Space Optimal Vertex Cover in Dynamic Streams
(with an overview of graph streaming)

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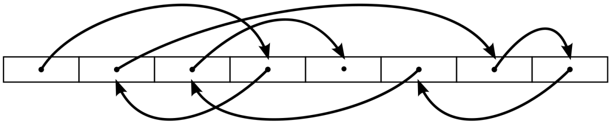
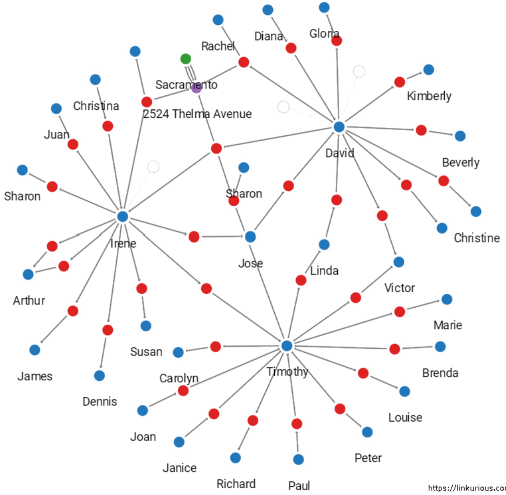
Joint work with Vihan Shah (Rutgers University)

Classic Setting

Assumption

Classical algorithms rely on the assumption that they have a random access to the input of the algorithm





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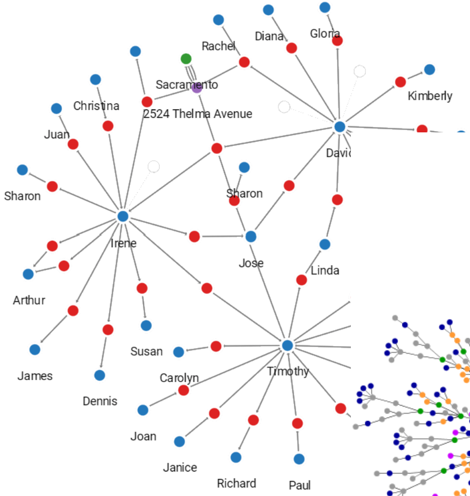
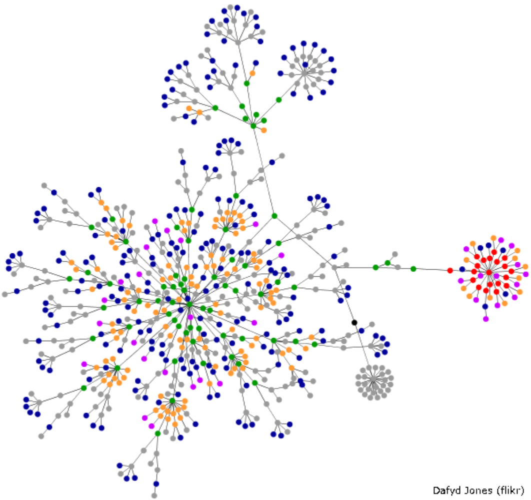
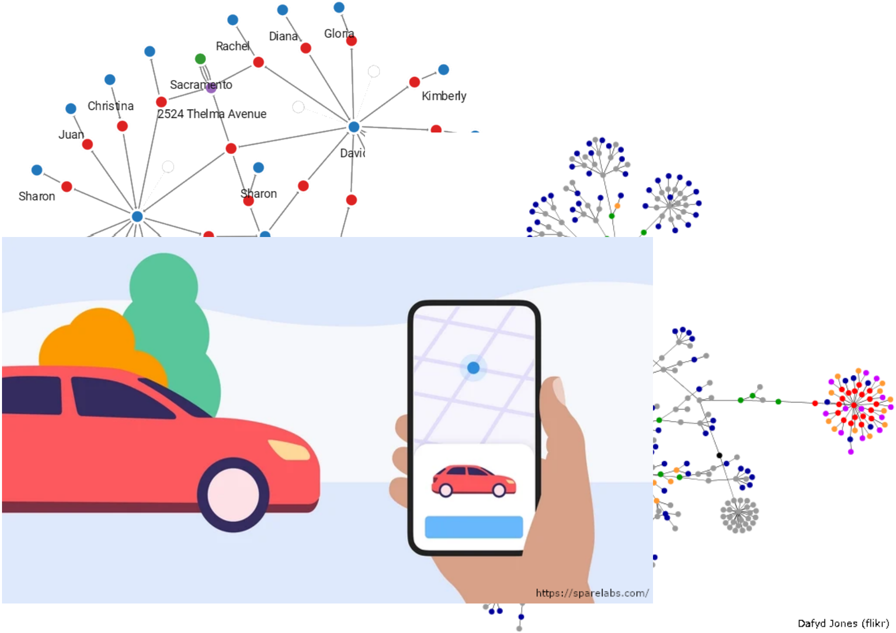




Dafyd Jones (flickr)

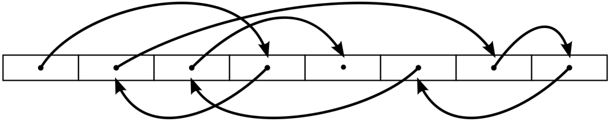


Dafyd Jones (flickr)

Dafyd Jones (flikr)

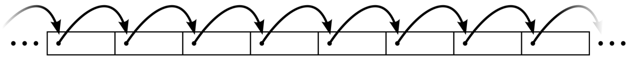
Assumption (Infeasible)

Classical algorithms rely on the assumption that they have a random access to the input of the algorithm



Definition (Graph Streaming)

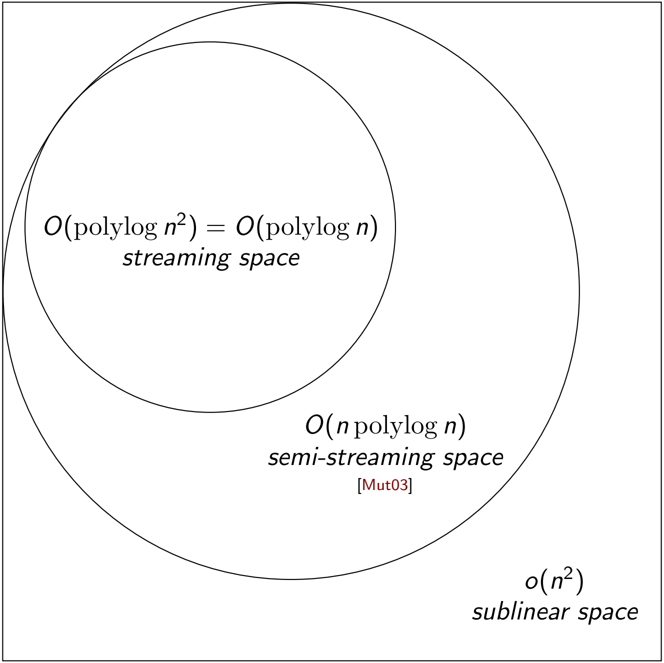
A n-vertex graph is presented as a sequence of edges to an algorithm uses space **sublinear** in the size of the input ($o(n^2)$).



