L10 Recursion

CHAPTER 19

Topics

Introduction to Recursion

A recursive function is a function that calls itself.

Recursive functions can be useful in solving problems that can be broken down into smaller or simpler subproblems of the same type.

A base case should eventually be reached, at which time the breaking down (recursion) will stop.

Recursive Functions

Consider a function for solving the count-down problem from some number `num` down to `0`:

- The base case is when `num` is already `0`: the problem is solved and we "blast off!"
- If `num` is greater than `0`, we count off `num` and then recursively count down from `num-1`

Recursive Functions

A recursive function for counting down to 0:

```
void countDown(int num)
{
  if (num == 0)
        cout << "Blastoff!";
    else
    {
        cout << num << ". . .";
        countDown(num-1); // recursive
    }                     // call
}
```

What Happens When Called?

If a program contains a line like `countDown(2);`

1. `countDown(2)` generates the output `2...`, then it calls `countDown(1)`
2. `countDown(1)` generates the output `1...`, then it calls `countDown(0)`
3. `countDown(0)` generates the output `Blastoff!`, then returns to `countDown(1)`
4. `countDown(1)` returns to `countDown(2)`
5. `countDown(2)` returns to the calling function

What Happens When Called?

first call to **`countDown`**
`num` is **2**

`countDown(1);`

second call to **`countDown`**
`num` is **1**

`countDown(0);`

third call to **`countDown`**
`num` is **0**

```
// no
// recursive
// call
```

OUTPUT:

`2...`

`1...`

`Blastoff!`

Stopping the Recursion

A recursive function should include a test for the **base cases**

In the sample program, the test is:

```
if (num == 0)
```

Each successive call should **take a step closer to the base case** to avoid an **infinite** number or recursive calls.

Stopping the Recursion

```
void countDown(int num)
{
  if (num == 0) // test
       cout << "Blastoff!";
    else
    {
     cout << num << "...\n";
       countDown(num-1); // recursive
  }                         // call
}
```

Stopping the Recursion

With each recursive call, the parameter controlling the recursion should **move closer to the base case**

Eventually, the parameter reaches the base case and the chain of recursive calls terminates

Stopping the Recursion

```
void countDown(int num)
{
    if (num == 0)          // base case
        cout << "Blastoff!";
    else
    {   cout << num << "...\n";
        countDown(num-1);
    }
}
```

Value passed to recursive call is closer to base case of num = 0.

What Happens When Called?

Each time a recursive function is called, a **new copy of the function runs, with new instances of parameters and local variables being created**

As **each copy finishes executing**, it **returns to the copy of the function that called it**

When the **initial copy finishes executing, it returns to the part of the program that made the initial call to the function**

Types of Recursion

Direct recursion

- a function **calls itself**

Indirect recursion

- **function A calls function B**, and **function B calls function A**. Or,
- function A calls function B, which calls …, which calls function A

2 The Recursive Factorial Function

The factorial of a nonnegative integer n is the product of all positive integers less or equal to n

Factorial of n is denoted by $n!$

The factorial of 0 is 1

$$0\,! = 1$$

$$n\,! = n \times (n\text{-}1) \times \ \ldots \times 2 \times 1 \quad \text{if} \quad n > 0$$

Recursive Factorial Function

Factorial of $\boldsymbol{n}$ can be expressed in terms of the factorial of $\boldsymbol{n}$**-1**

$$\mathbf{0\,!} = \mathbf{1}$$
$$\boldsymbol{n}\,! = \boldsymbol{n} \times (\boldsymbol{n}\mathbf{-1})\,!$$

Recursive function

```
int factorial(int n)
 { if (n == 0) return 1;
   else
       return n *factorial(n-1);
 }
```

3 The Recursive gcd Function

Greatest common divisor (gcd) of two integers x and y is the largest number that divides both x and y

The Greek mathematician Euclid discovered that

- If y divides x, then gcd(x, y) is just y
- Otherwise, the gcd(x, y) is the gcd of y and the remainder of dividing x by y

The Recursive gcd Function

```
int gcd(int x, int y)
{
   if (x % y == 0) //base case
       return y;
   else
       return gcd(y, x % y);
}
```

4 Solving Recursively Defined Problems

The natural definition of some problems leads to a recursive solution

Example: Fibonacci numbers:

