DAEDALUS: An economic-epidemiological model to optimize economic activity while containing the SARS-CoV-2 pandemic

Imperial College COVID-19 response team

Introduction

DAEDALUS is an economic-epidemiological model that calculates the outcomes (in terms of economic production, employment, infections, hospitalizations, and deaths) of a wide range of alternative SARS-CoV-2 control strategies. DAEDALUS can be used to determine optimal lockdown strategies differentiated by economic sector. It develops an optimal policy trajectory that recommends, as infections increase, which sectors to close last (partly or fully), and as infections decrease, which sectors to open first, whilst keeping hospital admissions within health system capacity. This is informed by the projections of DAEDALUS on the sectors of the economy that contribute most to economic production but have relatively modest infection risks in terms of interpersonal contact, amongst workers or consumers or both. This is not simply a case of prioritizing the individual sectors of the economy that contribute most to economic value relative to the spread of infection. There are important inter-dependencies between sectors, all of which rely to some extent on inputs from other sectors to produce their final outputs; a sector that is nominally opened may not be able to function properly if its supply chain is interrupted. DAEDALUS accommodates such interactions by relying on conventional Input-Output (IO) tables prepared as part of many systems of national accounts. DAEDALUS unites a deterministic Susceptible-Exposed-Infectious-Removed (SEIR) model of SARS-CoV-2 transmission that projects the spread of infection as sectors are opened and closed to varying degrees, reflecting sectoral heterogeneity in risks of infection between co-workers, non-workers in the community, and on the interface between these groups. This GitHub website publishes the main specification of DAEDALUS, key data and parameters that are used as inputs, and the computer code.

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Methods

The mathematical structure of DAEDALUS consists of two integrated parts: the economic model and the epidemiological model. The economic part of DAEDALUS organizes the economy into 63 discrete economic sectors. This gives rise to a set of economic production constraints that reflect the interdependencies between sectors. These are set out alongside a set of epidemiological constraints, which are modelled using a compartmental transmission model of SARS-CoV-2. We assume that the broad objective of the government is to put in place restrictions to non-essential economic activity that do the least damage to the economy, whilst limiting the spread of infection. For the purposes of exposition, we assume that the objective is to return as closely as possible to the economy as it was before the arrival of the epidemic, without consideration of possible permanent changes to firm structures, production processes or consumer demand that may be caused by the epidemic.

The population of the UK population (65,840,000) is divided into four age groups: pre-schooler, school-age children, working-age adults, and retired. Working-age adults who are in the labour force are working in one of the 63 economic sectors, as employed or self-employed workers. The total size of the workforce (30,550,016) is based on pre-pandemic data [1]. Adults in the labour force may temporarily not work (i.e. be economically inactive) if their sectors are fully or partially closed. The population of pre-schoolers aged 0-4 (4,064,198), pupils aged 5-19 (12,192,592), retired aged 65+ (13,005,432) and adults aged 20-64 who are not in the labour force, or registered as unemployed (6,027,762 pre-pandemic) are referred to as the "community". The number of individuals in the three non-working age community groups remains fixed, while the working-age community population is dependent on the current economic configuration, i.e. the extent to which economic sectors are open for production and economically inactive adults in the labour force join the working-age community population.

Economic model

The $N = 63$ economic sectors and 4 community groups play different roles in DAEDALUS. The productive sectors are engaged both in economic production and in the spread of infection, while the four unproductive sectors contribute only to the spread of infection. In the pre-pandemic world, the N economic sectors are fully operational and the final (consumption) product of each sector i contributes to the country's gross domestic product (GDP), as represented by its gross value added (GVA), including exports. However, in addition to final products for domestic consumption or export, sectors also create products that are used as intermediate inputs by other sectors in creating their own final products.

The flows between sectors are represented by a matrix Z. The components z_{ij} of Z indicate the monetary value of flows from sector i used as inputs to the production process of sector j. Column j of matrix Z indicates all the inputs used in the production process of sector j, and row i indicates all the products of sector i used as inputs by the other sectors. Therefore, the monetary value of total production of sector y_i can be represented by the N relationship $y_i = \sum_{j=1}^{N} z_{ij} + f_i$, which sums inputs produced for other sectors and final consumption products f_i by sector i. Let $a_{ij} = z_{ij}/y_j$ be the fraction of output of sector i used in sector j so that

$$y_i = \sum_{j=1}^{N} a_{ij} y_j + f_i, \quad i = 1, \ldots, N.$$

Intersectoral flows of products have been the subject of well-developed analytic frameworks represented in national accounts, most of which derive from the 1993 United Nations System of National Accounts [2]. They form the basis of the Input-Output (IO) tables that are central to DAEDALUS. Most sectors use both imported and domestically produced intermediate inputs from other sectors to produce their own goods and services. DAEDALUS considers only the impact of domestic production on SARS-CoV-19 transmission and assumes imports necessary for production continue to be available. The value of all flows is represented in 'basic' prices, which exclude the margins secured by producers [3]. The calculation of IO matrices is a major undertaking, so they are produced only periodically. The last full estimates for the UK economy refer to the calendar year 2016, and we are therefore constrained to using these estimates in our application. In the main specification of DAEDALUS, we assume a Leontief production function [3] for each sector, so that the proportionate contributions of inputs from other sectors, the workforce, and value-added remain unchanged whatever its level of operations. See economic data for more details on IO tables.

By solving this system of N equations in N unknowns, one can determine the (equilibrium) output of each sector y_i^* for $i = 1, \dots, N$ in the pre-pandemic world. Let $y_i^* = \frac{y_i^*}{w_i^*} w_i^* = g_i w_i^*$, where w_i^* is the pre-pandemic workforce of sector i, and $g_i = y_i^*/w_i^*$ is the output per worker (labour productivity) of sector $i = 1, \dots, N$ in a given period. For simplicity we assume that a_{ij} (interdependency between sectors), g_i (labour productivity) and final consumption is constant and held at pre-pandemic values. In reality, the pandemic may have a direct effect on the productivity of the workers in each sector resulting in a smaller or higher g_i in comparison to the pre-pandemic period. Productivity may decrease as some individuals work from home without childcare, become ill, are hospitalized or die; on the other hand, productivity may increase as a reduced number of workers is required to keep output at pre-pandemic levels. The aggregate impact on g_i is unknown and may vary by sector. Likewise, the demand for products and services may also be affected due to the reduction in income, the economic uncertainty and attempts to avoid contagion. These points are further discussed in the limitation section.

We assume that policy can influence the proportion $x_i^{min} \leq x_i \leq 1$ of individuals working in each sector in each decision period. When $x_i = 1$ sector i is fully open and productive at pre-pandemic levels, and when x_i^{min} sector i is closed except for the provision of essential goods and services. The value of x_i^{min} is given by x_i^{LD} the proportion of the sector open during lockdown March – May 2020 in the UK, which provides an indication of how extensively a sector can be closed for production. However, the estimates are generated from a non-random sample, and there is some evidence that the values may be too high. This is because the reporting of lockdown production actually relied on businesses being open to take the ONS survey [4-6]. To allow for misreporting regarding the precise observed lockdown values for x_i^{LD}, we use 80% of the lockdown values, i.e. $x_i^{min} = 0.8x_i^{LD}$. This effectively imposes a lower bound constraint on all scenarios that – with some flexibility - allows essential services to operate. The effective number of workers in each period in sector i is $w_i = x_i w_i^*.\, x_i^{min} x_i$

By controlling the proportion of individuals working in each sector and in each period, policy can keep the pandemic under control because infections at the workplace (between workers and between workers and customers) are reduced. However, by partially closing a sector, such policies also reduce the GVA contribution of each sector. During each period in the pandemic, the achieved output is $y_i = g_i\, w_i$. This implies that in DAEDALUS, the effect of the pandemic works through the reduction in the number of workers, and not reduced work productivity. Therefore, even if labour productivity remains unchanged,

$x_i^{min} y_i^* \leq g_i w_i \leq y_i^*$, $i = 1, \dots, N$, because closures of sectors reduces the number of individuals who work. We make this simplifying assumption because there is no empirical evidence on the relative impact of the pandemic's impact on numbers of individuals working versus work productivity by sector [7].

