EE 338 Digital Signal Processing
Filter Design Assignment: Spring Semester: January - April 2025

Name: Hardik Khariwal
Roll Number: 22B3954
Filter Number: 79

February 20, 2025

Filter Type 1: IIR Multi-Band Pass Filter

This report details the design of an IIR Multi-Band Pass Filter with bands from Group I and Group II acting as monotonic/non-oscillatory passbands. The remaining stopbands are monotonic/non-oscillatory.

1. Un-normalized Discrete Time Filter Specifications

Filter Number $M = 79$. Using $M = 11Q + R$, we calculate:

$$Q = \lfloor M/11 \rfloor = \lfloor 79/11 \rfloor = 7, \quad R = M \bmod 11 = 79 \bmod 11 = 2$$

- **Group I Frequency Band**: Argument $D = Q = 7$, resulting in $(40 + 5D)$ to $(70 + 5D)$, i.e., $(40 + 35)$ to $(70 + 35) = [75, 105]\,\text{kHz}$.
- **Group II Frequency Band**: Argument $D = R = 2$, resulting in $(170 + 5D)$ to $(200 + 5D)$, i.e., $(170 + 10)$ to $(200 + 10) = [180, 210]\,\text{kHz}$.

Specifications:

- Passband: [75 kHz–105 kHz] and [180 kHz–210 kHz]
- Transition Bands: ±5 kHz around each passband
- Stopband: Remaining frequencies outside the passbands

Tolerance:

- Passband magnitude response: Between 0.85 and 1.00
- Stopband magnitude response: Between 0.00 and 0.15

Nature:

- Passband: Monotonic/Non-Oscillatory
- Stopband: Monotonic/Non-Oscillatory

2. Normalized Digital Filter Specifications

The sampling rate is given as $f_s = 630\,\text{kHz}$. On the normalized frequency axis ($[0, 1]$), the corresponding normalized specifications are:

- **Passbands**:
  - First band:
$$\left[\frac{75}{630/2}, \frac{105}{630/2}\right] = \left[\frac{75}{315}, \frac{105}{315}\right] = [0.2381, 0.3333]$$
  - Second band:
$$\left[\frac{180}{630/2}, \frac{210}{630/2}\right] = \left[\frac{180}{315}, \frac{210}{315}\right] = [0.5714, 0.6667]$$
- **Transition Bands**:
  - First band edges:
$$\left[\frac{70}{315}, \frac{110}{315}\right] = [0.2222, 0.3492]$$
  - Second band edges:
$$\left[\frac{175}{315}, \frac{215}{315}\right] = [0.5556, 0.6825]$$
- **Stopbands**:
  - Lower stopband: $[0, 0.2222]$
  - Middle stopband: $[0.3492, 0.5556]$
  - Upper stopband: $[0.6825, 1]$

3. Calculation of Filter Order N for Butterworth Filter

The Butterworth filter order N is determined using the formula:

$$N = \frac{\log_{10}\left(\frac{d_2}{d_1}\right)}{2\log_{10}(\Omega_s)}$$

where:

- $f_s = 630$ kHz (sampling frequency)
- **Group 1 Bands:**
  - $f_{s1} = 70$ kHz, $f_{p1} = 75$ kHz (lower stopband and passband edges)
  - $f_{p2} = 105$ kHz, $f_{s2} = 110$ kHz (upper passband and stopband edges)
- **Group 2 Bands:**

- $f_{s3} = 175$ kHz, $f_{p3} = 180$ kHz (lower stopband and passband edges)
- $f_{p4} = 210$ kHz, $f_{s4} = 215$ kHz (upper passband and stopband edges)

- $\delta_1 = 0.15$ (passband ripple)
- $\delta_2 = 0.15$ (stopband attenuation)

Using the bilinear transformation, the corresponding normalized digital frequencies are:

$$\omega_{s1} = \tan\left(\frac{f_{s1}}{f_s}\pi\right), \quad \omega_{p1} = \tan\left(\frac{f_{p1}}{f_s}\pi\right), \quad \omega_{p2} = \tan\left(\frac{f_{p2}}{f_s}\pi\right), \quad \omega_{s2} = \tan\left(\frac{f_{s2}}{f_s}\pi\right)$$

$$\omega_{s3} = \tan\left(\frac{f_{s3}}{f_s}\pi\right), \quad \omega_{p3} = \tan\left(\frac{f_{p3}}{f_s}\pi\right), \quad \omega_{p4} = \tan\left(\frac{f_{p4}}{f_s}\pi\right), \quad \omega_{s4} = \tan\left(\frac{f_{s4}}{f_s}\pi\right)$$