`0, 1, 1, 2, 3, 5, 8, 13, 21, ...`

After the starting **`0, 1`**, each term is the sum of the two preceding terms

Recursive solution:

```
fib(n) = fib(n – 1) + fib(n – 2);
```

Base cases: **`n == 0, n == 1`**

Recursive Fibonacci Function

```
int fib(int n)
{
  if (n <= 0)              // base case
      return 0;
  else if (n == 1)         // base case
      return 1;
  else
         return fib(n - 1) + fib(n - 2);
}
```

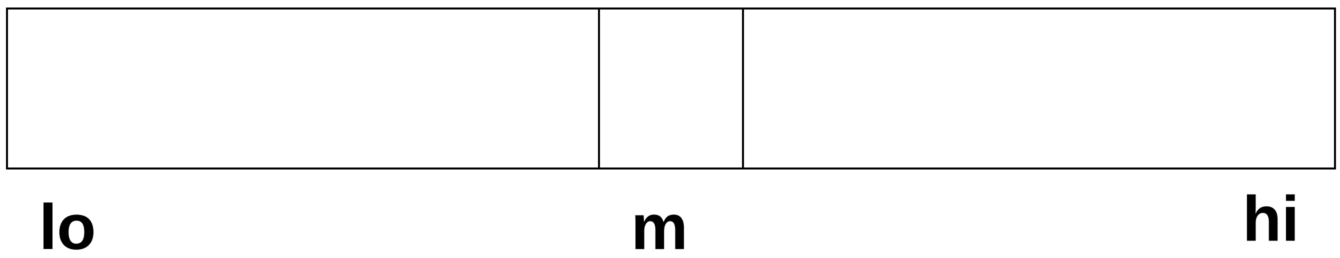
5 A Recursive Binary Search Function

Assume an array `a` that is sorted in ascending order, and an item `x`

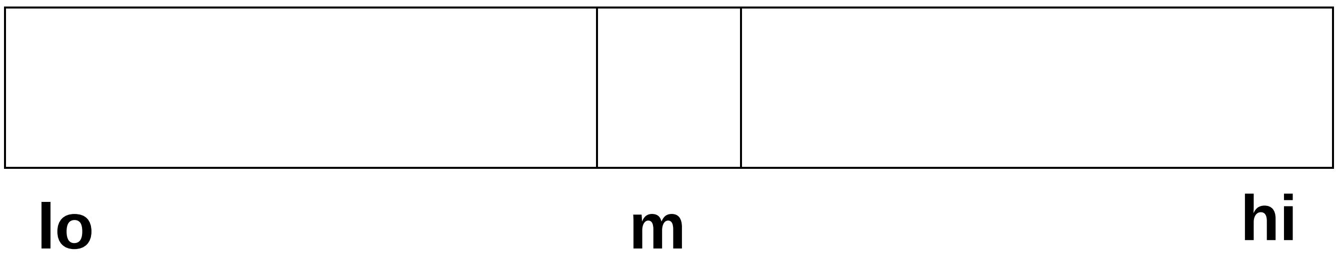
We want to write a function that searches for `x` within the array `a`, returning the index of `x` if it is found, and returning `-1` if `x` is not in the array

Recursive Binary Search

A recursive strategy for searching a portion of the array from index `lo` to index `hi` is to set `m` to index of the middle portion of array:



Recursive Binary Search



If `a[m] == x`, we found `x`, so return `m`

If `a[m] > x`, recursively search `a[lo..m-1]`

If `a[m] < x`, recursively search `a[m+1..hi]`

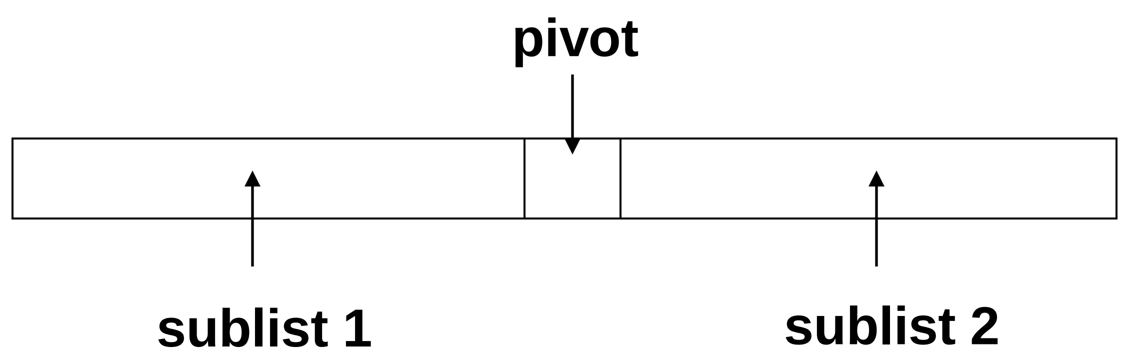
Recursive Binary Search

```
int bSearch(int a[],int lo,int hi,int x)
{
  int m = (lo + hi) /2;
  if(lo > hi) return -1;     // base
  if(a[m] == x) return m;    // base
  if(a[m] > x)
    return bsearch(a,lo,m-1,x);
  else
    return bsearch(a,m+1,hi,x);
}
```

