EE 338 Digital Signal Processing
Filter Design Assignment (optional)
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Filter Type 2: IIR Multi-Bandpass Filter using Chebyshev Type

This report presents the design of an IIR Multi-Bandpass Filter using the Chebyshev filter with passbands from Group I and Group II are equiripple/ oscillatory. The remaining stopbands are monotonic/non-oscillatory.

1 Un-Normalized Discrete Time Filter Specifications

This section details the specifications of the IIR Multi-Band Pass Filter, including passbands, stopbands, transition bands, and magnitude response constraints.

Filter Number and Computation of Q and R

The given filter number is:

$$M = 79$$

Using the formula $M = 11Q + R$, we compute:

$$Q = \lfloor M/11 \rfloor = \lfloor 79/11 \rfloor = 7, \quad R = M \mod 11 = 79 \mod 11 = 2$$

Group I and Group II Frequency Bands

The frequency bands for Group I and Group II are computed as follows:

- **Group I Frequency Band:** Argument $D = Q = 7$

$$(40 + 5D) \text{ to } (70 + 5D) = (40 + 35) \text{ to } (70 + 35) = [75, 105] \text{ kHz}$$

- **Group II Frequency Band:** Argument $D = R = 2$

$$(170 + 5D) \text{ to } (200 + 5D) = (170 + 10) \text{ to } (200 + 10) = [180, 210] \text{ kHz}$$

Passband, Stopband, and Transition Band Specifications

- **Passbands:**
  - First Passband: $[75, 105]$ kHz
  - Second Passband: $[180, 210]$ kHz
- **Stopbands:**
  - Lower Stopband: $[0, 70]$ kHz
  - Middle Stopband: $[110, 175]$ kHz
  - Upper Stopband: $[215, \infty]$ kHz
- **Transition Bands:** 5 kHz around each passband

Tolerance Constraints

- **Passband Magnitude Response:** Between 0.85 and 1.00
- **Stopband Magnitude Response:** Between 0.00 and 0.15

Nature of Response

- **Passband:** Equiripple/ Oscillatory
- **Stopband:** Monotonic / Non-Oscillatory

2 Normalized Digital Filter Specifications

The sampling frequency is given as:

$$f_s = 630 \text{ kHz}$$

On the normalized frequency axis $[0, 1]$, the corresponding **normalized frequency specifications** for the **Chebyshev Type I filter** are:

Passbands

- **First Band:**
$$\left[\frac{75}{\frac{630}{2}}, \frac{105}{\frac{630}{2}}\right] = \left[\frac{75}{315}, \frac{105}{315}\right] = [0.2381, 0.3333]$$
- **Second Band:**
$$\left[\frac{180}{\frac{630}{2}}, \frac{210}{\frac{630}{2}}\right] = \left[\frac{180}{315}, \frac{210}{315}\right] = [0.5714, 0.6667]$$

Transition Bands

- **First Band Edges:**

$$\left[\frac{70}{315}, \frac{110}{315}\right] = [0.2222, 0.3492]$$

- **Second Band Edges:**

$$\left[\frac{175}{315}, \frac{215}{315}\right] = [0.5556, 0.6825]$$

Stopbands

- **Lower Stopband:** $[0, 0.2222]$
- **Middle Stopband:** $[0.3492, 0.5556]$
- **Upper Stopband:** $[0.6825, 1]$

3 Calculation of Filter Order N for Chebyshev Filter

The Chebyshev filter order N is determined using the formula:

$$N = \frac{\cosh^{-1}\left(\sqrt{\frac{d_2}{d_1}}\right)}{\cosh^{-1}(\Omega_s)}$$

where:

- $f_s = 630$ kHz (sampling frequency)
- **Group 1 Bands:**
  - $f_{s1} = 70$ kHz, $f_{p1} = 75$ kHz (lower stopband and passband edges)
  - $f_{p2} = 105$ kHz, $f_{s2} = 110$ kHz (upper passband and stopband edges)
- **Group 2 Bands:**
  - $f_{s3} = 175$ kHz, $f_{p3} = 180$ kHz (lower stopband and passband edges)
  - $f_{p4} = 210$ kHz, $f_{s4} = 215$ kHz (upper passband and stopband edges)
- $\delta_1 = 0.15$ (passband ripple)
- $\delta_2 = 0.15$ (stopband attenuation)

