On Two-Pass Streaming Algorithms for Maximum Bipartite Matching

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Joint work with Dr Christian Konrad

Overview

1. Background
2. Our Work
   - Lower Bound
   - Algorithmic
3. Discussion

Matchings

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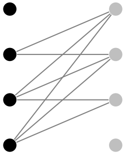

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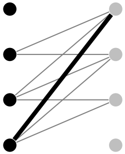

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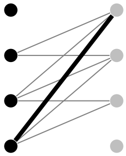

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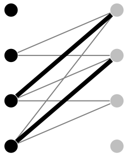

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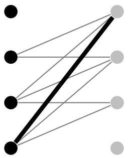

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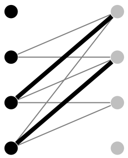

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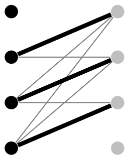

maximum matching

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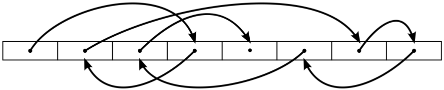
Finding a maximum matching:

- exact algorithms exists, i.e. Hopcroft-Karp [SWAT71];
- require random access to the graph's edges (infeasible requirement for massive graphs).

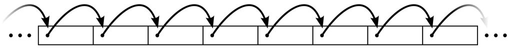


Semi-Streaming Model

Feigenbaum et al. [ICALP04]

A graph with n vertices is presented to an algorithm as a stream of edges where the storage space of the algorithm is bounded by $O(n \operatorname{polylog} n)$.

- Only allows **sequential access** to the graph.
- Algorithms with space $O(n \operatorname{polylog} n) = O(n\ (\log n)^{O(1)})$.
- Ideally with few passes of the stream.



Maximum Bipartite Matching Literature I

	Algorithmic	Lower Bound
one-pass	$\frac{1}{2}$ [folklore]	$\frac{1}{2} + 0.167$ [SODA12] $\frac{1}{2} + 0.132$ [SODA13] $\frac{1}{2} + 0.091$ [SODA21]
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Other strategies and techniques are required!

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[1] where the first pass finds a maximal matching, i.e., at least a $\frac{1}{2}$-approximation.

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- Uses a dense family of Rusza Szemeredi (RS) graphs which contains (many) near-perfect matchings. *This result may be of independent interest.*

Rusza Szemeredi Graphs

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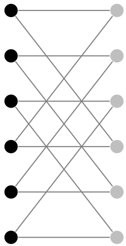

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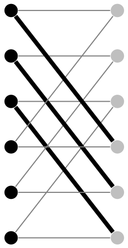

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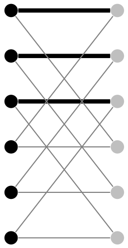

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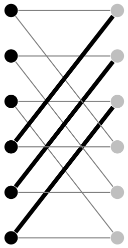

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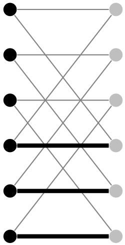

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Proposition ([SODA12])

There exists a bipartite RS graph with $N^{\Omega(\frac{1}{\log\log N})}$ induced matchings of size $\frac{N}{2} - \epsilon N$ where $N = |A| = |B|$ and $\epsilon > 0$.

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Note: This is a dense RS graph with $N^{1+\Omega(\frac{1}{\log\log N})} \supset O(N \operatorname{polylog} N)$ edges.

One-Pass Lower Bound Proof [SODA12]

Two-Party Communication Setup

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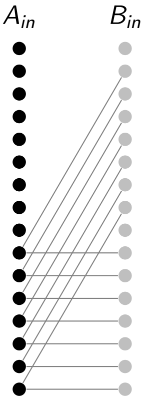

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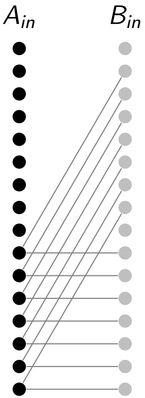

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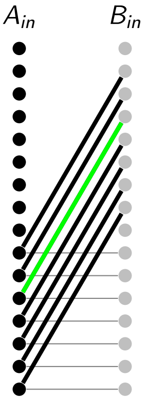

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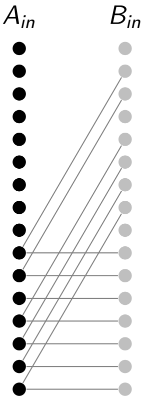

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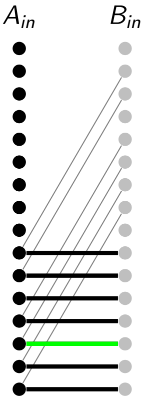

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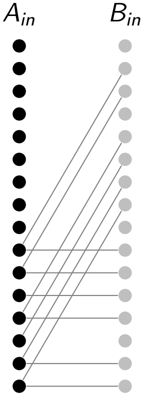

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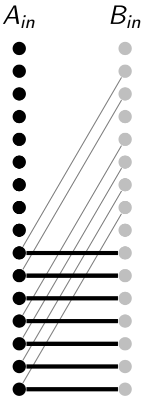

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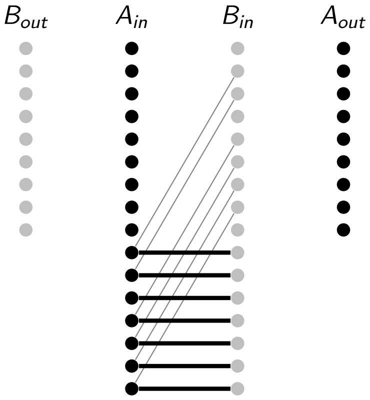

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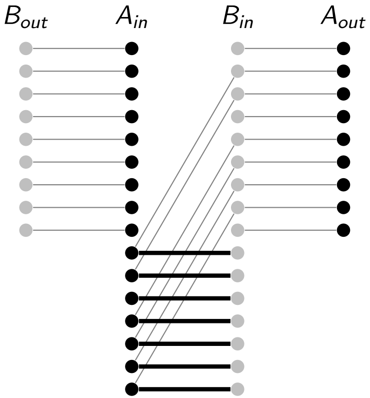

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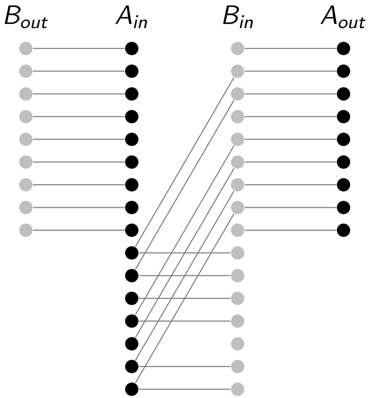

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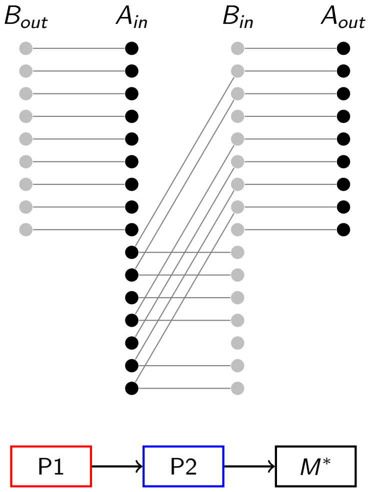

