A Unifying Class of Algorithms for Semi-Streaming Bipartite Maximum Matching

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Joint work with Dr Christian Konrad

Overview

1. Background
2. Our Work
   - Algorithmic
   - Impossibility
3. Discussion

Matchings

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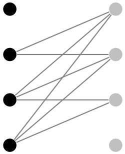

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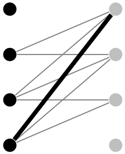

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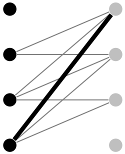

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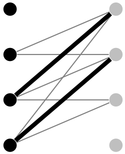

maximal matching

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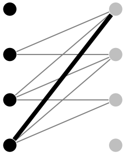
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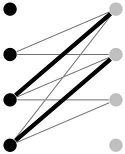
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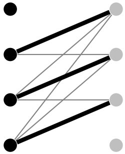
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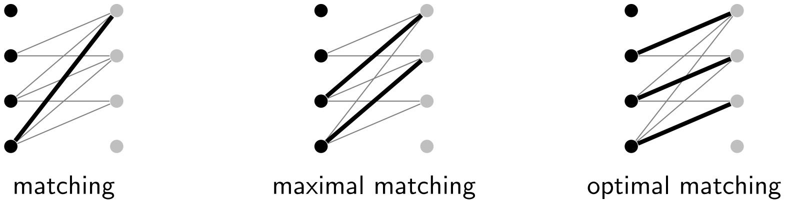
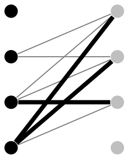


Matchings

Esfandiari et al. [ICDMW16]

A **semi-incomplete matching** is an extended matching in bipartite graphs where the vertices of one bipartition allows for degree $d > 1$.



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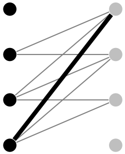

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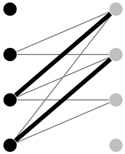

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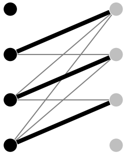

optimal matching

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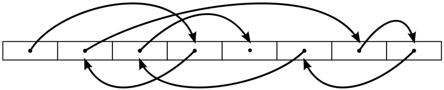
Finding a maximum (optimal) matching:

- exact algorithms exists, i.e. Hopcroft-Karp [SWAT71];
- require random access to the graph's edges (infeasible requirement for massive graphs).

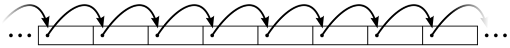


Semi-Streaming Model

Feigenbaum et al. [ICALP04]

A graph with n vertices is presented to an algorithm as a stream of edges where the storage space of the algorithm is bounded by $O(n \,\mathrm{polylog}\, n)$.

- Only allows **sequential access** to the graph.
- Algorithms with space $O(n \,\mathrm{polylog}\, n) = O(n\, (\log n)^{O(1)})$.
- Ideally with few passes of the stream.



Main Results

Two-Pass Bipartite Maximum (Optimal) Matching

Class of algorithms:

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Impossibility:

- In this class of algorithms, *(the first pass finds only a maximal matching)*, no better than a $\frac{2}{3} \approx \frac{1}{2} + 0.167$-approximation is possible in the semi-streaming model.

Related Work

	Algorithmic	Impossibility
one-pass	$\frac{1}{2}$ [ICALP04]	$\frac{1}{2} + 0.167$ [SODA12] $\frac{1}{2} + 0.132$ [SODA13] $\frac{1}{2} + 0.091$ [SODA21]
two-pass	$\frac{1}{2} + 0.019$ [APPROX12] $\frac{1}{2} + 0.083$ [ICDMW16] $\frac{1}{2} + 0.063$ [APPROX17] $\frac{1}{2} + 0.085$ [MFCS18] $\frac{1}{2} + 0.085$	$\frac{1}{2} + 0.167$[1]

[1]where the first pass finds a maximal matching, i.e., at least a $\frac{1}{2}$-approximation.

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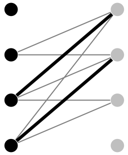

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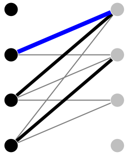

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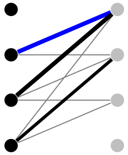

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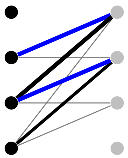

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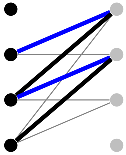

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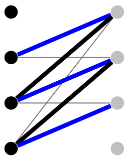

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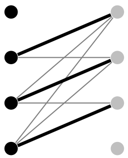

$|M_1| = 3$

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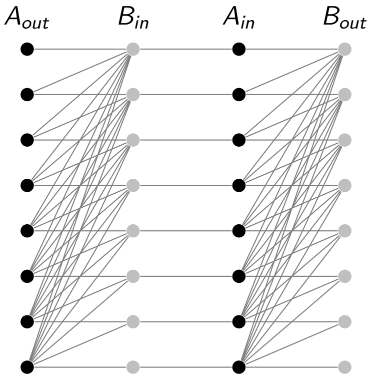

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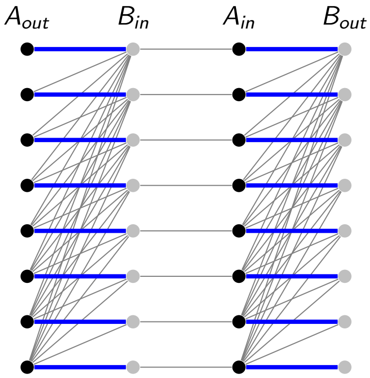
The goal of the algorithm is to find the maximum (optimal) matching M^*.



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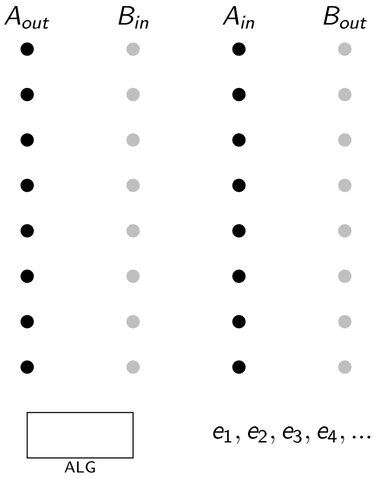

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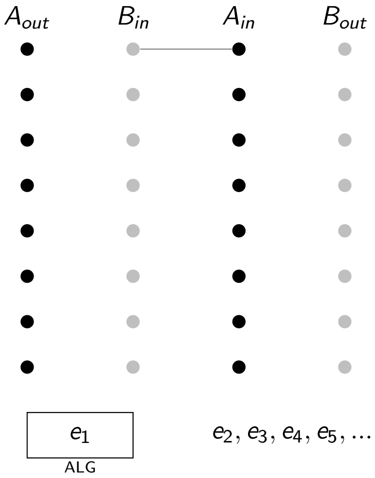