$o(n^2)$

sublinear space


$O(\text{polylog}\, n^2) = O(\text{polylog}\, n)$
streaming space
$o(n^2)$
sublinear space

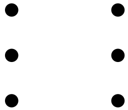

$O(\text{polylog}\, n^2) = O(\text{polylog}\, n)$
streaming space
$O(n\, \text{polylog}\, n)$
semi-streaming space
[Mut03]
$o(n^2)$
sublinear space

Insertion-Only [FKM+04]

Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM$^+$04]

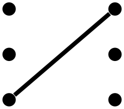


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM+04]

$\boxed{e_1}$



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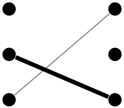
Dynamic [AGM12]

Insertion-Only [FKM+04]

e_1	e_2

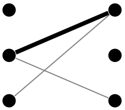


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM+04]

e_1	e_2	e_3

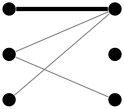


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM+04]

e_1	e_2	e_3	e_4

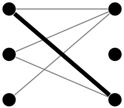


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM+04]

e_1	e_2	e_3	e_4	e_5

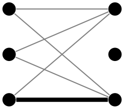


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM+04]

e_1	e_2	e_3	e_4	e_5	e_6



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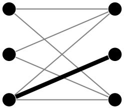
Dynamic [AGM12]

Insertion-Only [FKM+04]

e_1	e_2	e_3	e_4	e_5	e_6	e_7

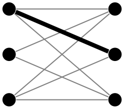


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM+04]

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8

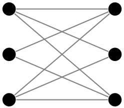


Sliding Window [CMS13]

Dynamic [AGM12]

Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



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Insertion-Only [FKM+04] (finite stream)

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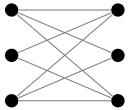

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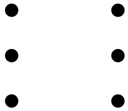

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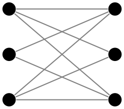
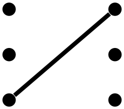
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



Sliding Window [CMS13]

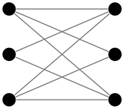
e_1



Dynamic [AGM12]

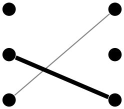
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



Sliding Window [CMS13]

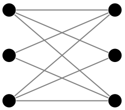
e_1	e_2



Dynamic [AGM12]

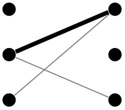
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



Sliding Window [CMS13]

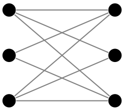
e_1	e_2	e_3



Dynamic [AGM12]

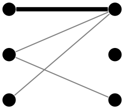
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



Sliding Window [CMS13]

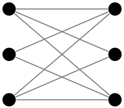
e_1	e_2	e_3	e_4



Dynamic [AGM12]

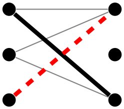
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



Sliding Window [CMS13]

e_1	e_2	e_3	e_4	e_5



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Insertion-Only [FKM+04] (finite stream)

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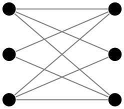
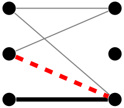


Sliding Window [CMS13]

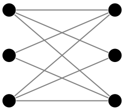
e_1	e_2	e_3	e_4	e_5	e_6



Dynamic [AGM12]

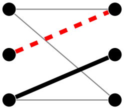
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



Sliding Window [CMS13]

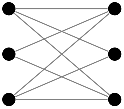
e_1	e_2	e_3	e_4	e_5	e_6	e_7



Dynamic [AGM12]

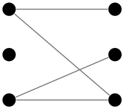
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



Sliding Window [CMS13] (infinite stream)

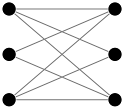
e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$



Dynamic [AGM12]

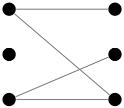
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$



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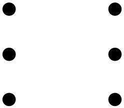
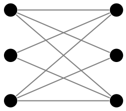


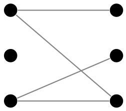
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



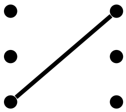
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$



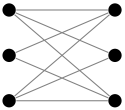
Dynamic [AGM12]

e_1



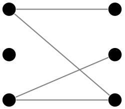
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



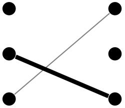
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$



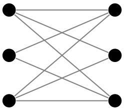
Dynamic [AGM12]

e_1	e_2



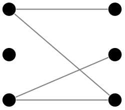
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



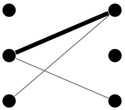
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$



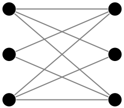
Dynamic [AGM12]

e_1	e_2	e_3



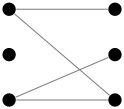
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



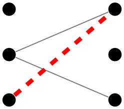
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$



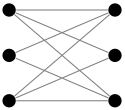
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$



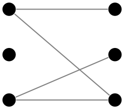
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



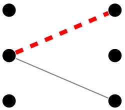
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	$\underline{e_4}$	$\underline{e_5}$	$\underline{e_6}$	$\underline{e_7}$	$\cdots$



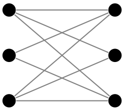
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$



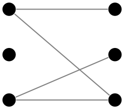
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



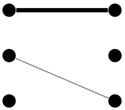
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$



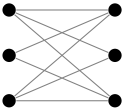
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4



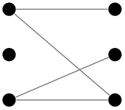
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



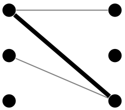
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$



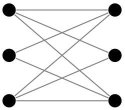
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4	e_5



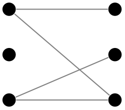
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



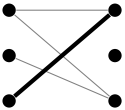
Sliding Window [CMS13] (infinite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$



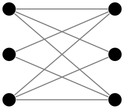
Dynamic [AGM12]

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4	e_5	e_1



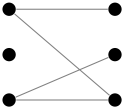
Insertion-Only [FKM+04] (finite stream)

e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8



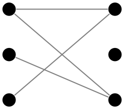
Sliding Window [CMS13] (infinite stream)

