Multi-Pass Graph Streaming Lower Bounds

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Based on joint works with Sepehr Assadi, Christian Konrad, Janani Sundaresan

Insertion-Only Graph Streaming Model

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A $(2n)$-vertex graph is presented as a sequence of edges to an algorithm

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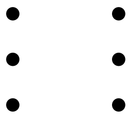

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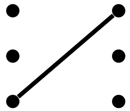
$\boxed{e_1}$



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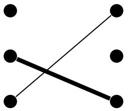

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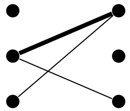

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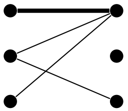
e_1	e_2	e_3	e_4



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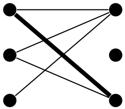

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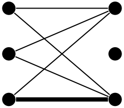

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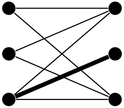

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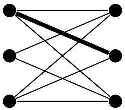

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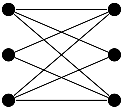

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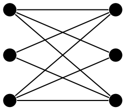

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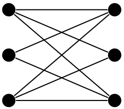

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Interesting Algorithms

- Use $O(n \operatorname{polylog} n)$ space (semi-streaming).
  - Many graph problems require $\Omega(n)$ space in one pass [FKM+04].
- Use one or more passes of the stream.

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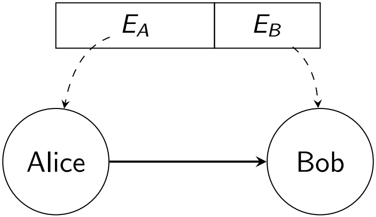

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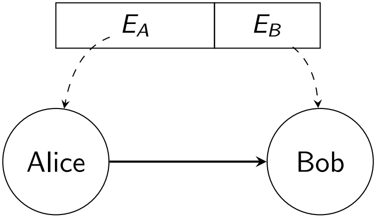

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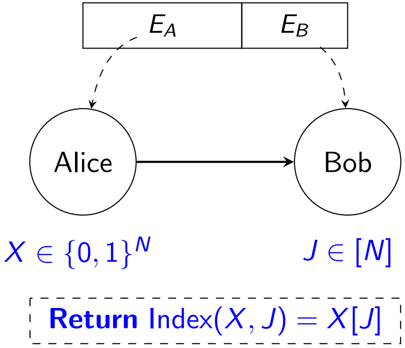

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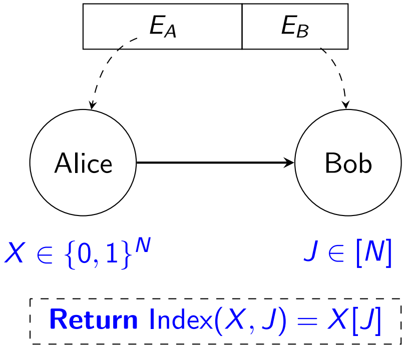

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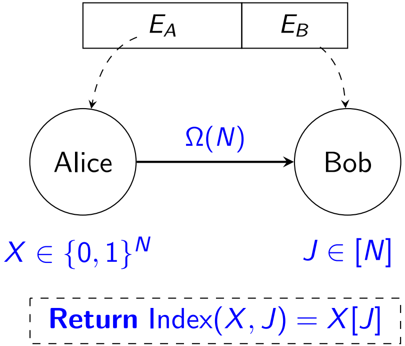

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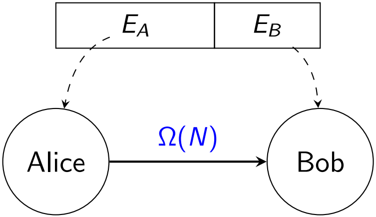

$X \in \{0,1\}^N$ $\quad$ $J \in [N]$

Return $\mathrm{Index}(X, J) = X[J]$

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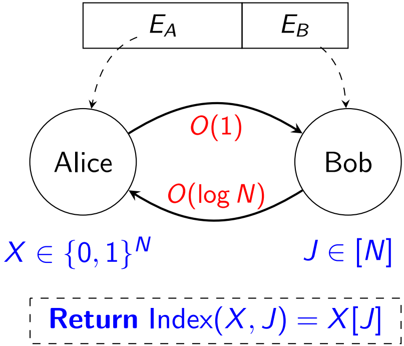

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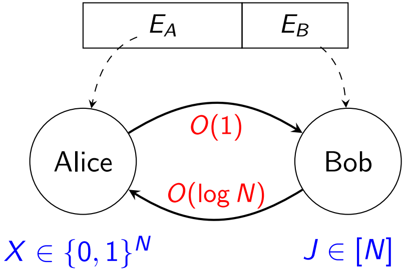

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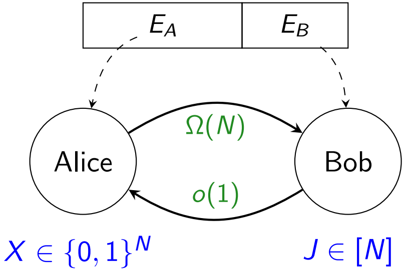

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2-Pass Streaming Lower Bounds (Our Results)

$\mathsf{HiddenStrings}_2$

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For $\varepsilon > 0$, any constant-error two-pass $(\frac{8}{9} + \varepsilon)$-approximation streaming algorithm for MBM requires $n^{1+\Omega(1/(\log\log n)^2)}$ space.

