Space Optimal Vertex Cover in Dynamic Streams
(with an overview of graph streaming)

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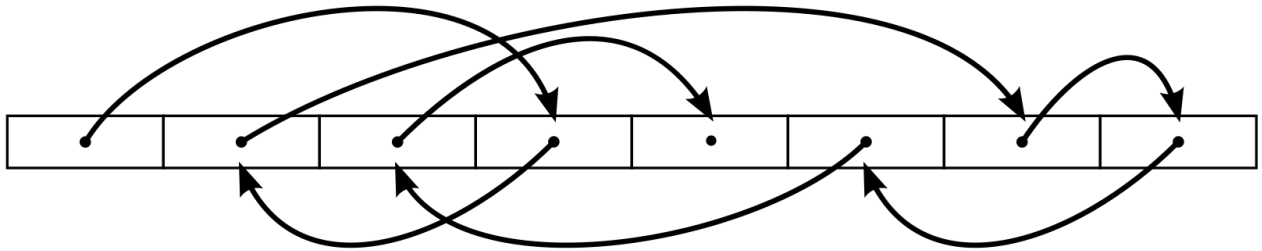
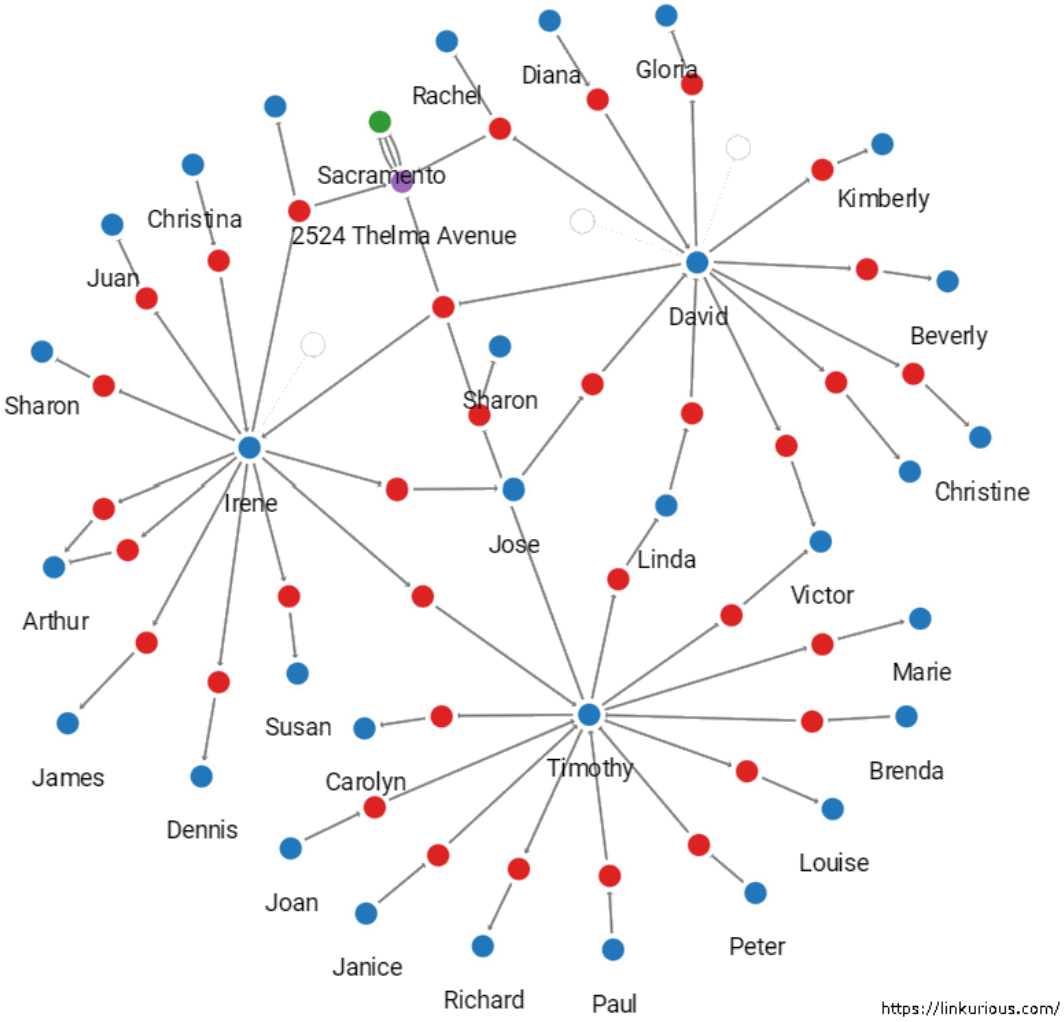
Joint work with Vihan Shah (Rutgers University)

Classic Setting

Assumption

Classical algorithms rely on the assumption that they have a random access to the input of the algorithm

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Sacramento
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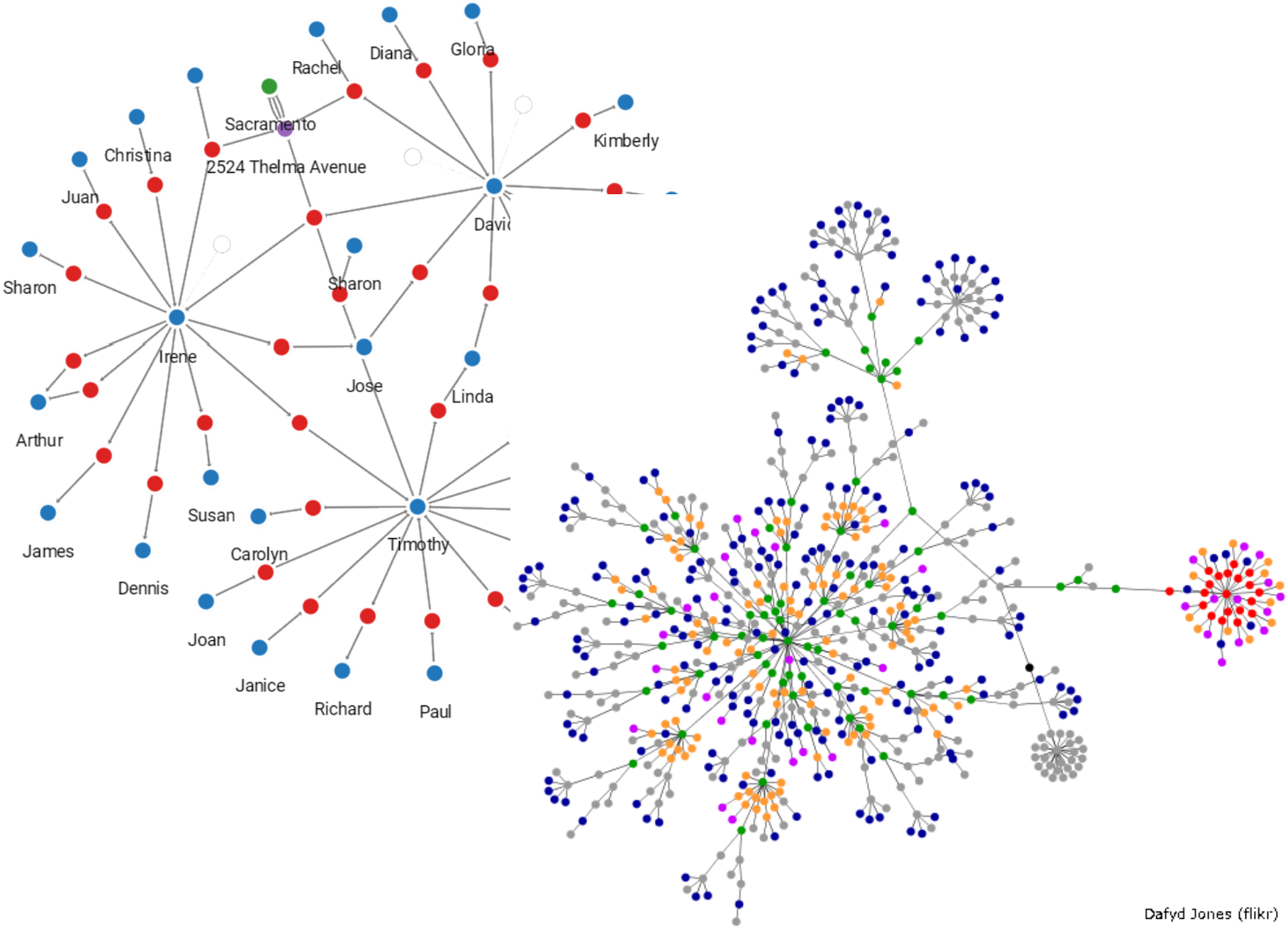
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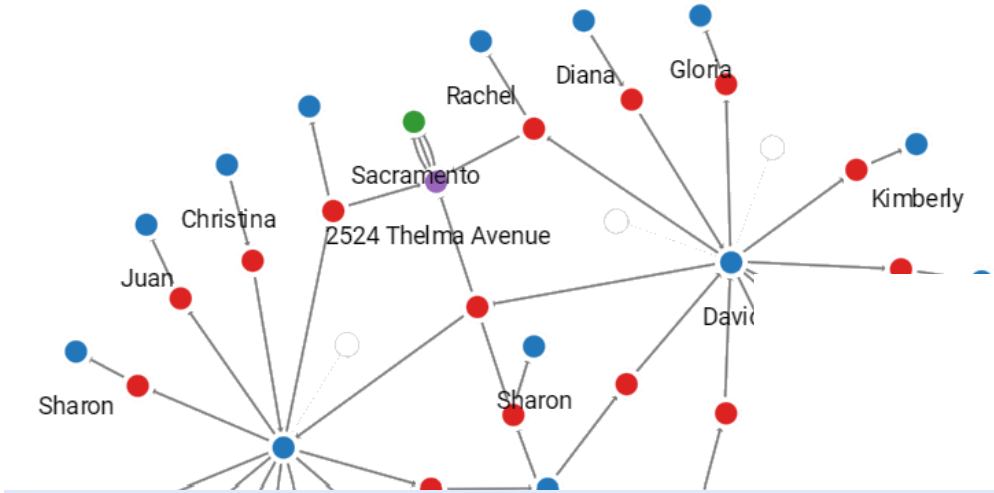
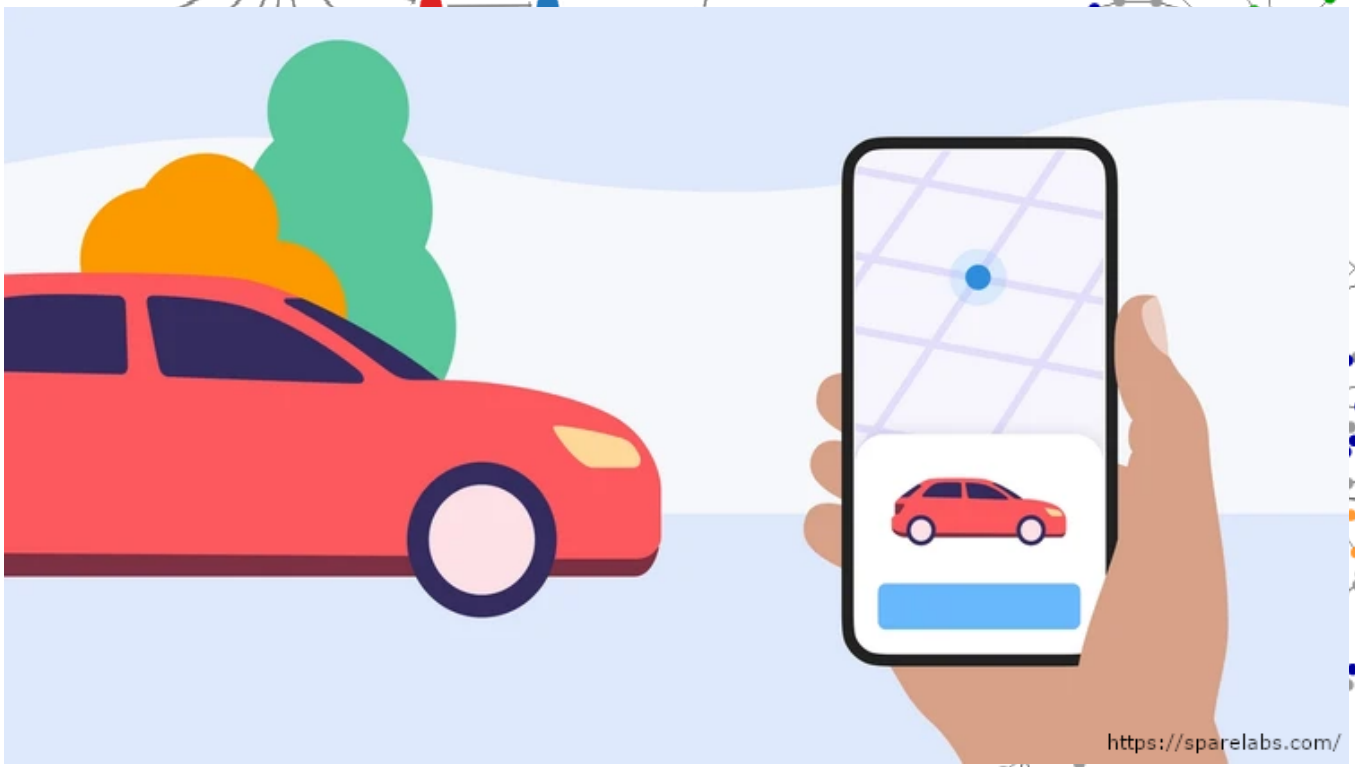
Dafyd Jones (flikr)





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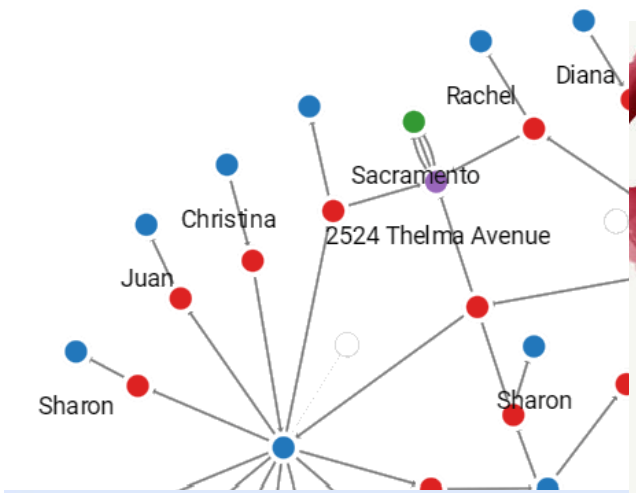
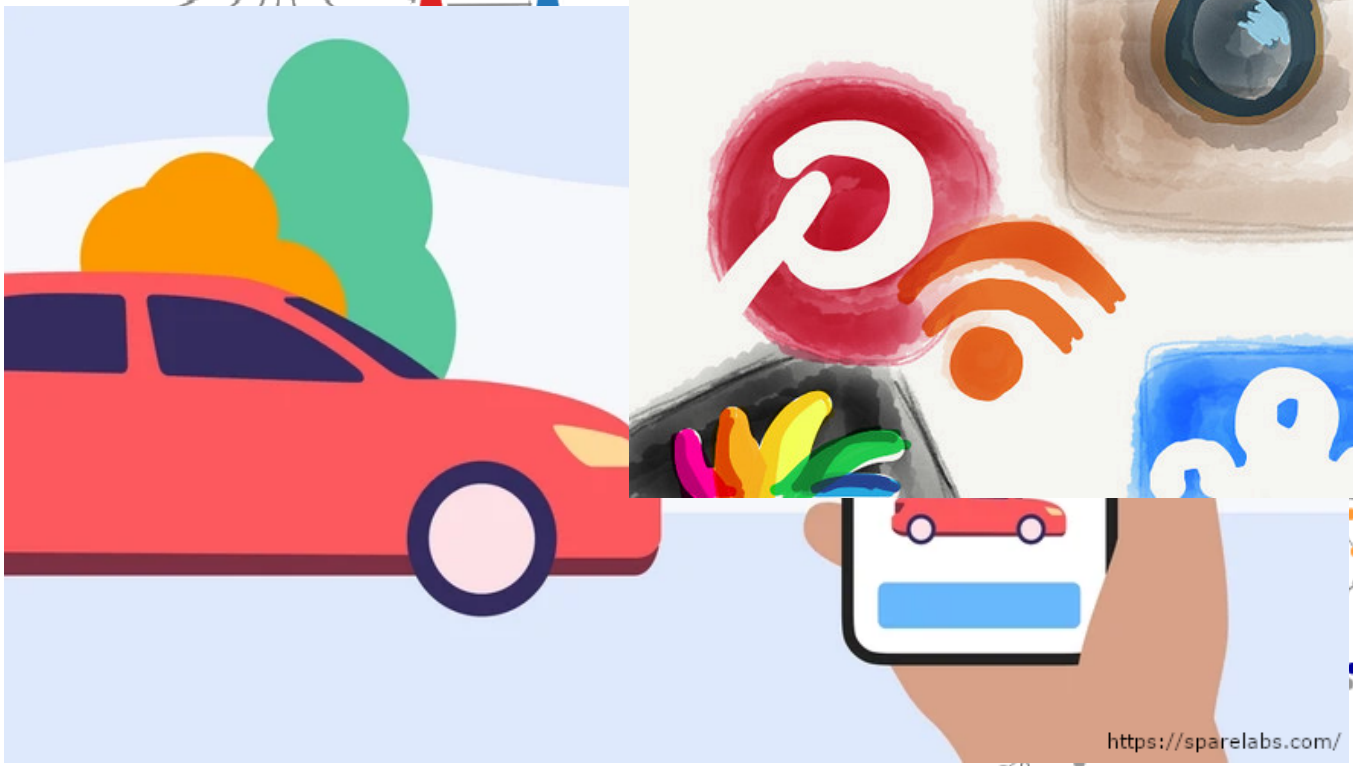
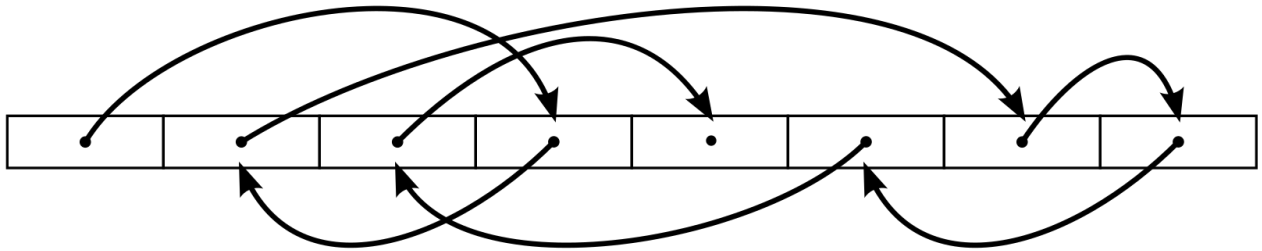




Dafyd Jones (flikr)

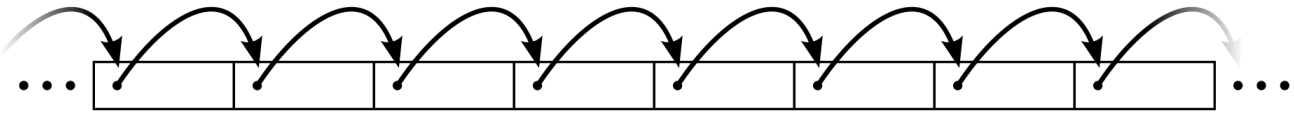
Assumption (Infeasible)

Classical algorithms rely on the assumption that they have a random access to the input of the algorithm

