An Unconditional Lower Bound for Two-Pass Streaming Algorithms for Maximum Matching Approximation

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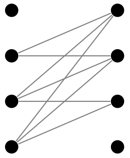
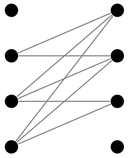
Maximum Bipartite Matching (MBM)

Definition

A bipartite **matching** is a subset of vertex disjoint edges of the graph. A **maximum matching** is a largest matching of the graph.





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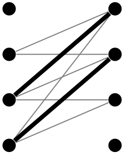
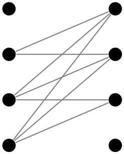


M

matching



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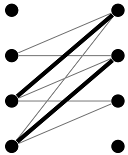

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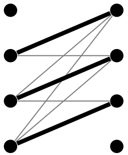

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Approximations

- M is a $(\frac{|M|}{|M^*|})$-approximate matching (e.g. $\frac{2}{3}$).

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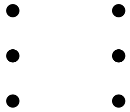

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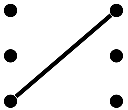
$\boxed{e_1}$



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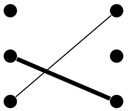

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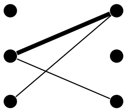

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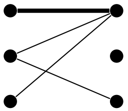

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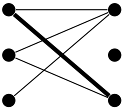

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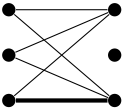

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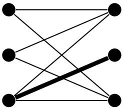

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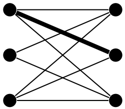

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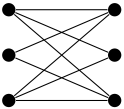

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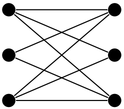

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- Store all edges with $O(n^2)$ space.

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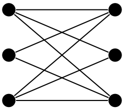

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Interesting Algorithms

- Use $O(n \operatorname{polylog} n)$ space (semi-streaming).
  - Many graph problems require $\Omega(n)$ space in one pass [FKM+04].
- Use one or more passes of the stream.

Approximate MBM in the Semi-Streaming Model

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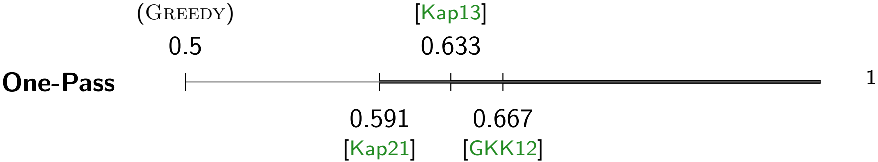
(Greedy)

0.5

One-Pass ⊢——————————— 1

Approximate MBM in the Semi-Streaming Model



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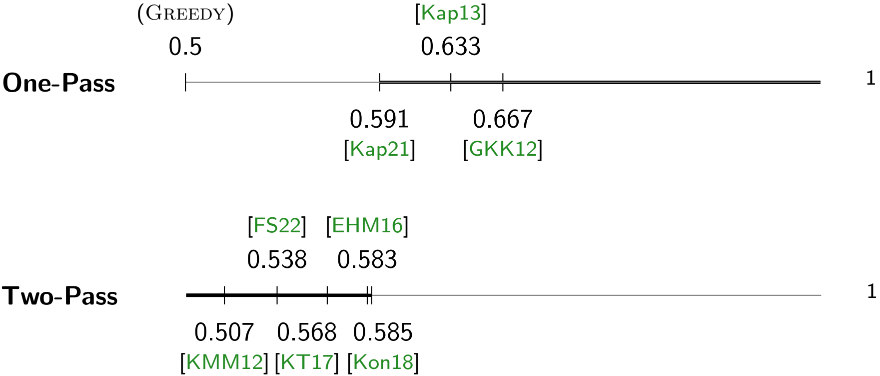

