



Overview of my research and related software packages

Khalil Al Handawi, PhD

Technical presentation

June 13, 2022



About me

- Khalil Al Handawi (*he/him/il*)
 - Experience with simulation based design
 - Experience with scientific computing
 - Experience with optimization and simulation research
- Born and raised in Abu Dhabi, United Arab Emirates
- Originally from Aleppo, Syria
- Moved to Montréal in 2017



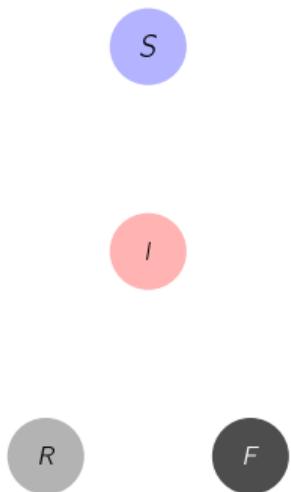
Optimization of public health models using artificial life models



Background: epidemiological models

What are compartmental epidemiological models?

S susceptible I infected R recovered F fatality



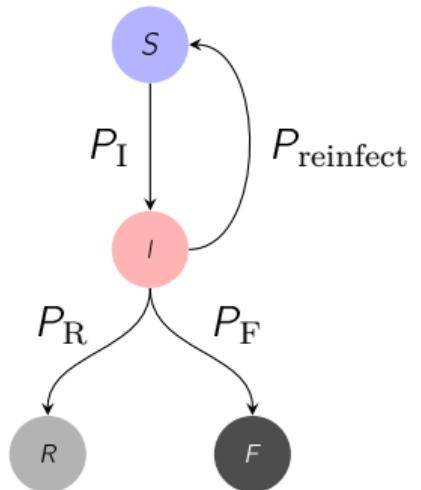


Background: epidemiological models

What are compartmental epidemiological models?

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- Described by a *stochastic* process

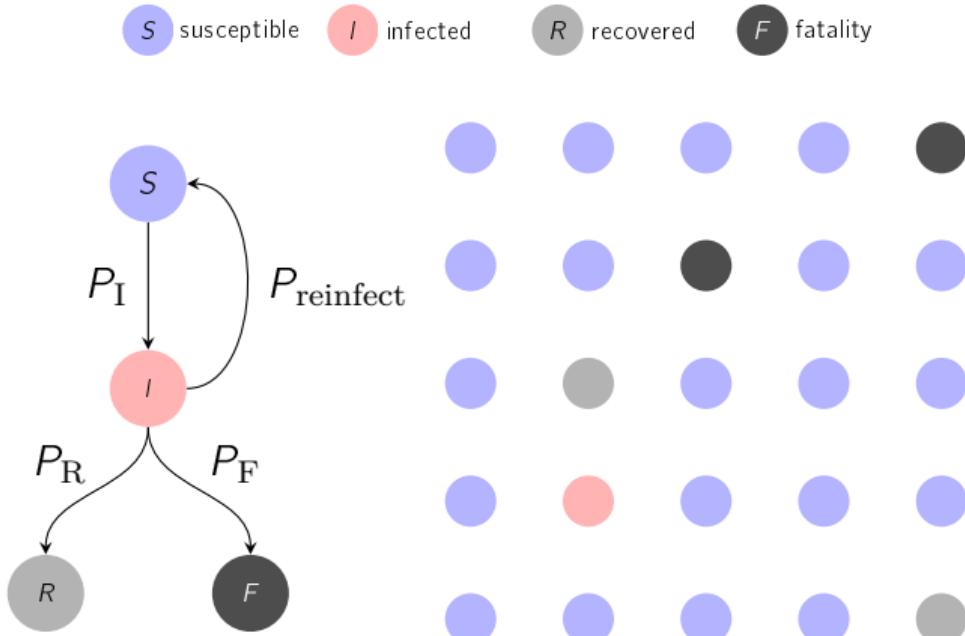




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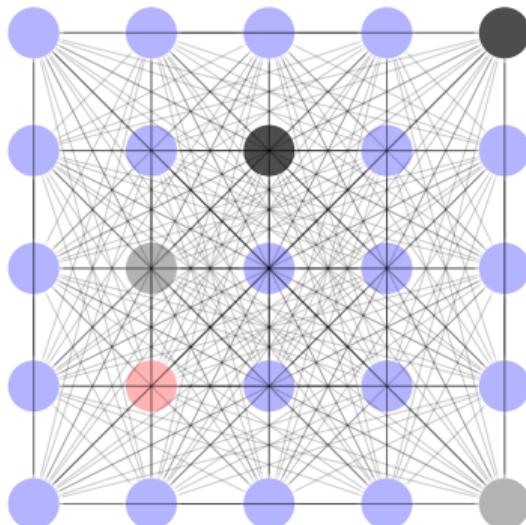
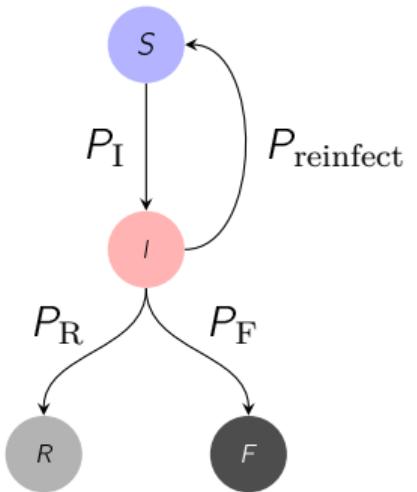


Background: epidemiological models

What are compartmental epidemiological models?

- Described by a *stochastic* process
- Assumes *homogenous* interaction

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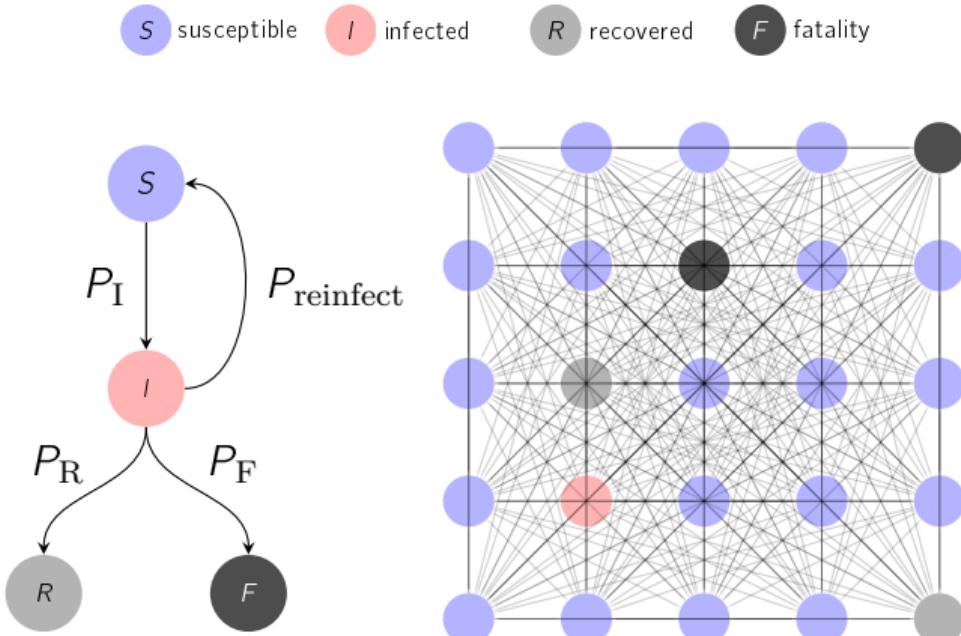




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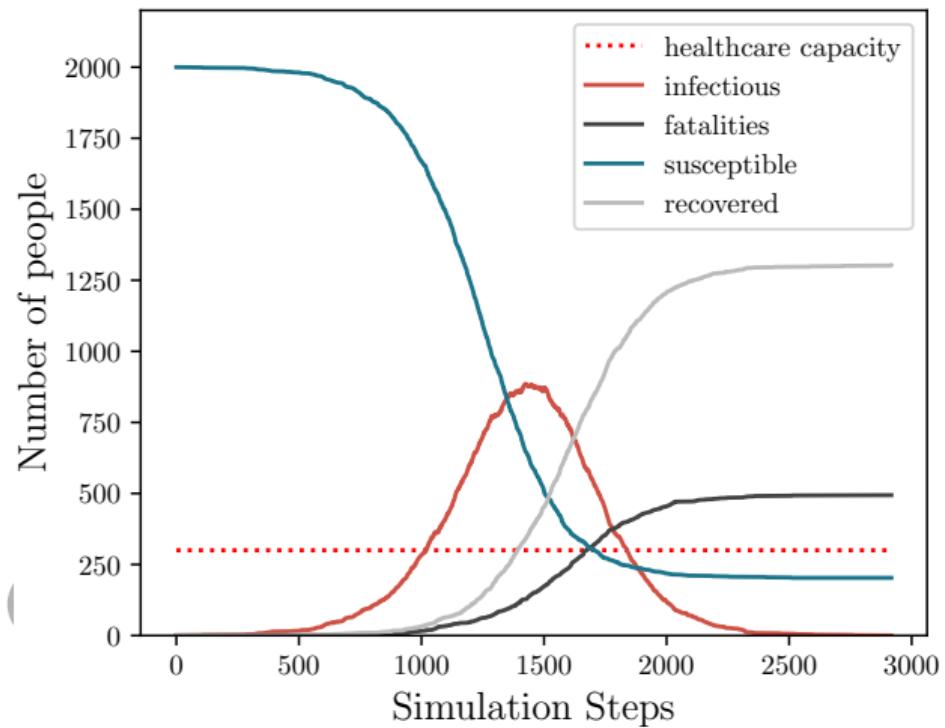
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$$\frac{dS}{dt} = -\frac{\beta IS}{N},$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I,$$

$$\frac{dR}{dt} = \gamma I,$$

where $N = S + I + R$, β controls infection spread, and γ controls recovery rate



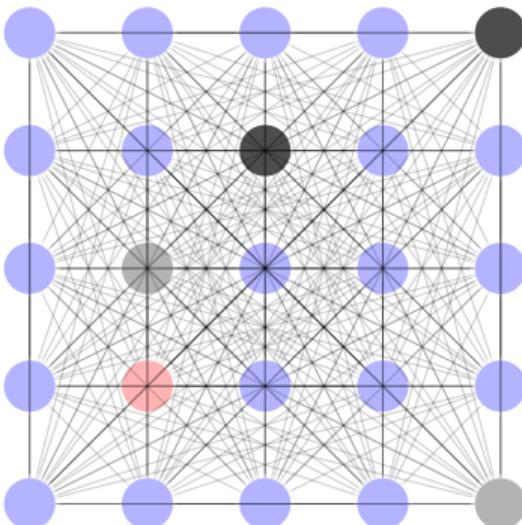
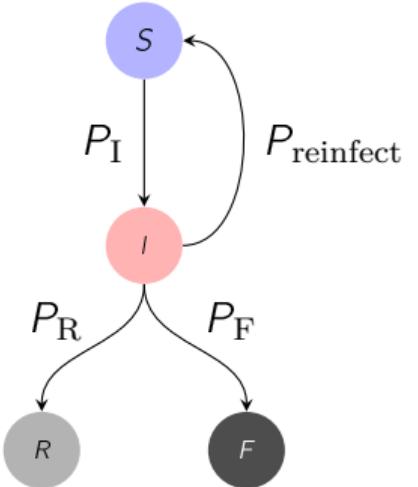


Background: epidemiological models

What are compartmental epidemiological models?

- ✓ Analytical solutions are available

S susceptible I infected R recovered F fatality



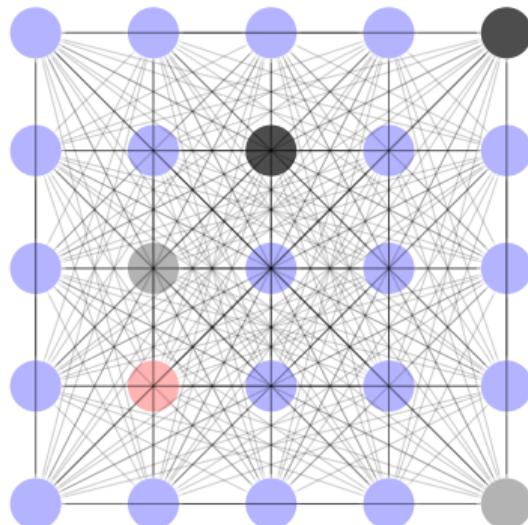
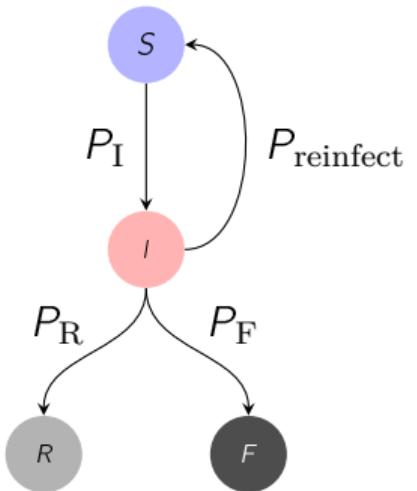


Background: epidemiological models

What are compartmental epidemiological models?

- ✓ Analytical solutions are available
- ✓ Captures large-scale population dynamics

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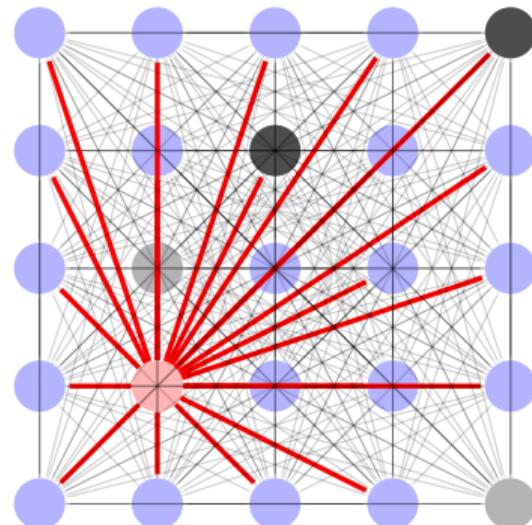
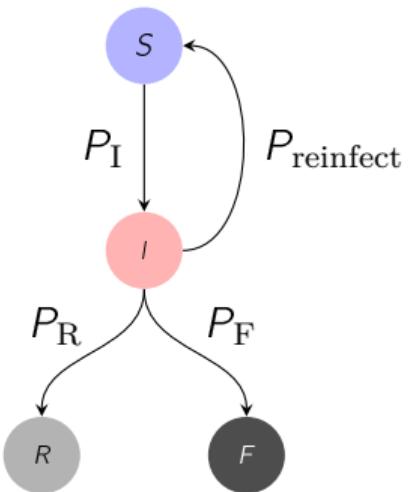


Background: epidemiological models

What are compartmental epidemiological models?

- ✓ Analytical solutions are available
- ✓ Captures large-scale population dynamics
- ✗ Does not account for *geography*

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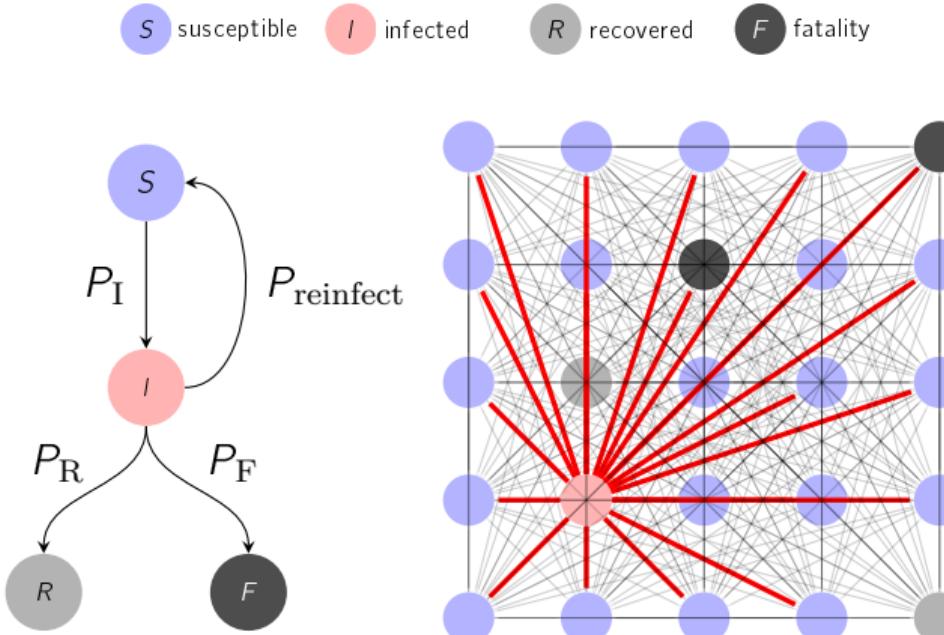




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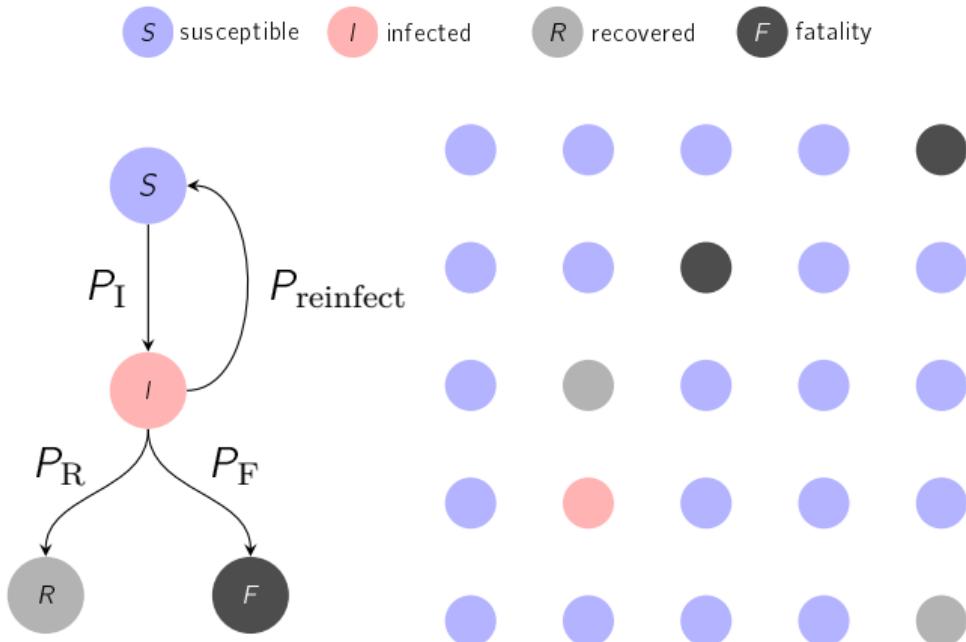
- ✓ Analytical solutions are available
- ✓ Captures large-scale population dynamics
- ✗ Does not account for *geography*
- ✗ Cannot model effect of intervention policies





Background: epidemiological models

What are agent-based epidemiological models?



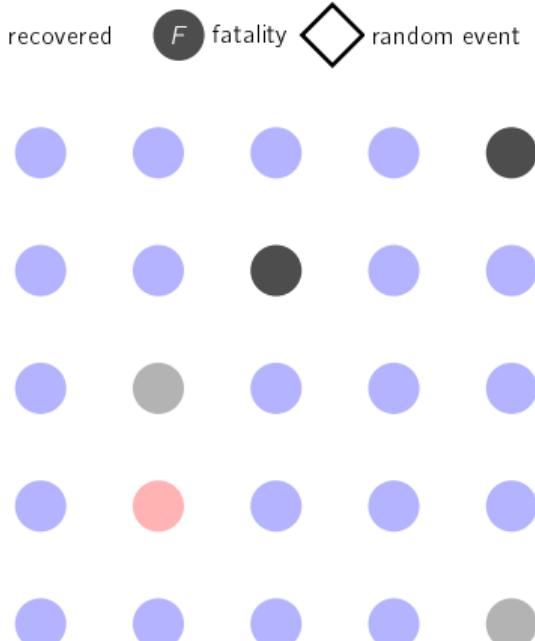
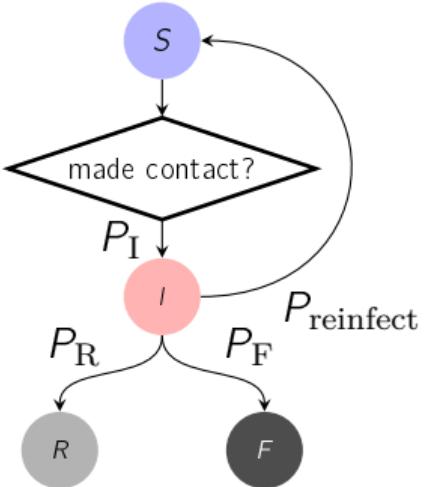


Background: epidemiological models

What are agent-based epidemiological models?

- Stochastic process

susceptible infected recovered fatality random event



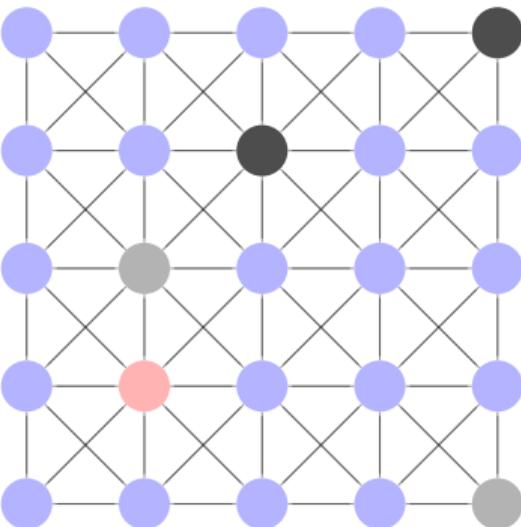
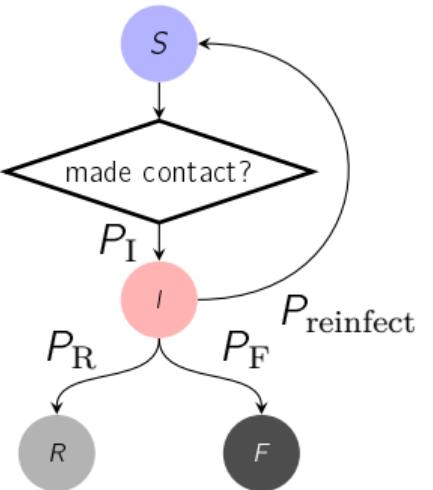


Background: epidemiological models

What are agent-based epidemiological models?

- Stochastic process
- Assume *heterogenous* interaction

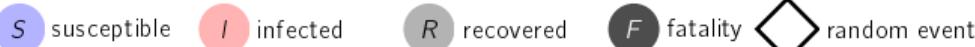
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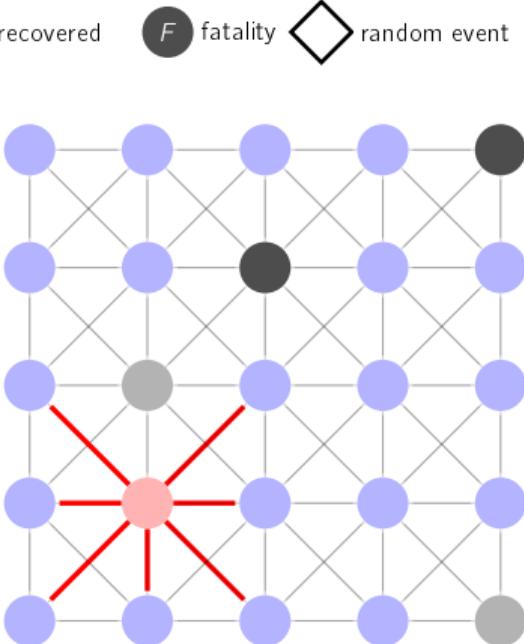
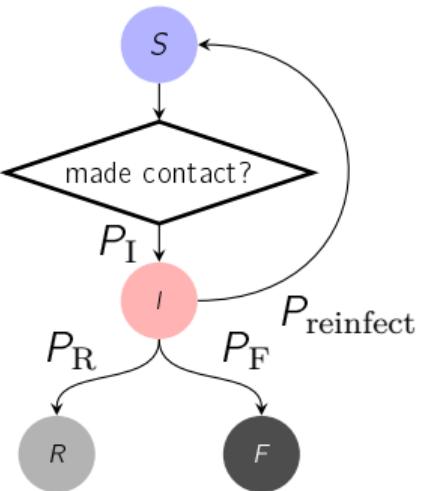


Background: epidemiological models

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Background: epidemiological models

What are agent-based epidemiological models?

Realization 1

- *Stochastic* process
- Assume *heterogenous* interaction
- Stochastic response



Background: epidemiological models

What are agent-based epidemiological models?

Realization 2

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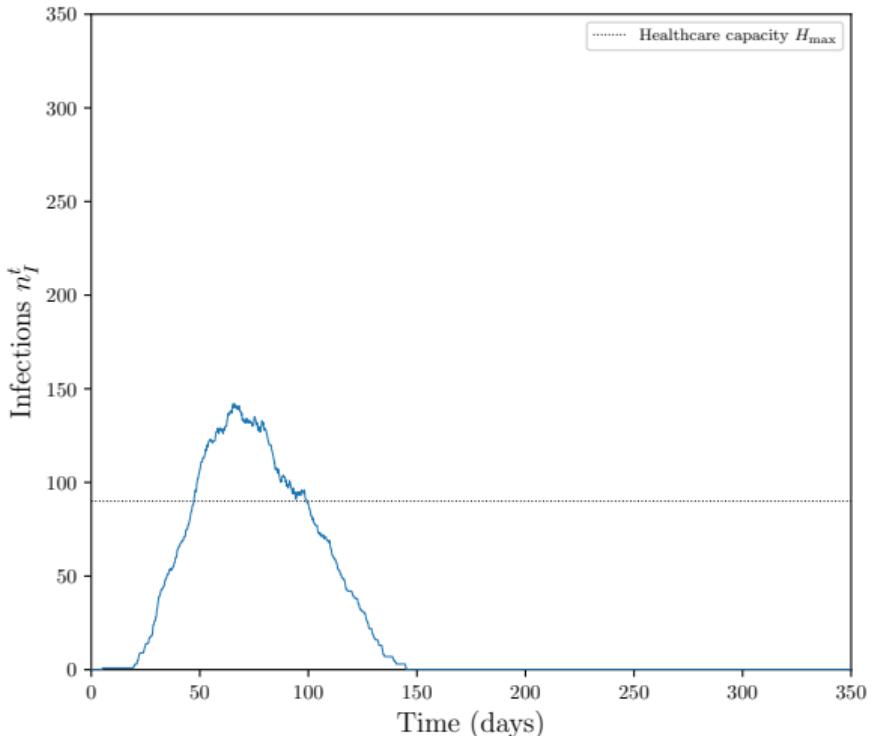


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Realization 1



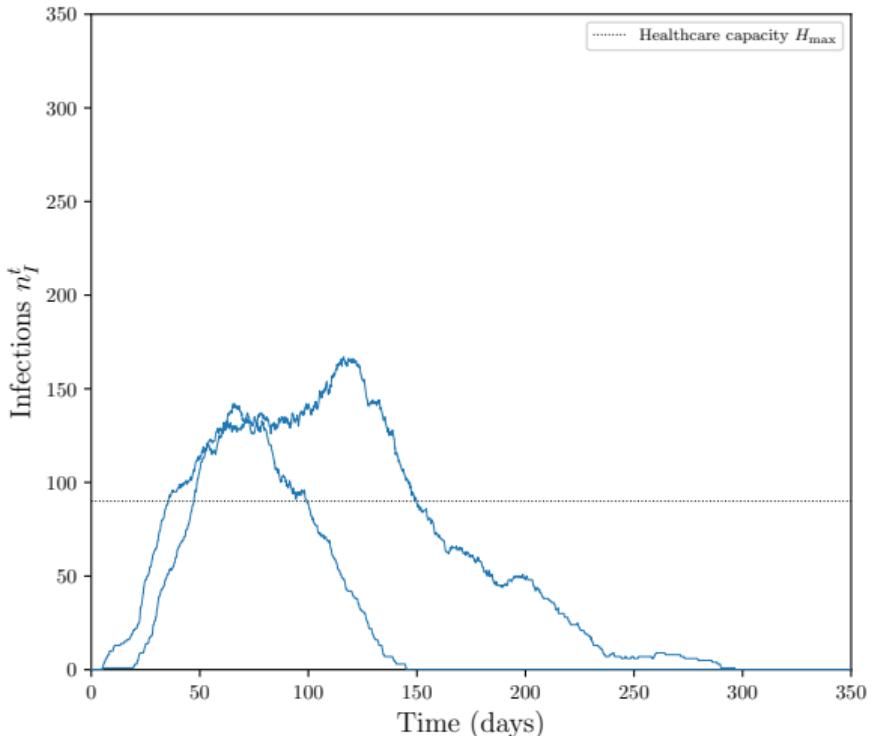


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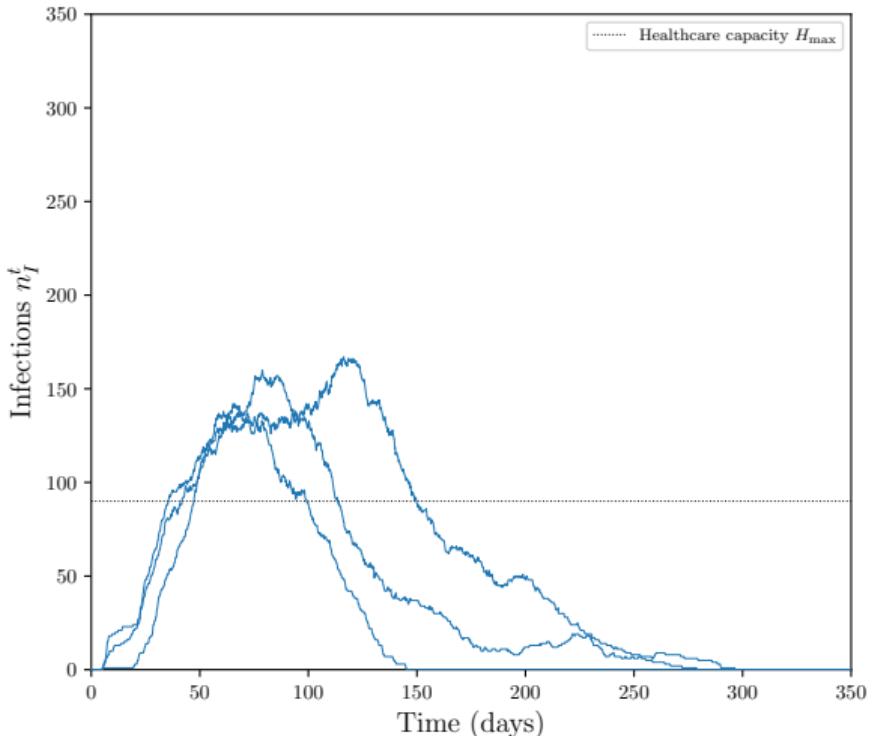




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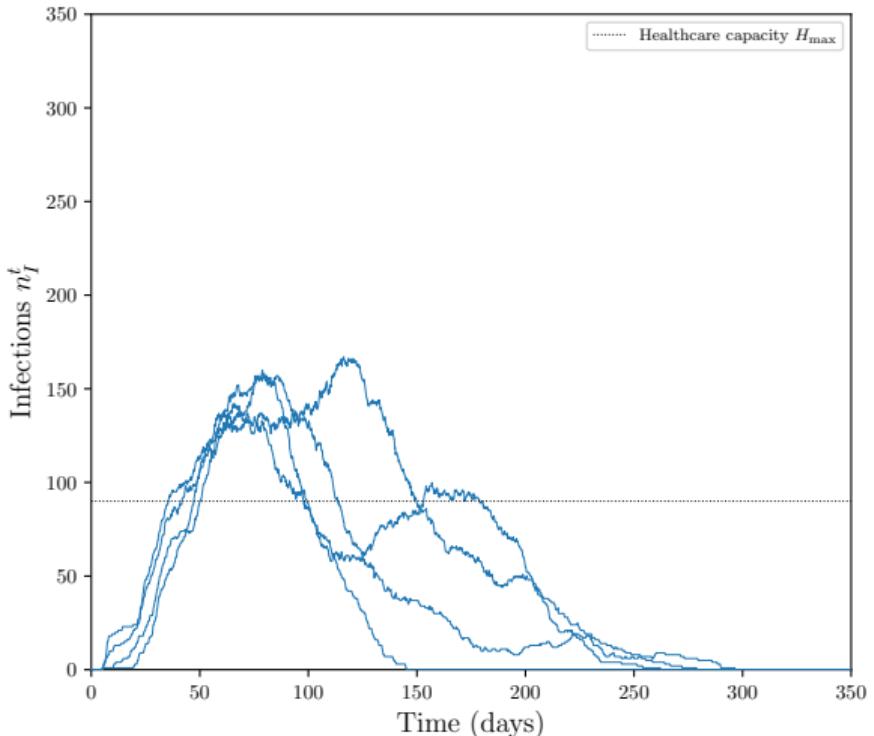




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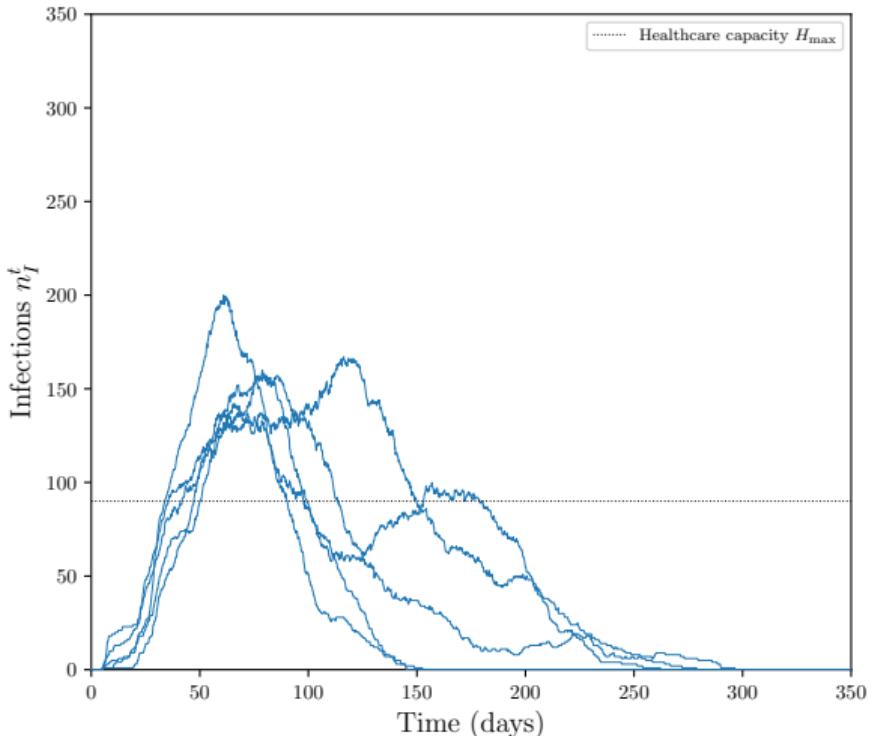




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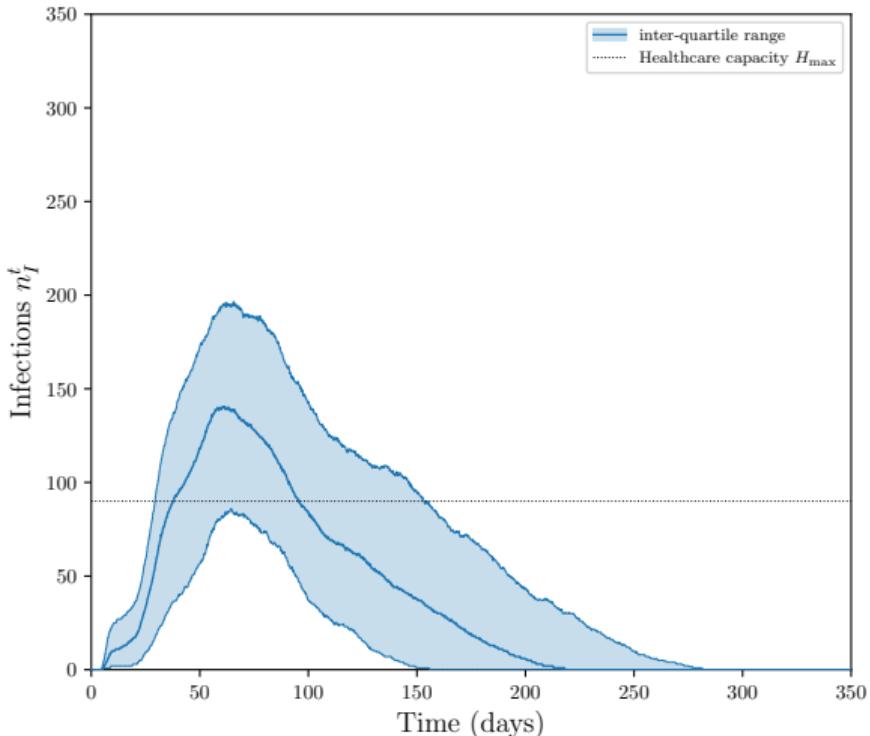




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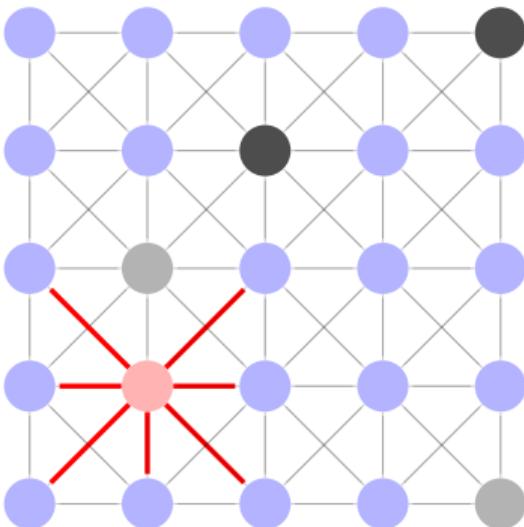
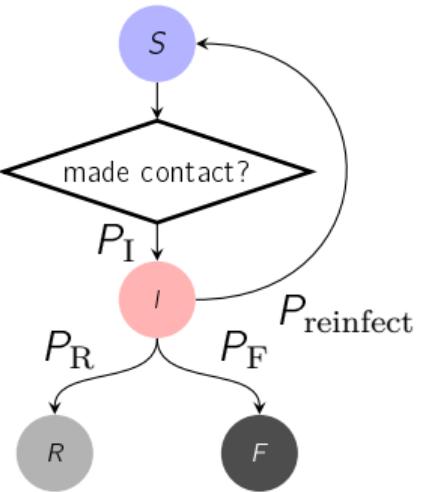


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What are agent-based epidemiological models?

