# Kalman Filter

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## 1 Introduction

Kalman Filter is used as a technique for filtering and prediction in linear Gaussian systems. The word linear here is of great importance here, as we will discuss in some time. In the Kalman filter, the belief (as studied in Bayes Filter) is represented by the mean, $\mu_t$, and the covariance, $\Sigma_t$, at any time $t$. **Kalman filer implements belief computation for continuous states. It is not applicable for discrete or hybrid states**.

The notation used here is the same as one used in Bayes Filter, *i.e.* $x_t$ represents state, $u_t$ and $z_t$ represent controls and measurements respectively. Note that these might be vectors, so we take column vectors as defaults.
Now, for Kalman Filter to work, we must have some constraints:

1. The state transition probability $p(x_t|u_t, x_{t-1})$ should be a linear function of its parameters with an added Gaussian noise, *i.e.*,
$$x_t = A_t x_{t-1} + B_t u_t + \epsilon_t \tag{1}$$
where $A$ and $B$ are matrices of respective dimensions and $\epsilon_t$ is the added noise at time $t$.

2. The measurement probability $p(z_t|x_t)$ should be a linear function of its parameters with added Gaussian noise. Therefore,
$$z_t = C_t x_t + \delta_t \tag{2}$$

3. The belief $bel(x_0)$ should be normally distributed with mean $\mu_0$ and covariance $\Sigma_0$

# 2 Algorithm

def Kalman_Filter($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$) :
$\overline{u_t} = A_t \mu_{t-1} + B_t u_t$
$\overline{\Sigma_t} = A_t \Sigma_{t-1} A_t^T + R_t$

$K_t = \overline{\Sigma}_t C_t^T (C_t \overline{\Sigma}_t C_t^T + Q_t)^{-1}$
$\mu = \overline{\mu} + K_t (z_t - C_t \overline{\mu_t})$
$\Sigma_t = (I - K_t C_t) \overline{\Sigma_t}$
return $\mu_t, \sigma_t$

## 2.1 Explanation

# 3 Derivation