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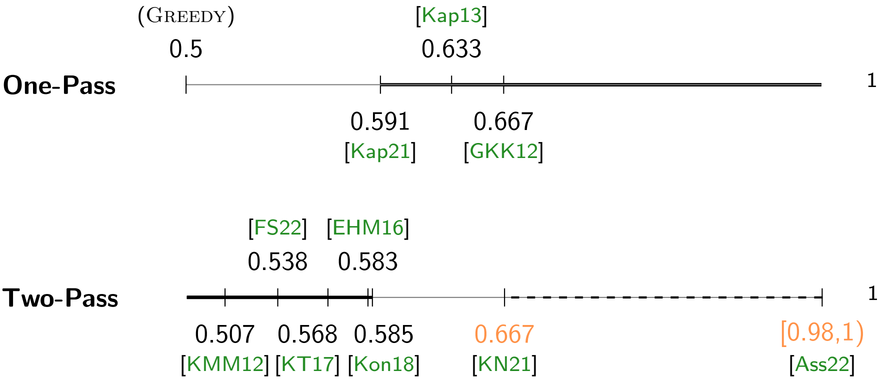

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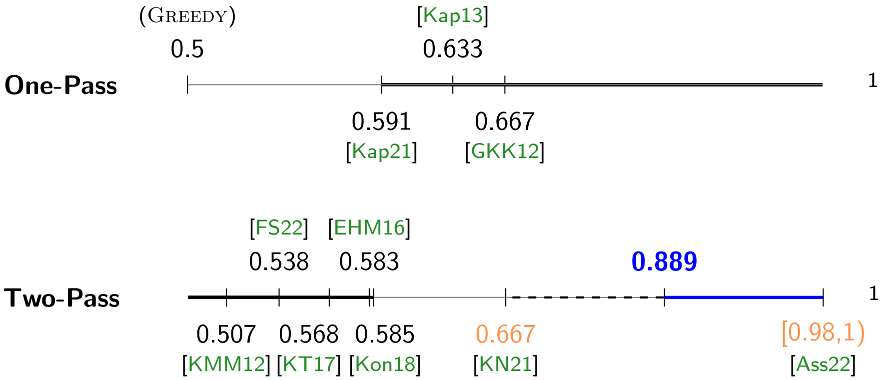

Our Result

For $\varepsilon > 0$, any constant-error two-pass $(\frac{8}{9} + \varepsilon)$-approximation streaming algorithm for MBM requires $n^{1+\Omega(1/(\log \log n)^2)}$ space.

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E_A	E_B

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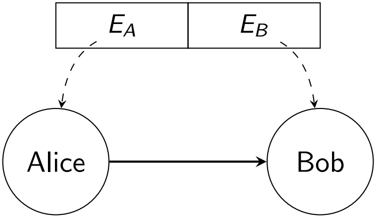

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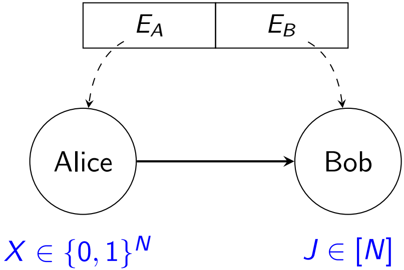

$\boxed{\textbf{Return } \text{Index}(X, J) = X[J]}$

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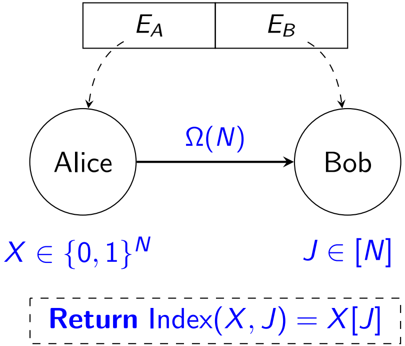

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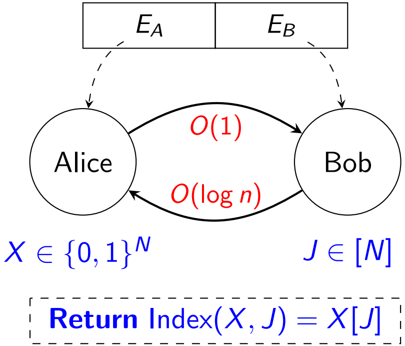

