

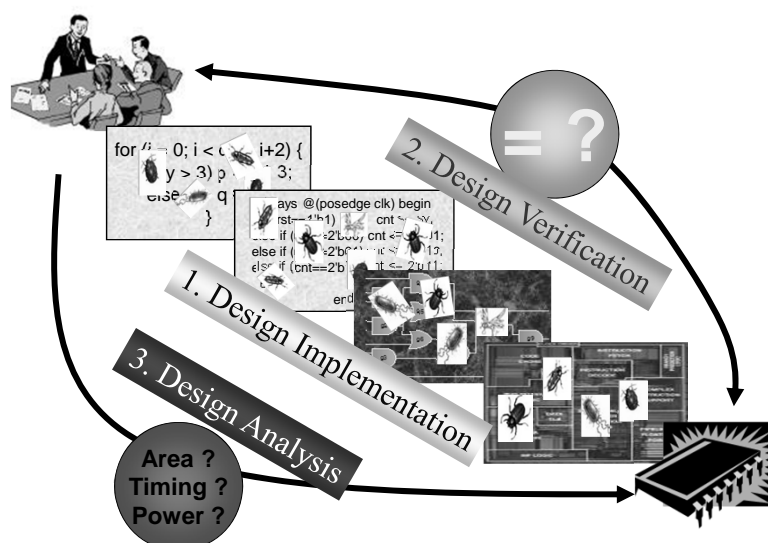
## Final Project

### ***FRAIG:*** ***Functionally Reduced And-Inverter Graph***

資料結構與程式設計  
Data Structure and Programming

Sep, 2012

## How is a “chip” designed nowadays?



## How to optimize a circuit?

### ◆ Area

- Reduce the number of gates
- Moreover, use the library cells with smaller sizes → but they will have weaker driving capability

### ◆ Timing

- Shorten the longest path
- Additionally, insert buffers and/or resize the cells to increase the driving capability

### ◆ Power

- Reduce the switching activities
- Moreover, shutdown the sub-circuit that is not currently used

## Optimization trade-offs

### ◆ In general, area, timing, power optimizations contradict with each other

### ◆ Moreover, different stages of design flow have different granularities and complexities for circuit optimization

- HDL (e.g. Verilog) // algorithm
- Gate (Boolean) // logic
- Schematic (transistor) // cell library
- Layout (wire length) // RC network

## A simplified view of circuit optimization

### ◆HDL (Verilog)

- Architectural and algorithmic optimizations

```
always @(posedge clk) begin
  if (rst==1'b1) cnt <= sv;
  else if (cnt==2'b00) cnt <= 2'b01;
  else if (cnt==2'b01) cnt <= 2'b10;
  else if (cnt==2'b10) cnt <= 2'b11;
  else cnt <= sv;
end
```

### ◆Gate (Boolean) What FRAIG focuses!!

- Minimize gate counts under reasonable timing and power constraints



### ◆Layout (transistor)

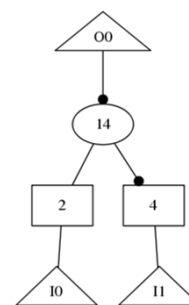
- Minimize wire length for timing and power optimizations with limited area overhead



## A simple example

### ◆sim05.aag

original:  
12 AIGs



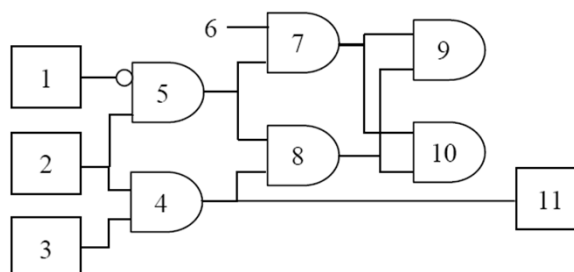
optimized:  
1 AIG

## Functionally Reduced AIG

1. Unused gate sweeping
2. Trivial optimization
3. Simplification by structural hash
4. FRAIG: Equivalence gate merging

## Unused Gate Sweeping

- ◆ Sweeping out those gates that are not reachable from POs.



## Unused Gate Sweeping

### ◆ Command: CIRSWEEP

- Can be called whenever necessary.
- After this command, all gates except for the unused PIs will be in the DFS list.
- Note: do not removed unused PIs.
- Note: be sure to update the reporting for "CIRPrint -FLoating".

