EEL 3040

Control System

Lab - 2

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Magnetic Levitation

Content

1. Objective :-

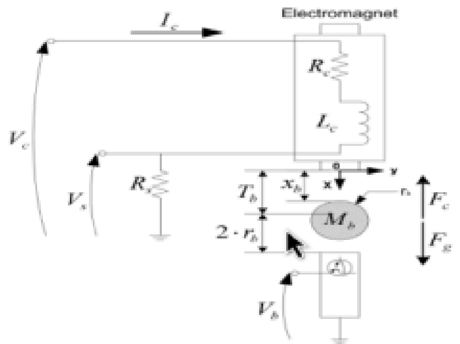
a) Aim :- The objective of this experiment is to model the magnetic levitation (MAGLEV) plant and to design a controller that levitates the ball from post and ball position and tracks a desired trajectory.

b) Software :- MATLAB, Magnetic levitation kit, Q8-USB, UPM-2405 amplifier.

c) Theory :-

The “MAGLEV” experiment consists of an electromagnet encased in a rectangular enclosure. One electromagnet pole faces a black post upon which a 2.54 cm diameter steel ball rests. The ball elevation from the post is measured using a sensor embedded in the post. The post is designed such that with the ball at rest on its surface, it is 14 mm from the face of the electromagnet.

2. Mathematical Model



(a) Electrical System-The coil used in the electromagnet has an inductor and a resistance. The voltage applied to the coil results in a current governed by the differential equation.

$$v(t) = i(t)(R_c + R_s) + L\frac{di(t)}{dt} \qquad \textbf{(i)}$$

Applying laplace transform on equation-1

$$G_c(s) = \frac{I(s)}{V(s)} = \frac{K_{c_{dc}}}{\tau_c s + 1}$$

$$\text{Where,} \qquad K_{c_{dc}} = \frac{1}{R_c + R_s}\ ,\ \tau_c = \frac{L_c}{R_c + R_s}$$

(b)Mechanical System-The force due to gravity applied to the ball is expressed by:

$$F_C + F_g = -\frac{1}{2}\frac{K_m I_c^{\,2}}{x_b^{\,2}} + M_b g \qquad \textbf{(ii)}$$

Applying Newton's second law of motion

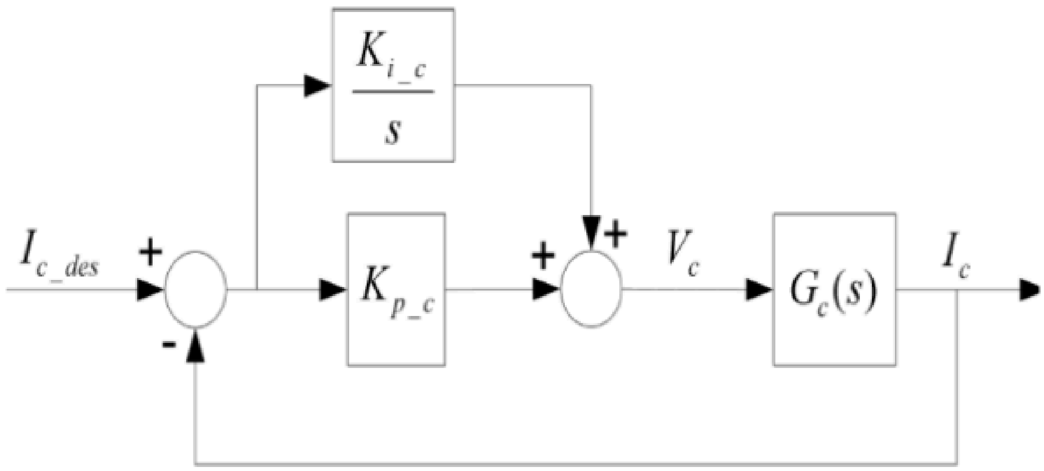
$$\frac{d^2 x_b}{dt^2} = g - \frac{1}{2}\frac{K_m I_c^{\,2}}{x_b^{\,2} M_b}$$

At equilibrium: $F_c + F_g = 0$

$$I_{c0} = \sqrt{\frac{2M_b g}{K_m}}\, x_{b0} \qquad \textbf{(iii)}$$

3. Controller Design

(a)Coil current controller design: pole placement-Prior to control the steel ball position, the current flowing through the electromagnet needs to be controlled. The electromagnet current control loop consists of a Proportional-plus-Integral (PI) closed-loop scheme.