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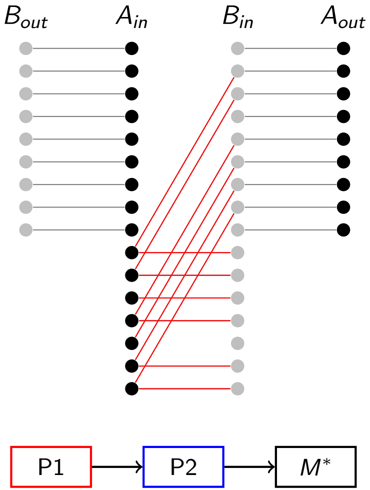

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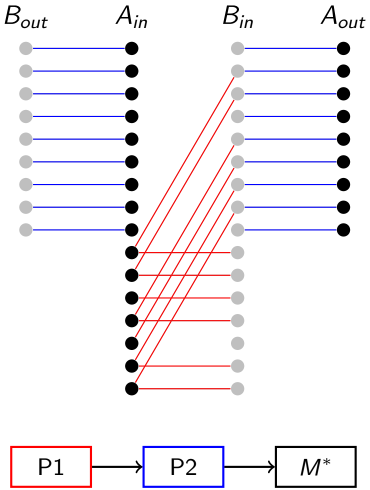

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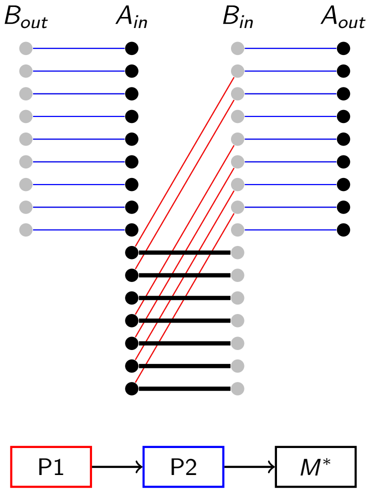

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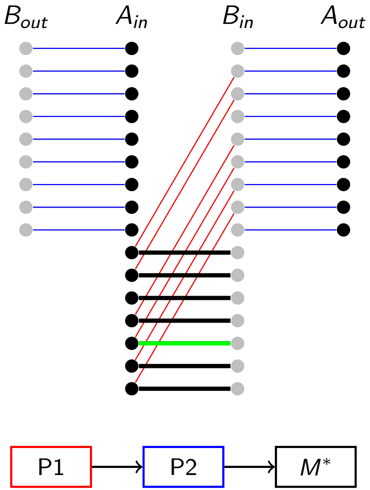

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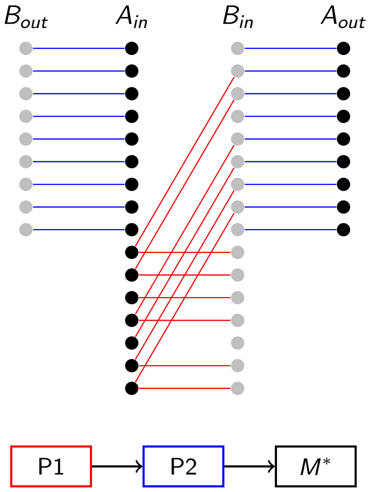
5. $(\frac{2}{3} + \epsilon)$-approx



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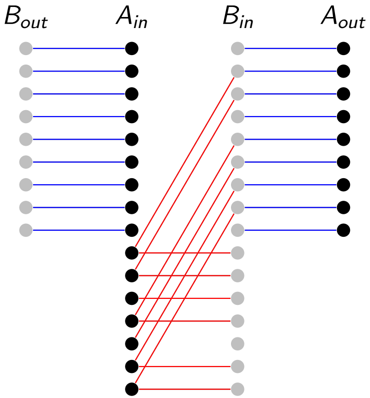

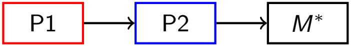

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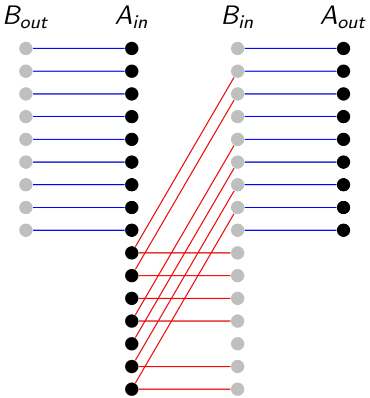
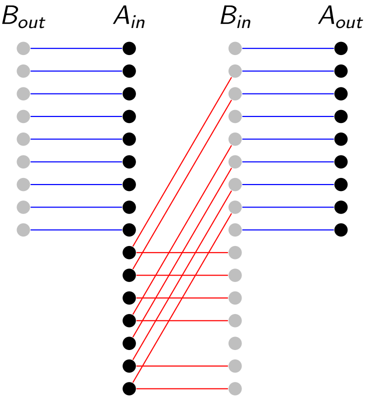
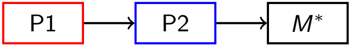


$$\boxed{P1} \longrightarrow \boxed{P2} \longrightarrow \boxed{M^*}$$

Two-Pass Lower Bound Proof

Goal: Give both players knowledge of a maximal matching without affecting the difficulty of the problem.





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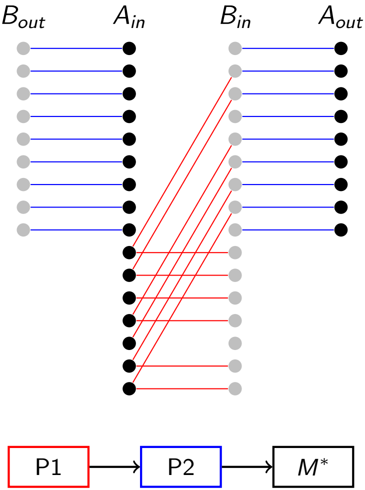
Outline:

- Do dense RS graphs contain perfect matchings?



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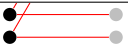

Proposition

There exists a bipartite RS graph with $N^{\Omega(\frac{1}{\log\log N})}$ induced matchings of size $\frac{N}{2} - \epsilon N$ where $N = |A| = |B|$ and $\epsilon > 0$ such that there are $N^{\Omega(\frac{1}{\log\log N})}$ disjoint near-perfect matchings, each of size $N - 2\epsilon N$.

P1 $\longrightarrow$ P2 $\longrightarrow$ M^*

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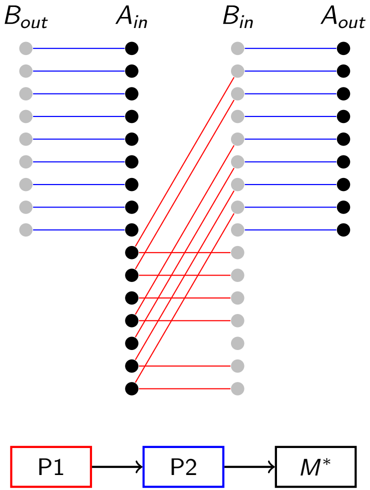

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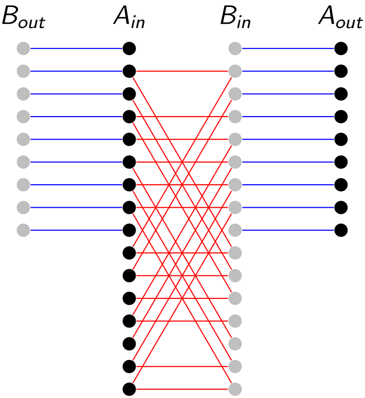