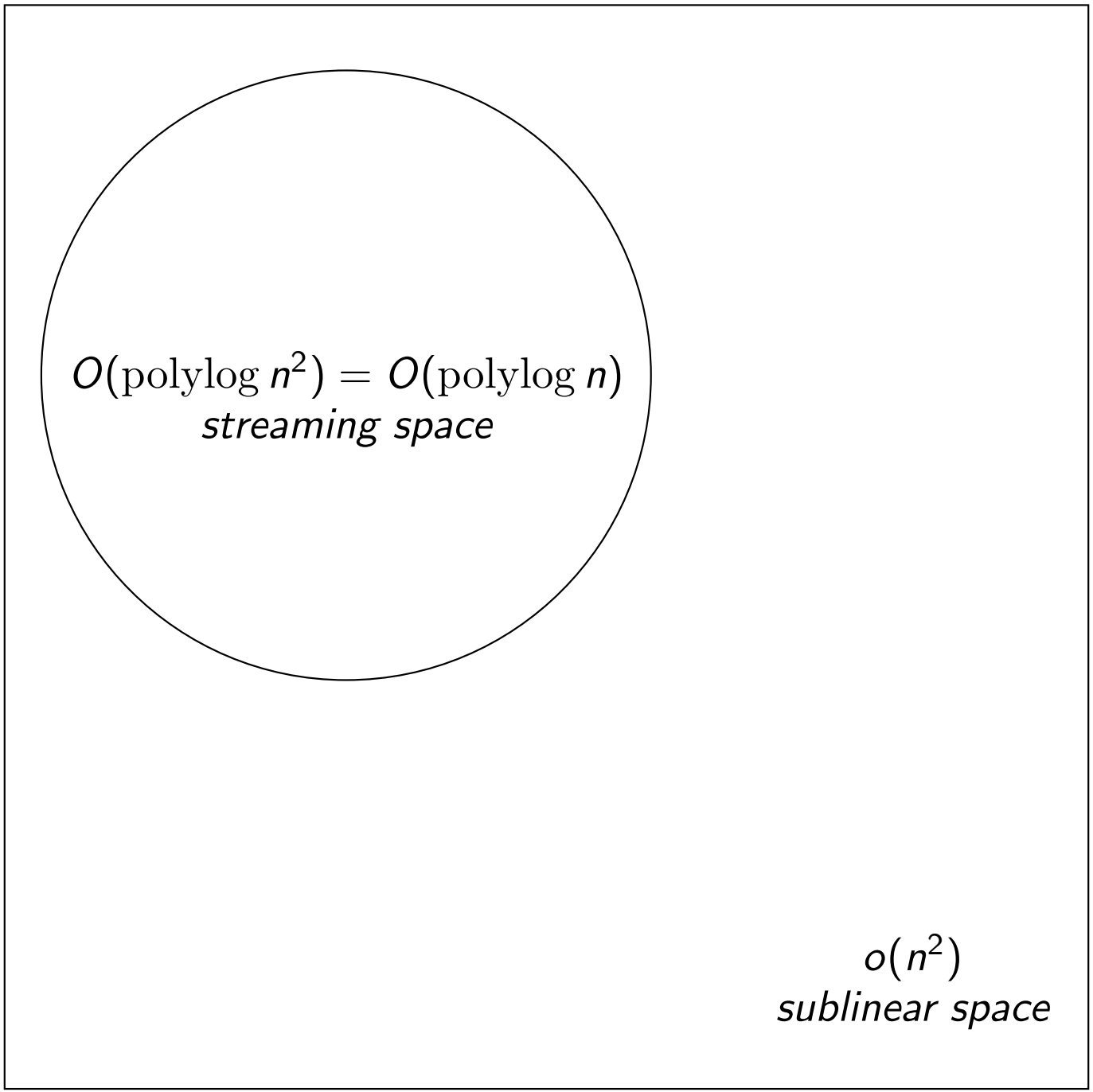


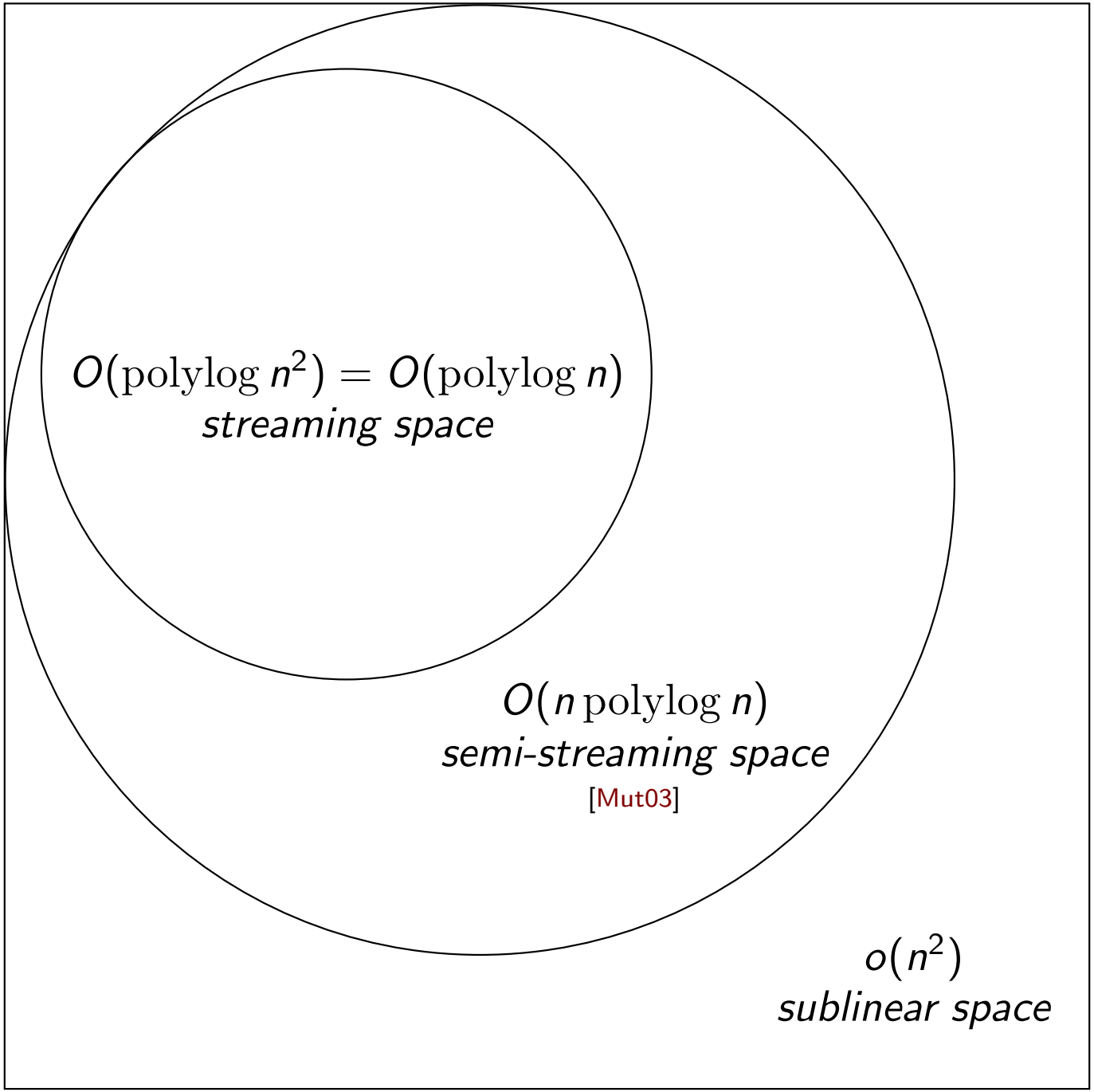
Definition (Graph Streaming)

A n-vertex graph is presented as a sequence of edges to an algorithm uses space **sublinear** in the size of the input ($o(n^2)$).



$o(n^2)$
sublinear space


$O(\mathrm{polylog}\, n^2) = O(\mathrm{polylog}\, n)$
streaming space
$o(n^2)$
sublinear space

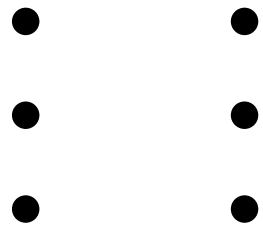

$O(\text{polylog}\, n^2) = O(\text{polylog}\, n)$
streaming space
$O(n\,\text{polylog}\, n)$
semi-streaming space
[Mut03]
$o(n^2)$
sublinear space

Insertion-Only [FKM+04]

Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM+04]



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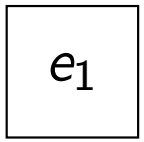
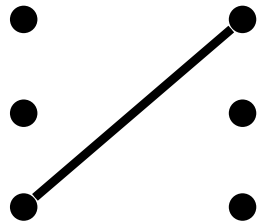
Insertion-Only [FKM+04]





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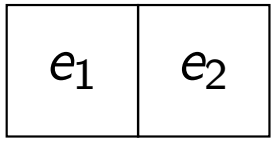
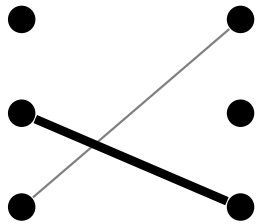


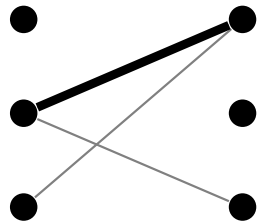


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM+04]

e_1	e_2	e_3



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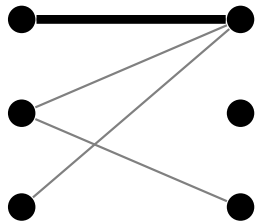
Insertion-Only [FKM+04]

e_1	e_2	e_3	e_4



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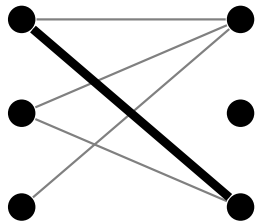
Insertion-Only [FKM+04]

e_1	e_2	e_3	e_4	e_5



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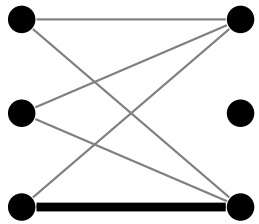
e_1	e_2	e_3	e_4	e_5	e_6



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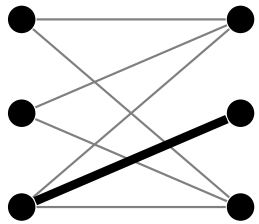
Dynamic [AGM12]

Insertion-Only [FKM+04]

e_1	e_2	e_3	e_4	e_5	e_6	e_7



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Dynamic [AGM12]

Insertion-Only [FKM+04]

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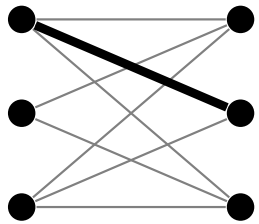
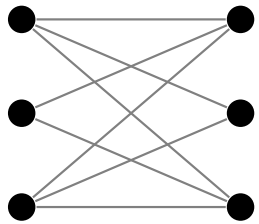


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



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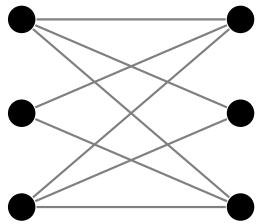
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



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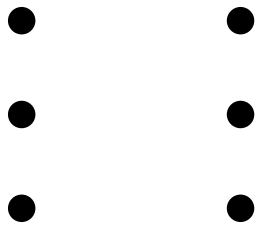

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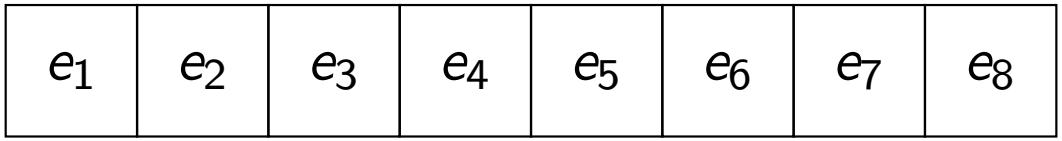

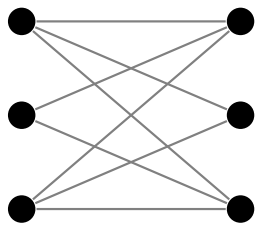

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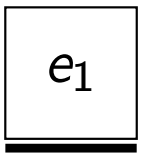

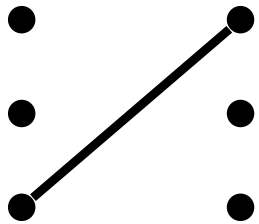

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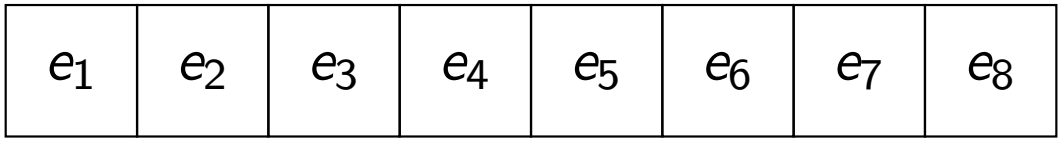

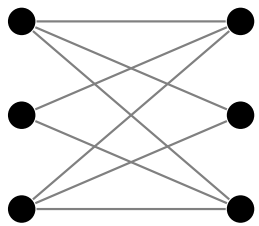

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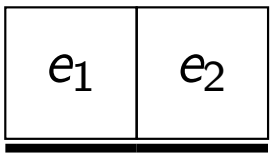
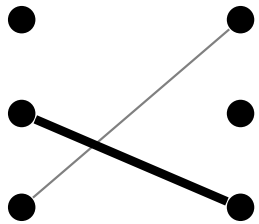
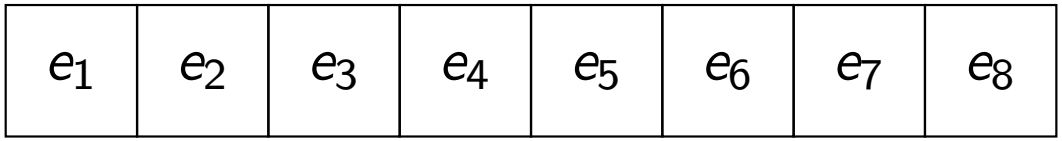
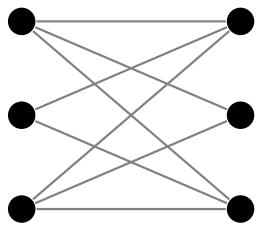




Dynamic [AGM12]

Insertion-Only [FKM+04] (finite stream)





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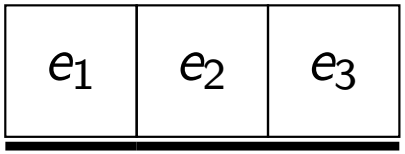
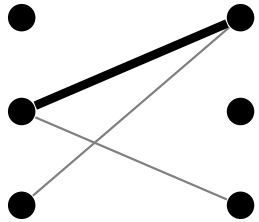
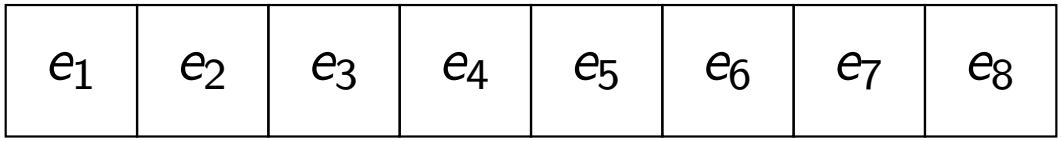
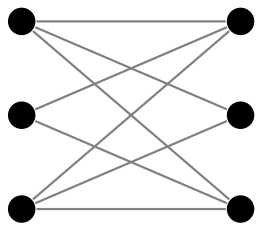




Dynamic [AGM12]

Insertion-Only [FKM+04] (finite stream)





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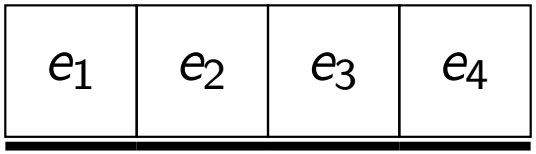
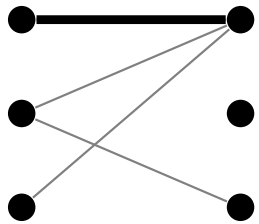
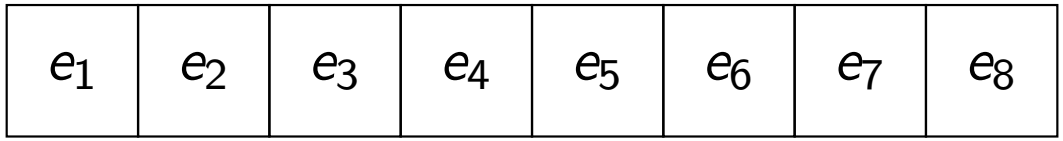
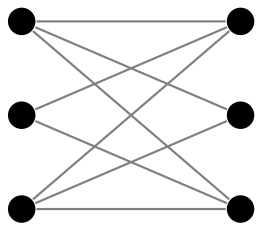




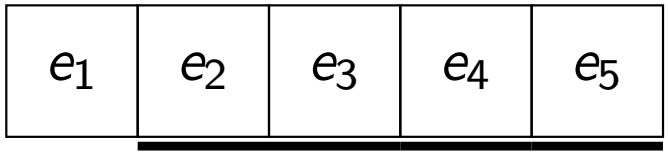
Dynamic [AGM12]

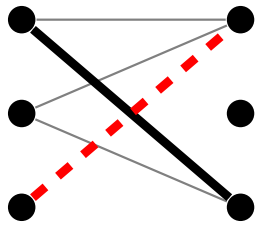
Insertion-Only [FKM+04] (finite stream)





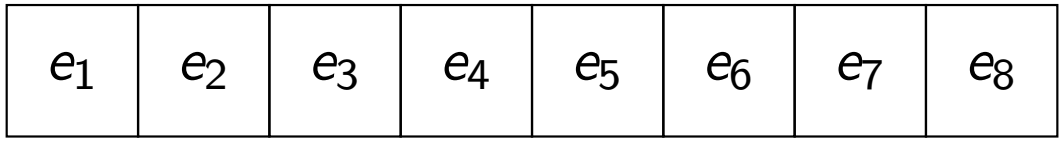
Sliding Window [CMS13]

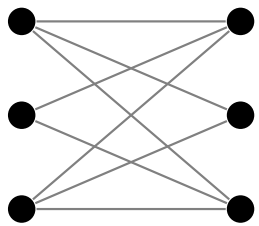




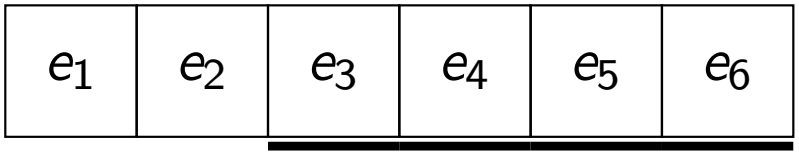
Dynamic [AGM12]

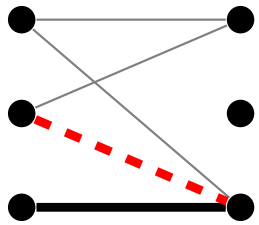
Insertion-Only [FKM+04] (finite stream)





Sliding Window [CMS13]

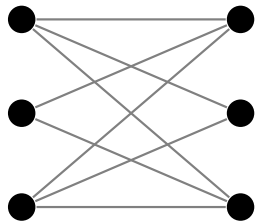




Dynamic [AGM12]

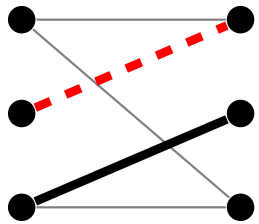
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



Sliding Window [CMS13]

e_1	e_2	e_3	e_4	e_5	e_6	e_7



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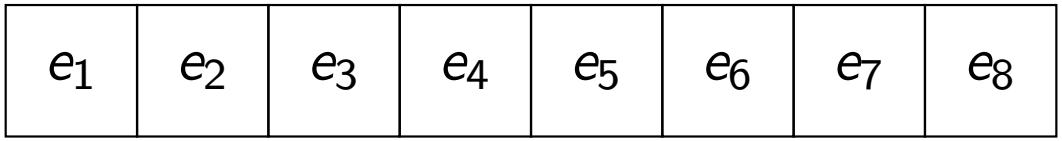
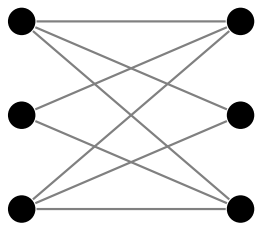
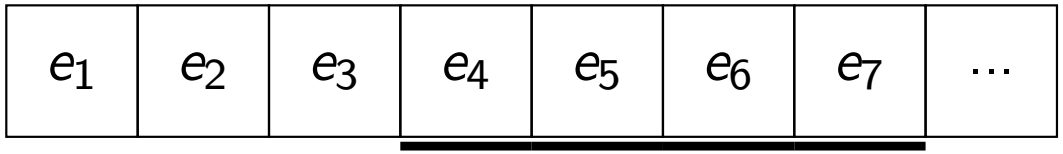
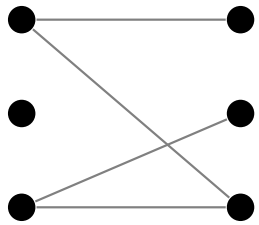




