(7) $$\frac{\partial I}{\partial t} = c\left(\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}\right)$$ → Isotropic heat eqⁿ (given)

$I(x, y, t)$

$$g(x, y) = \frac{1}{2\pi\sigma^2} e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)}$$

gaussian kernel

Initial Intensity distribution

Initial State $I(x, y, 0) = I_0(x, y)$

$$F\left(I(x,y,t)\right) = \hat{I}(u,v,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x,y,t) \cdot e^{j2\pi(ux+vy)}\, dx\, dy$$

$$\frac{\partial \hat{I}(u,v,t)}{\partial t} = c\left(-(2\pi u)^2 - (2\pi v)^2\right) \hat{I}(u,v,t)$$

$$\underbrace{\frac{\partial \hat{I}(u,v,t)}{\partial t}} = \underbrace{-c\left(4\pi^2 (u^2+v^2)\right)}_{\lambda} \hat{I}(u,v,t)$$

$$\boxed{\frac{\partial \hat{I}}{\partial t} = -\lambda \hat{I}}$$ → 1^{st} order linear ODE

$d = c4\pi^2(u^2+v^2)$

solution of this ODE is

$$\hat{I}(u,v,t) = \hat{I}(u,v,0) \cdot e^{-c4\pi^2(u^2+v^2)\cdot t}$$

this is what we get when a run PDE on image.

Now. ~~apply~~ convolve gaussian ~~on~~ with image

perform invers fourier transfor

$$I(x,y,t) = F^{-1}\left(\hat{I}(u,v,0) \cdot e^{-c4\pi^2(u^2+v^2)t}\right)$$

$$I(x,y,t) = F^{-1}\left(\hat{I}(u,v,0)\right) * F^{-1}\left(e^{-c4\pi^2(u^2+v^2)t}\right)$$

this can be interpreted as Image convolved with gaussian as fourier transform of gaussian is gaussian. which means fourier inverse of gaussian is also gaussian

$$g(x,y) = \frac{1}{2\pi\sigma^2} \cdot e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

$$G(u,v) = e^{-2\pi^2\sigma^2(u^2+v^2)}$$

we got is $e^{-c4\pi^2(u^2+v^2)}$

On comparing we got $c\cancel{4}^{2}\cancel{\pi^2} = \cancel{2}\cancel{\pi^2}\sigma^2$

$$\boxed{\sigma = \sqrt{2c}}$$