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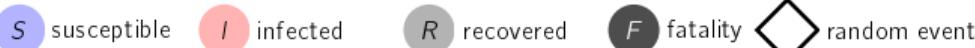
- ✓ Account for *geography* and *demographics*



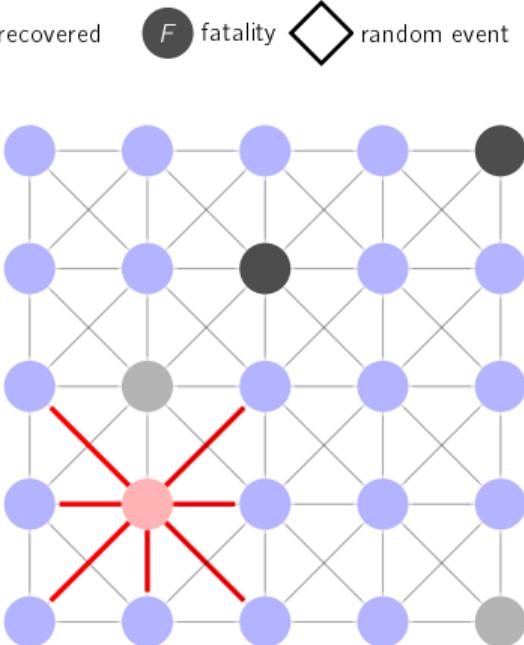
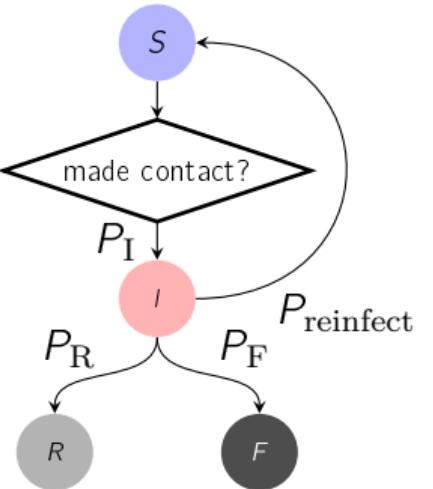


Background: epidemiological models

What are agent-based epidemiological models?



- ✓ Account for *geography* and *demographics*
- ✓ Describe local phenomena



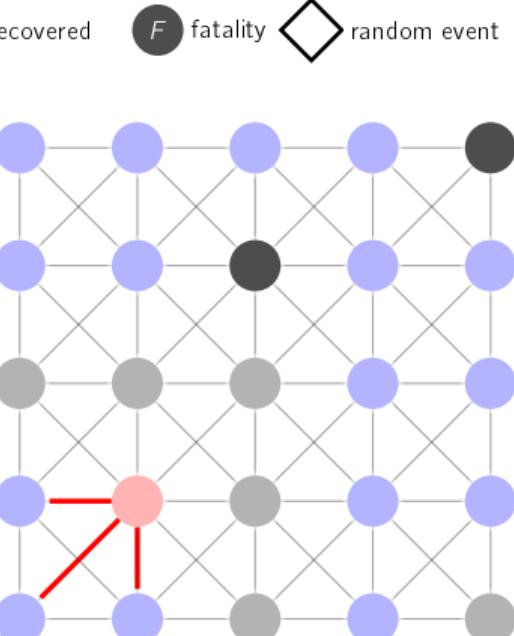
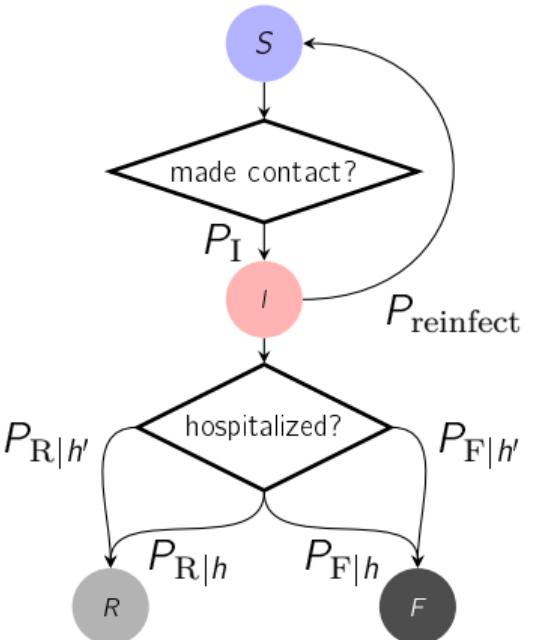


Background: epidemiological models

What are agent-based epidemiological models?

- ✓ Account for *geography* and *demographics*
- ✓ Describe local phenomena
- ✓ Can be used to model intervention policies

susceptible infected recovered fatality random event



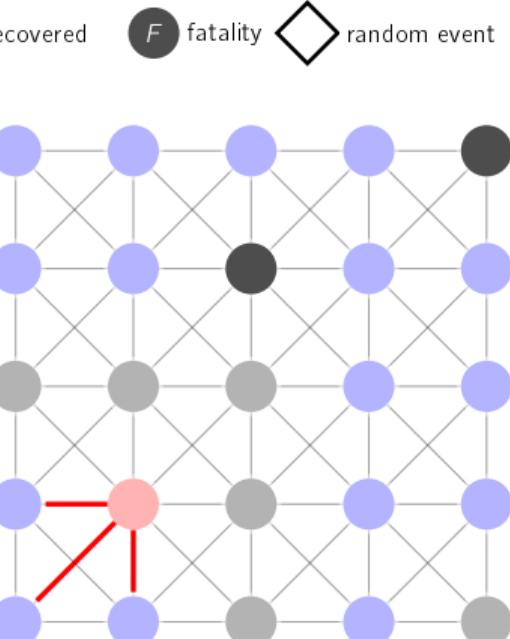
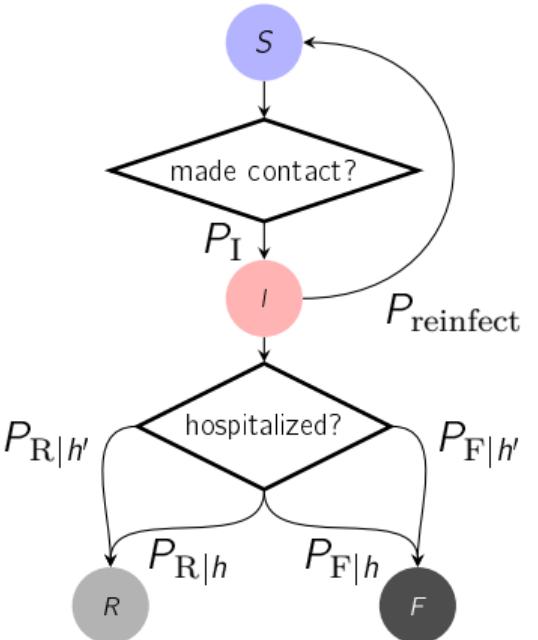


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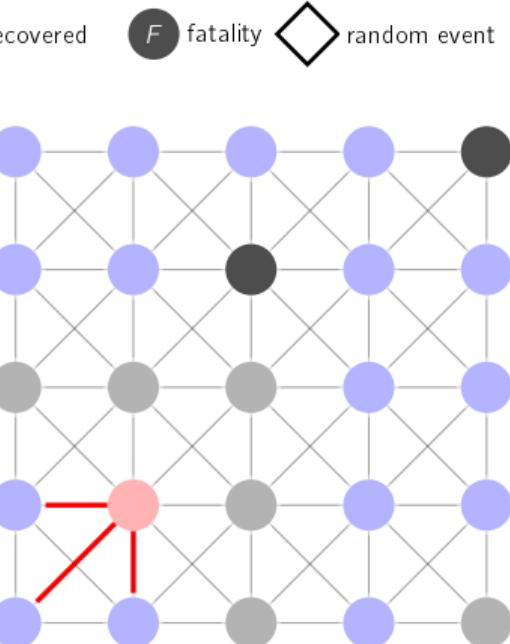
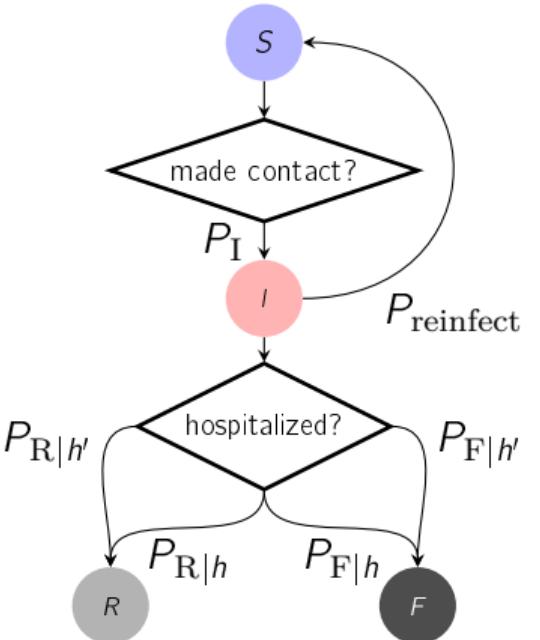


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- ✗ Computationally expensive

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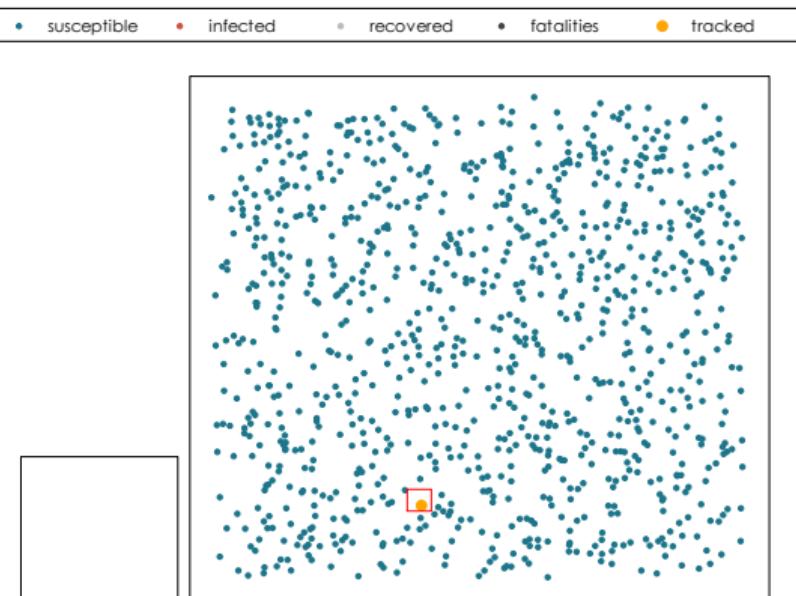




Public health policy-making problem formulation

What is the **cost** of public health interventions?

No interventions applied



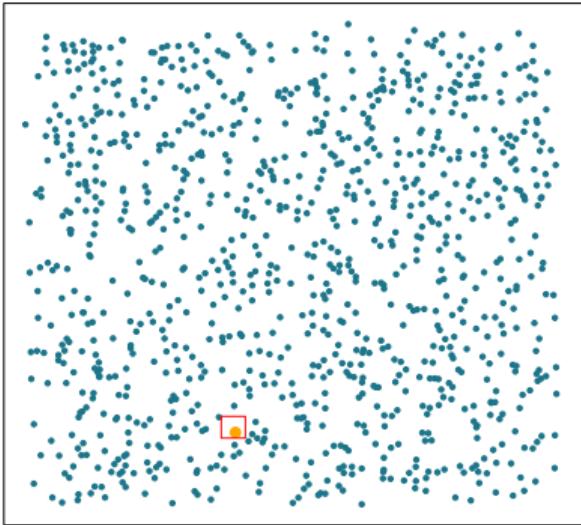
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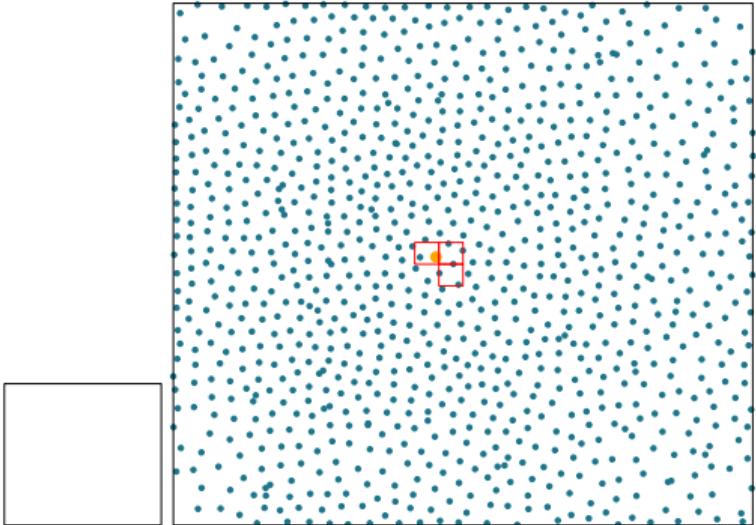
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• susceptible • infected • recovered • fatalities • tracked



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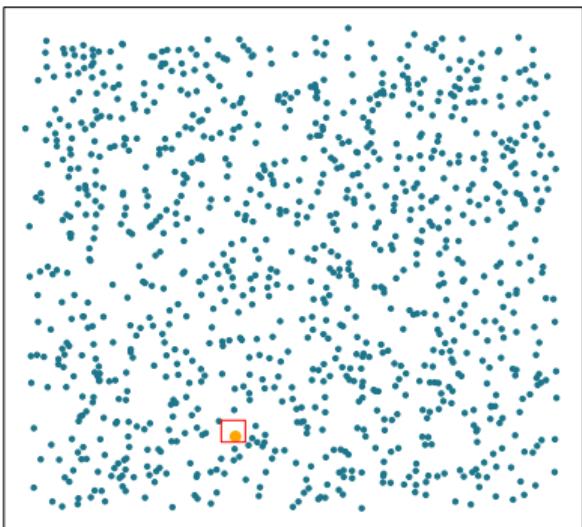
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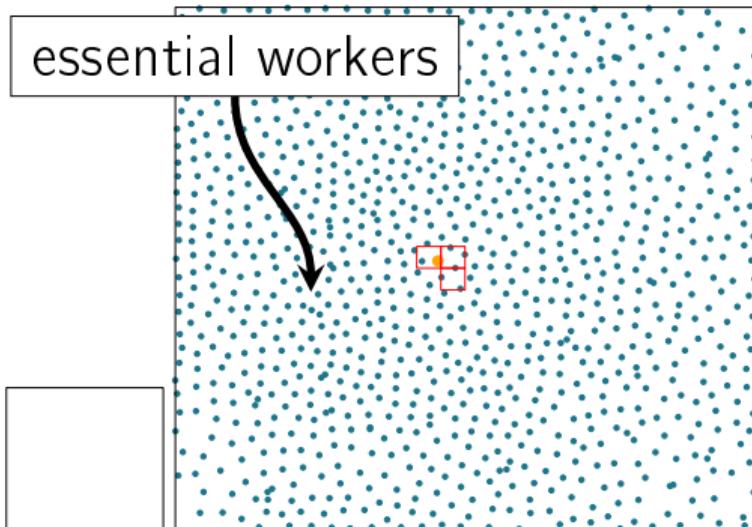
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with intervention

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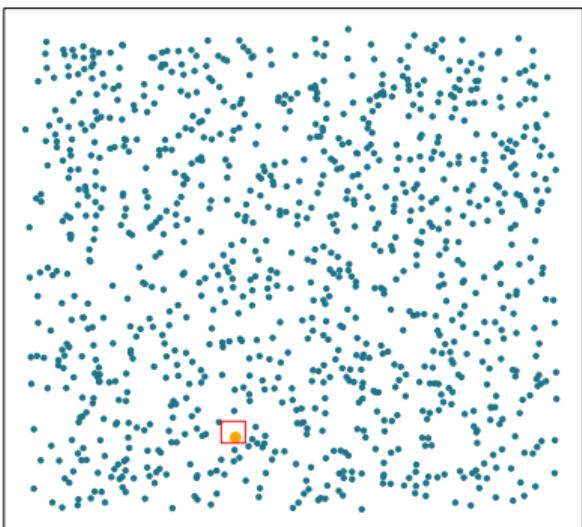
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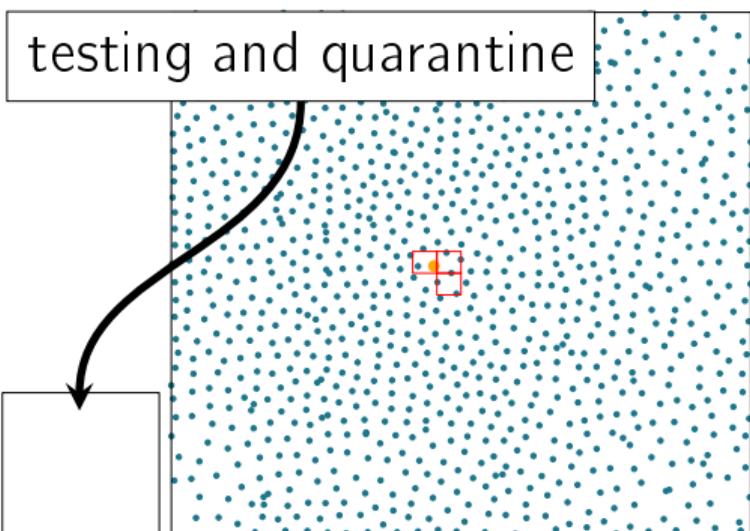
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testing and quarantine



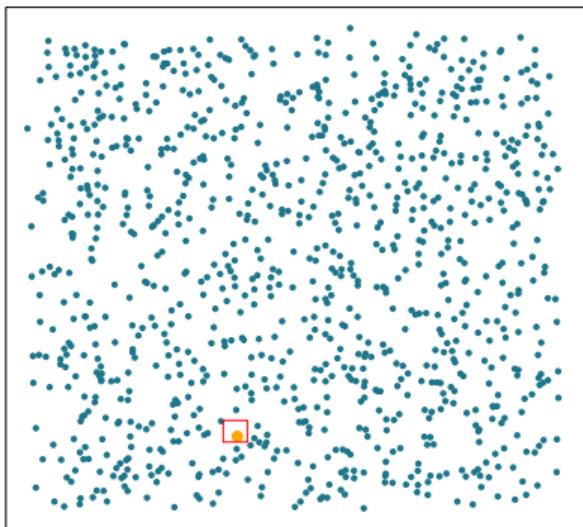
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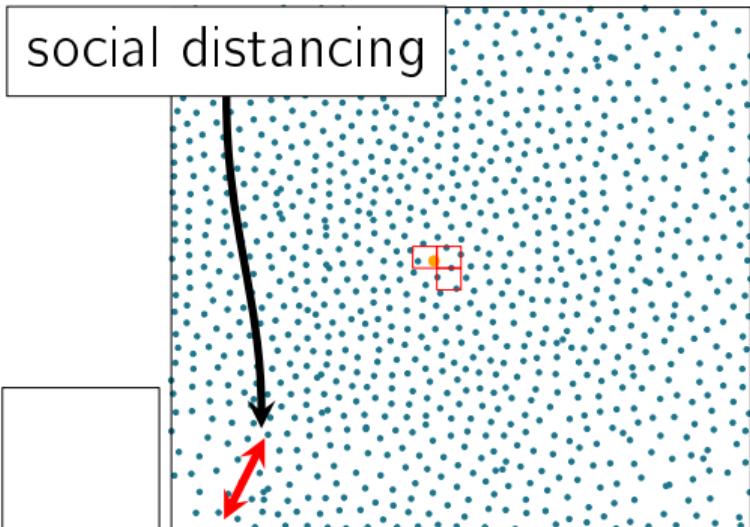
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social distancing





Public health policy-making problem formulation

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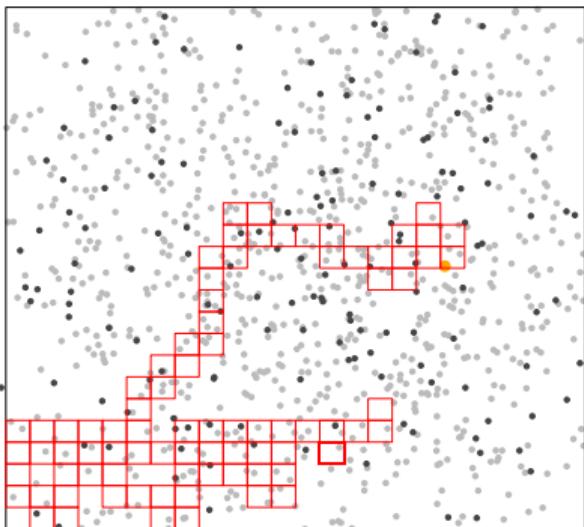
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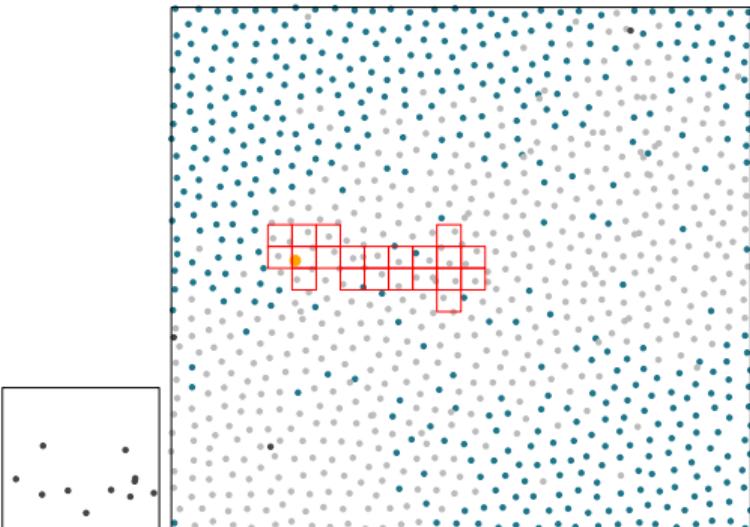
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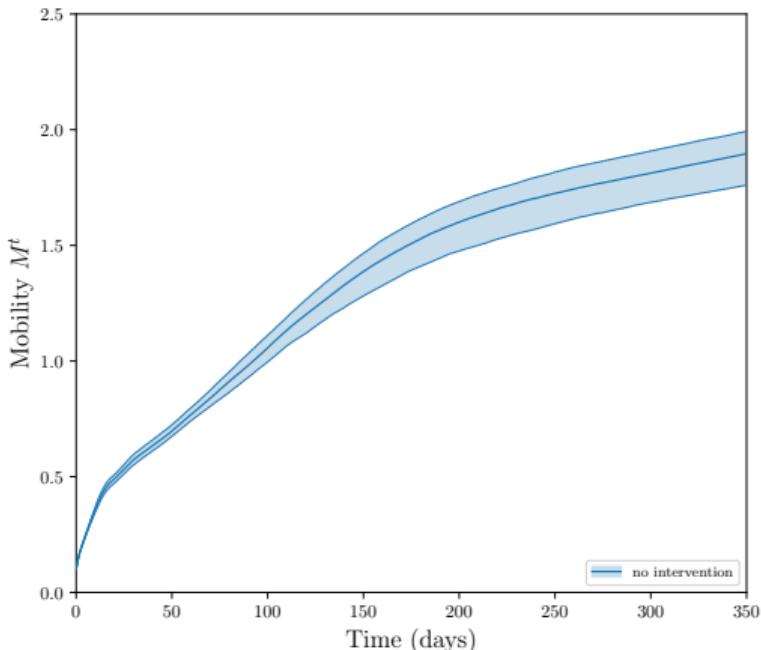
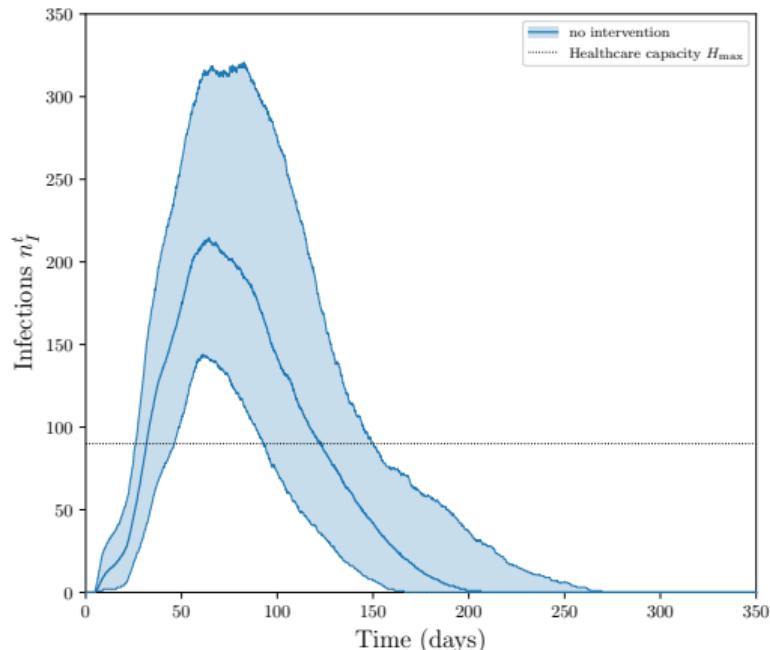
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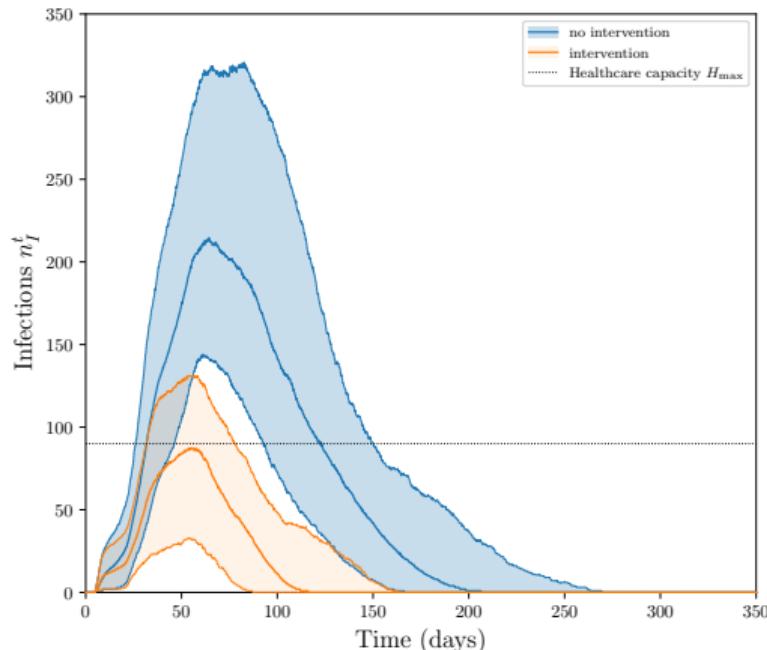




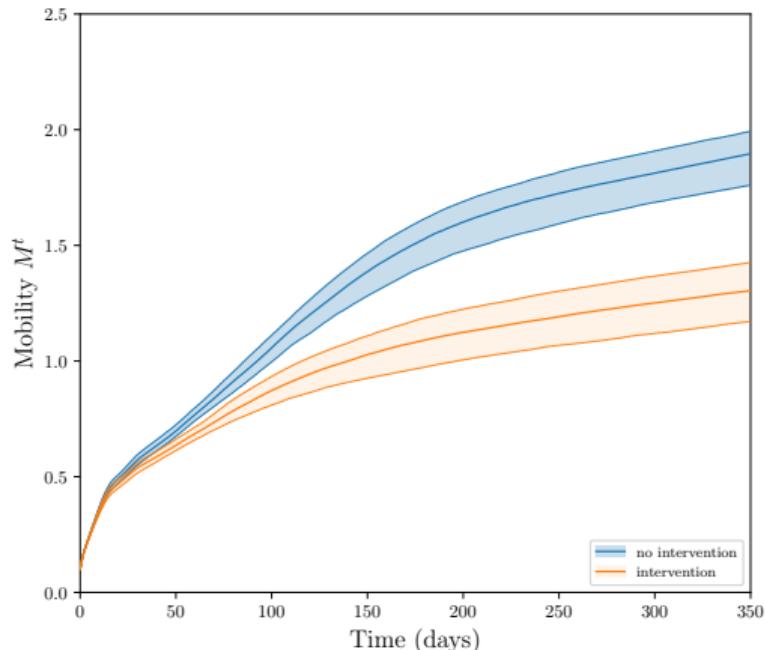
Public health policy-making problem formulation

What is the **cost** of public health interventions?

infections ↓



mobility ↓





Optimization problem

Objective and constraints

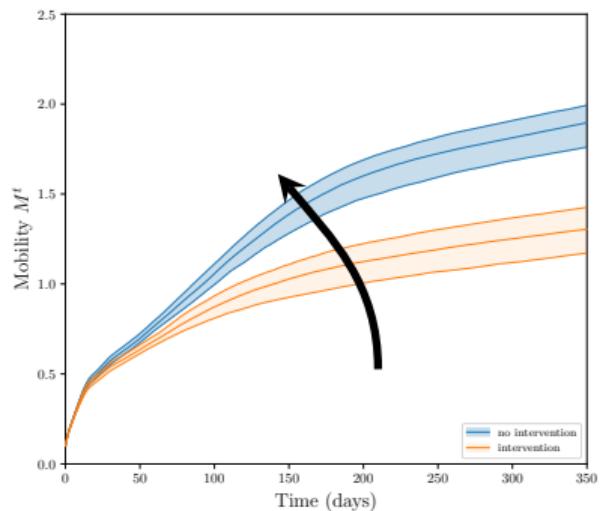
$$\min_x \quad f(x) = -M^T$$

subject to

where $x = [n_E, S_D, n_T]^T$

Design variables

- n_E : Number of essential workers
- S_D : Social distancing factor
- n_T : Number of tests daily



Optimization problem

No gradient information available, blackbox is expensive and noisy

Objective and constraints

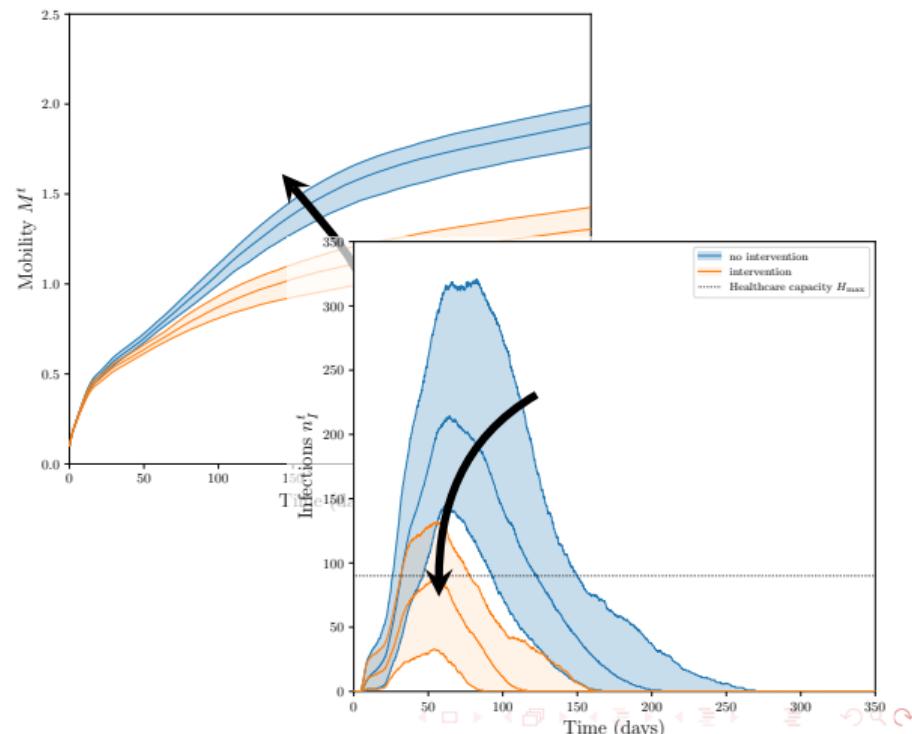
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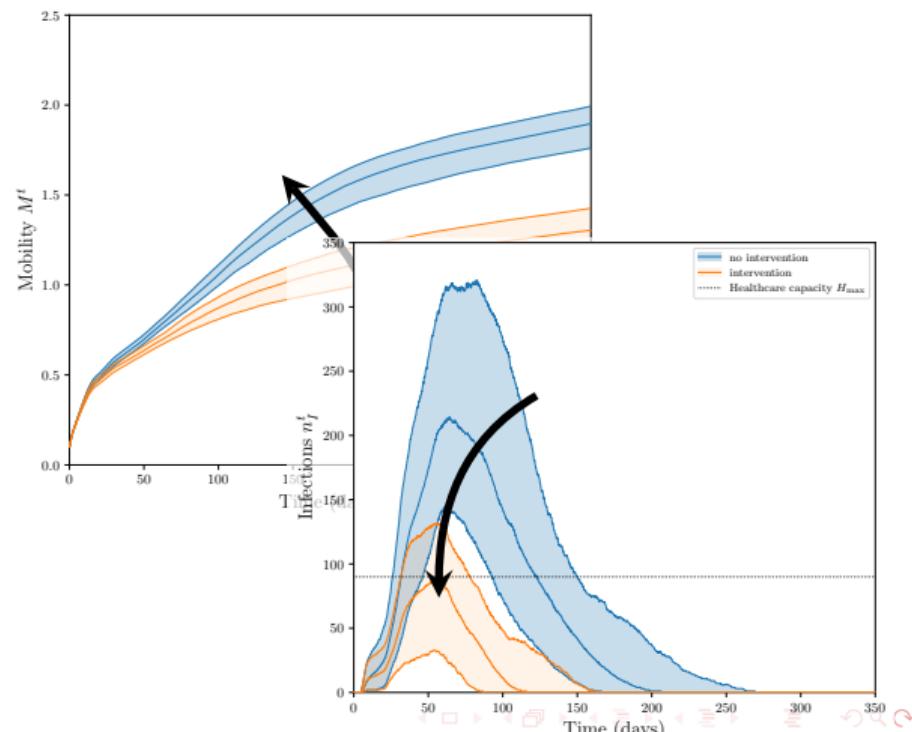
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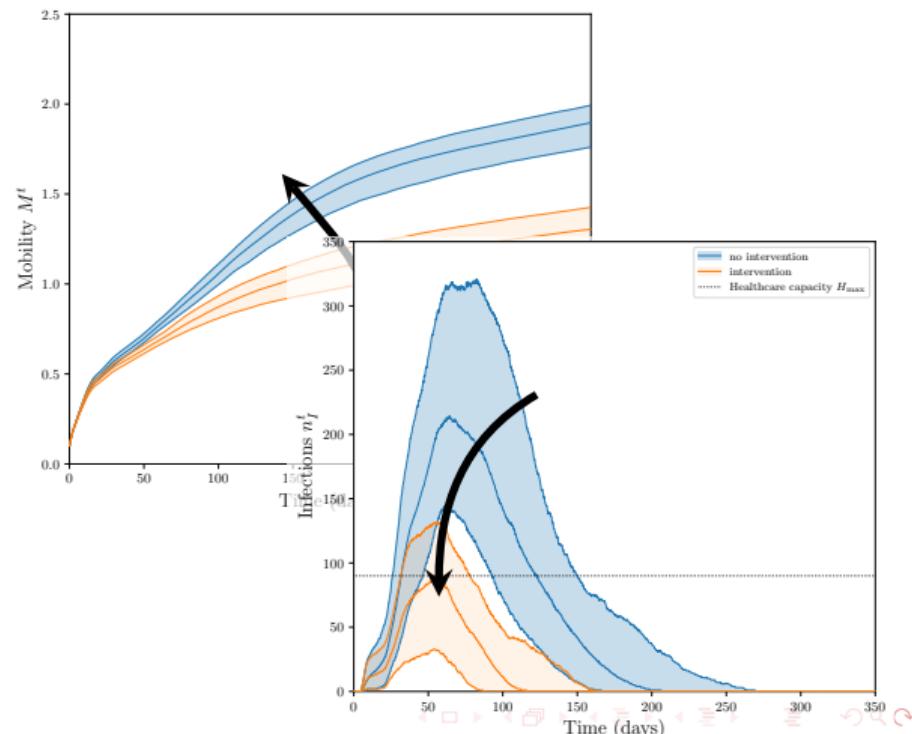
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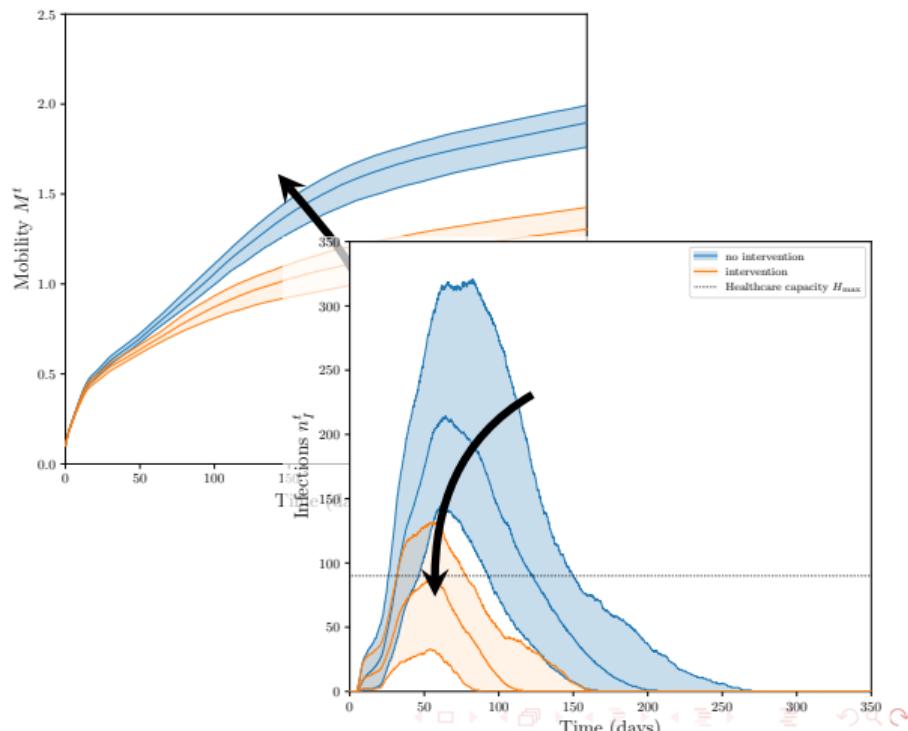
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Randomly seeded parameters

