

Abstract

This vignette provides an overview of calculating portfolio returns through time with an emphasis on the math used to develop the `Return.portfolio` function in **PerformanceAnalytics**. We first introduce some basic definitions, then give simple examples of computing portfolio returns in a prices and shares framework as well as a returns and weights framework. We then introduce `Return.portfolio` and demonstrate the function with a few examples.

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1 Basic definitions

Suppose we have a portfolio of N assets. The value of asset i , V_i , in the portfolio is defined as

$$V_i = \lambda_i * P_i$$

where:

λ_i is the number of shares of asset i

P_i is the price of asset i

The total portfolio value, V_P , is defined as

$$V_P = \sum_{i=1}^N V_i$$

The weight of asset i , w_i , in the portfolio is defined as

$$w_i = V_i / V_P$$

where:

V_i is the value of asset i

V_P is the total value of the portfolio

The portfolio return at time t , R_t , is defined as

$$R_t = \frac{V_{p_t} - V_{p_{t-1}}}{V_{p_{t-1}}}$$

V_{p_t} is the portfolio value at time t

2 Simple Example: Prices and Shares Framework

Suppose we have a portfolio of $N = 2$ assets, asset A and asset B. The prices for assets A and B are given as

```
> prices = cbind(c(5, 7, 6, 7),
+               c(10, 11, 12, 8))
> dimnames(prices) = list(paste0("t",0:3), c("A", "B"))
> prices

      A  B
t0 5 10
t1 7 11
t2 6 12
t3 7  8
```

We wish to form an equal weight portfolio, that is, form a portfolio where

$$w_i = \frac{1}{N} \text{ for } i \in 1, \dots, N.$$

Let $V_{P0} = 1000$ be the portfolio value at t_0 .

Step 1: Compute the number of shares of each asset to purchase.

$$\begin{aligned} w_i &= \frac{V_i}{V_P} \\ &= \frac{\lambda_i * P_i}{V_P} \end{aligned}$$

Solve for λ_i .

$$\lambda_i = \frac{w_i * V_P}{P_i}$$

$$\begin{aligned} \lambda_A &= \frac{w_A * V_{P0}}{P_A} = \frac{0.5 * \$1000}{\$5} = 100 \\ \lambda_B &= \frac{w_B * V_{P0}}{P_B} = \frac{0.5 * \$1000}{\$10} = 50 \end{aligned}$$

```
> V_P0 = 1000
> N = ncol(prices)
> w = rep(1 / N, N)
> lambda = w * V_P0 / prices["t0",]
> lambda
```

```
      A  B
100  50
```

Step 2: Compute the asset value and portfolio value for $t \in 0, \dots, 3$.

```

> # Compute the value of the assets
> V_assets <- matrix(0, nrow(prices), ncol(prices), dimnames=dimnames(prices))
> for(i in 1:nrow(prices)){
+   V_assets[i,] = prices[i,] * lambda
+ }
> V_assets

```

```

      A    B
t0 500 500
t1 700 550
t2 600 600
t3 700 400

```

```

> # Compute the value of the portfolio
> V_P = rowSums(V_assets)
> V_P

```

```

      t0    t1    t2    t3
1000 1250 1200 1100

```

Step 3: Compute the portfolio returns for $t \in 1, \dots, 3$.

```

> # Compute the portfolio returns
> R_t = diff(V_P) / V_P[1:3]
> R_t

```

```

      t1      t2      t3
0.25000000 -0.04000000 -0.08333333

```

Step 4: Compute the weights of each asset in the portfolio for $t \in 0, \dots, 3$

```

> weights = V_assets / V_P
> weights

```

```

      A      B
t0 0.5000000 0.5000000
t1 0.5600000 0.4400000
t2 0.5000000 0.5000000
t3 0.6363636 0.3636364

```

We have shown that calculating portfolio weights, values, and returns is simple in a prices and shares framework. However, calculating these metrics becomes more challenging in a weights and returns framework.

