Quiz 1: CS CS4640 Name ____________________

1. The image registration problem is to take a reference image, *ref*, a transformed image, *im*, and a set of corresponding points, *pts*, in the two images, and to determine T, the transform between the two images, as well as to produce a registered version of *im*, called *reg*, in the coordinate frame of *ref*.

Given the coordinates (x, y) in *ref* and (u, v) in *im*, T can map either direction; i.e., $T_f : (x, y) \rightarrow (u, v)$ or $T_b : (u, v) \rightarrow (x, y)$. Discuss the pros and cons of each of these maps with respect to producing *reg*.

Consider $T_f\ (x,y) \rightarrow (u,v)$

\+ the selected (x,y) locations are exactly where a value is needed

\- $T(x,y)$ may not be an exact (u,v) in im frame. So, a value must be interpolated from nearby points (if any)

Consider $T_b\ (u,v) \rightarrow (x,y)$

\+ Takes an exact known value from (u,v) in im frame to point in reg frame.

\- $T_b(u,v)$ may not be an exact grid point in reg frame and thus grid values may require calculation from nearby points.

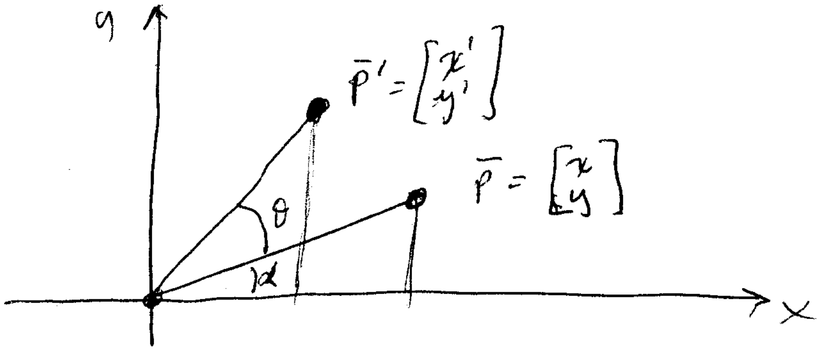
2. Give a simple geometric argument (with labeled diagram) why a rotational transform matrix has the form:

$$\begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

Hint: Consider $\bar{P} = [x, y]^T$ at angle α, and where it ends up after a rotation by θ. Use trig formulas:

$$cos(\alpha + \beta) = cos(\alpha)cos(\beta) - sin(\alpha)sin(\beta)$$

$$sin(\alpha + \beta) = sin(\alpha)cos(\beta) + cos(\alpha)sin(\beta)$$



let $|\bar{P}| = d$

$x = d \cos\alpha$

$y = d \sin\alpha$

Then $x' = d\cos(\alpha+\theta) = d\cos\alpha\cos\theta - d\sin\alpha\sin\theta = x\cos\theta - y\sin\theta$

$y' = d\sin(\alpha+\theta) = d\cos\alpha\sin\theta + d\sin\alpha\cos\theta = x\sin\theta + y\cos\theta$

$$\Rightarrow \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$