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runpriorfreeTS.m

```
% Runs prior free TS algorithm and returns regret and fraction of
 pulls.
%
% This code runs the prior free TS algorithm adapted to be used in our
% setting. For more information, on this algorithm see this paper:
%
% - https://arxiv.org/pdf/1209.3352v4.pdf
%
% For using the algorithm in this paper, we can concatenate all the
 arm
% parameters to get a theta, parameter (of interest) in R^{k*d}.
% The k actions at time period t, with context x_t are given by:
% A_t = {[X_t, 0, 0, ..., 0], [0, X_t, 0, 0, ..., 0], ... ,
% [0, 0, ..., X_t]}. After having this mapping, the algorithm builds
% confidence sets around theta and picks the action (corresponds to
 the
% same arm) that maximizes the optimistic reward.

% As updating priors (and the implementation of algorithm) requires
 the
% knowledge of noise parameters, if this parameter is not provided,
% we use observations to estimate such parameter.
%
```

Inputs:

```
k: Number of arms.
T: Time horizon.
d: Dimension of covariates.
b: A k*d matrix of arm parameters.
sigma_e: Standard deviation of noise (or subgaussianity parameter).
sigma_x: Standard deviation of covariates
    (used only for context generation for Gaussian contexts).
    This parameter is unused if noise and contexts are provided.
xmax: Maximum of l2-norm of covariates
    (used only for context generation). This parameter is unused if
    noise and contexts are provided.
delta: The probability that confidence intervals fail.
prior_scale: The scaling factor for the prior (for the original
 version
    set this equal to 1).
```

```
sigma_start: The noise parameter (or subgaussanity parameter) to start
with. If true sigma is provided, this parameter is not used.
use_true_sigma_e: Whether to use true noise parameter, sigma_e, for
    construction of confidence sets or not.
to_estimate_sigma_e: Whether to estimate sigma_e using observations.

 This is only effective if the true noise parameter is not provided.
verbose: Whether to print outputs or not.
varargin: Additional arguments. In particular, if these are not
    provided the noise and contexts will be generated according to
    Gaussian and truncated Gaussian distributions.
    In case they are provided,
 there should exactly be THREE additional
    arguments. The first one is contexts. The second one is a binary
    input, called noise_input. If noise_input = 1, this means the last
    argument will be noise e=(Y-X*beta). On the other hand, if
    noise_input = 0, then the last argument will be Y or
    rewards. Note that the noise should be T*1 while rewards should be
    T*k.
```

Outputs:

```
regret: Cumulative regret as a running sum over regret terms.
fractions: Fractions of pulls of different arms.
```

Code:

```
function [regret, fractions] = runpriorfreeTS(k, T, d, b, ...
    sigma_e, sigma_x, xmax, delta, prior_scale, ...
    sigma_start, use_true_sigma_e, to_estimate_sigma_e, verbose,
 varargin)

warning('off','all');

if nargin==13 % Context and noise are NOT provided, so generate those.
    % Noise is Gaussian with std sigma_e.
    e = randn(T,1)*sigma_e;
    noise_input = 1;

    % Contexts follow truncated gaussian distributions with l-infinity
 norm
    % at most xmax.
    X = max(-xmax, min(xmax, mvnrnd(zeros(d, 1), sigma_x, T)));
else
    X = varargin{1};
    noise_input = varargin{2};
    if(noise_input==1)
        e = varargin{3};
    else
        rewards = varargin{3};
    end
end
```

```
reward_vector = zeros(T, k);   % Vector of all (potential) rewards.
pull_ind = zeros(T, k);  % Binary indicator whether each is pulled.

regret = zeros(1, T);

cov_matrices = zeros(d, d, k); % Covariance matrices of Gaussians.

% Initialize all covariance matrices with I_d (they will be multiplied
 by v
% later).
for i=1:k
    cov_matrices(:, :, i) = eye(d);
end

mean_vectors = zeros(d, k);  % Means of Gaussian matrices.
XtopY = zeros(d, k);  % Running sum of reward times context.
sampled_vectors = zeros(d, k);  % Samples drawn according to
 posterior.
residuals = zeros(T, 1);  % Used for estimating the noise parameter.
sigma_e_hat = sigma_start;

for t=1:T
    x = X(t,:)';
    if(use_true_sigma_e==1)
        sigma_e_hat = sigma_e;
    end
     % First: update v.
     % Note that the dimension of contexts is now k*d.
     v = prior_scale*sigma_e_hat*sqrt(9*(k*d)*log(t/delta));

    % Second: draw samples.
    for i=1:k
        sampled_vectors(:, i) = mvnrnd(mean_vectors(:, i), ...
            v^2 * inv(cov_matrices(:, :, i)));
    end

    % Third: select which arm to play.
    [~, arm_pulled]=max(x' * sampled_vectors);
    pull_ind(t, arm_pulled)=1;

    % Fourth: compute reward and regret.
    if(noise_input==1)
        bx = b*x;
        ourreward = bx(arm_pulled);
        bestreward = max(bx);
    else
        ourreward = rewards(t, arm_pulled);
        bestreward = max(rewards(t,:));
    end

    if (t==1)
        regret(t) = bestreward - ourreward;
    else
        regret(t) = regret(t-1) + bestreward - ourreward;
```

```
    end

    % Fifth: update parameters.
    if(noise_input==1)
        reward_vector(t, arm_pulled) = ourreward + e(t);
    else
        reward_vector(t, arm_pulled) = rewards(t, arm_pulled);
    end

    cov_matrices(:, :, arm_pulled) = cov_matrices(:, :, arm_pulled)
 + ...
        x * x';
    XtopY(:, arm_pulled) = ...
        XtopY(:, arm_pulled) + reward_vector(t, arm_pulled) * x;
    mean_vectors(:, arm_pulled) = ...
        cov_matrices(:, :, arm_pulled) \ XtopY(:, arm_pulled);

    % Sixth: update estimate of sigma_e.
     if(to_estimate_sigma_e == 1)
        residuals(t) = reward_vector(t, arm_pulled) ...
            - x' * mean_vectors(:, arm_pulled);
        if (t>k*d+1)
            sigma_e_hat = sqrt(sum(residuals.^2)/(t-d));
        end
     end

    if(verbose==1)
        if (mod(t,500)==0)
            fprintf('PF-TS: t=%d, est. sigma_e = %f, sigma_e = %f.
 \n', ...
                t, sigma_e_hat, sigma_e);
        end
    end

end
fractions = mean(pull_ind);  %fraction of times each arm is pulled.
if(verbose==1)
    fprintf('PF-TS: Total parameter estimation error = %f. \n', ...
        norm(b - mean_vectors', 'fro'))
    fprintf('PF-TS: Fraction of pulls = %f. \n', fractions)
    fprintf('PF-TS: Total regret occured = %f. \n', regret(end))
end
end
```

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