Transfer Function: $T_c(s) = \frac{I_c(s)}{I_{des}(s)}$

Poles: $P_{c1,2} = -\ 235\ \pm\ 70i$

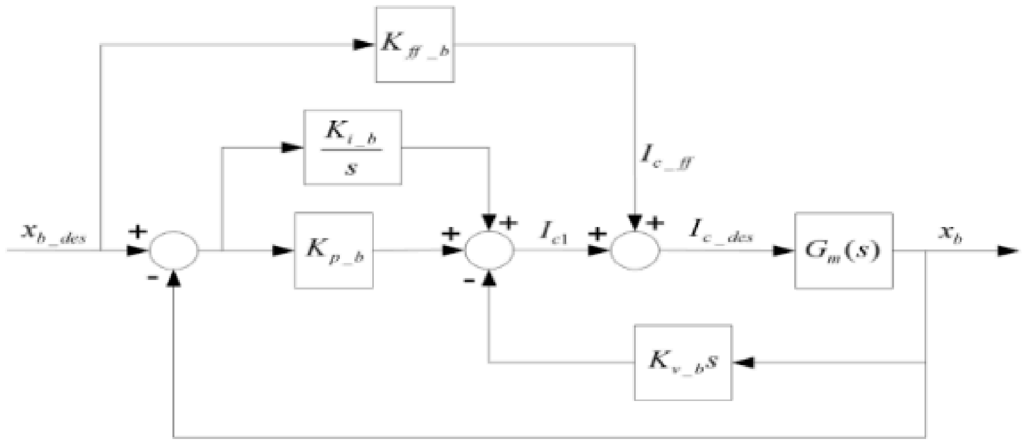
$$s^2 - (P_{c1} + P_{c2})s + P_{c1}P_{c2} = 0 \qquad \textbf{(iv)}$$

$$s^2 + 2\omega_n\zeta s + \omega_n^2 = 0 \qquad \textbf{(v)}$$

Comparing equation 4 and 5 we will get teh values of K_{pc} *and* K_{ic}.

The calculation can be verified in calculation sections below this section.

(b) Ball Position Controller Design: Pole Placement- The steel ball position is controlled by means of a Proportional-plus-Integral-plus-Velocity (PIV or PID) closed-loop scheme with the addition of a feed-forward action.



Transfer Function: $T_b(s) = \frac{x_b(s)}{x_{bdes}(s)}$

Poles: $P_{b1,2,3} = -2.5, -44, -51.6$

$$s^3 - \left(\sum_{i=1}^{3} P_{bi}\right)s^2 + \left(\sum_{i,j}^{3} P_{bi}P_{bj}\right)s - P_{b1}P_{b2}P_{b3} = 0 \qquad \textbf{(vi)}$$

$$s^3 - \frac{2gK_{vb}s^2}{I_{c0}} - \left(\frac{2g}{x_{b0}} + \frac{2gK_{pb}}{I_{c0}}\right)s - \frac{2gK_{ib}}{I_{c0}} = 0 \qquad \textbf{(vii)}$$

On comparing the above two quation we will get the values of K_{vb}, K_{pb}, K_{ib}

And we know that, $K_{ff} = \frac{I_{c0}}{x_{b0}}$

Where $x_{b0} = 6mm$, and I_{c0} can be calculated from equation-3.