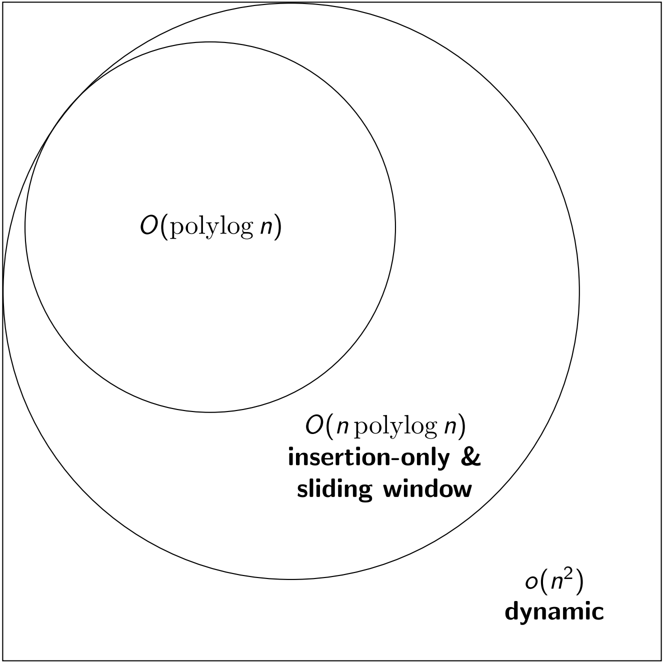
e_1	e_2	e_3	e_4	e_5	e_6	e_7	$\cdots$



Dynamic [AGM12] (finite stream)

e_1	e_2	e_3	$\overline{e_1}$	$\overline{e_3}$	e_4	e_5	e_1




$O(\mathrm{polylog}\, n)$
$O(n\, \mathrm{polylog}\, n)$
insertion-only &
sliding window
$o(n^2)$
dynamic

Simple and Powerful

Simple and Powerful

GREEDY Matching:

1. Add edge if neither endpoint is matched

Simple and Powerful

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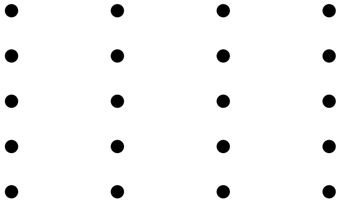

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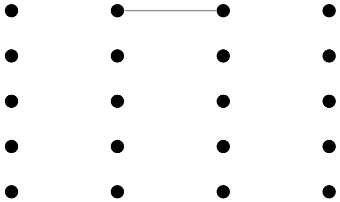

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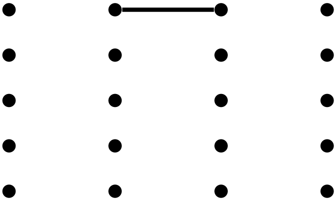

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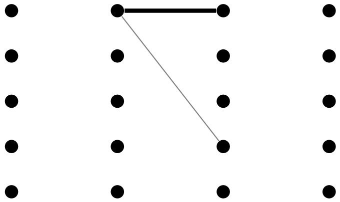

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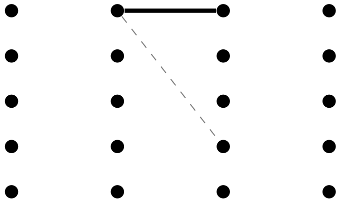

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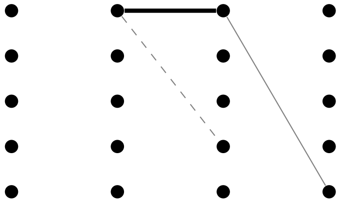

Simple and Powerful

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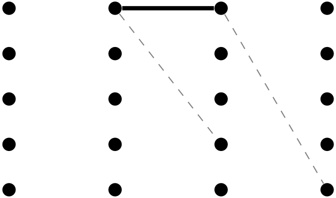
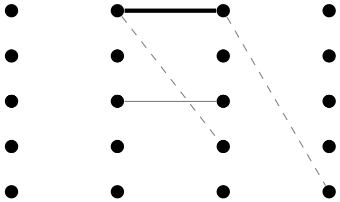


Simple and Powerful

GREEDY Matching:

1. Add edge if neither endpoint is matched



Simple and Powerful

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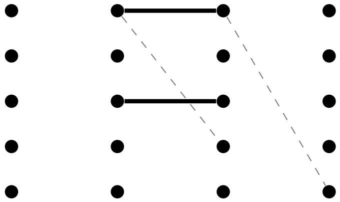
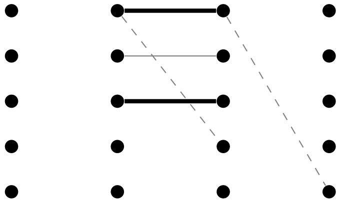


GREEDY Matching:

1. Add edge if neither endpoint is matched



Simple and Powerful

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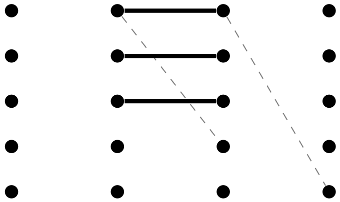

Simple and Powerful

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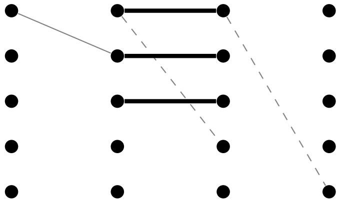

Simple and Powerful

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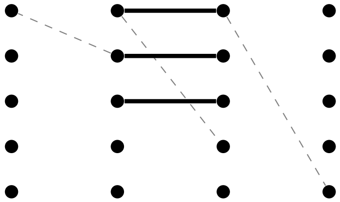
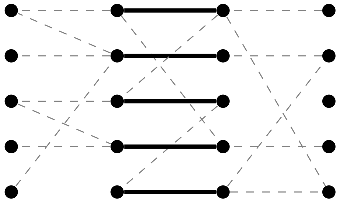


GREEDY Matching:

1. Add edge if neither endpoint is matched

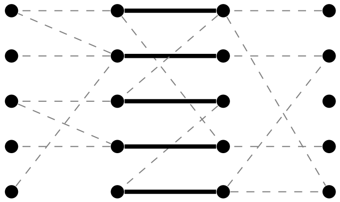


Simple and Powerful

GREEDY Matching:

1. Add edge if neither endpoint is matched

- Maximal

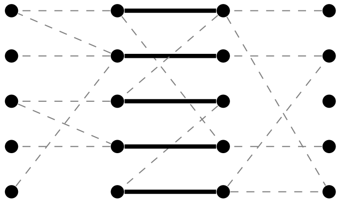


Simple and Powerful

GREEDY Matching:

1. Add edge if neither endpoint is matched

- Maximal
- 2-approximation



Simple and Powerful

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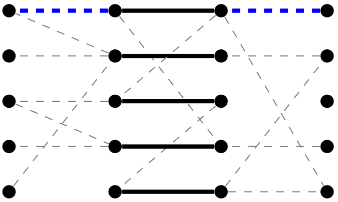
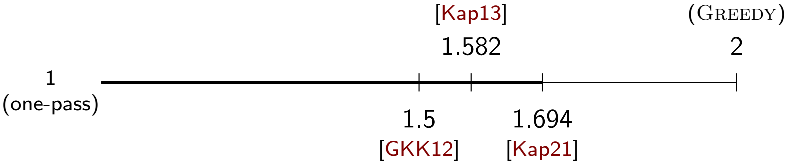
- 2-approximation



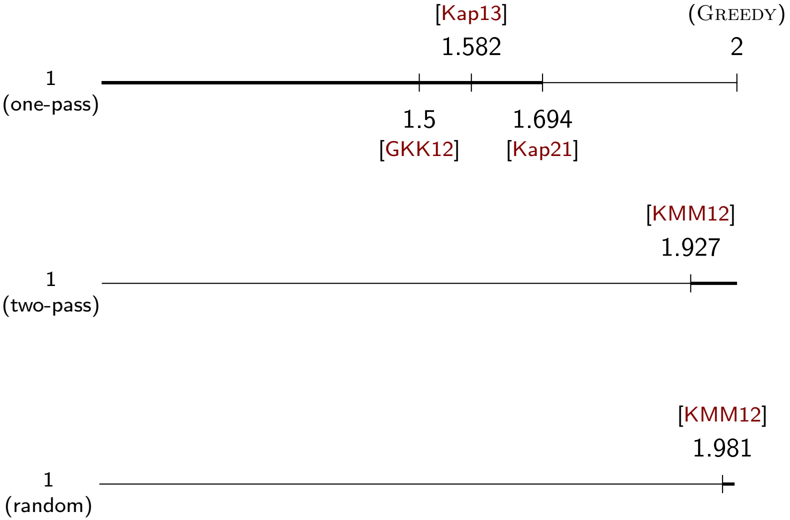
Insertion-Only

Insertion-Only

Insertion-Only



Insertion-Only



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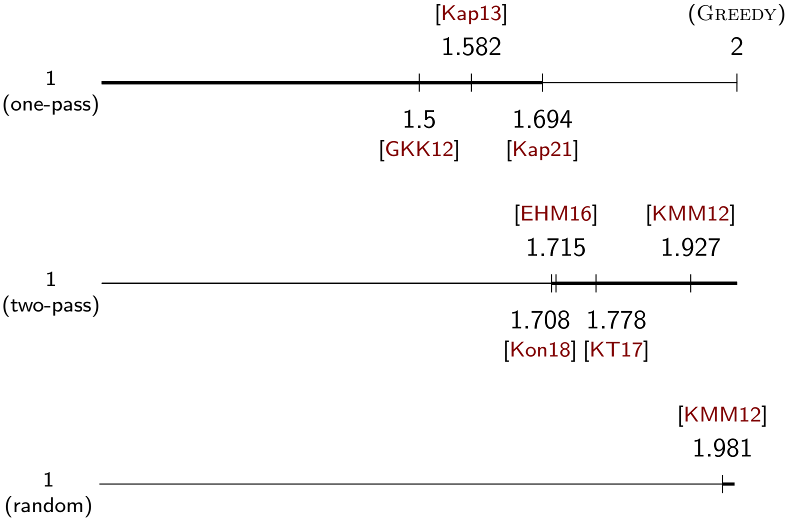
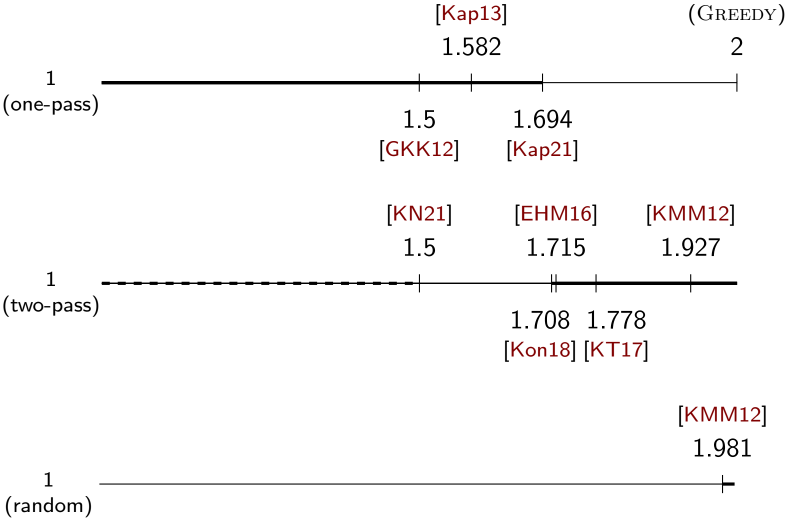
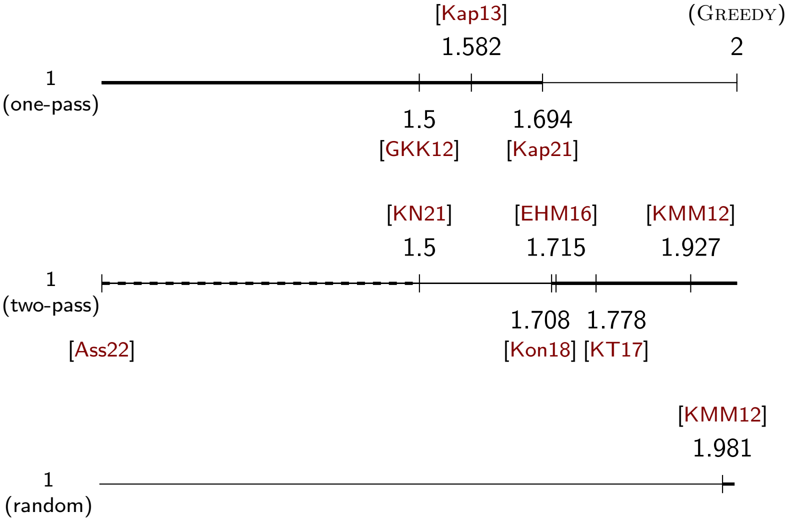


