(2) Correlation of 2D functions f and g is defined as

$$C(x,y) = (f \otimes g)(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x+t, y+s) \cdot g(t,s)\, dt\, ds$$

Continuous functions

correlation operator

→ Continuous Fourier transform of $f(x,y)$ is

$$F(f(x,y)) = F(U,V) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) \cdot e^{-j2\pi(ux+vy)} \cdot dx\, dy$$

using shift property of Fourier transform

$$F(f(x+t, y+s)) = F(U,V) \cdot e^{j2\pi(ut+vs)}$$

→ Fourier transform of $C(x,y) = (f \otimes g)(x,y)$ is

$$F(C(x,y)) = C(U,V) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(x,y) \cdot e^{-j2\pi(ux+vy)} dx\,dy$$

$$C(U,V) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x+t, y+s) \cdot g(t,s)\, dt\, ds\right] e^{-j2\pi(ux+vy)} \cdot dx\, dy$$

inter change limits

$$C(U,V) = \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x+t, y+s) \cdot e^{-j2\pi(ux+vy)} dx\,dy}_{F(U,V) \cdot e^{j2\pi(ut+vs)}}\right] \cdot g(t,s)\, dt\, ds\right]$$

$$C(U,V) = \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(U,V) \cdot e^{j2\pi(ut+vs)} \cdot g(t,s)\, dt\, ds\right]$$

$$C(u,v) = F(u,v) \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t,s) \cdot e^{j2\pi(ut+vs)} dt\,ds$$

$$C(u,v) = F(u,v) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(t,s) \cdot e^{-j2\pi(-ut-vs)} dt\,ds$$

$$\boxed{C(u,v) = F(u,v) \cdot G(-u,-v)}$$ final result

if f and g are real functions then

$$G(-u,-v) = G^*(u,v)$$

$$\Rightarrow \boxed{C(u,v) = F(u,v) \cdot G^*(u,v)}$$

Now for discrete

$$c[m,n] = (f \otimes g)[m,n] = \sum_{m'}\sum_{n'} f[m'+m, n'+n]\, g[m',n']$$

Correlation defn in discrete time

$$F\left(f[m,n]\right) = F[k,l] = \sum_{m}\sum_{n} f[m,n] \cdot e^{-j2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)}$$

2D DFT of correlation

$$F\left(c[m,n]\right) = C[k,l] = \sum_{m}\sum_{n} c[m,n] \cdot e^{-j2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)}$$

$$C[k,l] = \sum_{m}\sum_{n}\left(\sum_{m'}\sum_{n'} f[m'+m, n'+n] \cdot g[m',n']\right) \cdot e^{-j2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)}$$

Change order of summation

$$C[k,l] = \sum_{m'}\sum_{n'}\left(\sum_{m}\sum_{n} f[m'+m, n'+n] \cdot e^{-j2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)}\right) \cdot g[m',n']$$

As we know By shifting property of
2D Fourier transform

$$F\left(f[m'+m, n'+n]\right) = F[k,l] \cdot e^{j2\pi\left(k\frac{m'}{M} + l\frac{n'}{N}\right)}$$

use in above eqn.

$$C[k,l] = \sum_{m'}\sum_{n'} \underbrace{F[k,l]}_{\text{const.}} \cdot e^{j2\pi\left(k\frac{m'}{M} + l\frac{n'}{N}\right)} g[m',n']$$

$$C[k,l] = F[k,l] \cdot \sum_{m'}\sum_{n'} g[m',n'] \cdot e^{-j2\pi\left(-k\frac{m'}{M} - l\frac{n'}{N}\right)}$$

$$\boxed{C[k,l] = F[k,l] \cdot G[-k,-l]}$$ final Result

If g is real then

$$\boxed{C[k,l] = F[k,l] \cdot G^*[k,l]}$$