(Q) Annie and Alvie have agreed to meet between 5:00 pm and 6:00 pm. for dinner at a local health-food restraunt. Let X = annie's arrival time and Y = Alvie's arrival time. Suppose X and Y are independent with each uniformly distributed on interval $[5, 6]$.

(a) what is joint pdf of X and Y?
(b) What is probability that they both arrive b/w 5:15 and 5:45?
(c) If the first one to arrive will wait only 10min before leaving to eat elsewhere, what

what is the expected amount of time that the one who arrives first must wait for other $= E(|X-Y|) = ?$

is the prob that they have dinner at health-food restraunt ?

→ First doubt, why continuous?

- cause X can take values $5{\cdot}00, 5{\cdot}01, \ldots 5{\cdot}59, 6{\cdot}00$ and Y can take values $5{\cdot}00, 5{\cdot}01, \ldots 5{\cdot}59, 6{\cdot}00$
Instead of doing for all such we can consider it as continuous i.e. $x \in [5,6]$, $Y \in [5,6]$.

- It is given it follows uniform distribution
$\Rightarrow$ continuous.

→ Now since two variables given
$\Rightarrow$ joint pdf numerical.

(a) Since each follows uniform distribution

$$\Rightarrow f_X(x) = \begin{cases} 1/6-5, & x \in [5,6] \\ 0, & \text{otherwise.} \end{cases}$$

$$\Rightarrow f_Y(y) = \begin{cases} 1/6-5, & y \in [5,6] \\ 0, & \text{otherwise.} \end{cases}$$

$\therefore f(x,y) = f_X(x) \times f_Y(x)$
[cause given they are independent]

$$= \begin{cases} (1)(1), & x \in [5,6] \\ (0)\times(0), & \text{otherwise.} \end{cases}$$

(b) we need to find $P(5.15 \leq X \leq 5.45,\ 5.15 \leq y \leq 5.45)$

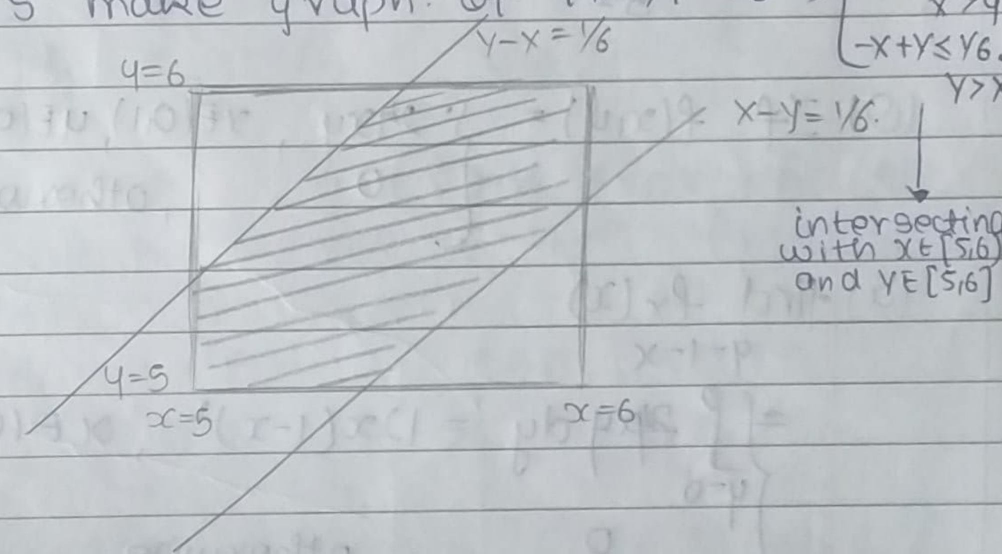
$$= \int_{5.15}^{5.45} \int_{5.15}^{5.45} f(x,y)\, dy\, dx.$$

$$= \int_{5.15}^{5.45} \int_{5.15}^{5.45} 1\, dy\, dx = \boxed{0.09}$$

(c) we need to find prob such that diff b/w arrival of X and y is $\leq$ 10 min $\left(= \frac{10}{60} \text{ hrs}\right)$

$= P(|X-Y| \leq 1/6)$ [∵ cause we don't know X arrives first or Y. Hence consider modulus.]

Now let's make graph of $|X-Y| \leq 1/6 \Rightarrow \begin{cases} X-Y \leq 1/6, & X > Y \\ -X+Y \leq 1/6, & Y > X \end{cases}$



∴ $P(|X-Y| \leq 1/6)$ = shaded region [cause $f(x,y) = 1$]

= (area of square) − (area of two triangles)

$$= (1)(1) - \left[\left(\frac{1}{2}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right) + \left(\frac{1}{2}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\right]$$

$= \boxed{\frac{11}{36}}$ — ans.

