An Unconditional Lower Bound for Two-Pass Streaming Algorithms for Maximum Matching Approximation

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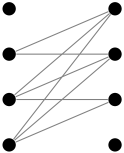
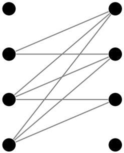
Maximum Bipartite Matching (MBM)

Definition

A bipartite **matching** is a subset of vertex disjoint edges of the graph. A **maximum matching** is a largest matching of the graph.





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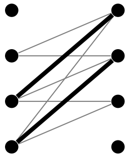
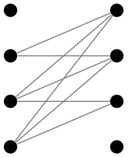


M
matching



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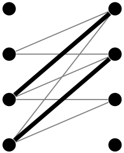

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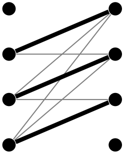

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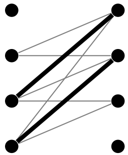

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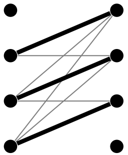

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Approximations

- M is a $(\frac{|M|}{|M^*|})$-approximate matching (e.g. $\frac{2}{3}$).

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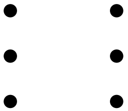

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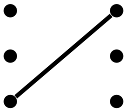
$\boxed{e_1}$



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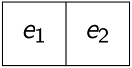

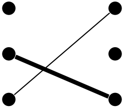

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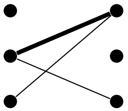

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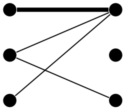

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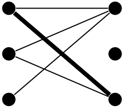

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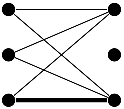

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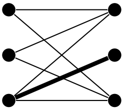

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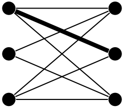

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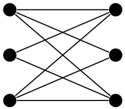

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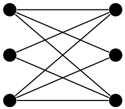

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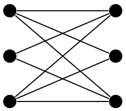

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Interesting Algorithms

- Use $O(n \operatorname{polylog} n)$ space (semi-streaming).
  - Many graph problems require $\Omega(n)$ space in one pass [FKM+04].
- Use one or more passes of the stream.

Approximate MBM in the Semi-Streaming Model

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(GREEDY)

0.5

One-Pass 1

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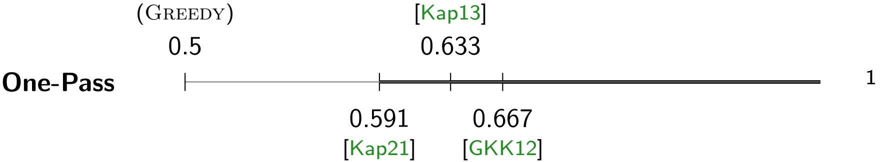

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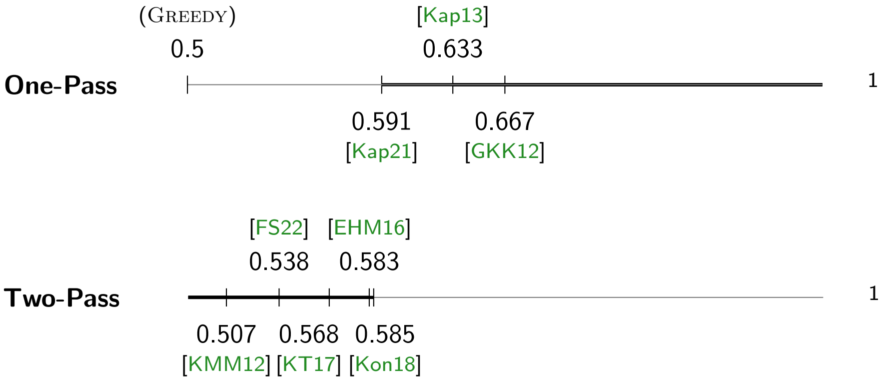

