

Report on

Online Learning for Min Sum Set Cover and Pandora's Box

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Abstract

This report presents an analysis and summary of the paper *Online Learning for Min Sum Set Cover and Pandora's Box* which provides a simple but effective skeleton for crafting online learning algorithms for the *Pandora's Box Problem*, the *Min Sum Set Cover Problem (MSSC)*, and other related problems in the field of *Stochastic optimization* [1]. Traditionally, these problems assume prior knowledge of value distributions. However, in this work, we explore an online learning approach where we are presented with information over multiple rounds.

Furthermore, the research, extends this framework to a bandit setting where only the values of the opened boxes are shown to the learner after each round. This extension also encompasses other alternatives of the above-mentioned problems [1].

1 Introduction

The two essential problems in *Stochastic Optimization* are the *Pandora's Box*, and *MSSC* problems. Introduced, by Weitzman [2] the *Pandora's Box* problem offers a general framework for optimizing decisions under uncertainty, thus the goal is to minimize the cost of uncovering information. *MSSC*, a variation of *Pandora's box*, deals with scenarios where values are either 0 or ∞ [3].

While the traditional formulation of these problems assumes access to known stochastic distributions [1], this study extends these frameworks to *online* settings, where data arrives sequentially without prior distributional knowledge.

The *online learning* perspective brings forth several challenges and opportunities. In this model, adversarial instances or independent scenarios are presented iteratively, and the algorithm must decide on the next action based on the current state and previous observations. The performance metric typically revolves around *regret*, measuring the algorithm's cumulative cost relative to an optimal offline policy [4].

The work's primary contributions are as follows:

1. Propose a computationally efficient online algorithm that achieves constant-competitive guarantees for both MSSC and Pandora's Box generalizations in full-information settings. This includes problems with complex constraints such as matroid bases [5].
2. Extend the analysis to *bandit settings*, where only partial feedback about the explored options is available [6]. Despite these limited observations, our algorithm maintains a logarithmic regret bound, providing robust performance guarantees.
3. Leveraging convex relaxations and online convex optimization techniques, demonstrate practical approximation strategies that bridge theoretical insights with computational efficiency [7].

The implications of these results are twofold. First, they broaden the applicability of MSSC and Pandora's Box to dynamic, real-world contexts where decisions must adapt to unfolding events. Second, the integration of online convex optimization frameworks highlights the role of mathematical tools in addressing combinatorial challenges [1].

The remainder of the paper is organized as follows. Section 2 presents the problem formulations and related work, Section 3 details the proposed methods, and Sections 4 and 5 provide theoretical analyses and experimental evaluations, respectively. It concludes with discussions on potential future extensions.

2 Problem Definitions

This section formalizes the two central problems studied: Min Sum Set Cover (MSSC) and Pandora's Box. We provide their definitions and highlight their significance in stochastic optimization and online learning.

2.1 Pandora's Box Problem

The Pandora's Box problem, introduced by Weitzman, involves n boxes, each containing an unknown value. Opening a box incurs a cost, and the goal is to minimize the sum of the inspection costs and the value of the chosen box. Formally:

- Let $B = \{1, 2, \dots, n\}$ be the set of boxes.
- Each box $i \in B$ has a value v_i drawn from an unknown distribution.
- Opening a box incurs a cost c_i , and the decision-maker observes v_i only after opening the box.
- The objective is to determine an optimal sequence of box openings and a stopping rule to minimize the total cost:

$$\text{Total Cost} = \sum_{i \in S} c_i + \min_{i \in S} v_i,$$

where S is the subset of opened boxes.

This problem is extended to scenarios with additional constraints:

- **Selection of k Boxes:** Find k boxes to minimize the sum of their values and inspection costs.
- **Matroid Constraints:** Select a subset of boxes forming a matroid basis to optimize the objective.

2.2 Min Sum Set Cover (MSSC) Problem

The MSSC problem is a combinatorial optimization task that seeks to minimize the total time at which elements are first covered by a sequence of sets. It is closely related to set cover problems and is defined as follows:

- Let $U = \{e_1, e_2, \dots, e_m\}$ be a set of elements and $\mathcal{S} = \{S_1, S_2, \dots, S_n\}$ be a collection of subsets of U such that $\bigcup_{i=1}^n S_i = U$.
- Each subset S_i can be chosen at a time t , and the covering time of an element $e \in U$ is the first time t when e belongs to a chosen subset.
- The objective is to find a sequence of subsets $S_{i_1}, S_{i_2}, \dots, S_{i_k}$ that minimizes the sum of the covering times of all elements:

$$\text{Total Cost} = \sum_{e \in U} \text{Covering Time}(e).$$

A special case of MSSC arises when the values in Pandora's Box are restricted to $\{0, \infty\}$, representing whether a scenario is covered or not. This case simplifies to deciding an ordering of subsets that minimizes the sum of first cover times.

2.3 Online Variants

In the online setting, the value vectors or scenarios are presented adversarially over T rounds:

- **Input:** At each round t , the learner observes an instance of the problem defined by the adversary.
- **Goal:** Develop strategies that minimize regret, defined as the difference in performance between the learner and the optimal offline solution over all rounds:

$$\text{Regret}(A, T) = \frac{1}{T} \sum_{t=1}^T (A(t) - \text{OPT}(t)).$$

By leveraging convex relaxations and online optimization techniques, these problems are extended to realistic feedback models like bandit settings, where only partial information about the outcomes is revealed after each decision.

3 Methodologies

Summarize the key approaches used in the paper, including:

- Convex relaxations for online learning.
- Online convex optimization frameworks.
- Rounding techniques for fractional to integral solutions.

4 Results

Detail the main results:

- Competitive ratios achieved for Pandora’s Box and MSSC.
- Extensions to bandit settings.
- Performance under matroid constraints.

5 Comparison with Previous Work

Discuss how the paper builds upon or improves results from prior studies. Highlight differences in techniques and results.

6 Applications and Future Work

Explain potential applications of these results in real-world scenarios. Propose future directions for research based on open questions identified in the paper.

7 Conclusion

Provide a concise summary of the main insights gained from the study and their significance in the field of online learning and optimization.

A Appendix

Include any additional proofs, explanations, or extended results here.

References

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