EE 338 Digital Signal Processing
Filter Design Assignment (optional)
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Filter Type 2: IIR Multi-Bandpass Filter using Chebyshev Type

This report presents the design of an IIR Multi-Bandpass Filter using the Chebyshev filter with passbands from Group I and Group II are equiripple/ oscillatory. The remaining stopbands are monotonic/non-oscillatory.

1 Un-Normalized Discrete Time Filter Specifications

This section details the specifications of the IIR Multi-Band Pass Filter, including passbands, stopbands, transition bands, and magnitude response constraints.

Filter Number and Computation of Q and R

The given filter number is:

$$M = 79$$

Using the formula $M = 11Q + R$, we compute:

$$Q = \lfloor M/11 \rfloor = \lfloor 79/11 \rfloor = 7, \quad R = M \mod 11 = 79 \mod 11 = 2$$

Group I and Group II Frequency Bands

The frequency bands for Group I and Group II are computed as follows:

- **Group I Frequency Band:** Argument $D = Q = 7$

$$(40 + 5D) \text{ to } (70 + 5D) = (40 + 35) \text{ to } (70 + 35) = [75, 105] \text{ kHz}$$

- **Group II Frequency Band:** Argument $D = R = 2$

$$(170 + 5D) \text{ to } (200 + 5D) = (170 + 10) \text{ to } (200 + 10) = [180, 210] \text{ kHz}$$

Passband, Stopband, and Transition Band Specifications

- **Passbands:**
  - First Passband: $[75, 105]$ kHz
  - Second Passband: $[180, 210]$ kHz
- **Stopbands:**
  - Lower Stopband: $[0, 70]$ kHz
  - Middle Stopband: $[110, 175]$ kHz
  - Upper Stopband: $[215, \infty]$ kHz
- **Transition Bands:** 5 kHz around each passband

Tolerance Constraints

- **Passband Magnitude Response:** Between 0.85 and 1.00
- **Stopband Magnitude Response:** Between 0.00 and 0.15

Nature of Response

- **Passband:** Equiripple/ Oscillatory
- **Stopband:** Monotonic / Non-Oscillatory

2 Normalized Digital Filter Specifications

The sampling frequency is given as:

$$f_s = 630 \text{ kHz}$$

On the normalized frequency axis $[0, 1]$, the corresponding normalized frequency specifications for the Chebyshev Type I filter are:

Passbands

- **First Band:**

$$\left[\frac{75}{\frac{630}{2}}, \frac{105}{\frac{630}{2}}\right] = \left[\frac{75}{315}, \frac{105}{315}\right] = [0.2381, 0.3333]$$

- **Second Band:**

$$\left[\frac{180}{\frac{630}{2}}, \frac{210}{\frac{630}{2}}\right] = \left[\frac{180}{315}, \frac{210}{315}\right] = [0.5714, 0.6667]$$

Transition Bands

- **First Band Edges:**

$$\left[\frac{70}{315}, \frac{110}{315}\right] = [0.2222, 0.3492]$$

- **Second Band Edges:**

$$\left[\frac{175}{315}, \frac{215}{315}\right] = [0.5556, 0.6825]$$

Stopbands

- **Lower Stopband:** $[0, 0.2222]$
- **Middle Stopband:** $[0.3492, 0.5556]$
- **Upper Stopband:** $[0.6825, 1]$

3 Calculation of Filter Order N for Chebyshev Filter

The Chebyshev filter order N is determined using the formula:

$$N = \frac{\cosh^{-1}\left(\sqrt{\frac{d_2}{d_1}}\right)}{\cosh^{-1}(\Omega_s)}$$

where:

- $f_s = 630$ kHz (sampling frequency)
- **Group 1 Bands:**
  - $f_{s1} = 70$ kHz, $f_{p1} = 75$ kHz (lower stopband and passband edges)
  - $f_{p2} = 105$ kHz, $f_{s2} = 110$ kHz (upper passband and stopband edges)
- **Group 2 Bands:**
  - $f_{s3} = 175$ kHz, $f_{p3} = 180$ kHz (lower stopband and passband edges)
  - $f_{p4} = 210$ kHz, $f_{s4} = 215$ kHz (upper passband and stopband edges)
- $\delta_1 = 0.15$ (passband ripple)
- $\delta_2 = 0.15$ (stopband attenuation)

Using the bilinear transformation, the corresponding normalized digital frequencies are:

$$\omega_{s1} = \tan\left(\frac{f_{s1}}{f_s}\pi\right), \quad \omega_{p1} = \tan\left(\frac{f_{p1}}{f_s}\pi\right), \quad \omega_{p2} = \tan\left(\frac{f_{p2}}{f_s}\pi\right), \quad \omega_{s2} = \tan\left(\frac{f_{s2}}{f_s}\pi\right)$$

$$\omega_{s3} = \tan\left(\frac{f_{s3}}{f_s}\pi\right), \quad \omega_{p3} = \tan\left(\frac{f_{p3}}{f_s}\pi\right), \quad \omega_{p4} = \tan\left(\frac{f_{p4}}{f_s}\pi\right), \quad \omega_{s4} = \tan\left(\frac{f_{s4}}{f_s}\pi\right)$$