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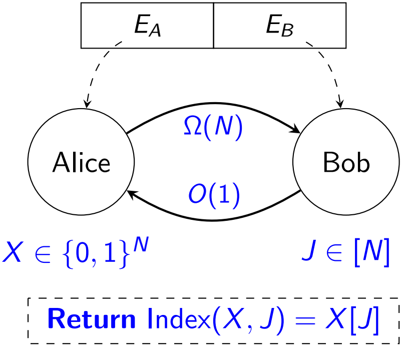
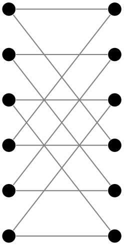


Info Cost Tradeoff [JRS09]

1. If $\mathrm{ICost}^B_{\mathcal{D}}(\pi) = O(1)$, then
2. $\mathrm{ICost}^A_{\mathcal{D}}(\pi) = \Omega(N)$

Ruzsa-Szeméredi (RS) Graphs



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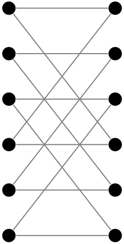
Definition (Induced Matching)

A matching whose vertex induced subgraph contains only its edges.



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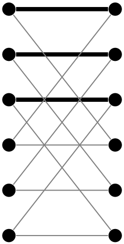

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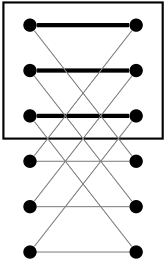

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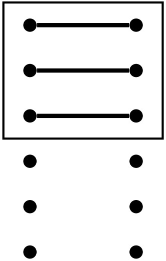

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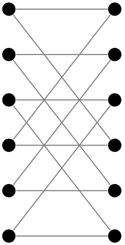

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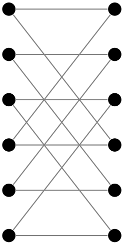

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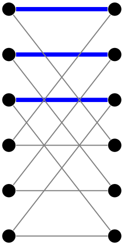

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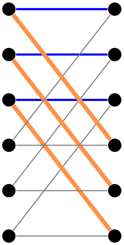

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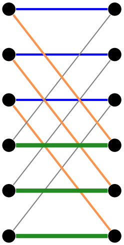

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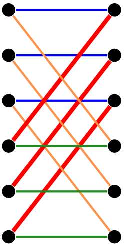

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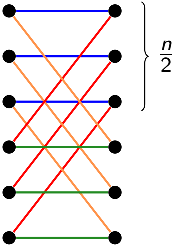

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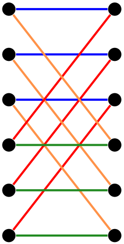
For constant $\delta > 0$, there exists a (bipartite) (r, t)-RS graph on $2n$ vertices where $r = (\frac{1}{2} - \delta) \cdot n$ and $t = n^{\Omega(1/\log\log n)}$.



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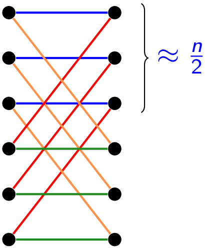

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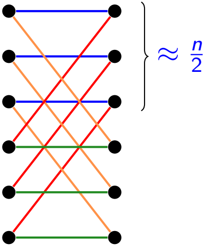

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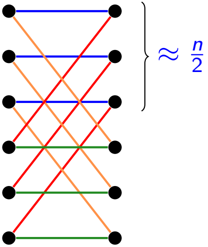

- $n^{\Omega(1/\log\log n)}$ many induced matchings.
- $\gg n \operatorname{polylog} n$ edges in the graph.

Two-Pass Hard Graph and Adversarial Stream

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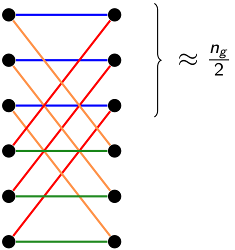
Global Graph



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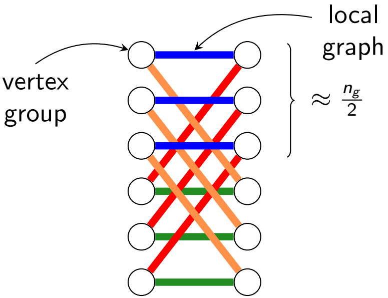