- Initial conditions
- Interactions, demographics





Optimization results

StoMADS, NOMAD¹, and genetic algorithms were used to solve the problem²

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[1] S. Le Digabel et al., 2018 , https://www.gerad.ca/nomad/Downloads/user_guide.pdf

[2] K. Al Handawi and M. Kokkolaras, 2021 , *IEEE Transactions on Emerging Topics in Computational Intelligence*



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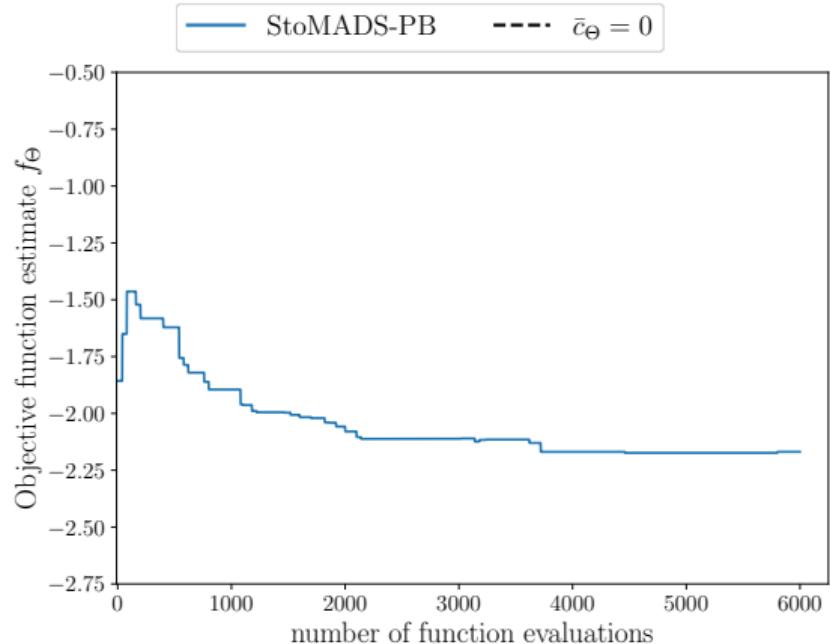
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sampling rate $n^k = 4$



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StoMADS, NOMAD¹, and genetic algorithms were used to solve the problem²

Objective and constraints

$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x) = -M^T]$$

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where $x = [n_E, S_D, n_T]^T$, Θ : realizations

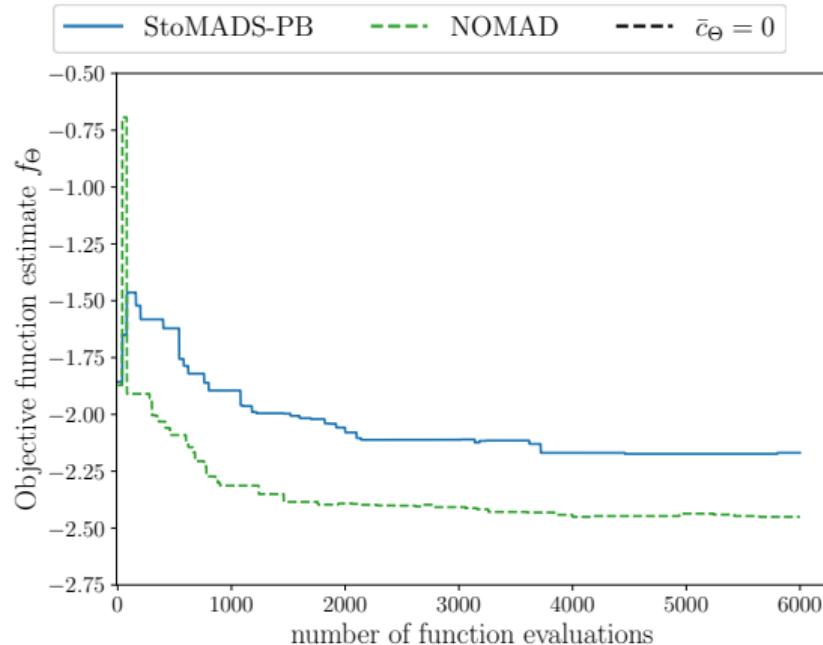
Design variables

- n_E : Number of essential workers
- S_D : Social distancing factor
- n_T : Number of tests daily

Randomly seeded parameters

- Initial conditions
- Interactions, demographics

sampling rate $n^k = 4$



[1] S. Le Digabel et al., 2018 , https://www.gerad.ca/nomad/Downloads/user_guide.pdf

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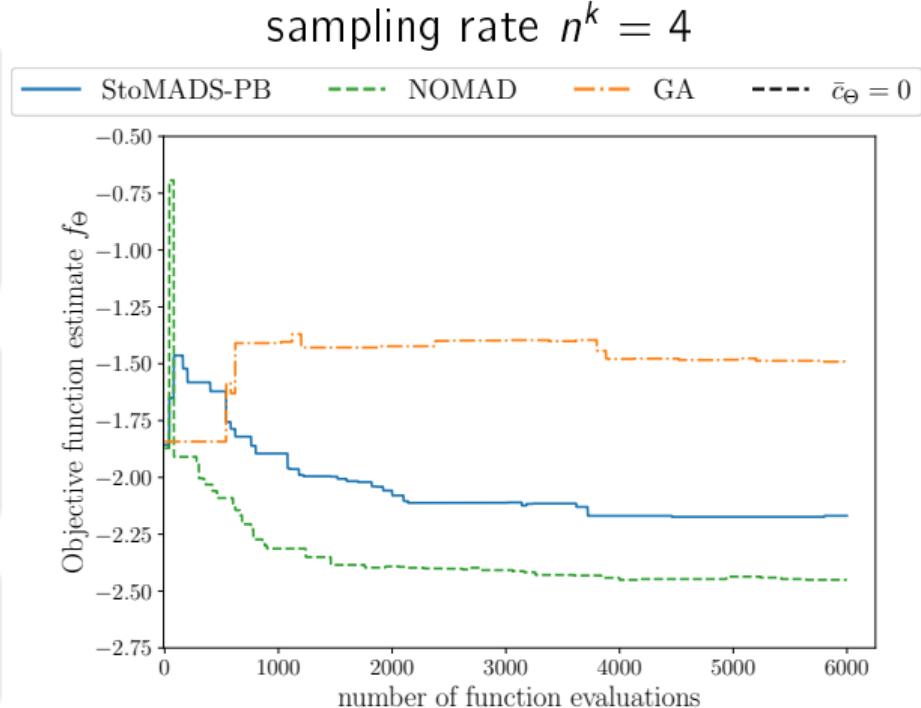
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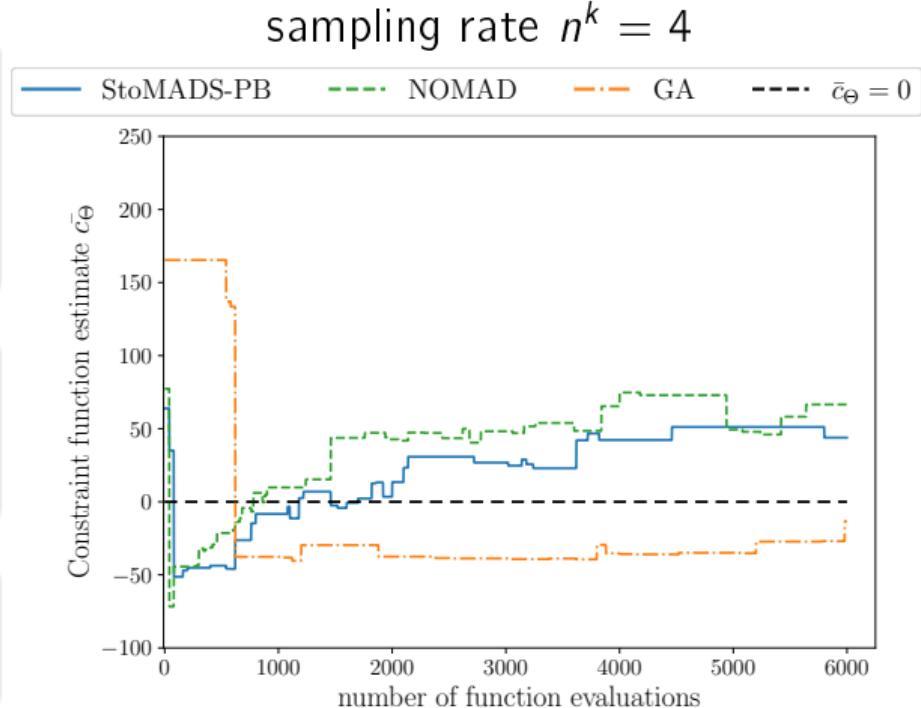
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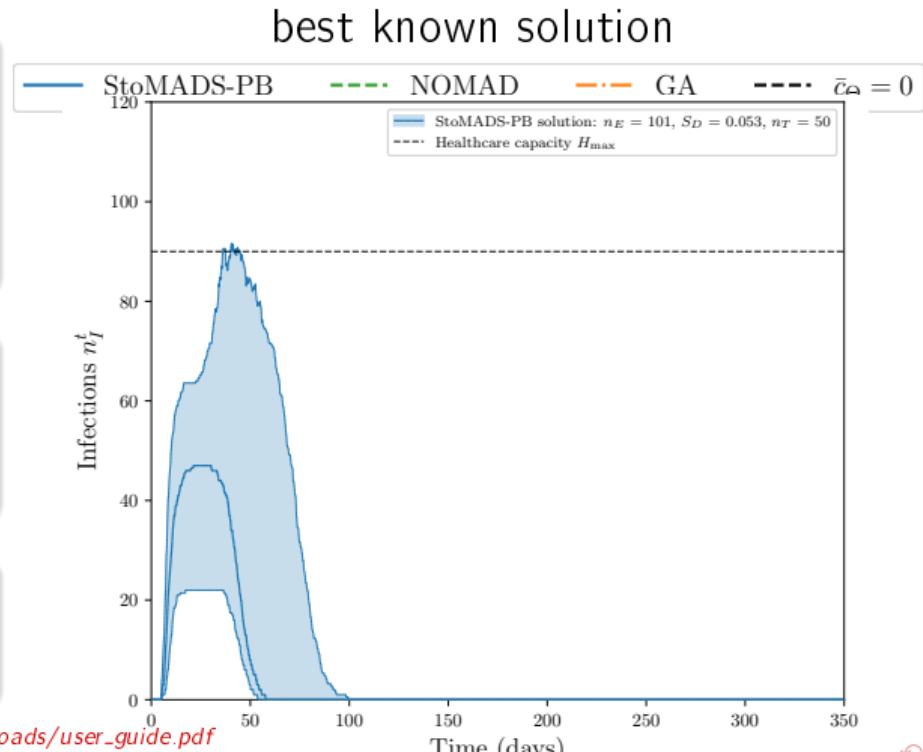
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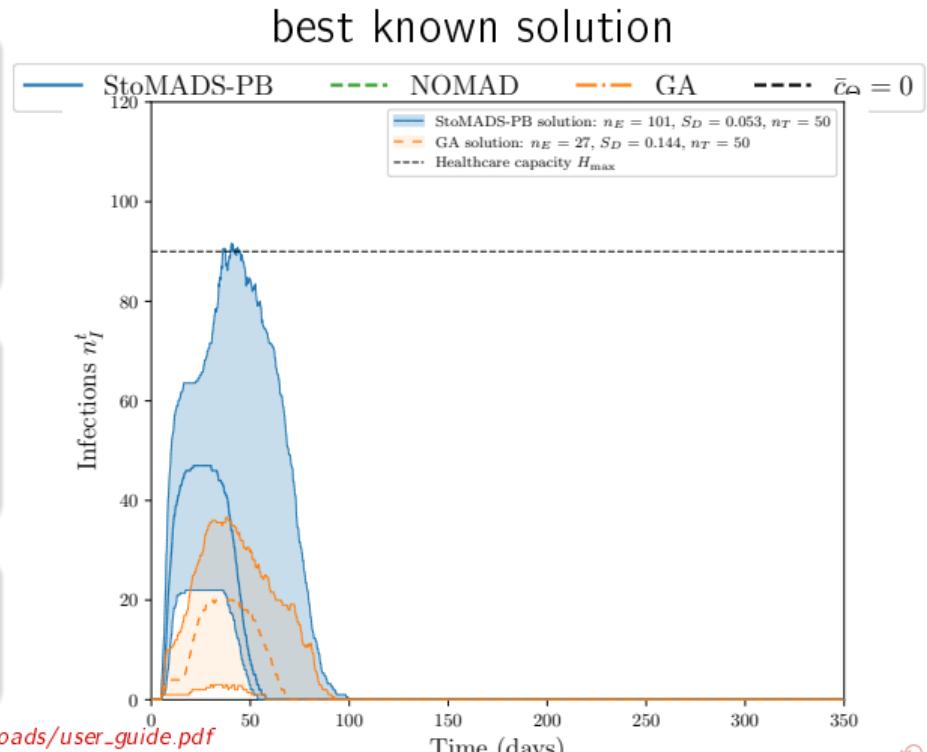
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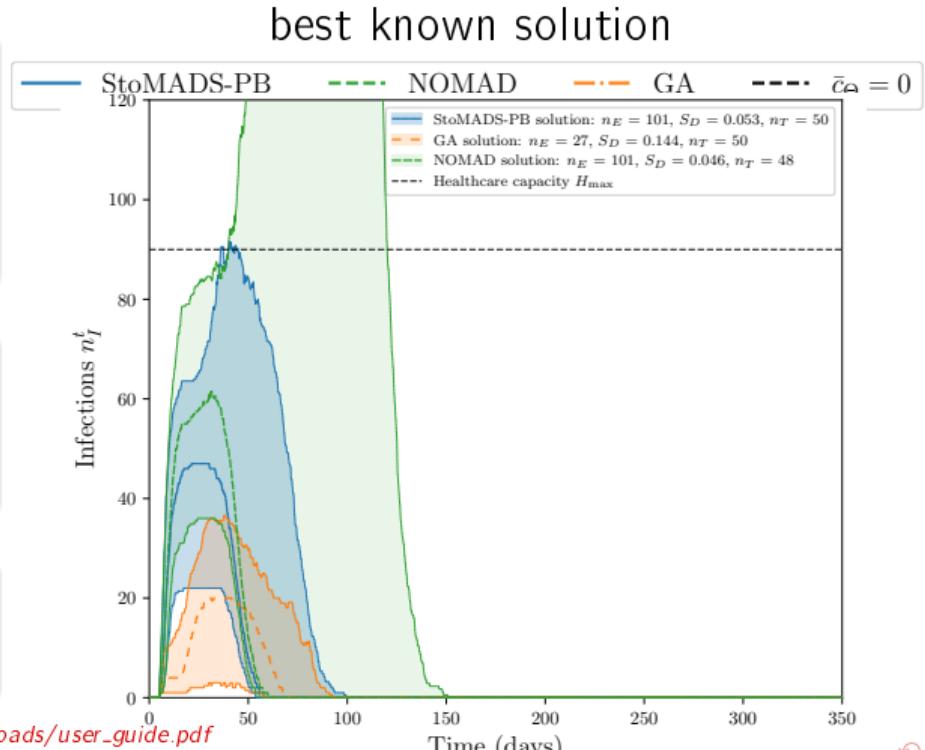
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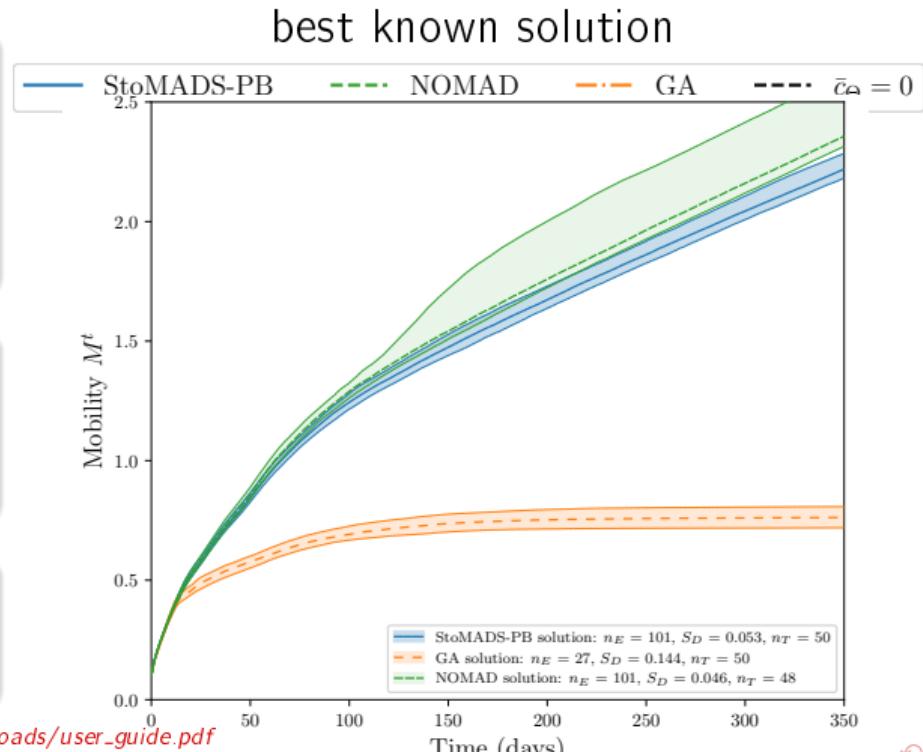
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Pseudo-code of ABM

Input: model parameters, demographics, random seed

Output: n_F^T , $n_{i,\max}$, M^T

```
1 [0] Initialization
2     set time frame  $t \leftarrow 0$ , initialize traits  $A^0$ , initialize mobility  $G^0 = 0$ 
3 [1] Update ABM
4     if  $t = 50$ , then set patient-zero as infected and seed the disease
5     loop over agents for  $i = 1, 2, \dots, n$  do
6         [2] Social interaction
7             for  $j = 1, 2, \dots, n$  do
8                 if  $j \neq i$  then compute  $f_{x,i,j}$ ,  $f_{y,i,j}$ 
9                 compute social interaction forces  $F_{x,i}^t$ ,  $F_{y,i}^t$ 
10    [3] Update model dynamics update velocities  $v_{x,i}^{t+1}$ ,  $v_{y,i}^{t+1}$ , update positions  $z_{x,i}^{t+1}$ ,  $z_{y,i}^{t+1}$ 
11    [4] Mobility
12        for  $jg = 1, 2, \dots, n_{\text{grids}}$  do
13            compute grid bounds  $l_{x,jg} = \frac{jg \bmod ng}{ng}$ ,  $l_{y,jg} = \frac{jg/ng}{ng}$ ,  $u_{x,jg} = \frac{(jg \bmod ng) + 1}{ng}$ , and  $u_{y,jg} = \frac{(jg/ng) + 1}{ng}$ 
14            if  $l_{x,jg} < z_{x,i}^{t+1} \leq u_{x,jg}$  and  $l_{y,jg} < z_{y,i}^{t+1} \leq u_{y,jg}$  then  $g_{i,jg}^{t+1} = 1$ 
15            else  $g_{i,jg}^{t+1} = g_{i,jg}^t$ 
16            compute mobility of agent  $i$   $\mu_i^t = \frac{1}{n_{\text{grids}}} \sum_{jg=1}^{n_{\text{grids}}} g_{i,jg}^t$ 
17 [6] Disease model determine infections, fatalities, and hospitalizations
18 [7] Termination
19 if  $t < T_{\max}$  then
20     set  $t \leftarrow t + 1$ , go to [1]
21     otherwise stop
```



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Input: model parameters, demographics, random seed

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5 loop over agents for $i = 1, 2, \dots, \bar{n}$ do

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7 for $j = 1, 2, \dots, \bar{n}$ do
8 if $j \neq i$ then compute f_x
9 compute social interaction forc

10 [3] Update model dynamics update

11 [4] Mobility

12 for $j_g = 1, 2, \dots, n_{\text{grids}}$ do
13 compute grid bounds $I_{x,j}$
14 if $I_{x,jg} < z_{x,i}^{t+1} \leq u_{x,jg}$ at
15 else $g_{i,jg}^{t+1} = g_{i,jg}^t$

16 compute mobility of agent i μ_i^t

17 [6] Disease model determine infection

18 [7] Termination

19 if $t < T_{\max}$ then
20 set $t \leftarrow t + 1$, go to [1]
21 otherwise stop

```
--global__ void CUDA_GPU::calc_force_m(float* diffs_x, float* diffs_y,  
float* atoms_x, float* atoms_y, float SD_factor, int N)  
{  
    int i(threadIdx.x + blockIdx.x * blockDim.x);  
    int j(threadIdx.y + blockIdx.y * blockDim.y);  
  
    if (i >= N || j >= N) {  
        return;  
    }  
  
    float distance = ((atoms_x[i] - atoms_x[j]) * (atoms_x[i] - atoms_x[j])  
    +  
        (atoms_y[i] - atoms_y[j]) * (atoms_y[i] - atoms_y[j]));  
    float distance3 = distance * distance * distance + 1e-15;  
  
    diffs_x[i + N * j] = -SD_factor * (atoms_x[i] - atoms_x[j]) / distance3;  
    diffs_y[i + N * j] = -SD_factor * (atoms_y[i] - atoms_y[j]) / distance3;  
}
```



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Input: model parameters, demographics, random seed

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[0] Initialization

set time frame $t \leftarrow 0$, initialize traits

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loop over agents for $i = 1, 2, \dots, \bar{n}$ do

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[4] Mobility

for $g_j = 1, 2, \dots, n_{\text{grids}}$ do
 compute grid bounds $l_{x,j}$
 if $l_{x,jg} < z_{x,i}^{t+1} \leq u_{x,jg}$ and
 else $g_{i,jg}^{t+1} = g_{i,jg}^t$

compute mobility of agent i μ_i^t

[6] Disease model determine infection

[7] Termination

if $t < T_{\max}$ then
 set $t \leftarrow t + 1$, go to [1]
otherwise stop

```
1    __global__ void CUDA_GPU::calc_tracking_matrix(float* G, float* G_track,
2        float* atoms_x, float* atoms_y, int n_grids, int N_rows, int N_cols)
3    {
4        int i(threadIdx.x + blockIdx.x * blockDim.x);
5        int j(threadIdx.y + blockIdx.y * blockDim.y);
6
7        if (i >= N_rows || j >= N_cols)
8        {
9            return;
10       }
11
12       float g1 = (float)(j % n_grids) / float(n_grids);
13       float g2 = (float)(j / n_grids) / float(n_grids);
14       float g3 = (float)((j % n_grids) + 1) / float(n_grids);
15       float g4 = (float)((j / n_grids) + 1) / float(n_grids);
16
17       bool check = (atoms_x[i] > g1) && (atoms_y[i] > g2) && (atoms_x[i] <= g3)
18       ) && (atoms_y[i] <= g4);
19
20       float t = (check) ? 1 : 0;
21       G[i + j * N_rows] += t; // update tracing matrix
22
23       float track = (check) ? 1 : G_track[i + j * N_rows];
24       G_track[i + j * N_rows] = track; // update tracking matrix
25   }
```

Open source GPU implementation of agent-based modeling¹



[1] Khalil Al Handawi, 2021 , https://github.com/khbalhandawi/COVID_SIM_GPU

K. Al Handawi

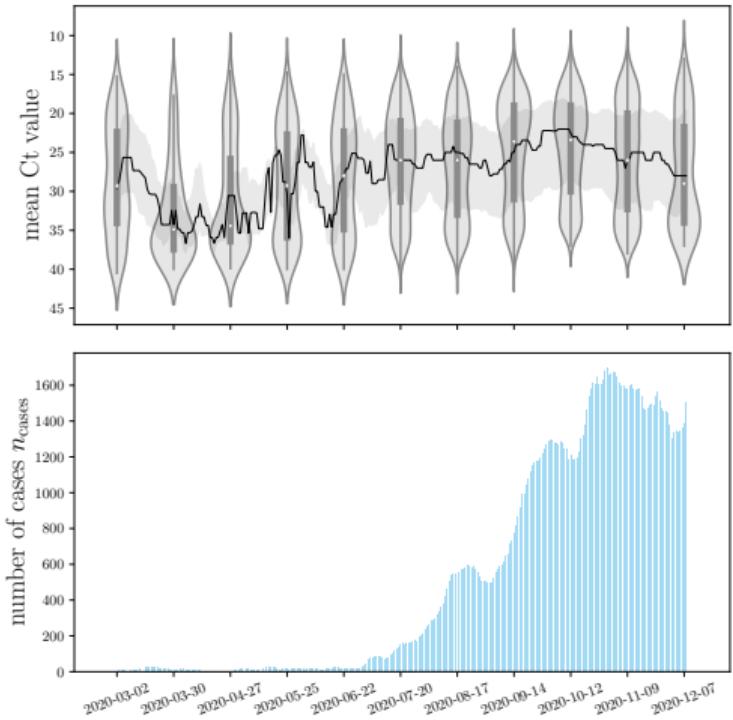
<https://khbalhandawi.github.io/>

COVID-19 incidence now-casting using machine learning



COVID-19 incidence forecasting

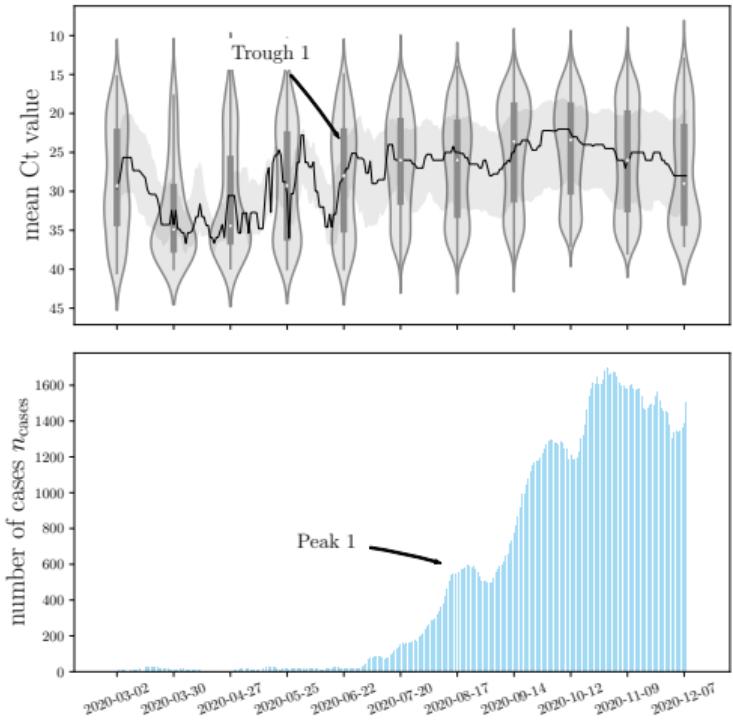
How to *design* a machine learning model for forecasting a time-series?





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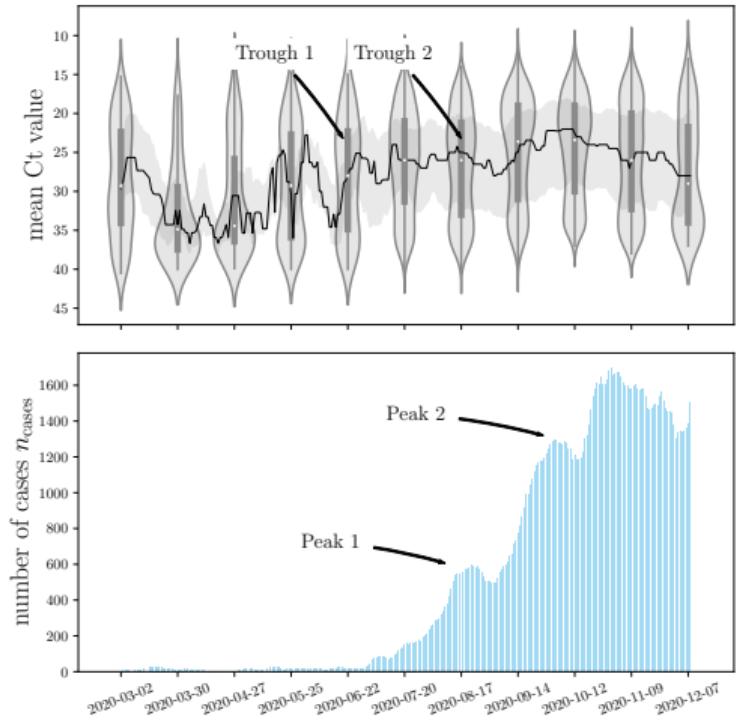
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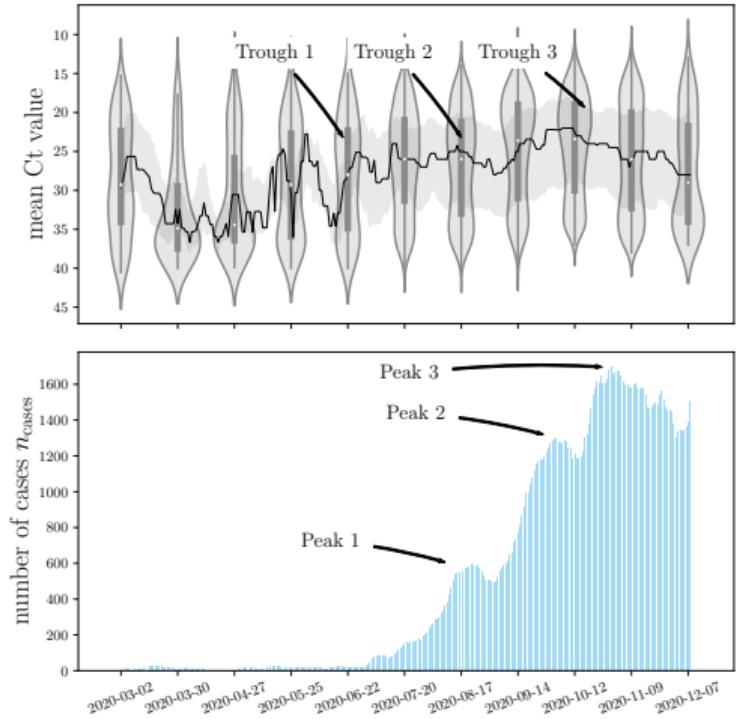
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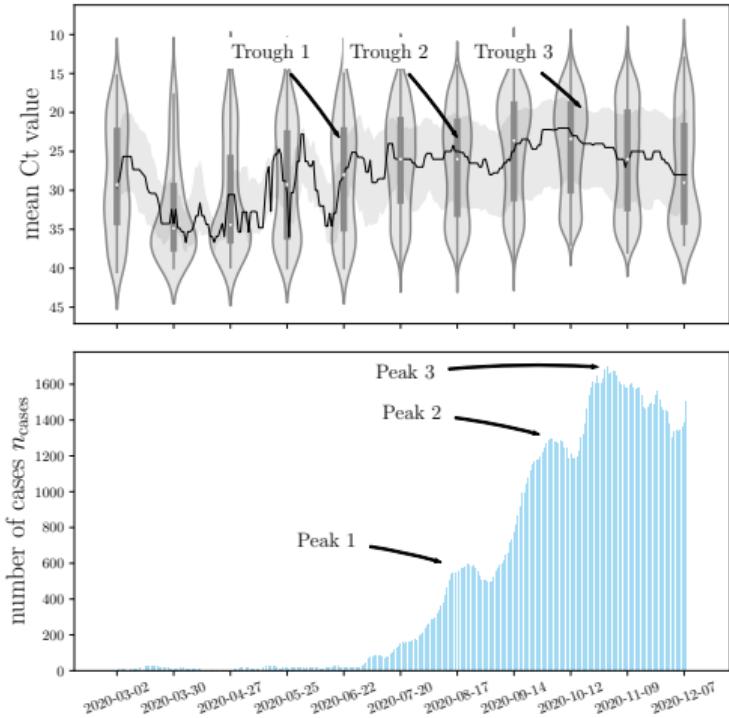
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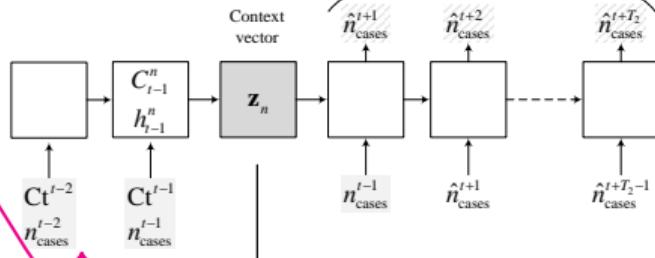
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Encoder

Decoder

Output window size T_2
for a weekly projection

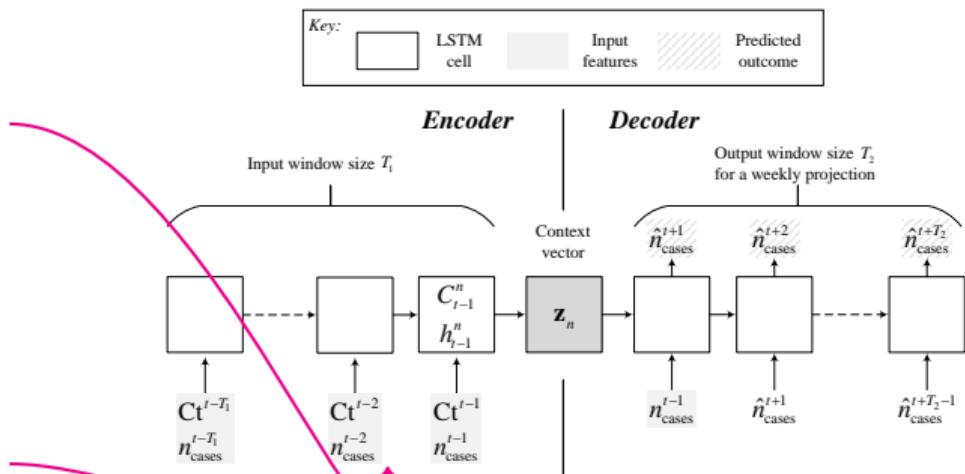
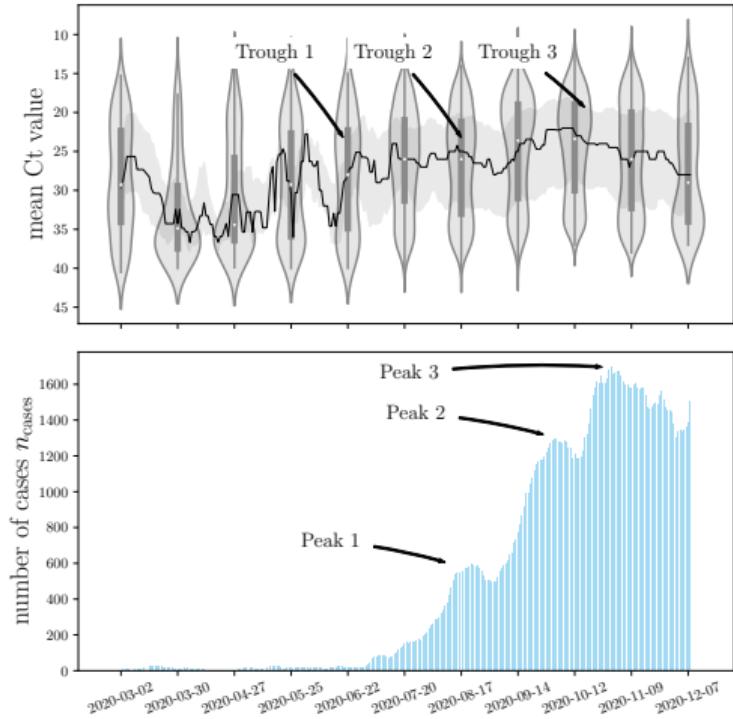


Seq2Seq model



COVID-19 incidence forecasting

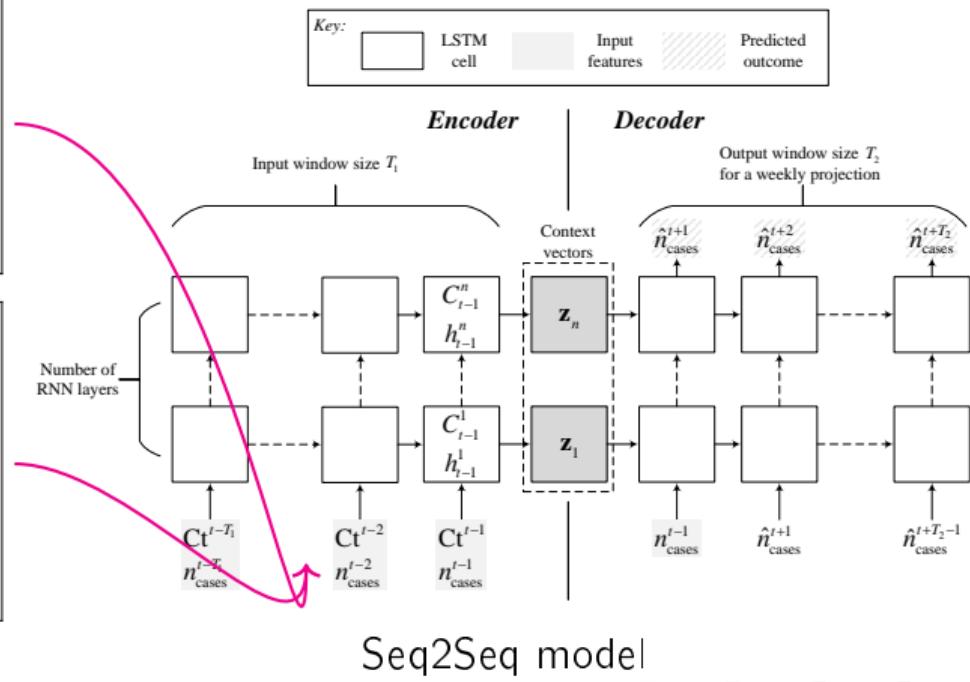
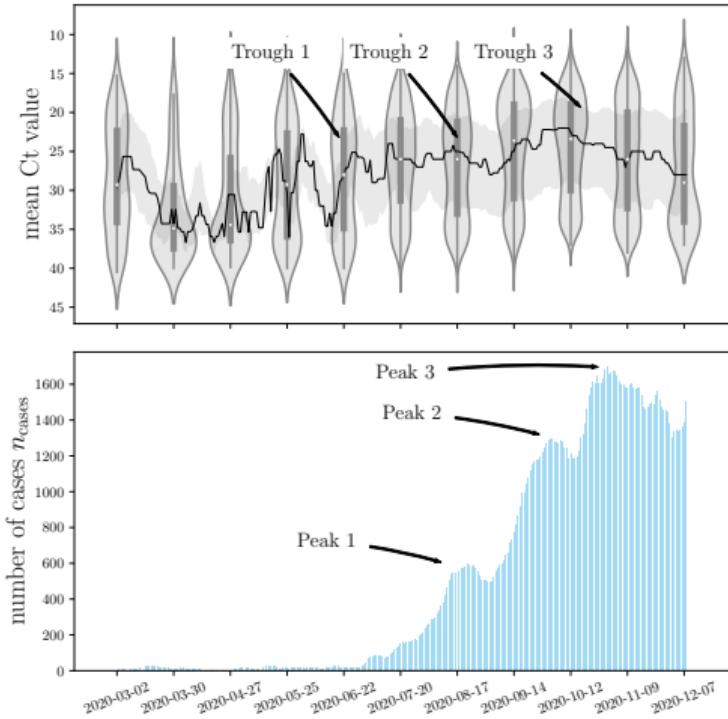
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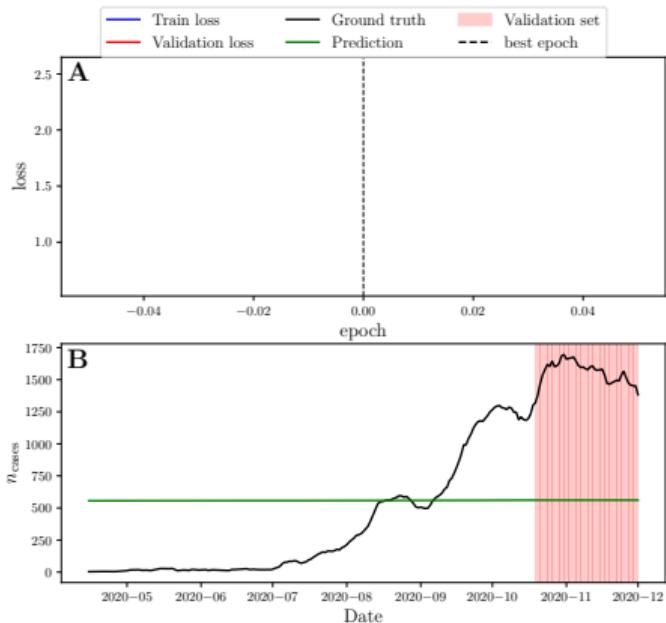