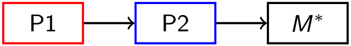

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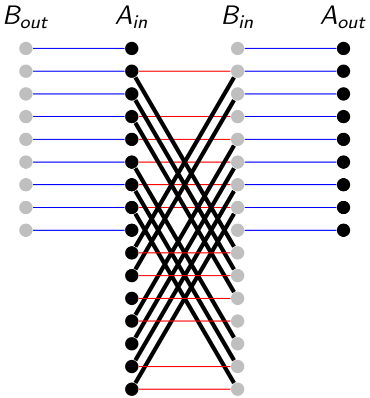

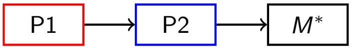

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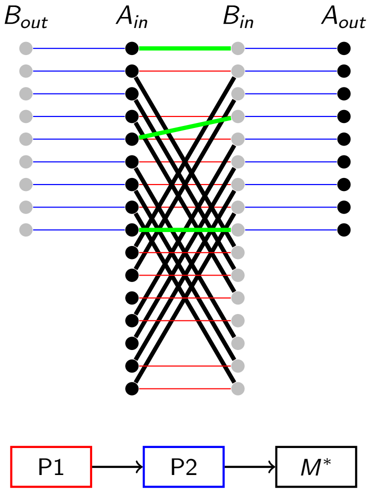

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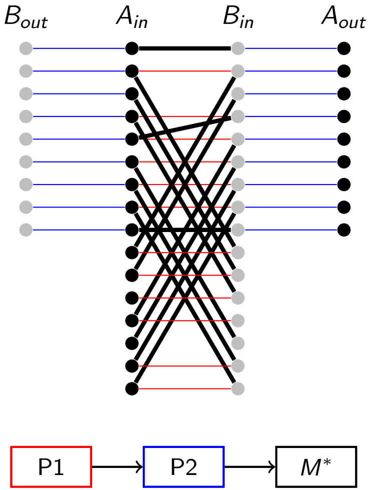
- We use this RS graphs which has a near-perfect matching of size $N - \epsilon' N$ in the construction.
- $(\frac{2}{3} + \epsilon)$-approx requires space $N^{1+\Omega(\frac{1}{\log \log N})} \supset O(N \operatorname{polylog} N)$.



Overview

1. Background
2. Our Work
   - Lower Bound
   - Algorithmic
3. Discussion

Algorithmic Result

Two-Pass Maximum Bipartite Matching

Algorithmic:

- Even a combination of the two dominant techniques in the area cannot beat the current state-of-the-art $2 - \sqrt{2} \approx \frac{1}{2} + 0.085$-approximation.

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- A novel meta algorithm that exactly achieves the current state-of-the-art.
- A family of hard-instance graphs shows the analysis is tight.

Algorithmic Approach

Class of algorithms:

1. finds a maximal matching M (first-pass);
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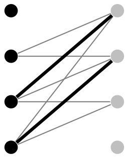

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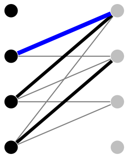

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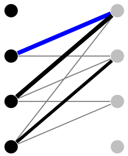

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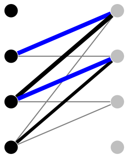

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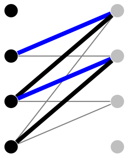

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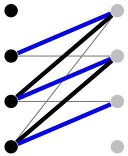

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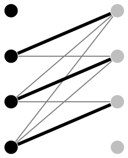

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Two-Pass Algorithm Summary

Using the two dominant techniques, parameterised by p and d:

1. subsampling with probability p [APPROX12] [MFCS18]
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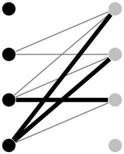

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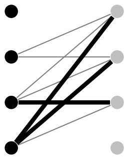

semi-incomplete matching

Let $G = (A, B, E)$ be any bipartite graph.

Let $\pi = e_1, e_2, \ldots$ be any stream of its edges.

First pass:

- $M \leftarrow \textsc{Greedy}(\pi)$

Second pass:

- subsample $M' \subseteq M$ with prob. p
- $H_L \leftarrow E \cap A_{out} \times B(M')$
- $H_R \leftarrow E \cap A(M') \times B_{out}$
- $M'_L \leftarrow \textsc{Greedy}_d(\pi_{H_L})$
- $M'_R \leftarrow \textsc{Greedy}_d(\pi_{H_R})$

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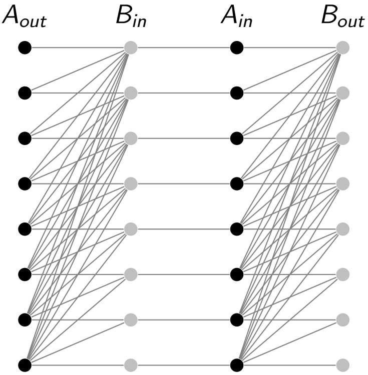

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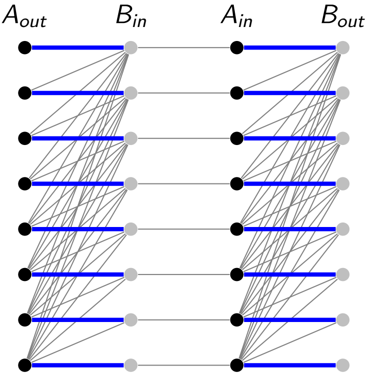
The goal of the algorithm is to find the maximum matching M^*.



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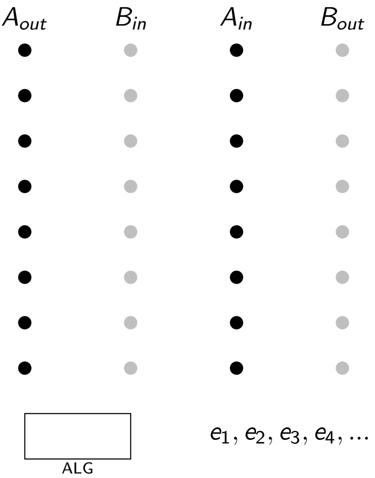

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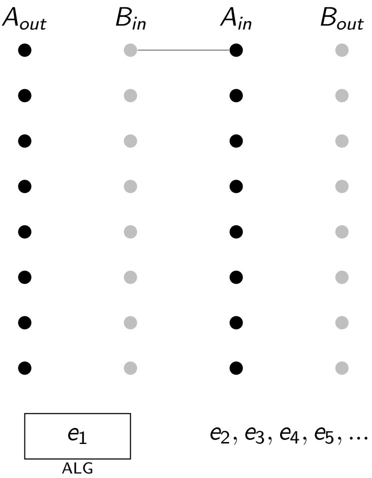

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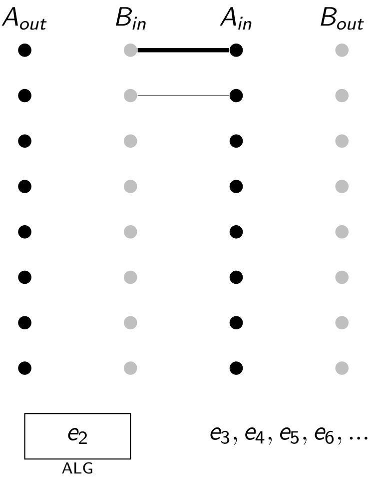

