Maximum Matching via Maximal Matching Queries

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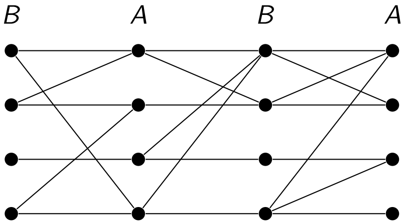
Preliminaries

In this talk, we consider bipartite graphs $G = (A, B, E)$.



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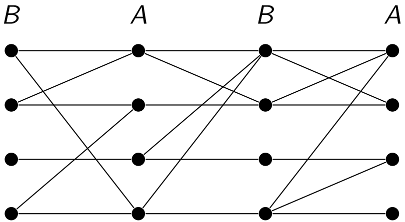

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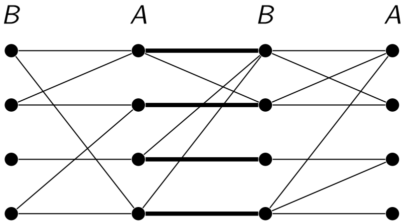

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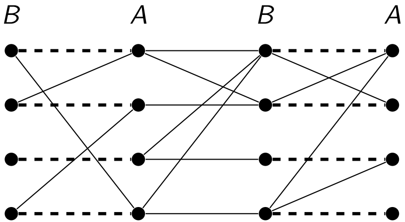

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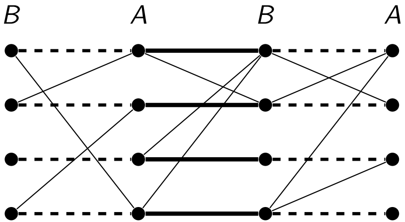

Matchings

A matching M is a subset of vertex-disjoint edges of a graph.

- **Maximal**: Every edge $e \in E \setminus M$ is incident to M.
- **Maximum**: Largest size, $\mu(G)$.
- Maximal matchings are 0.5-approximations of maximum matchings.

Edge Query Model

Algorithm's Goal

Return a **large matching** of the bipartite input graph $G = (A, B, E)$ using only **deterministic edge queries** to a **maximal matching oracle**.

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Algorithm
A & B

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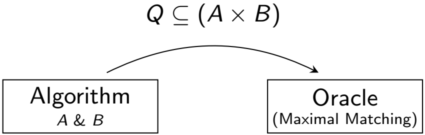

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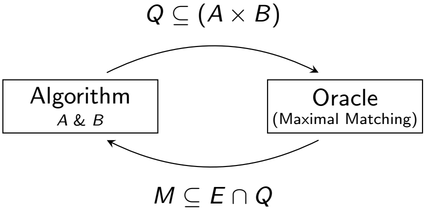

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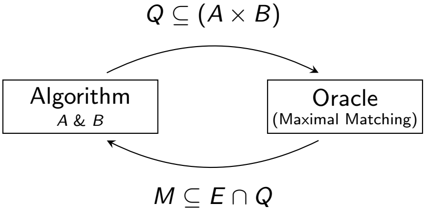
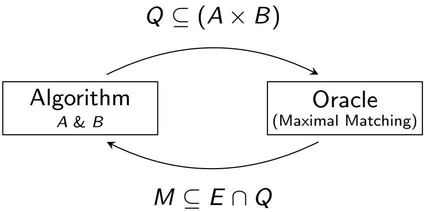


Motivation

Computing a maximal matching is **easy** in various computational models such as **data streaming** and Massively Parallel Computation.

Edge Query Model and Data Streaming



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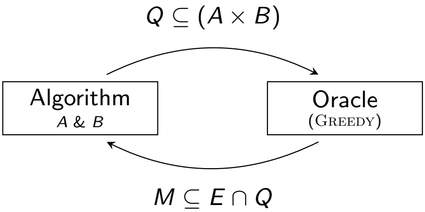

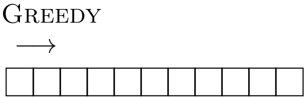

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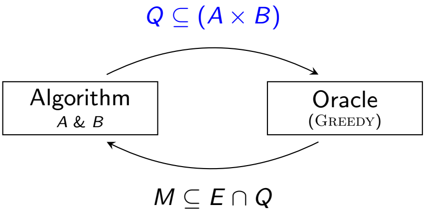

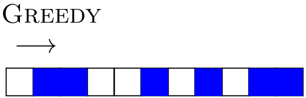

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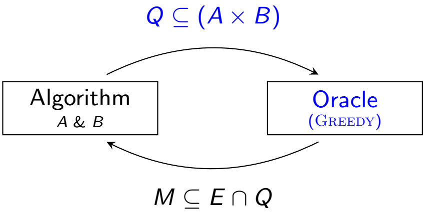

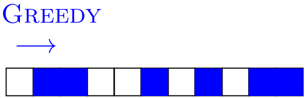

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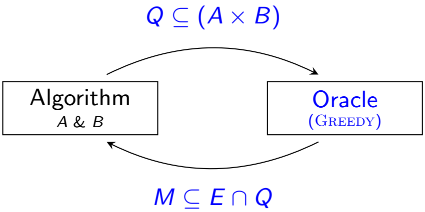

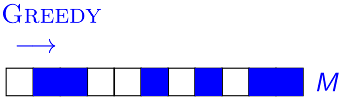

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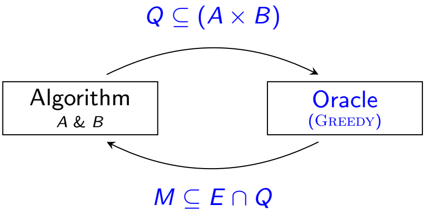

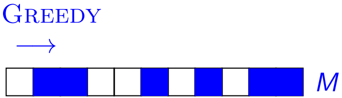

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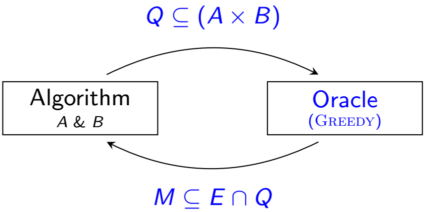

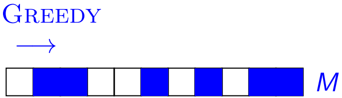

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Known Algorithms

1. 0.6-approximation **MBM** in 3-passes [KT17] (see also [KMM12, FKM+05]) – state-of-the-art is 0.611-approximation [FS22].
2. $(1-\epsilon)$-approximation **MBM** in $O(\frac{1}{\epsilon^2})$-passes [ALT21] – current state-of-the-art.