The bandwidth and center frequency for both groups are:

$$B_{w1} = \omega_{p2} - \omega_{p1}, \quad w_{01} = \sqrt{\omega_{p1} \cdot \omega_{p2}}$$

$$B_{w2} = \omega_{p4} - \omega_{p3}, \quad w_{02} = \sqrt{\omega_{p3} \cdot \omega_{p4}}$$

The equivalent lowpass stopband edges are computed as:

$$\Omega_{s1} = \frac{\omega_{s1}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s1}}, \quad \Omega_{s2} = \frac{\omega_{s2}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s2}}$$

$$\Omega_{s3} = \frac{\omega_{s3}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s3}}, \quad \Omega_{s4} = \frac{\omega_{s4}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s4}}$$

Taking the worst-case stopband frequency:

$$\Omega_{s1,\min} = \min(\Omega_{s1}, \Omega_{s2})$$

$$\Omega_{s2,\min} = \min(\Omega_{s3}, \Omega_{s4})$$

The values of d_1 and d_2 are computed from the given tolerances:

$$d_1 = \sqrt{\left(\frac{1}{(1-\delta_1)^2}\right) - 1}, \quad d_2 = \sqrt{\left(\frac{1}{\delta_2^2}\right) - 1}$$

Substituting these into the Chebyshev order equation:

$$N_1 = \left\lceil \frac{\cosh^{-1}\left(\sqrt{\frac{d_2}{d_1}}\right)}{\cosh^{-1}(\Omega_{s1,\min})} \right\rceil$$

$$N_2 = \left\lceil \frac{\cosh^{-1}\left(\sqrt{\frac{d_2}{d_1}}\right)}{\cosh^{-1}(\Omega_{s2,\min})} \right\rceil$$

From MATLAB calculations, the computed filter orders are:

$$N_1 = 5, \quad N_2 = 5$$

Thus, the Chebyshev bandpass filter order for Group 1 is determined to be $N_1 = 5$, and for Group 2, $N_2 = 5$.

4 IIR bandpass/stop Filter Design Process

(a) Analog Filter Specifications using Bilinear Transformation

To design the bandpass filter, the given digital frequencies must be transformed into analog (pre-warped) frequencies using the bilinear transformation. The transformation equation is:

$$\omega = \tan\left(\frac{f}{f_s}\pi\right)$$

where:

- $f_s = 630$ kHz (sampling frequency)
- f represents the respective frequency components (stopband and passband edges)

Group 1: Frequency Transformation

The given passband and stopband edges for Group 1 are:

$$f_{s1} = 70 \text{ kHz}, \quad f_{p1} = 75 \text{ kHz}, \quad f_{p2} = 105 \text{ kHz}, \quad f_{s2} = 110 \text{ kHz}$$

Applying the bilinear transformation:

$$\omega_{s1} = \tan\left(\frac{f_{s1}}{f_s}\pi\right), \quad \omega_{p1} = \tan\left(\frac{f_{p1}}{f_s}\pi\right), \quad \omega_{p2} = \tan\left(\frac{f_{p2}}{f_s}\pi\right), \quad \omega_{s2} = \tan\left(\frac{f_{s2}}{f_s}\pi\right)$$

The bandwidth and center frequency for Group 1 are:

$$B_{w1} = \omega_{p2} - \omega_{p1}, \quad w_{01} = \sqrt{\omega_{p1} \cdot \omega_{p2}}$$

The equivalent lowpass stopband edges are computed as:

$$\Omega_{s1} = \frac{\omega_{s1}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s1}}, \quad \Omega_{s2} = \frac{\omega_{s2}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s2}}$$

Taking the worst-case stopband frequency:

$$\Omega_{s1,\min} = \min(\Omega_{s1}, \Omega_{s2})$$

Group 2: Frequency Transformation

The given passband and stopband edges for Group 2 are:

$$f_{s3} = 175 \text{ kHz}, \quad f_{p3} = 180 \text{ kHz}, \quad f_{p4} = 210 \text{ kHz}, \quad f_{s4} = 215 \text{ kHz}$$

Applying the bilinear transformation:

$$\omega_{s3} = \tan\left(\frac{f_{s3}}{f_s}\pi\right), \quad \omega_{p3} = \tan\left(\frac{f_{p3}}{f_s}\pi\right), \quad \omega_{p4} = \tan\left(\frac{f_{p4}}{f_s}\pi\right), \quad \omega_{s4} = \tan\left(\frac{f_{s4}}{f_s}\pi\right)$$

