Graph Streaming and Maximum Matching (in a Few Passes)

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Graphs and Bipartite Graphs

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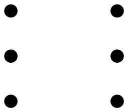

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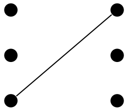

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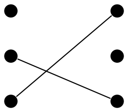

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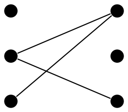

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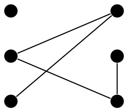

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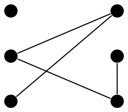

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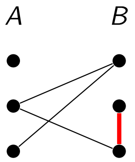

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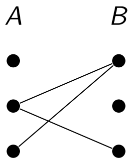

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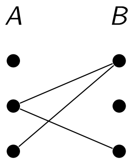

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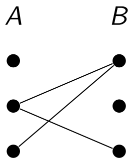

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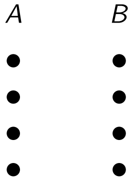

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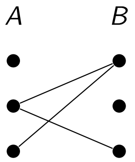

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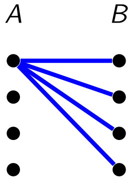

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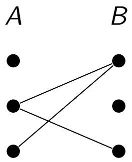

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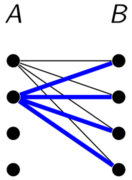

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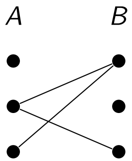

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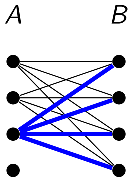

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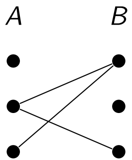

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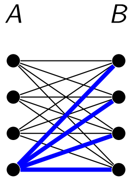

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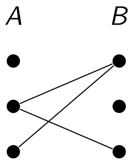

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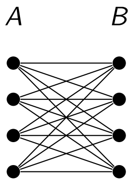

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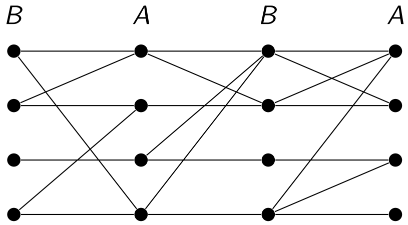

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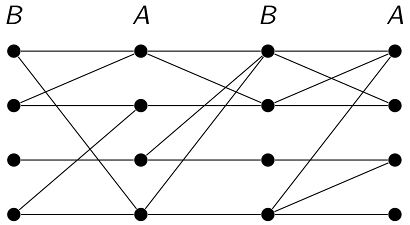

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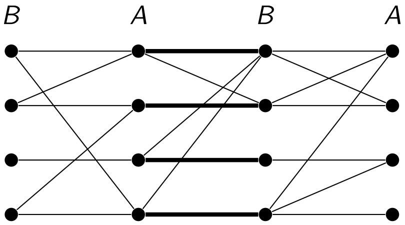

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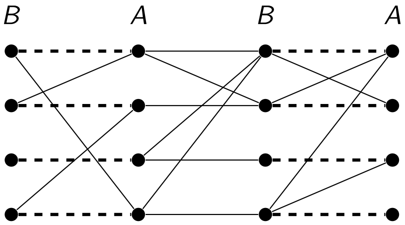

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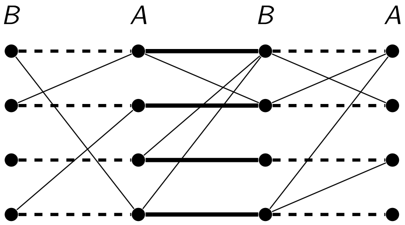

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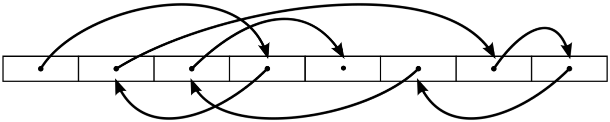
- Maximal matchings are 0.5-approximations of maximum matchings.

Traditional Model of Computation

Assumption

Algorithms have random access to the entire graph.



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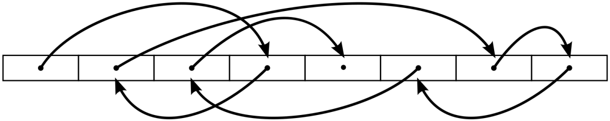
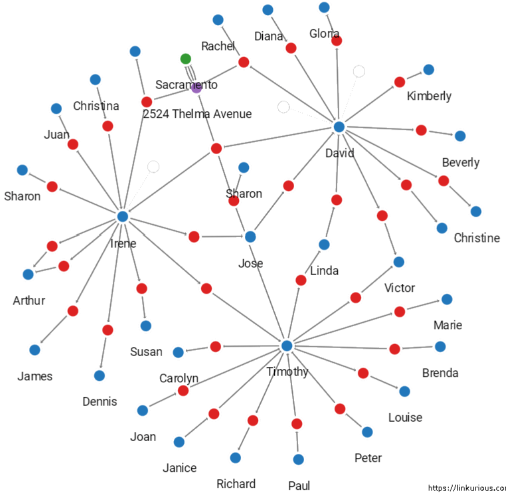


What if the input is extremely large?



