EEL 3040: Control Systems

Control of Multi-Agent Systems

Q 1 *Consider a system of N robots moving in a planar space. Let $(x_i(t), y_i(t))$ be the positional coordinates of the i^{th} robot and it moves with constant speed v_i in the direction $\theta_i(t)$ at any instant of time t. With these notations, the equations of motion of the i^{th} robot are given by*

$$\dot{x}_i(t) = v_i \cos\theta_i(t) \tag{1a}$$

$$\dot{y}_i(t) = v_i \sin\theta_i(t) \tag{1b}$$

$$\dot{\theta}_i(t) = u_i, \qquad i = 1, \dots, N, \tag{1c}$$

where time derivatives $\dot{x}_i, \dot{y}_i$ denote the speeds of the i^{th} robot along horizontal and vertical axes, respectively. Moreover, u_i is the control law which controls the rate of change $\dot{\theta}_i$ of the velocity direction. Suppose that the objective is to achieve consensus in the velocity directions θ_i of all the robots, starting from different velocity directions. To accomplish this, the control u_i can be designed as follows

$$u_i = K \sum_{j \in N(i)} (\theta_j(t) - \theta_i(t)), \quad i = 1, \dots, N, \tag{2}$$

where $N(i)$ is the set of the neighbors of the i^{th} robot and $K > 0$ is the controller gain. This form of the control law is usually referred to as agreement protocol *in the literature. Since the control law u_i of the i^{th} robot uses the information about the velocity directions $\theta_j(t)$ of the nearby robots, the interaction topology among the robots plays an important role, which we would understand through the following MATLAB exercise.*

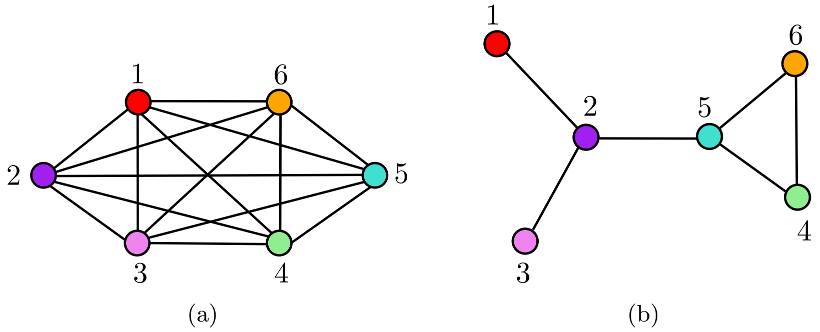

Figure 1: Interaction topology among robots.

a) *Considering three cases of the controller gains $K = 0.01, 0.1$ and 1, simulate the control protocol (2) for the six robots, starting from initial conditions given below, and sharing speed information according to iteration typology in Figure 1(a).*

Robot Number	*Initial Position $(x_i(0), y_i(0))$*	*Initial Velocity Direction $\theta_i(0)$*	*Speed v_i*
1	$(-10, -2)$	$0°$	*1.0*
2	$(4, -2)$	$30°$	*1.5*
3	$(-10, 10)$	$45°$	*2.0*
4	$(2, 5)$	$60°$	*2.5*
5	$(0, -5)$	$75°$	*3.0*
6	$(5, -7)$	$90°$	*3.5*

b) *Repeat the above exercise for the interaction topology in Figure 1(b) and compare the results for both scenarios in parts (a) and (b).*

c) *Where does the consensus in velocity directions occur in both the cases (a) and (b), and how do different values of the controller gains influence it? Justify your answer.*

d) *Simulate if there is no link between robots 2 and 5 in Figure 1(b).*

Q 2 *In* **Q 1** *above, suppose that the agents have different control gain, that is, the control gain of the* i^{th} *agent is* $K_i > 0$. *The control law* (2) *in this case becomes:*

$$u_i = K_i \sum_{j \in N(i)} (\theta_j(t) - \theta_i(t)), \quad i = 1, \ldots, N. \tag{3}$$

a) *Analyze your results under control law* (3) *and compare with control law* (2).

b) *Can we achieve the consensus at some pre-specified velocity direction by properly adjusting the control gains* K_i*? Justify using simulations.*