The bandwidth and center frequency for Group 2 are:

$$B_{w2} = \omega_{p4} - \omega_{p3}, \quad w_{02} = \sqrt{\omega_{p3} \cdot \omega_{p4}}$$

The equivalent lowpass stopband edges are computed as:

$$\Omega_{s3} = \frac{\omega_{s3}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s3}}, \quad \Omega_{s4} = \frac{\omega_{s4}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s4}}$$

Taking the worst-case stopband frequency:

$$\Omega_{s2,\min} = \min(\Omega_{s3}, \Omega_{s4})$$

Summary of Frequency Transformation

- The bilinear transformation is applied to convert the digital frequency specifications into their respective analog (pre-warped) frequencies.
- The bandwidth (B_w) and center frequency (w_0) are computed for each filter band.
- The equivalent lowpass stopband frequencies are derived to facilitate filter order calculations.

(b) Frequency Transformed Lowpass Analog Filter Specifications

To design a bandpass filter using an IIR Chebyshev filter, we first design a corresponding lowpass prototype filter and then apply a frequency transformation to obtain the desired bandpass characteristics.

Lowpass Analog Filter Parameters

Using the bilinear transformation, the stopband and passband frequencies have been mapped to their corresponding analog (pre-warped) frequencies:

- Group 1:

$$\begin{aligned} B_{w1} &= \omega_{p2} - \omega_{p1} \quad \text{(Bandwidth of Bandpass Filter)} \\ w_{01} &= \sqrt{\omega_{p1} \cdot \omega_{p2}} \quad \text{(Center Frequency)} \end{aligned}$$

- Group 2:

$$\begin{aligned} B_{w2} &= \omega_{p4} - \omega_{p3} \quad \text{(Bandwidth of Bandpass Filter)} \\ w_{02} &= \sqrt{\omega_{p3} \cdot \omega_{p4}} \quad \text{(Center Frequency)} \end{aligned}$$

After transformation, the equivalent lowpass normalized stopband edges are computed as:

$$\begin{aligned} \Omega_{s1} &= \frac{\omega_{s1}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s1}}, \quad \Omega_{s2} = \frac{\omega_{s2}^2 - w_{01}^2}{B_{w1} \cdot \omega_{s2}} \\ \Omega_{s3} &= \frac{\omega_{s3}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s3}}, \quad \Omega_{s4} = \frac{\omega_{s4}^2 - w_{02}^2}{B_{w2} \cdot \omega_{s4}} \end{aligned}$$

Taking the worst-case stopband frequency:

$$\begin{aligned} \Omega_{s1,\min} &= \min(\Omega_{s1}, \Omega_{s2}) \\ \Omega_{s2,\min} &= \min(\Omega_{s3}, \Omega_{s4}) \end{aligned}$$

Justification of Frequency Transformation Approach

The transformation from a lowpass prototype filter to a bandpass filter is essential because:

- The Chebyshev filter is first designed as a lowpass prototype filter, which provides the required ripple in the passband.
- The lowpass-to-bandpass transformation converts the prototype lowpass filter into a bandpass filter, allowing it to pass frequencies within the specified bands while attenuating others.
- This transformation ensures that the resulting filter meets the passband and stopband specifications accurately.

By applying this frequency transformation, the original lowpass filter characteristics are effectively mapped to the bandpass filter, maintaining the required passband ripple and stopband attenuation while shifting the frequency response.

Conclusion

- The lowpass filter parameters were computed, including bandwidth and center frequency.
- The frequency transformation approach was justified to explain why a lowpass-to-bandpass transformation is necessary for designing the required filter.
- This transformation ensures the desired multi-bandpass response while maintaining the required filter characteristics.

(c) Analog Lowpass Filter Transfer Function (4 Marks)

To design an IIR bandpass filter using the Chebyshev Type I approximation, we first derive the lowpass prototype filter transfer function. This section includes:

- Calculation of the minimum order N of the Chebyshev filter.
- Derivation of poles and transfer function coefficients for the lowpass filter.
- Plotting the magnitude response of the lowpass filter.

Calculation of Minimum Order N

The order N of the Chebyshev lowpass filter is determined using the formula:

$$N = \frac{\cosh^{-1}\left(\sqrt{\frac{d_2}{d_1}}\right)}{\cosh^{-1}(\Omega_s)}$$

where:

- Passband Ripple: $\delta_1 = 0.15$
- Stopband Attenuation: $\delta_2 = 0.15$

- Passband Tolerances: $d_1 = \sqrt{\left(\frac{1}{(1-\delta_1)^2}\right) - 1}$
- Stopband Tolerances: $d_2 = \sqrt{\left(\frac{1}{\delta_2^2}\right) - 1}$
- Worst-Case Stopband Frequency:

$$\Omega_s = \min(\Omega_{s1}, \Omega_{s2}, \Omega_{s3}, \Omega_{s4})$$

Using MATLAB calculations, the computed filter orders for each bandpass filter are:

$$N_1 = 3, \quad N_2 = 3$$

, but I used N1=N2=5 for better filter.