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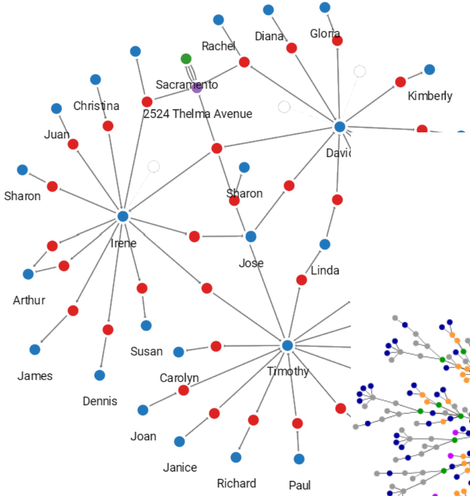
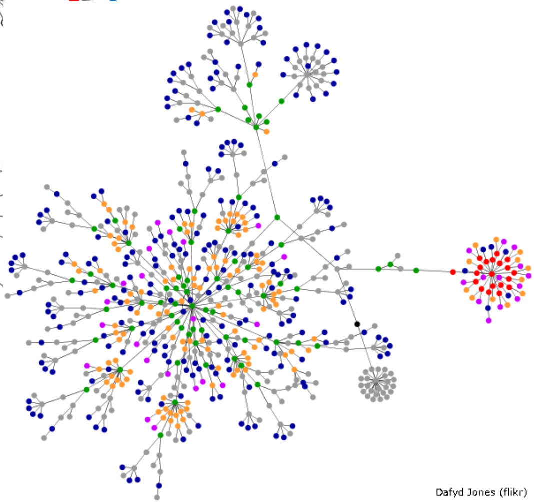
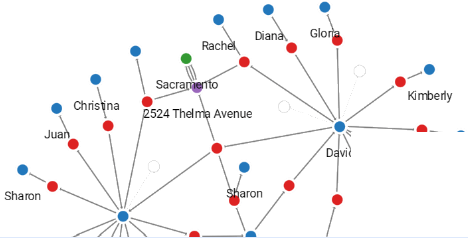
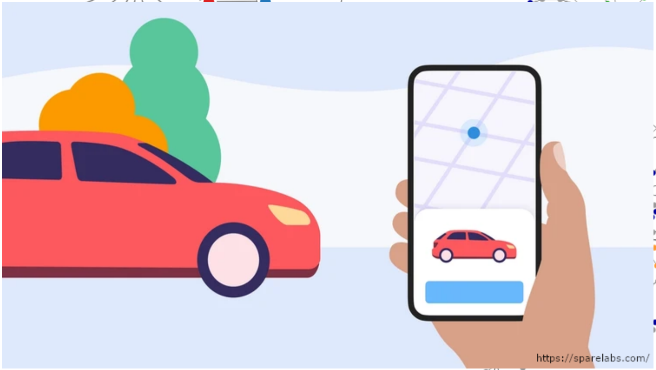




Dafyd Jones (flickr)





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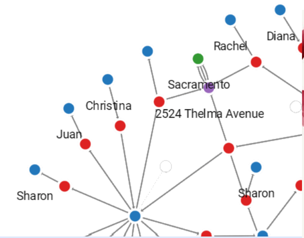

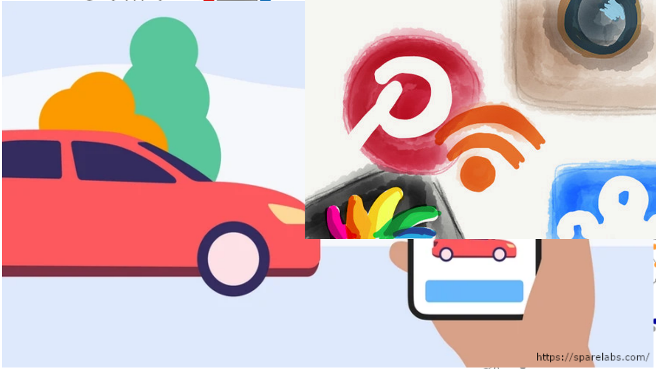

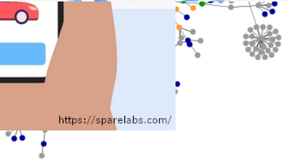

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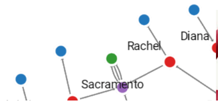

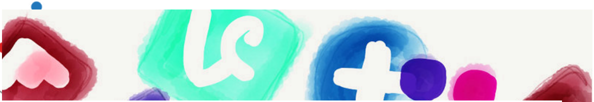

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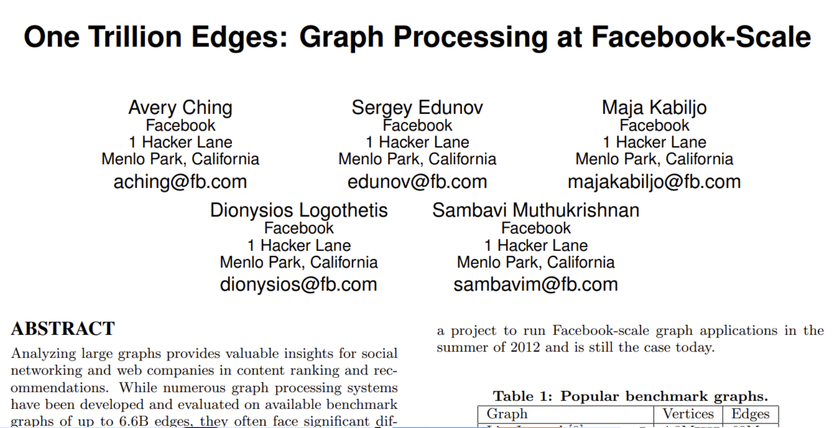

One Trillion Edges: Graph Processing at Facebook-Scale

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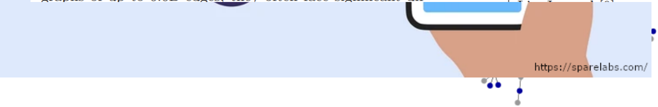
ABSTRACT

Analyzing large graphs provides valuable insights for social networking and web companies in content ranking and recommendations. While numerous graph processing systems have been developed and evaluated on available benchmark graphs of up to 6.6B edges, they often face significant dif-

a project to run Facebook-scale graph applications in the summer of 2012 and is still the case today.

Table 1: Popular benchmark graphs.

Graph	Vertices	Edges



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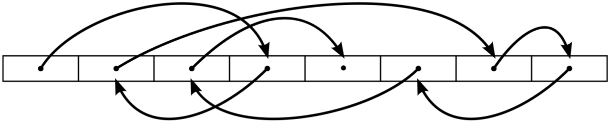
Massive Graphs

Assumption (Infeasible)

Algorithms have random access to the entire graph.



Graph Streaming (Model of Computation)

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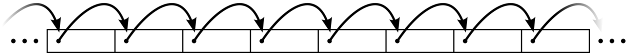
An n-vertex graph is presented as a sequence of edges to an algorithm.