Our Results

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Algorithm

0.625-approximation algorithm in **3-rounds** of the deterministic edge-query model.

Implies a **3-pass semi-streaming** algorithm for **MBM** (state-of-the-art – improving on 0.611 [FS22]).

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Lower Bounds

There **does not exist** a deterministic algorithm for **MM** (even for **MBM**) in the edge query model that achieves a better than

1. 0.5-approximation in **1 round**,
2. $(0.5 + o(1))$-approximation in **2 rounds**, and
3. $(0.625 + o(1))$-approximation in **3 rounds**.

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Algorithm is optimal!

Previous Related Work

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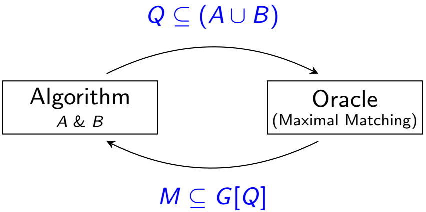
Vertex Query Model ([bKK20])



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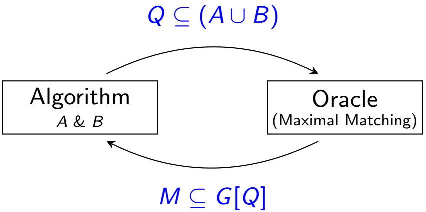


# Rounds	Vertex Query	Edge Query
1	0.5	0.5
2	0.5	$0.5 + o(1)$
3	0.6	$0.625 + o(1)$

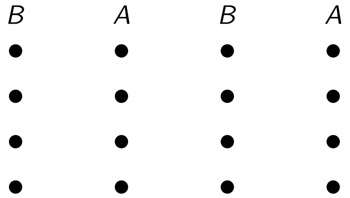
Algorithmic Idea

First Round

Find a maximal matching M in G by querying the complete graph $Q = A \times B$.

Subsequent Rounds

Find vertex-disjoint augmenting paths.



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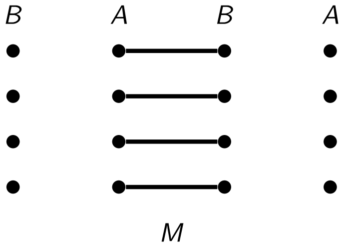

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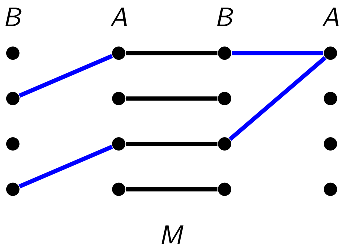

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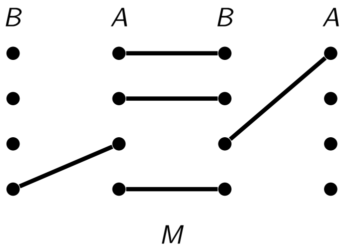

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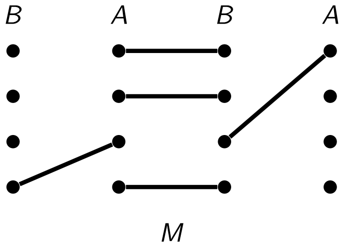

Finding length-3 augmenting paths

Simple Strategy

1. Find left wings
2. Extend with right wings

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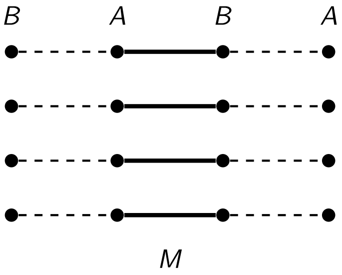

- $|M| = 0.5 \cdot \mu(G)$

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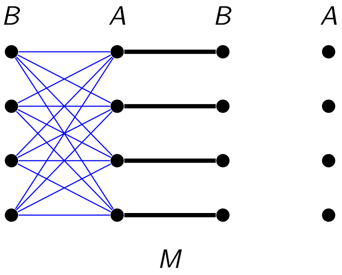

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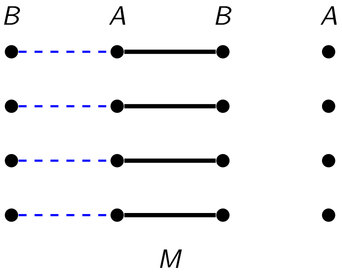

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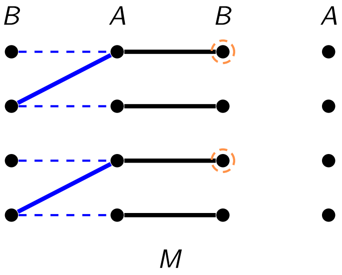

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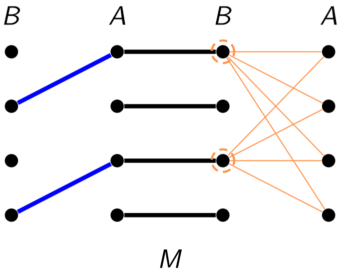

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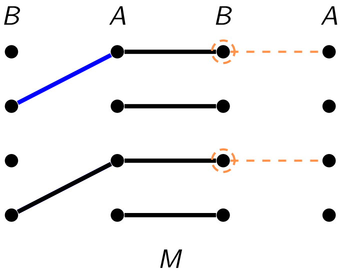

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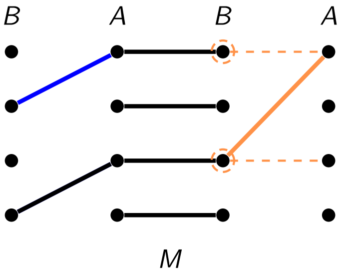

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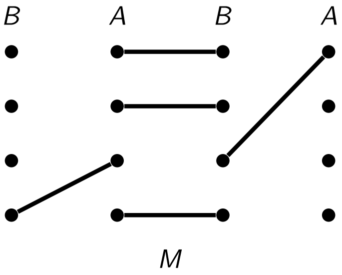

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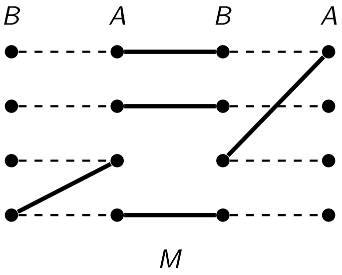

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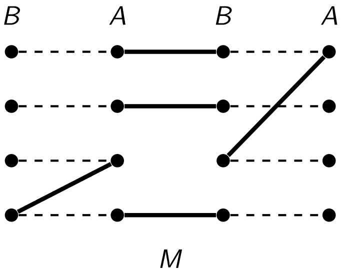