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Maximal Independent Set (MIS)

Any constant-error two-pass streaming algorithm for MIS requires $\Omega(n^{4/3-o(1)})$ space.

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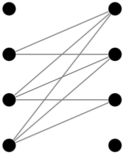
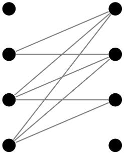
A bipartite **matching** is a subset of vertex disjoint edges of the graph. A **maximum matching** is a largest matching of the graph.





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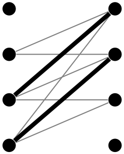
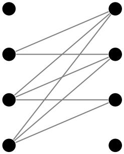


M
matching



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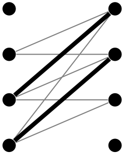

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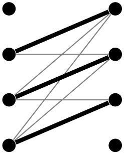

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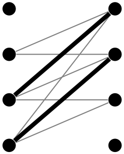

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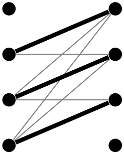

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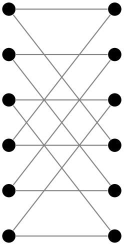
Approximations

- M is a $(\frac{|M|}{|M^*|})$-approximate matching (e.g. $\frac{2}{3}$).

Ruzsa-Szeméredi (RS) Graphs



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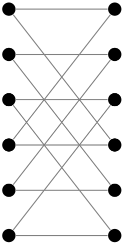
Definition (Induced Matching)

A matching whose vertex induced subgraph contains only its edges.



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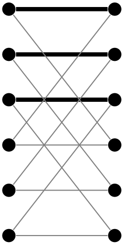

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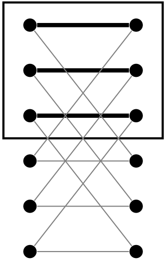

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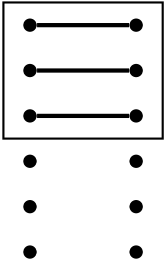

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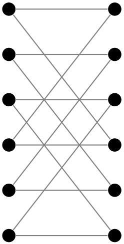

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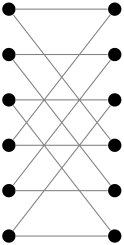

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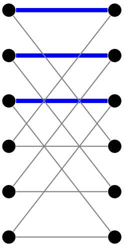

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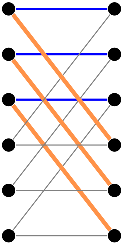

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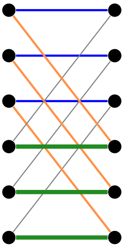

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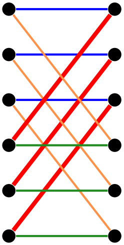

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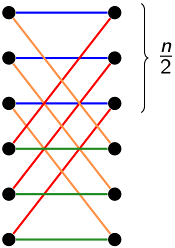

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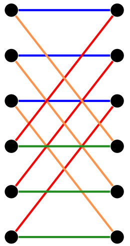

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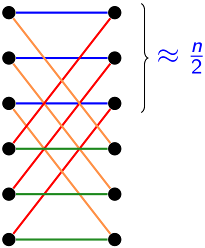

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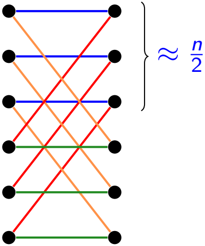
A matching whose vertex induced subgraph contains only its edges.

Definition ((r, t)-RS Graph)

A graph whose edges set is the union of t many edge-disjoint induced matchings of size r.

Proposition ([GKK12] (see also [FLN$^+$02]))

For constant $\delta > 0$, there exists a (bipartite) (r, t)-RS graph on $2n$ vertices where $r = (\frac{1}{2} - \delta) \cdot n$ and $t = n^{\Omega(1/\log\log n)}$.



- $n^{\Omega(1/\log\log n)}$ many induced matchings.

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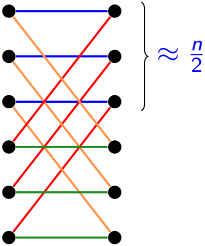
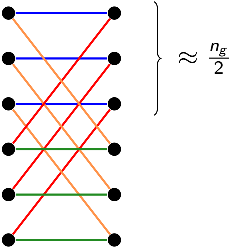


- $n^{\Omega(1/\log\log n)}$ many induced matchings.
- $\gg n \operatorname{polylog} n$ edges in the graph.

