Topic 11

Heap, Set and Map: How to store sorted data?

資料結構與程式設計

Data Structure and Programming

Sep, 2012

Linear Data Types

- In previous topic and Homework #5, we have learned linear data types like list and array
  - Tradeoffs between insert/delete/find operators
  - Memory overhead

  ➔ Constant time for “push_back()” or “push_front()” operation

- The best way to use linear data types is ---
  - Data are recorded in a linear sequence (i.e. only push_back or push_front is needed)
  - Linearly traverse each element (i.e. for(...; li++))
  - No “find”, “insert any”, nor “delete any”

Consider the Scenario...

- Suppose we are assigning jobs sequentially to several machines ---
  - One job to one machine and we record the accumulated runtime for each machine.
  - Our machine selection criteria is to “even out” the runtime of the machines.
  - In other words, we would like to pick the machine with least accumulated runtime for the next job

  ➔ Do we need to sort ALL the elements?

  ➔ Need a priority queue

Priority Queue

- An ADT that supports 2 operations
  - Insert
  - Delete min(or max)
- An element with arbitrary priority can be inserted to the queue
- At any time, it should take constant time to find the element with min(or max) priority and remove it from the list
  - Need to figure out which is the one with next lowest(highest) priority efficiently

Using List or Array?

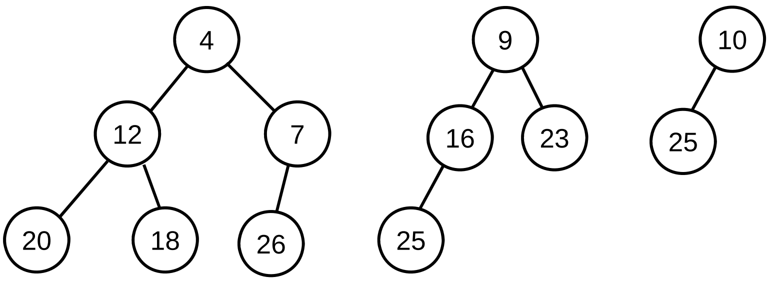
- Use linear ADT with an extra field to record the element with min(max) priority
  - Insert: O(1)
  - Delete min(max): O(n)

  (why?)

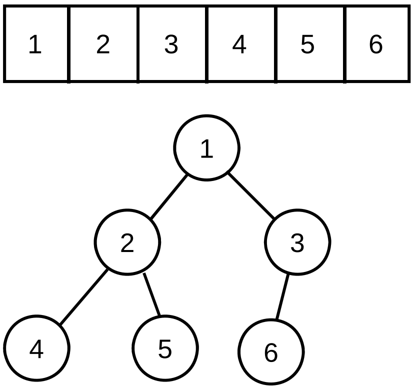
- As we learn before, O(n) is not good. We would prefer an ADT with O(log n) for both operations

Min (Max) Heap

- A complete binary tree in which the key value in each node is no larger (smaller) than its children



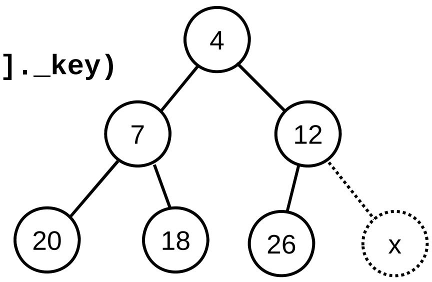
Remember that we can use array to implement a complete binary tree...



- Parent
  = child / 2
- Child
  = Parent * 2
  or Parent * 2 + 1

MinHeap Insertion

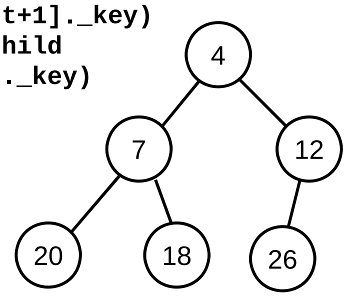
```
// Let n be the index of the last element
void MinHeap::insert(const T& x)
{
  int t = ++n; // next to the last
  while (t > 1) {
    int p = t / 2;
    if (x._key >= _heap[p]._key)
      break;
    _heap[t] = _heap[p];
    t = p;
  }
  _heap[t] = x;
}
```



What's the time complexity?

