

Space Optimal Vertex Cover in Dynamic Streams (with an overview of graph streaming)

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Joint work with Vihan Shah (Rutgers University)

- 1 Introduction
- 2 Streaming Models
- 3 Matchings in Graph Streams
- 4 Space Optimal Vertex Cover

1 Introduction

2 Streaming Models

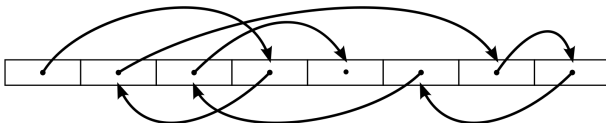
3 Matchings in Graph Streams

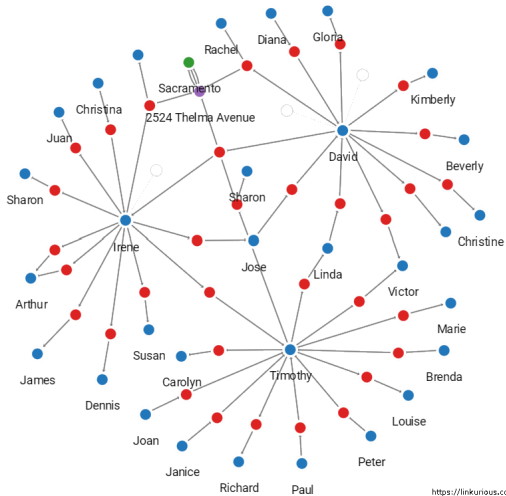
4 Space Optimal Vertex Cover

Classic Setting

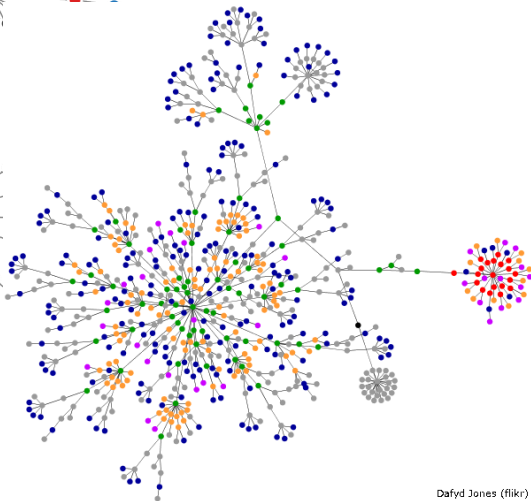
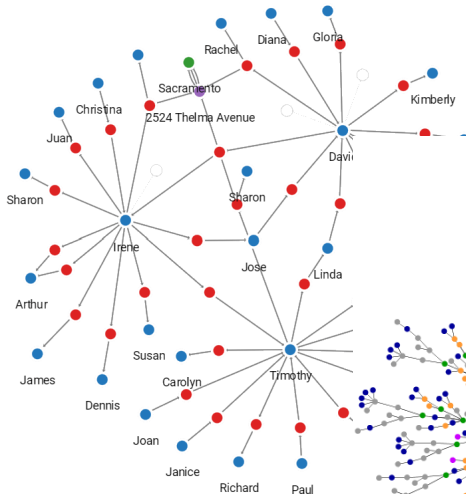
Assumption

Classical algorithms rely on the assumption that they have a random access to the input of the algorithm

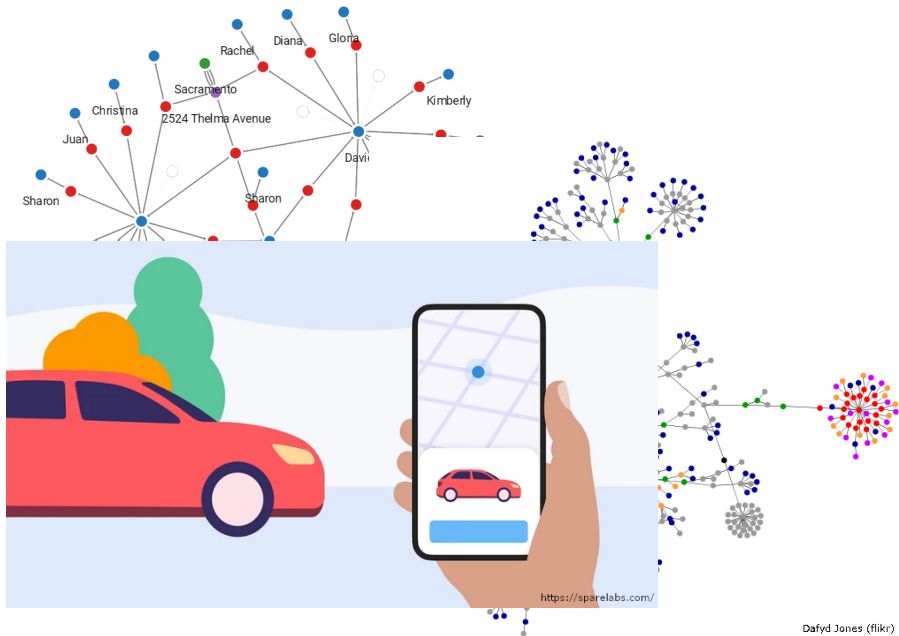




<https://linkurious.com/>



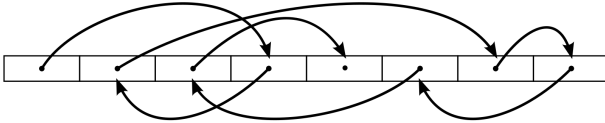
Dafyd Jones (flickr)





Assumption (Infeasible)

Classical algorithms rely on the assumption that they have a random access to the input of the algorithm

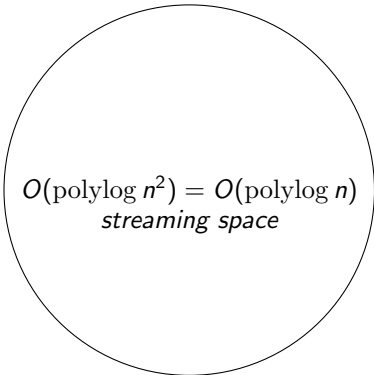


Definition (Graph Streaming)

A n -vertex graph is presented as a sequence of edges to an algorithm uses space **sublinear** in the size of the input ($o(n^2)$).

