Курсовая работа по математическому анализу. Вариант 22.

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Содержание

1 Задание 1

$$\int \operatorname{ctg}^5 x\,dx = \int \operatorname{ctg} x \left(\frac{1}{\sin^2 x} - 1\right)^2 dx = \int \operatorname{ctg} x \left(\frac{1}{\sin^4 x} - \frac{2}{\sin^2 x} + 1\right) dx =$$
$$= \int \frac{\operatorname{ctg} x}{\sin^4 x}\,dx - \int \frac{2\operatorname{ctg} x}{\sin^2 x}\,dx + \int \operatorname{ctg} x\,dx = \int \frac{\cos x}{\sin^5 x}\,dx - 2\int \frac{\cos x}{\sin^3 x}\,dx + \int \frac{\cos x}{\sin x}\,dx =$$
$$= \int \frac{d\sin x}{\sin^5 x} - 2\int \frac{d\sin x}{\sin^3 x} + \int \frac{d\sin x}{\sin x} = -\frac{1}{4\sin^4 x} + \frac{1}{\sin^2 x} + \ln|\sin x| + C.$$

2 Задание 2

$$\int \sqrt{\sin 2x}\cos x\,dx = \int \sqrt{\sin 2x}\cos x\,dx = \int \sqrt{\sin 2x}\,|\cos x|\operatorname{sgn}\cos x\,dx =$$
$$= \operatorname{sgn}\cos x \int \sqrt{\frac{\sin 2x(1+\cos 2x)}{2}}\,dx = \operatorname{sgn}\cos x \int \sqrt{\frac{\operatorname{tg} x}{1+\operatorname{tg}^2 x}\left(1 + \frac{1-\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x}\right)}\,dx =$$
$$= \operatorname{sgn}\cos x \int \frac{\sqrt{2\operatorname{tg} x}}{1+\operatorname{tg}^2 x}\,dx = \left|\begin{array}{l} t = \sqrt{\operatorname{tg} x} \\ t^2 = \operatorname{tg} x \\ d\operatorname{tg} x = 2t\,dt \\ \dfrac{dx}{\cos^2 x} = 2t\,dt \\ dx = 2t\cos^2 x\,dt \\ dx = \dfrac{2t}{t^4+1}\,dt \end{array}\right| = 2\sqrt{2}\operatorname{sgn}\cos x \int \frac{t^2}{(1+t^4)^2}\,dt = (*)$$

Применим метод Остроградского:

$$\int \frac{t^2}{(1+t^4)^2}\,dt = \frac{at^3+bt^2+ct+d}{t^4+1} + \int \frac{et^3+ft^2+gt+h}{t^4+1}\,dt$$
$$\frac{t^2}{(1+t^4)^2} = \left(\frac{at^3+bt^2+ct+d}{t^4+1}\right)' + \frac{et^3+ft^2+gt+h}{t^4+1}$$
$$\frac{t^2}{(1+t^4)^2} = -\frac{4t^3\left(at^3+bt^2+ct+d\right)}{\left(t^4+1\right)^2} + \frac{3at^2+2bt+c}{t^4+1} + \frac{et^3+ft^2+gt+h}{t^4+1}$$
$$\frac{t^2}{(1+t^4)^2} = \frac{t^6(f-a)+t^2(3a+f)+t^5(g-2b)+t(2b+g)+t^4(h-3c)+c+t^3(e-4d)+et^7+h}{\left(t^4+1\right)^2}$$
$$a = f = \frac{1}{4}$$

$$b = c = d = e = g = h = 0$$

$$\int \frac{t^2}{(1+t^4)^2}\,dt = \frac{t^3}{4\,(t^4+1)} + \frac{1}{4}\int \frac{t^2}{t^4+1}\,dt$$

$$\int \frac{t^2}{t^4+1}\,dt = \int \left(\frac{at+b}{t^2-\sqrt{2}t+1} + \frac{ct+d}{t^2+\sqrt{2}t+1}\right)dt =$$
$$= \int \frac{t^2\left(\sqrt{2}a+b-\sqrt{2}c+d\right) + t\left(a+\sqrt{2}b+c-\sqrt{2}d\right) + t^3(a+c)+b+d}{t^4+1}\,dt$$

$$a = \frac{1}{2\sqrt{2}}, b = 0, c = -\frac{1}{2\sqrt{2}}, d = 0$$

$$\int \left(\frac{t}{2\sqrt{2}\left(t^2-\sqrt{2}t+1\right)} - \frac{t}{2\sqrt{2}\left(t^2+\sqrt{2}t+1\right)}\right)dt =$$
$$= \frac{1}{2\sqrt{2}}\int \frac{t}{t^2-\sqrt{2}t+1}\,dt - \frac{1}{2\sqrt{2}}\int \frac{t}{t^2+\sqrt{2}t+1}\,dt =$$
$$= \frac{1}{2\sqrt{2}}\left(\frac{1}{2}\int \frac{d(t^2-\sqrt{2}t+1)}{t^2-\sqrt{2}t+1} - \frac{1}{2}\int \frac{d(t^2+\sqrt{2}t+1)}{t^2+\sqrt{2}t+1} + \right.$$
$$\left. + \frac{\sqrt{2}}{2}\int \frac{dt}{t^2-\sqrt{2}t+1} + \frac{\sqrt{2}}{2}\int \frac{dt}{t^2+\sqrt{2}t+1}\right) =$$
$$= \frac{1}{4\sqrt{2}}\left(\ln\left|\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right| + 2\,\mathrm{arctg}(\sqrt{2}t-1) + 2\,\mathrm{arctg}(\sqrt{2}t+1)\right)$$