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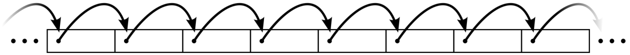

Insertion-only [FKM+04]

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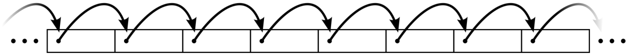

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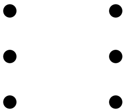

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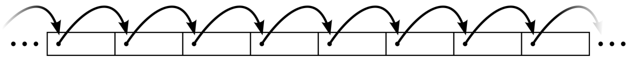
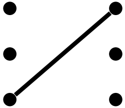


Insertion-only [FKM$^+$04]

$\boxed{e_1}$



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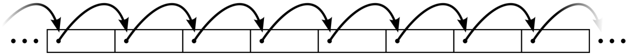

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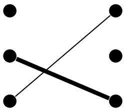
e_1	e_2



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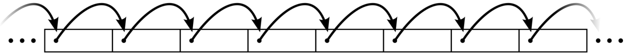

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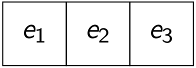

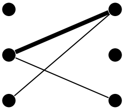

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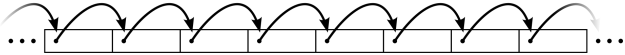
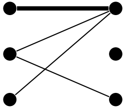


Insertion-only [FKM+04]

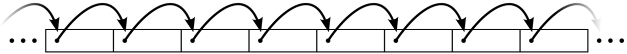
e_1	e_2	e_3	e_4



Graph Streaming (Model of Computation)

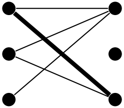
Definition

An *n*-vertex graph is presented as a sequence of edges to an algorithm.



Insertion-only [FKM+04]

e_1	e_2	e_3	e_4	e_5



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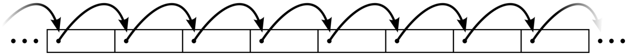
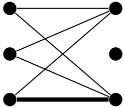


Insertion-only [FKM+04]

e_1	e_2	e_3	e_4	e_5	e_6



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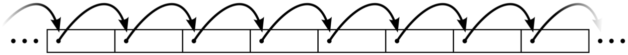

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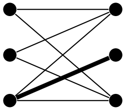

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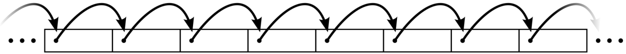

Insertion-only [FKM+04]

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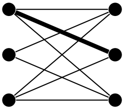

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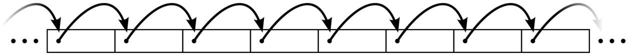

Insertion-only [FKM+04] (finite stream)

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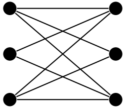

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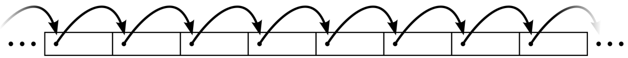

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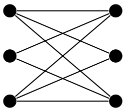

Trivial Algorithm

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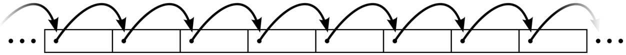

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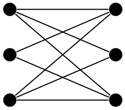

Trivial Algorithm

- Stores all edges using $O(n^2)$ space.

Interesting Algorithm

- Uses $O(n \operatorname{polylog} n)$ space.
- Uses one or more passes.

GREEDY: Simple and Powerful

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Greedy Matching:

1. Add edge if neither endpoint is matched

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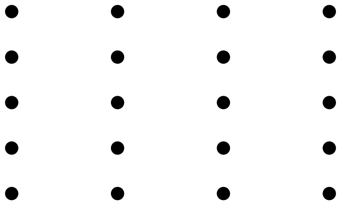

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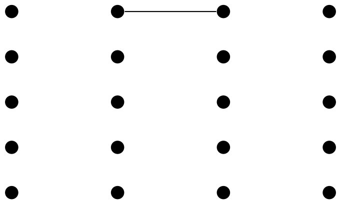

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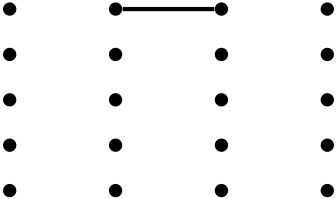

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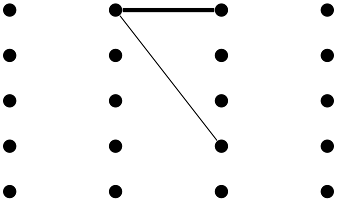

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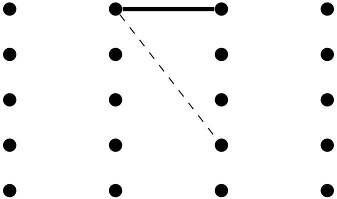

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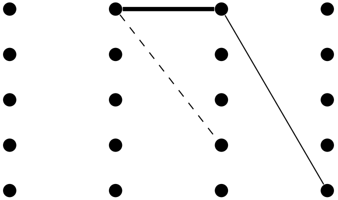

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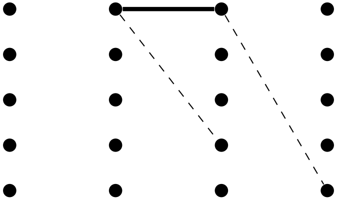

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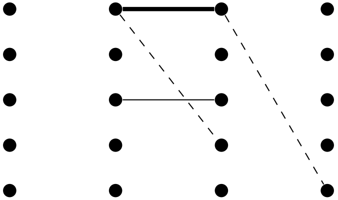

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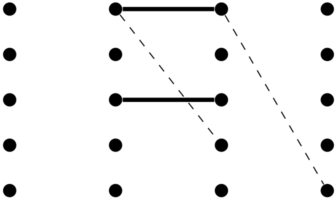

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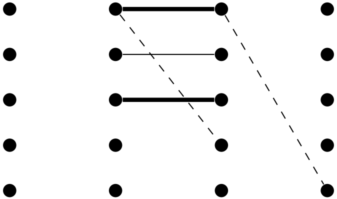

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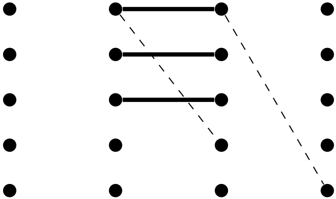

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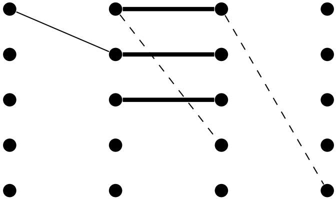

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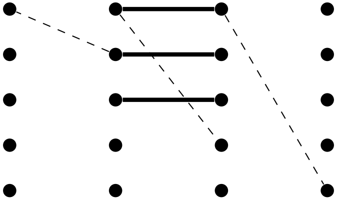

