Simulation Step 7D Automatic Static Allocation using CPP v2

July 15, 2024

1 Prelude

[1]:
```
import matplotlib as mpl
import matplotlib.pyplot as plt
import pulp
import math
import random
import pandas as pd
import numpy as np
import simpy
```

2 Utilities

2.1 Points and Distances

[2]:
```
def dist(p1, p2):
    (x1, y1) = p1
    (x2, y2) = p2
    return int(math.sqrt((x1-x2)**2+(y1-y2)**2))
```

2.2 PlotMap

[3]:
```
def label(i):
    return (label(i//26-1)+chr(65+i%26)) if i>25 else chr(65+i)
```

[4]:
```
def plotMap(G, T=[], P=[], w=None, frame=None,
            style='r-o', lw=1, ms=3,
            styleT='go', msT=3,
            styleP='b-o', lwP=2, msP=3,
            stylePT='go', msPT=7,
            styleW='ro', msW=9,
            text=None, grid=False, labels=False,
            size=4, scale=False):


    def round_down(x, level): return (x//level)*level
    def round_up(x, level): return (x//level+1)*level
```

```
    if frame is not None:
        V, E = frame
    else:
        V, E = G

    xmin = round_down(min([ x for (x, _) in V ]), 100)
    xmax = round_up(max([ x for (x, _) in V ]), 100)
    ymin = round_down(min([ y for (_, y) in V ]), 100)
    ymax = round_up(max([ y for (_, y) in V ]), 100)
    dx = xmax-xmin
    dy = ymax-ymin
    yoffset = (ymax-ymin)//10
    xoffset = (xmax-xmin)//20

    V, E = G

    fig = plt.gcf()
    fig.set_size_inches(size, size)
    plt.xlim(xmin-xoffset, xmax+xoffset)
    plt.ylim(ymin-yoffset, ymax+yoffset)

    if not grid:
        plt.axis('off')

    if frame is not None:
        for e in frame[1]:
            if e not in E:
                p1, p2 = e
                plt.plot( [ p1[0], p2[0] ],  [ p1[1], p2[1] ],
                          'k-', lw=0.5, ms=2)

    for e in E:
        p1, p2 = e
        plt.plot( [ p1[0], p2[0] ],
                  [ p1[1], p2[1] ],
                  style, lw=lw, ms=ms)
    if scale:
        # plot 1000m scale
        ybar = ymin-0.9*yoffset
        D = [ (xmin, ybar+50), (xmin, ybar), (xmin+1000, ybar), (xmin+1000,␣
↪ybar+50) ]
        plt.plot( [ d[0] for d in D ], [ d[1] for d in D ], 'k-', lw=0.5)
        plt.text(xmin+500, ymin-0.7*yoffset, '1000m' ,␣
↪horizontalalignment='center', size=8)

    if labels:
```

```
        for i in range(len(V)):
            x, y = V[i]
            plt.text(x+0.0150*dx, y-0.0350*dy, label(i), size=8)

    for t in T:
        plt.plot( [ t[0] ], [ t[1] ],
                  styleT, ms=msT)

    plt.plot( [ p[0] for p in P ],
              [ p[1] for p in P ],
              styleP, lw=lwP, ms=msP)

    for p in P:
        if p in T:
            plt.plot( [ p[0] ], [ p[1] ],
                      stylePT, ms=msPT)
    if w is not None:
        plt.plot( [ w[0] ], [ w[1] ],
                  styleW, ms=msW)
    if text is not None:
        plt.text(xmax, ymin-0.9*yoffset, text, horizontalalignment='right',␣
↪size=8)
    if grid:
        plt.grid()
    plt.show()
```

2.3 Add Targets

[5]:
```
def addTargets(M, T):
    V, E = M
    E = E.copy()
    V = V.copy()
    for t in T:
        minD = math.inf
        minE = None
        for e in E:
            P, Q = e
            distT = dist(P, t)+dist(t, Q)-dist(P, Q)
            if distT < minD:
                minD = distT
                minE = e
        P, Q = minE
        E.remove( (P, Q) )
        E.append( (P, t) )
        E.append( (t, Q) )
        V.append(t)
    return V, E
```

2.4 Generate Central Warehouse Location

[6]:
```
from statistics import median

def generateWarehouseLocation(M):
    V, _ = M
    xc = median([ x for (x, y) in V ])
    yc = median([ y for (x, y) in V ])
    cloc = (xc, yc)
    minloc = V[0]
    mindist = dist(minloc, cloc)
    for i in range(1, len(V)):
        d = dist(V[i], cloc)
        if d<mindist:
            minloc = V[i]
            mindist = dist(V[i], cloc)
    return minloc
```

2.5 Time Handling

Convention: In this project we measure simulation time in seconds. The simulation will start at 0:00. Time related methods will be added as they are needed.

`timestamp(t)` generates a timestamp string in the form `[dd] hh:mm:ss.d`

[7]:
```
def timestamp(t):
    t = round(t, 1)
    day = int(t)//(24*3600)
    t = t - day*24*3600
    hour = int(t)//3600
    t = t - hour*3600
    mins = int(t)//60
    t = t - mins*60
    secs = int(t)
    t = int(round((t-secs)*10,1))
    return f"[{day:2d}] {hour:02d}:{mins:02d}:{secs:02d}.{t:1d}"
```

[8]:
```
timestamp(24*3600*3+17*3600+615.1)
```

[8]: `'[ 3] 17:10:15.1'`

[9]:
```
timestamp(24*3600*12+3*3600+122.1)
```

[9]: `'[12] 03:02:02.1'`

[10]:
```
def day(now):
    return int(now//(24*3600))
```

[11]:
```python
def nextHour(env, hour):
    beginningOfDay = int(env.now//(24*3600))*24*3600
    timeOfDay = env.now-beginningOfDay
    if hour*3600 > timeOfDay:
        return hour*3600 - timeOfDay
    else:
        return hour*3600 + 24*3600 - timeOfDay
```

2.6 Plotting Routines

[12]:
```python
import scipy.stats as stats

def histPlot(data, title="", xlabel="",
             discrete=False, width=None, height=None):

    minx = min(data)
    maxx = max(data)
      = np.mean(data)
      = np.std(data)

    fig = plt.figure()
    fig.set_figwidth(width if width is not None else 4)
    fig.set_figheight(height if height is not None else 2.5)
    ax = fig.gca()

    if discrete:
        bins = [ i-0.5 for i in range(maxx+2) ]
        ax.xaxis.set_major_locator(mpl.ticker.MaxNLocator(integer=True))
        hist=plt.hist(data, bins=bins, rwidth=0.9, density=True)
    else:
        hist=plt.hist(data, density=True)
    plt.xlabel(xlabel)
    plt.ylabel('Density')
    plt.title(title)

    if discrete:
        poisson=stats.poisson( )
        x = [ i for i in range(maxx+1) ]
        y =[ poisson.pmf(i) for i in range(maxx+1) ]
        ax.plot(x, y, lw=1, color='red')
    else:
        x = np.linspace(minx, maxx, 100)
        y = [ stats.norm(loc= , scale= ).pdf(p) for p in x]
        ax.plot(x, y, lw=1, color='red')

    ax.axvline(x= , color='red')
    maxy = max(max(y), max(hist[0]))
```

```
    ax.text(maxx, maxy,
            f' ={ :2.2f}\n ={ :2.2f}',
            ha='right', va='top',
            color='red', fontsize=12)

    # ax.grid(True)
    plt.show()

def dailyPlot(data,
              title="", ylabel="",
              width=None, height=None):

    days = len(data)

    fig = plt.figure()
    fig.set_figwidth(width if width is not None else 6)
    fig.set_figheight(height if height is not None else 2)

    ax = fig.gca()
    diff = (max(data)-min(data)+1)*0.1
    ymin = math.floor(min(data))-diff
    ymax = math.ceil(max(data))+diff
    ax.set_xlim(-0.5, days-1+0.5)
    ax.set_ylim(ymin-0.5, ymax+0.5)
    ax.grid(True)

    ms = 2 if len(data)>=100 else 3
    lw = 0.5 if len(data)>=100 else 1

    x = np.arange(0, len(data))
    y = np.array([ y for y in data ])
    b, m = np.polynomial.polynomial.polyfit(x, y, 1)
    ax.xaxis.set_major_locator(mpl.ticker.MaxNLocator(integer=True))
    ax.yaxis.set_major_locator(mpl.ticker.MaxNLocator(integer=True))

    plt.plot(x, y, 'bo-', linewidth=lw, markersize=ms)
    plt.plot(x, m*x+b, 'r-')

    plt.xlabel('Day')
    plt.ylabel(ylabel)
    plt.title(title)
    plt.show()

def countPlot(A, B,
              title="", ylabel="",
              width=None, height=None):
```

```
    assert(len(A)==len (B))
    days = len(A)
    xmax = days-1
    ymax = A.max()

    fig = plt.figure()
    fig.set_figwidth(width if width is not None else 6)
    fig.set_figheight(height if height is not None else 4)
    ax = fig.gca()
    ax.set_xlim(-0.5, xmax+0.5)
    ax.set_ylim(0, ymax)

    def double(l, offset=0):
        return [] if l==[] else [l[0]+offset, l[0]+offset]+double(l[1:], offset)

    x = double([i for i in range(days)])+[days]
    xz = double([i+days*0.006 for i in range(days)])+[days]
    y = [0+ymax*0.006] + double(list(A), offset=ymax*0.006)
    z = [0] + double(list(B))

    ax.yaxis.set_major_locator(mpl.ticker.MaxNLocator(integer=True))
    ax.xaxis.set_major_locator(mpl.ticker.MaxNLocator(integer=True))

    for i in range(days):
        ax.fill_between([i, i+1],
                        [z[2*i+1], z[2*i+2]],
                        [y[2*i+1], y[2*i+2]], color='blue', alpha=0.2)

    lw = 1 if days>=50 or ymax>=100 else 2
    ax.plot(x, y, color='blue', lw=lw)
    ax.plot(xz, z, color='red', lw=lw)
    plt.xlabel('Day')
    plt.ylabel(ylabel)
    plt.title(title)
    ax.grid(True)
    plt.show()
```

3 Finding Shortest Path (as before)

[13]:
```
def pathLength(P):
    return 0 if len(P)<=1 else \
           dist(P[0], P[1])+pathLength(P[1:])
```

[14]:
```
def shortestPath(M, A, B):

    def h(p):
```

```
        return pathLength(p)+dist(p[-1],B)

    # candidates C are pairs of the path so far and
    # the heuristic function of that path,
    # sorted by the heuristic function, as maintained by
    # insert function
    def insert(C, p):
        hp = h(p)
        c = (p, hp)
        for i in range(len(C)):
            if C[i][1]>hp:
                return C[:i]+[c]+C[i:]
        return C+[c]

    V, E = M
    assert(A in V and B in V)
    C = insert([], [A])

    while len(C)>0:
        # take the first candidate out of the list of candidates
        path, _ = C[0]
        C = C[1:]
        if path[-1]==B:
            return path
        else:
            for (x, y) in E:
                if path[-1]==x and y not in path:
                    C = insert(C, path+[y])
                elif path[-1]==y and x not in path:
                    C = insert(C, path+[x])
    return None
```

4 Finding Short Delivery Route (as before)

4.1 Greedy Algorithm

[15]:
```
def FW(M):

    V, E = M

    n = len(V)
    d = [ [ math.inf for j in range(n) ] for i in range(n) ]
    p = [ [ None for j in range(n) ] for i in range(n) ]

    for (A, B) in E:
        a = V.index(A)
        b = V.index(B)
```

```
        d[a][b] = d[b][a] = dist(A, B)
        p[a][b] = [A, B]
        p[b][a] = [B, A]

    for i in range(n):
        d[i][i] = 0
        p[i][i] = [V[i]]

    for k in range(n):
        for i in range(n):
            for j in range(n):
                dk = d[i][k] + d[k][j]
                if d[i][j] > dk:
                    d[i][j] = dk
                    p[i][j] = p[i][k][:-1] + p[k][j]

    return d, p
```

[16]:
```
def createLoopG(M, T, timing=False):

    def makeLoop(L, V, P):
        loop = []
        for i in range(len(L)-1):
            A = L[i]
            B = L[i+1]
            a = V.index(A)
            b = V.index(B)
            sub = P[a][b]
            loop += sub if len(loop)==0 else sub[1:]
        return loop

    if timing:
        start_time = time.time()
        last_time = time.time()

    V, E = M
    D, P = FW(M)    # note these are the distances between all vertices in M
 ↪(and T)
    W = T[0]
    customers = T[1:]
    if len(T)==1:
        L = T
    elif len(T)<=3:
        L = T + [T[0]]
    else:
        L = T[:3]+[T[0]]
        T = T[3:]
```

```
        while len(T)>0:

            minExt = math.inf
            minInd = None
            selInd = None
            for k in range(len(T)):
                C = T[k]
                c = V.index(C)
                for i in range(0, len(L)-1):
                    A = L[i]
                    B = L[i+1]
                    a = V.index(A)
                    b = V.index(B)
                    ext = D[a][c] + D[c][b] - D[a][b]
                    if ext<minExt:
                        minExt, minInd, selInd = ext, i+1, k
            L = L[:minInd]+[T[selInd]]+L[minInd:]
            T = T[:selInd]+T[selInd+1:]

    if timing:
        print(f"createLoopG:    {time.time()-start_time:6.2f}s")

    return makeLoop(L, V, P)
```

5 Finding Optimal Delivery Route

5.1 Iterative Integer Programming

[17]:
```
def createTables(M, T):

    def reverse(P):
        return [ P[-i] for i in range(1,len(P)+1) ]

    def index(x, L):
        for i in range(len(L)):
            if x==L[i]:
                return i
        return None

    n = len(T)
    d = [ [ math.inf for t in T ] for t in T ]
    p = [ [ None for t in T ] for t in T ]
    for i in range(n):
        d[i][i] = 0
        p[i][i] = [ T[i] ]
    for i in range(n):
        for j in range(n):
```

```python
            if p[i][j] is None:
                s = shortestPath(M, T[i], T[j])
                d[i][j] = d[j][i] = pathLength(s)
                p[i][j] = s
                p[j][i] = reverse(s)
                for m in range(len(s)-1):
                    smi = index(s[m], T)
                    if smi is None:
                        continue
                    for l in range(m+1, len(s)):
                        sli = index(s[l], T)
                        if sli is None:
                            continue
                        sub = s[m:l+1]
                        if p[smi][sli] is None:
                            p[smi][sli] = sub
                            p[sli][smi] = reverse(sub)
                            d[smi][sli] = d[sli][smi] = pathLength(sub)
    return d,p
```

[18]:
```python
def roundtrips(x, n):

    def isElem(x, l):
        for i in range(len(l)):
            if l[i]==x:
                return True
        return False

    def startpoint(trips):
        for i in range(n):
            for t in trips:
                if isElem(i, t):
                    break
            else:
                return i

    def totalLength(trips):
        s=0
        for i in range(0, len(trips)):
            s += len(trips[i])-1
        return s

    trips = []
    while totalLength(trips)<n:
        start = startpoint(trips)
        trip = [ start ]
        i = start
```

```
        while len(trip) < n-totalLength(trips):
            for j in range(0, n):
                if pulp.value(x[i][j])==1:
                    trip.append(j)
                    i=j
                    break
            if pulp.value(x[trip[-1]][start])==1:
                trip.append(start)
                break
        trips.append(trip)
    return sorted(trips, key=lambda t: len(t), reverse=True)
```

