Applied Time Series Econometrics

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Example: Uhlig(2005)

```
library(VARsignR)
data(uhligdata)
```

The example is based on the original data set used by Uhlig (2005). The dataset contains monthly data on:

- real GDP
- GDP deflator
- commodity price index
- federal funds rate
- non-borrowed reserves
- borrowed reserves

The series that do not exist on monthly frequency were interpolated. All variables in the data set, except the FED funds rate, are the natural log of the original data series times 100.

Uhlig(2005): goal and assumptions

- The goal was to analyse the effect of an unanticipated monetary policy shock (one s.d. in i_t) on y_t and inflation.
- Uhlig (2005) assumed that an unanticipated innovation in the FED's policy rate:

```
* does not decrease the FED's policy rate for several
* does not increase commodity prices and inflation for

* does not increase non-borrowed reserves for several m
```

The 1st and the 6th variable of the model (real GDP and total reserves) remain unrestricted.

Constraints and estimation

$$Y_t = (y_t, defl_t, p_t, i_t, rnb_t, tr_t)$$

y	$defl_t$	p_t	i_t	rnb_t	tr_t
$\gtrless$	≤ 0	≤ 0	≥ 0	≤ 0	$\gtrless$

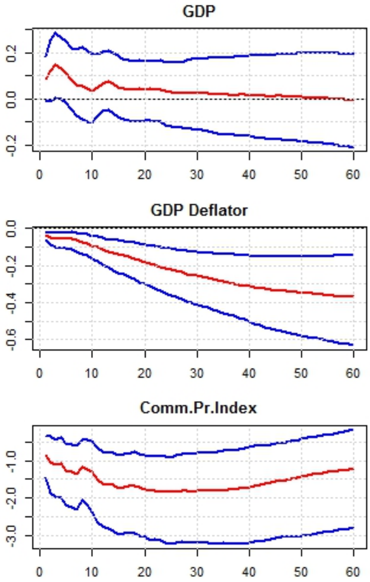
```
constr <- c(+4,-3,-2,-5)
model1 <- uhlig.reject(Y = uhligdata, nlags = 12,
                       nkeep = 1000, KMIN = 1,
                       KMAX = 6, constrained = constr,
                       constant = FALSE, steps = 60)
```

NB! Using other functions from the same package is also possible for sign identification.

Impulse response functions

```
irfs1 <- model1$IRFS
vl <- c("GDP","GDP Deflator","Comm.Pr.Index",
        "Fed Funds Rate","NB Reserves", "Total
        Reserves")
irfplot(irfdraws = irfs1, type = "median", labels = vl,
        save = FALSE, bands = c(0.16, 0.84), grid = TRUE,
        bw = FALSE)
```

Impulse response functions: graphs



Forward error variance decomposition

```
fevd1 <- model1$FEVDS
fevd.table <- fevdplot(fevd1, table = TRUE, label = vl,
                       periods = c(1,10,20,30,40,50,60))
print(fevd.table)
```

```
##       GDP GDP Deflator Comm.Pr.Index Fed Funds Rate NB Re
## 1   7.72          7.11          6.97          10.16
## 10  8.91          8.26          7.67          12.82
## 20 11.68          9.56          7.97          13.26
## 30 13.07         10.15          8.23          13.21
## 40 12.59         10.19          8.71          13.21
## 50 12.30         10.36          9.23          13.09
## 60 12.16         10.37          9.54          12.93
##    Total\n       Reserves
## 1                   7.77
## 10                  8.17
## 20                  9.37
## 30                  9.65
```

Sign restrictions: drawbacks

- Sign restrictions cannot generate a unique set of impulse responses (it is a set identification).
- Median line is not an IRF from a particular model.
- “The prior is uninformative about the angle of rotation, it can be highly informative for the objects about which the researcher intends to form an inference, namely the impulse response function. ... The users of these methods can in some cases end up performing hundreds of thosands of calculations, ostensibly analyzing the data, but in fact doing nothing more than generating draws from a prior distribution that they never acknowledged assuming”. (Baumeister and Hamilton, 2015)

Stability and unit roots

VAR in reduced form:

$$y_t = \mu + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + \upsilon_t, \qquad \upsilon_t \sim iid(0, \Omega)$$

The process is stable if:

$$|I - \Phi_1 z - \cdots - \Phi_p z^p| \neq 0 \text{ for } |z| \leq 1$$

If the polynomial has a unit root (i.e. the determinant is 0 for $z = 1$) then some or all variables are integrated.

Cointegration: idea(1)

- Assume that all variables in y_t are at most I(1). If the I(1) variables have common trends, it is possible that there are linear combinations of them are I(0). In that case they are cointegrated.
- Occasionally it is convenient to consider systems with both I(1) and I(0) variables. Then the concept of cointegration is extendend by calling any linear combinatoin that is I(0) a cointegration relation, although this terminology is not in the spirit of the original definition.
- The model on the previous slide is general and acommodates perfectly the variables with stochastic trends but now we want the cointegration relations appear explicitly.

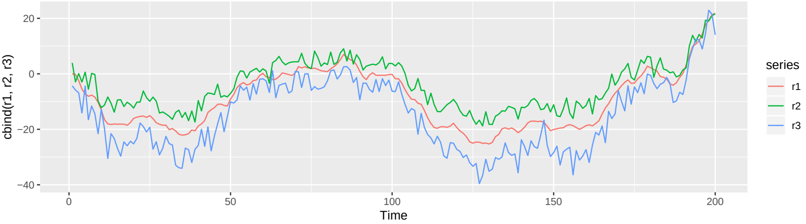
Cointegration: idea(2)

Simulate series:

$$\Delta r_{1,t} = 0.6\Delta r_{1,t-1} + u_{1,t}$$
$$r_{2,t} = 3 + 0.8 r_{1,t} + u_{2,t}$$
$$r_{3,t} = -4 + 1.2 r_{1,t} + u_{3,t},$$

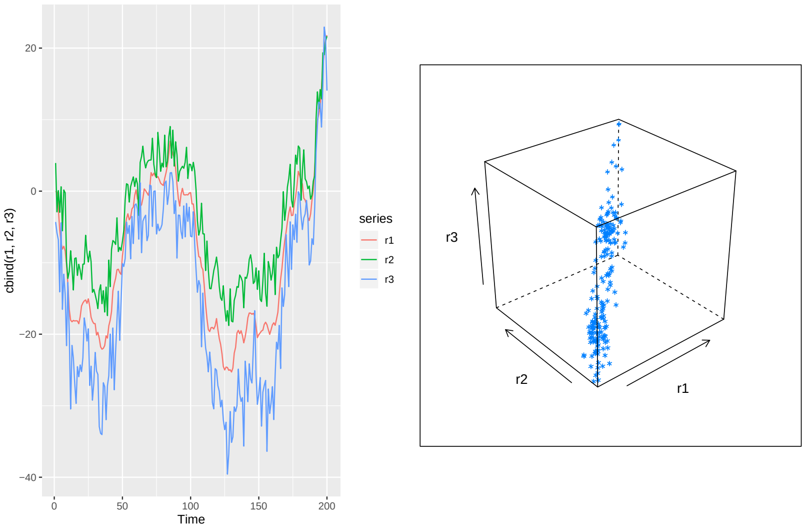
where $u_{i,t} \sim N(0, \sigma_i^2)$.

All three series are clearly nonstationary I(1).



Cointegration: idea(3)

Cointegrated I(1) series



Alternative representation

Let's remember the alternative form of AR(p) model useful for the ADF-test (slide 28 from Practice Session 8):

$$
\begin{aligned}
y_t &= \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \nu_t \\
y_t - y_{t-1} &= \mu + (\phi_1 + \phi_2 + \cdots + \phi_p - 1) y_{t-1} - \\
&\quad - (\phi_1 + \cdots + \phi_p) y_{t-1} + \phi_1 y_{t-1} + (\phi_2 + \cdots + \phi_p) y_{t-2} - \\
&\quad - (\phi_2 + \cdots + \phi_p) y_{t-2} + \phi_2 y_{t-2} + (\phi_3 + \cdots + \phi_p) y_{t-3} - \\
&\quad - (\phi_3 + \cdots + \phi_p) y_{t-3} + \phi_3 y_{t-3} + (\phi_4 + \cdots + \phi_p) y_{t-4} - \\
&\quad \ldots \\
&\quad - (\phi_{p-1} + \phi_p) y_{t-(p-1)} + \phi_{p-1} y_{t-(p-1)} + \phi_p y_{t-p} + \nu_t
\end{aligned}
$$

The equation can be written as:

$$
\Delta y_t = \mu + \beta y_{t-1} + \zeta_1 \Delta y_{t-1} + \zeta_2 \Delta y_{t-2} + \cdots + \zeta_{p-1} \Delta y_{t-(p-1)},
$$

where $\beta = \sum_{i=1}^{p} \phi_i - 1$, and $\zeta_j = -\sum_{i=j+1}^{p} \phi_i$.

We can do the same thing with vector endogenous variable.

VECM(p-1)

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_{p-1} \Delta y_{t-p+1} + \upsilon_t,$$

where $\Pi = -(I - \Phi_1 - \cdots - \Phi_p)$ and $\Gamma_i = -(\Phi_{i+1} + \cdots + \Phi_p)$ for $i = 1, \ldots, p-1$.

Δy_{t-j} is $I(0)$ for $j = 0, \ldots, p-1 \Rightarrow \Pi y_{t-1}$ must be $I(0)$ as well.

- If the VAR(p) process has unit roots, then matrix Π is singular $rank(\Pi) = r < n$.
- $\Pi = \alpha\beta'$, where $rank(\alpha) = rank(\beta) = r$ and α and β are matrices of full rank (skeleton decomposition of a matrix).

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Cointegration relations

$$\Pi y_{t-1} \text{ is } I(0)$$
$$\alpha\beta' y_{t-1} \text{ is } I(0) \qquad \times (\alpha'\alpha)^{-1}\alpha'$$
$$\beta' y_{t-1} \text{ is } I(0) \Rightarrow$$

- $\beta' y_{t-1}$ contains cointegrating relations
- There are $r = rank(\Pi)$ lineary independent cointegrating relations among the components of y $\Rightarrow$ r is a cointegrating rank of the system.
- β is a cointegration matrix, α is a loading matrix
- α and β are not unique as $\Pi = \alpha B(\beta B'^{-1})'$ for any nonsingular $(r \times r)$ matrix B. Some nonsample information is required to identify them uniquely.

Example of Π decomposition

$$\Pi y_{t-1} = \alpha\beta' y_{t-1} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{21} & \beta_{31} \\ \beta_{12} & \beta_{22} & \beta_{32} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \\ y_{3,t-1} \end{pmatrix} =$$

$$= \begin{pmatrix} \alpha_{11} ec_{1,t-1} + \alpha_{12} ec_{2,t-1} \\ \alpha_{21} ec_{1,t-1} + \alpha_{22} ec_{2,t-1} \\ \alpha_{31} ec_{1,t-1} + \alpha_{32} ec_{2,t-1}, \end{pmatrix}$$

where cointegration relations are:

$$ec_{1,t-1} = \beta_{11} y_{1,t-1} + \beta_{21} y_{2,t-1} + \beta_{31} y_{31,t-1}$$
$$ec_{2,t-1} = \beta_{12} y_{1,t-1} + \beta_{22} y_{2,t-1} + \beta_{31} y_{32,t-1}$$

Particular cases

- **Case 1.** If all variables are I(0), then $r = n$ and the process is stationary.
- **Case 2.** If $r = 0$, then the term Πy_{t-1} disappears in the VECM form and Δy_t has a stable VAR representation.
- **Case 3** All variables but one are $I(0), r = n - 1$
- **Case 4** $n - r$ unrelated I(1) variables and r $I(0)$ variables.

Rank estimation: idea

- Hence, the rank of matrix Π is crucial to determine the number of cointegration vectors.
- Johansen's procedure (Johansen, 1988) is based on a fact that the rank of a matrix is equal to the number of its non-zero eigenvalues.
- If an estimate of Π is known, it is possible to find its eigenvalues $\lambda_i \quad i = 1, \cdots, n$ and order them: $\lambda_1 > \lambda_2 > \cdots > \lambda_n$.

Literature

1. Lütkepohl, H., and Krätzig, M. (2004) Applied Time Series Econometrics, New York, Cambridge University Press
2. Martin, V., Hurn, S., and HarriS,D. (2013) Econometric Modelling with Time Series: Specification, Estimating and Testing, New York, Cambridge University Press
3. Pfaff, B. (2008) Analysis of Integrated and Cointegrated Time Series with R, New York, Springer