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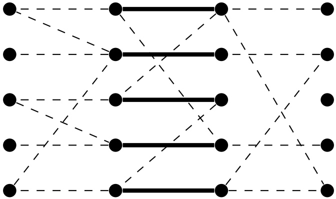

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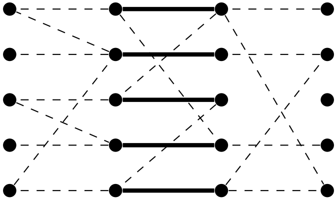
- Maximal



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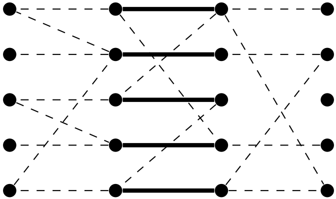

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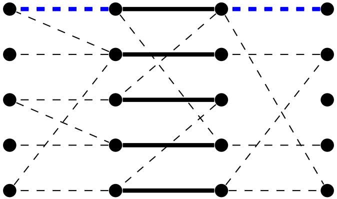

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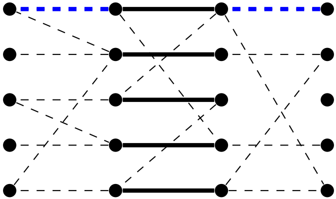


Space Used

Only the matching M is stored by the algorithm $\implies$ $O(n \log n)$ space since each edge requires $\Theta(\log n)$ space to store.

Approximate MBM using $O(n \operatorname{polylog} n)$ Space

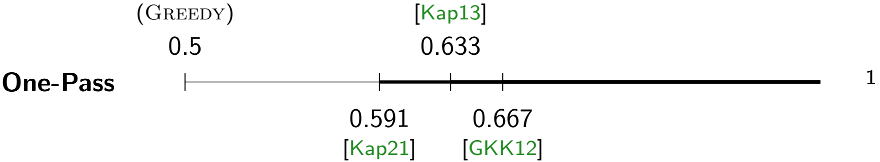
Approximate MBM using $O(n \operatorname{polylog} n)$ Space

(Greedy)

0.5

One-Pass 1

Approximate MBM using $O(n \operatorname{polylog} n)$ Space



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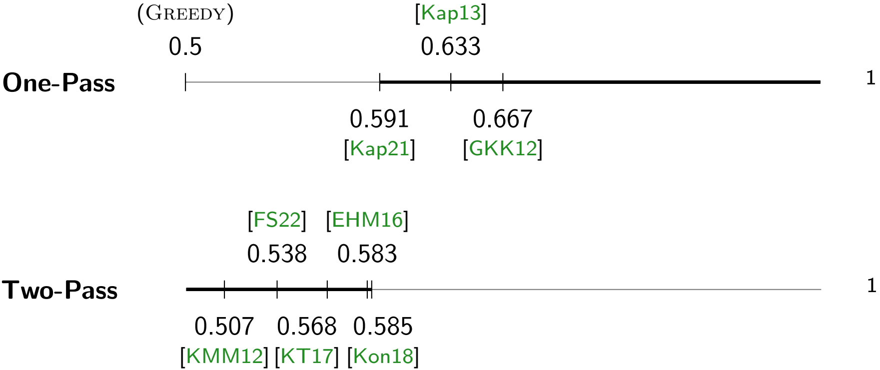

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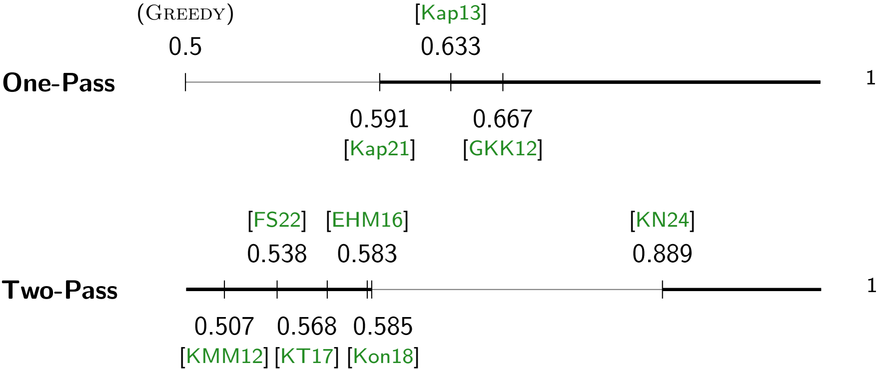

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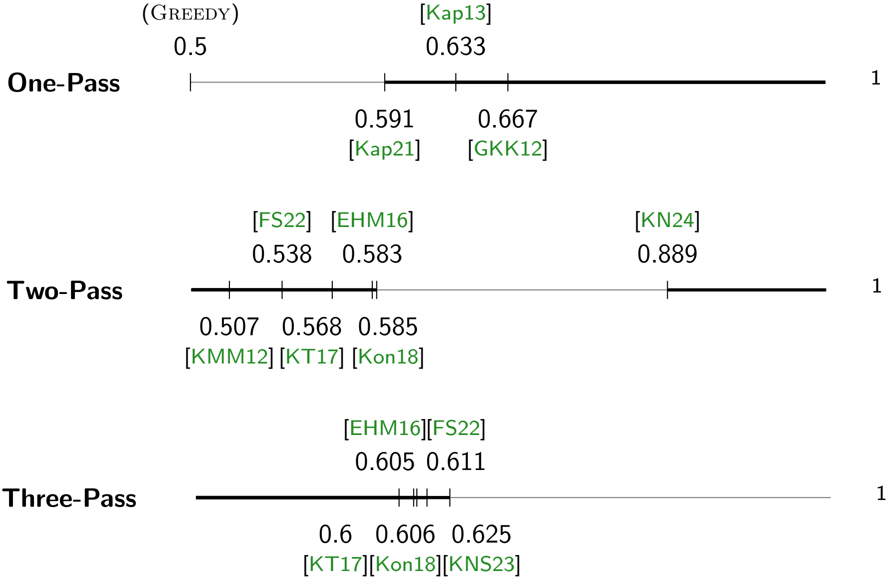

