MODAL ANALYSIS AND DESIGN OPTIMIZATION OF CANTILEVER

A PROJECT

Submitted in partial fulfilment of the requirement for Mechanical Vibration course

MASTER OF TECHNOLOGY

In

MACHINE DESIGN

Submitted by

Satya Prakash Gupta

(Enrolment No. 224103429)

Kantubhukta Hemanth Hari Kishore

(Enrolment No. 224103415)

Gaurav Soni

(Enrolment No. 224103413)

DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY

GUWAHATI 781039, ASSAM, INDIA

NOVEMBER-2022

ACKNOWLEDGEMENT

We would like to expresses my deep sense of indebtedness and gratitude to our respected course instructor **Prof. S.K. Dwivedi,** professor, Department of Mechanical Engineering IIT Guwahati for inspiring guidance, encouragement and invaluable help throughout the course of this work. Our association with him throughout the course, research activity and otherwise has been a great process of learning. Words are inadequate to express the great care and interest taken by him in all aspects of present work, without which this work would not have been presented in its present form. We thank you Sir, for your help, inspiration and blessings.

We express my sincere thanks to **Prof. T.G. Sitaram**, Director and **Prof. K.S.R. Krishna Murthy**, HOD, Department of Mechanical Engineering IIT Guwahati for providing us the necessary facilities in the department.

Submitting this project would have been a Herculean job, without the constant help, encouragement, support and suggestions from our **friends**. Who make our stay enjoyable in this great institute from very first day; we will relish your memories for years to come.

Finally, we want to express our gratitude to our **family members**, for their support for choices in all our life and their love, which has been a constant source of strength for everything we do.

(Satya Prakash Gupta)
224103429

(Kantubhukta Hemanth Hari Kishore)
224103415

(Gaurav Soni)
224103413

ABSTRACT

Cantilever beam like structures found many practical applications in engineering such as turbine blade, wings of aircraft and solid panels etc. Before application, we calculate natural frequencies of these structures to avoid resonance if material properties, boundary conditions and dimensions of structure are given. Severe vibrations are generated due to resonance, caused by interaction of inertial and elastic properties of material.

Here the mass of an object is considered distributed throughout the structure as a series of infinitely small elements. When a structure vibrates, each of these infinite number of elements move relative to each other in a continuous fashion. Hence these systems are called infinite-dimensional systems, continuous systems, or distributed-parameter systems.

For natural frequencies of cantilever beam, Analytical, FEA and experimental techniques (G.C.Makalke et al.2016). In this project, Analytical and FEA modal analysis of cantilever beam is done to investigate first six bending modes of vibration for cantilever beam along thickness direction and results are compared. First three modes are important because most of the energy is absorbed by them. Various cross section have been analysed to compare the results with original results also a crack has been introduced for analysis. An aerodynamic wing is modelled as a cantilever as modal analysis has been done. Best possible efforts have been made to achieve the desire objectives in limited resources.

CONTENTS

CONTENT	Page No
Acknowledgement	(2)
Abstract	(3)
Contents	(4)
List of Tables	(6)
List of Figures	(6)
Chapter 1	
Introduction	(8)
1.1 Cantilever	(8)
1.2 Modal Analysis	(9)
Chapter 2	
Literature Review	(11)
Objectives	(12)
Chapter 3	
Mathematical Modelling	(13)
3.1 Finite Element Analysis	(13)
3.2 Benefits of Finite Element Analysis	(14)
3.2.1 Digital Prototyping	(14)
3.2.2 Visualisation	(14)
3.2.3 Boundary Conditions	(14)

3.2.4 Accuracy (14)

3.3 Applications and Use Cases (14)

3.3.1 Structural Analysis (14)

3.3.2 Modal Analysis (15)

3.3.3 Thermal Analysis (15)

3.3.4 Computational Fluid Dynamics (15)

3.4 Mathematical Analysis (15)

Chapter 4

Design and Analysis (18)

4.1 CAD Model (18)

4.2 Structural Modifications (18)

4.3 Grid generation (19)

4.4 Material and Mesh Properties (20)

4.5 Grid Independency (21)

Chapter 5

Results and Discussions (22)

5.1 Validations (22)

Chapter 6

Conclusions (31)

Future Scope (32)

Resources (33)

LIST OF TABLES

4.1 Material Properties (20)

4.2 Mesh Properties (20)

5.1 Comparison of natural frequency (mild steel (22)

5.2 Comparison of natural frequency (aluminium) (22)

5.3 Natural frequency for various cross section of cantilever at different mode(25)

5.4 Comparison of natural frequency with and without crack at various modes (28)

5.5 Comparison of Aerodynamic wing with rectangular cantilever (29)

LIST OF FIGURES

1.1 A Cantilever Building (8)

1.2 Howrah Bridge in India (8)

3.1 Block Diagram of FEA (14)

3.2 A Cantilever Beam (16)

3.3 The beam under free vibration (16)

4.1 (a) Top view (b) Side view (c) Isometric view (18)

4.2 Crack at mid of cantilever (18)

4.3 Different cross sections of cantilever (19)

4.4 Aerodynamics wings (19)

4.5 Quad Finite Elements (19)

4.6 Grid Independency Test (21)

5.1 Mode Shapes (Aluminium) (23)

5.2 Mode Shapes (Mild Steel) (24)

5.3 Variation of natural frequencies with mode for various cross section (26)

5.4 Variation of natural frequencies with mode for various cross section (26)

5.5 Mode shapes at various cross sections corresponding to largest natural frequency (27)

5.6 Natural frequency at various mode with and without crack (28)

5.7 Mode shapes of cracked cantilever corresponding to largest frequency (28)

5.8 Comparison of natural frequency of aerodynamic with rectangular cantilever (29)

5.9 Mode shapes of aerodynamic wing as a cantilever (30)

CHAPTER 1

INTRODUCTION

1.1 Cantilever

A cantilever is a rigid structural element that extends horizontally and is supported at only one end. Typically it extends from a flat vertical surface such as a wall, to which it must be firmly attached. Like other structural elements, a cantilever can be formed as a beam, plate, truss, or slab.

When subjected to a structural load at its far, unsupported end, the cantilever carries the load to the support where it applies a shear stress and a bending moment.

Fig 1.1 A cantilever building.

Fig 1.2 Howrah Bridge in India

1.2 Modal Analysis

Modal analysis is the process of determining the inherent dynamic characteristics of a system in forms of natural frequencies, damping factors and mode shapes, and using them to formulate a mathematical model for its dynamic behaviour. The formulated mathematical model is referred to as the modal model of the system and the information for the characteristics are known as its modal data.

The dynamics of a structure are physically decomposed by frequency and position. This is clearly evidenced by the analytical solution of partial differential equations of continuous systems such as beams and strings. Modal analysis is based upon the fact that the vibration response of a linear time-invariant dynamic system can be expressed as the linear combination of a set of simple harmonic motions called the natural modes of vibration. This concept is akin to the use of a Fourier combination of sine and cosine waves to represent a complicated waveform. The natural modes of vibration are inherent to a dynamic system and are determined completely by its physical properties (mass, stiffness, damping) and their spatial distributions. Each mode is described in terms of its modal parameters: natural frequency, the modal damping factor and characteristic displacement pattern, namely mode shape. The mode shape may be real or complex. Each corresponds to a natural frequency. The degree of participation of each natural mode in the overall vibration is determined both by properties of the excitation source(s) and by the mode shapes of the system.

Modal analysis embraces both theoretical and experimental techniques. The theoretical modal analysis anchors on a physical model of a dynamic system comprising its mass, stiffness and damping properties. These properties may be given in forms of partial differential equations. An example is the wave equation of a uniform vibratory string established from its mass distribution and elasticity properties. The solution of the equation provides the natural frequencies and mode shapes of the string and its forced vibration responses. However, a more realistic physical model will usually comprise the mass, stiffness and damping properties in terms of their spatial distributions, namely the mass, stiffness and damping matrices. These matrices are incorporated into a set of normal differential equations of motion. The superposition principle of a linear dynamic system enables us to transform these equations into a typical eigenvalue problem. Its solution provides the modal data of the system. Modern finite element almost any linear dynamic structure and hence has greatly enhanced the capacity and scope of theoretical modal analysis. On the other hand, the rapid development over the last two decades of data acquisition and processing capabilities has given rise to major advances in the experimental realm of the analysis, which has become known as modal testing.