Insertion-Only



Insertion-Only



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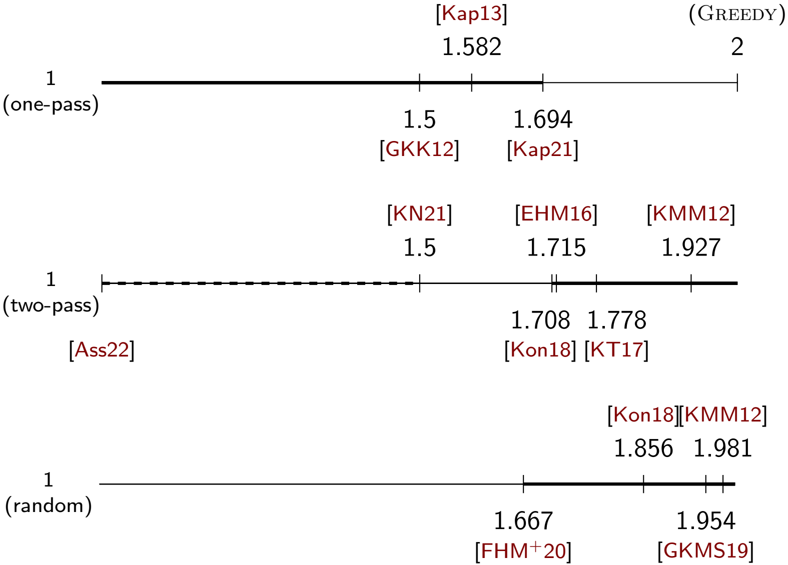
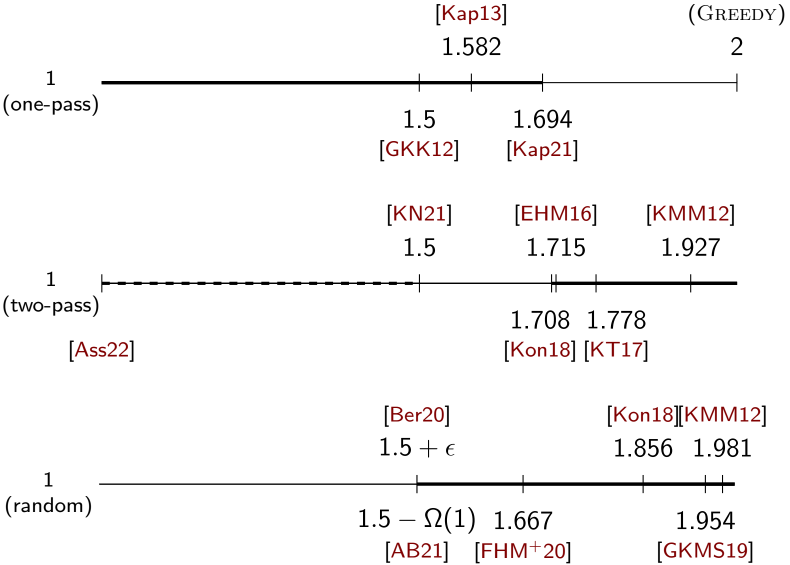
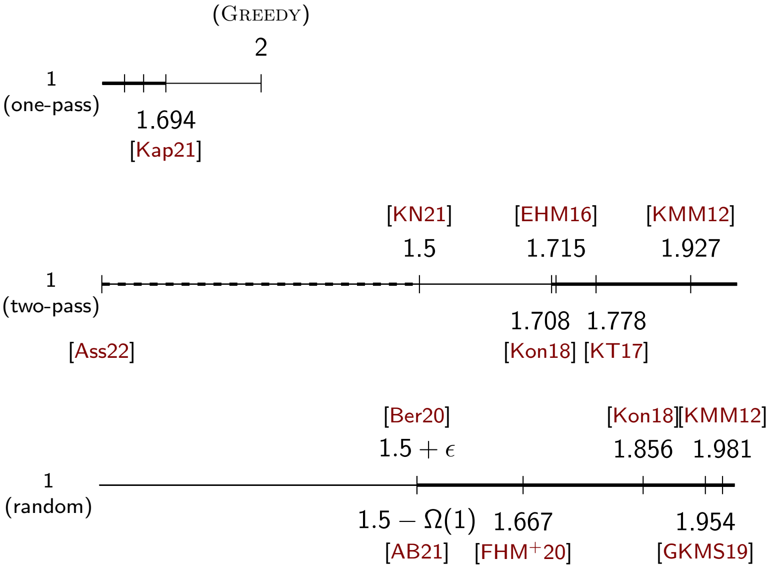


Insertion-Only



Insertion-Only



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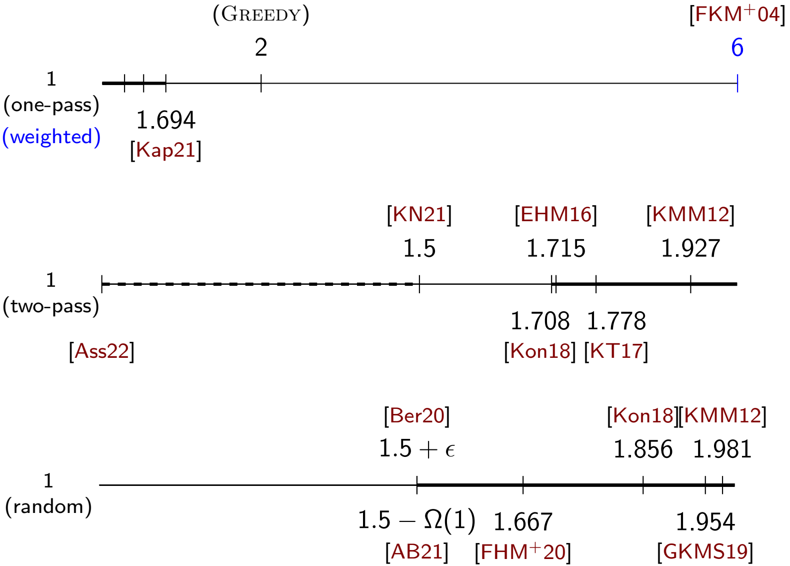
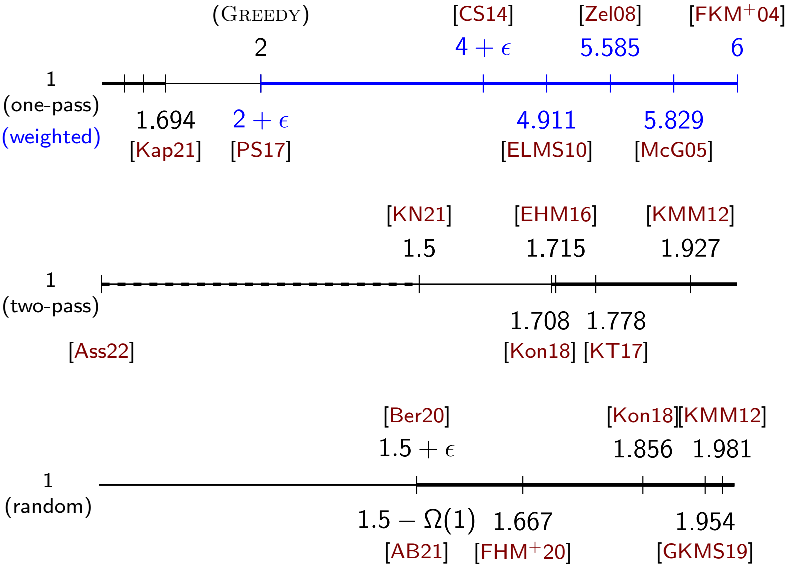