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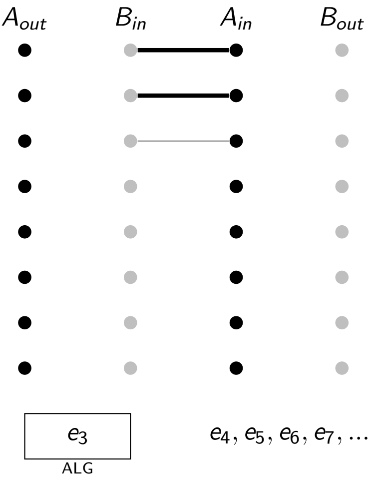

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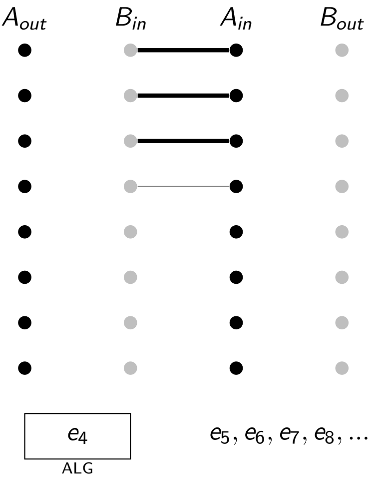

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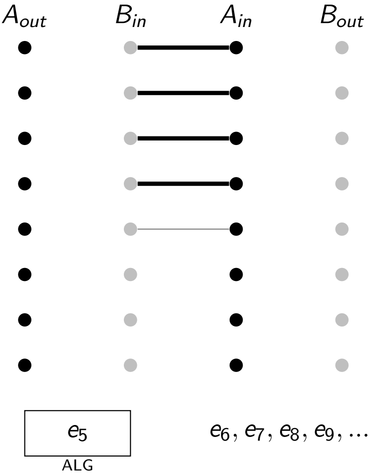

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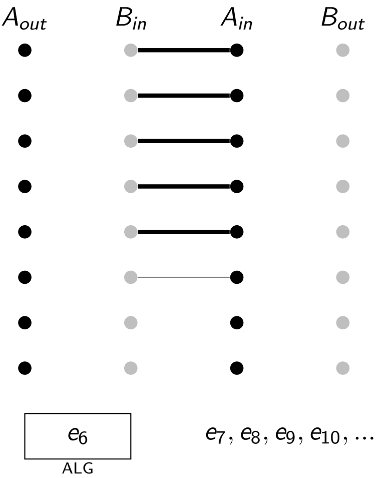

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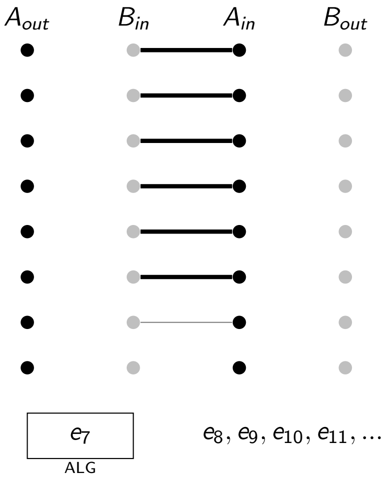

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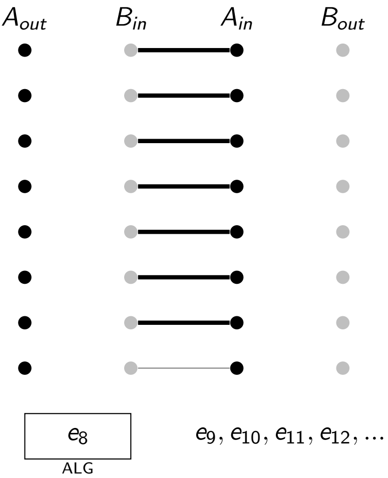

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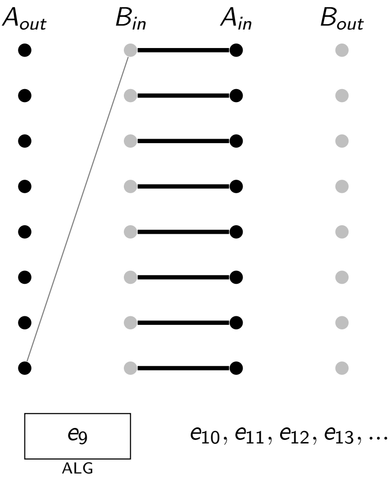

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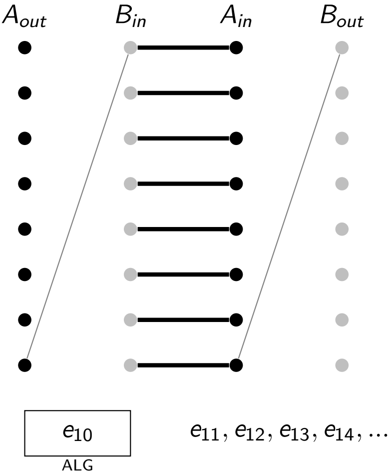

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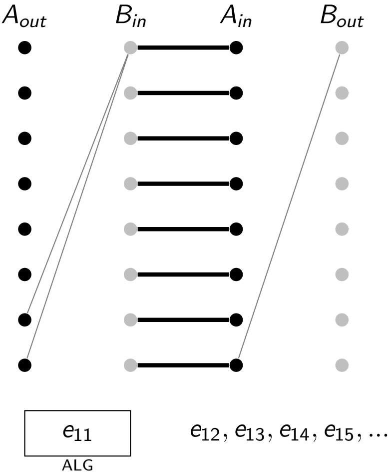

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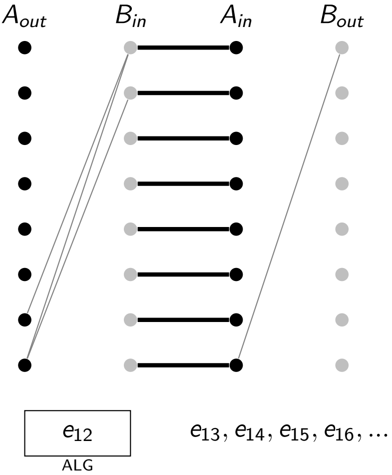

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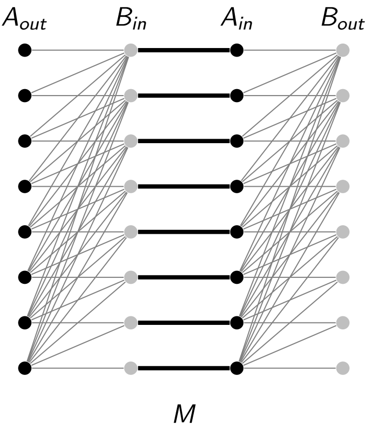

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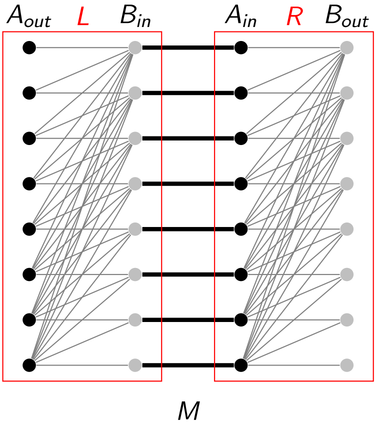