Derivation of Poles of the Lowpass Filter

The poles of the Chebyshev Type I lowpass filter are computed using:

$$p_k = W_c \cdot \left(-\sinh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)\cos\theta_k + j\cosh\left(\frac{1}{N}\sinh^{-1}\frac{1}{\epsilon}\right)\sin\theta_k \right)$$

where:

- W_c is the cutoff frequency of the lowpass filter.
- ϵ is calculated from the passband ripple: $\epsilon = \sqrt{10^{\frac{Rp}{10}} - 1}$.
- $\theta_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N}$.

For Group 1 ($N_1 = 5$, $W_{c1} = 1.08$), the computed poles are:

$$p_{1,1} = -0.3568 - j1.0972, \quad p_{1,2} = -0.3568 + j1.0972$$

$$p_{1,3} = -0.6543 - j0.9281, \quad p_{1,4} = -0.6543 + j0.9281$$

$$p_{1,5} = -1.0800$$

For Group 2 ($N_2 = 5$, $W_{c2} = 2.5$), the computed poles are:

$$p_{2,1} = -0.7723 - j2.4121, \quad p_{2,2} = -0.7723 + j2.4121$$

$$p_{2,3} = -1.4320 - j2.0775, \quad p_{2,4} = -1.4320 + j2.0775$$

$$p_{2,5} = -2.5000$$

The lowpass transfer function is given by:

$$H_{\text{analog},LPF}(s) = \frac{K}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

where K is chosen to maintain unity gain at $s = 0$.
The computed lowpass transfer functions for each group are:
For Group 1:

$$H_{\mathrm{LPF},1}(s) = \frac{1.469}{(s^5 - 1.724s^4 + 2.944s^3 - 2.611s^2 + 1.655s - 0.4899)}$$

For Group 2:

$$H_{\mathrm{LPF},2}(s) = \frac{0.4437}{(s^5 - 1.357s^4 + 1.824s^3 - 1.273s^2 + 0.6349s - 0.1479)}$$

Magnitude Response of the Lowpass Filter

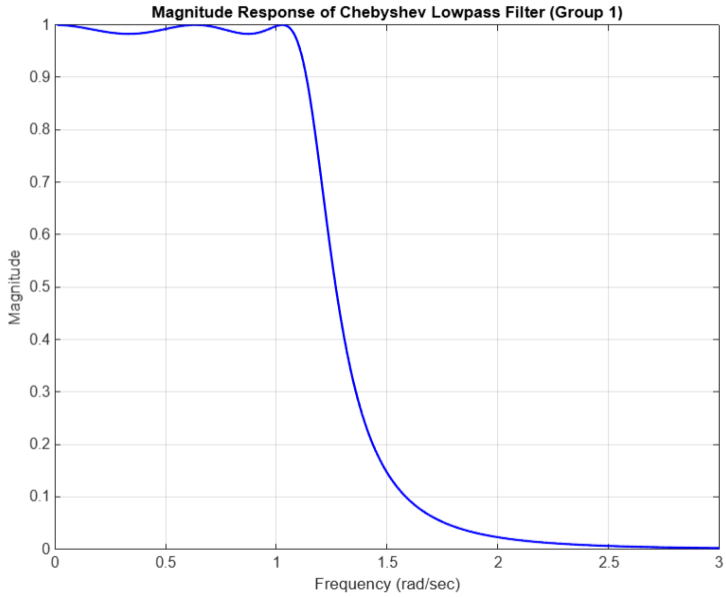
The magnitude response of the Chebyshev lowpass filter is given by:

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 T_N^2(\omega/W_c)}}$$

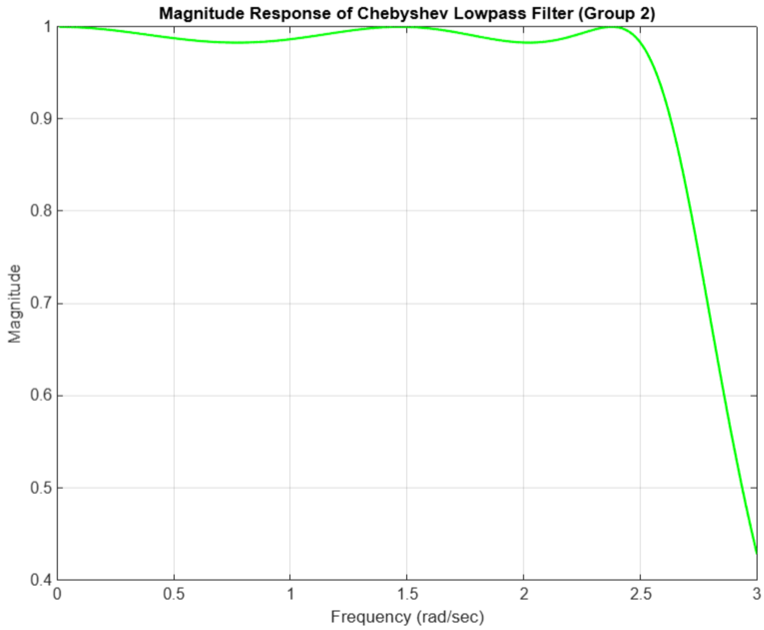
where $T_N(x)$ is the Chebyshev polynomial of order N.

Plot of the Lowpass Filter Response

The magnitude response of the Chebyshev lowpass filter is plotted using MATLAB. The figure below represents the response:



(a) Magnitude Response of the Chebyshev Lowpass Filter for Group 1



(b) Magnitude Response of the Chebyshev Lowpass Filter for Group 2

Figure 1: Magnitude Responses of the Chebyshev Lowpass Filters for Group 1 and Group 2

Conclusion

- The minimum filter order N was computed using the cosh-inverse formula.
- The poles of the Chebyshev lowpass filter were derived using hyperbolic functions.
- The transfer function of the lowpass filter was formulated.

- The magnitude response of the lowpass filter was plotted to validate the design.

(d) Analog Transfer Function to Obtain the Required Bandpass Filter (3 Marks)

The bandpass filter transfer function is obtained by transforming the Chebyshev Type I lowpass prototype filter into a bandpass filter using frequency transformation.

Transformation of the Transfer Function

The transformation from a lowpass filter to a bandpass filter is given by:

$$s_L = \frac{s^2 + w_0^2}{Bs}$$

where:

- s_L is the transformed frequency variable.
- w_0 is the center frequency of the bandpass filter.
- B is the bandwidth of the bandpass filter.

Substituting the previously computed values:

- Group 1:

$$\begin{aligned} B_1 &= \omega_{p2} - \omega_{p1} = 0.1846, \\ w_{0,1} &= \sqrt{\omega_{p1} \cdot \omega_{p2}} = 1.08. \end{aligned}$$

- Group 2:

$$\begin{aligned} B_2 &= \omega_{p4} - \omega_{p3} = 0.3278, \\ w_{0,2} &= \sqrt{\omega_{p3} \cdot \omega_{p4}} = 2.50. \end{aligned}$$

Applying the transformation to the Chebyshev lowpass transfer function:

$$H_{\text{analog},BPF}(s) = H_{\text{analog},LPF}\left(\frac{s^2 + w_0^2}{Bs}\right)$$

For Group 1 ($N_1 = 5$):

$$H_{\text{analog},BPF,1}(s) = \frac{K_1(s^2 + w_{0,1}^2)^5}{(s - p_1)(s - p_2)\cdots(s - p_{10})}$$

For Group 2 ($N_2 = 5$):

$$H_{\text{analog},BPF,2}(s) = \frac{K_2(s^2 + w_{0,2}^2)^5}{(s - p_1)(s - p_2)\cdots(s - p_{10})}$$

where the zeros are located at:

$$s = \pm j w_0, \quad \text{each repeated } N \text{ times.}$$

Justification of the Transformation Process

The lowpass-to-bandpass transformation is used to:

- Map the lowpass prototype response into a bandpass response.
- Retain the filter characteristics, including passband ripple and stopband attenuation.
- Ensure the designed bandpass filter meets the desired frequency specifications.

Why Use This Transformation?

