(5) Defn of DFT

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \cdot e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$F^*(u,v) = \left(\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \cdot e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)} \right)^*$$

$$= \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f^*(x,y) \cdot e^{j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

if $f(x,y)$ is Real then $f^*(x,y) = f(x,y)$

$$= \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \cdot e^{-j2\pi\left(-\frac{ux}{M} + \frac{-vx}{N}\right)}$$

$$\boxed{F^*(u,v) = F(-u,-v)}$$ Hence proved

As $F(-u,-v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(-\frac{ux}{M} - \frac{vy}{N}\right)}$

$$= F^*(u,v)$$

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given: $f(x,y)$ is Real & even

$$f(x,y) = f(-x,-y)$$

let $x' = -x$ & $y' = -y$

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \cdot e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$$

$$F(u,v) = \sum_{x'=0}^{M-1} \sum_{y'=0}^{N-1} f(-x',-y') \cdot e^{-j2\pi\left(-\frac{ux'}{M} - \frac{vy'}{N}\right)}$$

as $\underbrace{f(-x',-y') = f(x',y')}$ (given)

$$F(u,v) = \sum_{x'=0}^{M-1} \sum_{y'=0}^{N-1} f(x',y') \cdot e^{j2\pi\left(\frac{ux'}{M} + \frac{vy'}{N}\right)}$$

$$F(u,v) = F^*(u,v)$$

as previously proved $F^*(u,v) = F(-u,-v)$

sol $\boxed{F(u,v) = F(-u,-v)}$ Hence proved