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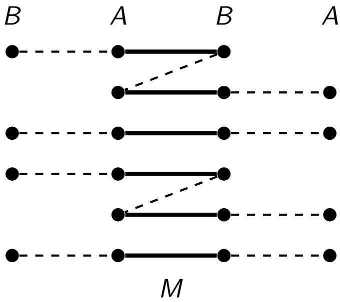

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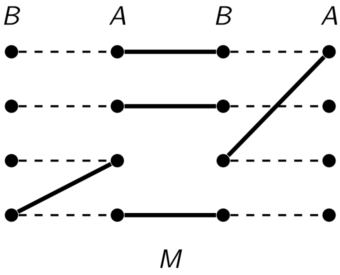

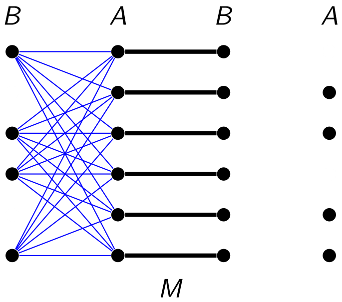

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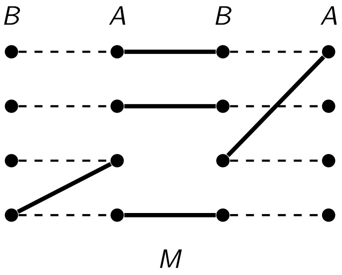

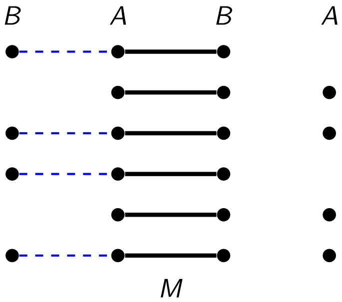

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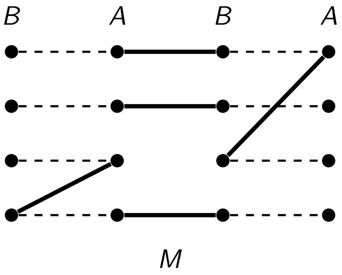

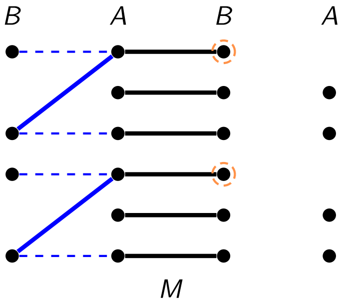

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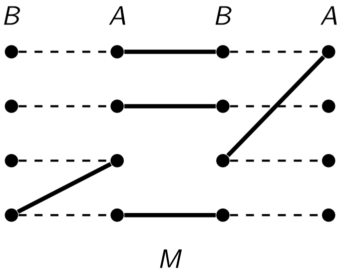

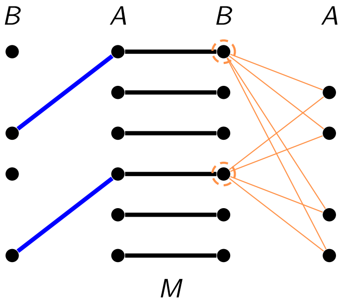

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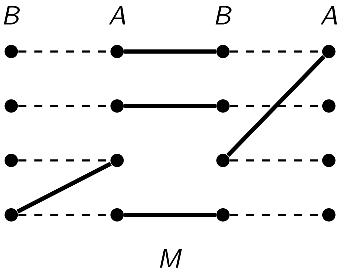

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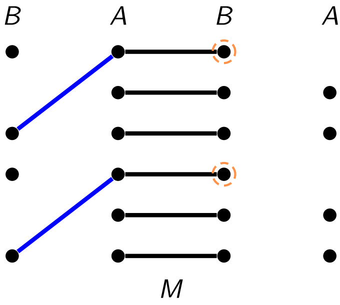

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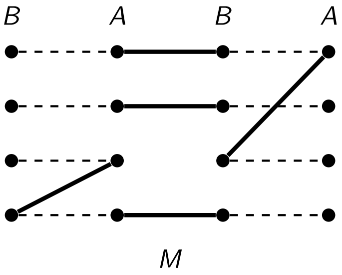

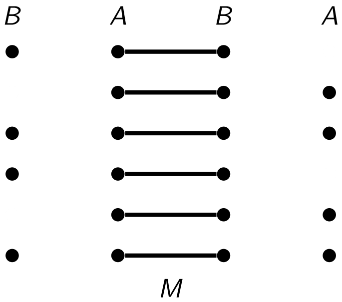

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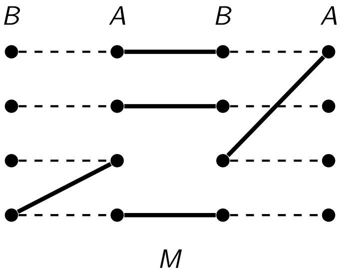

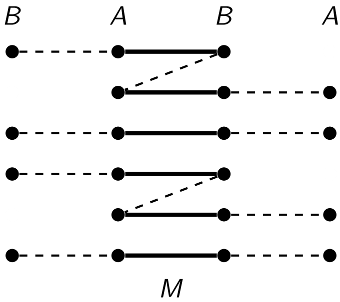

- $|M| = 0.5 \cdot \mu(G)$
- 0.625-approximation
- Not hard!

- $|M| = 0.6 \cdot \mu(G)$
- 0.6-approximation
- **Hard instance!**

Finding length-3 & length-5 augmenting paths

Our Strategy

1. Find left and right wings
2. Extend paths to either length-3 or length-5 augmenting paths

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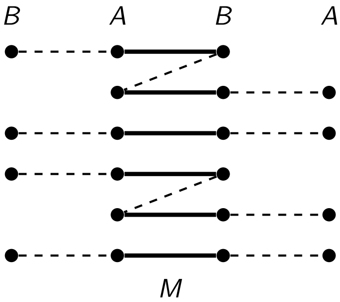

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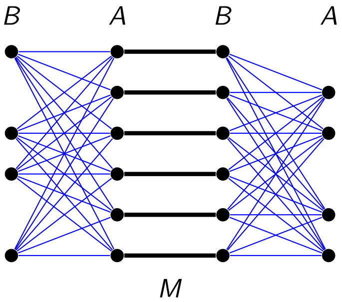

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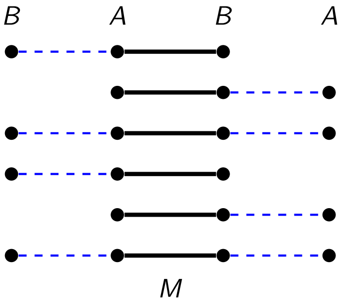