Two-Pass Hard Graph and Adversarial Stream

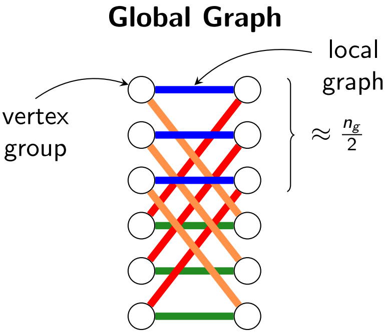
Two-Pass Hard Graph and Adversarial Stream

Global Graph



- $(2n_g)$-vertex (r_g, t_g)-RS graph.

Two-Pass Hard Graph and Adversarial Stream



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Two-Pass Hard Graph and Adversarial Stream

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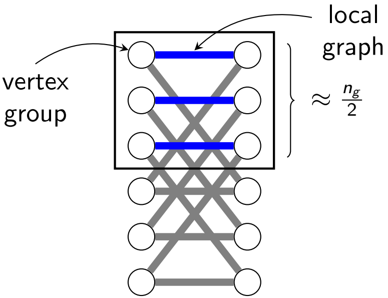

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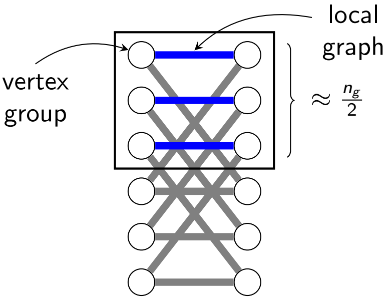

- $(2n_g)$-vertex (r_g, t_g)-RS graph.
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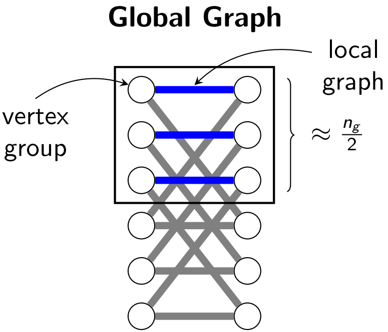

Q: What is the selector for MBM?

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Two-Pass Hard Graph and Adversarial Stream

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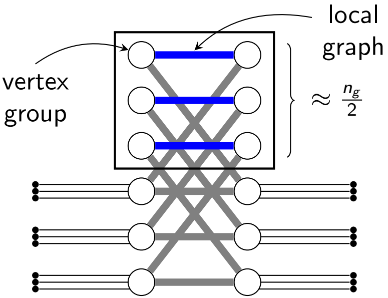

Q: What is the selector for MBM?

- $(2n_g)$-vertex (r_g, t_g)-RS graph.
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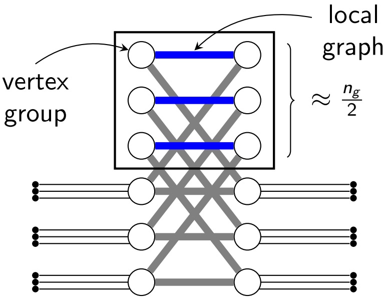
- $1/3^{\text{rd}}$ of the vertices are special.

Two-Pass Hard Graph and Adversarial Stream

Global Graph



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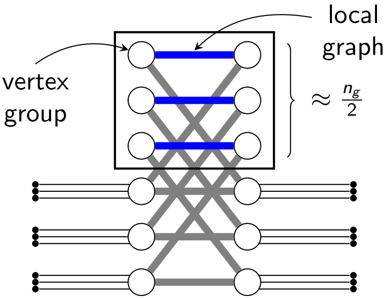

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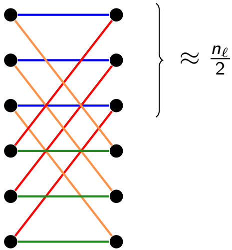

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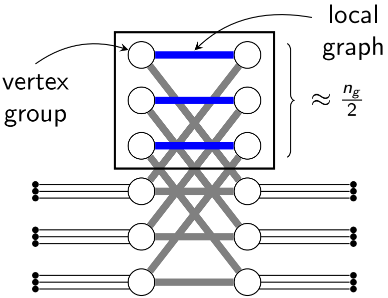

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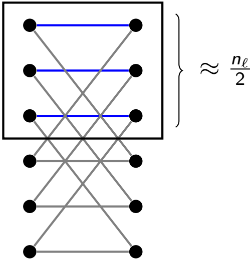

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Two-Pass Hard Graph and Adversarial Stream

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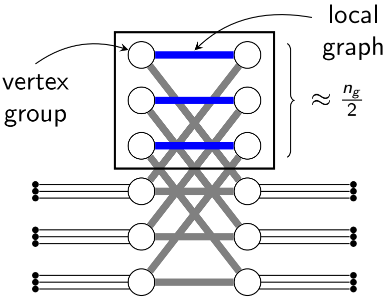

