(Q) An instructor has given a short quiz consisting of two parts. For a randomly selected student, let X be the number of points earned on the first part and Y the number of points earned on the second part. Suppose that the joint pmf of X and Y is given in the table below.

$p(x,y)$	0	5	10	15	$P_x(x)$
0	.02	.06	.02	.10	0.2
X 5	.04	.15	.20	.10	0.49
10	.01	.15	.14	.01	0.31
$P_y(y)$	0.07	0.36	0.36	0.21	

(a) If the score recorded in the grade book is the total number of points earned on the two parts, what is the expected recorded score ($E(X+Y)$)?

$$E(X+Y) = \sum_x \sum_y (x+y)\, p(x,y)$$

$= (0+0) \times p(0,0) + (0+5)\, p(0,5) + (0+10)\, p(0,10)$
$+ (0+15)\, p(0,15) + (5+0)\, p(5,0) + (5+5)\, p(5,5)$
$+ (5+10)\, p(5,10) + (5+15)\, p(5,15) + (10+0)\, p(10,0)$
$+ (10+5)\, p(10,5) + (10+10)\, p(10,10) + (10+15)\, p(10,15)$

$= 0.1 + 0.6\frac{1}{2} + 0.8 \cdot 4 \cdot 0.2 + 0.15 + 0.2 +$
$1.5 + 0.3 + 2 + 0.1 + 2.15 + 2.8 + 0.25$

$= \boxed{10.05}$ — ans.

(b) If the maximum of two scores are recorded, what is the expected score?

$$E(\text{Max}(x,y)) = \sum_{\forall x} \sum_{\forall y} \text{max}(x,y) \times p(x,y)$$

$$= \text{max}(0,0) \times 0.02 + \text{max}(0,5) \times 0.06 +$$
$$\text{max}(0,10) \times 0.02 + \text{max}(0,15) \times 0.10 +$$
$$\text{max}(5,0) \times 0.04 + \text{max}(5,5) \times 0.15 +$$
$$\text{max}(10,15) \times 0.01$$

$$= 0 \times 0.02 + 5 \times 0.06 + 10 \times 0.02 + 15 \times 0.10 +$$
$$5 \times 0.04 + 5 \times 0.15 + 10 \times 0.20 + 15 \times 0.10 +$$
$$10 \times 0.01 + 10 \times 0.15 + 10 \times 0.14 + 15 \times 0.01$$

(c) Calculate covariance of X and Y.
$$= E(x \times y) - E(x) \times E(y)$$

$$\therefore E(xy) = \sum \sum (xy)(p(x,y)) = (0 \times 0)(0.02) + (5 \times 0)(0.06)$$
$$+ \cdots (5 \times 0)(0.04) + (5 \times 5)$$
$$(0.15) + \cdots (10 \times 0)(0.01)$$
$$+ (10 \times 5)(0.15) + \cdots$$

$$\therefore E(x) = \sum x \times P_x(x)$$
$$= 0 \times 0.2 + 5 \times 0.49 + 10 \times 0.31$$
$$= 5.55$$

$$\therefore E(Y) = \sum y \times P_y(y)$$
$$= 0 \times 0.07 + 5 \times 0.36 + 10 \times 0.36 + 15 \times 0.21$$

(d) Calculate correlation of X and y
$$= \frac{\text{cov}(x,y)}{6x \times 6y} \longrightarrow \text{from c.}$$

$$6x = \sqrt{E(x^2) - (E(x))^2}$$
$$E(x^2) = \sum (x^2)(P_x(x))$$

$$6y = \sqrt{E(y^2) - (E(y))^2}$$
$$E(y^2) = \sum (y^2)(P_y(y))$$