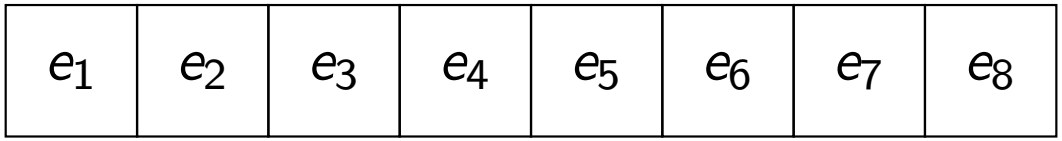
Sliding Window [CMS13] (infinite stream)

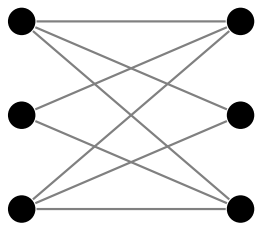




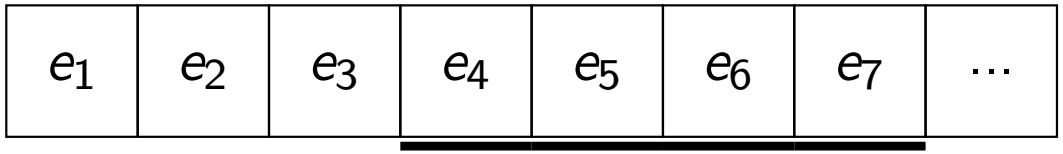
Dynamic [AGM12]

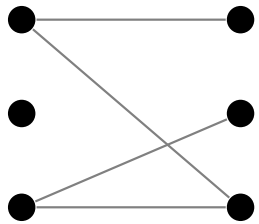
Insertion-Only [FKM+04] (finite stream)





Sliding Window [CMS13] (infinite stream)





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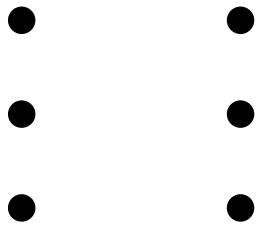

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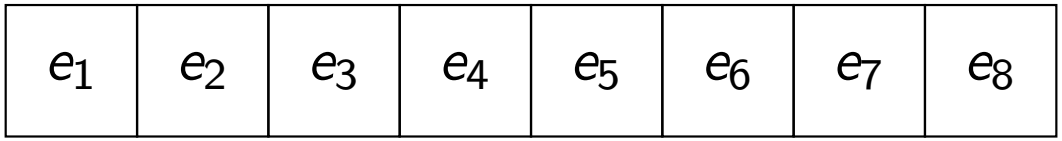
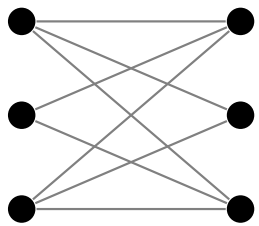
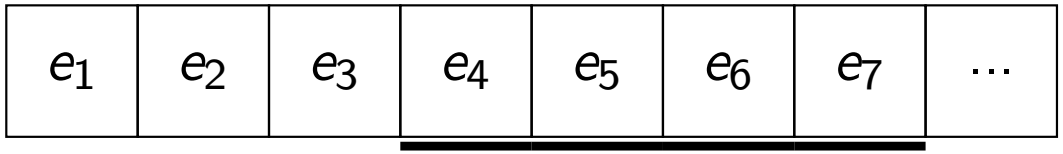
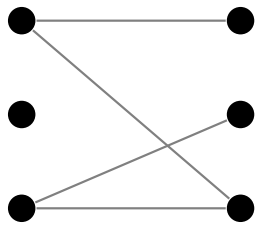




Sliding Window [CMS13] (infinite stream)





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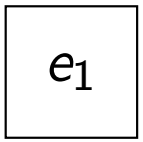
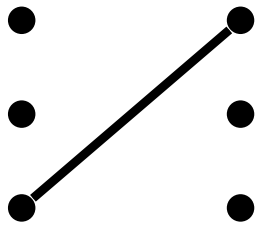
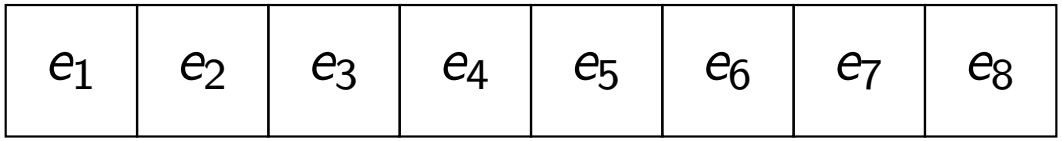
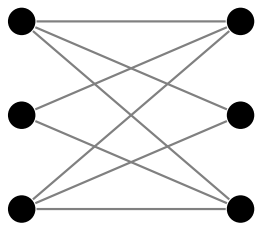


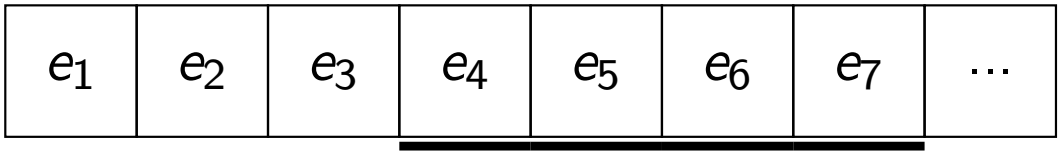


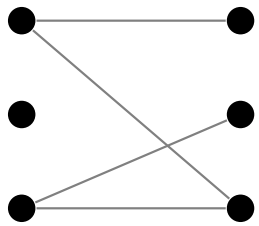
Insertion-Only [FKM+04] (finite stream)



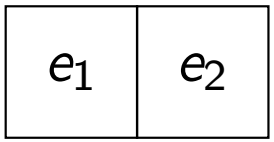


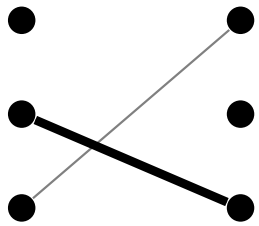
Sliding Window [CMS13] (infinite stream)





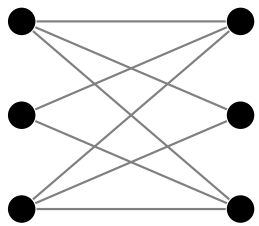
Dynamic [AGM12]





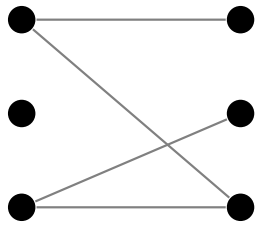
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



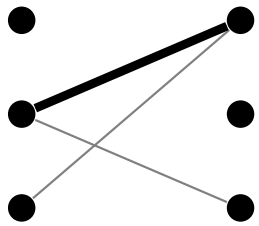
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...



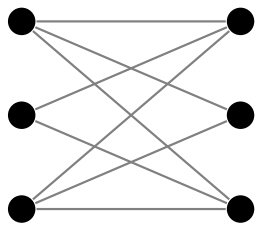
Dynamic [AGM12]

e_1	e_2	e_3



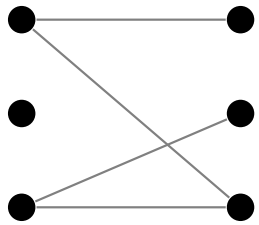
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



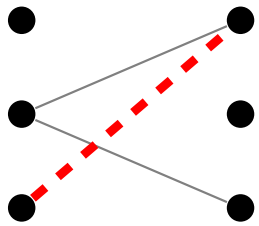
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...



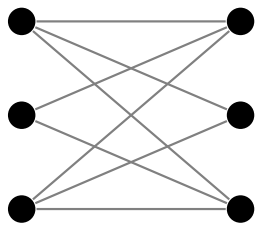
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$



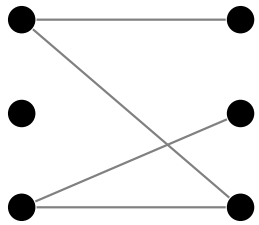
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



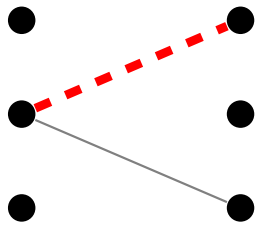
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...



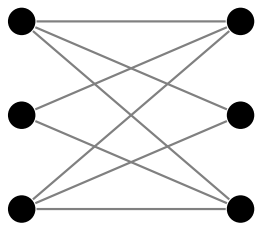
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$



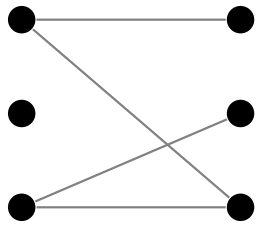
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



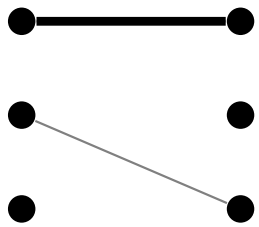
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...



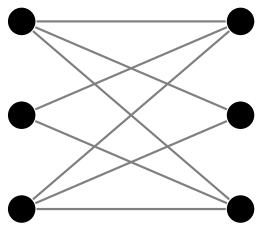
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4



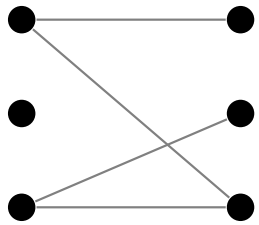
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



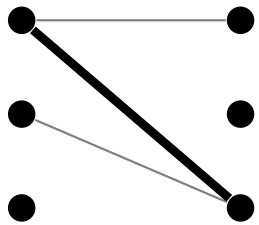
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...



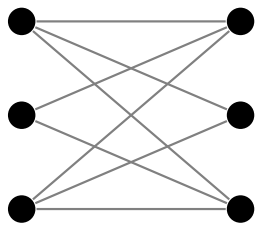
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4	e_5



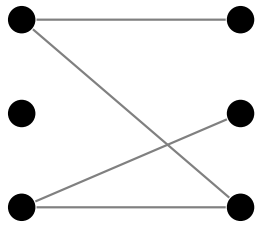
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



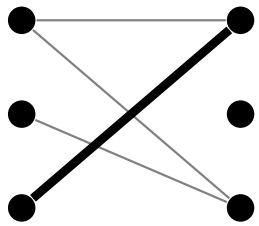
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...



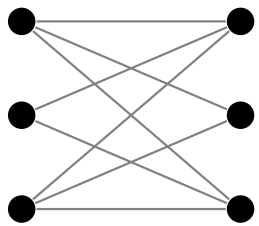
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4	e_5	e_1



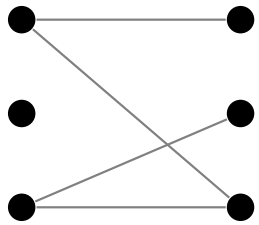
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



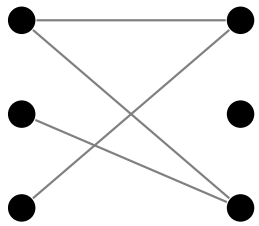
Sliding Window [CMS13] (infinite stream)

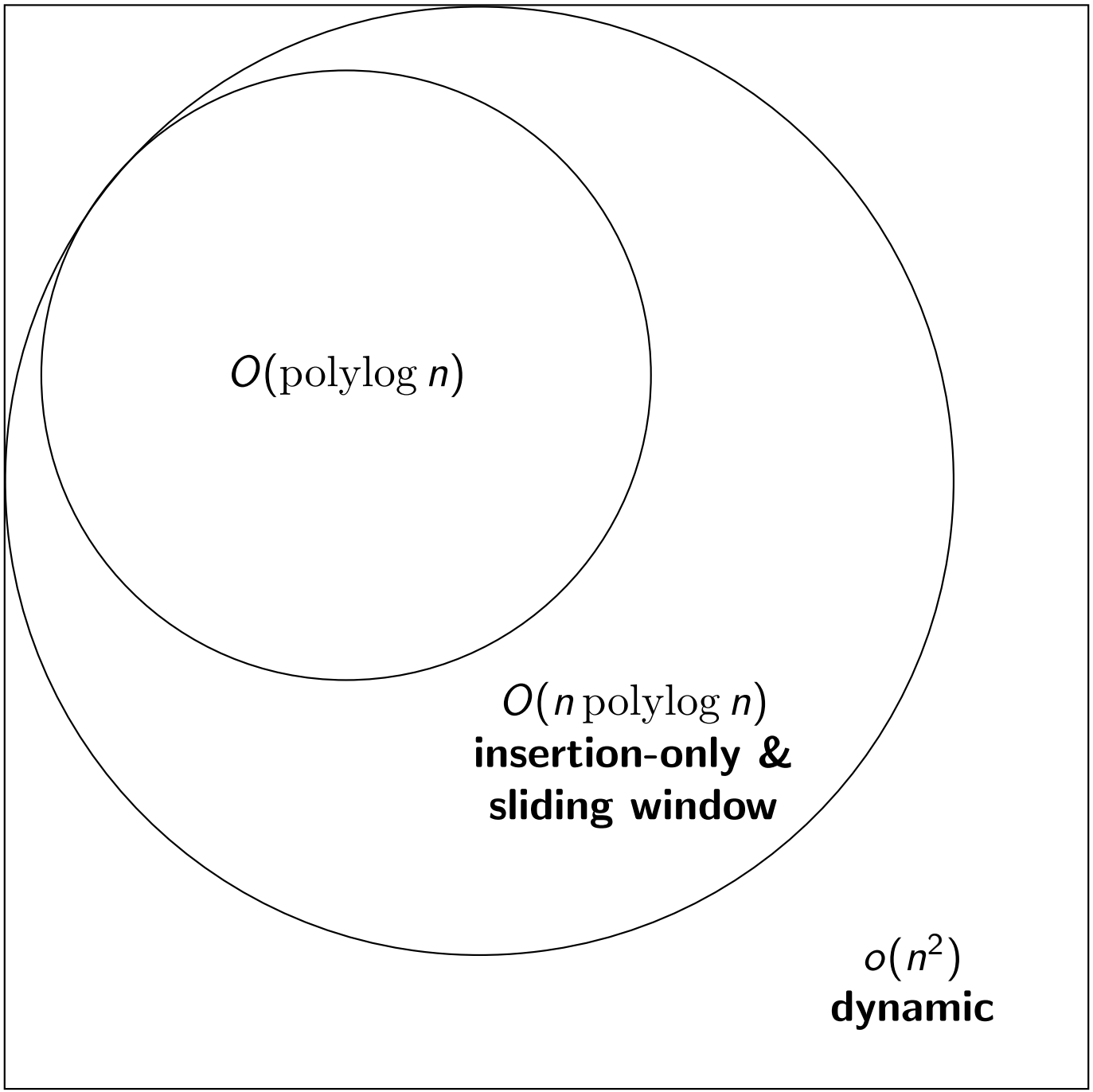
e_1	e_2	e_3	e_4	e_5	e_6	e_7	...



Dynamic [AGM12] (finite stream)

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4	e_5	e_1




$O(\text{polylog}\, n)$
$O(n\, \text{polylog}\, n)$
insertion-only &
sliding window
$o(n^2)$
dynamic

Simple and Powerful

Simple and Powerful

Greedy Matching:

1. Add edge if neither endpoint is matched

Simple and Powerful

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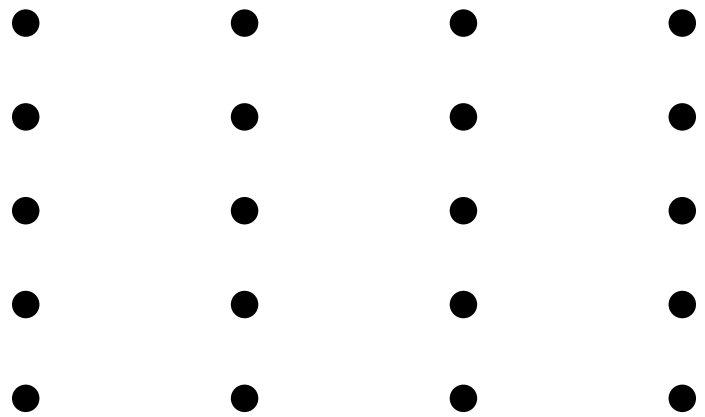

Simple and Powerful

GREEDY Matching:

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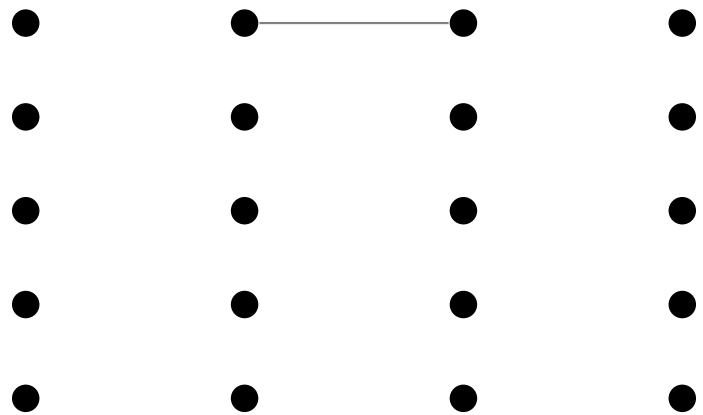

Simple and Powerful

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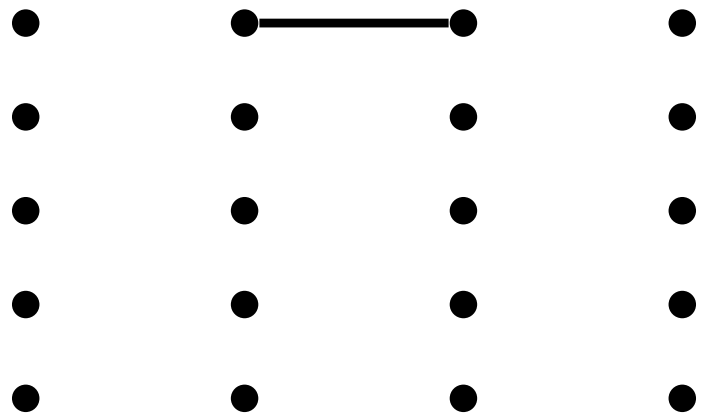

Simple and Powerful

Greedy Matching:

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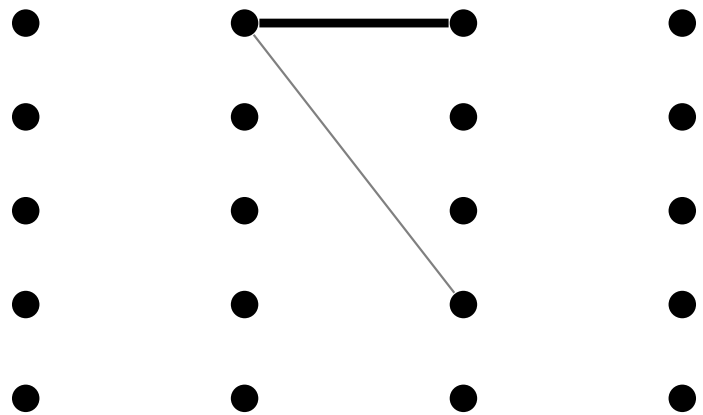

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Greedy Matching:

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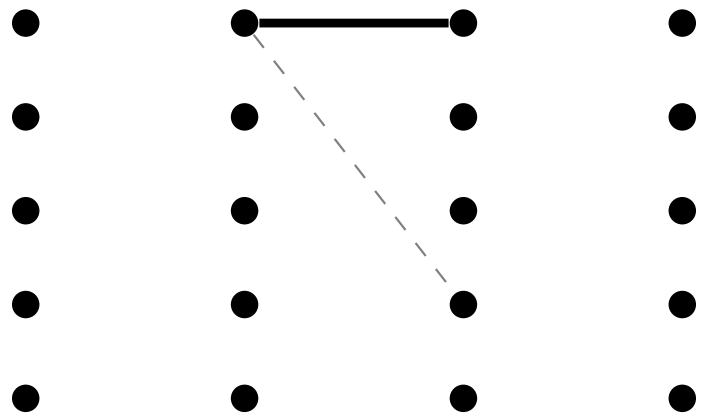