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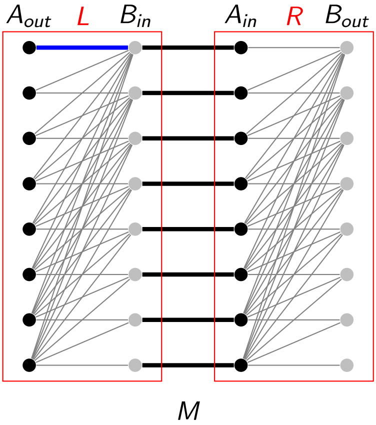

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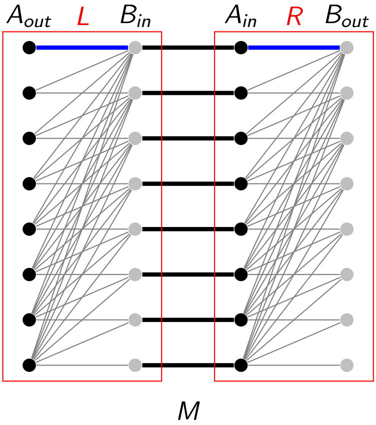

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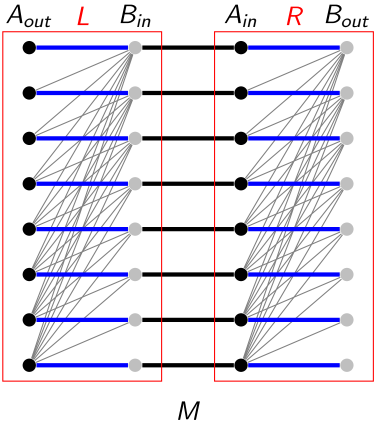

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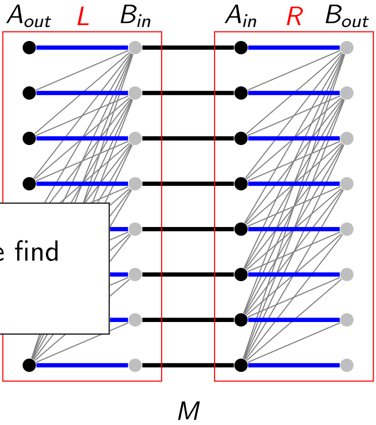
How can we find these?



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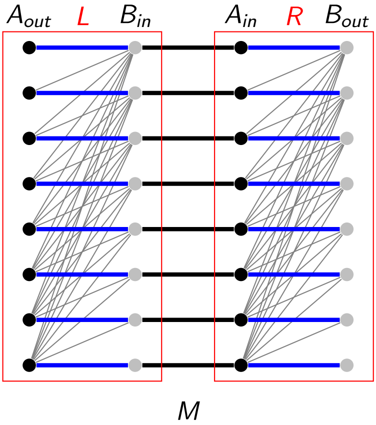

Two-Pass Algorithm

Let $G = (A, B, E)$ be a hard-instance (worst-case) graph.

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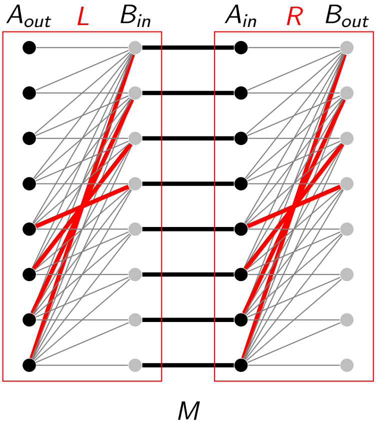

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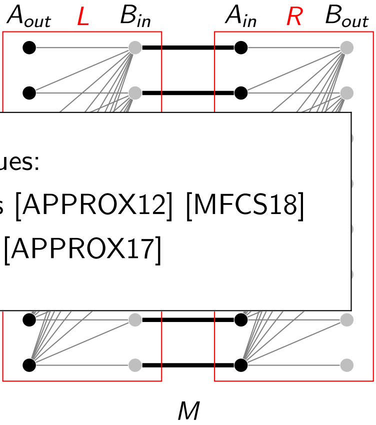
First

-



Using the two dominant techniques:

1. subsample the inner vertices [APPROX12] [MFCS18]
2. run $\textsc{Greedy}_d$ [ICDMW16] [APPROX17]

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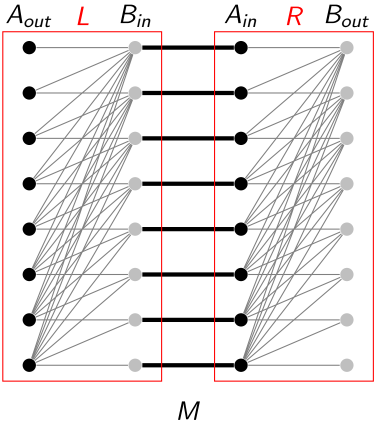

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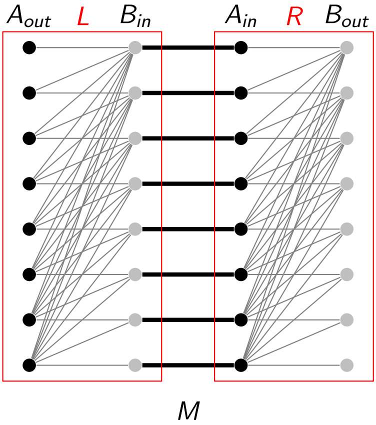

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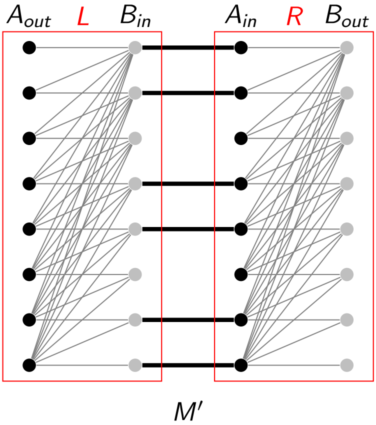
Second pass:

- subsample $M' \subseteq M$ with prob. p



$d = 3, \ p = 0.67$

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First pass:

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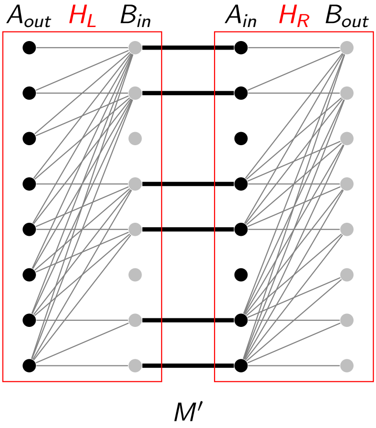
Second pass:

- subsample $M' \subseteq M$ with prob. p
- $H_L \leftarrow E \cap A_{out} \times B(M')$
- $H_R \leftarrow E \cap A(M') \times B_{out}$



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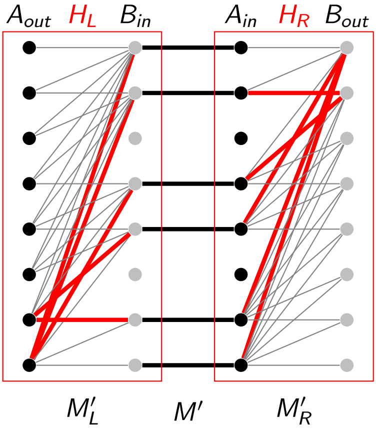

