EEL 3040

Control System

Lab - 3

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Ball & Beam

Content

1) Objective :-

a) Aim :-
The objective of the Ball and Beam experiment is to stabilise the ball to a desired position along the beam using the remote sensor unit. Using the proportional-derivative (PD) family, a cascade control system will be designed to meet a set of specifications.

b) Software :-
MATLAB, Quanser SRV02 unit, Quanser Ball and Beam module, Q8-USB, UPM-2405 amplifier and remote sensor unit.

C) Theory :-

The "Ball and Beam" system is a classic problem in control systems engineering that serves as an excellent platform for studying the principles of feedback control. The apparatus consists of a long beam that is pivoted at its centre, with a motor controlling the angle of the beam's tilt. A ball is free to roll along the length of the beam. The fundamental challenge is to design a control system that actively adjusts the beam's angle to position the ball at a specific point and maintain its stability. This system is inherently unstable because without active control, the ball will roll off either end of the beam .

The Primary Objective of this experiment is to design and implement a Cascade control system to stabilize the ball at a desired position. This involves:

- Developing a mathematical model of the Ball and Beam system from first principles.
- Deriving the system's transfer function.
- Designing a Proportional-Derivative (PD) controller for the outer loop (ball position) and a Proportional-Velocity (PV) controller for the inner loop (servo position).
- Simulating the controller's performance to ensure it meets predefined time-domain specifications for settling time, overshoot, and steady-state error.
- Implementing the controller on the physical hardware and evaluating its real-world performance.

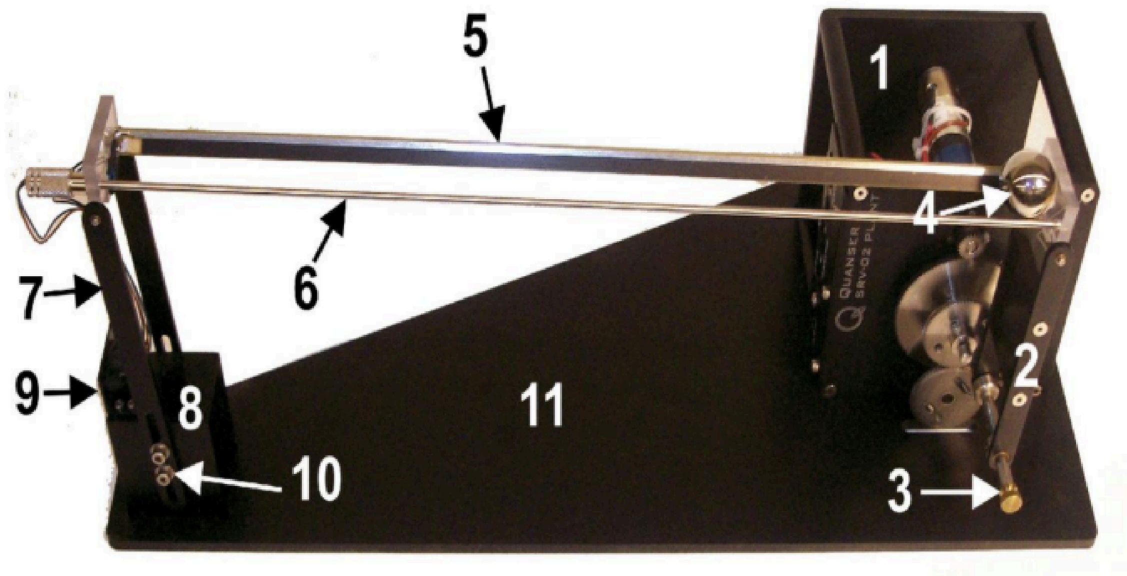


Figure 1: SRV02 Ball and Beam Module with components.

Mathematical Description Of the Model :-

1) Modeling of Ball and Beam system :-

The complete system can be modelled as two distinct connected in series . the SRV2 servo motor plant and the BB01 Ball and Beam plant .