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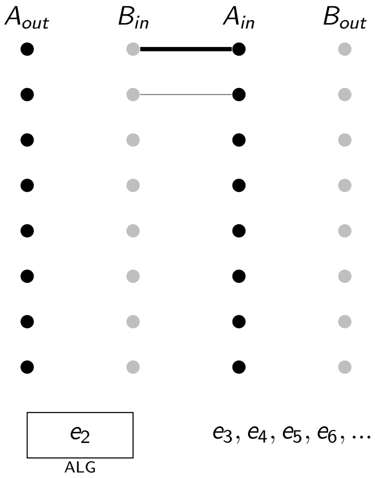

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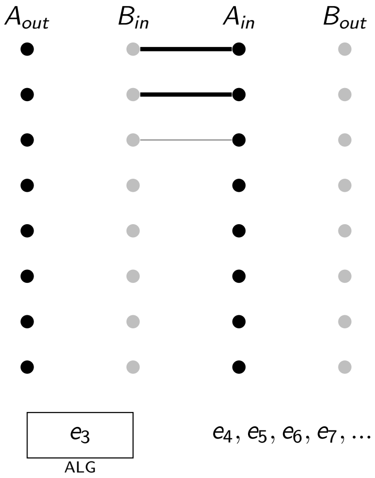

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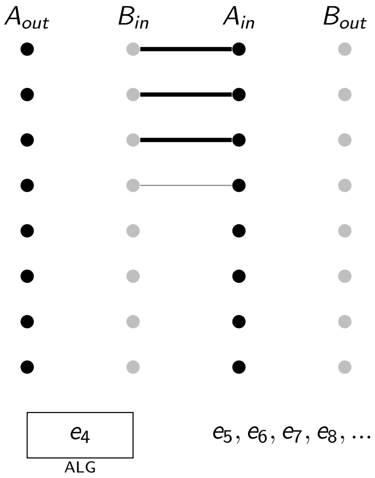

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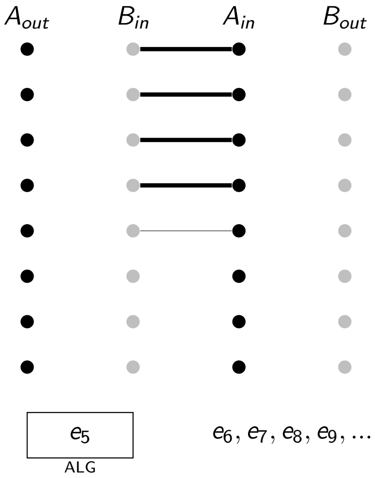

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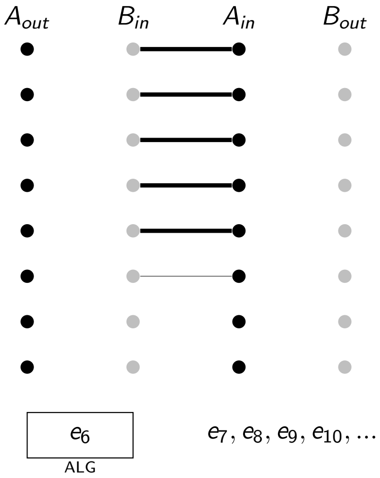

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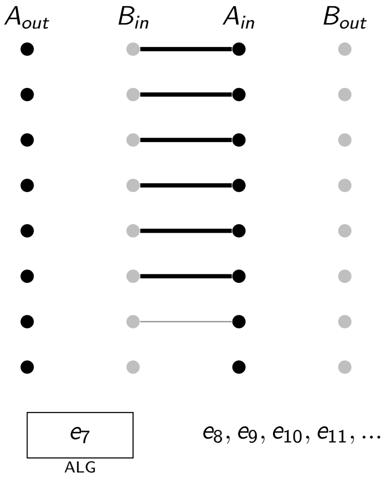

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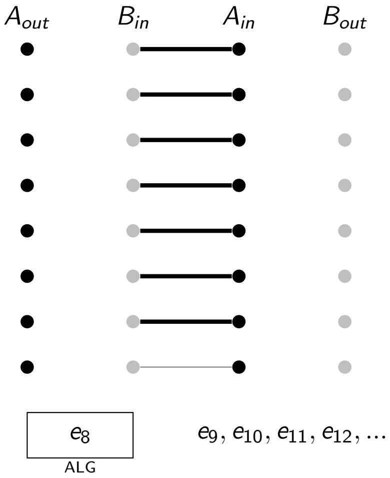

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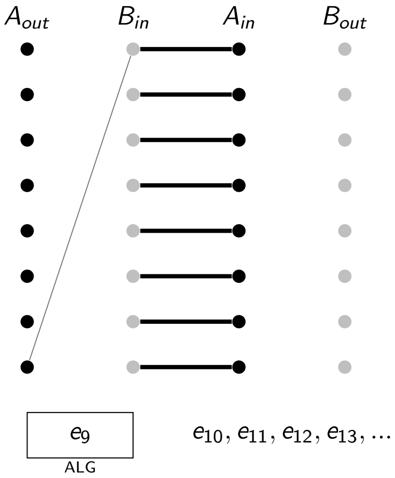

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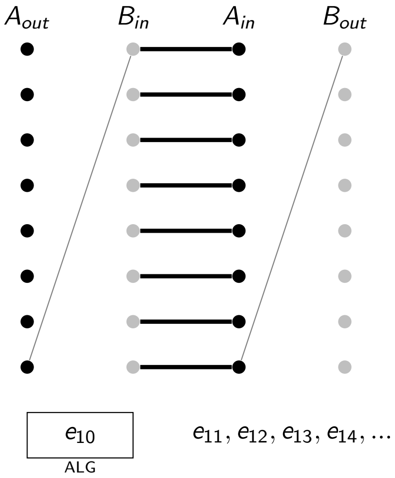

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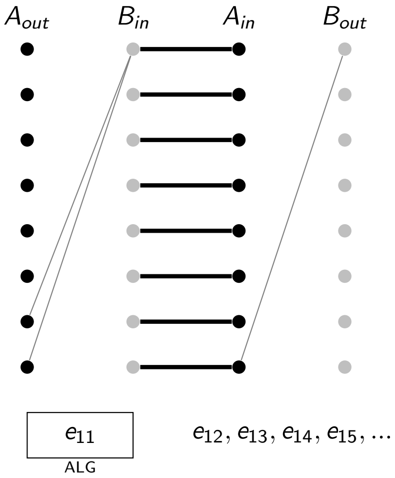

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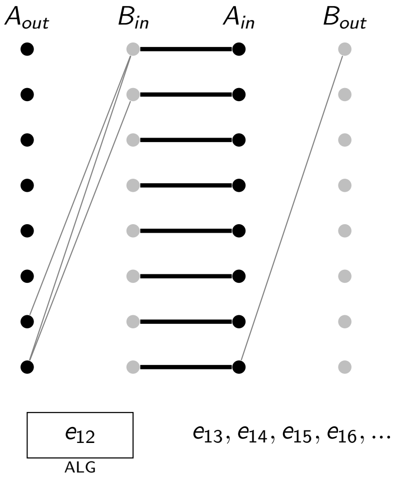