Static beam equation

The Euler–Bernoulli equation describes the relationship between the beam's deflection and the applied load as

$$\frac{d^2}{dx^2}\left(EI(x)\frac{d^2w}{dx^2}\right) = q$$

The curve w(x) describes the deflection of the beam in the z direction at some position x, q is a distributed load, in other words a force per unit length, E is the elastic modulus and I is the second moment of area of the beam's cross section. Where it is assumed that the centroid of the cross section occurs at y=z=0. Often, the product EI (known as the flexural rigidity) is a constant, so that

$$EI\frac{d^4w}{dx^4} = q(x)$$

CHAPTER 2

LITERATURE REVIEW

Mr. G. C. Mekalke etal. has suggested to know the natural frequencies of the coupled beam-mass system, in order to obtain a proper design of structural elements. This paper aims at determining the natural frequencies and mode shapes of a cantilever beam of different material and geometries with different methods like, Formulation, Softwares (ANSYS 12.0, DEWESoft) and Actual Experimentation.

Mr. Saju Joseph etal. has suggested a methodology for virtual experimental modal analysis(VEMA), for simulating an Experimental Modal Analysis (EMA) using a combination of two commercially available software tools, ADAMS and ME'scope. This paper used the methodology of Theoretical Route of Vibration, Experimental Route of Vibration, Modal Analysis of Cantilever beam, Modal analysis using Ansys and Modal analysis with MATLAB. Finally compared results of Ansys and Matlab.

Hong Hee Yoo etal. Has obtained , certain modal characteristic requirements such as maximum or minimum slope natural frequency loci are specified and the geometric shapes that satisfy the requirements. . Due to the rotational motion, the modal characteristics of cantilever structures often vary significantly. In this study, Formulation of optimisation problems and numerical results are employed to find the cross-section shape variations of rotating cantilever beams and compared with numerical results.

Yoonsoo Lee etal. has proposed a linear dynamic model for analysis. The accuracy of the proposed dynamic model was validated by comparing its transient analysis results to those obtained with a fully nonlinear multibody dynamic analysis program. The transient responses obtained with the proposed model are in good agreement with those obtained with a nonlinear dynamic analysis program. Hence he concluded that the prescribed model is useful for predicting the overall transient behaviour and natural frequencies of shaft-beam coupled system.

D. K. Agarwalla etal. has studied the diagnosis of the changes that allows the experimenter to identify the cracks in a structural element without aborting the system applications. The effect of an open crack on the modal parameters of the cantilever beam subjected to free vibration is analysed and the results obtained from the numerical method i.e. finite element method (FEM) and the experimental method are compared. He concluded that the results obtained from experiment have a very good agreement with the results obtained from FEM and the structure vibrates with more frequency in the presence of a crack away from the fixed end.

OBJECTIVES

- To find out the natural frequencies of cantilever at different modes.
- To study the effect of shape optimisation of cantilever on natural frequencies at different modes.
- To study the effect of cracks on the modal analysis of rectangular cantilever.
- To modal Aerodynamic wing as a cantilever and its modal analysis.

CHAPTER 3

MATHEMATICAL MODELLING

3.1 FINITE ELEMENT ANALYSIS

Finite element analysis (FEA) is used to predict how a model will react to forces in the real world. This analysis is a part of the product design cycle and examines the effects of forces such as heat, vibration and much more. FEA is carried out to optimize designs and understand their points of failure.

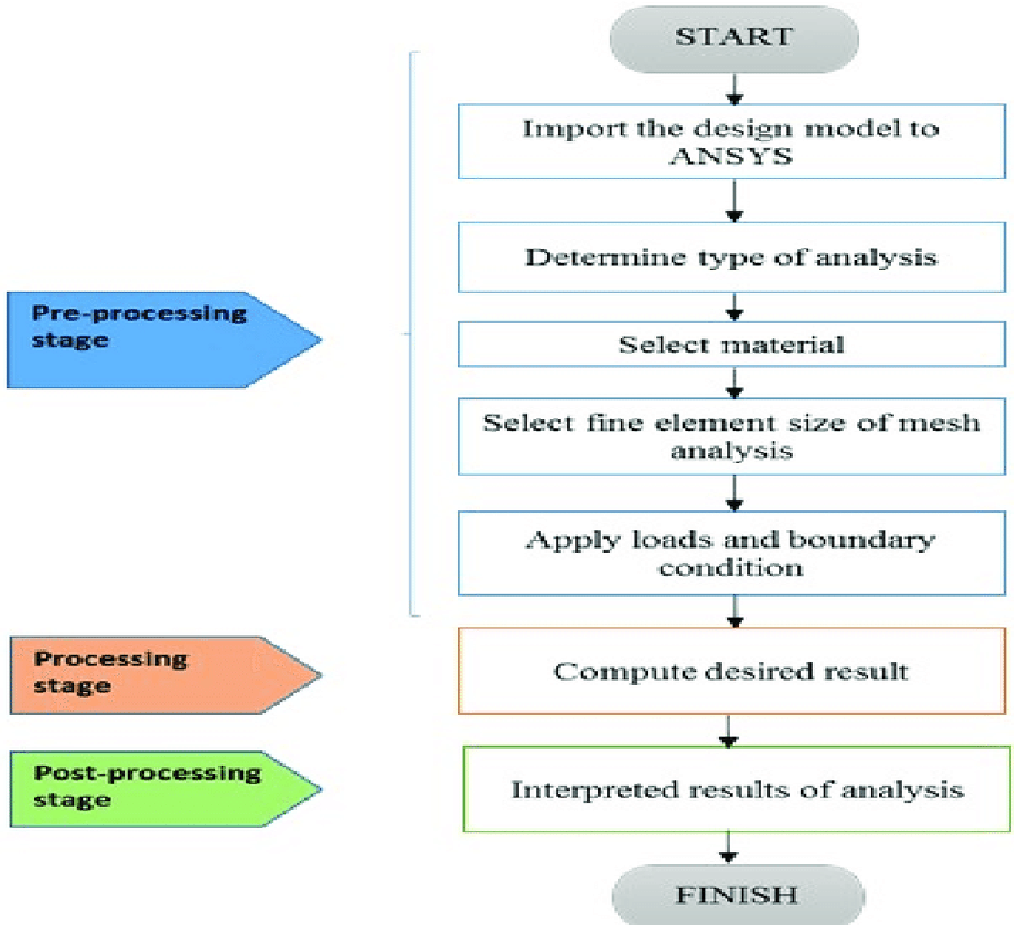


Fig 3.1 Block Diagram of FEA

3.2 Benefits of Finite Element Analysis

3.2.1 Digital prototyping

Creating a physical prototype to see how an object might react under real world stresses gets expensive. FEA allows you to digitally examine the stresses on the object and can let you run analysis on the digital prototype.

3.2.2 Visualization

When the interiors and exteriors of an object are modelled engineers can easily spot points of vulnerability and run studies on the model to visualize stresses and course correct design flaws as they see them.

3.2.3 Boundary Conditions

Boundary conditions such as point forces, distributed forces, thermal effects (such as temperature changes or applied heat energy), and positional constraints can be used as inputs in studies to see how the digital prototype responds to these conditions.

3.2.4 Accuracy

Calculations done by hand using a physical model could never match the accuracy of using software to model stresses; especially if the model is complex.

3.3 Applications and Use Cases

3.3.1. Structural Analysis

Structural analysis is the determination of the effects of static and dynamic loads on parts, assemblies, and mechanisms in order to avoid failure in the intended use.

3.3.2. Modal Analysis

Modal analysis is the determination of the effects of vibrations. This type of analysis uses the mass and rigidity of a structure to find at which points the component naturally resonates.

3.3.3. Thermal Analysis

Thermal analysis is a group of techniques that looks at how the physical properties of materials change with changes in temperature.

3.3.4. Computational Fluid Dynamics

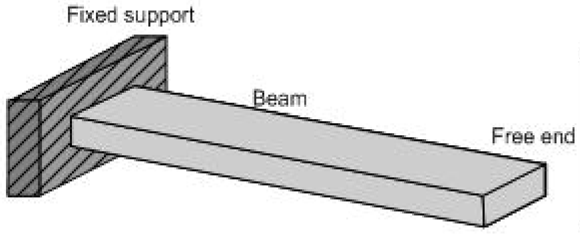
Computational fluid dynamics is a field of fluid mechanics that uses numerical analysis to study and visualize the flow of fluids in real-life.