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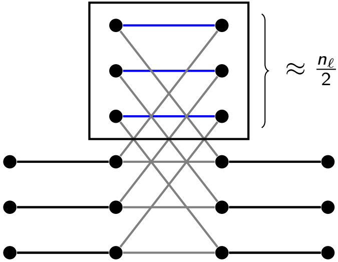

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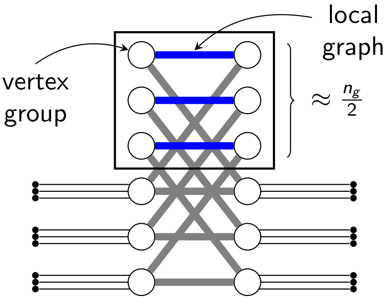

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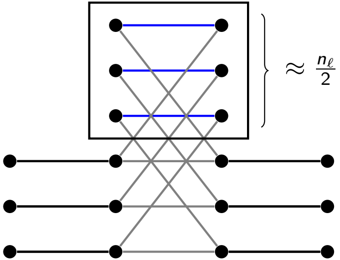

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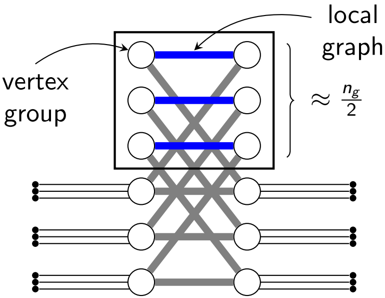
Two-Pass Hard Graph and Adversarial Stream

Global Graph



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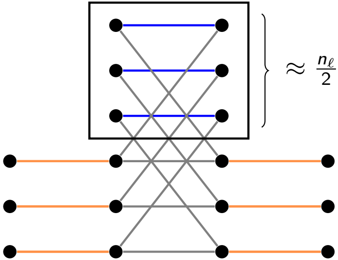

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Two-Pass Hard Graph and Adversarial Stream

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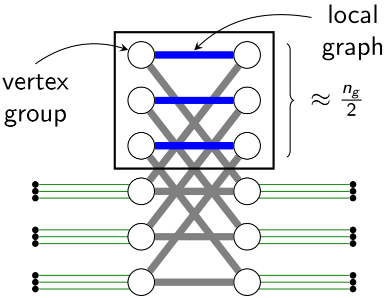

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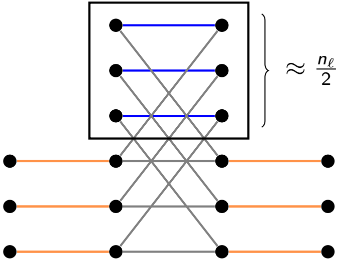

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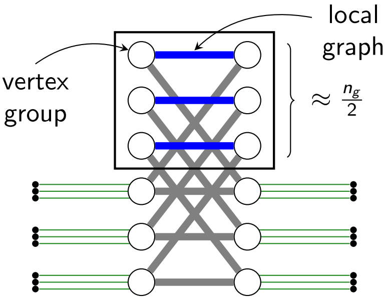

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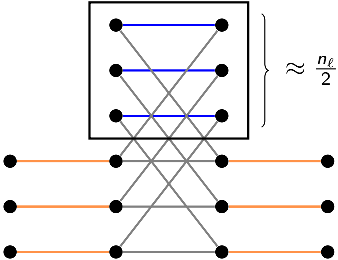

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local RS-graphs	local selectors	global selector

Two-Pass Hard Graph and Adversarial Stream

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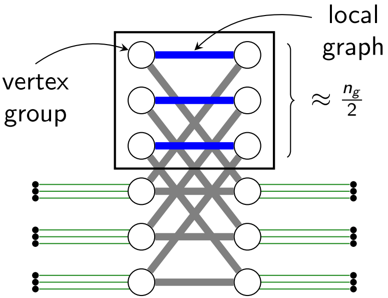

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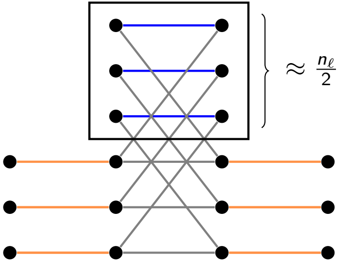

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$\mathcal{A}$

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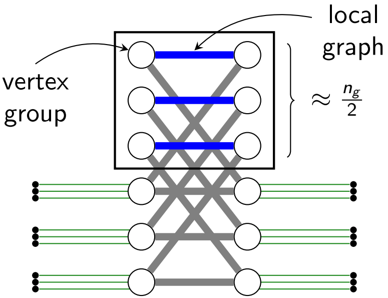
Global Graph



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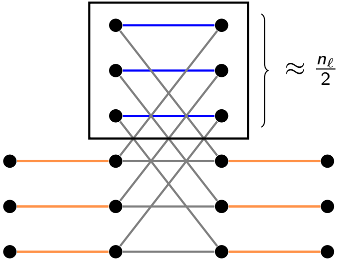