3 Example: Weights and Returns Framework

We will use the monthly returns during 1997 of the first 5 assets in the edhec dataset for the following example.

```

> library(PerformanceAnalytics)
> data(edhec)
> R = edhec["1997", 1:5]
> colnames(R) = c("CA", "CTAG", "DS", "EM", "EMN")
> R

```

	CA	CTAG	DS	EM	EMN
1997-01-31	0.0119	0.0393	0.0178	0.0791	0.0189
1997-02-28	0.0123	0.0298	0.0122	0.0525	0.0101
1997-03-31	0.0078	-0.0021	-0.0012	-0.0120	0.0016
1997-04-30	0.0086	-0.0170	0.0030	0.0119	0.0119
1997-05-31	0.0156	-0.0015	0.0233	0.0315	0.0189
1997-06-30	0.0212	0.0085	0.0217	0.0581	0.0165
1997-07-31	0.0193	0.0591	0.0234	0.0560	0.0247
1997-08-31	0.0134	-0.0473	0.0147	-0.0066	0.0017
1997-09-30	0.0122	0.0198	0.0350	0.0229	0.0202
1997-10-31	0.0100	-0.0098	-0.0064	-0.0572	0.0095
1997-11-30	0.0000	0.0133	0.0054	-0.0378	0.0041
1997-12-31	0.0068	0.0286	0.0073	0.0160	0.0066

Suppose that on 1996-12-31 we wish to form an equal weight portfolio such that the weight for asset i is given as:

$$w_i = \frac{1}{N} \quad \text{for } i \in 1, \dots, N$$

where N is equal to the number of assets.

```

> N = ncol(R)
> weights = xts(matrix(rep(1 / N, N), 1), as.Date("1996-12-31"))
> colnames(weights) = colnames(R)
> weights

```

	CA	CTAG	DS	EM	EMN
1996-12-31	0.2	0.2	0.2	0.2	0.2

There are two cases we need to consider when calculating the beginning of period (bop) value.

Case 1: The beginning of period t is a rebalancing event. For example, the rebalance weights at the end of 1996-12-31 take effect at the beginning of 1997-01-31. This means that the beginning of 1997-01-31 is considered a rebalance event.

The beginning of period value for asset i at time t is given as

$$V_{bop_{t,i}} = w_i * V_{t-1}$$

where w_i is the weight of asset i and V_{t-1} is the end of period (eop) portfolio value of the prior period.

Case 2: The beginning of period t is not a rebalancing event.

$$V_{bop_{t,i}} = V_{eop_{t-1,i}}$$

where $V_{eop_{t-1,i}}$ is the end of period value for asset i from the prior period.

The end of period value for asset i at time t is given as

$$V_{eop_{t,i}} = (1 + R_{t,i}) * V_{bop_{t,i}}$$

Here we demonstrate this and compute values for the periods 1 and 2. For the first period, $t = 1$, we need an initial value for the portfolio value. Let $V_0 = 1$ denote the initial portfolio value. Note that the initial portfolio value can be any arbitrary number. Here we use $V_0 = 1$ for simplicity.

```
> V_0 = 1
> bop_value = eop_value = matrix(0, 2, ncol(R))
```

Compute the values for $t = 1$.

```
> t = 1
> bop_value[t,] = coredata(weights) * V_0
> eop_value[t,] = coredata(1 + R[t,]) * bop_value[t,]
```

Now compute the values for $t = 2$.

```
> t = 2
> bop_value[t,] = eop_value[t-1,]
> eop_value[t,] = coredata(1 + R[t,]) * bop_value[t,]
```

It is seen that the values for the rest of the time periods can be computed by iterating over $t \in 1, \dots, T$ where $T = 12$ in this example.

The weight of asset i at time t is calculated as

$$w_{t,i} = \frac{V_{t,i}}{\sum_{i=0}^N V_{t,i}}$$

Here we compute both the beginning and end of period weights.

```
> bop_weights = eop_weights = matrix(0, 2, ncol(R))
> for(t in 1:2){
+   bop_weights[t,] = bop_value[t,] / sum(bop_value[t,])
+   eop_weights[t,] = eop_value[t,] / sum(eop_value[t,])
+ }
> bop_weights
```

```
      [,1]      [,2]      [,3]      [,4]      [,5]
[1,] 0.200000 0.200000 0.200000 0.200000 0.200000
[2,] 0.195839 0.2011419 0.1969808 0.2088446 0.1971937
```

```
> eop_weights
```

```
      [,1]      [,2]      [,3]      [,4]      [,5]
[1,] 0.1958390 0.2011419 0.1969808 0.2088446 0.1971937
[2,] 0.1936464 0.2023282 0.1947562 0.2147071 0.1945622
```

The portfolio returns at time t are calculated as

$$R_{P_t} = \frac{V_t - V_{t-1}}{V_{t-1}}$$

```
> V = c(V_0, rowSums(eop_value))
> R_P = diff(V) / V[1:2]
> R_P
```

```
[1] 0.03340000 0.02376201
```