[19]:
```
import time

def createLoop(M, T, timing=False):

    if timing:
        start_time = last_time = time.time()

    D, P = createTables(M, T)   # These are the distances between customers and␣
↪warehouse only

    if timing:
        print(f"createTables:   {time.time()-start_time:6.2f}s")
        last_time = time.time()

    n = len(T)
    # create variables
    x = pulp.LpVariable.dicts("x", ( range(n), range(n) ),
                            lowBound=0, upBound=1, cat=pulp.LpInteger)
    # create problem
    prob = pulp.LpProblem("Loop",pulp.LpMinimize)
    # add objective function
    prob += pulp.lpSum([ D[i][j]*x[i][j]
                            for i in range(n) for j in range(n) ])
    # add constraints
    constraints=0
    for j in range(n):
        prob += pulp.lpSum([ x[i][j] for i in range(n) if i!=j ]) ==1
    constraints += n
    for i in range(n):
        prob += pulp.lpSum([ x[i][j] for j in range(n) if i!=j ]) ==1
    constraints += n
    for i in range(n):
        for j in range(n):
            if i!=j:
                prob += x[i][j]+x[j][i] <= 1
```

```
                constraints += 1

    def cycles(k, n):
        if k==1:
            return [ [i] for i in range(0,n) ]
        else:
            sc=cycles(k-1, n)
            all=[]
            for c in sc:
                for i in range(0,n):
                    if c.count(i)==0:
                        all.append(c+[i])
            return all

    for k in range(3, 4):
        cycs=cycles(k,n)
        for c in cycs:
            c.append(c[0])
            prob+=pulp.lpSum([ x[c[i]][c[i+1]] for i in range(0,k)]) <= k-1
            constraints += 1

    # initialise solver
    solvers = pulp.listSolvers(onlyAvailable=True)
    solver = pulp.getSolver(solvers[0], msg=0, timeLimit=2)
    res = prob.solve(solver)

    if timing:
        print(f"Solver:         {time.time()-last_time:6.2f}s {constraints:6,d} Constraints")
        last_time = time.time()

    trips = roundtrips(x, n)
    while len(trips)>1:
        longest = max([ len(t) for t in trips ])
        for t in trips:
            if len(t)<longest:
                prob += pulp.lpSum([ x[t[i]][t[i+1]] + x[t[i+1]][t[i]]
                                        for i in range(0,len(t)-1) ]) <= len(t)-2
                constraints += 1
            else:
                longest = math.inf

        res = prob.solve(solver)

        if timing:
```

```
            print(f"Solver:         {time.time()-last_time:6.2f}s {constraints:
↪6,d} Constraints")
            last_time = time.time()

        trips = roundtrips(x, n)
    trip = trips[0]
    # print(trip)
    loop = []
    for k in range(len(trip)-1):
        sub = P[trip[k]][trip[k+1]]
        loop += sub if len(loop)==0 else sub[1:]

    if timing:
        print(f"createLoop:     {time.time()-start_time:6.2f}s")

    return loop
```

6 Static Route Assignment

6.1 Chinese Postman Problem

[20]:
```
import time

def solveCPP(M, PostOffice, n=1,
             maxLength=None, balance=None,
             timing=False, timeLimit=10):

    start_time = time.time()

    V, E = M

    postoffice = V.index(PostOffice)

    def edge(e): return label(V.index(E[e][0]))+label(V.index(E[e][1]))
    def edges(l): return ",".join([ edge(i) for i in range(len(l)) if l[i]==1 ])
    def path(p): return '-'.join([ label(v) for v in p ])

    Edges = [ edge(e) for e in range(len(E)) ]

    b = pulp.LpVariable.dicts("B", (range(n), range(len(V))), lowBound=0,␣
↪upBound=1, cat=pulp.LpInteger)

    y0 = pulp.LpVariable.dicts("Y0", (range(n), Edges), lowBound=0) # ,␣
↪upBound=len(E), cat=pulp.LpInteger)
    y1 = pulp.LpVariable.dicts("Y1", (range(n), Edges), lowBound=0) # ,␣
↪upBound=len(E), cat=pulp.LpInteger)
```

```
    yp0 = pulp.LpVariable.dicts("YP0", (range(n), Edges), lowBound=0) # , upBound=len(E), cat=pulp.LpInteger)
    yp1 = pulp.LpVariable.dicts("YP1", (range(n), Edges), lowBound=0) # , upBound=len(E), cat=pulp.LpInteger)
    yn0 = pulp.LpVariable.dicts("YN0", (range(n), Edges), lowBound=0) # , upBound=len(E), cat=pulp.LpInteger)
    yn1 = pulp.LpVariable.dicts("YN1", (range(n), Edges), lowBound=0) # , upBound=len(E), cat=pulp.LpInteger)

    e0 = pulp.LpVariable.dicts("E0", (range(n), Edges), lowBound=0, upBound=1, cat=pulp.LpInteger)
    e1 = pulp.LpVariable.dicts("E1", (range(n), Edges), lowBound=0, upBound=1, cat=pulp.LpInteger)
    dp0 = pulp.LpVariable.dicts("DP0", (range(n), Edges), lowBound=0, upBound=1, cat=pulp.LpInteger)
    dn0 = pulp.LpVariable.dicts("DN0", (range(n), Edges), lowBound=0, upBound=1, cat=pulp.LpInteger)
    dp1 = pulp.LpVariable.dicts("DP1", (range(n), Edges), lowBound=0, upBound=1, cat=pulp.LpInteger)
    dn1 = pulp.LpVariable.dicts("DN1", (range(n), Edges), lowBound=0, upBound=1, cat=pulp.LpInteger)

    def vars(l,x): return edges([ int(pulp.value(x[l][e])) for e in Edges ])
    def vals(l,x): return [ round(pulp.value(x[l][e]),1) for e in Edges ]

    prob = pulp.LpProblem("CPP", pulp.LpMinimize)

    def loopLength(l):
        return pulp.lpSum([ dist(v1,v2)*e0[l][e] for e in Edges for v1 in V for v2 in V if E[Edges.index(e)]==(v1, v2) ]) + \
               pulp.lpSum([ dist(v1,v2)*e1[l][e] for e in Edges for v1 in V for v2 in V if E[Edges.index(e)]==(v1, v2) ])

    prob += pulp.lpSum([ loopLength(l) for l in range(n) ])

    # avoid non-trivial loops
    # for l in range(n):
    #     prob += pulp.lpSum([ b[l][v] for v in range(len(V)) ]) >= 2, f"NT_{l}"

    if maxLength is not None:
        for l in range(n):
            prob += loopLength(l) <= maxLength, f"Max_{l}"

    if balance is not None:
        d = pulp.LpVariable.dicts("D", range(n), lowBound=0)
```

```python
        for l in range(n):
            prob += d[l] == loopLength(l), f"D_{l}"

        for l in range(n):
            prob += d[l] <= pulp.lpSum([ d[l] for l in range(n) ])*(1+balance)/n, f"DMAX_{l}"
            prob += d[l] >= pulp.lpSum([ d[l] for l in range(n) ])*(1-balance)/n, f"DMIN_{l}"

    # Constraint YE: Y-E Relationship [equivalent to (9)]
    BigM = 100*n*len(E)
    for e in Edges:
        for l in range(n):
            prob += y0[l][e] <= BigM*e0[l][e], f"YE0_{l}_{e}"
            prob += y1[l][e] <= BigM*e1[l][e], f"YE1_{l}_{e}"

    # Constraint C: counting [equivalent (8)]
    for v in range(len(V)):
        if v!=postoffice:
            for l in range(n):
                prob += pulp.lpSum([ yp0[l][e]+yp1[l][e] for e in Edges if E[Edges.index(e)][0]==V[v] ]) + \
                        pulp.lpSum([ yn0[l][e]+yn1[l][e] for e in Edges if E[Edges.index(e)][1]==V[v] ]) - \
                        pulp.lpSum([ yp0[l][e]+yp1[l][e] for e in Edges if E[Edges.index(e)][1]==V[v] ]) - \
                        pulp.lpSum([ yn0[l][e]+yn1[l][e] for e in Edges if E[Edges.index(e)][0]==V[v] ]) == -1*b[l][v], \
                        f"C_{l}_"+label(v)

    # Invariance Constraint F/B: Relation between variables in each loop every edge is passed this way or that way
    for e in Edges:
        for l in range(n):
            prob += dp0[l][e]+dn0[l][e] == e0[l][e], f"EF_{l}_{e}"
            prob += dp1[l][e]+dn1[l][e] == e1[l][e], f"EB_{l}_{e}"
    for e in Edges:
        for l in range(n):
            prob += yp0[l][e]+yn0[l][e] == y0[l][e], f"YF_{l}_{e}"
            prob += yp1[l][e]+yn1[l][e] == y1[l][e], f"YB_{l}_{e}"

    # Constraint E10: The duplicate edge is only used if the primary edge was used
    for e in Edges:
        for l in range(n):
            prob += e1[l][e] <= e0[l][e], f"E10_{l}_{e}"
```

```
    # Constraint O: Every edge is passed across all loops at least once [equivalent (2)]
    for e in Edges:
        prob += pulp.lpSum( [ e0[l][e]+e1[l][e] for l in range(n) ] ) >= 1, \
                f"O_{e}"

    # Constraint M: In each loop every edge is passed at most twice [equivalent (7)]
    for e in Edges:
        for l in range(n):
            prob += e0[l][e]+e1[l][e] <= \
              pulp.lpSum([ 2*b[l][v] for v in range(len(V)) if E[Edges.index(e)][0]==V[v] or E[Edges.index(e)][1]==V[v]]), \
                    f"M_{l}_{e}"

    # Constraint V: In each loop every vertex is entered as often as it is left [equivalent (3)]
    for v in range(len(V)):
        for l in range(n):
            prob += pulp.lpSum([ dp0[l][e]+dp1[l][e]-dn0[l][e]-dn1[l][e] for e in Edges if E[Edges.index(e)][0]==V[v] ]) + \
                pulp.lpSum([ dn0[l][e]+dn1[l][e]-dp0[l][e]-dp1[l][e] for e in Edges if E[Edges.index(e)][1]==V[v] ]) == 0, \
                    f"V_{l}_"+label(v)

    # print(prob)

    solvers = pulp.listSolvers(onlyAvailable=True)
    solver = pulp.getSolver(solvers[0], msg=False, timeLimit=timeLimit)
    status = prob.solve(solver)

    solverTime = time.time()-start_time

    if timing:
        print(f"Solver time: {solverTime:7.2f}s")

    if status!=1:
        print(pulp.LpStatus[status])
        return []

    if solverTime >= timeLimit:
        print("Solution possibly not optimal")

    print(f"Total Delivery Path Length: {pulp.value(prob.objective)}")
```

```
def solutions(l):

    DP0 = [ int(pulp.value(dp0[l][e])) for e in Edges ]
    DP1 = [ int(pulp.value(dp1[l][e])) for e in Edges ]
    DN0 = [ int(pulp.value(dn0[l][e])) for e in Edges ]
    DN1 = [ int(pulp.value(dn1[l][e])) for e in Edges ]

    E0 = [ int(pulp.value(e0[l][e])) for e in Edges ]
    E1 = [ int(pulp.value(e1[l][e])) for e in Edges ]

    # candidates C are the paths so far,
    def insert(C, p): return [p]+C

    def refine(c, v, e0=None, e1=None):
        assert(e0 is not None or e1 is not None)
        if e0 is not None:
            c0 = c[0].copy()
            c0[e0] = 0
            return (c0, c[1], c[2]+[V.index(v)])
        if e1 is not None:
            c1 = c[1].copy()
            c1[e1] = 0
            return (c[0], c1, c[2]+[V.index(v)])

    def isSolution(cand):
        return sum(cand[0])+sum(cand[1])==0

    C = [ (E0, E1, [ V.index(PostOffice) ]) ]
    solutions = []

    while len(C)>0:

        # take the first candidate out of the list of candidates
        cand, C = C[0], C[1:]

        if isSolution(cand):
            # solutions.append(cand[2])
            return [ cand[2] ]
        else:
            last = cand[2][-1]
            for e in range(len(E)):
                newCand = None
                if V.index(E[e][0]) == last:
                    ## print("edge", edge(e), 'forward extends', path(cand[2]))
                    # the edge begins in the current position
                    if cand[0][e] == 1 and DP0[e] == 1:
```

```
                            newCand = refine(cand, E[e][1], e0=e)
                        elif cand[1][e] == 1 and DP1[e] == 1:
                            newCand = refine(cand, E[e][1], e1=e)
                    elif V.index(E[e][1]) == last:   # the edge ends in the
 ↪current position
                        ## print("edge", edge(e), 'backwards extends',
 ↪path(cand[2]))
                        if cand[0][e] == 1 and DN0[e] == 1:
                            newCand = refine(cand, E[e][0], e0=e)
                        elif cand[1][e] == 1 and DN1[e] == 1:
                            newCand = refine(cand, E[e][0], e1=e)
                    if newCand is not None:
                        # if isSolution(newCand): return [ newCand[2] ]
                        C = insert(C, newCand)

        return solutions

    return [ ( [ path(p) for p in solutions(l) ],
               [ [ V[v] for v in p ] for p in solutions(l) ],
               pulp.value(loopLength(l)) )
            for l in range(n) ]
```

[21]:
```
def splitMap(fullMap, L, plot=False):
    Maps, Loops = [], []
    for l in range(len(L)):
        paths, loops, length = L[l]
        loop = loops[0]
        W = loop[0]

        V = []
        E = []
        for i in range(len(loop)-1):
            A, B = loop[i], loop[i+1]
            if A not in V:
                V.append(A)
            if (A, B) not in E and (B, A) not in E:
                E.append((A, B))
        subMap = V, E
        Maps.append(subMap)
        Loops.append(loop)

        if plot:
            #plotMap(fullMap, P=loop, w=W,
            #        labels=True, scale=True,
            #        text=f"Loop {l}: {int(length):7,d}m")

            plotMap(subMap, w=W, frame=fullMap, scale=True,
```

```
                    text=f"Map {l}: Full Roundtrip {int(length):,d}m")

    return Maps, Loops
```

`shortenLoop(L, T)` shortens a complete coverage loop `L` while adding new targets `T`

[22]:
```
def shortenLoop(L, M, T):
    for t in T:
        minD = math.inf
        minI = None
        for i in range(len(L)-1):
            P, Q = L[i], L[i+1]
            distT = dist(P, t)+dist(t, Q)-dist(P, Q)
            if distT < minD:
                minD = distT
                minI = i
        L = L[:minI+1]+[t]+L[minI+1:]
    success = True
    while success:
        success = False
        for i in range(len(L)-1):
            P = L[i]
            for j in range(i+1, len(L)):
                Q = L[j]

                if Q in T:
                    shortcut = shortestPath(M, P, Q)
                    if pathLength(shortcut)<pathLength(L[i:j+1]):
                        L = L[:i]+shortcut+L[j+1:]
                        success = True
                    break
                if Q==P:
                    L = L[:i+1]+L[j+1:]
                    success = True
                    break
            if success:
                break
    return L
```

6.2 Split Customers into Regions

[23]:
```
def pickRegion(C, Maps):
    options = []
    for m in range(len(Maps)):
        V, E = Maps[m]
        for (A, B) in E:
            if dist(A, C)+dist(C, B) - dist(A, B) <= 1:
```

```
                options.append(m)
    return random.choice(options)
```

[24]:
```python
def splitCustomers(C, Maps):
    return [ pickRegion(C[i], Maps) for i in range(len(C)) ]
```

[25]:
```python
def groupCustomersByRegions(C, Maps):
    regions = splitCustomers(C, Maps)
    n = len(Maps)
    CC = [ [] for i in range(n) ]
    for i in range(n):
        for c in range(len(C)):
            if regions[c]==i:
                CC[i].append(C[c])
    return CC
```

6.3 Integrated Planning Using CPP

[26]:
```python
def planCPP(M, W, C, n=1,
            maxLength=None, balance=None,
            timing=False, timeLimit=10, plot=False):
    L = solveCPP(M, W, n=n,
                 maxLength=maxLength, balance=balance,
                 timing=timing, timeLimit=timeLimit)
    MM, LL = splitMap(M, L)
    CC = groupCustomersByRegions(C, MM)

    if plot:
        for i in range(len(MM)):
            plotMap(MM[i], w=W, frame=M, scale=True, size=3,
                    text=f"Map {i}: Full Roundtrip {int(L[i][2]):,d}m")
            MMT = addTargets(MM[i], CC[i])
            path = shortenLoop(LL[i], MMT, [W] + CC[i])
            plotMap(MMT, T=CC[i], P=path, w=W, frame=M, lwP=1, msPT=3, size=3,
                    text=f"Driver {i:d}:  {len(CC[i]):d} Customers, "
                         f"max Roundtrip: {pathLength(path):,d}m")
    return MM, LL, CC
```

7 Class Recorder

We willl use a class Recorder as a reference point for capturing data during the simulation. There will be only one recorder. It will be created at the beginning of every simulation run. Every entity will carry a reference to the Recorder.

