FIR FILTER DESIGN

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Filter Number: 79

Contents

1 Problem Statement

Construct a discrete time filter which allows frequencies in two different bands. The filter is an FIR Multi-Band pass Filter. Bandpass filters have transition bands of 5kHz on either side of each passband. Both passband and stopband are monotonic. Bandstop filters have transition bands of 5kHz on either side of each stopband. The filter magnitude response has passband and stopband tolerances of 0.15 each. The given signal is bandlimited to 280 kHz and ideally sampled at 600 kHz.

2 Preliminaries

The filter number assigned: $M = 39$

$$M = 11Q + R \Rightarrow Q = 7, R = 2$$

The passband frequency ranges (in kHz) are given in two groups (for parameter D):

- Group I: (40 + 5D) to (70 + 5D), where $D = Q$
- Group II: (170 + 5D) to (200 + 5D), where $D = R$

Therefore, the passband ranges are:

- 75 kHz to 105 kHz
- 180 kHz to 210 kHz

Sampling rate is 630 kHz. Hence, the maximum frequency component is 280 kHz.
This filter is implemented using a **parallel cascade** structure:

- A bandpass filter with a passband from 75 kHz to 105 kHz
- A bandpass filter with a passband from 180 kHz to 210 kHz

The final FIR filter is obtained by summing the outputs of these two bandpass filters. This configuration allows the filter to pass two distinct frequency bands while attenuating all others.

3 Band-Pass Filter:Group-1

3.1 Unnormalized Discrete Time Filter Specifications

1. Passband: 75 to 105 kHz
2. Stopband: 0 to 70 kHz and 110 to 280 kHz
3. Passband Tolerance: 0.15
4. Stopband Tolerance: 0.15

3.2 Normalized Discrete Time Filter Specifications

Sampling rate = 630 kHz. On the normalized frequency axis, the sampling rate corresponds to 2π. Therefore, any frequency can be normalized as:

$$\omega = \frac{\Omega \cdot 2\pi}{\Omega_s}$$

$$\omega_{c1} = \frac{(f_{s1} + f_{p1}) \cdot 2\pi}{2 \cdot f_{\text{samp}}} = \frac{(70\,\text{kHz} + 75\,\text{kHz}) \cdot 2\pi}{2 \cdot 630\,\text{kHz}} = 0.575\,\text{rad}$$

$$\omega_{c2} = \frac{(f_{p2} + f_{s2}) \cdot 2\pi}{2 \cdot f_{\text{samp}}} = \frac{(105\,\text{kHz} + 110\,\text{kHz}) \cdot 2\pi}{2 \cdot 630\,\text{kHz}} = 0.861\,\text{rad}$$

- Passband: $0.723\,\text{rad}$ to $1.072\,\text{rad}$
- Stopband: 0 to $0.723\,\text{rad}$ and $1.072\,\text{rad}$ to $\pi\,\text{rad}$

3.3 Designing FIR Band-pass

Given $\delta = 0.15$:

$$A = -20\log_{10}(\delta) = 16.478\,\text{dB}$$

$$\beta = \begin{cases} 0, & \text{if } A < 21 \\ 0.5842 \cdot (A-21)^{0.4} + 0.07886 \cdot (A-21), & \text{if } 21 \leq A < 51 \\ 0.1102 \cdot (A - 8.7), & \text{if } A \geq 51 \end{cases}$$

Since $A < 21$, we use:

$$\beta = 0$$

The transition bandwidth is calculated as:

$$\Delta\omega_T = \frac{5 \times 10^3 \cdot 2\pi}{630 \times 10^3} = 0.0159\pi$$

Using the empirical formula for minimum filter order:

$$2N \geq \frac{A - 7.95}{2.285 \cdot \Delta\omega_T} = \frac{16.4782}{2.285 \cdot 0.0159\pi} \approx 145.75 \Rightarrow N \approx 74$$

Rounding up:

$$N_{\text{min}} = 74$$

Using trial and error for optimal performance, we finally select:

$$N = 99$$

The time domain coefficients were obtained by:

- Generating ideal impulse responses of two LPFs
- Subtracting them to get BPF
- Applying Kaiser Window using MATLAB

3.4 Figures

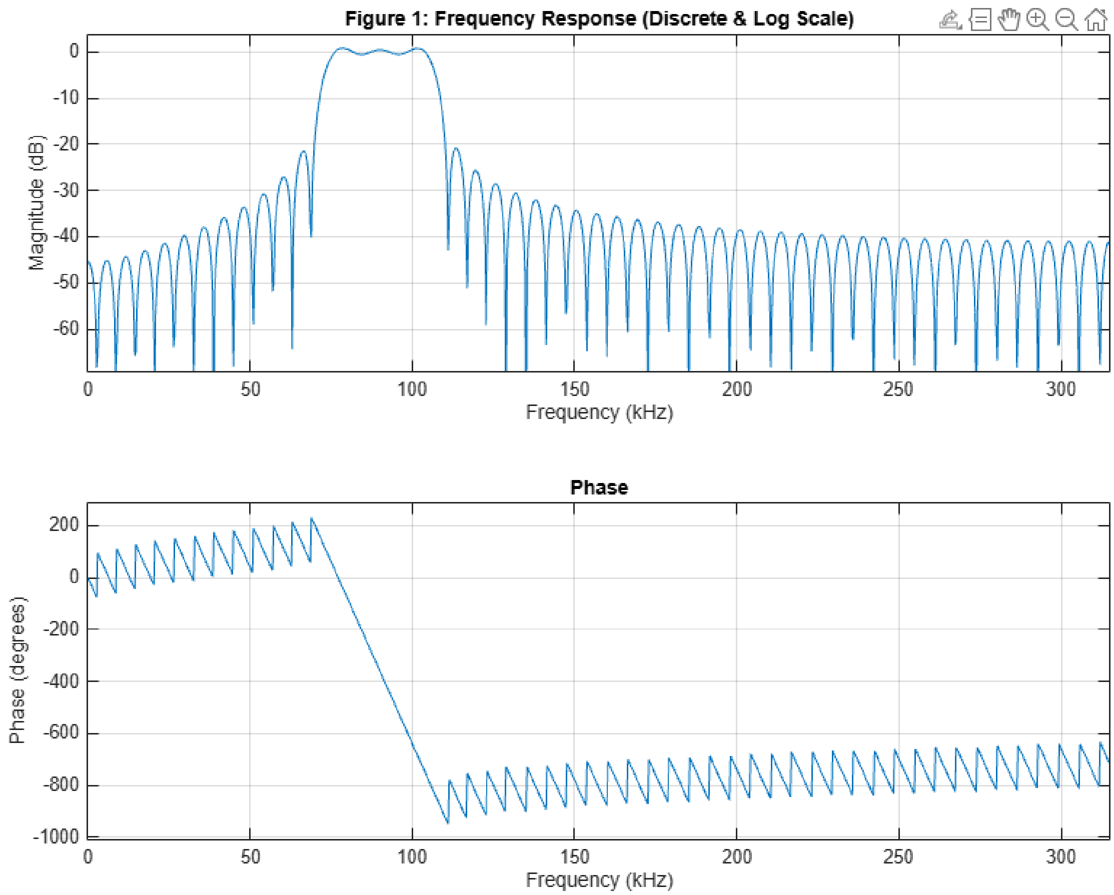


Figure 1: Frequency Response (Discrete and Log Scale)

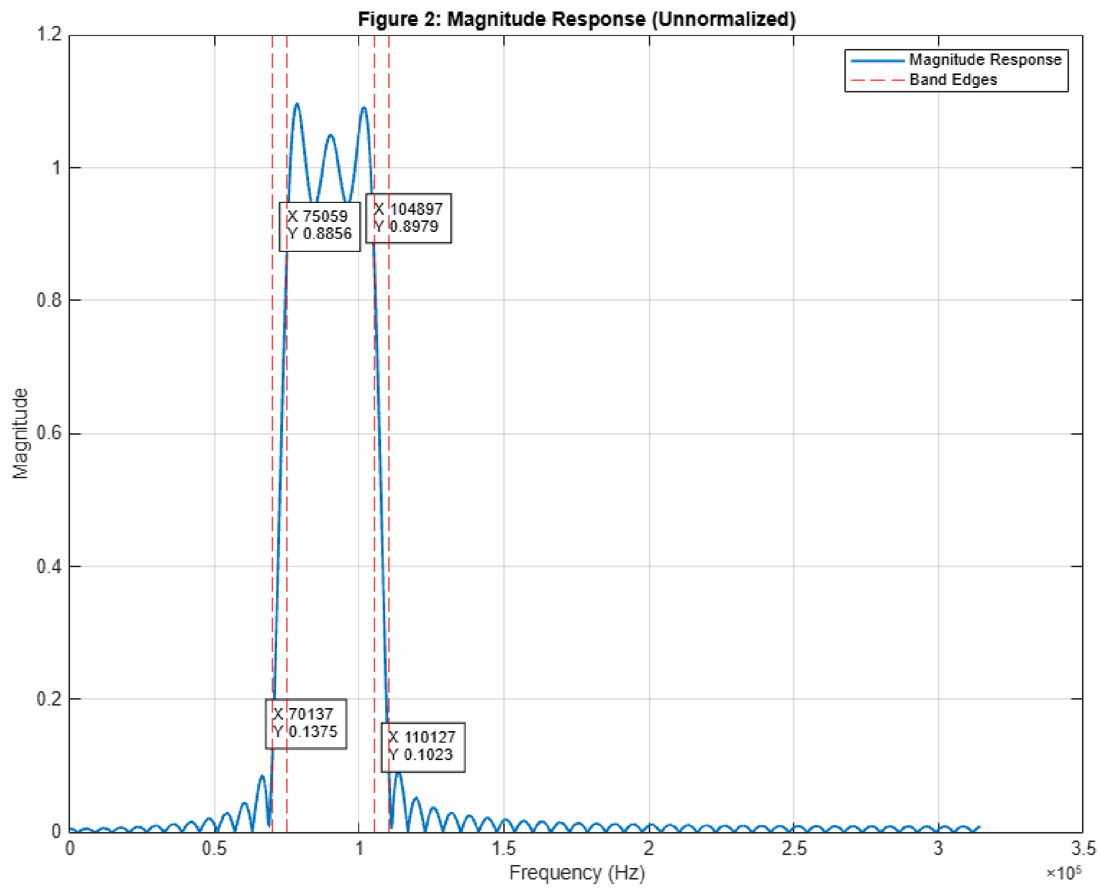


Figure 2: Magnitude Response (Unnormalized)

