Option Valuation Methods: Assignment 2

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1 Problem p-7.1

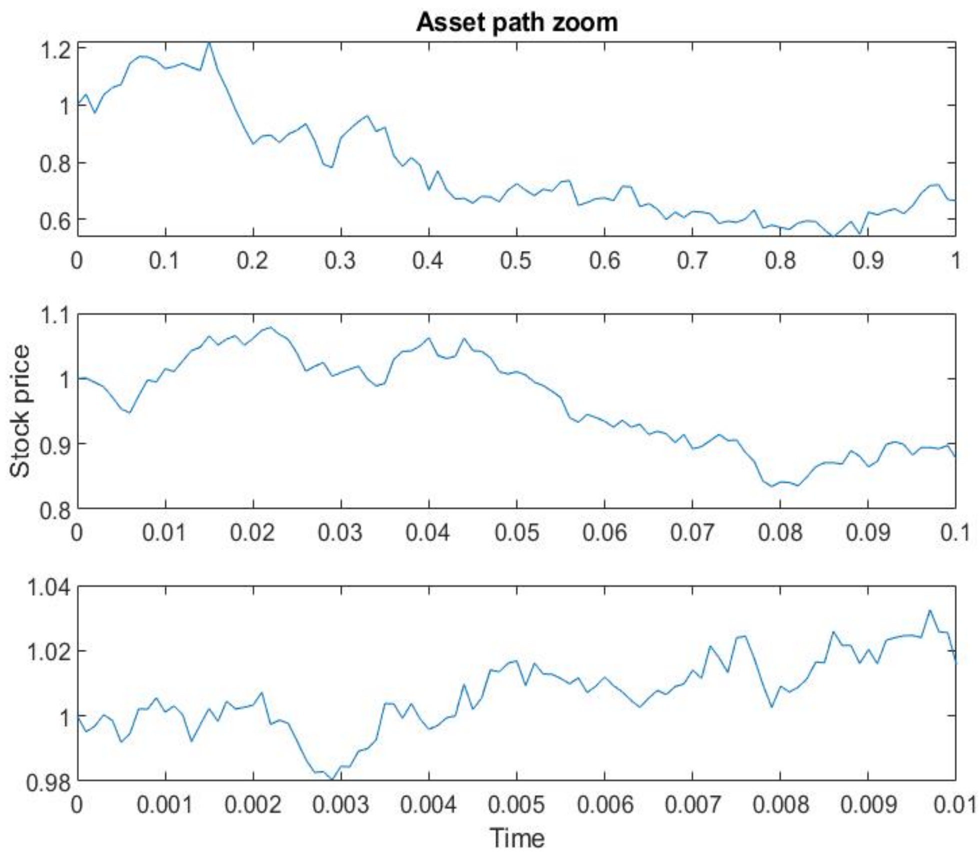

Figure 1: Timescale invariance of asset model

```
    randn('state',100);
%[0,1]
%Function parameters
S = 1; mu = 0.05; sigma = 0.5; L1 = 1e4; T1 = 1; dt1 = T1/L1; M = 1;
%Sets time range for Svals
tvals1 = [0:dt1:T];
%Calculates stock price over tvals1 with M rows, L1 columns
Svals1 = S*cumprod(exp((mu-0.5*sigma^2)*dt1 + sigma*sqrt(dt1)*randn(M,L1)),2);
Svals1 = [S*ones(M,1) Svals1]; % add initial asset price
%Takes each 100th element of Svals to plot 100 samples over interval as in
%fig 7.5
```

```
s1 = Svals1(:,1:100:end);
%Makes range for plot
t1 = 0:0.01:1;
%plot(t1,s1)


%[0,0.1]
L2 = 1e3; T2 = 0.1; dt2 = T2/L2; M = 1;
tvals2 = [0:dt2:T2];
Svals2 = S*cumprod(exp((mu-0.5*sigma^2)*dt2 + sigma*sqrt(dt2)*randn(M,L2)),2);
Svals2 = [S*ones(M,1) Svals2];
s2 = Svals2(:,1:10:end);
t2 = 0:0.001:0.1;
%plot(t2,s2)


%[0,0.01]
L3 = 1e2; T3 = 0.01; dt3 = T3/L3; M=1;
tvals3 = [0:dt3:T3];
Svals3 = S*cumprod(exp((mu-0.5*sigma^2)*dt3 + sigma*sqrt(dt3)*randn(M,L3)),2);
Svals3 = [S*ones(M,1) Svals3];
s3 = Svals3(:,1:1:end);
t3 = 0:0.0001:0.01;



subplot(3,1,1)
plot(t1,s1)
title("Asset path zoom")
subplot(3,1,2)
plot(t2,s2)
ylabel("Stock price")
subplot(3,1,3)
plot(t3,s3)
xlabel("Time")
```