[27]:
```python
import time

class Recorder:
```

```
    def __init__(self, env, M, Maps, W, C, D, days,
                 log=False, plot=False, timing=False):
        self.env = env
        self.M = M
        self.Maps = Maps
        self.W = W
        self.C = C
        self.D = D

        self.parcels = sum([ len(d) for d in D ])
        self.days = days
        self.drivers = len(Maps)

        self.log = log
        self.plot = plot

        # create a data frame for time records per working day
        self.daily = [ pd.DataFrame() for d in range(self.drivers) ]

        for driver in range(self.drivers):
            self.daily[driver]['begin work at'] = [None]*days
            self.daily[driver]['end work at'] = [None]*days
            self.daily[driver]['tour length'] = [None]*days

            self.daily[driver]['parcels left over'] = [0]*days
            self.daily[driver]['parcels arrived'] = [0]*days
            self.daily[driver]['parcels out for delivery'] = [0]*days
            self.daily[driver]['parcels returned from delivery'] = [0]*days
            self.daily[driver]['parcels delivered'] = [0]*days


        self.parcel = pd.DataFrame()
        self.parcel['arrived at'] = [None]*self.parcels
        self.parcel['delivered at'] = [None]*self.parcels

    def trace(self, event, driver=None):
        if self.log:
            prefix = "" if driver is None else f"D{driver.id:d}: "
            print(timestamp(self.env.now), prefix+event)

    def recordDriverBeginsWork(self, driver):
        self.trace("arrives for work", driver)
        self.daily[driver.id].at[day(self.env.now), 'begin work at'] =␣
↪int(round(self.env.now))

    def recordDriverEndsWork(self, driver):
```

```python
        self.trace("goes home", driver)
        self.daily[driver.id].at[day(self.env.now), 'end work at'] = int(round(self.env.now))

    def recordTourLength(self, driver, length):
        self.daily[driver.id].at[day(self.env.now), 'tour length'] = int(length)

    def recordParcelArrived(self, driver, parcel):
        self.trace(str(parcel)+" arr at delivery centre", driver)
        today = day(self.env.now)
        self.daily[driver.id].at[today, 'parcels arrived'] += 1
        self.parcel.at[parcel.i, 'arrived at'] = today

    def recordParcelOutForDelivery(self, driver, parcel):
        self.trace(str(parcel)+" out for delivery", driver)
        self.daily[driver.id].at[day(self.env.now), 'parcels out for delivery'] += 1

    def recordParcelReturnedFromDelivery(self, driver, parcel):
        self.trace(str(parcel)+" returned from delivery", driver)
        self.daily[driver.id].at[day(self.env.now), 'parcels returned from delivery'] += 1

    def recordParcelDelivered(self, driver, parcel):
        self.trace(str(parcel)+" delivered", driver)
        today = day(self.env.now)
        self.daily[driver.id].at[today, 'parcels delivered'] += 1
        self.parcel.at[parcel.i, 'delivered at'] = today

    def recordParcelsLeftOver(self, driver, n):
        self.trace(f"{n:d} parcels left over for next day", driver)
        self.daily[driver.id].at[day(self.env.now), 'parcels left over'] = n

    def finish(self):
        # simulation is finished for good
        # by removing the simulation environment we can
        # pickle recorder
        self.env = None

        for driver in range(self.drivers):
            self.daily[driver]['working time'] = (self.daily[driver]['end work at']-self.daily[driver]['begin work at'])//60
            self.daily[driver]['cost'] = self.daily[driver]['working time'].apply(lambda x: max(60, x*30/60))
            self.daily[driver]['cost'] += 0.08/1000*self.daily[driver]['tour length']
```

```
        self.parcel['delivery delay'] = self.parcel['delivered at']-self.
↪parcel['arrived at']

        self.totalDaily = pd.DataFrame()
        self.totalDaily['cum arrival'] = self.__Data('parcels arrived').cumsum()
        self.totalDaily['cum delivery'] = self.__Data('parcels delivered').
↪cumsum()

    # the Title() and Data() functions have been introduced
    # solely to maintain the code of the plot/hist methods below
    # as far as possible

    def __Title(self, title, driver):
        return ("Total " if driver is None else "") + \
                title + \
                " (" + f"{self.days:d} days" + \
                ("" if driver is None else f", Driver {driver:d} only") +")"

    def __Data(self, col, driver=None):
        if driver is None:
            total = pd.DataFrame()
            total['sum'] = [0] * self.days
            for d in range(self.drivers):
                total['sum'] += self.daily[d][col]
            return total['sum']
        else:
            return self.daily[driver][col]

    def histWorkingTime(self, driver=None):
        histPlot(self.__Data('working time', driver),
                 xlabel='Working Time [min]',
                 title=self.__Title('Daily Working Time', driver))

    def plotWorkingTime(self, driver=None):
        dailyPlot(self.__Data('working time', driver),
                  ylabel='Working Time [min]',
                  title=self.__Title('Daily Working Time', driver))

    def histTourLength(self, driver=None):
        histPlot(self.__Data('tour length', driver),
                 xlabel='Tour Length [m]',
                 title=self.__Title('Daily Tour Length', driver))

    def plotTourLength(self, driver=None):
        dailyPlot(self.__Data('tour length', driver),
                  ylabel='Tour Length [m]',
```

```
                  title=self.__Title('Daily Tour Length', driver))

    def histDailyCost(self, driver=None):
        histPlot(self.__Data('cost', driver),
                 xlabel='Cost [€]',
                 title=self.__Title('Daily Cost', driver))

    def plotDailyCost(self, driver=None):
        dailyPlot(self.__Data('cost', driver),
                  ylabel='Cost [€]',
                  title=self.__Title('Daily Cost', driver))

    def histParcelsArrived(self, driver=None):
        histPlot(self.__Data('parcels arrived', driver),
                 discrete=True,
                 xlabel='Parcels Arrived',
                 title=self.__Title('Daily Parcels Arrived', driver))

    def plotParcelsArrived(self, driver=None):
        dailyPlot(self.__Data('parcels arrived', driver),
                  ylabel='Parcels',
                  title=self.__Title('Parcels Arrived Daily', driver))

    def histParcelsOutForDelivery(self, driver=None):
        histPlot(self.__Data('parcels out for delivery', driver),
                 discrete=True,
                 xlabel='Parcels',
                 title=self.__Title('Parcels Daily out for Delivery', driver))

    def plotParcelsOutForDelivery(self, driver=None):
        dailyPlot(self.__Data('parcels out for delivery', driver),
                  ylabel='Parcels',
                  title=self.__Title('Parcels Daily out for Delivery', driver))

    def histParcelsReturnedFromDelivery(self, driver=None):
        histPlot(self.__Data('parcels returned from delivery', driver),
                 discrete=True,
                 xlabel='Parcels',
                 title=self.__Title('Parcels Daily Returned From Delivery', driver))

    def plotParcelsReturnedFromDelivery(self, driver=None):
        dailyPlot(self.__Data('parcels returned from delivery', driver),
                  ylabel='Parcels',
                  title=self.__Title('Daily Parcels Returned From Delivery', driver))
```

```
    def histParcelsDelivered(self, driver=None):
        histPlot(self.__Data('parcels delivered', driver),
                 discrete=True,
                 xlabel='Parcels',
                 title=self.__Title('Parcels Delivered Daily', driver))

    def plotParcelsDelivered(self, driver=None):
        dailyPlot(self.__Data('parcels delivered', driver),
                  ylabel='Parcels',
                  title=self.__Title('Parcels Delivered Daily', driver))

    def histParcelsLeftOver(self, driver=None):
        histPlot(self.__Data('parcels left over', driver),
                 discrete=True,
                 xlabel='Parcels',
                 title=self.__Title('Daily Left-Over Parcels', driver))

    def plotParcelsLeftOver(self, driver=None):
        dailyPlot(self.__Data('parcels left over', driver),
                  ylabel='Parcels',
                  title=self.__Title('Daily Left-Over Parcels', driver))

    def countPlot(self):
        countPlot(self.totalDaily['cum arrival'],
                  self.totalDaily['cum delivery'],
                  ylabel='Parcels',
                  title=f'Parcel Arrival/Delivery ({self.parcels:3,d} Parcels,␣
↪{self.days:d} Days)')

    def histParcelDeliveryDelay(self):
        histPlot(self.parcel['delivery delay'].dropna(),
                 discrete=True,
                 xlabel='Days',
                 title=f'Parcel Delivery Delay in Days ({self.parcels:3,d}␣
↪Parcels)')
```

8 Class Customer

[28]:
```
class Customer:

    def __init__(self, rec, id, location):
        self.rec = rec
        self.id = id
        self.location = location
        self.atHome = True
        self.answersDoor = False
```

```
        self.parcelsReceived = []
        rec.env.process(self.process())

    def __str__(self):
        return f"Customer {self.id:d} at {str(self.location):s}"

    def leaveHouse(self):
        assert(self.atHome and not self.answersDoor)
        # self.rec.trace(str(self)+" leaves house")
        self.atHome = False

    def returnHome(self):
        assert(not self.atHome)
        # self.rec.trace(str(self)+" returns home")
        self.atHome = True

    def answerDoor(self, driver):
        if self.atHome:
            answerTime = random.expovariate(1/AVG_TIME_ANSWER_DOOR)
            if answerTime < WAIT_TIME_IF_CUSTOMER_DOESNT_ANSWER_DOOR:
                yield self.rec.env.timeout(answerTime)
                self.rec.trace(str(self)+" answers door", driver)
                self.answersDoor = True
            else:
                yield self.rec.env.
↪timeout(WAIT_TIME_IF_CUSTOMER_DOESNT_ANSWER_DOOR)
                self.rec.trace(str(self)+" to slow to answer the door", driver)
                self.answersDoor = False
        else:
            yield self.rec.env.timeout(WAIT_TIME_IF_CUSTOMER_DOESNT_ANSWER_DOOR)
            self.rec.trace(str(self)+" not at home", driver)
            self.answersDoot = False

    def acceptParcel(self, driver, parcel):
        assert(self.answersDoor)
        self.parcelsReceived += [parcel]
        self.rec.recordParcelDelivered(driver, parcel)

    def signOff(self, driver):
        assert(self.answersDoor)
        self.rec.trace(str(self)+" signs off", driver)
        self.answersDoor = False

    def process(self):
        yield self.rec.env.timeout(nextHour(self.rec.env, 8))
        while day(self.rec.env.now)<self.rec.days:
            # in a refinement we may use random times
```

```
            self.leaveHouse()
            returnTime = 22 if random.random()<CUSTOMER_NOT_AT_HOME else 18
            yield self.rec.env.timeout(nextHour(self.rec.env, returnTime))
            self.returnHome()
            yield self.rec.env.timeout(nextHour(self.rec.env, 8))
```

9 Class Parcel

Parcels follow through a sequence of states: - processing - in transit (from manufacture to distribution centre) - arrived in distribution centre - ready for delivery - out for delivery - customer not present - returned to distribution centre - delivered

[29]:
```
class Parcel:

    def __init__(self, rec, i, day, cust):
        self.rec = rec
        self.i = i
        self.arrival = day
        self.cust = cust
        self.status = [  ] # status record and
        self.timing = [  ] # timing

    def __str__(self):
        return f"Parcel {self.i:d} (for cust {self.cust.id:d}"

    def index(self):
        return self.i

    def destination(self):
        return self.cust.location

    def __reg(self, state):
        self.status += [ state ]
        self.timing += [ self.rec.env.now ]

    def arrivedAtDeliveryCentre(self, driver):
        self.__reg('arr at delivery centre')
        self.rec.recordParcelArrived(driver, self)

    def outForDelivery(self, driver):
        self.__reg('out for delivery')
        self.rec.recordParcelOutForDelivery(driver, self)

    def returnFromDelivery(self, driver):
        self.__reg('return from delivery')
        self.rec.recordParcelReturnedFromDelivery(driver, self)
```

10 Class Driver

[30]:
```
class Driver:

    def __init__(self, rec, id, DC):
        self.rec = rec
        self.id = id
        self.DC = DC
        self.location = None
        self.parcels = None
        self.returns = None
        self.tour = None
        self.rec.env.process(self.process())

    # activity
    def __drive(self, target):
        assert(self.tour[0] == self.location)
        while self.location!=target:
            d = dist(self.location, self.tour[1])
            yield self.rec.env.timeout(d / AVG_SPEED)
            self.location = self.tour[1]
            self.tour = self.tour[1:]
        assert(self.tour[0] == self.location == target)

    def arriveForWork(self):
        self.location = self.DC.W
        self.parcels = []
        self.returns = []
        self.tour = [ self.DC.W ]
        # self.rec.trace("arrives for work", self)
        self.rec.recordDriverBeginsWork(self)

    def goesHome(self):
        self.location = None
        self.parcels = None
        self.returns = None
        self.tour = None
        # self.rec.trace("goes home", self)
        self.rec.recordDriverEndsWork(self)

    def leaveForDelivery(self, tour, parcels, addresses):
        self.tour, self.parcels = tour, parcels
        self.rec.trace(f"leaves for delivery "
                       f"of {len(parcels):d} parcels "
                       f"to {len(addresses):d} customers", self)
        self.rec.trace(f"Length of delivery tour: {pathLength(tour):,d}m", self)
        if self.rec.plot:
```

```
            plotMap(self.rec.Maps[self.id], frame=self.rec.M,
                    T=addresses, P=tour, w=tour[0],
                    text=f"Day {day(self.rec.env.now):d} D{self.id:d}:␣
↪{pathLength(tour):,d}m")

    def process(self):
        yield self.rec.env.timeout(nextHour(self.rec.env, 18))
        while day(self.rec.env.now)<self.rec.days:
            self.arriveForWork()

            ## chnge to deal with time limit
            startTime = self.rec.env.now

            tour, parcels, addresses = self.DC.sendForDelivery(self)
            if len(parcels)==0:
                self.rec.trace("Nothing to do today", self)
                self.rec.recordTourLength(self, 0)
            else:
                yield self.rec.env.timeout(PREP_TIME_PER_PARCEL*len(parcels))
                self.rec.recordTourLength(self, pathLength(tour))
                self.leaveForDelivery(tour, parcels, addresses)
                while len(self.parcels)>0:

                    ## change to deal with time limit
                    currentTime = self.rec.env.now
                    if currentTime-startTime>=self.DC.timeLimit:
                        self.rec.trace("Timelimit reached", self)
                        while len(self.parcels)>0:
                            self.returns += [self.parcels[0]]
                            self.parcels = self.parcels[1:]
                        break

                    # drive to customer
                    custLocation = self.parcels[0].destination()
                    cust = self.parcels[0].cust
                    self.rec.trace("drives to "+str(cust), self)
                    yield from self.__drive(custLocation)
                    self.rec.trace("arrived at "+str(cust), self)
                    # call at customer
                    yield from cust.answerDoor(self)

                    if cust.answersDoor:
                        while len(self.parcels)>0 and \
                                custLocation == self.parcels[0].destination():
                            cust.acceptParcel(self, self.parcels[0])
                            yield self.rec.env.timeout(random.expovariate(1/
↪AVG_TIME_HANDOVER))
```

```
                            self.parcels = self.parcels[1:]
                        cust.signOff(self)
                        yield self.rec.env.timeout(random.expovariate(1/
↪AVG_TIME_SIGNOFF))
                    else:
                        while len(self.parcels)>0 and  \
                                custLocation == self.parcels[0].destination():
                            self.returns += [self.parcels[0]]
                            self.parcels = self.parcels[1:]

                # return to delivery centre
                self.rec.trace("returns to delivery centre", self)
                yield from self.__drive(self.DC.W)
                self.rec.trace("arrived at delivery centre", self)

                for parcel in self.returns:
                    self.DC.returnFromDelivery(self, parcel)
                    yield self.rec.env.timeout(RETURN_TIME_PER_PARCEL)

            self.rec.recordParcelsLeftOver(self,
                                           len(self.returns)+
                                           len(self.DC.leftOver[self.id]))

            yield self.rec.env.timeout(DAY_END_PROCEDURE)

            self.goesHome()

            yield self.rec.env.timeout(nextHour(self.rec.env, 18))
```