3.4. Mathematical Analysis

For a cantilever beam subjected to free vibration, and the system is considered as continuous system in which the beam mass is considered as distributed along with the stiffness of the shaft, the equation of motion can be written as

$$\frac{d^2}{dx^2}\left(EI(x)\frac{d^2Y(x)}{dx^2}\right) = \omega^2 m(x)Y(x) \qquad (3.1)$$

Where, E is the modulus of rigidity of beam material, I is the moment of inertia of the beam cross-section, Y(x) is displacement in y direction at distance x from fixed end, ω is the circular natural frequency, m is the mass per unit length, $m = \rho A(x)$, ρ is the material density, x is the distance measured from the fixed end.



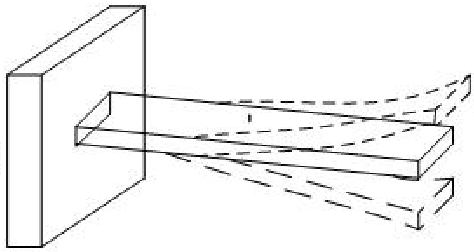


Fig 3.2 A Cantilever Beam **Fig 3.3** The beam under free vibration

Fig. 3.2 shows of a cantilever beam with rectangular cross section, which can be subjected to bending vibration by giving a small initial displacement at the free end; and Fig. 3.3 depicts of cantilever beam under the free vibration.

We have following boundary conditions for a cantilever beam (Fig. 3.2)

$$\text{at } x = 0, Y(x) = 0, \frac{dY(x)}{dx} = 0 \tag{3.2}$$

$$\text{at } x = l, \frac{d^2Y(x)}{dx^2} = 0, \frac{d^3Y(x)}{dx^3} = 0 \tag{3.3}$$

For a uniform beam under free vibration from equation (3.1), we get

$$\frac{d^4Y(x)}{dx^4} - \beta^4 Y(x) = 0 \tag{3.4}$$

with

$$\beta^4 = \frac{\omega^2 m}{EI}$$

The mode shapes for a continuous cantilever beam is given as

$$f_n(x) = A_n\left\{\left(\sin\beta_n L - \sinh\beta_n L\right)\left(\sin\beta_n x - \sinh\beta_n x\right) + \left(\cos\beta_n L - \cosh\beta_n L\right)\left(\cos\beta_n x - \cosh\beta_n x\right)\right\} \tag{3.5}$$

Where ,

$n = 1, 2, 3 \ldots \infty$ and $\beta_n L = n\pi$

A closed form of the circular natural frequency ω_n, from above equation of motion and boundary conditions can be written as,

