Q2. 1D convolution mask ⇒ $(w_0, w_1, w_2, w_3, w_4, w_5, w_6, w_7)$
let the 1D image be $f = (f_1, f_2 \ldots\ldots f_n)$

since convolution involves flipping the mask by 180°
the reversed mask $m' = (w_6, w_5, w_4 \ldots\ldots w_0)$

$$F = f * w = \sum_{j=0}^{6} w_j f_{i-j} = \sum_{j=0}^{6} f_j w_{i-j}$$

$F = W \times f$ where $W = \begin{bmatrix} w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & \cdots & 0 \\ 0 & w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & 0 \cdots & 0 \\ \vdots & & & & & & & & \\ 0 & \cdots & 0 & & & & w_6 w_5 & \cdots & w_0 \end{bmatrix}$

$(n \times 6) \times n$ matrix

$$\therefore F = \underbrace{\begin{bmatrix} w_6 & w_5 & w_4 & w_3 & \cdots & w_0 & \cdots & 0 \\ 0 & w_6 & w_5 & \cdots & & w_0 & \cdots & 0 \\ 0 & 0 & w_6 & \cdots & & & & \\ \vdots & & & & & & w_6 \; w_5 \cdots & w_0 \end{bmatrix}}_{W_{n \times 6 - n}} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix} \quad F_{n \times 1}$$

→ Thus we can convert the mask to the matrix as above to apply convolution to a 1D image as above.

Properties of the matrix structure
→ matrix has a Toeplitz matrix where each descending diagonal from left to right is a constant. It arise due to shift invariance property of convolution ensuring each image element is combined with same set of mask coefficients but shifted in position.

→ Shift invariance - matrix is inherently shift invariant meaning that a shift in the input signal results in corresponding shift in output.

Applications

→ The convolution matrix enhances images by systematically applying a filter across every pixel. It modifies each pixels value based on its neighbour allowing for effects like sharpening, blurring and edge detection.