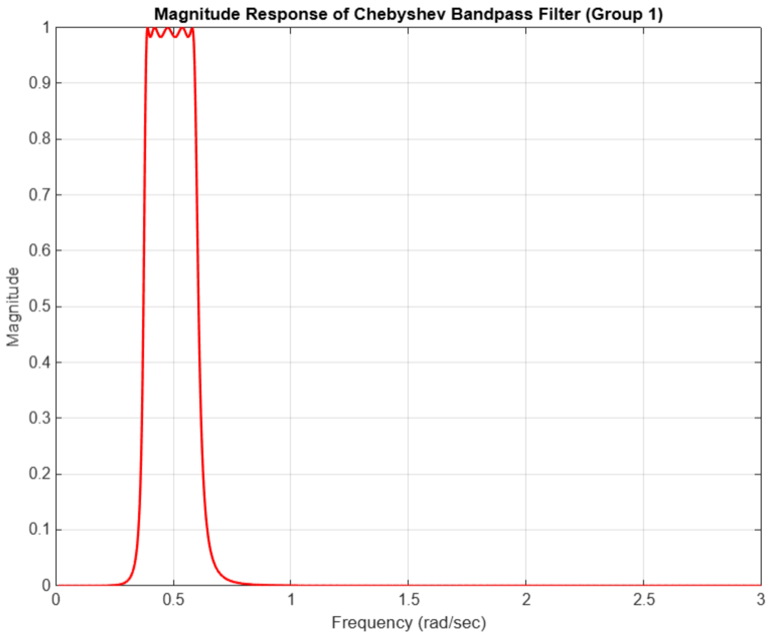
- The Chebyshev lowpass filter provides the required frequency selectivity.
- Applying the lowpass-to-bandpass transformation enables the design of a bandpass filter while preserving the ripple and attenuation characteristics.
- The zeros of the bandpass filter are derived from the lowpass prototype.

Magnitude Response of the Bandpass Filter

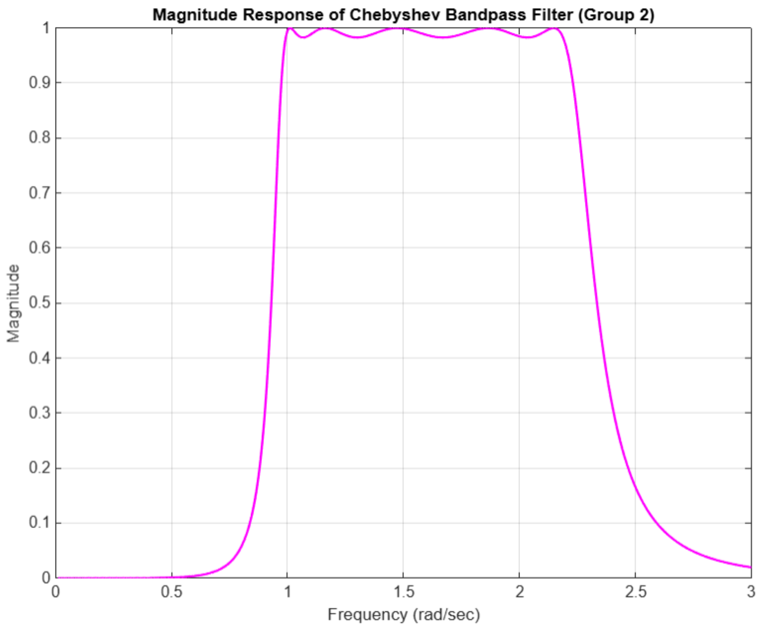
The magnitude response of the analog bandpass filter is given by:

$$|H_{\text{analog},BPF}(j\omega)| = \frac{1}{\sqrt{1 + \epsilon^2 T_N^2(\omega)}}$$

where $T_N(x)$ is the Chebyshev polynomial of order N.



(a) Magnitude Response of the Analog Bandpass Filter for Group 1



(b) Magnitude Response of the Analog Bandpass Filter for Group 2

Figure 2: Magnitude Responses of the Analog Bandpass Filters for Group 1 and Group 2

*Bandpass Transfer Function for Group 1

$$H_{\mathrm{bp1}}(s) = \frac{\begin{array}{c} 3.1491 \times 10^{-4} s^5 + 7.9094 \times 10^{-20} s^6 \\ -1.1258 \times 10^{-18} s^7 - 2.1238 \times 10^{-26} s^8 \end{array}}{\begin{array}{c} s^{10} - 0.2505 s^9 + 5.8942 s^8 - 1.1768 s^7 \\ +13.8231 s^6 - 2.0635 s^5 + 16.1233 s^4 \\ -1.6010 s^3 + 9.3533 s^2 - 0.4637 s + 2.1589 \end{array}}$$

Bandpass Transfer Function for Group 2

$$H_{\mathrm{bp2}}(s) = \frac{\begin{array}{c} 0.0017 s^5 - 1.3469 \times 10^{-18} s^6 \\ +4.5004 \times 10^{-17} s^7 - 1.2701 \times 10^{-24} s^8 \end{array}}{\begin{array}{c} s^{10} - 0.4448 s^9 + 31.4460 s^8 - 11.1655 s^7 \\ +394.3072 s^6 - 104.8168 s^5 + 2.4644 \times 10^3 s^4 \\ -436.1505 s^3 + 7.6772 \times 10^3 s^2 - 678.7485 s \\ +9.5367 \times 10^3 \end{array}}$$