- **SRV02 Servo Plant (Ps(s)):** This is the inner-loop system. The input is the motor voltage, Vm(s), and the output is the servo's load gear angle, θl(s). The transfer function, which was previously determined, is given by

- **BB01 Ball and Beam Plant (Pbb(s)):** This is the outer-loop system. Its input is the angle of the servo gear, θl(s), and its output is the linear position of the ball along the beam, X(s). The transfer function for this plant is derived in the following section.

Figure 5: Ball and Beam open-loop block diagram.

$$P_s(s) = \frac{\theta_l(s)}{V_m(s)}$$

$$P_s(s) = \frac{K}{(\tau s + 1)s}$$

where K is the steady-state gain and τ is the time constant of the motor.

The overall open-loop transfer function for the entire system is the product of the two individual plant transfer functions:

$$P(s) = P_{bb}(s)P_s(s)$$

The control strategy is a cascade control system. The outer loop uses a PD controller to calculate the desired beam angle (θd_l) required to move the ball to the desired position (Xd_l). This desired angle then becomes the setpoint for the inner PV control loop, which calculates the motor voltage (Vm_l) needed to achieve that angle.

2. Non-linear Equation of Motion:

The equation describing the motions of the ball, x, relative to the angle of the beam,

α, will be derived. Thus, the equation of motion will be of the form:

$$\frac{d^2x(t)}{dt^2} = f(\alpha(t))$$

Where is a nonlinear function. Applying Newton's Law of Motion, the sum of the

forces acting on the ball alongside the beam equals

$$m_b(\frac{d^2x(t)}{dt^2}) = \sum F$$

Where mb is the mass of the ball.

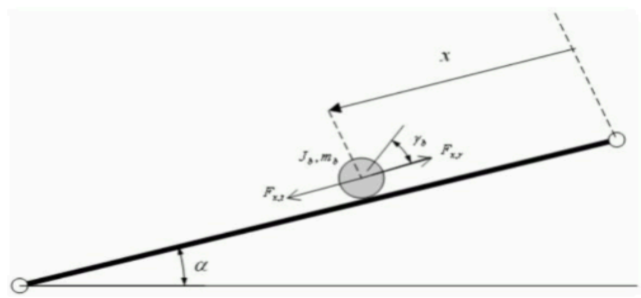


Figure 6(a): Free-body diagram of Ball and Beam.

Neglecting friction and viscous damping, the ball forces can can be represented by:

$$m_b(\frac{d^2x(t)}{dt^2}) = F_{x,t} - F_{x,r}$$

3. Adding SRV02 Dynamics:

The equation of motion representing the position of the ball relative the angle of the SRV02 load gear will be found. The obtained equation is nonlinear (includes a trigonometric term) and it will have to be linearized in order for the model to be used for control design.

$$J = \frac{2mr^2}{5}$$

4. Obtaining the Transfer Function :-

$$P(s) = P_{bb}(s)P_s(s)$$

3) Derivation and Calculation :-

1) Time Domain SpeciFication :-

The time-domain specifications for controlling the position of the SRV02 load . Shafts are :- the steady state error e_{ss} = 0 , peak time t_p = 0.15 sec and overshoot (p_o) = 5.0% .

Thus, when tracking and load shaft step reference , the transient response should have a peak time less than or equal to 0.15 sec and no steady state error . The specifications for controlling the position of the ball are: Steady-state error $e_{ss} \leq 0.005$ meter , setting time = 0.15 sec , time ts = 0.04 and overshoot (p_o) = 10.0% .
Given a step reference, the peak position of the ball should overshoot over 10% after the 3.5 sec

Consider the second order system as :-

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

2. Settling Time:

The response of a second-order system, y(t), when subjected to a unit step
reference, r(t). This response has a 5% settling time of 0.30 seconds. Thus, the response settles within 5% of its steady-state value, which is between 0.95 and 1.05, in 0.30 seconds. Settling time is defined as

$$\boxed{t_s = t_1 - t_0}$$

Where the initial step time is t0 and the time it takes to settle is t1. The settling time is given as-

$$\boxed{t_s = -\frac{ln(c_{ts}(1-\zeta^2)^{0.5})}{\zeta\omega_n}}$$

Where cts is the settling time percentage.