We assume that the decision maker has an outlook over a specific intervention horizon into the future (e.g. six months). Decisions on economic configuration are made at the beginning of regular periods during the intervention horizon (e.g. at the beginning of every month, or every two months), the decision points. Objective is to keep the economy as active as possible over the intervention horizon without breaching hospital capacity. At each decision point $\tau = 0, 1, 2, \dots, \mathrm{T}$ over the intervention horizon, the decision maker decides how much each sector is open in the next period. Precisely, the decision maker chooses $x_i^{min} \leq x_{i\tau} \leq 1$, for $i = 1, \dots, N$. The objective in each period is to maximise an objective-function which consists of the GDP.

We assume that over each period the domestic economy must be balanced. All necessary intermediate domestic inputs required by a sector must be available. That is

$$\sum_{\tau=0}^{\mathrm{T}} g_i w_i^* x_{i\tau} \geq \sum_{j=1}^{N} a_{ij} \left(\sum_{\tau=0}^{\mathrm{T}} g_j w_j^* x_{j\tau} \right).$$

This requires each sector to produce at least enough to satisfy the intermediate needs of other sectors that are open. The pandemic has likely led to changes in production processes in some sectors compared to the pre-pandemic world due to several factors, most notably disruptions in global supply chains [8, 9]. To allow for some flexibility in the economic configuration due to these factors, we do allow "excess" production of intermediate and final products - that is, over the intervention horizon, there is the opportunity for inventory or additional export if supply exceed demand. Output is however bound not to exceed pre-pandemic levels in any period, i.e. $x_{i\tau} \leq 1$.

Certain sub-sectors such as parts of healthcare, education and agriculture may be considered essential services that must remain open regardless of the consequences for disease transmission. We may also want to allow some sectors, such as health care, to expand beyond pre-pandemic levels. Keeping certain sectors at specified level of production can be introduced via additional constraints to the optimization.

Economic data

The economic data rely on the most recent 2016 Input-Output (IO) tables for 64 sectors prepared for the UK [10]. The IO table describes how products (and primary inputs) are used to produce further products and satisfy final demand. The IO table depicts inter-sector relationships within an economy, showing how output from one economic sector may become an input to another sector. In the IO table, column entries typically represent inputs to a sector, while row entries represent outputs from a given sector. Inputs and outputs are represented in monetary terms, not physical units. Each column of the table shows the value of inputs to each sector and each row represents the value of each sector's outputs [3]. This format therefore shows how dependent each sector is on every other sector, both as a customer of outputs from other sectors and as a supplier of inputs. Note that we have been constrained to use an IO table based on the sector to which a product under scrutiny belongs [10], rather than the sector of the industry in which it is produced. This is unlikely to materially affect the results. We consider 63 sectors (obtained from the

64 sector IO table) in this study, combining real estate services (sector 44) with imputed rents of owner-occupied dwellings (sector 45) because separate data were not available.

The information obtained from the IO table is summarized in **Table *1***. The first three columns list the high- mid- and lowest level sector designations. The fifth column shows the workforce of each sector as headcount, obtained from the ONS [1]. We assume the workforce to be constant at pre-pandemic levels over the intervention horizon although a certain proportion of the workforce may not work in any given period (non-active) depending on the extent to which a sector is closed. The first column of **Table *1*** shows the total workforce of each sector. The total output of each sector is measured at basic prices in £ million. The output includes the value of intermediate products and services used by the respective sector in production. The GVA of a sector is obtained by subtracting intermediate products from total output, including exports. GVA is measured in £ million; this denotes the value of all products and/or services that the sector adds to the economy over and above the intermediate products used in production. For the optimization we convert the annual GVA into a monthly value, by dividing the annual value by 12. This makes the simplifying assumption that the sector in the pre-pandemic world produces the same amount each period. Therefore, the GVA for sector j in the period τ is $x_{j\tau} g_j w_j^* \left(1 - \sum_{i=1}^{N} a_{ij}\right)$

The penultimate column of **Table *1*** gives the sectors' use of intermediate products as a percentage of their overall output (intermediate use excludes use of imports and taxes less subsidies on products). A higher value indicates that the sector has greater reliance on other sectors; for our analysis this implies that opening this sector is necessarily associated with greater opening of other sectors as well. The final column presents provision of intermediate products (row totals) as a percentage of total output, indicating the sector's reliance on other sectors as consumers, rather than final consumers (intermediate provision excludes exports and gross capital formation). The value of opening this relies heavily on other sectors being open to purchase its products and services. In summary, low values in the last two columns describe sectors that are relatively self-reliant and can be opened and closed with less consideration of activity levels in other sectors.

Table *2* provides changes in production and work-from-home during the lockdown period in the UK between March and May 2020 for the ONS Standard Industry Classification (SIC) sections and divisions. These values provide important parameters for the economic model. The number of infections contributed by each sector are influenced by the number of workers returning into the community, either working from home or economically inactive. Workers who work from home are exposed to (and contribute to) transmission only in the community. We assume that individuals working from home are not affected in their productivity, i.e. do not negatively affect the GVA contribution of their sector. We assume that the proportion of workers working from home during lockdown in spring 2020 stays constant over our intervention horizon [5].

The minimum economic configuration $0.8 * x_i^{LD}$ of each sector provides the lower bound estimate of how much a sector can be closed for production without endangering essential supplies. The lockdown productivity x_i^{LD} was obtained via a survey conducted as part of the monthly GDP calculation by the ONS [6]. We use 80% of the lockdown values, i.e. $x_i^{min} = 0.8 x_i^{LD}$, to allow for misreporting and changes in production processes in reaction to the pandemic and affecting the precise observed lockdown values for x_i^{LD}. We estimated DAEDALUS with x_i^{LD} and this economic configuration did not render a solution, i.e. the observed and surveyed lockdown economic configuration is evidently not economically feasible considering interdependencies between sectors as reported in the 2016 IO table. We applied the same value of these high-level sectors to all subsectors due to lack of more detailed data.

Table 1: Workforce, total output, gross value added, and intermediate use and provision for 63 sectors, United Kingdom 2016