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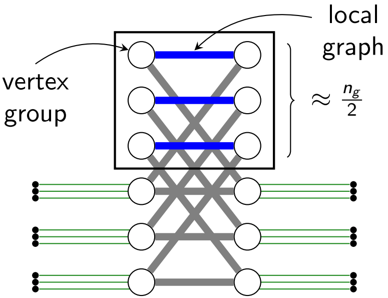
Two-Pass Hard Graph and Adversarial Stream

Global Graph



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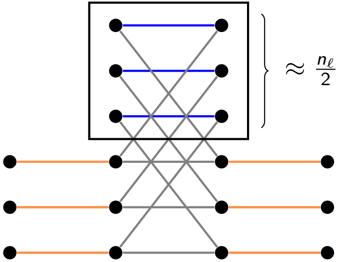

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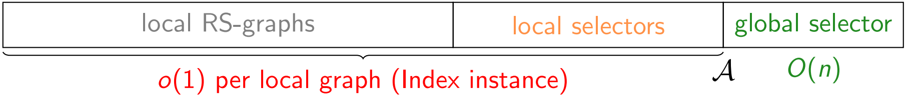

Two-Pass Hard Graph and Adversarial Stream

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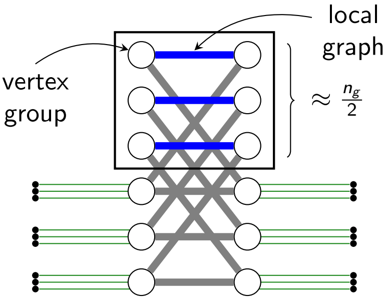

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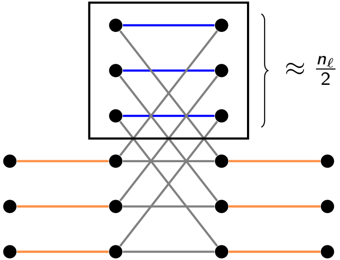

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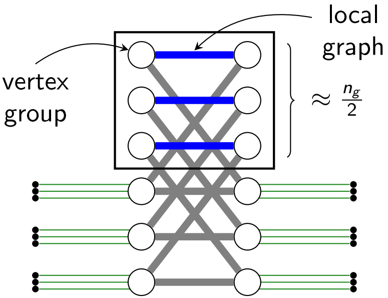
local RS-graphs	local selectors	global selector ✓

$o(1)$ per local graph (Index instance)

$\mathcal{A}$

Two-Pass Hard Graph and Adversarial Stream

Global Graph



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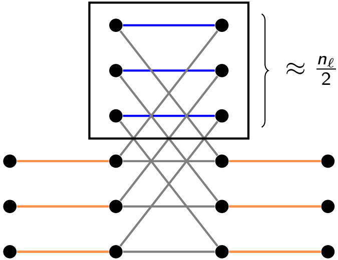

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Two-Pass Hard Graph and Adversarial Stream

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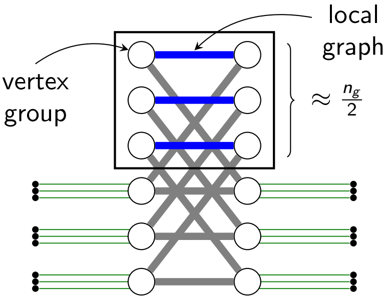

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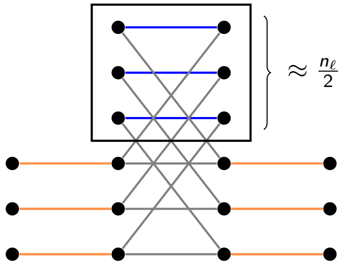

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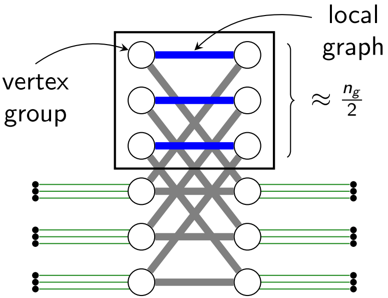
$o(1)$ per local graph (Index instance)

$\mathcal{A}$

Two-Pass Hard Graph and Adversarial Stream

Global Graph



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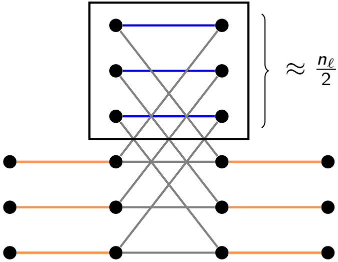

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	local RS-graphs	local selectors	global selector ✓
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Two-Pass Hard Graph and Adversarial Stream

