N.V. Imp.

in this chapter which distribution is followed is given by the question.

3. "Continuous Random Variable."

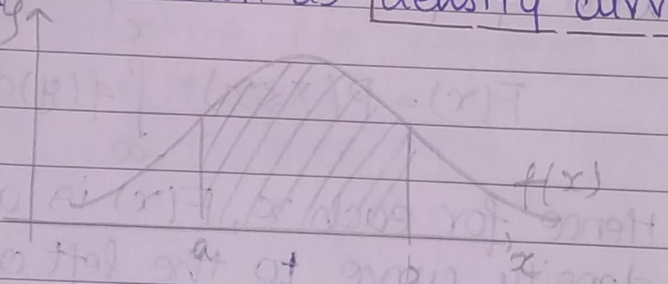
→ Let X be a continuous random variable. The probability density function (pdf) of X is a function f such that for any two no's a and b with a≤b

$$\boxed{\begin{array}{l} P(a\le X\le b) \\ \text{or } P(a<X<b) \end{array} = \left(\int_a^b f(x)\,dx\right) \Big/ \left(\int_{-\infty}^{\infty} f(x)\,dx\right) = \int_a^b f(x)\,dx}$$

$\int_{-\infty}^{\infty} f(x)\,dx = 1$ as prove below.

→ "AREA - PROPERTY."

$P(a\le x\le b)$, the probability that X assumes a value b/w a and b is equal to area under the probability density function f between the coordinates at $(x=a)$ and $(x=b)$. The graph of f is known as $\boxed{\text{density curve.}}$



→ Properties of prob distribution function (pdf)

→ probability density curve is always (+ve) cause probability always lies b/w 0 & 1

$f(x)\ge 0$ for all x.

→ $\int_{-\infty}^{\infty} f(x)\,dx = 1$ (total probability is always equal to 1)

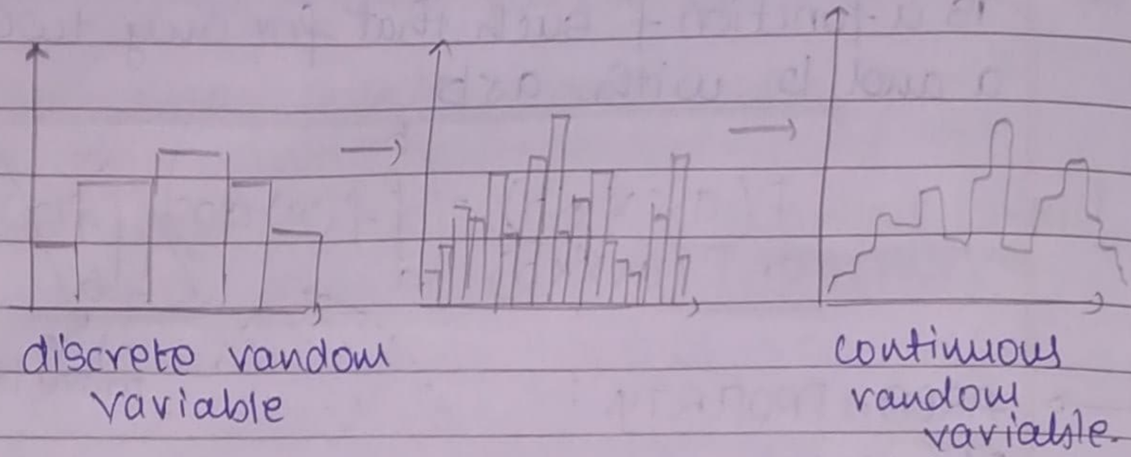
→ Probabilities associated with individual points are always zero.

$$P(X=c) = P(c\le X\le c) = \int_c^c f(x)\,dx = 0.$$

→ Consequently,

$$P(a \le x \le b) = P(a < x \le b) = P(a \le x < b) = P(a < x < b)$$

→ **PROBABILITY HISTOGRAM.**



→ **CUMULATIVE DISTRIBUTION FUNCTION. (cdf)**

The cumulative distribution function (cdf) of a continuous random variable X with pdf $f(x)$ is defined by

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(y)\,dy.$$

Hence for each x, $F(x)$ is area under the density curve to the left of x.