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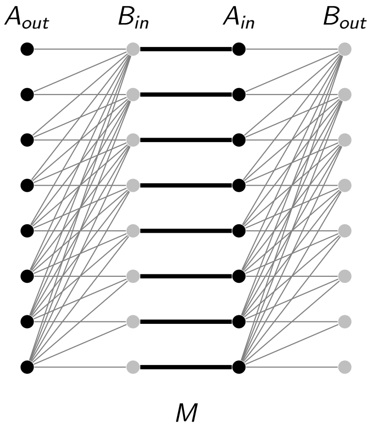

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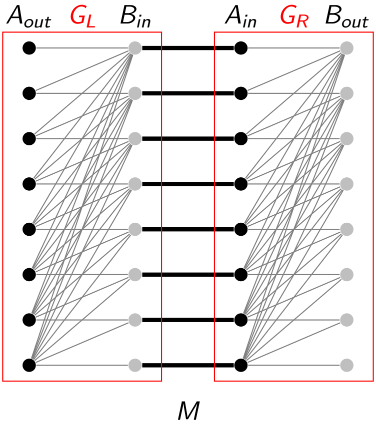

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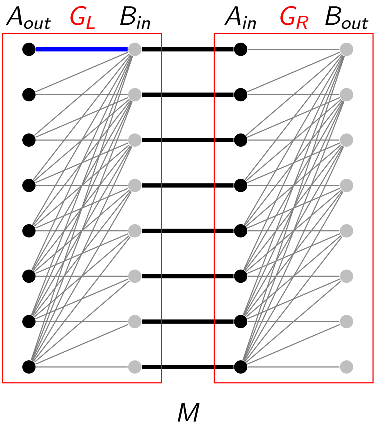

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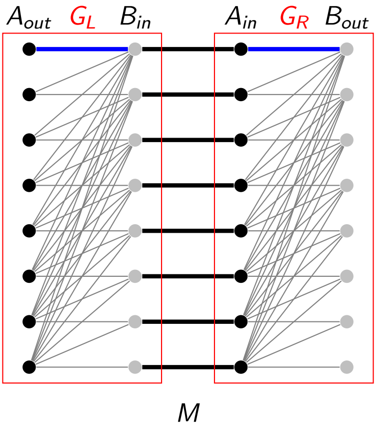

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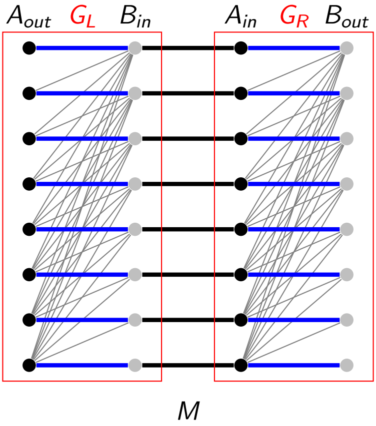

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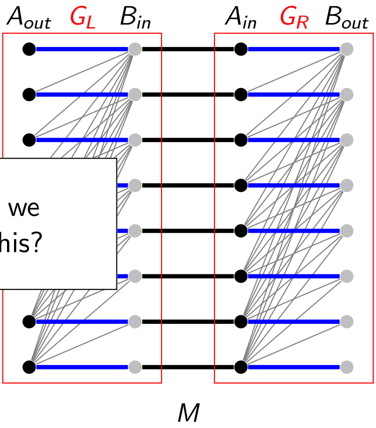

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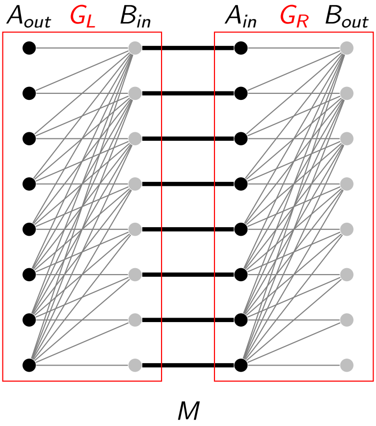

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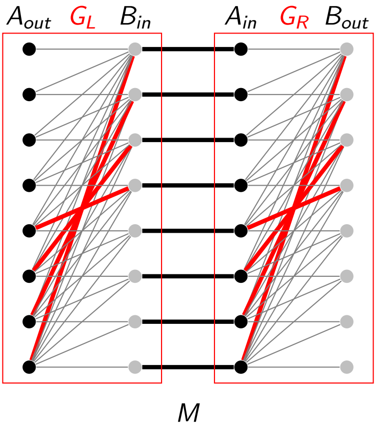

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Let π_G
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Using the two dominant techniques:

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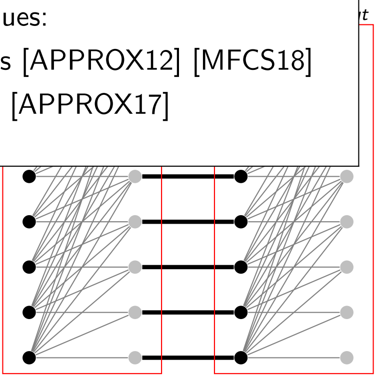

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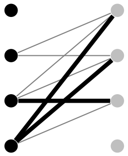

semi-incomplete matching

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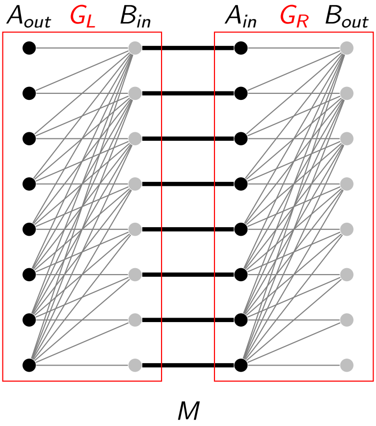

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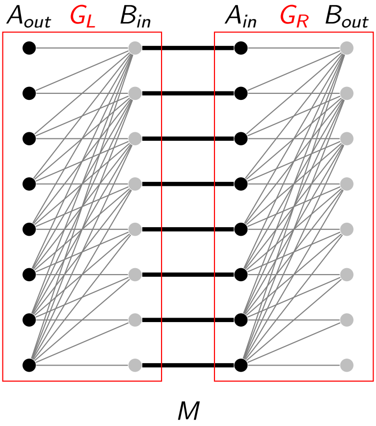

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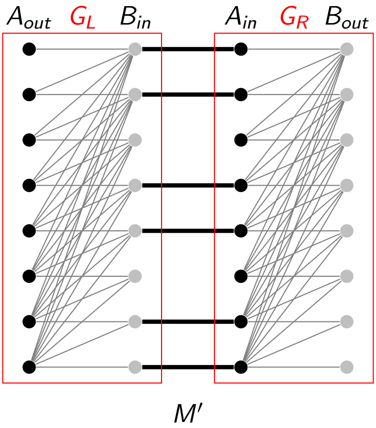