$$\omega_n = \frac{\lambda_i^2}{2\pi l^2}\sqrt{\frac{EI}{m}} \qquad (3.6)$$

CHAPTER 4

DESIGN AND ANALYSIS

4.1 CAD Model

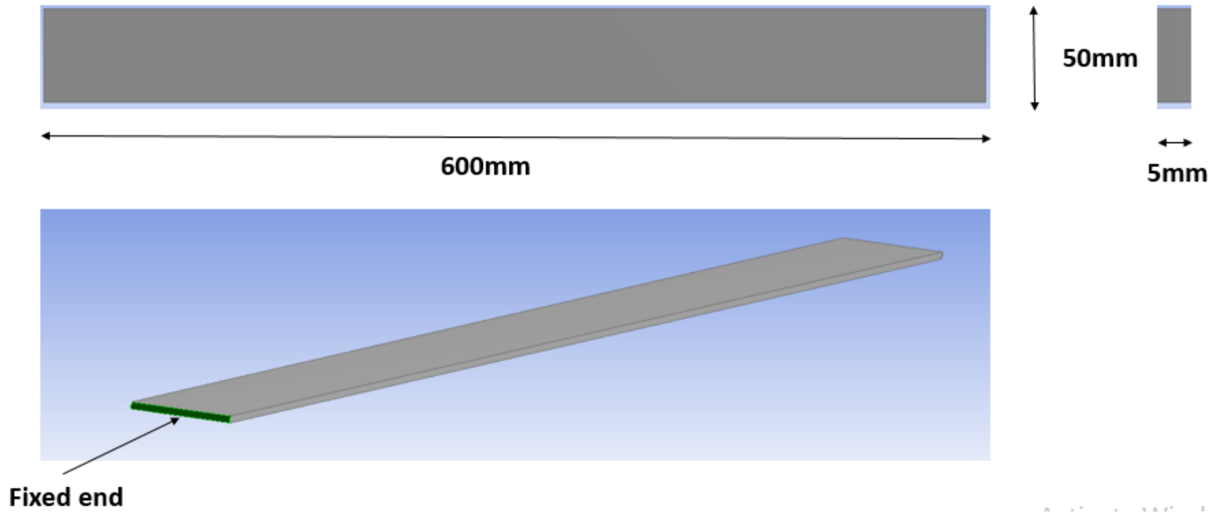


Fig 4.1 (a) Top view (b) side view (c) Isometric view

4.2. Structural Modifications

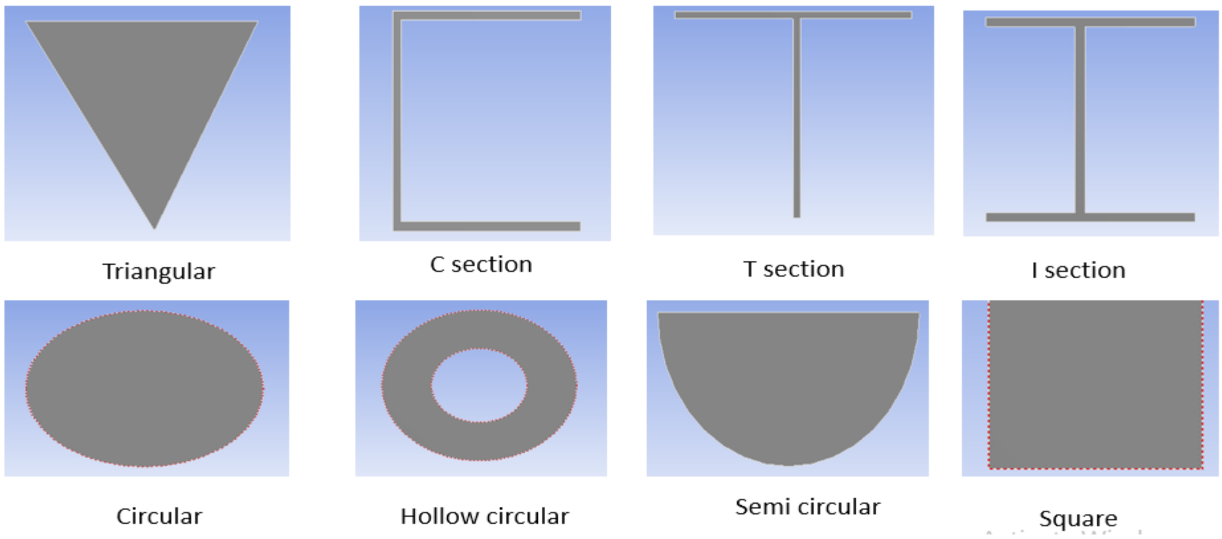


Fig 4.2 Different cross sections of cantilever

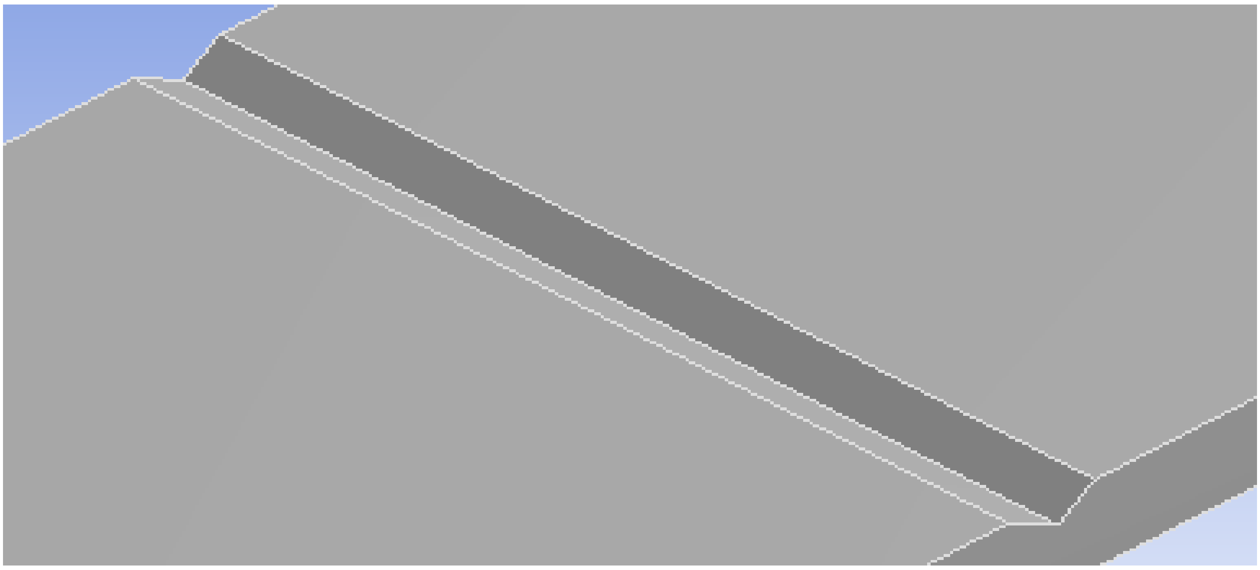


Fig 4.3 Crack at mid of cantilever

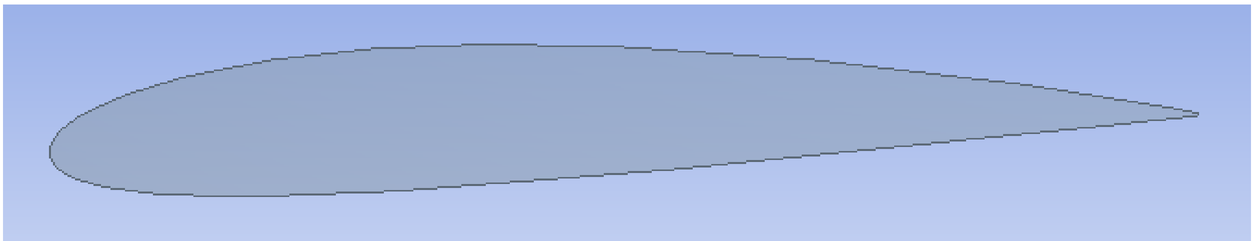


Fig 4.4 Aerodynamic wing

4.3 Grid Generation

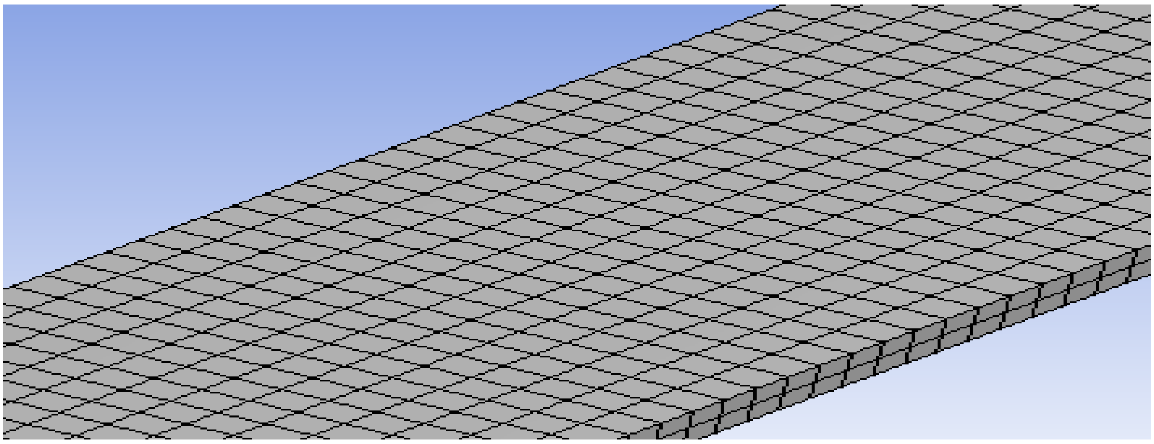


Fig 4.5 Quad Finite Elements

4.4 Material and Mesh properties

Table 4.1 Material Properties

Properties	Aluminum	Mild steel
Density (kg/m^3)	2700	7850
Young’s modulus (GPa)	69	210
Poisson’s ratio	0.334	0.303

Table 4.2 Mesh Properties

Elements	3900
Average aspect ratio	1.6
Skewness	1.3058 e-10
Orthogonality	1

4.5 Grid Independency

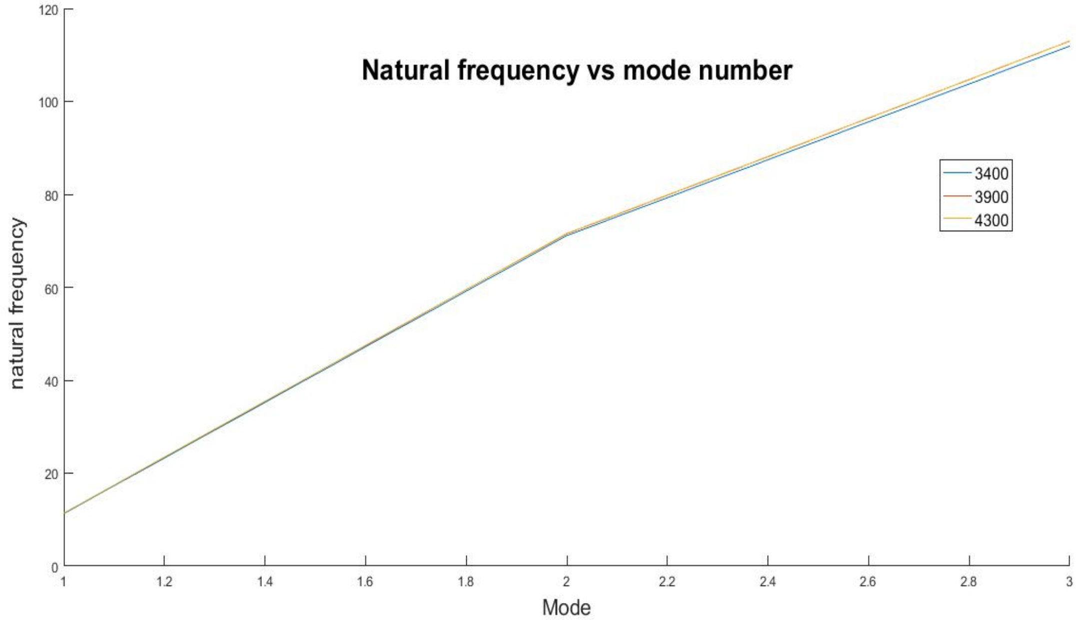


Fig 4.6 Grid Independency Test

First analysis was done on basis of 3.4k element and further it increased to 3.9k and then 4.3k elements. It was found that the variation of natural frequencies with mode from 3.9k to 4.3k was very less and error was 0.43 percent so analysis was carried out on basis of 3.9k elements.

CHAPTER 5

RESULTS AND DISCUSSIONS

5.1 Validations

Table 5.1 Comparison of natural frequency (Mild Steel)

Mode (i)	**Natural Frequency (Experimental)**	**Natural Frequency (Analytical)**	**Natural Frequency (FEA)**
1	11.071	11.422	11.667
2	69.206	71.587	73.084
3	201.810	200.668	204.63

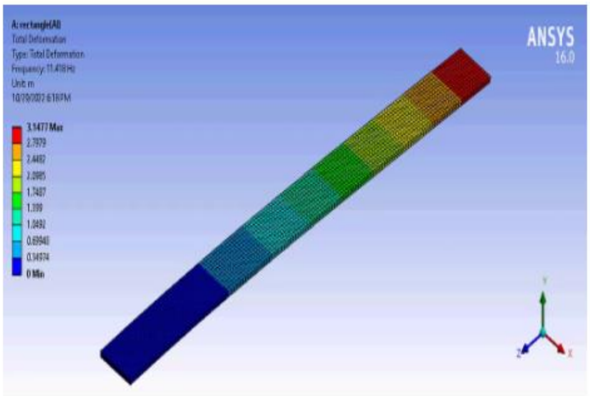
Table 5.2 Comparison of natural frequency (Aluminum)

Mode (i)	**Natural Frequency (Experimental)**	**Natural Frequency (Analytical)**	**Natural Frequency (FEA)**
1	11.219	11.604	11.418
2	71.983	72.717	71.519
3	203.113	203.631	200.26

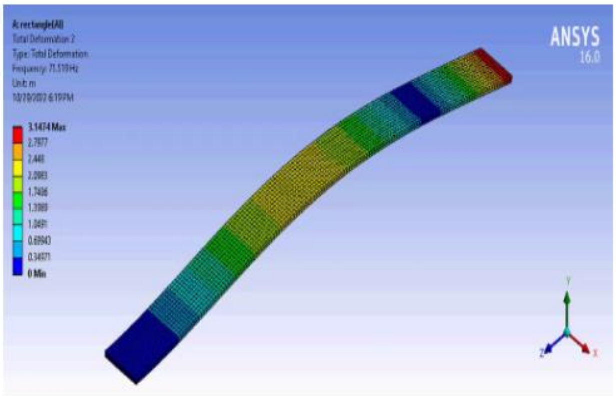
We selected Mild steel and Aluminum for our analysis and validated the analytical and numerical

Solutions with experimental results (G.C. Mekalke et al. 2016). It has been observed that the variation of natural frequencies with mode for both mild steel and aluminum is not much significant so for further analysis only mild steel is selected.