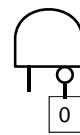
### ◆ In the previous example (cirp -fl):

- Before:
  - Defined but not used: 9 10
  - Gates with floating fanin: 7
- After: none

## Trivial optimization

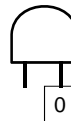
### 1. Fanin has constant 1

→ Replaced by the other fanin



### 2. Fanin has constant 0

→ Replaced with 0



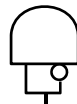
### 3. Identical fanins

→ Replaced with the (fanin+phase)



### 4. Inverted fanins

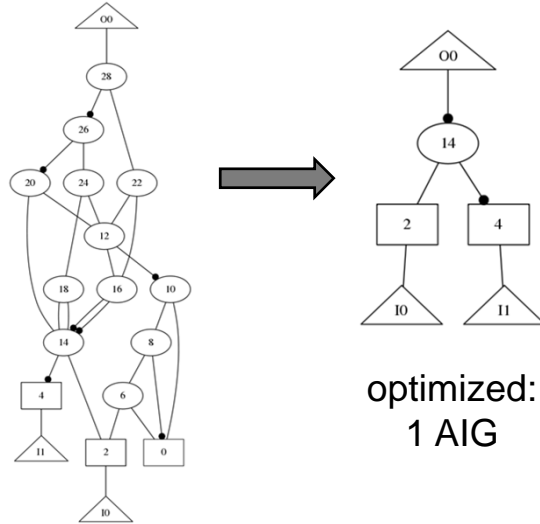
→ Replaced with 0



## A simple example

◆ sim05.aag

original:  
12 AIGs



optimized:  
1 AIG

## Trivial optimization

◆ Command: CIROPTimize

- Can be called whenever necessary
- Scan the DFS list and perform optimization ONCE. Don't repeatedly optimize the circuit. → The latter can be achieved by calling CIROPTimize multiple times.
- Don't perform optimization in CIRRead

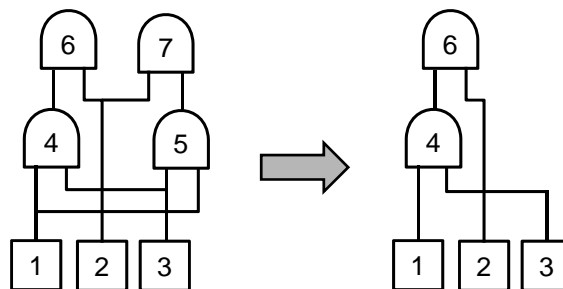
◆ Do not remove PIs / POs

◆ Should not create extra floating gates

- But some (floating) gates may disappear!
- But some gates (with side input = constant 0) may become "defined-but-not-used".

## Structural Hash (Strash)

### ◆ Example:



## Structural Hash (Strash)

### ◆ Problem: How to identify two AIG gates in a circuit that have same inputs?

[Method 1] Check for  $O(n^2)$  pairs of gates

[Method 2] For each gate, check its fanouts

- How many checks?

[Method 3] For each gate, create hash table <fanins, this gate>

- How many checks?

### ◆ We will pick method 3 in our project

- Must use “util/myHash.h” in your implementation

### ◆ Although it is possible to perform strash during circuit parsing, we choose to make “strash” a separate command. → CIRSTRash

### ◆ Note: Order matters!! You should merge from PIs to POs

## Structural Hash Algorithm

```
◆ Hash<HashKey, HashNode> hash;  
  // HashKey<GateType, List<Fanins> >  
  // HashNode = Gate*  
◆ for_each_gate_from_pi_to_po(gate, hash)  
  HashKey<...> k(...);  
  if (hash.check(k, mergeGate) == true)  
    mergeGate.merge(gate);  
  else hash.forceInsert(k, gate);
```

## Notes about CIRSTRash

- ◆ Perform strash only on gates in DFS list
  - Do not perform strash on gates which cannot be reached from POs
  - This is to avoid those unreachable gates appearing in DFS list
- ◆ It doesn't make sense to perform strash again before doing other optimizations
  - CIRSTRash cannot be repeated called



## Maintaining Netlist Consistency

- ◆ Once circuit is simplified, some gates may become invalid.
  - How to maintain the netlist consistency?
    1. Properly re-connect fanins/fanouts
    2. Properly release memory (if necessary)
    3. Properly update the lists in CirMgr  
(Note: PI/PO lists should never be changed)

## FRAIG: Merging equivalent gates

- ◆ Some gates are NOT structurally equivalent, but functionally equivalent.
  - Cannot be detected by strash
  - e.g.  $ab + c \equiv (a + c)(b + c)$
- ◆ How to know two gates are functionally equivalent?
  - By simulation? (If two gates have the same value)
    - ➔ No, equivalence requires “ALL input patterns” result in the same response
  - Need “formal (mathematical) proof”!!
    - ➔ But, whom to prove?  $O(n^2)$  pairs?
    - ➔ By simulation!!