The peak time and percentage overshoot equations are-

$$t_p = \frac{\pi}{\omega_n(1-\zeta^2)^{0.5}}$$

$$PO = 100e^{\left(-\frac{\pi\zeta}{(1-\zeta^2)^{0.5}}\right)}$$

3) Steady State error :-

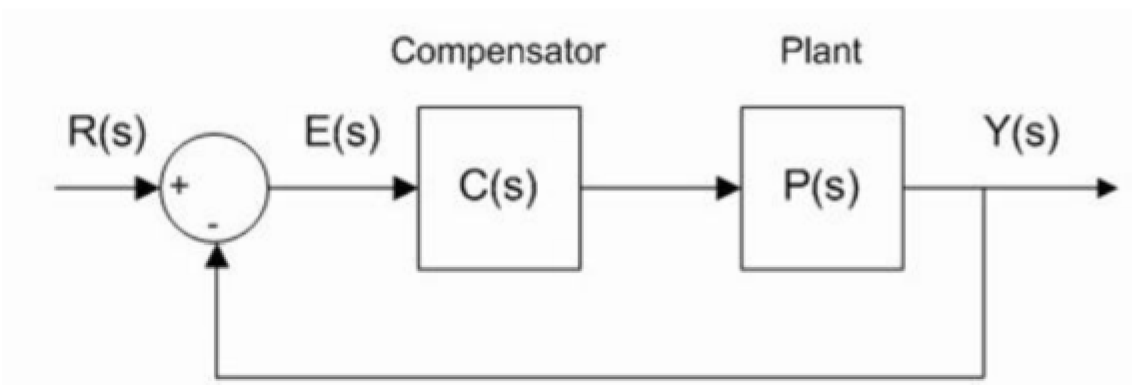


Figure 8: Unity feedback system.

The steady state error of the ball position is evaluated using a proportional compensator .

Find the steady state error of the ball and beam , pbb with the unity compensator C(s)=1 and a reference step of where R0 is the step Amplitude .

In this calculation the SRV02 dynamics are ignored and only the BB01 The plant is being considered.

Calculating the Model Gain (K_{bb}) : -

1) Calculating the model gain (K_{bb}) :-

$$K_{bb} = \frac{5g}{7} \cdot \frac{r_{arm}}{L_{beam}}$$

Given :-

$g = 9.81 \ m/s^2$

$r_{arm} = 2.54 \ cm$

$L_{beam} = 42.55 \ cm$

$$K_{bb} = \frac{5 \times 9.81 \times 0.0254}{7 \times 0.425}$$

$$\boxed{K_{bb} = 0.418 \ m/s^2}$$

4) Ball and Beam Cascade Control System :-

4.1) Inner Loop design :-SRV02 PV Position Controller:

In this section, the proportional-velocity (PV) controller gains are computed for the SRV02 when it is in the high-gear configuration and based on the specifications given in Section 2. The internal control loop is depicted in the block diagram shown in Figure 10.

