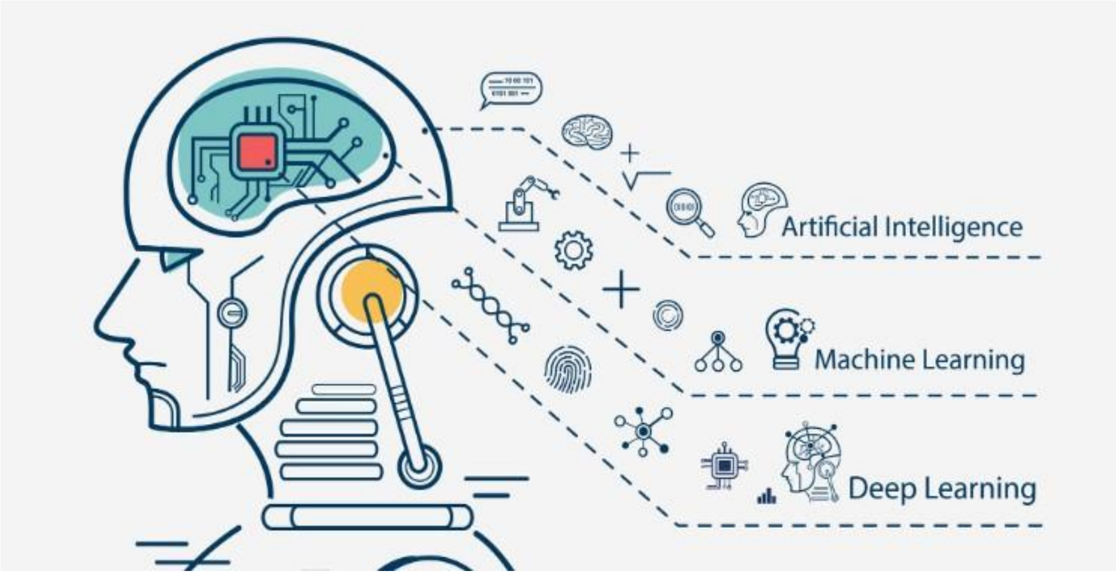
Image Enhancement



Introduction

- Intensity transformations

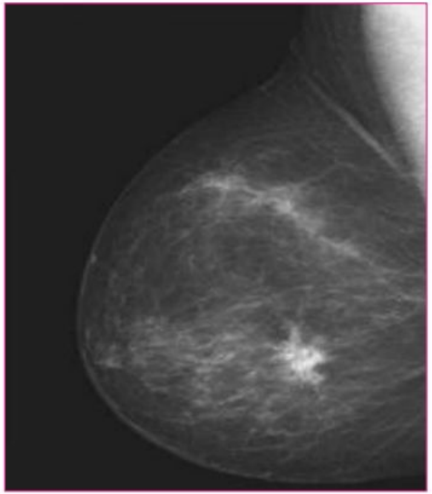
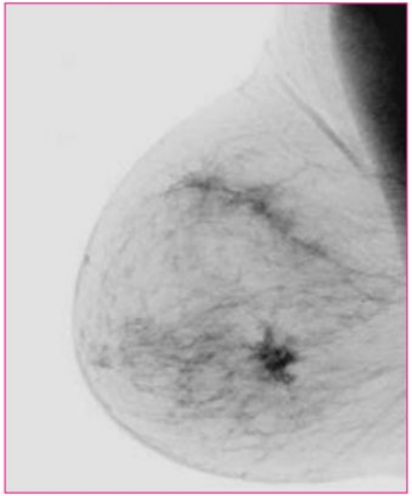
- Distribution transformation

SPATIAL DOMAIN TRANSFORMATION

- Transformations
  - intensity transformations
    - negatives
    - logs
    - power-law (gamma)
    - contrast stretching
    - level slicing
    - bit-plane slicing
  - distribution transformations
    - histogram equalization
- Spatial filtering
  - image filtering

Negatives

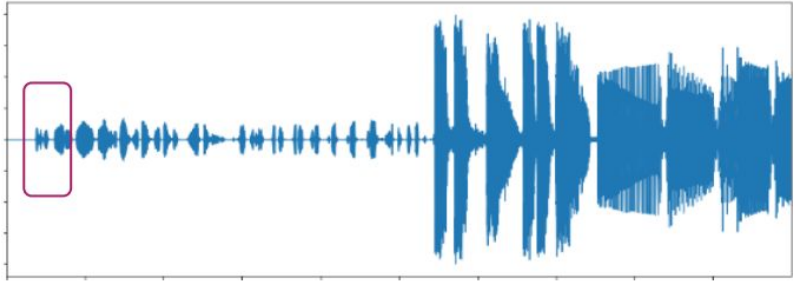
$$\boxed{s = L - 1 - r}$$

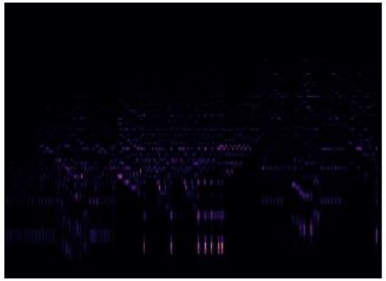



Logs

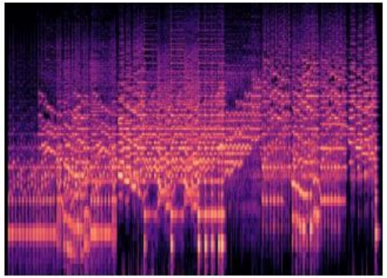
$$s = c \cdot log(1 + r)$$

- Log transformations
  - used to expand values of dark pixels
  - simultaneously compressing bright pixels
  - compresses dynamic range of images
    - Fourier spectrum