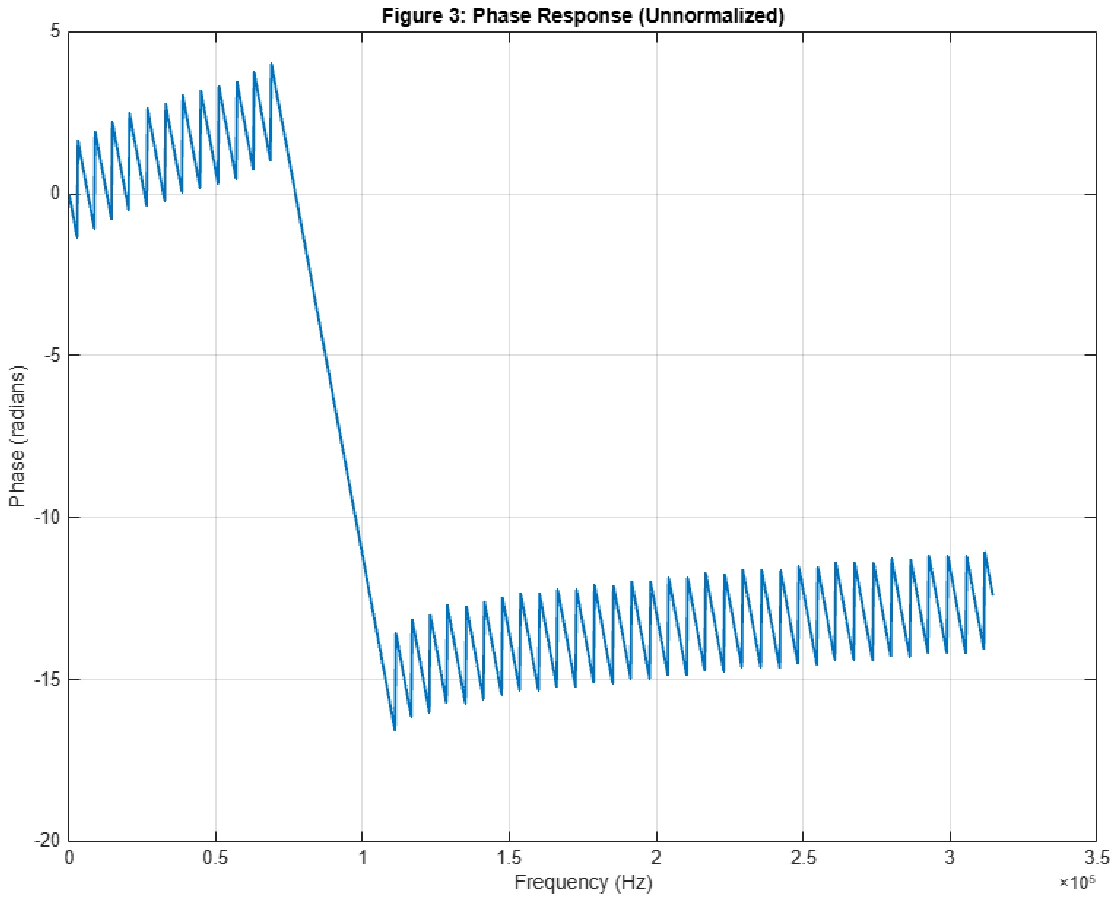


Figure 3: Phase Response (Unnormalized)