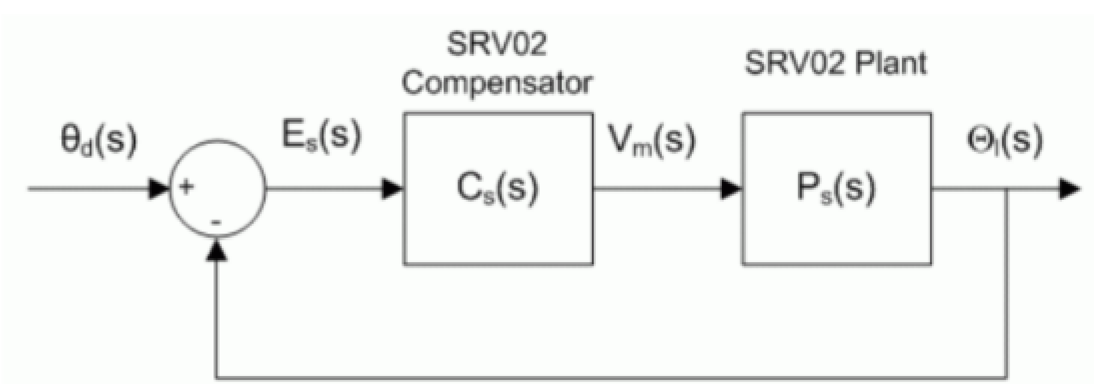


Figure 10: SRV02 closed-loop system.

SRVO2 Compensator: Proportional Velocity (PV) Compensator , compensator used to control the position of the SRV02 has the structure .

$$V_m(t) = k_p(\theta_d(t) - \theta_l(t)) - k_v(\dot{\theta}_d(t))$$

where kp is the proportional control gain, kv is the velocity control gain, is the Setpoint or the load angle and is the measured load shaft angle and The SRV2 is input voltage

2) Calculating inner loop (SRVO2) controller gains (k_p, k_v):-
first we have to calculate the Damping ratio (ζ) & the Natural frequency (w_n)

Given :-

$P_D = 5\%$

$t_p = 0.15$ sec

we know that,

$$P_D = 100 \exp\left(\frac{-\pi\zeta}{(1-\zeta^2)^{0.5}}\right)$$

$$\frac{5}{100} = \exp\left(\frac{-\pi\zeta}{(1-\zeta^2)^{0.5}}\right)$$

$$-\ln 20 = \frac{-\pi\zeta}{(1-\zeta^2)^{0.5}}$$

$$\boxed{\zeta = 0.69}$$

$$\rightarrow w_n = \frac{\pi}{t_p(1-\zeta^2)^{0.5}} = \frac{\pi}{0.15(1-(0.69)^2)^{0.5}}$$

$$\boxed{w_n = 28.86 \text{ rad/s}}$$

Now, using the eq^n given -

$$V_m(t) = K_P[\theta_d(t) - \theta_L(t)] - K_V(\dot{\theta}_L(t))$$

taking Laplace transformation.

$$V_m(s) = K_P[\theta_d(s) - \theta_L(s)] - K_V s\,\theta_L(s)$$

$$\frac{V_m(s)}{\theta_L(s)} = K_P\left[\frac{\theta_d(s)}{\theta_L(s)} - 1\right] - K_V$$

$$\frac{(\tau s+1)s}{K} = K_P[x-1] - K_V s \qquad \left[\frac{\theta_d(s)}{\theta_L(s)} = x\right]$$

$$\Rightarrow \frac{(\tau s+1)s}{K} + K_V s = K_P(x-1)$$

$$x = \frac{(\tau s+1)s + KK_V s + KK_P}{KK_P}$$

$$\frac{\theta_L(s)}{\theta_d(s)} = \frac{KK_P}{\tau s^2 + s + KK_V s + KK_P}$$

$$\frac{\theta_L(s)}{\theta_d(s)} = \frac{1.076\,K_P}{(0.02855)s^2 + (1.076K_V+1)s + (1.076)K_P}$$

$$= \frac{\left(\frac{1.076}{0.0285}\right)K_P}{s^2 + \left(\frac{1.076\,K_V+1}{0.02855}\right)s + \left(\frac{1.076}{0.02855}\right)K_P} \quad -(1)$$

& we know that, for SRO2

$$\text{Transfer function} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{(28.93)^2}{s^2 + 2(0.69)(28.93)s + (28.93)^2} \quad -(2)$$

comparing eqn ① & ②.

$$K_p = \left(\frac{0.0285}{1.76}\right)(28.93)^2$$

$$K_p = 13.55 \text{ V/rad}$$

&

$$K_v = \frac{(2)(0.689)(28.93)(0.0255) - 1}{1.76}$$

$$K_v = 0.077 \text{ Vs/rad}$$

$$\therefore \boxed{\begin{array}{l} K_p = 13.55 \text{ V/rad} \\ K_v = 0.077 \text{ Vs/rad} \end{array}}$$

4.2 Outer loop design :-

The inner loop that controls the position of the SRV02 load shaft is Complete and the servo dynamics are now considered negligible . Thus it is assumed that the desired load angle equals the actual load angle

$$\theta_l(t) = \theta_d(t)$$

The outer loop shown in the Fig will be used to control the ball on the beam.