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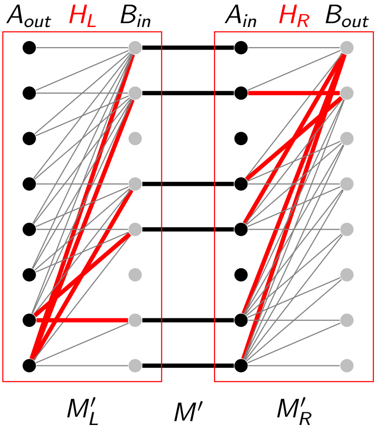

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$\mathbb{E}_{M'}[|M'_L|] = \frac{dp}{d+p} \cdot |M^*_L|$

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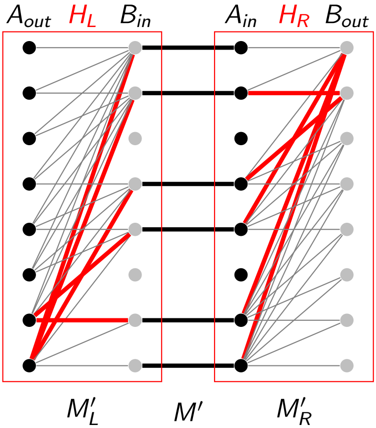

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$$\mathbb{E}_{M'}[|M'_L|] = \frac{dp}{d+p} \cdot |M^*_L|$$

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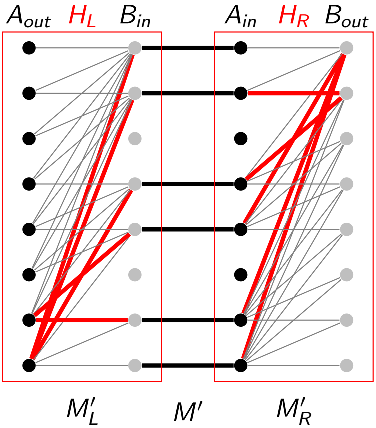
Two-Pass Algorithm

Analysis



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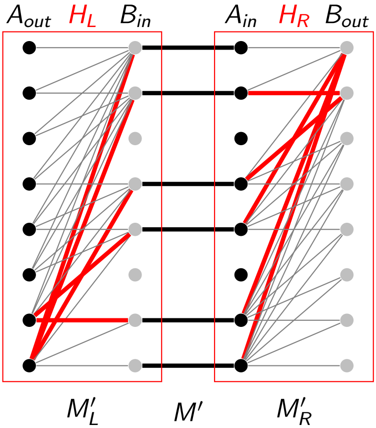
Two-Pass Algorithm

Analysis

$$|\mathcal{Q}| = (\frac{p}{d+p} - \frac{p}{2d}) \cdot |M^*|.$$



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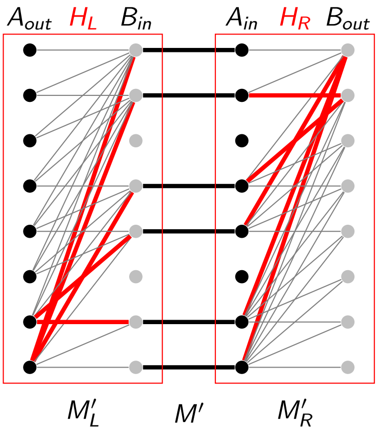
Analysis

$$|\mathcal{Q}| = \left(\frac{p}{d+p} - \frac{p}{2d}\right) \cdot |M^*|.$$

Therefore, the final matching is of size

$$\left(\frac{1}{2} + \frac{p}{d+p} - \frac{p}{2d}\right) \cdot |M^*|.$$



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$$\mathbb{E}_{M'}[|M'_L|] = \tfrac{dp}{d+p} \cdot |M^*_L|$$

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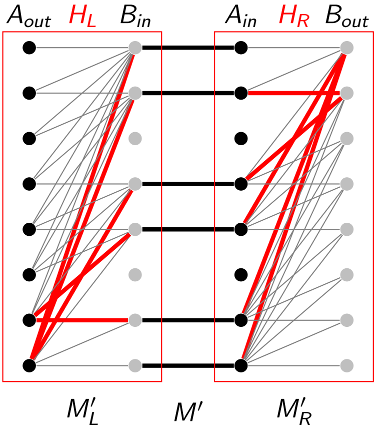
Analysis

$$|\mathcal{Q}| \geq (\frac{p}{d+p} - \frac{p}{2d}) \cdot |M^*|.$$

Therefore, the final matching is of size *at least*

$$(\frac{1}{2} + \frac{p}{d+p} - \frac{p}{2d}) \cdot |M^*|.$$



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$$\mathbb{E}_{M'}[|M'_L|] \geq \frac{dp}{d+p} \cdot |M^*_L|$$
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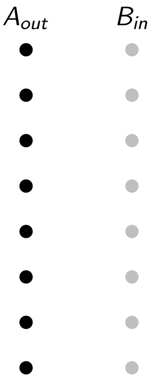
Main Proof Outline



Main Proof Outline

Setup:

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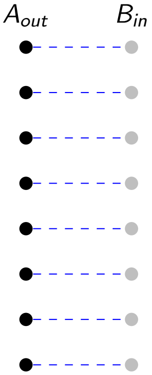

Main Proof Outline

Setup:

- any bipartite graph $G = (A, B, E)$ with a maximum matching M^* and any stream of edges π;
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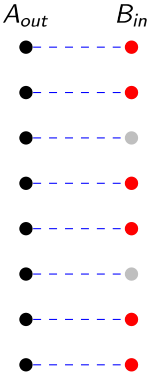

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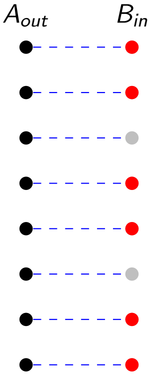

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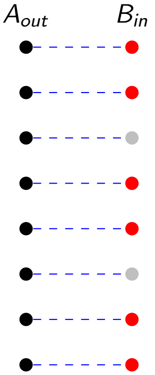

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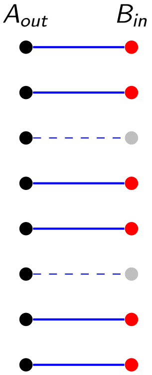

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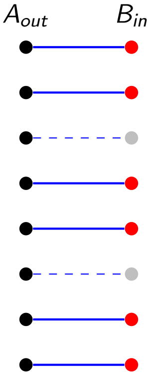

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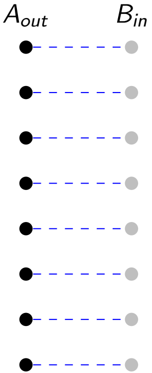

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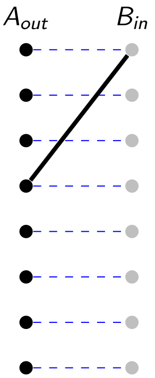

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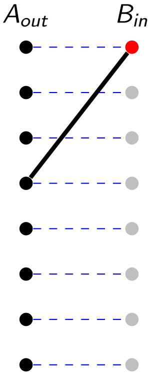

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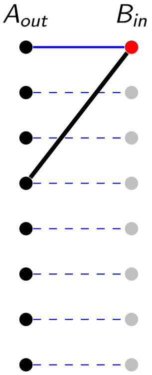

