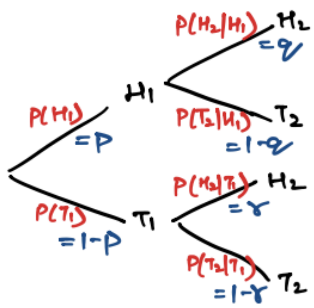
(Q) Toss two coins / Toss a coin twice, find all the probs in red

(a)



Sample Space		Given
(H_1H_2)	$\longrightarrow$	$P(H_1H_2) = 1/4$
(H_1T_2)	$\longrightarrow$	$P(H_1T_2) = 1/4$
(T_1H_2)	$\longrightarrow$	$P(T_1H_2) = 1/4$
(T_1T_2)	$\longrightarrow$	$P(T_1T_2) = 1/4$

$\therefore (p)(q) = 1/4 \qquad \therefore (1-p)(r) = 1/4$

$\therefore (p)(1-q) = 1/4 \qquad \therefore (1-p)(1-r) = 1/4$

$\Rightarrow$ on solving the equations you will get
$p = q = r = \frac{1}{2} \Rightarrow$ FAIR COIN

$$P\{H_1H_2, H_1T_2, T_1T_2\} = P\{H_1H_2\} + P\{H_1T_2\} + P\{T_1T_2\} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \boxed{\frac{3}{4} = \frac{\text{no. of elements in event set}}{\text{no. of elements in sample set}}}$$

Now let's test if the two coin tosses are independent of each other or not

Method$_1$ $\to$ to prove this we need to prove $P(H_1H_2) = P(H_1)\cdot P(H_2)$

$P(H_1H_2) = p \cdot q = \frac{1}{4}$

$P(H_1) = p = \frac{1}{2}$

$P(H_2) = P(H_1, H_2) + P(T_1H_2)$ (cause cases, you can get H is coin 2 by getting H or T in coin 1)
$= pq + (1-p)(r)$
$= \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}$

$\Rightarrow \frac{1}{4} = \frac{1}{4}$

$\Rightarrow$ yes they are independent events

Method$_2$ $\to$ to prove this we need to prove $P(H_2|H_1) = P(H_2)$

$P(H_2|H_1) = q = 1/2$ $\qquad P(H_2) = \frac{1}{2}$ (proved above)

$\Rightarrow$ yes they are independent

Method$_3$ $\to$ to prove this we need to prove $P(H_1|H_2) = P(H_1)$

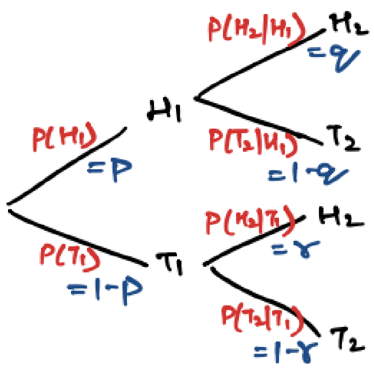
$P(H_1) = p = 1/2$

$P(H_1|H_2) = \frac{P(H_1H_2)}{P(H_2)}$ (bayes)

$= \frac{p\cdot q}{1/2} = \frac{1/4}{1/2} = \frac{1}{2}$

$\Rightarrow$ yes they are independent

(b)



Sample Space		Given
$(H_1 H_2)$	$\longrightarrow$	$P(H_1 H_2) = 1/3$
$(H_1 T_2)$	$\longrightarrow$	$P(H_1 T_2) = 1/3$
$(T_1 H_2)$	$\longrightarrow$	$P(T_1 H_2) = 1/3$
$(T_1 T_2)$	$\longrightarrow$	$P(T_1 T_2) = 0$

