# Pulse shaping for Digital Communication

EE340: Prelab Reading Material for Experiment 5

AUTUMN 2024

**Why Digital Communication:** In general, signals in the digital format are easier to store, transmit and process, compared to analog signals that are more susceptible to getting corrupted by noise and distortion. However, digital bit streams are not transmitted directly, particularly in case of wireless transmission.
In communications, digital signals need to be mapped to an analog waveform in order to be transmitted over the channel. The mapping process is accomplished in two steps:
(i) Mapping from source bits to complex symbols (also known as constellation points),
(ii) Mapping from complex symbols to analog pulse trains, which is studied in this part.

Data transmission systems that must operate in a bandwidth limited environment must contend with the fact that constraining the bandwidth of the transmitted signal necessarily increases the likelihood of a decoding error at the receiver. Bandwidth limited systems often employ pulse-shaping techniques that allow for bandwidth containment while minimizing the likelihood of errors at the receiver.

**THE RECTANGULAR PULSE:** The most basic information unit in a digital transmission scheme is a rectangular pulse. It has a defined amplitude, A, and defined duration, T. Such a pulse is shown in Figure 1, where A = 1, T = T$o$, with the pulse centered about the time origin at t = 0. Typically, a sequence of such pulses (each delayed by T seconds relative to the previous one) constitutes the transmission of information. The information, in this case, is encoded in the amplitude of the pulse. The simplest case is when a binary 0 is encoded as the absence of a pulse (A = 0) and a binary 1 is encoded as the presence of a pulse (A = constant). Since each pulse spans the period T, the maximum pulse rate is 1/T pulses per second, which leads to a data transmission rate of 1/T bits per second.
The remainder of this application note focuses on a single bipolar pulse for transmitting one bit at a time. That is, a logical 1 is represented by the presence of a pulse of unit amplitude and a logical 0 by the absence of a pulse (that is, zero amplitude).

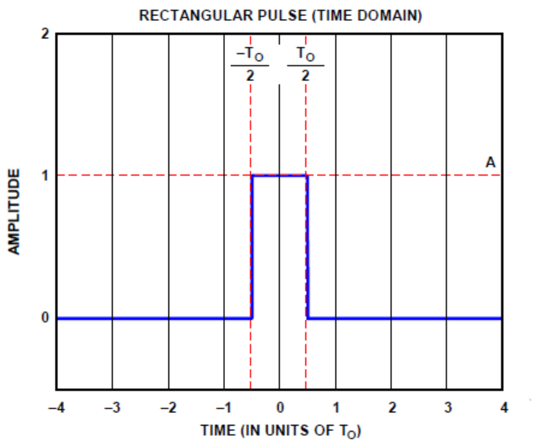


Figure 1: A Single Rectangular Pulse (T = T$o$, A = 1)

**SPECTRUM OF A RECTANGULAR PULSE:** The frequency content (or spectrum) associated with the pulse of Figure 1 is shown in Figure 2. The spectrum of the pulse is obtained by applying the Fourier transform to the time domain waveform of Figure 1. The shape of the spectrum is the well known sin(x)/x response, which is often referred to as the sinc response. The null points (where the spectral magnitude is zero) always occur at integer multiples of f$o$, which is the pulse (or symbol) rate. Therefore, the null points are solely determined by the pulse period, T. In theory, the nulls and peaks extend in frequency out to $\pm\infty$ with the peaks approaching zero magnitude. However, because the frequency span of Figure 2 is only $\pm 4$ f$o$, only four null points are evident on each side of the f = 0 line.

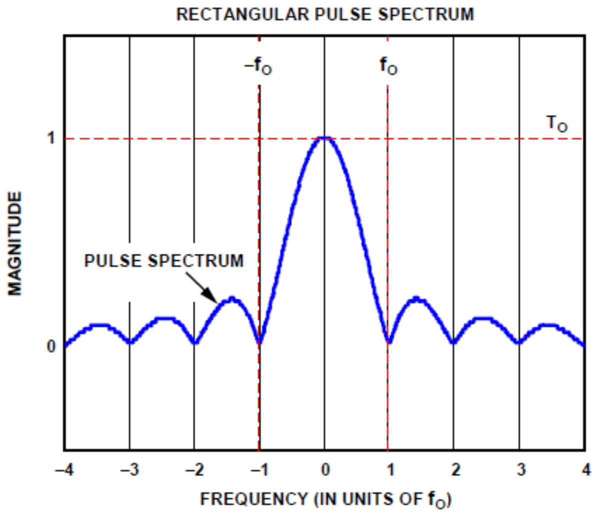


Figure 2: Spectrum of a Single Rectangular Pulse of Duration T$o$

**Pulse Shaping Filter:** As shown in Figure 2, the spectrum of a rectangular pulse spans infinite frequency .In many data transmission applications, the transmitted signal must be restricted to a certain bandwidth. This can be due to either system design constraints or government regulation. In such instances, the infinite bandwidth associated with a rectangular pulse is not acceptable. The bandwidth of the rectangular pulse can be limited, however, by forcing it to pass through a low-pass filter. The act of filtering the pulse causes its shape to change from purely rectangular to a smooth contour without sharp edges. Therefore, the act of filtering rectangular data pulses is often referred to as pulse shaping.