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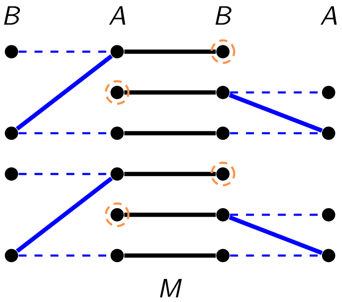

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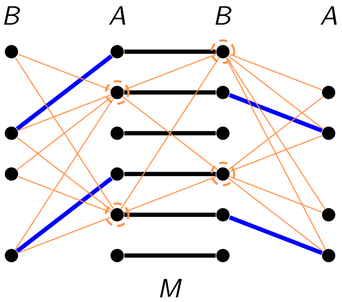

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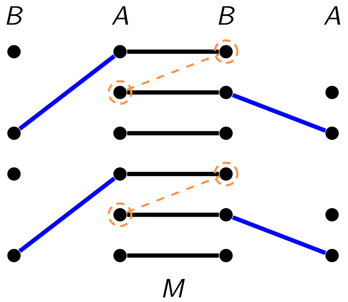

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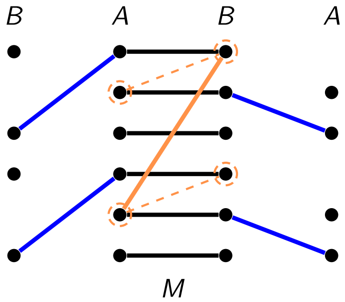

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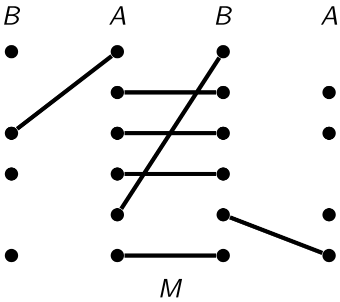

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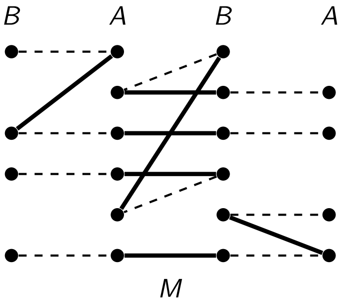

- $|M| = 0.6 \cdot \mu(G)$
- 0.7-approximation
- Not hard anymore!

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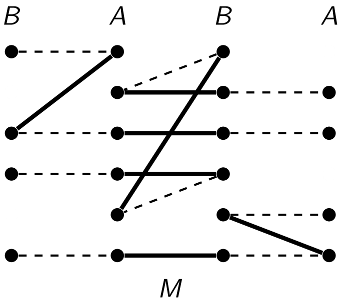

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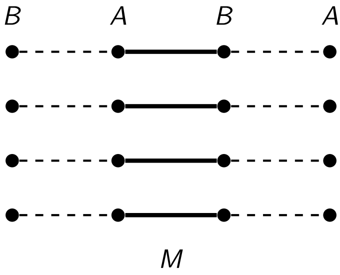

- $|M| = 0.5 \cdot \mu(G)$

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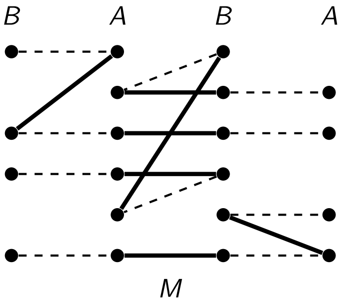

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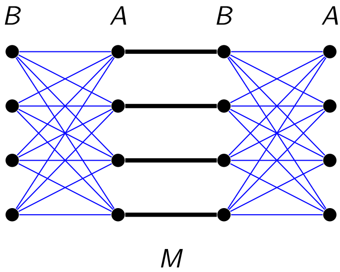

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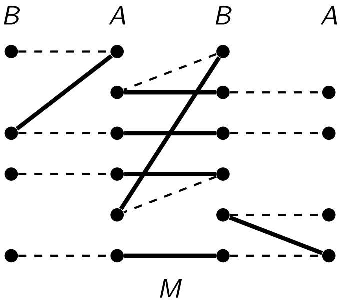

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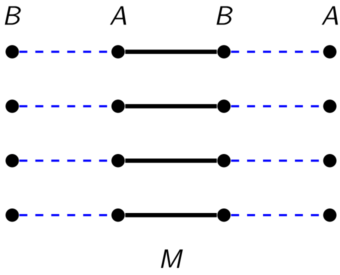

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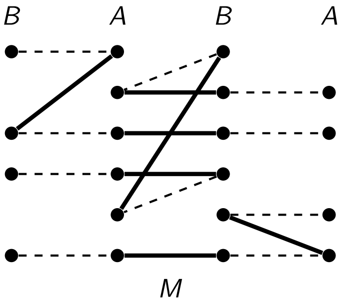

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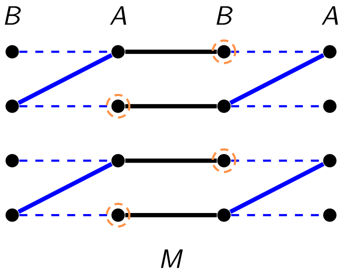

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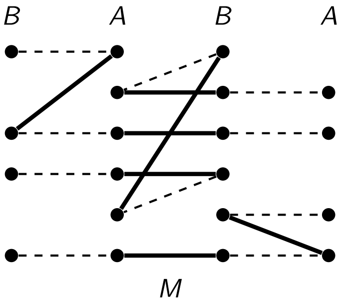

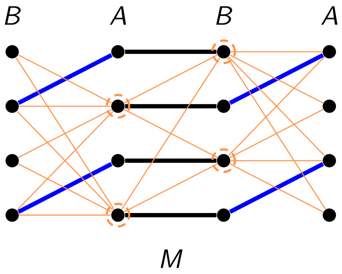

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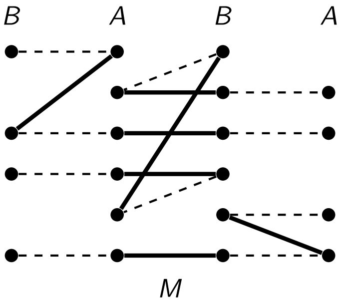