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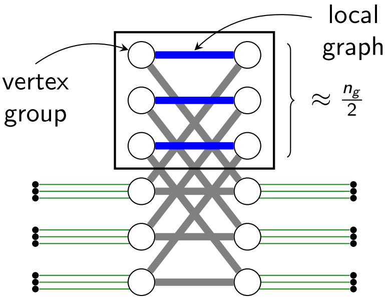

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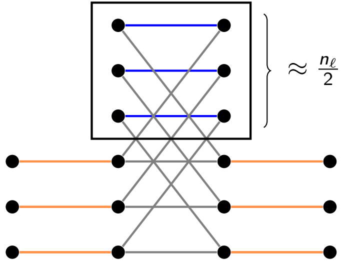

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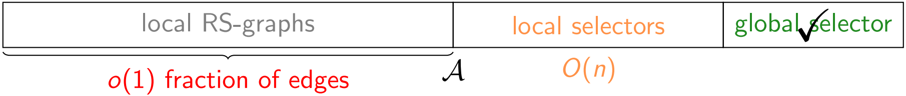
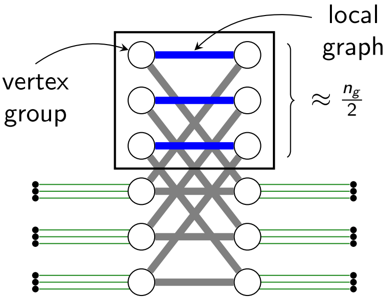
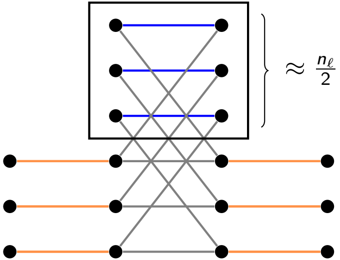


Two-Pass Hard Graph and Adversarial Stream





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local RS-graphs	local selectors ✓	global selector ✓

$\underbrace{\text{local RS-graphs}}_{o(1)\text{ fraction of edges}}$

$\mathcal{A}$

Two-Pass Hard Graph and Adversarial Stream

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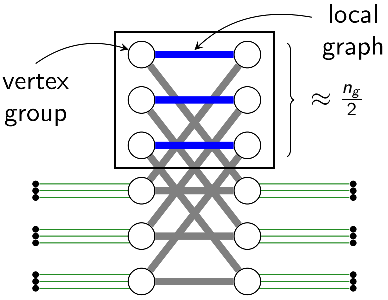

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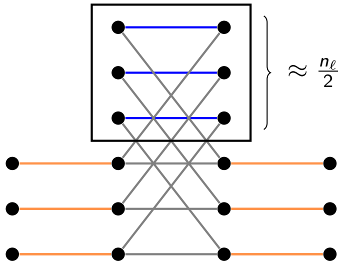
Local Graphs



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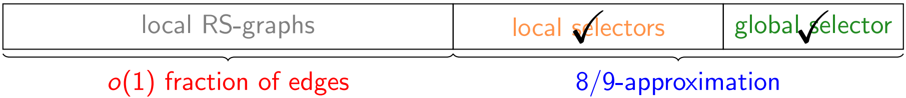

A New Communication Game (2 passes)

$\mathsf{HiddenStrings}_2$

A 3-player communication game for any integers $t_g, r_g \geq 1$ and $t_g \cdot r_g$ many independent instances of Index.

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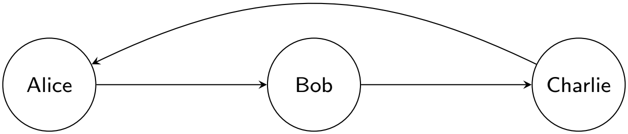
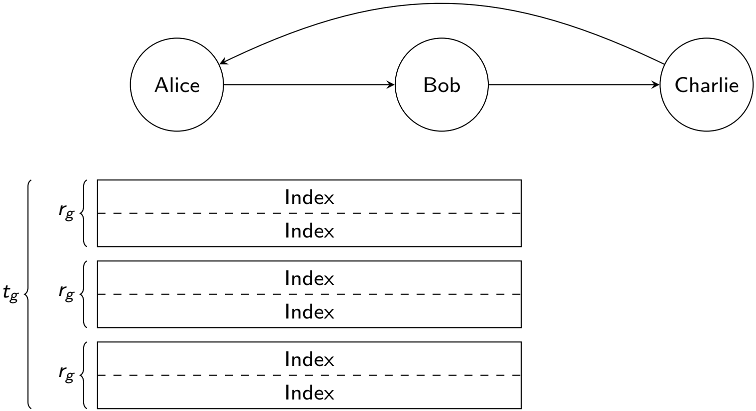


A New Communication Game (2 passes)

HiddenStrings$_2$

A 3-player communication game for any integers $t_g, r_g \geq 1$ and $t_g \cdot r_g$ many independent instances of Index.