Unfortunately, limiting the bandwidth of the rectangular pulse necessarily introduces a damped oscillation. That is, the rectangular pulse exhibits nonzero amplitude only during the pulse interval, whereas the smoothed (or filtered) pulse exhibits ripples both before and after the pulse interval. At the receiver, the ripples can lead to incorrect decoding of the data, because the ripples associated with one pulse interfere with the pulses before and after it. However, the choice of a proper filter can yield the desired bandwidth reduction while maintaining a time domain shape that does not interfere with the decoding process of the receiver.

This filter is the well-known raised cosine filter and its frequency response is given by

$$H(\omega) = \begin{cases} \tau & \omega \geq 0 \leq c \\ \tau * cos^2[\frac{\tau(\omega - c)}{r\alpha}] & \omega \geq c \leq d \\ 0 & \omega \geq d \end{cases}$$

where:
$\omega$ is radian frequency ($2\pi$f).
$\tau$ is the pulse period (equivalent to T$o$ in Figure 1).
$\alpha$ is the roll off factor.
c is equal to $\pi$ $(1 \;\; \alpha)/\tau$.
d is equal to $\pi$ $(1 + \alpha)/\tau$.

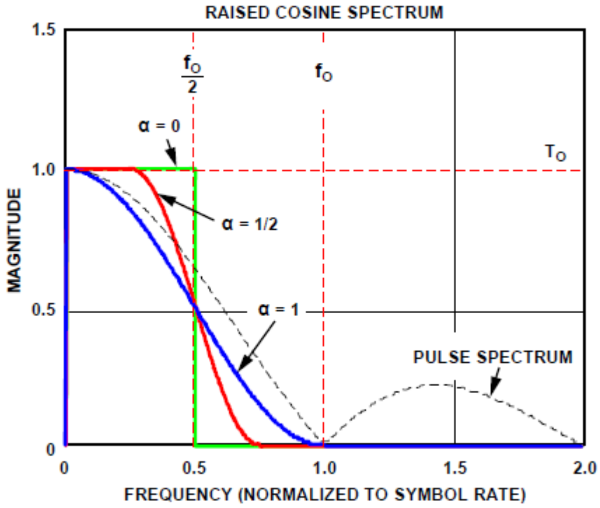


Figure 3: The Raised Cosine Frequency Response(normalized to $\tau = 1$)

The consequence of pulse shaping is that it distorts the shape of the original time domain rectangular pulse into a smoothly rounded pulse with damped oscillations (ripples) before and after the $\pm\frac{1}{2}$ T$o$ points. The ripples result from the convolution of the rectangular pulse with the raised cosine impulse response (convolution is the process of filtering in the time domain). The impulse response (time domain) of the raised cosine filter is shown in Figure 4 where the color scheme is the same as that used in Figure 3.However, the beauty of the raised cosine filter is that the zero crossings of the impulse response coincide with the midpoint of adjacent pulses. As long as the receiver makes its decision at the middle of each pulse interval, then the ripples from adjacent pulses are crossing through zero. Therefore, they do not interfere with the decision making process.

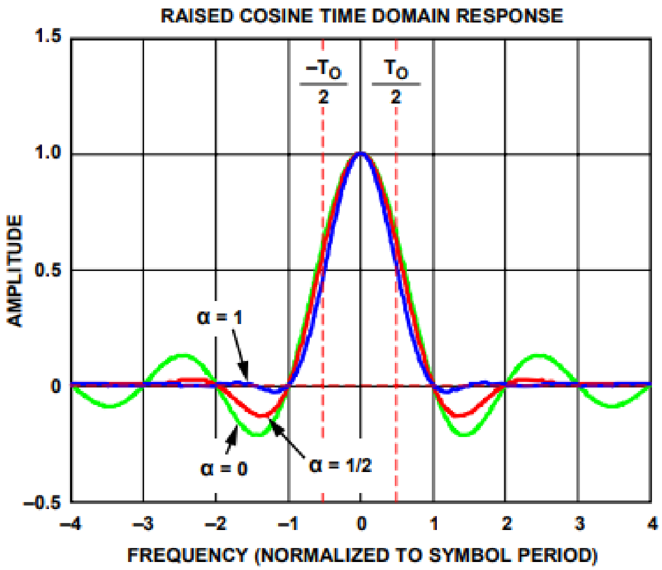


Figure 4: The Raised Cosine Time Domain Response

### EYE DIAGRAM

An eye diagram is used in electrical engineering to get a good idea of signal quality in the digital domain. The eye diagram takes its name from the fact that it has the appearance of a human eye. It is created simply by superimposing successive waveforms (The multiple copies, each delayed by a symbol duration from the previous copy)to form a composite image. The eye diagram is used primarily to look at digital signals for the purpose of recognizing the effects of distortion and finding its source.