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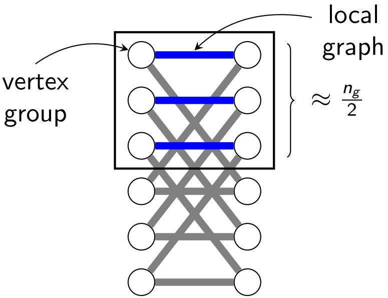
Two-Pass Hard Graph and Adversarial Stream

Global Graph



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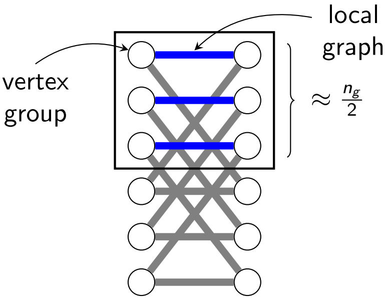

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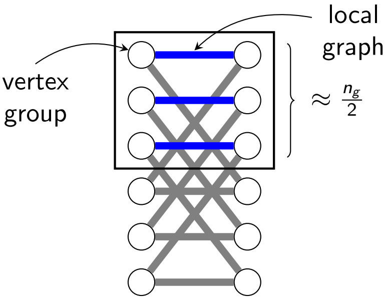

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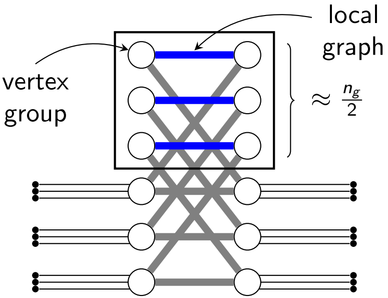

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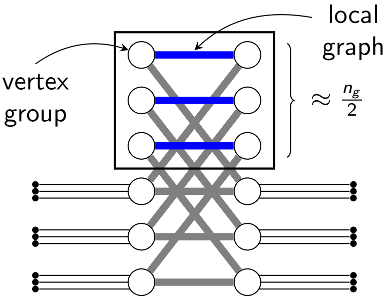

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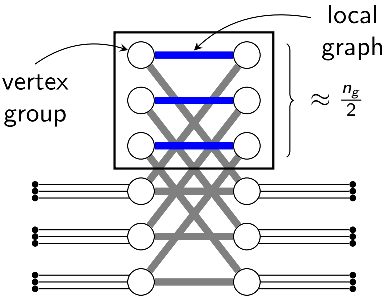

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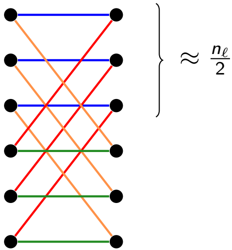

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Two-Pass Hard Graph and Adversarial Stream

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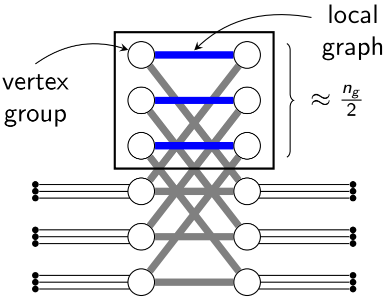

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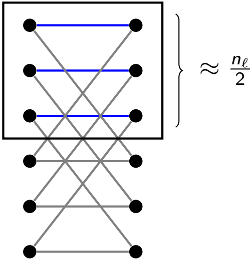

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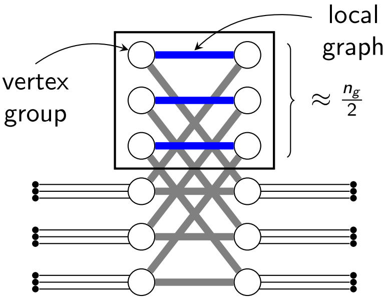

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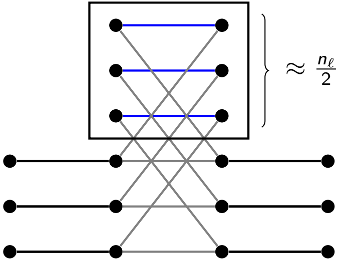

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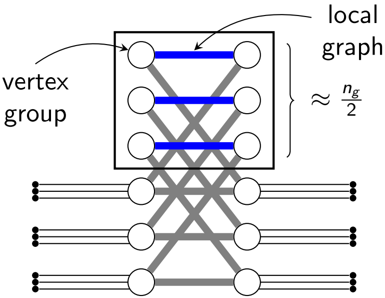