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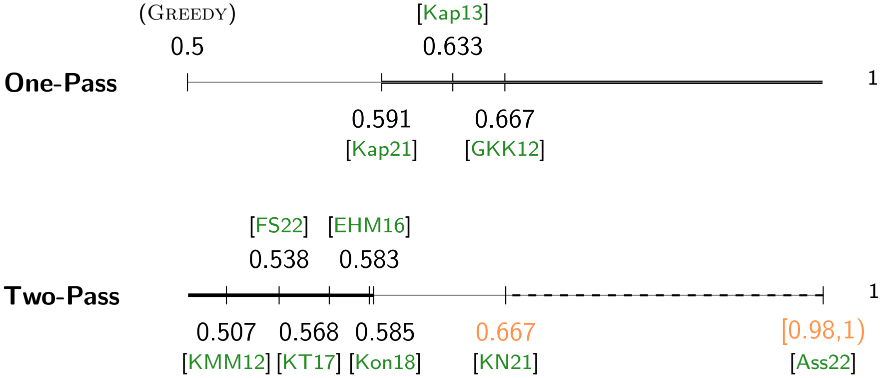

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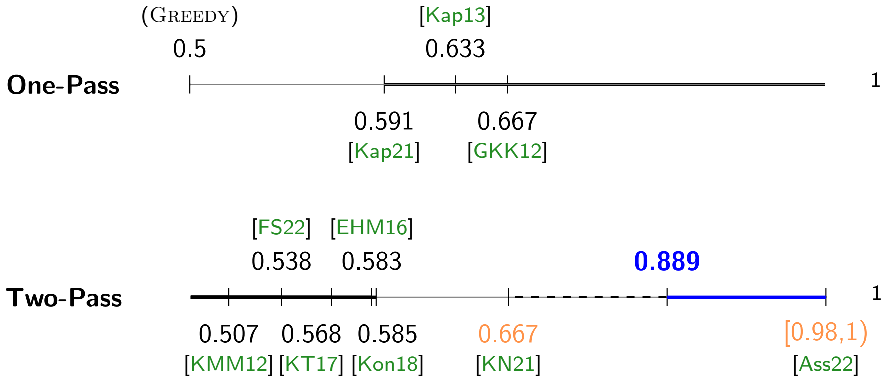

Our Result

For $\varepsilon > 0$, any constant-error two-pass $(\frac{8}{9} + \varepsilon)$-approximation streaming algorithm for MBM requires $n^{1+\Omega(1/(\log\log n)^2)}$ space.

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E_A	E_B

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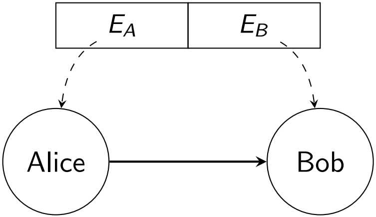

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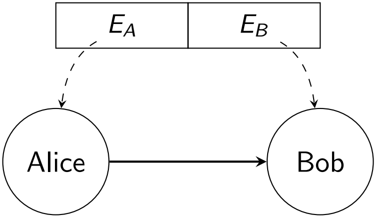

$X \in \{0,1\}^N$ $\qquad$ $J \in [N]$

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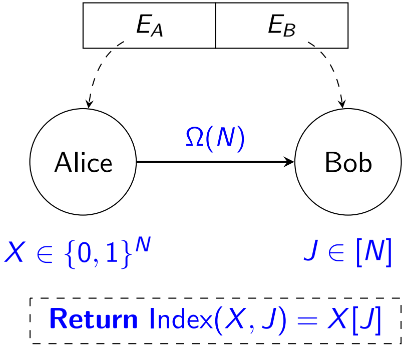

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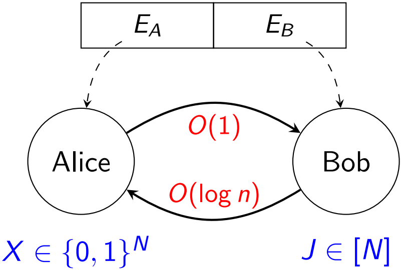

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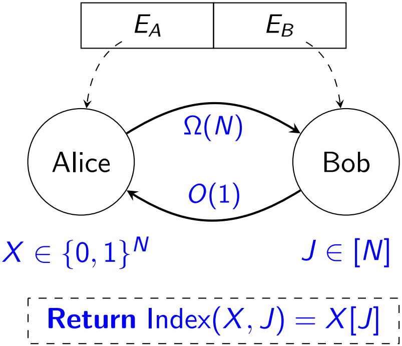
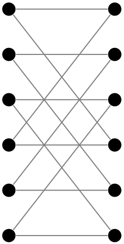