UK Standard Industry Classification (SIC)			Standard Industry Classification (SIC) description	Workforce (headcount)	Total output at basic prices (£ million)	Gross value added (£ million)[1]	Intermediate use as % output[2]	Intermediate provision as % output[3]
sections		divisions						
[A]	A	1	Crop and animal production, hunting and related service activities	275,986	22,863	10,055	0.59	0.62
[A]	A	2	Forestry and logging	18,131	1,344	477	0.44	0.87
[A]	A	3	Fishing and aquaculture	8,031	1,737	519	0.52	0.64
[B-E]	B	4	Mining and quarrying	96,622	28,797	10,075	0.47	0.50
[B-E]	C	5	Manufacture of food products, beverages and tobacco products	338,012	84,489	26,877	0.63	0.49
[B-E]	C	6	Manufacture of textiles, wearing apparel and leather products	70,706	9,830	5,231	0.39	0.19
[B-E]	C	7	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials	54,489	6,684	3,354	0.34	0.89
[B-E]	C	8	Manufacture of paper and paper products	36,998	11,303	4,669	0.38	0.80
[B-E]	C	9	Printing and reproduction of recorded media	85,513	8,908	4,130	0.33	0.91
[B-E]	C	10	Manufacture of coke and refined petroleum products	18,773	19,875	1,974	0.22	0.50
[B-E]	C	11	Manufacture of chemicals and chemical products	90,578	31,456	11,416	0.42	0.47
[B-E]	C	12	Manufacture of basic pharmaceutical products and pharmaceutical preparations	110,504	19,622	11,474	0.34	0.32
[B-E]	C	13	Manufacture of rubber and plastic products	120,505	20,414	7,533	0.37	0.65
[B-E]	C	14	Manufacture of other non-metallic mineral products	75,767	14,356	5,617	0.43	0.87
[B-E]	C	15	Manufacture of basic metals	62,633	14,587	4,170	0.40	0.84
[B-E]	C	16	Manufacture of fabricated metal products, except machinery and equipment	215,088	25,670	14,350	0.37	0.72
[B-E]	C	17	Manufacture of computer, electronic and optical products	144,594	17,490	12,605	0.38	0.44
[B-E]	C	18	Manufacture of electrical equipment	61,783	10,940	4,657	0.48	0.69
[B-E]	C	19	Manufacture of machinery and equipment not elsewhere classified	271,284	25,570	10,769	0.43	0.45
[B-E]	C	20	Manufacture of motor vehicles, trailers and semi-trailers	168,260	52,527	15,255	0.39	0.17
[B-E]	C	21	Manufacture of other transport equipment	170,185	27,662	8,975	0.61	0.34

[B-E]	C	22	Manufacture of furniture; other manufacturing	149,678	12,823	7,768	0.51	0.27
[B-E]	C	23	Repair and installation of machinery and equipment	202,188	24,204	11,058	0.41	0.93
[B-E]	D	24	Electricity, gas, steam and air conditioning supply	164,418	113,001	25,932	0.70	0.75
[B-E]	E	25	Water collection, treatment and supply	55,616	9,653	6,835	0.28	0.50
[B-E]	E	26	Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services	108,686	28,328	14,525	0.37	0.47
[F]	F	27	Construction	2,253,195	133,071	108,902	1.18	0.97
[G-I]	G	28	Wholesale and retail trade and repair of motor vehicles and motorcycles	472,742	48,558	29,882	0.39	0.40
[G-I]	G	29	Wholesale trade, except of motor vehicles and motorcycles	746,139	138,302	75,022	0.42	0.54
[G-I]	G	30	Retail trade, except of motor vehicles and motorcycles	2,764,702	137,525	86,436	0.34	0.18
[G-I]	H	31	Land transport and transport via pipelines	739,338	62,340	29,304	0.45	0.70
[G-I]	H	32	Water transport	33,849	14,406	5,878	0.28	0.20
[G-I]	H	33	Air transport	63,149	22,531	4,787	0.45	0.55
[G-I]	H	34	Warehousing and support activities for transportation	371,528	41,213	19,711	0.49	0.78
[G-I]	H	35	Postal and courier activities	289,632	23,003	12,752	0.38	0.89
[G-I]	I	36	Accommodation and food service activities	1,725,001	120,457	63,576	0.41	0.17
[J]	J	37	Publishing activities	166,948	10,497	11,680	0.77	0.13
[J]	J	38	Motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities	144,366	27,450	16,684	0.44	0.21
[J]	J	39	Telecommunications	160,904	45,181	28,715	0.24	0.47
[J]	J	40	Computer programming, consultancy and related activities; information service activities	764,447	64,417	53,962	0.44	0.80
[K]	K	41	Financial service activities, except insurance and pension funding	533,535	136,155	69,367	0.36	0.50
[K]	K	42	Insurance, reinsurance and pension funding, except compulsory social security	203,686	76,983	26,463	0.50	0.33
[K]	K	43	Activities auxiliary to financial services and insurance activities	479,385	35,271	22,066	0.29	0.27
[L]	L	44 & 45	Real estate activities & imputed rents of owner-occupied dwellings[4]	356,573	333,364	261,164	0.22	0.13
[M-N]	M	46	Legal and accounting activities; activities of head offices; management consultancy activities	1,065,069	97,552	65,533	0.30	0.69
[M-N]	M	47	Architectural and engineering activities; technical testing and analysis	590,015	33,836	20,766	0.51	0.69
[M-N]	M	48	Scientific research and development	119,815	14,800	20,527	1.20	0.50
[M-N]	M	49	Advertising and market research	187,546	37,152	23,946	0.30	0.85
[M-N]	M	50	Other professional, scientific and technical activities; veterinary activities	380,865	28,206	14,717	0.42	0.69

[M-N]	N	51	Rental and leasing activities	91,184	35,517	22,942	0.32	0.65
[M-N]	N	52	Employment activities	195,730	38,772	26,697	0.27	0.94
[M-N]	N	53	Travel agency, tour operator reservation service and related activities	96,562	24,212	9,581	0.57	0.32
[M-N]	N	54	Security and investigation activities; services to buildings and landscape activities; office administrative, office support and other business support activities	975,323	65,991	34,791	0.39	0.62
[O-Q]	O	55	Public administration and defence; compulsory social security	2,090,939	150,285	85,641	0.34	0.09
[O-Q]	P	56	Education	3,361,071	137,275	96,552	0.26	0.17
[O-Q]	Q	57	Human health activities	3,301,334	150,841	89,847	0.30	0.02
[O-Q]	Q	58	Social work activities	953,764	67,935	41,164	0.35	0.15
[R-T]	R	59	Creative, arts and entertainment activities; libraries, archives, museums and other cultural activities; gambling and betting activities	346,487	30,136	16,079	0.40	0.18
[R-T]	R	60	Sports activities and amusement and recreation activities	420,269	16,953	9,964	0.35	0.06
[R-T]	S	61	Activities of membership organisations	324,534	12,566	7,189	0.39	0.16
[R-T]	S	62	Repair of computers and personal and household goods	84,930	3,703	2,481	0.25	0.57
[R-T]	S	63	Other personal service activities	437,946	24,036	18,104	0.18	0.13
[R-T]	T	64	Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use	82,486	4,961	4,961	0.00	0.00

Notes: [1]we maximize monthly gross value added in the optimization, i.e. annual GVA divided by 12; [2]intermediate use excludes use of imports and excludes taxes (less subsidies) on products; [3]intermediate provision excludes exports and gross capital formation; [4]Sectors 44 and 45 are combined.

Table 2: Observed production and labour market changes during lockdown in the United Kingdom (March-May 2020)

Standard Industry Classification (SIC) sections	SIC sections	SIC divisions	Proportion working from home[1]	Proportion of sector open $(0.8x_i^{LD})$[3]
Agriculture [A]	A	1-3	0.47	0.94
Production [B-E]	B	4-26	0.47	0.84
	C		0.27	0.72
	D		0.47	0.85
	E		0.26	0.94
Construction [F]	F	27	0.36	0.56
Distribution, transport, hotels and restaurants [G-I]	G	28-36	0.362	0.65
	H		0.47	0.61
	I		0.17	0.09
Information and communication [J]	J	37-40	0.23	0.85
Financial and insurance [K]	K	41-43	0.47	0.96
Real estate [L]	L	44, 45	0.47	0.97
Professional and support activities [M-N]	M	46-54	0.86	0.79
	N		0.78	0.65
Government, health & education [O-Q]	O	55-58	0.47	0.1
	P		0.40	0.59
	Q		0.77	0.69
Other services [R-T]	R	59-64	0.3	0.49
	S		0.47	0.51
	T		0.57	0.51

Notes: [1,2]from the ONS 'Business Impact of COVID-19 Survey' (BICS) https://www.ons.gov.uk/economy/economicoutputandproductivity/output/datasets/businessimpactofcovid19surveybicsresults; the proportion working from home do not contribute to transmission at the workplace

[3] from the surveys conducted by the ONS for the monthly GDP estimate, see https://www.ons.gov.uk/economy/grossdomesticproductgdp/bulletins/gdpmonthlyestimateuk/latest; and https://www.ons.gov.uk/economy/grossdomesticproductgdp/methodologies/grossdomesticproductgdpqmi. We reduced surveyed lockdown values of production x_i^{LD} for all higher-level sectors except healthcare by 20% to account for misreporting and changes in production processes in reaction to the pandemic.