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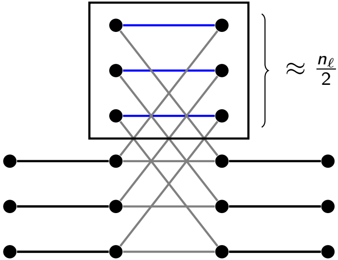

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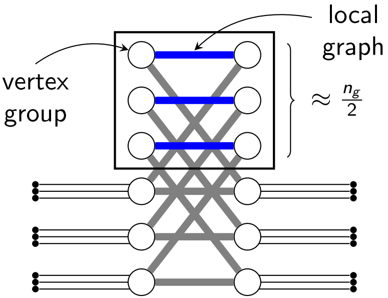

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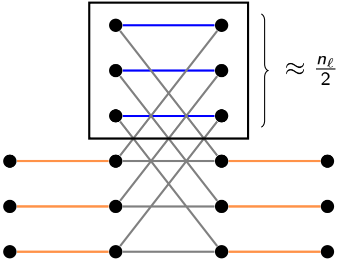

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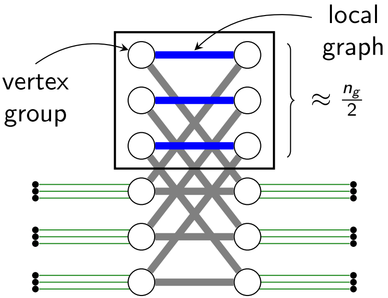

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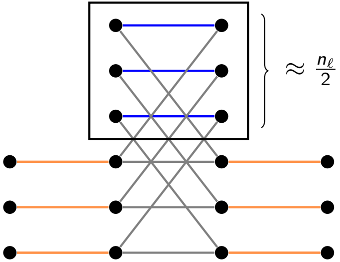

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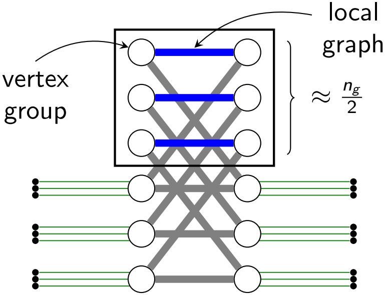

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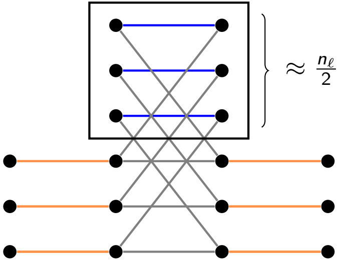

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local RS-graphs	local selectors	global selector

Two-Pass Hard Graph and Adversarial Stream

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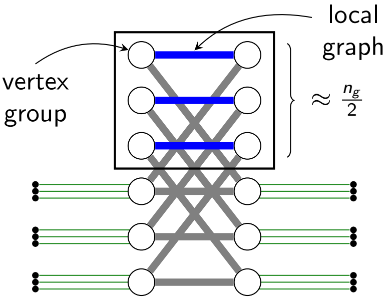

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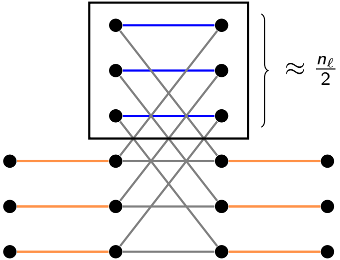

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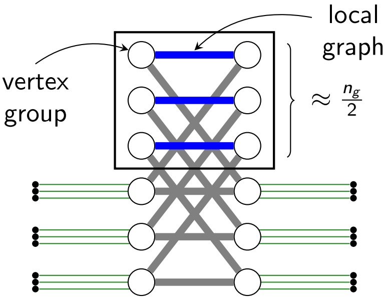
local RS-graphs	local selectors	global selector

