

1. Derive the following for random variables  $X$  and  $Y$

	$X = 1$	$X = 2$	$X = 3$
$Y = 1$	0.1	0.05	0.2
$Y = 2$	0.05	0.25	0.35

- a.  $\Pr(X = 1) = 0.1 + 0.05 = 0.15$   
 b.  $\Pr(X = 2 \cap Y = 1) = 0.25$   
 c.  $\Pr(X = 3 | Y = 2) = \frac{P(X \cap Y)}{\Pr(Y)} = \frac{0.35}{0.05+0.25+0.35} = 0.318$
2. Given 2 die  $D_1$  and  $D_2$  with  $\Omega = \{1,2,3,4,5,6\}$ , find the following
- a.  $\Pr(D_1 \text{ has a larger value than } D_2)$

	$D_1$ Values
$D_2 = 1$	$\{2,3,4,5,6\}$
$D_2 = 2$	$\{3,4,5,6\}$
$D_2 = 3$	$\{4,5,6\}$
$D_2 = 4$	$\{5,6\}$
$D_2 = 5$	$\{6\}$
$D_2 = 6$	$\{\}$

Since there are  $6^2$  possible combination of outcomes, and we see from the table that 15  $D_1$  values that are greater than  $D_2$  the answer is  $\frac{15}{36} = \frac{5}{12}$

- b. Expected value of the sum of  $D_1$  and  $D_2$

$$E[X] = \sum_{\omega \in \Omega} \omega * \Pr[X = \omega]$$

Sums	$D_1 = 1$	$D_1 = 2$	$D_1 = 3$	$D_1 = 4$	$D_1 = 5$	$D_1 = 6$
$D_2 = 1$	2	3	4	5	6	7
$D_2 = 2$	3	4	5	6	7	8
$D_2 = 3$	4	5	6	7	8	9
$D_2 = 4$	5	6	7	8	9	10
$D_2 = 5$	6	7	8	9	10	11
$D_2 = 6$	7	8	9	10	11	12

Value	2	3	4	5	6	7	8	9	10	11	12
$P[\text{Value}]$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$\begin{aligned}
 & 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + 4\left(\frac{3}{36}\right) + 5\left(\frac{4}{36}\right) + 6\left(\frac{5}{36}\right) + 7\left(\frac{6}{36}\right) + 8\left(\frac{5}{36}\right) + 9\left(\frac{4}{36}\right) + 10\left(\frac{3}{36}\right) \\
 & + 11\left(\frac{2}{36}\right) + 12\left(\frac{1}{36}\right) \\
 & = \frac{2 + 6 + 12 + 20 + 30 + 42 + 40 + 36 + 30 + 22 + 12}{36} = \frac{252}{36} = 7
 \end{aligned}$$

3. For  $f(X = x) = \begin{cases} \frac{1}{2} & \text{if } x \in [0,2] \\ 0 & \text{if } x \notin [0,2] \end{cases}$  what is  $\Pr(f(X = 1))$ ?

The probability that a continuous random variable equal any number is exactly 0

4. For  $D = \{-1, 7, 4\}$ , and  $M$  from  $\Omega = \{1, 3, 5\}$ .  $m$  pulled from  $f(x) = \frac{1}{6} \exp\left(-\frac{|m-x|}{3}\right)$ , and  $\Pr(M = 1) = 0.4, \Pr(M = 3) = 0.3$ , and  $\Pr(M = 5) = 0.3$ , find which model is most likely  
Given that  $\ln(\Pr(D | M)) = \sum_{x \in \Omega} \frac{-|m-x|}{3}$

$$M = 1$$

$$-\frac{|0.4 + 1|}{3} - \frac{|0.4 - 7|}{3} - \frac{|0.4 - 4|}{3} = -\frac{1.4 + 6.6 + 3.6}{3} = -\frac{8.4}{3}$$