→ **PROPERTIES OF CDF:**

1. $F(-\infty) = 0$ and $F(\infty) = 1$.
2. $F(x)$ increases smoothly as x increases
3. $P(a \le X \le b) = F(b) - F(a)$
4. $P(X > a) = 1 - F(a)$
5. $F'(x) = f(x)$, if $F'(x)$ exists.

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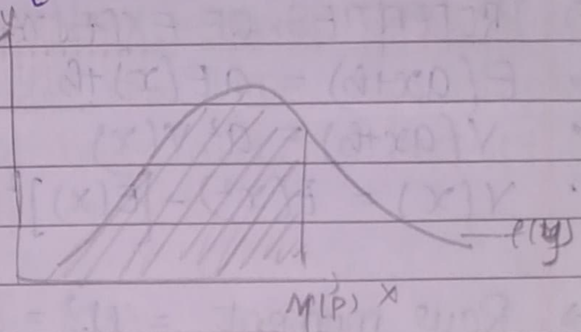
$(100P)^{th}$ PERCENTILE.

$(100P)^{th}$ percentile is a number which denotes the value of random variable and is present on the x-axis. It is denoted by $\eta(P)$

It is known that the area left side of the value $\eta(P) = P$



$$\therefore \boxed{\int_{-\infty}^{\eta(P)} f(y)\,dy = P = F(\eta(P))} \rightarrow \text{cdf}.$$

→ MEDIAN — $\eta(P) = 0.5$ (50^{th} percentile)
1st quartile — $\eta(P) = 0.25$
3rd quartile — $\eta(P) = 0.75$.

→ MODE —

The mode of a continuous random variable X with pdf 'f' is that value x^* for which f is largest $\Rightarrow$ do $f'(x) = 0$ and find value of x.

→ EXPECTATION —

$$E(X) = \mu_X = \int_{-\infty}^{\infty} (x)\, f(x)\,dx$$

$$E(h(x)) = \mu_{h(x)} = \int_{-\infty}^{\infty} h(x)\, f(x)\,dx.$$

→ VARIANCE

$$V(x) = \sigma_x^2 = E((X-\mu)^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx.$$

and the (+ve) square root of variance is called standard deviation.

→ PROPERTIES OF EXPECTATION & VARIANCE

- $E(ax+b) = aE(x)+b$
- $V(ax+b) = a^2 V(x)$
- $V(x) = E(x^2) - [E(x)]^2.$

→ Raw moment $(E(x^r))$ $= \mu_r' = \int_{-\infty}^{\infty} x^r f(x)\,dx.$ $[r=0,1,2\ldots]$

→ Central moment $(E(x-\mu)^r)$ $= \mu_r = \int_{-\infty}^{\infty} (x-\mu)^r f(x)\,dx.$ $[r=0,1,2\ldots]$

→ PROPERTIES OF MOMENTS

$\mu_1' = E(X)$

$\mu_2' = E(X^2)$

$\mu_1 = 0$

$\mu_2 = V(x) = E(x^2) - [E(X)]^2 = \mu_2' - (\mu_1')^2$

→ MOMENT GENERATING FUNCTION (MGF)

$$\boxed{M_x(t) = E(e^{tx}) = \int_{-\infty}^{+\infty} e^{tx} f(x)\,dx}$$

t = parameter.

we observe $\boxed{\mu_r' = \left.\frac{d^r}{dt^r} M_x(t)\right|_{t=0}}$

$\boxed{\begin{array}{l} P(X<a \text{ or } X>b) \\ = P(X<a) + P(X>b) \end{array}}$

$\boxed{P(X < a \text{ and } X>b) = P(\text{part intersecting between } a \,\&\, b)}$

→ PROPERTIES OF MGF.

1. $M_{ax+b}(t) = (e^{bt})\, M_x(at)$
2. $M_x(at) = M_{ax}(t)$.