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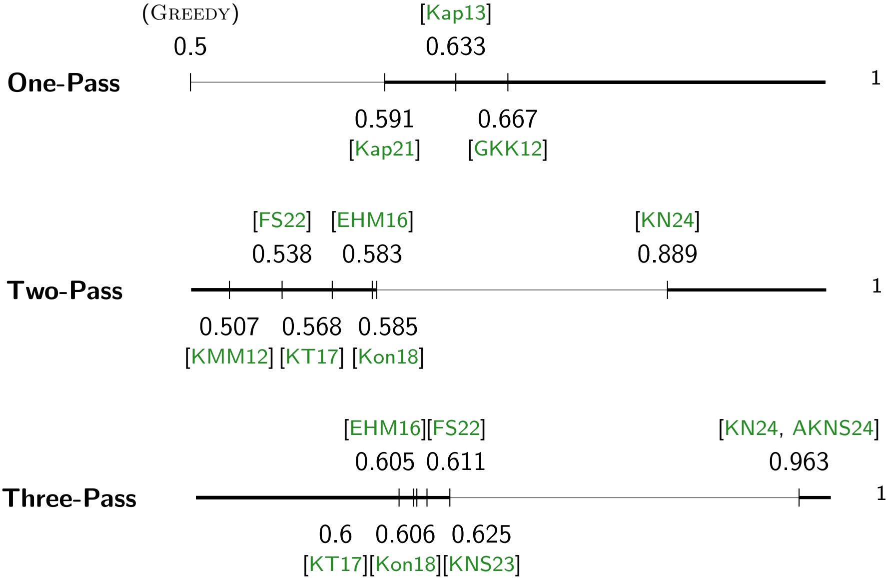


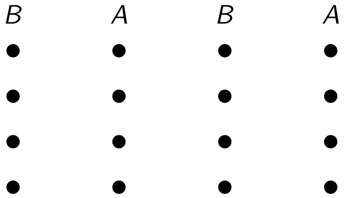
Three-Pass Algorithmic Idea

First Pass

Find a maximal matching M in G using GREEDY.

Subsequent Passes

Find vertex-disjoint augmenting paths (also using GREEDY).



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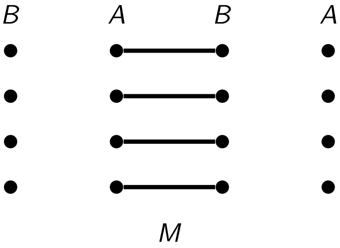

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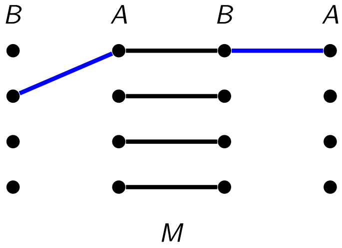

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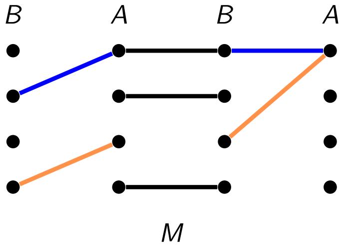

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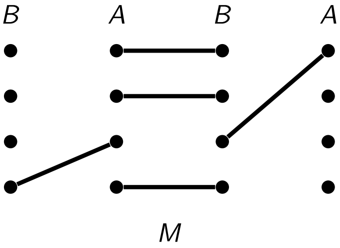

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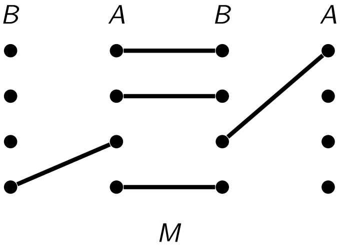

Finding length-3 augmenting paths [KT17]

Simple Strategy

1. Find left wings
2. Extend with right wings

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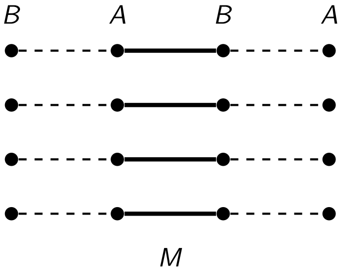

- $|M| = 0.5 \cdot \mu(G)$

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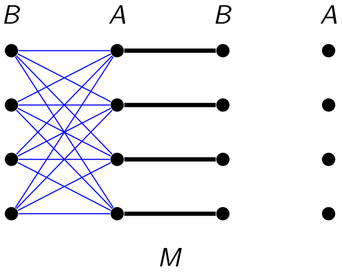

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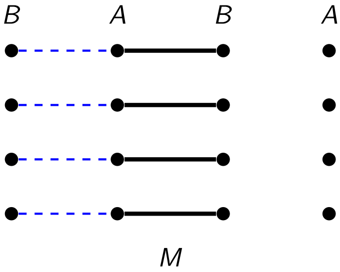

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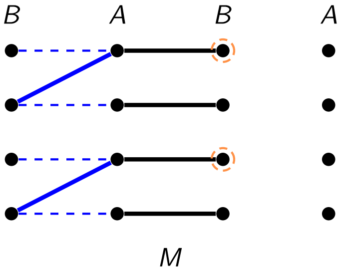

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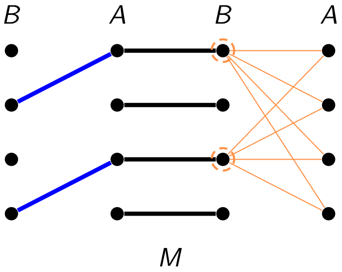

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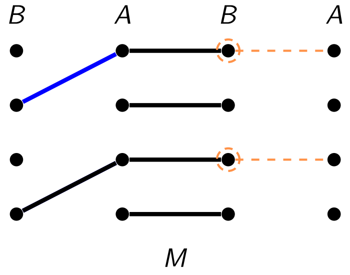

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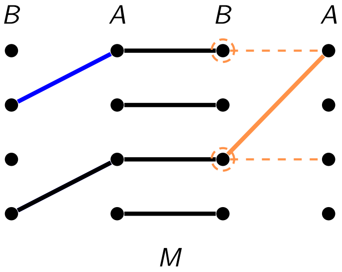

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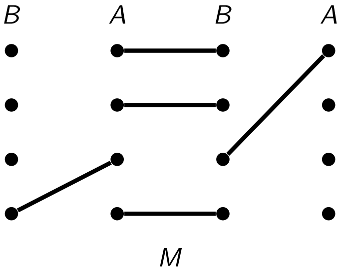

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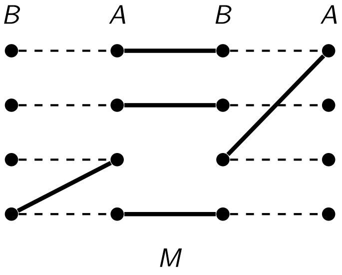

- $|M| = 0.5 \cdot \mu(G)$
- 0.625-approximation
- Not hard!

