

## **Assignment 1**

### **Conditional Probability**

1. Let A and B be two events with  $P(A) = 0.3$ ,  $P(B) = 0.4$  and  $P(A \cap B) = 0.2$ . Find the probability that:
  - (a) exactly one of the events A or B will occur;
  - (b) at least one of the events A or B will occur;
  - (c) none of A and B will occur.

### **Monty Hall Problem**

2. There are 3 doors with one door having an expensive car behind it and each of the other 2 doors having a goat behind them. Monty Hall, being the host of the game, knows what is behind each door. A contestant is asked to select one of the doors and he wins the item (car or goat) behind the selected door. The contestant selects one of the doors at random, and then Monty Hall opens one of the other two doors to reveal goat behind it (note that at least one of the other two doors has a goat behind it and Monty Hall knows the door having goat behind it). Monty Hall offers to trade the door that contestant has chosen for the other door that is closed. Should the contestant switch doors if his goal is to win the car? (This problem is based on the American television game show "Let's Make a Deal" hosted by Monty Hall.)  
Use Conditional probability to solve.

### **Bayes Theorem**

3. A bag contains 6 balls. 3 balls are drawn at random without replacement and are found to be red. What is the probability that all balls in the bag are red?

### **Random Variables (basics of probability mass function)**

4. Let X be a discrete random variable with the following PMF

$P_X(X) :-$

$$f(x) = \begin{cases} 0.1 & \text{for } x = 0.2 \\ 0.2 & \text{for } x = 0.4 \\ 0.2 & \text{for } x = 0.5 \\ 0.3 & \text{for } x = 0.8 \\ 0.2 & \text{for } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

- Find  $P(X < 0.5)$
- Find  $P(0.25 < X < 0.75)$
- Find  $P(X = 0.2 | X < 0.6)$

### Random Variables (basics of probability density function)

5. Let  $X$  be a random variable with distribution function

$$F(x) = \begin{cases} 0, & \text{if } x < 0 \\ \frac{2}{3}, & \text{if } 0 \leq x < 1 \\ \frac{7-6c}{6}, & \text{if } 1 \leq x < 2 \\ \frac{4c^2-9c+6}{4}, & \text{if } 2 \leq x \leq 3 \\ 1, & \text{if } x > 3 \end{cases},$$

$C$  is real constant

1. Find the value of  $C$
2. Find  $P(1 < X < 2)$ ;  $P(2 \leq X < 3)$ ;  $P(0 < X \leq 1)$ ;  $P(1 \leq X \leq 2)$ ;  $P(X \geq 3)$ ;

### Expectation Calculation

6. The random variable  $X$  is sampled from the uniform distribution over the interval  $[0,1]$  i.e., the probability density function of  $X$  looks like  $p(x) = 1$  if  $x \in [0,1]$  and  $p(x) = 0$  if  $x < 0$  or  $x > 1$ .

The r.v.  $Y$  is such that  $\mathbb{E}[X^2 + Y^2] = 1$  and  $\text{Var}[Y] = 5/9$ .  $Y$  need not be independent of  $X$  and may have different support than  $X$ .

Calculate the following

- $E[X]$

- $\text{Var}[X]$
- $E[Y]$
- $E[X + Y]$