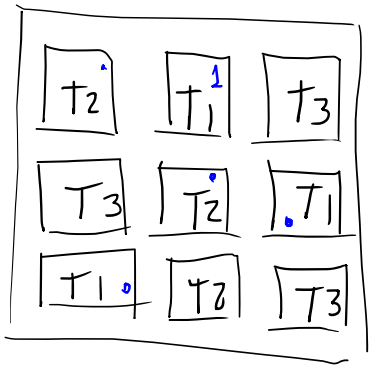
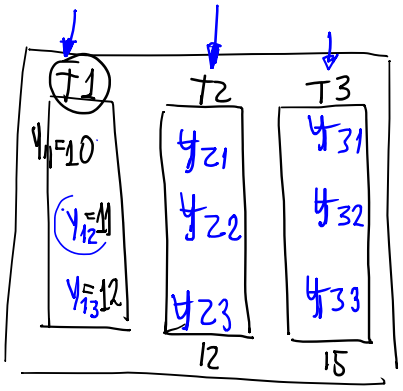
Repetición	Tratamientos			
	1	2	...	t
1	Y_{11}	Y_{21}	...	Y_{t1}
...	...	...	...	...
n_i	Y_{1n}	Y_{2n}	...	Y_{tn}
totales	$Y_{1.}$	$Y_{2.}$	...	$Y_{t.}$

$$\sum_{j=1}^{n} Y_{1j} = Y_{1.} \qquad \sum_{j=1}^{n} Y_{2j} = Y_{2.} \qquad \text{General } \sum_{j=1}^{n} Y_{ij} = Y_{i.}, \; i = 1,2,3,\ldots,t$$

$$\Rightarrow \sum_{i=1}^{t}\sum_{j=1}^{n} Y_{ij} = \sum_{i=1}^{t} Y_{i.} = Y_{..} = \text{Suma total.}$$





Variación total de los datos

Suma cuadrados total

$$\sum_{i=1}^{t}\sum_{j=1}^{n} (Y_{ij} - \bar{Y}_{..})^2$$

$$SCT_o = SC\text{entre tratamientos} + SC\text{dentro tratamientos}$$

$$\sum_{i=1}^{t}\sum_{j=1}^{n} (Y_{ij} - \bar{Y}_{..})^2 = \sum_{i=1}^{t}\sum_{j=1}^{n} \underbrace{(Y_{ij} - \bar{Y}_{..} + \bar{Y}_{i.} - \bar{Y}_{i.})}_{SCE}{}^2$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{n} \left[(\bar{Y}_{i.} - \bar{Y}_{..}) + (Y_{ij} - \bar{Y}_{i.})\right]^2$$

$$= \sum_{i=1}^{t}\sum_{j=1}^{n} \left[(\bar{Y}_{i.} - \bar{Y}_{..})^2 + 2(\bar{Y}_{i.} - \bar{Y}_{..})(Y_{ij} - \bar{Y}_{i.}) + (Y_{ij} - \bar{Y}_{i.})^2\right]$$

$$= \underbrace{\sum_{i=1}^{t}\sum_{j=1}^{n} (\bar{Y}_{i.} - \bar{Y}_{..})^2}_{\text{Variación entre tratamientos}} + \underbrace{\sum_{i=1}^{t}\sum_{j=1}^{n} (Y_{ij} - \bar{Y}_{i.})^2}_{\text{Variación dentro tratamiento}} + 2\sum_{i=1}^{t}\sum_{j=1}^{n} (\bar{Y}_{i.} - \bar{Y}_{..})(Y_{ij} - \bar{Y}_{i.})$$

$\sum_{j=1}^{n}(Y_{ij} - \bar{Y}_{i.}) = 0$

Error

$$\sum_{j=1}^{n} Y_{ij} - \sum_{j=1}^{n} \bar{Y}_{i.}$$

Error

en el termino $\sum_{j=1}^{n}(y_{ij} - \bar{y}_{i\cdot}) = \sum_{j=1}^{n} y_{ij} - \sum_{j=1}^{n} \bar{y}_{i\cdot}$