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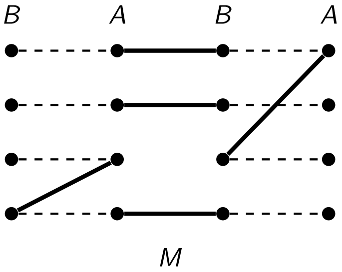

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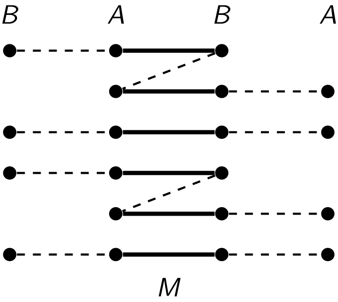

- $|M| = 0.6 \cdot \mu(G)$

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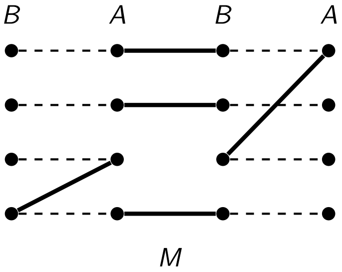
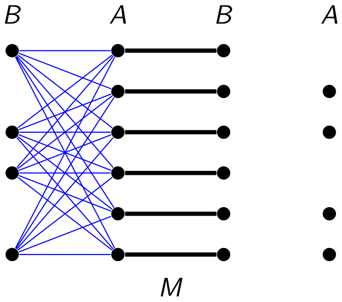


- $|M| = 0.5 \cdot \mu(G)$
- 0.625-approximation
- Not hard!



- $|M| = 0.6 \cdot \mu(G)$

Finding length-3 augmenting paths [KT17]

Simple Strategy

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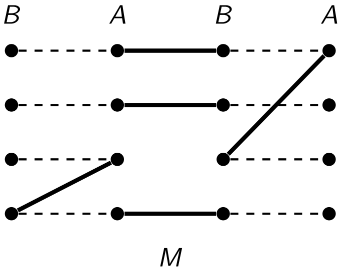

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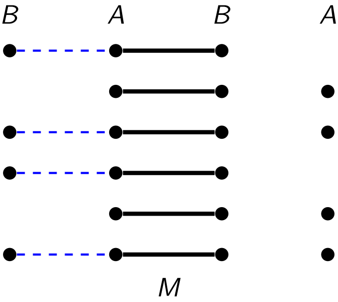

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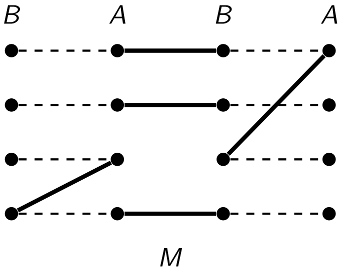

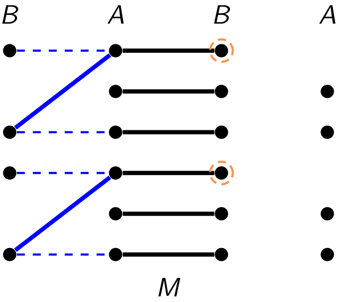

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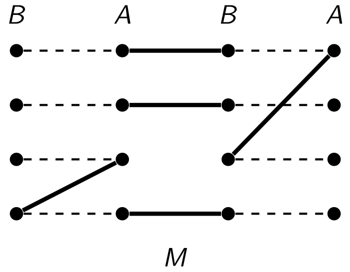

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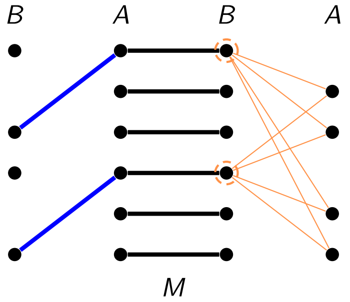

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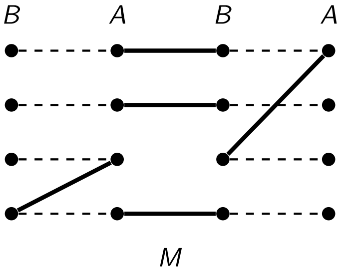

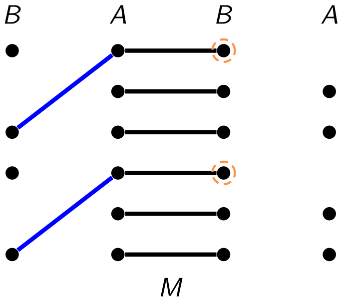

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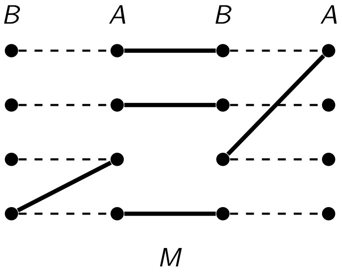

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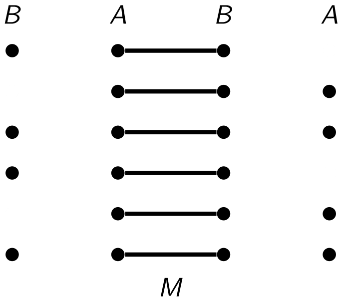

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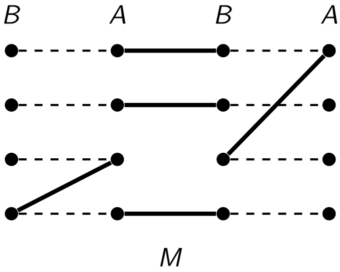

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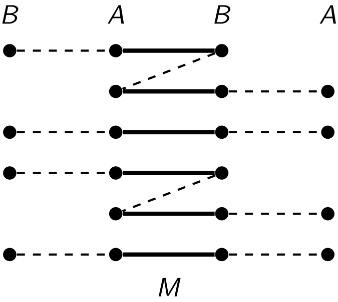

- $|M| = 0.6 \cdot \mu(G)$
- 0.6-approximation
- **Hard instance!**

Finding length-3 & length-5 augmenting paths [KNS23]