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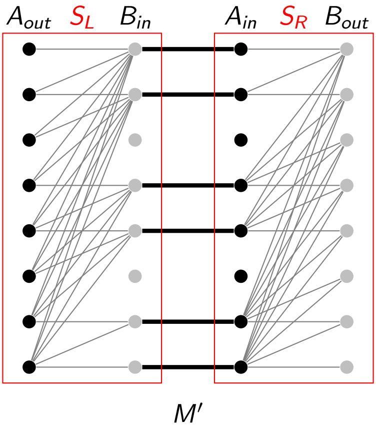

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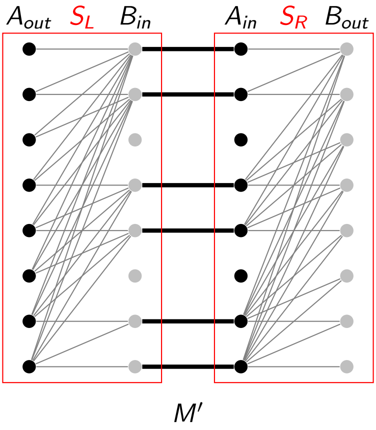

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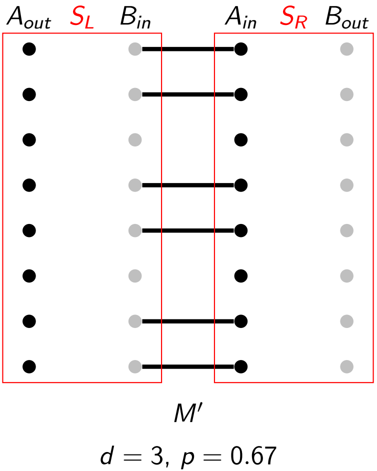

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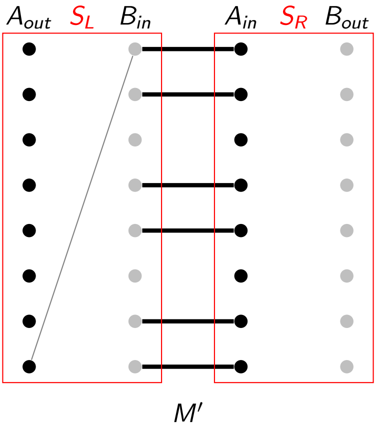

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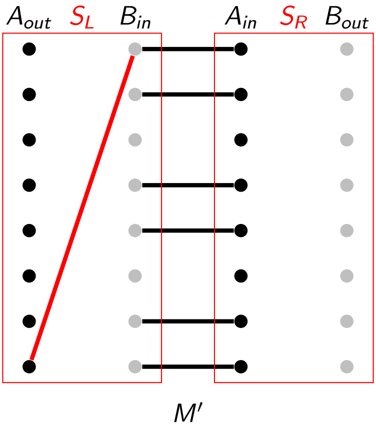

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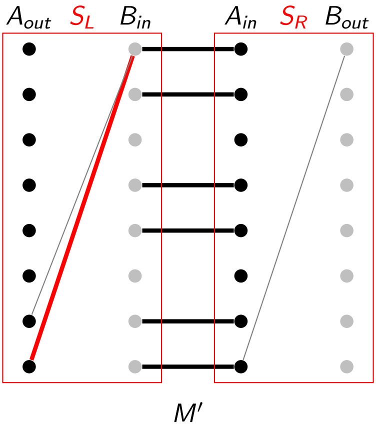

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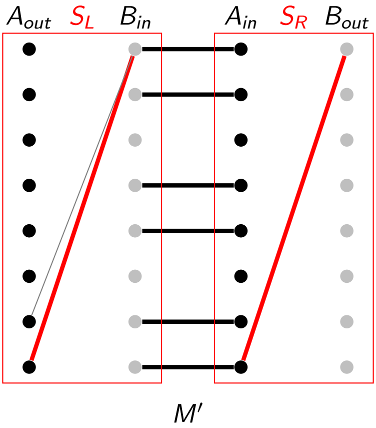

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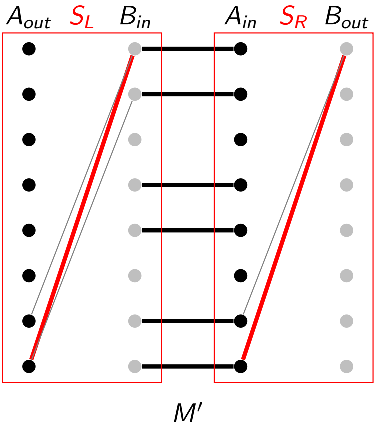

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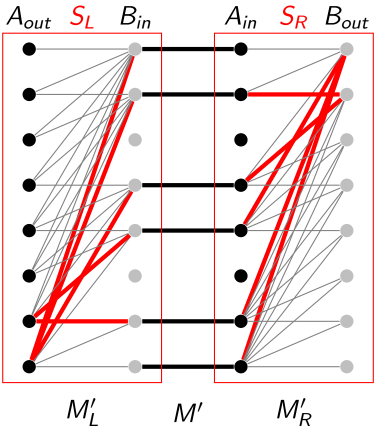

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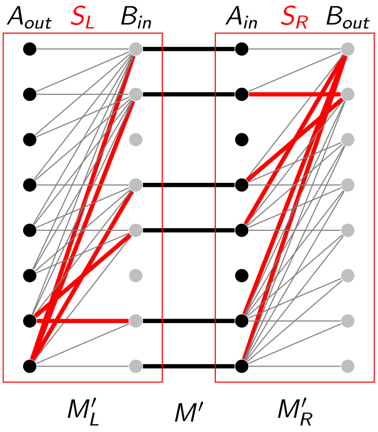

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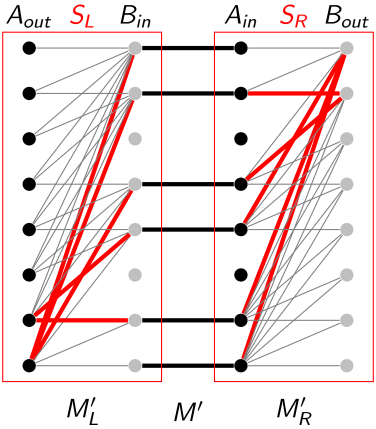

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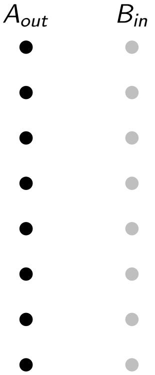
Main Theorem (Proof Outline)



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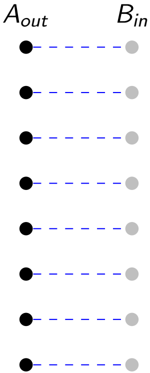

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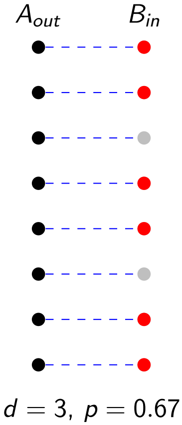