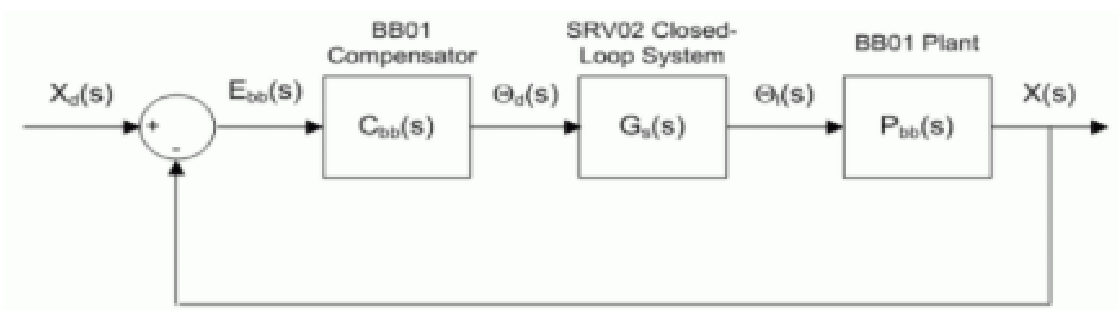


Figure 11: BB01 closed-loop system.

$$K_c = \frac{2\zeta\omega_n}{K_{bb}} \quad \text{and} \quad z = \frac{\omega_n^2}{K_{bb}K_c}$$

③ Calculating outer loop (Ball Position) controller gain :-
(BBO1) (K_p, Z) →

first, find ζ & ω_n →

$P_o = 10\%$

$t_s = 3.5\,s$

$C_{ts} = 0.04$

$$\Rightarrow \quad 10 = 100 \exp\left(\frac{-\pi\zeta}{(1-\zeta^2)^{0.5}}\right)$$

$$\boxed{\zeta = 0.591}$$

and, $$\boxed{t_s = \frac{-\ln\left(C_{ts}(1-\zeta^2)^{0.5}\right)}{\zeta\omega_n}}$$

$$\omega_n = \frac{-\ln\left(0.04(1-(0.591)^2)^{0.5}\right)}{0.591 \times 3.5}$$

$$\boxed{\omega_n = 1.66 \text{ rad/s}}$$

→ for finding K_c & z →

$$\theta_i(s) = zK_cX_d(s) - zK_cx(s) - sK_cx(s)$$

$$1 = zK_c\frac{X_d(s)}{\theta_i(s)} - zK_c\frac{x(s)}{\theta_i(s)} - sK_c\frac{x(s)}{\theta_i(s)}$$

$$1 = zK_c\frac{X_d(s)}{s^2x(s)}K_{bb} - zK_c\frac{K_{bb}}{s^2} - \frac{K_cK_{bb}s}{s^2}$$

$$s^2 = \frac{zK_cK_{bb}}{\left(\frac{x(s)}{x_d(s)}\right)} - zK_cK_{bb} - K_cK_{bb}s$$

$$s^2 + K_cK_{bb}s + zK_cK_{bb} = \frac{zK_cK_{bb}}{x}$$

$$x = \frac{zK_cK_{bb}}{s^2 + K_cK_{bb}s + zK_cK_{bb}} \quad \text{--- } (3)$$

$$\& \quad TF = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \text{--- } (4)$$