Simple and Powerful

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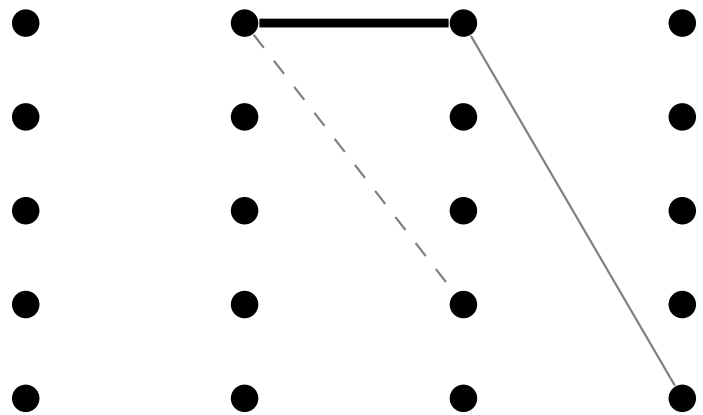
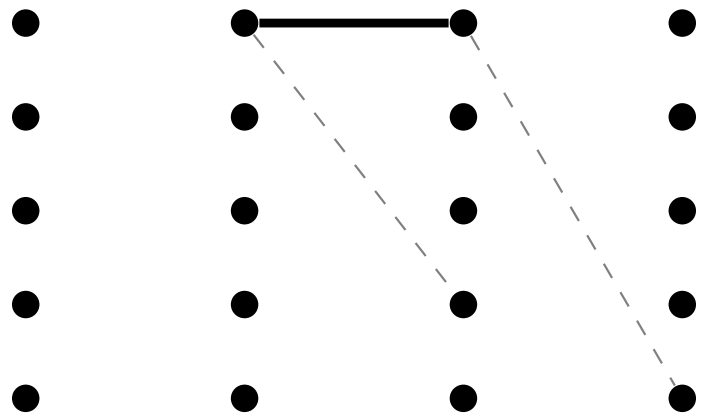


Simple and Powerful

Greedy Matching:

1. Add edge if neither endpoint is matched



Simple and Powerful

GREEDY Matching:

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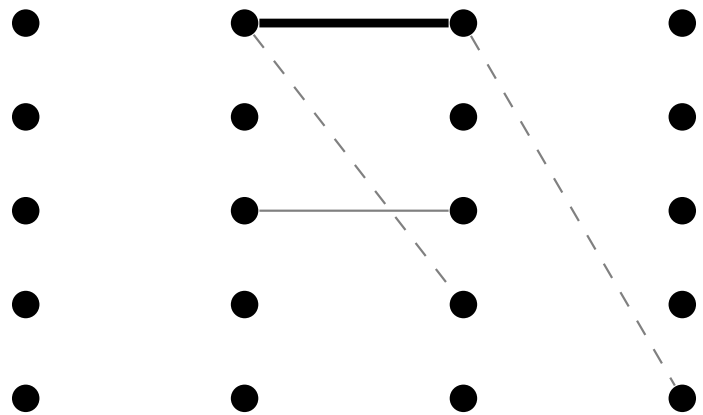

Simple and Powerful

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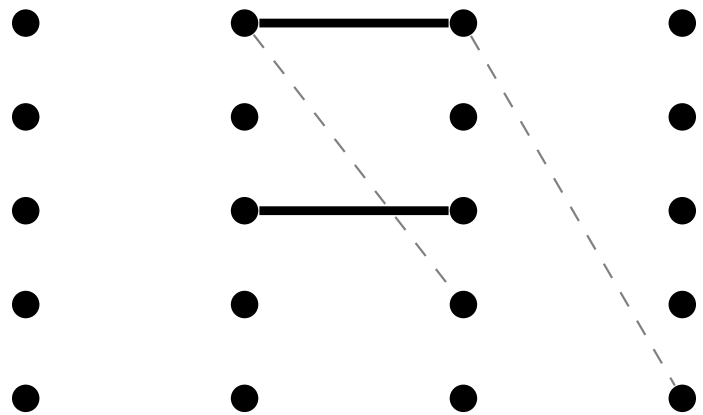

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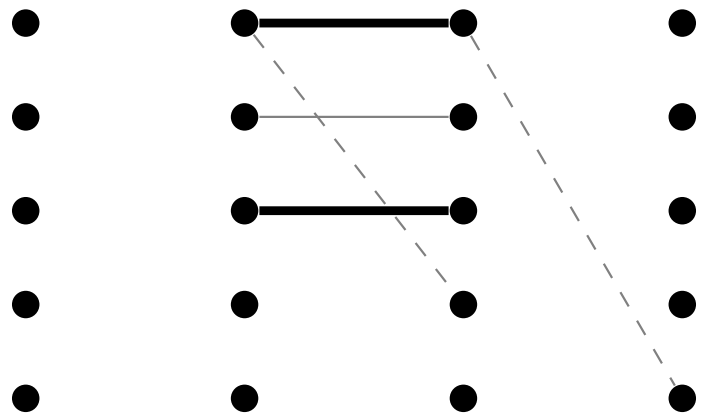

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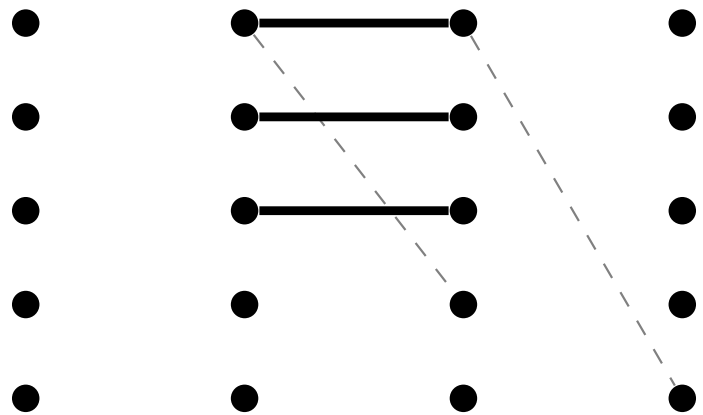

Simple and Powerful

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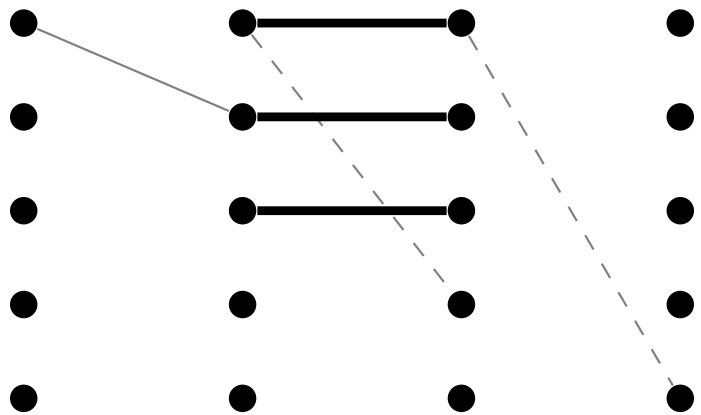

Simple and Powerful

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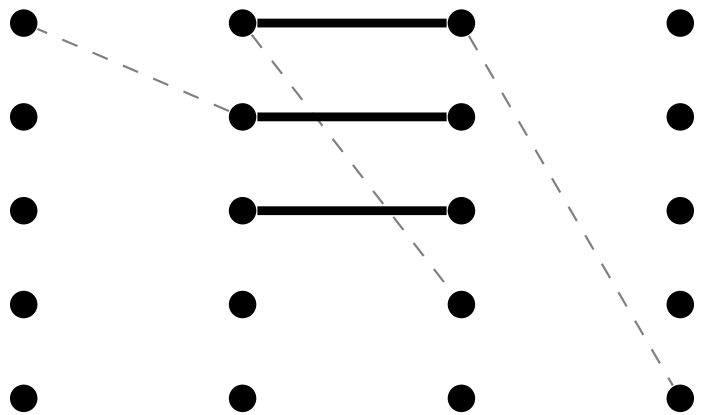
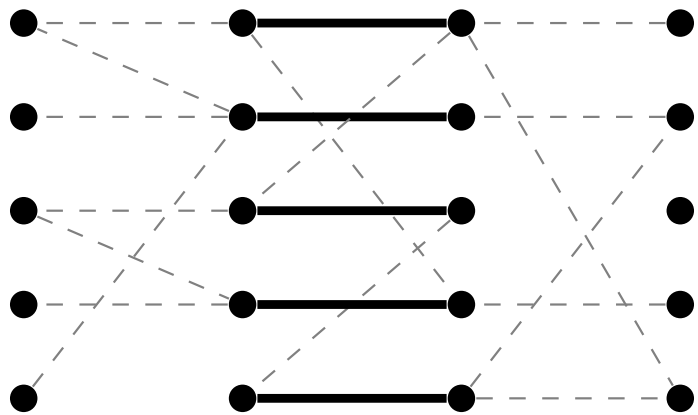


Simple and Powerful

GREEDY Matching:

1. Add edge if neither endpoint is matched

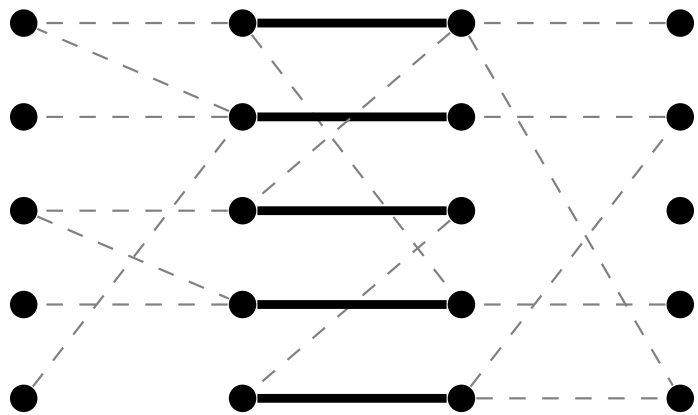


Simple and Powerful

GREEDY Matching:

1. Add edge if neither endpoint is matched

- Maximal

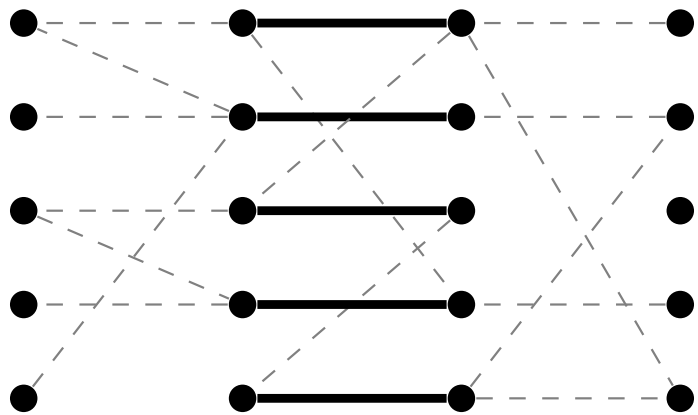


Simple and Powerful

Greedy Matching:

1. Add edge if neither endpoint is matched

- Maximal
- 2-approximation

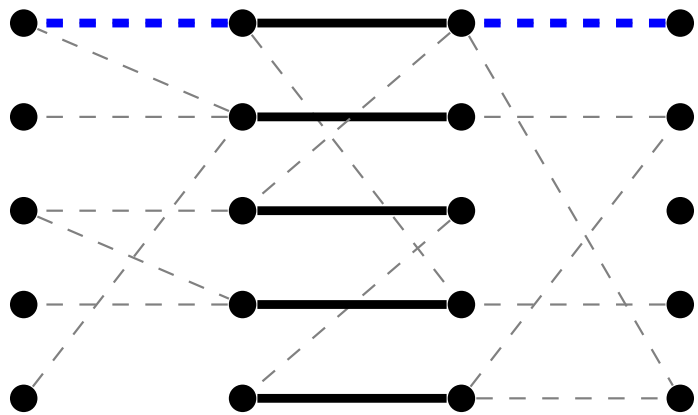


Simple and Powerful

Greedy Matching:

1. Add edge if neither endpoint is matched

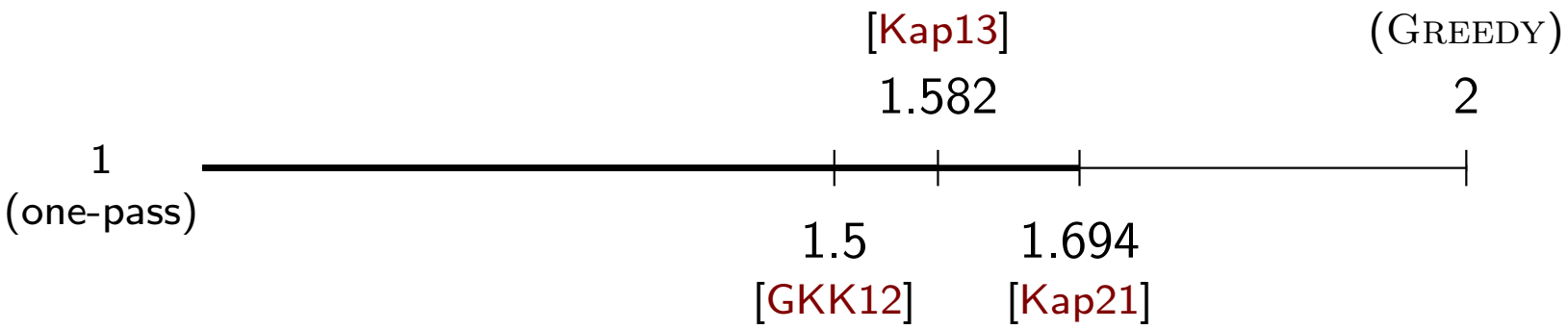
- Maximal
- 2-approximation



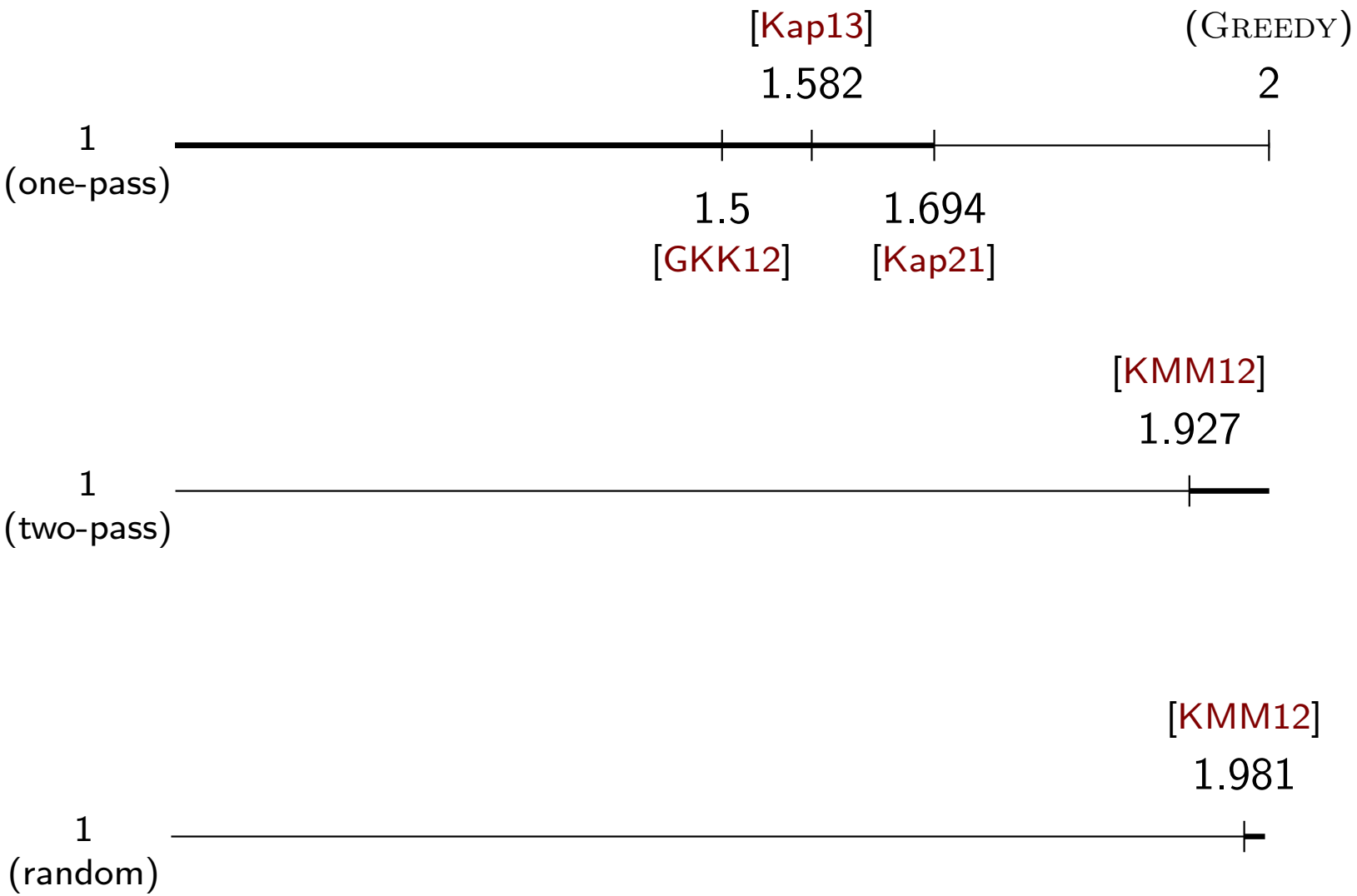
Insertion-Only

Insertion-Only

Insertion-Only



Insertion-Only



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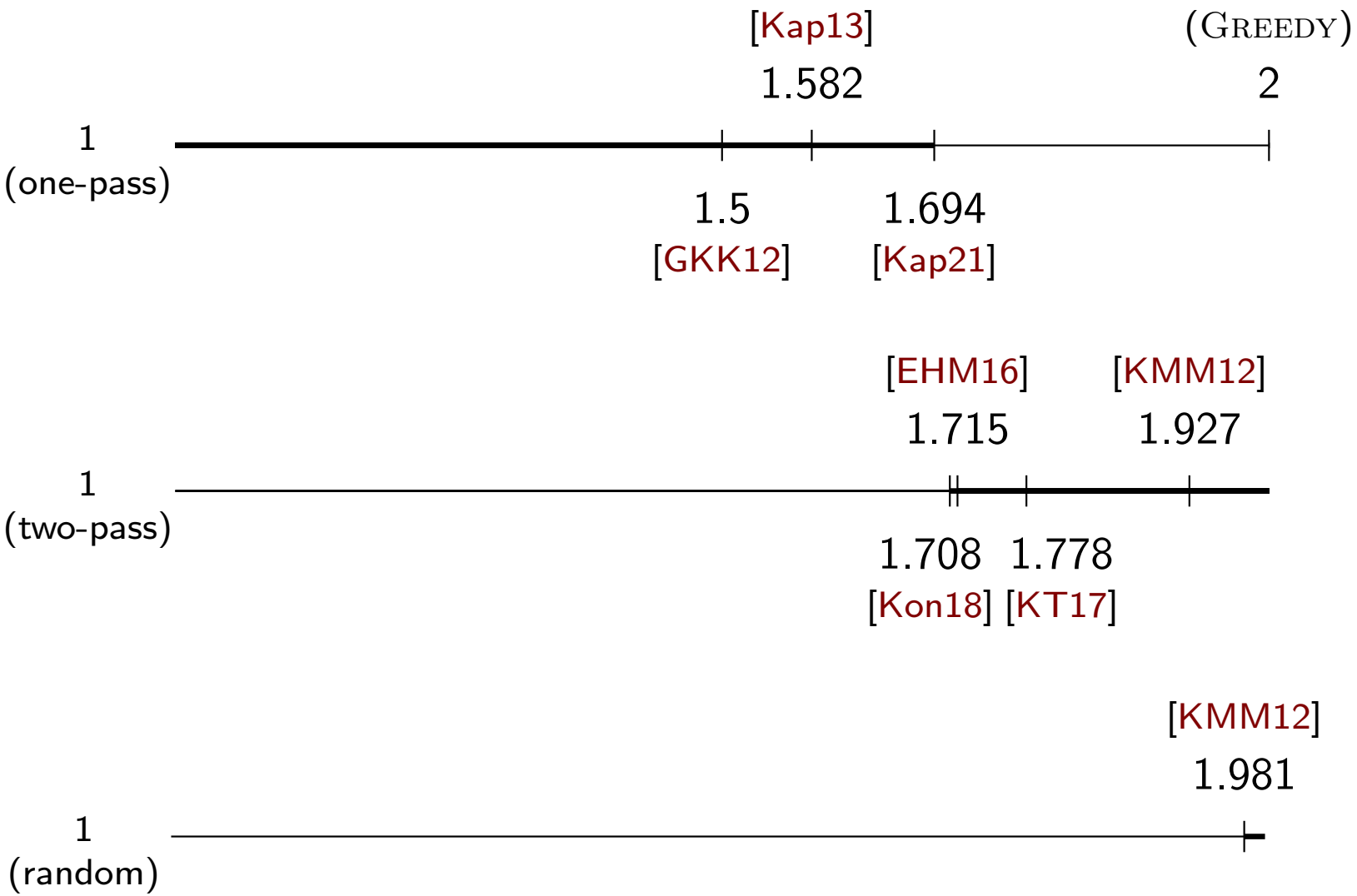
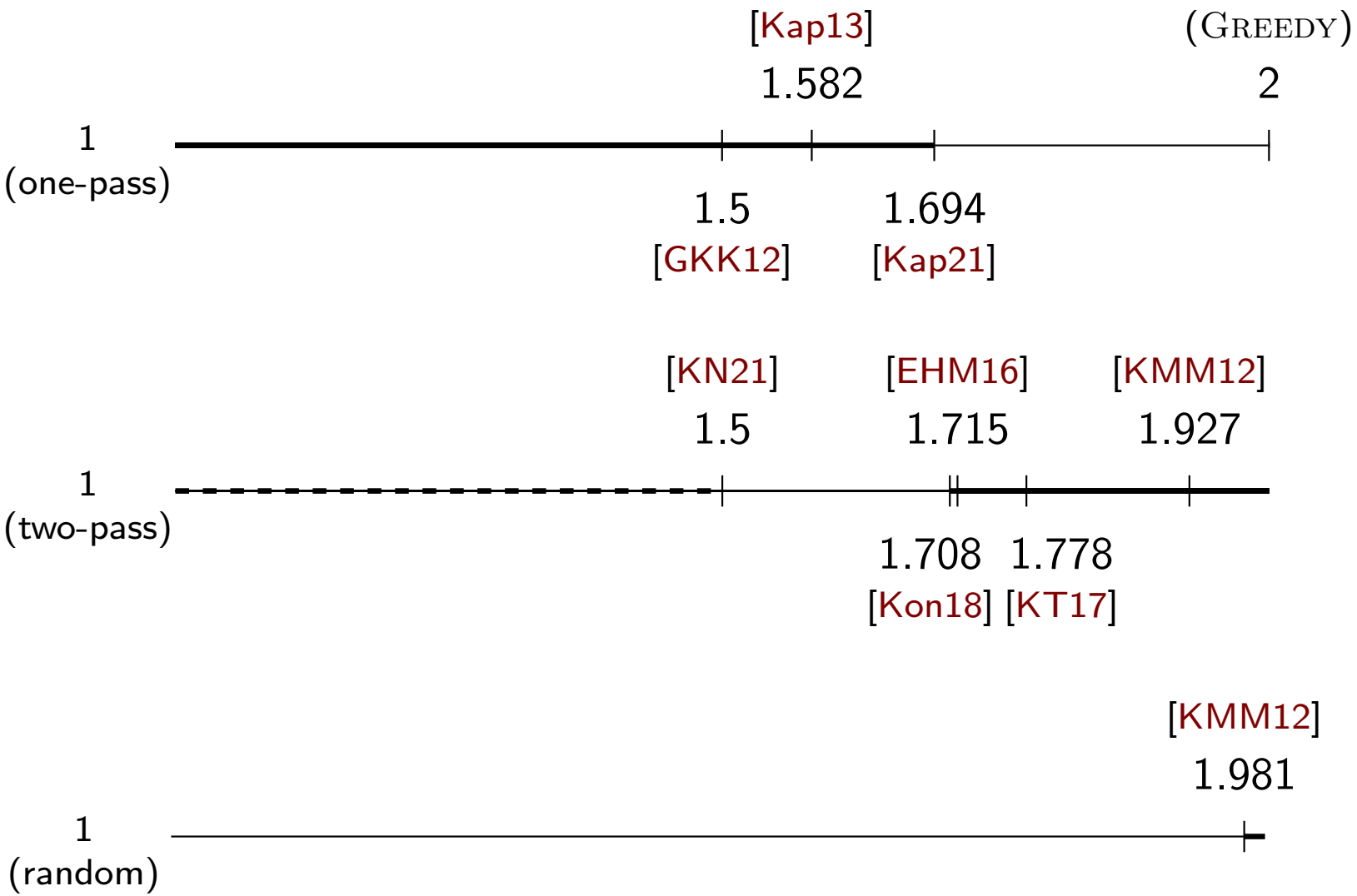
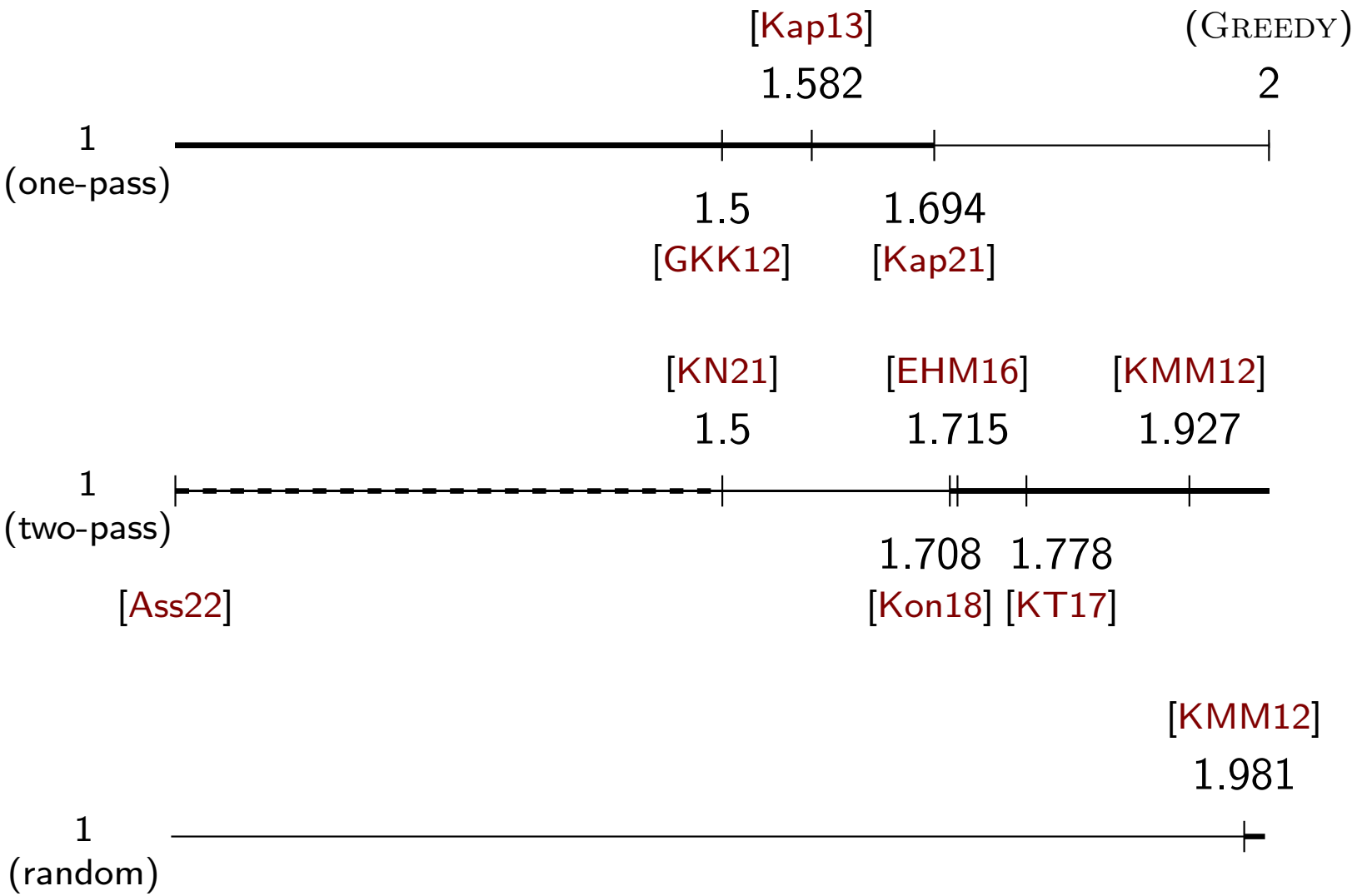