Without Log transform

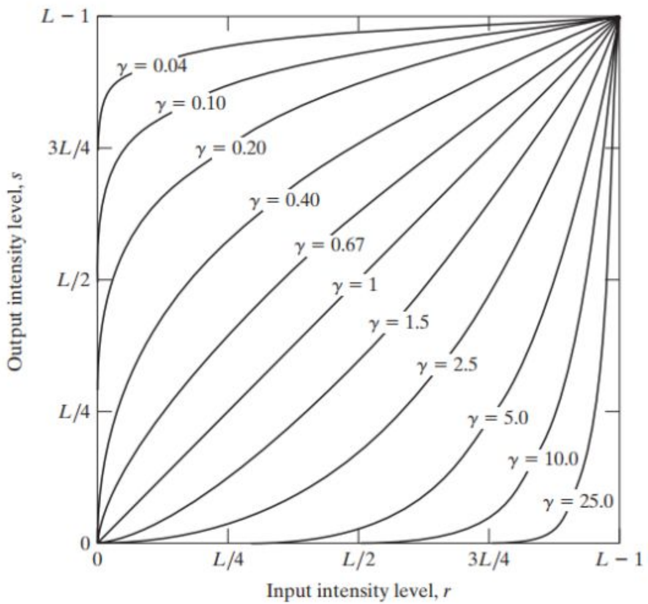


Log transform done on original image and thn, it has been scaled up to 255, to form the image matrix

Gammas

$$\boxed{s = c \cdot r^{\gamma}}$$

- Power-law transformations
  - sensors respond according to power law
    - CMOS, scanners, printing, displays
    - CRT: intensity to voltage response as power function ($\gamma' = 1.8\text{\textasciitilde}2.5$)
  - gamma correction
    - device dependent γ
    - γ variation also varies the color ratios
    - correct color reproduction needs knowledge of γ
  - gamma injection
    - post image processing for contrast manipulation



Gammas

$$s = c \cdot r^{\gamma}$$

- γ injection

Enhances Contrast

1.0

0.6

0.4

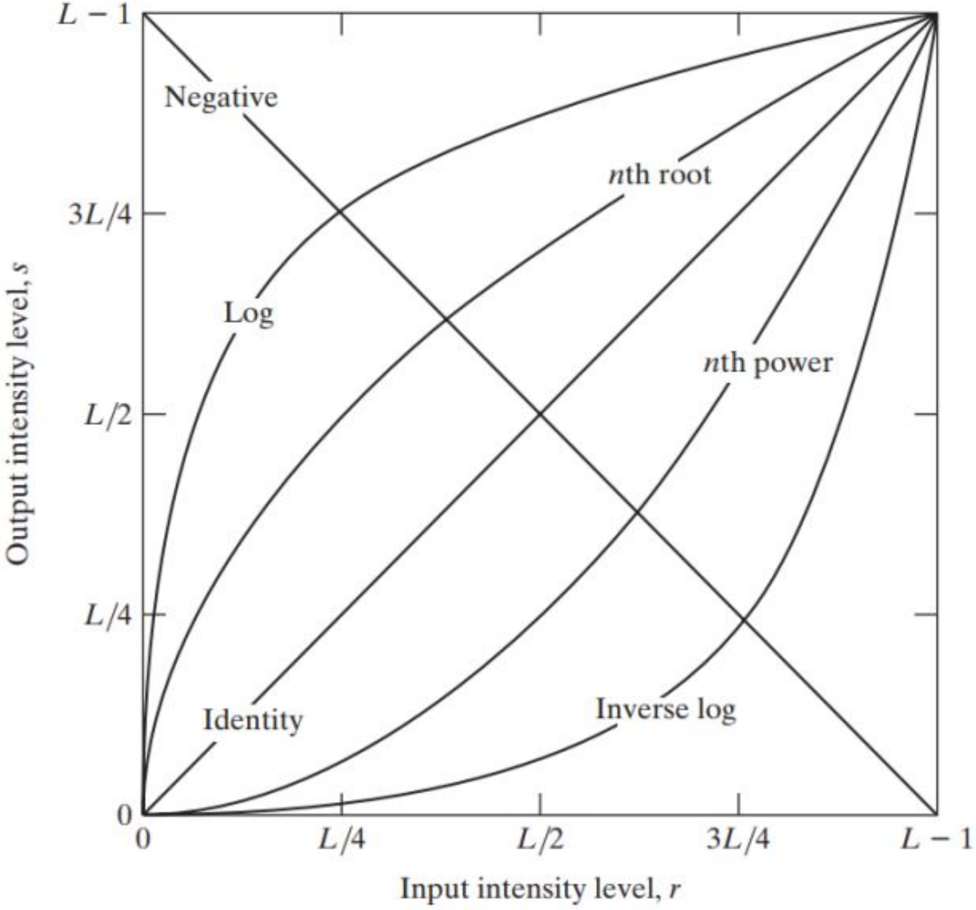
0.3

1.0

3.0

4.0

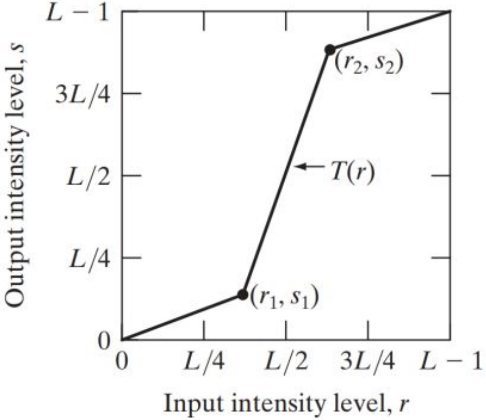
5.0



L − 1
3L/4
L/2
L/4
0
0
L/4
L/2
3L/4
L − 1
Input intensity level, r
Output intensity level, s
Negative
nth root
Log
nth power
Identity
Inverse log

Contrast stretching

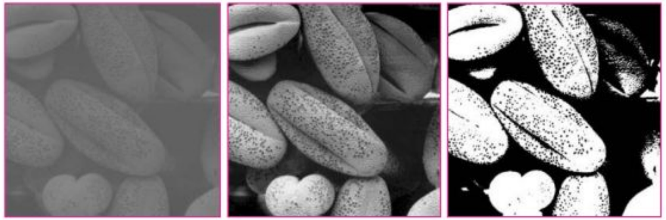
- Contrast
  - Low contrast images
    - due to poor illumination, low dynamic range sensors
    - wrong setting of lens aperture
  - full range stretching
    - $(r_1, s_1) = (r_{min}, 0)$
    - $(r_2, s_2) = (r_{max}, L-1)$
  - thresholding
    - $r_1 = r_2$, $s_1 = 0$, $s_2 = L-1$