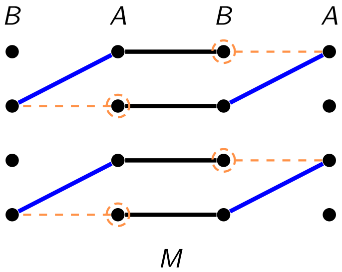

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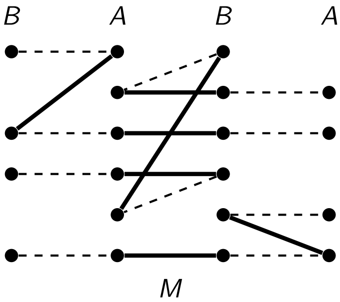

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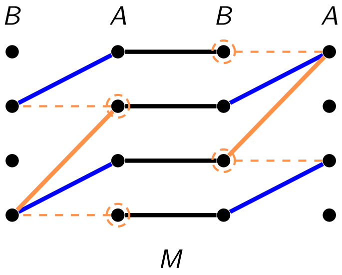

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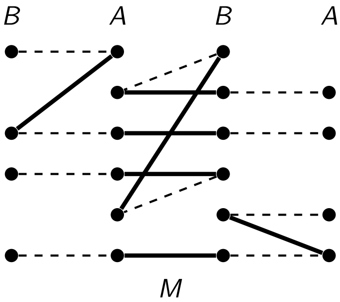

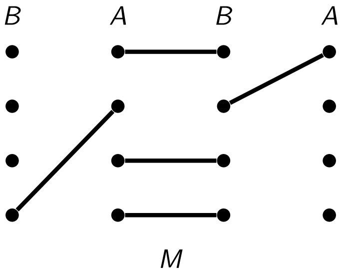

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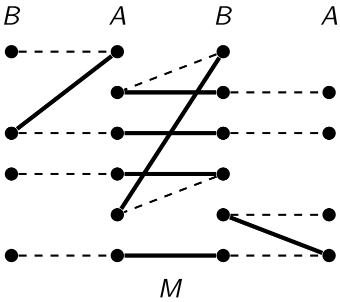

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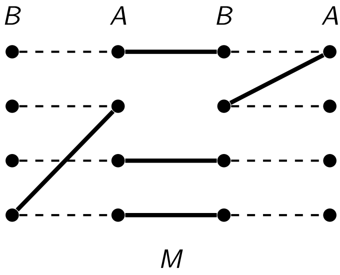

- $|M| = 0.5 \cdot \mu(G)$
- 0.625-approximation
- **Hard instance!**

Our Analysis

Main Lemma

Let $|M| = (0.5 + \epsilon) \cdot \mu(G)$ for $\epsilon \geq 0$, then our strategy finds

$$(0.125 - \frac{3}{4}\epsilon) \cdot \mu(G)$$

vertex-disjoint augmenting paths and the large matching found is of size

$$(0.625 + \frac{\epsilon}{4}) \cdot \mu(G).$$

This is tight for our algorithm.

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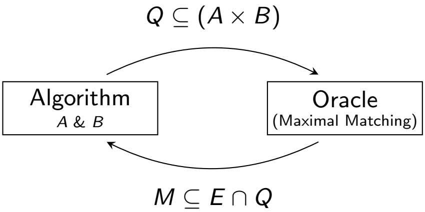
Semi-Streaming

Using GREEDY this immediately gives a 3-pass semi-streaming algorithm with the same guarantees.

Lower Bound Idea



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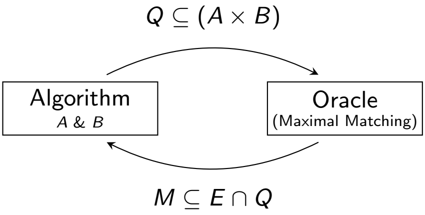
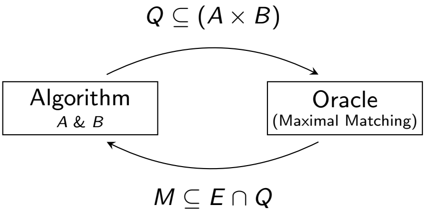
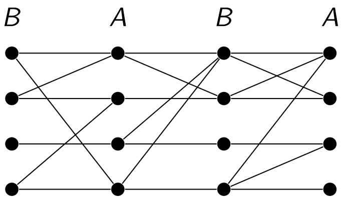


Observation

The algorithm learns about **edges** M and **non-edges** N of G.

Lower Bound Idea





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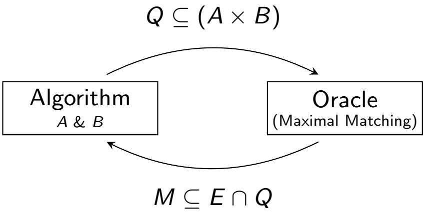

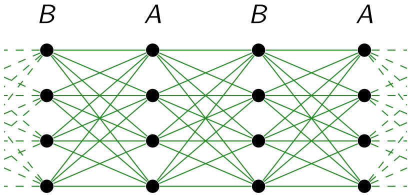

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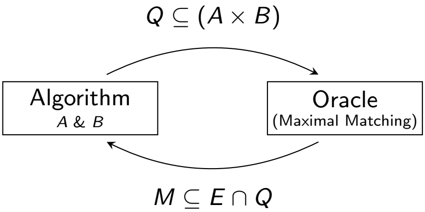

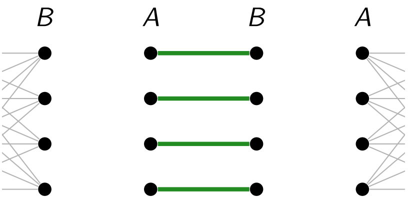

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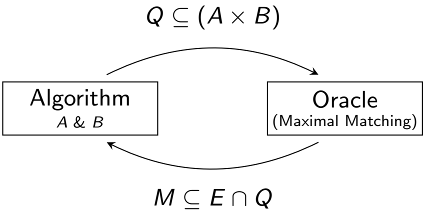

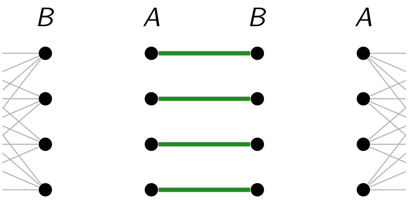

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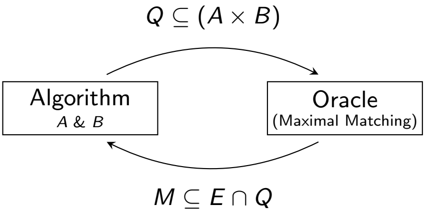
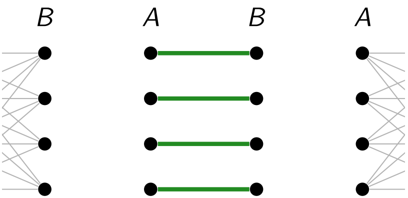
Main Idea

Lower Bound Idea





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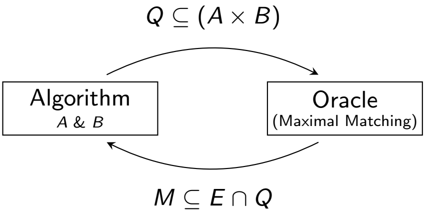
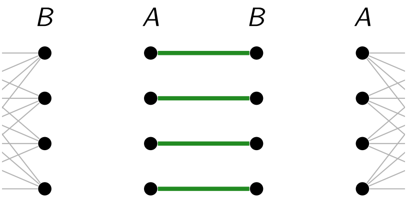
The algorithm learns about **edges** M and **non-edges** N of G.