Info Cost Tradeoff [JRS09]

1. If $\text{ICost}^B_{\mathcal{D}}(\pi) = O(1)$, then
2. $\text{ICost}^A_{\mathcal{D}}(\pi) = \Omega(N)$

Ruzsa-Szeméredi (RS) Graphs



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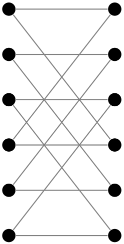

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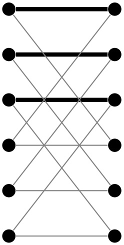

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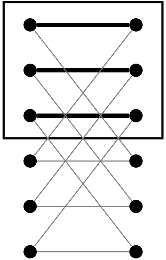

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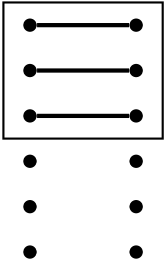

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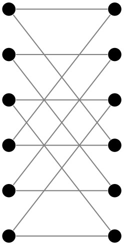

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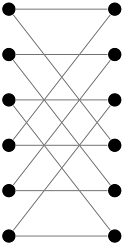

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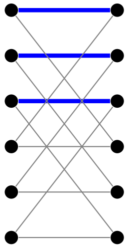

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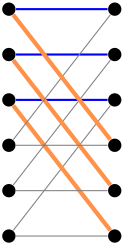

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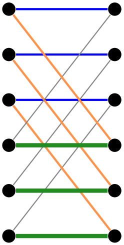

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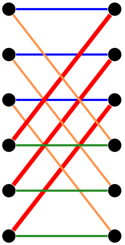

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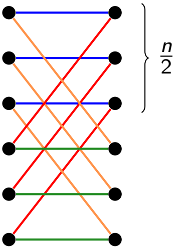

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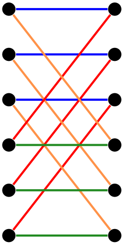
For constant $\delta > 0$, there exists a (bipartite) (r, t)-RS graph on $2n$ vertices where $r = (\frac{1}{2} - \delta) \cdot n$ and $t = n^{\Omega(1/\log\log n)}$.



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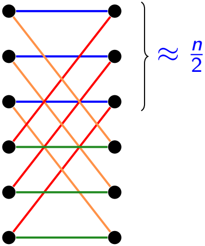

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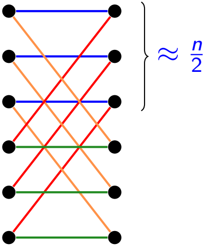

- $n^{\Omega(1/\log\log n)}$ many induced matchings.

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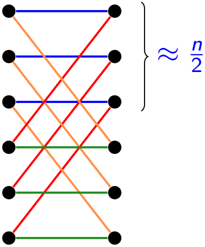
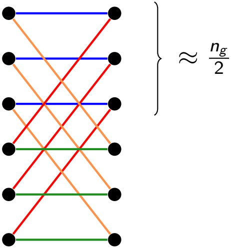


- $n^{\Omega(1/\log\log n)}$ many induced matchings.
- $\gg n \operatorname{polylog} n$ edges in the graph.

Two-Pass Hard Graph and Adversarial Stream

Two-Pass Hard Graph and Adversarial Stream

Global Graph



- $(2n_g)$-vertex (r_g, t_g)-RS graph.

Two-Pass Hard Graph and Adversarial Stream

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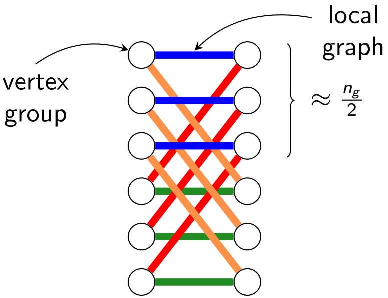

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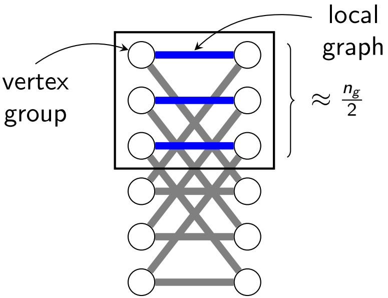