Using the bilinear transformation, the corresponding normalized digital frequencies are:

$$\omega_{s1} = \tan\left(\frac{f_{s1}}{f_s}\pi\right), \quad \omega_{p1} = \tan\left(\frac{f_{p1}}{f_s}\pi\right), \quad \omega_{p2} = \tan\left(\frac{f_{p2}}{f_s}\pi\right), \quad \omega_{s2} = \tan\left(\frac{f_{s2}}{f_s}\pi\right)$$

$$\omega_{s3} = \tan\left(\frac{f_{s3}}{f_s}\pi\right), \quad \omega_{p3} = \tan\left(\frac{f_{p3}}{f_s}\pi\right), \quad \omega_{p4} = \tan\left(\frac{f_{p4}}{f_s}\pi\right), \quad \omega_{s4} = \tan\left(\frac{f_{s4}}{f_s}\pi\right)$$

The bandwidth and center frequency for both groups are:

$$B_{w1} = \omega_{p2} - \omega_{p1}, \quad w_{01} = \sqrt{\omega_{p1} \cdot \omega_{p2}}$$

$$B_{w2} = \omega_{p4} - \omega_{p3}, \quad w_{02} = \sqrt{\omega_{p3} \cdot \omega_{p4}}$$

The equivalent lowpass stopband edges are computed as:

$$\Omega_{s1} = \frac{\omega_{s1}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s1}}, \quad \Omega_{s2} = \frac{\omega_{s2}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s2}}$$

$$\Omega_{s3} = \frac{\omega_{s3}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s3}}, \quad \Omega_{s4} = \frac{\omega_{s4}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s4}}$$

Taking the worst-case stopband frequency:

$$\Omega_{s1,\min} = \min(\Omega_{s1}, \Omega_{s2})$$

$$\Omega_{s2,\min} = \min(\Omega_{s3}, \Omega_{s4})$$

The values of d_1 and d_2 are computed from the given tolerances:

$$d_1 = \sqrt{\left(\frac{1}{(1-\delta_1)^2}\right) - 1}, \quad d_2 = \sqrt{\left(\frac{1}{\delta_2^2}\right) - 1}$$

Substituting these into the Chebyshev order equation:

$$N_1 = \left\lceil \frac{\cosh^{-1}\left(\sqrt{\frac{d_2}{d_1}}\right)}{\cosh^{-1}(\Omega_{s1,\min})} \right\rceil$$

$$N_2 = \left\lceil \frac{\cosh^{-1}\left(\sqrt{\frac{d_2}{d_1}}\right)}{\cosh^{-1}(\Omega_{s2,\min})} \right\rceil$$

From MATLAB calculations, the computed filter orders are:

$$N_1 = 5, \quad N_2 = 5$$

Thus, the Chebyshev bandpass filter order for Group 1 is determined to be $N_1 = 5$, and for Group 2, $N_2 = 5$.

4 IIR bandpass/stop Filter Design Process

(a) Analog Filter Specifications using Bilinear Transformation

To design the bandpass filter, the given **digital frequencies** must be transformed into **analog (pre-warped) frequencies** using the **bilinear transformation**. The transformation equation is:

$$\omega = \tan\left(\frac{f}{f_s}\pi\right)$$

where:

- $f_s = 630$ kHz (sampling frequency)
- f represents the respective frequency components (stopband and passband edges)

Group 1: Frequency Transformation

The given **passband and stopband edges** for **Group 1** are:

$$f_{s1} = 70 \text{ kHz}, \quad f_{p1} = 75 \text{ kHz}, \quad f_{p2} = 105 \text{ kHz}, \quad f_{s2} = 110 \text{ kHz}$$

Applying the bilinear transformation:

$$\omega_{s1} = \tan\left(\frac{f_{s1}}{f_s}\pi\right), \quad \omega_{p1} = \tan\left(\frac{f_{p1}}{f_s}\pi\right), \quad \omega_{p2} = \tan\left(\frac{f_{p2}}{f_s}\pi\right), \quad \omega_{s2} = \tan\left(\frac{f_{s2}}{f_s}\pi\right)$$

The **bandwidth and center frequency** for Group 1 are:

$$B_{w1} = \omega_{p2} - \omega_{p1}, \quad w_{01} = \sqrt{\omega_{p1} \cdot \omega_{p2}}$$

The **equivalent lowpass stopband edges** are computed as:

$$\Omega_{s1} = \frac{\omega_{s1}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s1}}, \quad \Omega_{s2} = \frac{\omega_{s2}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s2}}$$

Taking the worst-case stopband frequency:

$$\Omega_{s1,\min} = \min(\Omega_{s1}, \Omega_{s2})$$

Group 2: Frequency Transformation

The given **passband and stopband edges** for **Group 2** are:

$$f_{s3} = 175 \text{ kHz}, \quad f_{p3} = 180 \text{ kHz}, \quad f_{p4} = 210 \text{ kHz}, \quad f_{s4} = 215 \text{ kHz}$$

