

# Lab 4: Non-Linearity and its effects in communication systems

**EE340: Prelab Reading material for Experiment 4**

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# Non-Linear Systems

- Linear Systems: Satisfy superposition principle
- However, any practical system is non-linear (amount of non-linearity may vary)
- Non-linearity results in generation of “new frequency components” – i.e. frequency components that are not there at the input of the system.
- Memory-less non-linearity can be modeled as:

$$y(t) = a_1x(t) + a_2x^2(t) + a_3x^3(t) + a_4x^4(t)...$$

- Memory-less means present output depends only on the present input (also see Appendix – last slide)

# Effects of Non-Linearity

Consider a simplified non-linear system described by

$$y(t) = a_1x(t) + a_2x^2(t) + a_3x^3(t)$$

For  $x(t) = A\cos(\omega t)$ ,

$$y(t) = \frac{1}{2}a_2A^2 + (a_1 + \frac{3}{4}a_3A^2)A\cos(\omega t) + \frac{1}{2}a_2A^2\cos(2\omega t) + \frac{1}{4}a_3A^3\cos(3\omega t)$$

**Important observations:**

- **Second order non-linearity ( $a_2$  coefficient) :**

$$\frac{1}{2}a_2A^2(1 + \cos(2\omega t))$$

**Adds DC + 2<sup>nd</sup> harmonic**

- **Third order non-linearity ( $a_3$  coefficient) :**

$$(a_1 + \frac{3}{4}a_3A^2)A\cos(\omega t) + \frac{1}{4}a_3A^3\cos(3\omega t) \quad \text{Gain : } (a_1 + \frac{3}{4}a_3A^2)$$

**Adds 3<sup>rd</sup> harmonic**

**Gain becomes input amplitude (A) dependent. Also,  $a_3$  is generally negative  $\Rightarrow$  gain compression with increasing A**

# Second Order Non-Linearity

Consider a non-linear system described by

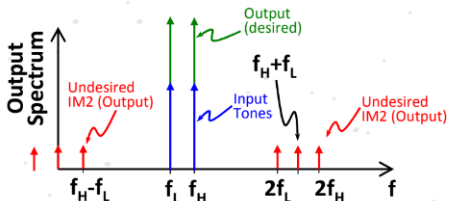
$$y(t) = a_1x(t) + a_2x^2(t)$$

For  $x(t) = A(\cos\omega_1t + \cos\omega_2t)$ ,

$$y(t) = a_2A^2 + a_1A(\cos(\omega_1t) + \cos(\omega_2t)) + a_2A^2\left(\frac{\cos(2\omega_1t) + \cos(2\omega_2t)}{2} + \cos((\omega_1 - \omega_2)t) + \cos((\omega_1 + \omega_2)t)\right)$$

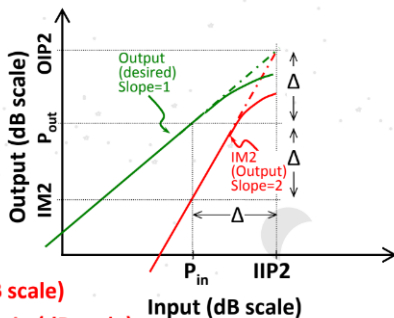
**The undesired spectral components generated due to the second order non-linearity coefficient  $a_2$  at frequencies  $0, 2\omega_1, 2\omega_2, (\omega_1 - \omega_2)$  and  $(\omega_1 + \omega_2)$  are called IM2 (Inter-Modulation products due to 2<sup>nd</sup> order non-linearity) components.**

## Observations:



$$\text{IIP2} = P_{\text{in}} + \Delta \text{ (dB scale)}$$

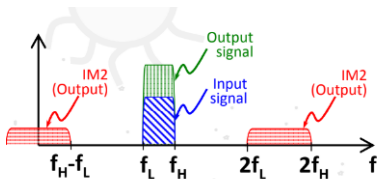
$$\text{OIP2} = P_{\text{in}} + \Delta + \text{Gain (dB scale)}$$



where

- 2nd-order intercept point (IP2) is where the asymptotes for the 2nd-order intermodulation product and the fundamental cross.
- IIP2 is the input power and OIP2 is the output power corresponding to the intercept point.
- $P_{\text{in}}$  is the given input signal's power.

## More Observations:

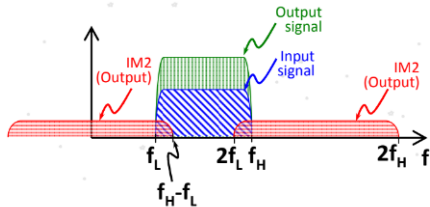


(a) Sub-Octave

**Sub-Octave:**  $f_H < 2f_L$

(i.e.  $BW < f_L$ )

- > No in-band IM2 distortion
- out-of-band IM2 components, can easily be filtered out
- > DC components can sometimes cause amplifier saturation



(b) Multi-Octave

**Multi-Octave:**  $f_H > 2f_L$

(i.e.  $BW > f_L$ )

- > In-band IM2 distortion present can't be filtered out
- > DC components may cause amplifier saturation

# Third Order Non-Linearity

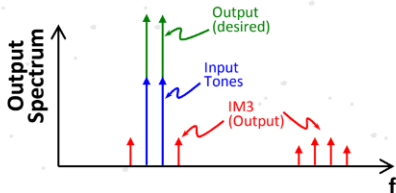
Consider a non-linear system described by

$$y(t) = a_1x(t) + a_3x^3(t); \quad \Rightarrow \text{For } x(t) = A \cos(\omega_1 t) + A \cos(\omega_2 t):$$