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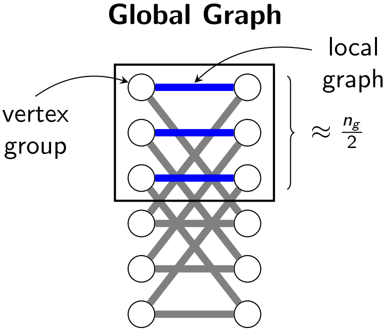
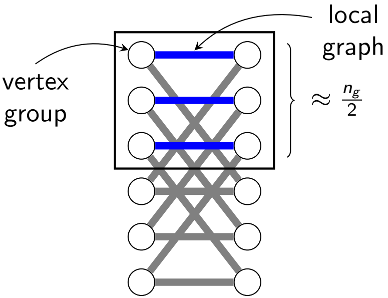
Two-Pass Hard Graph and Adversarial Stream



- $(2n_g)$-vertex (r_g, t_g)-RS graph.
- $\Theta(n_g)$ small hard instances

Two-Pass Hard Graph and Adversarial Stream

Global Graph



Q: What is the selector for MBM?

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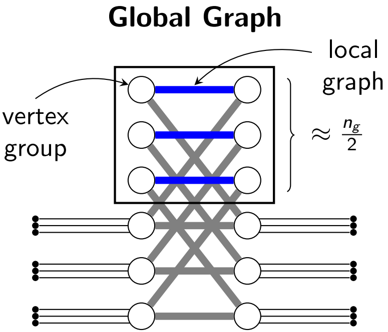
Two-Pass Hard Graph and Adversarial Stream



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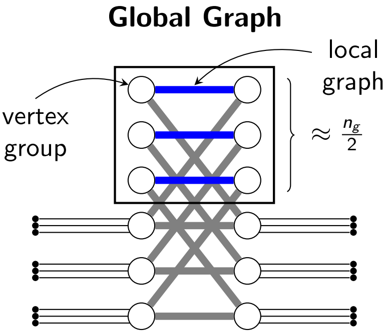
Two-Pass Hard Graph and Adversarial Stream



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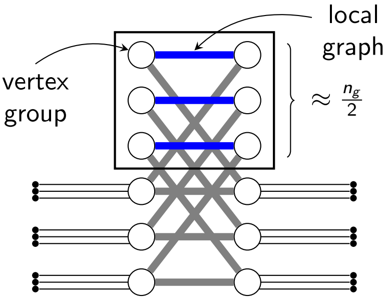

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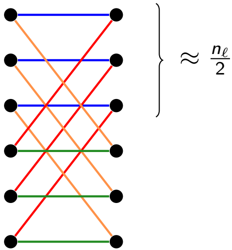

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Two-Pass Hard Graph and Adversarial Stream

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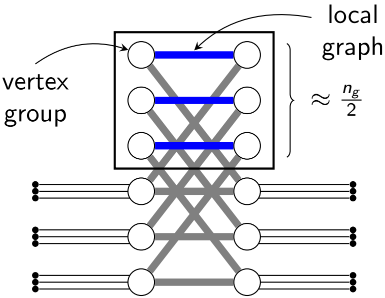

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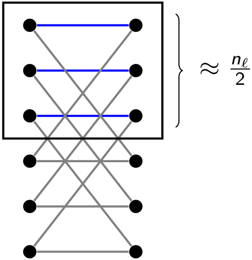

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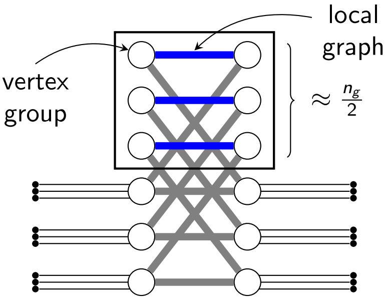

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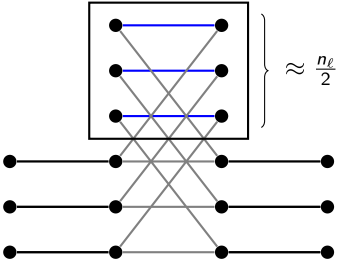