Epidemiological model

We now consider the modelling of transmission of SARS-CoV-2 infection in DAEDALUS. The partial or full opening of a sector increases the number of actively working adults. For most workers, working requires contact with colleagues and consumers, and therefore a risk of infection amongst the workforce, between workforce and consumers, and onward transmission to the general population when working adults move between economic sectors and the community. Furthermore, for some sectors there is increased transmission associated with contacts between consumers. DAEDALUS is sensitive to the different circumstances experienced by the workers and consumers associated with each sector.

The individuals in each productive sector and the four community groups are divided at each time step of the epidemiological model t into mutually exclusive "epidemiological" groups: susceptible, exposed (infected but not yet infectious), asymptomatic infectious, infectious with mild symptoms, infectious with severe symptoms ("influenza-like illness" or ILI), hospitalized, recovered, and dead. The number of individuals in each of these groups who are also member of sector *i* at time *t* are denoted respectively by $S_i(t)$, $E_i(t)$, $I_i^{asym}(t)$, $I_i^{mild}(t)$, $I_i^{ILI}(t)$, $H_i(t)$, $R_i(t)$, and $D_i(t)$, for $i = 1, \dots, N+4$. Notice that for each group *i* and time *t*, the total population of each group is the sum of the epidemiological groups:

$$w_i(t) = S_i(t) + E_i(t) + I_i^{asym}(t) + I_i^{mild}(t) + I_i^{ILI}(t) + H_i(t) + R_i(t) + D_i(t),$$

for all $t \in [t_\tau, t_{\tau+1})$, and all decision points $\tau = 0, \dots, \mathrm{T}$ at the start of each period. In each period $[t_\tau, t_{\tau+1})$, $\tau = 0, \dots, \mathrm{T}$ over the intervention horizon, the population of the epidemiological groups within each sector i change following a compartmental Susceptible-Exposed-Infectious-Removed (SEIR) model for all sectors $i = 1, \dots, N+4$. Transmissibility β is calibrated to a target R_0 with pre-lockdown contact patterns, using the next-generation operator eigenvalue method [11]. The force of infection (FOI) on sector i, $\lambda_i(t)$, and system Ordinary Differential Equations (ODEs) are given as follows

$$\dot{S}_i(t) = -S_i(t)\lambda_i(t)$$

$$\dot{E}_i(t) = S_i(t)\lambda_i(t) - \sigma E_i(t)$$

$$\lambda_i(t) = \beta \sum_{j=1}^{N+4} M_{ij} \frac{I_j(t)}{w_j}$$

$$I_i(t) = r\dot{I}_i^{asym}(t) + \dot{I}_i^{mild}(t) + \dot{I}_i^{ILI}(t)$$

$$\dot{I}_i^{asym}(t) = \sigma\left(1 - p_{sym}\right)E_i(t) - \gamma_1 I_i^{asym}(t)$$

$$\dot{I}_i^{mild}(t) = \sigma p_{sym}(1 - p_{ILI})E_i(t) - \gamma_1 I_i^{mild}(t)$$

$$\dot{I}_i^{ILI}(t) = \sigma p_{sym} p_{ILI} E_i(t) - \gamma_2 I_i^{ILI}(t) - h_i I_i^{ILI}(t)$$

$$\dot{H}_i(t) = h_i I_i^{ILI}(t) - \gamma_3 H_i(t) - \mu_i H_i(t)$$

$$\dot{D}_i(t) = \mu_i H_i(t)$$

$$\dot{R}_i(t) = \gamma_1 \left(I_i^{asym}(t) + I_i^{mild}(t)\right) + \gamma_2 I_i^{ILI}(t) + \gamma_3 H(t).$$

The first equation specifies the rate of decrease in susceptible individuals as proportional to the stock of susceptible individuals and the force of infection $\lambda_i(t)$, which is the rate at which susceptible individuals acquire the infection. The second equation states that the number of exposed individuals increases by the same amount the susceptible individual decreases minus a fraction of the stock of individuals already exposed, which progress to become infectious. The change in number of infectious individuals, whether asymptomatic or with mild or influenza-like illness (ILI) symptoms, is the sum of the number of individuals entering the group, assumed proportional to the number of exposed individuals, and the number of individuals exiting the group assumed proportional to the stock of infected. Transmission from asymptomatic individuals is reduced by a factor *r* relative to the other infectious compartments. A fraction of those infected are hospitalized, and a fraction of those hospitalized die, while the others recover. Note that hospitalizations arise from ILI cases only, and never from mild or asymptomatic infections. Similarly, our model allows only for deaths after hospitalization. The last equation describes the number of recovered individuals as a fraction of the individuals infected and hospitalized. We do not model transmission from hospitalized cases. At the start of each period $[t_\tau, t_{\tau+1}),\ \tau = 0, \ldots, \mathrm{T}$ the economic and the epidemiological groups get re-matched. We denote the limit of the various epidemiological groups as t tends to t_τ from the from the left as $S_i(t_\tau^-),\ E_i(t_\tau^-), I_i^{asym}(t_\tau^-), I_i^{mild}(t_\tau^-), I_i^{ILI}(t_\tau^-), H_i(t_\tau^-), R_i(t_\tau^-)$ and $D_i(t_\tau^-)$. These variables are given at time t_τ for all for $i = 1, \ldots, N + 4$ from their past developments.

We calibrate the following 4 parameters of DAEDALUS to the hospitalization data from the Office for National Statistics from 20th February to 31st July 2020 in England [12]: the basic reproductive number R_0; effectiveness of lockdown δ_{LD}; epidemic start time t_0 and lockdown onset t_{LD}. Model projections are then made using population data for the UK, in order to be consistent with the economic model. A modifier is a multiplicative factor applied to beta, used to capture the dampening impact of NPIs and individuals' behaviour on transmissions as represented by R_t. The modifier captures the combined effect of NPIs that are difficult to estimate empirically, including social distancing in social and work environments, facemasks, testing-and-tracing, shielding of the vulnerable, travel restrictions, and limits to social gatherings. The interventions may be government-mandated or adopted by individuals voluntarily. We use the calibrated value δ_{LD} of the modifier over the lockdown period as an upper bound estimate of the NPI's impact. For most forward projections, we adjust the modifier to reflect less stringent NPIs and weaker compliance in the post-lockdown period.

Additional parameter estimates are aligned with Imperial College's Real-time Model (RTM). A full list of DAEDALUS's parameters is given in ***Table 3***. Transitions between epidemiological compartments are governed by rates, and therefore assume exponentially distributed periods for serial interval, time to hospitalisation and hospital stay. We output all epidemiological variables, including disease incidence, hospital occupancy, and cumulative deaths. These quantities then inform our epidemiological constraints and our comparison between model scenarios.

