(1) clean Image denoted by $I(x,y)$
Additive Noise denoted by $N(x,y)$
where noise is 0-mean gaussian distribution
with std $=6$.

$$I_{noisy}(x,y) = I(x,y) + N(x,y)$$

$$\text{PDF of noise} = P_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{n^2}{2\sigma^2}}$$

→ we need to find PDF of $I_{noisy}(i)$; which is sum of two random variables $P_N(n)$ [Noise Random Var], $P_I(i)$ [clean Image Random Var]

→ for any Intensity value; PDF of noisy image is convolution of PDF of clean image and PDF of noise.

→ Let PDF of clean image intensity be $P_I(i)$; noise image be $P_{I_{noise}}(i_{noisy})$

(a) $$\boxed{P_{I_{noisy}}(i_{noisy}) = \int_{-\infty}^{\infty} P_I(i)\, P_N(i_{noisy} - i)\, di}$$; $P_{I_{noisy}}(x) = P_I(x) * P_N(x)$

$*$ → convolution

(b) In Image Processing, Gaussian smoothing involves convolution an image with a Gaussian kernel to reduce noise or details.

→ In context of PDF of noisy image ⇒ Convolution with $P_N(n)$ effectively "spread out" or "smooths" the intensity values;

⇒ Thus; adding gaussian noise to an image has same effect on its intensity distribution as performing Gaussian blur.

① (c) uniform Noise distribution

PDF of $N(x,y)$ is $P_N(n)$

$$P_N(n) = \begin{cases} \frac{1}{2r} & \text{if } -r \leq n \leq r \\ 0 & \text{otherwise} \end{cases}$$

$$I_{noisy} = I(x,y) + N(x,y)$$

$\Rightarrow$ PDF of noisy image; $P_{I_{noisy}}(i_{noisy})$

$$P_{I_{noisy}}(i_{noisy}) = P_N(i_{noisy}) * P_I(i_{noisy})$$

$$\left[P_{I_{noisy}}(i_{noisy}) = \int_{-\infty}^{\infty} \underbrace{P_N(i_{noisy}-i)}\cdot P_I(i)\, di \right]$$

$\rightarrow$ $P_N(i_{noisy}-i)$ is non-zero only for

$$i_{noisy} - r \leq i \leq i_{noisy} + r$$

$\Rightarrow$ Convolving with a uniform distribution has effect of Box Blurring or Rectangular smoothing where intensity values spreads uniformly over the range r.