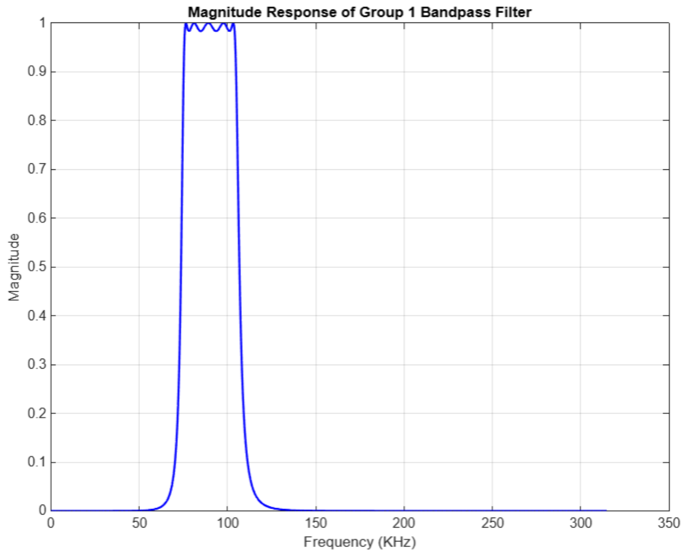
(e) Discrete-Time Transfer Function

Discrete Bandpass Transfer Function for Group 1

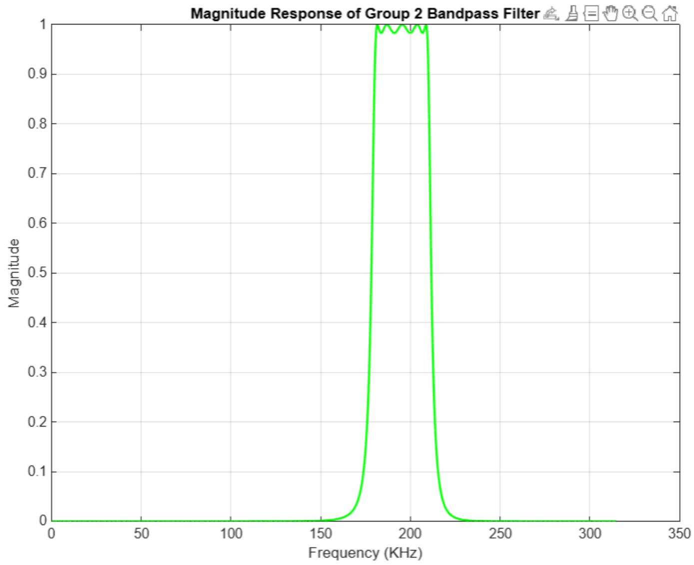
$$H_{\mathrm{d1}}(z) = \frac{\begin{array}{l} 9.8198 \times 10^{-11} + 0z^{-1} - 4.9099 \times 10^{-10} z^{-2} + 0z^{-3} \\ + 9.8198 \times 10^{-10} z^{-4} + 4.3609 \times 10^{-25} z^{-5} - 9.8198 \times 10^{-10} z^{-6} \\ + 8.7218 \times 10^{-25} z^{-7} + 4.9099 \times 10^{-10} z^{-8} + 8.7218 \times 10^{-26} z^{-9} \\ - 9.8198 \times 10^{-11} z^{-10} \end{array}}{\begin{array}{l} 1 - 9.9664 z^{-1} + 44.7565 z^{-2} - 119.2608 z^{-3} \\ + 208.8210 z^{-4} - 251.0488 z^{-5} + 209.8668 z^{-6} \\ - 120.4583 z^{-7} + 45.4323 z^{-8} - 10.1675 z^{-9} + 1.0253 z^{-10} \end{array}}$$