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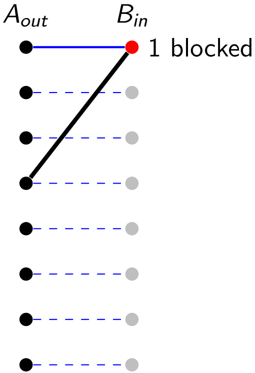

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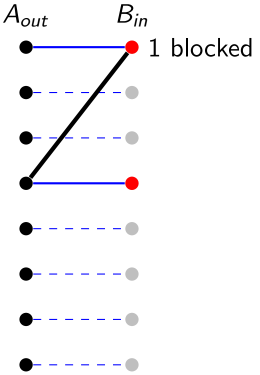

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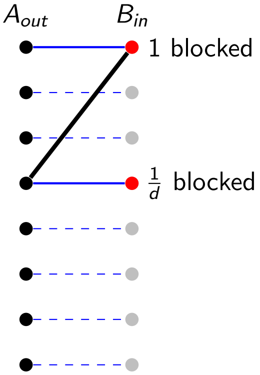

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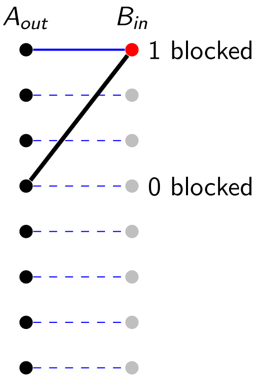

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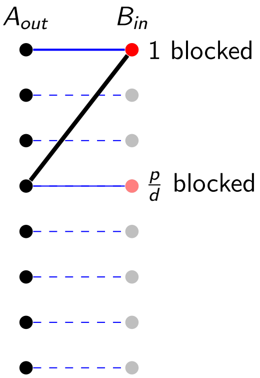

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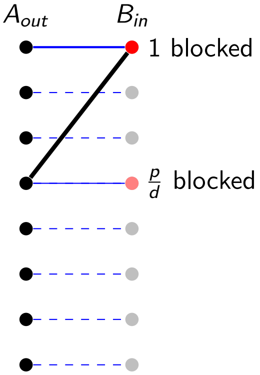

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$$\mathbb{E}[|M^*_{B'}|] \leq (1 + \tfrac{p}{d})\mathbb{E}[|M|]$$

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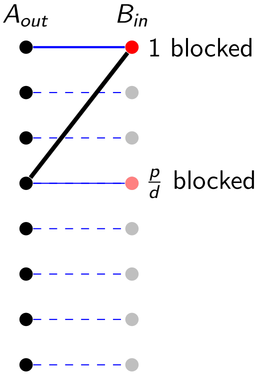

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$p \cdot |M^*| \leq (1 + \frac{p}{d})\mathbb{E}[|M|]$

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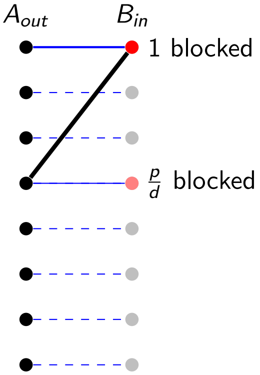

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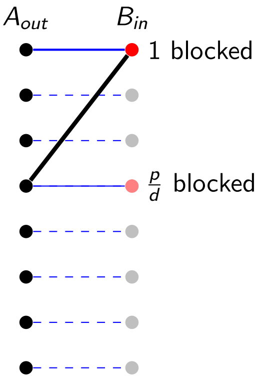
- Formalised using Wald's Equation.



$d = 3,\ p = 0.67$

$$\mathbb{E}[|M|] \geq \frac{dp}{d+p} \cdot |M^*|$$

Optimal Algorithms

The final matching returned is always at least of size

$$(\frac{1}{2} + \frac{p}{d+p} - \frac{p}{2d}) \cdot |M^*|.$$

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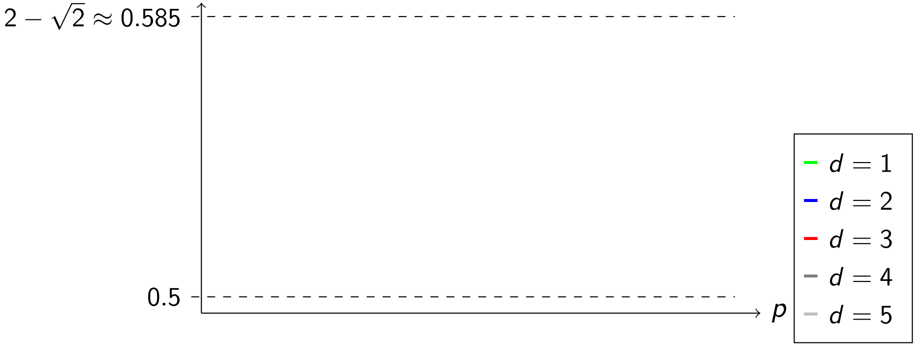

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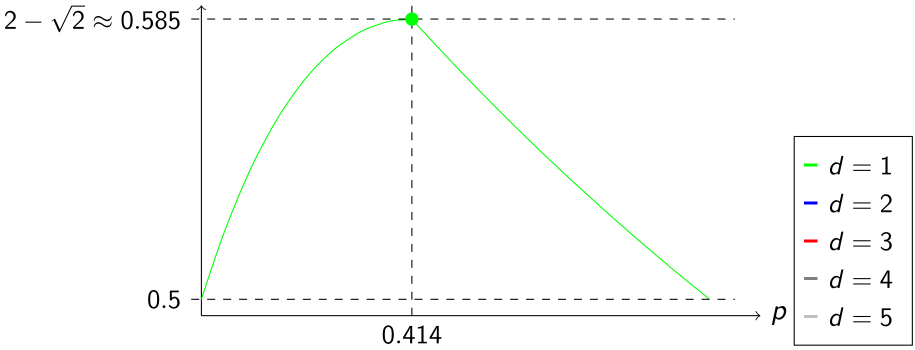

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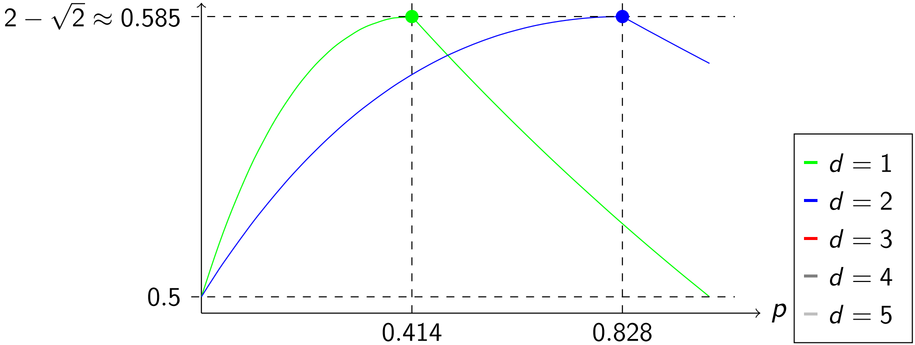

