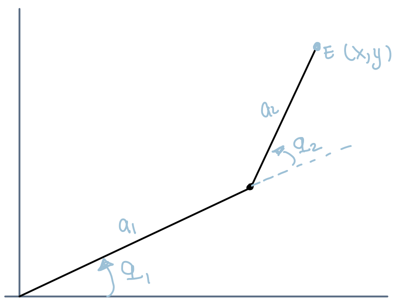
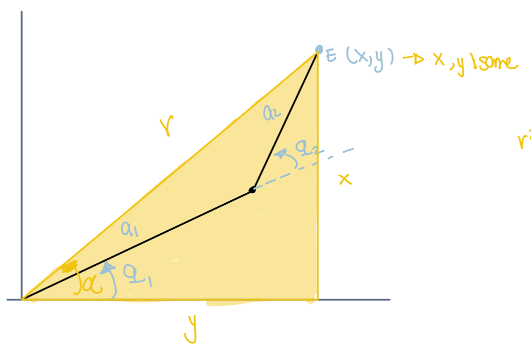
Inverse kinematics of a 2-joint robot arm using geometry:



} Original problem.

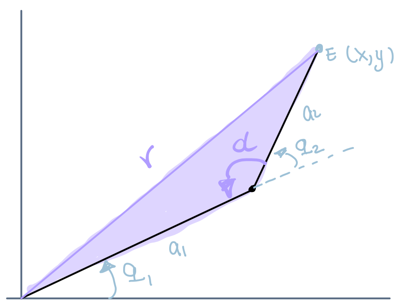
(1)



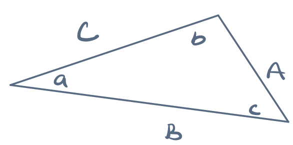
$r^2 = x^2 + y^2$ → pitagora.

$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$

(2) lets get q_2:



using cosine law we can get α.



$A^2 = B^2 + C^2 - 2BC\cos a$

$\Rightarrow \quad r^2 = a_1^2 + a_2^2 - 2a_1a_2 \cos\alpha$

↳ recall $r^2 = x^2 + y^2$

$x^2 + y^2 = a_1^2 + a_2^2 - 2a_1a_2\cos\alpha$

$\cos\alpha = \dfrac{a_1^2 + a_2^2 - x^2 - y^2}{2a_1a_2}$

↳ from the shaded triangle we can see that:

$\alpha + q_2 = 180°$

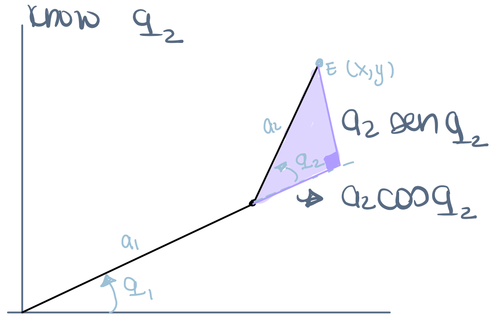
$\alpha + q_2 = \pi$

$q_2 = \pi - \alpha$

↳ $\cos q_2 = -\cos\alpha$

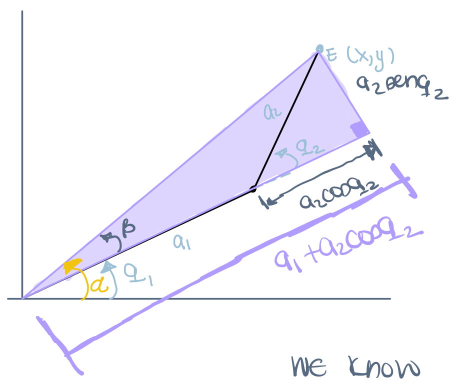
$\cos q_2 = \dfrac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1a_2}$

③ We know q_2

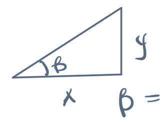


$$\operatorname{sen} q_2 = \sqrt{1-\cos^2 q_2}$$

④



to get β:



$\beta = \tan^{-1}\left(\frac{y}{x}\right)$

$$\beta = \tan^{-1}\left(\frac{a_2 \operatorname{sen} q_2}{a_1 + a_2 \cos q_2}\right)$$

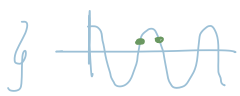
we know α from step #1:

$$q_1 = \alpha - \beta$$

$$\Rightarrow q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{a_2 \operatorname{sen} q_2}{a_1 + a_2 \cos q_2}\right)$$

$$\cos q_2 = \underbrace{\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}}$$

has 2 positive solutions; one positive & one negative.



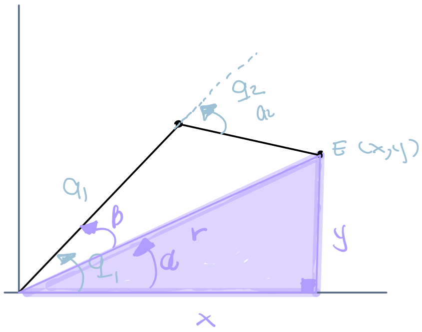
↳ if we only want the positive solution → $q_2 = \cos^{-1}\left(\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}\right)$

→ based on our link 2 position it is positive.

↳ if we want the negative solution → $q_2 = -\cos^{-1}\left(\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}\right)$

↳ better to have arctan.

*Link #2 negative q_2 :



$$q_2 = -\cos^{-1}\left(\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}\right)$$

$$\alpha = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\beta = \tan^{-1}\left(\frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2}\right)$$

$$q_1 = \alpha + \beta$$

$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) + \tan^{-1}\left(\frac{a_2 \sin q_2}{a_1 + a_2 \cos q_2}\right)$$