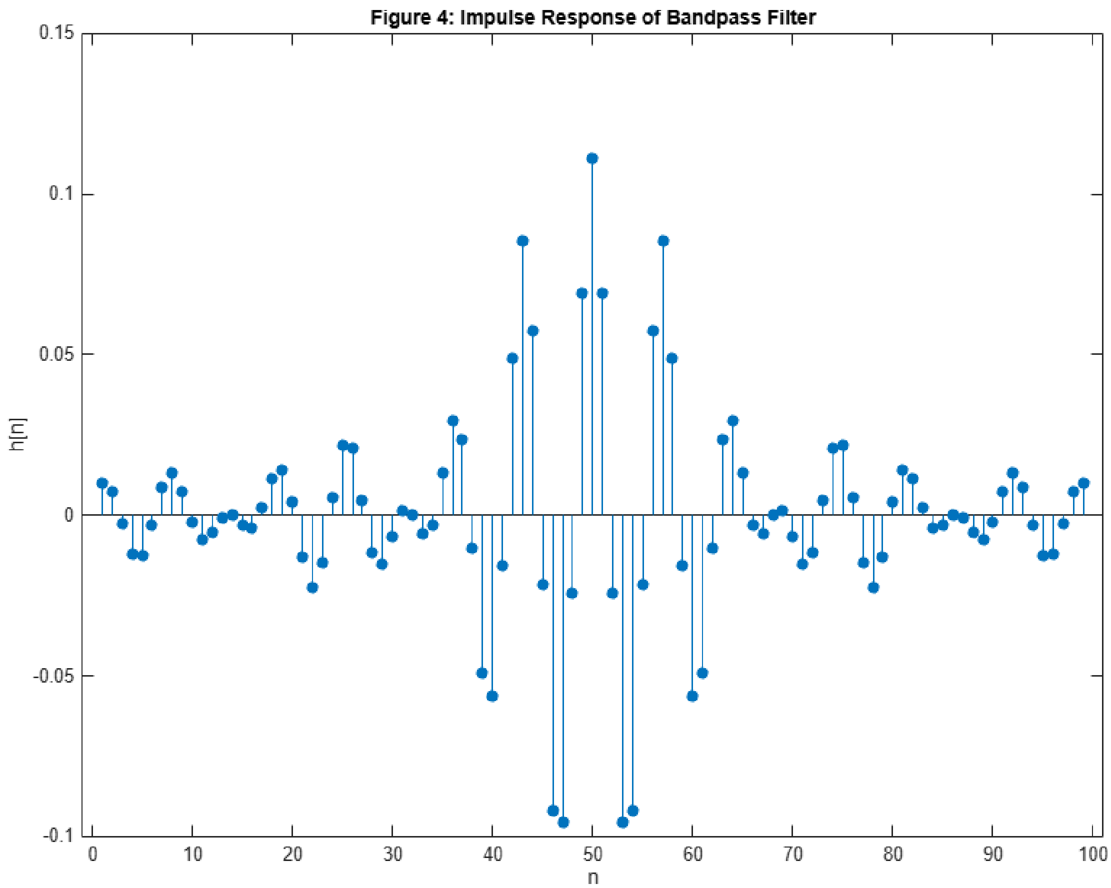


Figure 4: Impulse Response of Bandpass Filter

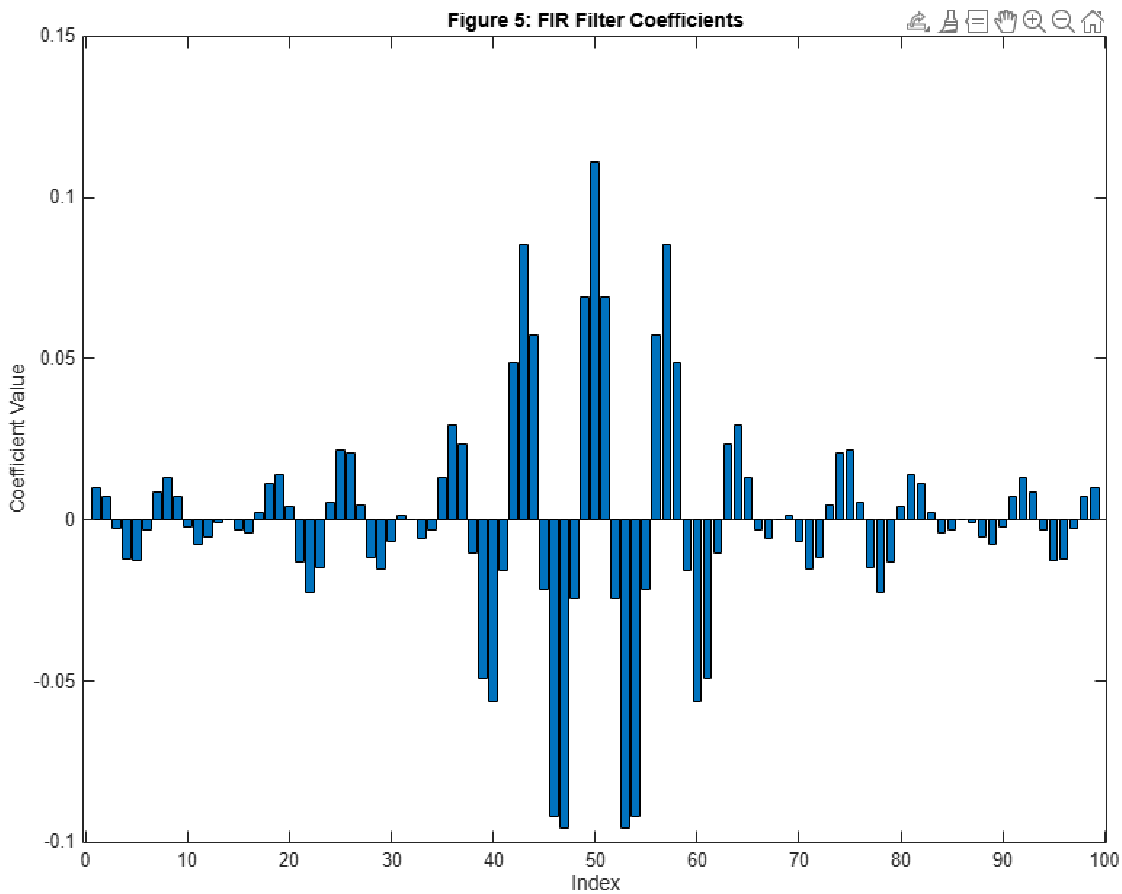


Figure 5: FIR Filter Coefficients

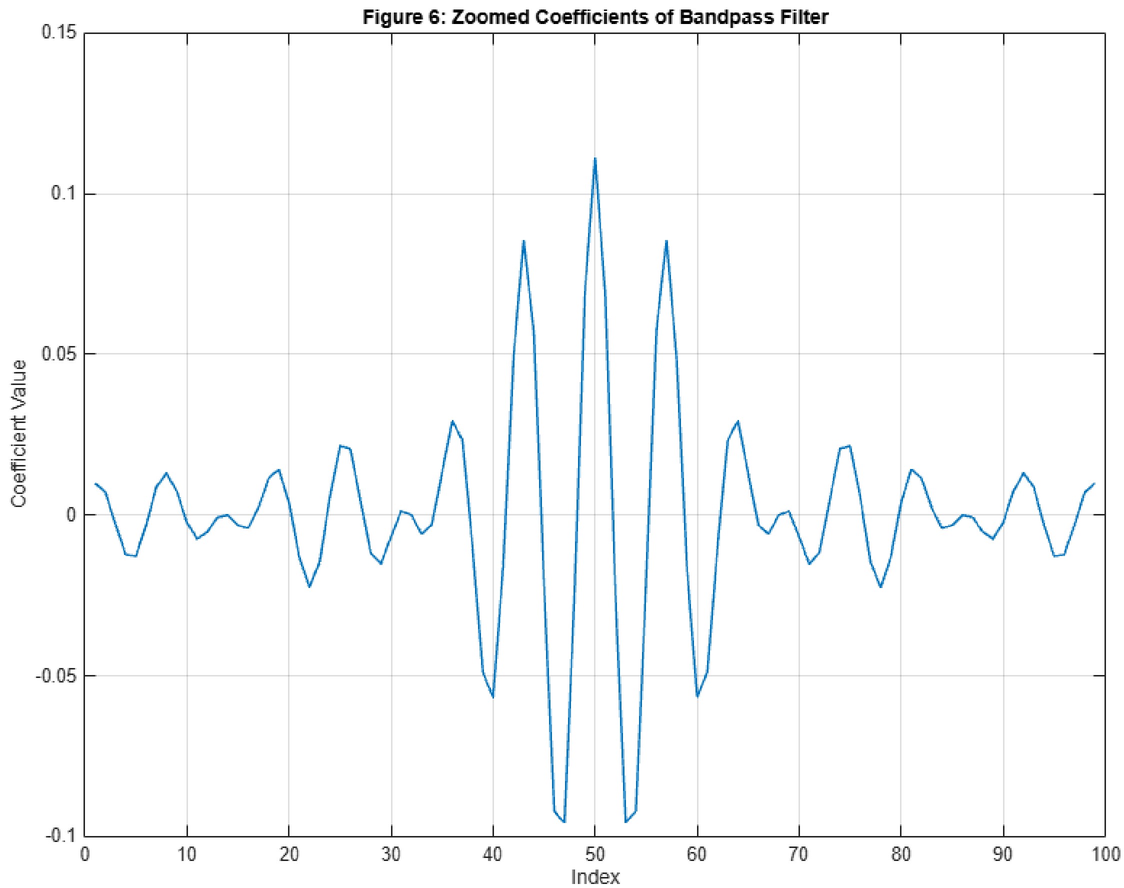


Figure 6: Zoomed Coefficients of Bandpass Filter

```
N = 99
FIR Filter Coefficients(Group-1):
  Columns 1 through 14

    0.0100    0.0072   -0.0028   -0.0123   -0.0127   -0.0032    0.0087    0.0131    0.0074   -0.0023   -0.0074   -0.0052   -0.0007   -0.0000

  Columns 15 through 28

   -0.0032   -0.0040    0.0021    0.0115    0.0142    0.0041   -0.0129   -0.0224   -0.0147    0.0054    0.0216    0.0207    0.0047   -0.0116

  Columns 29 through 42

   -0.0152   -0.0068    0.0013   -0.0000   -0.0059   -0.0030    0.0132    0.0292    0.0234   -0.0102   -0.0490   -0.0565   -0.0157    0.0489

  Columns 43 through 56

    0.0855    0.0573   -0.0217   -0.0922   -0.0956   -0.0242    0.0689    0.1111    0.0689   -0.0242   -0.0956   -0.0922   -0.0217    0.0573

  Columns 57 through 70

    0.0855    0.0489   -0.0157   -0.0565   -0.0490   -0.0102    0.0234    0.0292    0.0132   -0.0030   -0.0059   -0.0000    0.0013   -0.0068

  Columns 71 through 84

   -0.0152   -0.0116    0.0047    0.0207    0.0216    0.0054   -0.0147   -0.0224   -0.0129    0.0041    0.0142    0.0115    0.0021   -0.0040

  Columns 85 through 98

   -0.0032   -0.0000   -0.0007   -0.0052   -0.0074   -0.0023    0.0074    0.0131    0.0087   -0.0032   -0.0127   -0.0123   -0.0028    0.0072

  Column 99

    0.0100
```

Figure 7: BandPass Impulse Response Value (Group1)

4 Band-Pass Filter:Group-2

4.1 Unnormalized Discrete Time Filter Specifications

1. Passband: 180 to 210 kHz
2. Stopband: 0 to 175 kHz and 215 to 280 kHz
3. Passband Tolerance: 0.15
4. Stopband Tolerance: 0.15

4.2 Normalized Discrete Time Filter Specifications

Sampling rate = 630 kHz. On the normalized frequency axis, the sampling rate corresponds to 2π. Therefore, any frequency can be normalized as:

$$\omega = \frac{\Omega \cdot 2\pi}{\Omega_s}$$

$$\omega_{c1} = \frac{(f_{s1} + f_{p1}) \cdot 2\pi}{2 \cdot f_{\text{samp}}} = \frac{(175\,\text{kHz} + 180\,\text{kHz}) \cdot 2\pi}{2 \cdot 630\,\text{kHz}} = 1.770\,\text{rad}$$

$$\omega_{c2} = \frac{(f_{p2} + f_{s2}) \cdot 2\pi}{2 \cdot f_{\text{samp}}} = \frac{(210\,\text{kHz} + 215\,\text{kHz}) \cdot 2\pi}{2 \cdot 630\,\text{kHz}} = 2.119\,\text{rad}$$

- Passband: 1.770 rad to 2.119 rad
- Stopband: 0 to 1.770 rad and 2.119 rad to π rad

4.3 Designing FIR Band-pass

Given $\delta = 0.15$:

$$A = -20\log_{10}(\delta) = 16.478\,\text{dB}$$

$$\beta = \begin{cases} 0, & \text{if } A < 21 \\ 0.5842 \cdot (A-21)^{0.4} + 0.07886 \cdot (A-21), & \text{if } 21 \leq A < 51 \\ 0.1102 \cdot (A - 8.7), & \text{if } A \geq 51 \end{cases}$$

Since $A < 21$, we use:

$$\beta = 0$$

The transition bandwidth is calculated as:

$$\Delta\omega_T = \frac{5 \times 10^3 \cdot 2\pi}{630 \times 10^3} = 0.0159\pi$$

Using the empirical formula for minimum filter order:

$$2N \geq \frac{A - 7.95}{2.285 \cdot \Delta\omega_T} = \frac{16.4782}{2.285 \cdot 0.0159\pi} \approx 147.63 \Rightarrow N \approx 75$$

Rounding up:

$$N_{\min} = 75$$

Using trial and error for optimal performance, we finally select:

$$N = 95$$

The time domain coefficients were obtained by:

- Generating ideal impulse responses of two LPFs
- Subtracting them to get BPF
- Applying Kaiser Window using MATLAB