$= y_{i\cdot} - n\bar{y}_{i\cdot}$

$= y_{i\cdot} - n\cdot\frac{y_{i\cdot}}{n}$

$= y_{i\cdot} - y_{i\cdot}$

$= 0$

	Fuentes Variación	gl	Sumas Cuadrados	Cuadrados Medios	F
Tratami ⇒	Entre tratam.	$t-1$	$\sum\sum(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$	$CMtrat = \frac{SCtrat}{t-1}$	$F_c = \frac{\frac{SCtrat}{t-1}}{\frac{SCError}{t(n-1)}} = \frac{\frac{\chi^2}{(t-1)}}{\frac{\chi^2}{t(n-1)}}$
Error ⇒	dentro trat.	$\equiv N-t$ $t(n-1)$	$\sum\sum(y_{ij} - \bar{y}_{i\cdot})^2$	$CMError = \frac{SCError}{t(n-1)} = \sigma^2$	
	Total	$\equiv N-1$ $tn-1$	$\sum\sum(y_{ij} - \bar{y}_{\cdot\cdot})^2$		

ojo: $N = t \times n$

$F_c(t-1;\ t(n-1)\ gl)$

$y_{ij} = \mu + \tau_i + \varepsilon_{ij}$ modelo lineal

Parametros μ, τ_i

Estimación por minimos cuadrados.

$Q = \sum_{i=1}^{t}\sum_{j=1}^{n}\varepsilon_{ij}^2 = \sum_{i=1}^{t}\sum_{j=1}^{n}(y_{ij} - \underbrace{\mu - \tau_i}_{\mu_i})^2$

$\frac{\partial Q}{\partial \mu} = 2\sum_{i=1}^{t}\sum_{j=1}^{n}(y_{ij} - \mu - \tau_i)\frac{\partial(-\mu)}{\partial \mu}^{-1} = -2\sum_{i=1}^{t}\sum_{j=1}^{n}(y_{ij} - \mu - \tau_i) = 0$

$\sum\sum y_{ij} = \sum\sum \mu + \sum_{1}^{n}\sum \tau_i$

$$\frac{\partial}{\partial \mu} \sum_{i=1}^{t}\sum_{j=1}^{n} (y_{ij} - \mu - \tau_i)^2 = \frac{\partial}{\partial \mu} 0$$

$$\sum_i \sum_j y_{ij} = \sum_i \sum_j \mu + \sum_i \sum_j \tau_i$$

$$\sum\sum y_{ij} = N\mu + n\sum_{i=1}^{t} \tau_i \quad (A)$$

$$\frac{\partial Q}{\partial \tau_i} = 2\sum\sum (y_{ij} - \mu - \tau_i) \frac{\partial(-\tau_i)}{\partial(\tau_k)}$$

$$\begin{cases} -1, & i = k \\ 0, & i \neq k \end{cases} \Big\} \delta_{ik}$$

$$= 2\sum_{i=1}^{t}\sum_{j=1}^{n} (y_{ij} - \mu - \tau_i)\,\delta_{ik}$$

$$= -2\sum_{j=1}^{n} (y_{ij} - \mu - \tau_i) = 0$$

$$\sum_{j=1}^{n} y_{ij} = \sum_{j=1}^{n} \mu + \sum_{j=1}^{n} \tau_i \quad , \; i = 1,2,3,4,5,\ldots,t$$

$$\frac{y_{i\cdot}}{n} = \frac{n\mu}{n} + \frac{n\tau_i}{n}$$

$$\bar{y}_{i\cdot} = \mu + \tau_i \quad \ldots (B)$$ Ecuación normal

Promedio del i-ésimo tratamiento = media global + Efect. tratami

R.L.S

$y = \beta_0 + \beta_1 x_1 + \varepsilon$ modelo

$\hat{y} = b_0 + b_1 x_1$ Ecuación

$$\hat{y}_{ij} = \mu + \tau_i = \bar{y}_{i\cdot}$$

Imponemos restricciones $\sum_{i=1}^{t} n\tau_i = 0$ ó $n\sum_{i=1}^{t} \tau_i = 0$

Reemplazando restricción en (A)

$$\sum\sum y_{ij} = N\mu + n\sum_{i=1}^{t} \tau_i$$

$$\frac{\sum\sum y_{ij}}{N} = \mu$$

$$\boxed{\bar{y}_{\cdot\cdot} = \hat{\mu}}$$

Ahora en B

$$\bar{y}_{i\cdot} = \hat{\mu} + \tau_i \quad (B)$$

$$\bar{Y}_{i\cdot} = \hat{\mu} + \hat{\tau}_i \quad \cdots \quad (B)$$

$$\hat{\tau}_i = \bar{Y}_{i\cdot} - \hat{\mu}$$

$$\hat{\tau}_i = \bar{Y}_{i\cdot} - \bar{Y}_{\cdot\cdot}$$

Por tanto

$$\hat{Y}_{ij} = \hat{\mu} + \hat{\tau}_i$$

$$= \bar{Y}_{\cdot\cdot} + \bar{Y}_{i\cdot} - \bar{Y}_{\cdot\cdot}$$

$$\hat{Y}_{ij} = \bar{Y}_{i\cdot}$$