C) **Calculation :-**

→ Electrical system.

$$G_c(S) = \frac{I(S)}{V(S)} = \frac{K_{dc}}{\tau_c(S)+1}.$$

where. $K_{dc} = \frac{1}{R_c+R_s}$

$\tau = L_c / R_c + R_s.$

$$K_{d(s)} = \frac{1}{R_c+R_s} = \frac{1}{10+1} = \underline{0.909}$$

$$\tau = \frac{L_c}{R_c+R_s} = \frac{412.5}{1000} \cdot (0.909) = 0.0375$$

$$G_c(S) = \frac{0.909}{(0.0375)S+1}$$

$$G_c(S) = \frac{1}{(0.4125)S + 11.0011}$$

→ Mechanical system :-

$$F_c + F_g = -\frac{1}{2}\frac{K_m I_c^2}{x_b^2} + M_b g$$

$$\ddot{x}_b = g - \frac{1}{2}\frac{K_m I_c^2}{x_b^2 M_b}$$

$x_b = x_{b1} + x_{b0}$, $I_c = I_{c0} + I_{c1}$

$F_c = -F_g$ ⟶ (equilibrium)

$x_{b0} = 6 \times 10^{-3}\ m$

$$I_{c0} = \sqrt{\frac{2m_{bg}}{K_m}} \cdot x_{b0}$$

$m_{bg} = 0.068\ kg$

$(g = 9.8\ m/s^2)$

$$I_{c0} = \sqrt{\frac{2 \times 0.068 \times 9.8}{K_m}} \cdot x_{b0}$$

$\therefore x_{b0} = 6 \times 10^{-3}\ m$

$$\boxed{I_{c0} = 0.8572\ A}$$

→ Electromech. model.

$$\frac{\partial^2}{\partial t^2} x_{b1} = -\frac{1}{2}\frac{K_m I_{c0}^2}{m_b x_{b0}^2} + g + \frac{K_m I_{c0}^2 x_{b1}}{m_b x_{b0}^2} - \frac{K_m I_{c0} I_c}{m_b x_{b0}^2}$$

$$\frac{x_{b1}(s)}{I_{c1}(s)} = G_{b1}(s) = \frac{-K_{bdc}\omega_b^2}{s^2 - \omega_b^2}$$

here, $K_{bdc} = \frac{x_{b0}}{I_{c0}}$ & $\omega_b = \sqrt{\frac{2g}{x_{b0}}}$

$$K_{bdc} = \frac{6 \times 10^{-3}}{0.85762}$$

$$\omega_b = \sqrt{\frac{2g}{x_{b0}}}$$

$$\boxed{K_{bdc} = 6.996 \times 10^{-3}}$$

$$\omega_b = \sqrt{\frac{2 \times 9.8}{6 \times 10^{-3}}}$$

$$\boxed{\omega_b = 57.1834}$$

$$P_{c1,2} = -235 \pm 70i$$

$$s^2 - (P_{c1} + P_{c2})s + P_{c1}P_{c2} = 0.$$

→ Block diagram :-

(1)

$$G(s) = \frac{I_c}{I_{cdes}}$$

$$I_{cdes} - I_c = \left(K_p + \frac{K_i}{s}\right)(G(s))$$

$$I_{cdes} = (I_{cdes} - I_c)\left(K_p + \frac{K_i}{s}\right) G(s).$$

$$(I_{cdes} - I_c)\left(\frac{K_{ic}}{s} + K_{pc}\right) = V_c$$

$$I_c = \left(\frac{K_{ic}}{s} + K_p\right) G(s)\,!$$

$$G_c(s) = \frac{I_c}{V_c} = \frac{G(s)}{I_{cdes} - I_c}$$

$$I_{cdes} = I_1 + 1 \quad \text{—(I)}$$

&

$$1 + \left(K_p + \frac{K_i}{s}\right)(G(s)) = 0 \quad \text{—(II)}.$$

Putting (P_1 & P_2) in $G(s)$

$$1 + \left(K_p + \frac{K_i}{-235 \pm 70i}\right)\left(\frac{1}{0.415(-235+70i) + 11.011}\right) = 0$$

$$1 + \left[\frac{K_p + K_i}{-235 - 70i}\right]\left(\frac{1}{0.415(-235+70i) + 11.011}\right) = 0$$

solving above eqn

$$\boxed{\begin{aligned} K_{pc} &= 182.876 \\ K_{ic} &= 24801.56 \end{aligned}}$$