Insertion-Only



Sliding Window

[CMS13]

$3 + \epsilon$

1

Sliding Window

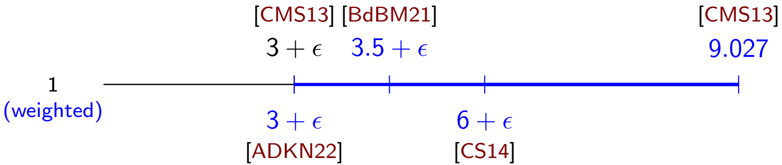
[CMS13]

$3 + \epsilon$

1 ———————————┤

(weighted)

Sliding Window



Dynamic (Insertion-Deletion)

Goal:

- Find an α-approximation

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Goal:

- Find an α-approximation

[AKLY16]
$\Omega(\frac{n^{2-o(1)}}{\alpha^3})$

[Kon15]
$O(\frac{n^2}{\alpha^2} \log^4 n)$

$\Omega(\frac{n^{1.5}}{\alpha^4})$
[Kon15]

$\Omega(\frac{n^2}{\alpha^3})$
[DK20]

$O(\frac{n^2}{\alpha^3} \log^4 n)$
[CCE$^+$16, AKLY16]

Dynamic (Insertion-Deletion)

Goal:

- Find an α-approximation

$\Omega(\frac{n^{1.5}}{\alpha^4})$ [Kon15]

[AKLY16] $\Omega(\frac{n^{2-o(1)}}{\alpha^3})$

[AS22] $O(\frac{n^2}{\alpha^3})$

$\Omega(\frac{n^2}{\alpha^3})$ [DK20]

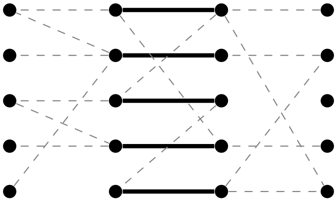
$O(\frac{n^2}{\alpha^3}\log^4 n)$ [CCE+16, AKLY16]

[Kon15] $O(\frac{n^2}{\alpha^2}\log^4 n)$

Relation to Vertex Cover

GREEDY Matching:

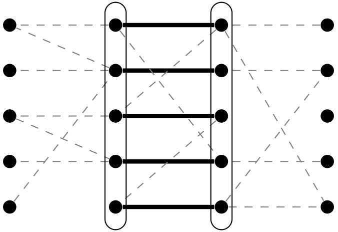
- Maximal
- 2-approximation



Relation to Vertex Cover

GREEDY Vertex Cover:

- 2-approximation



Relation to Vertex Cover (Results)

	matching	**vertex cover**
insertion-only	[1.694, 2]	
sliding window	$[1, 3+\epsilon]$	
dynamic	$\Theta(\frac{n^2}{\alpha^3})$	

Relation to Vertex Cover (Results)

	matching	vertex cover
insertion-only	[1.694, 2]	[1, 2] GREEDY
sliding window	$[1,\ 3+\epsilon]$	
dynamic	$\Theta(\frac{n^2}{\alpha^3})$	

Relation to Vertex Cover (Results)

	matching	**vertex cover**
insertion-only	$[1.694,\ 2]$	$[1,\ 2]$ GREEDY
sliding window	$[1,\ 3+\epsilon]$	$[1,\ 3+\epsilon]$ [Sub21]
dynamic	$\Theta(\frac{n^2}{\alpha^3})$	

Relation to Vertex Cover (Results)

	matching	vertex cover
insertion-only	[1.694, 2]	[1, 2] GREEDY
sliding window	$[1, 3 + \epsilon]$	$[1, 3 + \epsilon]$ [Sub21]
dynamic	$\Theta(\frac{n^2}{\alpha^3})$	$\Theta(\frac{n^2}{\alpha^2})$ [DK20] & this work

Relation to Vertex Cover (Results)

	matching	**vertex cover**
insertion-only	[1.694, 2]	[1, 2] GREEDY
sliding window	$[1, 3+\epsilon]$	$[1, 3+\epsilon]$ [Sub21]
dynamic	$\Theta(\frac{n^2}{\alpha^3})$	$\Theta(\frac{n^2}{\alpha^2})$ [DK20] & this work

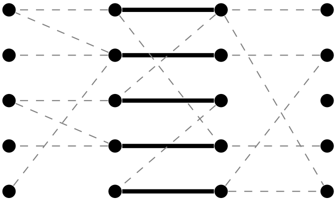
All UBs use GREEDY in some way!

What makes dynamic graph streams hard?

What makes dynamic graph streams hard?

GREEDY Matching:

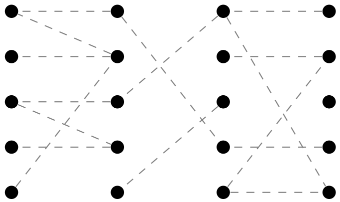
- Maximal
- 2-approximation



What makes dynamic graph streams hard?

GREEDY Matching:

- **Not** Maximal
- **0**-approximation



What makes dynamic graph streams hard?