(Q) A college professor never finishes his lecture before the end of the hour and always finishes his lectures within 2 min after the hour. Let the time that elapses b/w the end of hour and the end of lect and suppose pdf of X is

$$f(x) = \begin{cases} Kx^2 & , 0 \le x \le 2 \\ 0 & , \text{otherwise.} \end{cases}$$

(a) Find value of K.

$$\int_0^2 Kx^2 = 1 \Rightarrow \frac{1}{K} = \frac{8}{3} \Rightarrow \boxed{K = 3/8}$$

(b) What is the probability that the lecture ends within one min of the end of the hr?

$$= P(0<x<1) = \int_0^1 f(x)\,dx$$

$$= \int_0^1 Kx^2\,dx = \boxed{\frac{1}{8}}$$

(c) What is the probability that lecture continues the hr for b/w 60 and 90 sec

$$P(1<x<1.5) = \int_1^{1.5} f(x)\,dx = \int_1^{1.5} Kx^2\,dx. \boxed{= 0.2969}$$

prob mass func → discrete variable → $p(x)$

prob density function → continuous variable → $f(x)$

(d) Prob that the lect continues for at-least 90 sec beyound the end of hour

$= P(X \geq 90sec) = P(x \geq 1.5sec)$

$= \int_{1.5}^{2} \cancel{f(x)}dx + \int_{2}^{\infty} \cancel{f(x)} dx$

Kx^2 $\quad$ 0

$= \int_{1.5}^{2} Kx^2 dx = \boxed{0.5781}$

(Q) Let X denote the amount of time a book on two-hour reserve is actually checked out, and suppose cdf is

$$F(x) = \begin{cases} 0, & x<0 \\ x^2/4, & x \in [0,2) \\ 1, & x \geq 2 \end{cases}$$

(a) Calculate $P(X \leq 1)$

$= P(-\infty \leq X \leq 1)$

$= F(1) - F(-\infty)$

$= \left(\frac{1^2}{4}\right) - 0 = \boxed{1/4}$

(b) Calculate $P(1/2 \leq X \leq 1)$

$= F(1) - F(0.5)$

$= 1^2/4 - (1/2)^2/4$

$= 1/4 - 1/4/4$

$= \boxed{3/16}$

(c) $P(X \geq 1.5)$

$= P(1.5 < X < \infty)$

$= P(\infty) - P(1.5) = 1 - \frac{1.5^2}{4} = \boxed{7/16}$

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→ if asked (90)th percentile
$F(\eta(P)) = 90/100$.

(d) The median checkout duration
$\Rightarrow P = 0.5 = F(\eta(P))$

$\Rightarrow 1/2 = \frac{[\eta(P)]^2}{4} \Rightarrow \boxed{\eta(P) = 1.41}$

Remember $\eta(p)$ always belongs to the interval.

(e) The density function $f(x) = ?$
→ if f^n not continuous ⇒ not differentiable.

continuity questionable at 0 and 2.
at $x=0 \rightarrow (0)$ and $(0) \rightarrow x^2/4$ ⇒ cont.
at $x=2 \rightarrow \frac{x^2}{4} = 1$ and 1 ⇒ cont.

⇒ function $F(x)$ continuous everywhere.

$$\therefore f(x) = F'(x) = \begin{cases} 0, & x<0 \\ x/2, & x \in [0,2) \\ 0, & x \geq 2 \end{cases}$$

(f) Calculate $E(X)$ and $V(x)$.

$$\therefore E(x) = \int x f(x)\,dx = \int_{-\infty}^{0} x \times 0\,dx + \int_0^2 x \times \frac{x}{2} + \int_2^{\infty} x \times 0\,dx$$