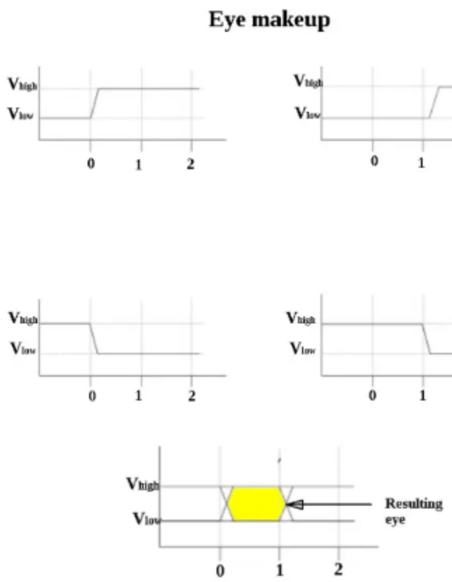


Figure 5: Here, the bit sequences 011, 001, 100, and 110 are superimposed over one another to obtain the example eye diagram.

It is important to realize what is shown in an eye diagram and what is not shown. In digital transmission, a succession of ones and zeroes flows to the receiver. The transmission can consist of a long series of ones, a long series of zeroes, a regular or irregular sequence that repeats periodically, a quasi-random series or any combination. The eye diagram will reveal whether everything works as intended or if there are faults that garble the transmission, causing, for example, the reception of a zero when a one has been sent.

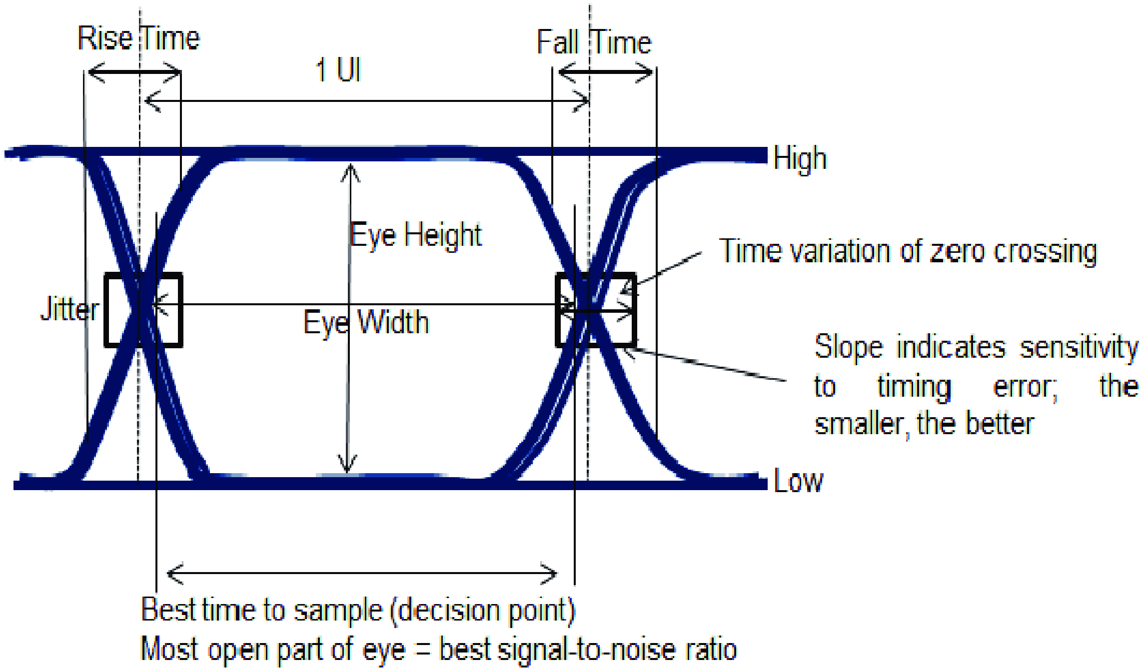


Figure 6: Interpretation of eye diagram

## References


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- https://www.testandmeasurementtips.com/basics-eye-diagrams/