Controller design

$$T_D(s) = \frac{x_b(s)}{x_{b\text{-}des}(s)}.$$

$$P_{1,2,3} = -2.5, -44, -51.6 \text{ rad/s}$$

$$\Rightarrow s^3 - \left(\sum_{i=1}^{3} P_{bi}\right)s^2 + \left(\sum_{i \neq j}^{3} P_{bi}P_{bj}\right)s + \prod_{i=1}^{3} P_{bi} = 0$$

$$= s^3 - 2g\frac{K_{vb}s^2}{I_{co}} - \left(\frac{2g}{x_{bo}} + \frac{2g K_{pb}}{I_{co}}\right)s - \frac{2g K_{ib}}{I_{co}} = 0$$

Comparing above two eqn.

$$98.1 = \frac{2(9.81)K_{vb}}{0.857} \Rightarrow \boxed{K_{vb} \cong 4}$$

$$2.5 \times 44 + 51.6 \times 44 + 51.6 \times 2.5 = \frac{-2(9.81)}{6\,mm} - \frac{2(9.81)K_{pb}}{0.857}$$

$$\Rightarrow 110 + 2270.4 + 129 = \frac{-19.62}{6} - \frac{19.62 K_{pb}}{0.851}$$

$$\boxed{K_{pb1} = -252.61}$$

$$-2.5 \times 44 \times 51.6 = \frac{-2 \times 9.81 \times K_{ib}}{0.857}.$$

$$\boxed{K_{ib} = -248.09}$$

$$K_{ff} = \frac{I_{co}}{x_{bo}} = \frac{0.857}{6 \times 10^{-3}}$$

$$\boxed{K_{ff} = 142.9336}$$

4. Assignment

(a) Important Formula's

$$PO = exp(-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi)100 \quad , \quad T_s = \frac{4}{\zeta\omega_n}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \quad , \quad P_{c1,2} =- \zeta\omega_n \pm \sqrt{\zeta^2 - 1}$$

$$T_r = \frac{\pi-\beta}{\omega_d} \quad , \quad \beta = tan^{-1}(\frac{\sqrt{1-\zeta^2}}{\zeta})$$