$$M = 3$$

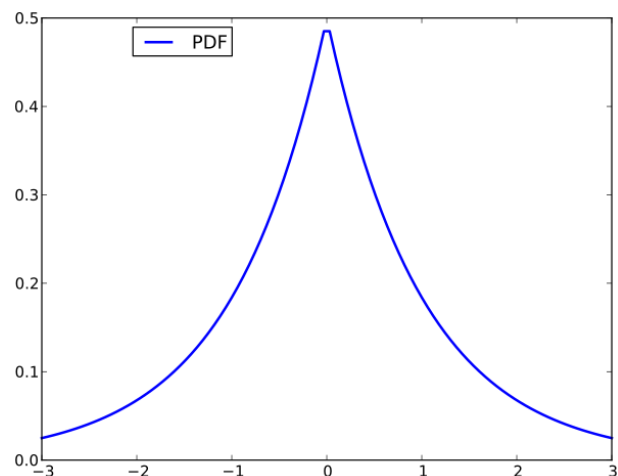
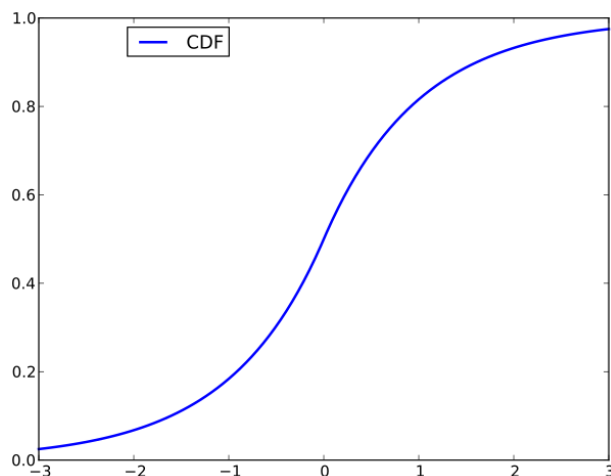
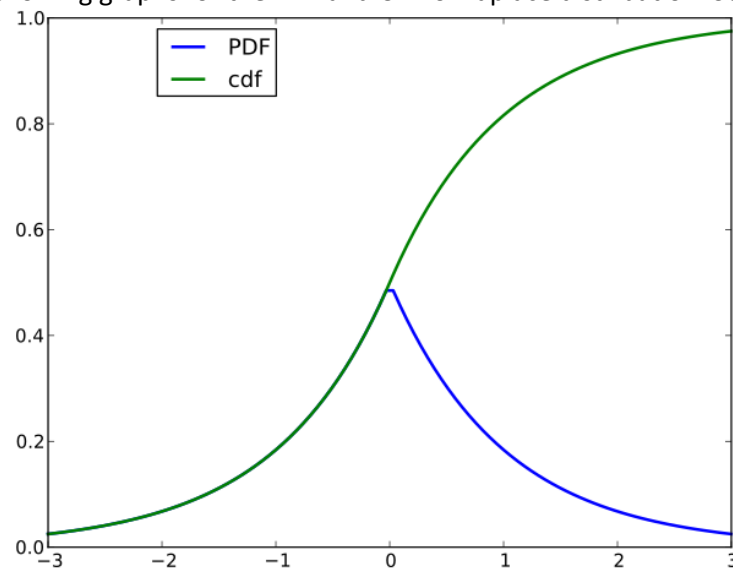
$$-\frac{|0.3 + 1|}{3} - \frac{|0.3 - 7|}{3} - \frac{|0.3 - 4|}{3} = -\frac{1.3 + 6.7 + 3.7}{3} = -\frac{11.7}{3}$$

$$M = 5$$

$$-\frac{|0.3 + 1|}{3} - \frac{|0.3 - 7|}{3} - \frac{|0.3 - 4|}{3} = -\frac{1.3 + 6.7 + 3.7}{3} = -\frac{11.7}{3}$$

$M = 1$  is the most likely model

5. Generated following graphs for the PDF and CDF of Laplace distribution. See code below



```
import matplotlib as mpl
mpl.use('svg')
import matplotlib.pyplot as plt
import matplotlib.mlab as mlab
from scipy.stats import laplace as l
import numpy as np
import math

x = np.linspace(-3,3,100)

plt.plot(x, l.pdf(x), linewidth=2.0, label='PDF')
plt.plot(x, l.cdf(x), linewidth=2.0, label='CDF')

plt.legend(bbox_to_anchor=(.35,1))
plt.savefig('cdf.svg', bbox_inches='tight')
```