Training the model

There are several challenges associated with hyperparameter optimization

- The number of epochs can be tuned using *early stopping*
- This is a form of *regularization* to reduce over-fitting



Effect of number of epochs on testing loss



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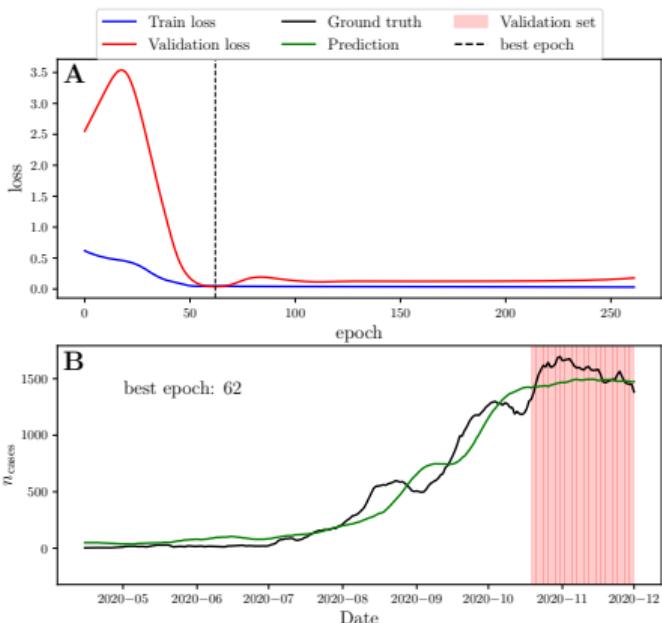


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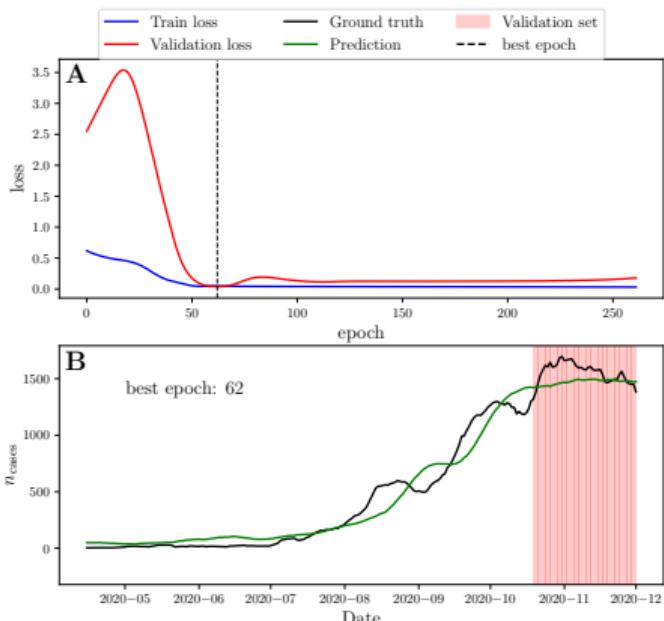
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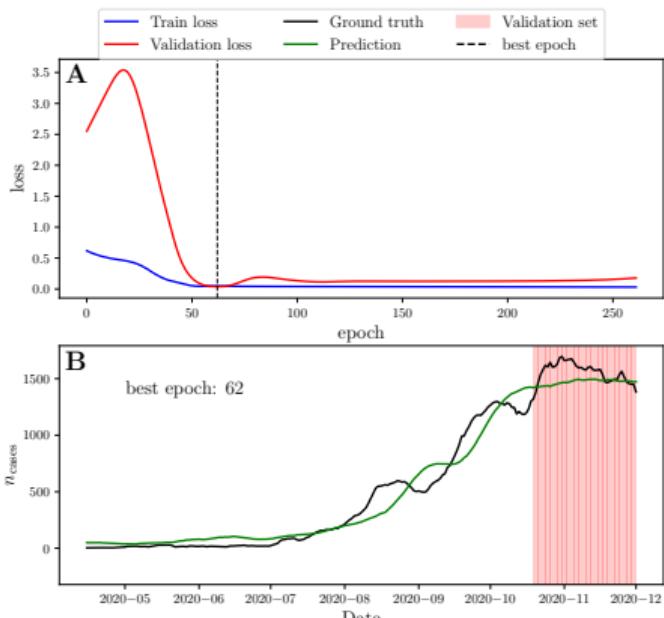
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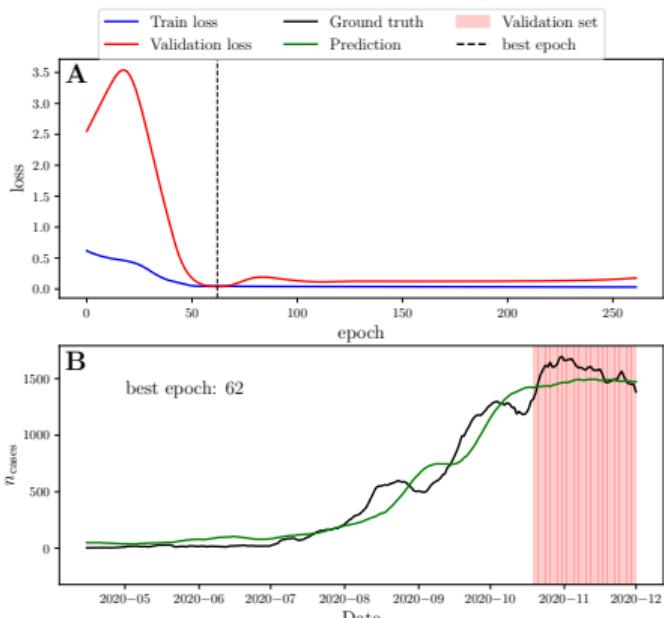
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3) Backpropagation is **stochastic**

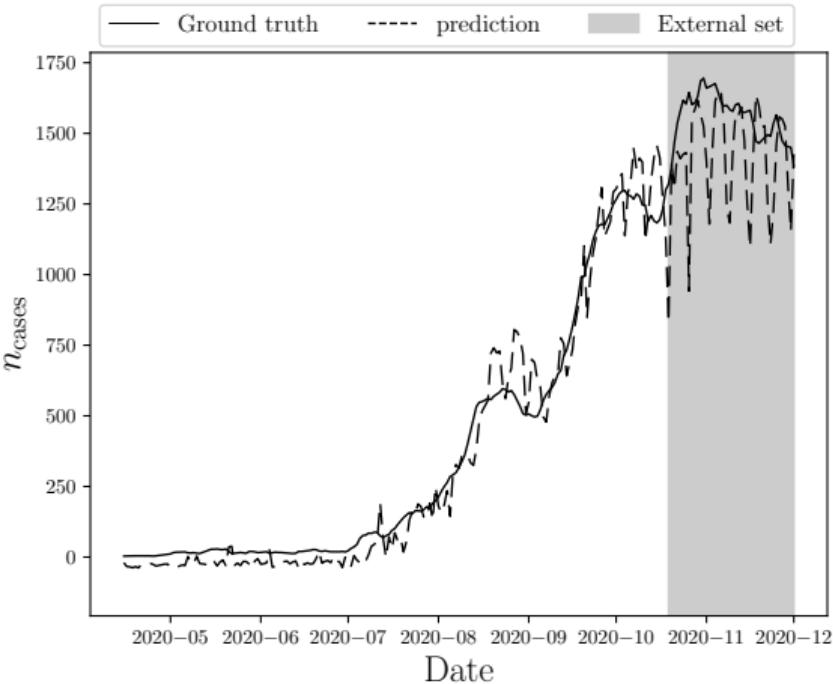
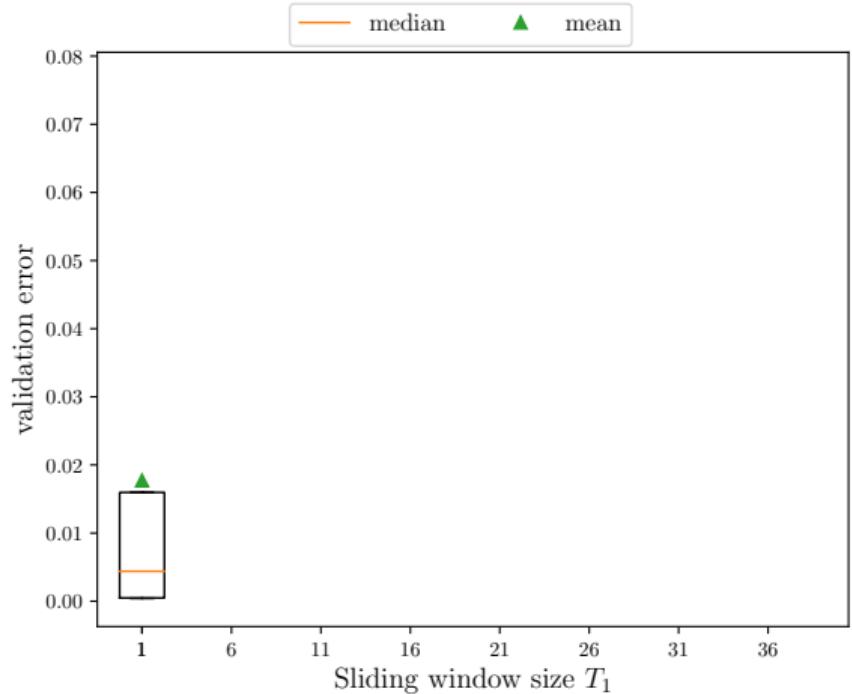


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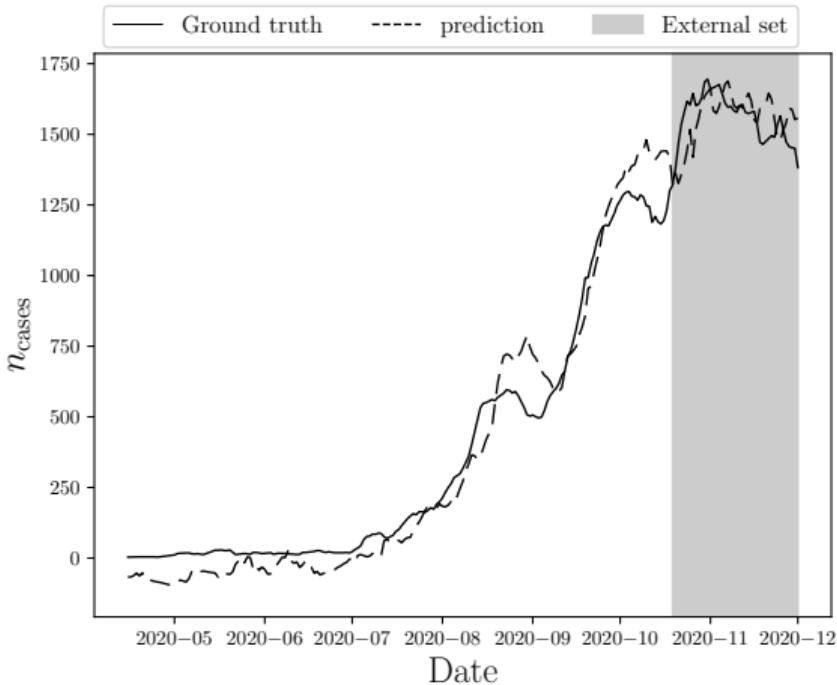
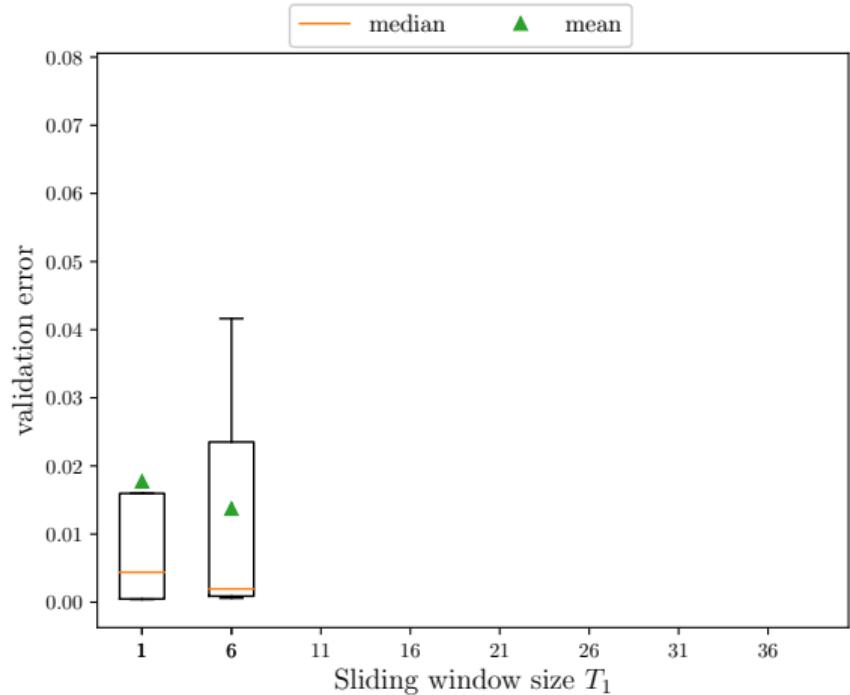
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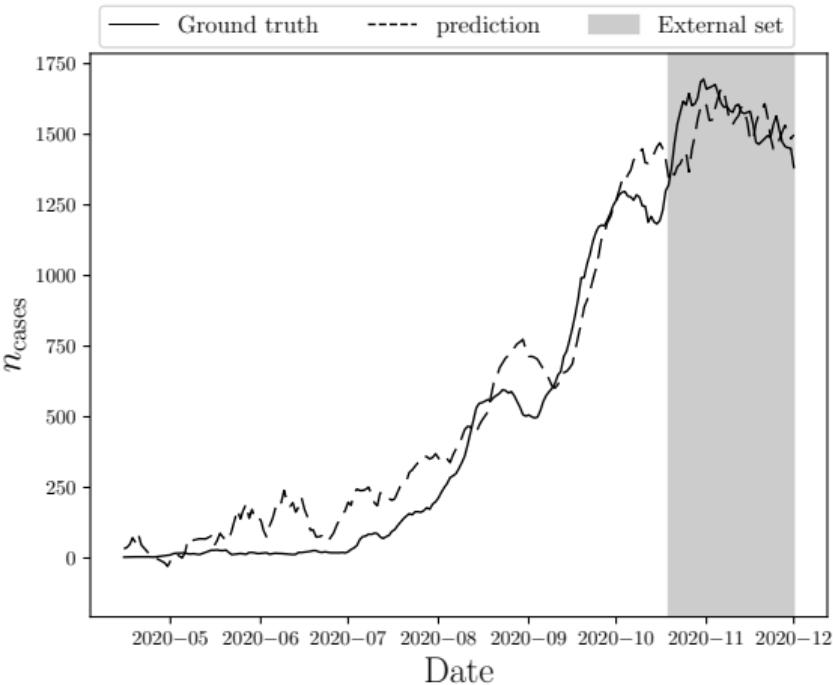
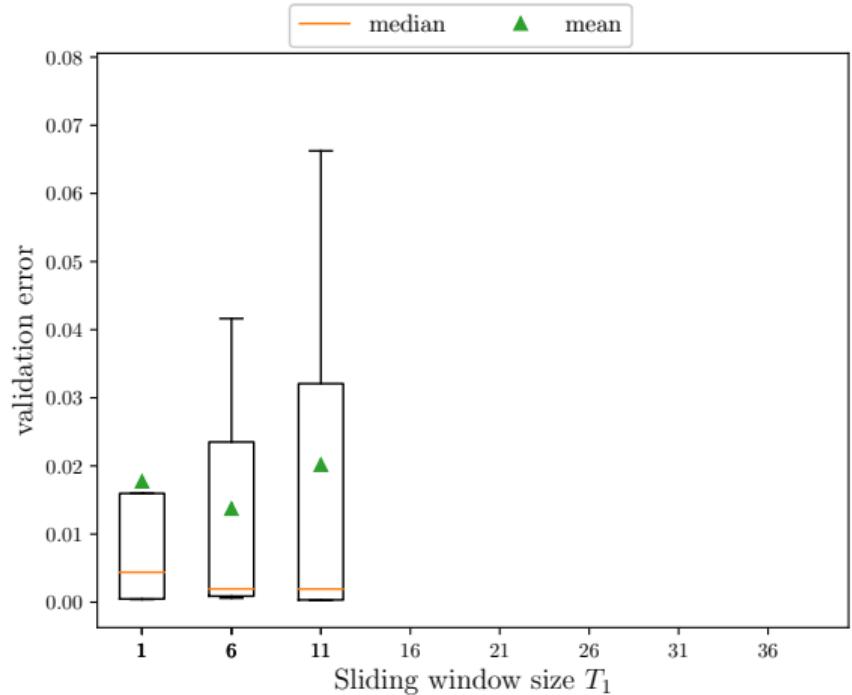
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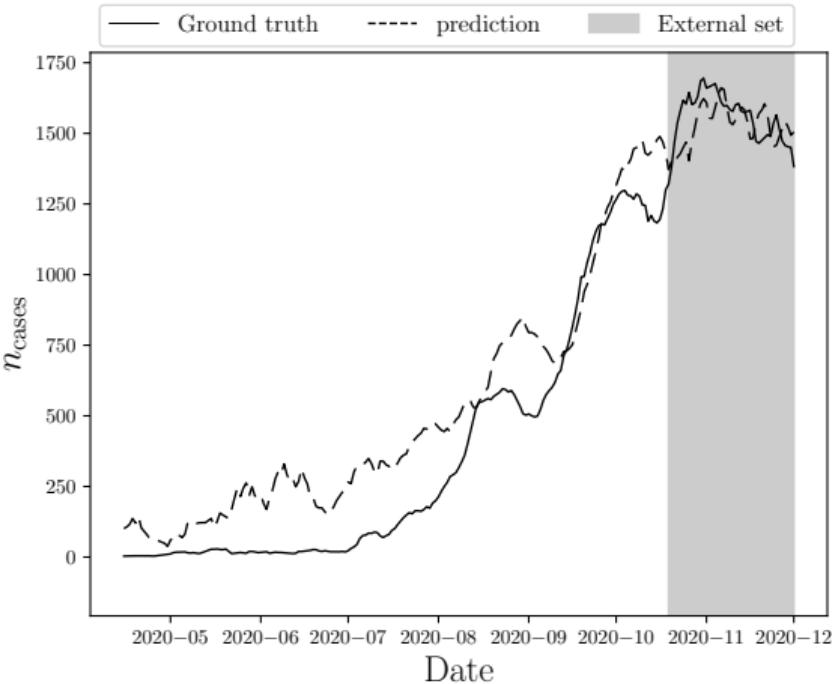
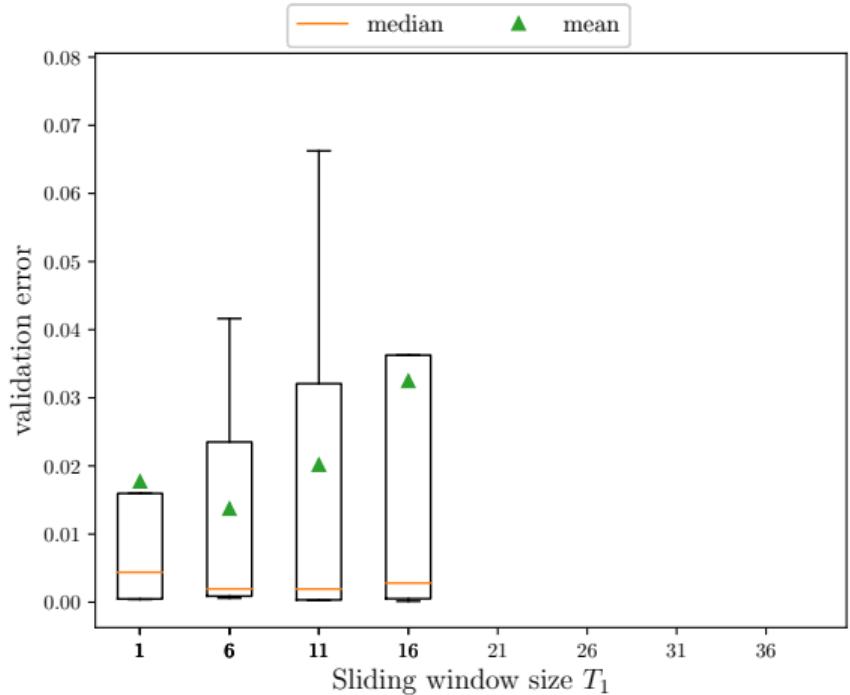
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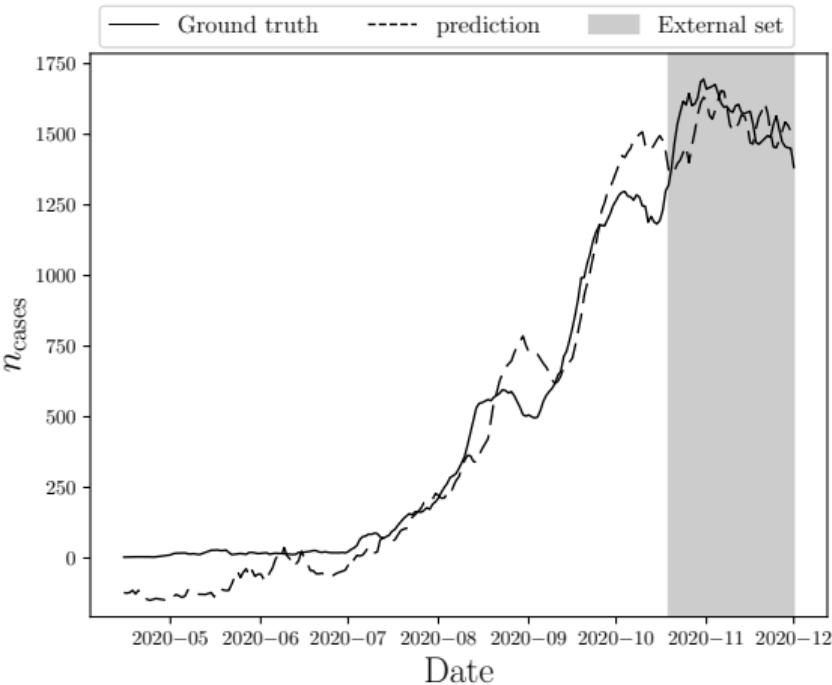
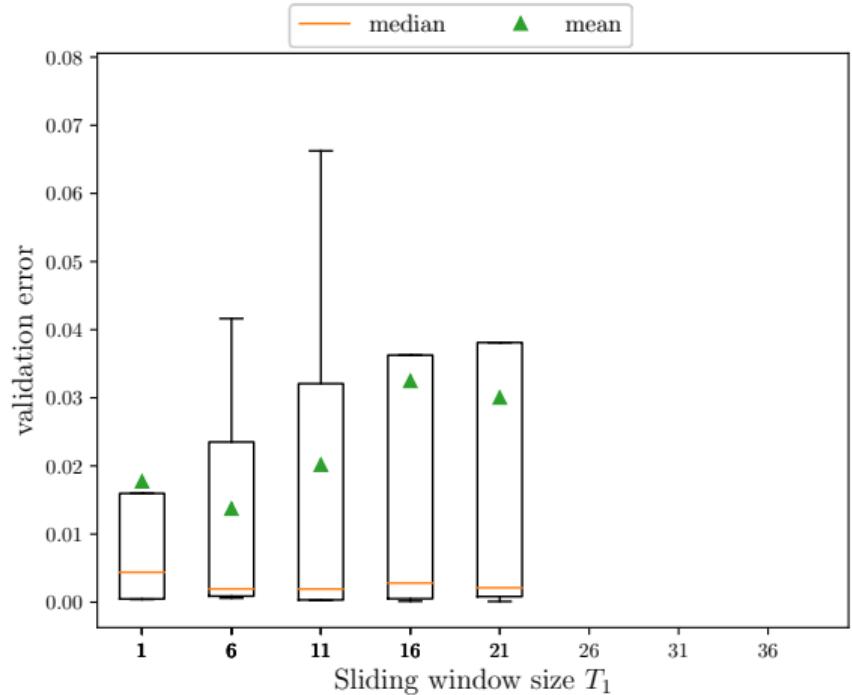
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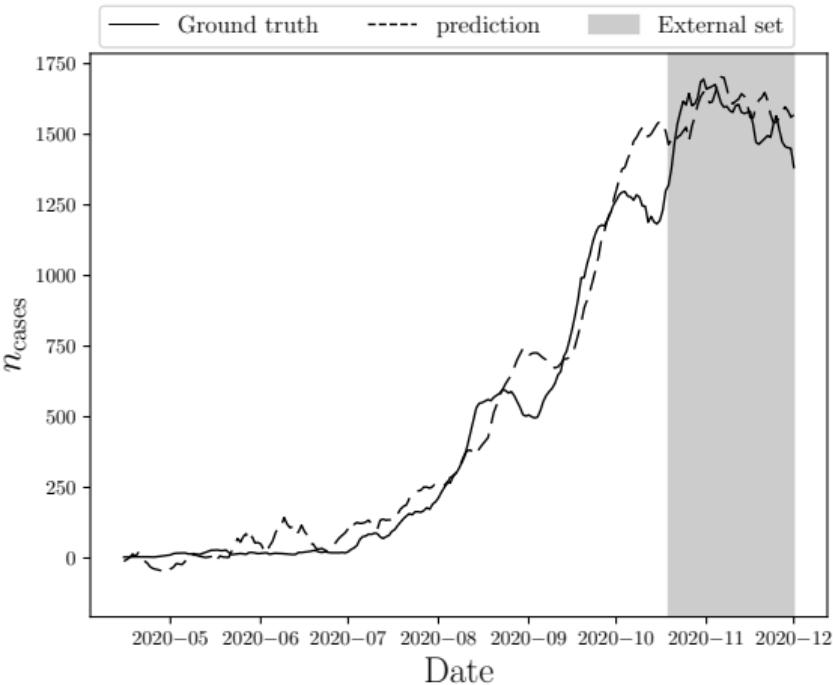
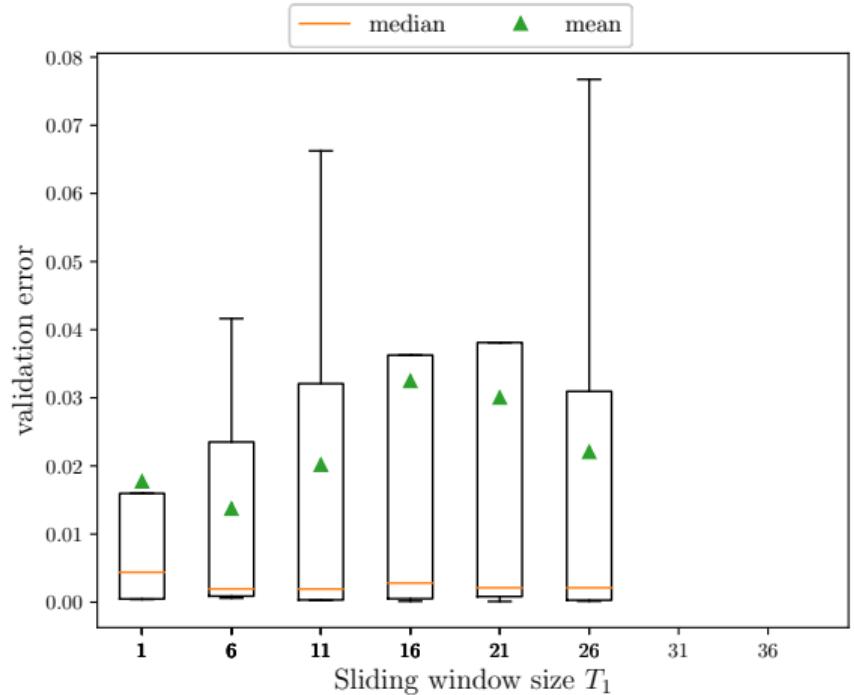
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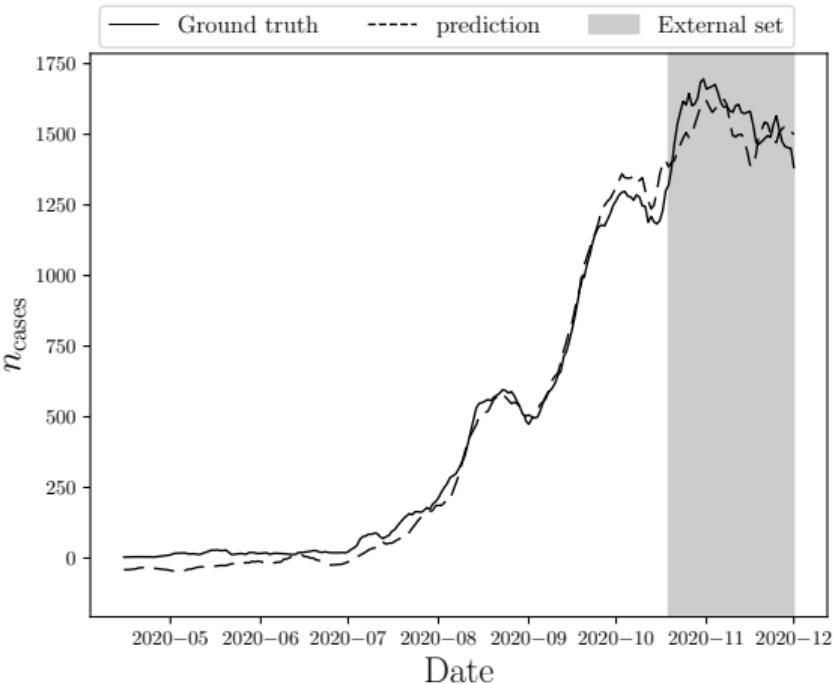
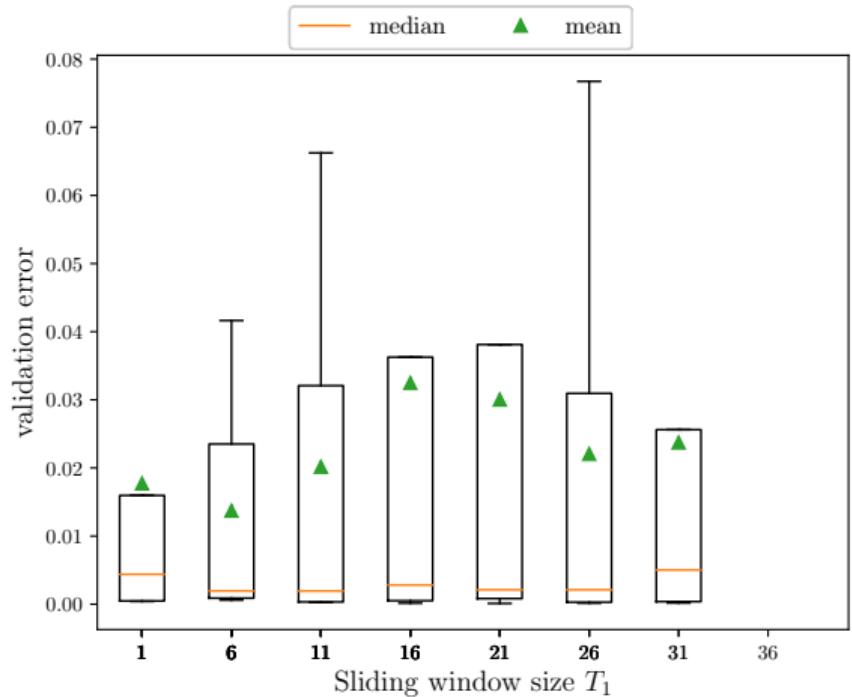
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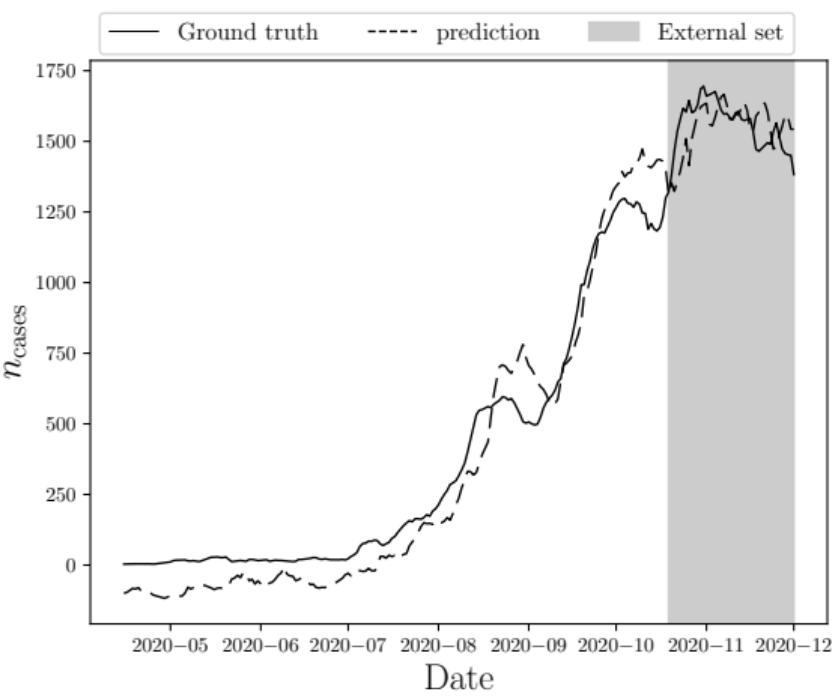
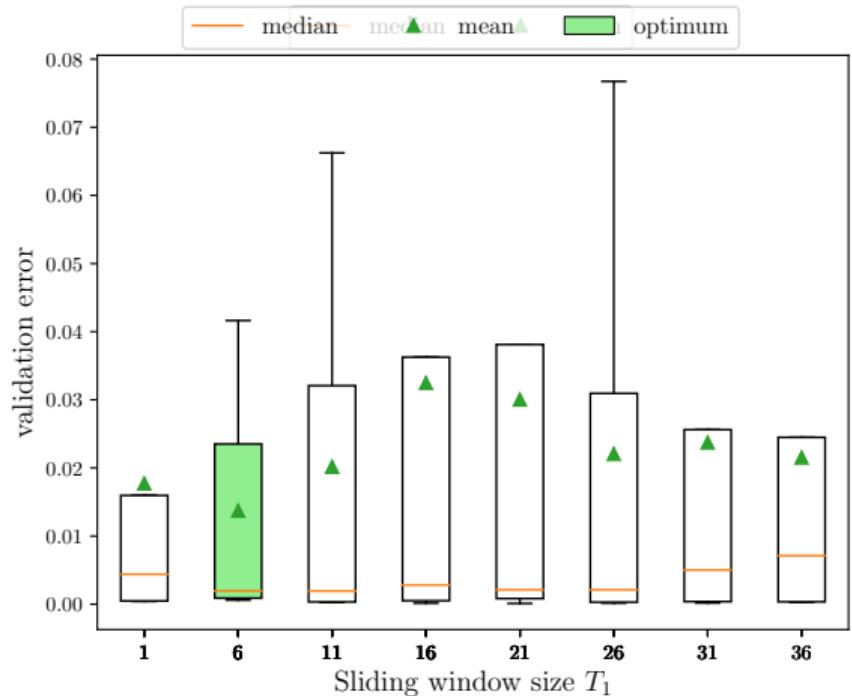
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We can use StoMADS to solve such hyperparameter optimization problems¹



