

Trial wave function is

$$\psi_T(\mathbf{r}_1, \mathbf{r}_2) = C \exp\left(-\alpha\omega(r_1^2 + r_2^2)/2\right) \exp\left(\frac{ar_{12}}{1 + \beta r_{12}}\right).$$

The local energy is defined as

$$E_L(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\psi_T} H \psi_T.$$

In our case the Hamiltonian is

$$H = \sum_{i=1}^2 -\frac{1}{2} \nabla_i^2 + \frac{1}{2} \omega^2 r_i^2 + \frac{1}{r_{12}}.$$

The complicated part will be the laplacian.

$$\frac{\partial^2 \psi_T}{\partial x_1^2} = \psi_T \left[ \frac{\partial^2}{\partial x_1^2} \left( -\alpha\omega(r_1^2 + r_2^2)/2 + \frac{ar_{12}}{1 + \beta r_{12}} \right) + \left( \frac{\partial}{\partial x_1} \left( -\alpha\omega(r_1^2 + r_2^2)/2 + \frac{ar_{12}}{1 + \beta r_{12}} \right) \right)^2 \right]$$

Note that

$$\frac{\partial}{\partial x_1} - \alpha\omega(r_1^2 + r_2^2)/2 = -\alpha\omega x_1,$$

and

$$\begin{aligned} \frac{\partial}{\partial x_1} \frac{ar_{12}}{1 + \beta r_{12}} &= a \left( \frac{1}{1 + \beta r_{12}} - \frac{r_{12}\beta}{(1 + \beta r_{12})^2} \right) \frac{\partial r_{12}}{\partial x_1} \\ &= \frac{a}{(1 + \beta r_{12})^2} \frac{\partial r_{12}}{\partial x_1}. \end{aligned}$$

Now

$$\frac{\partial r_{12}}{\partial x_1} = \frac{\partial}{\partial x_1} \left( (x_1 - x_2)^2 + (y_1 - y_2)^2 \right)^{\frac{1}{2}} = \frac{x_1 - x_2}{r_{12}}.$$

So

$$\left( \frac{\partial}{\partial x_1} \left( -\alpha\omega(r_1^2 + r_2^2)/2 + \frac{ar_{12}}{1 + \beta r_{12}} \right) \right)^2 = \alpha^2 \omega^2 x_1^2 - \frac{2\alpha\omega a x_1 (x_1 - x_2)}{(1 + \beta r_{12})^2 r_{12}} + \frac{a^2 (x_1 - x_2)^2}{(1 + \beta r_{12})^4 r_{12}^2}.$$

And

$$\frac{\partial^2}{\partial x_1^2} \left( -\alpha\omega(r_1^2 + r_2^2)/2 + \frac{ar_{12}}{1 + \beta r_{12}} \right) = -\alpha\omega + \frac{a}{(1 + \beta r_{12})^2 r_{12}} - \frac{2a\beta(x_1 - x_2)^2}{(1 + \beta r_{12})^3 r_{12}^2} - \frac{a(x_1 - x_2)^2}{(1 + \beta r_{12})^2 r_{12}^3}$$

Because the  $\psi_T$  is symmetric under  $1 \leftrightarrow 2$  and  $x \leftrightarrow y$ :

$$\frac{1}{\psi_T} \sum_i \nabla_i^2 \psi_T = \alpha\omega(r_1^2 + r_2^2) - 4\alpha\omega + \frac{2a^2}{(1 + \beta r_{12})^4} + \frac{4a}{(1 + \beta r_{12})^2 r_{12}} - \frac{4a\beta}{(1 + \beta r_{12})^3} - \frac{2a}{(1 + \beta r_{12})^2 r_{12}} - \frac{2\alpha\omega a r_{12}}{(1 + \beta r_{12})^2}$$

and finally:

$$\frac{1}{\psi_T} \sum_i \nabla_i^2 \psi_T = \alpha\omega(r_1^2 + r_2^2) - 4\alpha\omega + \frac{2a}{(1 + \beta r_{12})^2} \left[ \frac{a}{(1 + \beta r_{12})^2} + \frac{1}{r_{12}} - \frac{2\beta}{(1 + \beta r_{12})} - \alpha\omega r_{12} \right]$$

Using this expression we have the local energy as

$$E_L(\mathbf{r}_1, \mathbf{r}_2) = \left[ \frac{1}{2} (1 - \alpha)\omega(r_1^2 + r_2^2) + 2\alpha\omega \right] - \frac{a}{(1 + \beta r_{12})^2} \left[ \frac{a}{(1 + \beta r_{12})^2} + \frac{1}{r_{12}} - \frac{2\beta}{(1 + \beta r_{12})} - \alpha\omega r_{12} \right] + \frac{1}{r_{12}},$$

where the second term is from the Jastrow-factor and the third is the interaction term.