The global effective reproductive number and the end of the last decision period R_{end} is of particular importance as it is the second epidemiological constraint in the optimization. We use the Eigenvalue approach to calculate R_0 and R_{end} following the standard approach for populations with host heterogeneities [see box 3.1, p. 60 in 11, 13]. All sectors and non-working groups have specific reproductive numbers, but it is assumed that incidence increases at the same exponential rate (i.e. the dynamics become slaved) from a certain point after the initial seeding, i.e. the start of the pandemic. R_{end} for the entire population lies between the values calculated for each group and will generally be greater than the weighted average because we assume that individuals belonging to certain groups are more likely to interact with each other than with individuals belonging to other groups (assortative mixing). We calculate R_0 and R_{end} from the distribution of infection across groups in the region of slaved dynamics, where the behaviour is independent of the initial seeds. This slaved distribution provides a natural weighting for the number of secondary cases generated by a primary case in each group.

Table 3: Epidemiological parameters

Parameter	Explanation	Value	Reference
R_0	Basic reproductive number	2.7274	Fitted (ONS hospitalisations)
t_{LD}	Start of first lockdown period in the UK (lockdown onset)	86.3881 (day of year) 26th March	Fitted (ONS hospitalisations)
δ_{LD}	Effectiveness of first lockdown in reducing transmission	0.5007	Fitted (ONS hospitalisations)
$T_E = 1/\sigma$	Latent period	4.6	RTM
$T_R^1 = 1/\gamma_1$	Infectious period (asymptomatic/mild)	2.1	RTM
$T_R^2 = 1/\gamma_2$	Infectious period (ILI)	5.0454 (0-4) 5.0772 (5-19) 7.0522 (20-64) 15.3280 (65+)	RTM
$T_R^3 = 1/\gamma_3$	Duration of hospital stay (recovery)	12.9849 (0-4) 12.9849 (5-19) 13.2208 (20-64) 18.5462 (65+)	RTM
r	Relative infectiousness of asymptomatic cases	0.6667	RTM
p_1	Proportion of cases that are symptomatic	0.66	RTM
p_2	Proportion of symptomatic cases that exhibit ILI	4.2403 (0-4) 4.1375 (5-19) 3.0049 (20-64) 2.7306 (65+)	RTM
h	Hospitalisation rate (determined from T_R^2, proportion of ILI cases hospitalised, and time to hospitalisation)	0.0018 (0-4) 0.0030 (5-19) 0.0582 (20-64) 0.1348 (65+)	RTM
μ	Death rate (determined from T_R^3 and duration of hospital stay leading to death)	0.0018 (0-4) 0.0018 (5-19) 0.0034 (20-64) 0.0290 (65+)	RTM

Note: RTM is Imperial College's 'Real-time Model' of SARS-CoV-2 transmission for the UK [14].

At time t_τ^-, the number of workers of sector i who are working is determined by the extent to which sector i is allowed to open and given by $x_{i\tau-1}w_i^*$, and the number of workers who are not working by $(1 - x_{i\tau-1})w_i^*$. Although this is the same number as at the start of the period, the composition of the population in the epidemiological groups has changed. Choosing $x_{i\tau}$ changes the initial values of the transmission model at the start of each period $[t_\tau, t_{\tau+1})$, $\tau = 0, \dots, \mathrm{T}$. We now make clear how this is done.

At time t_τ the government's decision variables change from $x_{i\tau-1}$ to $x_{i\tau}$. The active workers in sector *i* at time t_τ are

$$x_{i\tau} w_i^* = x_{i\tau-1} w_i^* + (x_{i\tau} - x_{i\tau-1}) w_i^*.$$

If $x_{i\tau} - x_{i\tau-1} = 0$ nothing changes for sector i and production is the same compared to the previous period.

If $x_{i\tau} - x_{i\tau-1} < 0$, then the sector's production is reduced compared to the previous period. The number of those working in sector i decreases and $(x_{i\tau} - x_{i\tau-1}) w_i^*$ of them join the group of non-working adults for that period. Hence, $x_{i\tau} w_i^* = \frac{x_{i\tau}}{x_{i\tau-1}} (x_{i\tau-1} w_i^*)$. We assume that each epidemiological group in sector i is reduced by the same amount $\frac{x_{i\tau}}{x_{i\tau-1}}$. The remaining fraction $1 - \frac{x_{i\tau}}{x_{i\tau-1}}$ is added to the corresponding epidemiological group of the non-working adults for period $[t_\tau, t_{\tau+1})$ at time t_τ.

If $x_{i\tau} - x_{i\tau-1} > 0$, then the sector's production is increased compared to the previous period. The number of those working in sector i increases with $(x_{i\tau} - x_{i\tau-1}) w_i^*$ and some or all of those workers who did not work the previous period are now joining sector i. The number of individuals in group N+4 changes at time t_τ, so it will be indexed by τ. The fraction $\chi_{i\tau} = (x_{i\tau} - x_{i\tau-1}) w_i^* / w_{N+4}(t_\tau^-)$ of $w_{N+4}(t_\tau^-)$ enters sector i at the start of period $[t_\tau, t_{\tau+1})$.

So, assuming all non-working adults have the same chance of being employed in sector i, the initial values of the transmission model for the N productive sectors for the period $[t_\tau, t_{\tau+1})$ are

$$S_i(t_\tau) = \begin{cases} \frac{x_{i\tau}}{x_{i\tau-1}} S_i(t_\tau^-) & x_{i\tau} < x_{i\tau-1} \\ S_i(t_\tau^-) + \chi_{i\tau} S_{N+4}(t_\tau^-) & x_{i\tau} > x_{i\tau-1} \end{cases},$$

$$E_i(t_\tau) = \begin{cases} \frac{x_{i\tau}}{x_{i\tau-1}} E_i(t_\tau^-) & x_{i\tau} < x_{i\tau-1} \\ E_i(t_\tau^-) + \chi_{i\tau} E_{N+4}(t_\tau^-) & x_{i\tau} > x_{i\tau-1} \end{cases},$$

$$I_i^{asym}(t_\tau) = \begin{cases} \frac{x_{i\tau}}{x_{i\tau-1}} I_i^{asym}(t_\tau^-) & x_{i\tau} < x_{i\tau-1} \\ I_i^{asym}(t_\tau^-) + \chi_{i\tau} I_{N+4}^{asym}(t_\tau^-) & x_{i\tau} > x_{i\tau-1} \end{cases},$$

$$I_i^{mild}(t_\tau) = \begin{cases} \frac{x_{i\tau}}{x_{i\tau-1}} I_i^{mild}(t_\tau^-) & x_{i\tau} < x_{i\tau-1} \\ I_i^{mild}(t_\tau^-) + \chi_{i\tau} I_{N+4}^{mild}(t_\tau^-) & x_{i\tau} > x_{i\tau-1} \end{cases},$$

$$I_i^{ILI}(t_\tau) = \begin{cases} \frac{x_{i\tau}}{x_{i\tau-1}} I_i^{ILI}(t_\tau^-) & x_{i\tau} < x_{i\tau-1} \\ I_i^{ILI}(t_\tau^-) + \chi_{i\tau} I_{N+4}^{ILI}(t_\tau^-) & x_{i\tau} > x_{i\tau-1} \end{cases},$$

$$H_i(t_\tau) = \begin{cases} \frac{x_{i\tau}}{x_{i\tau-1}} H_i(t_\tau^-) & x_{i\tau} < x_{i\tau-1} \\ H_i(t_\tau^-) + \chi_{i\tau} H_{N+4}(t_\tau^-) & x_{i\tau} > x_{i\tau-1} \end{cases},$$

$$D_i(t_\tau) = \begin{cases} \frac{x_{i\tau}}{x_{i\tau-1}} D_i(t_\tau^-) & x_{i\tau} < x_{i\tau-1} \\ D_i(t_\tau^-) + \chi_{i\tau} D_{N+4}(t_\tau^-) & x_{i\tau} > x_{i\tau-1} \end{cases},$$

for $i = 1, \ldots, N$. The values for each epidemiological group vary depending on whether production is increased or decreased in the respective period. Notice that when we move workers between working and non-working, we assume that we move the same proportion of all epidemiological groups including the infected, the hospitalised and the dead. This accounts for reduced transmission due to absence. Also, the effect of deaths in the working population is negligible owing to the low death rate in under 65's.