11 Class Delivery Centre

[31]:
```
class DeliveryCentre:

    def __init__(self, rec, M, Maps, W, C, D,
                 limit, timeLimit):
        self.rec = rec
        self.M = M
        self.Maps = Maps
        self.W = W
        self.C = C
        self.D = D

        self.limit = limit
        self.timeLimit = timeLimit

        self.overhang = []   # list of parcels to be delivered next day
```

```python
        # generate and initialise all customers
        self.customers = [ Customer(rec, i, C[i]) for i in range(len(C)) ]

        # generate and initialise all drivers
        self.drivers = [ Driver(rec, i, self) for i in range(len(Maps)) ]

        self.PARCELS = []    # registry of all the parcels processed

        rec.env.process(self.process())

    def __accept(self, d, parcel):
        custLoc = parcel.destination()

        assert(0<=d<len(self.Maps))

        ## chnge to deal with time limit estimate
        timeEstimate = \
            len(self.parcels[d])*(PREP_TIME_PER_PARCEL + AVG_TIME_HANDOVER) \
            + len(self.dest[d])*(AVG_TIME_ANSWER_DOOR + AVG_TIME_SIGNOFF)

        if custLoc not in self.dest[d]:

            targets = [self.W] + self.dest[d] + [custLoc]

            if self.rec.plot:
                plotMap(self.Maps[d], T=targets,  w=self.W, frame=self.M,
                    size=2, text=f"Driver {d:d}: accept Parcel")

            MT = addTargets(self.Maps[d], targets)
            SH = createLoopG(MT, targets)

            ## chnge to deal with time limit estimate
            if pathLength(SH)<self.limit and \
                timeEstimate + pathLength(SH)/AVG_SPEED + \
                PREP_TIME_PER_PARCEL + AVG_TIME_ANSWER_DOOR + \
                AVG_TIME_HANDOVER + AVG_TIME_SIGNOFF <= self.timeLimit:

                self.parcels[d].append(parcel)
                self.dest[d] += [custLoc]
                self.tour[d] = SH
            else:
                self.leftOver[d].append(parcel)

            ## chnge to deal with time limit estimate
        elif timeEstimate + pathLength(self.tour[d])/AVG_SPEED + \
                PREP_TIME_PER_PARCEL + AVG_TIME_HANDOVER <= self.timeLimit:
```

```
            self.parcels[d].append(parcel)
        else:
            self.leftOver[d].append(parcel)

    def sendForDelivery(self, driver):
        d = driver.id
        parcels = []
        tour = self.tour[d]
        addresses = []

        # pick parcels in sequence to be delivered
        for i in range(1, len(tour)-1):
            dest = tour[i]
            for p in self.parcels[d]:
                if p.destination() == dest and p not in parcels:
                    parcels += [p]
                    p.outForDelivery(driver)
                    if dest not in addresses:
                        addresses += [dest]

        # what cant go out goes straigt for next day
        for p in self.leftOver[d]:
            self.overhang.append(p)

        return tour, parcels, addresses

    def returnFromDelivery(self, driver, parcel):
        parcel.returnFromDelivery(driver)
        self.overhang.append(parcel)

    def getInventory(self):
        return len(self.overhang)

    def process(self):
        for day in range(len(self.D)):
            yield self.rec.env.timeout(nextHour(self.rec.env, 17.00))
            # make plan how to split workload for the day
            regions = splitCustomers(self.C, self.Maps)

            # initialise the workload for all drivers
            self.leftOver = [ [] for driver in self.drivers ]   # list of␣
↪parcels that can't go out today
            self.parcels = [ [] for driver in self.drivers ]    # list of␣
↪parcels scheduled for delivery
            self.dest = [ [] for driver in self.drivers ]       # list of␣
↪unique customer destinations
```

```
            self.tour = [ [self.W] for driver in self.drivers ] # tour planned␣
↪for delivery

            # process overhang from previous day (if any)
            for p in self.overhang:
                driverId = regions[p.cust.id]
                self.__accept(driverId, p)
            self.overhang = []

            # generate the parcels newly arriving this day
            for c in self.D[day]:
                parcel = Parcel(self.rec, len(self.PARCELS),
                                day, self.customers[c])
                self.PARCELS.append(parcel)
                driverId = regions[c]
                parcel.arrivedAtDeliveryCentre(self.drivers[driverId])
                self.__accept(driverId, parcel)
```

12 Simulation

12.1 Parameters from Specification

The hard time limit for the driver. When this time limit is reached on a delivery tour, the driver is supposed to return immediately

[32]:
```
DELIVERY_TIME_LIMIT = 3*3600 # 3 hours
```

The proportion of customers that for whatever are not at home or return home late

[33]:
```
CUSTOMER_NOT_AT_HOME = 0.1  # 10%
```

The maximum bike range. This is passed as parameter to the Delivery Centre and taken into account for the daily tour planning

[34]:
```
BIKE_RANGE = 40000
```

The time required for driving is based on the distance between way points at an average speed of 15km/h.

[35]:
```
AVG_SPEED = 15/3.6
```

The **cumulative preparation time** (route planning and sorting of the parcels in the delivery order and packing the cargo-bike) is assumed to be 50 sec per parcel to be delivered.

[36]:
```
PREP_TIME_PER_PARCEL = 50
```

Additional assumption: The time to **process returned parcels** in the delivery centre is 30 sec per parce.

[37]:
```
RETURN_TIME_PER_PARCEL = 30
```

The average time to answer the door.

[38]:
```
AVG_TIME_ANSWER_DOOR = 40
```

[39]:
```
WAIT_TIME_IF_CUSTOMER_DOESNT_ANSWER_DOOR = 60
```

[40]:
```
AVG_TIME_HANDOVER = 10
AVG_TIME_SIGNOFF = 10
```

[41]:
```
DAY_END_PROCEDURE = 600
```

12.2 Generate Input Data

[42]:
```
def generateDeliveries(p, C, days, seed=0):
    ## p is the average number of parcels per day per customer
    ## C is the number of customers to be served
    ## days is the number of days for which data are to be generated.
    random.seed(seed)
    deliveries = [ [ ] for _ in range(days) ]
    for c in range(C):
        arr = 0
        while True:
            arr += random.expovariate(p)
            day = int(arr)
            if day>=days:
                break
            deliveries[day].append(c)
    return deliveries
```

12.3 Simulation Routine

[43]:
```
def simulation(M, Maps, W, C, p=0.2, days=10, seed=0,
               log=False, plot=False, timing=False):

    random.seed(seed)
    D = generateDeliveries(p, len(C), days, seed)

    env = simpy.Environment()
    rec = Recorder(env, M, Maps, W, C, D, days,
                   log=log, plot=plot, timing=timing)

    print(f"Simulating delivery of {sum([len(d) for d in D]):d} parcels "
          f"over {len(D):d} days to {len(C):d} customers "
          f"using {len(Maps):d} drivers")

    # initialise all customer processes
```

```
    # initialises delivery center and creates all parcels
    DC = DeliveryCentre(rec, M, Maps, W, C, D,
                        BIKE_RANGE, DELIVERY_TIME_LIMIT)

    env.run()

    rec.finish()

    if DC.getInventory()>0:
        print(f"Delivery Centre Inventory at the end of last day: {DC.
↪getInventory():d} parcels")

    return rec
```

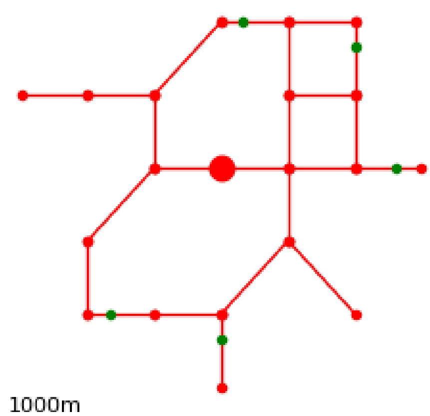
12.4 Testing

12.4.1 Simple Test Case

[44]:
```
import pickle
with open('data/simpleData.pickled', 'rb') as f:
    MS, CS = pickle.load(f)
```

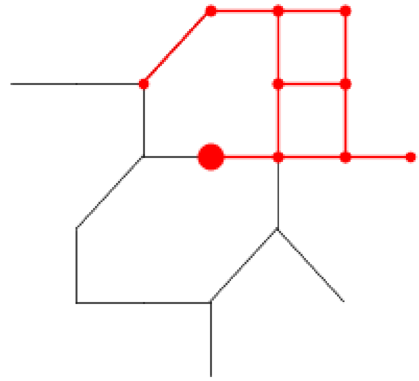
[45]:
```
WS = generateWarehouseLocation(MS)
```

[46]:
```
plotMap(MS, T=CS, w=WS, scale=True, size=3)
```

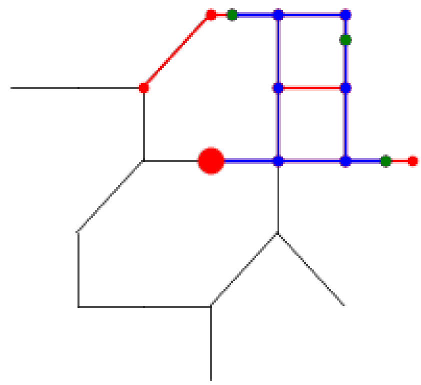


[47]:
```
MMS, LLS, CCS = planCPP(MS, WS, CS, n=2, balance=0.2, timing=True, plot=True)
```

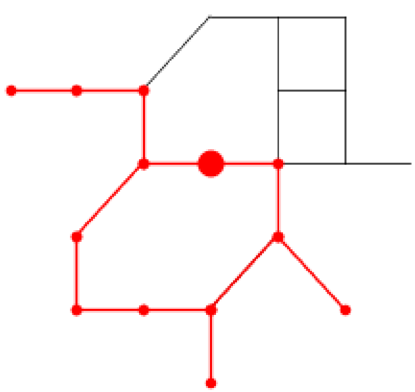
```
Solver time:   10.10s
Solution possibly not optimal
Total Delivery Path Length: 51074.0
```



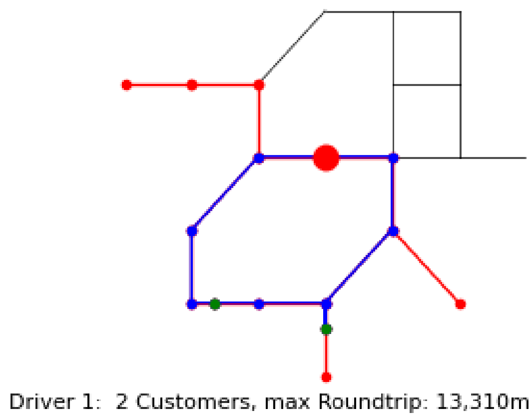
1000m Map 0: Full Roundtrip 23,558m



Driver 0: 3 Customers, max Roundtrip: 14,876m



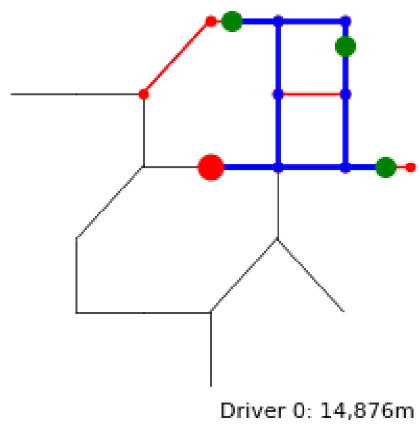
1000m Map 1: Full Roundtrip 27,516m



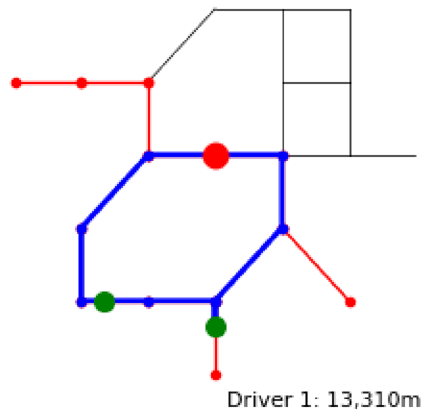
Comparison with Greedy Solution using Static Allocation shows equal performance:

[48]:
```
for i in range(len(MMS)):
    print(f"Driver {i:d}")
    MTS = addTargets(MMS[i], CCS[i])
    path = createLoopG(MTS, [WS] + CCS[i])
    plotMap(MTS, T=CCS[i], P=path, w=WS, frame=MS,
            size=3, text=f"Driver {i:d}: {pathLength(path):,d}m")
```

```
Driver 0
```



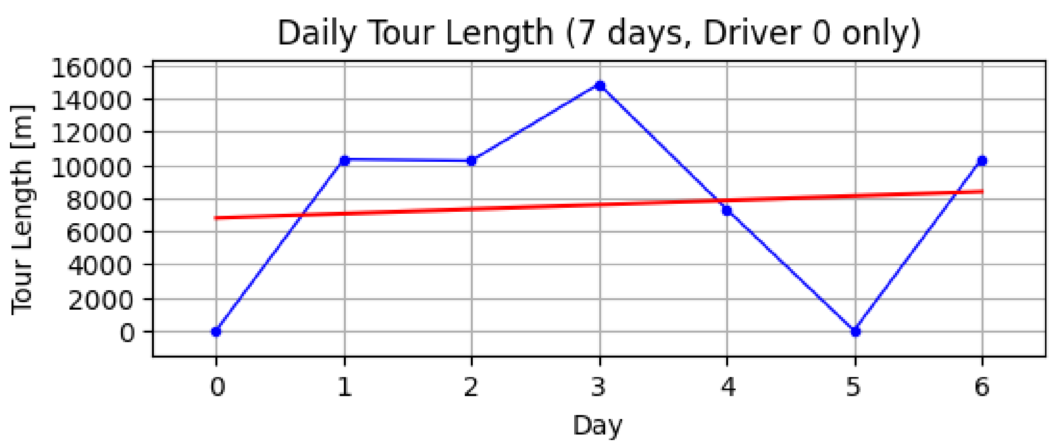

```
Driver 1
```

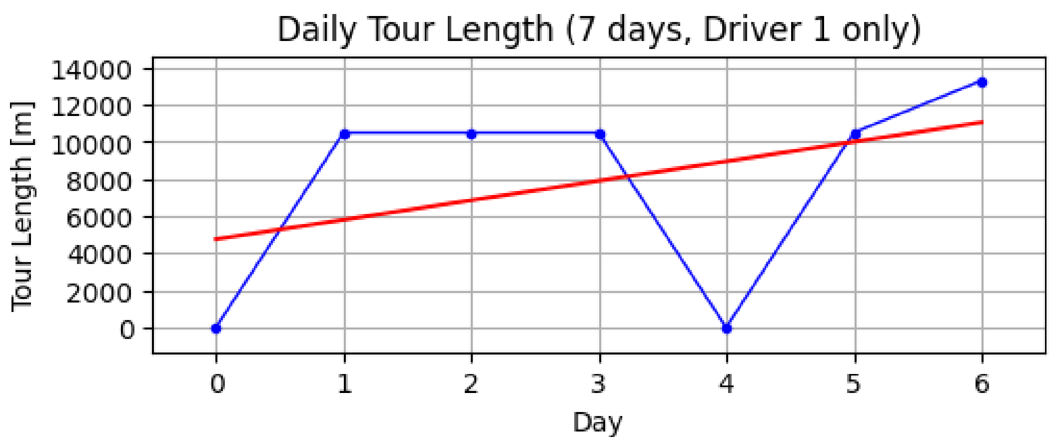


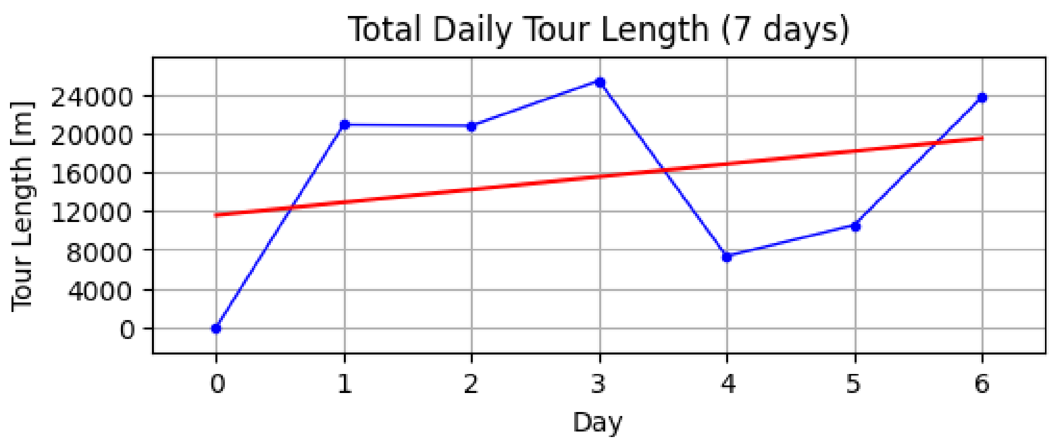

```
[49]: rec1 = simulation(MS, MMS, WS, CS, p=0.3, days=7)
```

```
Simulating delivery of 12 parcels over 7 days to 5 customers using 2 drivers
```