## FEC Pairs

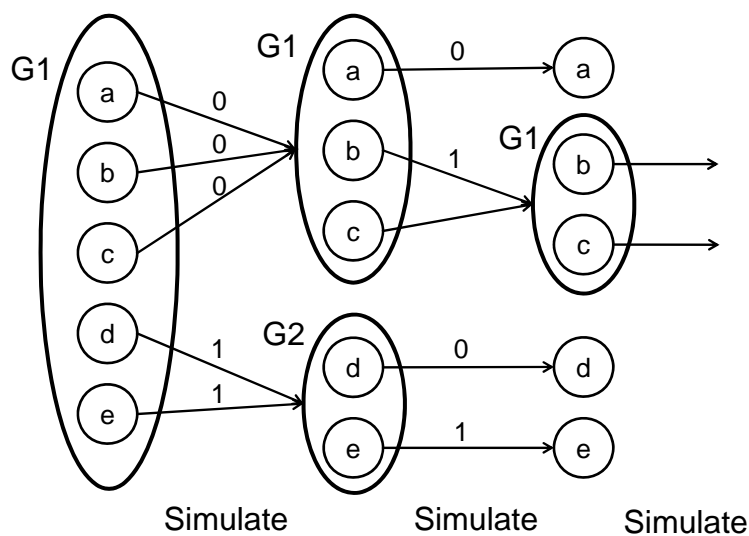
### ◆ Functionally Equivalent Candidate (FEC)

- For all simulated patterns, if two signals always have the same response, they are very likely to be equivalent.

### ◆ Properties

- Two signals can be separated if they have different simulation values for ANY input pattern
- Two paired signals can be separated by simulation, but two separated signals won't get paired again
  - Singleton signal won't be in any FEC pair anymore

## Identify FECs by Simulation



## Identify FECs by Simulation

1. Initial: put all the signals in ONE FEC group.
2. Add this FEC group into fecGrps (list of FEC groups)
3. Randomly simulate the entire miter
4. for\_each(fecGrp, fecGrps):  
    Hash<SimValue, FECGroup> newFecGrps;  
    for\_each(gate, fecGrp)  
        grp = newFecGrps.check(gate);  
        if (grp != 0) // existed  
            grp.add(gate);  
        else createNewGroup(newFecGrps, gate);  
    CollectValidFecGrp(newFecGrps, fecGrp, fecGrps);
5. Repeat 3-4 until no new FEC Group can be identified, or efforts exceed certain limit.

## Simulation Algorithm

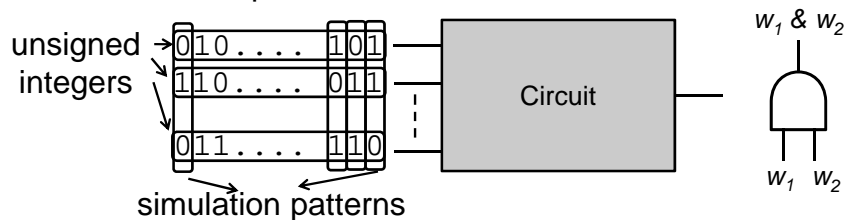
- ◆ All-gate simulation:  
Perform simulation for each gate on the DFS list
  - void CirMgr::simulate() {  
    for\_each\_gate(gate, \_dfsList) gate->simulate(); }
- ◆ Event-driven simulation:  
Perform simulation only if any of the fanins changes value
  - void CirMgr::simulate() {  
    for\_each\_PO(po, \_dfsList) po->simulate(); }
  - bool CirAigGate::simulate() {  
    Recursively simulate each fanin.  
    If (no fanin has value change) return false;  
    Simulate this gate;  
    if (value changed) return true;  
    return false;  
}

## Simulation algorithm trade-offs

- ◆ All-gate simulation or event-driven?
- ◆ Evaluation
  - By operator? By if-else? By table lookup?
- ◆ To detect FEC pairs, how many simulation patterns are enough?
  - Stop if no new FEC pair is found?
  - (Dynamically) Controlled by “#failTimes”
- ◆ Patterns
  - Single pattern? Parallel pattern?