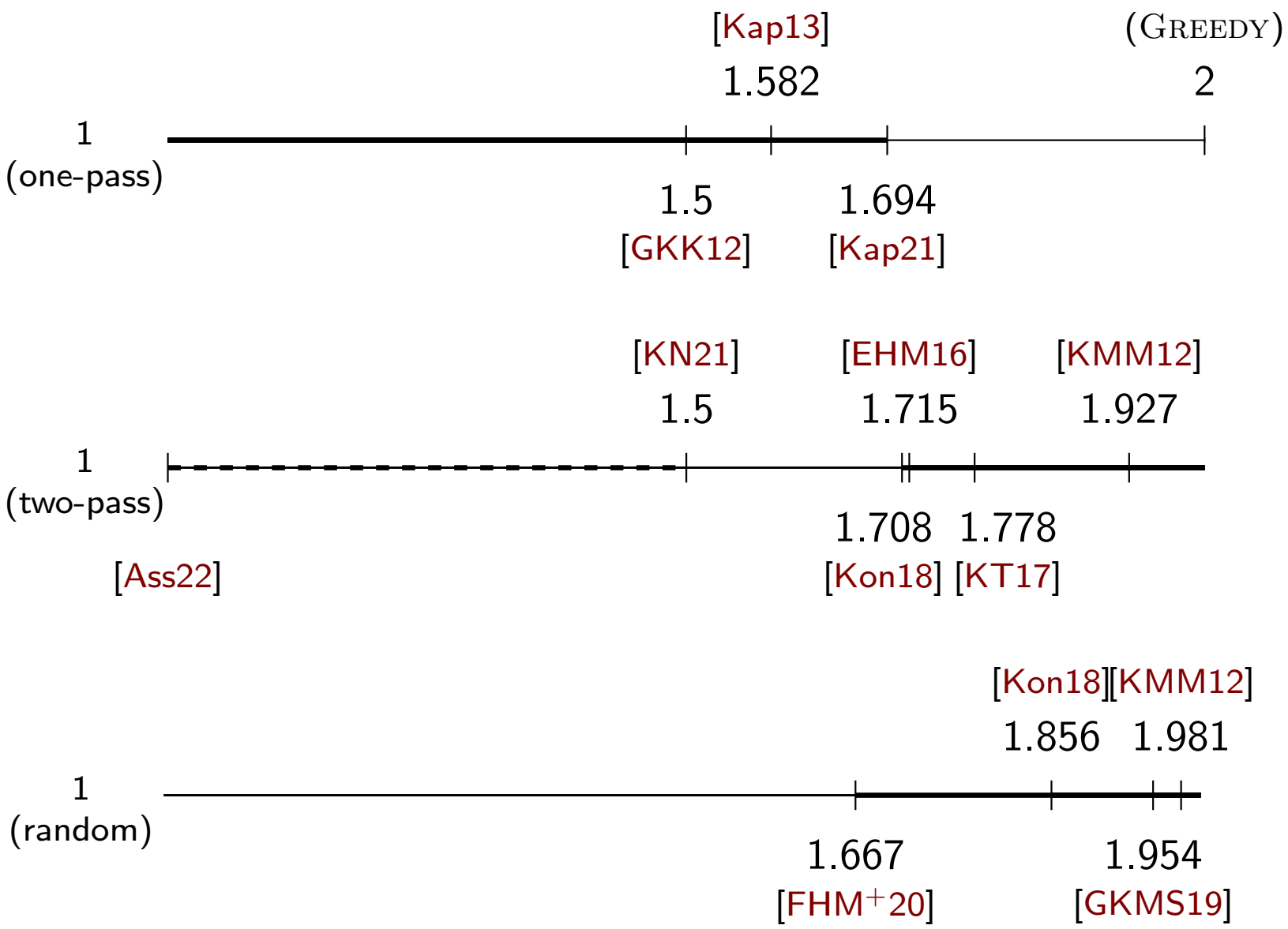
Insertion-Only



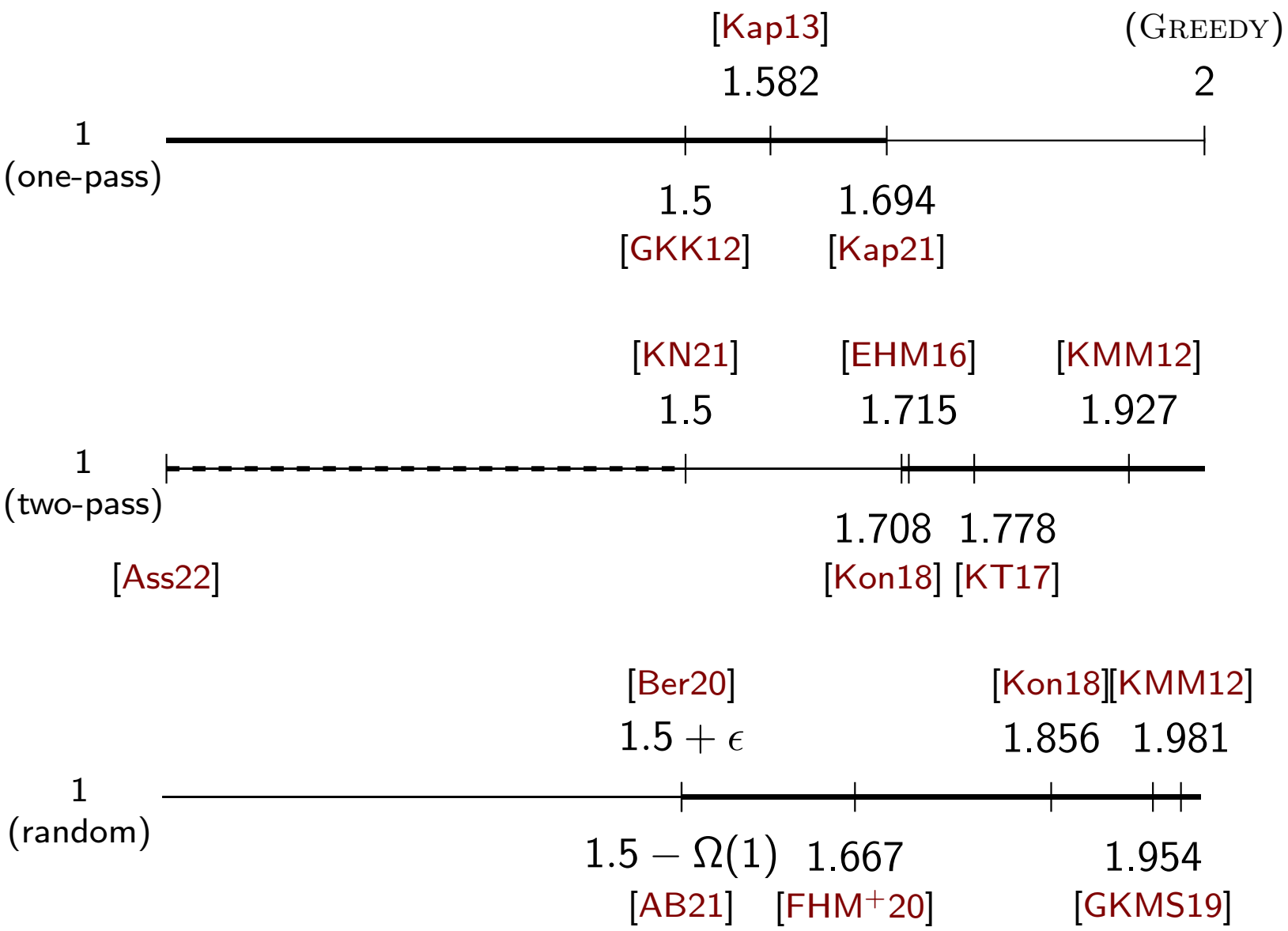
Insertion-Only



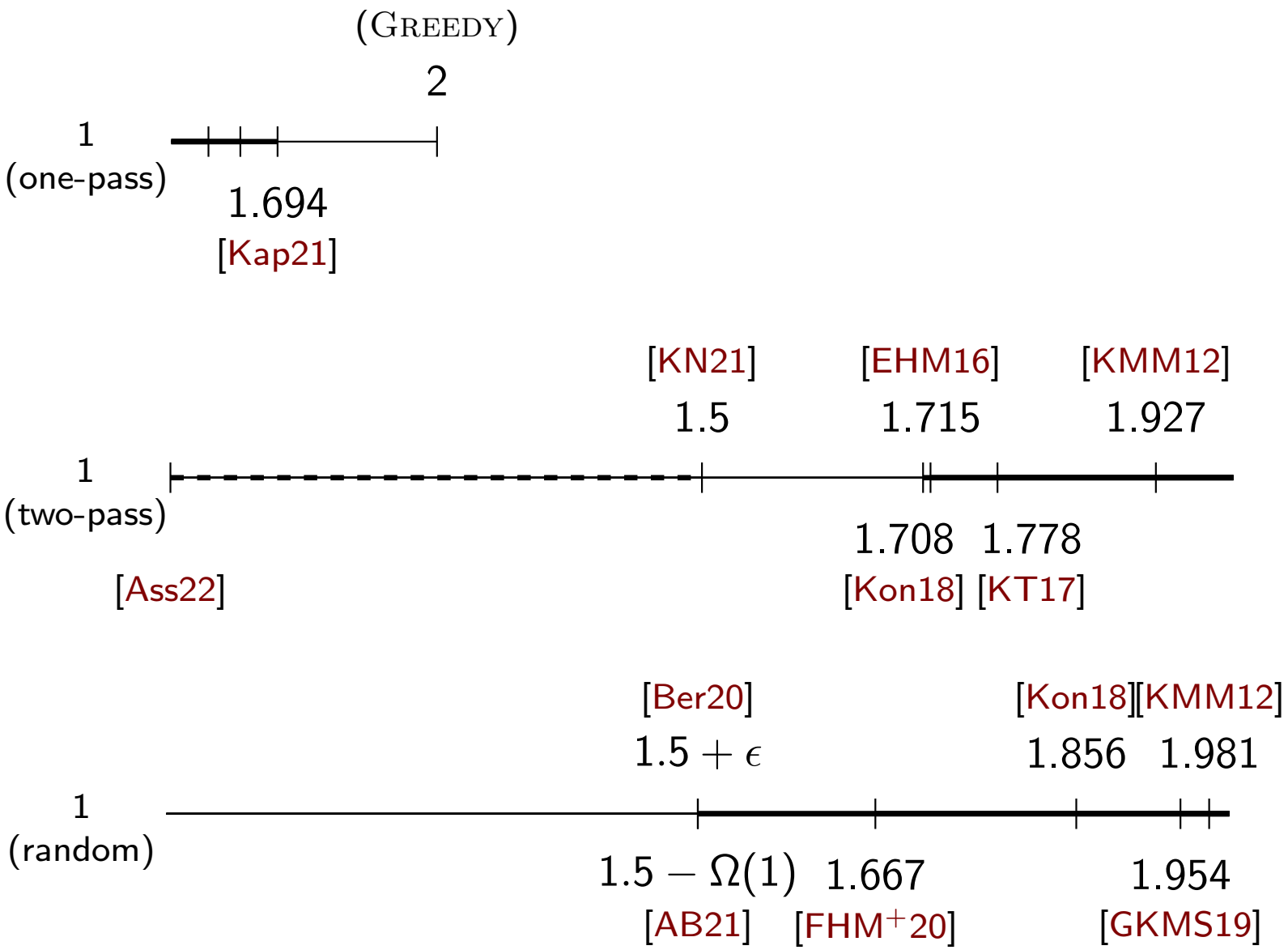
Insertion-Only



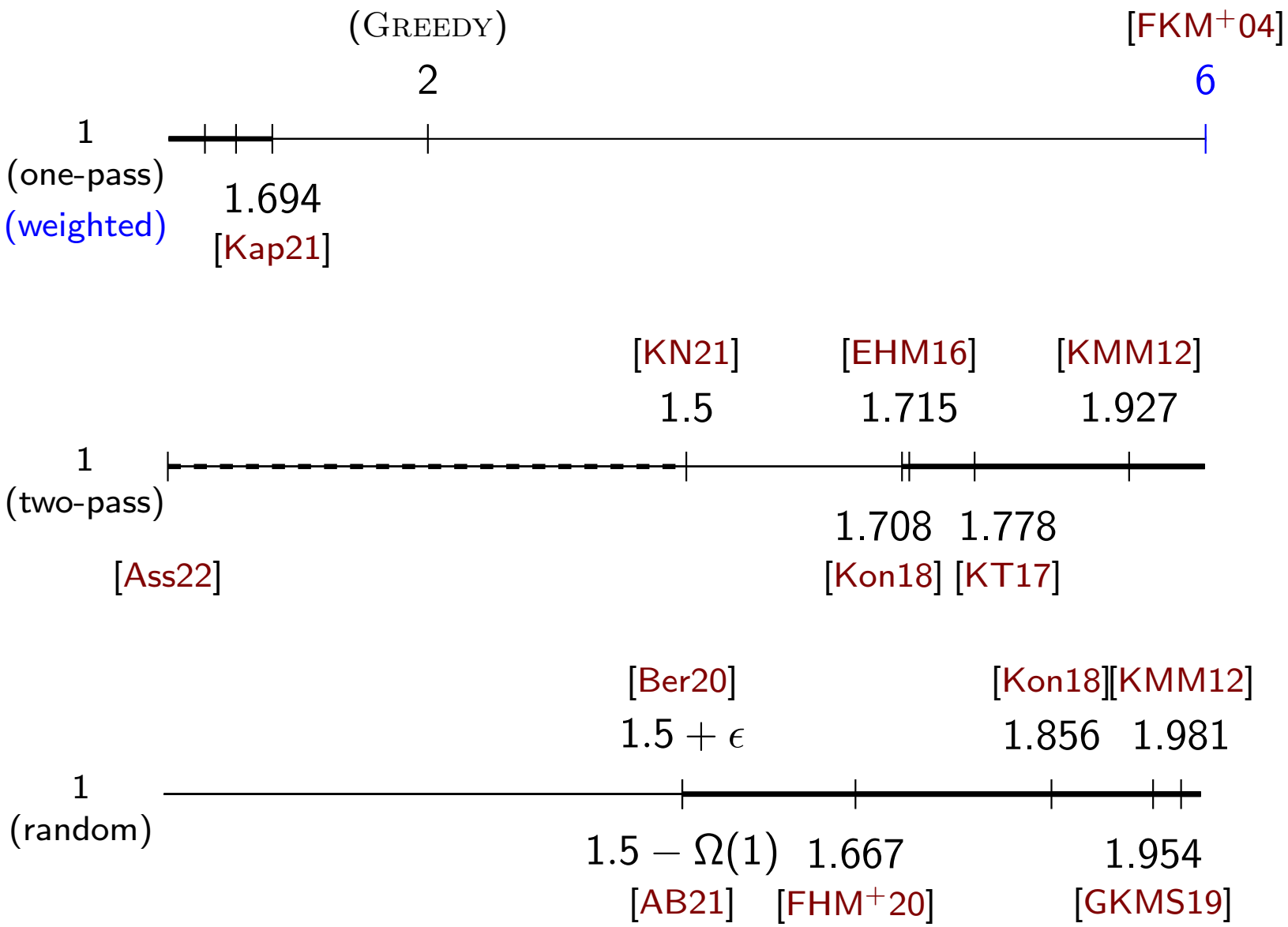
Insertion-Only



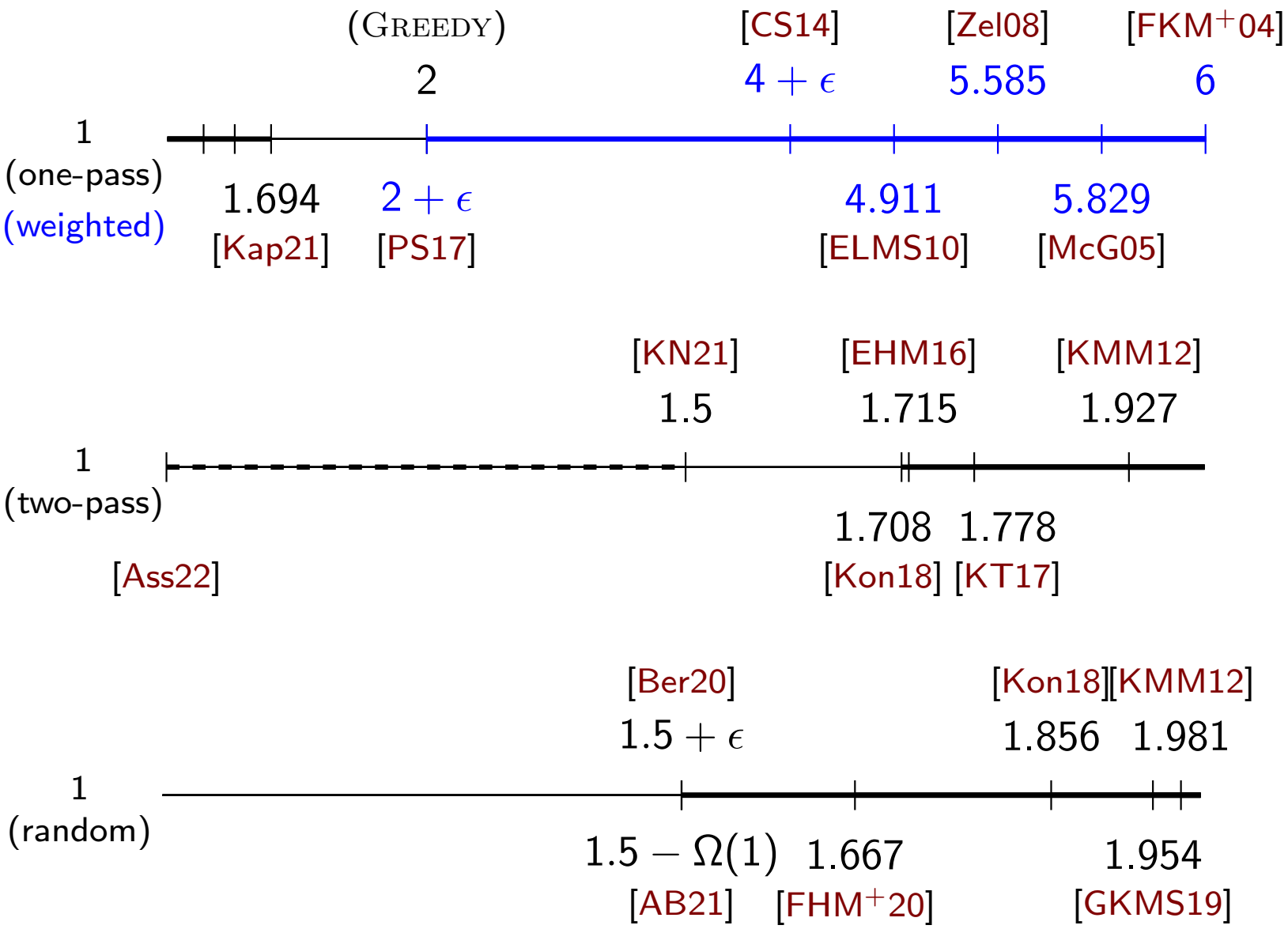
Insertion-Only



Insertion-Only



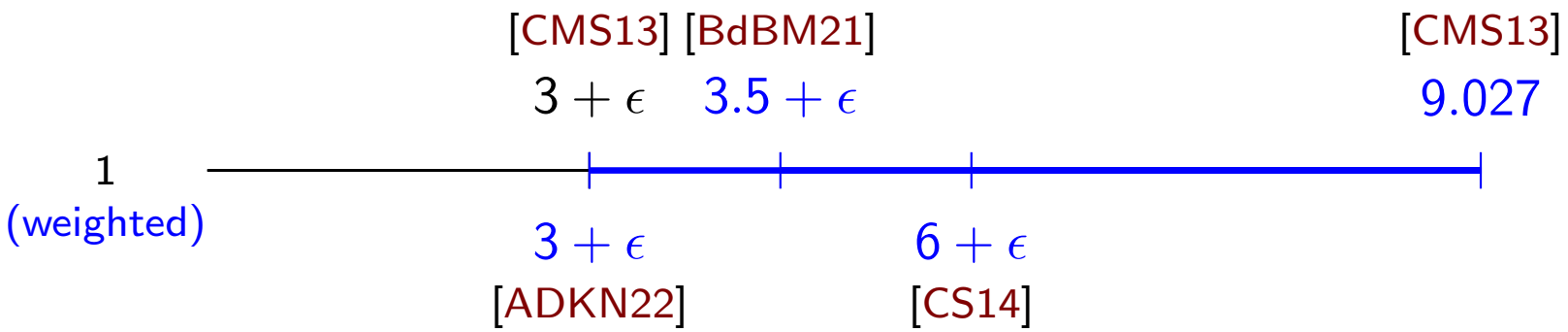
Insertion-Only



Sliding Window

Sliding Window

Sliding Window



Dynamic (Insertion-Deletion)

Goal:

- Find an α-approximation

Dynamic (Insertion-Deletion)

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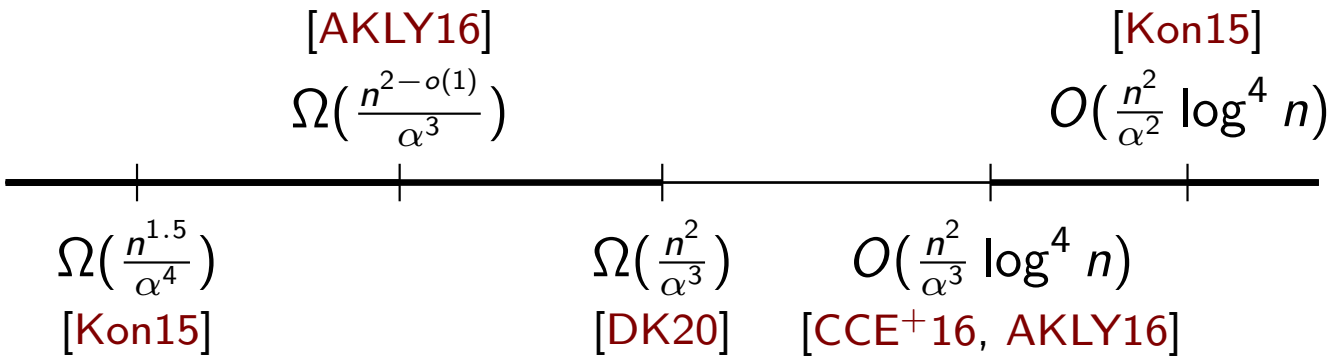
Dynamic (Insertion-Deletion)

Goal:

- Find an α-approximation



Dynamic (Insertion-Deletion)

Goal:

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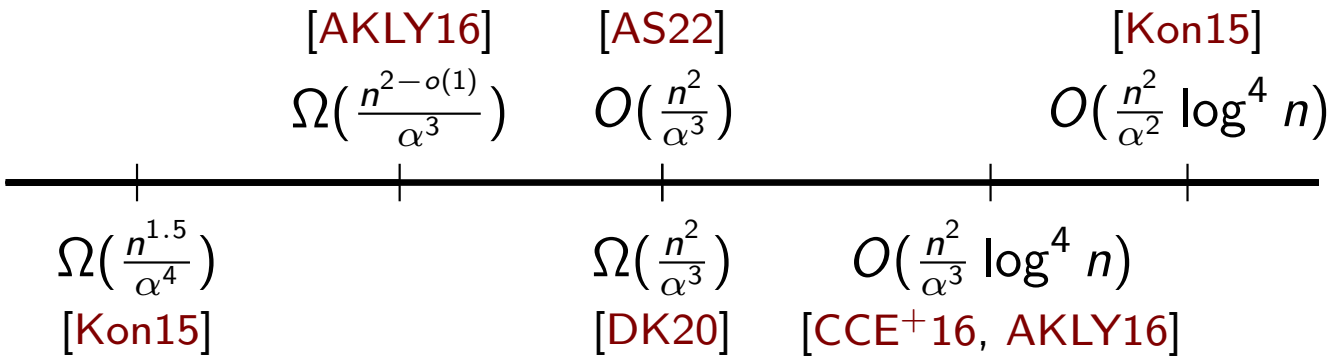
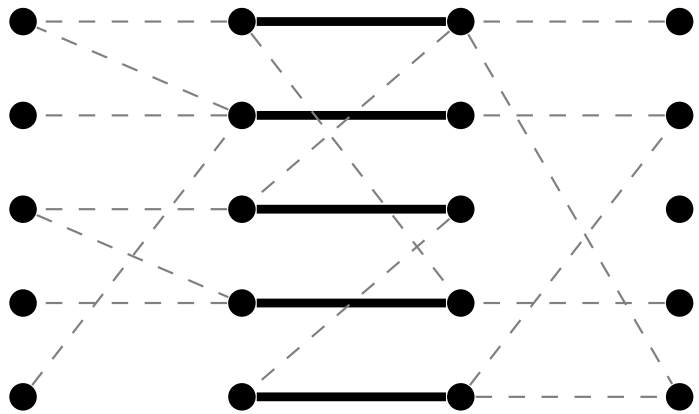


Relation to Vertex Cover

Greedy Matching:

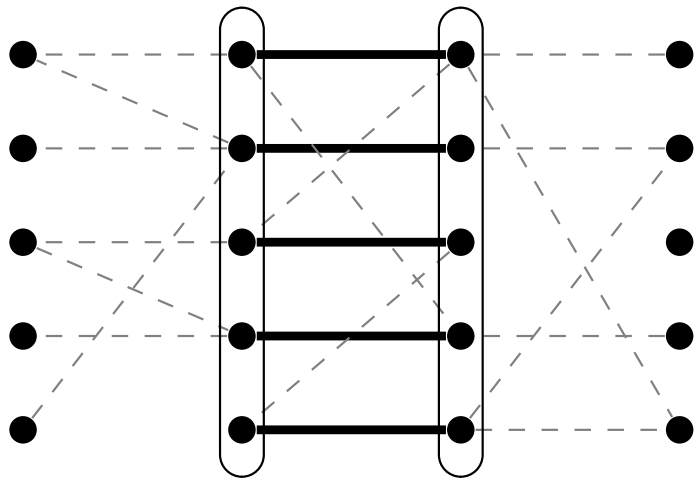
- Maximal
- 2-approximation



Relation to Vertex Cover

Greedy Vertex Cover:

- 2-approximation



Relation to Vertex Cover (Results)

	matching	**vertex cover**
insertion-only	$[1.694, 2]$	
sliding window	$[1, 3+\epsilon]$	
dynamic	$\Theta(\frac{n^2}{\alpha^3})$	

Relation to Vertex Cover (Results)

	matching	**vertex cover**
insertion-only	[1.694, 2]	[1, 2] GREEDY
sliding window	$[1, 3+\epsilon]$	
dynamic	$\Theta(\frac{n^2}{\alpha^3})$	

Relation to Vertex Cover (Results)

	matching	**vertex cover**
insertion-only	$[1.694, 2]$	$[1, 2]$ GREEDY
sliding window	$[1, 3+\epsilon]$	$[1, 3+\epsilon]$ [Sub21]
dynamic	$\Theta(\frac{n^2}{\alpha^3})$	

Relation to Vertex Cover (Results)

	matching	**vertex cover**
insertion-only	$[1.694, 2]$	$[1, 2]$ GREEDY
sliding window	$[1, 3+\epsilon]$	$[1, 3+\epsilon]$ [Sub21]
dynamic	$\Theta(\frac{n^2}{\alpha^3})$	$\Theta(\frac{n^2}{\alpha^2})$ [DK20] & this work

Relation to Vertex Cover (Results)

	matching	**vertex cover**
insertion-only	$[1.694, 2]$	$[1, 2]$ GREEDY
sliding window	$[1, 3+\epsilon]$	$[1, 3+\epsilon]$ [Sub21]
dynamic	$\Theta(\frac{n^2}{\alpha^3})$	$\Theta(\frac{n^2}{\alpha^2})$ [DK20] & this work

All UBs use GREEDY in some way!

What makes dynamic graph streams hard?

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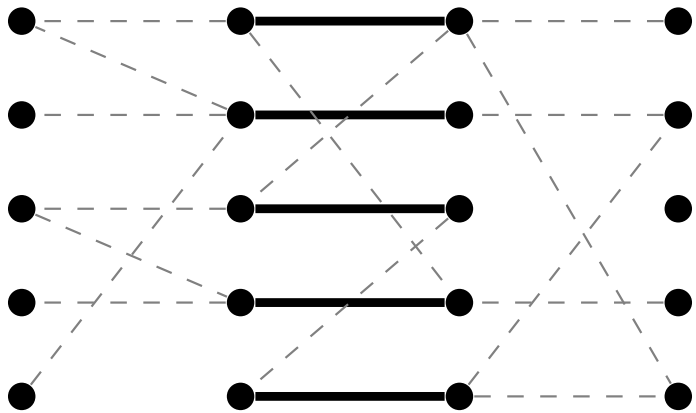
Greedy Matching:

- Maximal
- 2-approximation



What makes dynamic graph streams hard?

Greedy Matching:

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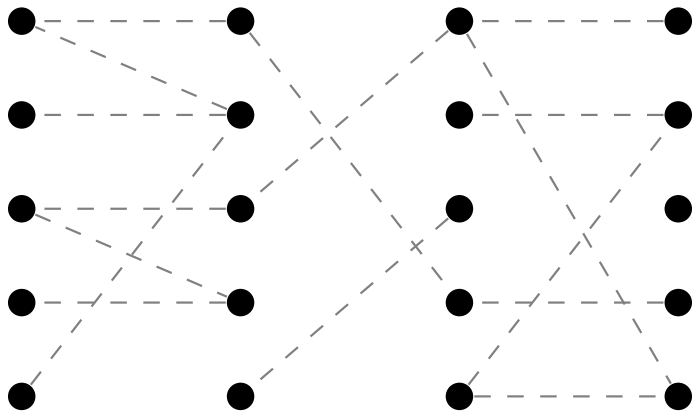

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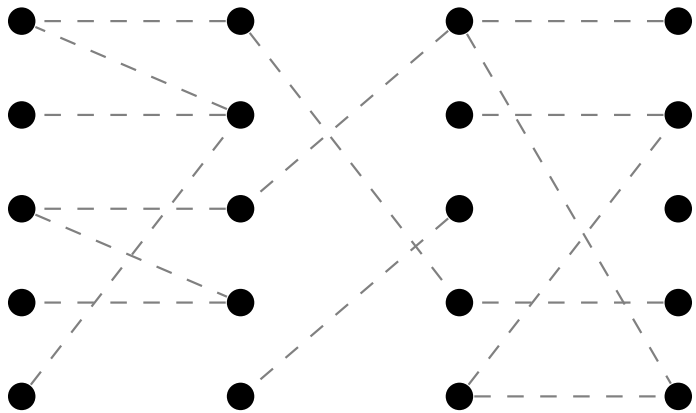
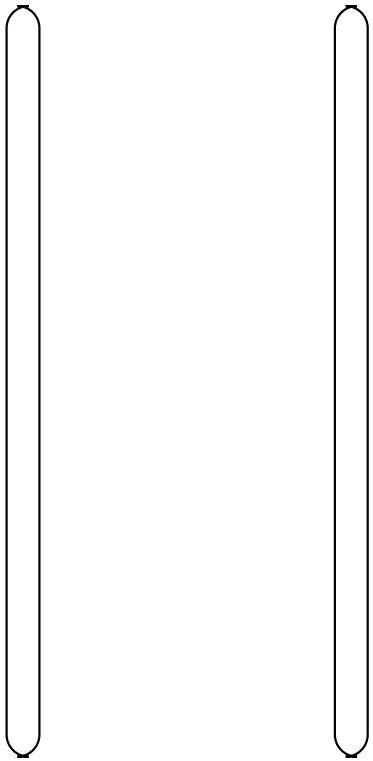


Note: Deterministically returning a single edge requires $\Omega(n^2)$ bits of space for dense graphs.

What techniques are used?