4.4 Figures

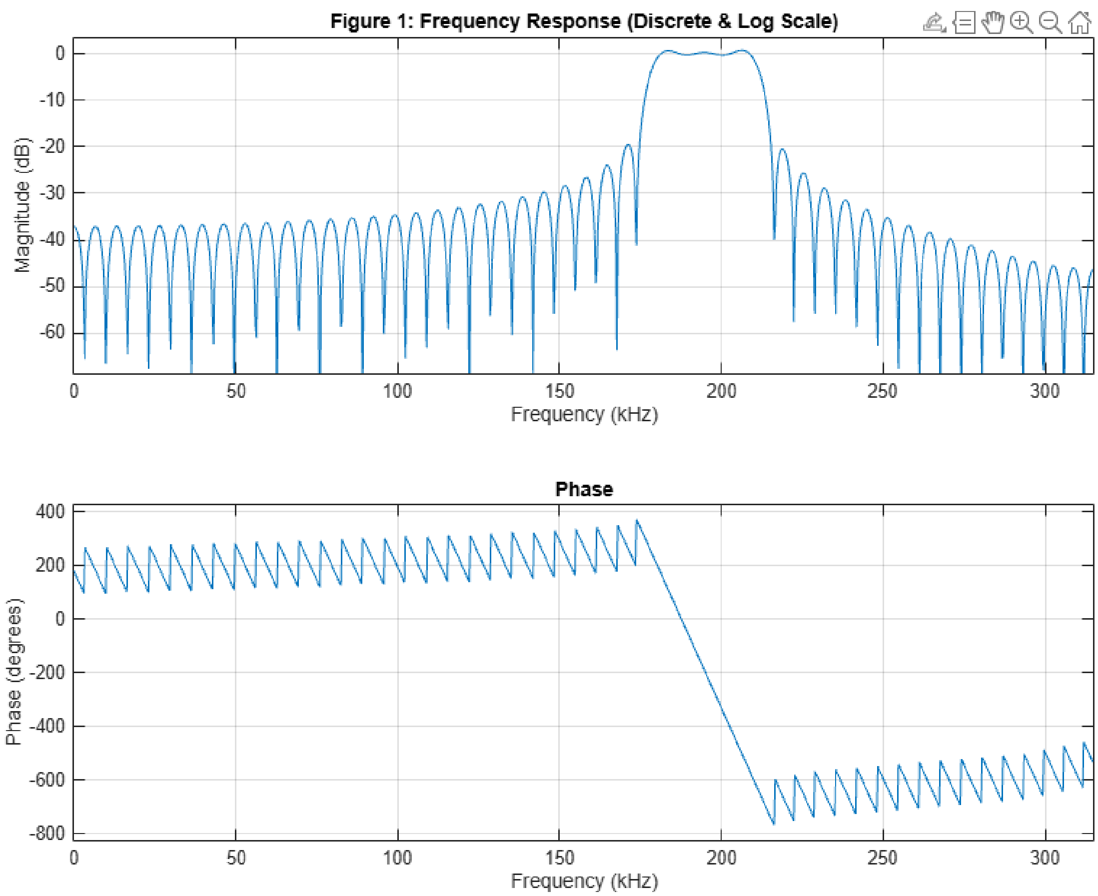


Figure 8: Frequency Response (Discrete and Log Scale)

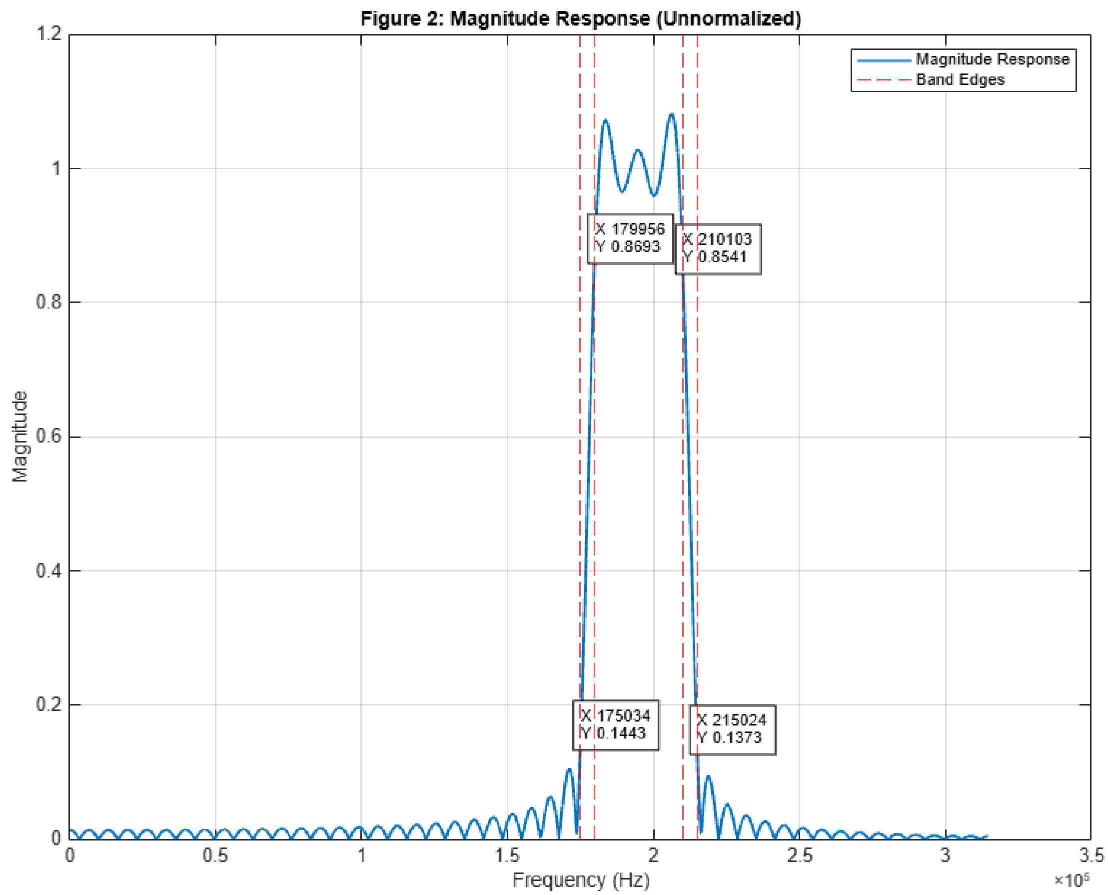


Figure 9: Magnitude Response (Unnormalized)

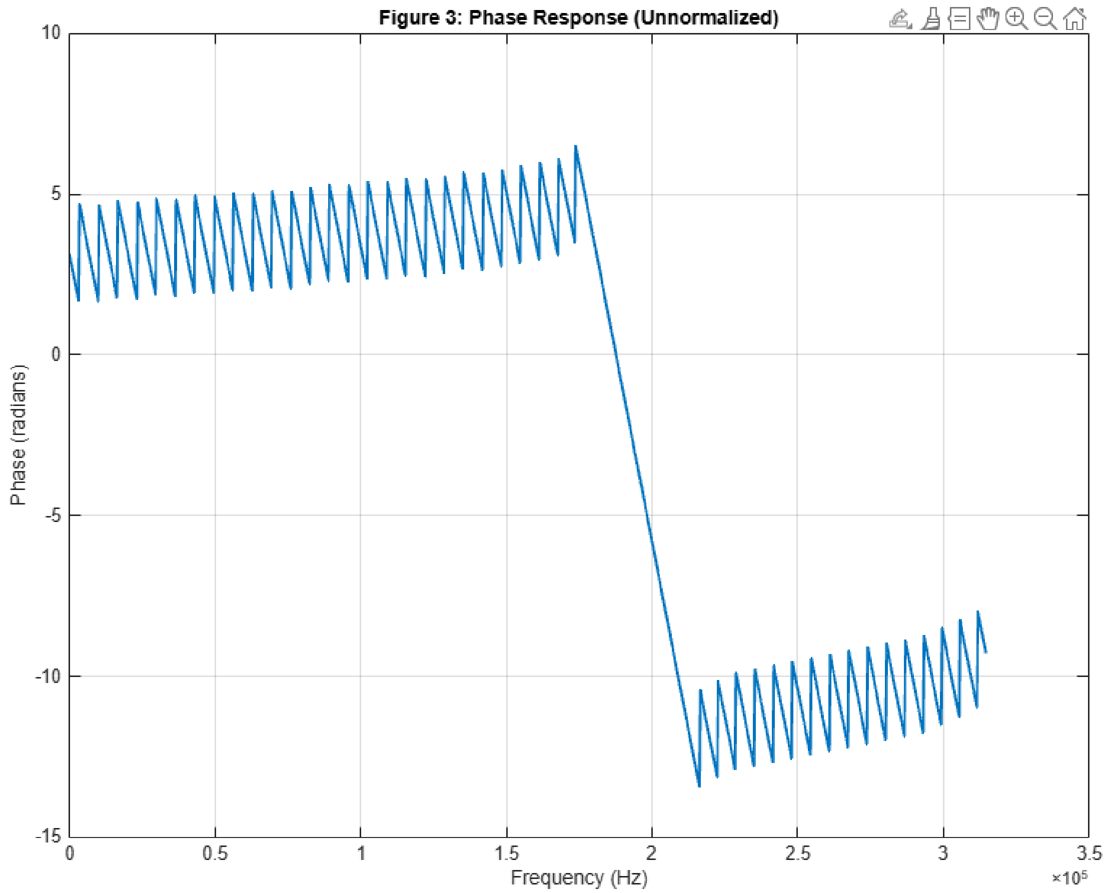


Figure 10: Phase Response (Unnormalized)