GREEDY Matching:

- **Not** Maximal
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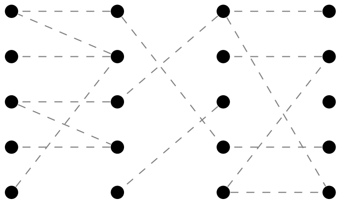
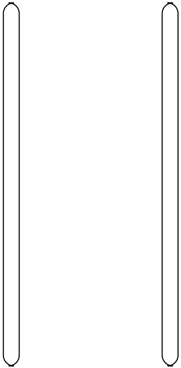


Note: Deterministically returning a single edge requires $\Omega(n^2)$ bits of space for dense graphs.

What techniques are used?



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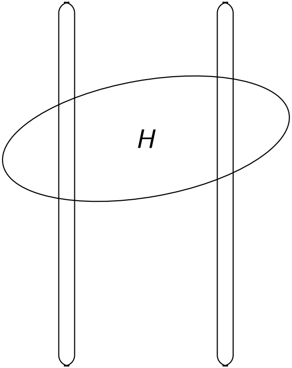
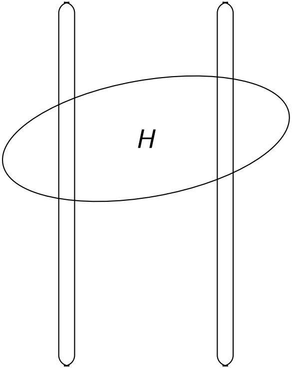


For a subgraph H of G with m edges:

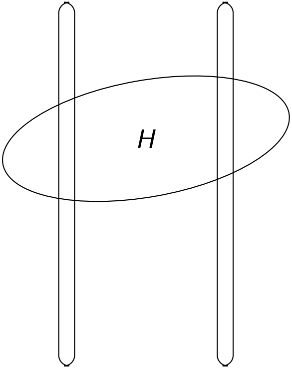
What techniques are used?



For a subgraph H of G with m edges:

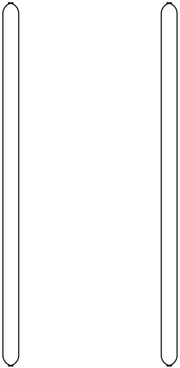
1. Check if H is empty
   - A counter using $\Theta(\log m)$ bits

What techniques are used?

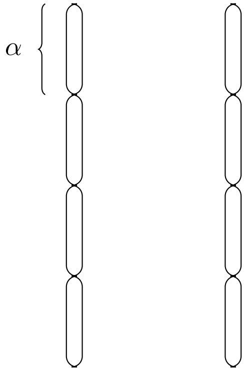


For a subgraph H of G with m edges:

1. Check if H is empty
   - A counter using $\Theta(\log m)$ bits
2. Retrieve an edge if one is present
   - An L_0-sampler using $\Theta(\log^3 n)$ bits
   - Neighbourhood sampler

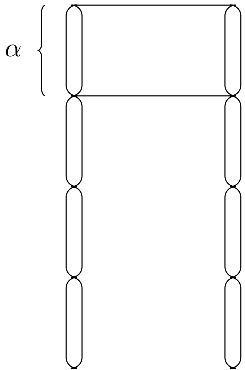


α-Approx Det. Dynamic Vertex Cover [DK20]



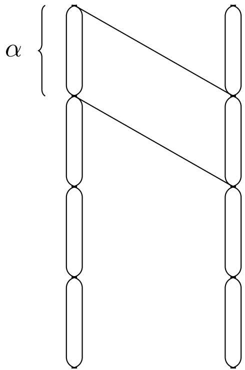
1. Vertex groups of size α
   - about $\frac{n}{\alpha}$ groups

α-Approx Det. Dynamic Vertex Cover [DK20]



1. Vertex groups of size α
   - about $\frac{n}{\alpha}$ groups
2. Check if there is an edge between each pair of groups
   - about $\frac{n^2}{\alpha^2}$ pairs

α-Approx Det. Dynamic Vertex Cover [DK20]



1. Vertex groups of size α
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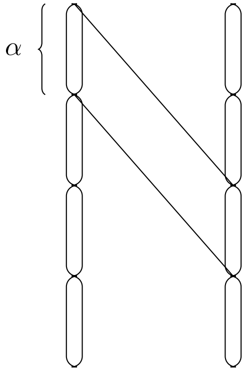
α-Approx Det. Dynamic Vertex Cover [DK20]



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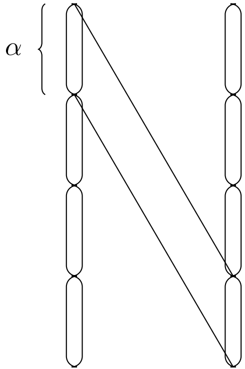

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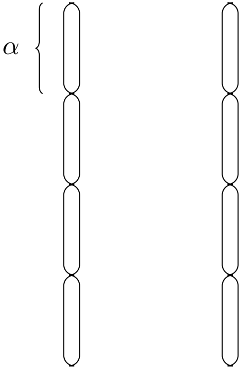

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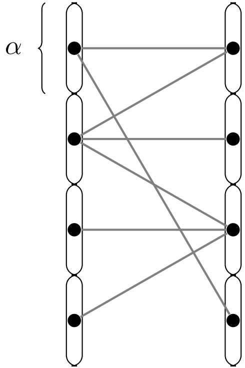

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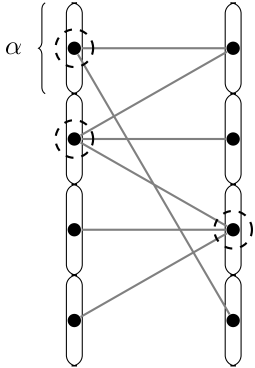

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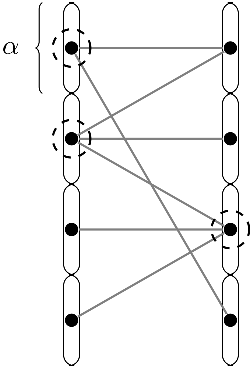

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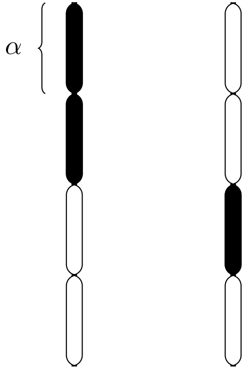

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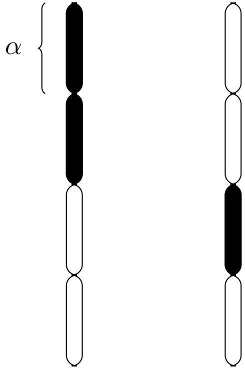