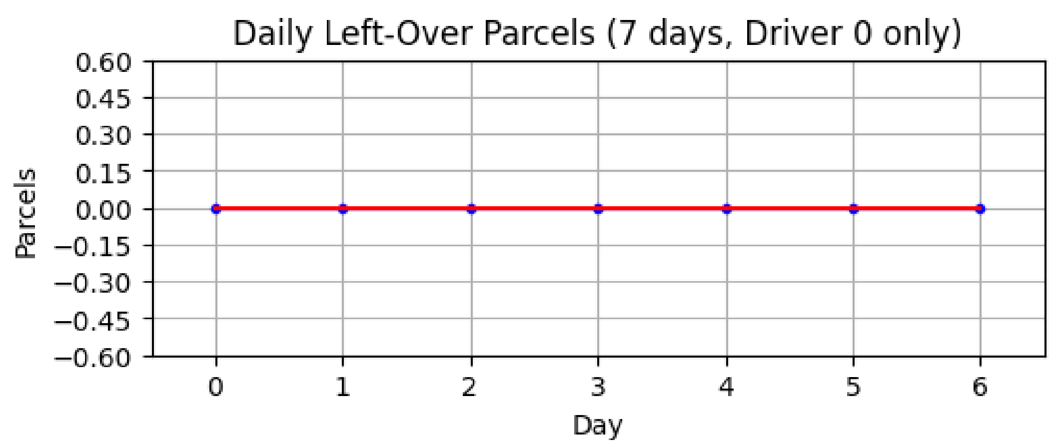
```
[50]: for i in range(rec1.drivers):
          rec1.plotTourLength(i)
      rec1.plotTourLength()
```



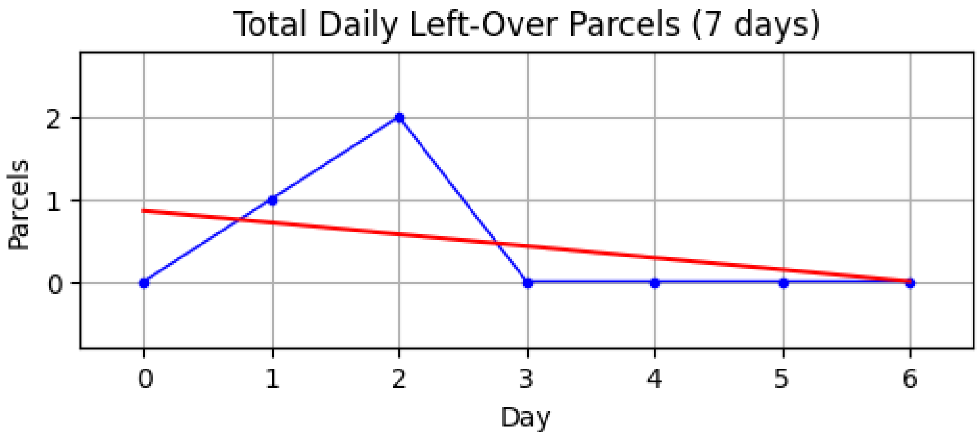




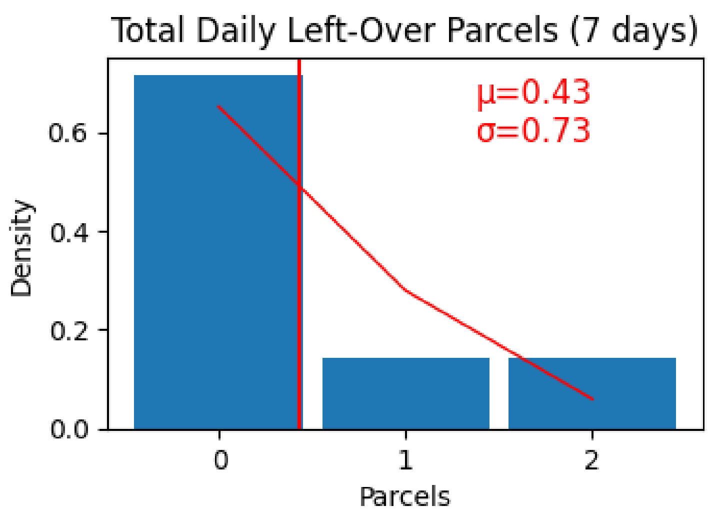
[51]:
```
rec1.plotParcelsLeftOver(0)
```



[52]:
```
rec1.plotParcelsLeftOver()
```



[53]:
```
rec1.histParcelsLeftOver()
```

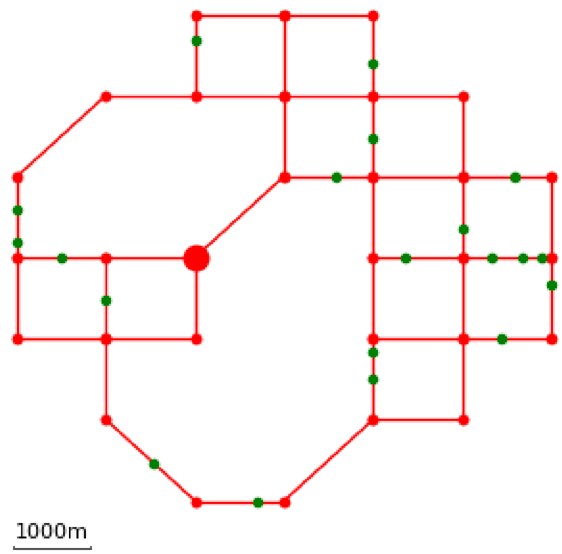


12.4.2 Stable Base Case

[54]:
```
import pickle
with open('data/testData.pickled', 'rb') as f:
    MT, CT = pickle.load(f)
```

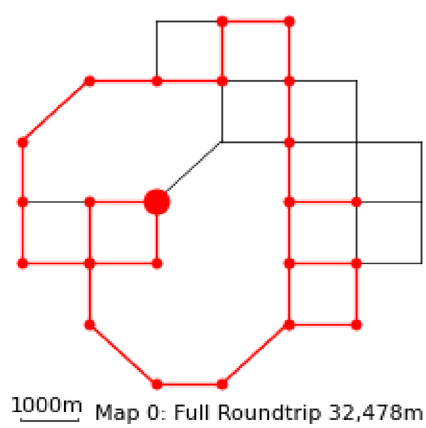
[55]:
```
WT = generateWarehouseLocation(MT)
```

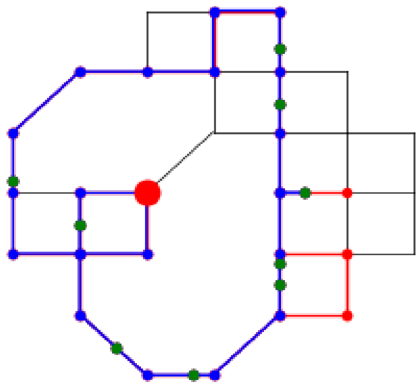
[56]:
```
plotMap(MT, T=CT, w=WT, scale=True)
```



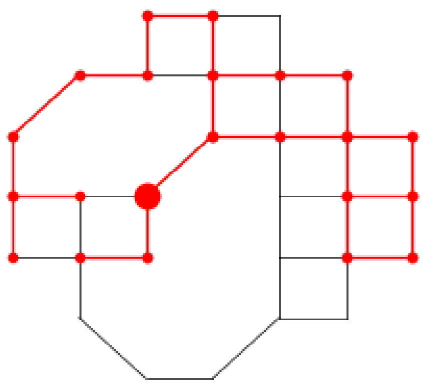
[57]:
```
MMT, LLT, CCT = planCPP(MT, WT, CT, n=2, balance=0.2, timing=True, plot=True)
```

```
Solver time:   10.19s
Solution possibly not optimal
Total Delivery Path Length: 75782.0
```

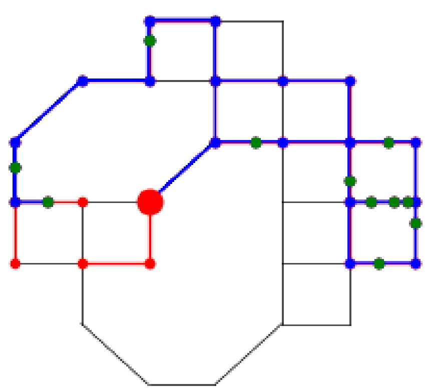




Driver 0: 9 Customers, max Roundtrip: 26,399m



1000m Map 1: Full Roundtrip 43,304m

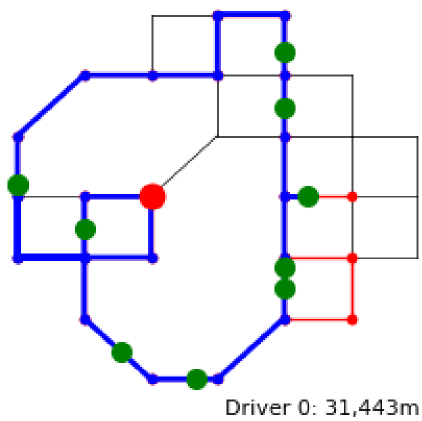


Driver 1: 11 Customers, max Roundtrip: 35,240m

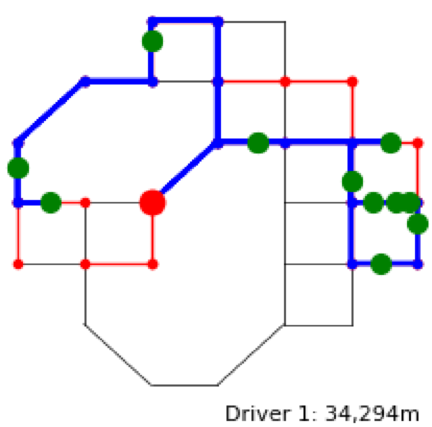
Comparison with Greedy Solution using Static Allocation

[58]:
```
for i in range(len(MMT)):
    print(f"Driver {i:d}")
    MMTT = addTargets(MMT[i], CCT[i])
    path = createLoopG(MMTT, [WT] + CCT[i])
    plotMap(MMTT, T=CCT[i], P=path, w=WT, frame=MT,
            size=3, text=f"Driver {i:d}: {pathLength(path):,d}m")
```

```
Driver 0
```



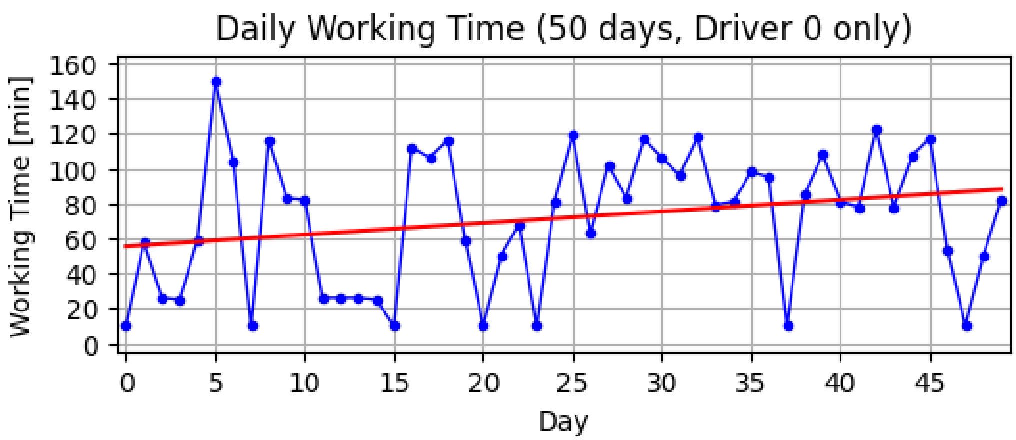

```
Driver 1
```

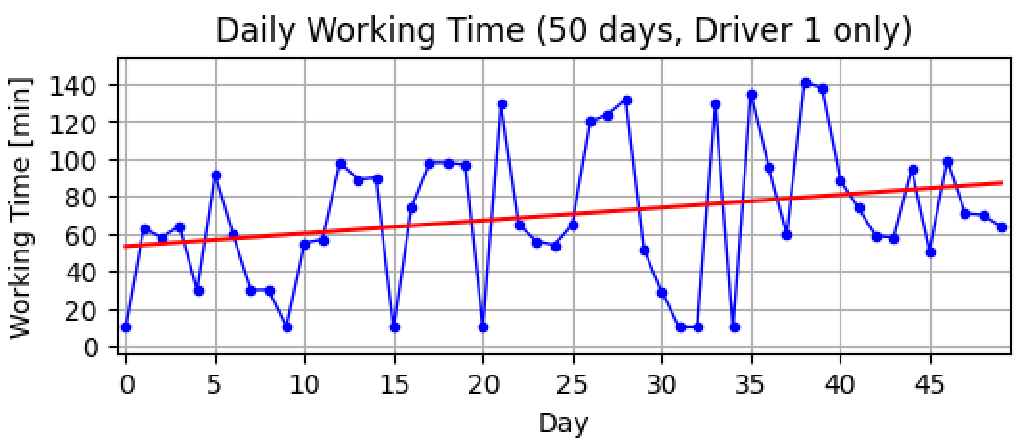


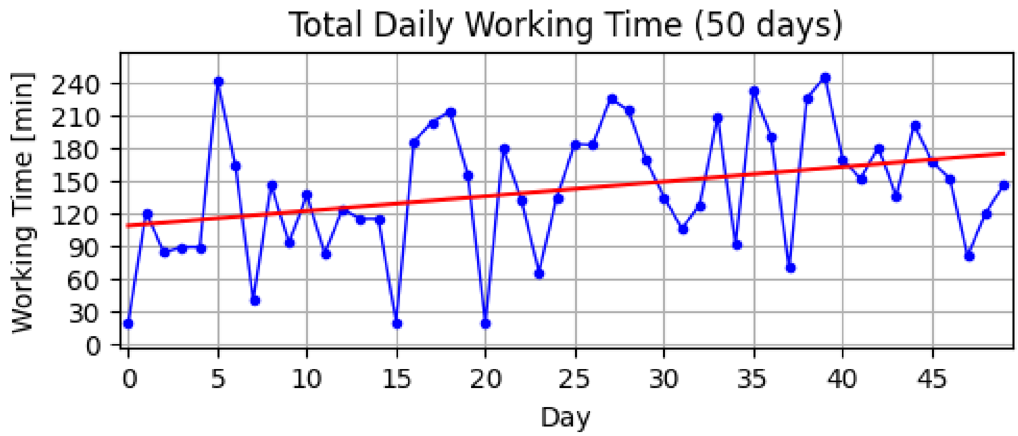
[59]:
```
rec2 = simulation(MT, MMT, WT, CT, p=0.15, days=50)
```

```
Simulating delivery of 134 parcels over 50 days to 20 customers using 2 drivers
Delivery Centre Inventory at the end of last day: 2 parcels
```

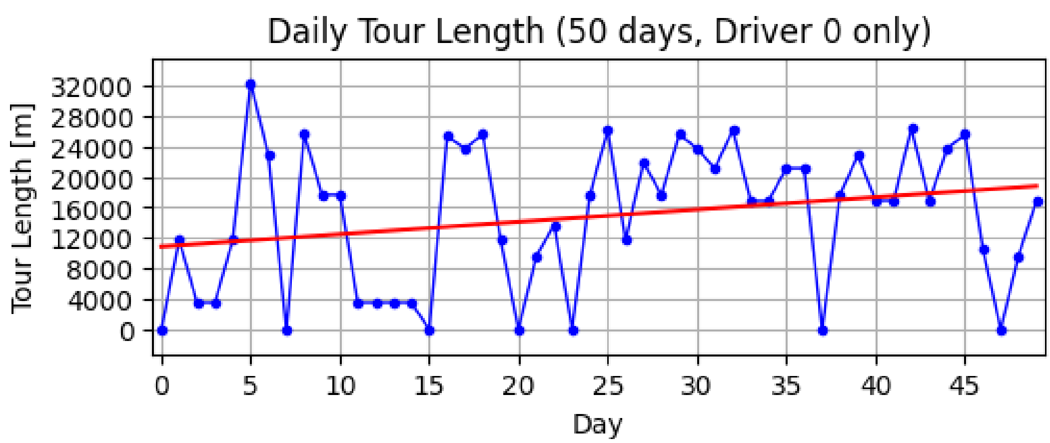
[60]:
```
for i in range(rec1.drivers):
    rec2.plotWorkingTime(i)
rec2.plotWorkingTime()
```