70~140 Contrast streching 0~255

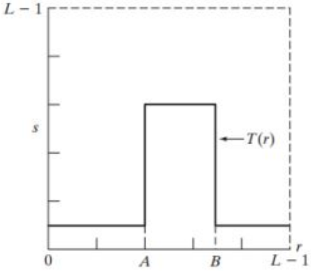
pixel ranges

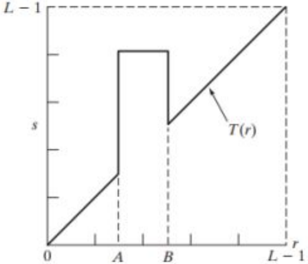
SEM image of pollen grains

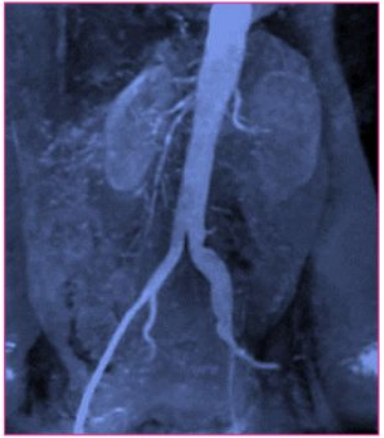


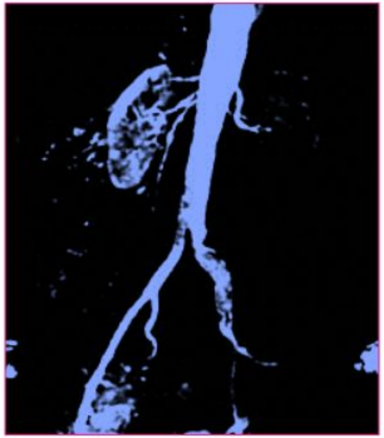
Level slicing

- Intensity levels
  - local thresholding, stretching
  - enhancing only specific intensities
    - e.g. detecting water, wetland in sat. images



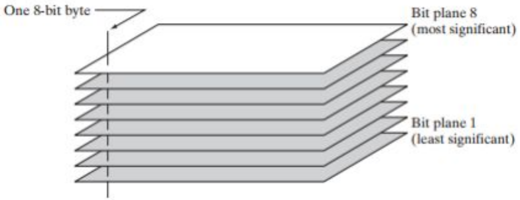






Bitplane slicing

- Bitplanes
  - contribution of each bit for total image appearance
  - gives clue for a compression



- slicing

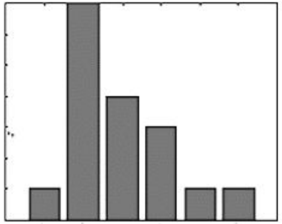
- reconstruction

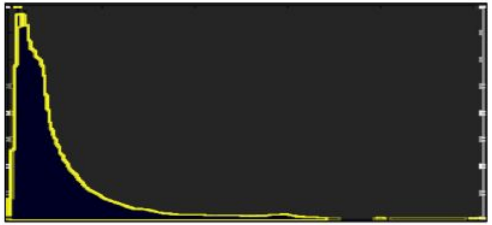
- bitplanes (8+7)
- bitplanes (8+7+6)
- bitplanes (8+7+6+5)

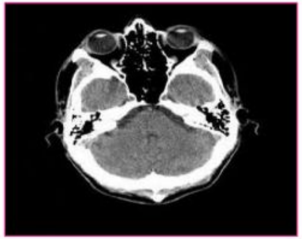
Histograms

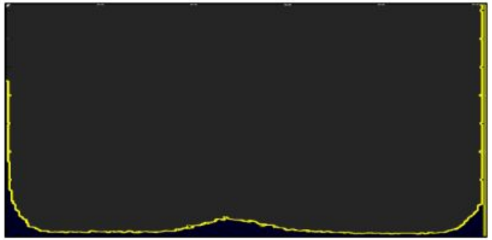
- distribution of discrete intensities
  - distribution is also discrete