Main Idea

- Find a hard instance for any sequence of queries $Q_1, Q_2 \ldots$

Lower Bound Idea





Observation

The algorithm learns about **edges** M and **non-edges** N of G.

Main Idea

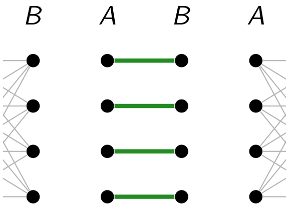
- Find a hard instance for any sequence of queries $Q_1, Q_2 \ldots$
- For any query Q_i, the information committed is a subset of $\tilde{M}_i$ and $\tilde{N}_i$ (up to isomorphisms)

One, Two & Three Query Rounds

One Round **Two Rounds** **Three Rounds**

One, Two & Three Query Rounds

One Round | **Two Rounds** | **Three Rounds**



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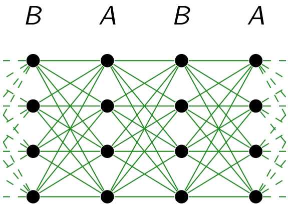

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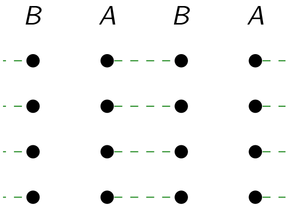

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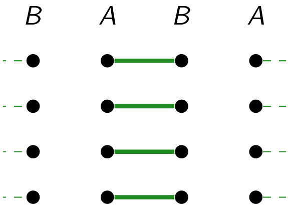

Two Rounds

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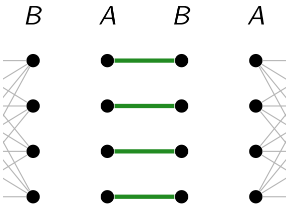

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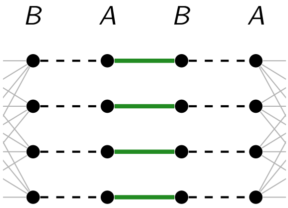

0.5
approx LB

Two Rounds

Three Rounds

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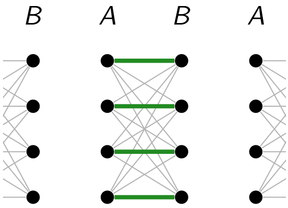

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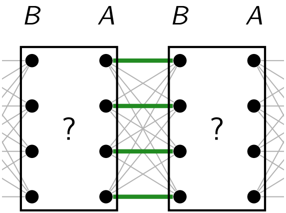

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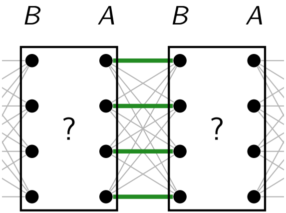

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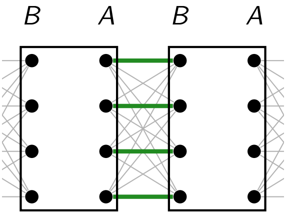
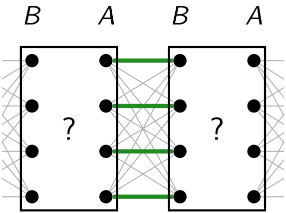


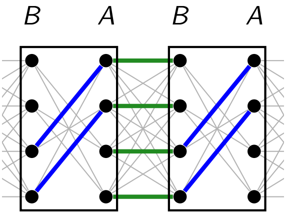
Three Rounds

One, Two & Three Query Rounds

One Round



Two Rounds

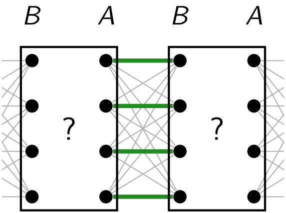


Three Rounds

0.5
approx LB

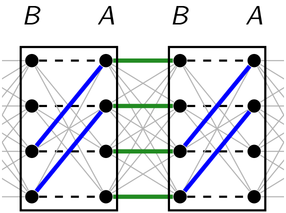
One, Two & Three Query Rounds

One Round



0.5
approx LB

Two Rounds

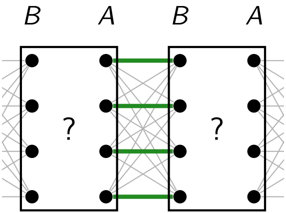


$0.5 + o(1)$
approx LB

Three Rounds

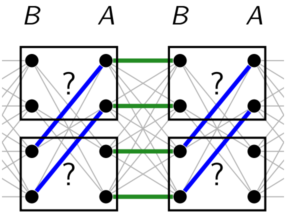
One, Two & Three Query Rounds

One Round



0.5
approx LB

Two Rounds

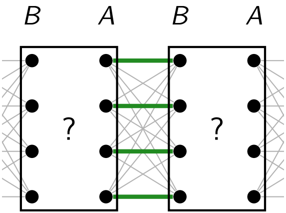


$0.5 + o(1)$
approx LB

Three Rounds

One, Two & Three Query Rounds

One Round



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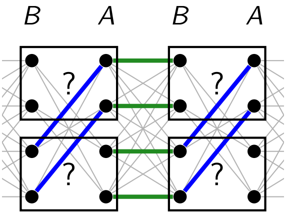
approx LB

Two Rounds



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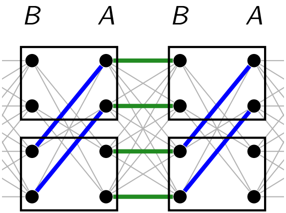
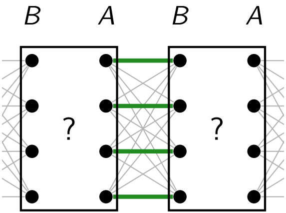
Three Rounds