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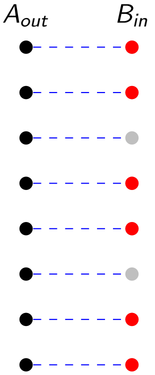

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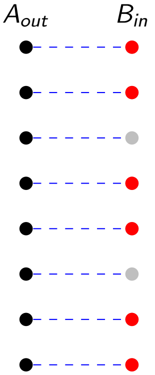

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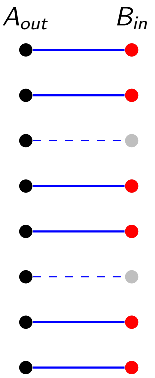

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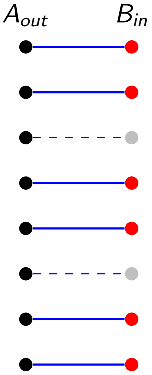

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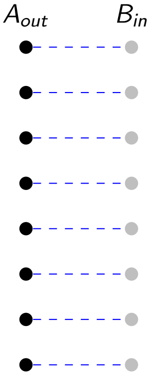

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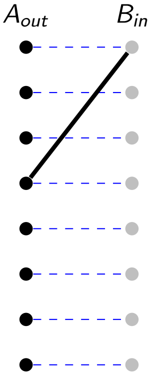

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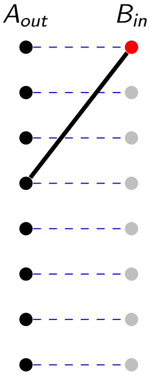

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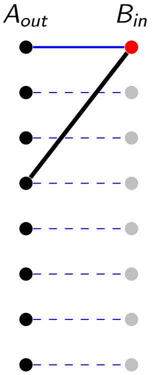

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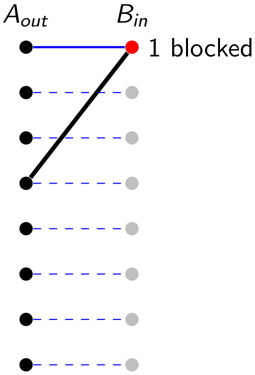

$d = 3,\ p = 0.67$

Main Theorem (Proof Outline)

Setup:

- any bipartite graph $G = (A, B, E)$ with a maximum (optimal) matching M^*;
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Proof:

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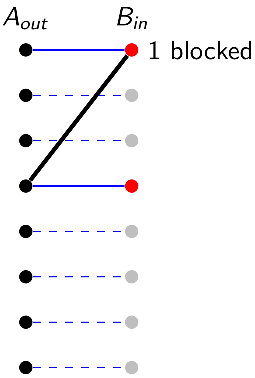

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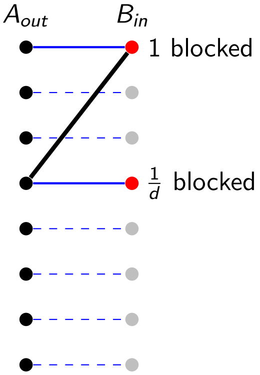

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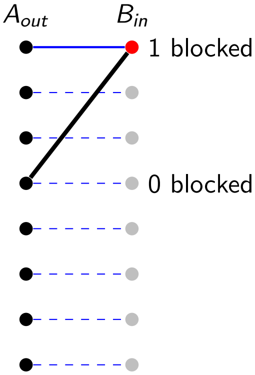

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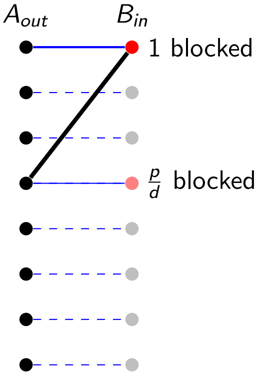

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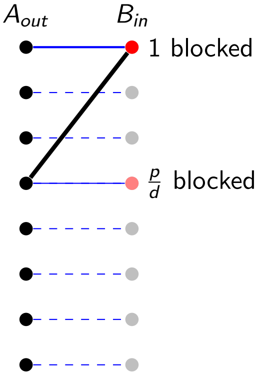

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$$\mathbb{E}[|M^*_{B'}|] \leq (1 + \tfrac{p}{d})\mathbb{E}[|M|]$$

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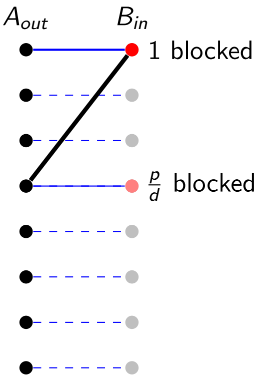

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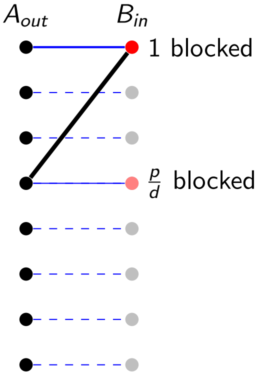

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$$\mathbb{E}[|M|] \geq \frac{dp}{d+p} \cdot |M^*|$$

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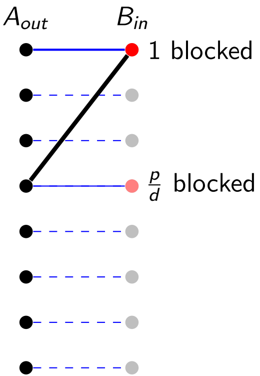
- Formalised using Wald's Equation.



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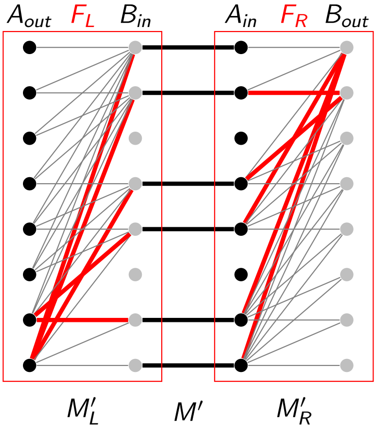
Optimal Algorithms I



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$$\mathbb{E}_{M'}[|M'_L|] = \frac{dp}{d+p} \cdot |M^*_L|$$

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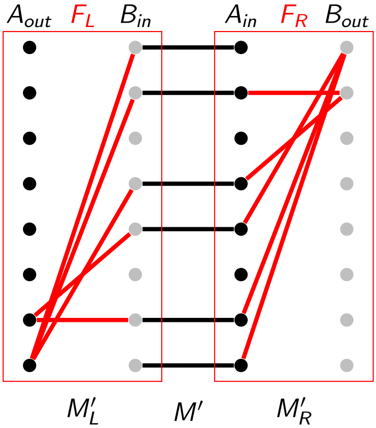

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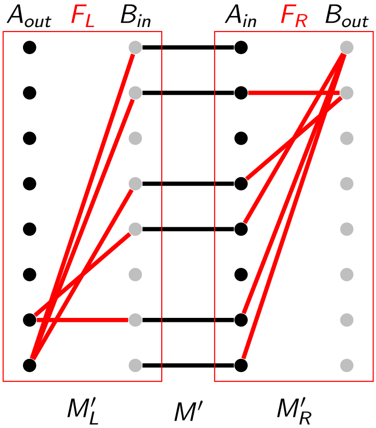
Optimal Algorithms I

Analysis:

We find at least $(\frac{1}{2} + \frac{p}{d+p} - \frac{p}{2d}) \cdot |M^*|$ edges in our final matching.