The contribution of asset i at time t is calculated as

$$contribution_{t,i} = \frac{V_{eop_{t,i}} - V_{bop_{t,i}}}{\sum_{i=1}^N V_{bop_{t,i}}}$$

```
> contribution = matrix(0, 2, ncol(R))
> for(t in 1:2){
+   contribution[t,] = (eop_value[t,] - bop_value[t,]) / sum(bop_value[t,])
+ }
> contribution
```

```
      [,1]      [,2]      [,3]      [,4]      [,5]
[1,] 0.002380000 0.007860000 0.003560000 0.01582000 0.003780000
[2,] 0.002408819 0.005994027 0.002403166 0.01096434 0.001991657
```

Note that contribution can also be calculated as

$$contribution_{t,i} = R_{t,i} * w_{t,i}$$

4 Return.portfolio Examples

```
> args(Return.portfolio)

function (R, weights = NULL, wealth.index = FALSE, contribution = FALSE,
  geometric = TRUE, rebalance_on = c(NA, "years", "quarters",
    "months", "weeks", "days"), value = 1, verbose = FALSE,
  ...)
NULL
```

If no **weights** are specified, then an equal weight portfolio is computed. If **rebalance_on=NA** then a buy and hold portfolio is assumed. See **?Return.portfolio** for a detailed explanation of the function and arguments.

```
> # Equally weighted, buy and hold portfolio returns
> Return.portfolio(R)
```

```
      portfolio.returns
1997-01-31      0.03340000
1997-02-28      0.02376201
1997-03-31     -0.001413340
1997-04-30      0.003678107
1997-05-31      0.017767315
1997-06-30      0.025914372
1997-07-31      0.036969968
```

1997-08-31	-0.005005540
1997-09-30	0.022080944
1997-10-31	-0.012352423
1997-11-30	-0.003843547
1997-12-31	0.012936194

```
> # Equally weighted, rebalanced quarterly portfolio returns
> Return.portfolio(R, rebalance_on="quarters")
```

	portfolio.returns
1997-01-31	0.033400000
1997-02-28	0.023762011
1997-03-31	-0.001413340
1997-04-30	0.003680000
1997-05-31	0.017660872
1997-06-30	0.025452430
1997-07-31	0.036500000
1997-08-31	-0.005136602
1997-09-30	0.022049167
1997-10-31	-0.010780000
1997-11-30	-0.002621013
1997-12-31	0.012985944

```
> # Equally weighted, rebalanced quarterly portfolio returns.
> # Use verbose=TRUE to return additional information
> # including asset values and weights
> Return.portfolio(R, rebalance_on="quarters", verbose=TRUE)
```

\$returns

	portfolio.returns
1997-01-31	0.033400000
1997-02-28	0.023762011
1997-03-31	-0.001413340
1997-04-30	0.003680000
1997-05-31	0.017660872
1997-06-30	0.025452430
1997-07-31	0.036500000
1997-08-31	-0.005136602
1997-09-30	0.022049167
1997-10-31	-0.010780000
1997-11-30	-0.002621013
1997-12-31	0.012985944

\$contribution

	CA	CTAG	DS	EM	EMN
1997-01-31	0.002380000	0.0078600000	0.0035600000	0.015820000	0.0037800000
1997-02-28	0.002408819	0.0059940275	0.0024031662	0.010964341	0.0019916567
1997-03-31	0.001510442	-0.0004248891	-0.0002337074	-0.002576485	0.0003112995
1997-04-30	0.001720000	-0.0034000000	0.0006000000	0.002380000	0.0023800000
1997-05-31	0.003135294	-0.0002938187	0.0046568428	0.006351596	0.0038109577

1997-06-30	0.004252156	0.0016336242	0.0043610924	0.011874480	0.0033310776
1997-07-31	0.003860000	0.0118200000	0.0046800000	0.011200000	0.0049400000
1997-08-31	0.002635527	-0.0096662672	0.0029028423	-0.001344834	0.0003361293
1997-09-30	0.002444218	0.0038748559	0.0070493383	0.004659301	0.0040214532
1997-10-31	0.002000000	-0.0019600000	-0.0012800000	-0.011440000	0.0019000000
1997-11-30	0.000000000	0.0026626352	0.0010847819	-0.007205240	0.0008368108
1997-12-31	0.001392218	0.0058170647	0.0014782579	0.002942265	0.0013561387