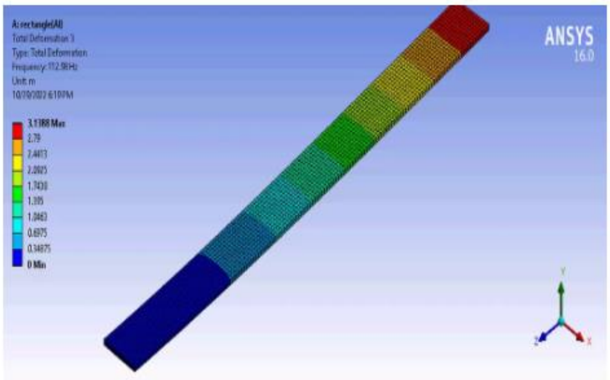
Mode 1



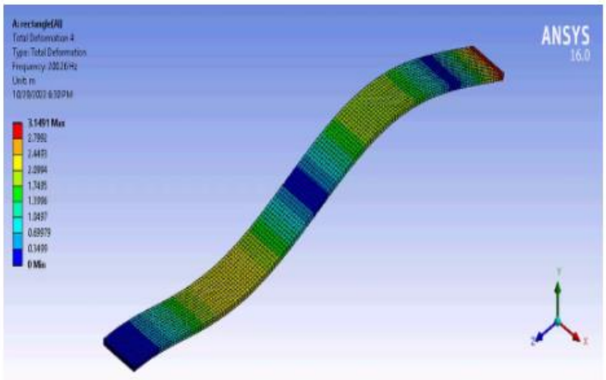
Mode 2



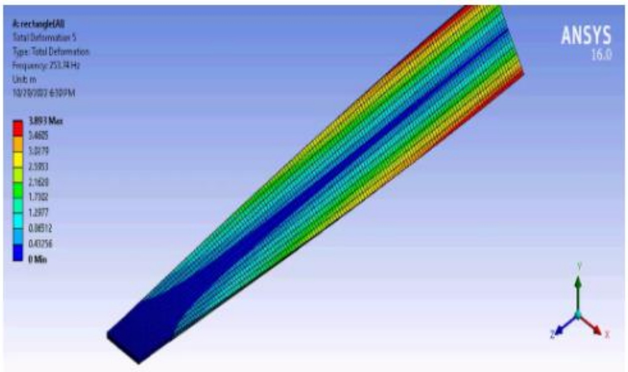
Mode 3



Mode 4



Mode 5



Mode 6

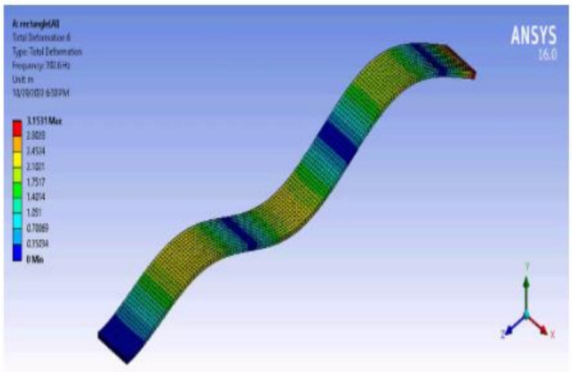


Fig 5.1 Mode shapes (Aluminum)

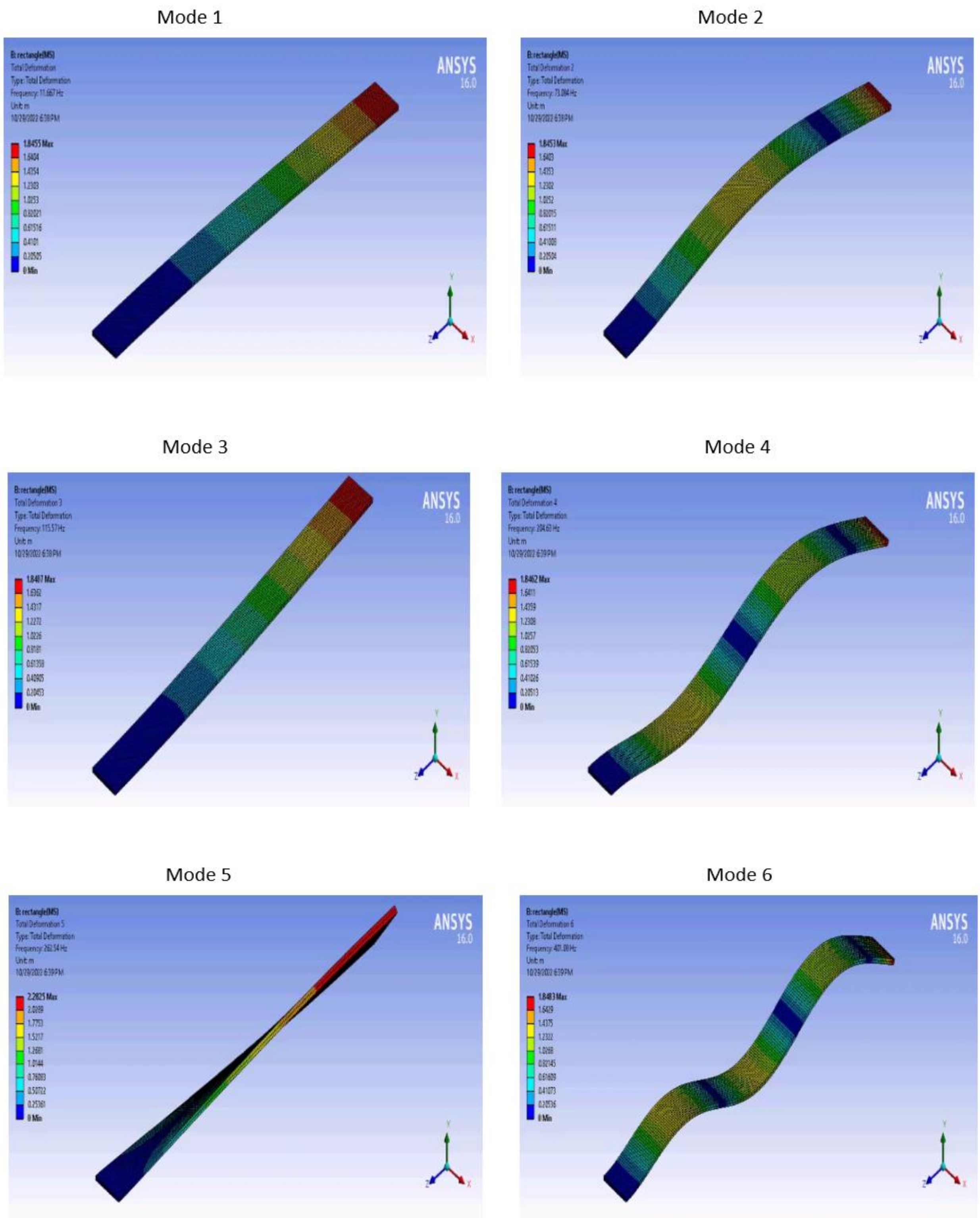


Fig 5.2 Mode Shapes (Mild Steel)

Table 5.3 Natural frequency for various cross section of cantilever at different mode

Mode 1

Cross section	Natural Frequency(HZ)
Rectangle	**11.667**
Square	36.738
Triangle	34.661
C section	68.634
T section	52.766
I section	**77.42**
Semi circular	26.851
Circular	35.889
Hollow circular	44.641