Hyperparameter tuning

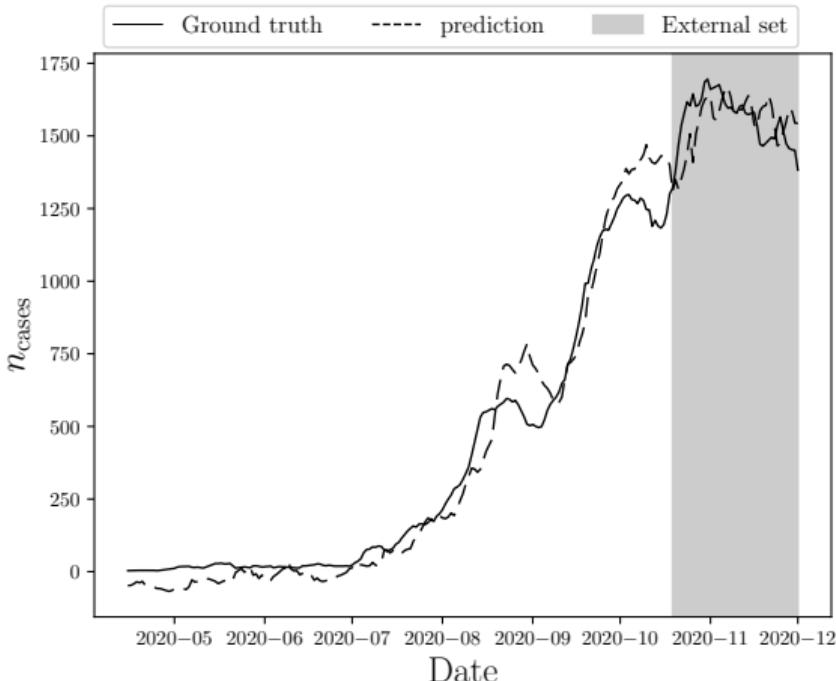
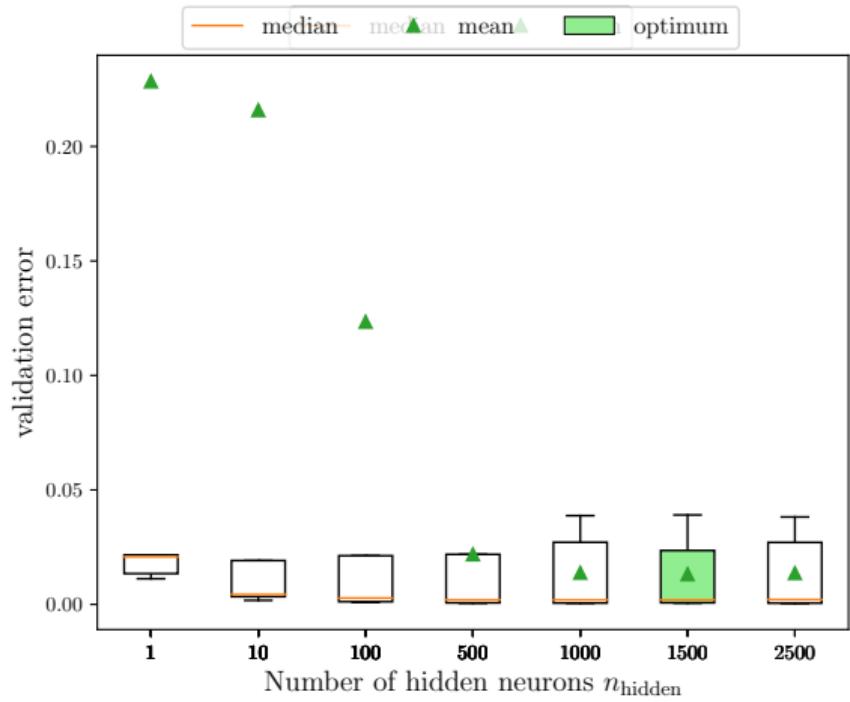
We can use StoMADS to solve such hyperparameter optimization problems¹





Hyperparameter tuning

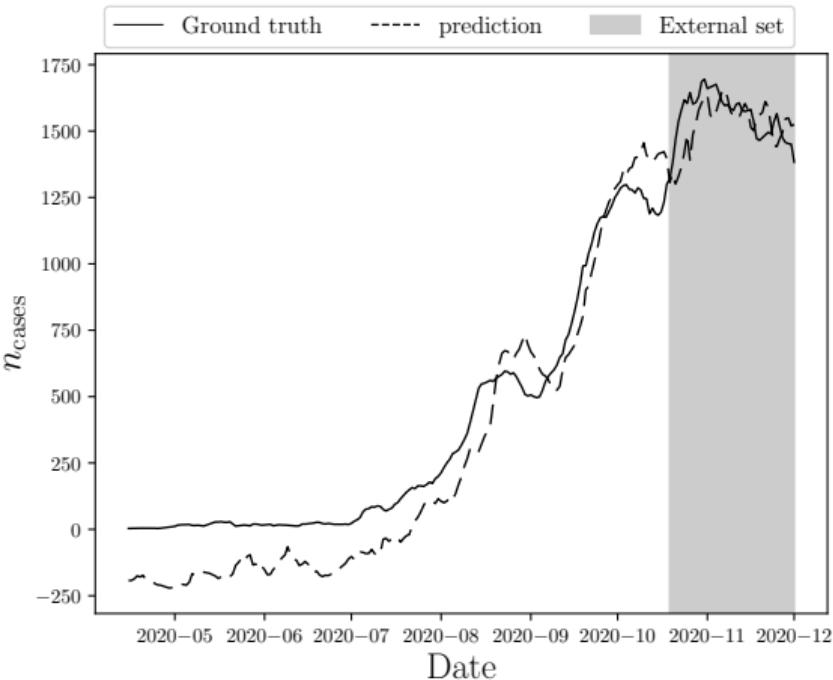
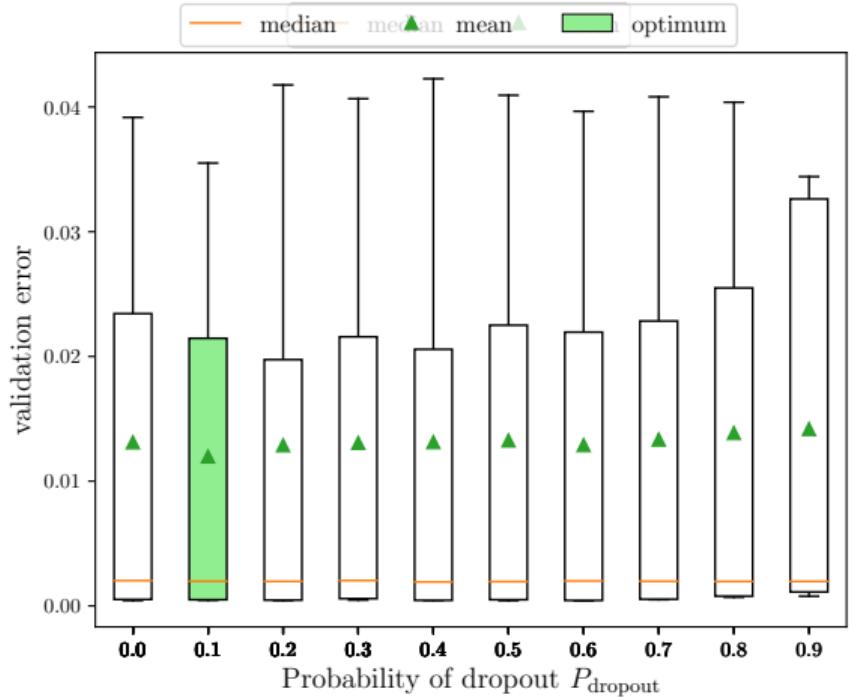
We can use StoMADS to solve such hyperparameter optimization problems¹





Hyperparameter tuning

We can use StoMADS to solve such hyperparameter optimization problems¹



Hyperparameter tuning

We can use StoMADS to solve such hyperparameter optimization problems¹

Objective and constraints

$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x) = \text{error}_{\text{CV}}]$$

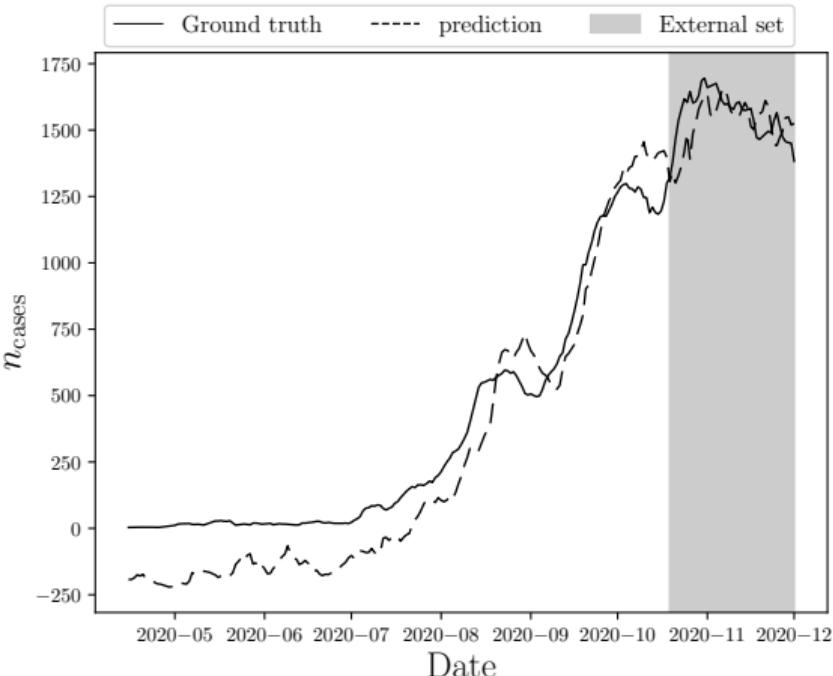
where Θ : realizations

Design variables (\mathbf{x})

- T_1 : Input dimension
- n_{hidden} : Number of hidden neurons
- P_{dropout} : Probability of dropout, etc.

Randomly seeded parameters

- Initial weights
- Gradient descent steps

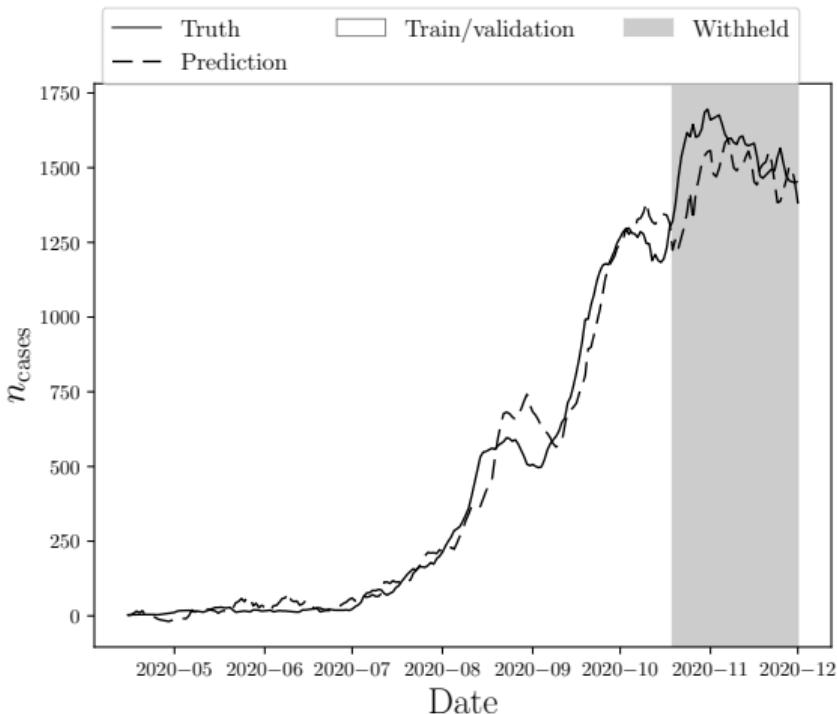




Results: Optimal hyperparameters

Optimal hyperparameters for the *Seq2Seq* model:

Hyperparameter	Value
Sliding window size	T_1
Number of hidden neurons	n_{hidden}
Probability of dropout	P_{dropout}
Number of hidden layers	n_{hidden}
Teacher forcing probability	P_{teacher}
Learning rate	l_{rate}
batch size	b_{size}

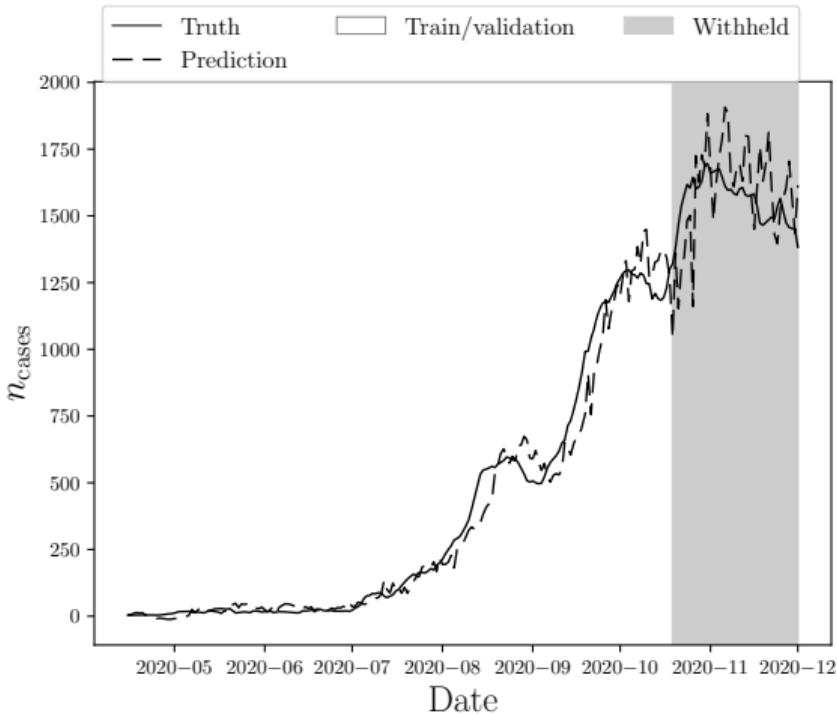


Results: Optimal hyperparameters



Optimal hyperparameters for the *support vector machine regression (SVR)* model:

Hyperparameter	Value
Sliding window size	T_1
Ridge factor	λ
Margin of tolerance	ϵ
Stopping criteria tolerance	ϵ_{tol}
Learning rate	l_{rate}



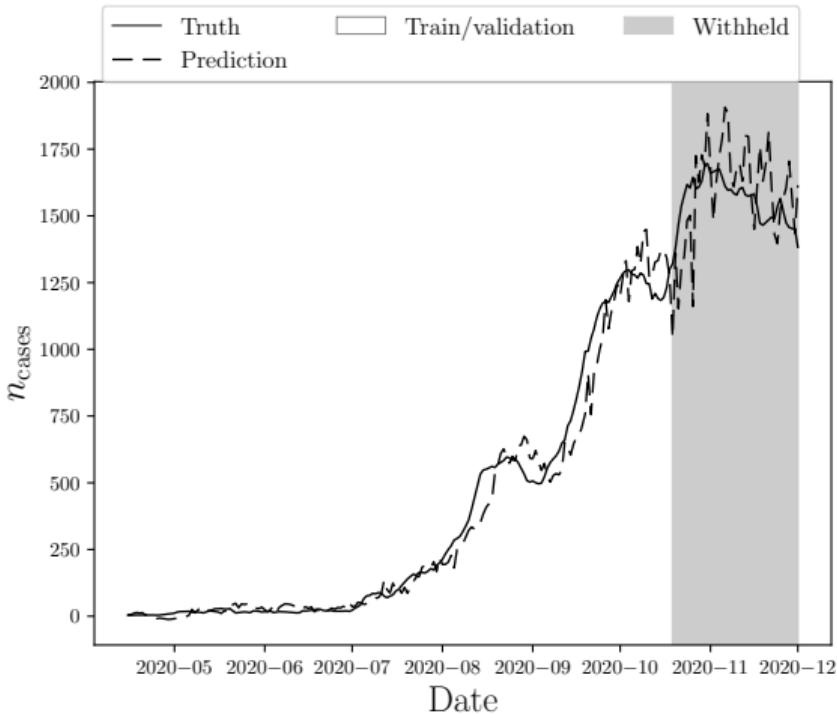
Results: Optimal hyperparameters



Optimal hyperparameters for the *support vector machine regression (SVR)* model:

Hyperparameter	Value
Sliding window size	T_1
Ridge factor	λ
Margin of tolerance	ϵ
Stopping criteria tolerance	ϵ_{tol}
Learning rate	l_{rate}

Support vector machine models have
deterministic performance

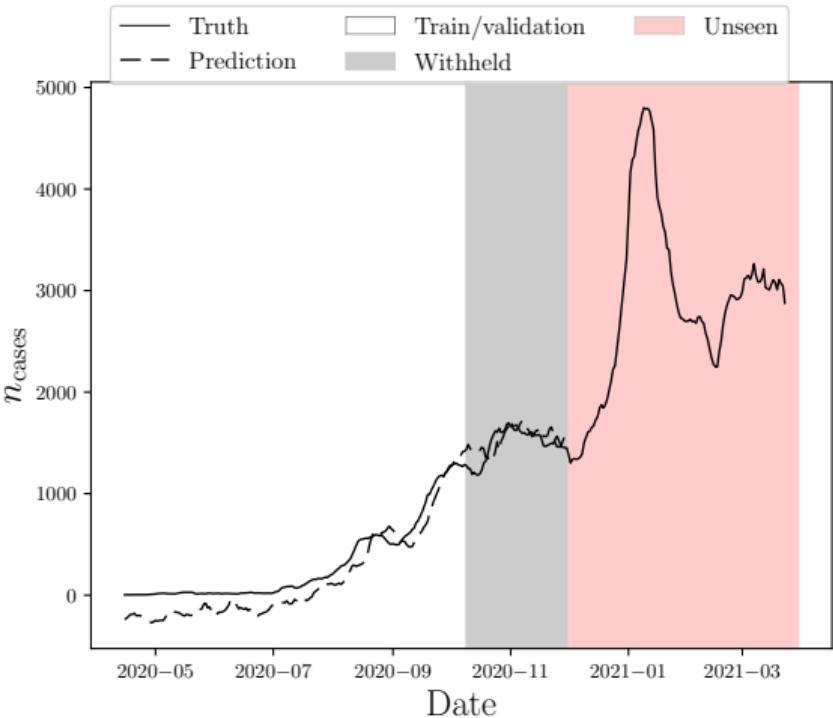




Results: Prospective validation

Performance of models on **unseen** data (first 4 months of 2021):

Model	Test error
Seq2Seq	0.571
Long short term memory (LSTM) cell	0.326
feedforward neural network	0.255
Support vector machine	0.168
Gradient boosting	1.444
Linear regression	0.160

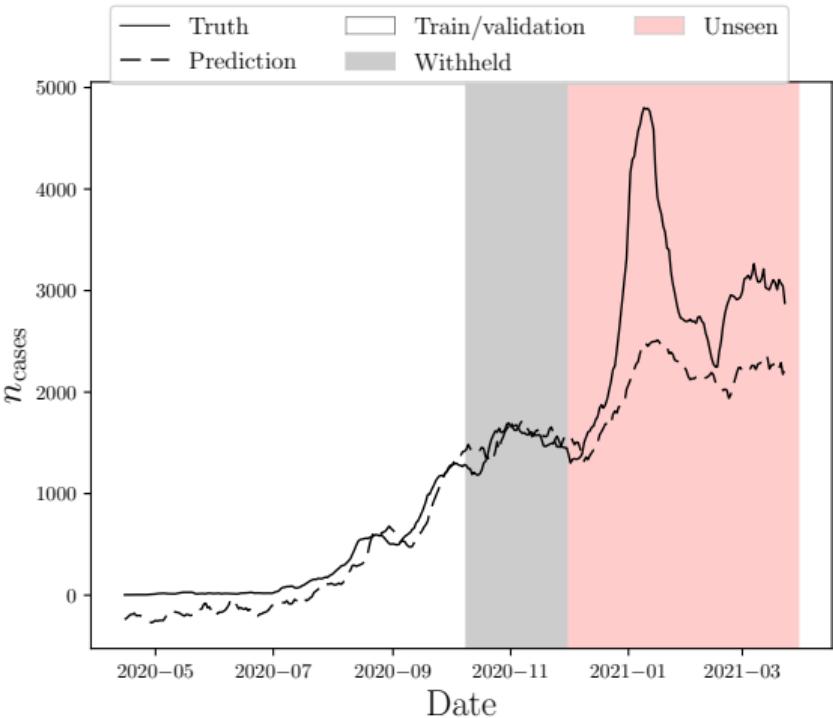




Results: Prospective validation

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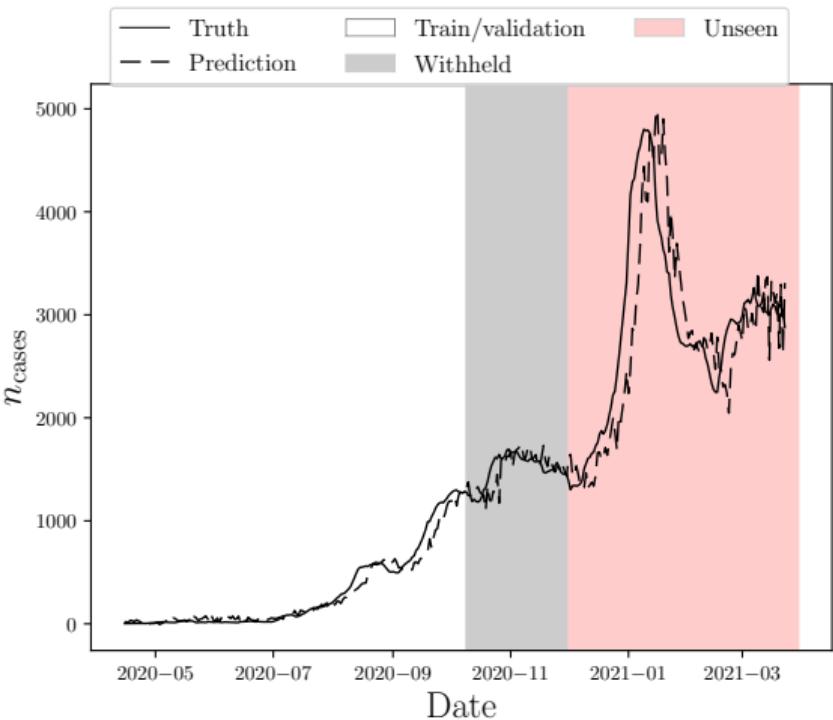




Results: Prospective validation

Performance of models on **unseen** data (first 4 months of 2021):

Model	Test error
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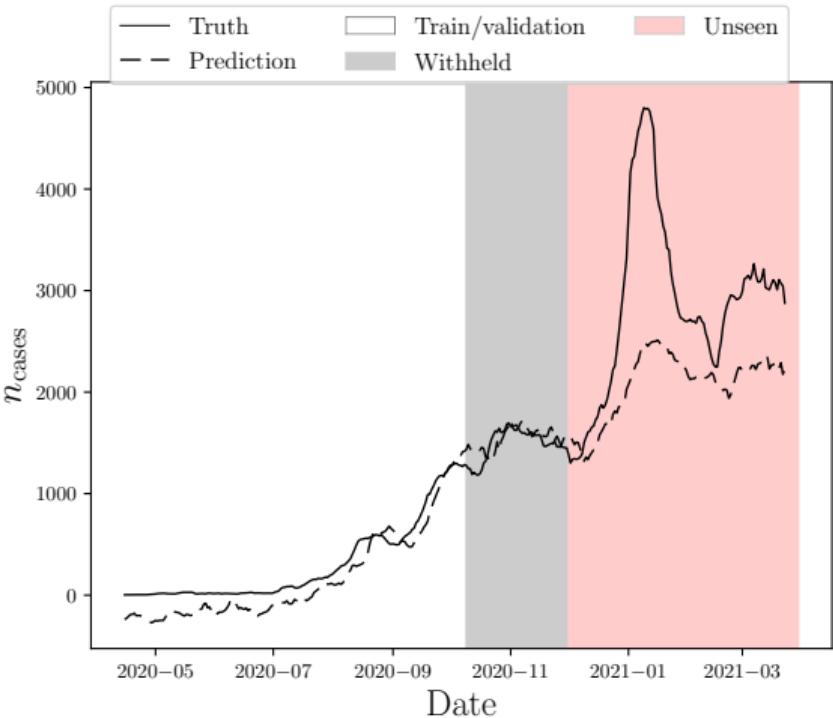




Results: Prospective validation

Effect of increasing number of training days (Adding 1 month of data):

Model	Test error
Seq2Seq	0.571
Long short term memory (LSTM) cell	0.326
feedforward neural network	0.255
Support vector machine	0.168
Gradient boosting	1.444
Linear regression	0.160

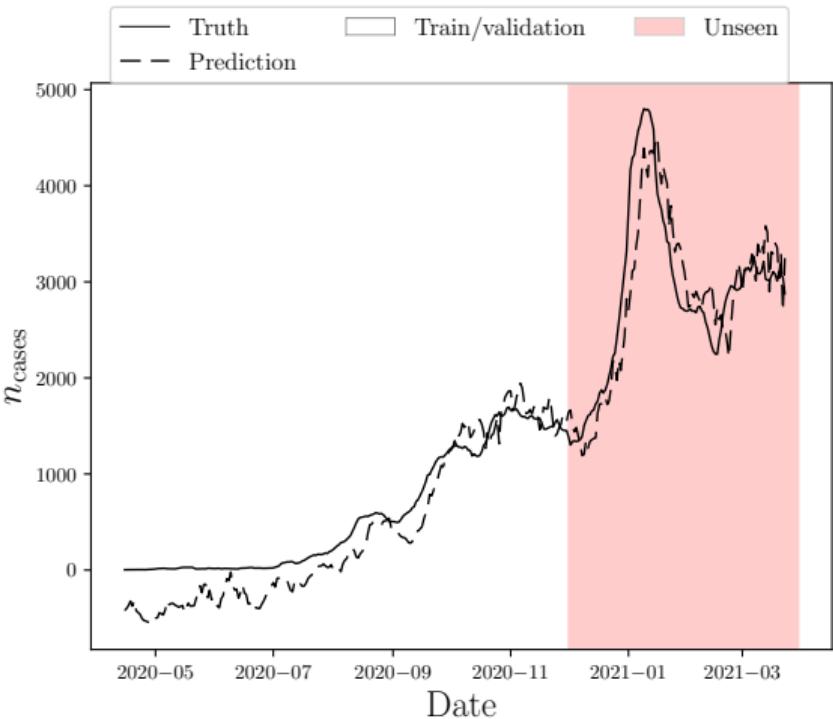




Results: Prospective validation

Effect of increasing number of training days (Adding 1 month of data):

Model	Test error
Seq2Seq	0.106
Long short term memory (LSTM) cell	0.326
feedforward neural network	0.255
Support vector machine	0.168
Gradient boosting	1.444
Linear regression	0.160

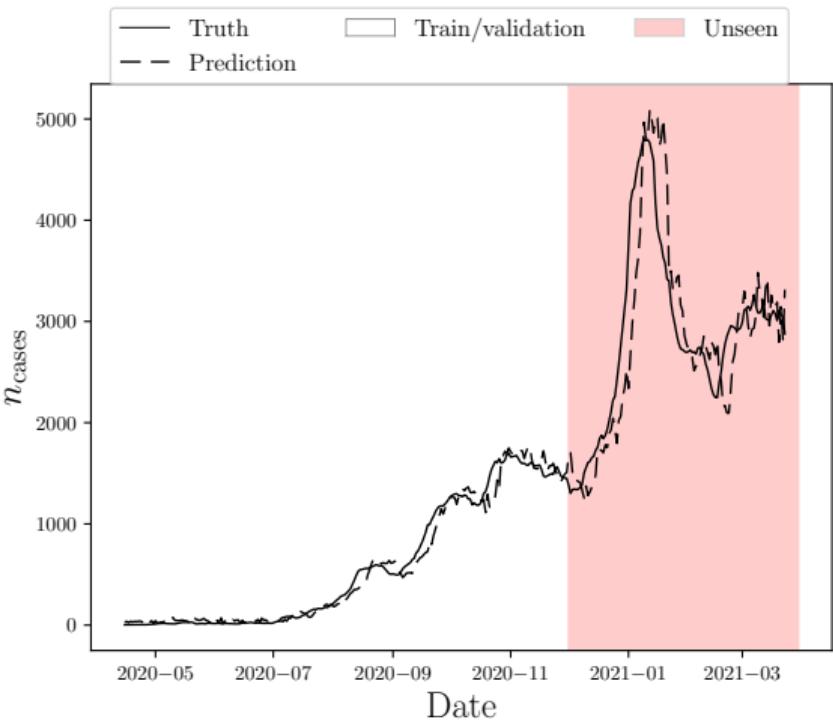




Results: Prospective validation

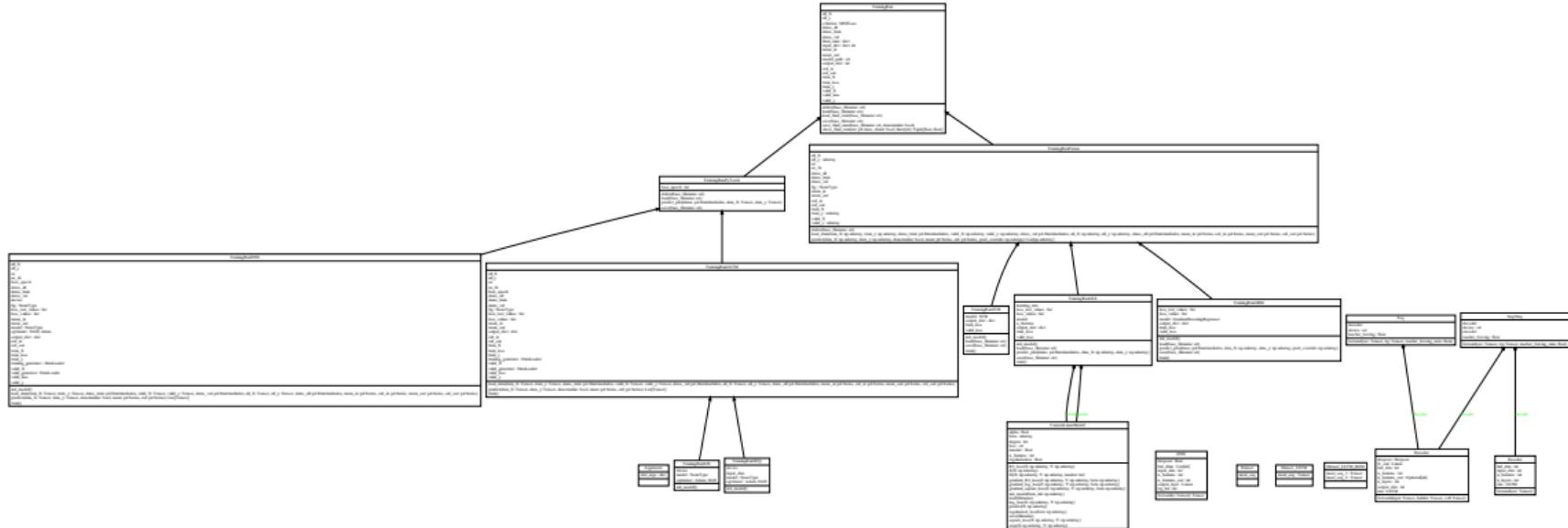
Effect of increasing number of training days (Adding 1 month of data):

Model	Test error
Seq2Seq	0.106
Long short term memory (LSTM) cell	0.326
feedforward neural network	0.255
Support vector machine	0.140
Gradient boosting	1.444
Linear regression	0.160



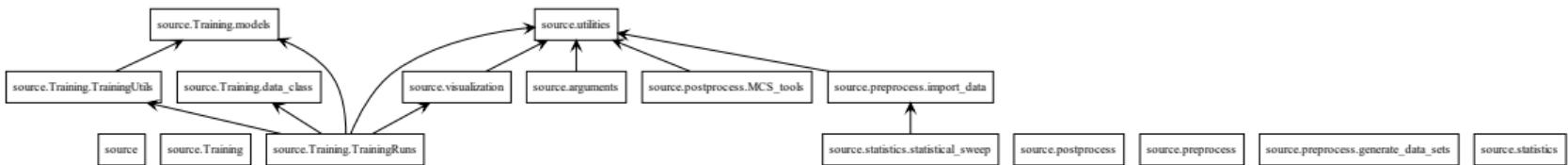


UML diagram of code





UML diagram of code

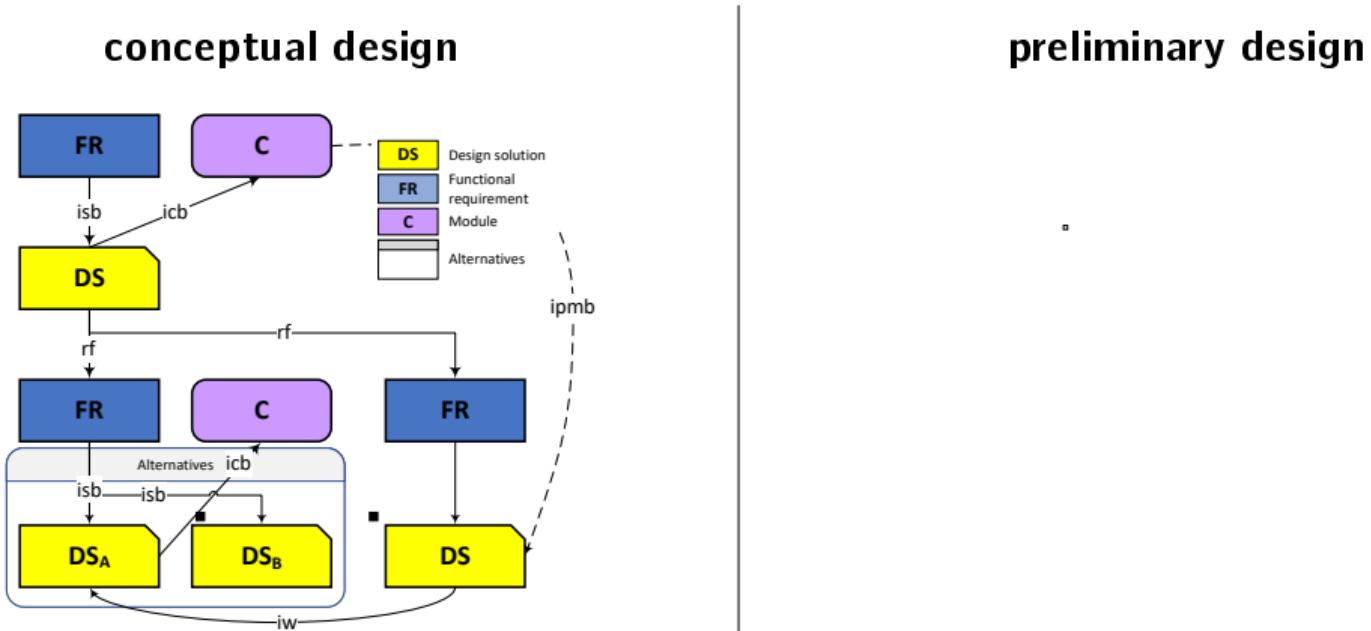


Ongoing work: Design space exploration



Design space exploration at the conceptual level

Consider a functional model (FM) of a product (conceptual design)

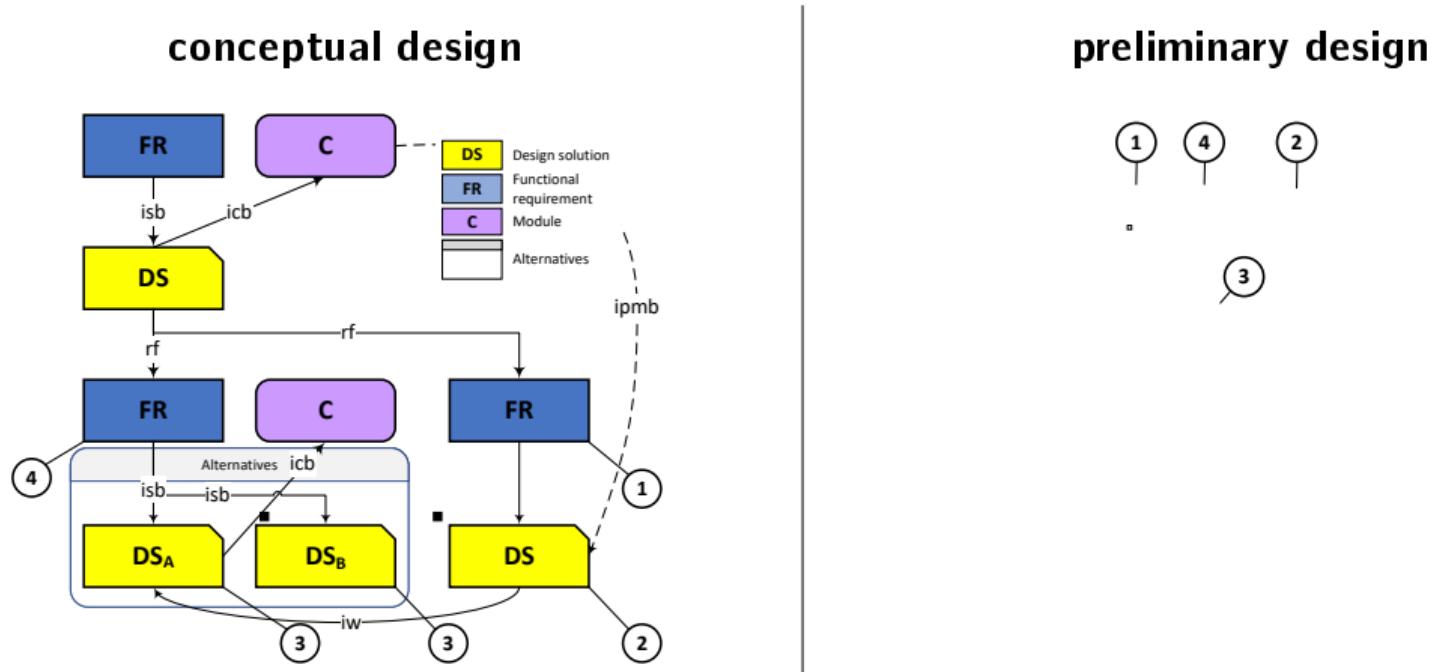


[1] N. Ahmad et al., 2013 , *Research in Engineering Design*
[2] A. Brahma and D. C. Wynn, 2020 , *Research in Engineering Design*



Design space exploration at the conceptual level

hyperlinks¹ used in multi-domain modelling

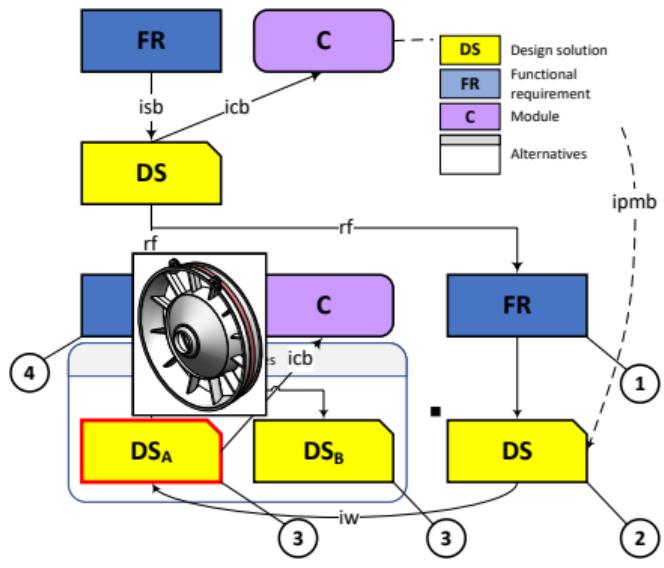


[1] N. Ahmad et al., 2013 , *Research in Engineering Design*
[2] A. Brahma and D. C. Wynn, 2020 , *Research in Engineering Design*

