2.34375$

$$\left.\begin{aligned} x_r &= 8.9218 \\ M(x_r) &= -3.4375 \\ M(x_u) &= -0.546875 \end{aligned}\right\} 9^{th}$$

$M(x_l) = 2.34375$

$$\left.\begin{aligned} x_r &= 8.9140625 \\ M(x_r) &= 0.8984 \\ M(x_u) &= -0.5468 \end{aligned}\right\} 10^{th}$$

Hence $x = 8.91$ feet is where moment $= 0$

$M(x_l) =$ ---

⑤

Question no. 03.

$$0 = 1 - \frac{Q^2}{gA_c^3} \cdot B \quad — (1)$$

$Q = 20 m^3/s$
$g = 9.81 \ m/s^2$
$B = 3 + y$ $\qquad A_c = 3y + y^2/2$

a) Bisection Method

$x_l = 0.5$, $x_u = 2.5$

Substiting values in (1) we get

$$0 = 1 - \frac{20^2}{9.81 \times \left[3y + y^2/2\right]^3} \times (3+y)$$

$$f(y) = 1 - \frac{40.77}{(3y + y^2/2)^3} \times (3+y) \quad — (2)$$

as $x_l = 0.5$ and $x_u = 2.5$

$f(x_l) = -32.26$
$f(x_u) = 0.81$] by putting in eq (2)

using bisection method,

$$x_r = \frac{0.5 + 2.5}{2} = 1.5 \quad , \quad f(x_r) = -0.03$$

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Error = 66.67%.

Now, as $f(x_r) f(x_e) > 0$, This means solution lies b/w x_r and x_u.

So, $x_l = x_r$, $x_u = x_u$

Iteration table.

No.	x_l	x_u	x_r	$f(x_l)$	$f(x_u)$	$f(x_r)$	Ea
1	0.5	2.5	1.5	-32.258	0.8130	-0.0309	66.6
2	1.5	2.5	2	-0.03095	0.81303	0.60181	25
3	1.5	2	1.75	-0.03095	0.60181	0.378909	14.285
4	1.5	1.75	1.625	-0.03095	0.378909	0.206928	7.6923
5	1.5	1.625	1.5625	-0.03095	0.2069	0.09795	4
6	1.5	1.5625	1.53125	-0.03095	0.09795	0.0362	2.04
7	1.5	1.53125	1.515625	"	0.03626	0.0033	1.03
8	1.5	1.515625	1.507813	"	0.003384	-0.01359	0.51

Hence after 8 iterations, we obtain

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$$\boxed{X_r = 1.5078}$$

with error $= 0.52\%$.

b) Using False-Position Method:-

Formula

$$x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)} \quad \text{--- (3)}$$

$x_r = 2.4508$ by putting $x_l = 0.5$, $x_u = 2.5$ in eq (3)

$f(x_r) = 0.7999$

As now $f(x_r) f(x_l) < 0$ Hence root lies between x_l and x_r, for next we tak

$$x_l = x_l$$

$$x_u = x_r.$$

8)

Iteration table:-

No.	X_l	X_u	X_r	$f(x_l)$	$f(x_u)$	$f(x_r)$	Ea
1	0.5	2.5	2.450	-32.258	0.813032	0.7998	66.6
2	0.5	2.45	2.403	-32.258	0.79987	0.7861	66.1
3	0.5	2.4036	2.3151	-16.1291	0.78612	0.7569	65.5
4	0.5	2.315	2.2337	″	0.7569	0.7256	64.4
5	0.5	2.33	2.1591	″	0.7256	0.6921	63.4
6	0.5	2.159	2.0908	″	0.6921	0.6569	62.3
7	0.5	2.090	2.0286	″	0.6569	0.6503	62.3
8	0.5	2.02	1.9719	″	0.6203	0.5825	61.4
9	0.5	1.9719	1.9206	″	0.5825	0.5441	60.[illegible]
10	0.5	1.906	$\boxed{1.874}$	″	0.5441	0.5054	58.6

$$\boxed{X_r = 1.8743}$$

Error = 58.6%

1

Question no 4

$$V = \pi h^2 \frac{(3R-h)}{3} \quad \text{--- } (1)$$

$R = 3m \qquad V = 30m^3$

False position Method.

Formula :-

$$x_{n+1} = \frac{x_{n-1} \cdot f(x_n) - x_n \, f(x_{n-1})}{f(x_n) - f(x_{n-1})} \quad \text{--- } (2)$$

putting values in eq (1)

$$30 = \frac{\pi h^2 (3(3) - h)}{3}$$

$$\boxed{f(h) = \pi h^2 (9-h) - 90}$$

$x_0 = 0$ and $x_1 = 3$

using eq (2) at $n = 1$

$$x_2 = \frac{x_0 \cdot f(x_1) - x_1 \cdot f(x_0)}{f(x_1) - f(x_0)}$$

where
"x" is equivalent to "h"

$$h_2 = \frac{(0(79.64)) - 3(-90)}{79.64 - (-90)}$$

$$\left.\begin{aligned} h_2 &= 1.59159 \\ f(h_2) &= -31.0454 \end{aligned}\right\} 1^{st} \text{ iteration}$$

using same formula,

$$h_3 = \frac{h_1 \cdot f(h_2) - h_2 \cdot f(h_1)}{f(h_2) - f(h_1)}$$

$$h_3 = \frac{(3) \cdot (-31.0454) - (1.59159)(79.64)}{-31.0434 - (79.646)}$$

$$\left.\begin{aligned} h_3 &= 1.9866 \\ f(h_3) &= -3.0437 \end{aligned}\right\} 2^{nd} \text{ iteration}$$