2 Problem 8.10

2.1 Verification

The Black-Scholes PDE is defined as Equation 1. To verify if a particular equation is a solution to the PDE, this equation must be plugged into Equation 1 and should result in the same right hand side as the original PDE, which is 0 in this case.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + r\frac{\partial V}{\partial S} S - rV = 0 \tag{1}$$

From problem 8.10 the following solutions have to be verified:

.

$$V(S,t) = S \tag{2}$$

.

$$V(S,t) = e^{rt} \tag{3}$$

As can be seen from Equation 1, each of the proposed solutions have to be partially derived to both t and S such that these can be plugged into the equation. First, for Equation 2 it can be derived that

$$\frac{\partial V}{\partial t} = 0$$
$$\frac{\partial^2 V}{\partial S^2} = 0$$
$$\frac{\partial V}{\partial S} = 1$$

Plugging these values into the PDE according to Equation 4 confirms that Equation 2 is indeed a valid solution.

$$0 + 0 + rS - rS = 0 \tag{4}$$

The same can be done for Equation 3 which results in the same conclusion:

$$\frac{\partial V}{\partial t} = re^{rt}$$
$$\frac{\partial^2 V}{\partial S^2} = 0$$
$$\frac{\partial V}{\partial S} = 0$$
$$re^{rt} + 0 + 0 - re^{rt} = 0$$

2.2 Financial explanation

The two solutions can be interpreted as two extreme values in which for Equation 2 the value of the option is always equal to the stock value and thus the option can be replicated by always owning exactly one stock and no cash. Equation 3 on the other hand can be replicated by owning 1 in cash and letting this grow according to the compounding interest e^{rt} and owning 0 stocks. Since the PDE is linear [1] the principle of superposition can be applied and thus the linear combination of Equation 2 and Equation 3 is also a solution to the PDE. From this, the conclusion can be drawn that any value of the option $V(S,t)$ can be replicated by a portfolio with x amount of stocks S and y amount of cash that grows by e^{rt}.

[1] https://pdfs.semanticscholar.org/a4c0/35299e75a394c7c9ba65a0f1b60f94ff5f82.pdf

3 Problem 3

3.1 Student number asset simulation

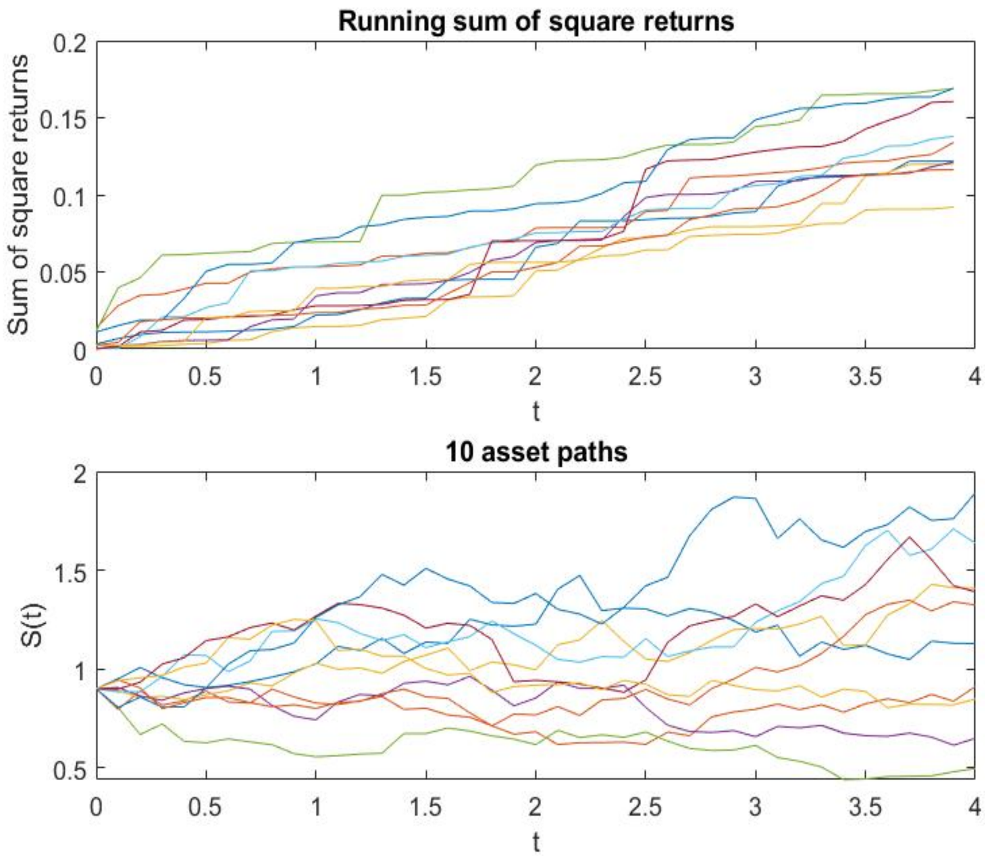

Figure 2: Upper figure: running sum of square returns using 10 asset paths. Lower figure: 10 asset paths

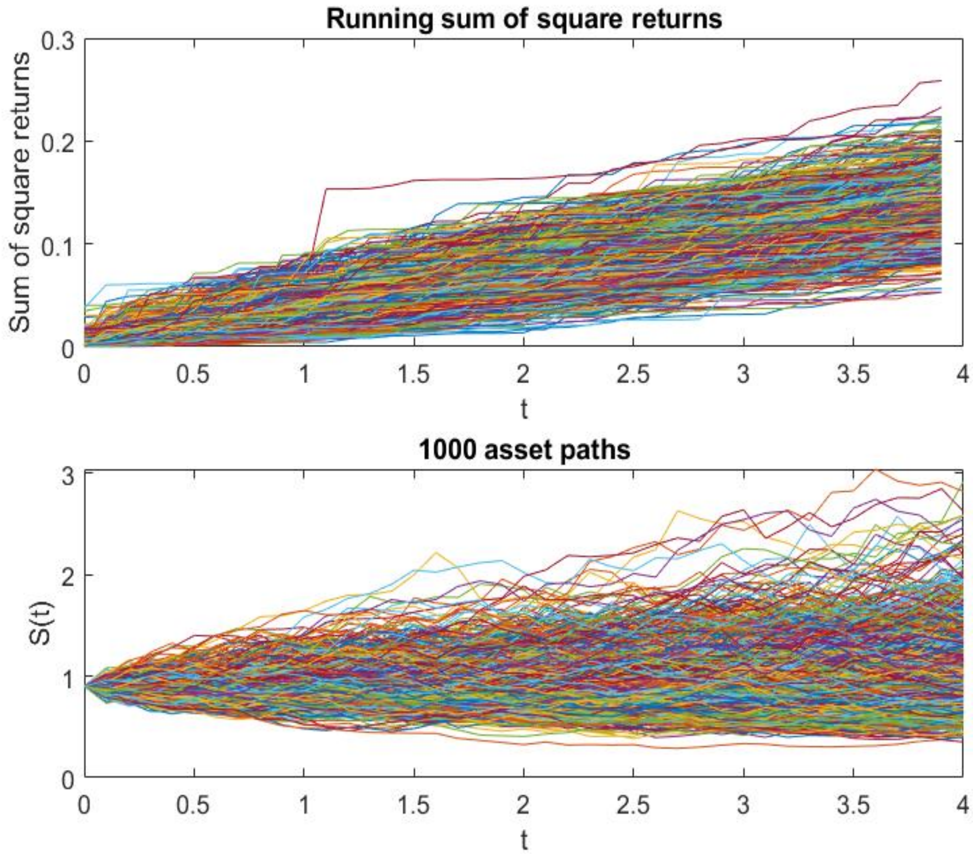

Figure 3: Upper figure: running sum of square returns using 1000 asset paths.
Lower figure: 1000 asset paths

```
    %CH07 Program for Chapter 7
%
% Plot discrete sample paths
randn('state',100)
clf
%%%%%%%%% Problem parameters %%%%%%%%%%%
S = 0.9; mu = 0.04537882; sigma = mu*4; L = 40; T = 4; dt = T/L; M = 1000;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
tvals = [0:dt:T];
tvalsSum = [0:dt:T-dt];
Svals = S*cumprod(exp((mu-0.5*sigma^2)*dt + sigma*sqrt(dt)*randn(M,L)),2);
Svals = [S*ones(M,1) Svals]; % add initial asset price
```

```
%Initializes divisor removing last element from Svals since
%we do not need this element for the difference operator
divisor = Svals(:,1:end-1);
%Computes square return
SquareReturn = (diff(Svals,1,2)./divisor).^2;

%Computes sum of all squarereturns
SumOfSquares = cumsum(SquareReturn,2);
subplot(2,1,1);
plot(tvalsSum, SumOfSquares)
title('Running sum of square returns')
xlabel('t')
ylabel('Sum of square returns')
subplot(2,1,2);
plot(tvals,Svals)
title(sprintf('%d asset paths', M))
xlabel('t'), ylabel('S(t)')
```

Since we have a dt of merely 0.1, it can be observed that the graphs in the plots still deviate quite a lot from each other due to uncertainty. By taking smaller time steps dt, the uncertainty could be made smaller $\rightarrow 0$ when dt $\rightarrow \infty$.