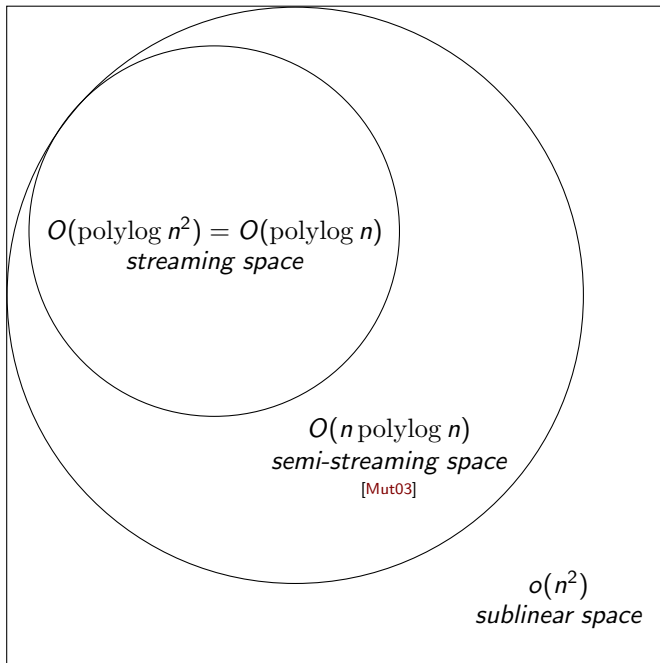


$o(n^2)$
sublinear space



$O(\text{polylog } n^2) = O(\text{polylog } n)$
streaming space

$o(n^2)$
sublinear space



1 Introduction

2 Streaming Models

3 Matchings in Graph Streams

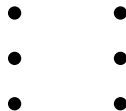
4 Space Optimal Vertex Cover

Insertion-Only [FKM⁺04]

Sliding Window [CMS13]

Dynamic [AGM12]

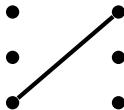
Insertion-Only [FKM⁺04]



Sliding Window [CMS13]

Dynamic [AGM12]

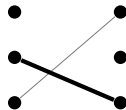
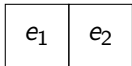
Insertion-Only [FKM⁺04]



Sliding Window [CMS13]

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Insertion-Only [FKM⁺04]

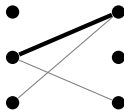


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM⁺04]

e_1	e_2	e_3
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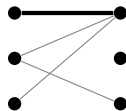


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM⁺04]

e_1	e_2	e_3	e_4
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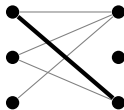


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM⁺04]

e_1	e_2	e_3	e_4	e_5
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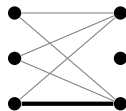


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM⁺04]

e_1	e_2	e_3	e_4	e_5	e_6
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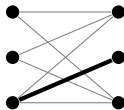


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM⁺04]

e_1	e_2	e_3	e_4	e_5	e_6	e_7
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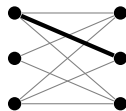


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM⁺04]

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
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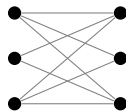


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
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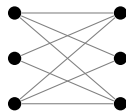


Sliding Window [CMS13]

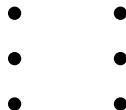
Dynamic [AGM12]

Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
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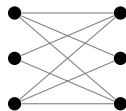
Sliding Window [CMS13]



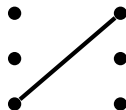
Dynamic [AGM12]

Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
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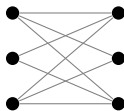
Sliding Window [CMS13]



Dynamic [AGM12]

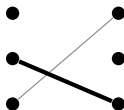
Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
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Sliding Window [CMS13]

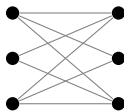
e_1	e_2
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Dynamic [AGM12]

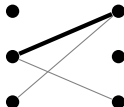
Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
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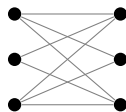
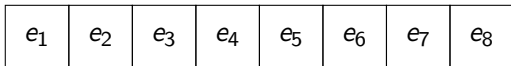
Sliding Window [CMS13]

e_1	e_2	e_3
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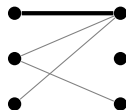
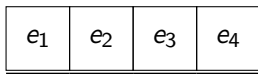


Dynamic [AGM12]

Insertion-Only [FKM⁺04] (finite stream)

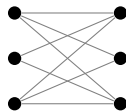
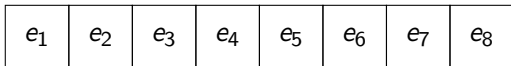


Sliding Window [CMS13]

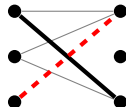
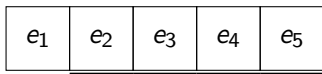


Dynamic [AGM12]

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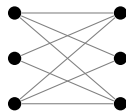
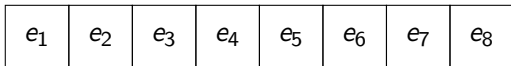


Sliding Window [CMS13]

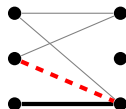
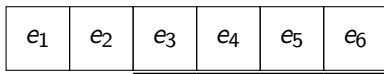


Dynamic [AGM12]

Insertion-Only [FKM⁺04] (finite stream)

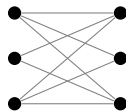
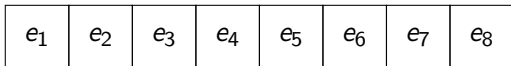


Sliding Window [CMS13]

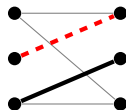
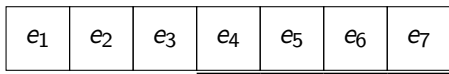


Dynamic [AGM12]

Insertion-Only [FKM⁺04] (finite stream)

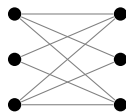
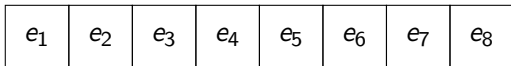


Sliding Window [CMS13]

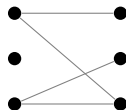
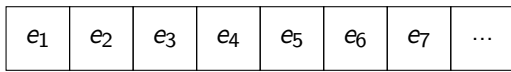


Dynamic [AGM12]

Insertion-Only [FKM⁺04] (finite stream)



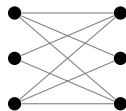
Sliding Window [CMS13] (infinite stream)



Dynamic [AGM12]

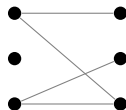
Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
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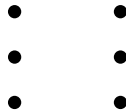


Sliding Window [CMS13] (infinite stream)

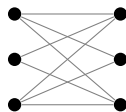
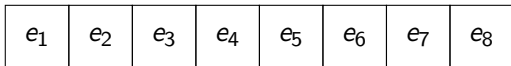
e_1	e_2	e_3	e_4	e_5	e_6	e_7	...
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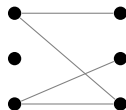
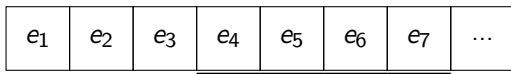
Dynamic [AGM12]



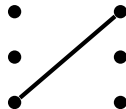
Insertion-Only [FKM⁺04] (finite stream)



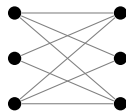
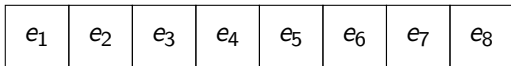
Sliding Window [CMS13] (infinite stream)



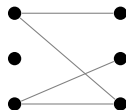
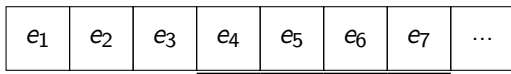
Dynamic [AGM12]