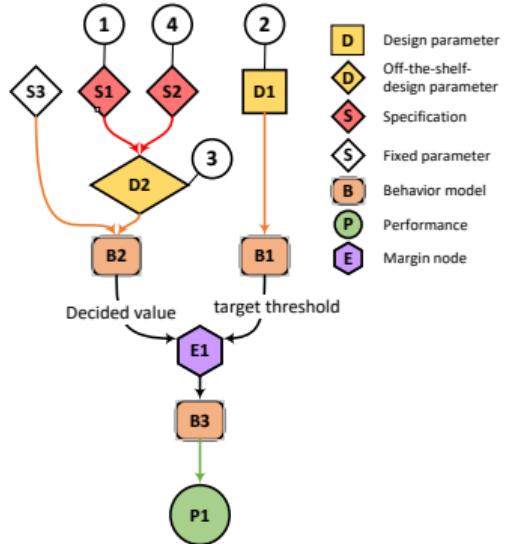
Design space exploration at the conceptual level

to link FM → margin value analysis²

conceptual design



preliminary design



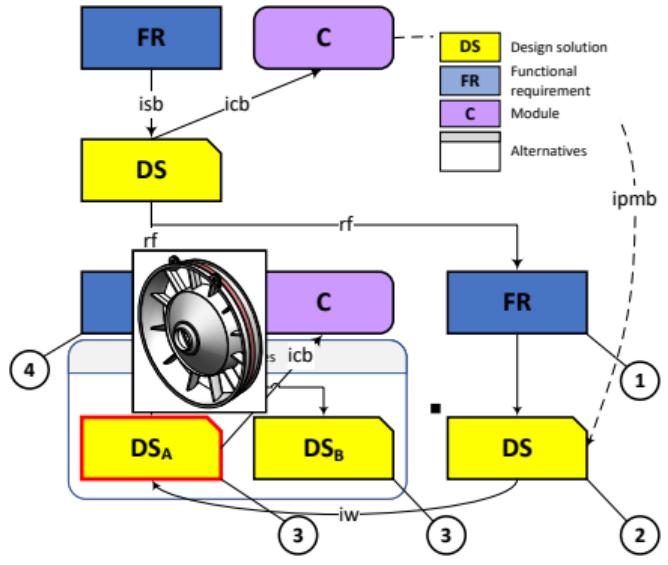
[1] N. Ahmad et al., 2013 , *Research in Engineering Design*
[2] A. Brahma and D. C. Wynn, 2020 , *Research in Engineering Design*

Design space exploration at the conceptual level

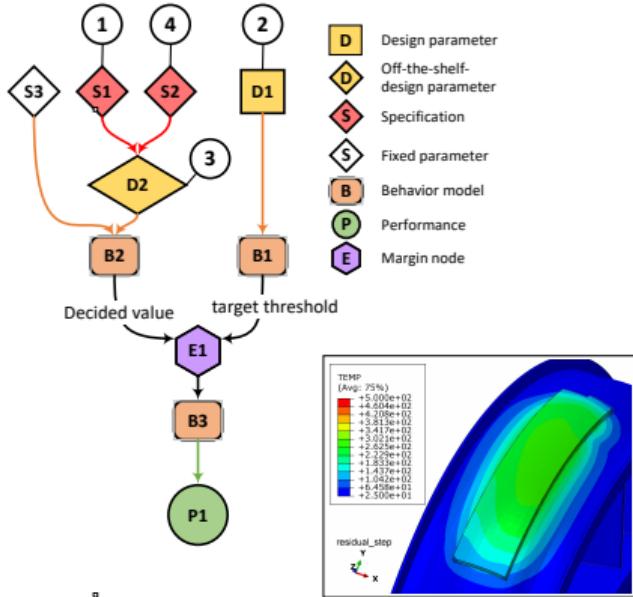


different concepts have different preliminary designs

conceptual design



preliminary design



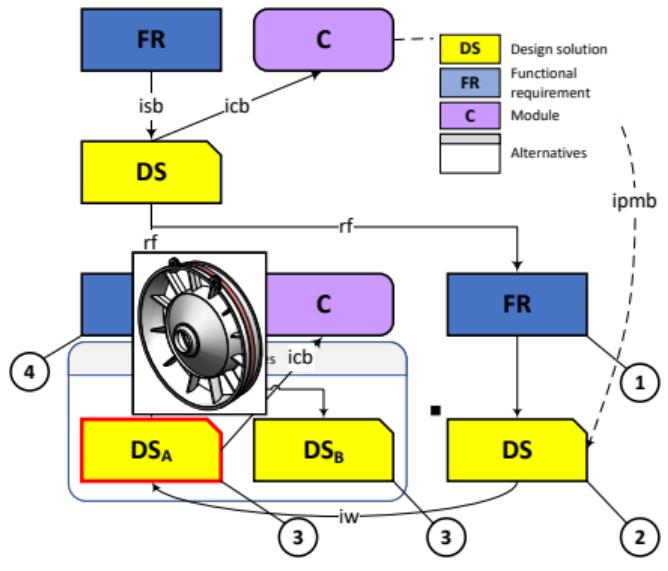
- [1] N. Ahmad et al., 2013 , *Research in Engineering Design*
[2] A. Brahma and D. C. Wynn, 2020 , *Research in Engineering Design*



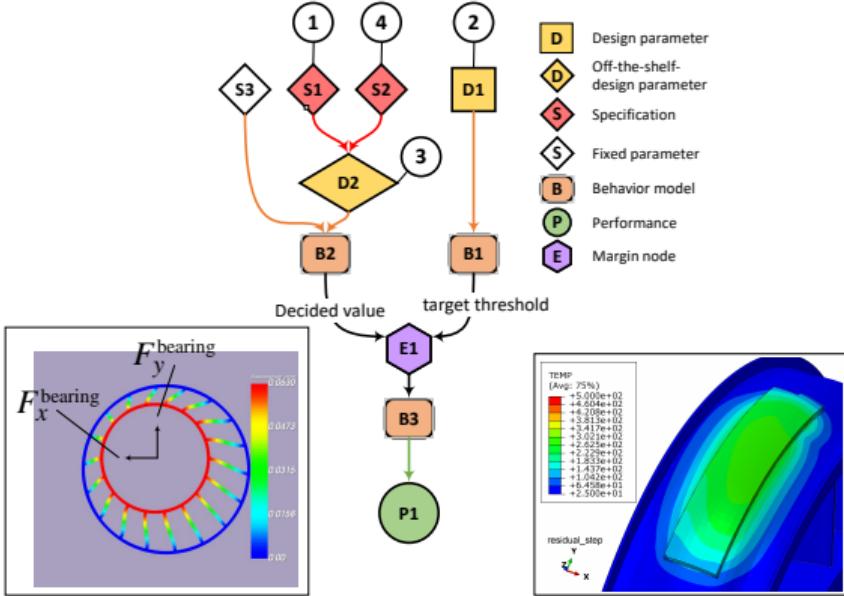
Design space exploration at the conceptual level

different concepts have different preliminary designs

conceptual design



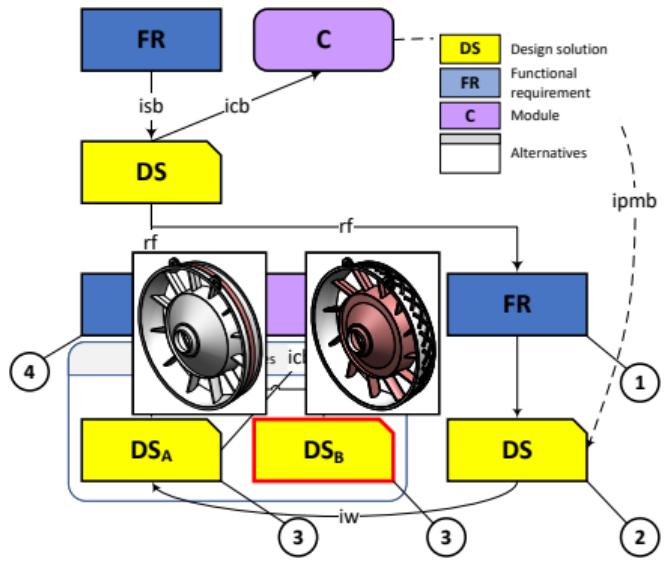
preliminary design



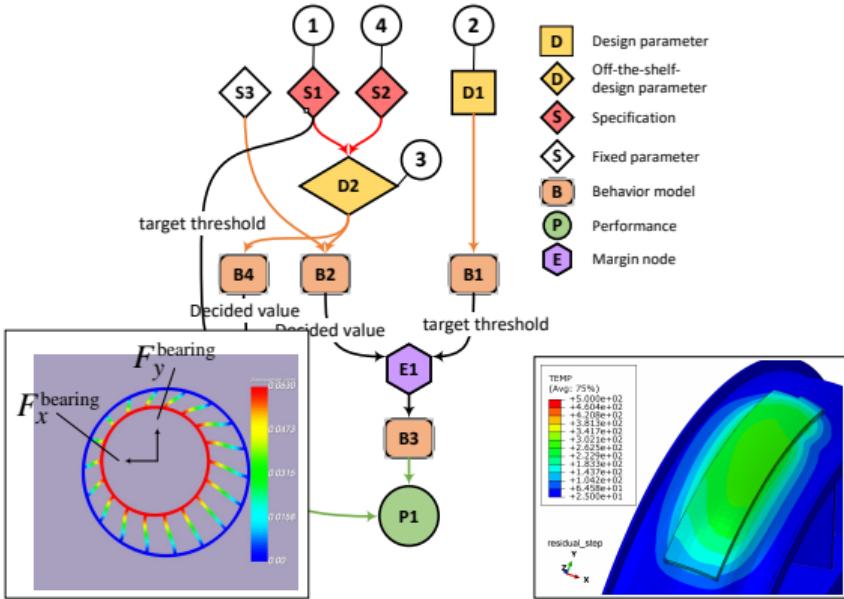
Design space exploration at the conceptual level

different concepts have different preliminary designs

conceptual design



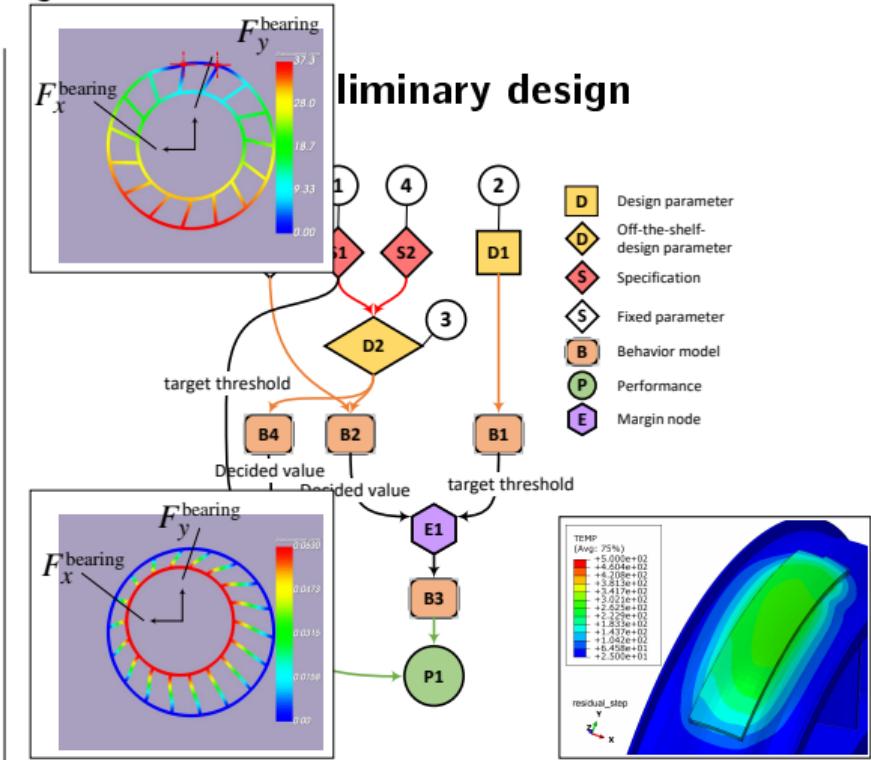
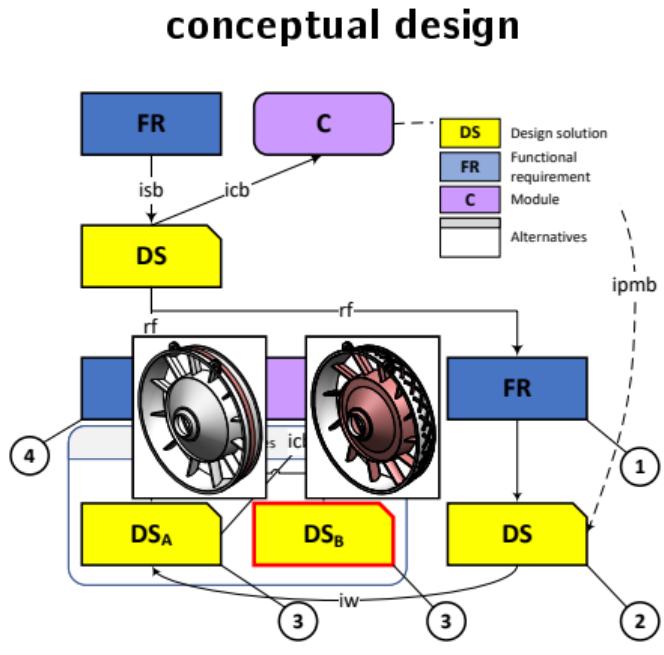
preliminary design



Design space exploration at the conceptual level



different concepts have different preliminary designs

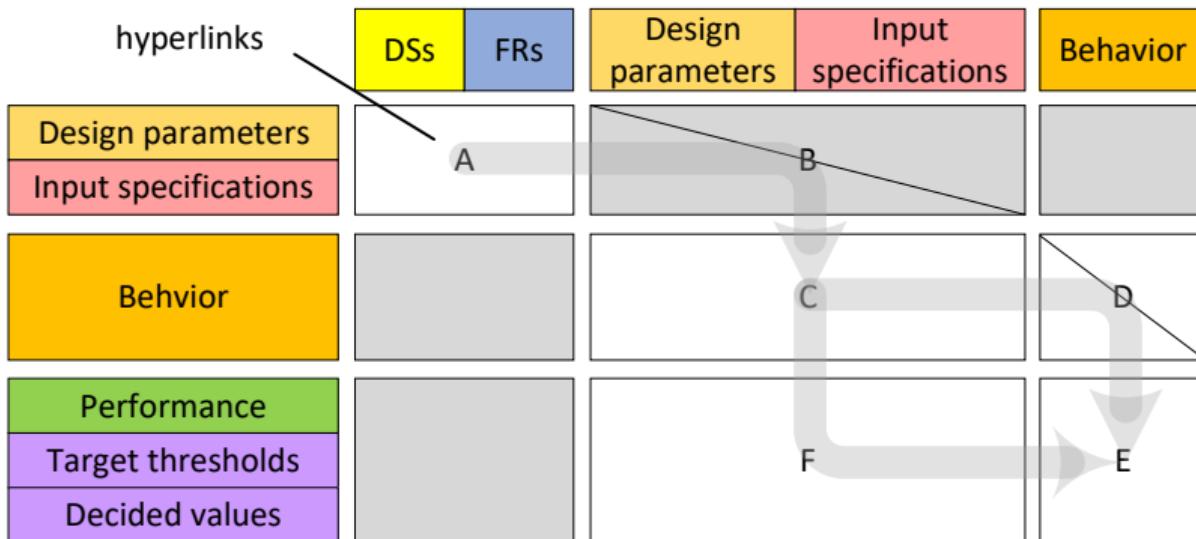


- [1] N. Ahmad et al., 2013 , *Research in Engineering Design*
[2] A. Brahma and D. C. Wynn, 2020 , *Research in Engineering Design*
K. Al Handawi

Design space exploration at the conceptual level



Multi-domain modelling (MDM) matrix for change propagation management can be used to represent hyperlinks¹





Overview of turbine rear frame design

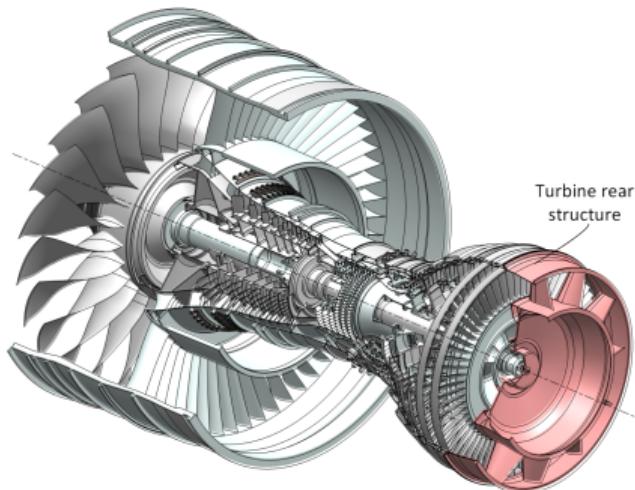
The turbine rear structure (TRS) sits aft of the turbine

Design parameters:

Input Specifications:

Design margins:

Performance parameters:



Trent 900 aeroengine



Overview of turbine rear frame design

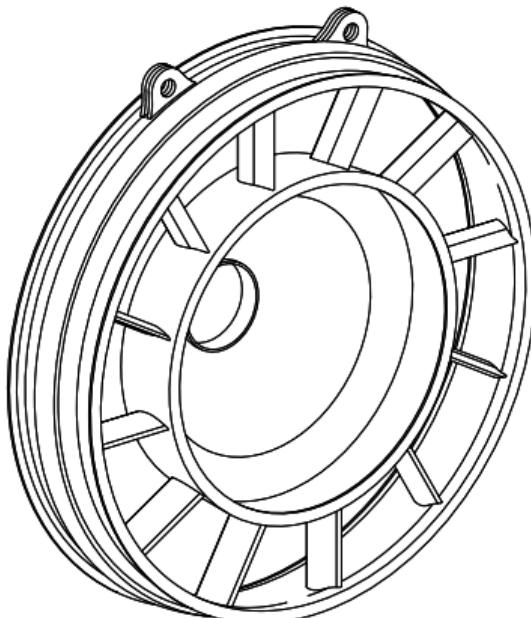
The turbine rear structure (TRS) sits aft of the turbine

Design parameters:

Input Specifications:

Design margins:

Performance parameters:



Turbine rear structure



Overview of turbine rear frame design

We consider the design of a single strut

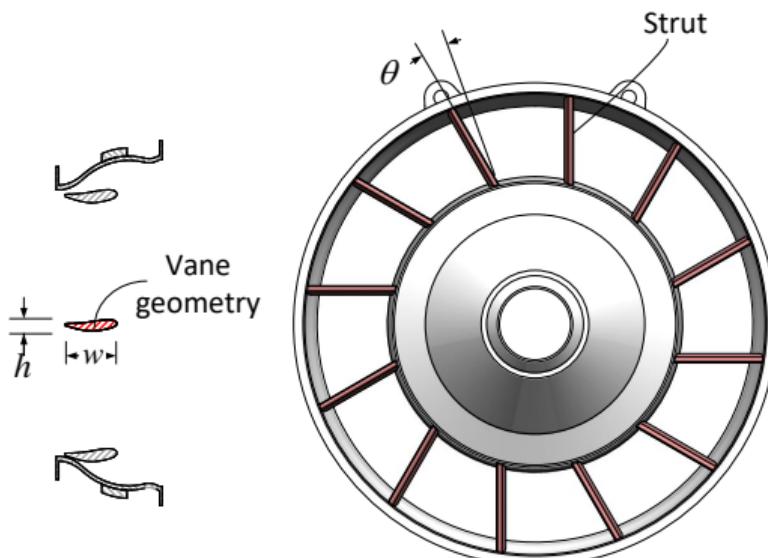
Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Input Specifications:

Design margins:

Performance parameters:



Turbine rear structure: cross-sectional view



Overview of turbine rear frame design

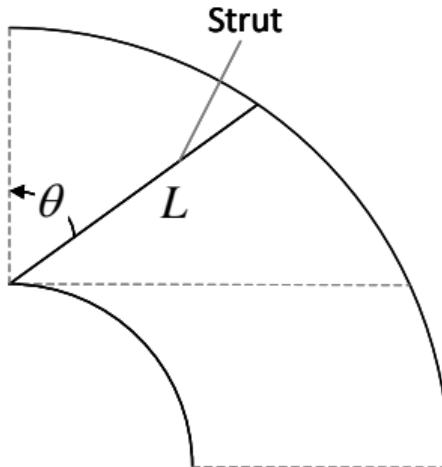
Variable thermal loads are expected

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Input Specifications:

Design margins:



Performance parameters:

Thermal expansion



Overview of turbine rear frame design

Which cause thermal expansion

Design parameters:

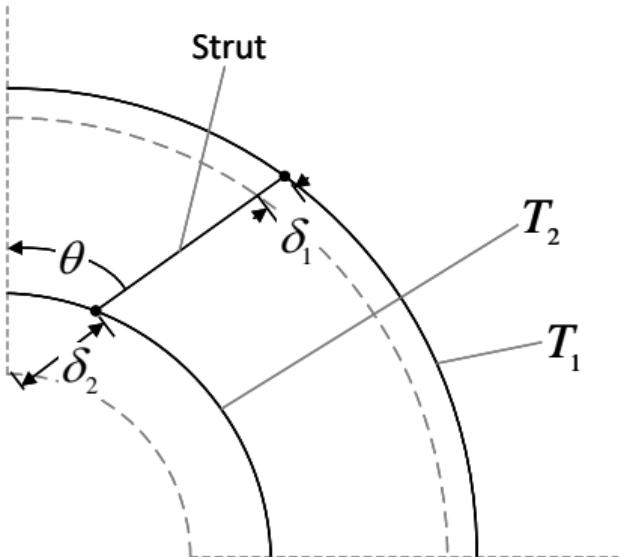
- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Input Specifications:

- Nacelle temperature T_1 (\downarrow)
- Gas surface temperature T_2 (\uparrow)

Design margins:

Performance parameters:



Thermal expansion



Overview of turbine rear frame design

Which cause thermal expansion and stress on the strut

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

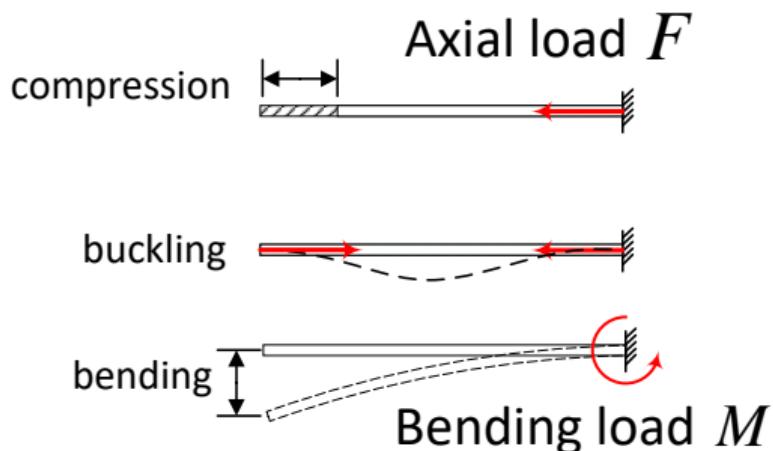
Input Specifications:

- Nacelle temperature T_1 (\downarrow)
- Gas surface temperature T_2 (\uparrow)

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:



Load cases



Overview of turbine rear frame design

Which cause thermal expansion and stress on the strut

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Input Specifications:

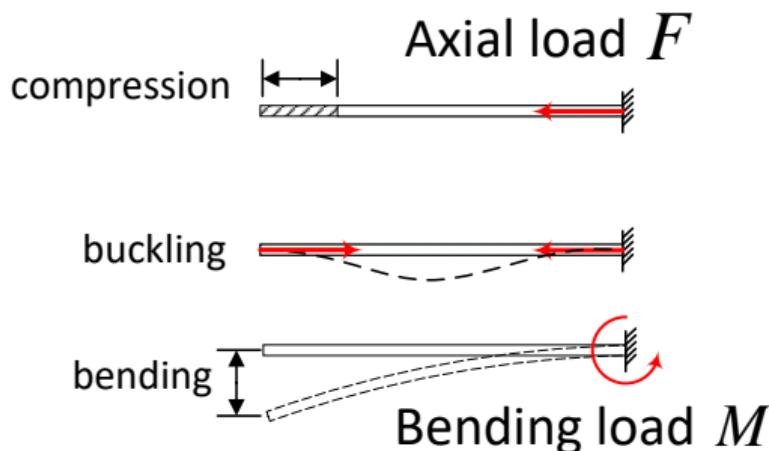
- Nacelle temperature T_1 (\downarrow)
- Gas surface temperature T_2 (\uparrow)

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:

- Weight



Load cases



Overview of turbine rear frame design

The entire problem can be represented compactly as an MDM

Design parameters:

- D_1 vane width w
- D_2 vane height h
- D_3 lean angle θ
- D_4 Material - yield stress σ_y (off-the shelf)

Input Specifications:

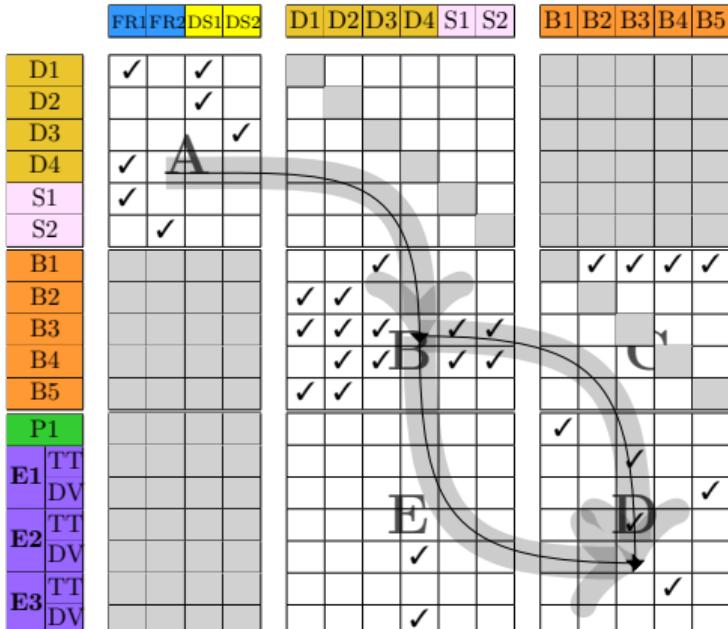
- S_1 Nacelle temperature T_1 (\downarrow)
- S_2 Gas surface temperature T_2 (\uparrow)

Design margins:

- E_1 compression $\sigma_y - \sigma_a \geq 0$
- E_2 bending $\sigma_y - \sigma_m \geq 0$
- E_3 buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:

- P_1 Weight





Calculating impact of excess margin on performance

Need to construct a response surface

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Input Specifications:

- Nacelle temperature T_1 (\downarrow)
- Gas surface temperature T_2 (\uparrow)

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:

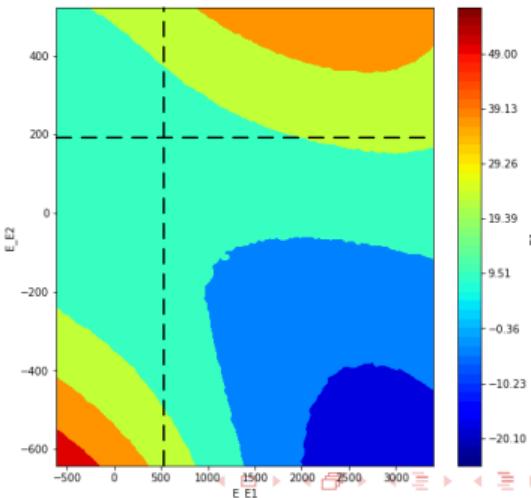
- Weight

Need to find a function f that translates
excess to performance

$$\hat{\mathbf{p}} = f(\mathbf{e})$$

$$\mathbf{p} = [p_1, p_2, \dots, p_j]^T$$

$$\mathbf{e} = [e_1, e_2, \dots, e_m]^T$$





Calculating impact of excess margin on performance

Need to construct a response surface

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Input Specifications:

- Nacelle temperature T_1 (\downarrow)
- Gas surface temperature T_2 (\uparrow)

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:

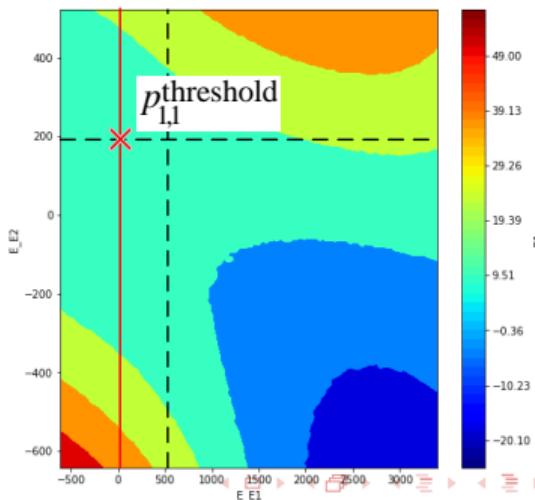
- Weight

Need to find a function f that translates
excess to performance

$$\hat{\mathbf{p}} = f(\mathbf{e})$$

$$\mathbf{p}_1^{\text{threshold}} = [p_1, p_2, \dots, p_j]^T$$

$$\mathbf{e} = [0, e_2, \dots, e_m]^T$$





Calculating impact of excess margin on performance

Need to construct a response surface

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Input Specifications:

- Nacelle temperature T_1 (\downarrow)
- Gas surface temperature T_2 (\uparrow)

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:

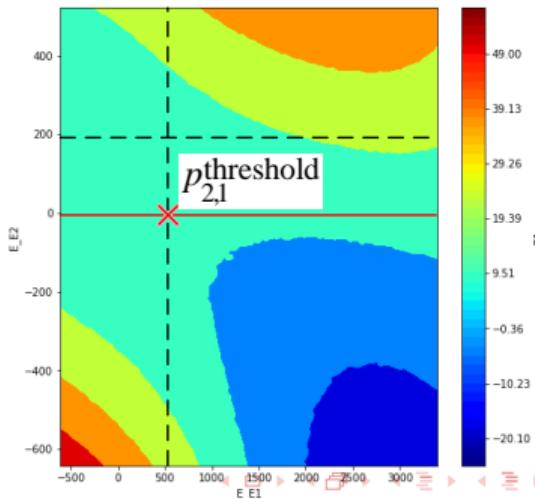
- Weight

Need to find a function f that translates
excess to performance

$$\hat{\mathbf{p}} = f(\mathbf{e})$$

$$\mathbf{p}_2^{\text{threshold}} = [p_1, p_2, \dots, p_j]^T$$

$$\mathbf{e} = [e_1, 0, \dots, e_m]^T$$





Calculating impact of excess margin on performance

Need to construct a response surface

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Input Specifications:

- Nacelle temperature T_1 (\downarrow)
- Gas surface temperature T_2 (\uparrow)

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:

- Weight

Need to find a function f that translates
excess to performance

$$\text{Impact} = \begin{matrix} \text{weight} \\ \text{buckling} \\ \text{compression} \\ \text{bending} \end{matrix} \begin{bmatrix} 0.545 \\ 0.111 \\ -1.011 \end{bmatrix}$$



Calculating deterioration (allowable change in specifications)

We calculate the maximum allowable deterioration in the **specifications**

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Gradually change each input specifications until any margin is consumed

Input Specifications:

- Nacelle temperature T_1 (\uparrow)
- Gas surface temperature T_2 (\downarrow)

$$\mathbf{s} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \begin{bmatrix} 450 \\ 425 \end{bmatrix}$$

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

$$\mathbf{e} = \begin{array}{ll} \text{buckling} & 597.440 \\ \text{compression} & 72.6904696 \\ \text{bending} & 451.9918707 \end{array}$$

Performance parameters:

- Weight



Calculating deterioration (allowable change in specifications)

We calculate the maximum allowable deterioration in the **specifications**

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Gradually change each input specifications until any margin is consumed

Input Specifications:

- Nacelle temperature T_1 (\uparrow)
- Gas surface temperature T_2 (\downarrow)

$$\mathbf{s} = \begin{matrix} T_1 \uparrow \\ T_2 \end{matrix} \begin{bmatrix} 454.5 \\ 425 \end{bmatrix}$$

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

$$\mathbf{e} = \begin{matrix} \text{buckling} \\ \text{compression} \\ \text{bending} \end{matrix} \begin{bmatrix} 488.404 \\ 0 \\ 450.4889005 \end{bmatrix}$$

Performance parameters:

- Weight



Calculating deterioration (allowable change in specifications)

We calculate the maximum allowable deterioration in the **specifications**

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Gradually change each input specifications until any margin is consumed

Input Specifications:

- Nacelle temperature T_1 (\uparrow)
- Gas surface temperature T_2 (\downarrow)

$$\mathbf{s} = \frac{T_1}{T_2 \downarrow} \begin{bmatrix} 450 \\ 420.75 \end{bmatrix}$$

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

$$\mathbf{e} = \begin{array}{ll} \text{buckling} & 488.404 \\ \text{compression} & 0 \\ \text{bending} & 450.4889005 \end{array}$$

Performance parameters:

- Weight



Calculating deterioration (allowable change in specifications)

We calculate the maximum allowable deterioration in the **specifications**

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Gradually change each input specifications until any margin is consumed

Input Specifications:

- Nacelle temperature T_1 (\uparrow)
- Gas surface temperature T_2 (\downarrow)

$$\mathbf{s}^{\max} = \begin{cases} T_1 \uparrow & [454.5] \\ T_2 \downarrow & [420.75] \end{cases}$$

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:

- Weight



Calculating deterioration (allowable change in specifications)

We calculate the maximum allowable deterioration in the **specifications**

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Gradually change each input specifications until any margin is consumed

Input Specifications:

- Nacelle temperature T_1 (\uparrow)
- Gas surface temperature T_2 (\downarrow)

$$\text{deterioration} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}$$

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:

- Weight



Calculating deterioration (allowable change in specifications)

We calculate the change absorption capability as a function of deterioration

Design parameters:

- vane width w
- vane height h
- lean angle θ
- Material - yield stress σ_y (off-the shelf)

Change absorption is calculated using the following equation:

$$\text{absorption}_{mi} = \frac{t_{mi}^{\text{new}} - t_{mi}^{\text{nominal}}}{t_{mi}^{\text{nominal}} \times \text{deterioration}_i}$$

Input Specifications:

- Nacelle temperature T_1 (\downarrow)
- Gas surface temperature T_2 (\uparrow)

for each margin node m and specification i

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

$$\text{absorption} = \begin{matrix} T_1 & T_2 \\ \text{buckling} & \begin{bmatrix} 2.79 & 3.87 \end{bmatrix} \\ \text{compression} & \begin{bmatrix} 1.98 & 3.04 \end{bmatrix} \\ \text{bending} & \begin{bmatrix} 2.58 & 3.66 \end{bmatrix} \end{matrix}$$

Performance parameters:

- Weight



Constructing the margin value plot (MVP)

Using “impact” and “absorption” we can construct a tradespace (MVP)

Design parameters:

- vane width
- vane height
- lean angle
- Material - yield stress σ_y (off-the shelf)

Input Specifications:

- Nacelle temperature T_1 (\downarrow)
- Gas surface temperature T_2 (\uparrow)

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:

- Weight

$$\text{Impact} = \begin{matrix} & \text{weight} \\ \text{buckling} & [0.545] \\ \text{compression} & [0.111] \\ \text{bending} & [-1.011] \end{matrix}$$

$$\text{absorption} = \begin{matrix} & T_1 & T_2 \\ \text{buckling} & [2.79 & 3.87] \\ \text{compression} & [1.98 & 3.04] \\ \text{bending} & [2.58 & 3.66] \end{matrix}$$



Constructing the margin value plot (MVP)

Using “impact” and “absorption” we can construct a tradespace (MVP)

Design parameters:

- vane width
- vane height
- lean angle
- Material - yield stress σ_y (off-the shelf)

We aggregate across each performance and specification

Input Specifications:

- Nacelle temperature T_1 (\downarrow)
- Gas surface temperature T_2 (\uparrow)

$$\text{Impact} = \begin{matrix} \text{buckling} & [0.545] \\ \text{compression} & [0.111] \\ \text{bending} & [-1.011] \end{matrix}$$

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

$$\text{absorption} = \begin{matrix} \text{buckling} & [3.33] \\ \text{compression} & [2.51] \\ \text{bending} & [3.12] \end{matrix}$$

Performance parameters:

- Weight



Constructing the margin value plot (MVP)

Using “impact” and “absorption” we can construct a tradespace (MVP)

Design parameters:

- vane width $w = 6.78$
- vane height $h = 125.60$
- lean angle $\theta = 14.6$
- Material - yield stress σ_y (off-the shelf)

Design A: $d = 2.70$

Input Specifications:

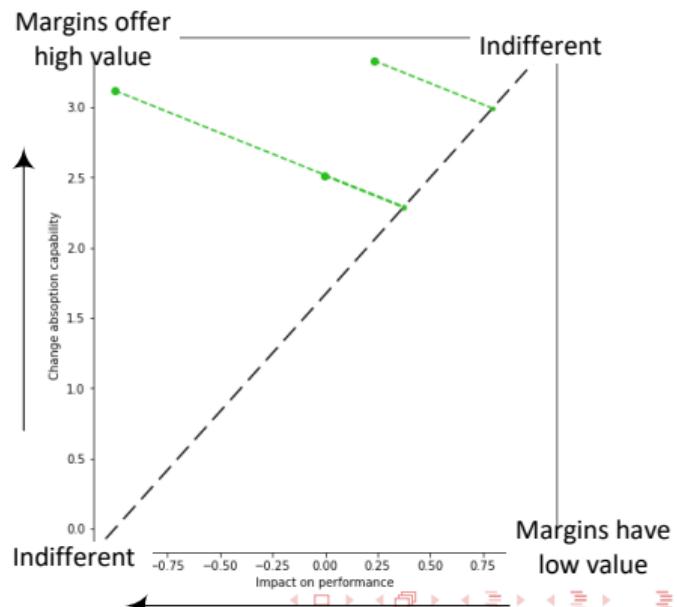
- Nacelle temperature $T_1 (\downarrow)$
- Gas surface temperature $T_2 (\uparrow)$