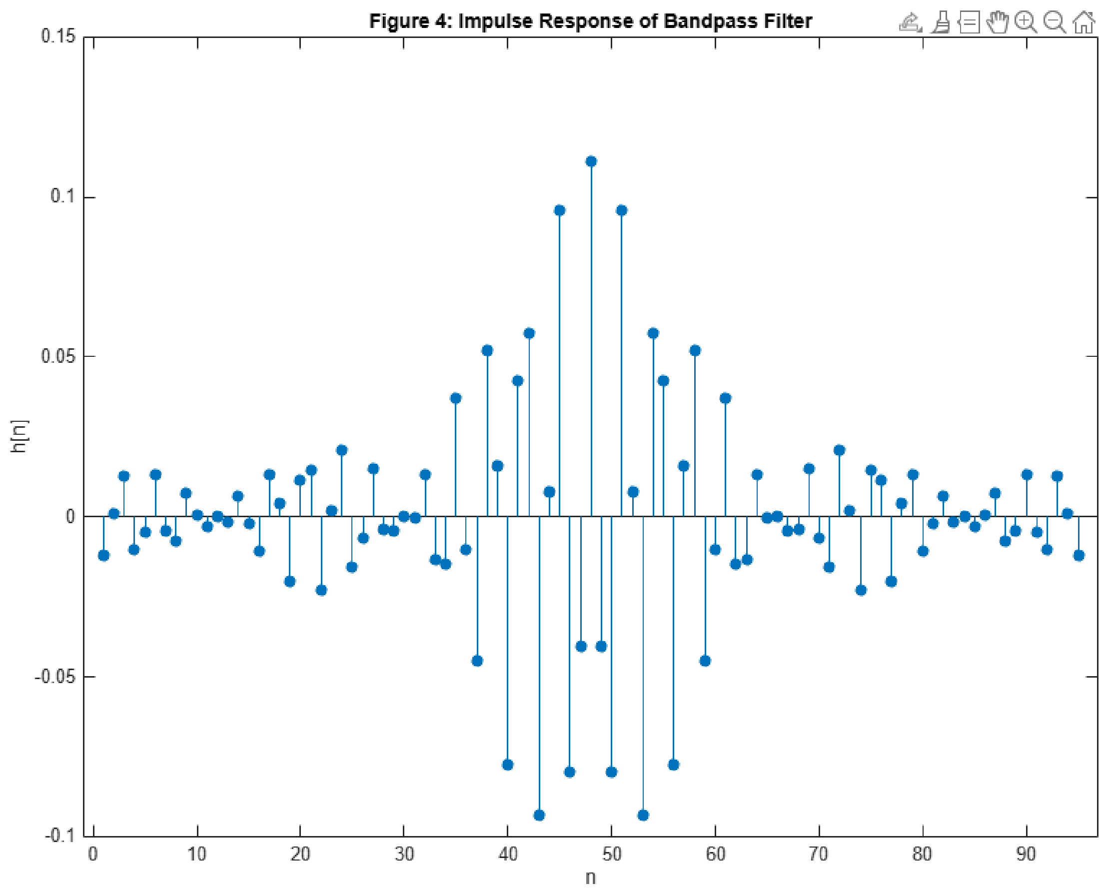


Figure 11: Impulse Response of Bandpass Filter

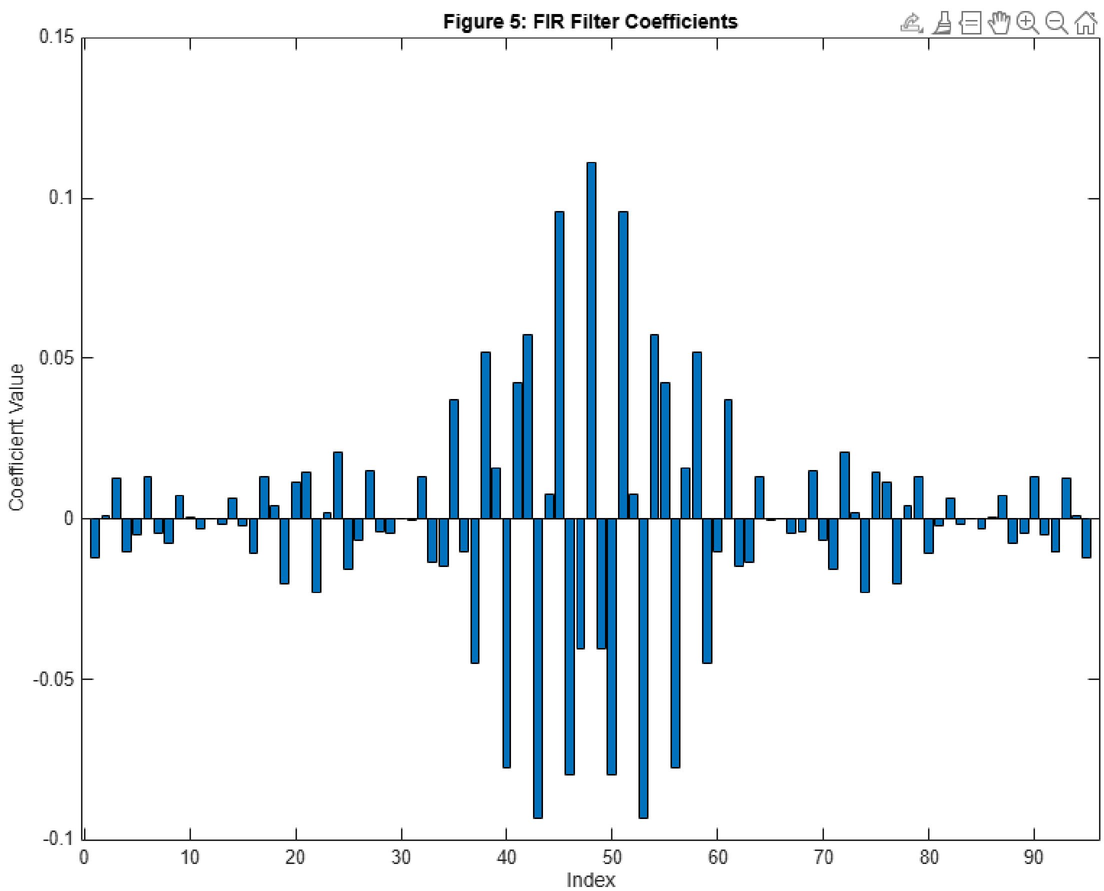


Figure 12: FIR Filter Coefficients

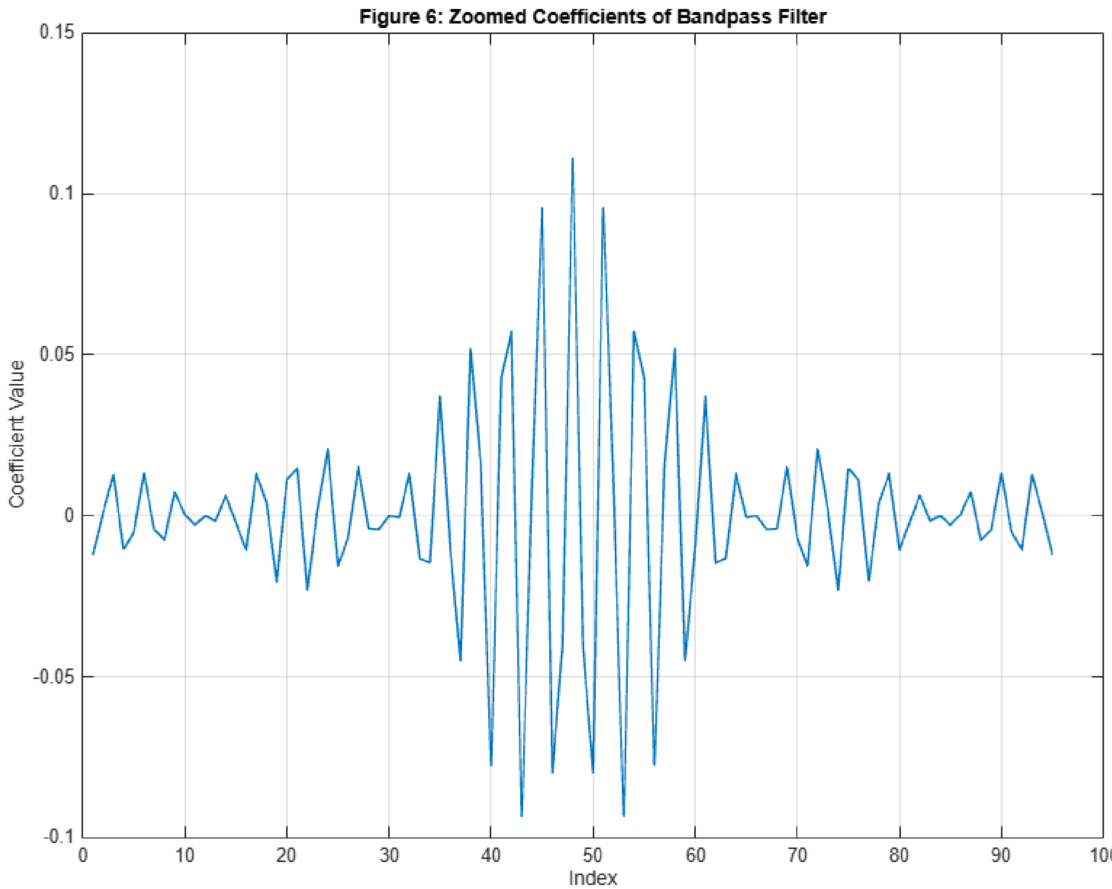


Figure 13: Zoomed Coefficients of Bandpass Filter

```
N = 95
FIR Filter Coefficients(Group-2):
  Columns 1 through 14

   -0.0122    0.0010    0.0127   -0.0104   -0.0051    0.0131   -0.0043   -0.0075    0.0074    0.0004   -0.0029   -0.0000   -0.0016    0.0063

  Columns 15 through 28

   -0.0021   -0.0106    0.0130    0.0041   -0.0204    0.0112    0.0147   -0.0230    0.0018    0.0207   -0.0155   -0.0068    0.0152   -0.0040

  Columns 29 through 42

   -0.0043   -0.0000   -0.0005    0.0130   -0.0132   -0.0146    0.0371   -0.0102   -0.0449    0.0518    0.0157   -0.0775    0.0427    0.0573

  Columns 43 through 56

   -0.0932    0.0076    0.0956   -0.0798   -0.0404    0.1111   -0.0404   -0.0798    0.0956    0.0076   -0.0932    0.0573    0.0427   -0.0775

  Columns 57 through 70