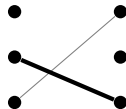
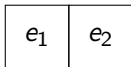
Insertion-Only [FKM⁺04] (finite stream)



Sliding Window [CMS13] (infinite stream)

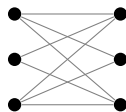


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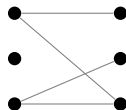
Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
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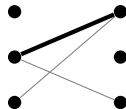
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...
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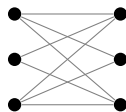
Dynamic [AGM12]

e_1	e_2	e_3
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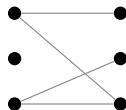
Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
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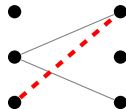
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...
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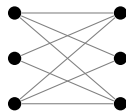
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$
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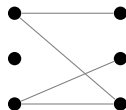
Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
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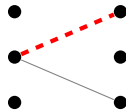
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...
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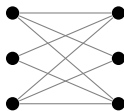
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$
-------	-------	-------	------------------	------------------



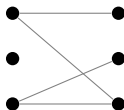
Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
-------	-------	-------	-------	-------	-------	-------	-------



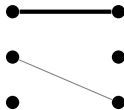
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...
-------	-------	-------	-------	-------	-------	-------	-----



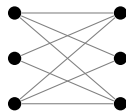
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4
-------	-------	-------	------------------	------------------	-------



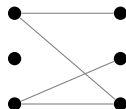
Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
-------	-------	-------	-------	-------	-------	-------	-------



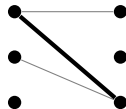
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...
-------	-------	-------	-------	-------	-------	-------	-----



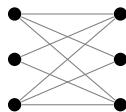
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4	e_5
-------	-------	-------	------------------	------------------	-------	-------



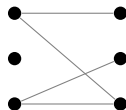
Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
-------	-------	-------	-------	-------	-------	-------	-------



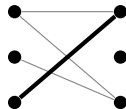
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	...
-------	-------	-------	-------	-------	-------	-------	-----



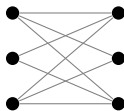
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4	e_5	e_1
-------	-------	-------	------------------	------------------	-------	-------	-------



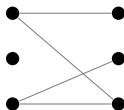
Insertion-Only [FKM⁺04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
-------	-------	-------	-------	-------	-------	-------	-------



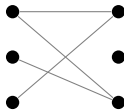
Sliding Window [CMS13] (infinite stream)

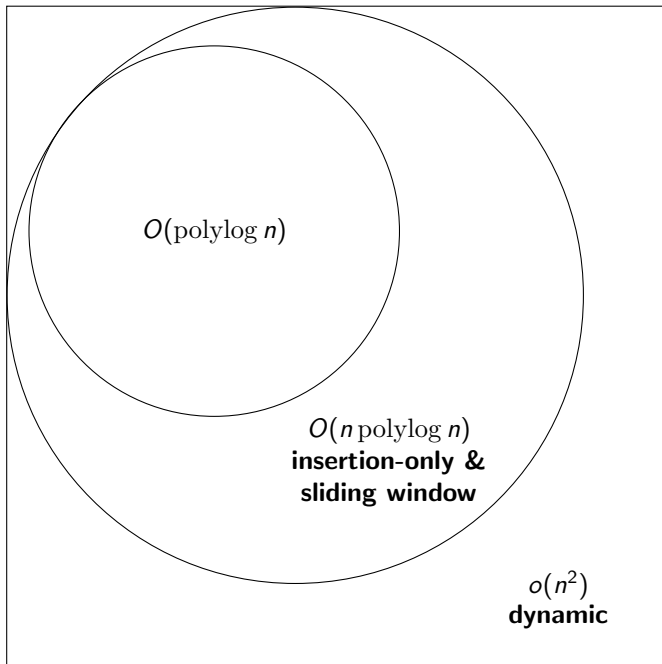
e_1	e_2	e_3	e_4	e_5	e_6	e_7	...
-------	-------	-------	-------	-------	-------	-------	-----



Dynamic [AGM12] (finite stream)

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4	e_5	e_1
-------	-------	-------	------------------	------------------	-------	-------	-------





- 1 Introduction
- 2 Streaming Models
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- 4 Space Optimal Vertex Cover

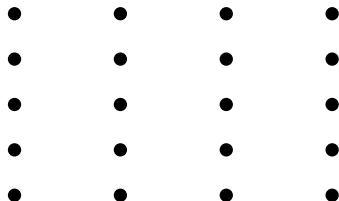
Simple and Powerful

GREEDY Matching:

- ① Add edge if
neither endpoint
is matched

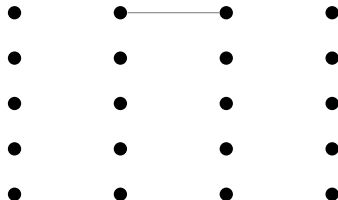
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is matched



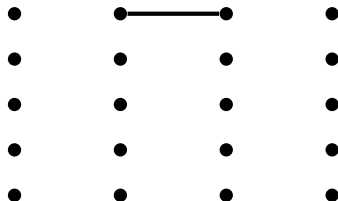
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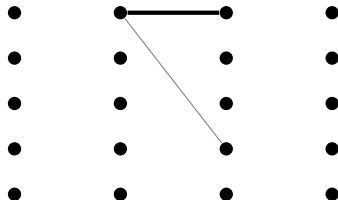
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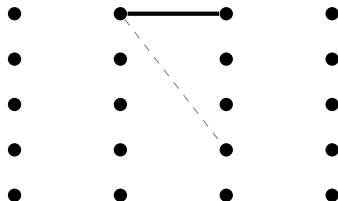
GREEDY Matching:

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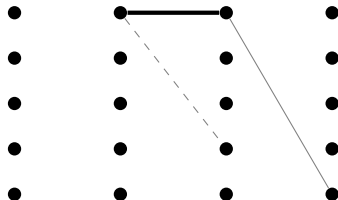
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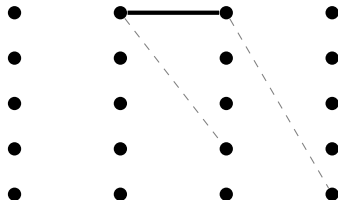
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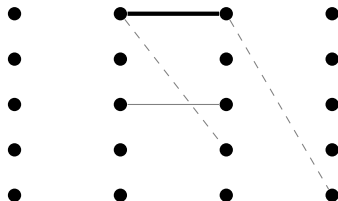
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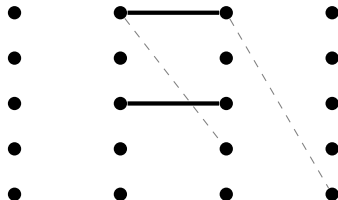
GREEDY Matching:

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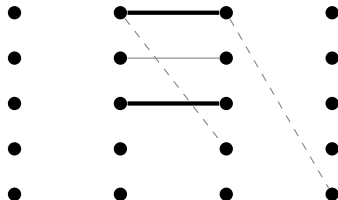
GREEDY Matching:

- ① Add edge if
neither endpoint
is matched



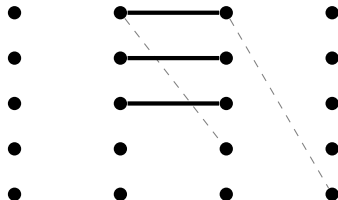
GREEDY Matching:

- ① Add edge if
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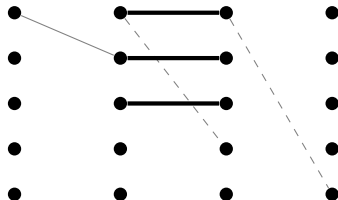
GREEDY Matching:

- ① Add edge if
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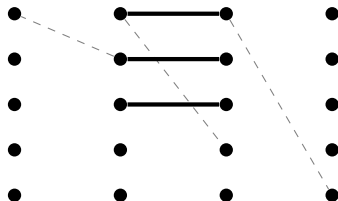
GREEDY Matching:

- 1 Add edge if neither endpoint is matched



GREEDY Matching:

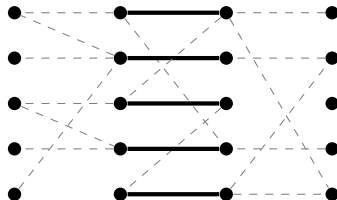
- ① Add edge if neither endpoint is matched



Simple and Powerful

GREEDY Matching:

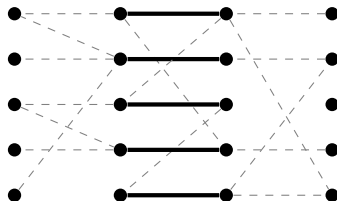
- ① Add edge if neither endpoint is matched



Simple and Powerful

GREEDY Matching:

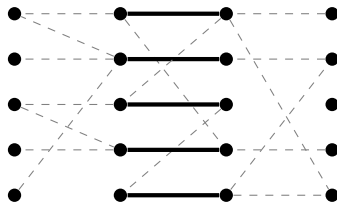
- ① Add edge if neither endpoint is matched
- Maximal



Simple and Powerful

GREEDY Matching:

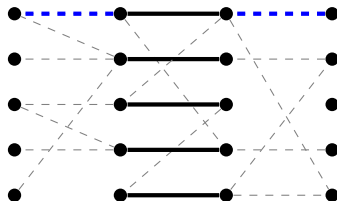
- ① Add edge if neither endpoint is matched
- Maximal
- 2-approximation



Simple and Powerful

GREEDY Matching:

- ① Add edge if neither endpoint is matched
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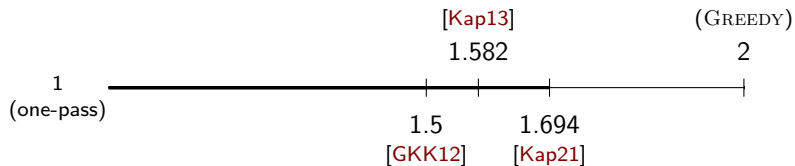