$\mathcal{A}$

Two-Pass Hard Graph and Adversarial Stream

Global Graph



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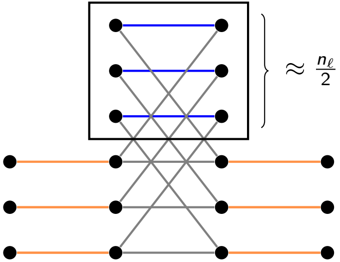

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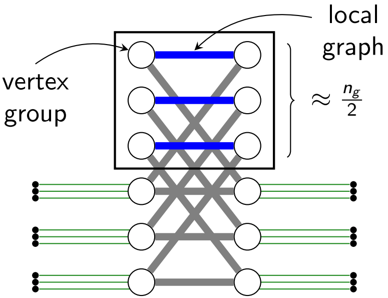
Two-Pass Hard Graph and Adversarial Stream

Global Graph



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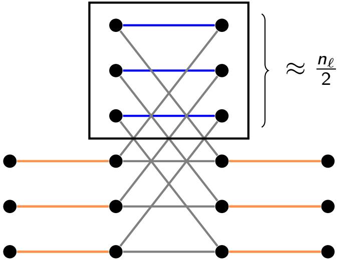

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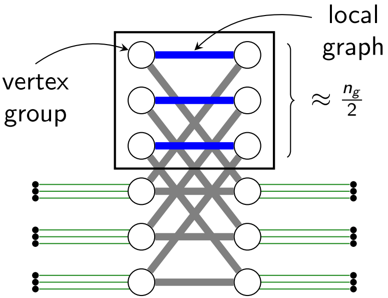
$o(1)$ per local graph (Index instance) $\quad \mathcal{A} \quad O(n)$

Two-Pass Hard Graph and Adversarial Stream

Global Graph



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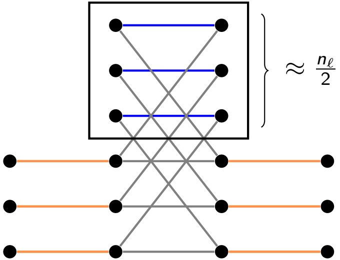

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Two-Pass Hard Graph and Adversarial Stream

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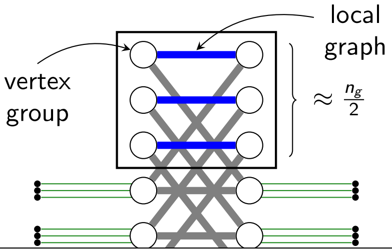

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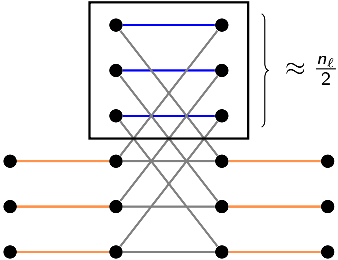

Info Cost Tradeoff [JRS09]

1. If $\mathrm{ICost}^{B}_{\mathcal{D}}(\pi) = O(1)$, then
2. $\mathrm{ICost}^{A}_{\mathcal{D}}(\pi) = \Omega(N)$

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$\mathcal{A}$

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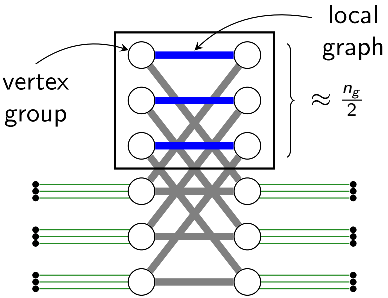

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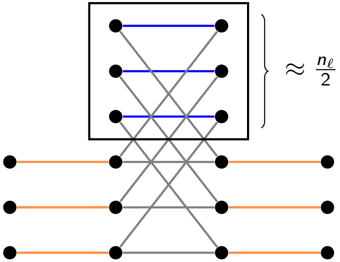

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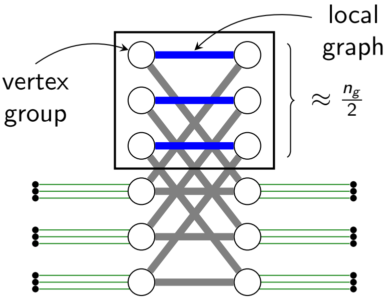