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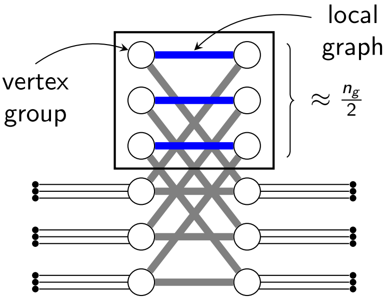
Two-Pass Hard Graph and Adversarial Stream

Global Graph



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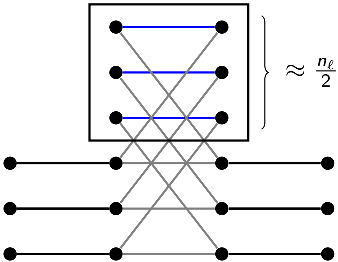

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Two-Pass Hard Graph and Adversarial Stream

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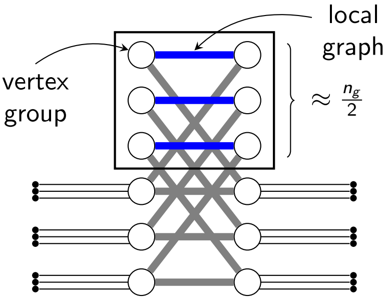

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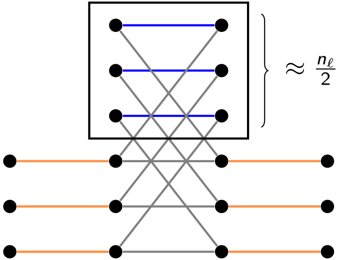

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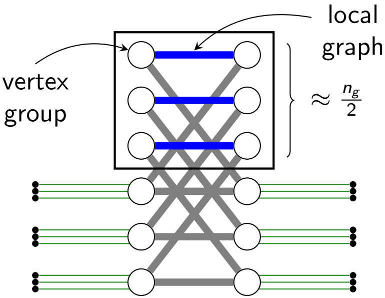

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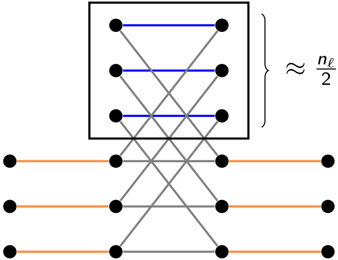

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Two-Pass Hard Graph and Adversarial Stream

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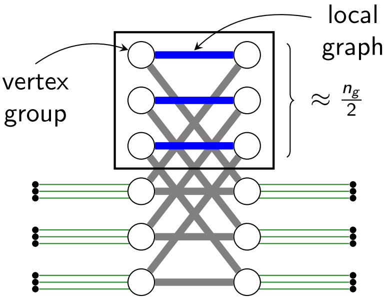

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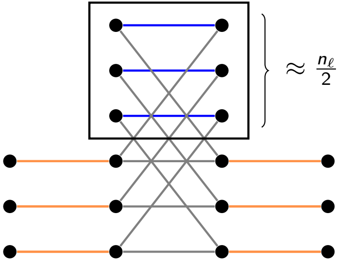

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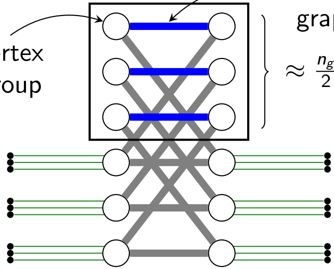
local RS-graphs	local selectors	global selector

Two-Pass Hard Graph and Adversarial Stream

Global Graph



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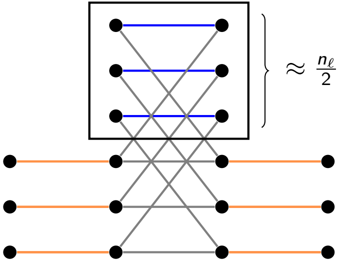

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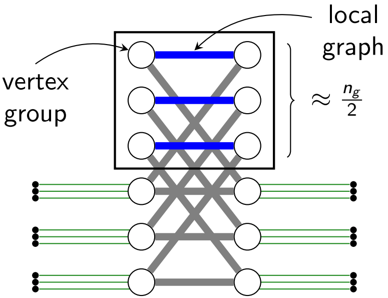
local RS-graphs	local selectors	global selector