Insertion-Only

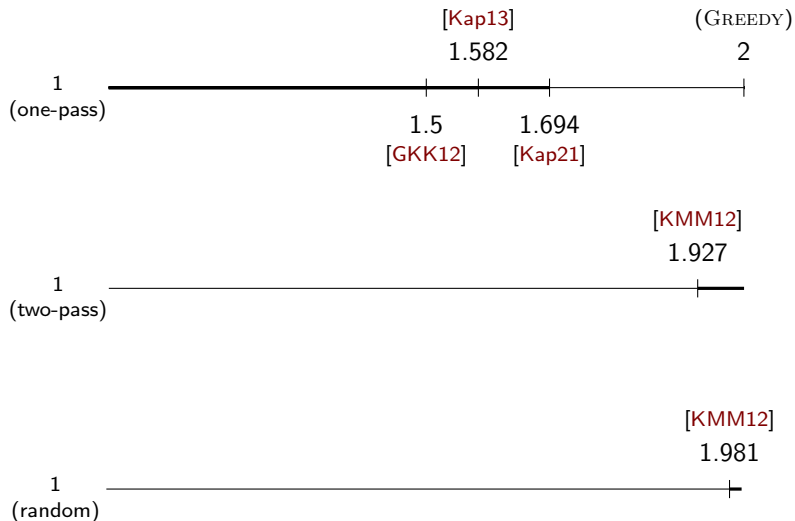
Insertion-Only



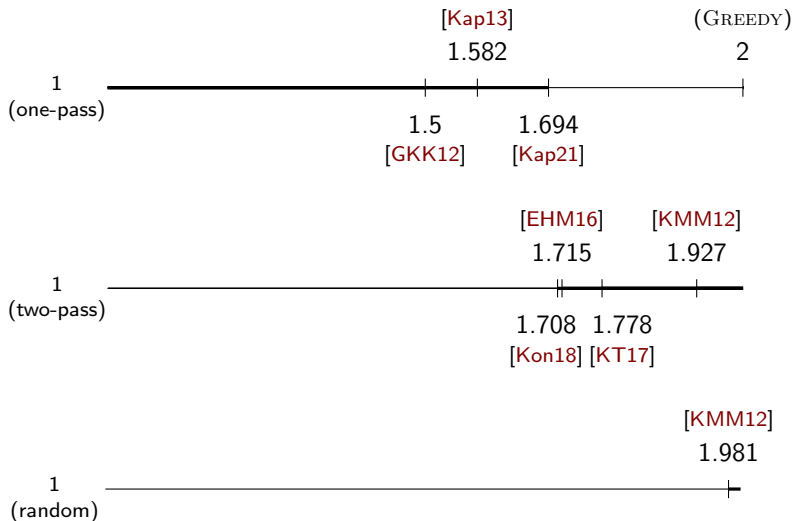
Insertion-Only



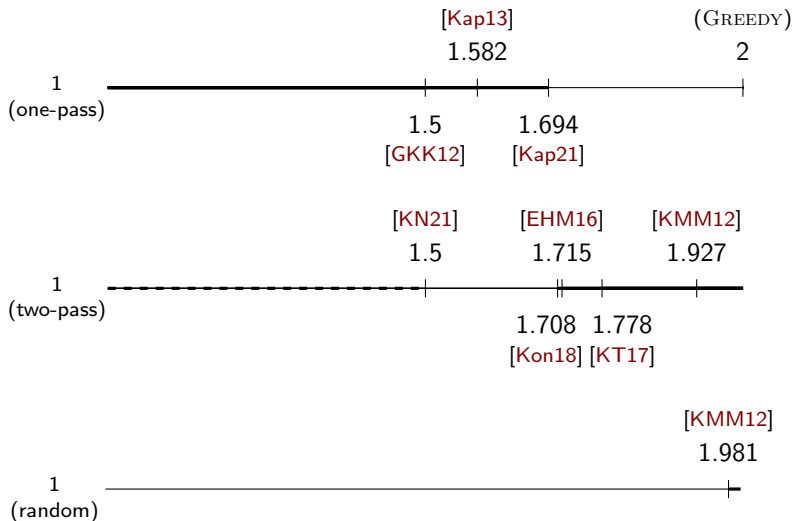
Insertion-Only



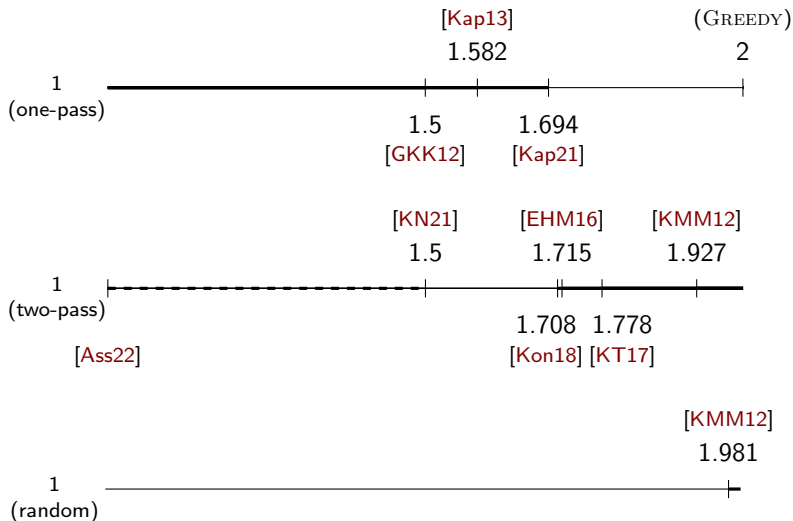
Insertion-Only



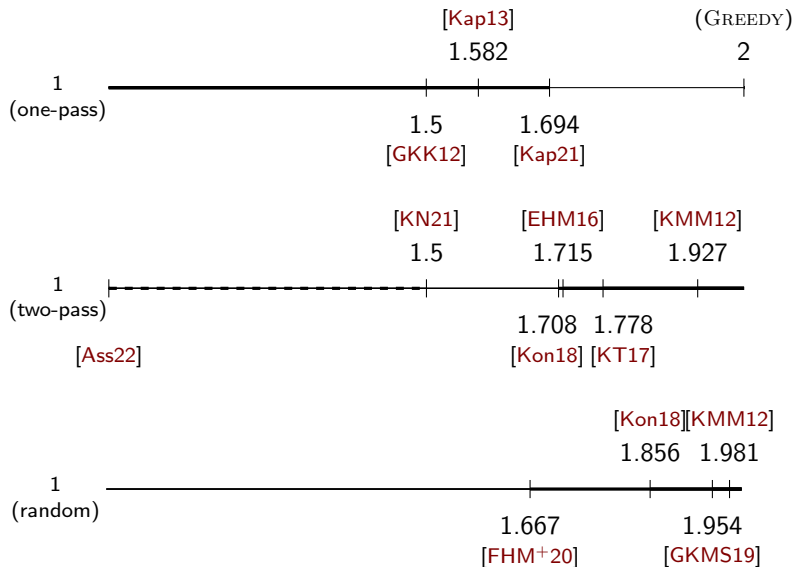
Insertion-Only



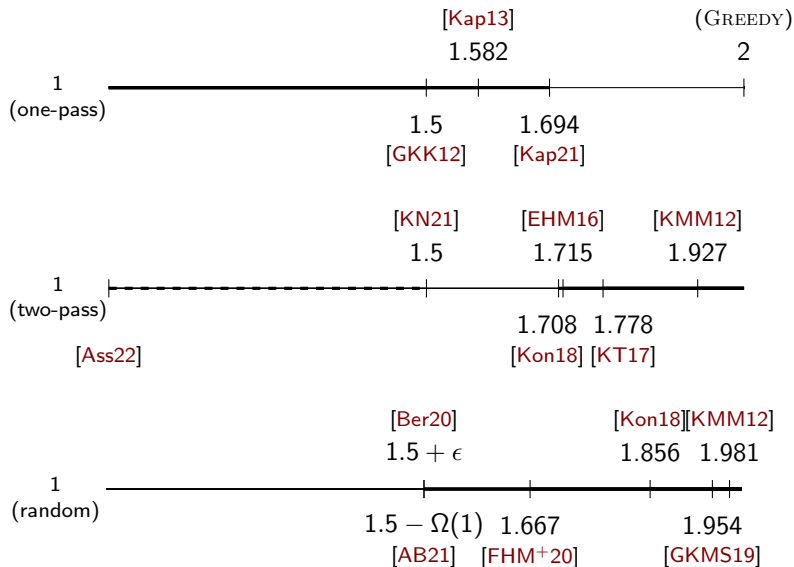
Insertion-Only



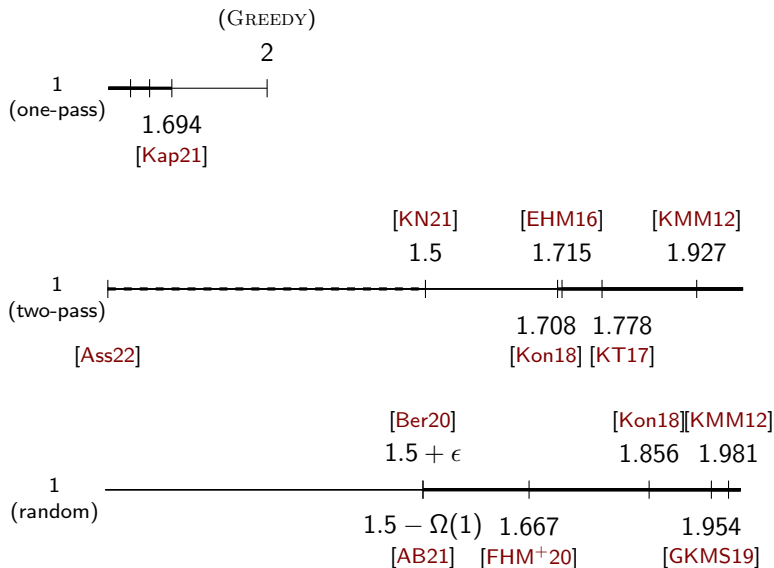
Insertion-Only



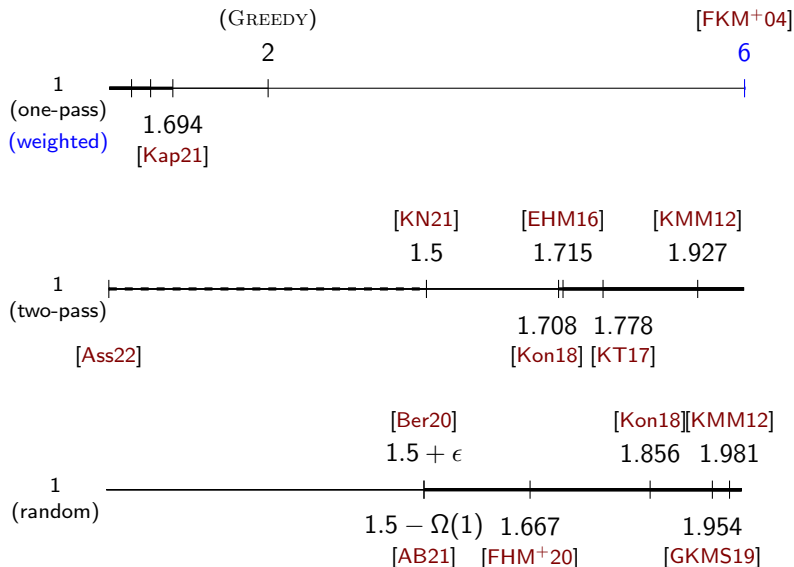
Insertion-Only



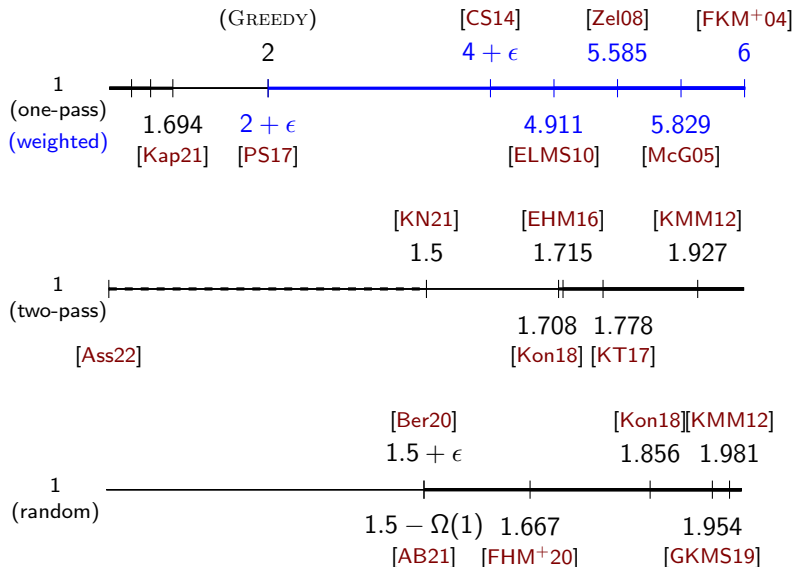
Insertion-Only



Insertion-Only



Insertion-Only



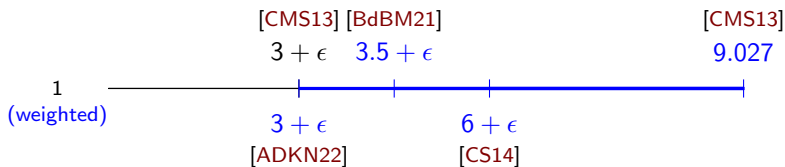
Sliding Window



Sliding Window



Sliding Window



Dynamic (Insertion-Deletion)

Goal:

- Find an α -approximation

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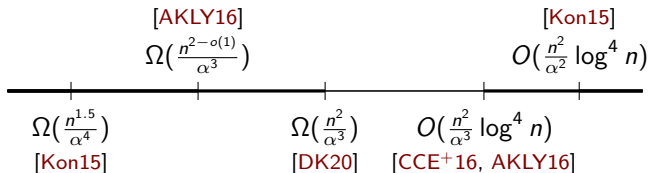
- Find an α -approximation



Dynamic (Insertion-Deletion)

Goal:

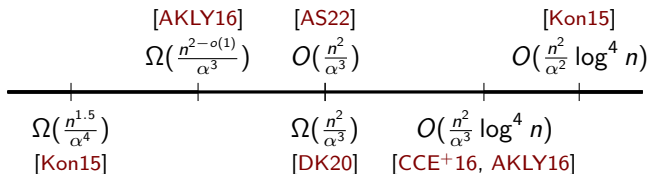
- Find an α -approximation



Dynamic (Insertion-Deletion)

Goal:

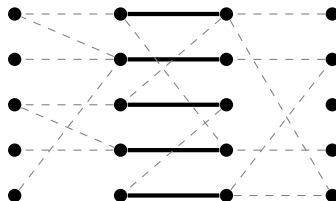
- Find an α -approximation



Relation to Vertex Cover

GREEDY Matching:

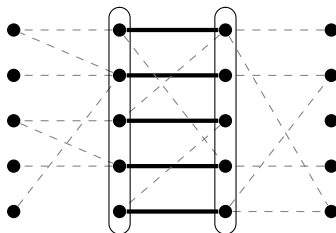
- Maximal
- 2-approximation



Relation to Vertex Cover

GREEDY Vertex Cover:

- 2-approximation