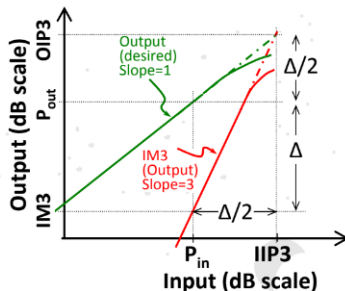
$$y(t) = A \left( a_1 + \frac{9a_3A^2}{4} \right) (\cos(\omega_1 t) + \cos(\omega_2 t)) + \frac{1}{4} a_3 A^3 (\cos(3\omega_1 t) + \cos(3\omega_2 t))$$

$$+ \frac{3}{4} a_3 A^3 [\cos((2\omega_1 - \omega_2)t) + \cos((2\omega_1 + \omega_2)t) + \cos((2\omega_2 - \omega_1)t) + \cos((2\omega_2 + \omega_1)t)]$$

Observations:



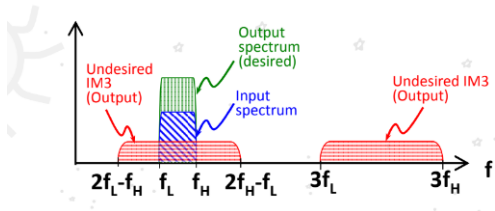
Generates in-band/adjacent band, out-of-band components, but no DC



$$IIP3 = P_{in} + \Delta/2 \text{ (dB scale)}$$

$$OIP3 = P_{in} + \Delta/2 + \text{Gain (dB scale)}$$

# Third Order Non-Linearity



- The figure above shows the undesired spectrum generated by  $3^{rd}$  order non-linearity (i.e. due to non-zero  $a_3$  coefficient).
- The undesired spectrum generated is called IM3 component, i.e. Inter-Modulation products due to  $3^{rd}$  order non-linearity component.
- Due to  $3^{rd}$  or odd order non-linearities (unlike  $2^{nd}$  or even order non-linearities), part of the spectrum is in-band and hence CANNOT be removed by filtering even for narrow-band inputs.
- Therefore, effects of  $3^{rd}$  (or odd) order non-linearities are more difficult to remove in general (than of even order non-linearities).



# Calculation Of Intercept Points

Suppose we use a signal with two tones of equal strength, then the  $n$ th-order intercept point is given by

$$IP_n = \frac{nP_{in} - P_{IM}}{n - 1} \quad (1)$$

where

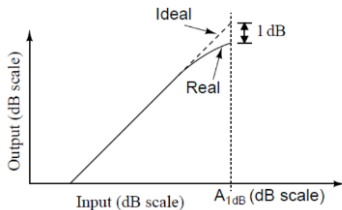
- $IP_n$  is the  $n$ th-order intercept point
- $P_{in}$  is the input signal strength
- $P_{IM}$  is the power of the intermodulation distortion (IMD) product

For the calculation of IIP $_n$  and OIP $_n$ , refer to the sections till 4.3 in [Calculations of intercept points](#)

# Compression Point and Jamming

**1-dB compression point:** Amplitude ( $A_{-1dB}$ ) at which gain decreases by 1-dB (without interferer) because  $a_3$  is "almost always" negative.

$$20 \log \left[ \frac{(a_1 + \frac{3}{4} a_3 A_{-1dB}^2) A_{-1dB}}{a_1 A_{-1dB}} \right] = -1dB \text{ i.e. } A_{-1dB} \approx 0.40 \sqrt{\frac{|a_1|}{|a_3|}}$$



$$\text{Compression point in dB} = 20 \log(A_{-1dB}) = IIP3(\text{in dB}) - 9.6dB$$

where

$$IIP3(\text{in dB}) = 20 \log \left( \sqrt{\frac{4|a_1|}{3|a_3|}} \right)$$

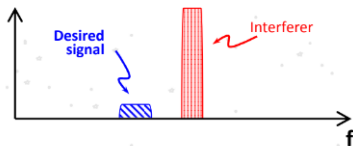
$$OIP3(\text{in dB}) = IIP3(\text{in dB}) + 20 \log(|a_1|)$$

## Jamming / Blocking / Desensitization

For  $x(t) = A\cos(\omega t) + B\cos(\omega_1 t)$ ,

where  $A\cos(\omega t)$  is the desired signal and  $B\cos(\omega_1 t)$  is the interferer,

$$y(t) = (a_1 + \frac{3}{4}a_3A^2 + \frac{3}{2}a_3B^2)A\cos(\omega t) + \text{other terms}$$



- Therefore, if interferer amplitude  $B \gg A$ , the receiver is jammed
- The transmitter can jam the receiver if they are operating concurrently, for example in full duplex systems (and isolation is poor)

# APPENDIX: Real Systems are not memory-less or linear: Non-linear dynamical behaviour

## Transfer function of a dynamic non-linear system

- Very complex, commonly expressed as the Volterra series

$$y(t) = a_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_0^{\infty} \dots \int_0^{\infty} a_n(\tau_1, \tau_2, \dots, \tau_n) x(t-\tau_1) x(t-\tau_2) \dots x(t-\tau_n) d\tau_1 d\tau_2 \dots d\tau_n$$

$a_n$  is called the  $n^{th}$  order Volterra kernel.

Therefore, a 2nd order dynamic non-linear system can be modeled as

$$y(t) = a_0 + \int_0^{\infty} a_1(\tau_1) x(t-\tau_1) d\tau_1 + \frac{1}{2} \int_0^{\infty} \int_0^{\infty} a_2(\tau_1, \tau_2) x(t-\tau_1) x(t-\tau_2) d\tau_1 d\tau_2$$