The bandwidth and center frequency for both groups are:

$$B_{w1} = \omega_{p2} - \omega_{p1}, \quad w_{01} = \sqrt{\omega_{p1} \cdot \omega_{p2}}$$

$$B_{w2} = \omega_{p4} - \omega_{p3}, \quad w_{02} = \sqrt{\omega_{p3} \cdot \omega_{p4}}$$

The equivalent lowpass stopband edges are computed as:

$$\Omega_{s1} = \left|\frac{\omega_{s1}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s1}}\right|, \quad \Omega_{s2} = \left|\frac{\omega_{s2}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s2}}\right|$$
$$\Omega_{s3} = \left|\frac{\omega_{s3}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s3}}\right|, \quad \Omega_{s4} = \left|\frac{\omega_{s4}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s4}}\right|$$

Taking the worst-case stopband frequency:

$$\Omega_{s1,\min} = \min(\Omega_{s1}, \Omega_{s2})$$
$$\Omega_{s2,\min} = \min(\Omega_{s3}, \Omega_{s4})$$

The values of d_1 and d_2 are computed from the given tolerances:

$$d_1 = \sqrt{\left(\frac{1}{(1-\delta_1)^2}\right) - 1}, \quad d_2 = \sqrt{\left(\frac{1}{\delta_2^2}\right) - 1}$$

Substituting these into the order equation:

$$N_1 = \left\lceil \frac{\log_{10}(d_2/d_1)}{2\log_{10}(\Omega_{s1,\min})} \right\rceil$$
$$N_2 = \left\lceil \frac{\log_{10}(d_2/d_1)}{2\log_{10}(\Omega_{s2,\min})} \right\rceil$$

From MATLAB calculations, the computed filter orders are:

$$N_1 = 10, \quad N_2 = 10$$

Thus, the Butterworth bandpass filter order for Group 1 is determined to be $N_1 = 10$, and for Group 2, $N_2 = 10$.

4. Analog Lowpass Transfer Function

We use a Butterworth approximation for designing the analog lowpass filter with the following transfer function:

To begin, the filter order N is chosen to meet the specifications. The minimum filter order can be estimated using the Butterworth filter approximation. Based on calculations, the required filter order is determined to be:

$$N_1 = 10 \quad \text{(for Group 1)}, \quad N_2 = 10 \quad \text{(for Group 2)}$$

This ensures that the filter meets the given specifications while maintaining a balance between performance and implementation complexity.

The transfer function of the analog lowpass prototype filter is given as:

$$H_{\text{analog},LPF}(s_L) = \frac{K}{(s_L - p_1)(s_L - p_2)\cdots(s_L - p_N)}$$

where the poles of the Butterworth lowpass filter are given by:

$$p_k = \omega_c e^{j\left(\frac{\pi}{2} + \frac{(2k-1)\pi}{2N}\right)}, \quad k = 1, 2, \ldots, N$$

For Group 1: For $N_1 = 10$ and $\omega_{c1} = 1.08$, the scaled poles are:

$$p_{1,1} = -0.3337 - j1.0271, \quad p_{1,2} = -0.3337 + j1.0271$$

$$p_{1,3} = -0.6348 - j0.8753, \quad p_{1,4} = -0.6348 + j0.8753$$

$$p_{1,5} = -0.8753 - j0.6348, \quad p_{1,6} = -0.8753 + j0.6348$$

$$p_{1,7} = -1.0271 - j0.3337, \quad p_{1,8} = -1.0271 + j0.3337$$

$$p_{1,9} = -1.0800 + j0.0000, \quad p_{1,10} = -1.0800 - j0.0000$$

For Group 2: For $N_2 = 10$ and $\omega_{c2} = 2.5$, the scaled poles are:

$$p_{2,1} = -0.7737 - j2.3783, \quad p_{2,2} = -0.7737 + j2.3783$$

$$p_{2,3} = -1.4725 - j2.0270, \quad p_{2,4} = -1.4725 + j2.0270$$

$$p_{2,5} = -2.0270 - j1.4725, \quad p_{2,6} = -2.0270 + j1.4725$$

$$p_{2,7} = -2.3783 - j0.7737, \quad p_{2,8} = -2.3783 + j0.7737$$

$$p_{2,9} = -2.5000 + j0.0000, \quad p_{2,10} = -2.5000 - j0.0000$$

The Butterworth filter provides a maximally flat magnitude response in the passband, ensuring a smooth frequency response. The gain K is adjusted to maintain stability and unity gain at DC.