For pre-schoolers, school children and retired, the dynamics of the epidemic continue from the end values reached in the previous period. That is, $S_i(t_\tau^+) = S_i(t_\tau^-)$, $E_i(t_\tau) = E_i(t_\tau^-)$, $I_i^{asym}(t_\tau) = I_i^{asym}(t_\tau^-)$, $I_i^{mild}(t_\tau) = I_i^{mild}(t_\tau^-)$, $I_i^{ILI}(t_\tau) = I_i^{ILI}(t_\tau^-)$, $H_i(t_\tau) = H_i(t_\tau^-)$, $D_i(t_\tau) = D_i(t_\tau^-)$ for $i = N+1, N+2, N+3$.

For the non-working adults, the initial conditions at t_τ capture the fact that the productive sectors are partially closed and that a fraction of the workers may become temporarily non-working adults at t_τ, $\tau = 0, \ldots, \mathrm{T}$:

$$S_{N+4}(t_\tau) = \left(1 - \sum_{i:x_{i\tau}-x_{i\tau-1}>0} \chi_{i\tau}\right) S_{N+4}(t_\tau^-) + \sum_{i:x_{i\tau}-x_{i\tau-1}<0} \left(1 - \frac{x_{i\tau}}{x_{i\tau-1}}\right) S_i(t_\tau^-),$$

$$E_{N+4}(t_\tau) = \left(1 - \sum_{i:x_{i\tau}-x_{i\tau-1}>0} \chi_{i\tau}\right) E_{N+4}(t_\tau^-) + \sum_{i:x_{i\tau}-x_{i\tau-1}<0} \left(1 - \frac{x_{i\tau}}{x_{i\tau-1}}\right) E_i(t_\tau^-),$$

$$I_{N+4}^{asym}(t_\tau) = \left(1 - \sum_{i:x_{i\tau}-x_{i\tau-1}>0} \chi_{i\tau}\right) I_{N+4}^{asym}(t_\tau^-) + \sum_{i:x_{i\tau}-x_{i\tau-1}<0} \left(1 - \frac{x_{i\tau}}{x_{i\tau-1}}\right) I_i^{asym}(t_\tau^-),$$

$$I_{N+4}^{mild}(t_\tau) = \left(1 - \sum_{i:x_{i\tau}-x_{i\tau-1}>0} \chi_{i\tau}\right) I_{N+4}^{mild}(t_\tau^-) + \sum_{i:x_{i\tau}-x_{i\tau-1}<0} \left(1 - \frac{x_{i\tau}}{x_{i\tau-1}}\right) I_i^{mild}(t_\tau^-),$$

$$I_{N+4}^{ILI}(t_\tau) = \left(1 - \sum_{i:x_{i\tau}-x_{i\tau-1}>0} \chi_{i\tau}\right) I_{N+4}^{ILI}(t_\tau^-) + \sum_{i:x_{i\tau}-x_{i\tau-1}<0} \left(1 - \frac{x_{i\tau}}{x_{i\tau-1}}\right) I_i^{ILI}(t_\tau^-),$$

$$H_{N+4}(t_\tau) = \left(1 - \sum_{i:x_{i\tau}-x_{i\tau-1}>0} \chi_{i\tau}\right) H_{N+4}(t_\tau^-) + \sum_{i:x_{i\tau}-x_{i\tau-1}<0} \left(1 - \frac{x_{i\tau}}{x_{i\tau-1}}\right) H_i(t_\tau^-),$$

$$D_{N+4}(t_\tau) = \left(1 - \sum_{i:x_{i\tau}-x_{i\tau-1}>0} \chi_{i\tau}\right) D_{N+4}(t_\tau^-) + \sum_{i:x_{i\tau}-x_{i\tau-1}<0} \left(1 - \frac{x_{i\tau}}{x_{i\tau-1}}\right) D_i(t_\tau^-).$$

Let $H(t)$ be the number of individuals in the country who are hospitalized in at each t, and $\underline{H}$ be the country's hospital capacity – which for simplicity is assumed constant over the intervention horizon.

The first epidemiological constraint demands that the number hospitalized does not exceed capacity at any one time. That is,

$$H(t) \leq \underline{H} \qquad \text{(A)}$$

for all t. The number of individuals hospitalized $H(t)$ is the sum of the individuals in each productive sector and in the community who are hospitalized, $H_i(t)$:

$$H(t) = \sum_{i=1}^{N+4} H_i(t). \qquad \text{(A)}$$

The second epidemiological constraint requires $R_{end} \leq 1$ (for calculation of R_{end} see above). This is to ensure that the epidemic is under control at the end of the planning horizon, leaving a manageable 'legacy' for future periods.

Hospital capacity

Hospital capacity is an important variable in DAEDALUS because it is the main constraint to the optimization. At any point in time, the number of patients with COVID-19 and other conditions must not exceed the number of available beds. We do not distinguish between critical care and general & acute beds for simplicity and summed them for an estimate of total capacity. This is reasonable as hospitalised COVID-19 patients occupy full spectrum of severity, from those admitted for observation, through non-invasive oxygen support and mechanical ventilation up to those requiring highly complex Extra Corporeal Membrane Oxygenation. The treatment of COVID-19 patients is changing with new insights into optimal treatment, which may mean that critical care needs are changing over our intervention horizon. Lastly, hospitals have converted some general & acute wards into critical care wards to cope with the surge in demand over spring 2020 which makes a clear distinction between critical and general & acute care difficult. We make three alternative assumptions on emergency spare hospital capacity available for the treatment of COVID-19 patients (see ***Table 4***).

Table 4: Assumption on emergency spare hospital capacity for the treatment of COVID-19 in the United Kingdom, 2020

Capacity scenario	**capacity (total beds)**	**Explanation and reference**
Occupancy two-thirds	12,000	Bed occupancy assumed to be decreased by one-third (-6000) of the maximum occupancy observed in April 2020
Occupancy first peak	18,000	Maximum observed bed occupancy by COVID-19 patients during the first peak in April 2020
Occupancy one-third more	24,000	Bed occupancy assumed to be increased by one-third (+6000) of the maximum occupancy observed in April 2020

Contact matrices

Contact matrices define the mean number of contacts per day reported between groups of individuals, for example between age-groups. Contact matrices form the foundation of force-of-infection (FOI) and next-generation operators in heterogeneous epidemic models. Contact survey data allow derivation of the average number of contacts made while performing specific activities. Analyses using data from one of the landmark studies in the field {SIMID group, 2020 #96} focus on contacts made in the household, during transport, at work and other settings. In epidemiological studies, contact survey data are used to construct contact matrices, also referred to as Who-Acquires-Infection-From-Whom (WAIFW) matrices. There exists an extensive literature deriving coefficients for heterogeneous mixing with respect to age and geography. However, few studies estimated contacts structures with respect to economic sector. For the purpose of this study, we account for the heterogeneity in contact rates between sectors. We synthesise a contact matrix based on a contact survey conducted in 2012 in France [15]. While more recent contact surveys have been conducted, to our knowledge, this is the only survey that includes sector-specific and work-related information of respondents. The industry sectors as reported by the respondents of the French survey were translated to the industry sectors of the UK IO table (***Table 4***). Due to lack of more detailed data, we had to use the same contact rate for all subsectors.