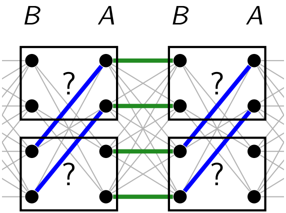
One, Two & Three Query Rounds

One Round



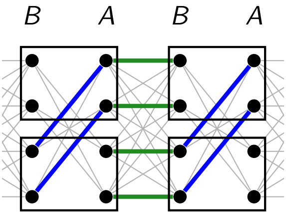
0.5
approx LB

Two Rounds



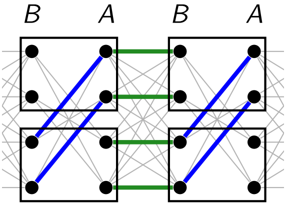
$0.5 + o(1)$
approx LB

Three Rounds



(more involved)

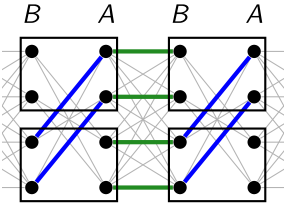
Three Round Proof Sketch I



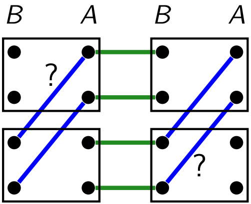
Part A

Part B

Three Round Proof Sketch I

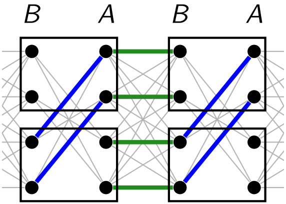


Part A

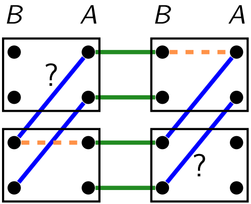


Part B

Three Round Proof Sketch I

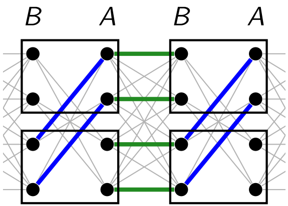


Part A



Part B

Three Round Proof Sketch I



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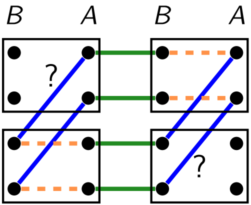

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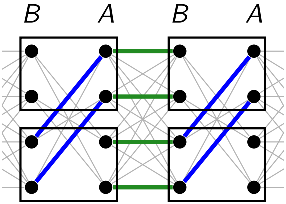

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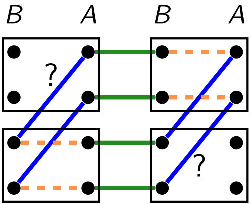

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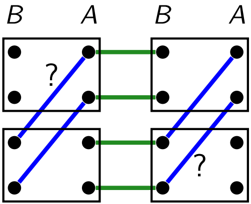
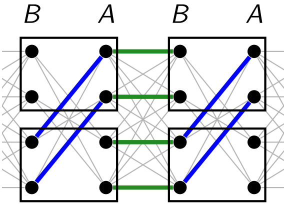
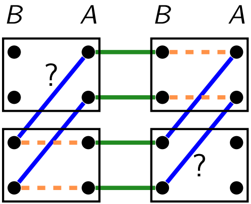


Three Round Proof Sketch I



Part A



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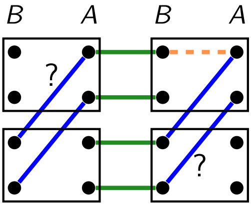
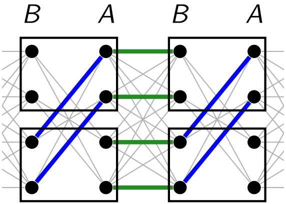


Three Round Proof Sketch I



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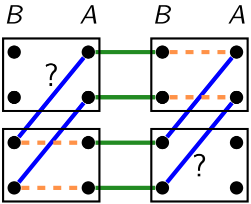

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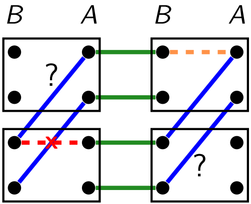
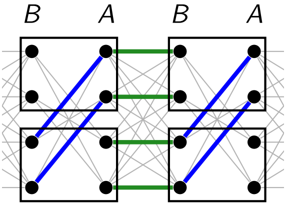


Three Round Proof Sketch I



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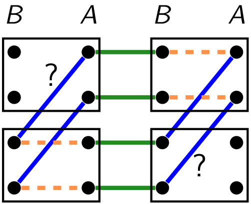

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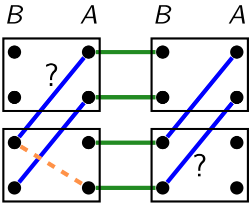
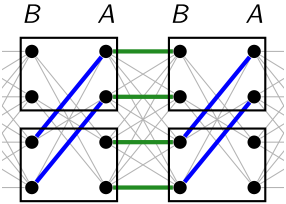
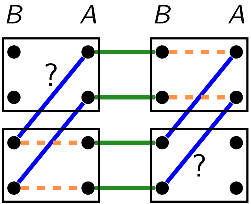


Three Round Proof Sketch I



Part A



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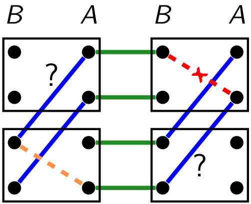
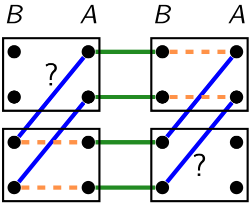


Three Round Proof Sketch II

Part A



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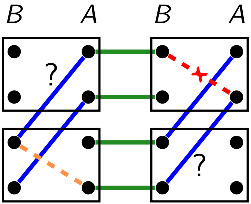
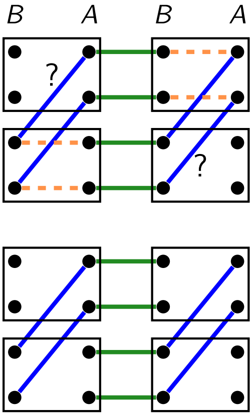


Three Round Proof Sketch II

<u>Part A</u>



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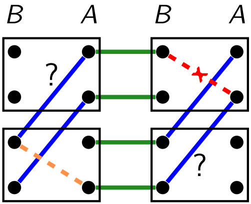

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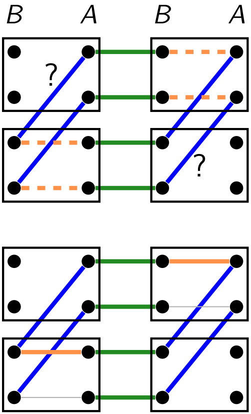