4	1	3	2
3	1	1	1
0	1	5	2
1	1	2	2



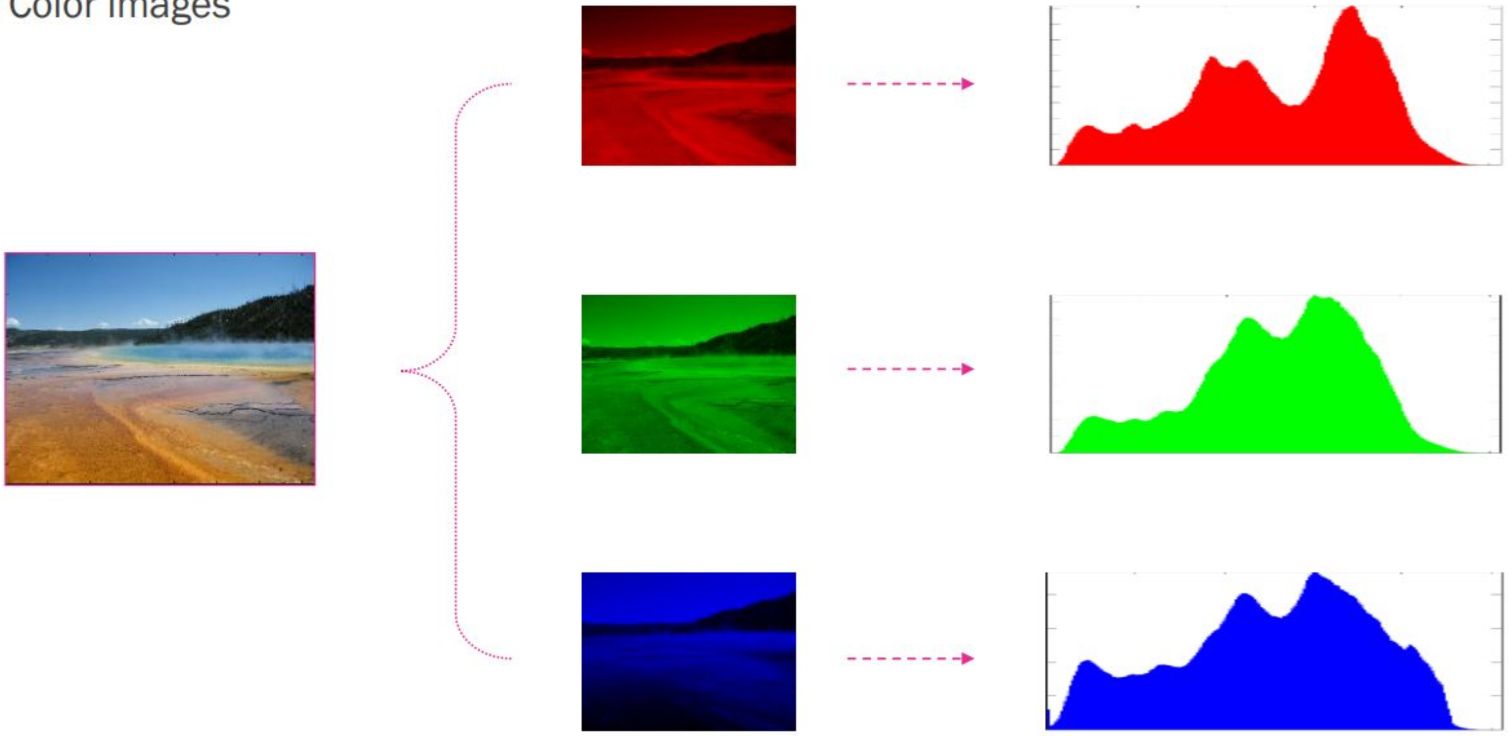




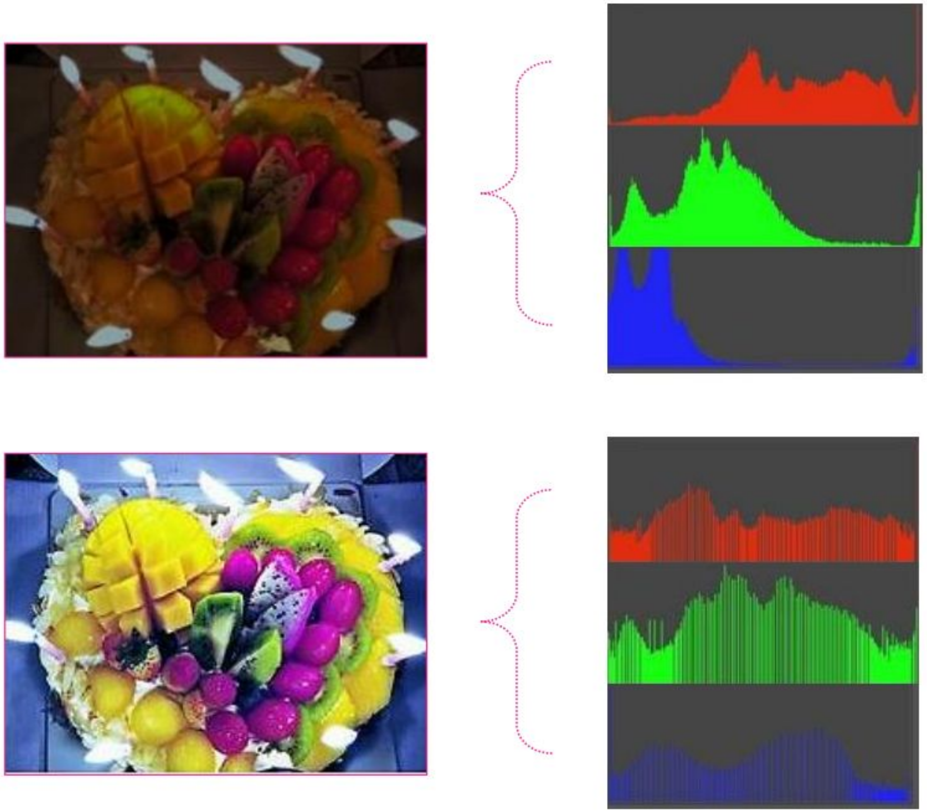


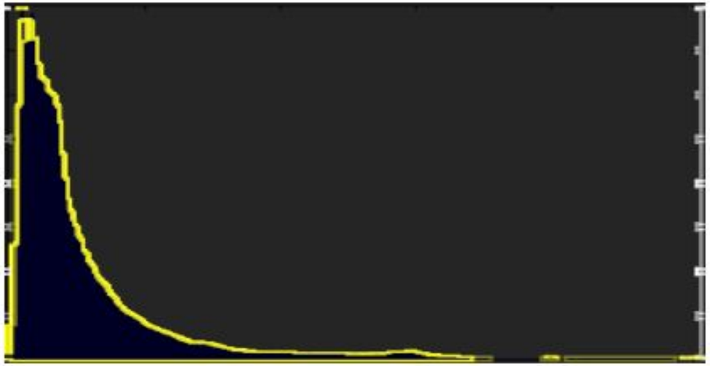
Histograms

- Color images



Histogram equalization



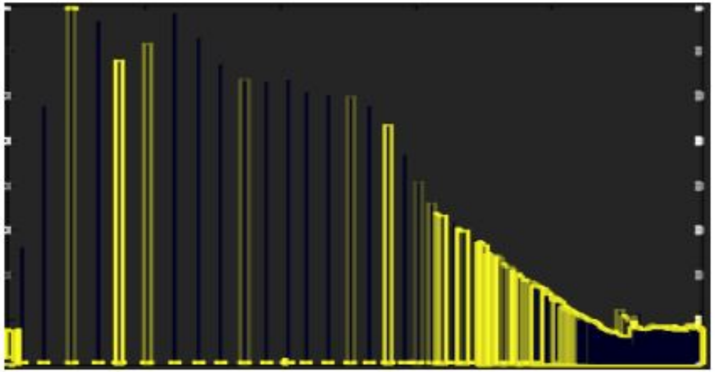
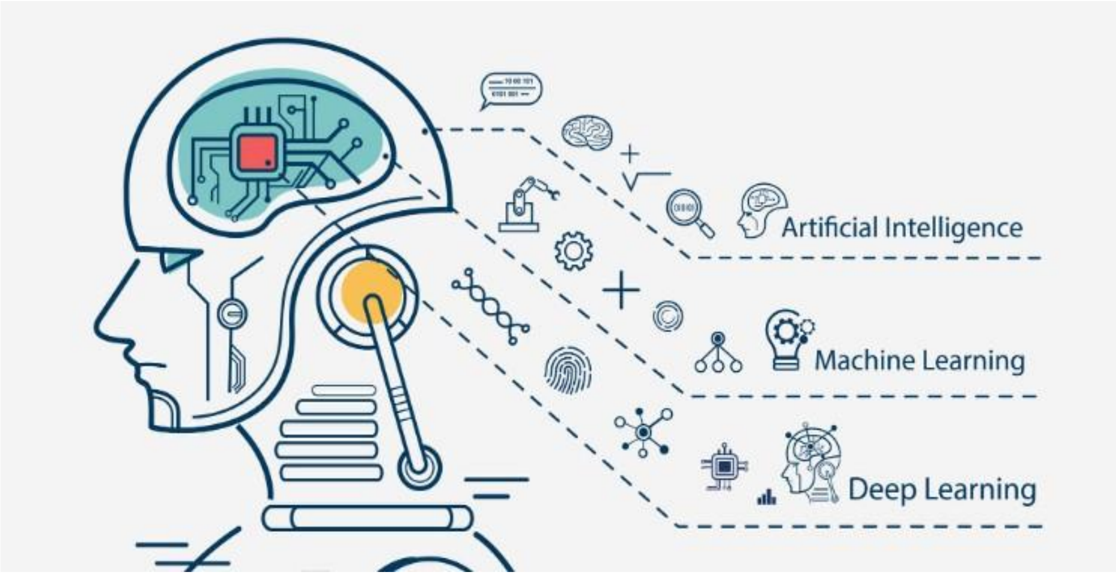
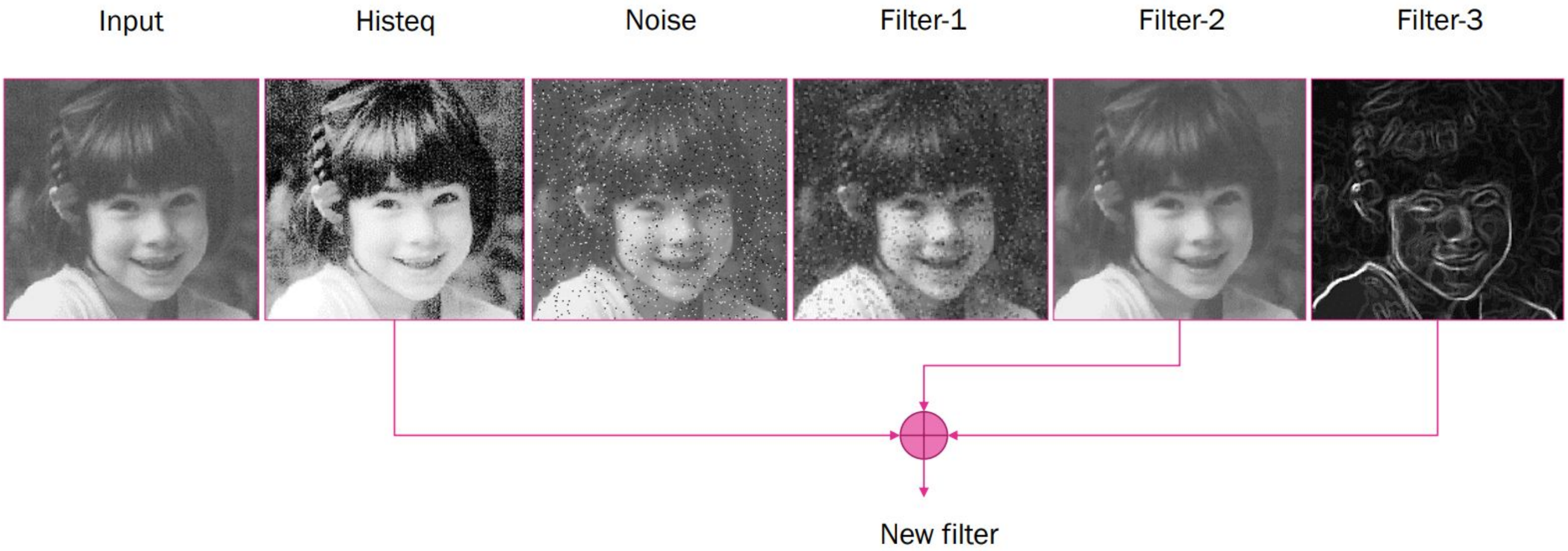


Image Filtering



Introduction



Linearity

- Operations
  - linear
    - additivity

$$\mathcal{T}[a \cdot f_1(x,y) + b \cdot f_2(x,y)] = \mathcal{T}[a \cdot f_1\ (x,y)] + \mathcal{T}[b \cdot f_2\ (x,y)]$$

    - homogeneity

$$\mathcal{T}[a \cdot f_1(x,y)] = a \cdot \mathcal{T}[f_1(x,y)]$$

  - non-linear
    - not satisfying above

- Examples
  - linear
    - negatives
  - non-linear
    - gammas