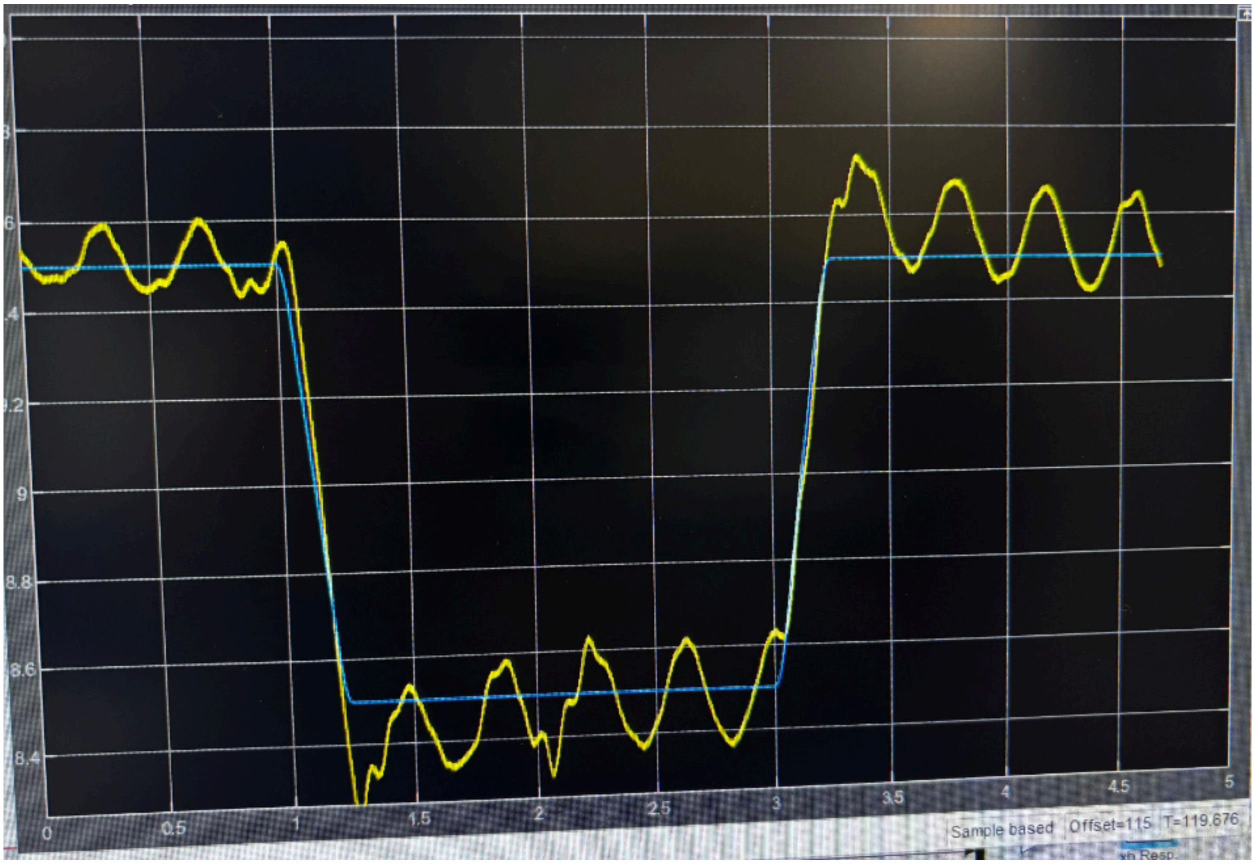
(b) Controller gains : Value's Table

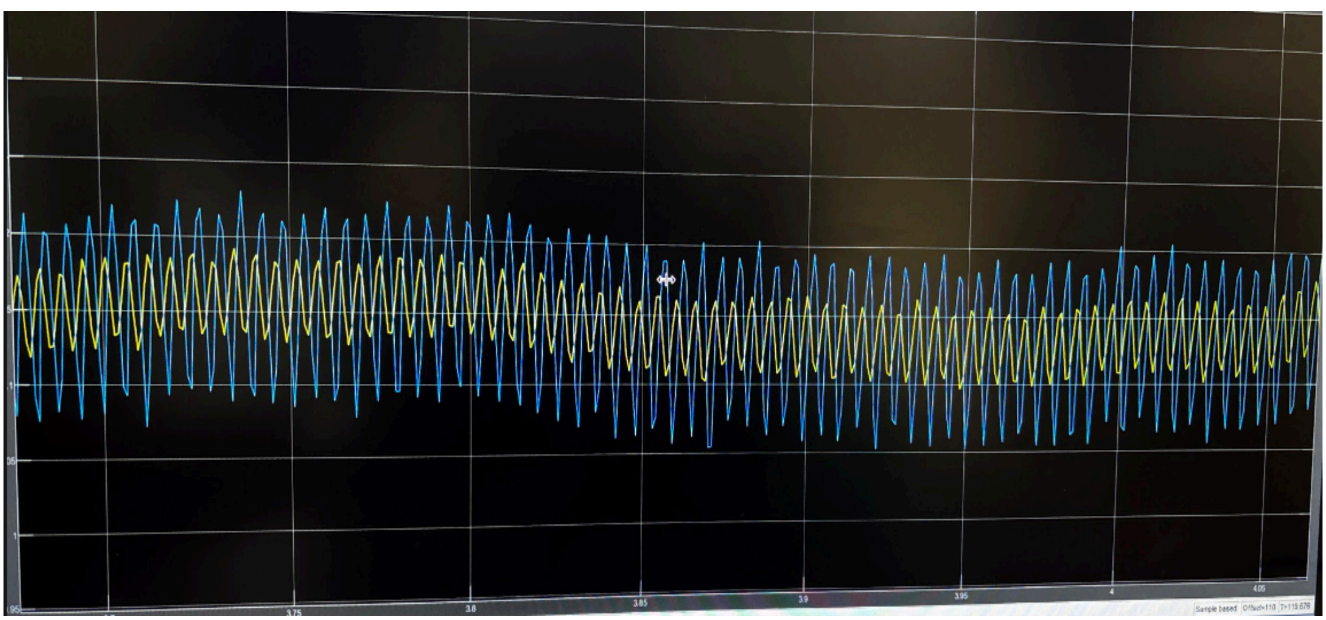
Controller Gain	Values
K_{pc}	182.876
K_{ic}	24801.56
K_{ffb}	142.9336
K_{pb}	-252.61
K_{ib}	-248.09
K_{vvb}	-4.24828