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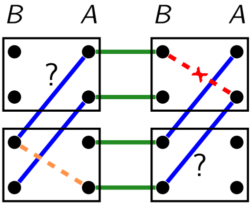
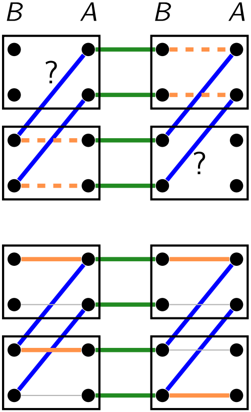


Three Round Proof Sketch II

Part A



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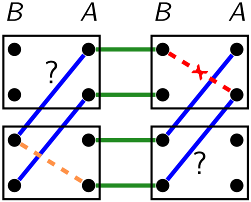
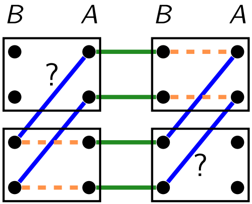
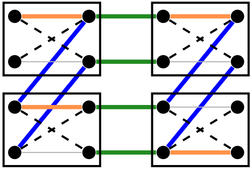


Three Round Proof Sketch II

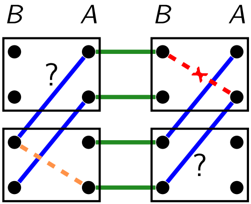
Part A





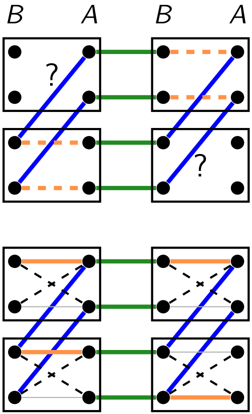
$0.625 + o(1)$
approx LB

Part B



Three Round Proof Sketch II

Part A



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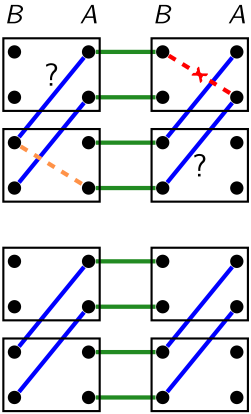

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Three Round Proof Sketch II

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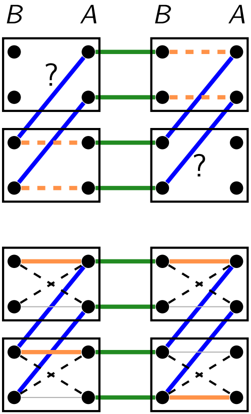

$0.625 + o(1)$
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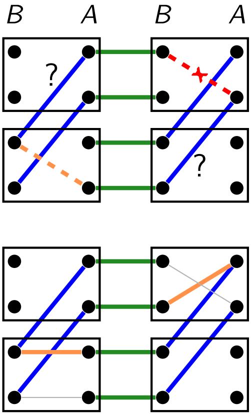
Part B



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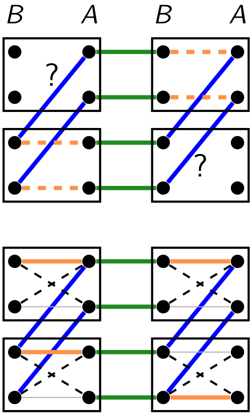

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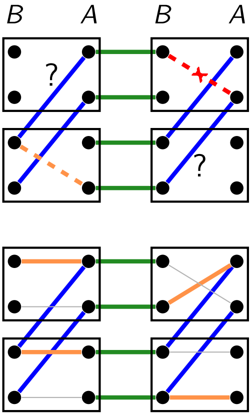
Part B



Three Round Proof Sketch II

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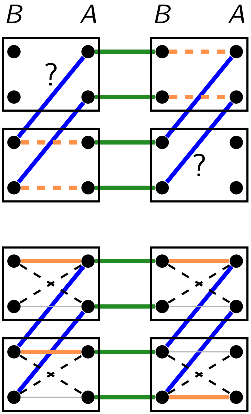
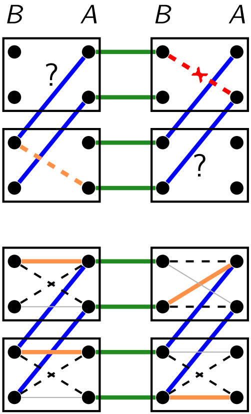


$0.625 + o(1)$
approx LB

Part B



$0.625 + o(1)$
approx LB

Summary

Lower Bounds

There **does not exist** a deterministic algorithm for **MM** (even for **MBM**) in the edge query model that achieves a better than

1. 0.5-approximation in **1 round**,
2. $(0.5 + o(1))$-approximation in **2 rounds**, and
3. $(0.625 + o(1))$-approximation in **3 rounds**.

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Algorithm

0.625-approximation algorithm in **3-rounds** of the deterministic edge-query model.

Open Questions

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Do randomised query algorithms allow us to improve on our results?

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Semi-Streaming

Is there a 3-pass semi-streaming algorithms for **MBM** that improves on our 0.625-approximation algorithm?

Open Questions

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Do randomised query algorithms allow us to improve on our results?

Adaptivity

Can we obtain better query algorithms if we allow multiple non-adaptive queries per round?

Semi-Streaming

Is there a 3-pass semi-streaming algorithms for **MBM** that improves on our 0.625-approximation algorithm?

Thank You!

References I

Sepehr Assadi, S. Cliff Liu, and Robert E. Tarjan, *An auction algorithm for bipartite matching in streaming and massively parallel computation models*, 4th Symposium on Simplicity in Algorithms, SOSA 2021, Virtual Conference, January 11-12, 2021 (Hung Viet Le and Valerie King, eds.), SIAM, 2021, pp. 165–171.

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- Joan Feigenbaum, Sampath Kannan, Andrew McGregor, Siddharth Suri, and Jian Zhang, *On graph problems in a semi-streaming model*, Theor. Comput. Sci. **348** (2005), no. 2-3, 207–216.

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Sagar Kale and Sumedh Tirodkar, *Maximum matching in two, three, and a few more passes over graph streams*, Approximation, Randomization, and Combinatorial Optimization. Algorithms and Techniques, APPROX/RANDOM 2017, August 16-18, 2017, Berkeley, CA, USA (Klaus Jansen, José D. P. Rolim, David Williamson, and Santosh S. Vempala, eds.), LIPIcs, vol. 81, Schloss Dagstuhl - Leibniz-Zentrum für Informatik, 2017, pp. 15:1–15:21.