Our Strategy

1. Find left and right wings
2. Extend paths to either length-3 or length-5 augmenting paths

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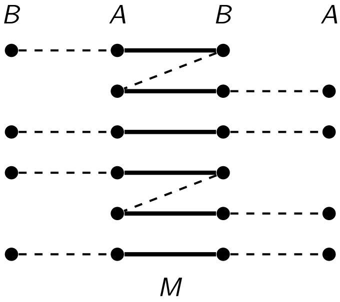

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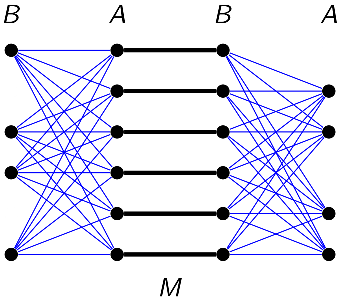

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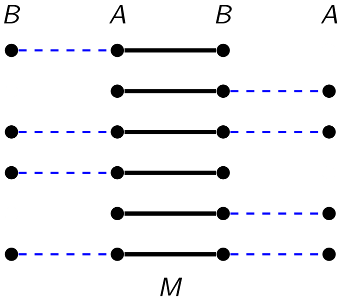

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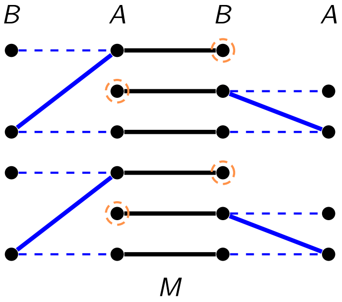

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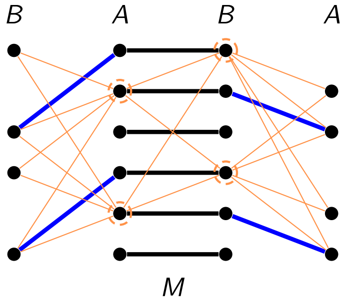

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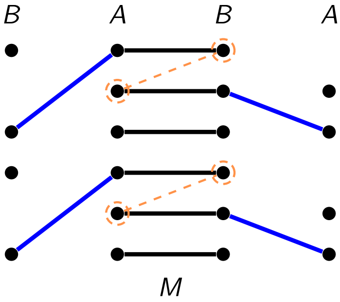

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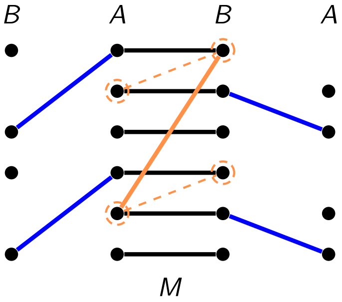

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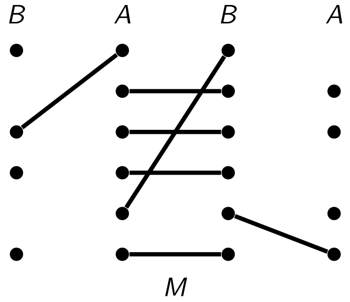

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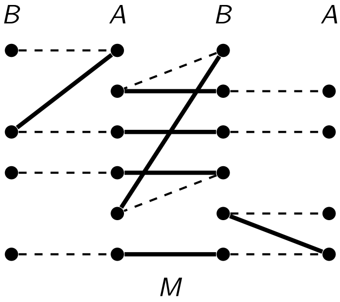

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- 0.7-approximation
- Not hard anymore!

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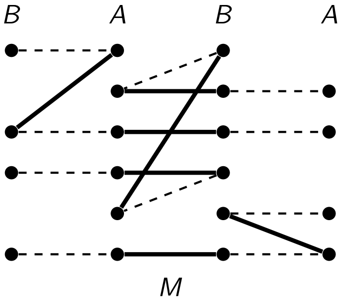

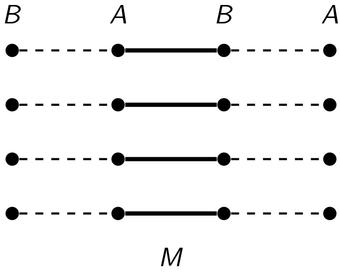

- $|M| = 0.6 \cdot \mu(G)$
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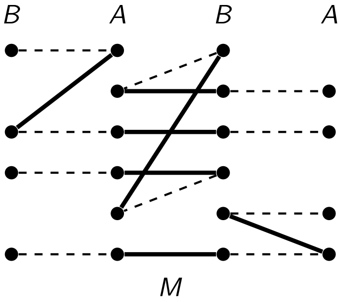

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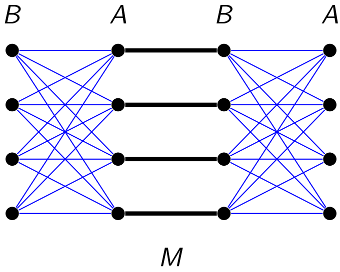

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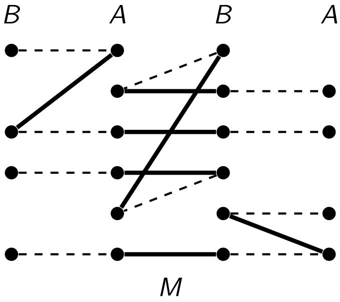

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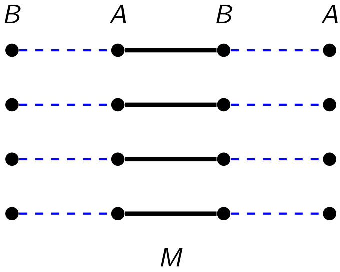

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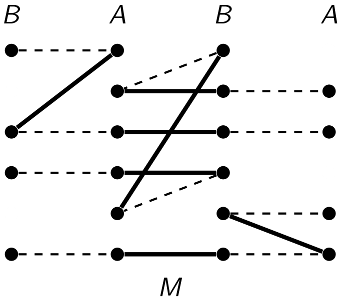

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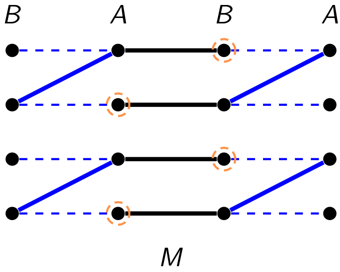