## Parallel-Pattern Simulation for FEC Identification

- ◆ Note: The overhead in simulation speed by parallel-pattern simulation is very small.
  - Most of the programming languages (e.g. C/C++) support “bit-wise” operations (e.g. &, |, ~ in C).
- ◆ Idea
  - Using 32- or 64-bit unsigned integer to pack 32 or 64 patterns into a word



## How many patterns to parallelize?

◆ In practice, max parallelization will lead to the best simulation performance

- Use the max “unsigned int” to store the parallel patterns (e.g. `size_t` in C/C++)

[Discussion]

◆ Can we go beyond 32/64 bits?

- e.g. 1024-bit

◆ What are the pros and cons?

◆ How about the FEC detection rate?

## FRAIG flow

### 1. Simulation

- a) Put all signals in the same group
- b) Simulate the circuit. If two signals have different simulation results, separate them into different groups
- c) Repeat (b) until no more signals can be distinguished, or the simulation efforts exceed a preset limit
- d) Collect the functionally equivalence candidate (FEC) pairs

## FRAIG flow

2. For each FEC pair, call Boolean Satisfiability (SAT) engine to prove their equivalence
  1. If they are equivalent, merge them together  
→ remove one of them
  2. If they are NOT equivalent, acquire the counter-example (CEX) to distinguish them
  3. Repeat until all the FEC pairs have been proved, or enough CEXes (2.2) have been collected → Repeat simulation

## Boolean Satisfiability Problem

- ◆ Given a Boolean function  $f(X)$ , find an input assignment  $X = A$  such that  $f(A) = 1$ .
  - Satisfiable: if such an assignment is found
  - Unsatisfiable: if no assignment is possible
    - i.e. All assignments make  $f(X) = 0$
  - Undecided: can't find a satisfying assignment, but haven't exhaust the search
- ◆ Complexity?
  - First proven NP-complete problem by Dr. S. Cook in 1971 (Turing Award winner)

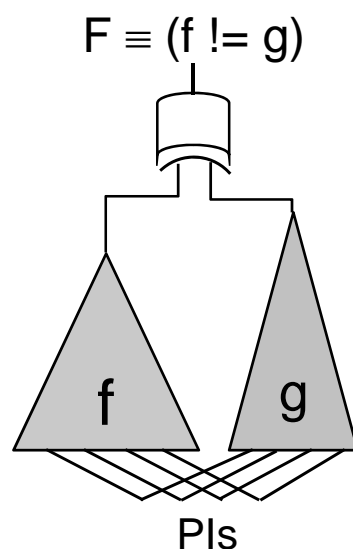
## How to prove the equivalence of FEC gates?

◆ In general, given two Boolean functions,  $f$ ,  $g$ , how to check if they are equivalent?

◆ Remember:

- SAT proves things by contraposition
  - By showing that it is *impossible* to find an assignment to make  $f \neq g$ .
  - Create a SAT problem  $F \equiv (f \neq g)$ , showing that it is unsatisfiable.
  - Note:  $f \neq g \rightarrow$  an XOR gate

## How to prove the equivalence of FEC gates?



If UNSAT  $\rightarrow f = g$

If SAT  $\rightarrow f \neq g$  with an input assignment  $A$  that can potentially distinguish other pairs

## In short...

1. Simulation identifies a group of FEC pairs
  2. For each FEC pair, say (f, g), call SAT engine to check if (f != g) is satisfiable
  3. If UNSAT  $\rightarrow f = g \rightarrow f$  can replace g
  4. If SAT
    - $\rightarrow$  collect the pattern that witness (f != g)
    - $\rightarrow$  simulate again to see if it can distinguish other FEC pairs
  5. Repeat 2 ~ 4
- $\rightarrow$  So the remaining problems are: How to create SAT proof instance? How to call SAT engine?