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:

- Weight





Constructing the margin value plot (MVP)

Using “impact” and “absorption” we can construct a tradespace (MVP)

Design parameters:

- vane width $w = 11.5$
- vane height $h = 113.33$
- lean angle $\theta = 18.0$
- Material - yield stress σ_y (off-the shelf)

Input Specifications:

- Nacelle temperature $T_1 (\downarrow)$
- Gas surface temperature $T_2 (\uparrow)$

Design margins:

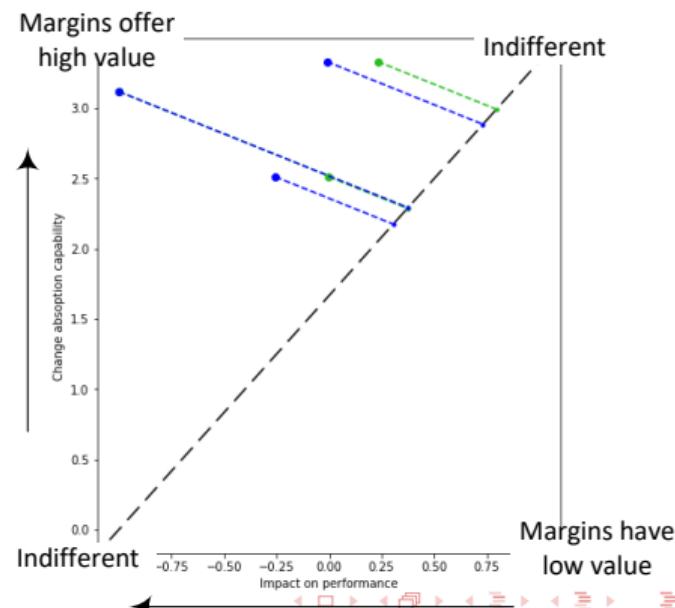
- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

Performance parameters:

- Weight

Design A: $d = 2.70$

Design B: $d = 3.13$





Constructing the margin value plot (MVP)

Using “impact” and “absorption” we can construct a tradespace (MVP)

Design parameters:

- vane width $w = 5.42$
- vane height $h = 106.14$
- lean angle $\theta = 15.7$
- Material - yield stress σ_y (off-the shelf)

Input Specifications:

- Nacelle temperature $T_1 (\downarrow)$
- Gas surface temperature $T_2 (\uparrow)$

Design margins:

- compression $\sigma_y - \sigma_a \geq 0$
- bending $\sigma_y - \sigma_m \geq 0$
- buckling $F_{\text{buckling}} - F_a \geq 0$

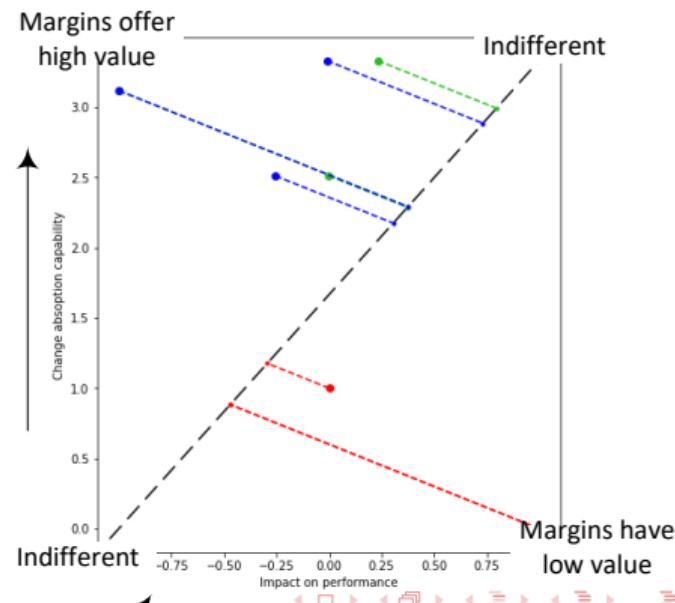
Performance parameters:

- Weight

Design A: $d = 2.70$

Design B: $d = 3.13$

Design C: $d = -3.77$





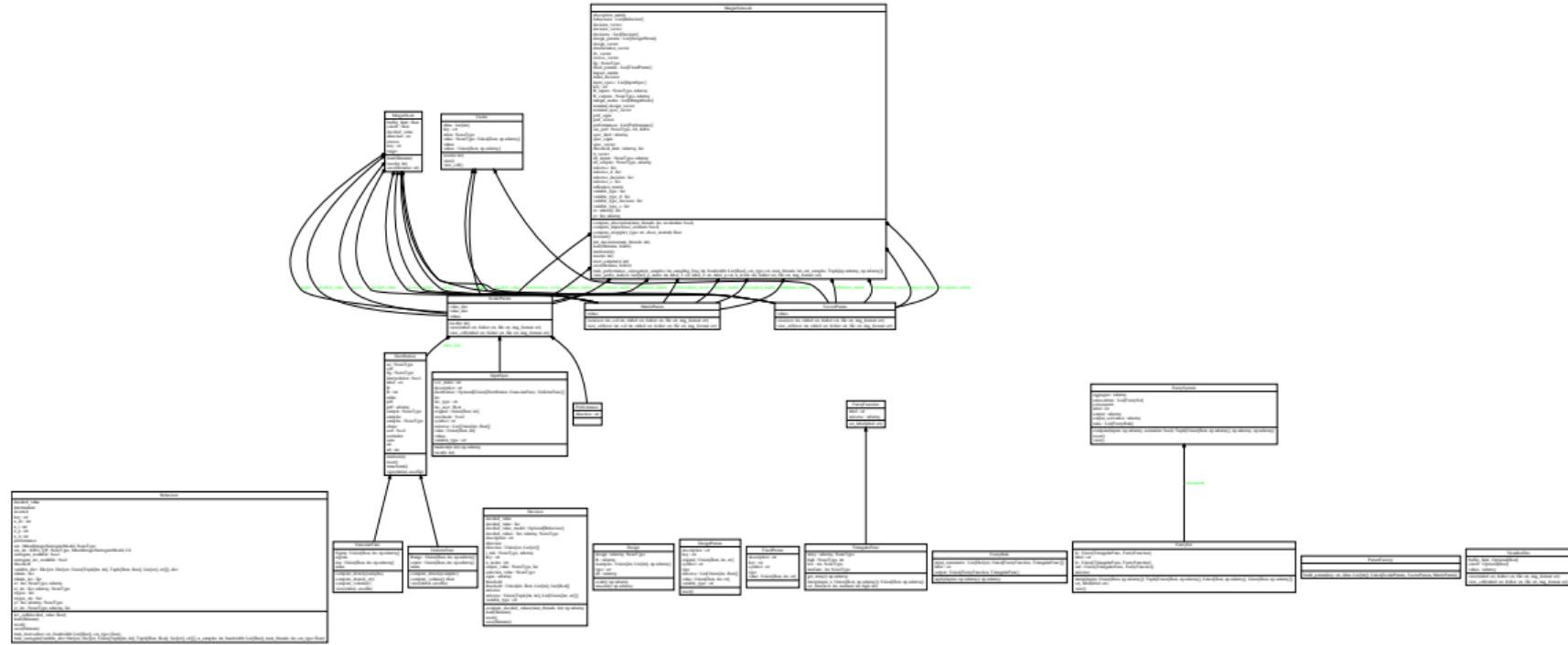
Pseudo-code of MVM

Input: set of possible choices \mathcal{D} , chosen design $c \in \mathcal{D}$, the MAN given by analytic functions, models, and surrogates, n_{samples} , Joint PDF $F_S(s)$

Output: estimated mean impact and change absorption $\bar{\alpha}, \bar{i}$

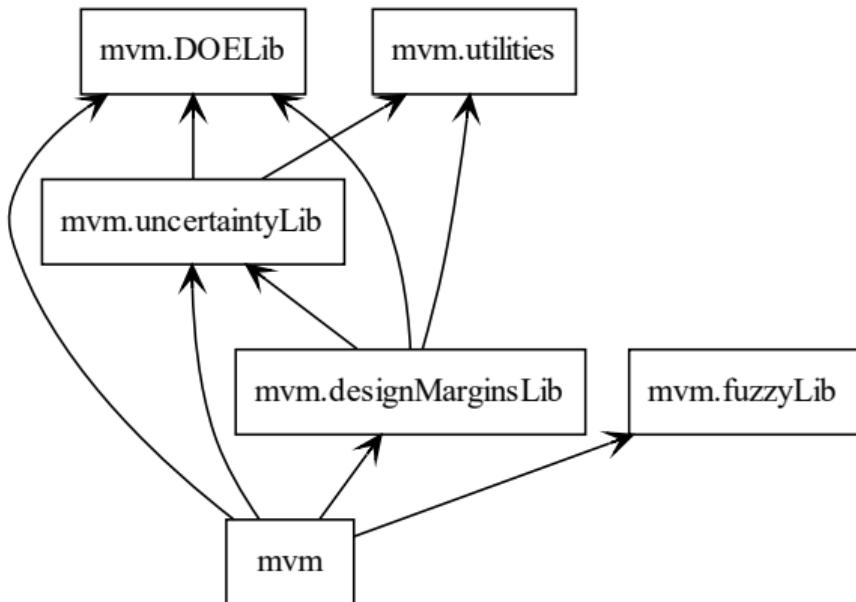
- [0] Initialization: set counter $i_{\text{sample}} \leftarrow 0$, initialize aggregate absorption and impact $\bar{\alpha} \in \mathbb{R}^e$ and $\bar{i} \in \mathbb{R}^e$
- [1] Monte-carlo simulation: draw sample input specifications $S \in \mathbb{R}^n \sim F_S(s)$
- [2] Calculate threshold, decided, excess, and performance values: $t = f(S, c)$, $d = g(c)$, $e = (t - d)^{\text{excess}^T}$, $p = h(S, c)$
- [3] Calculate allowable deterioration
 - loop over input specifications for $k = 1, 2, \dots, n$ do
 - perform a line search along the k th direction to identify the root of: $\min((f(x, c) - d)^{\text{excess}^T}) = 0$, $s'_k \leftarrow x^*$, $s_k^{\text{limit}} \leftarrow s'_k$
- [4] Calculate margin value metrics
 - initialization tensors and vectors, $I \in \mathbb{R}^{m \times e} \leftarrow 0$ and $A \in \mathbb{R}^{n \times e} \leftarrow 0$, $v \in \mathbb{R}^n \leftarrow 0$, $\bar{a} \in \mathbb{R}^e \leftarrow 0$, and $\bar{i} \in \mathbb{R}^e \leftarrow 0$
 - loop over margin nodes for $i = 1, 2, \dots, e$ do
 - [a] Calculate impact on performance
 $d^{\text{threshold}} \leftarrow d$, $d_i^{\text{threshold}} \leftarrow t_i$, $c_i^{\text{threshold}} = g^{-1}(d^{\text{threshold}})$, $p_i^{\text{threshold}} = h(S, c_i^{\text{threshold}})$
 - loop over performance parameters for $j = 1, 2, \dots, m$ do
 $i_{i,j} \leftarrow (p_j - p_{j,i}^{\text{threshold}})I_j^p / p_{j,i}^{\text{threshold}}$
 - [b] Calculate change absorption capability
 - loop over input specifications for $k = 1, 2, \dots, n$ do
 - calculate deterioration $v_k = (s_k^{\text{limit}} - s_k) / s_k$
 - $a_{i,k} \leftarrow (f_i(s_k^{\text{limit}}, c) - f_i(S, c)) / (f_i(S, c)v_k)$
 - [c] Aggregate metrics: $\bar{a}_i \leftarrow \frac{1}{n} \sum_{k=1}^n a_{i,k}$, $\bar{i}_i \leftarrow \frac{1}{m} \sum_{k=1}^m i_{i,k}$
 - [5] sum of monte-carlo samples: $\bar{\alpha} \leftarrow \bar{\alpha} + \bar{a}$, $\bar{i} \leftarrow \bar{i} + \bar{i}$
 - [6] Termination, if $i_{\text{sample}} < n_{\text{samples}}$ then
 - set $i_{\text{sample}} \leftarrow i_{\text{sample}} + 1$, go to [1]
 - otherwise $\bar{\alpha} \leftarrow \bar{\alpha} / n_{\text{samples}}$ and $\bar{i} \leftarrow \bar{i} / n_{\text{samples}}$, stop

UML diagram of mvmlib¹





UML diagram of mvmlib¹



Thank you for your time



Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

Objective and constraints

$$\min_x f(x)$$

$$\text{subject to } c(x) \leq 0$$

where x : variables

- SEARCH step and POLL step



Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

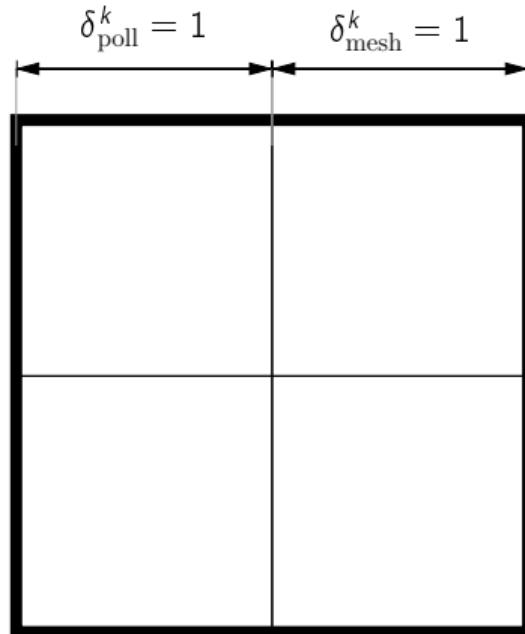
Objective and constraints

$$\min_x f(x)$$

$$\text{subject to } c(x) \leq 0$$

where x : variables

- SEARCH step and POLL step
- we focus on the POLL step





Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

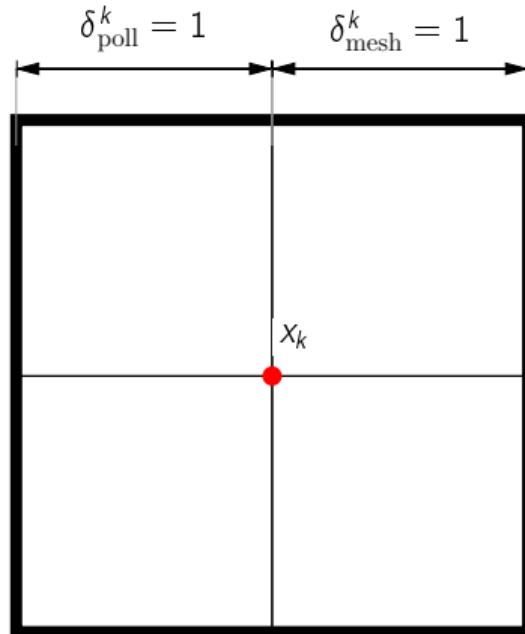
Objective and constraints

$$\min_x f(x)$$

$$\text{subject to } c(x) \leq 0$$

where x : variables

- SEARCH step and POLL step
- we focus on the POLL step
- incumbent solution: x_k





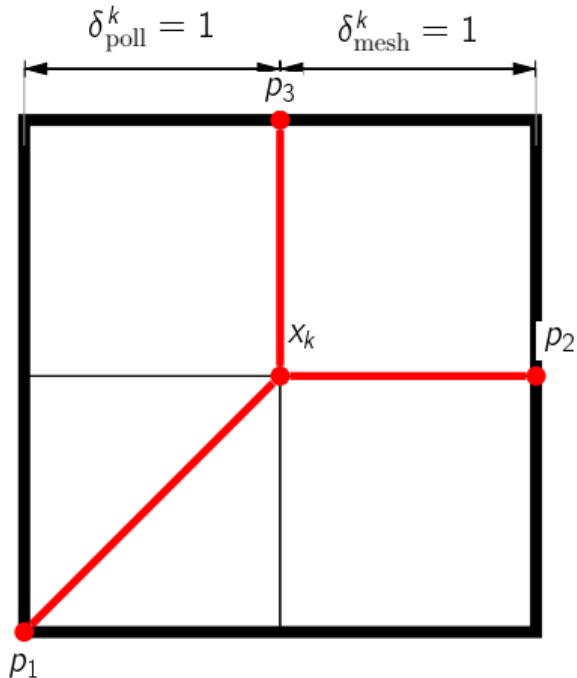
Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

Objective and constraints

$$\begin{array}{ll} \min_x & f(x) \\ \text{subject to} & c(x) \leq 0 \\ \text{where} & x : \text{variables} \end{array}$$

- SEARCH step and POLL step
- we focus on the POLL step
- incumbent solution: x_k
- trail points: $\{p_1, p_2, \dots, p_k\}$





Overview of mesh adaptive direct search

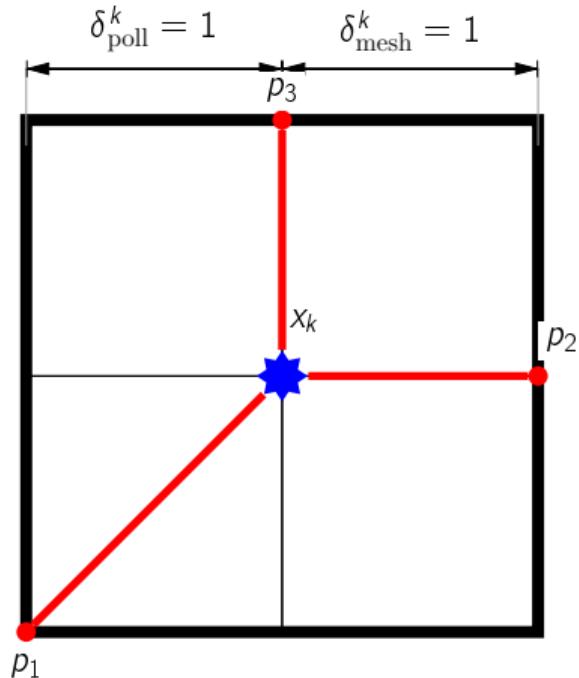
No gradient information available, blackbox is expensive¹

Objective and constraints

$$\begin{array}{ll} \min_x & f(x) \\ \text{subject to} & c(x) \leq 0 \\ \text{where} & x : \text{variables} \end{array}$$

- SEARCH step and POLL step
- we focus on the POLL step
- incumbent solution: x_k
- trail points: $\{p_1, p_2, \dots, p_k\}$

Poll **failure**





Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

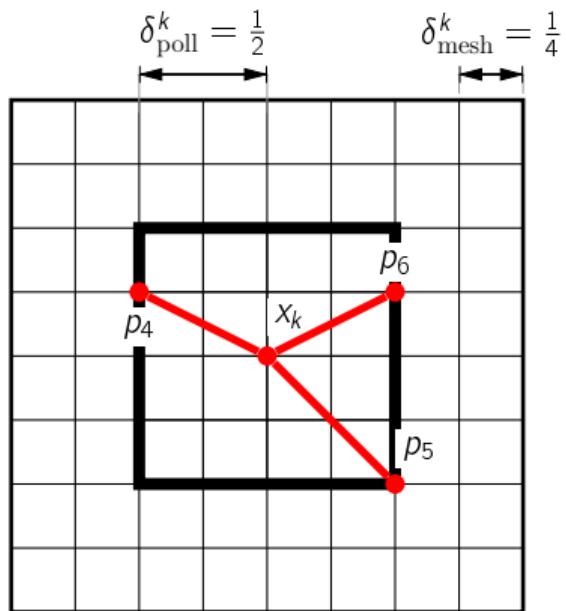
Objective and constraints

$$\min_x f(x)$$

$$\text{subject to } c(x) \leq 0$$

where x : variables

- SEARCH step and POLL step
- we focus on the POLL step
- incumbent solution: x_k
- trail points: $\{p_1, p_2, \dots, p_k\}$





Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

Objective and constraints

$$\min_x f(x)$$

$$\text{subject to } c(x) \leq 0$$

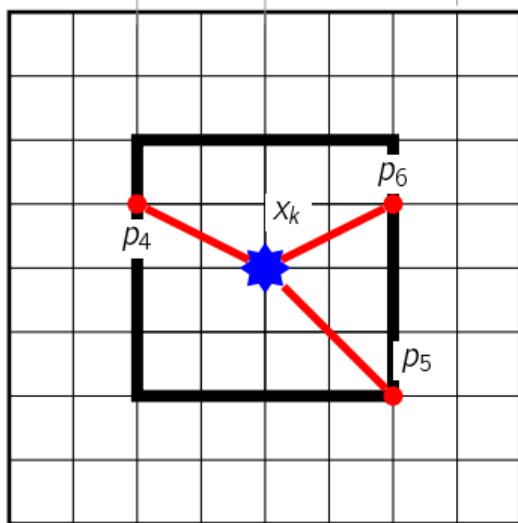
where x : variables

- SEARCH step and POLL step
- we focus on the POLL step
- incumbent solution: x_k
- trail points: $\{p_1, p_2, \dots, p_k\}$

Poll **failure**

$$\delta_{\text{poll}}^k = \frac{1}{2}$$

$$\delta_{\text{mesh}}^k = \frac{1}{4}$$





Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

Objective and constraints

$$\min_x f(x)$$

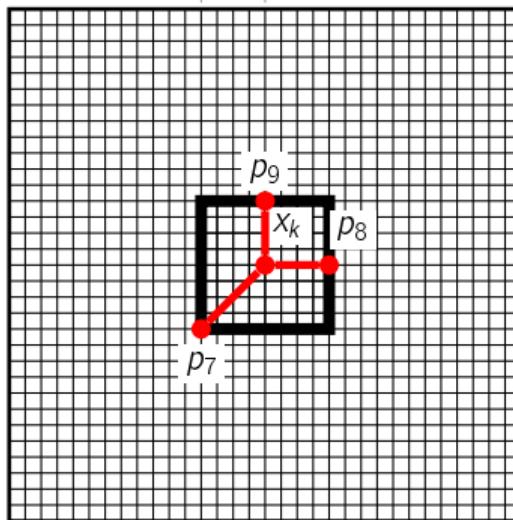
$$\text{subject to } c(x) \leq 0$$

where x : variables

- SEARCH step and POLL step
- we focus on the POLL step
- incumbent solution: x_k
- trail points: $\{p_1, p_2, \dots, p_k\}$

$$\delta_{\text{poll}}^k = \frac{1}{4}$$

$$\delta_{\text{mesh}}^k = \frac{1}{16}$$





Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

Objective and constraints

$$\min_x f(x)$$

$$\text{subject to } c(x) \leq 0$$

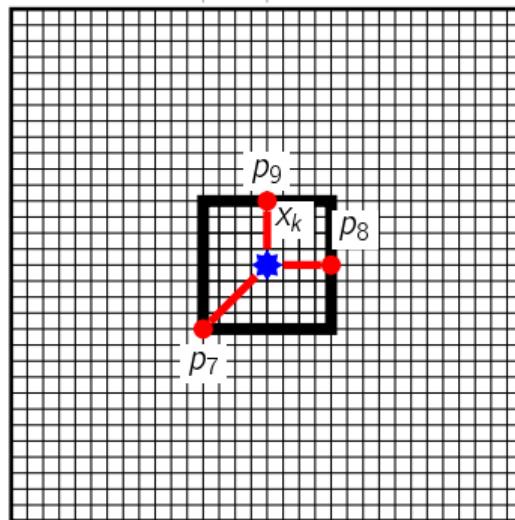
where x : variables

- SEARCH step and POLL step
- we focus on the POLL step
- incumbent solution: x_k
- trail points: $\{p_1, p_2, \dots, p_k\}$

Poll **failure**

$$\delta_{\text{poll}}^k = \frac{1}{4}$$

$$\delta_{\text{mesh}}^k = \frac{1}{16}$$





Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

Objective and constraints

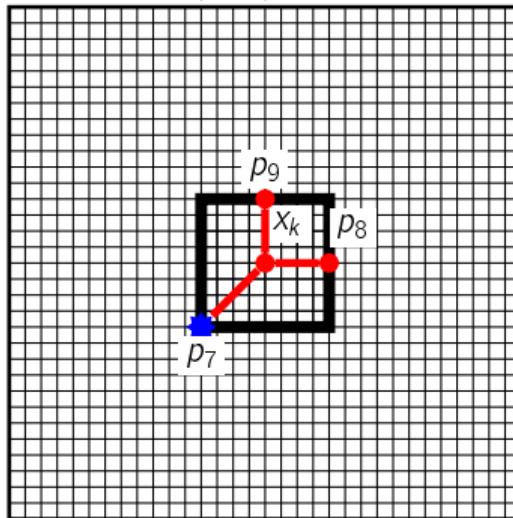
$$\begin{array}{ll} \min_x & f(x) \\ \text{subject to} & c(x) \leq 0 \\ \text{where} & x : \text{variables} \end{array}$$

- SEARCH step and POLL step
- we focus on the POLL step
- incumbent solution: x_k
- trail points: $\{p_1, p_2, \dots, p_k\}$
- successful trial point p_7

Poll **success**

$$\delta_{\text{poll}}^k = \frac{1}{4}$$

$$\delta_{\text{mesh}}^k = \frac{1}{16}$$





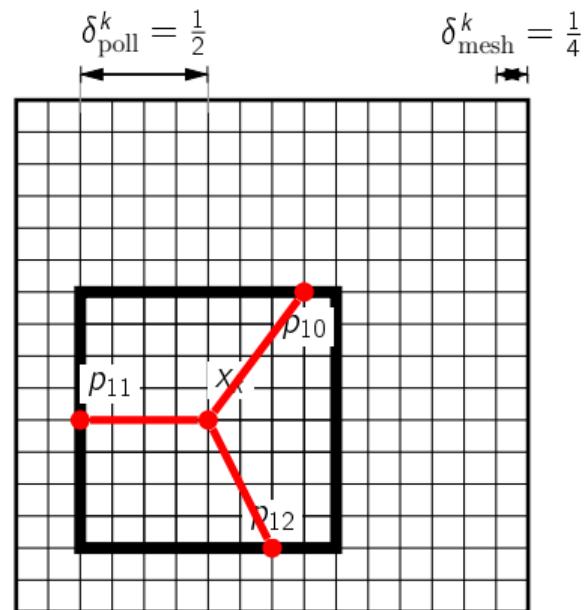
Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

Objective and constraints

$$\begin{array}{ll} \min_x & f(x) \\ \text{subject to} & c(x) \leq 0 \\ \text{where} & x : \text{variables} \end{array}$$

- SEARCH step and POLL step
- we focus on the POLL step
- incumbent solution: x_k
- trail points: $\{p_1, p_2, \dots, p_k\}$
- successful trail point p_7





Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

Objective and constraints

$$\min_x f(x)$$

$$\text{subject to } c(x) \leq 0$$

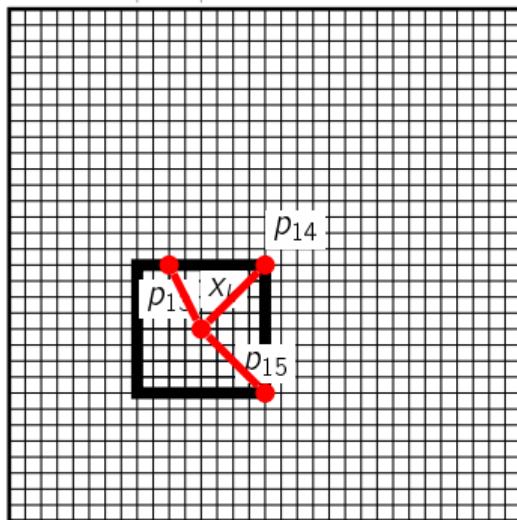
where x : variables

- SEARCH step and POLL step
- we focus on the POLL step
- incumbent solution: x_k
- trail points: $\{p_1, p_2, \dots, p_k\}$
- successful trail point p_7

Poll **failure**

$$\delta_{\text{poll}}^k = \frac{1}{4}$$

$$\delta_{\text{mesh}}^k = \frac{1}{16}$$





Overview of mesh adaptive direct search

No gradient information available, blackbox is expensive¹

Objective and constraints

$$\begin{array}{ll} \min_x & f(x) \\ \text{subject to} & c(x) \leq 0 \\ \text{where} & x : \text{variables} \end{array}$$

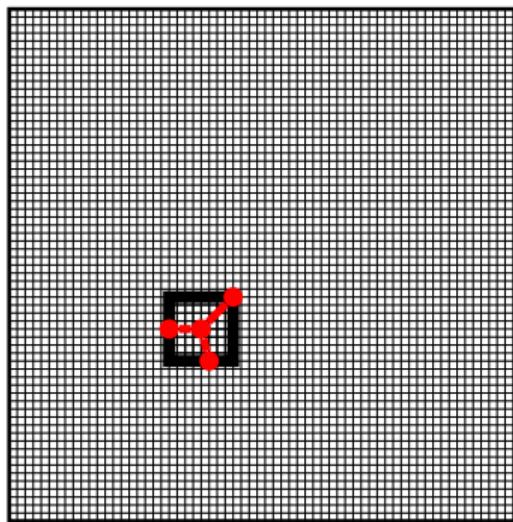
- SEARCH step and POLL step
- we focus on the POLL step
- incumbent solution: x_k
- trail points: $\{p_1, p_2, \dots, p_k\}$
- successful trail point p_7

Poll **failure**

$$\delta_{\text{poll}}^k = \frac{1}{8}$$



$$\delta_{\text{mesh}}^k = \frac{1}{64}$$





Overview of stochastic mesh adaptive direct search

No gradient information available, blackbox is expensive and **noisy**¹

Objective and constraints

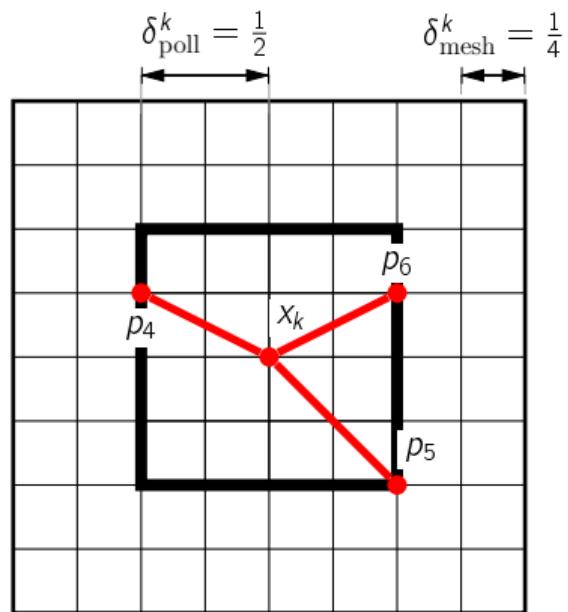
$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x)]$$

$$\text{subject to} \quad c(x) = \mathbb{E}_{\Theta} [c_{\Theta}(x)] \leq 0$$

where x : variables Θ : realizations

- Constructs estimates of objective:

$$f^k = \frac{1}{n^k} \sum_{i=1}^{n^k} f_{\Theta_{0,i}}(x_k)$$





Overview of stochastic mesh adaptive direct search

No gradient information available, blackbox is expensive and **noisy**¹

Objective and constraints

$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x)]$$

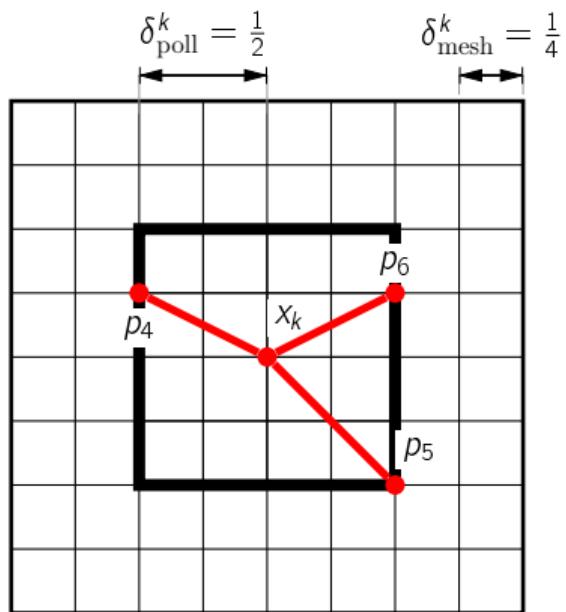
$$\text{subject to} \quad c(x) = \mathbb{E}_{\Theta} [c_{\Theta}(x)] \leq 0$$

where x : variables Θ : realizations

- Constructs estimates of objective:

$$f^k = \frac{1}{n^k} \sum_{i=1}^{n^k} f_{\Theta_{0,i}}(x_k)$$

- n^k is the sampling rate





Overview of stochastic mesh adaptive direct search

No gradient information available, blackbox is expensive and **noisy**¹

Objective and constraints

$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x)]$$

$$\text{subject to} \quad c(x) = \mathbb{E}_{\Theta} [c_{\Theta}(x)] \leq 0$$

where x : variables Θ : realizations

- Constructs estimates of objective:

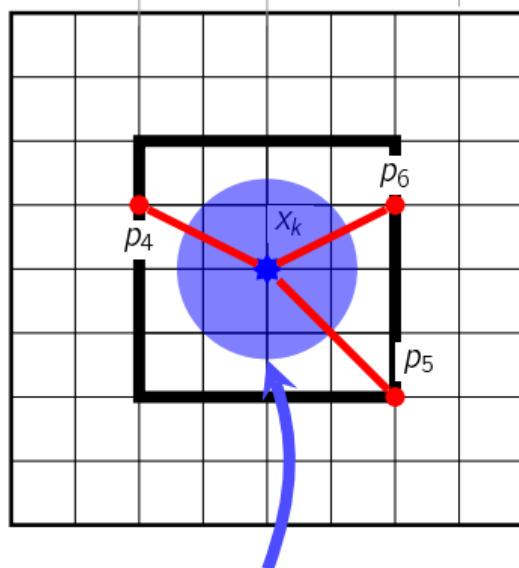
$$f^k = \frac{1}{n^k} \sum_{i=1}^{n^k} f_{\Theta_{0,i}}(x_k)$$

- n^k is the sampling rate
- Tracks uncertainty interval $\mathcal{I}(\delta_{\text{poll}}^k)$ in the estimate

Poll **certain failure**

$$\delta_{\text{poll}}^k = \frac{1}{2}$$

$$\delta_{\text{mesh}}^k = \frac{1}{4}$$



$$\{x : f_s^k - f_0^k \in \mathcal{I}\}$$



Overview of stochastic mesh adaptive direct search

No gradient information available, blackbox is expensive and **noisy**¹

Objective and constraints

$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x)]$$

$$\text{subject to} \quad c(x) = \mathbb{E}_{\Theta} [c_{\Theta}(x)] \leq 0$$

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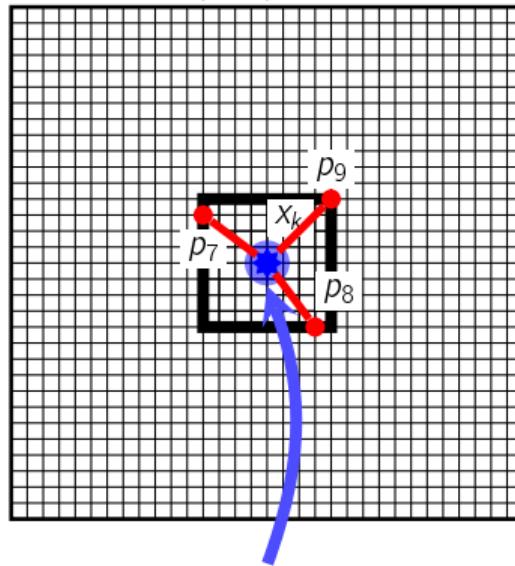
$$f^k = \frac{1}{n^k} \sum_{i=1}^{n^k} f_{\Theta_{0,i}}(x_k)$$

- n^k is the sampling rate
- Tracks uncertainty interval $\mathcal{I}(\delta_{\text{poll}}^k)$ in the estimate

Poll **certain failure**

$$\delta_{\text{poll}}^k = \frac{1}{4}$$

$$\delta_{\text{mesh}}^k = \frac{1}{16}$$



$$\{x : f_s^k - f_0^k \in \mathcal{I}\}$$



Overview of stochastic mesh adaptive direct search

No gradient information available, blackbox is expensive and **noisy**¹

Objective and constraints

$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x)]$$

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- Constructs estimates of objective:

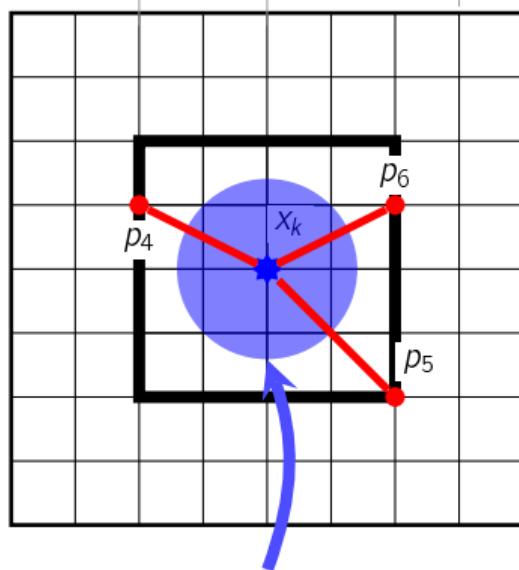
$$f^k = \frac{1}{n^k} \sum_{i=1}^{n^k} f_{\Theta_{0,i}}(x_k)$$

- n^k is the sampling rate
- Tracks uncertainty interval $\mathcal{I}(\delta_{\text{poll}}^k)$ in the estimate

Poll **certain failure**

$$\delta_{\text{poll}}^k = \frac{1}{2}$$

$$\delta_{\text{mesh}}^k = \frac{1}{4}$$



$$\{x : f_s^k - f_0^k \in \mathcal{I}\}$$



Overview of stochastic mesh adaptive direct search

No gradient information available, blackbox is expensive and **noisy**¹

Objective and constraints

$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x)]$$

$$\text{subject to} \quad c(x) = \mathbb{E}_{\Theta} [c_{\Theta}(x)] \leq 0$$

where x : variables Θ : realizations

- Constructs estimates of objective:

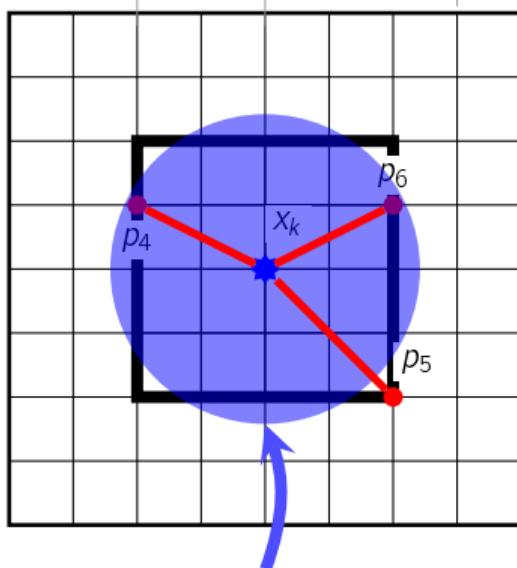
$$f^k = \frac{1}{n^k} \sum_{i=1}^{n^k} f_{\Theta_{0,i}}(x_k)$$

- n^k is the sampling rate
- Tracks uncertainty interval $\mathcal{I}(\delta_{\text{poll}}^k)$ in the estimate

Poll **uncertain failure**

$$\delta_{\text{poll}}^k = \frac{1}{2}$$

$$\delta_{\text{mesh}}^k = \frac{1}{4}$$



$$\{x : f_s^k - f_0^k \in \mathcal{I}\}$$



Overview of stochastic mesh adaptive direct search

No gradient information available, blackbox is expensive and **noisy**¹

Objective and constraints

$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x)]$$

$$\text{subject to} \quad c(x) = \mathbb{E}_{\Theta} [c_{\Theta}(x)] \leq 0$$

where x : variables Θ : realizations

- Constructs estimates of objective:

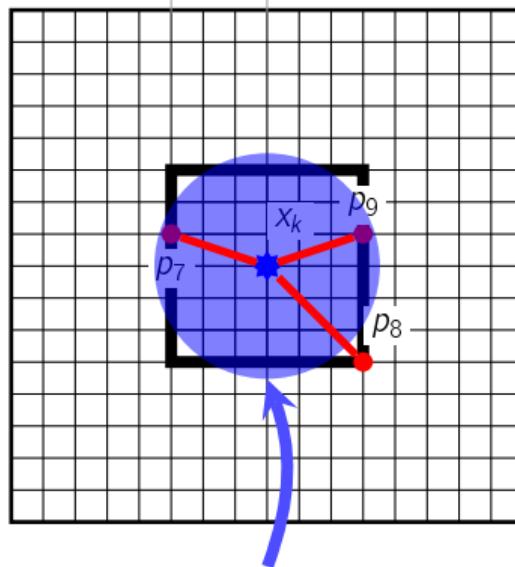
$$f^k = \frac{1}{n^k} \sum_{i=1}^{n^k} f_{\Theta_{0,i}}(x_k)$$

- n^k is the sampling rate
- Tracks uncertainty interval $\mathcal{I}(\delta_{\text{poll}}^k)$ in the estimate

Poll **uncertain failure**

$$\delta_{\text{poll}}^k = \frac{3}{8}$$

$$\delta_{\text{mesh}}^k = \frac{9}{64}$$





Overview of stochastic mesh adaptive direct search

No gradient information available, blackbox is expensive and **noisy**¹

Objective and constraints

$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x)]$$

$$\text{subject to} \quad c(x) = \mathbb{E}_{\Theta} [c_{\Theta}(x)] \leq 0$$

where x : variables Θ : realizations

- Constructs estimates of objective:

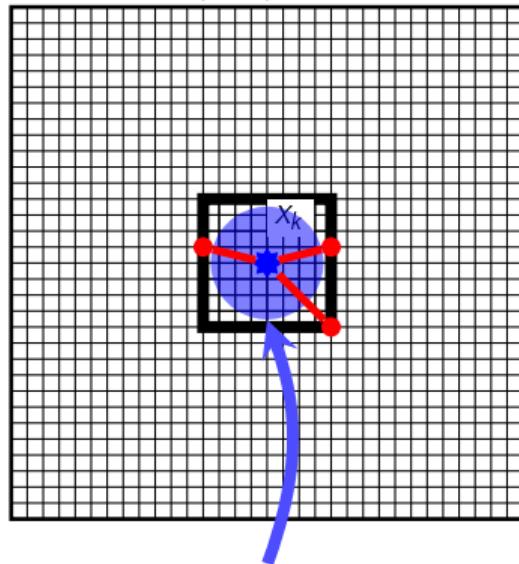
$$f^k = \frac{1}{n^k} \sum_{i=1}^{n^k} f_{\Theta_{0,i}}(x_k)$$

- n^k is the sampling rate
- Tracks uncertainty interval $\mathcal{I}(\delta_{\text{poll}}^k)$ in the estimate

Poll **certain failure**

$$\delta_{\text{poll}}^k = \frac{1}{4}$$

$$\delta_{\text{mesh}}^k = \frac{1}{16}$$



$$\{x : f_s^k - f_0^k \in \mathcal{I}\}$$



Overview of stochastic mesh adaptive direct search

No gradient information available, blackbox is expensive and **noisy**¹

Objective and constraints

$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x)]$$

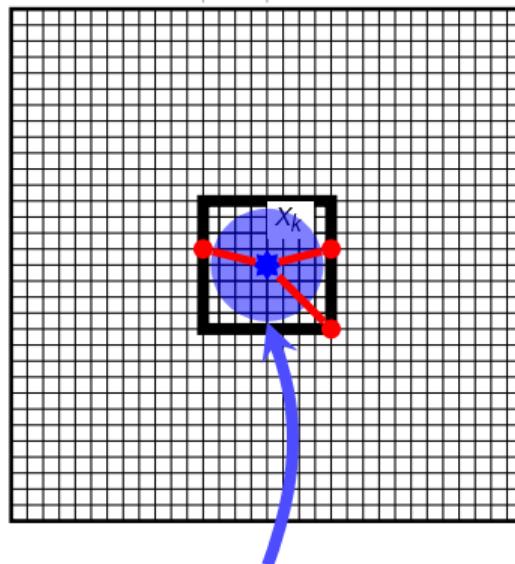
$$\text{subject to} \quad c(x) = \mathbb{E}_{\Theta} [c_{\Theta}(x)] \leq 0$$

where x : variables Θ : realizations

Poll **certain failure**

$$\delta_{\text{poll}}^k = \frac{1}{4}$$

$$\delta_{\text{mesh}}^k = \frac{1}{16}$$



$$\{x : f_s^k - f_0^k \in \mathcal{I}\}$$

[1] C. Audet et al., 2021, *Computational Optimization and Applications*

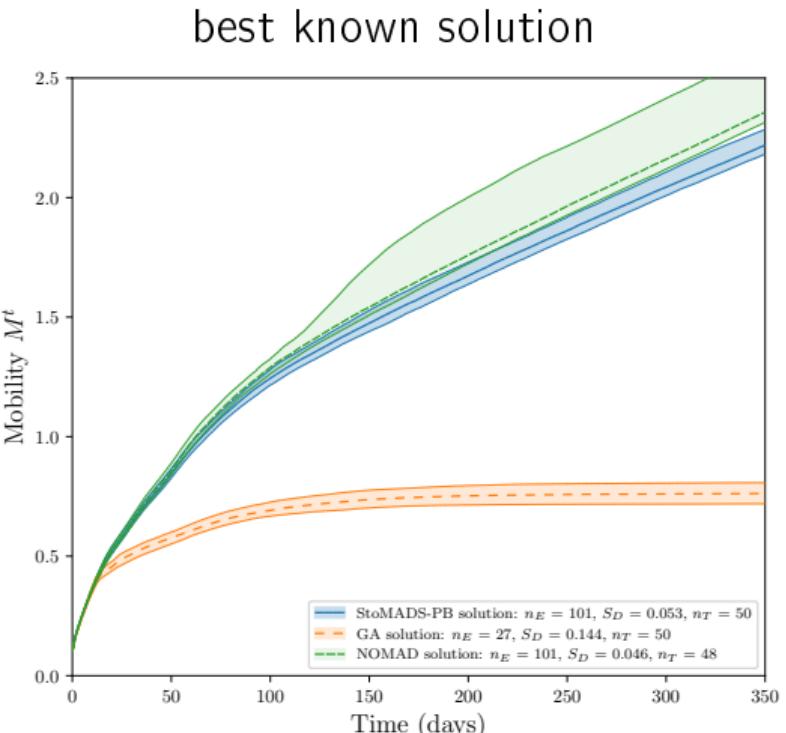
[2] K. J. Dzahini et al., 2022, *Mathematical Programming*



Conclusion

Formulated and solved public health policy-making problems

- Identified a public health policy that favored

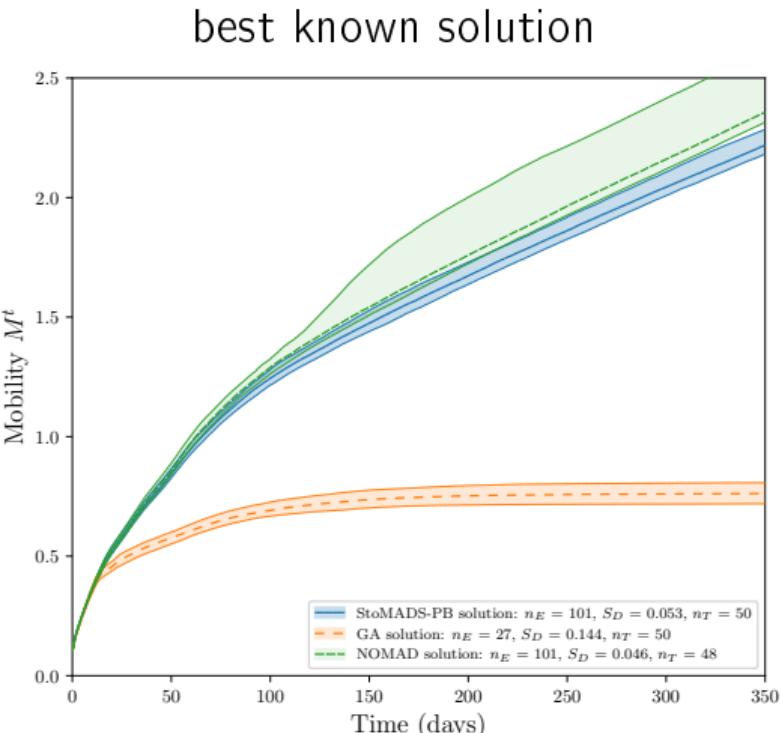




Conclusion

Formulated and solved public health policy-making problems

- Identified a public health policy that favored
 - High testing capacity n_T

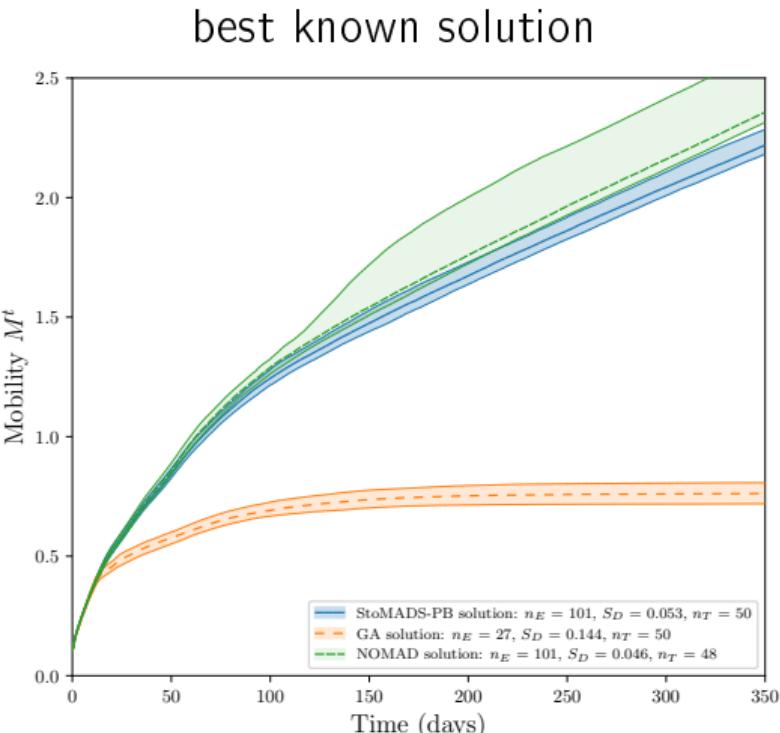




Conclusion

Formulated and solved public health policy-making problems

- Identified a public health policy that favored
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 - Large number of essential workers n_E

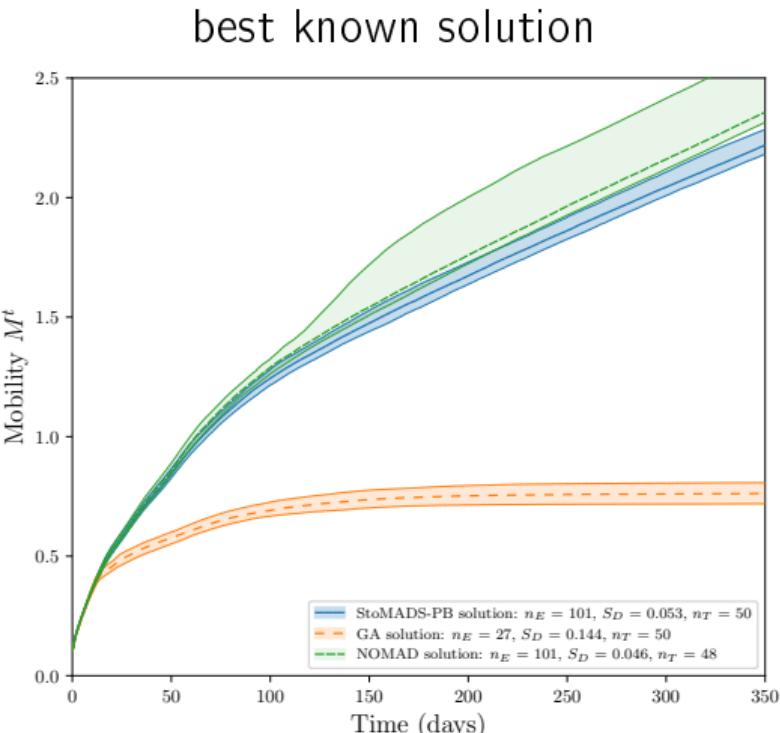




Conclusion

Formulated and solved public health policy-making problems

- Identified a public health policy that favored
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 - Large number of essential workers n_E
 - Modest social distancing S_D

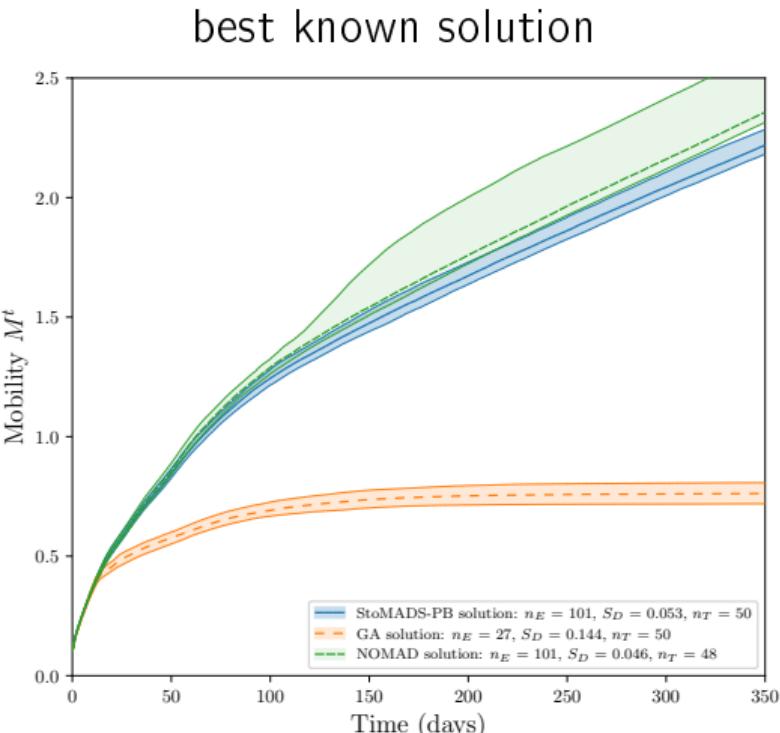




Conclusion

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- Identified a public health policy that favored
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- StoMADS outperformed GAs and NOMAD on active constraints





Conclusion

Formulated and solved public health policy-making problems

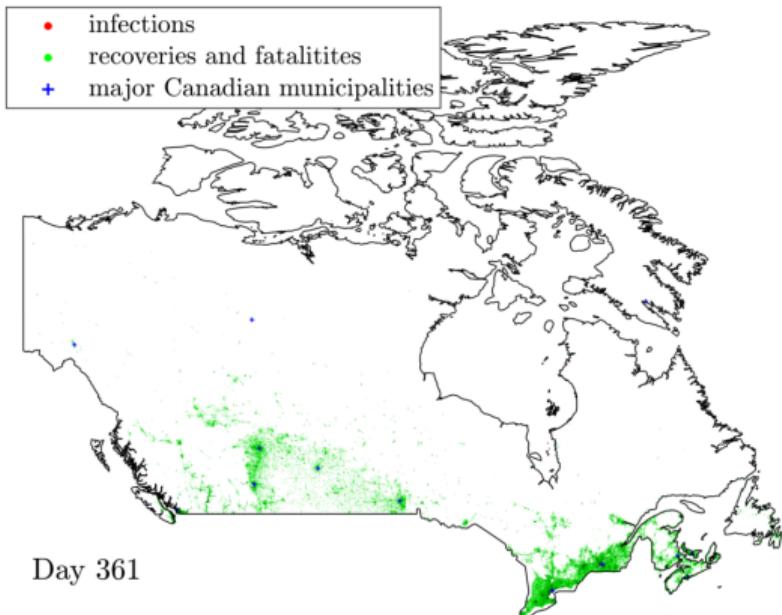
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Conclusion

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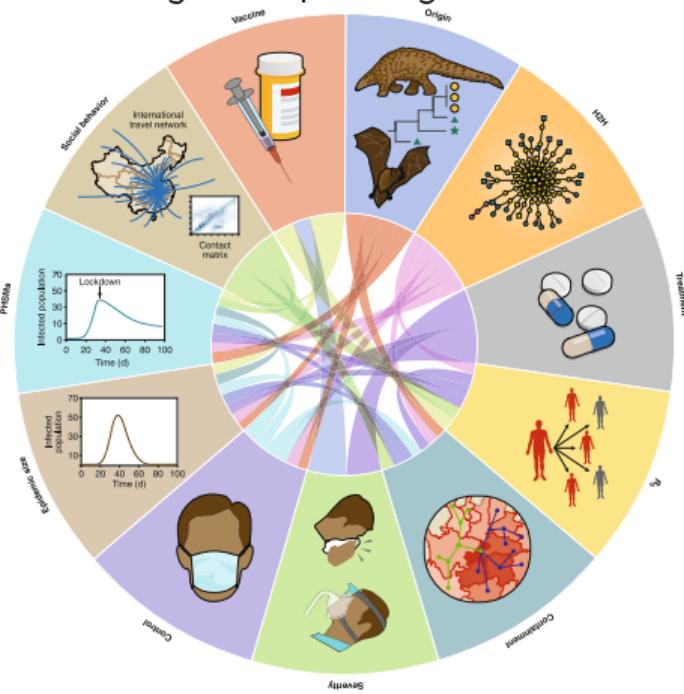
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Background: COVID-19 forecasting targets

Forecasting novel epidemics is a multidisciplinary field involving multiple *targets*¹
Inputs for forecasting epidemic size:



Nowcasting targets



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Forecasting novel epidemics is a multidisciplinary field involving multiple *targets*¹
Inputs for forecasting epidemic size:

- Growth rate indicators



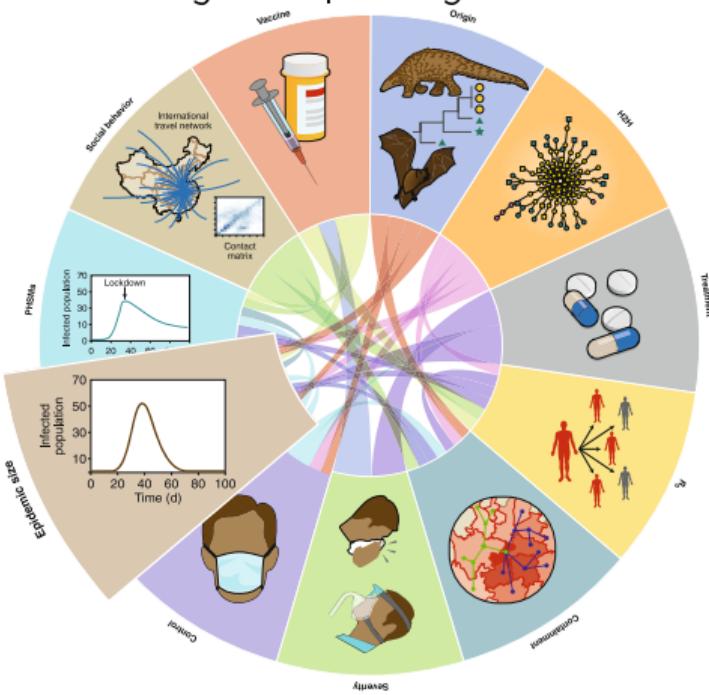
Nowcasting targets



Background: COVID-19 forecasting targets

Forecasting novel epidemics is a multidisciplinary field involving multiple *targets*¹
Inputs for forecasting epidemic size:

- Growth rate indicators
- Historical incidence rate data



Nowcasting targets

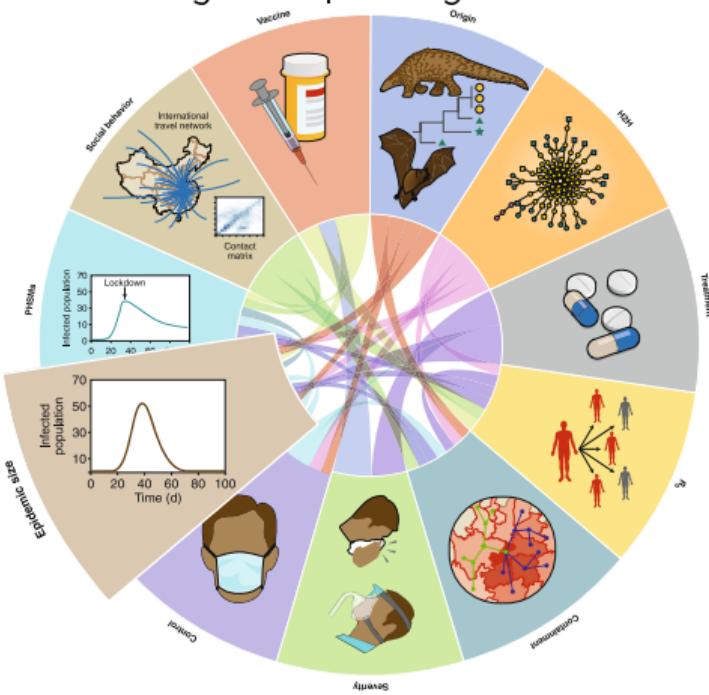
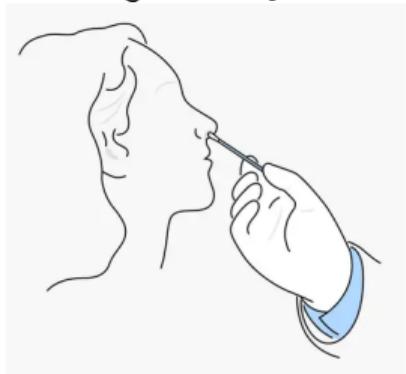
[1] J. T. Wu et al., 2021, *Nature Medicine*



Background: COVID-19 forecasting targets

Forecasting novel epidemics is a multidisciplinary field involving multiple *targets*¹
Inputs for forecasting epidemic size:

- Growth rate indicators
- Historical incidence rate data
- Serologic assays



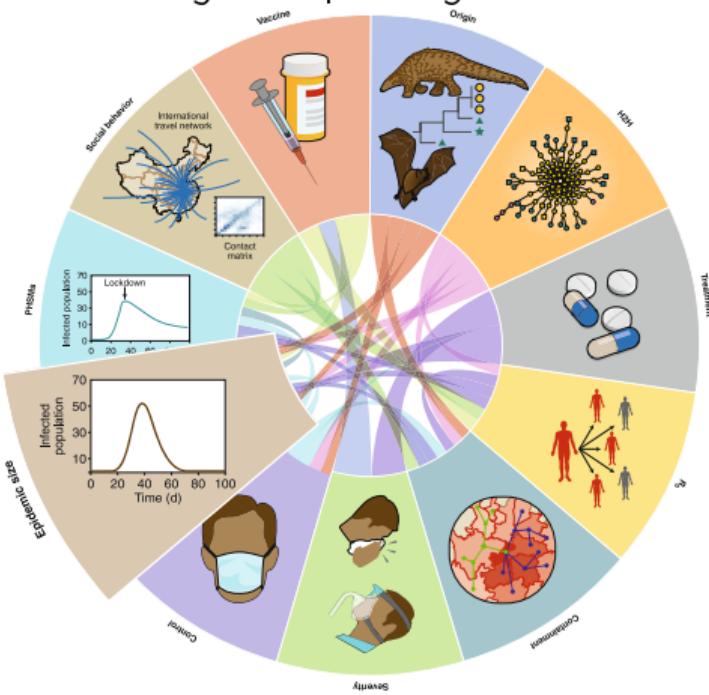
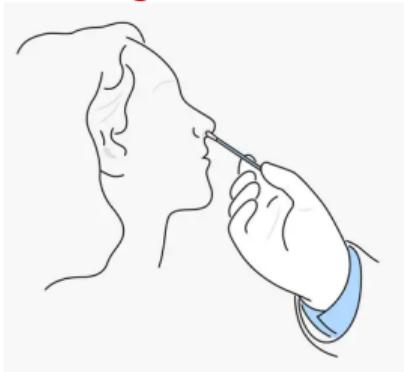
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Nowcasting targets



Conclusion and future directions

Model discovery and development facilitated by hyperparameter optimization



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Model discovery and development facilitated by hyperparameter optimization

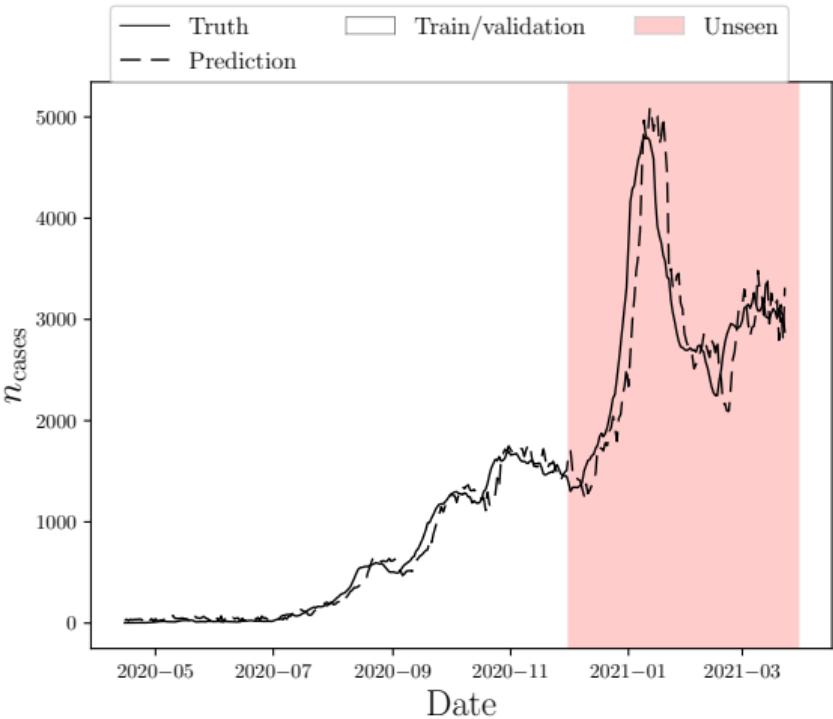
- Cycle Threshold (C_t) is a useful feature for incidence projection

Conclusion and future directions



Model discovery and development facilitated by hyperparameter optimization

- Cycle Threshold (C_t) is a useful feature for incidence projection
- Model that generalizes well on unseen data

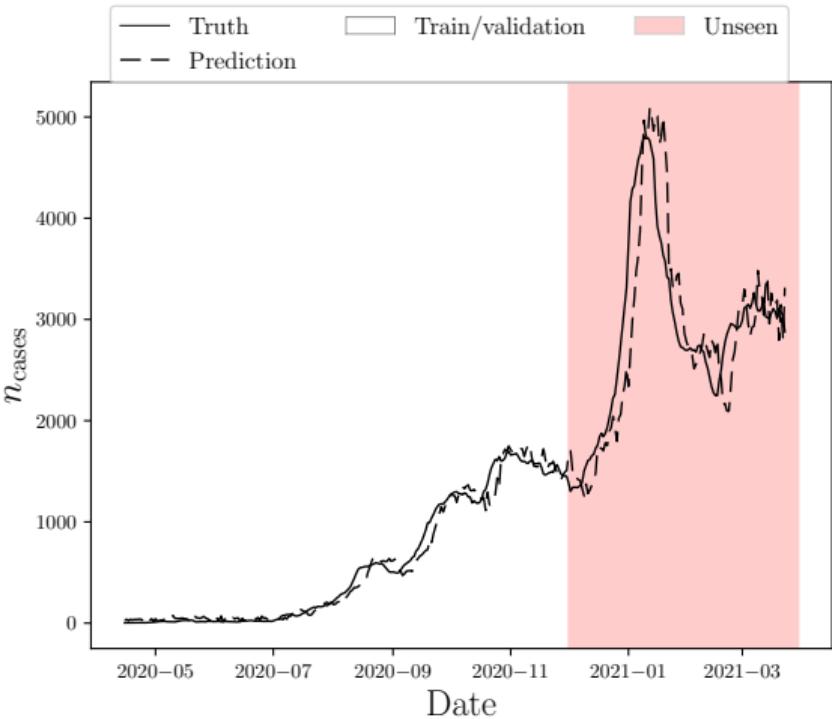


Conclusion and future directions



Model discovery and development facilitated by hyperparameter optimization

- Cycle Threshold (C_t) is a useful feature for incidence projection
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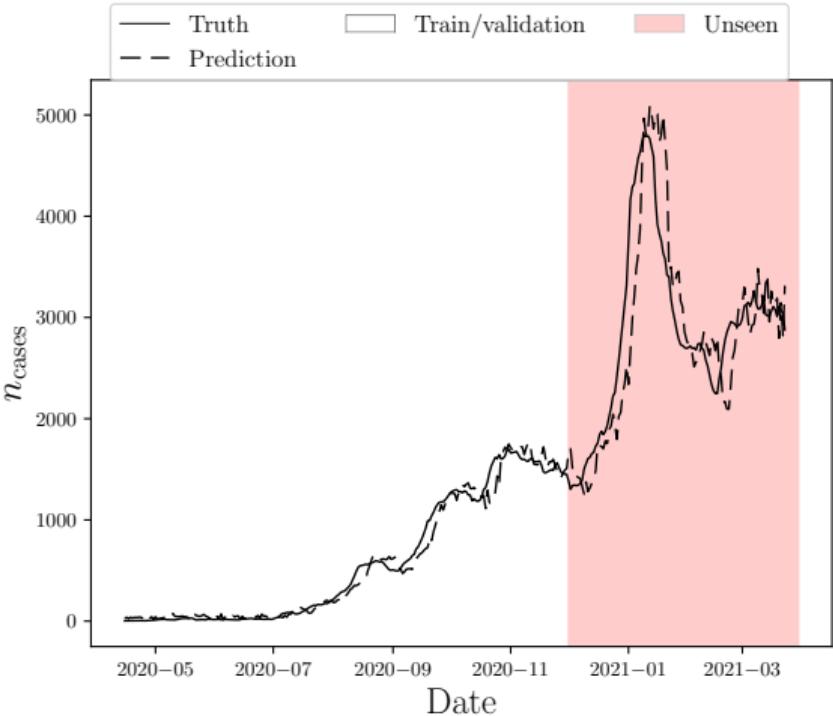


Conclusion and future directions



Model discovery and development facilitated by hyperparameter optimization

- Cycle Threshold (C_t) is a useful feature for incidence projection
- Model that generalizes well on unseen data
- Simpler models perform well when historical data is limited
- Works well on other datasets¹





StoMADS can be improved to solve a wide variety of HPO problems

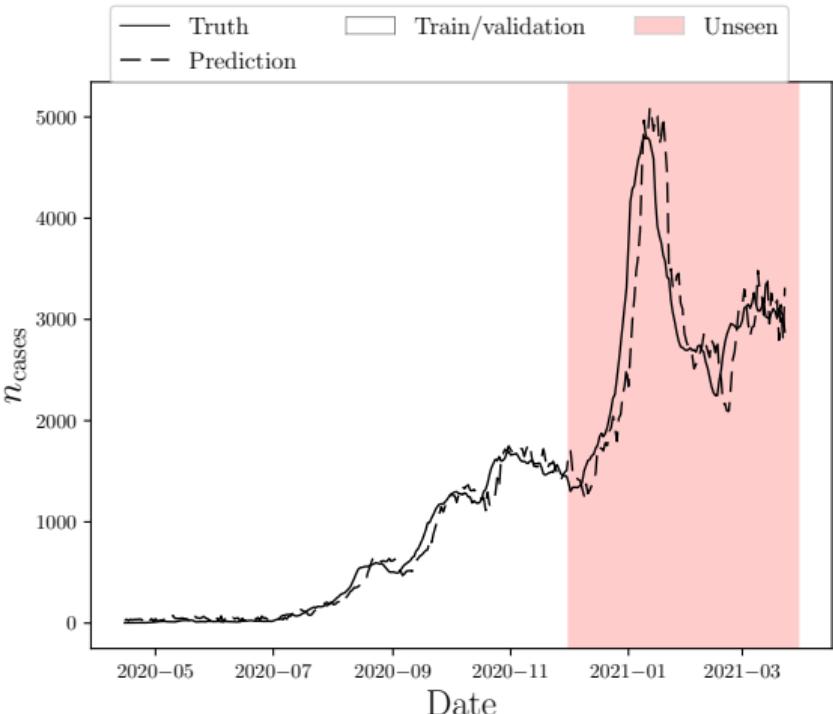
- Can be used to meet deployment targets

Objective and constraints

$$\min_x \quad f(x) = \mathbb{E}_{\Theta} [f_{\Theta}(x) = \text{error}_{\text{CV}}]$$

subject to $c(x) = \text{inference time} - \text{threshold} \leq 0$

where Θ : realizations



Conclusion and future directions

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- Can be used to meet deployment targets

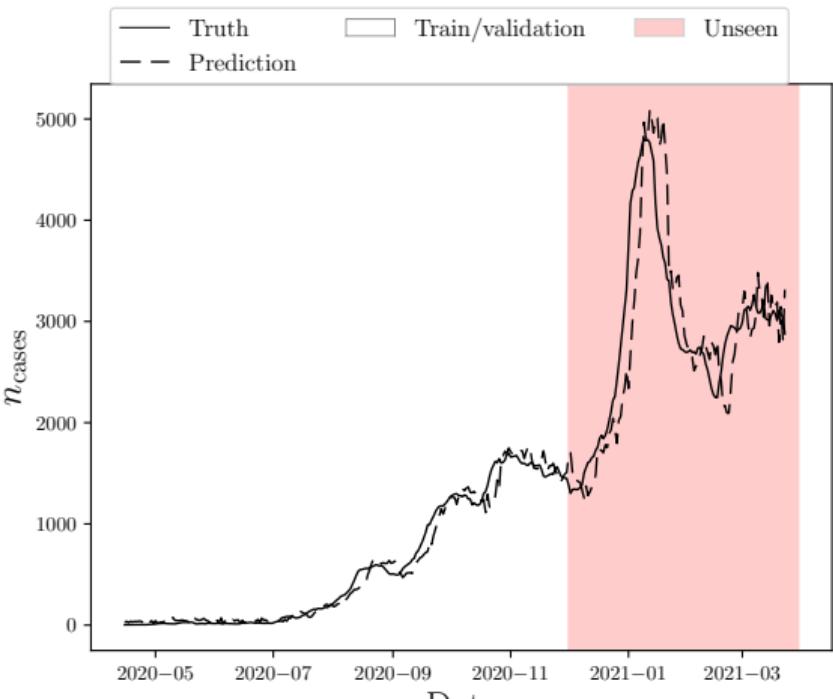
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[1] D. Lakhmiri et al., 2019, *ACM Transactions on Mathematical Software*

[2] J. Bergstra et al., 2011, *International conference on neural information processing systems*



StoMADS can be improved to solve a wide variety of HPO problems

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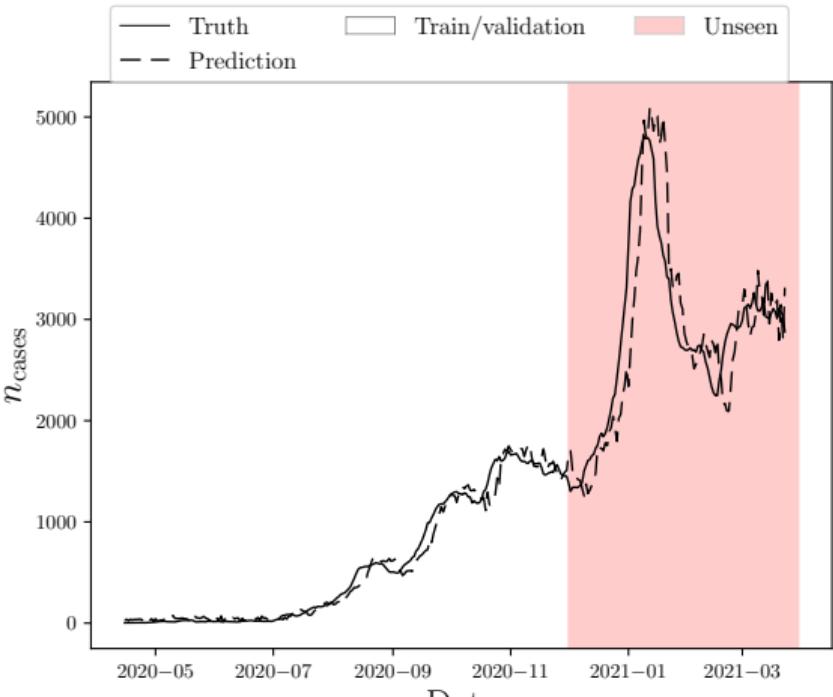
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- Should be benchmarked against HyperNOMAD¹, Bayesian optimization²
- Mixed variable version is needed



[1] D. Lakhmiri et al., 2019, *ACM Transactions on Mathematical Software*

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