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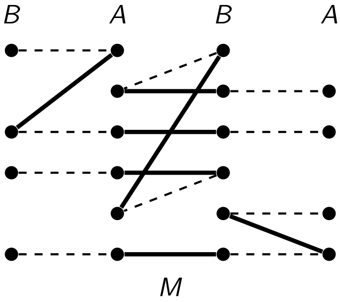

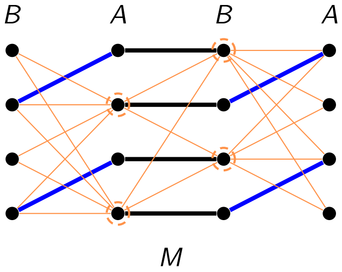

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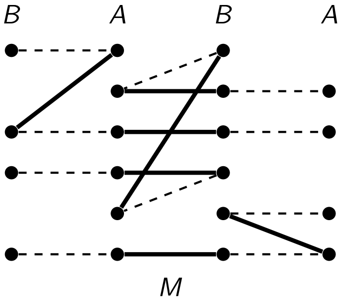

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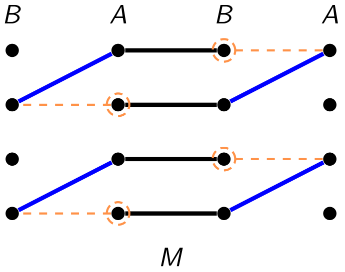

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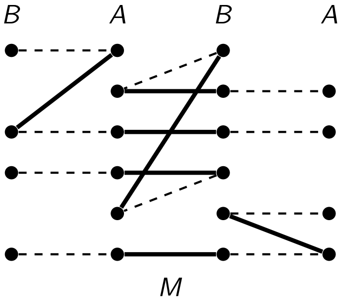

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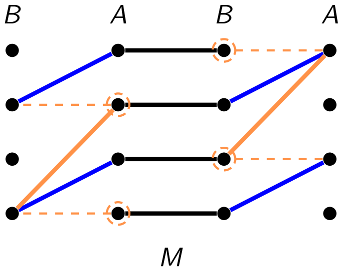

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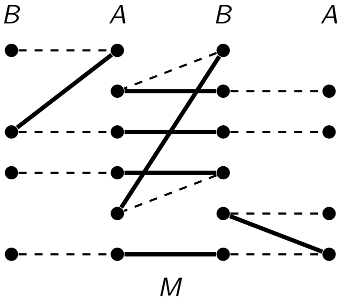

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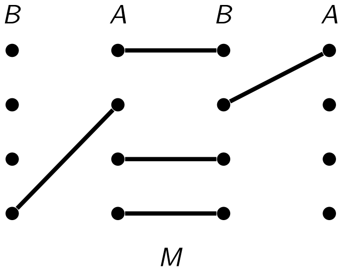

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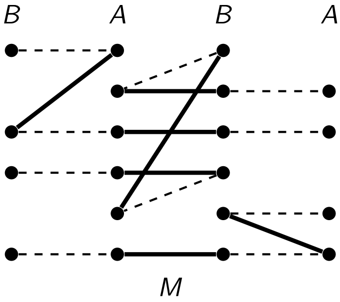

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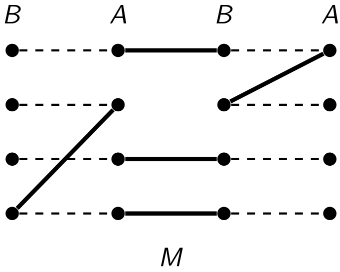


- $|M| = 0.5 \cdot \mu(G)$
- 0.625-approximation
- **Hard instance!**

Our Analysis [KNS23]

Main Lemma

Let $|M| = (0.5 + \epsilon) \cdot \mu(G)$ for $\epsilon \geq 0$, then our strategy finds

$$(0.125 - \frac{3}{4}\epsilon) \cdot \mu(G)$$

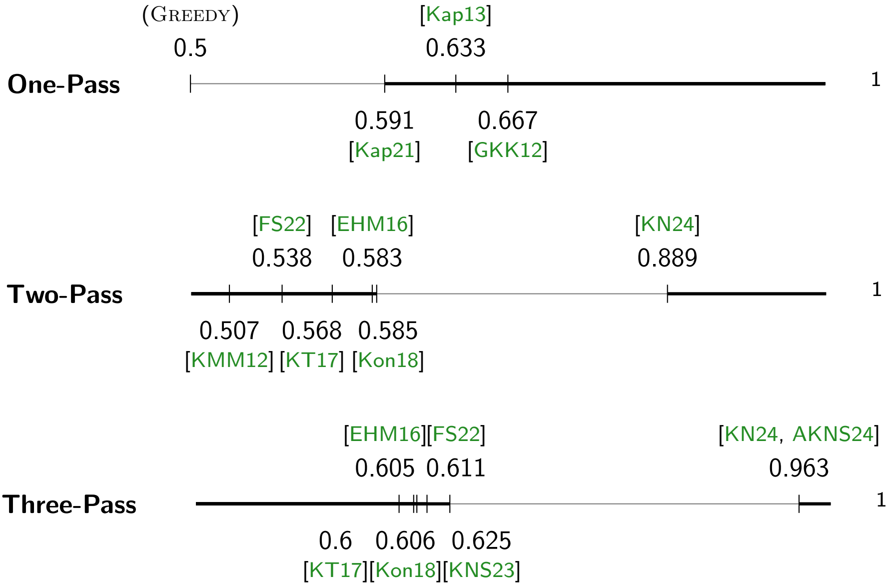
vertex-disjoint augmenting paths and the large matching found is of size

$$(0.625 + \frac{\epsilon}{4}) \cdot \mu(G).$$

Space Used

$\textsc{Greedy}$ only stores $O(n)$ edges in each pass $\implies$ $O(n \log n)$ space.

Conclusion


One-Pass
(GREEDY)
0.5
[Kap13]
0.633
0.591
[Kap21]
0.667
[GKK12]
1
Two-Pass
[FS22]
0.538
[EHM16]
0.583
[KN24]
0.889
0.507
[KMM12]
0.568
[KT17]
0.585
[Kon18]
1
Three-Pass
[EHM16][FS22]
0.605 0.611
[KN24, AKNS24]
0.963
0.6
[KT17]
0.606
[Kon18]
0.625
[KNS23]
1

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