Correlation & Convolution

- Correlation
  - measures similarity between the two signals
  - windowed signal (kernel) is not reversed
  - sliding vectors dot product
  - orthogonal signals are uncorrelated

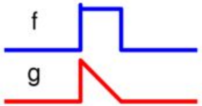
- Convolution
  - measure the effect of one signal on the another
  - windowed signal (kernel) is reversed
    - for symmetric kernels convolution = correlation

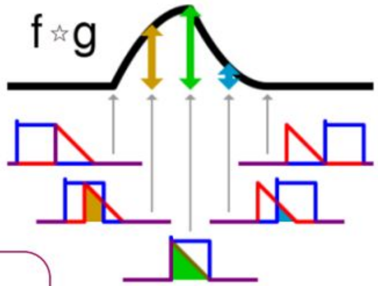
‘ ✯, ⊛, ✲, ✺, ✶, ✷, ✭, ✬, ✫, ☆, ✡, ✰, ✱, ✳, ✪ ’

Correlation & Convolution

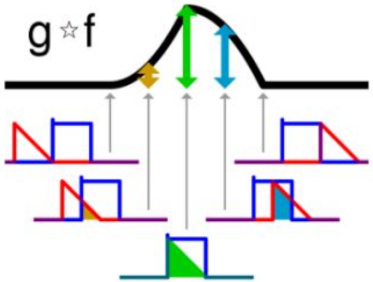
- Correlation

Start with the first signals last point in contact with second signal. Keep the second signal fixed and move the first signal over it. Now see the area enclosed by them and plot it.



