(5) 1D Ramp Image $I(x) = cx + d$

(1) Filtering. 0-mean gaussian filter [std - σ]

$$G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

$$J(x) = I(x) * G(x)$$

$$J(x) = \int_{-\infty}^{\infty} I(t) \cdot G(x-t)\, dt$$

$$J(x) = \int_{-\infty}^{\infty} \frac{(ct+d)}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-t)^2}{2\sigma^2}}\, dt$$

$$J(x) = \int_{-\infty}^{\infty} \frac{ct}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-t)^2}{2\sigma^2}}\, dt + \underbrace{\int_{-\infty}^{\infty} \frac{d}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-t)^2}{2\sigma^2}}\, dt}_{\text{same as } d}$$

$\therefore$ $\underbrace{J(x) = cx + d}_{\text{same as original image}}$; $\therefore$ Using property of gaussian distrn

$$\left[\int_{-\infty}^{\infty} t \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-t)^2}{2\sigma^2}}\, dt = x \right]$$

③ (2) Filtering By bilateral filter

definition of this filter

$$B(x) = \frac{1}{W(x)} \cdot \int_{-\infty}^{\infty} I(t) \cdot e^{-\frac{(x-t)^2}{2\sigma_s^2}} \cdot e^{-\frac{(I(x)-I(t))^2}{2\sigma_r^2}} \, dt$$

parameter given: $\sigma_s \to$ spatial std

$\sigma_r \to$ range std

$$W(x) = \int_{-\infty}^{\infty} e^{-\frac{(x-t)^2}{2\sigma_s^2}} \cdot e^{-\frac{(I(x)-I(t))^2}{2\sigma_r^2}} \, dt$$

Normalization factor

→ Put $I(x) = cx + d$ in it

$$B(x) = \frac{1}{W(x)} \int_{-\infty}^{\infty} (ct+d) \, e^{-\frac{(x-t)^2}{2\sigma_s^2}} \cdot e^{-\frac{c^2(x-t)^2}{2\sigma_r^2}} \, dt$$

exponents: $\frac{(x-t)^2 (\sigma_r^2 + c^2\sigma_s^2)}{2\sigma_s^2\sigma_r^2} = \frac{2}{\alpha}$

$$B(x) = \frac{1}{W(x)} \underbrace{\int_{-\infty}^{\infty} (ct+d) \, e^{-\frac{(x-t)^2\alpha}{2}}}_{\text{same as gaussian}} \cdot \sqrt{\frac{2\pi}{\alpha}} \cdot dt$$

let $\left[\alpha = \frac{1}{\sigma_s^2} + \frac{1}{\sigma_r^2}\right]$ $\Rightarrow W(x) = \sqrt{\frac{2\pi}{\alpha}}$

$$B(x) = \frac{\cancel{\sqrt{\frac{2\pi}{\alpha}}}\,(cx+d)}{\cancel{W(x)}}$$

$$\boxed{B(x) = cx + d}$$ → same as original Image