Mode 2

Cross section	Natural Frequency(HZ)
Rectangle	**73.667**
Square	36.738
Triangle	37.39
C section	108.42
T section	**124.54**
I section	115.92
Semi circular	50.722
Circular	35.89
Hollow circular	44.642

Mode 3

Cross section	Natural Frequency(HZ)
Rectangle	**115.57**
Square	229.5
Triangle	216.52
C section	**262.23**
T section	162.35
I section	151.65
Semi circular	167.96
Circular	224.25
Hollow circular	278.01

Mode 4

Cross section	Natural Frequency(HZ)
Rectangle	204.63
Square	229.5
Triangle	233.47
C section	328.34
T section	**171.33**
I section	**473.29**
Semi circular	315.91
Circular	224.25
Hollow circular	278.01

Mode 5

Cross section	Natural Frequency(HZ)
Rectangle	**262.54**
Square	639.33
Triangle	603.14
C section	601.61
T section	282.38
I section	527.45
Semi circular	468.91
Circular	624.94
Hollow circular	**770.87**

Mode 6

Cross section	Natural Frequency(HZ)
Rectangle	**401.08**
Square	639.33
Triangle	649.92
C section	617.48
T section	407.87
I section	833.37
Semi circular	**875.92**
Circular	624.94
Hollow circular	770.9

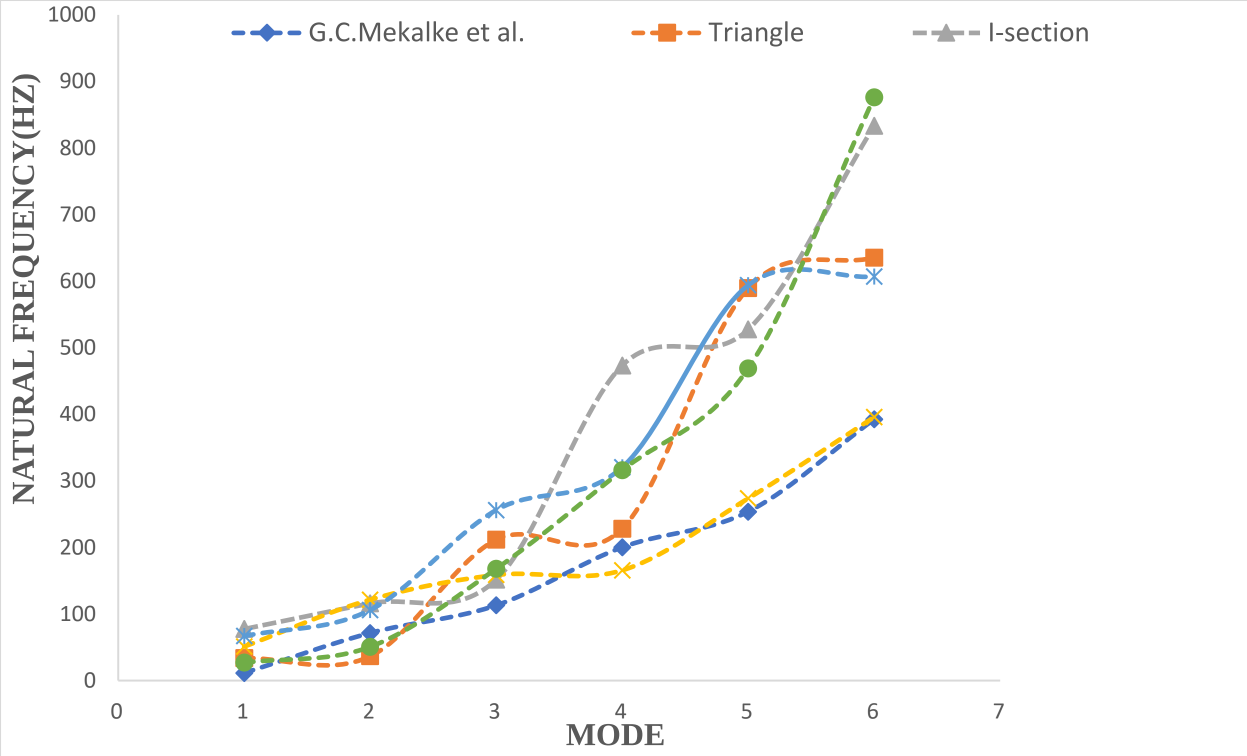


Fig 5.3 Variation of natural frequencies with mode for various cross section

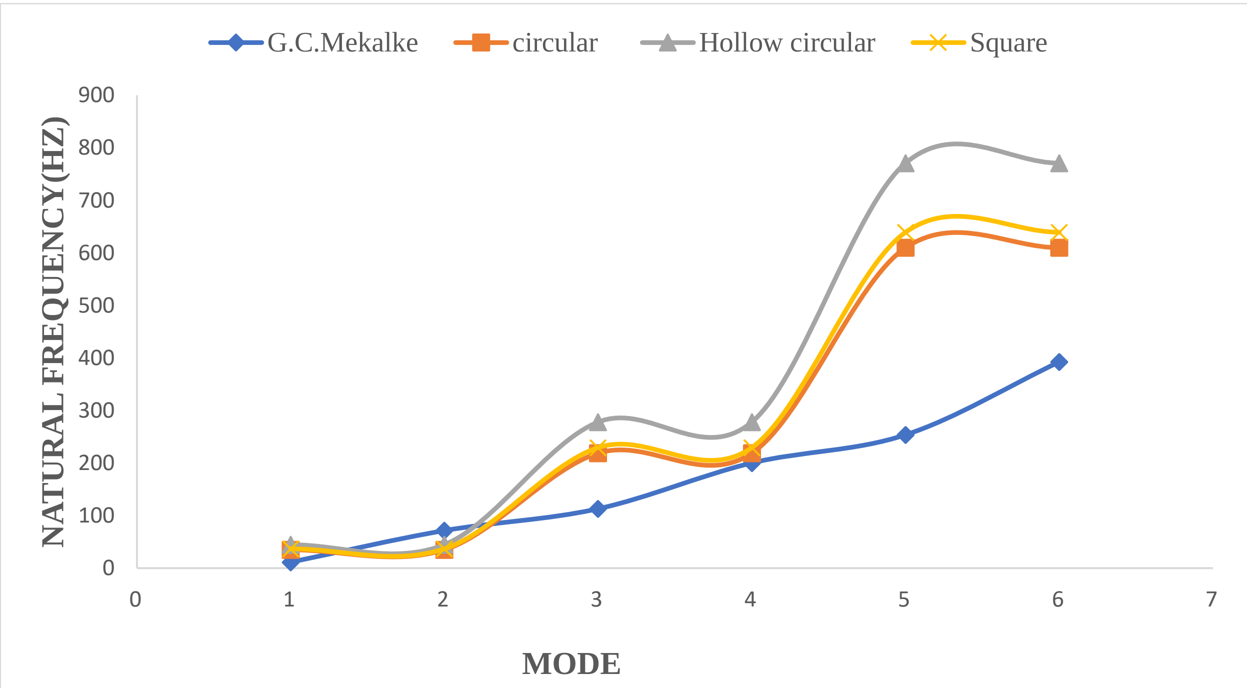
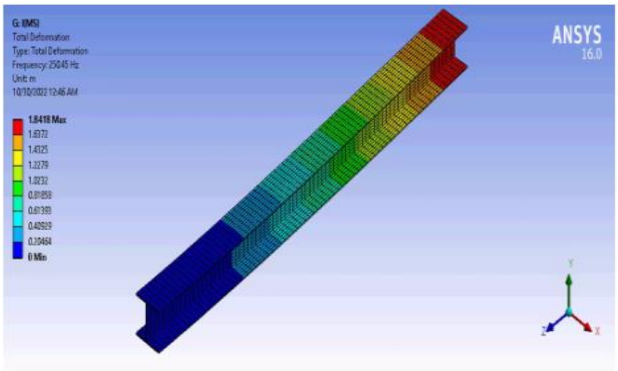


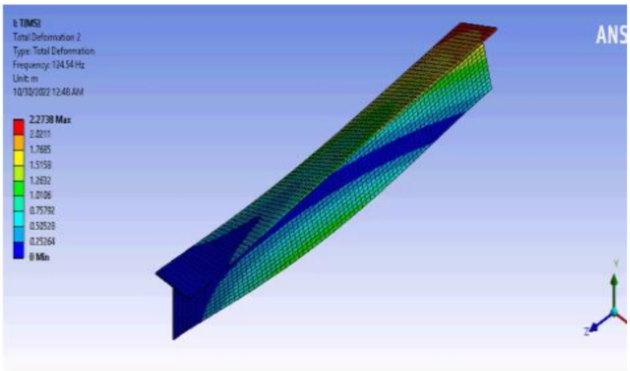
Fig 5.4 Variation of natural frequencies with mode for various circular cross section