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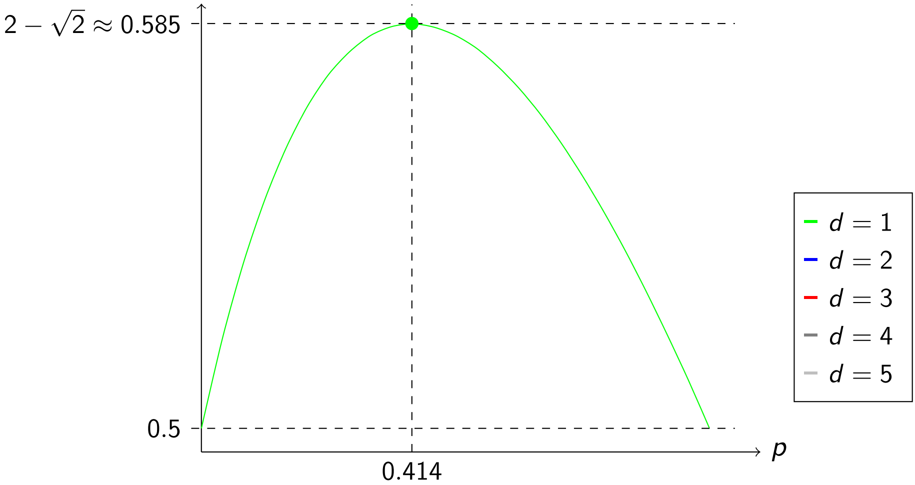
Optimal Algorithms II



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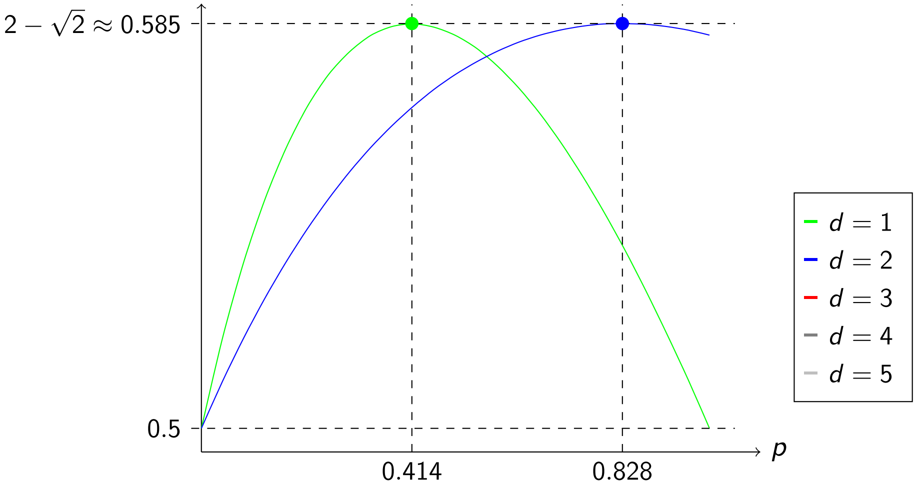

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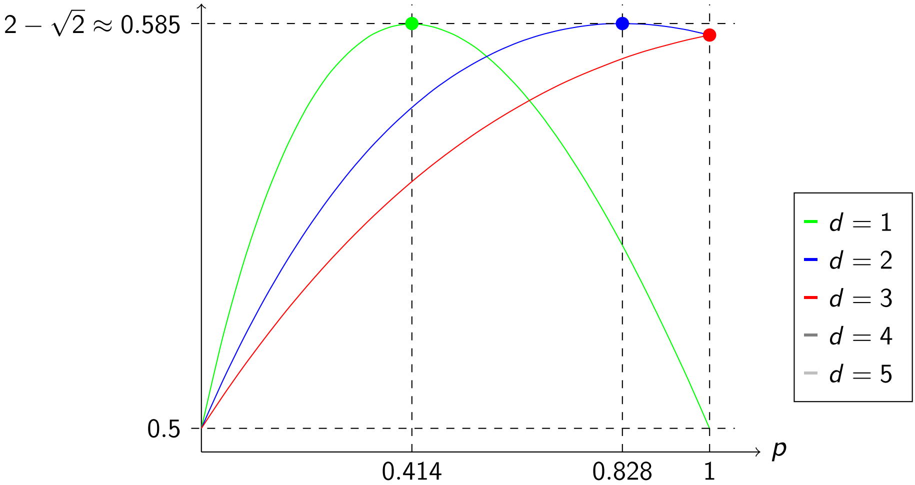

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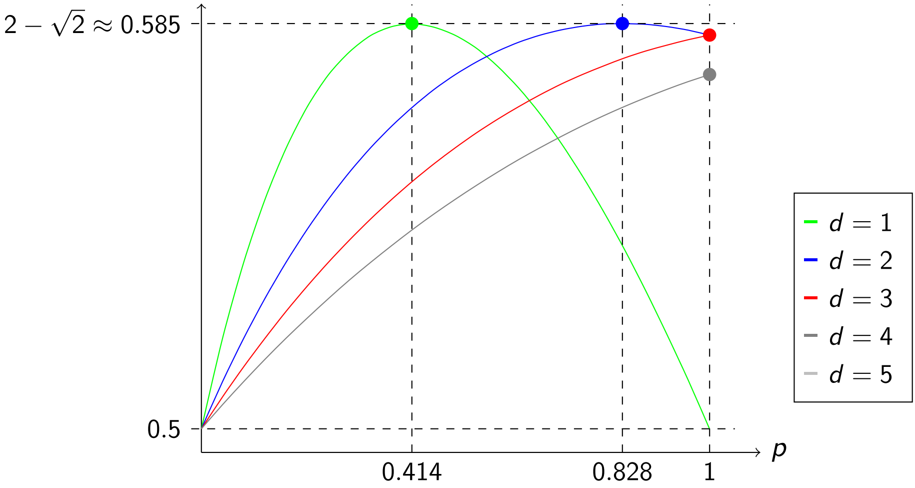

