

Insertion sort

Implementation:

```
void insertion(int ara[],int n)
{
    int i,j,x;
    for(i=0;i<n;i++)
    {
        x=ara[i];
        j=i-1;
        while(j>=0 && ara[j]>x)
            ara[j+1]=ara[j],j--;
        ara[j+1]=x;
    }
}
```

Analysis:

We should analyze the algorithm from it's source code:

	Cost	Time
1. void insertion(int ara[],int n)		
2. {		
3. int i,j,x;		
4. for(i=0;i<n;i++)	c1	n
5. {		
6. x=ara[i];		c2
7. j=i-1;	c3	n-1
8. while(j>=0 && ara[j]>x)	c4	$\frac{n(n+1)}{2}$
9. ara[j+1]=ara[j],j--;	c5	$\frac{n(n+1)}{2}$
10. ara[j+1]=x;	c6	n-1
11. }		
12. }		

Time function will be

$$f(t) = c_1n + c_2(n-1) + c_3(n-1) + c_4\frac{n(n+1)}{2} + c_5\frac{n(n+1)}{2} + c_6(n-1)$$

Best case occurs when array is already sorted

1	2	3	5	7
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In this situation c_4 will execute for $(n-1)$ time and c_5 will execute for 0 time

So function will be,

$$\begin{aligned} f(t) &= c_1 * n + c_2(n-1) + c_3(n-1) + c_4(n-1) + c_5 * 0 + c_6(n-1) \\ &= n(c_1 + c_2 + c_3 + c_4 + c_6) - (c_2 + c_3 + c_4 + c_6) \end{aligned}$$

Which look like $y = an - b$, it is a linear equation

Worst case occurs when the array is reversely sorted,

7	5	3	2	1
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Then time function will be,

$$\begin{aligned} f(t) &= c_1n + c_2(n-1) + c_3(n-1) + c_4 \frac{n(n+1)}{2} + c_5 \frac{n(n+1)}{2} + c_6(n-1) \\ &= \frac{c_1+c_2}{2}n^2 + \frac{2(c_1+c_2+c_3+c_6)+c_4+c_5}{2}n - (c_2 + c_3 + c_6) \end{aligned}$$

Which look like $y = an^2 + bn + c$, it is a quadratic equation.

Time complexity:

Best Case:

We know for best case, $y = an - b$

So, $\Omega(n)$

Worst Case:

For worst case, $y = an^2 + bn + c$

So, $O(n^2)$