Relation to Vertex Cover (Results)

	matching	vertex cover
insertion-only	$[1.694, 2]$	
sliding window	$[1, 3 + \epsilon]$	
dynamic	$\Theta(\frac{n^2}{\alpha^3})$	

Relation to Vertex Cover (Results)

	matching	vertex cover
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Relation to Vertex Cover (Results)

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Relation to Vertex Cover (Results)

	matching	vertex cover
insertion-only	$[1.694, 2]$	$[1, 2]$ GREEDY
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All UBs use GREEDY in some way!

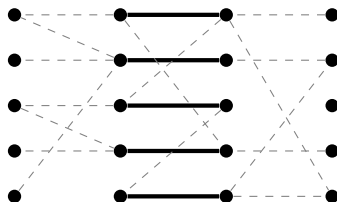
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What makes dynamic graph streams hard?

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GREEDY Matching:

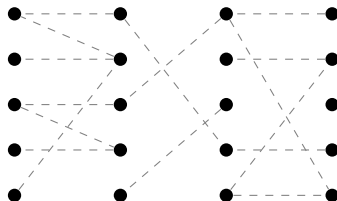
- Maximal
- 2-approximation



What makes dynamic graph streams hard?

GREEDY Matching:

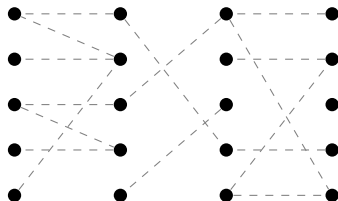
- **Not** Maximal
- **0**-approximation



What makes dynamic graph streams hard?

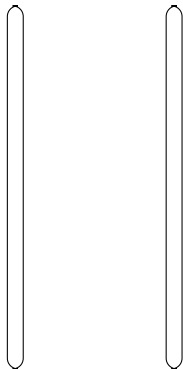
GREEDY Matching:

- **Not** Maximal
- **0**-approximation

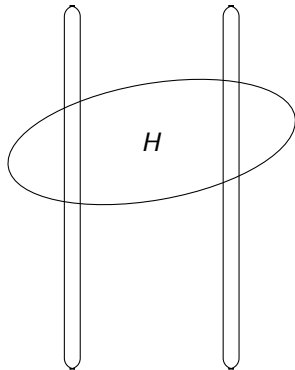


Note: Deterministically returning a single edge requires $\Omega(n^2)$ bits of space for dense graphs.