Delete Min Element

```
T& MinHeap::deleteMin()
{
  T ret = _heap[1];
  int p = 1, t = 2 * p;
  while (t <= n) {
    if (t < n) // has right child
      if (_heap[t]._key > _heap[t+1]._key)
        ++t; // to the smaller child
    if (_heap[n]._key < _heap[t]._key)
      break;
    _heap[p] = _heap[t];
    p = t;
    t = 2 * p;
  }
  _heap[p] = _heap[n--];
   return ret;
}
```



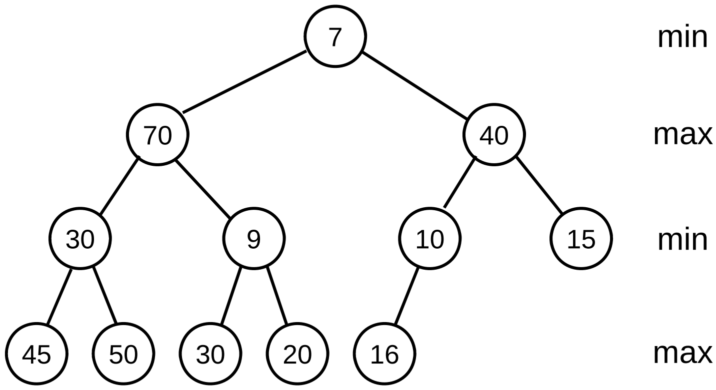
What's the time complexity?

Min(Max) Heap

- Simple implementation (just an array)
- Good insertion and deleteMin complexity
  - O(log n) vs. O(n)

What if you want to
delete min AND delete max?

Min-Max Heap

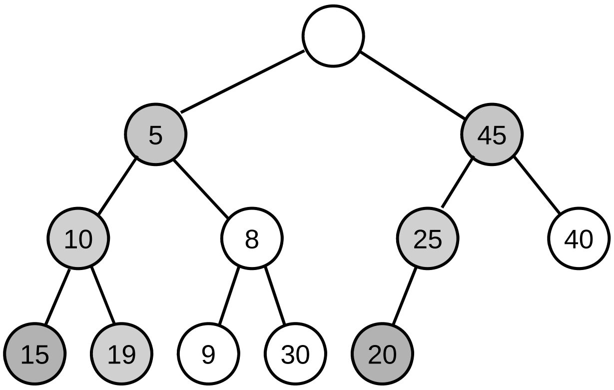


- Insert, delete min, delete max: all O(log n) (why?)

Deap

- Double-ended heap
  1. The root contains no element
  2. The left subtree is a min heap
  3. The right subtree is a max heap
  4. Let i be any node in the left subtree. Let j be the corresponding node in the right subtree. If such a j node does not exist, then let j be the corresponding parent of i.
     ➔ The key in node i is less than or equal to that in j.

Deap Example



- Insert, delete min, delete max: all O(log n)   (why?)
  - But faster than min-max heap by a constant factor
  - Algorithm is simpler

(FYI) More Varieties of Heaps

- Leftist heap
  - Also support “combine(heap1, heap2)” in O(log n)
- Binomial heap
  - Certain individual operations may take O(n)
  - But the amortized complexity is either O(1) or O(log n)
- Fibonacci heap
  - Complexity similar to binomial heap
  - But also support
    - decreaseKey(node): O(1)
    - delete(node): O(log n)

Heap Operations Supported in STL

- STL does not have a “heap” class
  - Instead, it support several operations that can operate on “array” like data structure
- Operations
  - void make_heap(first, last[, comp]);
  - void push_heap(first, last[, comp]);
  - void pop_heap(first, last[, comp]);
  - void sort_heap(first, last[, comp]);
  - bool is_heap(first, last[, comp]);

  ➔ *fist, last: RandomAccessIterator*

  ➔ *comp: StrictWeakOrdering (optional)*

Summary: Heap Structures

- Pros:
  1. Good complexity of “insert”, “delete min(max)”, ... operations
  2. Simple data structure (low memory overhead)
  3. Simpler algorithms (than BST)
- Con
  1. Data are not sorted

     ➔ Still have O(n) for “find” operation

Review: Binary Search Trees

- Binary Search Trees (BSTs)
  - Left subtree <= this <= right subtree
  - Complexity depends on the height of the tree
  - Worst case: can be degenerated as a tree with height O(n)
- Balanced BSTs
  - The heights of left subtree and right subtree are somewhat balanced
    - Height ~ O(log n)
  - Examples: AVL, 2-3, 2-3-4, red-black, splay trees
  - Algorithms for their operations are complicated

ADT for Sorted Data Structures

- A sorted abstract data type has the following operations in O(log n)

```
template <class T>
class SortedAdt
{
public:
   virtual iterator insert(const T&) = 0;
   // Note: this is different from
   //       erase(pos)
   virtual void     erase(const T&) = 0;
   virtual iterator find(const T&) = 0;
};
```

Why Pure Virtual Functions?