$\mathcal{A}$

Two-Pass Hard Graph and Adversarial Stream

Global Graph



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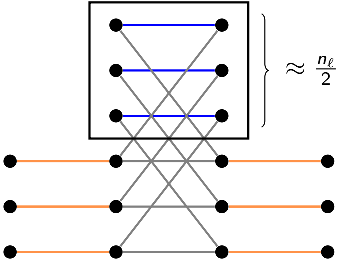

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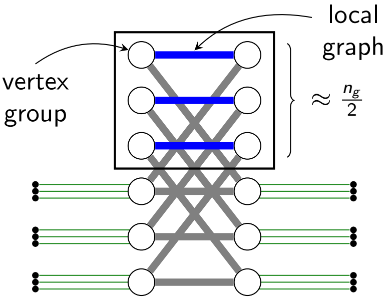
	local RS-graphs	local selectors	global selector
$\mathcal{A}$	$\gg\gg n \operatorname{polylog} n$	$\gg n \operatorname{polylog} n$	$O(n)$

Two-Pass Hard Graph and Adversarial Stream

Global Graph



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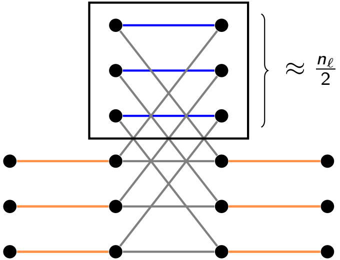

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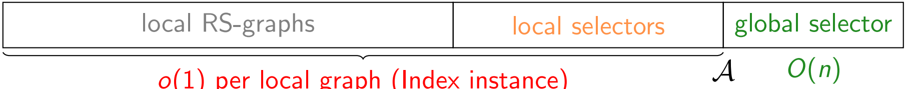
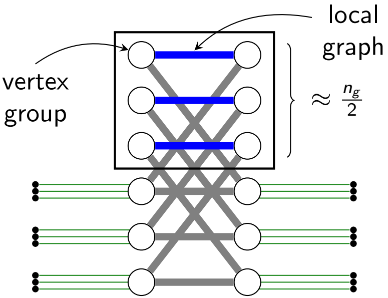


Two-Pass Hard Graph and Adversarial Stream

Global Graph



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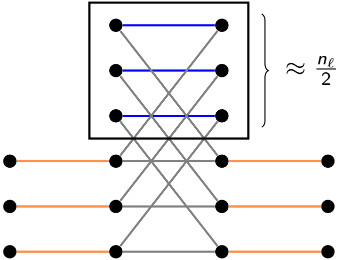

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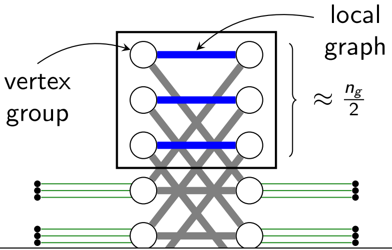
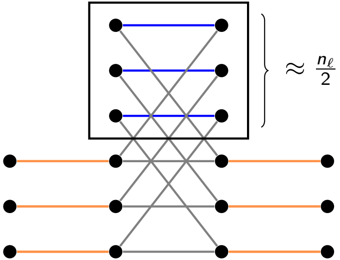


Local Graphs



Info Cost Tradeoff [JRS09]

1. If $\text{ICost}^{B}_{\mathcal{D}}(\pi) = O(1)$, then
2. $\text{ICost}^{A}_{\mathcal{D}}(\pi) = \Omega(N)$

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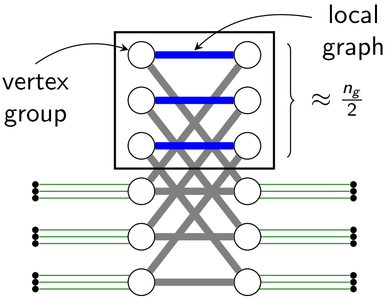
local RS-graphs	local selectors	global selector ✓

$\underbrace{\text{local RS-graphs} \quad \text{local selectors}}_{o(1) \text{ per local graph (Index instance)}}$