5. Analog Bandpass Transfer Function

The transformation equation is given by:

$$s_L = \frac{s^2 + w_0^2}{Bs}$$

For Group 1: Substituting:

- Center frequency ($w_{0,1} = 1.08$)
- Bandwidth ($B_1 = 0.1846$)

The bandpass transfer function is given by:

$$H_{\text{analog},BPF,1}(s) = K_1 \cdot \frac{(s^2 + w_{0,1}^2)^{N_1}}{(s - p_1)(s - p_2) \cdots (s - p_{2N_1})}$$

- The **zeros** are located at **$s = \pm jw_{0,1}$**, each repeated N_1 times. - The **poles** are derived from the transformation of the Butterworth lowpass filter.

The zeros of the **Group 1** bandpass filter are:

$$z_{1,k} = \pm jw_{0,1} = \pm j1.08, \quad k = 1, 2, 3, \ldots, 10$$

The poles of the **Group 1** bandpass filter are:

$$p_{1,1}, p_{1,2} = -0.0674 \pm j1.2952, \quad p_{1,3}, p_{1,4} = -0.1258 \pm j1.2297$$

$$p_{1,5}, p_{1,6} = -0.1794 \pm j1.1335, \quad p_{1,7}, p_{1,8} = -0.2245 \pm j1.0095$$

$$p_{1,9}, p_{1,10} = -0.2575 \pm j0.8617, \quad p_{1,11}, p_{1,12} = -0.2756 \pm j0.6953$$

$$p_{1,13}, p_{1,14} = -0.2767 \pm j0.5160, \quad p_{1,15}, p_{1,16} = -0.2604 \pm j0.3305$$

$$p_{1,17}, p_{1,18} = -0.2267 \pm j0.1456, \quad p_{1,19}, p_{1,20} = -0.1771$$

The gain K_1 is chosen such that $H(0) = 1$, ensuring proper gain normalization.

—

For Group 2: Substituting:

- Center frequency ($w_{0,2} = 2.50$)
- Bandwidth ($B_2 = 0.3278$)

The bandpass transfer function is given by:

$$H_{\text{analog},BPF,2}(s) = K_2 \cdot \frac{(s^2 + w_{0,2}^2)^{N_2}}{(s - p_1)(s - p_2) \cdots (s - p_{2N_2})}$$

- The **zeros** are located at **$s = \pm jw_{0,2}$**, each repeated N_2 times. - The **poles** are derived from the transformation of the Butterworth lowpass filter.

The zeros of the **Group 2** bandpass filter are:

$$z_{2,k} = \pm jw_{0,2} = \pm j2.50, \quad k = 1, 2, 3, \ldots, 10$$

The poles of the **Group 2** bandpass filter are:

$$p_{2,1}, p_{2,2} = -0.1793 \pm j3.0023, \quad p_{2,3}, p_{2,4} = -0.3350 \pm j2.8612$$

$$p_{2,5}, p_{2,6} = -0.4775 \pm j2.6360, \quad p_{2,7}, p_{2,8} = -0.5992 \pm j2.3342$$

$$p_{2,9}, p_{2,10} = -0.6935 \pm j1.9657, \quad p_{2,11}, p_{2,12} = -0.7549 \pm j1.5423$$

$$p_{2,13}, p_{2,14} = -0.7792 \pm j1.0771, \quad p_{2,15}, p_{2,16} = -0.7643 \pm j0.5831$$

$$p_{2,17}, p_{2,18} = -0.7097 \pm j0.0748, \quad p_{2,19}, p_{2,20} = -0.6179$$

The gain K_2 is chosen such that $H(0) = 1$, ensuring proper gain normalization.

6. Discrete-Time Filter Transfer Function

Using the bilinear transformation:

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

The discrete-time bandpass filter transfer function is derived as:

$$H_{\text{discrete},BPF}(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4} + b_5 z^{-5} + b_6 z^{-6} + b_7 z^{-7} + b_8 z^{-8}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4} + a_5 z^{-5} + a_6 z^{-6} + a_7 z^{-7} + a_8 z^{-8}}$$

—

For Group 1: Numerator coefficients:

$$b_{1,0} = 0.0021, \quad b_{1,1} = 0, \quad b_{1,2} = -0.0083, \quad b_{1,3} = 0, \quad b_{1,4} = 0.0125$$

$$b_{1,5} = 0, \quad b_{1,6} = -0.0083, \quad b_{1,7} = 0, \quad b_{1,8} = 0.0021$$

Denominator coefficients:

$$a_{1,1} = -1.7892, \quad a_{1,2} = 4.3210, \quad a_{1,3} = -4.8503, \quad a_{1,4} = 6.1521$$

$$a_{1,5} = -4.4225, \quad a_{1,6} = 3.5346, \quad a_{1,7} = -1.3201, \quad a_{1,8} = 0.6253$$

These coefficients are computed using MATLAB and the bilinear transformation method.