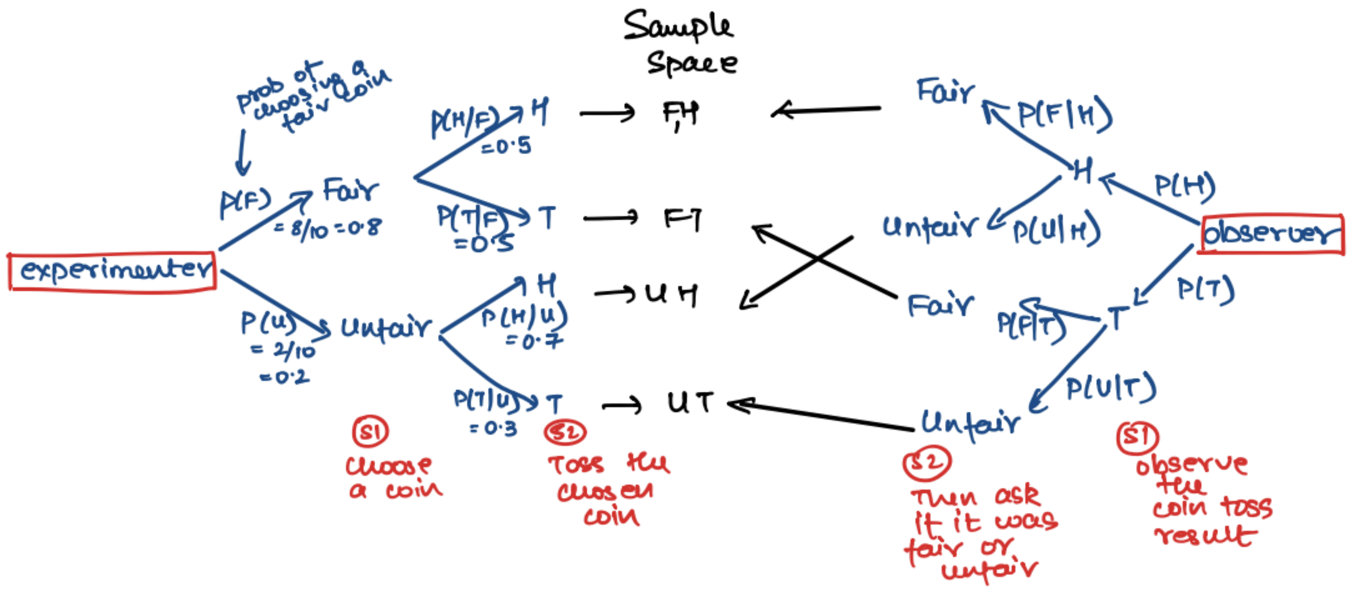
$\therefore \ (p)(q) = 1/3 \qquad , \ (1-p)(r) = 1/3$

$\therefore \ p(1-q) = 1/3 \qquad , \ (1-p)(1-r) = 0$

$\Rightarrow p = 2/3 , \ q = 1/2 , \ r = 1 \ \Rightarrow$ BIASED COIN

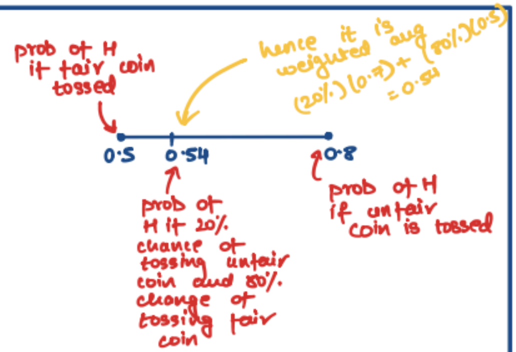
$$P\{H_1H_2, H_1T_2, T_1T_2\} = P\{H_1H_2\} + P\{H_1T_2\} + P\{T_1T_2\}$$
$$= \frac{1}{3} + \frac{1}{3} + 0 = \boxed{\frac{2}{3} \neq \frac{\text{no. of elements in event set}}{\text{no. of elements in sample set}}}$$

(Q) Assume a bag has 8 fair coins and 2 unfair coins with $P(H) = 0.7$. We randomly choose a coin & toss it once. Given that coin landed H, what is the prob that it is a fair coin??



$$P(\text{coin landed } H) = P(F,H) + P(U,H) \quad [\because \text{sum cause cases}]$$
$$= P(H|F)\cdot P(F) + P(H|U)\cdot P(U)$$
$$= (0.8)(0.5) + (0.2)(0.7)$$
$$= 0.4 + 0.14$$
$$= 0.54$$