Applying the bilinear transformation:

$$\omega_{s3} = \tan\left(\frac{f_{s3}}{f_s}\pi\right), \quad \omega_{p3} = \tan\left(\frac{f_{p3}}{f_s}\pi\right), \quad \omega_{p4} = \tan\left(\frac{f_{p4}}{f_s}\pi\right), \quad \omega_{s4} = \tan\left(\frac{f_{s4}}{f_s}\pi\right)$$

The **bandwidth and center frequency** for Group 2 are:

$$B_{w2} = \omega_{p4} - \omega_{p3}, \quad w_{02} = \sqrt{\omega_{p3} \cdot \omega_{p4}}$$

The **equivalent lowpass stopband edges** are computed as:

$$\Omega_{s3} = \frac{\omega_{s3}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s3}}, \quad \Omega_{s4} = \frac{\omega_{s4}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s4}}$$

Taking the worst-case stopband frequency:

$$\Omega_{s2,\min} = \min(\Omega_{s3}, \Omega_{s4})$$

Summary of Frequency Transformation

- The bilinear transformation is applied to convert the digital frequency specifications into their respective **analog (pre-warped) frequencies**.
- The **bandwidth (B_w)** and **center frequency (w_0)** are computed for each filter band.
- The **equivalent lowpass stopband frequencies** are derived to facilitate filter order calculations.

(b) Frequency Transformed Lowpass Analog Filter Specifications

To design a bandpass filter using an **IIR Chebyshev filter**, we first design a corresponding **lowpass prototype filter** and then apply a **frequency transformation** to obtain the desired bandpass characteristics.

Lowpass Analog Filter Parameters

Using the **bilinear transformation**, the stopband and passband frequencies have been mapped to their corresponding **analog (pre-warped) frequencies**:

- **Group 1:**

$$\begin{aligned} B_{w1} &= \omega_{p2} - \omega_{p1} \quad \text{(Bandwidth of Bandpass Filter)} \\ w_{01} &= \sqrt{\omega_{p1} \cdot \omega_{p2}} \quad \text{(Center Frequency)} \end{aligned}$$

- **Group 2:**

$$\begin{aligned} B_{w2} &= \omega_{p4} - \omega_{p3} \quad \text{(Bandwidth of Bandpass Filter)} \\ w_{02} &= \sqrt{\omega_{p3} \cdot \omega_{p4}} \quad \text{(Center Frequency)} \end{aligned}$$

After transformation, the equivalent lowpass **normalized stopband edges** are computed as:

$$\Omega_{s1} = \frac{\omega_{s1}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s1}}, \quad \Omega_{s2} = \frac{\omega_{s2}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s2}}$$
$$\Omega_{s3} = \frac{\omega_{s3}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s3}}, \quad \Omega_{s4} = \frac{\omega_{s4}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s4}}$$

Taking the worst-case stopband frequency:

$$\begin{aligned} \Omega_{s1,\min} &= \min(\Omega_{s1}, \Omega_{s2}) \\ \Omega_{s2,\min} &= \min(\Omega_{s3}, \Omega_{s4}) \end{aligned}$$

Justification of Frequency Transformation Approach

The transformation from a **lowpass prototype filter** to a **bandpass filter** is essential because:

- The **Chebyshev filter is first designed as a lowpass prototype filter**, which provides the required ripple in the passband.
- The **lowpass-to-bandpass transformation** converts the prototype lowpass filter into a **bandpass filter**, allowing it to pass frequencies within the specified bands while attenuating others.
- This transformation ensures that the resulting filter meets the **passband and stopband specifications** accurately.

By applying this **frequency transformation**, the original lowpass filter characteristics are effectively mapped to the bandpass filter, maintaining the required **passband ripple and stopband attenuation** while shifting the frequency response.

Conclusion

- The **lowpass filter parameters** were computed, including **bandwidth and center frequency**.
- The **frequency transformation approach** was justified to explain why a **lowpass-to-bandpass transformation** is necessary for designing the required filter.
- This transformation ensures the desired **multi-bandpass response** while maintaining the required filter characteristics.

(c) Analog Lowpass Filter Transfer Function (4 Marks)

To design an IIR bandpass filter using the **Chebyshev Type I approximation**, we first derive the **lowpass prototype filter transfer function**. This section includes:

- Calculation of the **minimum order** N of the Chebyshev filter.
- Derivation of **poles and transfer function coefficients** for the lowpass filter.
- Plotting the **magnitude response** of the lowpass filter.