By comparing eqⁿ (3) & (4)

$$K_c = \frac{2\zeta\omega_n}{K_{bb}} \quad \& \quad z = \frac{\omega_n^2}{K_{bb}K_c}$$

$$K_c = \frac{2(0.591)(1.66)}{(0.418)} \qquad z = \frac{(1.66)^2}{(0.418)(K_c)}$$

$$\boxed{K_c = 4.686}$$

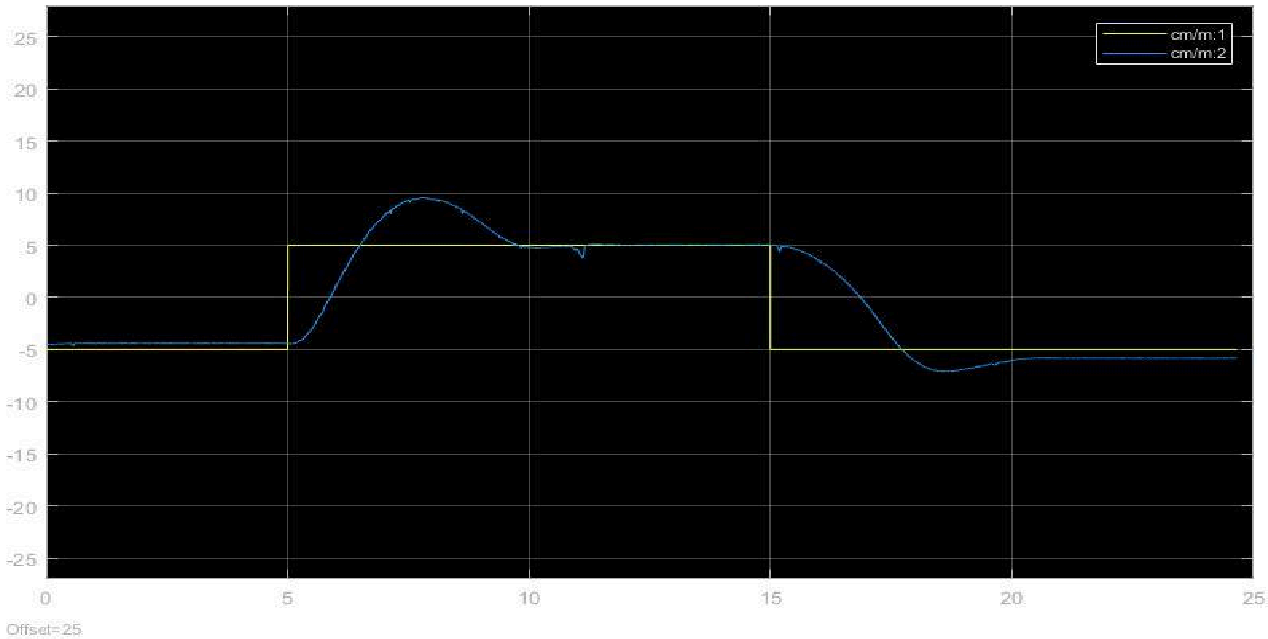
$$z = \frac{(1.66)^2}{(0.418)(4.686)}$$

$$\boxed{z = 1.406}$$

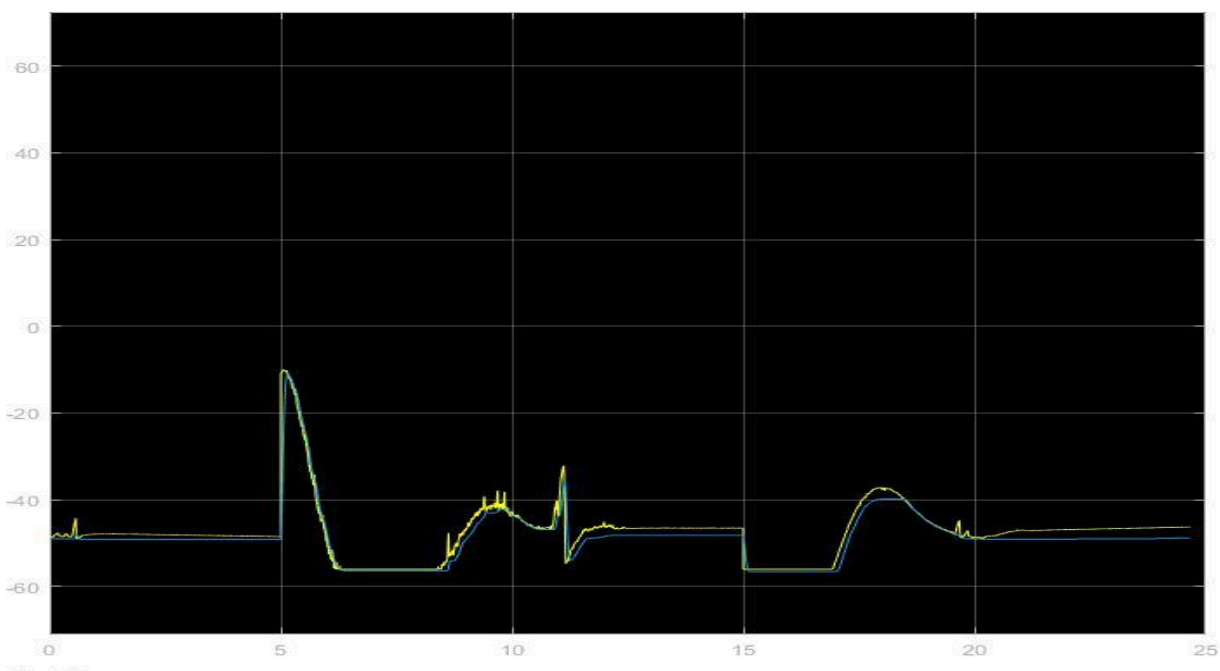
Graphs :-

1) Graphs for the inner loop :-

a) Displacement :-

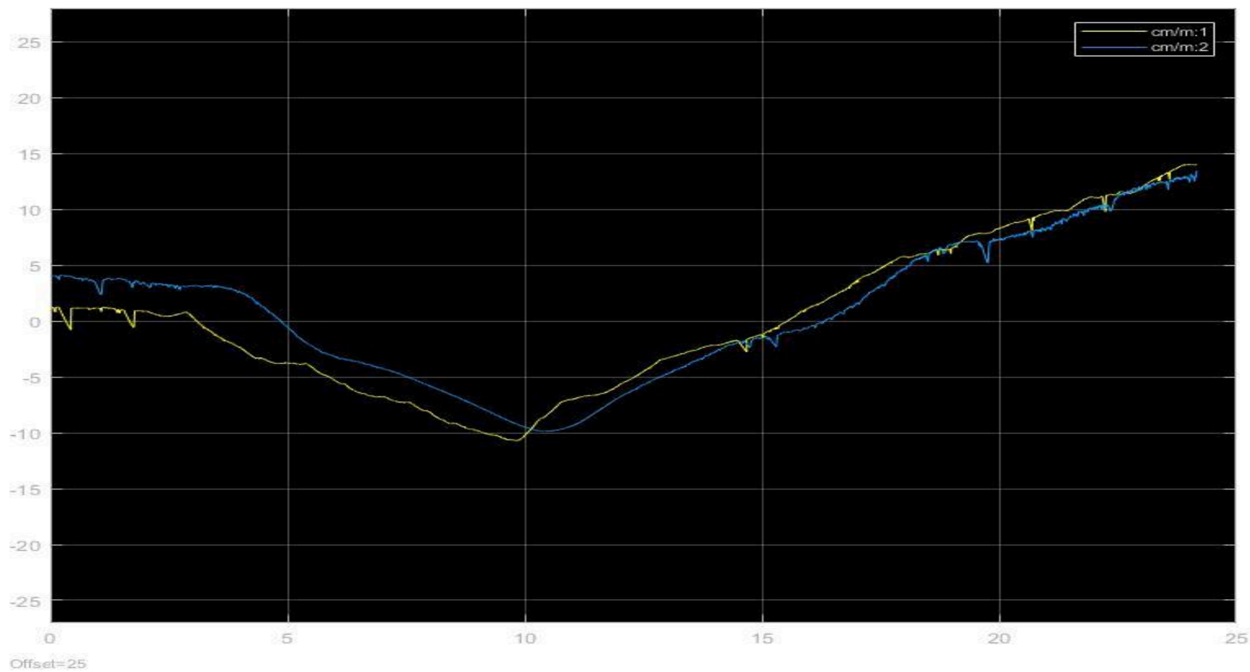


b) Theta :-