What techniques are used?

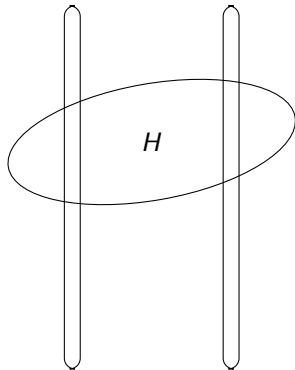


What techniques are used?



For a subgraph H of G with m edges:

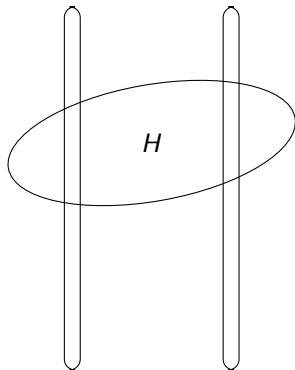
What techniques are used?



For a subgraph H of G with m edges:

- ① Check if H is empty
 - A counter using $\Theta(\log m)$ bits

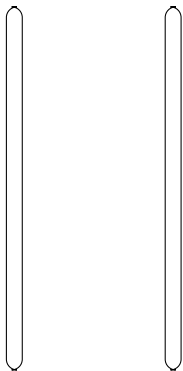
What techniques are used?



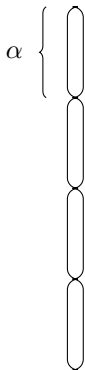
For a subgraph H of G with m edges:

- ① Check if H is empty
 - A counter using $\Theta(\log m)$ bits
- ② Retrieve an edge if one is present
 - An L_0 -sampler using $\Theta(\log^3 m)$ bits
 - Neighbourhood sampler

α -Approx Det. Dynamic Vertex Cover [DK20]



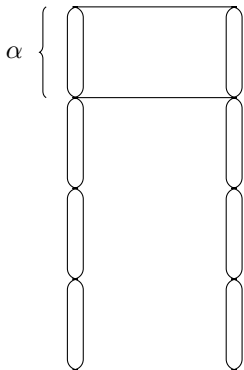
α -Approx Det. Dynamic Vertex Cover [DK20]



① Vertex groups of size α

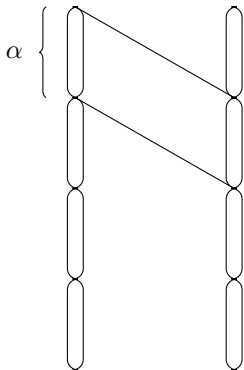
- about $\frac{n}{\alpha}$ groups

α -Approx Det. Dynamic Vertex Cover [DK20]



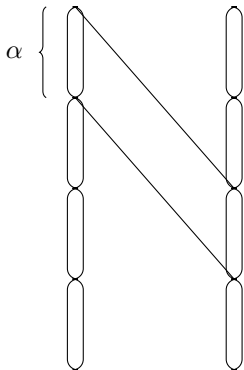
- ① Vertex groups of size α
 - about $\frac{n}{\alpha}$ groups
- ② Check if there is an edge between each pair of groups
 - about $\frac{n^2}{\alpha^2}$ pairs

α -Approx Det. Dynamic Vertex Cover [DK20]



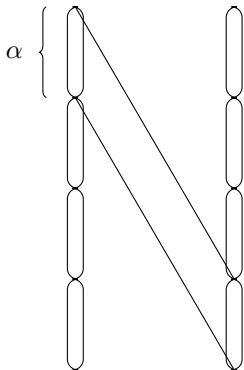
- ① Vertex groups of size α
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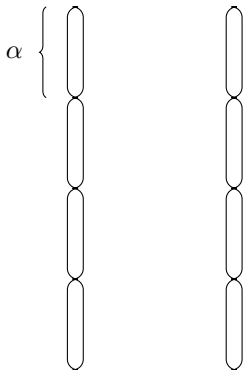
- ① Vertex groups of size α
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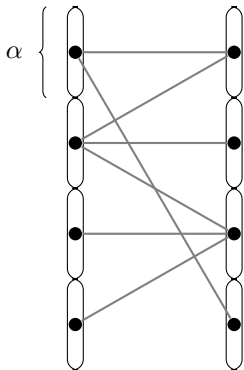
- ① Vertex groups of size α
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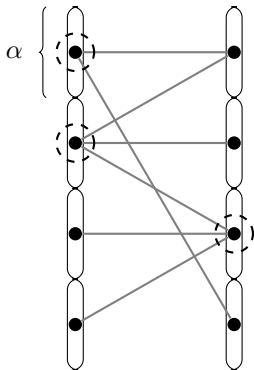
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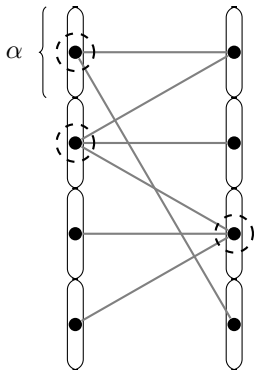
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α -Approx Det. Dynamic Vertex Cover [DK20]



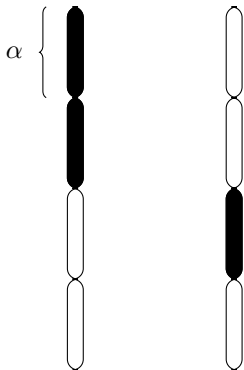
- ① Vertex groups of size α
 - about $\frac{n}{\alpha}$ groups
- ② Check if there is an edge between each pair of groups
 - about $\frac{n^2}{\alpha^2}$ pairs
- ③ Return vertices of the group-level vertex cover

α -Approx Det. Dynamic Vertex Cover [DK20]



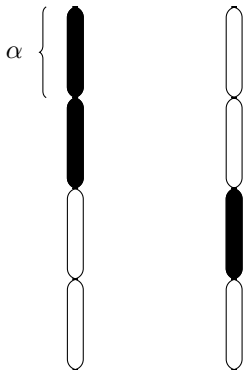
- ① Vertex groups of size α
 - about $\frac{n}{\alpha}$ groups
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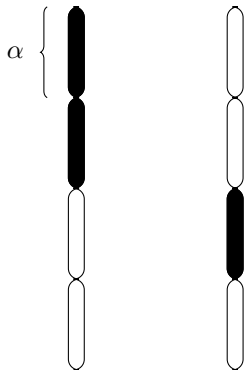
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Space: $O(\frac{n^2}{\alpha^2})$ counters, each using $O(\log \alpha)$ bits. Hence, $O(\frac{n^2}{\alpha^2} \log \alpha)$ bits.