3.2 Crypto asset simulation

XRP data was used to plot the open prices over a year time from 20-02-2017 up till 20-02-2018. As can be seen from both graphs, the coin is very volatile. From the sum of square returns plot it can be observed that its value is still very unpredictable (i.e. not likely to be linear as $\delta t \rightarrow \infty$) unlike the running sum of square returns that follow from assets driven by geometric Brownian motion. This implies that the asset XRP cannot be modelled as if it were driven by geometric Brownian motion.

```
formatIn = 'dd-mm-yyyy'; %Specifies format of date
startDateString = '20-02-2017'; %Start date
endDateString = '20-02-2018'; %End date
startDate = datenum(startDateString, formatIn); %Converts date to matlab-readable date
endDate = datenum(endDateString, formatIn); %idem


flip = flipud(rippleprice); %table is initially set from now to past
%flip upside down to have idx(1) = oldest date idx(end) = newest date

dailyPriceOpen = flip.Open(1297:end,:); %using open prices to plot the
%price path of XRP
```

```
%Sum of square returns
divisor = dailyPriceOpen(1:end-1,:);
SquareReturn = (diff(dailyPriceOpen,1,1)./divisor).^2;
SumOfSquares = cumsum(SquareReturn);

%plotting
subplot(2,1,1)
xDate = linspace(startDate,endDate,366); %range for x axis in days
plot(xDate,dailyPriceOpen)
xlabel('date')
ylabel('Price')
title('Asset path of XRP for open prices for 1 year')
datetick('x','mm-yyyy'); % years on x-axis
subplot(2,1,2)
plot(xDate(1:end-1),SumOfSquares)
title('Sum of square returns for XRP for 1 year')
datetick('x','mm-yyyy');
xlabel('date')
ylabel('Sum of square returns')
```

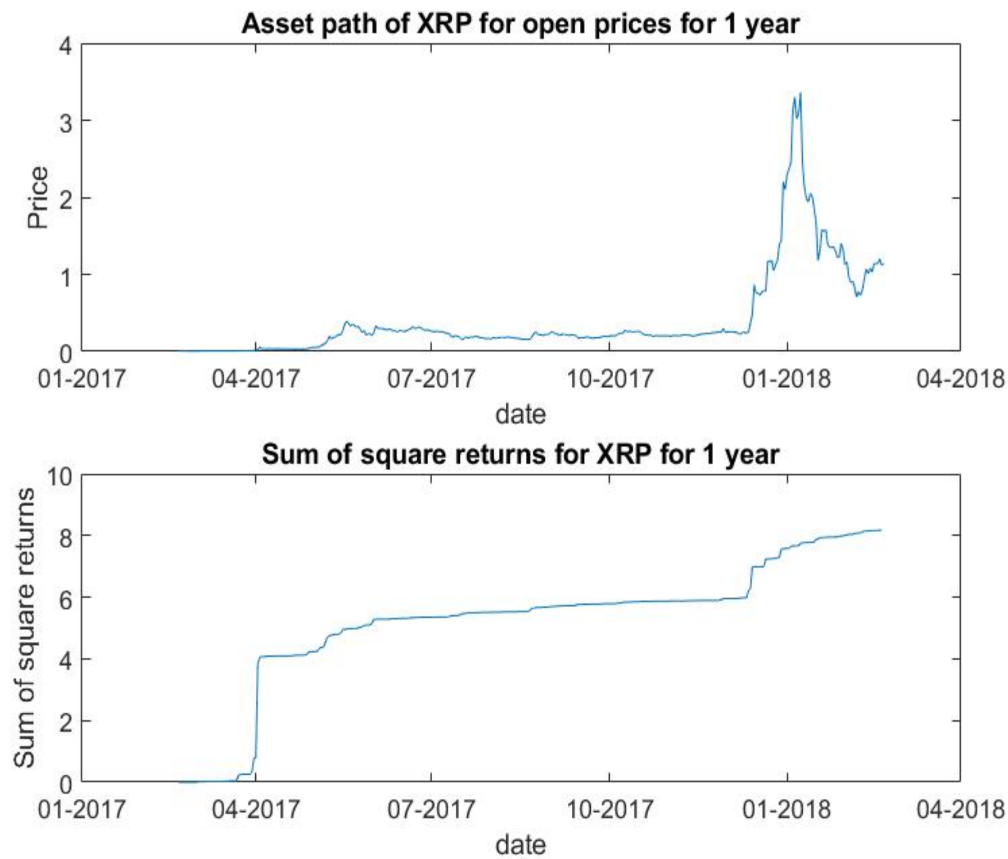

Figure 4: Upper figure: Open price path for XRP for one year Lower figure: Sum of square returns for XRP for one year