(4) Dimension of Image 201×201

$x \rightarrow M = 201$ (dimension in x direction)

$y \rightarrow N = 201$ (dimension in y direction)

$$f(x,y) = \begin{cases} 255 & \text{if } x = 101 \\ 0 & \text{otherwise} \end{cases}$$

x is column index
y is row index

→ $f(x,y)$ is only 255 if $x = 101$, otherwise 0.

→ $f(x,y)$ can be realized using delta function

So. $f(x,y) = 255\,\delta(x-101$

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\cdot e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}\,dx\,dy$$

(continuous)

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)\cdot e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

(discrete)

for $x \neq 101 \rightarrow f(x,y) = 0$

$$\text{So } F(u,v) = \sum_{x=0}^{201}\sum_{y=0}^{201} f(x,y)\cdot e^{-j2\pi\left(\frac{ux}{201}+\frac{vy}{201}\right)}$$

$$F(u,v) = \sum_{y=0}^{201} 255\cdot e^{-j2\pi\left(\frac{101u}{201}+\frac{vy}{201}\right)}$$

$$F(u,v) = 255\cdot e^{-j2\pi\left(\frac{101u}{201}\right)}\cdot\sum_{y=0}^{201} e^{-j2\pi\frac{vy}{201}}$$

$$g(v) = \sum_{y=0}^{201} e^{-j2\pi\frac{vy}{201}} = \begin{cases} 201 & \text{for } v=0 \\ 0 & \text{otherwise} \end{cases}$$

$$F(u,v) = \begin{cases} 255\cdot 201\cdot e^{-j2\pi\frac{101u}{201}} & \text{for } v=0 \\ 0 & \text{otherwise} \end{cases}$$