Prob of drawing 3 cards = (Prob of drawing 1st) × (P of drawing 2nd) × (P of drawing 3rd)

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$$= \left(\frac{{}^{13}C_1}{{}^{52}C_1}\right)\left(\frac{{}^{13}C_1}{{}^{52}C_1}\right)\left(\frac{{}^{13}C_1}{{}^{52}C_1}\right)$$

(Q) If 3 cards are selected ($\boxed{\text{together}}$) at random. What is the prob that all 3 are hearts in a deck of 52 cards?

$\therefore$ $n(3 \text{ from } 52) = n(S) = {}^{52}C_3$

$\therefore$ $n(3 \text{ from } 13 \text{ hearts}) = n(E) = {}^{13}C_3$

$\therefore$ $\boxed{P(E) = \frac{n(E)}{n(S)}} = \boxed{\frac{{}^{13}C_3}{{}^{52}C_3}}$

(Q) same as above but, one by one without replacement $= \frac{{}^{13}C_1 \times {}^{12}C_1 \times {}^{11}C_1}{{}^{52}C_1 \times {}^{51}C_1 \times {}^{50}C_1}$

(Q) same as above, one by one with replacement $= \frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_1 \times {}^{52}C_1 \times {}^{52}C_1}$

(Q) 4 - 40W bulbs, 5 - 60W bulbs, 6 - 75W bulbs.
Suppose 3 bulbs are randomly selected.
$\Rightarrow n(S) = {}^{15}C_3$

(a) P(exactly two are 75W)
$\therefore n(E) = ({}^{6}C_2)({}^{9}C_1)$
$\therefore P(E) = \frac{{}^{6}C_2 \times {}^{9}C_1}{{}^{15}C_3}$

(b) P(all three have same rating)
$\therefore n(E) = {}^{4}C_3 + {}^{5}C_3 + {}^{6}C_3$
$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{{}^{4}C_3 + {}^{5}C_3 + {}^{6}C_3}{{}^{15}C_3}$

(c) P(one bulb of each type selected)
$\therefore n(E) = ({}^{4}C_1)({}^{5}C_1)({}^{6}C_1)$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{({}^{4}C_1)({}^{5}C_1)({}^{6}C_1)}{{}^{15}C_3}$

Imp (d) Suppose now that bulbs are to be selected "one by one" until a 75W bulb is found. What is the probability that it is necessary to examine at least six bulbs? → obvio without replacement

$\Rightarrow$ 75W bulb not found till 5th bulb + 75W bulb not found till 6th bulb + 75W bulb not found till 7th bulb

+ 75W bulb not found till 8th bulb + 75W bulb not found till 9th bulb + ~~75W bulb not found till 10th bulb~~

$= n(E)$

cause after 9 bulbs only 6 75W bulbs left so 10th bulb will be 75W.

$\therefore\ n(E) = \text{Total} - \Big[$ 7SW found in 1st attempt + 7SW found in 2nd attempt + 7SW found in 3rd attempt + 7SW found in 4th attempt + 7SW found in 5th attempt $\Big]$

$$= {}^{15}C_3 - \left[{}^6C_1 + ({}^9C_1)({}^6C_1) + ({}^9C_1)({}^8C_1)({}^6C_1) + ({}^9C_1)({}^8C_1)({}^7C_1)({}^6C_1) + ({}^9C_1)({}^8C_1)({}^7C_1)({}^6C_1)({}^6C_1) \right]$$

$$\therefore\ P(E) = \frac{n(E)}{n(S)} = \frac{n(E)}{\cancel{{}^{15}C_3}}$$

$\longrightarrow$ since one by one this will also change

$$\therefore\ P(E) = 1 - \left[\frac{{}^6C_1}{{}^{15}C_1} + \frac{({}^9C_1)}{({}^{15}C_1)} \times \frac{({}^6C_1)}{({}^{14}C_1)} + \frac{({}^9C_1)({}^8C_1)}{({}^{15}C_1)({}^{14}C_1)} \times \frac{({}^6C_1)}{({}^{13}C_1)} + \frac{({}^9C_1)({}^8C_1)({}^7C_1)}{({}^{15}C_1)({}^{14}C_1)({}^{13}C_1)} \times \frac{({}^6C_1)}{({}^{12}C_1)} + \frac{({}^9C_1)({}^8C_1)({}^7C_1)({}^6C_1)}{({}^{15}C_1)({}^{14}C_1)({}^{13}C_1)({}^{12}C_1)} \times \frac{{}^6C_1}{{}^{11}C_1} \right]$$