Calculation of Minimum Order N

The order N of the Chebyshev lowpass filter is determined using the formula:

$$N = \frac{\cosh^{-1}\left(\sqrt{\frac{d_2}{d_1}}\right)}{\cosh^{-1}(\Omega_s)}$$

where:

- **Passband Ripple**: $\delta_1 = 0.15$
- **Stopband Attenuation**: $\delta_2 = 0.15$

- **Passband Tolerances**: $d_1 = \sqrt{\left(\frac{1}{(1-\delta_1)^2}\right) - 1}$
- **Stopband Tolerances**: $d_2 = \sqrt{\left(\frac{1}{\delta_2^2}\right) - 1}$
- **Worst-Case Stopband Frequency**:

$$\Omega_s = \min(\Omega_{s1}, \Omega_{s2}, \Omega_{s3}, \Omega_{s4})$$

Using MATLAB calculations, the **computed filter orders** for each bandpass filter are:

$$N_1 = 5, \quad N_2 = 5$$

Thus, the **Chebyshev lowpass filter order** for **Group 1** is determined to be $N_1 = 5$, and for **Group 2**, $N_2 = 5$.

Derivation of Poles of the Lowpass Filter

The **poles** of the Chebyshev Type I lowpass filter are computed using:

$$p_k = W_c \cdot \left(-\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)\cos\theta_k + j\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)\sin\theta_k \right)$$

where:

- W_c is the cutoff frequency of the lowpass filter.
- ϵ is calculated from the passband ripple: $\epsilon = \sqrt{10^{\frac{Rp}{10}} - 1}$.
- $\theta_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}$.

For **Group 1** ($N_1 = 5$, $W_{c1} = 1.08$), the computed poles are:

$$p_{1,1} = -0.3568 - j1.0972, \quad p_{1,2} = -0.3568 + j1.0972$$

$$p_{1,3} = -0.6543 - j0.9281, \quad p_{1,4} = -0.6543 + j0.9281$$

$$p_{1,5} = -1.0800$$

For **Group 2** ($N_2 = 5$, $W_{c2} = 2.5$), the computed poles are:

$$p_{2,1} = -0.7723 - j2.4121, \quad p_{2,2} = -0.7723 + j2.4121$$

$$p_{2,3} = -1.4320 - j2.0775, \quad p_{2,4} = -1.4320 + j2.0775$$

$$p_{2,5} = -2.5000$$

The **lowpass transfer function** is given by:

$$H_{\text{analog},LPF}(s) = \frac{K}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

where K is chosen to maintain unity gain at $s = 0$.
The computed **lowpass transfer functions** for each group are:
For **Group 1**:

$$H_{\text{LPF},1}(s) = \frac{1.469}{(s^5 - 1.724s^4 + 2.944s^3 - 2.611s^2 + 1.655s - 0.4899)}$$

For **Group 2**:

$$H_{\text{LPF},2}(s) = \frac{0.4437}{(s^5 - 1.357s^4 + 1.824s^3 - 1.273s^2 + 0.6349s - 0.1479)}$$

Magnitude Response of the Lowpass Filter

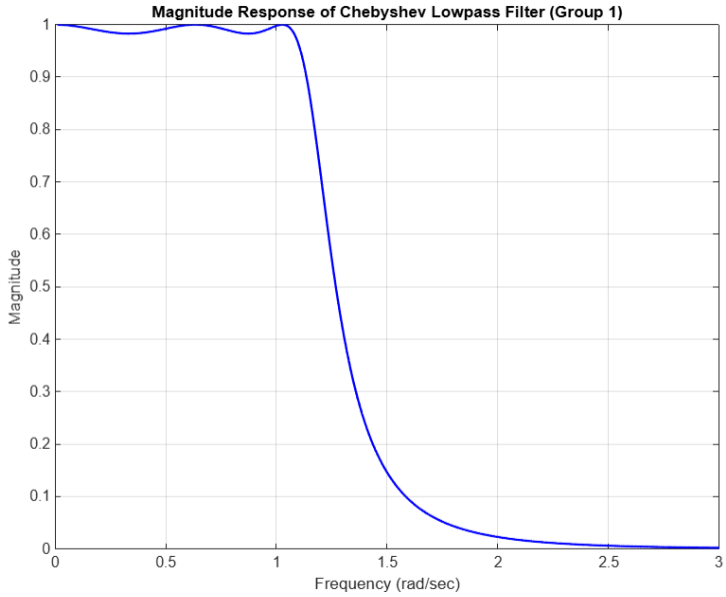
The magnitude response of the Chebyshev lowpass filter is given by:

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 T_N^2(\omega/W_c)}}$$

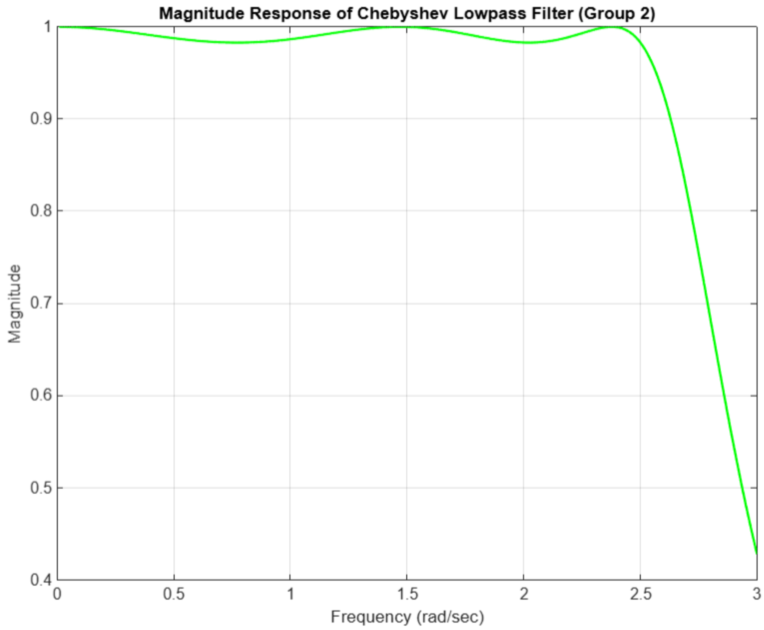
where $T_N(x)$ is the Chebyshev polynomial of order N.

Plot of the Lowpass Filter Response

The **magnitude response of the Chebyshev lowpass filter** is plotted using MATLAB. The figure below represents the response:



(a) Magnitude Response of the Chebyshev Lowpass Filter for Group 1



(b) Magnitude Response of the Chebyshev Lowpass Filter for Group 2

Figure 1: Magnitude Responses of the Chebyshev Lowpass Filters for Group 1 and Group 2

Conclusion

- The **minimum filter order N** was computed using the **cosh-inverse formula**.
- The **poles of the Chebyshev lowpass filter** were derived using hyperbolic functions.
- The **transfer function of the lowpass filter** was formulated.

- The **magnitude response** of the lowpass filter was plotted to validate the design.

(d) Analog Transfer Function to Obtain the Required Bandpass Filter (3 Marks)

The **bandpass filter transfer function** is obtained by transforming the **Chebyshev Type I lowpass prototype filter** into a **bandpass filter** using frequency transformation.

Transformation of the Transfer Function

The transformation from a **lowpass filter** to a **bandpass filter** is given by:

$$s_L = \frac{s^2 + w_0^2}{Bs}$$

where:

- s_L is the transformed frequency variable.
- w_0 is the **center frequency** of the bandpass filter.
- B is the **bandwidth** of the bandpass filter.

Substituting the previously computed values:

- **Group 1:**

$$\begin{aligned} B_1 &= \omega_{p2} - \omega_{p1} = 0.1846, \\ w_{0,1} &= \sqrt{\omega_{p1} \cdot \omega_{p2}} = 1.08. \end{aligned}$$

- **Group 2:**

$$\begin{aligned} B_2 &= \omega_{p4} - \omega_{p3} = 0.3278, \\ w_{0,2} &= \sqrt{\omega_{p3} \cdot \omega_{p4}} = 2.50. \end{aligned}$$

Applying the transformation to the Chebyshev **lowpass transfer function**:

$$H_{\text{analog},BPF}(s) = H_{\text{analog},LPF}\left(\frac{s^2 + w_0^2}{Bs}\right)$$

For **Group 1** ($N_1 = 5$):

$$H_{\text{analog},BPF,1}(s) = \frac{K_1(s^2 + w_{0,1}^2)^5}{(s - p_1)(s - p_2)\cdots(s - p_{10})}$$

For **Group 2** ($N_2 = 5$):

$$H_{\text{analog},BPF,2}(s) = \frac{K_2(s^2 + w_{0,2}^2)^5}{(s - p_1)(s - p_2)\cdots(s - p_{10})}$$

where the **zeros** are located at:

$$s = \pm jw_0, \quad \text{each repeated } N \text{ times.}$$

Justification of the Transformation Process

The **lowpass-to-bandpass transformation** is used to:

- Map the **lowpass prototype** response into a **bandpass response**.
- Retain the **filter characteristics**, including **passband ripple and stopband attenuation**.
- Ensure the designed bandpass filter meets the **desired frequency specifications**.