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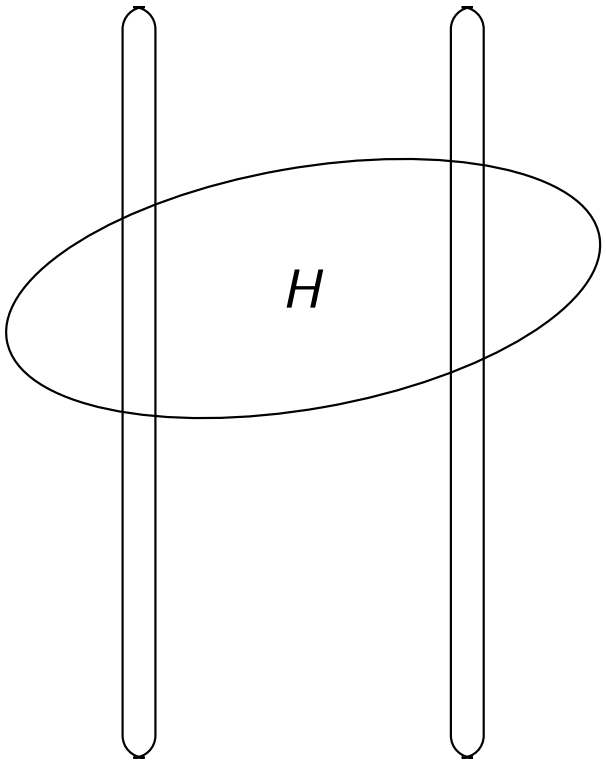

For a subgraph H of G with m edges:

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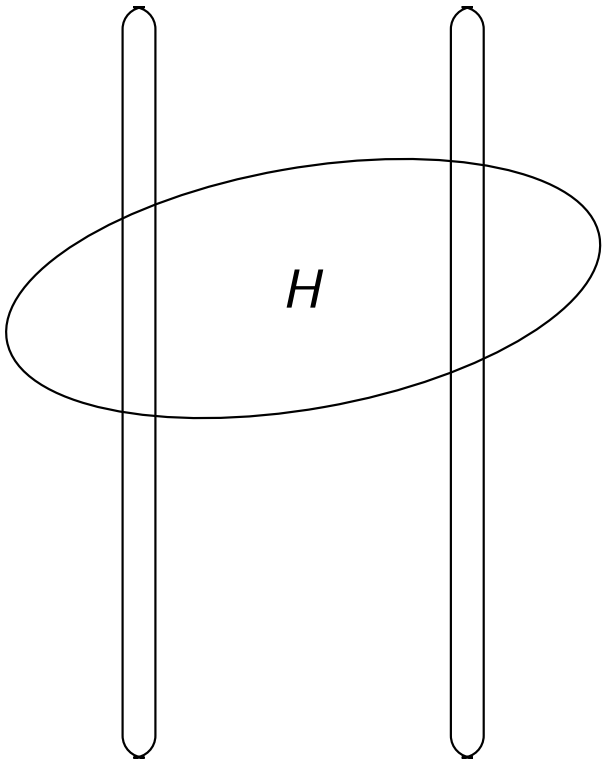

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1. Check if H is empty
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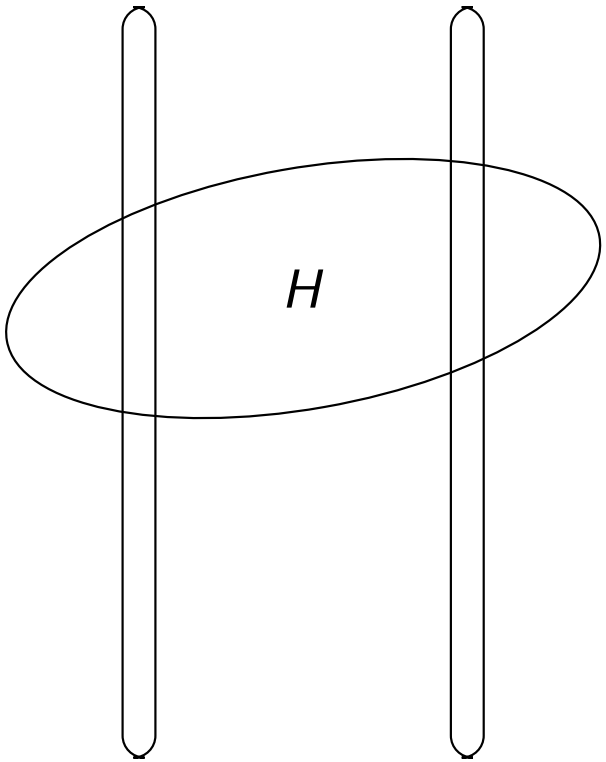

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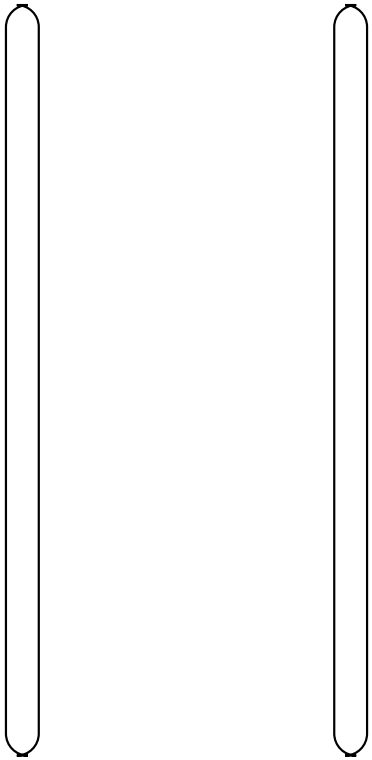
α-Approx Det. Dynamic Vertex Cover [DK20]



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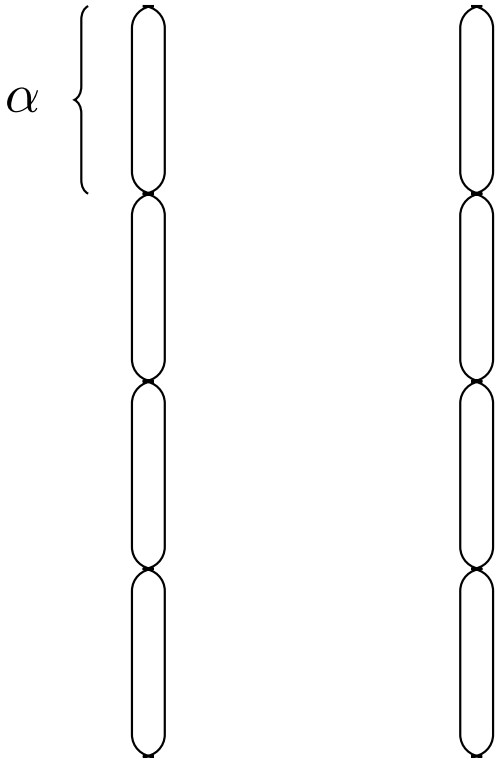

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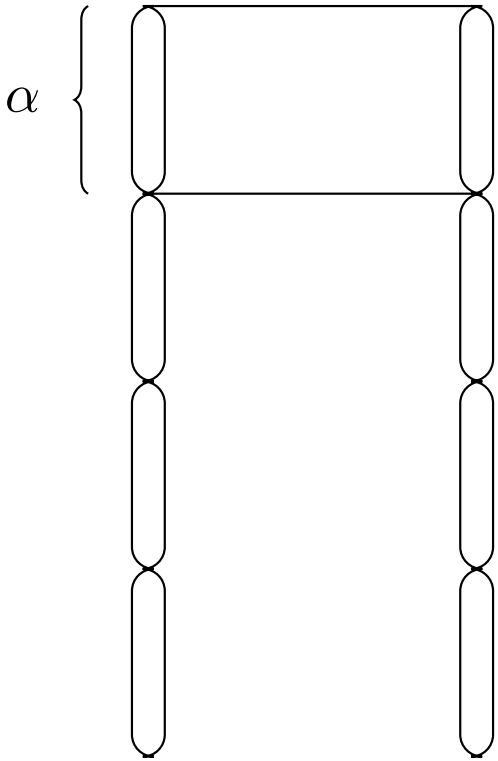

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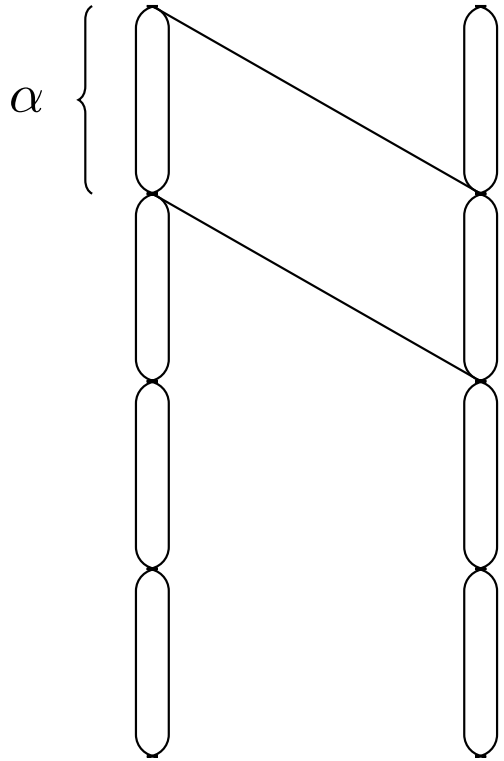

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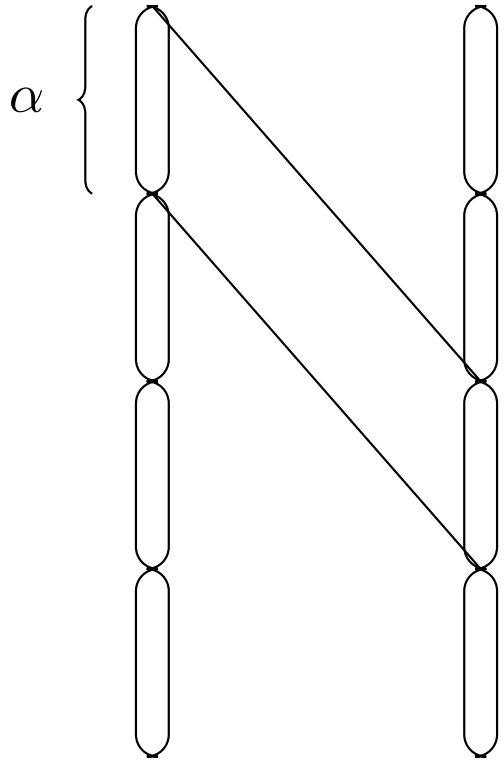

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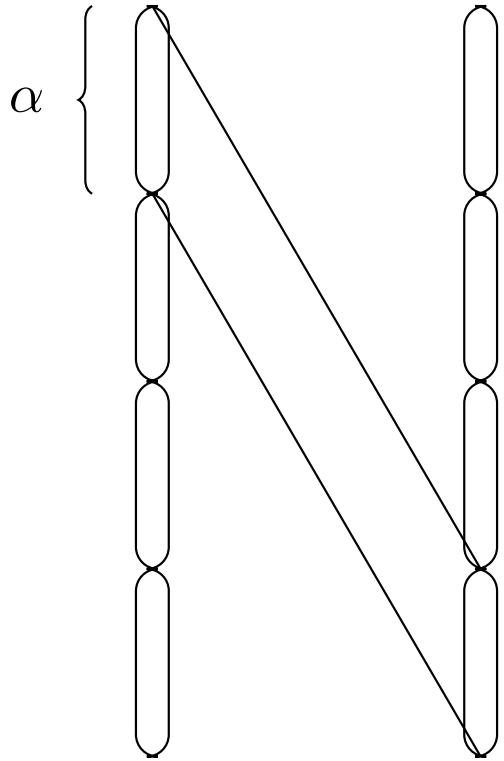

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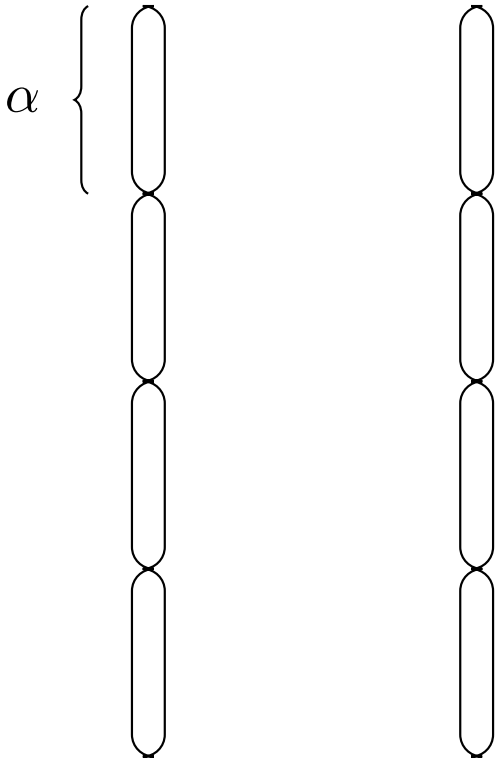

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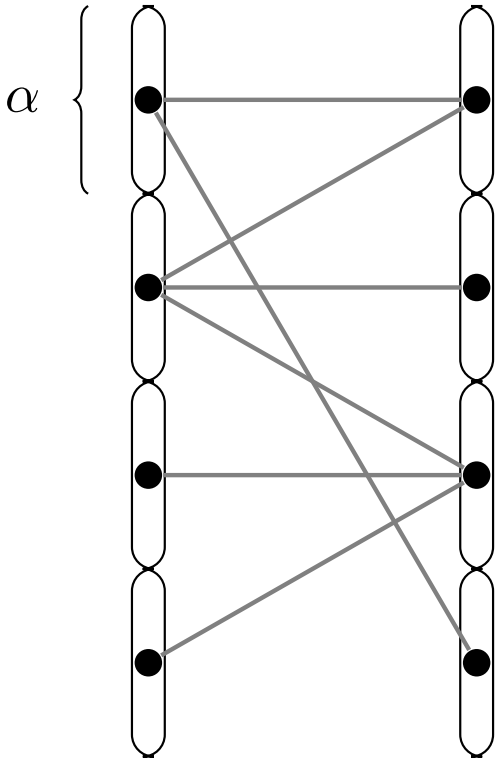

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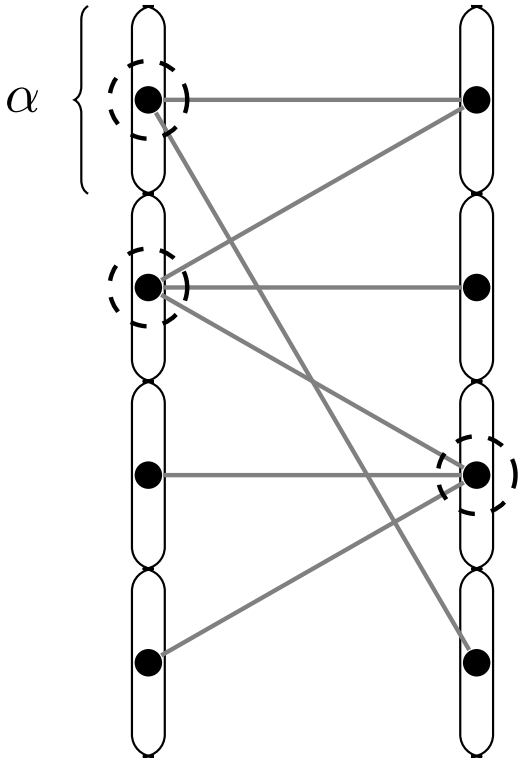

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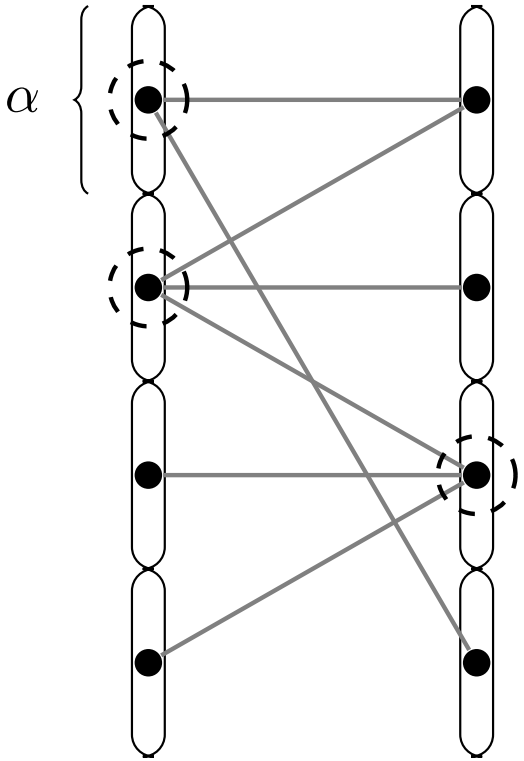

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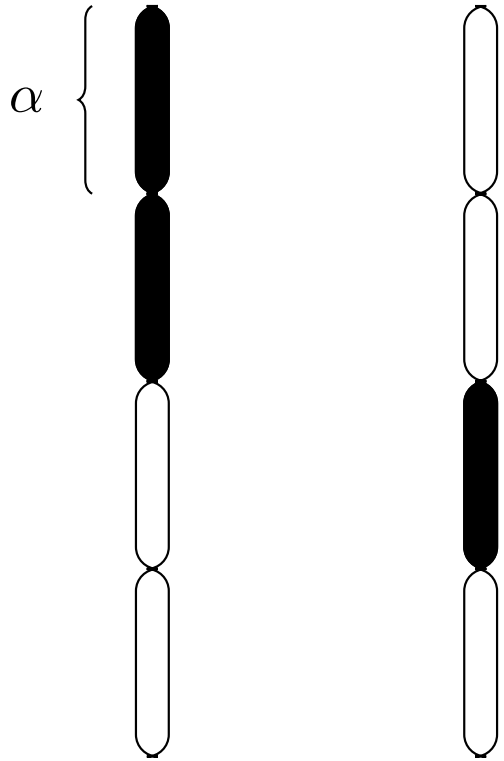

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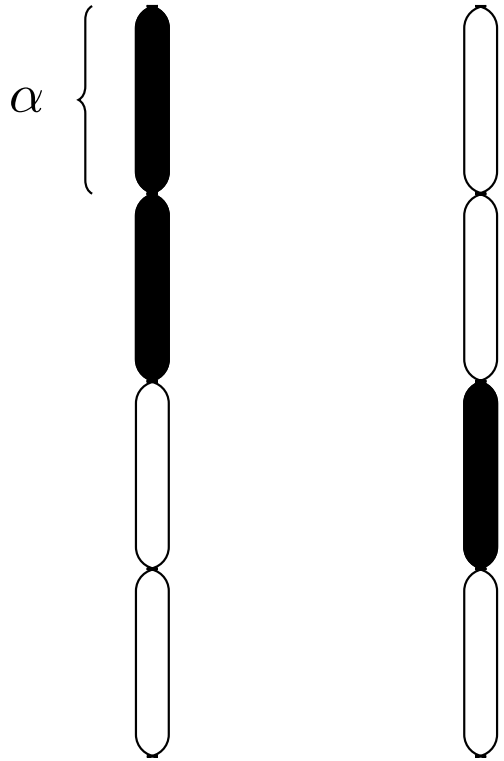

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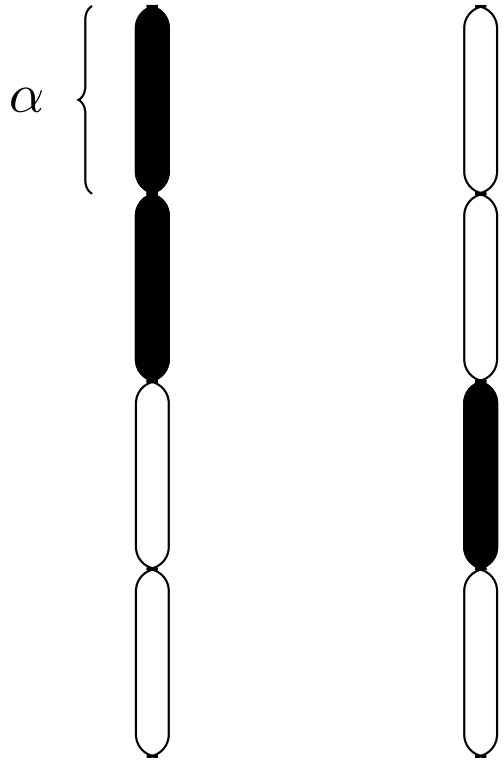

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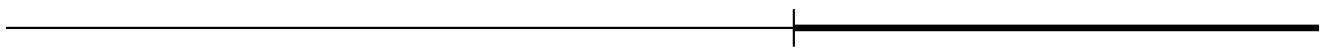

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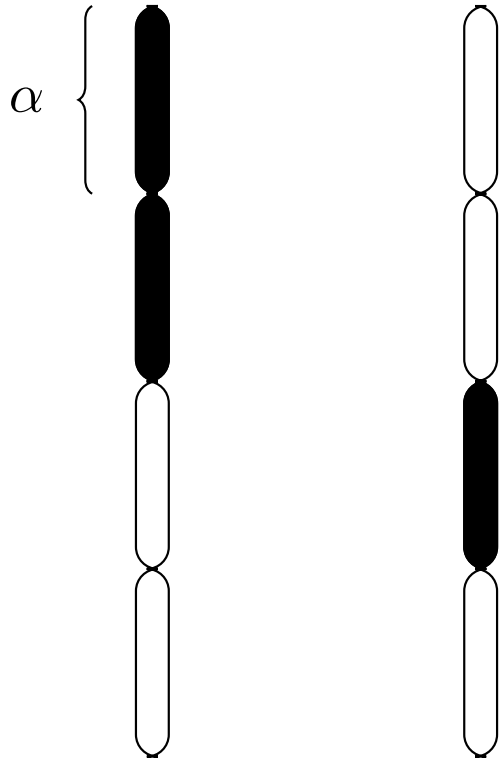