    0.0157    0.0518   -0.0449   -0.0102    0.0371   -0.0146   -0.0132    0.0130   -0.0005   -0.0000   -0.0043   -0.0040    0.0152   -0.0068

  Columns 71 through 84

   -0.0155    0.0207    0.0018   -0.0230    0.0147    0.0112   -0.0204    0.0041    0.0130   -0.0106   -0.0021    0.0063   -0.0016   -0.0000

  Columns 85 through 95

   -0.0029    0.0004    0.0074   -0.0075   -0.0043    0.0131   -0.0051   -0.0104    0.0127    0.0010   -0.0122
```

Figure 14: BandPass Impulse Response Value (Group2)

5 Net Filter Response

The final FIR filter is designed using the parallel method by combining two separate FIR band-pass filters:

- **Group 1 (G1):** Band-pass filter from 75 kHz to 105 kHz
- **Group 2 (G2):** Band-pass filter from 180 kHz to 210 kHz

The two filters are added in parallel to create a multiband FIR filter.
As a result, the final filter allows the following frequency bands to pass:

- 75 kHz to 105 kHz
- 180 kHz to 210 kHz

5.1 Final Net Multiband Pass FIR Filter Results

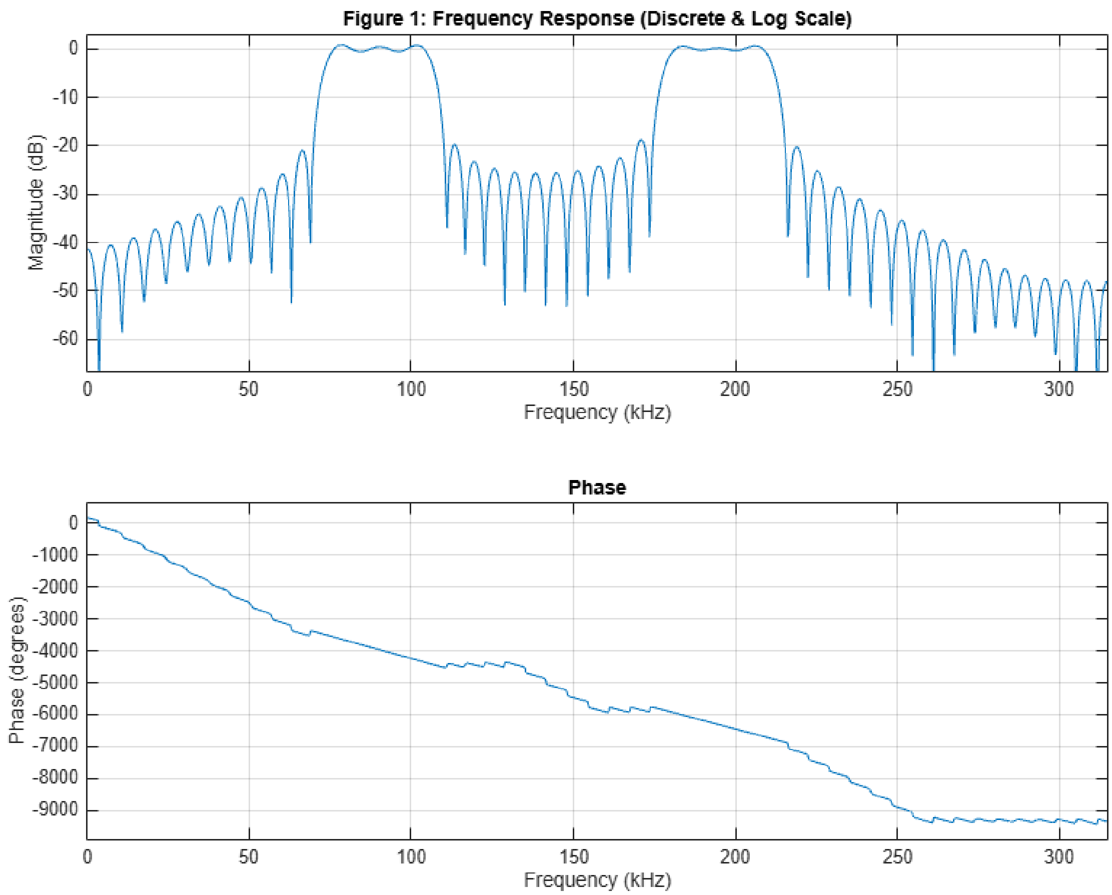


Figure 15: Frequency Response (Discrete and Logarithmic Scale) of Final Multiband FIR Filter