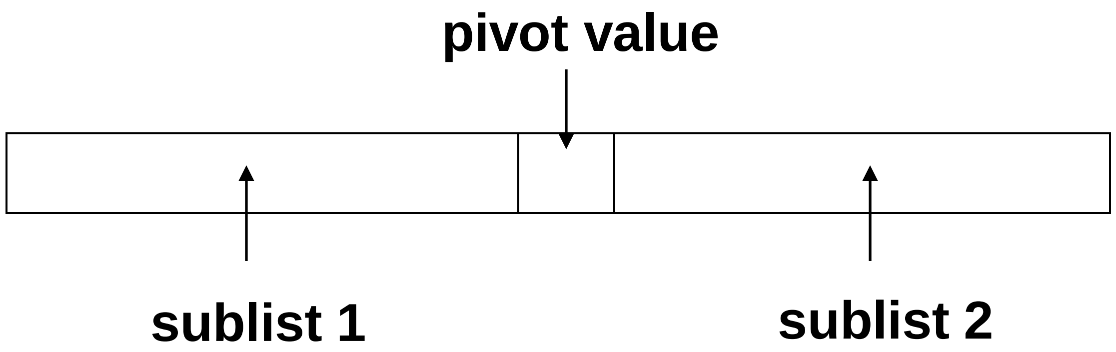
6 The QuickSort Algorithm

Recursive algorithm that can sort an array

First, determine an element to use as pivot_value:



The QuickSort Algorithm



Then, values are shifted so that elements in sublist1 are < pivot and elements in sublist2 are >= pivot

Algorithm then recursively sorts sublist1 and sublist2

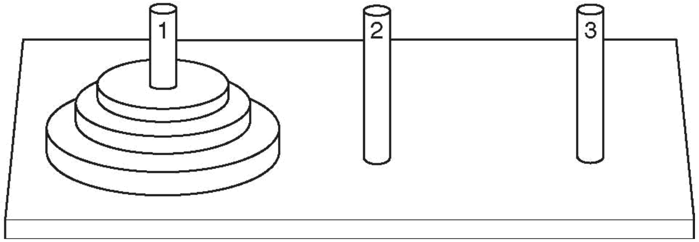
Base case: sublist has size <=1

7 The Towers of Hanoi

Setup: 3 pegs, one has n disks on it, the other two pegs empty. The disks are arranged in increasing diameter, top→ bottom

Objective: move the disks from peg 1 to peg 3, observing

- only one disk moves at a time
- all remain on pegs except the one being moved
- a larger disk cannot be placed on top of a smaller disk at any time



The Towers of Hanoi

How it works:

n=1	Move disk from peg 1 to peg 3. Done.
n=2	Move top disk from peg 1 to peg 2. Move remaining disk from peg 1 to peg 3. Move disk from peg 2 to peg 3. Done.

Outline of Recursive Algorithm

If n==0, do nothing (base case)

If n>0, then

a. Move the topmost n-1 disks from peg1 to peg2
b. Move the n^{th} disk from peg1 to peg3
c. Move the n-1 disks from peg2 to peg3

end if

8 Exhaustive and Enumeration Algorithms

Enumeration algorithm: generate all possible combinations

Example:

- All possible ways to make change for a certain amount of money
- Find all the possible passwords with a certain pattern.

Exhaustive algorithm: search a set of combinations to find an optimal one

Example:

- Change for a certain amount of money that uses the fewest coins

14.8 Recursion vs. Iteration

Benefits (+), disadvantages(-) for recursion:

- +Natural formulation of solution to certain problems
- +Results in shorter, simpler functions
- ◦ May not execute very efficiently

Benefits (+), disadvantages(-) for iteration:

- +Executes more efficiently than recursion
- ◦ May not be as natural as recursion for some problems