$\mathcal{A}$

Two-Pass Hard Graph and Adversarial Stream

Global Graph



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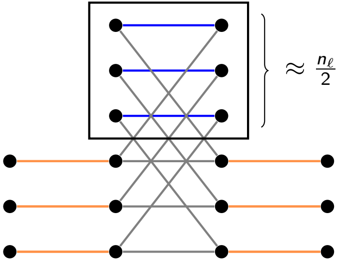

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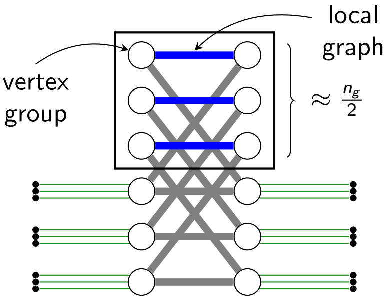

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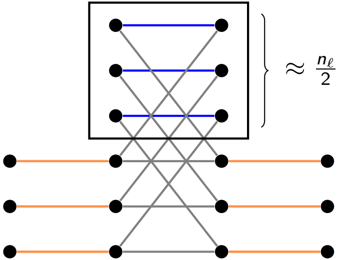

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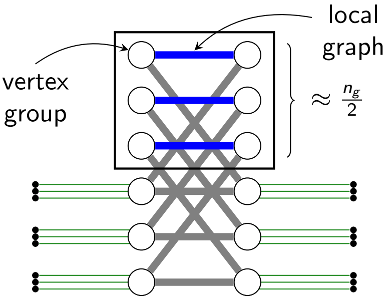

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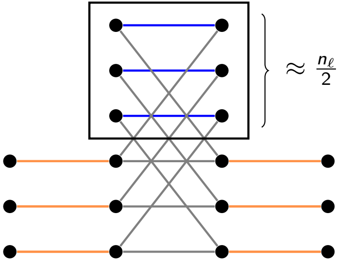

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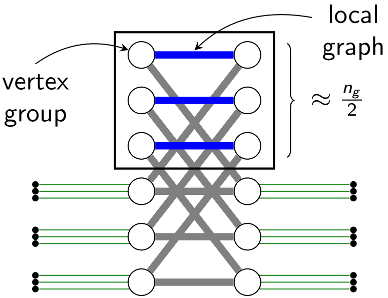
local RS-graphs	local selectors	global selector ✓

$\underbrace{\text{local RS-graphs}}_{o(1) \text{ fraction of edges}}$ $\mathcal{A}$ $O(n)$

Two-Pass Hard Graph and Adversarial Stream

Global Graph



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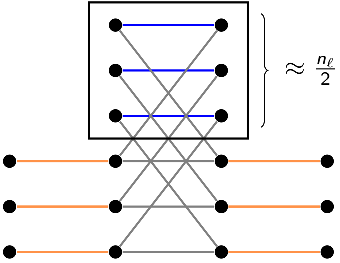

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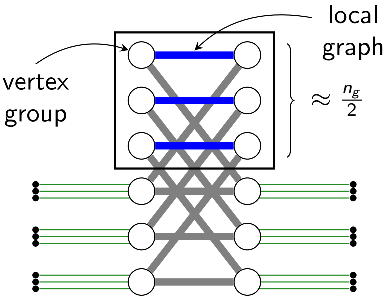
$\mathcal{A}$

Two-Pass Hard Graph and Adversarial Stream

Global Graph



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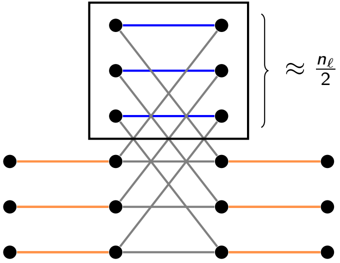

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local RS-graphs	local selectors ✓	global selector ✓

$o(1)$ fraction of edges — 8/9-approximation

Next Steps

Our Result

For $\varepsilon > 0$, any constant-error two-pass $(\frac{8}{9} + \varepsilon)$-approximation streaming algorithm for MBM requires $n^{1+\Omega(1/(\log\log n)^2)}$ space.

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Multiple Passes

- [AS23] recently showed that $(1-\varepsilon)$-approximations require $\Omega(\log 1/\varepsilon)$ passes (conditional)
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Open Questions

1. Narrow the gap for either the 1-pass, 2-pass or multi-pass settings.
2. How well do these ideas work for other graph problems?

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Thank you!

References I

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