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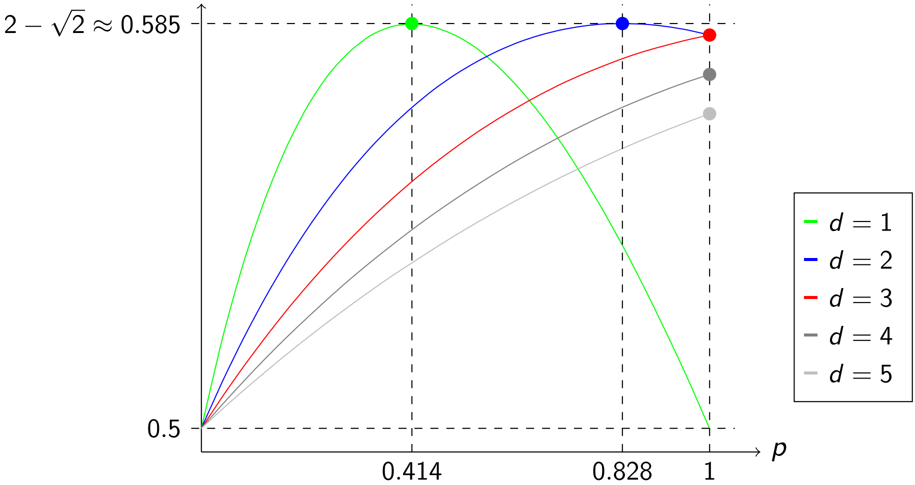

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Two-Party Communication Setup

$$\boxed{\text{Alice}} \longrightarrow \boxed{\text{Bob}} \longrightarrow \boxed{M^*}$$

Proving Impossibility Results

Two-Party Communication Setup

$$\boxed{\text{Alice}} \longrightarrow \boxed{\text{Bob}} \longrightarrow \boxed{M^*}$$

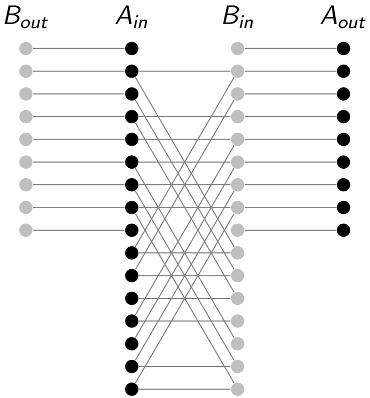
Goal:

- to bound the size of the message from Alice to Bob needed to output a large matching.

One-Pass Impossibility Proof [SODA12]

Outline:

1. A family of graphs constructed from very dense Rusza-Szemeredi (RS) graphs.



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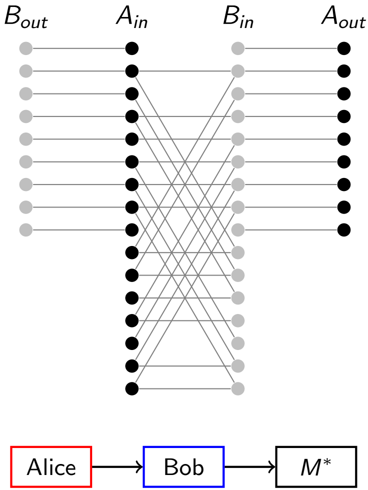

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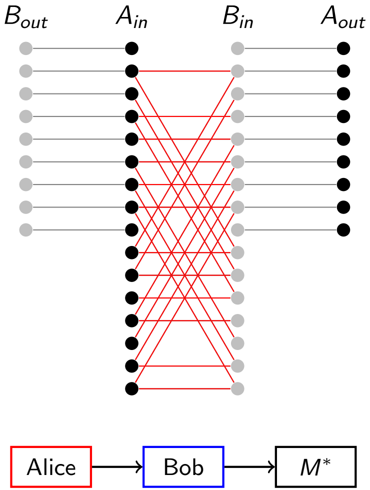

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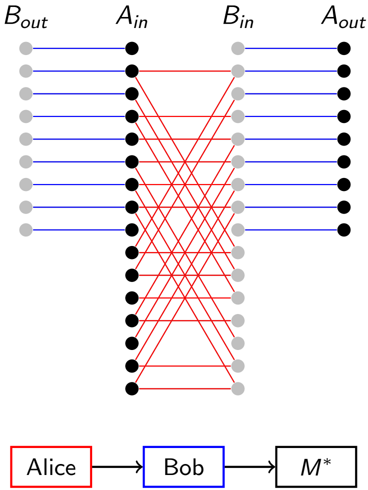

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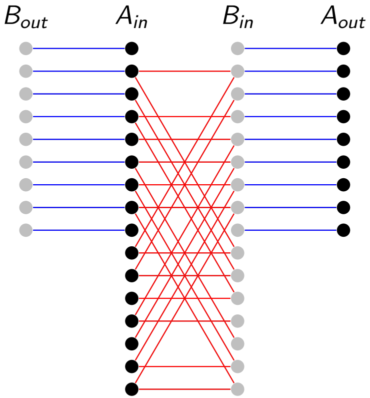
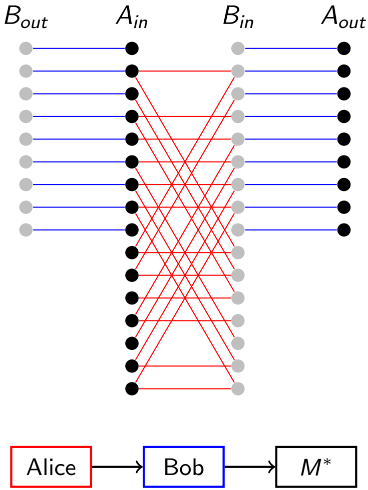


Two-Pass Impossibility Proof

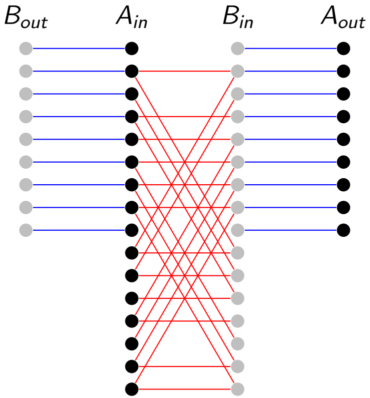
Class of algorithms:

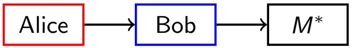
1. finds a maximal matching M (first-pass);
2. increases the size of the matching M (second-pass).



Two-Pass Impossibility Proof

Goal: Give both Alice and Bob knowledge of a maximal matching without affecting the difficulty of the problem.