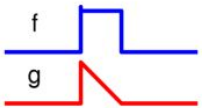


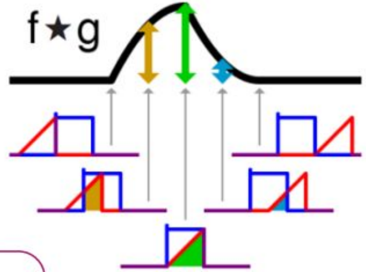
$$R(x) = f(x) \star g(x)$$
$$R(x) = \int_{-\infty}^{\infty} f(z) g(x+z) dz$$



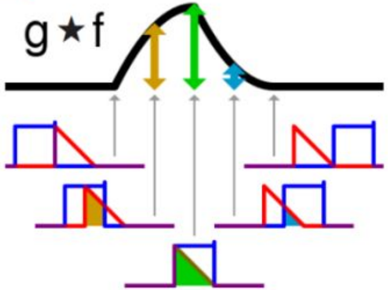
- Convolution

Reverse the second signal. Now place the last point of second signal in contact with the first point of first signal. Keep the first signal fixed. And keep moving the reversed second signal. Plot the area enclosed.





$$G(x) = f(x) * g(x)$$
$$G(x) = \int_{-\infty}^{\infty} f(z) g(x-z) dz$$



credit: wikipedia

Correlation & Convolution

- 2D correlation
  - cross-correlation
  - filtering algos internally use it
    - w need to be appropriately reflected before filtering

$$(w \star f)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

W is the kernel. F is the image function

- 2D convolution
  - $w \rightarrow m \times n$
  - $a = \frac{m-1}{2}, b = \frac{n-1}{2}$
    - a, b are assumed to be odd integers
    - note the kernels do not depend on (x, y)

$$(w \bigstar f)(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

Correlation & Convolution

$$(w \star f)(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

$$(w \bigstar f)(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$

Property	**Correlation**	**Convolution**
Commutative	—	$f \bigstar g = g \bigstar f$
Associative	—	$f \bigstar (g \bigstar h) = (f \bigstar g) \bigstar h$
Distributive	$f \star (g+h) = (f \star g) + (f \star h)$	$f \bigstar (g+h) = (f \bigstar g) + (f \bigstar h)$

Filtering

- Image filtering
  - spatial filtering
  - convolving a kernel with an image
  - filtering: $g(x,y) = (w \star f)(x,y)$

$$g(x,y) = w_3 \star w_2 \star w_1 \star f(x,y)$$

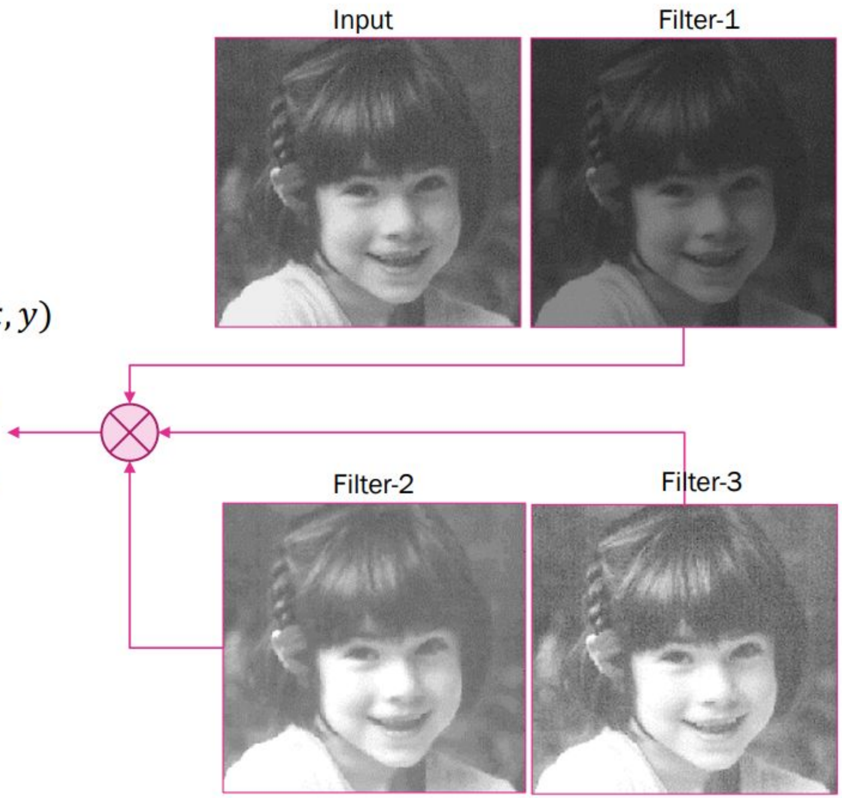
- Multistage filtering
  - filtering the filtered
  - use properties
    - commutative & associative