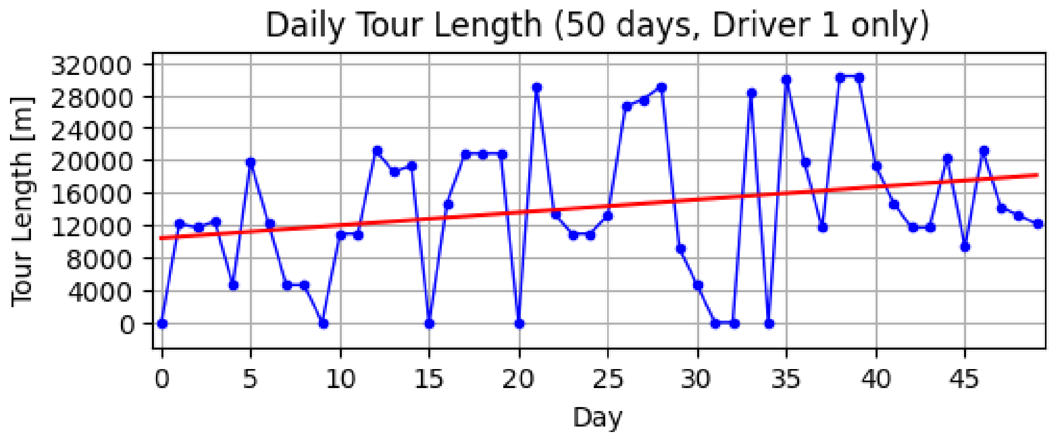


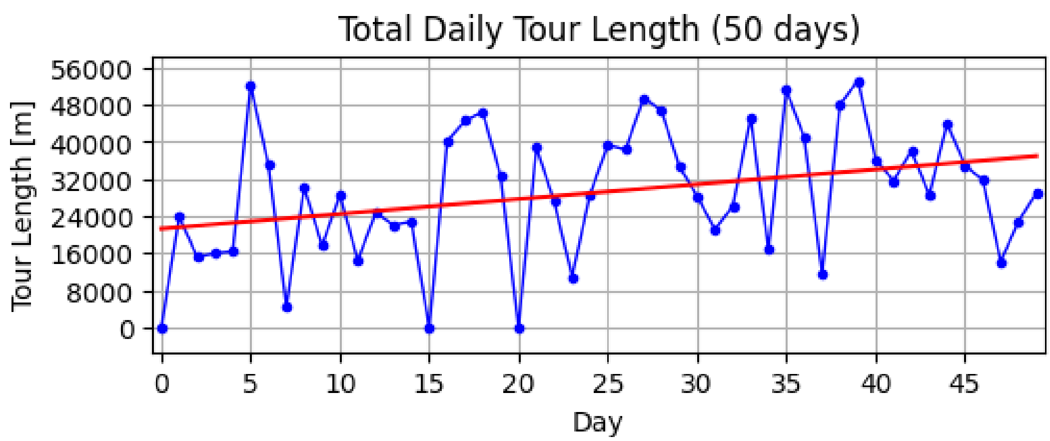




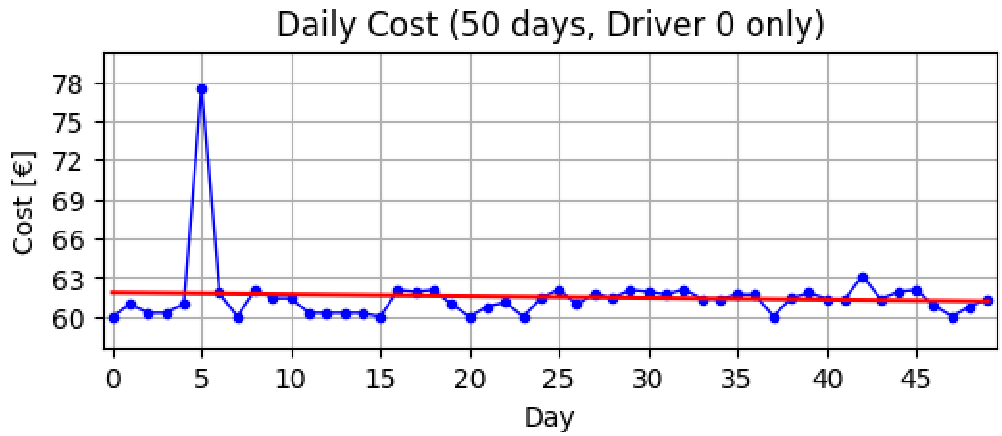
[61]:
```
for i in range(rec1.drivers):
    rec2.plotTourLength(i)
rec2.plotTourLength()
```

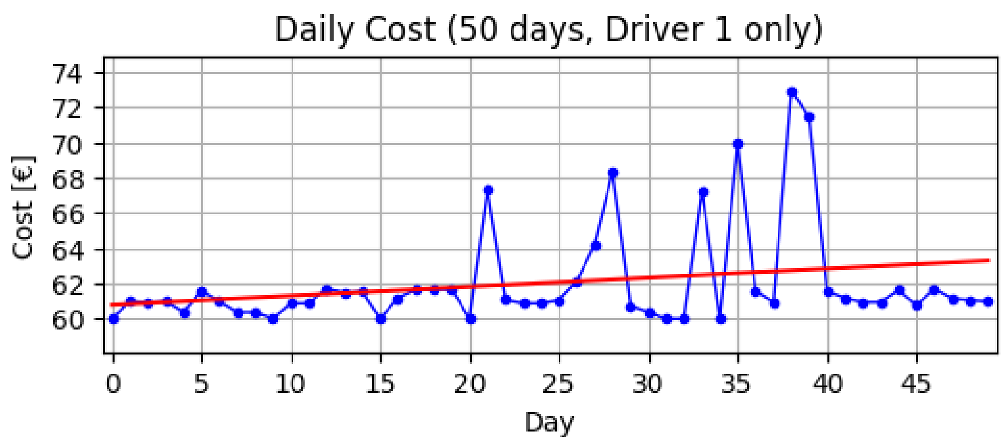


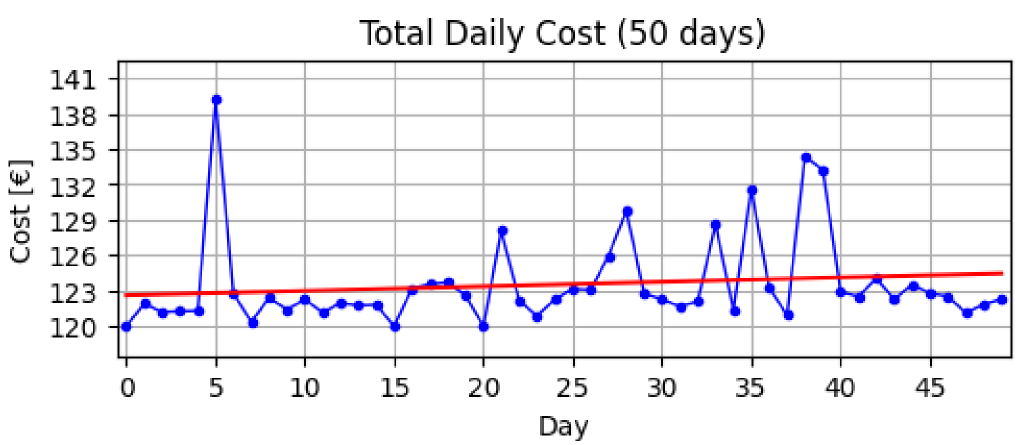




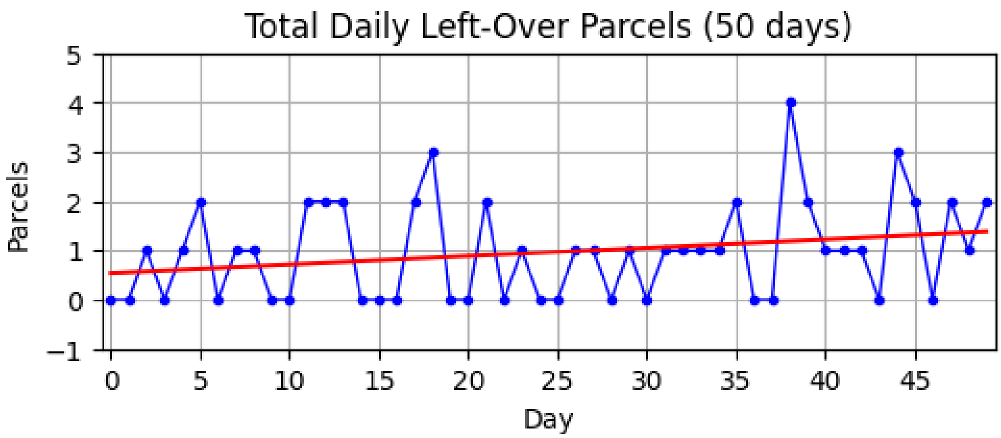
[62]:
```
for i in range(rec1.drivers):
    rec2.plotDailyCost(i)
rec2.plotDailyCost()
```


Daily Cost (50 days, Driver 0 only)
Cost [€]
Day
78
75
72
69
66
63
60
0
5
10
15
20
25
30
35
40
45

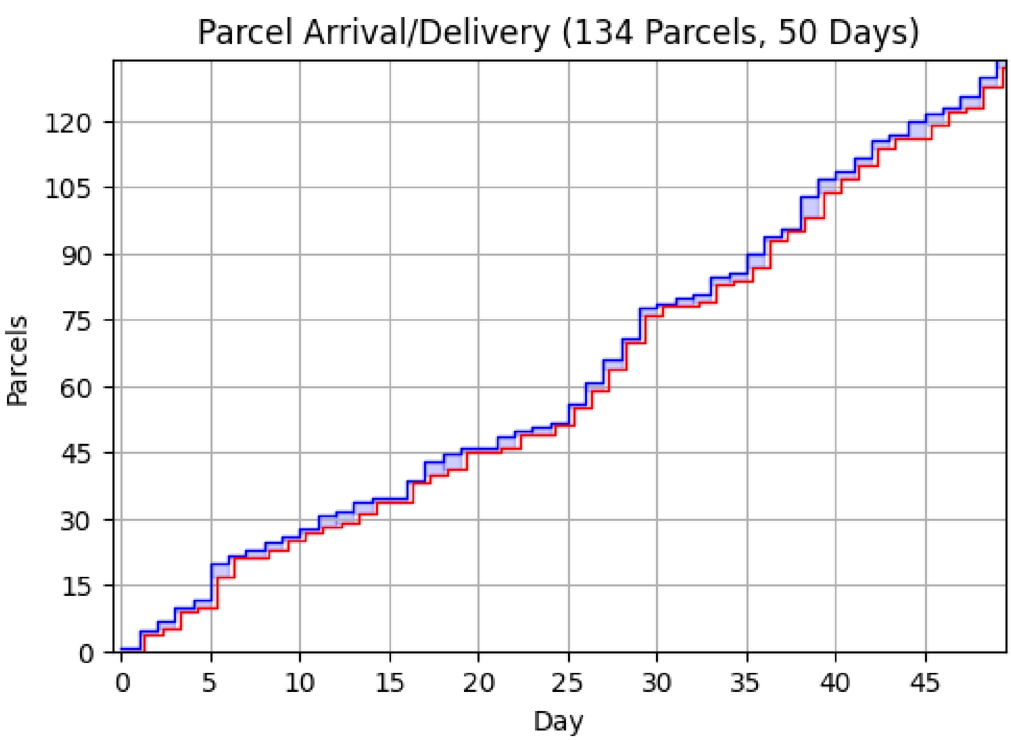

Daily Cost (50 days, Driver 1 only)
Cost [€]
Day
74
72
70
68
66
64
62
60
0
5
10
15
20
25
30
35
40
45


Total Daily Cost (50 days)
Cost [€]
Day
141
138
135
132
129
126
123
120
0
5
10
15
20
25
30
35
40
45

[63]:
```
rec2.plotParcelsLeftOver()
```



[64]:
```
rec2.countPlot()
```

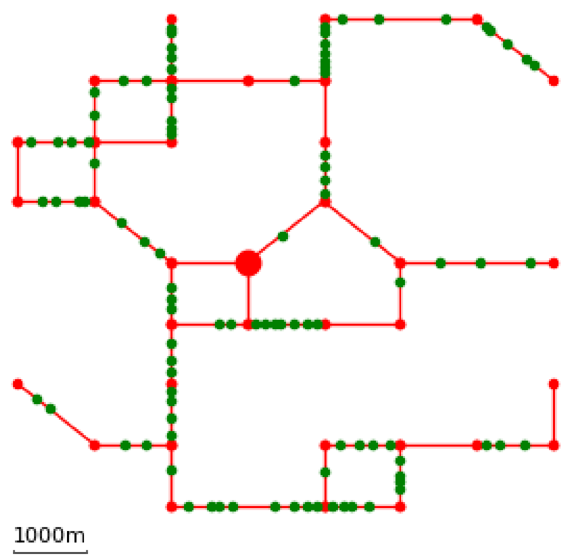


12.4.3 High Demand System

[65]:
```python
import pickle
with open('data/data.pickled', 'rb') as f:
    M, C = pickle.load(f)
```

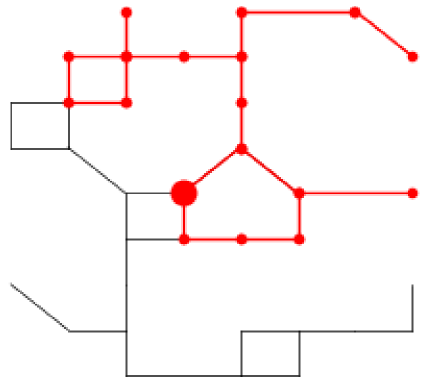
[66]:
```python
W = generateWarehouseLocation(M)
```

[67]:
```python
plotMap(M, T=C, w=W, scale=True)
```

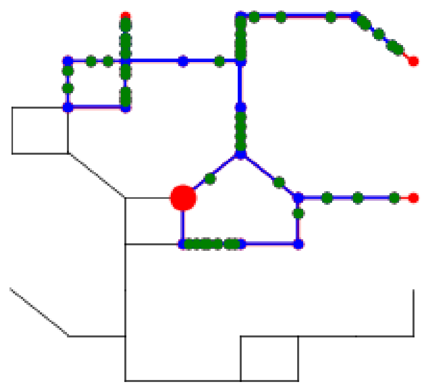


[68]:
```python
MM, LL, CC = planCPP(M, W, C, n=2,
                     maxLength=40000, balance=0.1, timeLimit=40,
                     timing=True, plot=True)
```

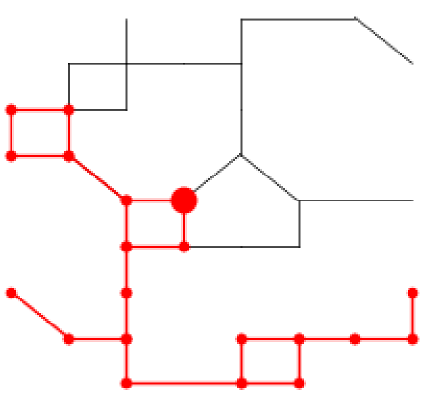
```
Solver time:   40.31s
Solution possibly not optimal
Total Delivery Path Length: 73472.0
```