(c) Conclusion:

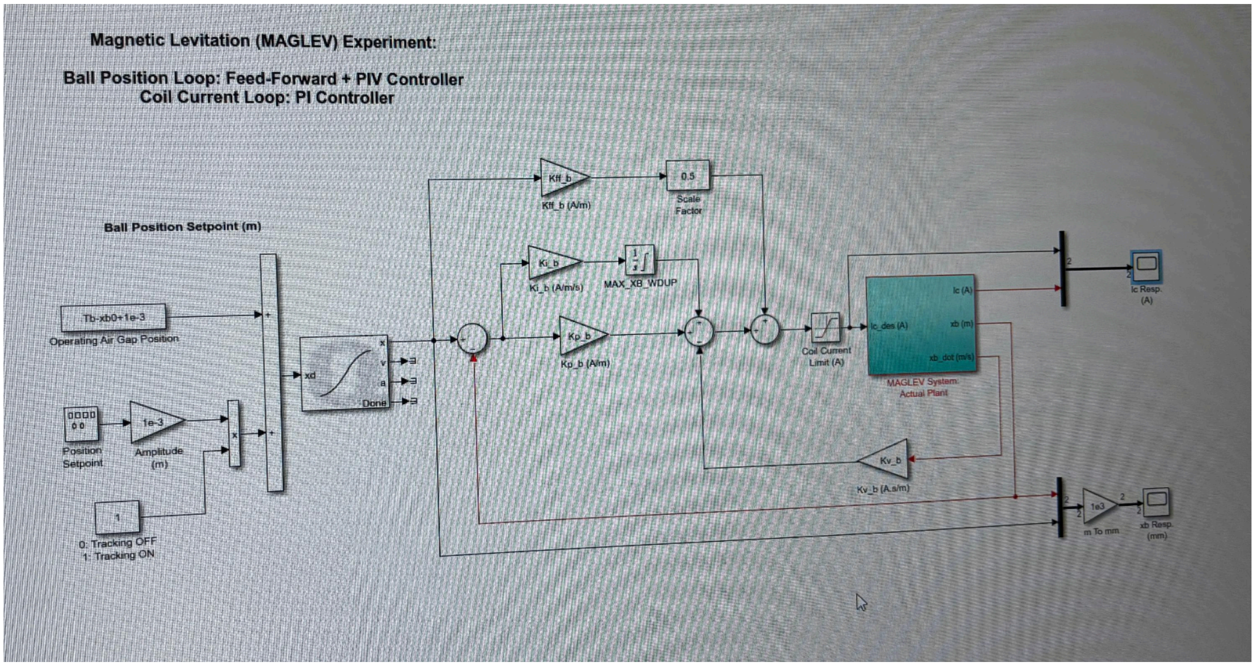
The laboratory experiment results demonstrate that by determining and utilizing the necessary constants for the control system, we can achieve precise positioning of the ball (x_b) within the system, even when varying the desired position. This remarkablecapability is a testament to the effectiveness of control systems in minimizing errors and ensuring accurate positioning.

Graphs :-





Ckt Diagram :-



Magnetic Levitation (MAGLEV) Experiment:
Ball Position Loop: Feed-Forward + PIV Controller
Coil Current Loop: PI Controller
Ball Position Setpoint (m)
Tb-xb0+1e-3
Operating Air Gap Position
Position Setpoint
1e-3
Amplitude (m)
0: Tracking OFF
1: Tracking ON
xd
Done
Kff_b
Kff_b (A/m)
0.5
Scale Factor
Ki_b
Ki_b (A/m/s)
MAX_XB_WDUP
Kp_b
Kp_b (A/m)
Coil Current Limit (A)
Ic_des (A)
Ic (A)
xb (m)
xb_dot (m/s)
MAGLEV System: Actual Plant
Kv_b
Kv_b (A.s/m)
Ic Resp. (A)
1e3
m To mm
xb Resp. (mm)