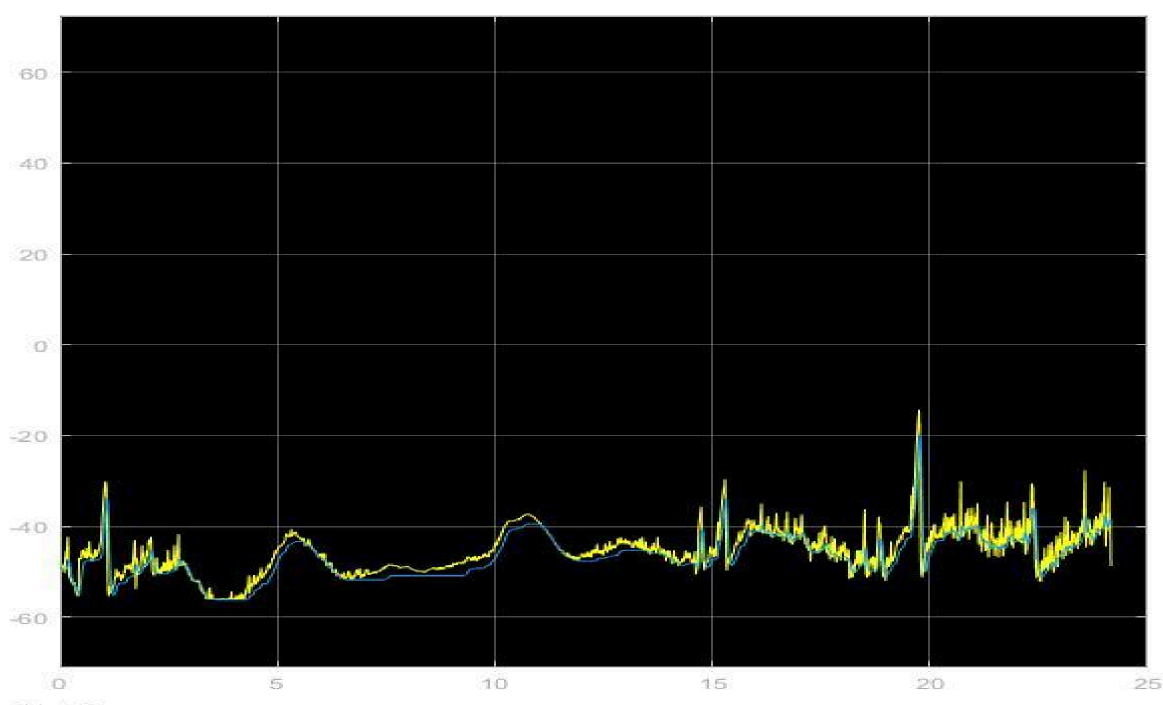


2) Graphs for the Outer loop :-

a) Displacement :-



b) Theta :-



Observations and Conclusion :-

This experiment demonstrated the full process of designing, simulating, and implementing a cascade control system to stabilize the unstable Ball-and-Beam setup. The target holding the ball at a chosen position while meeting time-domain requirements was successfully met.

A mathematical model identified the system as a double integrator, leading to a PV controller for the inner servo loop and a PD controller for the outer position loop. Simulations confirmed the design, but real-world testing required iterative tuning to handle friction, noise, and parameter mismatches. Adding a small integral term removed steady-state error, and the final controller achieved stable and reliable performance.

The experiment highlighted how modeling guides controller design, simulations verify it, and hardware testing refines it offering valuable insight into turning theory into practice.
