Multipath Propagation and Equalization

EE340: Prelab Reading Material for Experiment 7

AUTUMN 2024

Multipath propagation and Equalization are concepts often encountered in wireless communication systems and digital signal processing. They relate to the challenges posed by signal propagation in environments where signals can take multiple paths to reach the receiver. These multiple paths can lead to signal distortion and interference, which need to be mitigated for reliable communication.

1 Multipath Propagation

Multipath propagation occurs when a transmitted signal reaches the receiver via multiple paths due to reflections, diffractions, and scattering in the environment, as shown in Figure 1. This can result from various physical obstacles and reflections of buildings, trees, or other objects in the transmission path. Multipath propagation can lead to several issues, including:

- **Signal Fading**: The signals arriving via different paths can interfere constructively or destructively, causing variations in signal strength known as fading. This can lead to signal dropout or poor signal quality.
- **Inter-symbol Interference (ISI)**: Since the various paths are of different lengths, they arrive at the receiver with different delays. When multiple delayed versions of a signal arrive at the receiver with different phases, they can interfere with each other, causing ISI. So the symbols (data or information) transmitted through a channel overlap in time, making it difficult for the receiver to distinguish one symbol from another thus, ISI can degrade the quality of the received signal and make it challenging to correctly decode the transmitted data

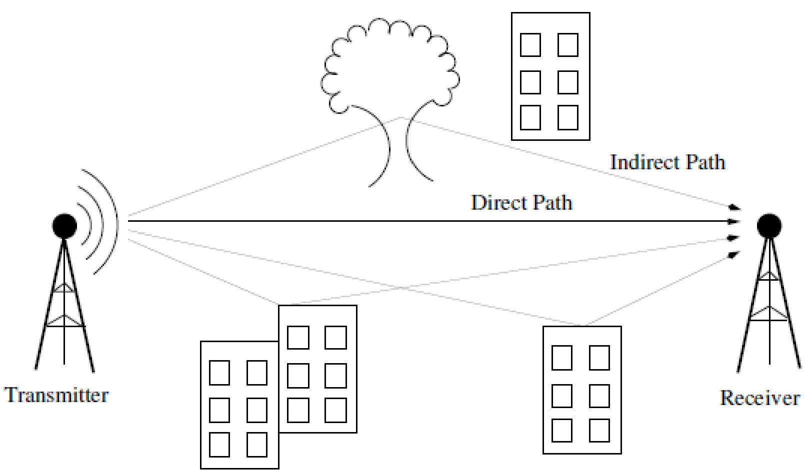

Figure 1: Multipath Propagation in a Wireless System

To model the multipath propagation at the receiver, consider the signal received through the direct path to be an impulse of unit amplitude. All the other subsequent signals that arrive at the receiver after suffering reflections and refractions can be modeled as time-delayed impulses with amplitude less than unity. Hence, the addition of a signal and its time-delayed versions obtained at the receiver input results in ISI. We can use the z-transform to get a system transfer function for this multipath model, as shown in Figure 2, where the coefficients of the transfer function are called as taps.

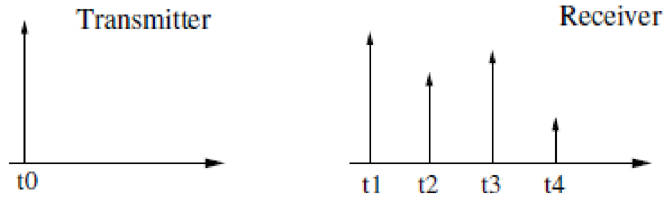

Figure 2: A Multipath Model

2 Equalization

Equalization is a technique used to mitigate the effects of multipath propagation and ISI. It involves modifying the received signal to compensate for the distortion introduced by the channel.

An equalizer is a signal processing block that is used to reduce the effect of ISI on the transmitted symbols. Hence, they are designed in such a way that they have the inverse transfer function of the multipath model.
Let the transfer function H(z) of the multipath model be

$$H(z) = 1 + \sum_{i=1}^{n} a_i z^{-i}$$

where a_i is the channel tap coefficient for ith tap. Then, the equalizer transfer function E(z) is,

$$E(z) = \frac{1}{H(z)}$$

$$E(z) = 1 + \sum_{j=1}^{\infty} b_j z^{-j}$$

where b_j represent the equalizer's tap coefficient for jth tap and such an equalizer is called feed-forward equalizer.