For h_4:-

$$h_4 = \frac{h_2 \cdot f(h_3) - h_3 f(h_2)}{f(h_3) - f(h_2)}$$

$$h_4 = \frac{(1.59159)(-3.0437) - 1.9866(-31.045)}{-3.0437 - (-31.0454)}$$

$$\left.\begin{aligned} h_4 &= 2.02953 \\ f(h_4) &= 0.19977 \end{aligned}\right\} 3^{rd} \text{ iteration.}$$

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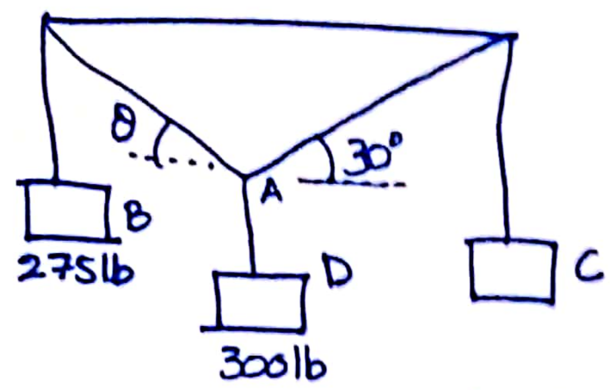
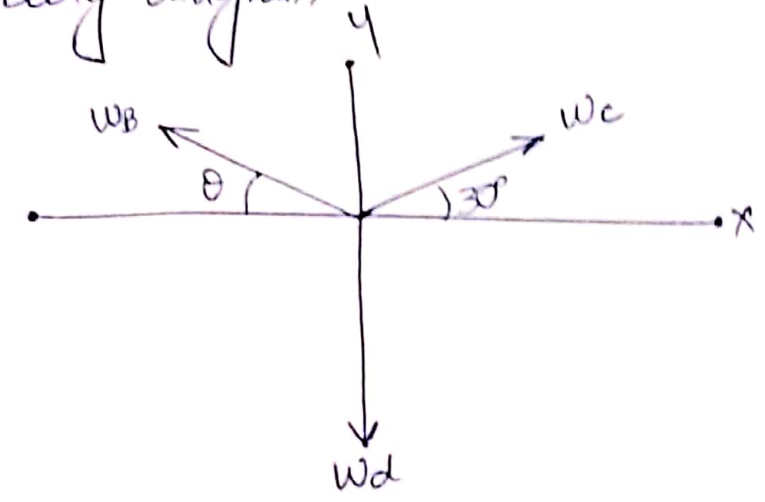
Question # 05



Newton Raphson to find θ.

Answer

Freebody diagram



$\Sigma F_x = 0$

$$W_c \cos 30° - 275 \cos\theta = 0$$

$\Sigma F_y = 0$

$$W_c \sin 30° + 275 \sin\theta - 300 = 0$$

Solving both above equations

$$W_c = 240 lb$$

$$\theta = 40.9°$$

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Question#06

Use data to find following,

1- jam density
2- speed at max. flow
3- density at max. flow
4- capacity
5- r^2 .

$$U_s = c \ln \frac{k_j}{k}$$,

Expanding above eq.

$$U_s = c\left[\ln k_j - \ln k\right]$$

$$U_s = -c \ln k + c \ln k_j$$

let $U_s = Y$ and $\ln k = X$

$-c = m$ $c \ln k_j = C$

we get

$$Y = mX + C$$

plotting given data we get

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$Y = -30.001x + 149.93$

Comparing it to greenberg's Model

$Y = U_s$, $c \ln k_j = 149.93$

$m = -30.001 = -c$

we get

$c = 30.001$

$c \ln k_j = 149.93$

$\boxed{k_j = 148.04}$ — For part 1

Flow : $Q = ku$

$= kc \ln \frac{k_j}{k}$

For max. flow , $\frac{ds}{dk} = 0$

$\frac{d}{dk}(kc \ln k_j/k) = 0$

$\frac{d}{dk}(kc \ln k_j - kc \ln k) = 0$

$c \ln k_j - c \ln k - c = 0$

$c \ln k_j/k = c$

$k_j/k = e$

$k_j/e = k$

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$k_j = 148.04$ vpm

$$\therefore \boxed{K_o = \frac{148.04}{e} = 54.46}$$

put this back in greenberg's model

$$V_o = c \ln k_j/k$$

$$\boxed{V = 30.001 \text{mph}}$$

Capacity (Q_{max}) = maxflow

$= U_o K_o$

$= 30.001 \times 54.46$

$$\boxed{= 1633.88 \text{vph}}$$