(Q) Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction of three successive vehicle.

(a) Calculate sample space. — use tree diagram.

S = { (LLL), (LLS), (LLR), (LSL), (LSS), (LSR),
(LRL), (LRS), (LRR), (SLL), (SLS), (SLR),
(SSL), (SSS), (SSR), (SRL), (SRR), (SRS),
(RLL), (RLS), (RLR), (RSL), (RSS), (RSR),
(RRL), (RRS), (RRR), }

$\underline{3}$ $\underline{3}$ $\underline{3}$ => 27 elements in sample space.

(b) list all outcomes in event A that all three vehicles go in same direction.

$A = \{ (L, L, L), (S, S, S), (R, R, R) \}$

(c) list all outcomes in event B that all three vehicles take different direction

$B = \{ (L, R, S), (L, S, R), (R, L, S), (R, S, L), (S, R, L), (S, L, R) \}$

(d) list all outcomes in event C that exactly two vehicles go in same direction.

$C = \{ (L, R, R), (L, S, S), (R, L, L), (R, S, S), (S, L, L), (S, R, R), (R, L, R), (S, L, S), (L, S, L), (R, S, R), (S, R, S), (L, R, L), (R, R, L), (SSL), (LLR), (SSR), (LLS), (RRS) \}$

(e) List all outcomes in event D that exactly two vehicles go right direction.

$D = \{ (RRL), (RRS), (LRR), (SRR), (RLR), (RSR) \}$

Note:- an event may be a subset that includes the $\boxed{\text{entire sample space}}$ S or a subset of S called $\boxed{\text{null set}}$ and denoted by symbol ϕ, which contains no elements at all.

The empty-set ϕ is called $\boxed{\text{impossible-event}}$ while subset S is called $\boxed{\text{certain-event}}$

Example 2.1. *Experiments involving coin tosses:*

(a). *Toss a coin once.*

$S = \{H, T\}$ $\quad |\mathcal{E}| = 2^{|S|} = 2^2 = 4$

$\{H\} \longrightarrow$ "Coin lands H"

$\{H, T\} \longrightarrow$ "tossed the coin".

(b). *Toss the coin two times.*

$S = \{HH, TH, HT, TT\}$

$|\mathcal{E}| = 2^{|S|} = 2^4 = 16$

$\{HT, TH\} \longrightarrow$ "alternating heads and tails"

"a head and a tail was observed"

"exactly one head"

"Exactly one tail"

"Not both of the same type".

ie not the event $\{HH, TT\}$

(c). *Toss the coin 10 times.*

$S = \{HHH \cdots H, HH \cdots HT, \ldots\}$

$|S| = 2^{10} = 1024$

(d). *Keep tossing the coin until the coin lands heads.*

$S = \{H, TH, TTH, TTTH, \ldots\}$

Example 2.2. *Experiments involving die rolls:*

(a). Roll a six-sided die once.

$\{1,2,3,4,5,6\}$

(b). Roll the die two times.

$$S=\left\{\begin{array}{l}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(2,1),(2,2),\ldots,(2,6)\\ \vdots\\(6,1),(6,2),\ldots,(6,6)\end{array}\right\} \qquad |\mathcal{E}|=2^{36}$$

(c). Keep rolling the die until the die rolls to an even number.

$$S=\left\{\begin{array}{l}(2),(4),(6),(1,2),(3,2),(5,2),(1,4),(3,4),(5,4),(1,6),(3,6),(5,6)\\(1,1,2),(1,1,4),\ldots\end{array}\right\}$$

Example 2.3. *Consider the following two step experiment: In step 1, we toss a coin; In step 2, we roll a 6-sided die.*

$$S=\left\{\begin{array}{l}(H,1),(H,2),\ldots,(H,6)\\(T,1),(T,2),\ldots,(T,6)\end{array}\right\}$$

"Coin lands to heads" $\longrightarrow \{(H,1),(H,2),(H,3),\ldots,(H,6)\}$

"Die rolls to even number" $\longrightarrow \{(H,2),(H,4),(H,6),(T,2),(T,4),(T,6)\}$

"Die rolls to a 2" $\longrightarrow \{(H,2),(T,2)\}$

→ since sample space is small & you can manually count, no need to use PnC.

(Q) Computers of six faculty members in a certain department are to be replaced. Two of the faculty members have selected laptop machines and other four have chosen desktop machine. Suppose that only two of the setups can be done on a particular day and the two computers to be setup are randomly selected from six nubered as 1, 2, 3, 4, 5, 6. Assume that computer 1 and 2 are laptops and rest desktops.

(a) Calculate sample space.

$\therefore S = \{(1,2), (23), (13), (14), (15), (16), (24), (25), (26), (34), (35), (36), (45), (46), (56)\}$

(b) Prob that both are laptops

$\therefore E = \{(1,2)\}$

$\therefore P(E) = \dfrac{n(E)}{n(S)} = \boxed{\dfrac{1}{15}}$

(c) Prob that ~~both~~ atleast one are desktops

$E = \{all - (1,2)\} = S - \{1,2\}$

$\therefore P(E) = \dfrac{n(E)}{n(S)} = \boxed{\dfrac{14}{15}}$

(d) Prob that both are desktops.

$E = \{(3,4), (3,5), (36), (45), (46), (5,6)\}$

$\therefore P(E) = n(E)/n(S) = \boxed{6/15}$

(e) Prob that atleast one of each comp is chosen.

$E = \{(2,3)\ (1,3)\ (14)\ (15)\ (16)\ (24), (25)\ (26)\}$

if already prob(s) are given → use set theory/venn diagram to solve

since sample space is small, no need to use ^nC_r.

(Q) An individual is presented with three different glasses of cola, labeled C, D, P. He is asked to taste all three and then list them in order of preference. Suppose that same cola has been put into all three glasses.

(a) What are simple events in exp and what prob would you assign to it.

$\therefore$ S = simple events = sample space = $\{(C,D,P), (CPD), (PDC), (PCD), (DPC), (DCP)\}$

$\therefore$ Since all cola's are same
$\Rightarrow$ all have same prob i.e. (1/6).

(b) Prob that C is ranked first?

$$\boxed{P(E) = \frac{n(E)}{n(S)} = \frac{\text{no of elements in event}}{\text{no. of elements in Sample space.}}}$$

$\therefore$ Event $E = \{(C,P,D), (C,D,P)\}$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \boxed{\frac{1}{3}}$ - ans

(c) Prob that C is ranked first and D is

Following are some standard notations

S	Experiment was performed.
$\emptyset$	Experiment was not performed.
A	Performed experiment and outcome in A
A^c	Outcome not in A ; A has not happened.

S	$P(S) = 1$
$\emptyset$	$P(\phi) = 0$
A	$P(A) = P(\text{Coin lands heads})$
A^c	$P(A^c) = P(\text{coin does not land head})$

Probability Formulas are equivalent to Set Theory Formula, I have shown one here below, rest you can derive yourself

$$\boxed{\begin{aligned} n(A') &= 1 - n(A) \\ \Rightarrow P(A') &= 1 - P(A) \end{aligned}}$$