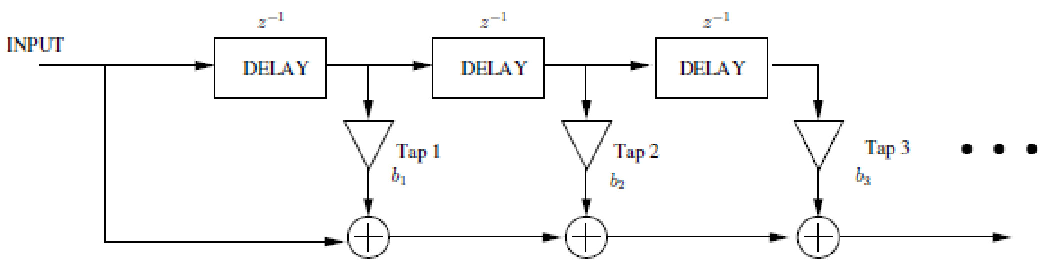

Figure 3: A Feed-Forward Equalizer

The channel tap coefficients (a_i s) can vary slowly with time due to changes in atmosphere or due to moving objects. In such cases, the equalizer tap coefficients (b_j s) also need to adapt to the time-varying channel. In general, adaptive equalizer algorithms are used as the channel coefficients are difficult to predict in advance (particularly for wireless channels). The most popular among them are Least Mean Square (LMS) algorithm and Constant Modulus Algorithm (CMA).

2.1 Least Mean Square Algorithm

LMS is a linear adaptive algorithm that minimizes the mean square error (i.e. tap coefficients bj s are updated/adjusted by minimizing the $E[|error|^2]$). Let d[n] be the training sequence that is used for adjusting the taps/coefficients before random data/sequence arrives (can you think of the reason why the adaptive mechanism is not performed when random data arrives), u[n] be the input to the receiver, y[n] be the output of the receiver as shown in Figure 4.
Then, before the random data arrives, the taps/coefficients in the LMS algorithm are updated by the equation

$$b_j[n+1] = b_j[n] + \mu u[n] e^*[n]$$

where $e[n] = d[n] - y[n]$, μ is the step size of the LMS algorithm and determines the convergence rate. Too small a step size will make the algorithm take a lot of iterations, while too large a step size may diverge the weight taps (can you think of the reason)?

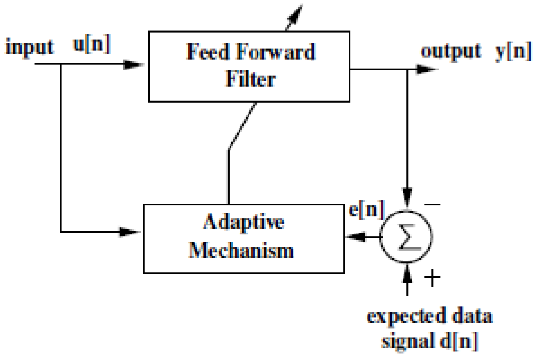

Figure 4: Least Mean Square Algorithm

2.2 Constant Modulus Algorithm (CMA)

The CMA equalizer is an adaptive equalizer that works well when the signal has a constant modulus, i.e., when the signal constellation points lie on a circle (such as QPSK or 8-PSK). Multipath effects distort the received signal and its constellation diagram. The equalizer tries to ensure that the signal samples from the equalizer output lie on a circle. This is done by minimizing the dispersion of the equalizer output y[n] around a circular contour with a predefined radius R (for n th symbol), which is termed as a cost function and given by equation.

$$J[n] = E[||y[n]|^2 - R^2|^2]$$

where $y[n] = y_I[n] + jy_Q[n]$ and $y_I[n]$, $y_Q[n]$ are the in-phase and quadrature-phase components of the signal obtained at the equalizer output for nth symbol, and R is the radius of the circle. Let u[n] be the signal to be equalized, and coefficient of kth tap of the adaptive filter taps with L taps be $b_k[n]$, then the equalizer output and error are given by:

$$y[n] = \sum_{j=0}^{L-1} b_j[n] u[n-j] = B_n^H U_n$$

$$e[n] = |y[n]|^2 - R^2$$

Then, the updated equation of the filter coefficients for a CMA equalizer is given by:

$$b_j[n+1] = b_j^*[n] - \mu e[n] y^*[n] u[n]$$

where μ is the step factor that is to be carefully selected. Usually, minimizing the difference ensures that the equalizer has compensated for the multipath effects added by the channel, and as a result, output samples lie on the desired constellation. It should be noted that if there are any phase and frequency offsets, the points may appear anywhere on a circle, and hence, a carrier frequency and phase synchronization block (such as a Costas loop) is required to remove these offsets to obtain the desired constellation plot.

Generally, some training sequences (symbol sequences known to the transmitter) are used for the initial adjustment of tap coefficients. However, CMA algorithm can be used for "blind-adaptation," which means no training sequences are required (can you think of the reason?).