\$BOP.Weight

	CA	CTAG	DS	EM	EMN
1997-01-31	0.2000000	0.2000000	0.2000000	0.2000000	0.2000000
1997-02-28	0.1958390	0.2011419	0.1969808	0.2088446	0.1971937
1997-03-31	0.1936464	0.2023282	0.1947562	0.2147071	0.1945622
1997-04-30	0.2000000	0.2000000	0.2000000	0.2000000	0.2000000
1997-05-31	0.2009804	0.1958792	0.1998645	0.2016380	0.2016380
1997-06-30	0.2005734	0.1921911	0.2009720	0.2043800	0.2018835
1997-07-31	0.2000000	0.2000000	0.2000000	0.2000000	0.2000000
1997-08-31	0.1966811	0.2043608	0.1974723	0.2037627	0.1977231
1997-09-30	0.2003458	0.1956998	0.2014097	0.2034629	0.1990818
1997-10-31	0.2000000	0.2000000	0.2000000	0.2000000	0.2000000
1997-11-30	0.2042013	0.2001981	0.2008855	0.1906148	0.2041002
1997-12-31	0.2047379	0.2033939	0.2025011	0.1838916	0.2054756

\$EOP.Weight

	CA	CTAG	DS	EM	EMN
1997-01-31	0.1958390	0.2011419	0.1969808	0.2088446	0.1971937
1997-02-28	0.1936464	0.2023282	0.1947562	0.2147071	0.1945622
1997-03-31	0.1954330	0.2021890	0.1947978	0.2124308	0.1951493
1997-04-30	0.2009804	0.1958792	0.1998645	0.2016380	0.2016380
1997-05-31	0.2005734	0.1921911	0.2009720	0.2043800	0.2018835
1997-06-30	0.1997416	0.1890138	0.2002366	0.2108869	0.2001210
1997-07-31	0.1966811	0.2043608	0.1974723	0.2037627	0.1977231
1997-08-31	0.2003458	0.1956998	0.2014097	0.2034629	0.1990818
1997-09-30	0.1984151	0.1952691	0.2039618	0.2036323	0.1987216
1997-10-31	0.2042013	0.2001981	0.2008855	0.1906148	0.2041002
1997-11-30	0.2047379	0.2033939	0.2025011	0.1838916	0.2054756
1997-12-31	0.2034876	0.2065290	0.2013644	0.1844387	0.2041802

\$BOP.Value

	CA	CTAG	DS	EM	EMN
1997-01-31	0.2000000	0.2000000	0.2000000	0.2000000	0.2000000
1997-02-28	0.2023800	0.2078600	0.2035600	0.2158200	0.2037800
1997-03-31	0.2048693	0.2140542	0.2060434	0.2271506	0.2058382
1997-04-30	0.2112921	0.2112921	0.2112921	0.2112921	0.2112921
1997-05-31	0.2131092	0.2077001	0.2119260	0.2138065	0.2138065
1997-06-30	0.2164337	0.2073886	0.2168638	0.2205414	0.2178474
1997-07-31	0.2213080	0.2213080	0.2213080	0.2213080	0.2213080
1997-08-31	0.2255792	0.2343873	0.2264866	0.2337012	0.2267743

1997-09-30	0.2286020	0.2233008	0.2298159	0.2321588	0.2271598
1997-10-31	0.2332393	0.2332393	0.2332393	0.2332393	0.2332393
1997-11-30	0.2355716	0.2309535	0.2317465	0.2198980	0.2354550
1997-12-31	0.2355716	0.2340252	0.2329980	0.2115858	0.2364204

\$EOP.Value

	CA	CTAG	DS	EM	EMN
1997-01-31	0.2023800	0.2078600	0.2035600	0.2158200	0.2037800
1997-02-28	0.2048693	0.2140542	0.2060434	0.2271506	0.2058382
1997-03-31	0.2064673	0.2136047	0.2057962	0.2244247	0.2061675
1997-04-30	0.2131092	0.2077001	0.2119260	0.2138065	0.2138065
1997-05-31	0.2164337	0.2073886	0.2168638	0.2205414	0.2178474
1997-06-30	0.2210221	0.2091514	0.2215698	0.2333548	0.2214419
1997-07-31	0.2255792	0.2343873	0.2264866	0.2337012	0.2267743
1997-08-31	0.2286020	0.2233008	0.2298159	0.2321588	0.2271598
1997-09-30	0.2313909	0.2277221	0.2378595	0.2374752	0.2317484
1997-10-31	0.2355716	0.2309535	0.2317465	0.2198980	0.2354550
1997-11-30	0.2355716	0.2340252	0.2329980	0.2115858	0.2364204
1997-12-31	0.2371735	0.2407183	0.2346988	0.2149712	0.2379808