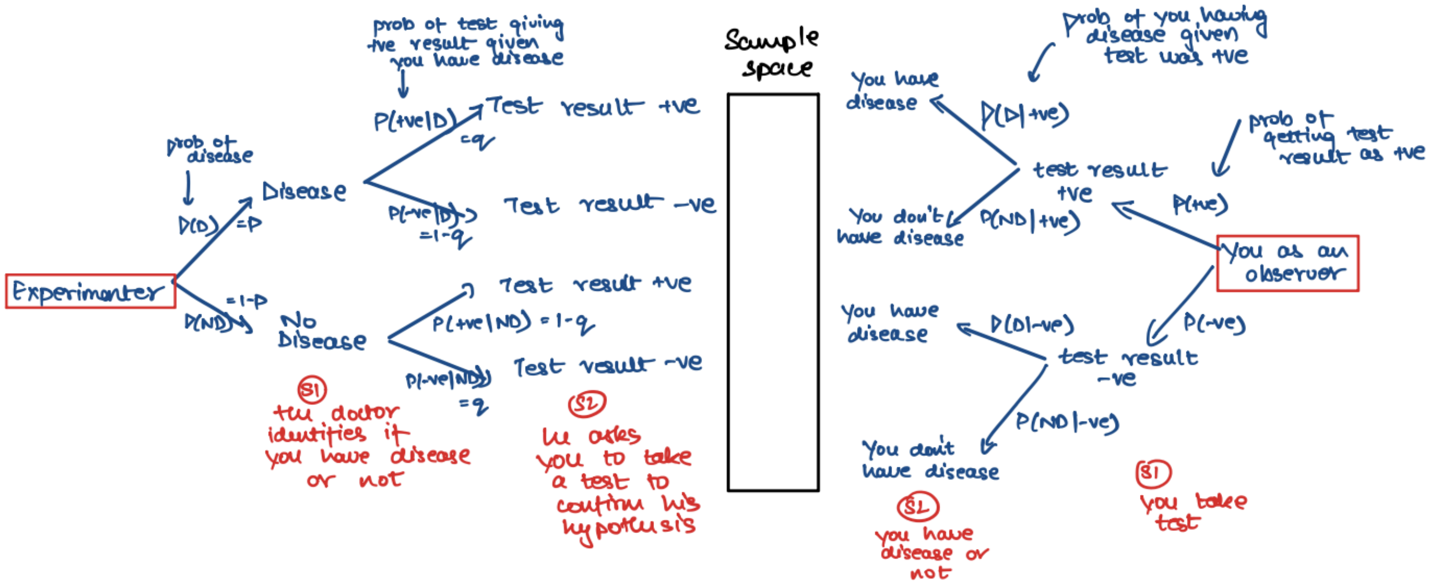
Observation :



$\therefore$ P (tossing a fair coin given Head is observed) $= P(F|H)$

$$= \frac{P(F,H)}{P(H)}$$
$$= \frac{(0.8)(0.5)}{0.54} \quad (\text{from part a})$$
$$= 0.71$$

(10) Disease Testing



$$\therefore P(D|+ve) = \frac{P(D,+ve)}{P(+ve)} \quad [\because \text{Bayes Theorem}]$$

$$= \frac{P(D,+ve)}{P(D,+ve) + P(ND,+ve)}$$

$$= \frac{P(+ve|D)\cdot P(D)}{P(+ve|D)\cdot P(D) + P(+ve|ND)\cdot P(ND)}$$

$$= \frac{pq}{pq + (1-p)(1-q)}$$

$$\therefore P(ND|-ve) = \frac{P(ND,-ve)}{P(-ve)}$$

$$= \frac{P(ND,-ve)}{P(ND,-ve) + P(D,-ve)}$$

$$= \frac{P(-ve|ND)\cdot P(ND)}{P(-ve|ND)\cdot P(ND) + P(-ve|D)\cdot P(D)}$$

$$= \frac{(q)(1-p)}{(q)(1-p) + (1-q)(p)}$$

(Q) 70% of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared

→ let mutually exclusive and exhaustive events be

(m1) $A = \{$ has emergency locator $\}$

$B = \{$ does not has emergency locator $\}$

special event $E = \{$ light aircraft has disappeared $\}$

(m2) $A = \{$ the aircraft that disappear while in flight is subsequently discovered $\}$,

$B = \{$ the aircraft that dissapear while in flight is not discovered $\}$

$$P(E'/B) = \frac{P(E' \cap B)}{P(B)} = \frac{P(B) - P(B \cap E)}{P(B)} = 1 - P(E/B)$$

$$\Rightarrow P(E/B) = 1 - P(E'/B).$$

logic

special event $E = \{$aircraft has an emergency locator$\}$

$\Rightarrow E' = \{$not has emergency locator$\}$

let opt M2 and process, u would get same answer with M1 also.......

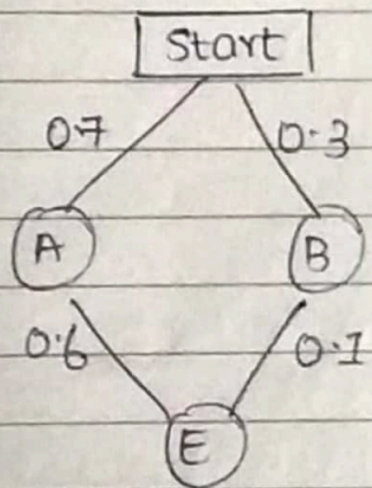
$P(A) = 0.7$, $\Rightarrow P(B) = 1 - P(A) = 1 - 0.7 = 0.3$

$P(E/A) = 0.6$, $\boxed{P(E'/B) = 0.9 \quad \text{or} \quad P(E/B) = 0.1.}$

(a) If it has an emergency locator, find probability that it will not be discovered.

