Space Optimal Vertex Cover in Dynamic Streams

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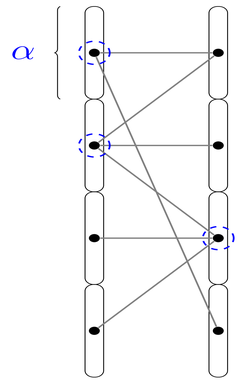
Preliminaries

- Graph $G = (V, E)$ given as an adversarial dynamic stream.
- The input cannot be a multi-graph.

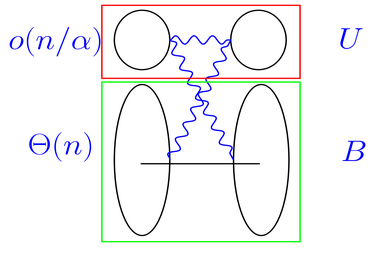
Previous work

- Upper bound of $\tilde{O}(n^2/\alpha^2)$ bits [DK20].
- Lower bound of $\Omega(n^2/\alpha^2)$ bits [DK20].

Algorithm of [DK20]



Hard Instance



Main Results

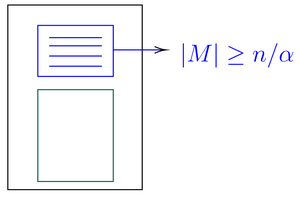
There is a randomised dynamic streaming algorithm that with high probability outputs an α-approximation to the minimum vertex cover using $O(n^2/\alpha^2)$ bits of space given any $\alpha \leq n^{1-\delta}$ for any constant $\delta > 0$.

Match-Or-Sparsify Lemma

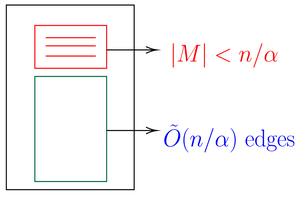
This algorithm uses $O(n^2/\alpha^2)$ bits of space and whp outputs a matching M that satisfies at least one of the following conditions (see also [AS22]):

- **Match-case:** $|M| \geq n/\alpha$;
- **Sparsify-case:** The induced subgraph on vertices not matched by M, has at most $\widetilde{O}(n/\alpha)$ edges.

Match Case



Sparsify Case

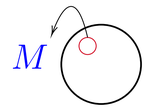


Grouping

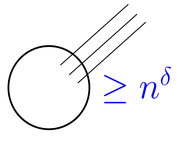
- Create n/α groups of size roughly α using a $O(\log n)$ wise independent hash function.

Group Types

- Simple



- Residual



- Clean: Remaining groups (for group level vertex cover).

Algorithm for small opt

- Run algorithm of [CCE+16] for exact vertex cover when opt $\leq k$.
- Set $k = n/\alpha \log^2 n$.
- Space: $O(k^2 \log^4 n) = O(n^2/\alpha^2)$.

References

[AS22] Sepehr Assadi and Vihan Shah. An asymptotically optimal algorithm for maximum matching in dynamic streams. In Mark Braverman, editor, *13th Innovations in Theoretical Computer Science Conference, ITCS 2022, January 31 - February 3, 2022, Berkeley, CA, USA*, volume 215 of *LIPIcs*, pages 9:1–9:23. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2022.

[CCE+16] Rajesh Chitnis, Graham Cormode, Hossein Esfandiari, MohammadTaghi Hajiaghayi, Andrew McGregor, Morteza Monemizadeh, and Sofya Vorotnikova. Kernelization via sampling with applications to finding matchings and related problems in dynamic graph streams. In Robert Krauthgamer, editor, *Proceedings of the Twenty-Seventh Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2016, Arlington, VA, USA, January 10-12, 2016*, pages 1326–1344. SIAM, 2016.

[DK20] Jacques Dark and Christian Konrad. Optimal lower bounds for matching and vertex cover in dynamic graph streams. In Shubhangi Saraf, editor, *35th Computational Complexity Conference, CCC 2020, July 28-31, 2020, Saarbrücken, Germany (Virtual Conference)*, volume 169 of *LIPIcs*, pages 30:1–30:14. Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2020.