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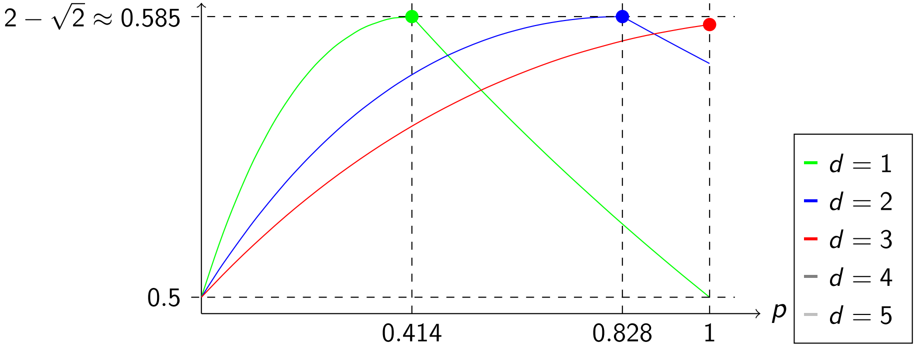

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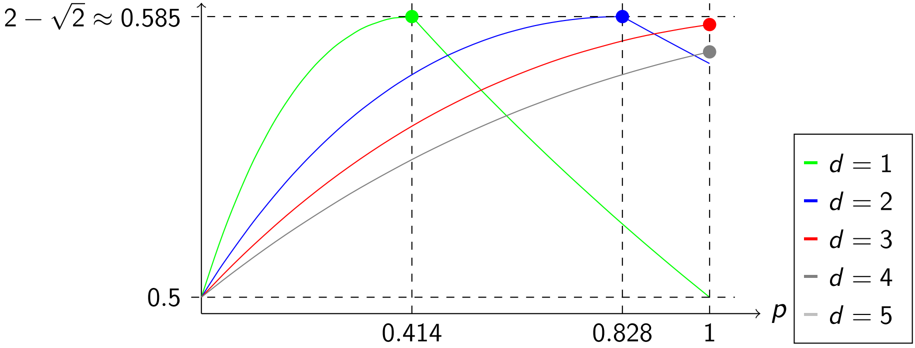

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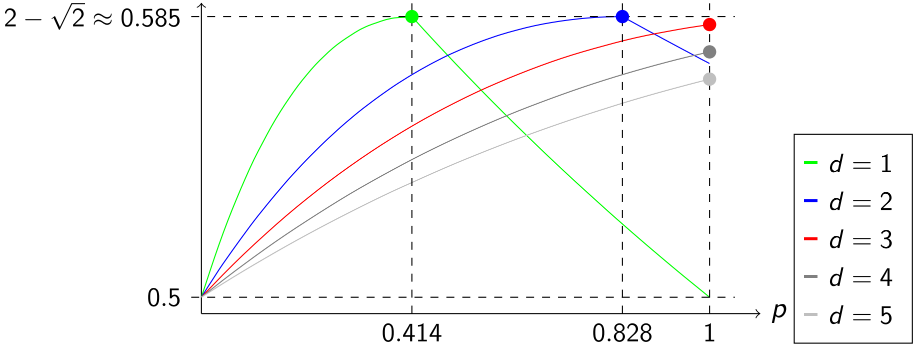

Overview

Conclusion

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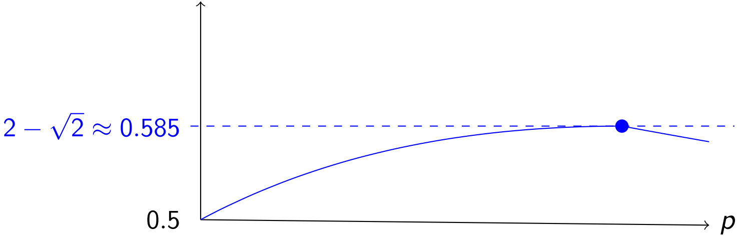
- Our two-pass algorithm unifies the dominant techniques used, achiving the current state-of-the-art.



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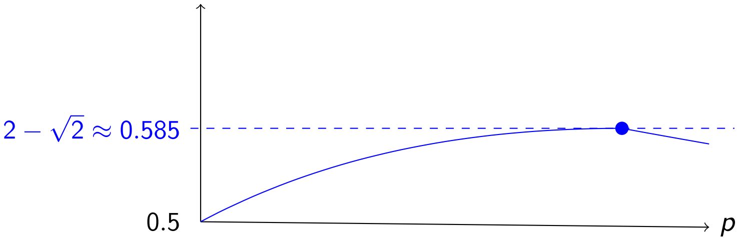
- Our two-pass algorithm unifies the dominant techniques used, achiving the current state-of-the-art.
- For appropriate settings of d and p, we can find Konrad's [MFCS18] and Esfandiari et al.'s [ICDMW16] algorithms.



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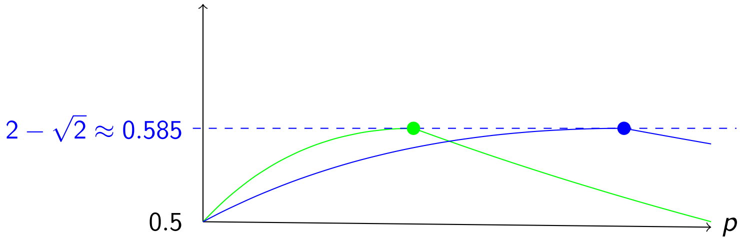

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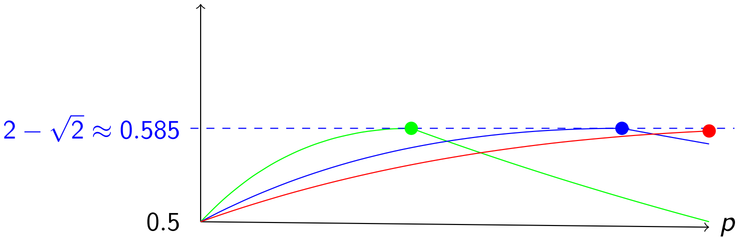

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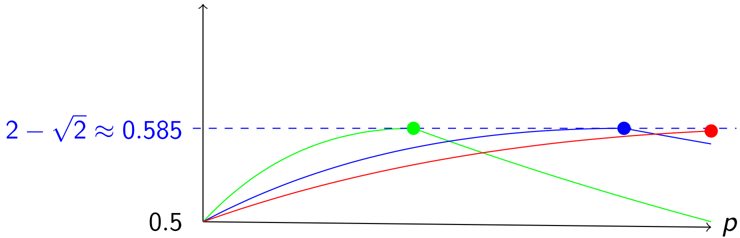

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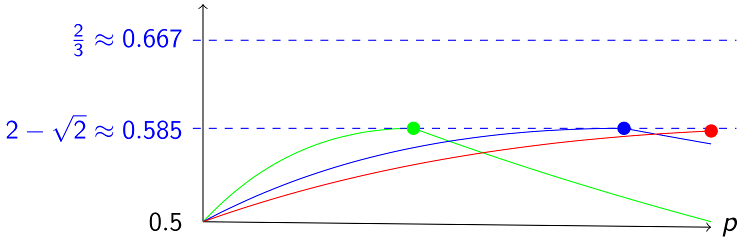
- Our hard-instance family of graphs proves that the analysis is tight.
- We reduced the gap of possibility with this class of algorithms to $[0.585, 0.667]$.



Open Questions

- Can we extend other one-pass lower bounds to improve the two-pass result? I.e. Kapralov's [SODA21].
- Is there a way to do better by finding more than just a maximal matching in the first-pass?
- Can we beat a $\frac{1}{2}$-approximation in just one-pass?

Thank You