$$= P(B/E) = \frac{P(B \cap E)}{P(E)} = \frac{P(E/B) \times P(B)}{P(E)} = \frac{(0.1)(0.3)}{0.45}$$

$$= \boxed{0.067} \text{ ans}$$

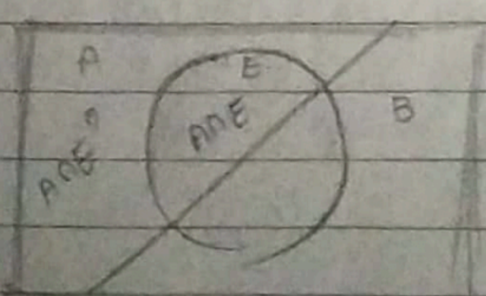


$$\therefore P(E) = (0.7)(0.6) + (0.3)(0.1)$$

$$= \boxed{0.45}$$

(b) If it does not has an emergency locator, what is prob that it will be discovered.

$$= P(A/E') = \frac{P(A \cap E')}{P(E')} = \frac{P(E'/A) \times P(A)}{P(E')}$$

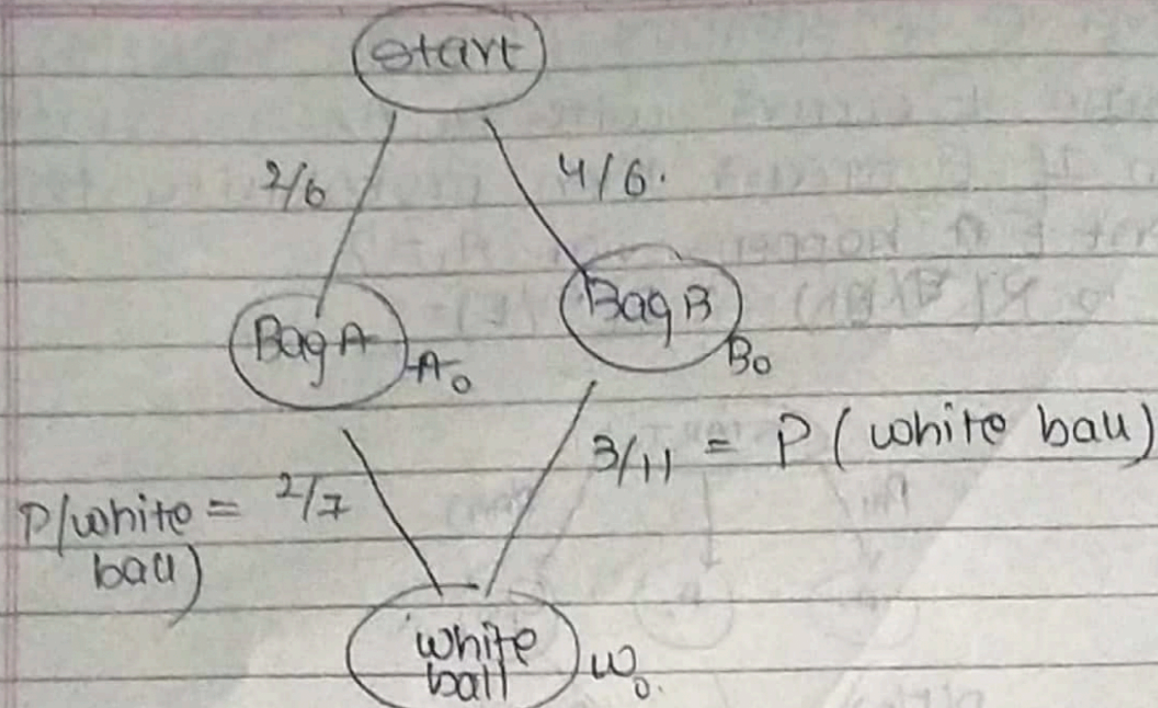


$$= \frac{P(A) - P(A \cap E)}{1 - P(E)}$$

$$= \frac{(0.7) - P(E/A) \times P(A)}{1 - 0.45}$$

$$= \boxed{0.51} \text{ ans.}$$

(Q) Bag A → 2 white & 5 black.
Bag B → 3 white & 8 red.
we cast a die if 1,2 occurs then balls from A is drawn or else from B. If drawn ball is white. Find probability that it came from bag A.



$$\therefore P(A_0/w_0) = \frac{P(A_0) \times P(w_0/A_0)}{P(w_0)}$$

$$= \frac{(2/6)(2/7)}{\left(\frac{2}{6} \times \frac{2}{7}\right) + \left(\frac{4}{6} \times \frac{3}{11}\right)} \quad \text{— ans.}$$