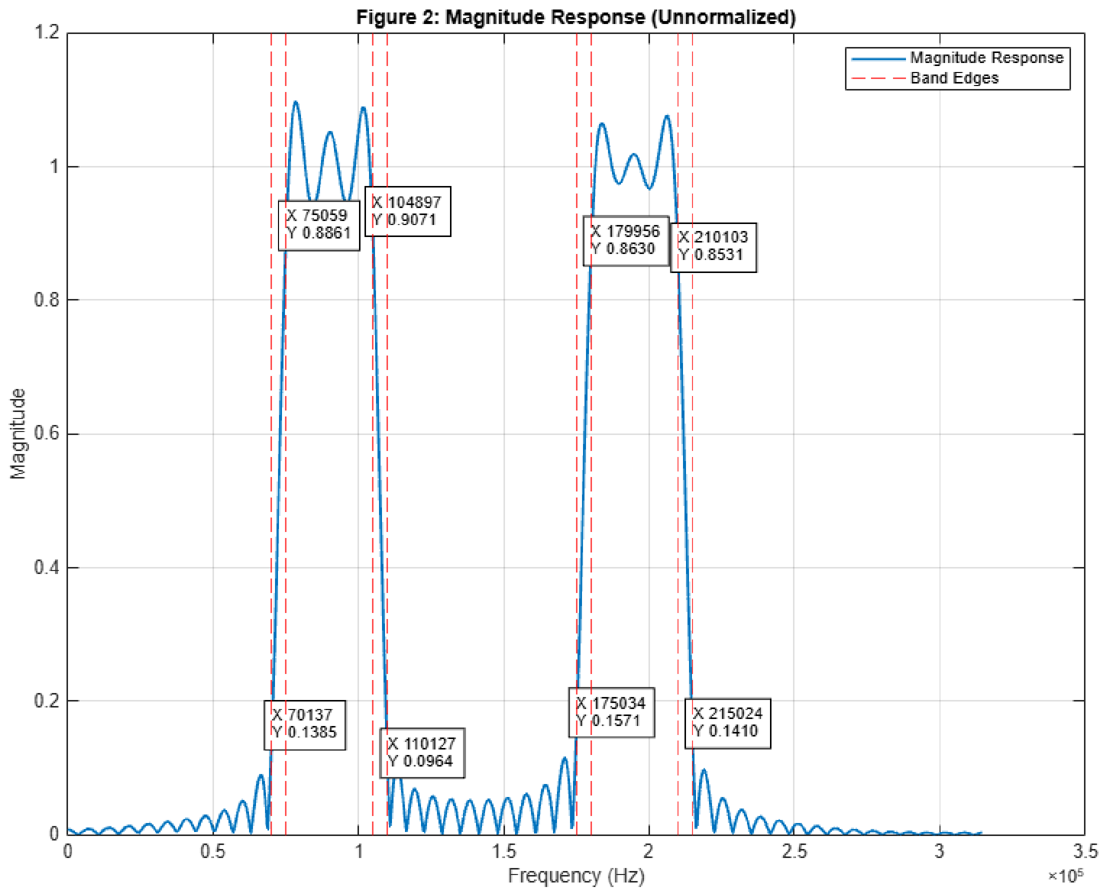


Figure 16: Unnormalized Magnitude Response of Final FIR Filter

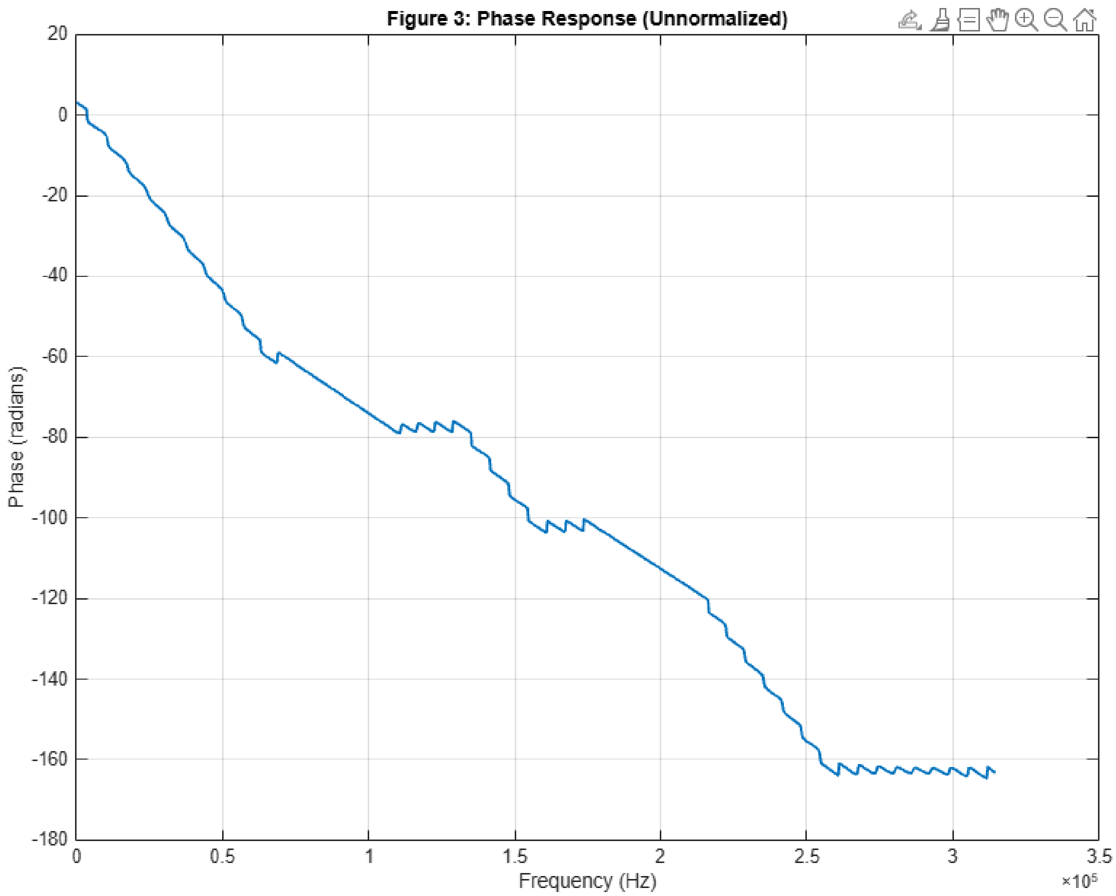


Figure 17: Unnormalized Phase Response of Final FIR Filter

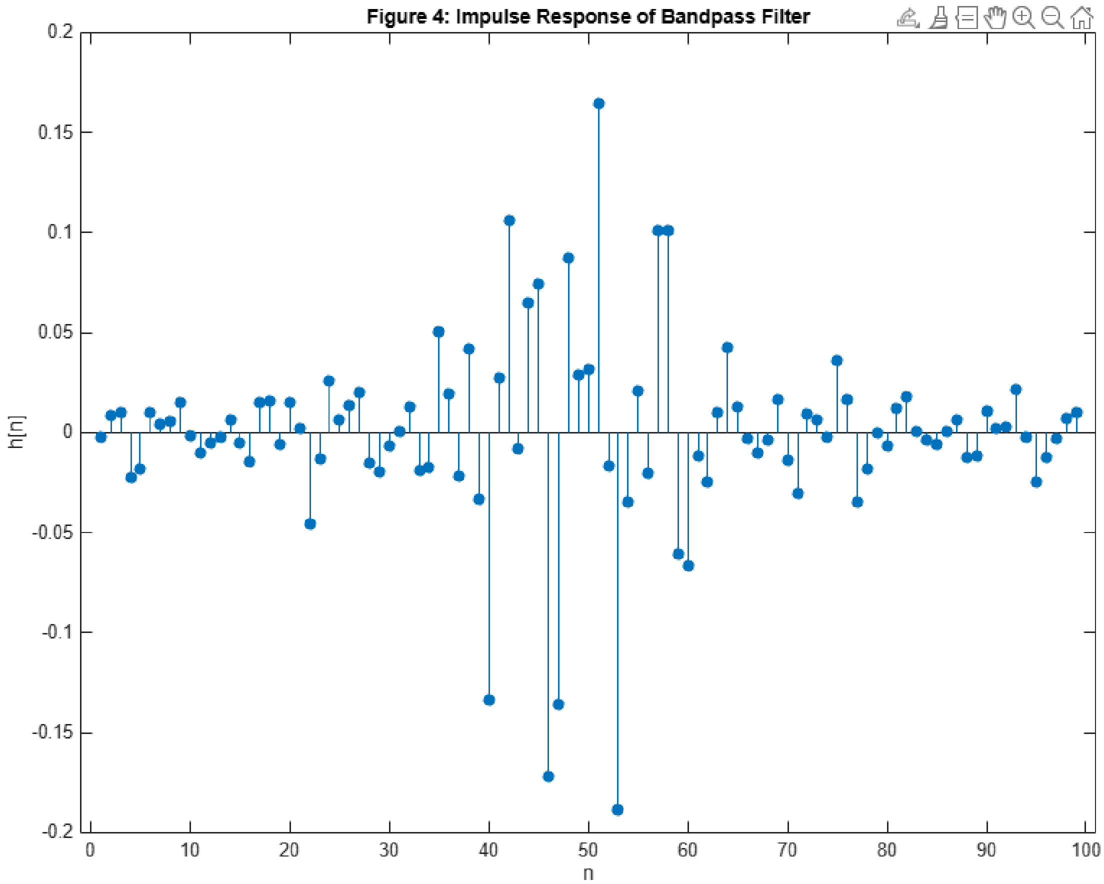


Figure 18: Impulse Response of Final Multiband Bandpass FIR Filter

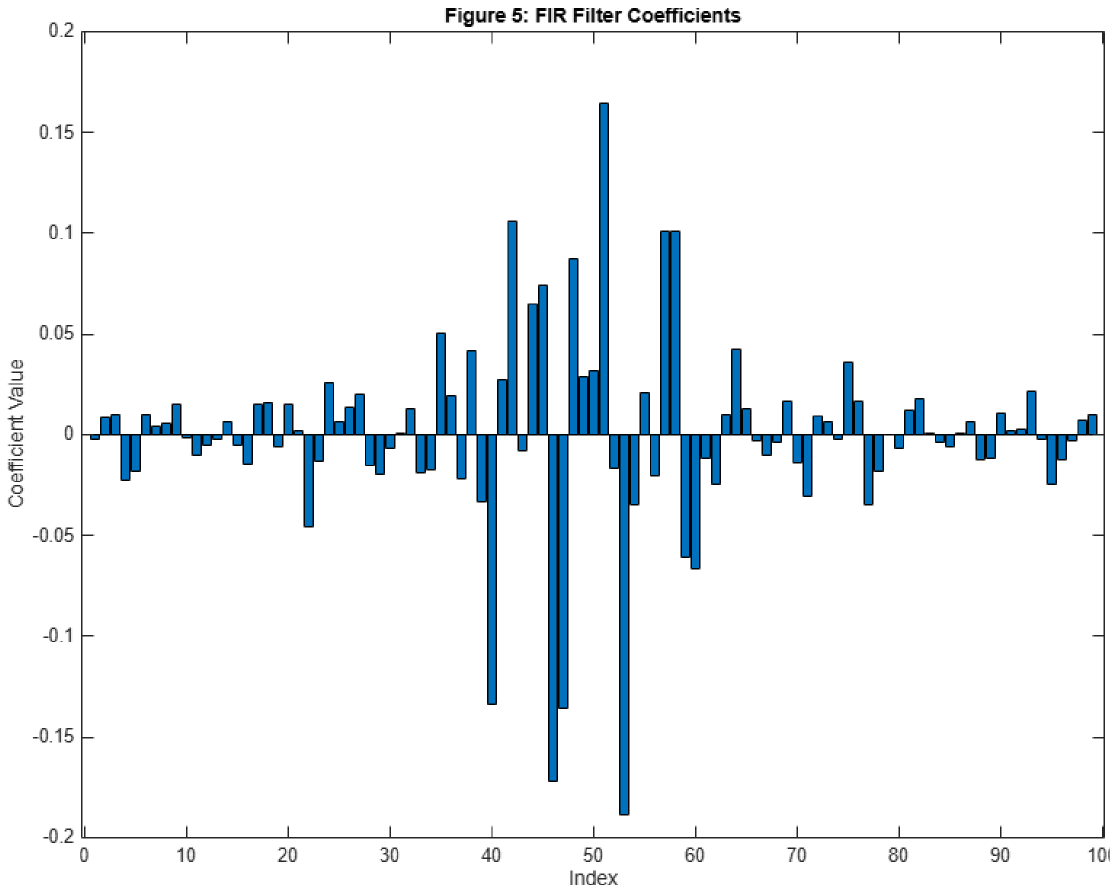


Figure 19: FIR Filter Coefficients of Final Multiband Filter

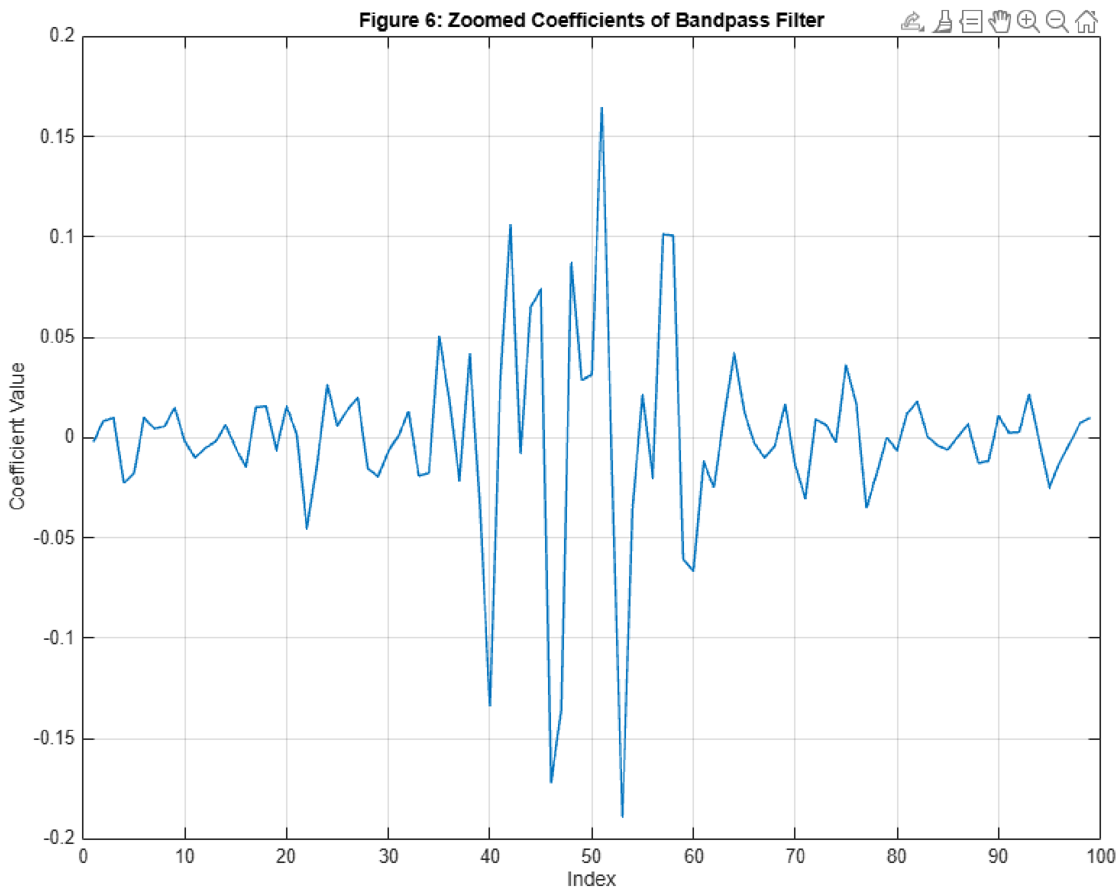


Figure 20: Zoomed View of FIR Filter Coefficients

```
FIR Filter Coefficients(Final-filter):
  Columns 1 through 14

   -0.0022    0.0082    0.0099   -0.0227   -0.0178    0.0100    0.0043    0.0056    0.0148   -0.0018   -0.0102   -0.0052   -0.0022    0.0063

  Columns 15 through 28

   -0.0053   -0.0146    0.0151    0.0156   -0.0062    0.0153    0.0018   -0.0454   -0.0129    0.0261    0.0060    0.0139    0.0199   -0.0156

  Columns 29 through 42

   -0.0194   -0.0068    0.0008    0.0130   -0.0191   -0.0176    0.0503    0.0190   -0.0215    0.0416   -0.0333   -0.1340    0.0270    0.1062

  Columns 43 through 56

   -0.0077    0.0649    0.0739   -0.1720   -0.1360    0.0869    0.0285    0.0313    0.1645   -0.0166   -0.1888   -0.0349    0.0210   -0.0202

  Columns 57 through 70

    0.1012    0.1007   -0.0607   -0.0667   -0.0119   -0.0248    0.0102    0.0422    0.0127   -0.0030   -0.0101   -0.0040    0.0165   -0.0136

  Columns 71 through 84

   -0.0307    0.0091    0.0065   -0.0023    0.0363    0.0166   -0.0351   -0.0183    0.0001   -0.0065    0.0120    0.0179    0.0006   -0.0040

  Columns 85 through 98

   -0.0060    0.0004    0.0067   -0.0127   -0.0117    0.0109    0.0023    0.0027    0.0214   -0.0022   -0.0249   -0.0123   -0.0028    0.0072

  Column 99

    0.0100
```

Figure 21: Impulse Response Value Detail for Final Multiband FIR Filter

5.2 Magnitude Response at Key Frequencies

Frequency (Hz)	Magnitude
70,000	0.1385
75,000	0.8861
105,000	0.9071
110,000	0.0964
175,000	0.1471
180,000	0.8630
210,000	0.8531
215,000	0.1410

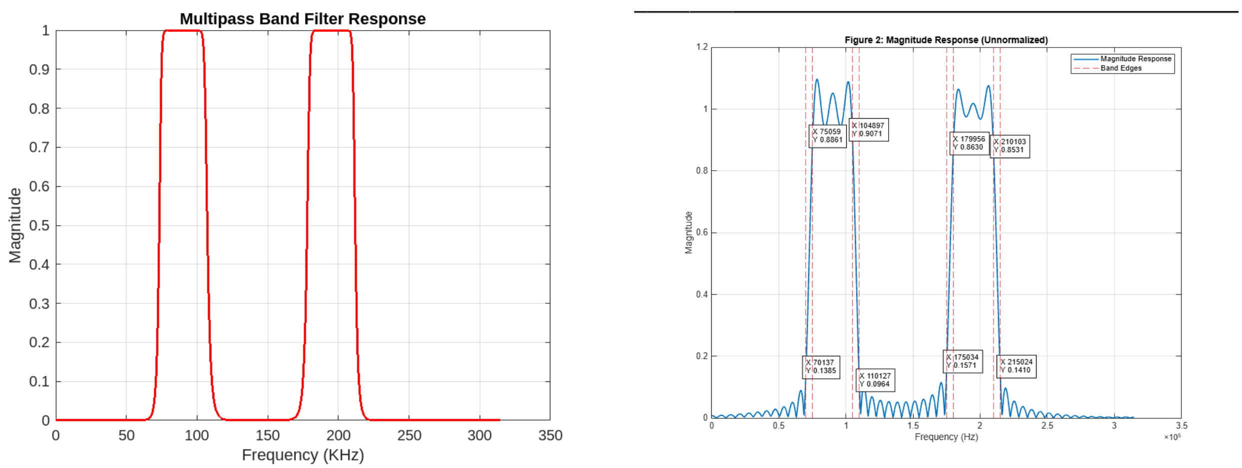
Table 1: Measured Magnitudes at Critical Frequencies for Final FIR Filter

