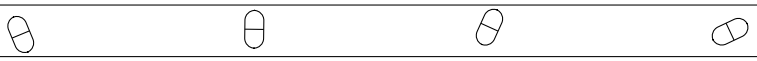

ℙROBABILISTIC 𝕋HINKING

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8-Paracetamol

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1 Royal revenge

Edmond Dantes is on a maze consisting of the integer lattice points, $G = \{(x, y) : 0 \leq x, y \leq n\}$. He starts off at the origin $(0, 0)$ and he wants to reach his fiancee Mercedes up at (n, n). At every second he can take one step to the right moving from the point (u, v) to $(u + 1, v)$ or a step above to $(u, v + 1)$. However, there is a river running between the lines $y = x$ and $y = x + 1$, so if he goes above the diagonal he will fall into the river. Let P_n denote the number of paths from $(0, 0) \mapsto (n, n)$ that do not cross the diagonal i.e. the path never goes above the line $y = x$.

Your mission should you choose to accept it, is to write a program that computes the value of $P_n (\mathrm{mod}(10^9 + 7))$ for $n = 1, 2, \ldots 100$.
(*Hint: It is easy to make a recurrence for the answer. However, if you try to write a recursive function to do this, then you run the risk of exploding your personal computational gizmo. This approach will compute the same answer repeatedly. You might want to store all the answers that you compute in a matrix and look up the answer again whenever needed.*)

Bonus: What is the asymptotic behaviour for P_n? Can you come up with a closed-form expression for P_n?