$$(*) = \frac{1}{8}\,\mathrm{sgn}\cos x\left(\frac{4\sqrt{2}t^3}{t^4+1} + \ln\left|\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right| + 2\,\mathrm{arctg}(\sqrt{2}t-1) + 2\,\mathrm{arctg}(\sqrt{2}t+1)\right) + C,$$

где $t = \sqrt{\mathrm{tg}\, x}$.

3 Задание 3

$$\int \frac{2x^3+7x^2+7x-1}{(x+2)^2(x^2-x+1)}\,dx = \int \frac{A}{(x+2)^2} + \frac{B}{x+2} + \frac{Cx+D}{x^2-x+1}\,dx =$$
$$= \int \frac{x^3(B+C) + x^2(A+B+4C+D) + x(-A-B+4C+4D) + A+2B+4D}{(x+2)^2\,(x^2-x+1)}\,dx = (*)$$

$$\begin{cases} B+C=2 \\ A+B+4C+D=7 \\ -A-B+4C+4D=7 \\ A+2B+4D=-1 \end{cases}$$

$$A=-\frac{3}{7}, B=\frac{6}{49}, C=\frac{92}{49}, D=-\frac{10}{49}$$

$$(*)=\int \frac{2(46x-5)}{49\left(x^2-x+1\right)}+\frac{6}{49(x+2)}-\frac{3}{7(x+2)^2}\,dx=-\frac{3}{7}\int \frac{1}{(x+2)^2}\,dx+$$
$$+\frac{6}{49}\int \frac{1}{x+2}\,dx+\frac{2}{49}\int \frac{46x-5}{x^2-x+1}\,dx=(**)$$

$$\int \frac{dx}{(x+2)^2}=-\frac{1}{x+2}$$
$$\int \frac{dx}{x+2}=\ln|x+2|$$

$$\int \frac{46x-5}{x^2-x+1}\,dx=\int \frac{23(2x-1)+18}{x^2-x+1}\,dx=23\int \frac{d(x^2-x+1)}{x^2-x+1}+18\int \frac{dx}{x^2-x+1}=$$
$$=23\ln|x^2-x+1|+18\int \frac{dx}{(x-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}=23\ln|x^2-x+1|+12\sqrt{3}\operatorname{arctg}\frac{2x-1}{\sqrt{3}}$$

$$(**)=\frac{3}{7}\cdot\frac{1}{x+2}+\frac{6}{49}\ln|x+2|+\frac{2}{49}\left(23\ln|x^2-x+1|+12\sqrt{3}\operatorname{arctg}\frac{2x-1}{\sqrt{3}}\right)+C.$$

4 Задание 4

$$\int \frac{dx}{(\sin x+\cos x+1)^2}=\int \frac{dx}{\left(\frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}}+\frac{1-\operatorname{tg}^2\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}}+1\right)^2}=\frac{1}{4}\int\left(\frac{1+\operatorname{tg}^2\frac{x}{2}}{\operatorname{tg}\frac{x}{2}+1}\right)^2 dx=$$

$$=\left|\begin{array}{l} t=\operatorname{tg}\dfrac{x}{2}\\ \dfrac{x}{2}=\operatorname{arctg}t\\ \dfrac{dx}{2}=\dfrac{dt}{1+t^2}\\ dx=\dfrac{2dt}{1+t^2}\end{array}\right|=\frac{1}{4}\int\left(\frac{1+t^2}{1+t}\right)^2\frac{2}{1+t^2}\,dt=\frac{1}{2}\int\frac{1+t^2}{(1+t)^2}\,dt=\frac{1}{2}\int\left(1-\frac{2t}{(1+t)^2}\right)dt=$$
$$=\frac{t}{2}-\int\frac{t}{(1+t)^2}\,dt=\frac{t}{2}-\int\frac{dt}{1+t}+\int\frac{dt}{(1+t)^2}=\frac{t}{2}-\ln|1+t|-\frac{1}{1+t}=$$
$$=\frac{1}{2}\operatorname{tg}\frac{x}{2}-\ln|1+\operatorname{tg}\frac{x}{2}|-\frac{1}{1+\operatorname{tg}\frac{x}{2}}+C.$$

5 Задание 5

$$\int \frac{\sqrt[3]{1+\sqrt[5]{x^4}}}{x^2\,\sqrt[15]{x}}\,dx$$