5.3 Comparison with IIR Filter from Mid-Semester Exam

The final multiband FIR filter design differs from the IIR filter implemented during the mid-semester take-home component in the following ways:

- **Filter Type:** The FIR filter is non-recursive and uses a finite impulse response, while the IIR filter is recursive with infinite impulse response.
- **Stability:** FIR filters are inherently stable. The IIR filter required careful pole placement to ensure stability.

- **Phase Response:** The FIR filter has a linear phase, making it suitable for applications requiring phase preservation. The IIR filter had a nonlinear phase response.
- **Computational Complexity:** The FIR filter has higher computational complexity due to more coefficients, while the IIR filter achieved similar performance with fewer coefficients.
- **Group Delay:** FIR filters introduce more delay because of the longer impulse response. The IIR filter had lower group delay.
- **Implementation:** The FIR filter used window-based design, while the IIR filter was designed using Butterworth/Chebyshev approximation and bilinear transformation.
- **Transition Band:** The FIR design offers sharper control in multiband regions, though at the cost of increased order. The IIR filter had broader transition bands.



(a) IIR Filter Magnitude Response

(b) FIR Filter Magnitude Response

Figure 22: Comparison of Magnitude Responses: IIR vs FIR Filters

Criteria	**FIR Filter**	**IIR Filter**
Filter Type	Non-recursive (FIR)	Recursive (IIR)
Stability	Always Stable	Conditionally Stable
Phase Response	Linear Phase	Nonlinear Phase
Group Delay	Higher	Lower
Magnitude Ripple	Minimal	May have ripple (Butterworth/Chebyshev)
Design Method	Windowed Design	Bilinear Transform (Analog Prototype)
Complexity (No. of Coefficients)	High	Low
Multiband Support	Precise Band Control	Limited without high-order design

Table 2: Comparison Between Final FIR Filter and Mid-Semester IIR Filter

Overall, the FIR filter design provides better frequency control and phase characteristics at the cost of increased computational demand.