(eq:-) Two ballpoint pens are selected at random from a box that contains 3 blue pens, 2 red pens and 3 green pens. If X is the number of blue pens selected and Y is the number of red pens selected, then (DISCRETE RANDOME VARIABLE)

(M1) TABULAR REPRESENTATION

$\therefore$ X = no. of blue pens selected = 0/1/2

$\therefore$ Y = no. of red pens selected = 0/1/2

X and Y can respectively takes these values.

X = 0 ⇒ no blue pen selected
= 1 ⇒ 1 blue pen selected
= 2 ⇒ 2 blue pen selected
= 3 ⇒ 3 blue pen selected ⟶ case not possible here
x cannot take value = 3

Y = 0 ⇒ 0 red pen selected
= 1 ⇒ 1 red pen selected
= 2 ⇒ 2 red pen selected

(X, Y) = (0,1) ⇒ no blue + 1 red.
= (1,1) ⇒ 1 blue + 1 red.
= (2,0) ⇒ 2 blue + 0 red.
= (2,1) ⇒ 2 blue + 1 red.

case not possible cause leads to selection of 3 pens but we need to select only 2.
⇒ P((2,1)) = 0.

Y=y \ X=x	0	1	2
0	$^3C_2/^8C_2$	$^3C_1 \times ^3C_1/^8C_2$	$^3C_2/^8C_2$
1	$^2C_1\,^3C_1/^8C_2$	$^2C_1\,^3C_1/^8C_2$	0
2	$^2C_2/^8C_2$	0	0

0 red + 0 blue ⇒ 2 from green

1 red + 1 green.

2 red.

1 blue + 1 green.

2 blue.

1 red + 1 blue.

(M2) FORMULA APPROACH

The joint PMF of (X, Y):

$$P(X=x; Y=y) = p(x,y) = \frac{(^3C_x)(^2C_y)(^3C_{2-x-y})}{(^8C_2)}$$

$[x \geq 0;\ x \leq 2;\ y \leq 2;\ y \geq 0]$

- Now when we have joint probability distribution, we also have individual probability distribution called $\boxed{\text{MARGINAL PROBABILITY DISTRIBUTION}}$

(iii) TABULAR REPRESENTATION.

$X=x$ / $Y=y$	0	1	2	Total $P_Y(y)$
0	3/28	9/28	3/28	15/28
1	3/14	3/14	0	3/7
2	1/28	0	0	1/28
Total $P_X(x)$	5/14	15/28	3/28	

sum = 1

sum = 1.

Hence individual probability of random variable X is p(x) known as marginal pmf of X and can be written as.

$X=x$	0	1	2
$P_X(x)$	5/14	15/28	3/28

Hence individual probability of random variable Y is p(y) known as marginal pmf of Y and can be written as.

$Y=y$	0	1	2
$P_Y(y)$	15/28	3/7	1/28

similarly he can play in exam $\rightarrow$ $X =$ gamma
$Y =$ exponential ....

10) Let X denote the number of Canon-digital camera sold during a particular week by a certain store. The pmf of X is

$X=x$	0	1	2	3	4
$p(x)$	0.1	0.2	0.3	0.25	0.15

Sixty percent of all customers who purchase these cameras also buy an extended warranty. Let Y denote the number of purchases during this week who buy an extended warranty.

(a) Calculate $P(X=4, Y=2)$?

Now we can clearly see Y is dependent on X cause Y = no. of purchases which includes warranty, so to purchase warranty first you need to purchase i.e. X.

$\rightarrow$ Y takes values $0, 1, 2, \ldots X$.

$\therefore$ $P(Y/X) = \dfrac{P(Y, X)}{P_X(x)} \Rightarrow P(Y=2/X=4) = \dfrac{P(X=4; Y=2)}{P_X(x=4)}$

$\rightarrow$ Y follows Binomial distribution (obviously)

$\Rightarrow Y \sim B(n, p) \Rightarrow Y \sim B(X, 0.60)$

$\therefore P(X=4, Y=2) = P(Y=2 / X=4) \times P(X=4)$

$= \left\{ \binom{4}{2} (0.6)^2 (0.4)^2 \right\} (0.15)$

$= \boxed{0.0518}$ — ans.

(b) Calculate $P(X=Y)$ (same as a)

$= P(X=0, Y=0) + P(X=1, Y=1) + P(X=2, Y=2) + P(X=3, Y=3) + P(X=4, Y=4)$

$= (1)(0.1) + (0.6)(0.2) + (0.6)^2(0.3) + (0.6)^3(0.25) + (0.6)^4(0.3)$

$= \boxed{0.4014}$

(c) Determine the joint PMF of X and Y and then the marginal PMF of Y.

In option (a) we wrote for $X=4, Y=2$. In this case we need to write it for $X=x, Y=y$.

$$\therefore p(x,y) = P(Y=y \mid X=x) * P(X=x)$$

$$= \left[\binom{x}{y}(0.6)^y(0.4)^{x-y}\right] * P_X(x), \quad x \in [0,4], \; y \in [0,x].$$

Marginal pmf of Y is given by

$$P_Y(y) = \sum_{x=0}^{4} p(x,y)$$

$$= \sum_{x=0}^{4} \binom{x}{y}(0.6)^y(0.4)^{x-y} * P_X(x)$$