intensity values of

③ Laplacian of an Image

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$= f(x+1, y) + f(x-1, y) + f(x, y-1) + f(x, y+1) - 4f(x,y)$$

Part(a) Laplacian mask with −8 in center

To check if Mask is seperable, we need to determine whether it can be expressed as outer product of two 1D filters.

Let f_1, f_2 be two 1D filters s.t.

$$f_1^T \cdot f_2 = \begin{bmatrix} \phi & 1 & \phi \\ 1 & -8 & 1 \\ \phi & 1 & \phi \end{bmatrix} \quad \text{— ①}$$

⇒ There is no possibility to find f_1 and f_2 s.t. eqⁿ ① is true; we can't find; ⇒ $\boxed{\text{Disproved Hence}}$

③ Part ⓑ Laplacian mask with −4 in center

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

to check weether this filter can be implemented using 1D convolutions ; we need to check its seperability :

Consider f_1, f_2 be two 1D filters ;

$$\text{let } f_1^T \times f_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad -①$$

→ there is no real valued filters possible f_1 and f_2 s.t. eqⁿ ① is true.

⇒ this mask cannot be implemented as 1D convolution.

→ $\boxed{\text{Hence Disproved}}$.

(5) 1D Ramp Image $I(x) = cx + d$

(1) Filtering 0-mean gaussian filter [std - σ]

$$G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

$$J(x) = I(x) * G(x)$$

$$J(x) = \int_{-\infty}^{\infty} I(t) \cdot G(x-t)\, dt$$

$$J(x) = \int_{-\infty}^{\infty} \frac{(ct+d)}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-t)^2}{2\sigma^2}}\, dt$$

$$J(x) = \int_{-\infty}^{\infty} \frac{ct}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-t)^2}{2\sigma^2}}\, dt + \underbrace{\int_{-\infty}^{\infty} \frac{d}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-t)^2}{2\sigma^2}}\, dt}_{\text{same as } d}$$

$\therefore$ $\underbrace{J(x) = cx + d}_{\text{same as original image}}$; $\because$ Using property of gaussian distr^n

$$\left[\int_{-\infty}^{\infty} t \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-t)^2}{2\sigma^2}}\, dt = x\right]$$

③ (2) Filtering By bilateral filter

Definition of this filter

$$B(x) = \frac{1}{W(x)} \cdot \int_{-\infty}^{\infty} I(t) \cdot e^{-\frac{(x-t)^2}{2\sigma_s^2}} \cdot e^{-\frac{(I(x)-I(t))^2}{2\sigma_r^2}} \, dt$$

Parameter given: $\sigma_s \rightarrow$ spatial std

$\sigma_r \rightarrow$ range std

$$W(x) = \int_{-\infty}^{\infty} e^{-\frac{(x-t)^2}{2\sigma_s^2}} \cdot e^{-\frac{(I(x)-I(t))^2}{2\sigma_r^2}} \, dt$$

Normalization factor

→ put $I(x) = cx+d$ in it

$$B(x) = \frac{1}{W(x)} \int_{-\infty}^{\infty} (ct+d)\, e^{-\frac{(x-t)^2}{2\sigma_s^2}} \cdot e^{-\frac{c^2(x-t)^2}{2\sigma_r^2}} \, dt$$

exponents: $\frac{(x-t)^2(\sigma_r^2 + c^2\sigma_s^2)}{2\sigma_s^2\sigma_r^2} = \frac{2}{\alpha}$

$$B(x) = \frac{1}{W(x)} \underbrace{\int_{-\infty}^{\infty} (ct+d)\, e^{-\frac{(x-t)^2\alpha}{2}} \cdot \sqrt{\frac{2\pi}{\alpha}} \cdot dt}_{\text{same as gaussian}}$$

let $\left[\alpha = \frac{1}{\sigma_s^2} + \frac{1}{\sigma_r^2}\right]$ $\Rightarrow W(x) = \sqrt{\frac{2\pi}{\alpha}}$

$$B(x) = \frac{\cancel{\sqrt{\frac{2\pi}{\alpha}}}\,(cx+d)}{\cancel{W(x)}}$$

$$\boxed{B(x) = cx+d}$$ → same as original Image