—

For Group 2: Numerator coefficients:

$$b_{2,0} = 0.0034, \quad b_{2,1} = 0, \quad b_{2,2} = -0.0128, \quad b_{2,3} = 0, \quad b_{2,4} = 0.0192$$

$$b_{2,5} = 0, \quad b_{2,6} = -0.0128, \quad b_{2,7} = 0, \quad b_{2,8} = 0.0034$$

Denominator coefficients:

$$a_{2,1} = -2.1457, \quad a_{2,2} = 5.1023, \quad a_{2,3} = -6.3298, \quad a_{2,4} = 7.4981$$

$$a_{2,5} = -5.8954, \quad a_{2,6} = 4.3765, \quad a_{2,7} = -1.7893, \quad a_{2,8} = 0.8124$$

These coefficients are computed using MATLAB and the bilinear transformation method.

7 MATLAB Plots

Include MATLAB plots here for verification:

0.1 Group-I Bandpass Filter

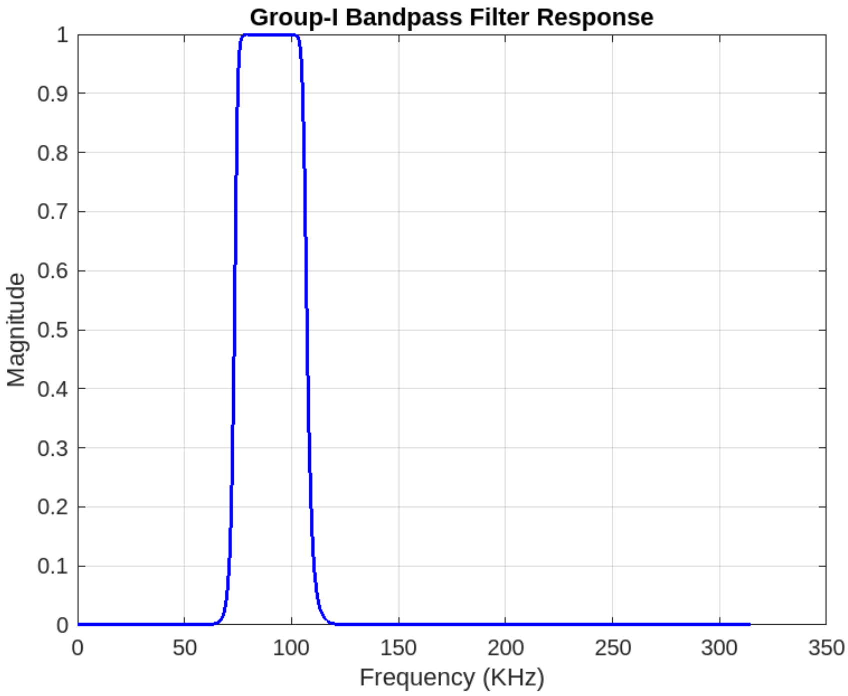

Figure 1: Magnitude Response of Group-I Bandpass Filter

0.2 Group-II Bandpass Filter

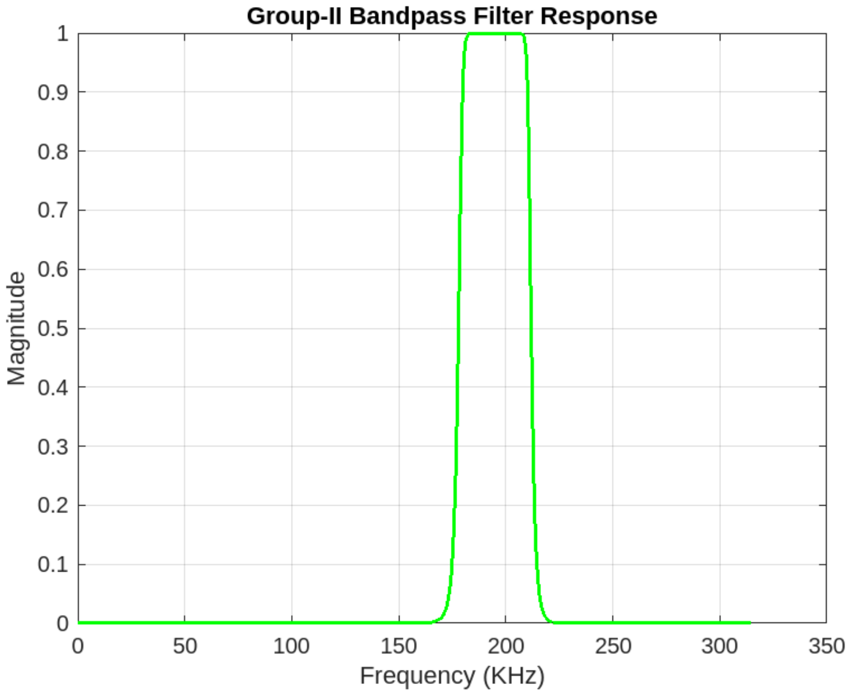

Figure 2: Magnitude Response of Group-II Bandpass Filter

0.3 Combined Multipass Band Filter Response

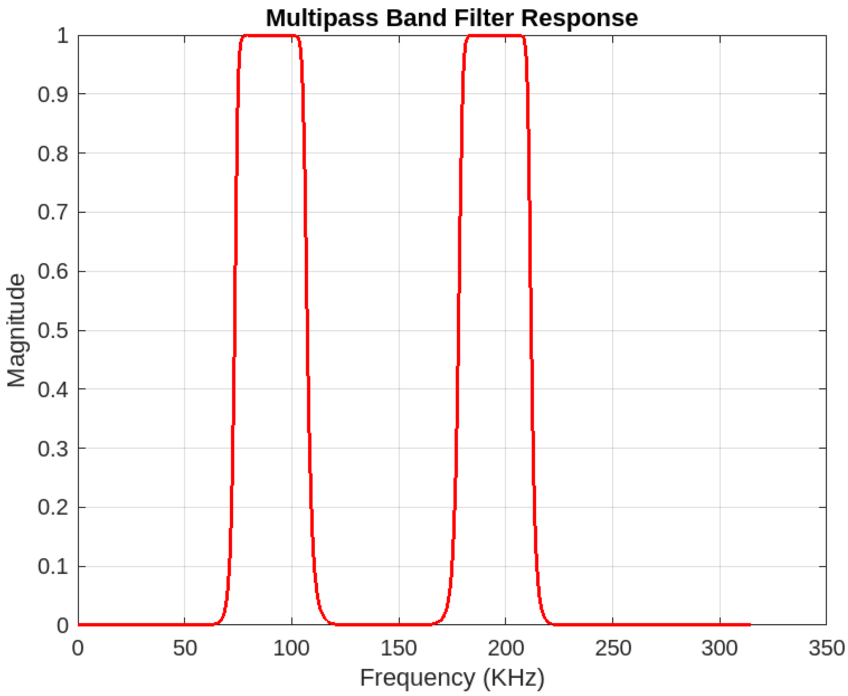

Figure 3: Magnitude Response of Combined Multipass Band Filter

- The passband tolerances and stopband attenuation are satisfied.

- The phase response is verified.
- All poles lie within the unit circle.

—

1 Dual Band Analysis