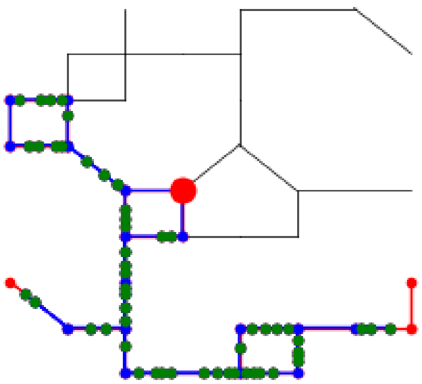
1000m Map 0: Full Roundtrip 35,676m



Driver 0: 47 Customers, max Roundtrip: 33,976m



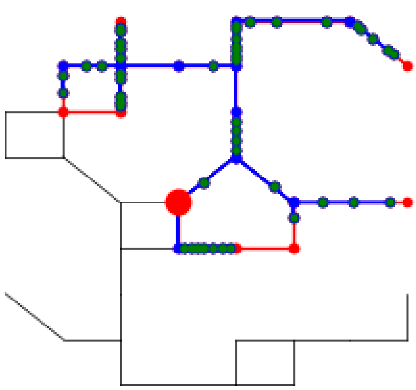
1000m Map 1: Full Roundtrip 37,796m



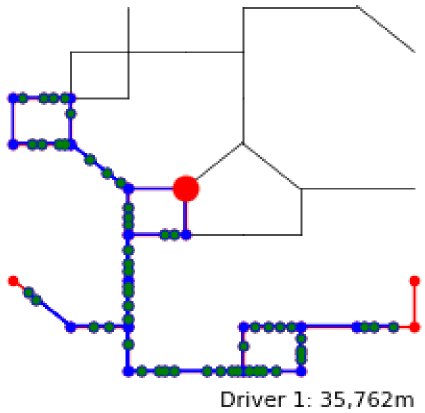
Comparison with Greedy Solution using Static Allocation shows noticeable better performance:

```
[69]: for i in range(len(MM)):
          print(f"Driver {i:d}")
          MMT = addTargets(MM[i], CC[i])
          path = createLoopG(MMT, [W] + CC[i])
          plotMap(MMT, T=CC[i], P=path, w=W, frame=M,
                  size=3, msPT=2, lwP=1,
                  text=f"Driver {i:d}: {pathLength(path):,d}m")
```

```
Driver 0
```



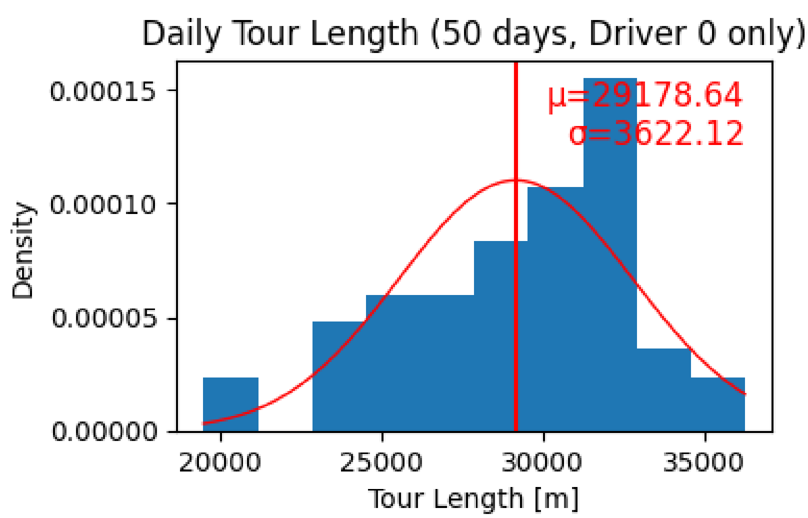

```
Driver 1
```

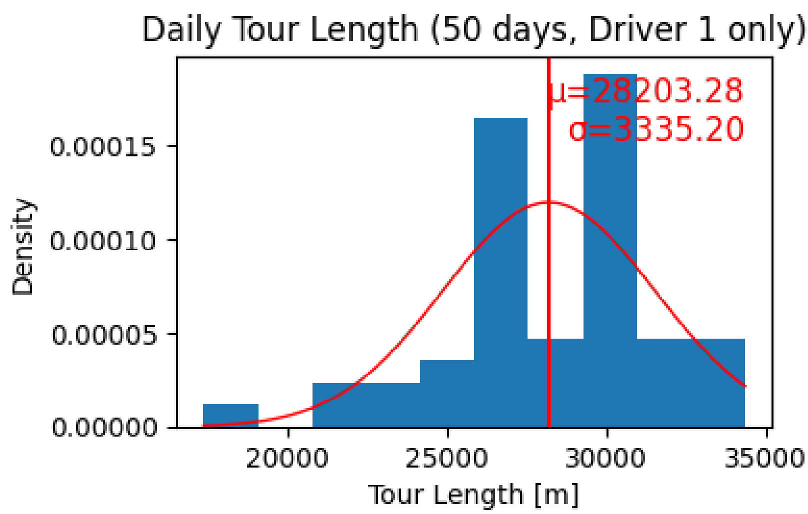


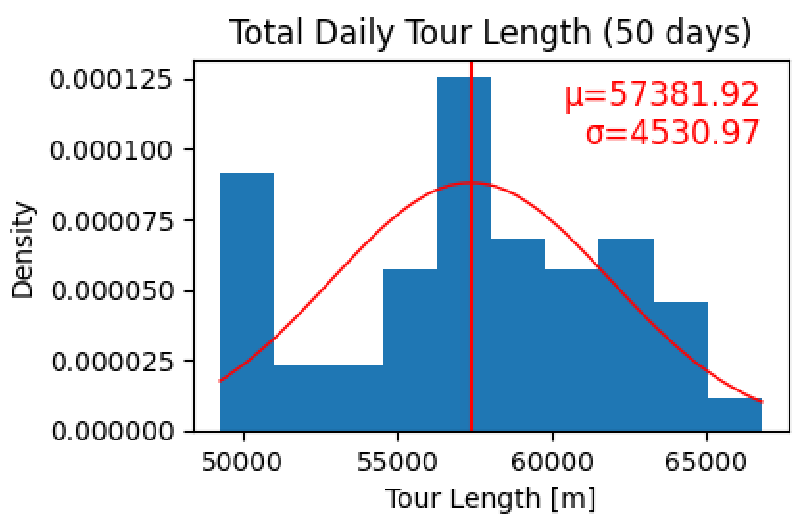

```
[70]: rec5 = simulation(M, MM, W, C, p=0.25, days=50)
```

```
Simulating delivery of 1249 parcels over 50 days to 100 customers using 2
drivers
Delivery Centre Inventory at the end of last day: 9 parcels
```

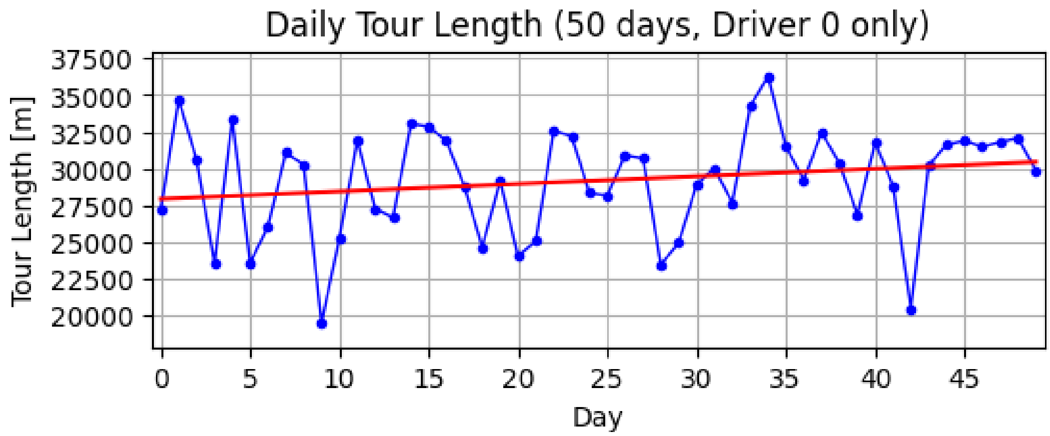
```
[71]: for i in range(rec5.drivers):
          rec5.histTourLength(i)
      rec5.histTourLength()
```

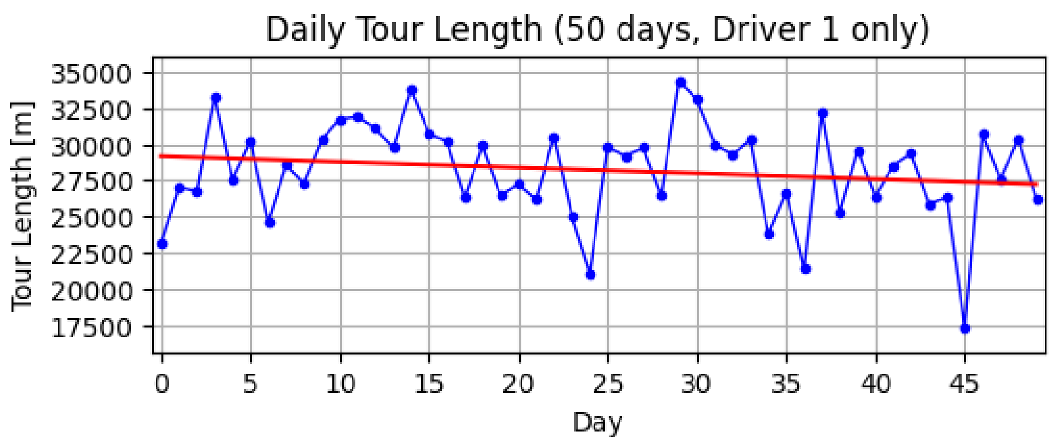


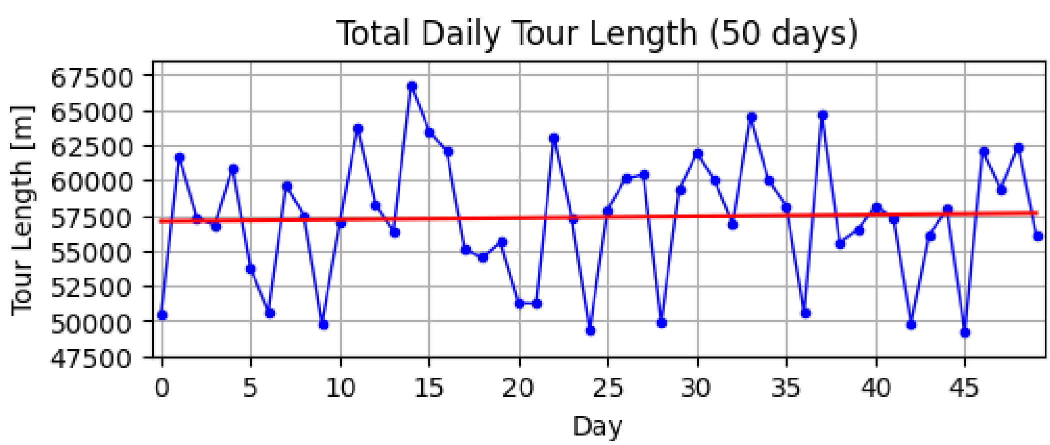




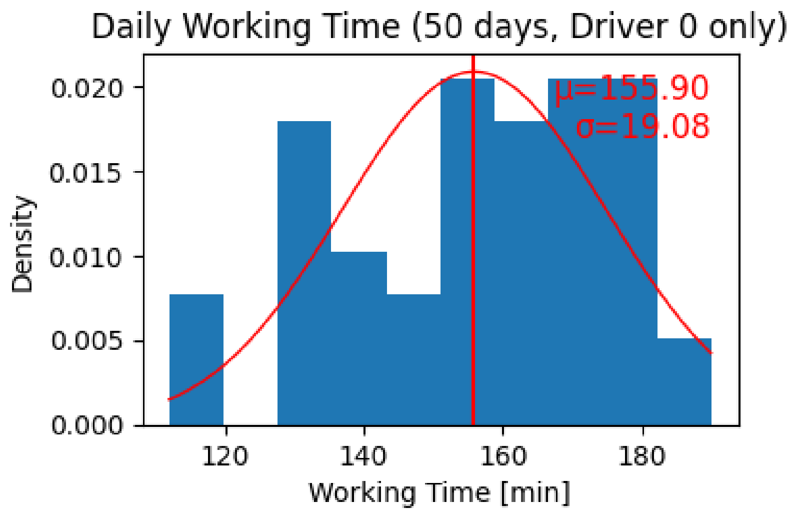
[72]:
```
for i in range(rec5.drivers):
    rec5.plotTourLength(i)
rec5.plotTourLength()
```

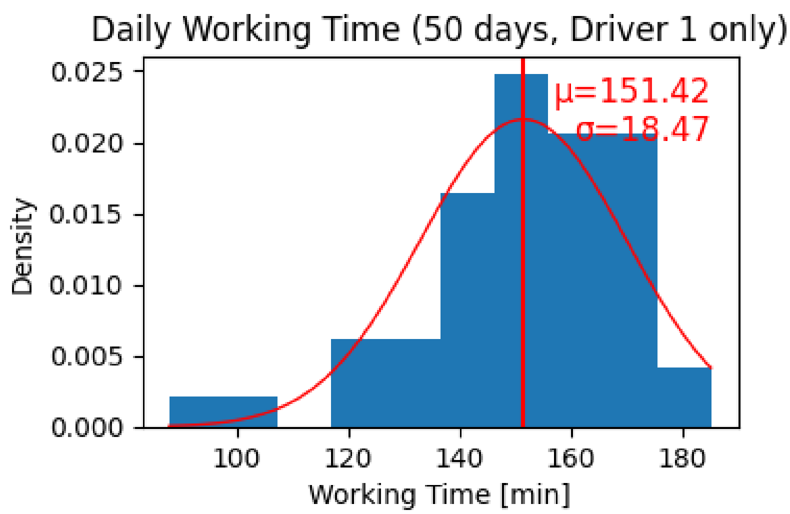

Daily Tour Length (50 days, Driver 0 only)
Tour Length [m]
Day

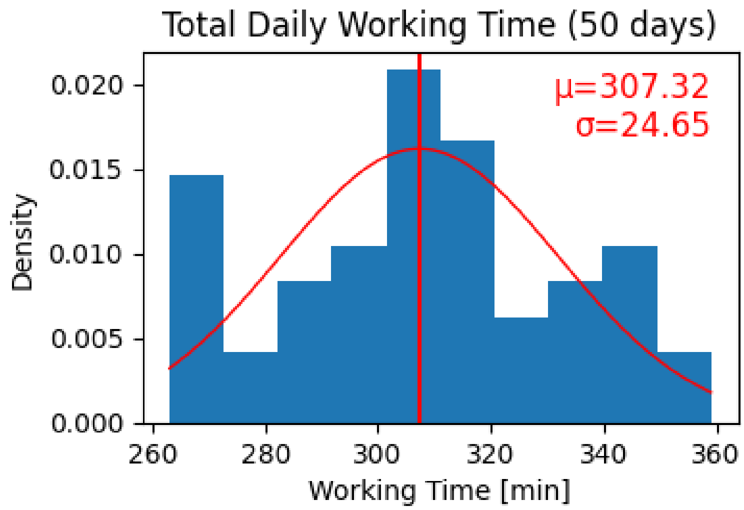

Daily Tour Length (50 days, Driver 1 only)
Tour Length [m]
Day


Total Daily Tour Length (50 days)
Tour Length [m]
Day

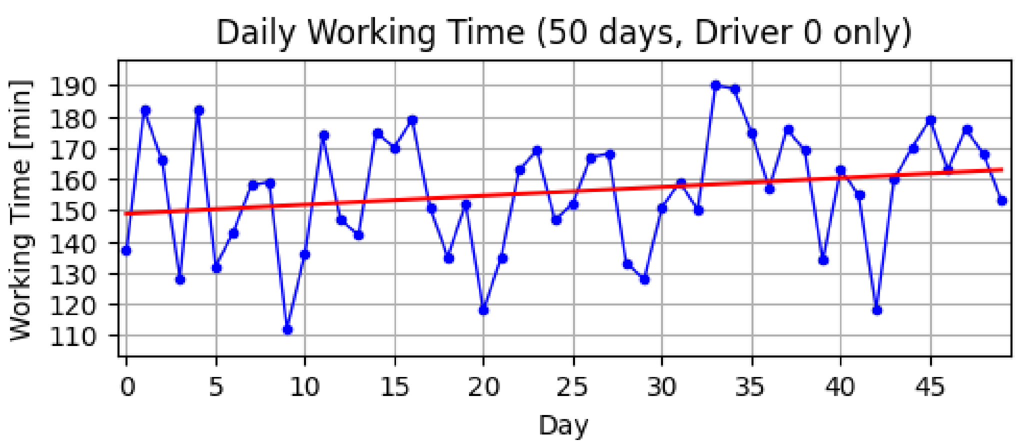
[73]:
```
for i in range(rec5.drivers):
    rec5.histWorkingTime(i)
rec5.histWorkingTime()
```