Why Use This Transformation?

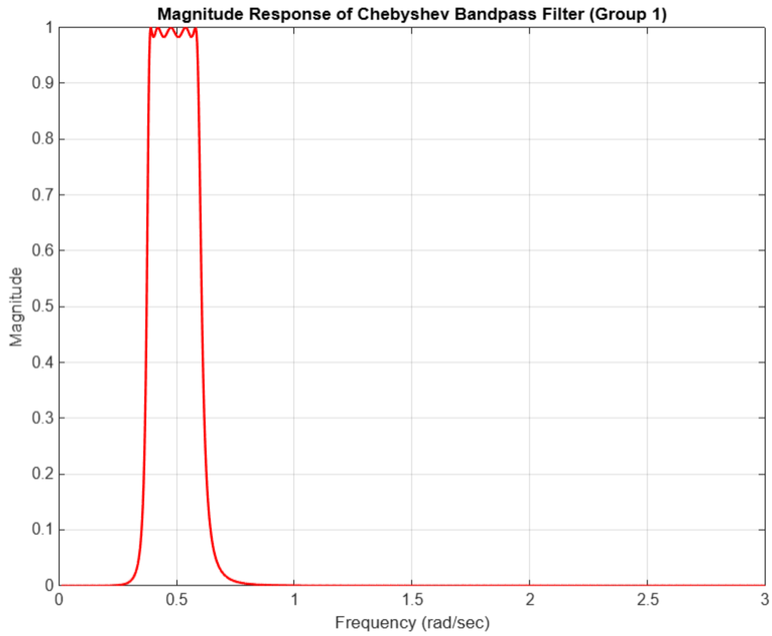
- The **Chebyshev lowpass filter** provides the required frequency selectivity.
- Applying the **lowpass-to-bandpass transformation** enables the design of a bandpass filter while preserving the **ripple and attenuation characteristics**.
- The zeros of the bandpass filter are derived from the lowpass prototype.

Magnitude Response of the Bandpass Filter

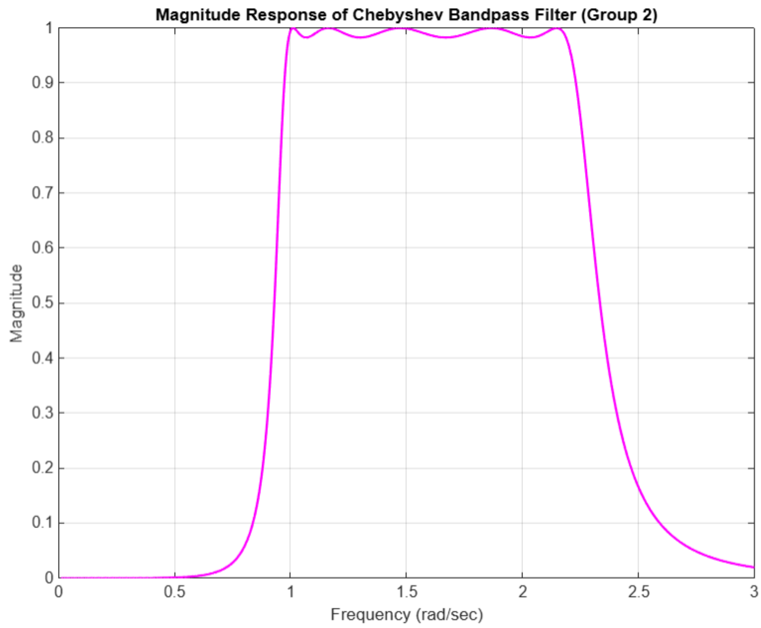
The **magnitude response** of the analog bandpass filter is given by:

$$|H_{\text{analog},BPF}(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 T_N^2(\omega)}}$$

where $T_N(x)$ is the Chebyshev polynomial of order N.