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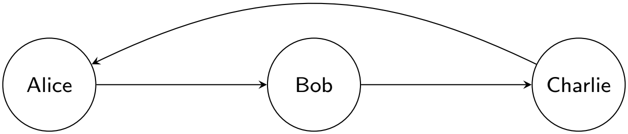

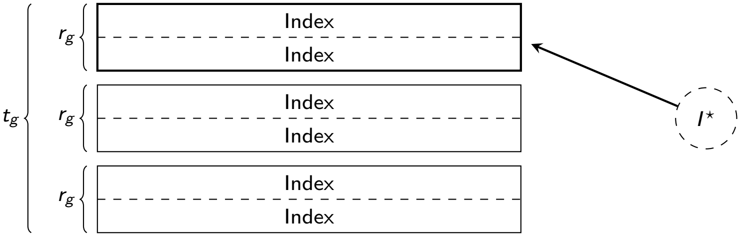

2-Pass Streaming Lower Bounds

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For $\varepsilon > 0$, any constant-error two-pass $(\frac{8}{9} + \varepsilon)$-approximation streaming algorithm for MBM requires $n^{1+\Omega(1/(\log\log n)^2)}$ space.

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Maximal Independent Set (MIS)

Any constant-error two-pass streaming algorithm for MIS requires $\Omega(n^{4/3-o(1)})$ space.

A New Communication Game (p passes)

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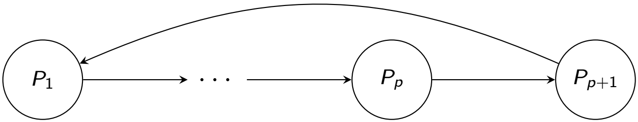

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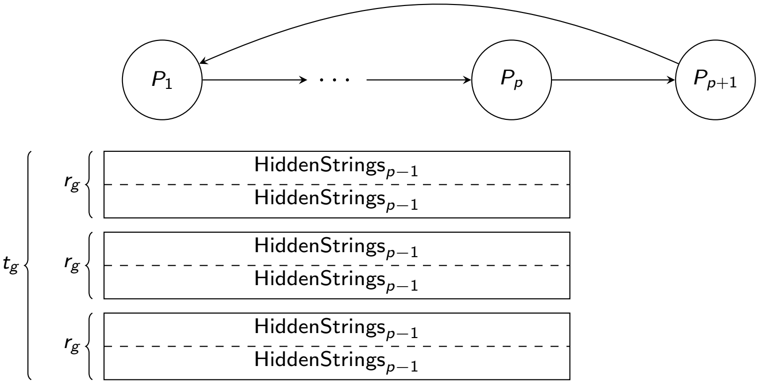

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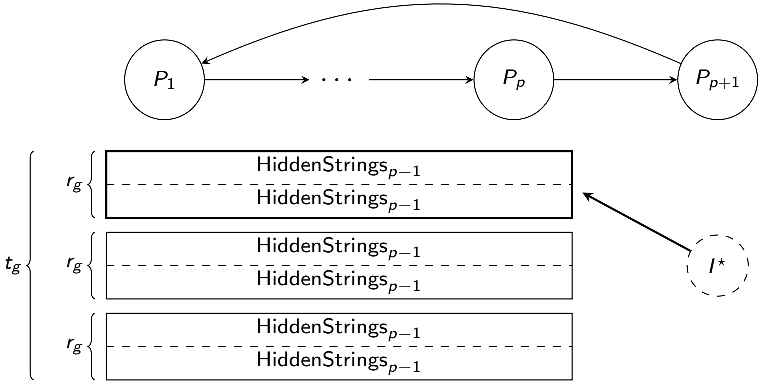

Multi-Pass Semi-Streaming Lower Bounds

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Any constant-error semi-streaming algorithm for MIS requires $\Omega(\log \log n)$ passes. **Optimal!**

Next Steps

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- Algorithms require either $O(1/\varepsilon^2)$ or $O((1/\varepsilon) \cdot \log n)$ passes.
- Narrowing the gap even for the 1-pass or 2-pass settings.

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General Questions

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2. Are there other settings where HiddenStrings is useful?

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Thank you!

References I

Sepehr Assadi and Janani Sundaresan, *Hidden permutations to the rescue: Multi-pass semi-streaming lower bounds for approximate matchings*, 64th IEEE Annual Symposium on Foundations of Computer Science, FOCS 2023, Santa-Cruz, CA, USA, November 6 - 9, 2023, IEEE, 2023.

Sepehr Assadi, *A two-pass (conditional) lower bound for semi-streaming maximum matching*, Proceedings of the 2022 ACM-SIAM Symposium on Discrete Algorithms, SODA 2022, Virtual Conference / Alexandria, VA, USA, January 9 - 12, 2022 (Joseph (Seffi) Naor and Niv Buchbinder, eds.), SIAM, 2022, pp. 708–742.