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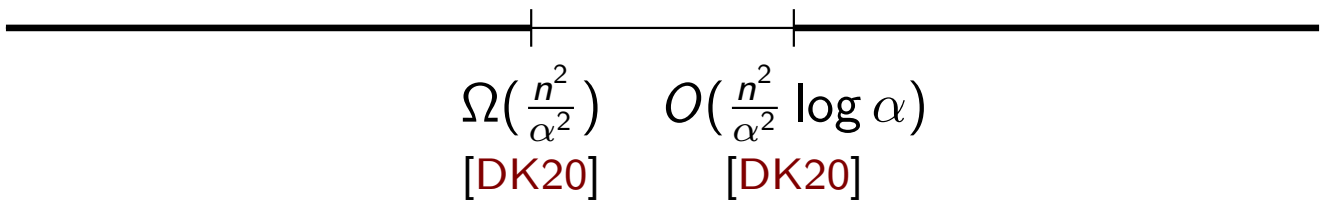

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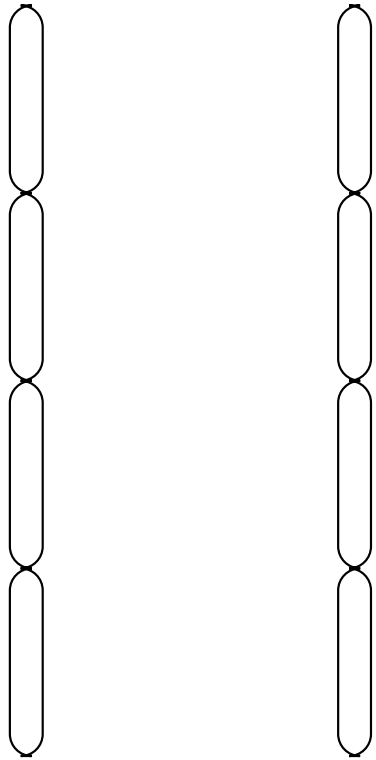

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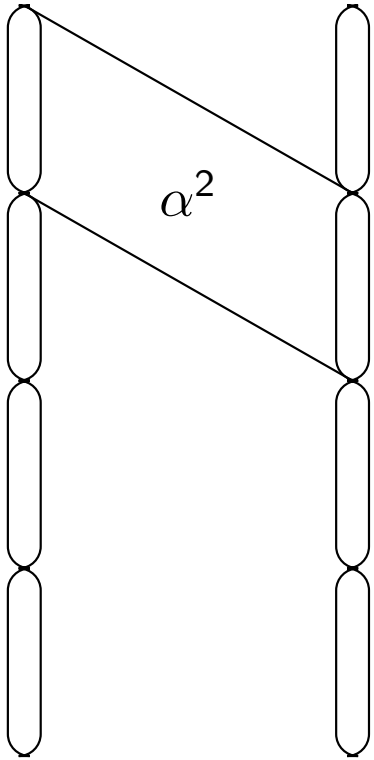

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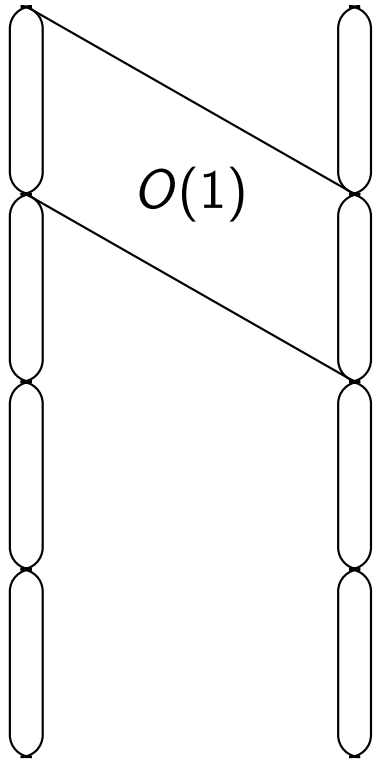

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How?

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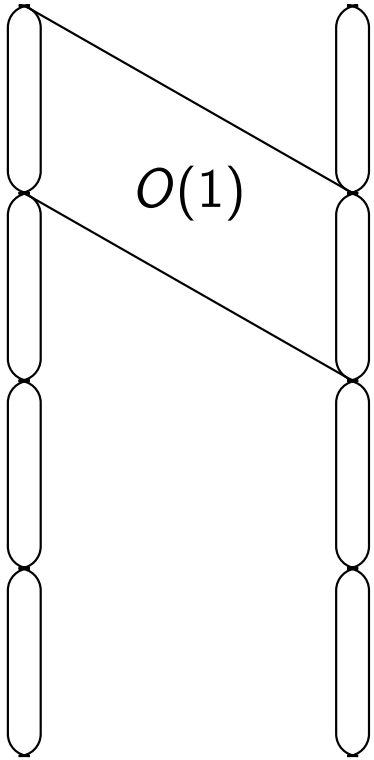
- Randomly partition



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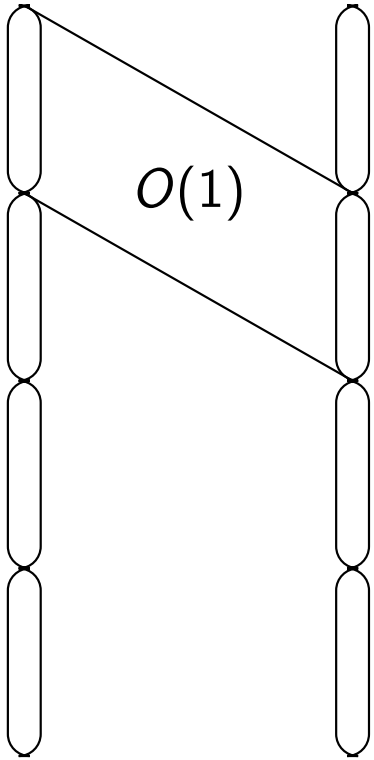

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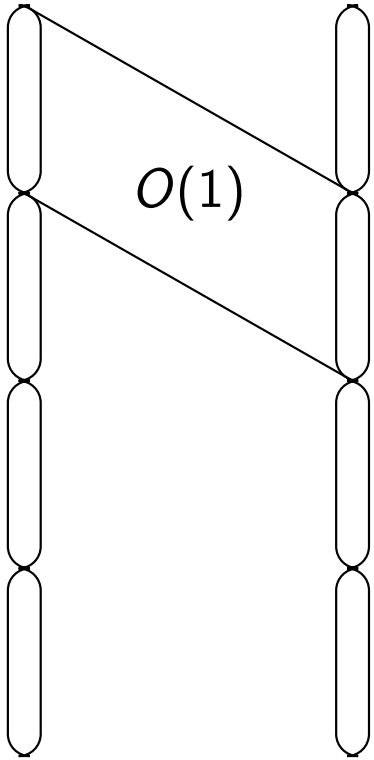

Sparsness properties of Greedy

Lemma

Let G be a n-vertex graph with m edges and let M_s be a Greedy *matching on $s \leq m$ uniform randomly sampled edges. Then, for $G_R = G[V \setminus V(M_s)]$,*

$$\Delta(G_R) \leq \frac{C \cdot m \cdot \log n}{s}.$$

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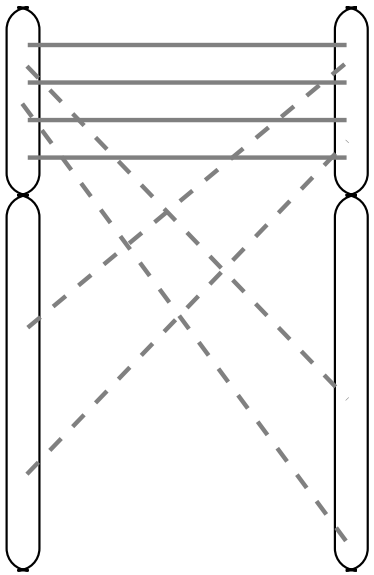

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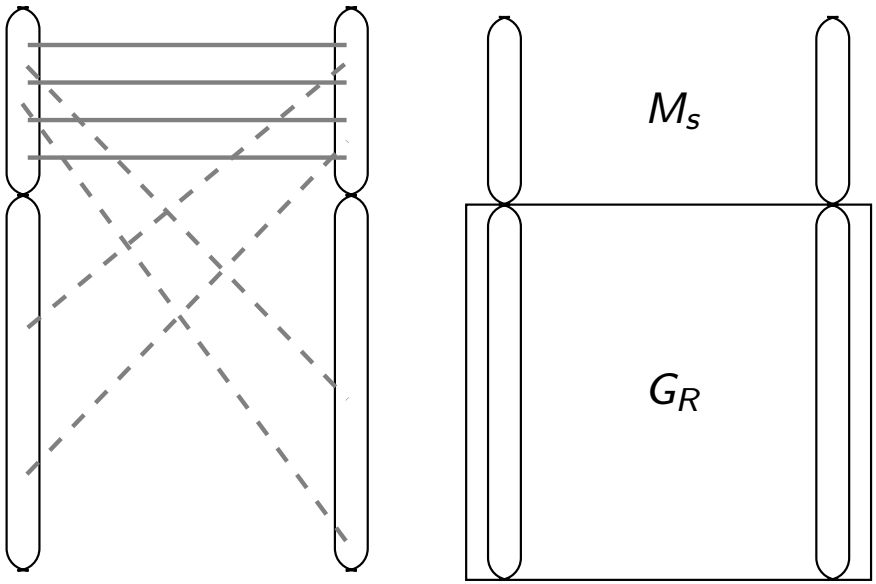

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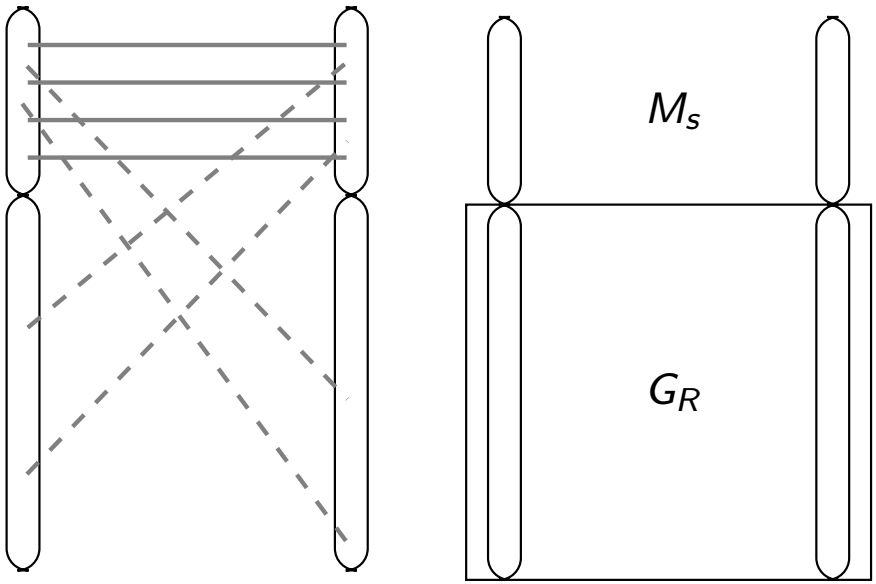


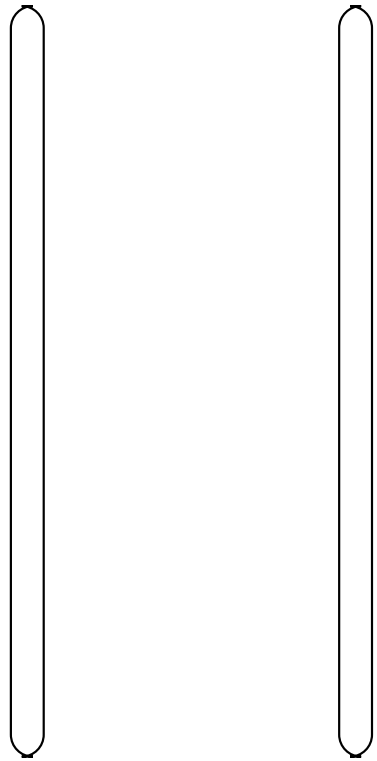
Proof (sketch).

For any vertex $v \in G$,

- $v \in G_R$
- $\deg_{G_R}(v) > \frac{C \cdot m \cdot \log n}{s}$

both do not occur w.h.p. □

Algorithm



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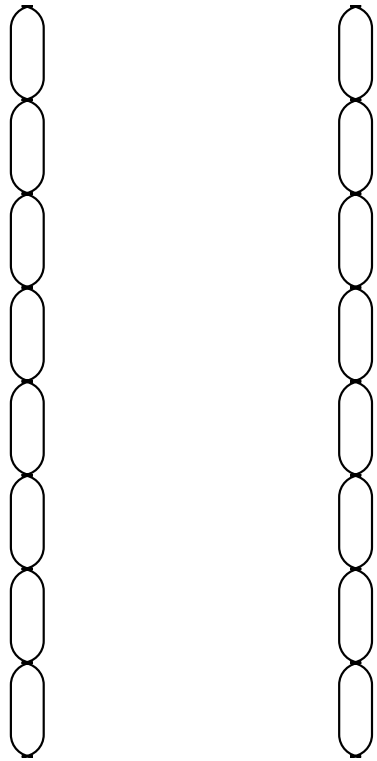

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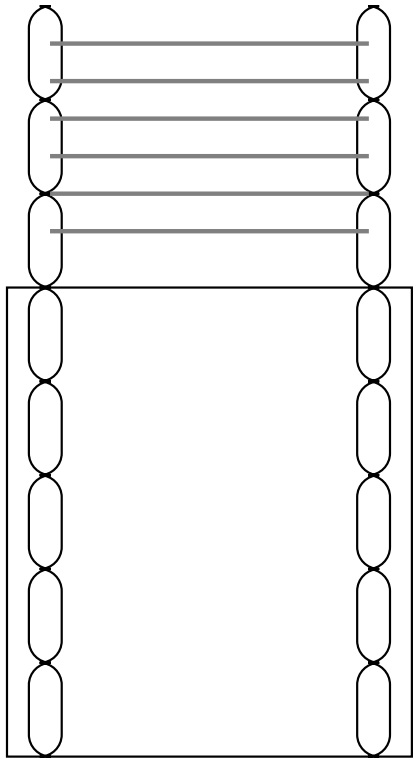

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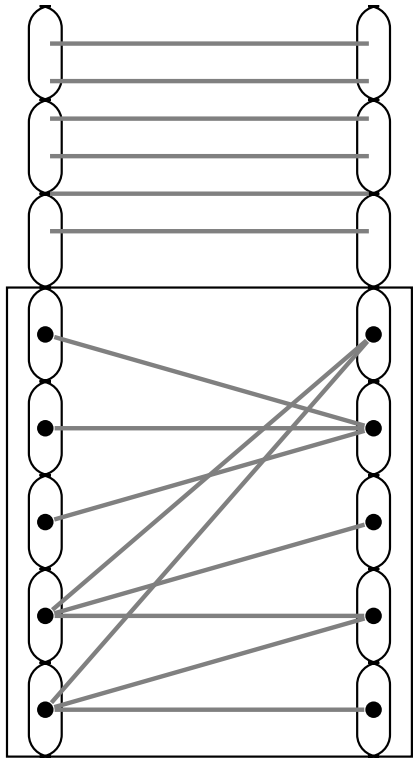
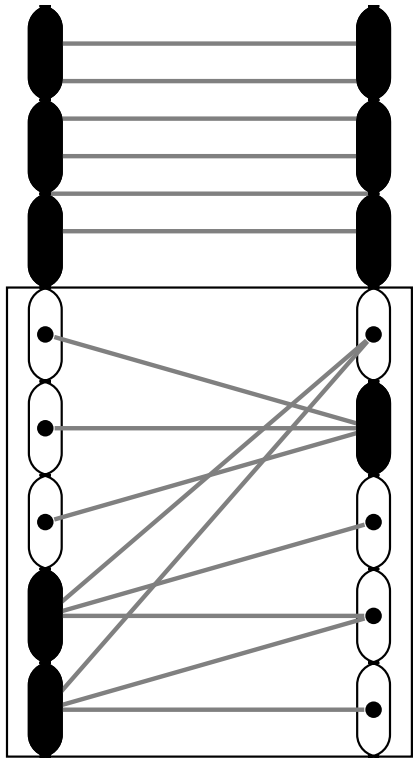
3. Check if an edge is present between pairs and compute group-level vertex cover



Algorithm

1. Randomly partition vertices ($\frac{n}{\alpha}$ groups)
2. Sample $O(\frac{n^2}{\alpha^2 \log^3 n})$ edges randomly and run GREEDY on them to get M
3. Check if an edge is present between pairs and compute group-level vertex cover
4. Return vertices of the covering groups including those with matched vertices



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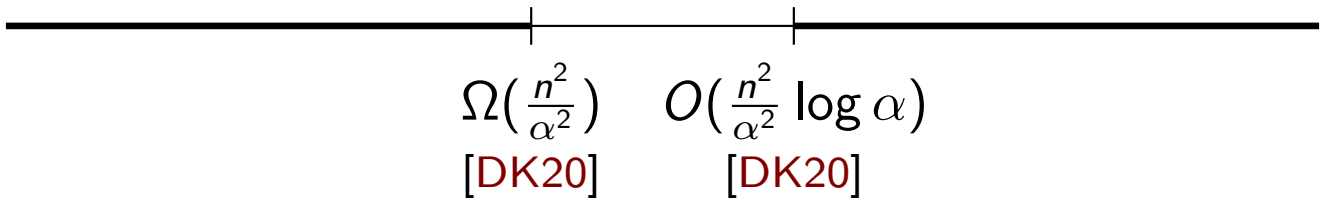

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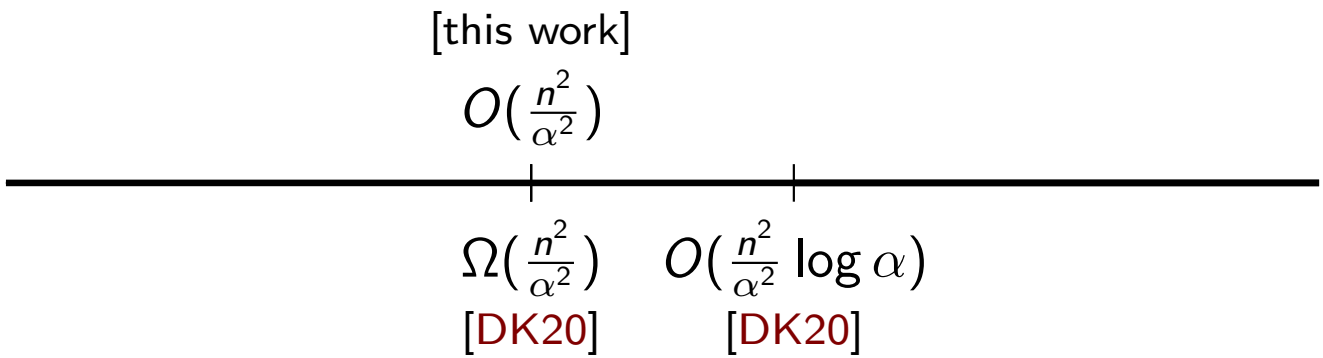

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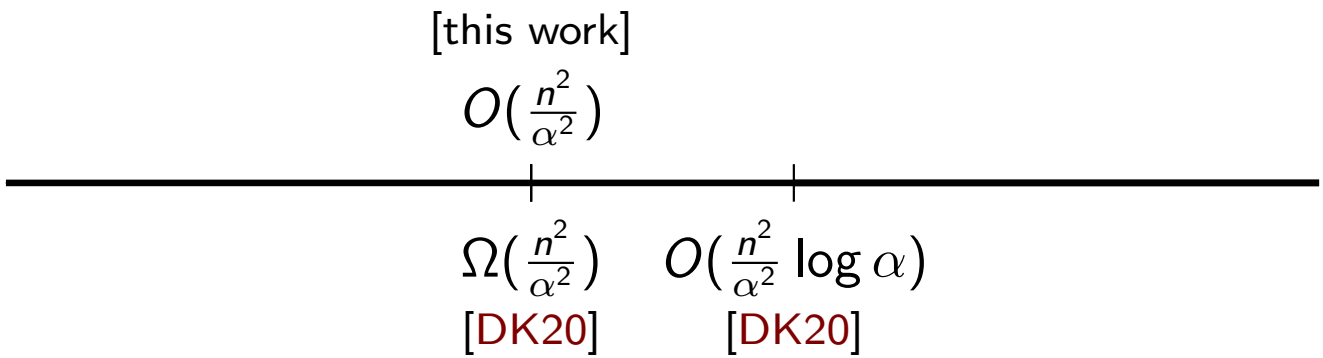

Open Questions:

- Deterministic techniques or LB instead
- Other problems like dominating set and spectral sparsification

Thank You

Questions?

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