Quantum State Tomography via Compressed Sensing

Gross, Liu, Flammia, Becker, Eisert

Brief paper review

March 14, 2025

The Problem with Traditional Tomography

Quantum Tomography

- Reconstruct unknown quantum states (density matrices)
- Critical for verifying quantum hardware

Exponential Scaling:

$$d = 2^n$$
$$\text{Parameters} = d^2 - 1$$

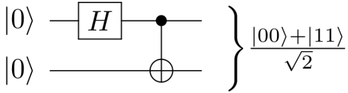
Example

10 qubits $\Rightarrow$ 1,048,575 parameters
$\mathcal{O}(d^2)$ measurements needed

Low-Rank States

Structured States

- Bell / GHZ states
  (quantum entanglement)



- Noisy pure states

10-qubit Example

Rank 3 state: 30,000 vs 1,000,000 parameters

Low-Rank Advantage

$$\text{Parameters} = \mathcal{O}(rd)$$
$$\text{vs } \mathcal{O}(d^2)$$

Compressed Sensing Framework

Step 1: Measurements

- Random Pauli measurements: $\sigma_\mu = \otimes_{i=1}^{n} \sigma_{\mu_i}$
- Sample complexity: $m = \mathcal{O}(rd \log^2 d)$

Step 2: Optimization

$$\begin{aligned} \min_{\sigma} \; & \|\sigma\|_{\mathrm{tr}} \\ \text{s.t. } & \mathcal{R}(\sigma) = \mathcal{R}(\rho) \end{aligned}$$

Nuclear norm promotes low-rank solutions

Theoretical Guarantees

Theorem (Gross et al.)

For rank-r states:
$m = \mathcal{O}(rd \log^2 d)$ *measurements suffice*
Success probability: $1 - e^{-\mathcal{O}(m)}$

Robustness

- Stable against depolarizing noise
- Error scaling: $\mathcal{O}(\epsilon\sqrt{rd})$

Tomography Quantities

- No prior assumptions needed
- Rank estimation via singular value thresholding

Experimental Demonstration

99% fidelity with 30% of Pauli measurements

Fast Post-Processing

Hybrid Method

- $\mathcal{O}(r \log d)$ operators
- $\mathcal{O}(d)$-time completion

Result

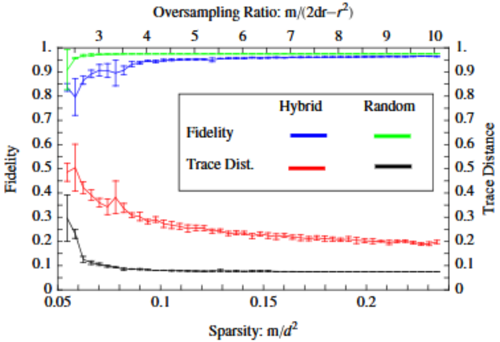
8-qubit state:

1 min runtime

95% fidelity

Applications

- Verify entangled states
- Gate benchmarking



Limitations & Open Questions

Challenges

- Exponential scaling remains
- Optimal measurement sets?

Future Work

- ML for adaptive measurements
- Hardware implementations

Conclusion

Conclusion: Scalable tomography is possible thanks to low-rank structure

Feel free to ask your questions!

D. Gross, Y.-K. Liu, S. T. Flammia, S. Becker, J. Eisert "Quantum State Tomography via Compressed Sensing", Physical Review Letters (2010)