Mode 1



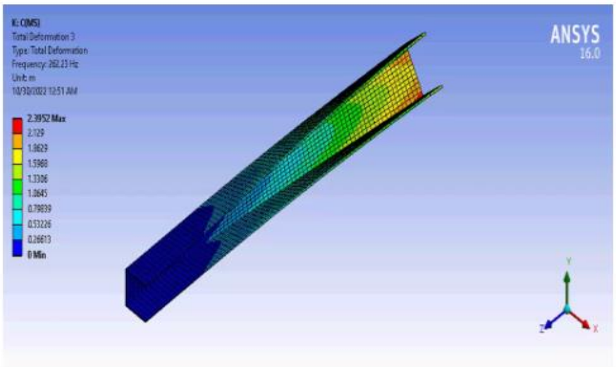
I section

Mode 2



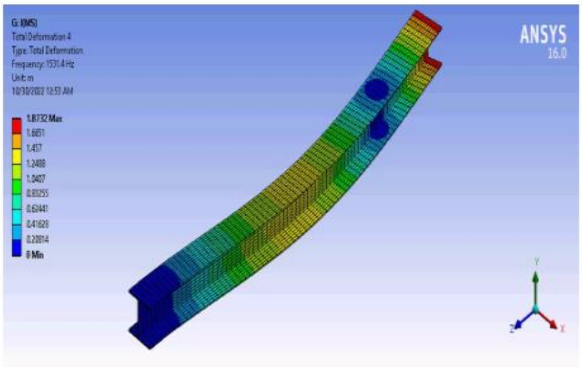
T section

Mode 3



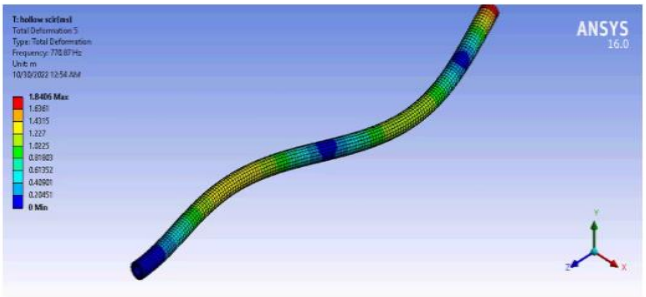
C section

Mode 4



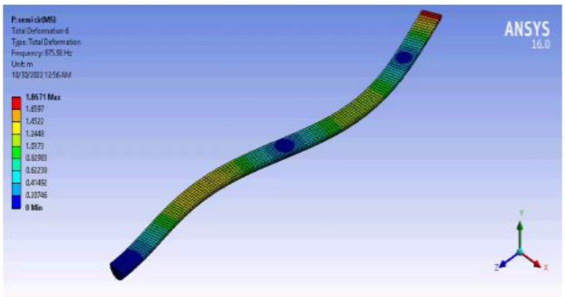
I section

Mode 5



Hollow circular

Mode 6



Semi circular

Fig.5.5. Mode shapes at various cross section corresponding to largest natural frequency

Table 5.4 Comparison of natural frequencies with and without crack at various modes

Mode	Natural frequency (HZ) without crack	Natural frequency (HZ) with crack
1	11.667	11.373
2	73.084	70.084
3	115.57	112.73
4	204.63	199.8
5	262.54	256.84
6	401.08	389.08

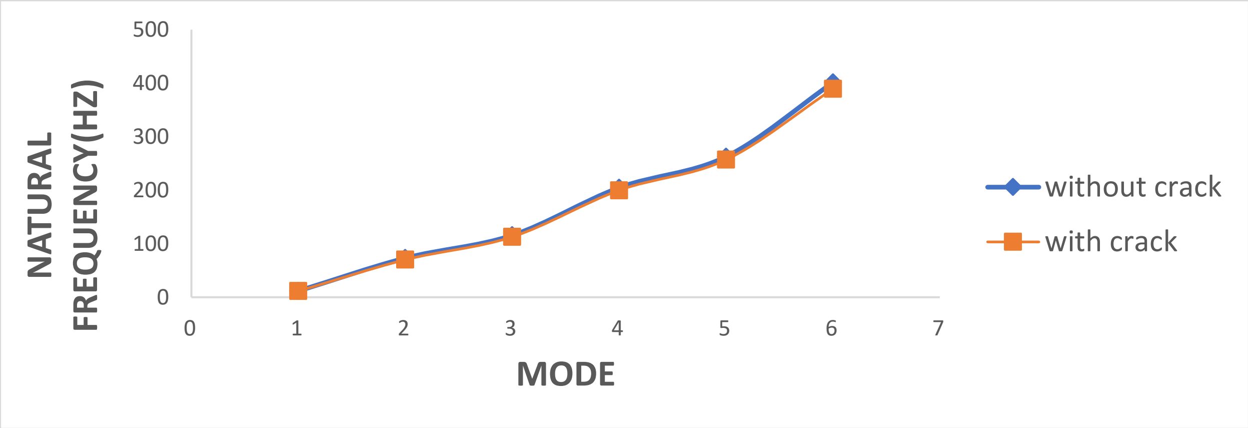


Fig 5.6 Natural frequency at various mode with and without crack

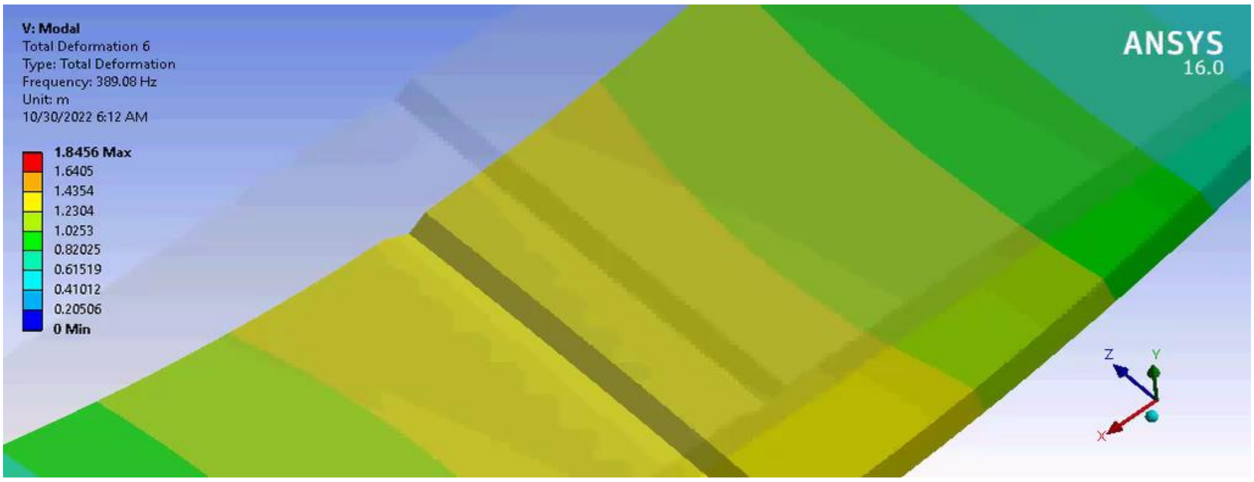


Fig 5.7 Mode shape of cracked cantilever corresponding to largest frequency

Table 5.5 Comparison of Aerodynamic wing with rectangular cantilever

Mode	Natural frequency (HZ) of rectangular cantilever	Natural frequency (HZ) with crack
1	11.667	12.373
2	73.084	70.084
3	115.57	112.73
4	204.63	199.8
5	262.54	256.84
6	401.08	389.08