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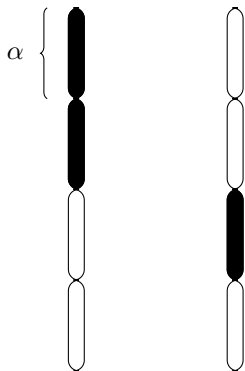
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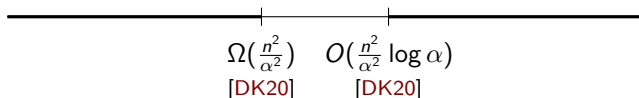
[DK20]

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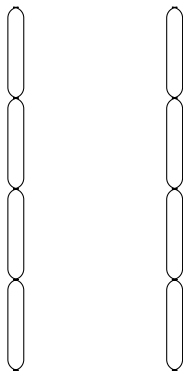


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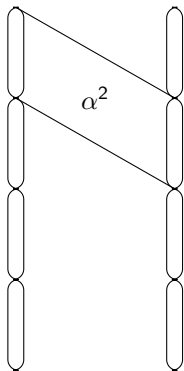
What's the issue?



What's the issue?

Problem:

- Counters use $O(\log \alpha)$ bits.
- Each counter counts upto α^2 edges.



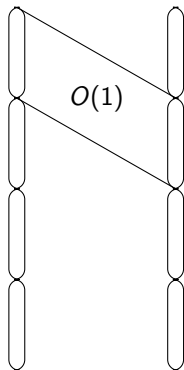
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- Counters to use $O(1)$ bits.
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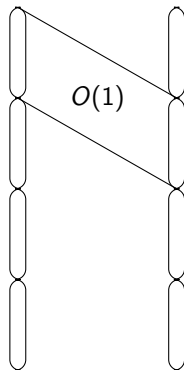
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- Sparse graph
- Randomly partition



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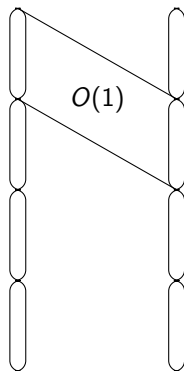
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How?

- Sparse graph
- Randomly partition (easy)



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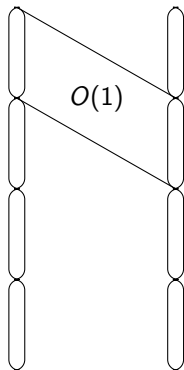
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Goal:

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How?

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- Randomly partition (easy)



Sparsness properties of GREEDY

Lemma

Let G be a n -vertex graph with m edges and let M_s be a GREEDY matching on $s \leq m$ uniform randomly sampled edges. Then, for $G_R = G[V \setminus V(M_s)]$,

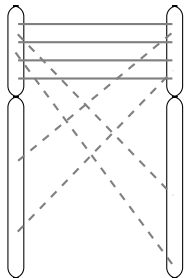
$$\Delta(G_R) \leq \frac{C \cdot m \cdot \log n}{s}.$$

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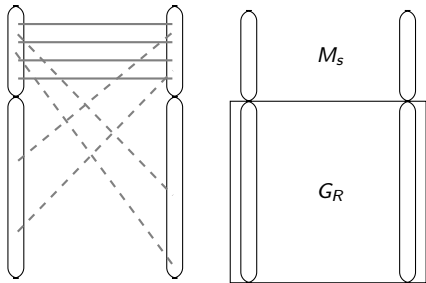


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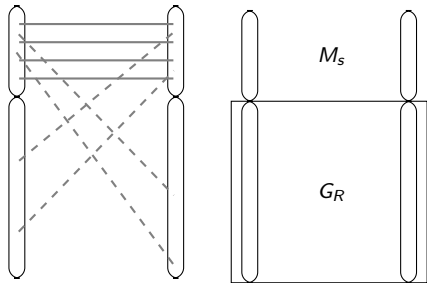


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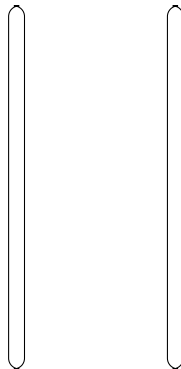
Proof (sketch).

For any vertex $v \in G$,

- $v \in G_R$
- $\deg_{G_R}(v) > \frac{C \cdot m \cdot \log n}{s}$

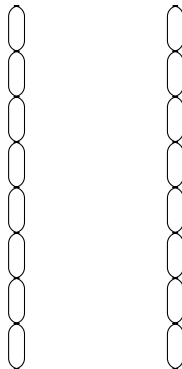
both do not occur w.h.p. □

Algorithm



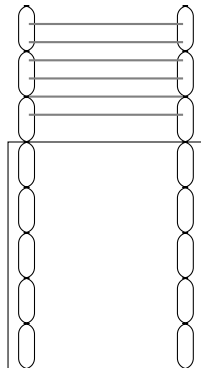
Algorithm

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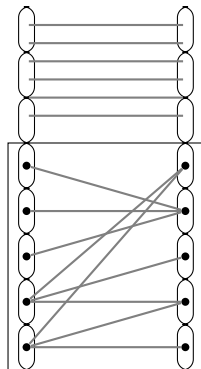
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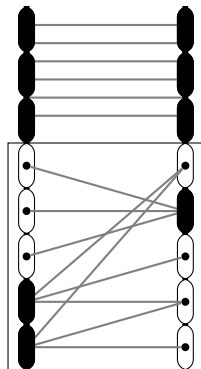
Algorithm

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- ③ Check if an edge is present between pairs and compute group-level vertex cover
- ④ Return vertices of the covering groups including those with matched vertices



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Extension:

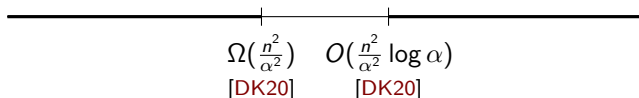
- Non-uniform sampling instead of uniform sampling, i.e., using neighbourhood edge sampling [AS22]
- Results are an average degree bound instead of max degree bound
- Full range, i.e., $\alpha \ll n$

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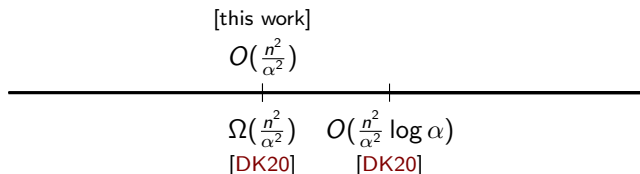


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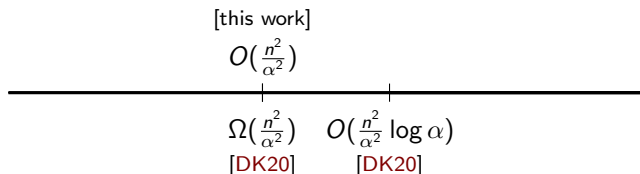


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Open Questions:

- Deterministic techniques or LB instead
- Other problems like dominating set and spectral sparsification

Thank You

Questions?

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