- class SortAdt will be a abstract class (i.e. cannot instantiate any object of this class)
- We can realize SortAdt with different data structures
  - e.g. class SortAdtImp1 : public SortAdt { ... };
  - Must implement “insert”, “delete” and “find” functions

Sorted ADT in STL

- Also classified as “Associative Containers”

1. set
2. multiset
3. map
4. multimap

class set in STL

- To store elements in a set
  - e.g. { 2, 3, 5, 7, 9 }

- set<Key[, Compare, Alloc]>
  - class Key: element type
  - class Compare: how the elements are compared (optional; default = less<Key>)
  - class Alloc: used for internal memory management (optional; default = alloc)

Member Functions in class set

1. iterator begin() const;
   iterator end() const;
2. pair<iterator, bool> insert(const value_type& x);
   iterator insert(iterator pos, const value_type& x);
   void insert(InputIterator, InputIterator);
3. void erase(iterator pos);
   size_type erase(const key_type& k);
   void erase(iterator first, iterator last);
4. iterator find(const key_type& k) const;
5. size_type count(const key_type& k) const;
6. iterator lower_bound(const key_type& k) const;
   iterator upper_bound(const key_type& k) const;
   pair<iterator, iterator> equal_range(const key_type& k) const;

Other Functions for class set

1. includes
   - Check if one set is included in another
2. set_union
3. set_intersection
4. set_difference
5. set_symmetric_difference
   - (A – B) U (B – A)

class multiset in STL

- Unlike “set”, where elements with same value are stored only once, in multiset, they can be stored repeatedly
  - e.g. { 2, 3, 5, 5, 6, 7, 7, 7 }
- multiset<Key[, Compare, Alloc]>
  - class Key: element type
  - class Compare: how the elements are compared (optional; default = less<Key>)
  - class Alloc: used for internal memory management (optional; default = alloc)

class map in STL

- In many applications, data are associated with keys (or id's)
  - For example, (id, student record)
  - e.g. { (Mary, 90), (John, 85), (Sam, 71) ... }
- class map<Key, Data[, Compare, Alloc]>
  - class Key: compare data type
  - class Data: value type
  - class Compare: how the elements are compared (optional; default = less<Key>)
  - class Alloc: used for internal memory management (optional; default = alloc)

Example of using class map (1)

```
map<string, unsigned>  scoreMap;
scoreMap[“Mary”] = 90;
scoreMap[“John”] = 85;
scoreMap[“Sam”] = 71;
unsigned maryScore = scoreMap[“Mary”];
cout << “Mary’s score = “ << maryScore << endl;
map<string, unsigned>::iterator mi;
mi = scoreMap.find(“John”);
if (mi != scoreMap.end())
  cout << “John’s score = “ << (*mi).second << endl;
```

➔ How about “map<const char*, unsigned>”?

Comments about map::operator []

- Since operator[] might insert a new element into the map, it can't possibly be a const member function.
- Note that the definition of operator[] is extremely simple: m[k] is equivalent to (*((m.insert(value_type(k,data_type()))).first)).second.
  - value_type = pair<Key, Data>
  - insert(value_type) returns a pair<map::iterator, bool>
- Strictly speaking, this member function is unnecessary: it exists only for convenience.

http://www.sgi.com/tech/stl/Map.html

Bad example of using class map

```
map<const char*, unsigned> mmm;
map<const char*, unsigned>::iterator mi;
char buf[1024];
cin >> buf; mmm[buf] = 10;
cin >> buf; mmm[buf] = 20;
cin >> buf; unsigned s1 = mmm[buf];
cout << buf << “ = “ << s1 << endl;
cin >> buf; unsigned s2 = mmm[buf];
cout << buf << “ = “ << s2 << endl;
```

Example of using class map (2)

```
string str;
for (int i = 0; i < 5; ++i) {
  cin >> str; mm.insert(pair<string, int>(str, i));
}
while (1) {
  cin >> str;
  map<string, int>::iterator mi = mm.find(str);
  if (mi == mm.end()) {
    cout << "Not found!!" << endl;
    break;
  }
  cout << (*mi).first << " = " << (*mi).second << endl;
}
```


Conclusion: Set and Map

- "set" and "map" are useful data structures when we need to perform efficient "insert", "erase", and "find" operations
  - Usually implemented by balanced binary search trees
  - Implementation efforts can be high
  - Using STL may be a good choice
- Remember, unbalanced BSTs may not be a bad choice
  - Most randomly inserted BSTs are somewhat balanced
- Remember, there's no free lunch
  - Overhead in insert (vs. push_back)
  - If we don't need to do "erase" or "find" during insertions... (what alternative?)