50.2) Proposition: For TELCV), il V = null T @ range T, then T must be diagonalizable. Counterexample: If V is not brite dimension, then Trannot be diagonalisable. Let V= 1= . Let the stordard basis for V be P, Pe, ... e.g. P, = (1,0,0,...) let vev: v = a, l, + 02 t2 + 02 t2 + aq tq + Let T= a2 (3+ a4 (4+... Hence no! T = Span (P1, P2) range T = span (t , (t , ...) hull $T \oplus range T = V$.
But range T is infinite dimensional $\Rightarrow V$ is infinite dimensional $\Rightarrow T$ as another diagonalized. 50.1) Suppose TELCV) is alragonalizable Then let Pi,..., In be the basis such that MCT?
Hence Pi,..., Pn one eigenvectors of T. is diagonal, Tex= xiek for all k=1,...,n.

We know that Pi, ..., Pr are linearly independent. For the vectors TPx = xx Px where xx =0, Px = range T. Alternaturely, Por vectors TPx = xx Px where xx=0, Px = noll T.

Hence all vectors to ..., In can be partitioned into two sets: range T and null T.

No vector in Pi, ..., Pin exists In both range Tond null T.

Since Pi, ..., Pin Brim a basis in V, this also means that

tange T null T = 803.

Hence we showed that ep..., en which toms a bours of V con be partitioned into ranger and null T, with range TA null T = 20%

> V= null T & roge T.

5C.B) TELCES) dim E(B, T) = ct => there exists a linearly independent list of vectors T-vk = 8-vk for k=1,...,4. Suppose T-2I is not invertible, then 2 is an eigenvalue. Let us be the eigenvalue corresponding to 2. Herce vs is independent of vi,..., va, thus vi,..., us Borm a basis of IF5 Using the fundamental thin of linear maps, dim IFE = dim range T+dim null T 5 = dim range T+dim null T. Since Vi,..., Us are all eigenvectors of T with non-zoo eigenvalues, and vi,..., Us also forms the basis of IES, NUILT = 803 Herce V= E(8,T)+E(2,T) > T does not have eigendure with value 6 and T-6I is injective. The same argument applies for the cone where 6 is an eigenvalue of T, instead of 2. Heree effer T-2I or T-67 is invertible 5C.14) Tele(6) s.t. I-41, & where Tois = 60, and T does not have a diagonal matrix. Let T (x,y, 2) = (6x, 7y, 62+y) M(T) = 6 7 6 アセニアナ $6x = \lambda R \Rightarrow 6$ can be an eigenvalue $74 = \lambda y \Rightarrow 7$ can be an eigenvalue. · 62+4=>2. y= (1-6)z 1 1 1 1 ,s not diagonalisable since there are only 2 experied as of T.

8C.8) TESCV). TIS invertible it it constant term in minimal polynomial of TIS

Pt. Assuming that T is invertible,

By lemma 5.27, Thust how an upper triangular matrix w.r.t some basis, Since T is invertible, by lemma 5.20, the entires along the diagonals of the matrix must be non-zero.

Flore there must exist dim V eigenvectors for T, since by lemma 5.32, the entires in the diagonals of the upper triangular representation of T are the eigenvalues

By lemma 6.49, the eigenvalues must be the soots of the minimal polynomial.

Since none of the eigenvalues of V are non-zero, zero carnot be a root of the minimal polynomial. Thus there must exist a constant term in the minimal polynomial so zero cannot be a root

Now, assume that the minimal polynomical has a non-zero constant term. Then by the Purdamental theorem of algebra, there exists a soot in the minimal

This pot annot be 0 since the minimal polynomial has a nonzero contact term. Since the roots of the minimal polynomial are the eigenvalues of T, and 0 is not an eigenvalue of T, $null <math>T=\xi 03$ and T is invertible

Q1) Example of
$$T \in L(C^3)$$
 whose minimal polynomial is $(7.42)^2$.

$$T(x,y,z) = (-2x,-2y,-2y-2z)$$

$$(T+2)^2 = (T+2)(T+2+2y)$$

$$= (T+2)(T+2+2y)$$

$$= (T+2)((-2x,-2y,-2y-2z)+(2x,2y,2z))$$

$$= (T+2)((0,0,-2y)+(0,0,-4y)+(0,0,-4y)$$

$$= (0,0,0)$$

$$T(x,y,z) = (-2x,-2y,-2y-2z)$$
has the minimal polynomial $(7.42)^2$

c) a) eigenvalues:
$$-2$$
b) $i, -i, 3$

$$T = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 3 \end{bmatrix}$$

$$T = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

$$Tv = \lambda v$$

$$T \begin{bmatrix} \frac{2}{3} & \frac{2}{3} &$$

$$3C = \lambda C \implies \text{eigenvalues} : (2)$$

$$3d = \lambda d$$
Characteristic polynomial: $(2-2)(2-3)^3 = 0$







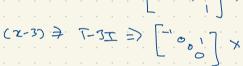


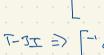
 $(z-3)^2 \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$

 $(x-3)^2(x-2) \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

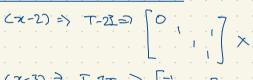












 $(\chi-2)(\chi-3) \rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} X$





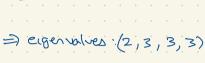














Cx-3) (x-2) (5 minimal polynomial