Table 5: Mapping of French into UK Business Sectors as defined in the UK IO tables

French Business Sector	UK Business Sector
Agriculture, sylviculture, pêche	Agriculture [A]
Industrie agricole et alimentaire	Production [B-E]
Autre industrie*	Other services [R-T] + Information & communication [J]
Énergie	Production [B-E]
Construction	Construction [F]
Commerce	Distribution, transport, hotels and restaurants [G-I]
Activités financières et immobilières**	Financial and insurance [K] + real estate [L]
Services aux entreprises	Professional and support activities [M-N]
Services aux personnes*	Other services [R-T]
Éducation, santé, action social	Government, health & education [O-Q]
Administration	Professional and support activities [M-N]

We constructed a contact matrix M as the sum of 3 matrices: A (community contacts), B (worker-to-worker contacts) and C (consumer-to-worker contacts, see **Table *5***). Opening of sectors, including hospitality and education, increases transmissions as informed by contact rates in matrices B and C. Workers and consumers carry infection back into the community, which increases transmission as informed by rates in A. Refining the matrices A, B and C in this study is fundamental to estimating DAEDALUS.

Table 6: Construction of contact matrices from survey data

Contact matrix	Definition and inclusion/exclusion criteria
A: **community contacts**	• Any contact made at home, in a vehicle or other private place, retail outlet, public transport, leisure facilities, with loved ones in a closed place (“Chez des proches en lieux clos”), open place (park, street) • Disaggregated by age group (0 – 4; 5 – 18; 19 – 63; 64+) • Consumer-consumer contacts are added with respect to the proportion the hospitality and education sectors are opened
B: **Worker-to-worker contacts**	• Contacts made at work (office, studio, etc.) and which are reported to be made (almost) every day, or a few times per week • Disaggregated by sector • Individuals who stated that they are in employment • Individuals who are of working age (19 – 64)
C: **Consumers-to-worker contacts**	• Contacts made at work (office, studio, etc.) and which are reported to be a few times per month, a few times per year or less often, for the first time • Disaggregated by sector • Individuals who stated that they are in employment • Individuals who are of working age (19 – 64) • If more than 20 contacts are made by the individual, the survey respondent could state the total number of contacts made instead of listing all individual contacts. If this was the case, this number was used instead of the sum of individual contacts made

Community contacts (matrix A) are any contacts made that are unrelated to the workplace (see **Table *5***). This includes contacts in the household, during travel to and from the workplace and non-work-related travel, outside spaces, leisure activities (e.g. meeting friends), retail outlets (e.g. supermarkets), and contacts made in the hospitality or service sectors. When sectors are partially or fully opened, we account for additional transmission risk from contacts between consumers in matrix A. As such, in addition to household contacts, contacts are being made when consuming products or services from specific sectors. Opening the hospitality sector, for example, will increase the community transmission as consumers meet in pubs and restaurants. **Table *6*** shows the average contact rate for the community. The row sums of the community matrix A are equal, and the columns are weighted by the size of the workforce (measured in headcounts) in each sector. The value of row sums depends on the extent to which given sectors are open. If all economic sectors were fully closed, the total contact is reduced to roughly 40% of total contact when all sectors are open.

Table 7: Average daily contacts in the community (A), worker-to-worker (B) and consumer-to-worker (C) by high-level industry sector grouping

	Matrix A: Community contacts	**Matrix B:** Worker-to-worker contacts	**Matrix C:** Consumer-to-worker contacts
Community			
0 – 4 years old	4.6		
5 – 18 years old	6.0		
19 – 64 years old	3.2		
64+ years old	1.7		
Economic sectors			
Agriculture [A]	3.2	2	1
Production [B-E]	3.2	7.5	3
Construction [F]	3.2	2.8	1
Distribution, transport, hotels and restaurants [G-I]	3.2	3	14
Information and communication [J]	3.2	4.5	2
Financial and insurance [K]	3.2	7.5	1.3
Real estate [L]	3.2	7.5	1.3
Professional and support activities [M-N]	3.2	3	2.3
Government, health & education [O-Q]	3.2	1	20
Other services [R-T]	3.2	3	2

Note: the same contact rates were applied to all subsectors.

Students are ‘consumers’ of education services. In the education sector, we account for the number of contacts between students going to school or university. School contacts are estimated separately in two age groups (pre-school age: 0 – 4; school age: 5 – 18). Similarly, hospitality consumer contacts are estimated considering age-heterogeneity in hospitality contacts. See **Table *7*** for the age-adjusted average number of contacts in the education and hospitality sectors. Opening of schools and universities will affect the row sum of the school-age children group. The community contact rates were adjusted to that the mean rate across all age groups is equal to 3.5, as taken from survey data.

Table 8: Average number of contacts in the hospitality and education sectors, adjusted by age group.

Age group	Contact rate
Hospitality sector	
0 - 4	0.0
5 - 18	0.5
19 - 64	1.5
64+	1.5
Education sector	
0-4	4
5-18	8

Worker-to-worker contacts (matrix B) describes the at-work contacts in sectors, i.e. the number of contacts per day reported by an individual actively working in the same sector (**Table *5***). Here "actively working" refers to one period, i.e. from one decision point to the next-. Matrix B is diagonal owing to lack of data regarding between-sector contacts. Worker-worker contacts are defined by those contacts recorded to have happened at work and frequently (reported as a contact made almost every day). At work contacts at low frequency are classified as worker-consumer contacts. At-home working is considered, and community contact rates apply for contacts between working household member. We assume that transport contacts only add to the infection risk if the sector is open and the workers travel to and from their workplace.

Consumer-to-worker contacts (matrix C) describes contacts experienced by workers from consumers. As for A, the columns are weighted by sector population, though the row sums are sector specific. Contacts experienced by workers from consumers are defined by those contacts recorded to have happened at work less frequently than every day (i.e. recorded as *a few times a* week, *a few times a* month, *a few times a year or less often*, or *for the first* time). We account for a reduction in transmission risk due to NPIs or working from home via a reduction in effective contact rates. Furloughed and unemployed workers are considered as non-working.

Optimization model

The objective of DAEDALUS in its main specification is to maximize the GDP of the economy as represented by the GVA created by the sectors of the economy that are operational – either wholly or in part. Aggregate GVA is maximized subject to the constraints that hospital capacity is not breached at any point in time, and the global effective reproductive number R_t is smaller than one at the end of the optimization horizon, i.e. $R_{end} \leq 1$. This leaves the country in a reasonable position with respect to incidence and hospitalizations. If required, a subsequent optimization can be implemented as new data become available, policy objectives change, or we approach the end of the intervention horizon at the end .

For a single period, we assume decision variables $x_{j\tau}$ that indicate the proportion of each sector *j* that is operational in periodτ, and therefore take the value between x_j^{min} and 1. A partial opening $x_{j\tau}$ of sector *j* in periodτ creates an output for sector j equal to $x_{j\tau}\, g_j w_j^*$. In order to calculate the GVA for sector j we need to subtract the value of the intermediate products used by sector j, which are given by $\sum_{i=1}^{N} a_{ij} x_{j\tau}\, g_j w_j^*$. Considering the interdependency of the economic sectors, the level of transmissions and the hospital capacity specified above, DAEDALUS optimises an objective function. The optimization problem can be represented as:

$$max \sum_{j=1}^{N} \sum_{\tau=0}^{\mathrm{T}} x_{j\tau} g_j w_j^* \left(1 - \sum_{i=1}^{N} a_{ij}\right)$$

Subject to

$$\sum_{i=1}^{N+4} H_i(t) \leq \underline{H}$$

$$R_{end} \leq 1$$

$$\sum_{\tau=0}^{5} g_i w_i^* x_{i\tau} \geq \sum_{j=1}^{\mathrm{T}} a_{ij} \left(\sum_{\tau=0}^{5} g_j w_j^* x_{j\tau}\right)$$

$$0 \leq x_{i\tau} \leq 1;\;\; i = 1 \dots n, \tau = 0, \dots, \mathrm{T}$$

where $g_j w_j^* \left(1 - \sum_{i=1}^{N} a_{ij}\right)$ is the pre-pandemic GVA associated with sector *j*. The first two epidemiological constraints set upper limits on the extent of hospitalizations and transmissions. First, the quantities $\sum_{i=1}^{N+4} H_i(t)$ represent the number of hospitalized patients in decision period τ. They depend on the state of opening of the economic sectors as defined by transmission model. The amount $\underline{H}$ represents the (static) hospital capacity. Second, R_{end} represents the global effective reproductive number at the end of the intervention horizon and the last decision period T. The constraint ensures that hospitalizations are not going to breach available capacity right after the intervention horizon. Constraints three and four represent economic constraints. Constraint three reflects the need for a sector to produce at least enough to satisfy the intermediate needs of other sectors that are open in periodτ, and constraint four to produce no more than the maximum level produced before the pandemic.