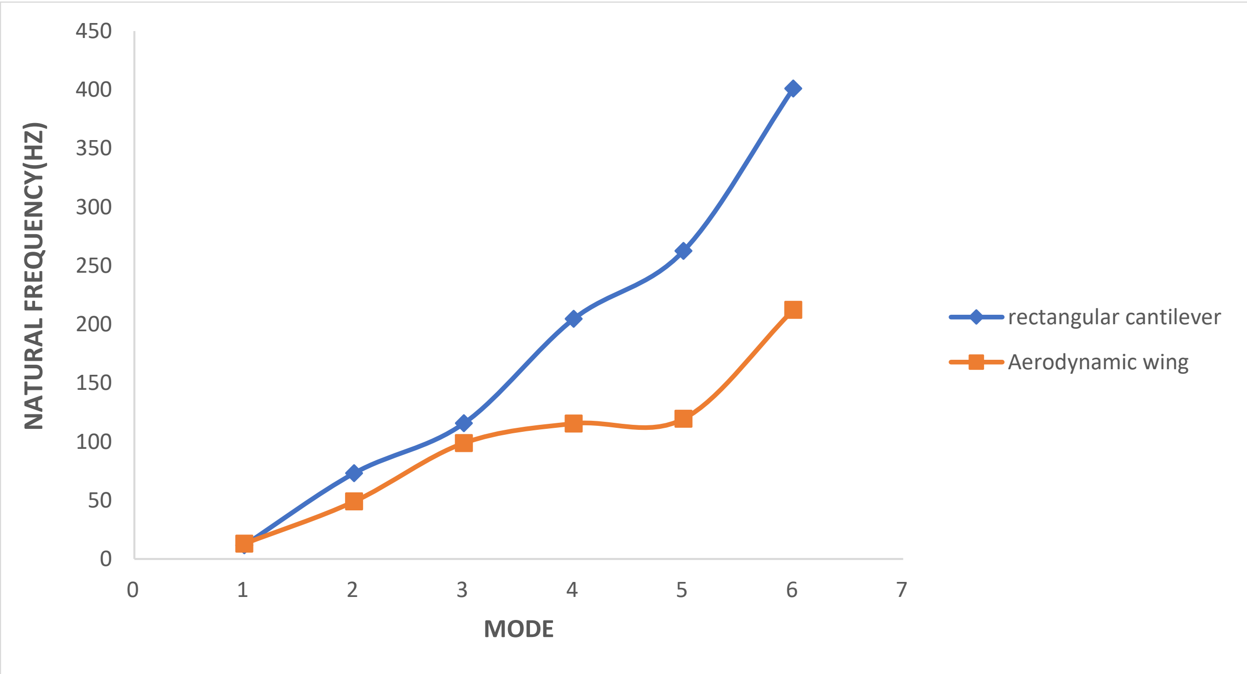


Fig 5.8 Comparison of natural frequency of aerodynamic wing with rectangular cantilever

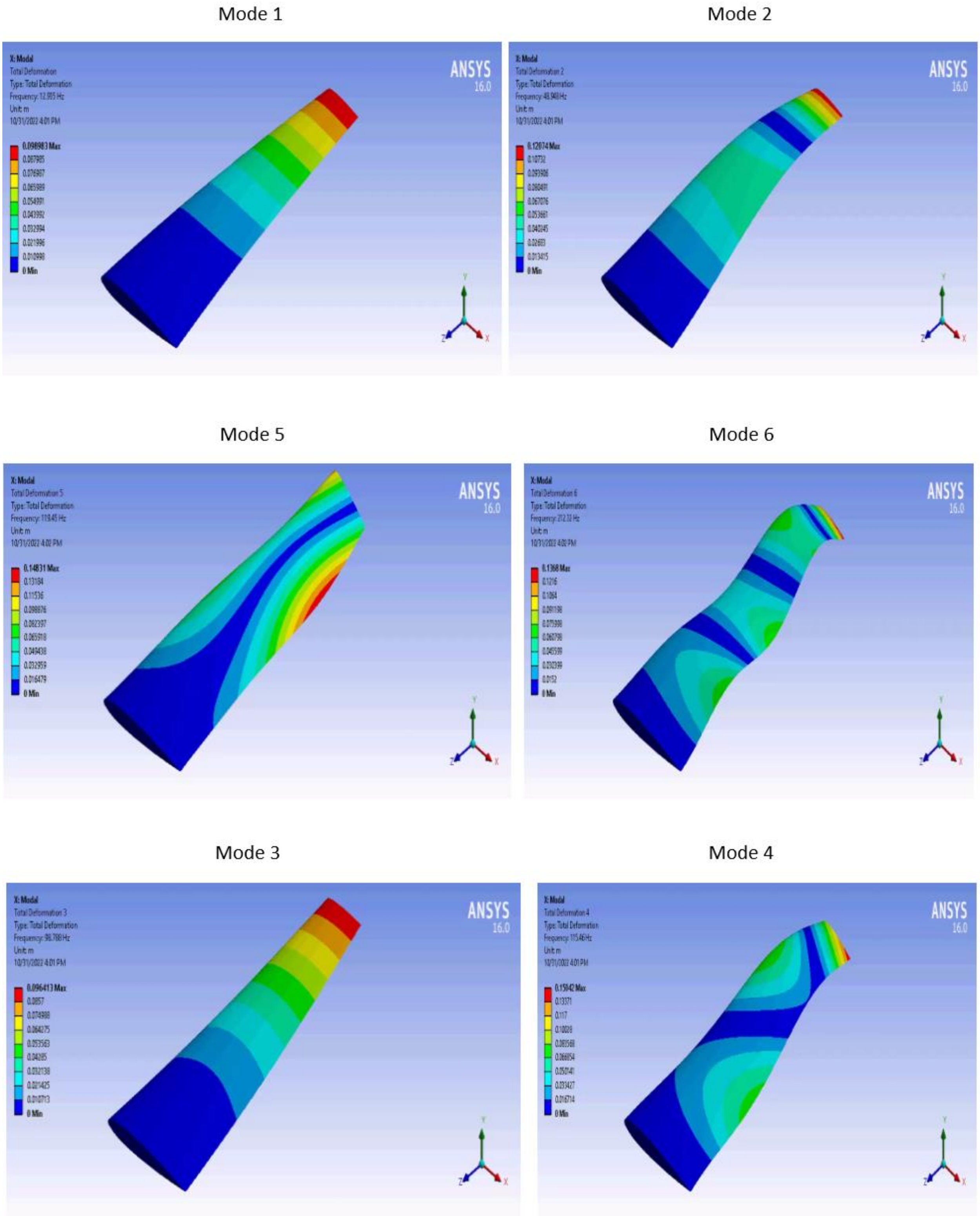


Fig 5.9 Mode shapes of aerodynamic wing as a cantilever

CHAPTER 6

CONCLUSIONS

- Modal analysis of a rectangular cross section cantilever has been done and analytical and numerical results have been validated with experimental results of G.C. Mekalle et al.(2016).
- Shape optimization has been carried out for modal analysis at various cross section.
- A V shaped notch has been introduced at mid of cantilever and obtained results have been compared with original cantilever.
- Aerodynamic wing has been modeled as a cantilever beam and modal analysis has been done.
- At mode 1 rectangle has minimum natural frequency and I section has maximum value.
- At mode 2 rectangle has minimum natural frequency and T section has maximum value.
- At mode 3 rectangle has minimum natural frequency and C section has maximum value.
- At mode 4 T section has minimum natural frequency and I section has maximum value.
- At mode 5 rectangle has minimum natural frequency and Hollow circular has maximum value.
- At mode 6 rectangle has minimum natural frequency and semicircular has maximum value.
- Natural frequencies of cracked cantilever are lower compared to cantilever without crack and variation is almost 3 to 4 percent at every mode.
- Natural frequency of aerodynamic wing has significant variation except at first mode.

FUTURE SCOPE

- In the study only Mild steel and Aluminium are used but analysis can be done for composite material and analysis can be done as multi-layered structures.
- Crack is introduced to only rectangular cross section but it can be introduced to various cross section.
- In the study stiffener has not been used but it may be used for further analysis.
- Study for aerodynamic wing with one camber position and thickness has been done but it can be studied for various combination.

RESOURCES

1. D. K. Agarwalla, D. R. Parhi, 2013, Effect of Crack on Modal Parameters of a Cantilever Beam Subjected to Vibration.

2. Hong Hee Yoo, jung Eun Cho, 2006, Modal analysis and shape optimization of rotating cantilever beams.

3. R. Southwell, F. Gough, The free transverse vibration of airscrew blades, British A.R.C. Reports and Memoranda, No. 766, 1921.

4. D. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley, Longman, Reading, MA, 1989.

5. Mr. G. C. Mekalke, Mr. A. V. Sutar, February 2016, Modal Analysis of Cantilever Beam for Various Cases and its Analytical and Fea Analysis.

6. “Vibration analysis of a cantilever beam by using F.F.T analyzer”; Prof. S.S. Kadam, Vaziri, International Journal of Advanced Engineering Technology;2013.