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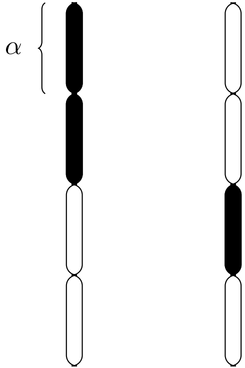

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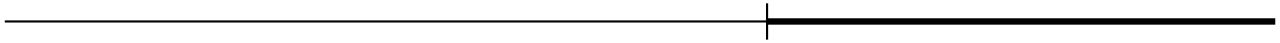

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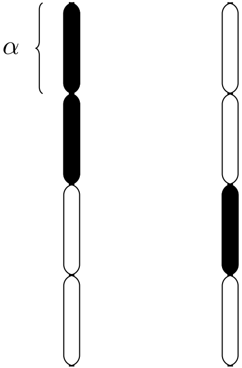

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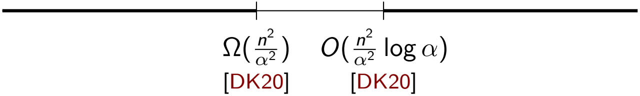

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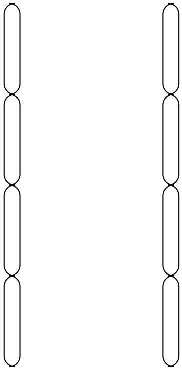

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- Counters use $O(\log \alpha)$ bits.
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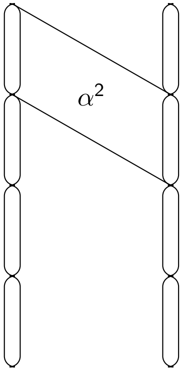

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- Counters to use $O(1)$ bits.
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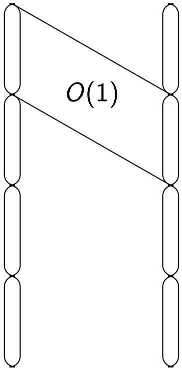

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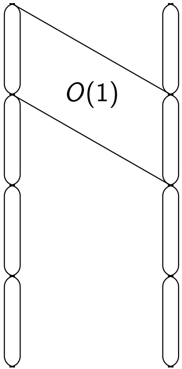
How?

- Sparse graph
- Randomly partition



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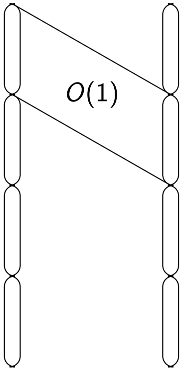
How?

- Sparse graph
- Randomly partition (easy)



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Goal:

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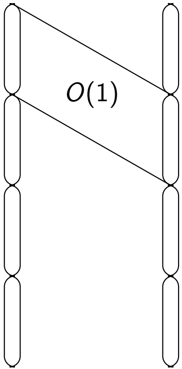
How?

- Sparse graph (GREEDY!)
- Randomly partition (easy)



Sparsness properties of GREEDY

Lemma

Let G be a n-vertex graph with m edges and let M_s be a GREEDY *matching on $s \leq m$ uniform randomly sampled edges. Then, for $G_R = G[V \backslash V(M_s)]$,*

$$\Delta(G_R) \leq \frac{C \cdot m \cdot \log n}{s}.$$

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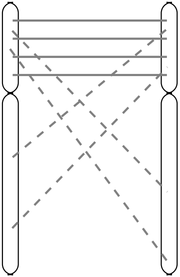

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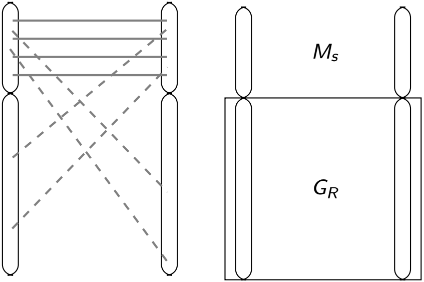

Sparsness properties of Greedy

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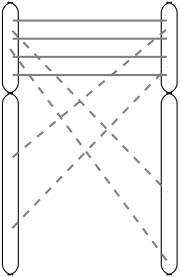
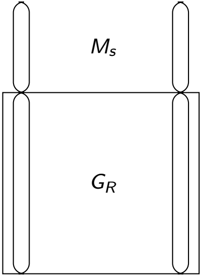




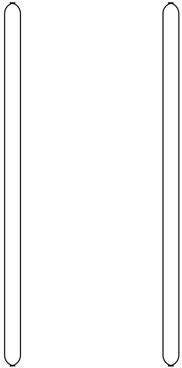
Proof (sketch).

For any vertex $v \in G$,

- $v \in G_R$
- $\deg_{G_R}(v) > \frac{C \cdot m \cdot \log n}{s}$

both do not occur w.h.p. □

Algorithm



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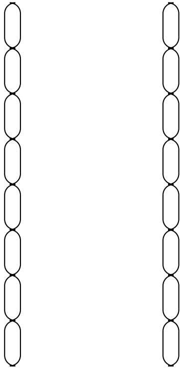

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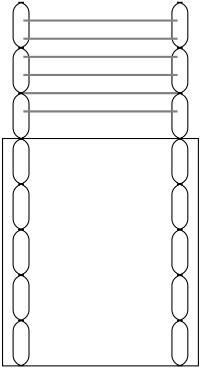
2. Sample $O(\frac{n^2}{\alpha^2 \log^3 n})$ edges randomly and run GREEDY on them to get M



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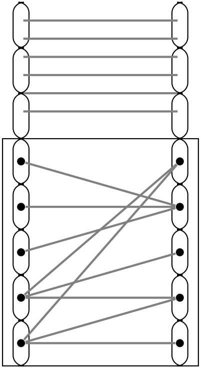

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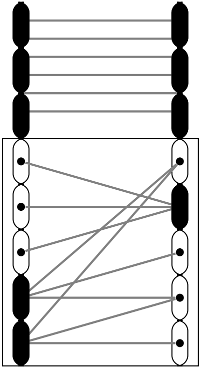
4. Return vertices of the covering groups including those with matched vertices



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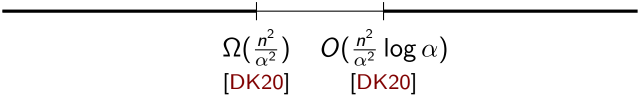

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Open Questions:

- Deterministic techniques or LB instead
- Other problems like dominating set and spectral sparsification

Thank You

Questions?

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