$$(w_3 \star w_2 \star w_1) \star f(x,y)$$

$$(w_1 \star w_2 \star w_3) \star f(x,y)$$

$$g(x,y) = w \star f(x,y)$$

$$w = w_1 \star w_2 \star w_3$$



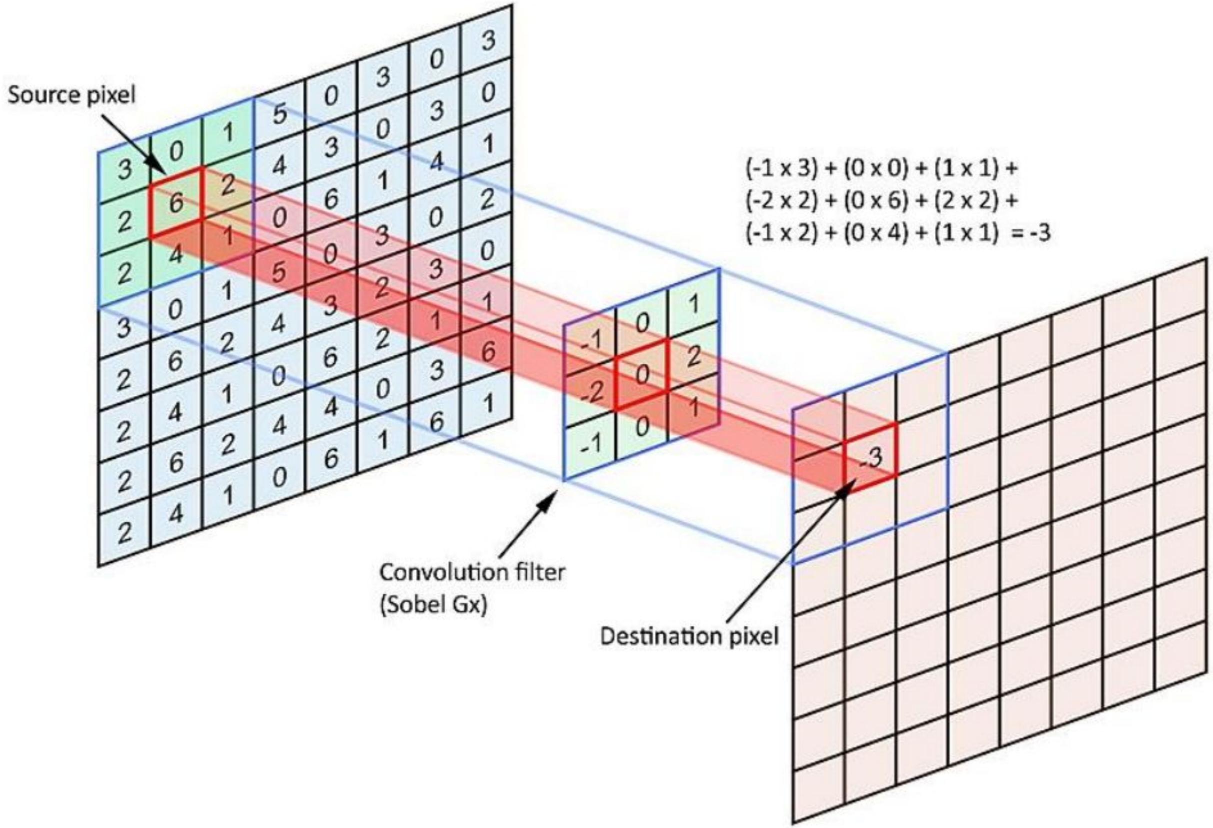
Filtering

- Filter
  - kernel, mask, window, template
  - $w(i,j)$ or $k(i,j)$ $\forall\ i,j \in N_K,\ K$- kernel size
    - K : determine neighbourhood of operation
    - $w(i,j)$: filter coefficients – determine nature of the filter

- Nature of a filter
  - neighbour interactions
    - filter coefficients define severity of interaction
  - smoothing
  - sharpening
  - noise handling capacity

Filtering

- Paddings
  - zero
  - mirror
  - replicate



3. Mirror padding

- values outside the boundary of the image are obtained by mirror-reflecting the image across its border

- It is more applicable when the areas near the border contain image details

13	12	11	12	13	14	15	14	13
8	7	6	7	8	9	10	9	8
3	2	1	2	3	4	5	4	3
8	7	6	7	8	9	10	9	8
13	12	11	12	13	14	15	14	13
18	17	16	17	18	19	20	19	18
13	12	11	12	13	14	15	14	13
8	7	6	7	8	9	10	9	8

mirror padding

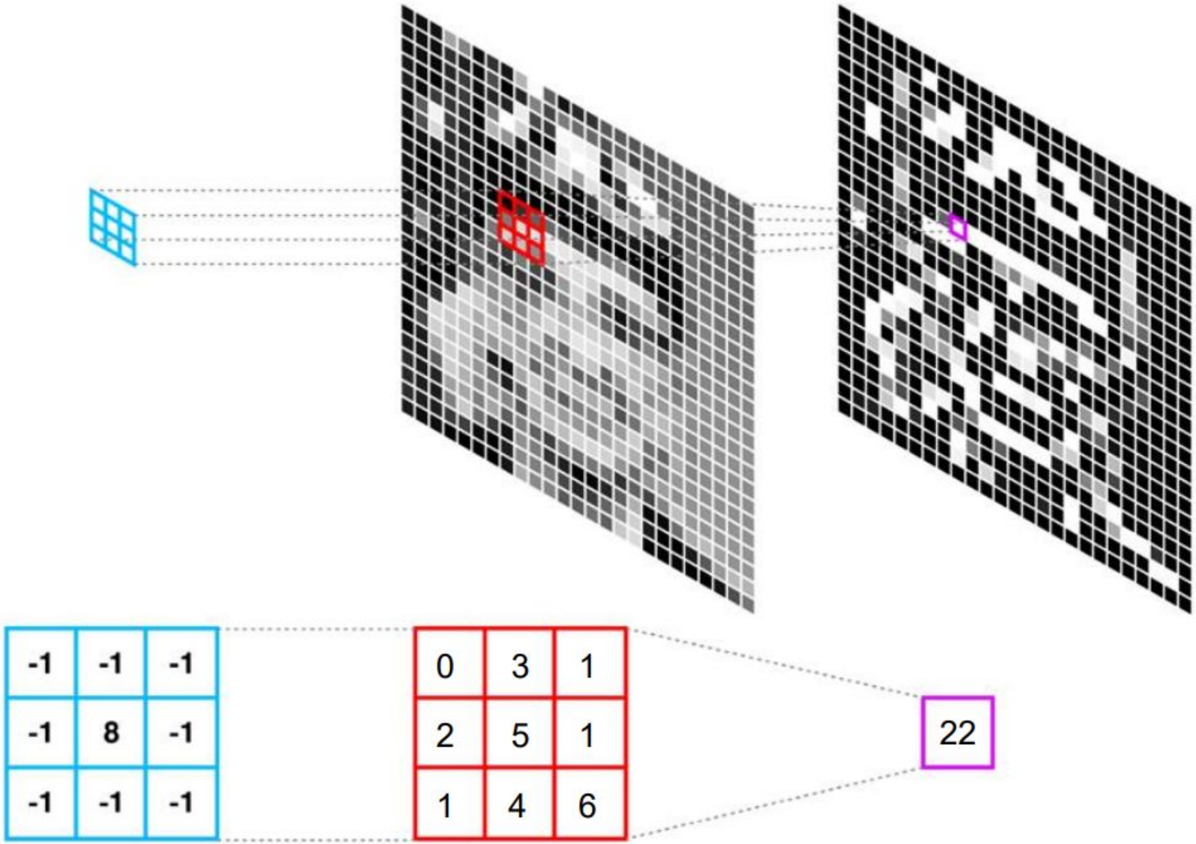
3	5	1
3	6	1
4	7	9

No padding

5	3	3	5	1	1	5
5	3	3	5	1	1	5
6	3	3	6	1	1	6
7	4	4	7	9	9	7
7	4	4	7	9	9	7

(1, 2) replication padding

Filtering



Filtering

- Separable kernels
  - a kernel in a matrix form can be represented as outer product of two vectors
  - $w = uv^T$
    - $u \in m \times 1$
    - $v \in n \times 1$
    - sq. kernels $w = uu^T, \; w \in m \times m$