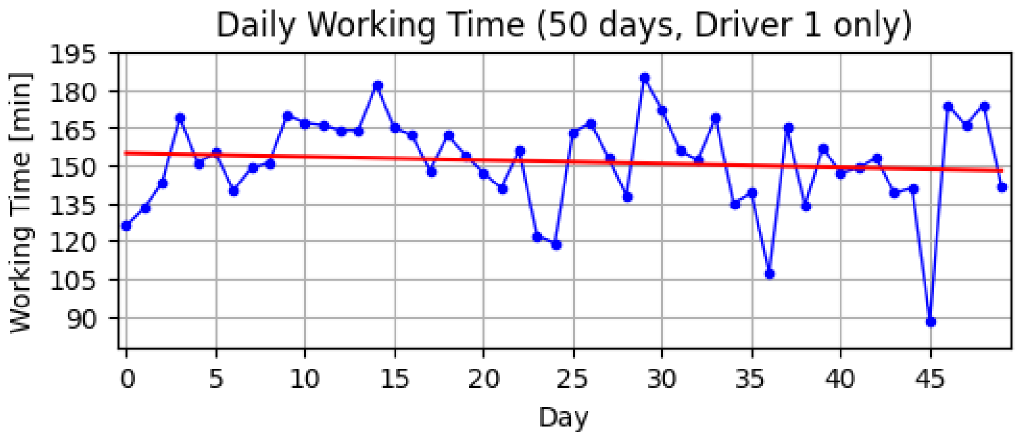


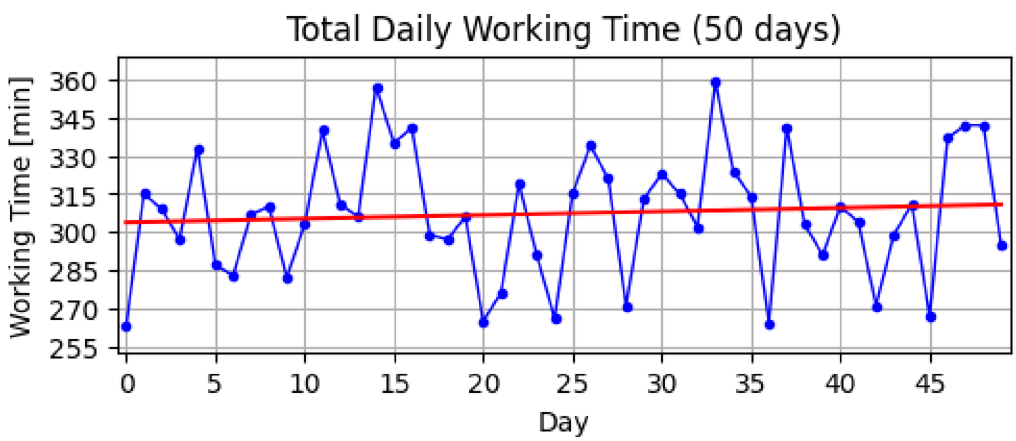




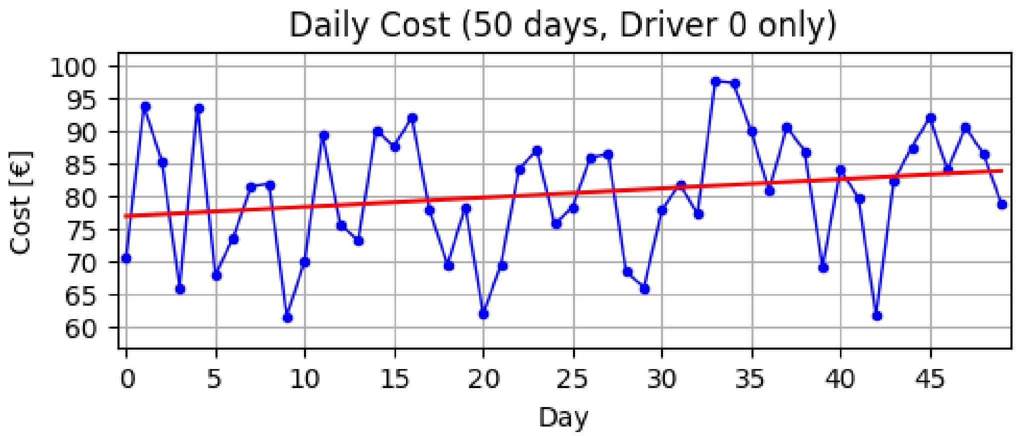
[74]:
```
for i in range(rec5.drivers):
    rec5.plotWorkingTime(i)
rec5.plotWorkingTime()
```

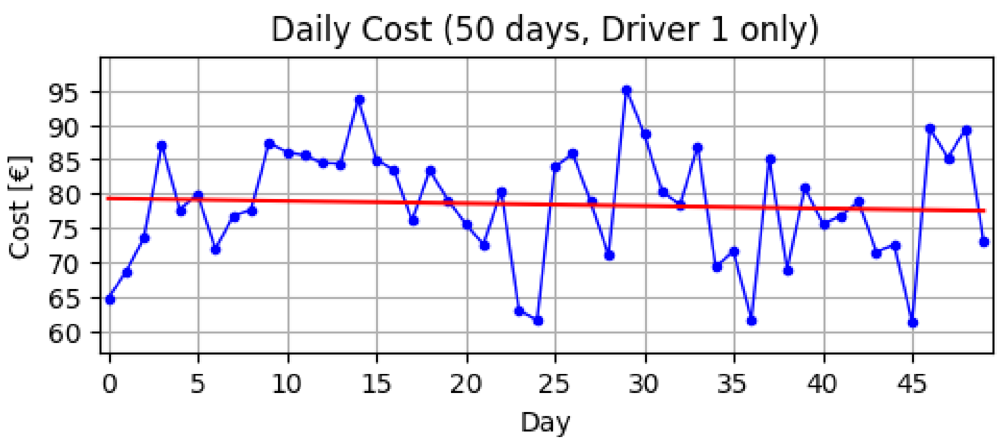


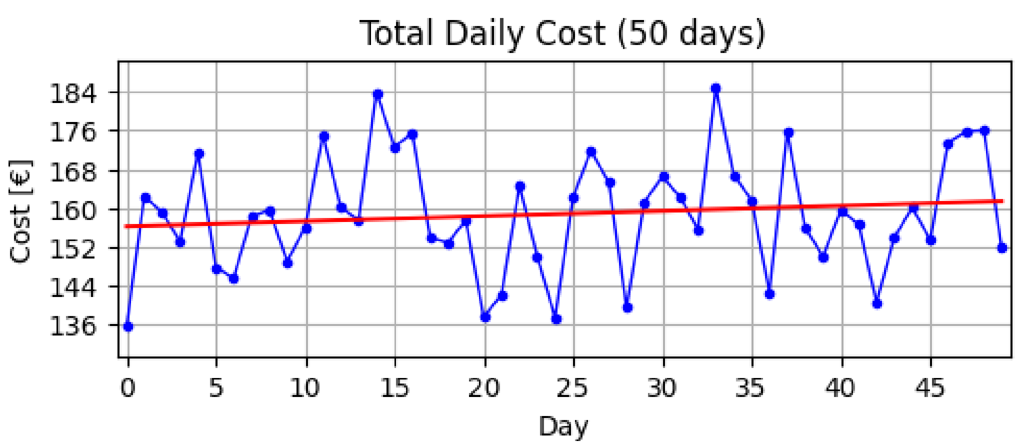




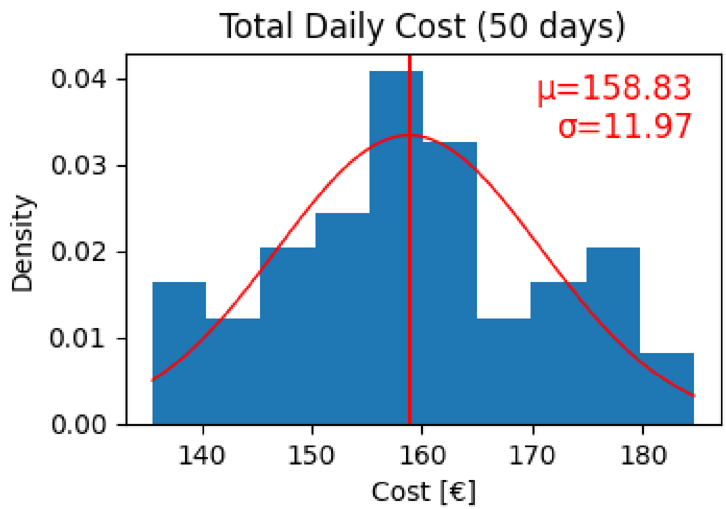
[75]:
```
for i in range(rec5.drivers):
    rec5.plotDailyCost(i)
rec5.plotDailyCost()
```


Daily Cost (50 days, Driver 0 only)
Cost [€]
Day

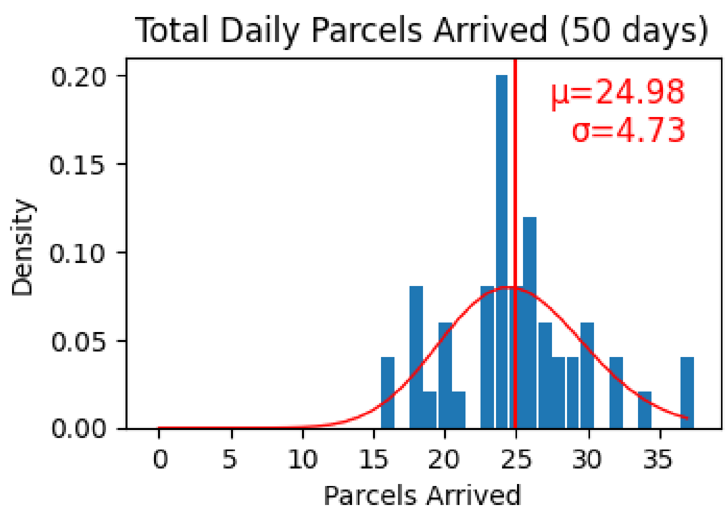

Daily Cost (50 days, Driver 1 only)
Cost [€]
Day


Total Daily Cost (50 days)
Cost [€]
Day

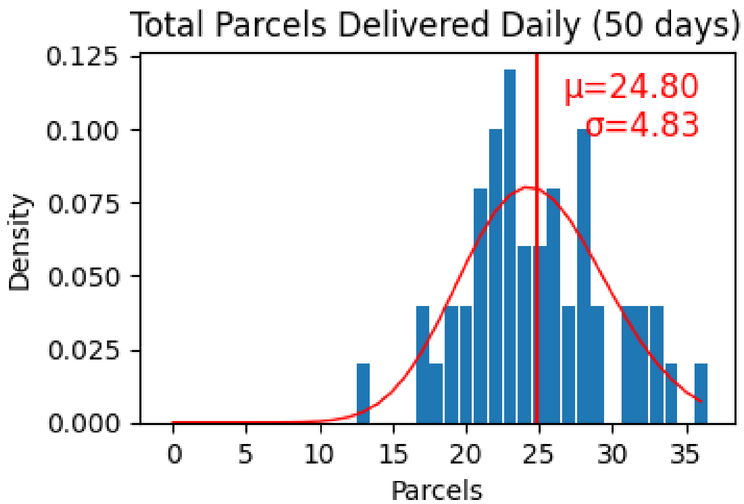
[76]:
```
rec5.histDailyCost()
```



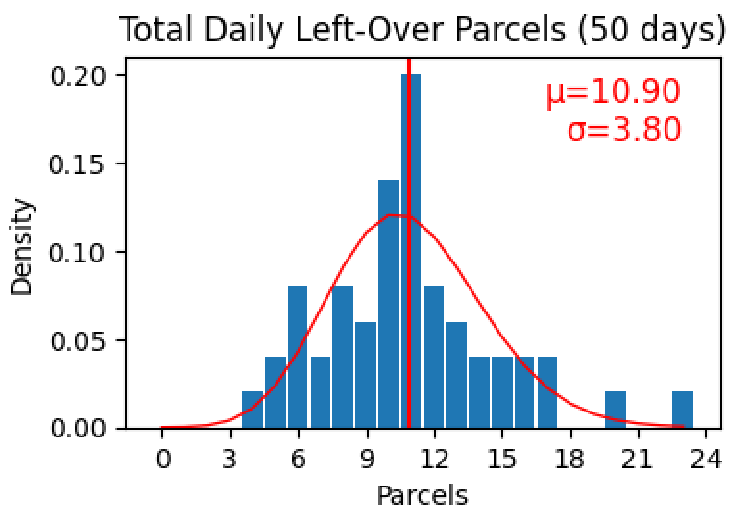
[77]:
```
rec5.histParcelsArrived()
```



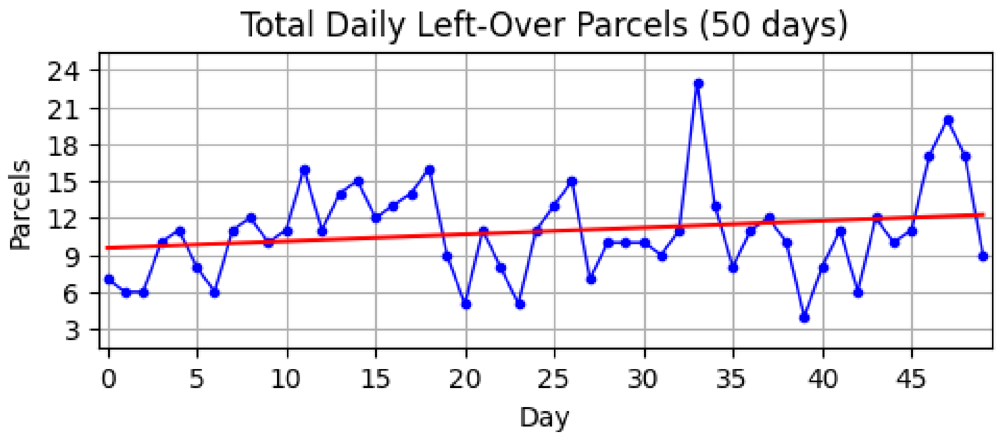
[78]:
```
rec5.histParcelsDelivered()
```



[79]:
```
rec5.histParcelsLeftOver()
```

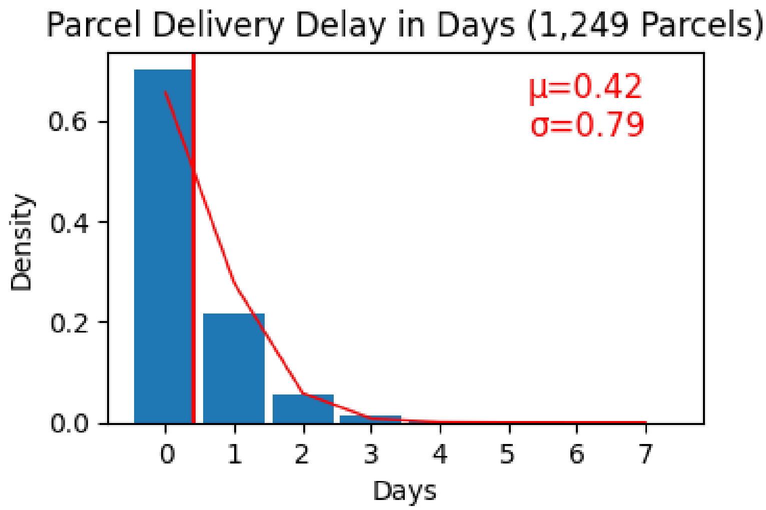


[80]:
```
rec5.plotParcelsLeftOver()
```



[81]:
```
rec5.countPlot()
```

[82]:
```
rec5.histParcelDeliveryDelay()
```


Parcel Delivery Delay in Days (1,249 Parcels)
μ=0.42
σ=0.79
Density
0.0
0.2
0.4
0.6
0
1
2
3
4
5
6
7
Days

[ ]: