Applied Time Series Econometrics

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December 3, 2019

SVAR with long-run restrictions: example

The most known example of the SVAR model with long-run restrictions is from the study by Blanchard and Quah (1989). Their model is a bivariate VAR for ΔQ (growth rate of output) and u_t (unemployment rate).

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta Q_t \\ u_t \end{pmatrix} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} + A_1 \begin{pmatrix} \Delta Q_{t-1} \\ u_{t-1} \end{pmatrix} + \cdots + \\ + A_p \begin{pmatrix} \Delta Q_{t-p} \\ u_{t-p} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_t^{as} \\ \varepsilon_t^{ad} \end{pmatrix}$$

Assmption: aggregate demand shocks have only transitory effects on the level of output. It means that the long-run effect of ε_t^{ad} on ΔQ is equal to zero:

$$\Psi(1,2) = 0$$

2 variables $\Rightarrow$ Ψ has dimension 2×2.The assumption makes Ψ lower triangular.

Blanchard and Quah(1989) model

```r
var_bq <- VAR(bq_ts, p = 8, type = "const")
svar_bq = BQ(var_bq)
svar_bq[["B"]] # or svar_bq$B
```

```
##             dQ          u
## dQ 0.07467792 -0.9296093
## u  0.21980221  0.2082399
```

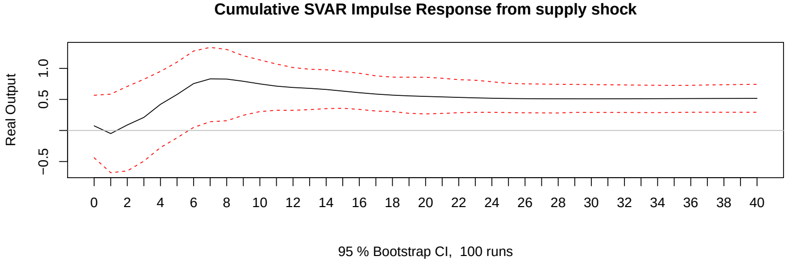
```r
svar_bq[["LRIM"]] # or svar_bq$LRIM
```

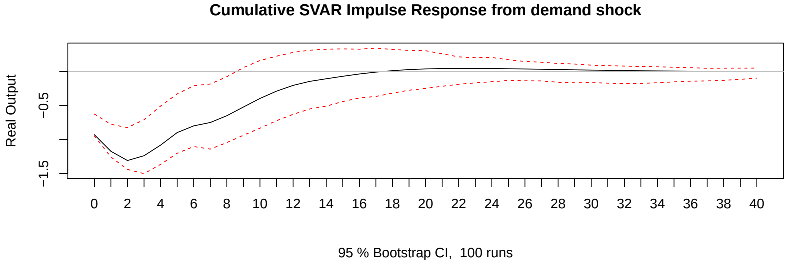
```
##              dQ        u
## dQ 0.518604141 0.000000
## u  0.007847413 4.043684
```

IRFs for BQ(1989) model: code

```
irf_bq_QQ <- irf(svar_bq, n.ahead = 40, cumulative =
                   TRUE,response = "dQ",impulse ="dQ")
irf_bq_uQ <- irf(svar_bq, n.ahead = 40, cumulative =
                   TRUE,response = "dQ",impulse = "u")
par(mfrow = c(2,1))
plotIRF(irf_bq_QQ, vlabels= "Real Output",
        slabels="supply shock")
plotIRF(irf_bq_uQ, vlabels = "Real Output",
        slabels= "demand shock")
```

IRFs for BQ(1989) model: graph

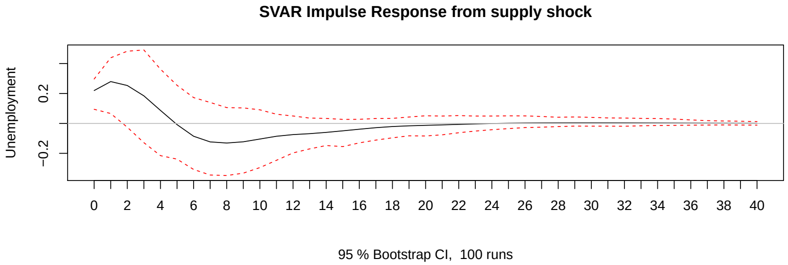


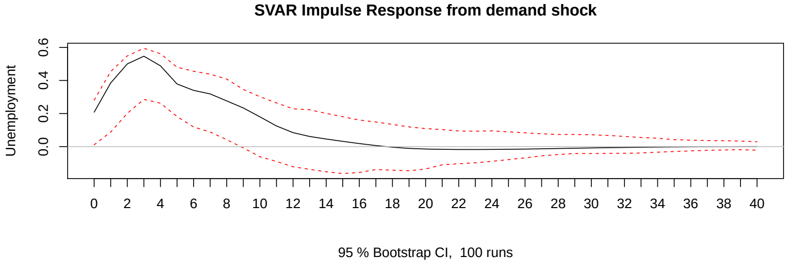


IRFs for BQ(1989) model: code(2)

```
irf_bq_Qu <- irf(svar_bq, n.ahead = 40,
                 response = "u",impulse ="dQ")
irf_bq_uu <- irf(svar_bq, n.ahead = 40,
                 response = "u",impulse = "u")
par(mfrow = c(2,1))
plotIRF(irf_bq_Qu, vlabels = "Unemployment",
        slabels = "supply shock")
plotIRF(irf_bq_uu, vlabels = "Unemployment",
        slabels = "demand shock")
```

IRFs for BQ(1989) model: graph(2)





FEVD for Blanchard-Quah model

```
fevd_bq <- fevd(svar_bq, n.ahead = 12)
fevd_bq
```

```
## $dQ
##                dQ         u
##  [1,] 0.006411946 0.9935881
##  [2,] 0.022255946 0.9777441
##  [3,] 0.040914220 0.9590858
##  [4,] 0.054972433 0.9450276
##  [5,] 0.092693239 0.9073068
##  [6,] 0.110173642 0.8898264
##  [7,] 0.132653559 0.8673464
##  [8,] 0.136643574 0.8633564
##  [9,] 0.135503117 0.8644969
## [10,] 0.134572642 0.8654274
## [11,] 0.134098229 0.8659018
## [12,] 0.133771206 0.8662288
##
```

Sign restrictions (1)

A structural VAR model can be specified as:

$$B_0 y_t = \lambda + B_1 y_{t-1} + \cdots + B_p y_{t-p} + \varepsilon_t, \qquad \varepsilon_t \sim iidN(0, \Sigma), \quad (1)$$

where Σ is diagonal. Reduced-form VAR for the model (1) is:

$$y_t = \mu + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + u_t, \qquad u_t \sim iidN(0, \Omega)$$

Recursive identification implied that:

- B_0 is a lower triangular matrix,
- Σ is identity matrix.

With these assumptions we obtained:

$$\upsilon_t = B_0^{-1} \varepsilon_t$$
$$E(\upsilon_t \upsilon_t') = B_0^{-1} E(\varepsilon_t \varepsilon_t')(B_0^{-1})'$$
$$\Omega = B_0^{-1} (B_0^{-1})'$$

Sign restrictions (2)

Q: What if the structural model was different?

Alternative specification:

$$\tilde{B}_0 y_t = \tilde{\lambda} + \tilde{B}_1 y_{t-1} + \cdots + \tilde{B}_p y_{t-p} + \omega_t, \qquad \omega_t \sim iidN(0, I), \quad (2)$$

where $\tilde{B}_i = QB_i, \tilde{\lambda} = Q\lambda$ and Q is an orthogonal matrix: $Q' = Q^{-1}$ so that $QQ' = Q'Q = I$.

Write the reduced-form model for (2):

$$QB_0 y_t = Q\lambda + QB_1 y_{t-1} + \cdots + QB_p y_{t-p} + \omega_t$$
$$B_0 y_t = \lambda + B_1 y_{t-1} + \cdots + B_p y_{t-p} + \underbrace{Q'\omega_t}_{\varepsilon_t}$$
$$y_t = \mu + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + u_t,$$

where $u_t = (QB_0)^{-1}\omega_t, \quad \Phi_i = B_0^{-1}B_i, \quad \mu = B_0^{-1}\lambda$

Sign restrictions (3)

Find covariance matrix of new reduced-form residuals:

$$
\begin{aligned}
E(u_t u_t') &= (QB_0)^{-1} E(\omega_t \omega_t')((QB_0)^{-1})' \\
&= B_0^{-1} Q' Q (B_0^{-1})' = B_0^{-1} (B_0^{-1})' = \Omega
\end{aligned}
$$

The baseline and an alternative model has the same reduced-form representaton, so they are observationally equvalent with $\varepsilon_t = Q'\omega_t$.

we cannot discriminate ω_t with respect to ε_t.

Sign restrictions: example

In case of two variables the matrix Q can be written in a simple way:

$$Q = \begin{pmatrix} cos\gamma & -sin\gamma \\ sin\gamma & cos\gamma \end{pmatrix} \quad 0 \leq \gamma \leq \pi$$

$$\omega_t = \begin{pmatrix} cos\gamma & -sin\gamma \\ sin\gamma & cos\gamma \end{pmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} = \begin{pmatrix} \varepsilon_{1t} cos\gamma - \varepsilon_{2t} sin\gamma \\ \varepsilon_{1t} sin\gamma + \varepsilon_{2t} cos\gamma \end{pmatrix}$$

$$\begin{aligned} \tilde{B}_0 &= QB_0 = \\ &= \begin{pmatrix} cos\gamma & -sin\gamma \\ sin\gamma & cos\gamma \end{pmatrix} \begin{pmatrix} b_{0,1,1} & 0 \\ b_{0,2,1} & b_{0,2,2} \end{pmatrix} \\ &= \begin{pmatrix} b_{0,1,1} cos\gamma - b_{0,2,1} sin\gamma & -b_{0,2,2} sin\gamma \\ b_{0,1,1} sin\gamma + b_{0,2,1} cos\gamma & b_{0,2,2} cos\gamma \end{pmatrix} \end{aligned}$$

Sign restrictions: idea(3)

- If $\gamma = 0$, then $\tilde{B}_0$ is lower triangular (y_1 is "more exogenous" than y_2).
- If $\gamma = \frac{\pi}{2}$, then $\tilde{B}_0$ is upper triangular (y_2 is "more exogenous" than y_1).
- Other structural shocks are identified by choosing alternative values of γ within the range $0 \leq \gamma \leq \pi$.
- Sign restriction approach is an approach of generating alternative models using the orthonormal rotation and selecting those models that generate impulse responses consistent with economic theory.

Sign restriction algorithm

Step 1. Estimate a VAR and construct VMA parameters, C_i, compute B_0^{-1} so that $B_0^{-1}(B_0^{-1})' = \Omega$.

Step 2. Draw a value of γ from $0 \leq \gamma \leq \pi$ and compute $Q, \tilde{B}_0, \tilde{\Psi}_i$.

Step 3. Compute a finite number of impulse responses (for example, IRF for 4 periods for quarterly data).

Step 4. If all IRF have the correct sign, select the model, otherwise discard it.

Step 5. Repeat steps 2 to 4 and generate other models that satisfy the restrictions.

Literature

1. Lütkepohl, H., and Krätzig, M. (2004) Applied Time Series Econometrics, New York, Cambridge University Press
2. Martin, V., Hurn, S., and HarriS,D. (2013) Econometric Modelling with Time Series: Specification, Estimating and Testing, New York, Cambridge University Press