$$w = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{c} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \mathbf{r} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- Advantage: separable kernels
  - computationally fast
    - outer product of vectors is same as their 2D conv

$$\mathbf{c}\,\mathbf{r}^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = w$$

Filtering

- Box filter
  - smoothing filter
  - lowpass filter
  - averaging filter

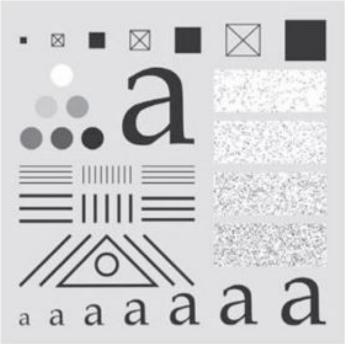
- Use cases
  - random noise reduction
    - reducing sharp transitions in intensity
    - favours blurring along perpendicular directions
  - reduce aliasing
    - smoothing prior to resampling
  - reduce quantization noise
    - reduce false contours of intensities
  - essential in composite filtering
    - multistage filters

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Filtering

- Box filter
  - smoothing filter
  - lowpass filter
  - averaging filter

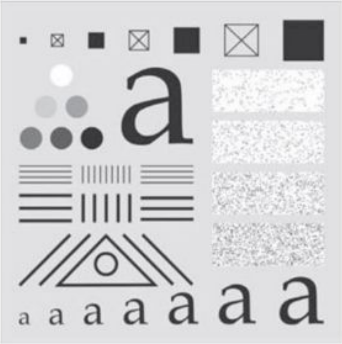
input



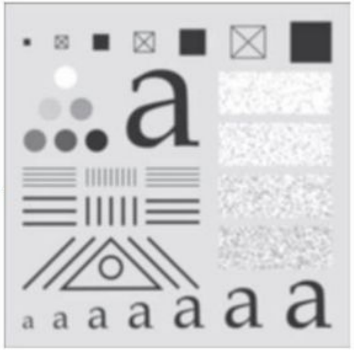
$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

m represents Kernel size

m=3



m=11



m=21

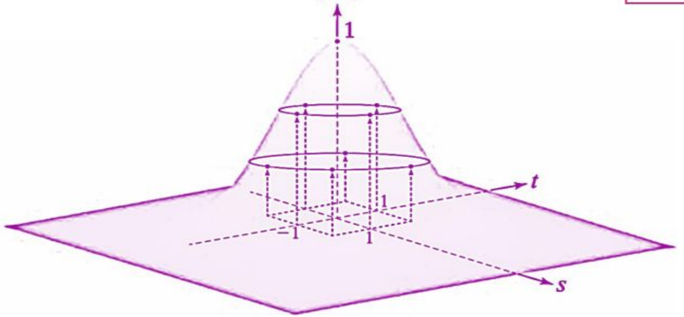
Filtering

- Gaussian filter
  - smoothing filter
  - defocused lens approximators
  - isotropic
    - response is independent of orientation
    - circularly symmetric

$$w(s,t) = G(s,t) = Ke^{-\frac{s^2+t^2}{2\sigma^2}}$$

$\frac{1}{4.8976} \times$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679



difference $m_{85} - m_{43}$

m=85 σ=7 Gauss

m=43 σ=7 Gauss

m=21 box

Filtering

- Padding effects

  m=187 σ=31 Gauss

  image 1024x1024

zero | mirror | replicate

- Relative size effect

  m=187 σ=31 Gauss

  image 4096x4096

  m=745 σ=124 Gauss

input | m=187 σ=31 | m=745 σ=124

Filtering

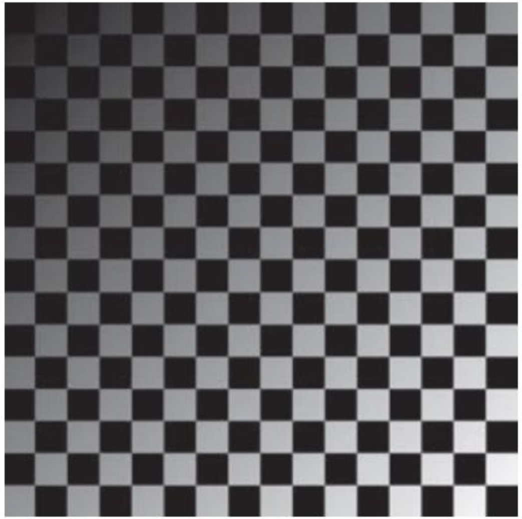
- Relevant region extraction

filtering

thresholding

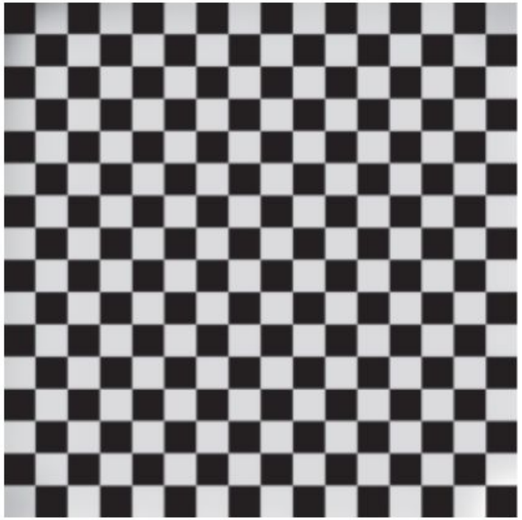
Filtering

- Shading correction



filtering

scaling



THANK YOU