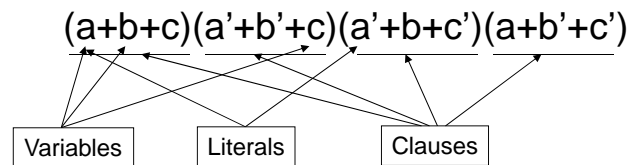
## Boolean Satisfiability Engine

- ◆ A engine (i.e. a program/library/function) that can prove or disprove a Boolean Satisfiability problem
  - Called a “SAT engine” or “SAT solver”
- ◆ A well-studied CS problem, but was once generally thought as an intractable problem.
  - Many practical, powerful, and brilliant ideas were brought up by EDA researchers in early 2000  $\rightarrow$  Orders of improvement
  - $\rightarrow$  Made a revolutionary change on the applications of SAT



## Creating Proof Instance

- ◆ Proof instance: the formula under proof
- ◆ Conjunctive Normal Form (CNF)
  - Most modern SAT engines represent the proof instances in CNF
  - Actually a “product of sum” representation



- To be satisfied, all the clauses need to be 1

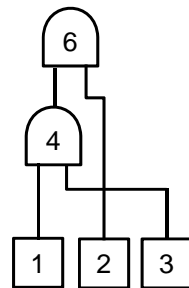
## Converting circuit to CNF

- ◆ Each gate is assigned a variable ID
- ◆ Each gate is converted to a set of CNF clauses based on its fanin variables
  - $g = \text{AND}(a, b)$ 
    1.  $a = 0 \rightarrow g = 0$                        $(a + !g)$
    2.  $b = 0 \rightarrow g = 0$                        $(b + !g)$
    3.  $a = 1 \ \& \ b = 1 \rightarrow g = 1$      $(!a + !b + g)$
- ◆ To solve  $(f = 1)$ , add a  $(f)$  clause
  - SAT engine is to check if all the clauses can be satisfiable at the same time.

## Converting circuit to CNF

### ◆ Example:

SAT [6] = 1



$$(1 + !4)(3 + !4)(!1 + !3 + 4) \\ (4 + !6)(2 + !6)(!2 + !4 + 6) \\ (6)$$

## Calling SAT engine

- ◆ Create a solver object
- ◆ Add clauses → proof instance
- ◆ (optional) Set proof limits
- ◆ Solve()!!
- ➔ We provide a SAT interface in “sat.h”
  
- ◆ (FYI) Incremental SAT
  - Reuse the partial learned information

## Using SAT to prove FEC pair

1. Create a solver object

```
satSolver solver;  
solver.initialize();
```
2. Create CNF for the circuit
  - For each gate in the circuit, create a variable for it

```
solver.newVar();
```
  - For each gate in the circuit, create CNF clauses for it

```
solver.addAigCNF(v, v1, ph1, v2, ph2);
```
  - Remember to take care CONST gate
3. Create the proof instance for  $F \equiv (f \neq g)$ 
  - Add clauses for F

```
solver.addXorCNF(FVar, fVar, fPh, gVar, gPh);
```
  - Call SAT to prove

```
solver.assumeRelease();  
solver.assumeProperty(newV, true);  
bool isSat = solver.assumpSolve();  
getSatAssignment(solver, patterns);
```

## Notes about FEC proof

- ◆ Order matters!!
  - Proving from PIs to POs can greatly reduce the proof effort
- ◆ Don't waste SAT-generated patterns (for  $f \neq g$ )
  - Pack them for parallel pattern simulation
- ◆ Many FEC pairs are actually  $(f, 1)$  or  $(f, 0)$ .
  - Should we do anything special for them?
- ◆ It's OK to skip some proofs. (Why?)
  - Skip it or limit the proof effort (e.g. #conflicts)
- ◆ Incremental SAT
- ◆ Balance between simulation and proof efforts

## Some advices

- ◆ Please do not fall into 軍備競賽...
  - Although it is possible you can implement a version that is 10X faster than mine...
- ◆ It's OK that you CANNOT finish the project.
  - I don't expect too many people can finish the project.
  - Think: 你的電子學有拿 100 分嗎?
- ◆ Please DO NOT spend 80% time for 20% points
  - e.g. parser error message, circuit optimization
- ◆ Always keep your code simple and straight!!
  - Always modularize your code
  - Compile and test from time to time

## References

- ◆ Functionally Reduced And-Inverter-Graph
  - [http://www.eecs.berkeley.edu/~alanmi/publications/2005/tech05\\_fraigs.pdf](http://www.eecs.berkeley.edu/~alanmi/publications/2005/tech05_fraigs.pdf)
- ◆ A System for Sequential Synthesis and Verification
  - <http://www.eecs.berkeley.edu/~alanmi/abc/>
- ◆ SAT solver
  - <http://www.satcompetition.org/>
  - <http://www.princeton.edu/~chaff/publication/DAC2001v56.pdf>
  - [http://www.princeton.edu/~chaff/publication/cade\\_cav\\_2002.pdf](http://www.princeton.edu/~chaff/publication/cade_cav_2002.pdf)