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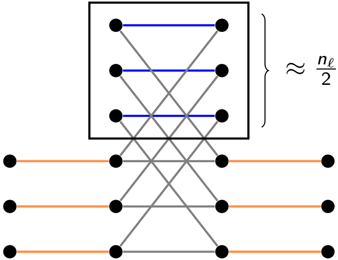

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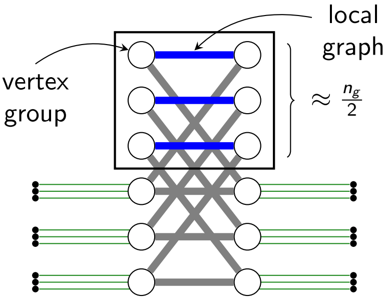

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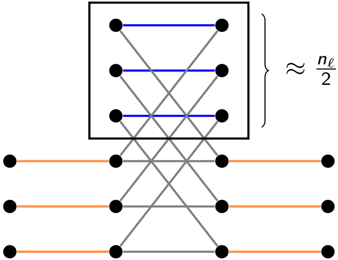

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Two-Pass Hard Graph and Adversarial Stream

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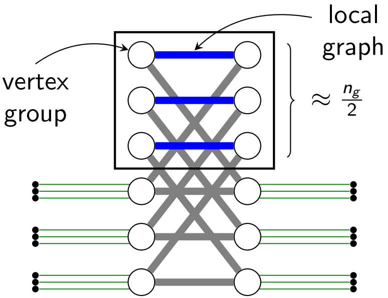

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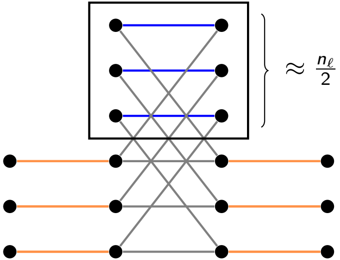

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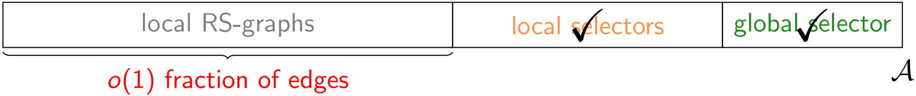

Two-Pass Hard Graph and Adversarial Stream

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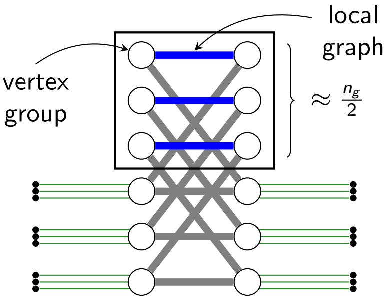

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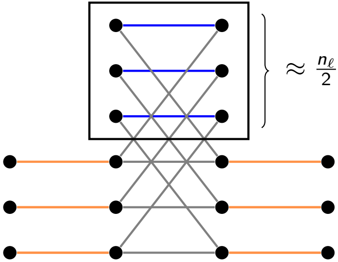

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Next Steps

Our Result

For $\varepsilon > 0$, any constant-error two-pass $(\frac{8}{9} + \varepsilon)$-approximation streaming algorithm for MBM requires $n^{1+\Omega(1/(\log\log n)^2)}$ space.

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- [AS23] recently showed that $(1-\varepsilon)$-approximations require $\Omega(\log 1/\varepsilon)$ passes (conditional)
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Open Questions

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Thank you!

References I

Sepehr Assadi and Janani Sundaresan, *Hidden permutations to the rescue: Multi-pass semi-streaming lower bounds for approximate matchings*, 64th IEEE Annual Symposium on Foundations of Computer Science, FOCS 2023, Santa-Cruz, CA, USA, November 6 - 9, 2023, IEEE, 2023.

Sepehr Assadi, *A two-pass (conditional) lower bound for semi-streaming maximum matching*, Proceedings of the 2022 ACM-SIAM Symposium on Discrete Algorithms, SODA 2022, Virtual Conference / Alexandria, VA, USA, January 9 - 12, 2022 (Joseph (Seffi) Naor and Niv Buchbinder, eds.), SIAM, 2022, pp. 708–742.

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