Corrigendum to "Characteristics are Covariances: A Unified Model of Risk and Return"

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1 December 2023—arbitrage portfolios

1.1 Changes to the paper

In the published version of "Characteristics are Covariances: A Unified Model of Risk and Return" (*Journal of Financial Economics*, December 2019, 134(3): 237–254) we report mistaken empirical results for the "arbitrage" portfolios, in KPS Table 7 (we use "KPS" to refer to tables in the publication, as opposed to this erratum). Our code erroneously *over* lagged—that is, the weight w_{t-1} used Z_{t-2} (in the code) instead of the proper Z_{t-1} (as correctly written on page 517 in Section 4.5.3). For bringing this to our attention in the code `IPCA_empirical_GBGA_outofsample.m` we thank the research team of Luca Pezzo, Raja Velu, Lei Wang, and Zhaoque (Chosen) Zhou. An updated version of the code now resides in the Dropbox folder whose link is available at `https://sethpruitt.net/downloads/`, a folder where all our code has been publicly available since before the paper's publication. **To reiterate, *only* KPS Table 7 changes and the error had no effect on any other results**.

The corrected results are in Table 1. Notably these annualized Sharpe ratios are above one and reach as high as 3. Therefore two statements we make in the text should be altered—we discuss them in the order of the page they appear.

On page 504 it is stated:

Table 1: Corrected KPS Table 7, IPCA pure-alpha portfolios

This table reports the out-of-sample annualized Sharpe ratios for a portfolio designed to exploit characteristic-based mispricing estimated from Γ_α in the unrestricted IPCA model. It corrects the mistaken numbers reported in KPS Table 7.

	K					
	1	2	3	4	5	6
Sharpe ratio	1.42	1.82	2.40	3.00	2.91	2.71

> However, we find that pure-alpha compensation is far smaller than that earned from harvesting factor risk premia.

This is no longer true because the pure-alpha portfolio Sharpe ratio is not "far smaller" than the tangency portfolio Sharpe ratio reported in KPS Table 6, when $K \geq 3$. Indeed, we find the Sharpe ratios of comparable size.

On page 517 we discuss KPS Table 7 and state:

> Pure-alpha Sharpe ratios range from 0.55 to 1.07, and are substantially smaller than the factor risk premium portfolios reported in Table 6... And, with these Sharpe ratios, they appear far from consistent with the typical "near arbitrage" interpretation of mispricing-based alpha.

Obviously now the numerical range is 1.42 to 3.00 and so these are more consistent to the typical "near arbitrage" interpretation of mispricing-based alpha.

1.2 Support for the qualitative argument in KPS

Our qualitative conclusion in KPS was that an investor trading factor risk does not gain a significant amount by trading pure-alpha. After correcting the above mistakes, we continue to argue this is the case.

To demonstrate this, we calculate two notions of a tangency portfolio. The first is the out-of-sample tangency portfolio attainable for an investor using only the restricted K-factor IPCA model with alphas fixed at zero (these are exactly the results in the second row of Panel A of KPS Table 6). The second uses the K factors from the restricted model, but also gives the investor access to the pure-alpha portfolio implied by the unrestricted model. As Table 2 shows, inclusion of the pure-alpha portfolio has no material benefit for out-of-sample performance for $K > 3$.

Table 2: Tangency portfolios, restricted factors and pure-alpha portfolio

This table reports the out-of-sample annualized Sharpe ratios for two notions of a tangency portfolio. The first is the out-of-sample tangency portfolio attainable for an investor using only the restricted K-factor IPCA model with alphas fixed at zero. The second uses the K factors from the restricted model, but also gives the investor access to the pure-alpha portfolio implied by the unrestricted model.

	K					
	1	2	3	4	5	6
Factors	0.62	0.62	2.51	3.09	3.89	4.05
Factors + pure-alpha	1.57	1.90	2.72	3.13	3.10	3.59

These findings support the statement (hidden in the ellipsis above) on KPS page 517, albeit using a novel notion of the tangency portfolio:

> While pure-alpha portfolios have the virtue of factor neutrality, they do not appear particularly attractive in terms of mean-variance efficiency.

Out-of-sample performance *degrades* when we add the pure-alpha portfolio to the factor space (of a sufficiently rich dimension), implying it adds nothing useful to investors' mean-variance objective.

2 June 2020—Typos

Table 3, Panel C, row SMB, column 1 has a typo: it should read "29.7" instead of "2.97"

Footnote 26 should read "at least 12 non-missing months" instead of 60. If we *instead* require 60 months, the results are identical.

Given the root-N consistency of the factor estimates, we filtered the data to look only at months with at least 100 cross-sectional observations. If we reduce this filter to 6 observations (so that $Z_t'Z_t$ has rank >=6 every month) we pick up 12 more months of data. Our results are unchanged.

We thank Philip Bold for alerting us to these.