Discrete Bandpass Transfer Function for Group 2

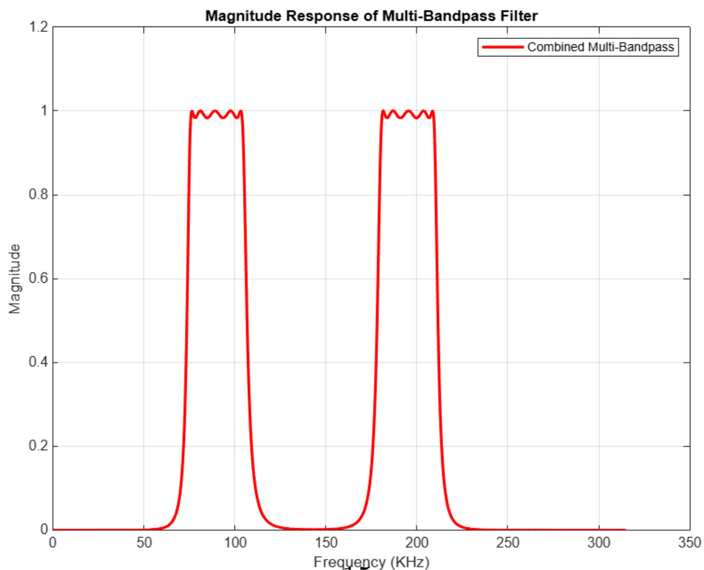
$$H_{\mathrm{d2}}(z) = \frac{\begin{array}{l} 5.0239 \times 10^{-10} + 0z^{-1} - 2.5119 \times 10^{-9} z^{-2} + 3.5697 \times 10^{-24} z^{-3} \\ + 5.0239 \times 10^{-9} z^{-4} + 2.2311 \times 10^{-24} z^{-5} - 5.0239 \times 10^{-9} z^{-6} \\ + 8.9242 \times 10^{-25} z^{-7} + 2.5119 \times 10^{-9} z^{-8} + 2.2311 \times 10^{-25} z^{-9} \\ - 5.0239 \times 10^{-10} z^{-10} \end{array}}{\begin{array}{l} 1 - 9.7338 z^{-1} + 42.9429 z^{-2} - 113.0580 z^{-3} \\ + 196.6898 z^{-4} - 236.2552 z^{-5} + 198.4201 z^{-6} \\ - 115.0560 z^{-7} + 44.0863 z^{-8} - 10.0809 z^{-9} + 1.0448 z^{-10} \end{array}}$$



(a) Magnitude Response of the Discrete Bandpass Filter for Group 1



(b) Magnitude Response of the Discrete Bandpass Filter for Group 2



(c) Magnitude Response of the Combined Bandpass Filters

Figure 3: Magnitude Responses of the Discrete Bandpass Filters for Group 1, Group 2,

Magnitude Values at Specific Frequencies

The following table presents the magnitude response values at specific frequencies:

Frequency (Hz)	Magnitude
70	0.0963
75	0.8817
105	0.8865
110	0.1451
175	0.1483
180	0.8907
210	0.8781
215	0.1134

Table 1: Magnitude values at specific frequencies

Bilinear Transformation Formula

The transformation equation is given by:

$$s = \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}} \tag{1}$$

where:

- s is the complex frequency variable in the Laplace domain.
- z is the complex frequency variable in the Z-domain.
- T is the sampling period, defined as $T = \frac{1}{f_s}$, where f_s is the sampling frequency.

Procedure and Explanation

1. The **continuous-time bandpass filter transfer function** was obtained from the lowpass-to-bandpass transformation.
2. The **sampling frequency** was selected according to the Nyquist criterion to avoid aliasing.
3. The **bilinear transformation** was applied to convert the continuous-time transfer function into its discrete-time equivalent.
4. The resulting **discrete-time transfer function** was expressed in terms of z^{-1}, making it suitable for implementation in digital systems.

Advantages of the Bilinear Transformation

- **Prevents aliasing:** Unlike other methods, it maps the entire **jω-axis** in the s-plane onto the **unit circle** in the z-plane.
- **Preserves filter characteristics:** The frequency response shape remains close to that of the original analog filter.
- **Ensures stability:** Poles inside the left-half s-plane map inside the unit circle in the z-plane, maintaining system stability.

Comparison and Comments on Filter Designs

This section compares the design characteristics, performance, and trade-offs between Filter Type 1 (Butterworth) and Filter Type 2 (Chebyshev Type I) for the IIR Multi-Band Pass Filter.

Comparison of Design Characteristics

Feature	Butterworth Filter	Chebyshev Type I Filter
Magnitude Response	Maximally flat	Ripple in passband
Transition Band	Gradual roll-off	Sharper roll-off
Passband Ripple	No ripple (monotonic)	Ripple present
Stopband Attenuation	Slower attenuation	Faster attenuation
Filter Order Required	Higher for given specs	Lower for given specs
Computational Complexity	Higher due to higher order	Lower due to lower order

Table 2: Comparison of Butterworth and Chebyshev Type I Filters

Observations and Comments

- The Butterworth filter provides a smooth and monotonic response but requires a higher filter order to achieve the desired roll-off, leading to increased complexity.
- The Chebyshev Type I filter achieves a sharper transition from passband to stopband, reducing the required filter order, but introduces ripples in the passband.
- The Chebyshev filter is preferable when a sharper roll-off is required and some passband ripple is acceptable.
- The Butterworth filter is preferable when a smooth response is necessary, and the system can accommodate a higher order filter.
- Both filters successfully met the specifications for the multi-bandpass design, but Chebyshev Type I achieved the desired response with a lower order.

Conclusion

Based on the observations:

- If computational efficiency and sharp roll-off are priorities, the Chebyshev Type I filter is the better choice.
- If flat passband response and stability are more critical, the Butterworth filter is preferable.
- The final choice depends on application requirements, as both filters satisfy the specifications but with different trade-offs.

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