Q4. $\nabla^2 I(x,y) \rightarrow$ second spatial derivative of image

$$\nabla^2 I(x,y) = f(x+1) + f(x-1) - 2f(x)$$

$\Rightarrow$ when Laplacian is positive it indicates that the pixel at x is surrounded by pixels of lower intensity which occurs when bright objects on darker backgrounds.

lets consider a simple image $I = [10, 10, 10, 100, 10, 10, 10]$

$\nabla^2 I(0) = 90 - 10\times2 + 10 = 0$

$\nabla^2 I(1) = 10 - 2(10) + 10 = 0$

$\nabla^2 I(2) = 10 - 2(10) + 100 = 90$

$\nabla^2 I(3) = 10 - 2(100) + 10 = -180$

$\nabla^2 I(4) = 10 - 2(10) + 100 = 90$

$\nabla^2 I(5) = 0]$

a. $\nabla^2 I = [0, 80, -180, 0, 0, 0, 0]$

$$I + \alpha \nabla^2 I = \begin{bmatrix} 10 \\ 10 \\ 10 \\ 100 \\ 10 \\ 10 \\ 10 \end{bmatrix} + \alpha \begin{bmatrix} 0 \\ 0 \\ 90 \\ -180 \\ 90 \\ 0 \\ 0 \end{bmatrix}$$

α is small

Let $\alpha = 0.1$

$$I' = \begin{bmatrix} 10 \\ 10 \\ 10+9 = 19 \\ 82 \\ 19 \\ 10 \\ 10 \end{bmatrix}$$

we see that the contrast along the edges decreases and the value at edge in I viz $100 - 10 = 90$ while in I_2 is $(82 - 19) = 63$ and thus contrast in I' decreases as compared to I leading to blurring

b. Applying the operation of I'

$$I'' \rightarrow I' + \alpha \nabla^2 I' \qquad I' = \begin{bmatrix} 10 \\ 10 \\ 19 \\ 82 \\ 19 \\ 10 \\ 10 \end{bmatrix}$$

$\nabla^2 I(0) = 0$
$\nabla^2 I(1) = 9$
$\nabla^2 I(2) = 54$
$\nabla^2 I(3) = -44$
$\nabla^2 I(4) = 54$
$\nabla^2 I(5) = 9$
$\nabla^2 I(6) = 0$

$$I'' \Rightarrow \begin{bmatrix} 10 \\ 10 \\ 19 \\ 82 \\ 19 \\ 10 \\ 10 \end{bmatrix} + 0.1 \begin{bmatrix} 0 \\ 9 \\ 54 \\ -44 \\ 54 \\ 9 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 10.9 \\ 24.4 \\ 77.6 \\ 24.4 \\ 10.9 \\ 10 \end{bmatrix}$$

→ we see when we apply the operation the values between the pixel at the edges decreases leading to blurring between the edge values.
we may thus speculate that if we apply the same operation for a large number of times it leads to blurring between the edges due to decrease in contrast.

c. If we apply $I(x,y) \leftarrow I(x,y) - \alpha \nabla^2 I(x,y)$ a large no. of times

$$I'(x,y) = I(x,y) - \alpha \nabla^2 I(x,y)$$

$$= \begin{bmatrix} 10 \\ 10 \\ 10 \\ 100 \\ 10 \\ 10 \\ 10 \end{bmatrix} - 0.1 \begin{bmatrix} 0 \\ 0 \\ 90 \\ -180 \\ 90 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 1 \\ 118 \\ 1 \\ 10 \\ 10 \end{bmatrix}$$

We see here that the contrast increases along the edges and value at the edge in I' is $(118-1) = 117$ and in I is 90.
→ Thus similarly if we apply the same operation for a large no. of iteration it causes the contrast at the edge to increase thus leading to image sharpening