Applied Time Series Econometrics

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Short resumé about SVAR models

A structural VAR model may be represented as follows:

$$B_0 y_t = \lambda + B_1 y_{t-1} + \cdots + B_p y_{t-p} + \varepsilon_t, \qquad \varepsilon_t \sim iidN(0, \Sigma),$$

where y_t is a $n \times 1$ vector of time series.

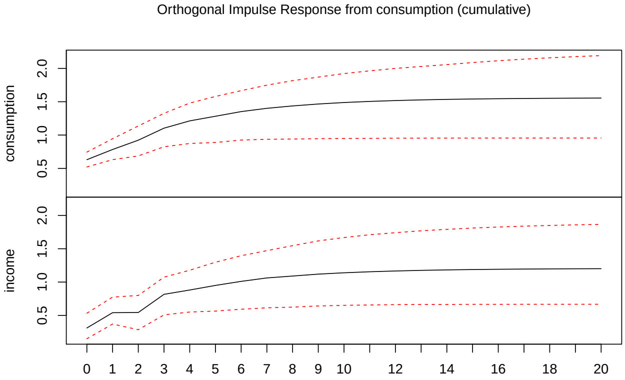
- We want to determine $n(1 + pn + n)$ parameters of B_i matrices and n parameters of Σ.
- We cannot estimate SVAR directly because of contemporenous relations.
- We need to estimate a reduced-form VAR and then "restore" the parameters of the structural form.

Short résumé about SVAR models(2)

- The number of parameters in the structural form is greater than the number of estimates from the reduced form.
- So, we need additional restrictions to have an exactly identified VAR.
- What kind of restrictions exist in economic applcations? * recursive scheme restrictions (Choleski identification) * short-run restrictions * long-run restrictions * sign restrictions * explicit prior distributions in Bayesian econometrics * identification through heteroscedasticity

Impulse response functions: code

```
irf3 <- irf(var3, impulse = "consumption", response =
              c("consumption","income"), n.ahead = 20,
            cumulative = TRUE)
plot(irf3)
```



Forward error variance decomposition: code

```r
fevd(var3, n.ahead = 4)
```

```
## $consumption
##      consumption      income
## [1,]   1.0000000 0.000000000
## [2,]   0.9975504 0.002449636
## [3,]   0.9837082 0.016291810
## [4,]   0.9845215 0.015478538
##
## $income
##      consumption    income
## [1,]   0.1324366 0.8675634
## [2,]   0.1817073 0.8182927
## [3,]   0.1815947 0.8184053
## [4,]   0.2473056 0.7526944
```

Change of ordering

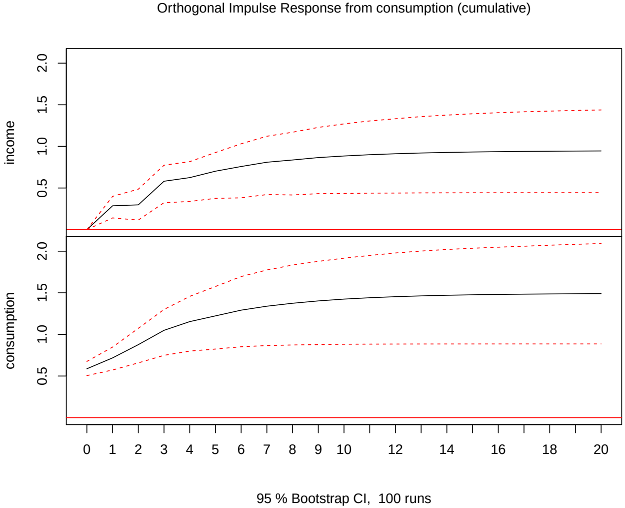
The results change if we change ordering of the variables.

```
usconsumption2 <- usconsumption[, c("income",
                                      "consumption")]
head(usconsumption2,4)
```

```
##            income consumption
## 1970 Q1  0.496540   0.6122769
## 1970 Q2  1.736460   0.4549298
## 1970 Q3  1.344881   0.8746730
## 1970 Q4 -0.328146  -0.2725144
```

```
var3a <- VAR(usconsumption2, p = 3, type = "const")
irf3a <- irf(var3a, impulse = "consumption", response =
               c("consumption","income"), n.ahead = 20,
             cumulative = TRUE) plot(irf3a)
```

IRF with alternative ordering



FEVD with alternative ordering

```
fevd(var3a, n.ahead = 4)
```

```
## $income
##          income consumption
## [1,] 1.0000000  0.00000000
## [2,] 0.9006141  0.09938587
## [3,] 0.9005221  0.09947791
## [4,] 0.8204873  0.17951270
##
## $consumption
##          income consumption
## [1,] 0.1324366   0.8675634
## [2,] 0.1421420   0.8578580
## [3,] 0.1351174   0.8648826
## [4,] 0.1321322   0.8678678
```

Short-run restrictions

- Matrix B_0 must not necessarily be lower triangular. Applying restrictions on B_i, we may try to take into account the economic reasoning.

Example: A four-variable macroeconomic model consisting of the logarithm of output, the interest rate, the logarithm of prices and the logarithm of money: $y_t = [\ln x_t, r_t, \ln p_t, \ln m_t]$. The model is:

$$\ln\left(\frac{p_t}{p_{t-1}}\right) = b_1(\ln x_t - \varepsilon_{as,t}) \qquad \text{Aggregate supply}$$

$$\ln x_t = -b_2\left(r_t - \ln\left(\frac{p_t}{p_{t-1}}\right) - \varepsilon_{is,t}\right) \qquad \text{IS equation}$$

$$\ln m_t - \ln p_t = b_3 \ln x_t - b_4 r_t - \varepsilon_{md,t} \qquad \text{Money demand}$$

$$\ln m_t = \varepsilon_{ms,t} \qquad \text{Money supply}$$

SVAR with short-run restrictions: example

$y_t = [\ln q_t, r_t, \ln p_t, \ln m_t]$

$$\begin{aligned}
\ln q_t - b_1^{-1} \ln(p_t) &= -b_1^{-1} \ln p_{t-1} + b_1^{-1} \varepsilon_{as,t} \\
b_2^{-1} \ln q_t + r_t - \ln p_t &= -\ln p_{t-1} + \varepsilon_{is,t} \\
b_3 \ln q_t - b_4 r_t + \ln p_t - \ln m_t &= \varepsilon_{md,t} \\
\ln m_t &= \varepsilon_{ms,t}
\end{aligned}$$

$$B_0 = \begin{pmatrix} 1 & 0 & -b_1^{-1} & 0 \\ b_2^{-1} & 1 & -1 & 0 \\ b_3 & -b_4 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad B_1 = \begin{pmatrix} 0 & 0 & -b_1^{-1} & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

VAR with short-run restriction: estimation

$$\upsilon_t = B_0^{-1}\varepsilon_t$$
$$E(\upsilon_t\upsilon_t') = B_0^{-1}E(\varepsilon_t\varepsilon_t')(B_0^{-1})'$$
$$\Omega = B_0^{-1}\Sigma(B_0^{-1})'$$

- But now B_0^{-1} is not necesserily so we can not make use of a Choleski decomposition.
- A special numerical algorithm (a scoring algorithm of Amisano and Giannini(1997)) is commonly used with an alternative of direct maximisation of log-likelihood.

Types of short-run restrictions (1)

- To estimate a SVAR model with short-run restrictions we can use the `SVAR()` function from the `vars` package.
- We should define A matrix (*Amat*) or/and B matrix (*Bmat*) in a model with the following notations:
  $Ay_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + \lambda + B\varepsilon_t, \qquad \varepsilon \sim (0, I_n)$

The most popular types of SR restrictions are:

1. A-model: at least $\frac{n(n-1)}{2}$ restrictions are imposed on A matrix and $B = I$.
2. B-model: at least $\frac{n(n-1)}{2}$ restrictions are imposed on B matrix and $A = I$.
3. AB-model: at least $n^2 + \frac{n(n-1)}{2}$ restrictions are imposed on A and B matrix

Types of short-run restrictions (2)

$$
\begin{aligned}
Ay_t &= A_1 y_{t-1} + \ldots + A_p y_{t-p} + \lambda + B\varepsilon_t \\
y_t &= A^{-1}A_1 y_{t-1} + \ldots + A^{-1}A_p y_{t-p} + A^{-1}\lambda + A^{-1}B\varepsilon_t \\
\upsilon_t &= A^{-1}B\varepsilon_t \\
A\upsilon_t &= B\varepsilon_t
\end{aligned}
$$

NB! So the model can be equivalently written both in terms of original variables (y_t) and in terms of reduced-form residuals.

SVAR with short-run restrictions: example

A simple IS-LM model from Pagan(1995) describes three variables: q_t (output), r_t (interest rate), m_t (real money) and can be written in the structural form as:

$$
\begin{aligned}
\upsilon_t^q &= -a_{12}\upsilon_t^r + b_{11}\varepsilon_t^{IS} && \text{IS curve} \\
\upsilon_t^r &= -a_{21}\upsilon_t^q - a_{23}\upsilon_t^m + b_{22}\varepsilon_t^{LM} && \text{inverse LM curve} \\
\upsilon_t^m &= b_{33}\varepsilon_t^m && \text{money supply rule}
\end{aligned}
$$

This system can be represented as an AB model as:

$$
\begin{pmatrix} 1 & a_{12} & 0 \\ a_{21} & 1 & a_{23} \\ 0 & 0 & 1 \end{pmatrix} \upsilon_t = \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix} \varepsilon_t
$$

For $n = 3$ the number of restriction is $n^2 + \frac{n(n-1)}{2} = 12$

SVAR example: defining Amat

```r
var_qrm <- VAR(qrm_ts, p = 4, type = "both")
amat <- diag(3)
amat[1,2] = NA
amat[2,1] = NA
amat[2,3] = NA
amat
```

```
##      [,1] [,2] [,3]
## [1,]    1   NA    0
## [2,]   NA    1   NA
## [3,]    0    0    1
```

SVAR example: defining Bmat

```r
bmat <- diag(3)
diag(bmat) = NA
bmat
```

```
##      [,1] [,2] [,3]
## [1,]   NA    0    0
## [2,]    0   NA    0
## [3,]    0    0   NA
```

```r
svar_islm <- SVAR(var_qrm, Amat = amat, Bmat = bmat)
```

SVAR example: A and B estimates

```
svar_islm[["A"]] # or svar_islm$A
```

```
##             output int_rate  money
## output      1.0000 -0.03976 0.0000
## int_rate   -0.1442  1.00000 0.7321
## money       0.0000  0.00000 1.0000
```

```
svar_islm[["B"]] # or svar_islm$B
```

```
##              output int_rate    money
## output     0.006859 0.000000 0.000000
## int_rate   0.000000 0.008762 0.000000
## money      0.000000 0.000000 0.005674
```

Impulse response functions for an AB model

Q: Why do we need to know $\hat{A}$ and $\hat{B}$ matrices?

A: To be able to calculate IRF and FEVD!

$$Ay_t = \lambda + A_1 y_{t-1} + \ldots + A_p y_{t-p} + B\varepsilon_t$$
$$y_t = A^{-1}\lambda + A^{-1}A_1 y_{t-1} + \ldots + A^{-1}A_p y_{t-p} + A^{-1}B\varepsilon_t$$
$$y_t = \mu + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \ldots + \Phi_p y_{t-p} + \upsilon_t$$

VMA representation of the reduced-form VAR(p) is:

$$y_t = \tilde{\mu} + C_0\upsilon_t + C_1\upsilon_{t-1} + C_2\upsilon_{t-2}\ldots$$

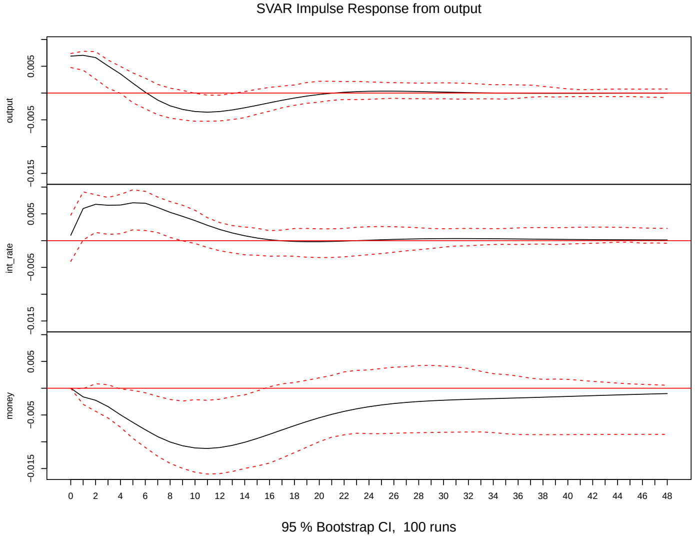
As $\upsilon_t = A^{-1}B\varepsilon_t$ then:

$$y_t = \tilde{\mu} + C_0A^{-1}B\varepsilon_t + C_1A^{-1}B\varepsilon_{t-1} + C_2A^{-1}B\varepsilon_{t-2}\ldots$$
$$y_t = \tilde{\mu} + \Psi_0\varepsilon_t + \Psi_1\varepsilon_{t-1} + \Psi_2\varepsilon_{t-2}\ldots,$$

where $\Psi_i = C_iA^{-1}B$

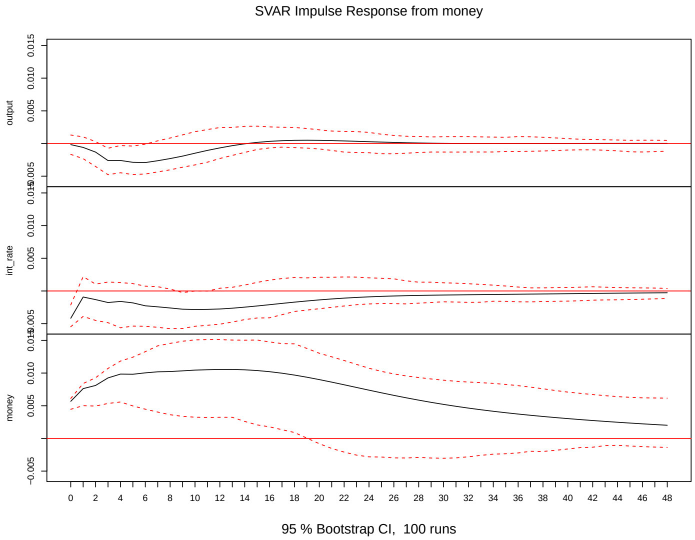
Impulse response functions: IS shock

```
irf_o <- irf(svar_islm, n.ahead = 48, impulse = "output")
plot(irf_o)
```



Impulse response functions: money supply shock

```r
irf_m <- irf(svar_islm, n.ahead = 48, impulse = "money")
plot(irf_m)
```



FEVD for IS-LM model

```r
fevd(svar_islm, n.ahead = 12)
```

```
## $output
##         output int_rate     money
##  [1,] 0.9969 0.002572 0.0005782
##  [2,] 0.9910 0.005081 0.0039255
##  [3,] 0.9291 0.057052 0.0138852
##  [4,] 0.8581 0.096026 0.0458421
##  [5,] 0.7918 0.139304 0.0689055
##  [6,] 0.7101 0.197147 0.0927925
##  [7,] 0.6325 0.255516 0.1120317
##  [8,] 0.5769 0.300168 0.1228901
##  [9,] 0.5430 0.329689 0.1273045
## [10,] 0.5257 0.346999 0.1272537
## [11,] 0.5207 0.354980 0.1243280
## [12,] 0.5232 0.356424 0.1203803
##
## $int_rate
```

Long-run restrictions: idea

- Sometimes it is desirable to impose a prior knowledge that some shocks do not have any long-run effects. This is achieved by setting the respective elements of the long-run impact matrix equal to zero.

$$
\begin{aligned}
\Psi &= \Psi_0 + \Psi_1 + \Psi_2 + \cdots = \\
&= (I - \Phi_1 - \cdots - \Phi_p)^{-1} A^{-1} B = \\
&= F A^{-1} B, \qquad \text{where } F = (I - \Phi_1 - \cdots - \Phi_p)^{-1} \\
\Psi \Psi' &= F A^{-1} B B' A'^{-1} F'
\end{aligned}
$$

Long-run restrictions: computation

$$\upsilon_t = A^{-1}B\varepsilon_t, \qquad \varepsilon \sim iid(0, I)$$
$$E(\upsilon_t\upsilon_t') = A^{-1}BE(\varepsilon_t\varepsilon_t')B'A'^{-1}$$
$$\Omega = A^{-1}BB'A'^{-1}$$
$$\Psi\Psi' = F\Omega F'$$

The estimation is easy if Ψ is assumed to be lower triangular.

$$\hat{\Psi} = \hat{F}chol(\hat{\Omega}) \qquad \hat{A}^{-1}\hat{B} = \hat{F}^{-1}\hat{\Psi}$$

NB! This procedure works only in stationary VAR models because $(I - \Phi_1 - \cdots - \Phi_p)^{-1}$ does not exist otherwise.

Literature

1. Martin, V., Hurn, S., and HarriS,D. (2013) Econometric Modelling with Time Series: Specification, Estimating and Testing, New York, Cambridge University Press
2. Lütkepohl, H., and Krätzig, M. (2004) Applied Time Series Econometrics, New York, Cambridge University Press