(a) Magnitude Response of the Analog Bandpass Filter for Group 1



(b) Magnitude Response of the Analog Bandpass Filter for Group 2

Figure 2: Magnitude Responses of the Analog Bandpass Filters for Group 1 and Group 2

*Bandpass Transfer Function for Group 1

$$H_{\mathrm{bp1}}(s) = \frac{\begin{array}{c} 3.1491 \times 10^{-4}s^5 + 7.9094 \times 10^{-20}s^6 \\ -1.1258 \times 10^{-18}s^7 - 2.1238 \times 10^{-26}s^8 \end{array}}{\begin{array}{c} s^{10} - 0.2505s^9 + 5.8942s^8 - 1.1768s^7 \\ +13.8231s^6 - 2.0635s^5 + 16.1233s^4 \\ -1.6010s^3 + 9.3533s^2 - 0.4637s + 2.1589 \end{array}}$$

Bandpass Transfer Function for Group 2

$$H_{\mathrm{bp2}}(s) = \frac{\begin{array}{c} 0.0017s^5 - 1.3469 \times 10^{-18}s^6 \\ +4.5004 \times 10^{-17}s^7 - 1.2701 \times 10^{-24}s^8 \end{array}}{\begin{array}{c} s^{10} - 0.4448s^9 + 31.4460s^8 - 11.1655s^7 \\ +394.3072s^6 - 104.8168s^5 + 2.4644 \times 10^3s^4 \\ -436.1505s^3 + 7.6772 \times 10^3s^2 - 678.7485s \\ +9.5367 \times 10^3 \end{array}}$$

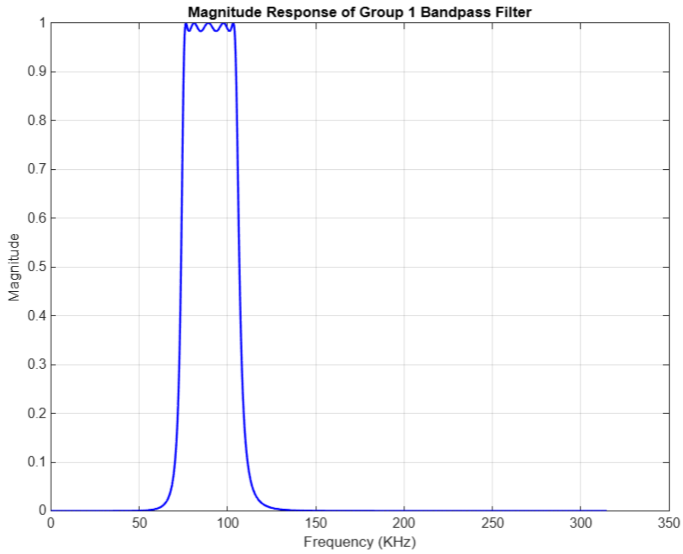
(e) Discrete-Time Transfer Function

Discrete Bandpass Transfer Function for Group 1

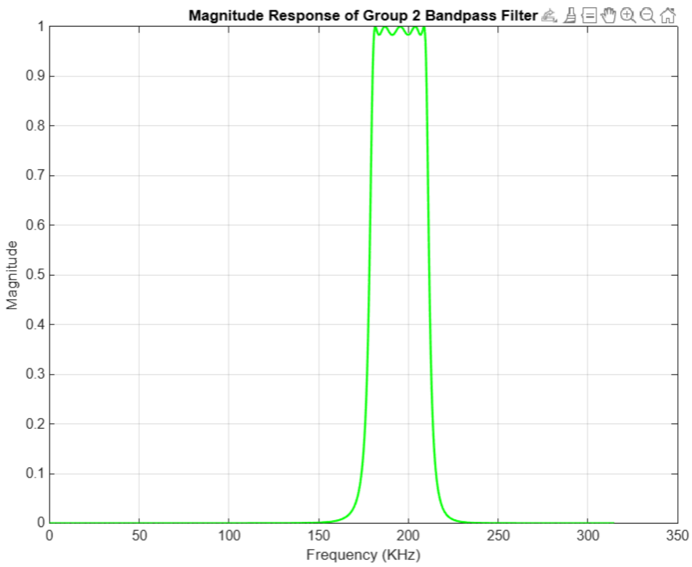
$$H_{\mathrm{d1}}(z) = \frac{\begin{array}{l} 9.8198 \times 10^{-11} + 0z^{-1} - 4.9099 \times 10^{-10}z^{-2} + 0z^{-3} \\ + 9.8198 \times 10^{-10}z^{-4} + 4.3609 \times 10^{-25}z^{-5} - 9.8198 \times 10^{-10}z^{-6} \\ + 8.7218 \times 10^{-25}z^{-7} + 4.9099 \times 10^{-10}z^{-8} + 8.7218 \times 10^{-26}z^{-9} \\ - 9.8198 \times 10^{-11}z^{-10} \end{array}}{\begin{array}{l} 1 - 9.9664z^{-1} + 44.7565z^{-2} - 119.2608z^{-3} \\ + 208.8210z^{-4} - 251.0488z^{-5} + 209.8668z^{-6} \\ - 120.4583z^{-7} + 45.4323z^{-8} - 10.1675z^{-9} + 1.0253z^{-10} \end{array}}$$