Other policy constraints can be easily introduced into DAEDALUS. For example, employment can be maximized instead of GDP, or certain sectors can be forced to stay open on the expense of others. This would require an additional constraint which specifies that a specific sector remains at a certain operational level in every period over the intervention horizon. For example, if the education sector $i = 56$ is to stay open at least 70%, then a third economic constraint would be added to DAEDALUS as $x_{56\tau} \geq 0.7, \tau = 0, \dots, \mathrm{T}$. DAEDALUS can also calculate the outcomes (in terms of GDP, employment, infections, hospitalizations and deaths) of scenarios that do not attempt to maximize GDP. For example, an evaluation of the closure of all non-essential businesses (national lockdown) simply sets all $x_{i\tau}$ at the observed lockdown values $x_i^{min} = x_i^{LD}$ or the adjusted minimum lockdown values $x_i^{min} = 0.8x_i^{LD}$, meaning only essential production is open. An evaluation of the unmitigated pandemic sets $x_{i\tau} = 1$, meaning the economy functions as in pre-pandemic times with resulting high levels in infections. We would assume that demand for products is as observed in pre-pandemic times, which is probably unrealistic. It would be reasonable to assume a voluntary behaviour change that dampens transmission dynamics (and demand) to some extent. Another adaption could be introduced by allowing only incremental changes in sector activity at each decision point or accommodating seasonality on both epidemiological and economic sides of DAEDALUS.

The multiperiod DAEDALUS model has serious computational demands. The number of decision variables is the product of the number of periods and the number of sectors under consideration i.e. 63* T. The number of linear (economic) constraints is 2*63* T because they are applied to each time period. For a scenario where certain sectors are open there are additional constraints, one for each sector and period. Although fewer in number at only $\mathrm{T} + 1$, the epidemiological constraints are non-linear and much more computationally demanding. Optimizations use 'Global search' with derivative-based base algorithm fmincon in MATLAB's global optimization toolbox.

Limitations

DAEDALUS has important limitations. First, assuming that the a_{ij} are constant requires each sector to use inputs in fixed proportions according to a production function of the Leontief form [3]. Although, some of the inputs are necessary to produce a given output, others are incidental. A few authors have suggested ways of getting around these limitations using a network approach [16, 17]. We have allowed for this effect in a simplified way by setting the minimum required opening of each sector x_i^{min} at 80% of observed average production levels during the lockdown in the UK between March and May 2020.

The assumption that labour productivity g_i is constant for each sector may also be questioned. For example, there is likely to be some change in productivity from parents working from home without childcare, and from measures to reduce the spreading of infection in the workplace [7, 18]. The behavioural modifier will capture social distancing interventions at the workplace and in the community, but this is a crude way to model the impacts of NPIs. These factors become important when we wish to model with more granularity sector productivity and the disease transmission associated with economic activity within a sector. Furthermore, we do not at this stage explicitly model workforce requirements, or the impact of the pandemic on labour supply.

We are retaining a general form for the GVA functions $g_i(.)$. Data limitations preclude use of anything other than use of linear $g_i(.)$ functions, although it should be noted that – to some extent – our disaggregation of sectors into 63 sub-sectors can accommodate different production processes and levels of productivity within a broad sector. Assuming the most productive parts of a sector are opened up first, these functions should in principle exhibit diminishing returns as a sector is opened up.

The assumption that the final demand for goods and services is constant may be too strong. Firstly, the uncertainty about the economic situation in general - and future income in particular - affects the level of consumption. Uncertainty about future income may encourage individuals to consume a smaller fraction of their current income and save more or to postpone purchases of certain types of goods. Consumption levels may bounce back quickly provided the households' long-term income is not affected. However, if household incomes are permanently reduced through unemployment or reduced output for self-employed running a business, household consumption will also be permanently reduced. These income effects are heterogeneous across households, time, sectors, and types of goods. Secondly, individual preferences may change. The desire to avoid spreading or acquiring the infection for themselves and others may draw individuals away from certain goods in favour of substitutes. Similarly, the unavailability of certain goods could increase the demand of alternative goods. Some of these changes may be reversed as soon as the risks associated with the pandemic are sufficiently low or the original product becomes available again. But some changes could lead to changes in preferences and be permanent.

DAEDALUS focuses on domestic production and assumes imports can remain (proportionately) available as before. In practice imports might - to some extent - act as substitutes for lost domestic production, or alternatively they might become unavailable. Changes to such assumptions could readily be treated explicitly with minor modifications to DAEDALUS, once economic data on such effects become available.

Each sector is open as a proportion of the pre-pandemic level of production or equivalently employment. However, the adults who are working in each period may be in any of the seven epidemiological groups. This means that some of them will not be able to work. We assume that the sectors keep the level of productivity irrespective of sickness absences, which implies that in the short term the sectors can mobilize other inputs to make up for lost production due to sickness of workers.

The government's decision problem could be formulated as an optimal control problem, in which continuous choices about sector activity can be used over the entire planning horizon. However, the practical difficulty of implementing policy changes continuously renders such an approach infeasible. Similarly, we do not consider reactive closures of some or all sectors, for example, in response to increases in hospitalizations. Purpose of our model is to plan optimal scenarios in advance. It is of course possible to define a new intervention horizon at any point in the current intervention horizon, and change course. This would be a sensible approach if new and improved data become available. The epidemiological model can readily accommodate the discontinuities in disease exposure caused by discrete changes in economic activity and can thus be used to assess the infection consequences of these choices over time. The important requirement when a sector changes size is to augment its active workforce from the general community when increases in workforce are required (or return workers to the community when a reduction in the active workforce is signalled).

The main limitations of the epidemiological part of DAEDALUS are limitations of the estimated contact matrices. We do not account for duration of contact due to lack of data; we also do not track infection history across sectors. This means we do not consider the sector in which individuals worked when they re-enter the community, or transmission events in the community as workers re-enter their sectors. We also do not consider contacts from workers to consumers, (just consumers to workers) but we consider this negligible compared to community transmission, and again we are faced with an absence of data. As an optimization model, DAEDALUS is deterministic. Sensitivity analyses on specific parameters of interest and the delta modifier can be undertaken.

The education sector groups both schools and universities, though the induced community transmission when this sector is open occurs only in 5-18-year-olds. Moreover, the GDP contributions of schools and universities differ significantly. More refined data with respect to contact patterns and GDP would allow is to disaggregate the education sector accordingly. Finally, our assumption that the modifier δ takes on a specific value informed by the calibration over the lockdown period is somewhat arbitrary. The modifier is a global estimate for the effectiveness of a highly heterogeneous set of interventions. The effect of this value on the reproductive number is consistent with case data to date, though contact tracing data would allow is to refine our estimate.

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