(d) Calculate $P(5.2 \le y \le 5.7)$

$$= \int_{5.2}^{5.7} f_Y(y)\,dy$$

$$= \int_{5.2}^{5.7} 1\,dy = (5.7)-(5.2) = \boxed{0.5}$$

(e) Calculate $P(5.2 \le X \le 5.7)$

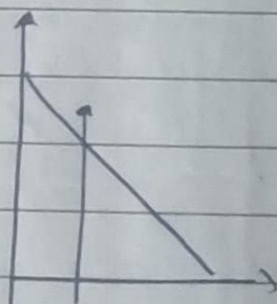
$$= \int_{5.2}^{5.7} f_X(x)\,dx$$

$$= \int_{5.2}^{5.7} 1\,dx = 5.7 - 5.2 = \boxed{0.5}$$

(Q) If $f(x,y) = \begin{cases} 24xy, & x \in (0,1),\ y \in (0,1),\ x+y \le 1 \\ 0, & \text{otherwise.} \end{cases}$

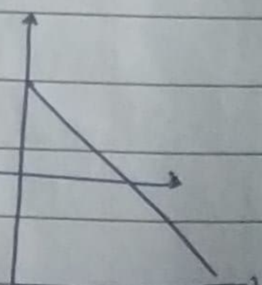
(a) find $f_X(x)$

$$= \begin{cases} \int_{y=0}^{y=1-x} 24xy\,dy = 12x(1-x)^2, & x \in (0,1) \\ 0, & \text{otherwise} \end{cases}$$



(b) find $f_Y(y)$

$$= \begin{cases} \int_{x=0}^{x=1-y} 24xy\,dy = 12y(1-y)^2, & y \in (0,1) \\ 0, & \text{otherwise.} \end{cases}$$



(Q) You have two light bulbs for a particular lamp. let X = lifetime of Ist bulb and Y = lifetime of 2nd bulb. (both in 1000s of hrs). Suppose that X and Y are independent and that each has an exponential distribution with parameter ($\lambda = 1$).

(a) what is joint pdf of X and y.?
(b) Prob that each bulb lasts atmost 1000 hours?
(c) Prob that totol lifetime of two bulbs is atmost 2?
(d) Prob that total lifetime is between 1 and 2?

$X = 1000, 2000, 3000 \ldots$ hrs $Y = 1000, 2000, 3000 \ldots$ hrs

(a) Same logic as previous quoestion

$$f_X(x) = e^{-x} \quad , x \geq 0$$
$$f_Y(y) = e^{-y} \quad , y \geq 0.$$

∴ Since x and y, both are independent of each other $\Rightarrow f(x,y) = f_X(x)\, f_Y(y)$

$$\Rightarrow f(x,y) = e^{-(x+y)} \quad , x \geq 0, y \geq 0.$$

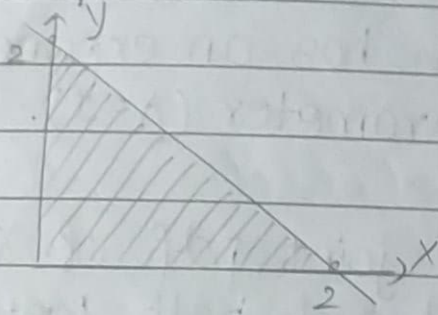
(b) we need to calculate $P(0 \leq X \leq 1, 0 \leq Y \leq 1)$

$$= \int_0^1 \int_0^1 e^{-(x+y)} \, dx\, dy.$$

cause actually $P(0 \leq X \leq 1000)$ but X's are in 1000.

(c) $P(X+Y \leq 2)$

let see intersection of $X+Y \leq 2$ and $x \in (0,\infty)$ and $y \in (0,\infty)$.



$\therefore P(X+Y \leq 2) =$ area of shaded region

$= \cancel{(1/2)(2)(2)}$

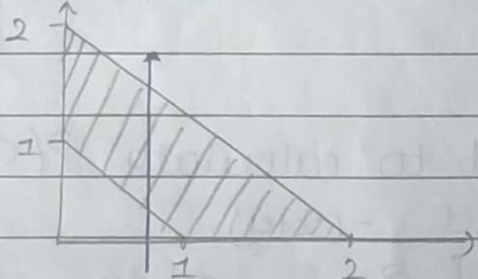
$= \cancel{\boxed{2}}$

$$= \int_0^2 \int_{y=0}^{y=2-x} f(x,y)\, dy\, dx = \int_0^2 \int_0^{2-x} e^{-(x+y)}\, dy\, dx$$

$$= \boxed{1-3e^{-2}}$$

(d) $P(1 \leq X+Y \leq 2)$

let us see intersection of $1 \leq X+Y \leq 2$ and $x \in (0,\infty)$ and $y \in (0,\infty)$



$\therefore P(1 \leq X+Y \leq 2) =$ area of shaded area

$= P(X+Y \leq 2) - P(X+Y \leq 1)$

$$= \int_0^2 \int_0^{2-x} e^{-(x+y)}\, dy\, dx - \int_0^1 \int_0^{1-x} e^{-(x+y)}\, dy\, dx$$

or $$\int_0^2 \int_{1-x}^{2-x} e^{-(x+y)}\, dy\, dx.$$