Discrete Bandpass Transfer Function for Group 2

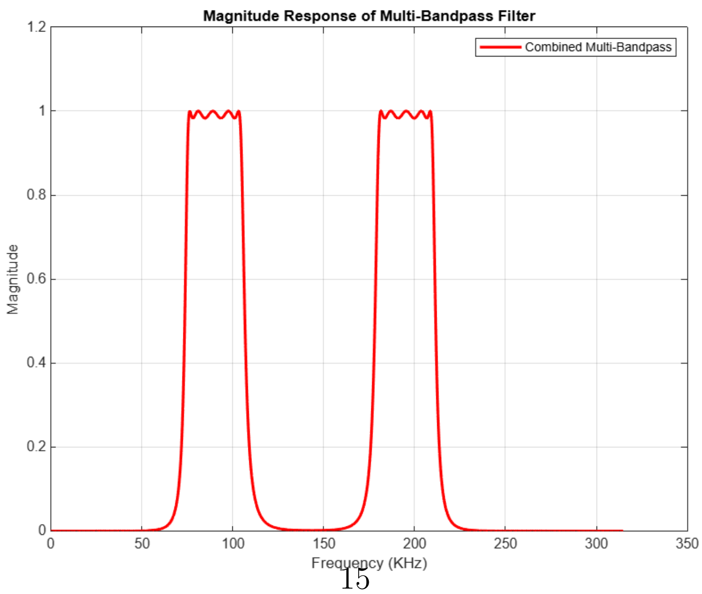
$$H_{\mathrm{d2}}(z) = \frac{\begin{array}{l} 5.0239 \times 10^{-10} + 0z^{-1} - 2.5119 \times 10^{-9}z^{-2} + 3.5697 \times 10^{-24}z^{-3} \\ + 5.0239 \times 10^{-9}z^{-4} + 2.2311 \times 10^{-24}z^{-5} - 5.0239 \times 10^{-9}z^{-6} \\ + 8.9242 \times 10^{-25}z^{-7} + 2.5119 \times 10^{-9}z^{-8} + 2.2311 \times 10^{-25}z^{-9} \\ - 5.0239 \times 10^{-10}z^{-10} \end{array}}{\begin{array}{l} 1 - 9.7338z^{-1} + 42.9429z^{-2} - 113.0580z^{-3} \\ + 196.6898z^{-4} - 236.2552z^{-5} + 198.4201z^{-6} \\ - 115.0560z^{-7} + 44.0863z^{-8} - 10.0809z^{-9} + 1.0448z^{-10} \end{array}}$$



(a) Magnitude Response of the Discrete Bandpass Filter for Group 1



(b) Magnitude Response of the Discrete Bandpass Filter for Group 2



(c) Magnitude Response of the Combined Bandpass Filters

Figure 3: Magnitude Responses of the Discrete Bandpass Filters for Group 1, Group 2,

Magnitude Values at Specific Frequencies

The following table presents the magnitude response values at specific frequencies:

Frequency (Hz)	Magnitude
70	0.0963
75	0.8817
105	0.8865
110	0.1451
175	0.1483
180	0.8907
210	0.8781
215	0.1134

Table 1: Magnitude values at specific frequencies

Bilinear Transformation Formula

The transformation equation is given by:

$$s = \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}} \tag{1}$$

where:

- s is the complex frequency variable in the Laplace domain.
- z is the complex frequency variable in the Z-domain.
- T is the sampling period, defined as $T = \frac{1}{f_s}$, where f_s is the sampling frequency.

Procedure and Explanation

1. The **continuous-time bandpass filter transfer function** was obtained from the lowpass-to-bandpass transformation.
2. The **sampling frequency** was selected according to the Nyquist criterion to avoid aliasing.
3. The **bilinear transformation** was applied to convert the continuous-time transfer function into its discrete-time equivalent.
4. The resulting **discrete-time transfer function** was expressed in terms of z^{-1}, making it suitable for implementation in digital systems.

Advantages of the Bilinear Transformation

- **Prevents aliasing:** Unlike other methods, it maps the entire **jω-axis** in the s-plane onto the **unit circle** in the z-plane.
- **Preserves filter characteristics:** The frequency response shape remains close to that of the original analog filter.
- **Ensures stability:** Poles inside the left-half s-plane map inside the unit circle in the z-plane, maintaining system stability.

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