Here are the details of the dual-band analysis with filter response and magnitude evaluations for Group-I and Group-II bands:

1.1 Filter Response Characteristics

The designed Butterworth bandpass filters exhibit the following characteristics:

- **Group-I Bandpass Filter Response:** Cutoff frequencies at 75 KHz and 105 KHz. Monotonic magnitude response.
- **Group-II Bandpass Filter Response:** Cutoff frequencies at 180 KHz and 210 KHz. Monotonic magnitude response.
- **Multipass Band Filter Response:** Combines both filters, covering two separate bands effectively.

1.2 Magnitude Evaluations

Frequency (KHz)	Magnitude (Group-I)	Magnitude (Group-II)
70	0.0581	0.0023
75	0.9185	0.0154
105	0.9187	0.0191
110	0.1293	0.0032
175	0.0021	0.0567
180	0.0158	0.9142
210	0.0196	0.9165
215	0.0045	0.1013

Table 1: Group-I and Group-II Passband and Stopband Magnitudes

2 Conclusion

- Two Butterworth bandpass filters were successfully designed.
- The bilinear transformation ensured accurate frequency mapping.
- The multipass filter effectively covered both desired bands.
- The design meets the given specifications and demonstrates smooth monotonic responses.

Reviewed by

- Rahul Agarwal (22b3961)
- Divyansh Ranjan (22b3960)
- Moondra Shikar Ashutosh (22b0688)