F (F

(6)

$F(f(t)) = F(\nu) = \int_{-\infty}^{\infty} f(t) \cdot e^{-i\nu t}\, dt$ Fourier transfo

$F^{-1}(F(\nu)) = f(t) = \int_{-\infty}^{\infty} F(\nu) \cdot e^{i\nu t}\, d\nu$ Inverse fourier transform

↳ we can also change variable $t = \nu$, $\nu = t$ (No effect of changing var.) as it is symmetric

$F(f(\nu)) = F(t) = \int_{-\infty}^{\infty} f(\nu) \cdot e^{-i\nu t}\, d\nu$

$F^{-1}(F(t)) = f(\nu) = \int_{-\infty}^{\infty} F(t) \cdot e^{i\nu t}\, dt$

let so. $F(F(f(t))) = g(t)$

$F(f(t)) = F(\nu)$

$g(t) = F\left(F(\nu)\right)$

$$g(t) = \int_{-\infty}^{\infty} F(\nu) \cdot e^{-i\nu t} \cdot d\nu$$

$g(t) = f(-t)$

$$F\left(F\left(f(t)\right)\right) = f(-t) \quad \text{—} \ ①$$

we have to find $F(F(\underbrace{F(F(f(t)))}))$

$F\left(F\left(f(-t)\right)\right)$.

from eqⁿ ①

Now $F\left(F\left(f(-t)\right)\right) = f\left(-(-t)\right)$

$= f(t)$

$$\Rightarrow \boxed{F(F(F(F(f(t))))) = f(t)}$$ Hence proved

Practical uses of $F(F(f(t))) = f(-t)$

↳ Applying fourier transform on its frequency spectrum will directly give you the function (flipping the sign of argument)

If f^n is even, then $F(F(t)) = f(t)$
great use

If f^n is odd, then $F(F(t)) = -f(t)$

$F(F(F(F(t)))) = f(t)$