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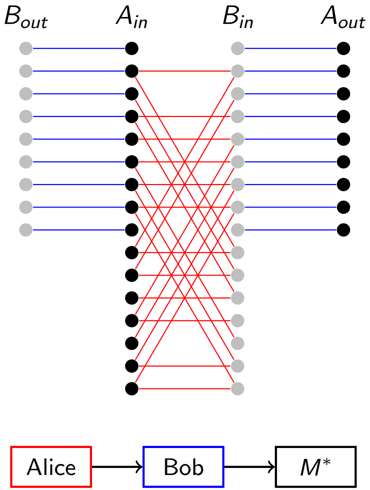

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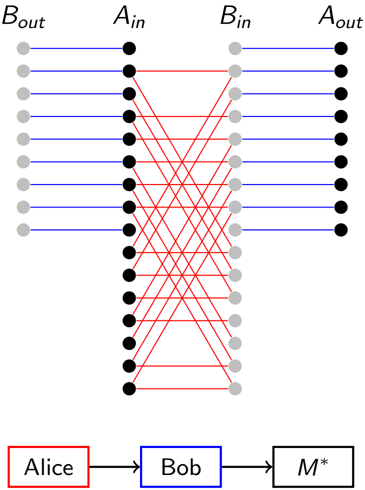

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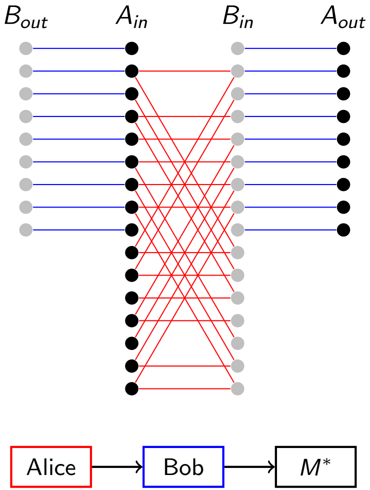

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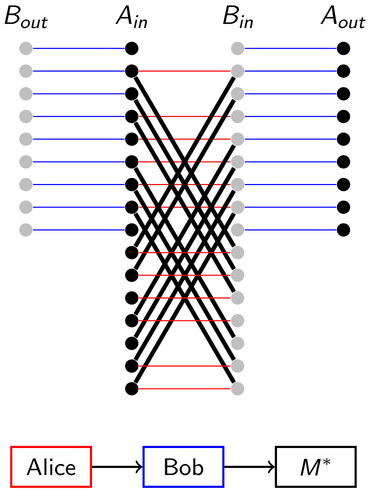

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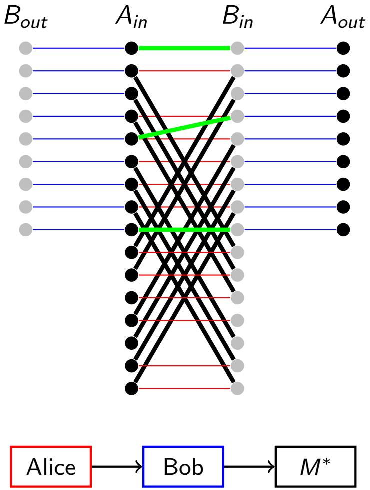

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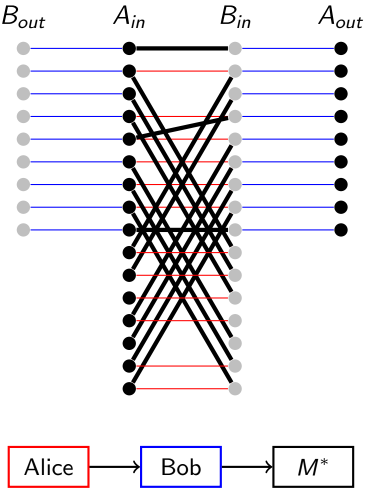

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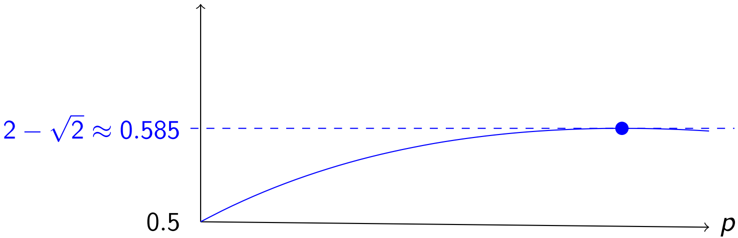

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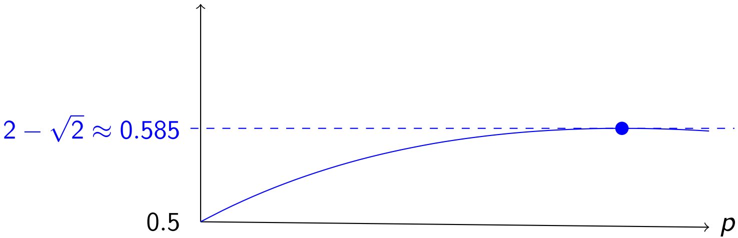

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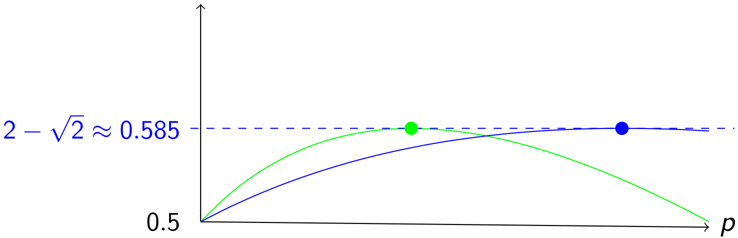

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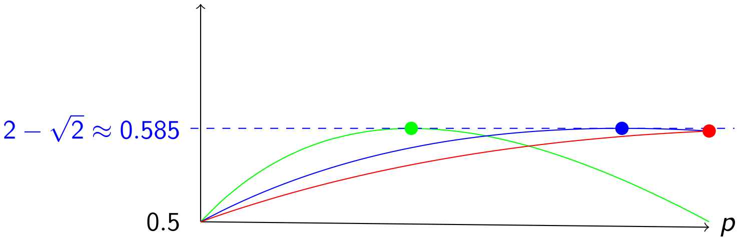

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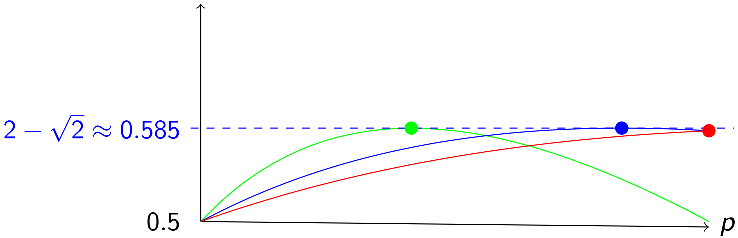

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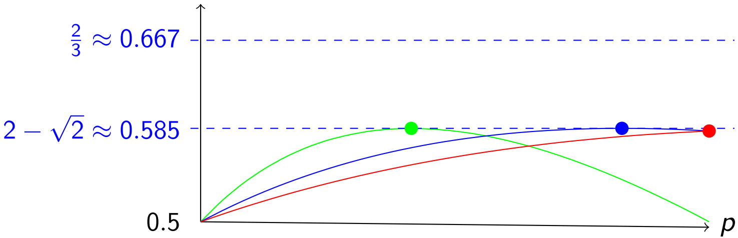
- Our hard-instance graph proves that our analysis is tight.
- We reduced the gap of possibility with this class of algorithms to $[0.585, 0.667]$.



Open Questions

- Can we extend other one-pass impossibility results to improve the two-pass bound? I.e. Kapralov's [SODA21].
- Is there a way to do better by finding more than just a maximal matching in the first-pass?
- Can we beat a $\frac{1}{2}$-approximation in just one-pass?

Thank You