$$= \int_0^2 \frac{x^2}{2}\,dx = \boxed{4/3} \text{ -ans}$$

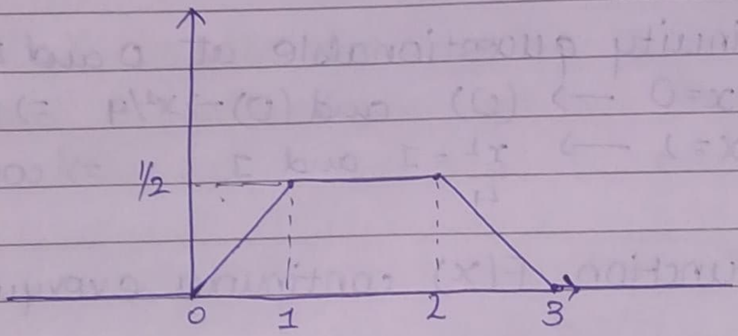
$$\therefore E(x^2) = \int_0^2 x^2 \times \frac{x}{2}\,dx = \left(\frac{1}{2}\right)\left(\frac{1}{\cancel{4}}\right)(\cancel{4})(4) = \boxed{2}$$

$$\therefore V(x) = E(x^2) - [E(x)]^2 = 2 - \left(\frac{4}{3}\right)^2 = \boxed{0.4714} = \sigma_x^2$$

** cummulative.

(Q) Find the distribution function of the random variable whose pdf is given by

$$f(x) = \begin{cases} x/2 & , 0 < x \le 1 \\ 1/2 & , 1 < x \le 2 \\ 3 - x/2 & , 2 < x < 3 \\ 0 & ; \text{elsewhere.} \end{cases}$$



$$F(x) = \begin{cases} \boxed{0} & , x < 0 \\ 0 + \int_0^x \frac{x}{2}\,dx = \boxed{\frac{x^2}{4}} & , 0 \le x < 1 \\ \frac{(1)^2}{4} + \int \frac{1}{2}\,dx = \boxed{\frac{(x-1)}{2} + \frac{1}{4}} & , 1 \le x < 2 \\ \left(\frac{2-1}{2} + \frac{1}{4}\right) + \int_2^3 \left(\frac{3-x}{2}\right) dx & , 2 \le x < 3 \\ = \boxed{\frac{-x^2 + 6x - 5}{4}} & \\ \frac{-3^2 + 6 \times 3 - 5}{4} + 0 = \boxed{1} & , x \ge 3 \end{cases}$$

atmost means — less than or equal to

atleast means — more than or equal to.

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(Q) Find mean and variance using MGF for pdf $= \frac{x}{2}$, $x \in (0, 2)$

$\therefore$ MGF $= E(e^{tx}) = \int (e^{tx}) f(x)\,dx$

$\therefore$ MGF $= \int_0^2 (e^{tx})\left(\frac{x}{2}\right) dx = \left(\frac{1}{2t^2}\right)\left(e^{2t}(2t-1)+1\right)$

$\left[\frac{x}{2}\int e^{tx} - \int \frac{1}{2}\int e^{tx}\right]_0^2$

$\therefore E(X) = \left.\frac{d}{dt} M_x(t)\right|_{t=0} = 4/3$

$\therefore E(X^2) = \left.\frac{d^2}{dt^2} M_x(t)\right|_{t=0} = 2$

$\therefore$ Variance $(V(X))$
$= E(X^2) - (E(X))^2$
$= 2 - \frac{16}{9} = 2/9$

(Q) Let X be the temperature in °C at which a certain chemical rxn takes places, and let Y be temp in °F (so $Y = 1.8X + 32$)

(a) If median of X distribution is μ, show that $1.8\mu + 32$ is median of Y distribution.

$\therefore F_X(\mu) = 0.5$ — give $\Rightarrow P_X(X \leq \mu) = 0.5$

$F_Y(1.8\mu + 32) = 0.5$ — to prove

assume $F_Y(1.8\mu + 32) = P_Y(Y \leq 1.8\mu + 32)$

$= P_Y(1.8X + \cancel{32} \leq 1.8\mu + \cancel{32})$

$= P_Y(X \leq \mu)$

$= 0.5$

Hence proved.

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NOTE now similarly for any $(100p)^{th}$ percentile we can prove this and mean and median also satisfies this

	Mean	Median	$(100p)^{th}$ percentile
x	μ	ν	λ
$y = ax + b$	$a\mu + b$	$a\nu + b$	$a\lambda + b$.

Hence we can have so different continuous pdf as long as it satisfies those 3 criterias discussed on page I. but there are some special ones due to their special properties.