References II

Hossein Esfandiari, MohammadTaghi Hajiaghayi, and Morteza Monemizadeh, *Finding large matchings in semi-streaming*, IEEE International Conference on Data Mining Workshops, ICDM Workshops 2016, December 12-15, 2016, Barcelona, Spain (Carlotta Domeniconi, Francesco Gullo, Francesco Bonchi, Josep Domingo-Ferrer, Ricardo Baeza-Yates, Zhi-Hua Zhou, and Xindong Wu, eds.), IEEE Computer Society, 2016, pp. 608–614.

Joan Feigenbaum, Sampath Kannan, Andrew McGregor, Siddharth Suri, and Jian Zhang, *On graph problems in a semi-streaming model*, Automata, Languages and Programming: 31st International Colloquium, ICALP 2004, Turku, Finland, July 12-16, 2004. Proceedings (Josep Díaz, Juhani Karhumäki, Arto Lepistö, and Donald Sannella, eds.), Lecture Notes in Computer Science, vol. 3142, Springer, 2004, pp. 531–543.

References III

Eldar Fischer, Eric Lehman, Ilan Newman, Sofya Raskhodnikova, Ronitt Rubinfeld, and Alex Samorodnitsky, *Monotonicity testing over general poset domains*, Proceedings on 34th Annual ACM Symposium on Theory of Computing, May 19-21, 2002, Montréal, Québec, Canada (John H. Reif, ed.), ACM, 2002, pp. 474–483.

Moran Feldman and Ariel Szarf, *Maximum matching sans maximal matching: A new approach for finding maximum matchings in the data stream model*, Approximation, Randomization, and Combinatorial Optimization. Algorithms and Techniques, APPROX/RANDOM 2022, September 19-21, 2022, University of Illinois, Urbana-Champaign, USA (Virtual Conference) (Amit Chakrabarti and Chaitanya Swamy, eds.), LIPIcs, vol. 245, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2022, pp. 33:1–33:24.

References IV

Ashish Goel, Michael Kapralov, and Sanjeev Khanna, *On the communication and streaming complexity of maximum bipartite matching*, Proceedings of the Twenty-Third Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2012, Kyoto, Japan, January 17-19, 2012 (Yuval Rabani, ed.), SIAM, 2012, pp. 468–485.

Rahul Jain, Jaikumar Radhakrishnan, and Pranab Sen, *A property of quantum relative entropy with an application to privacy in quantum communication*, J. ACM **56** (2009), no. 6, 33:1–33:32.

Michael Kapralov, *Better bounds for matchings in the streaming model*, Proceedings of the Twenty-Fourth Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2013, New Orleans, Louisiana, USA, January 6-8, 2013 (Sanjeev Khanna, ed.), SIAM, 2013, pp. 1679–1697.

References V

———, *Space lower bounds for approximating maximum matching in the edge arrival model*, Proceedings of the 2021 ACM-SIAM Symposium on Discrete Algorithms, SODA 2021, Virtual Conference, January 10 - 13, 2021 (Dániel Marx, ed.), SIAM, 2021, pp. 1874–1893.

Christian Konrad, Frédéric Magniez, and Claire Mathieu, *Maximum matching in semi-streaming with few passes*, Approximation, Randomization, and Combinatorial Optimization. Algorithms and Techniques - 15th International Workshop, APPROX 2012, and 16th International Workshop, RANDOM 2012, Cambridge, MA, USA, August 15-17, 2012. Proceedings (Anupam Gupta, Klaus Jansen, José D. P. Rolim, and Rocco A. Servedio, eds.), Lecture Notes in Computer Science, vol. 7408, Springer, 2012, pp. 231–242.

References VI

Christian Konrad and Kheeran K. Naidu, *On two-pass streaming algorithms for maximum bipartite matching*, Approximation, Randomization, and Combinatorial Optimization. Algorithms and Techniques, APPROX/RANDOM 2021, August 16-18, 2021, University of Washington, Seattle, Washington, USA (Virtual Conference) (Mary Wootters and Laura Sanità, eds.), LIPIcs, vol. 207, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2021, pp. 19:1–19:18.

Christian Konrad, *A simple augmentation method for matchings with applications to streaming algorithms*, 43rd International Symposium on Mathematical Foundations of Computer Science, MFCS 2018, August 27-31, 2018, Liverpool, UK (Igor Potapov, Paul G. Spirakis, and James Worrell, eds.), LIPIcs, vol. 117, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2018, pp. 74:1–74:16.

References VII

Sagar Kale and Sumedh Tirodkar, *Maximum matching in two, three, and a few more passes over graph streams*, Approximation, Randomization, and Combinatorial Optimization. Algorithms and Techniques, APPROX/RANDOM 2017, August 16-18, 2017, Berkeley, CA, USA (Klaus Jansen, José D. P. Rolim, David Williamson, and Santosh S. Vempala, eds.), LIPIcs, vol. 81, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2017, pp. 15:1–15:21.