EM 314 – NUMERICAL METHODS
ASSIGNMENT 03

MALITHTHA K.H.H.
E/15/220
SEMESTER 04
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01) $P(x) \simeq \ln(x)$ $\quad [1,4]$

x	1	2	3	4
$y = \ln(x)$	0	0.6931	1.0986	1.3863

$$l_i(x) = \prod_{\substack{j=0 \\ i \neq j}}^{n} \frac{x - x_j}{x_i - x_j}$$

$$l_0(x) = \left(\frac{x-2}{1-2}\right)\left(\frac{x-3}{1-3}\right)\left(\frac{x-4}{1-4}\right)$$

$$= -\tfrac{1}{6}(x-2)(x-3)(x-4) \quad \text{——— (1)}$$

$$l_1(x) = \left(\frac{x-1}{2-1}\right)\left(\frac{x-3}{2-3}\right)\left(\frac{x-4}{2-4}\right)$$

$$= \frac{1}{2}(x-1)(x-3)(x-4) \quad \text{——— (2)}$$

$$l_2(x) = \left(\frac{x-1}{3-1}\right)\left(\frac{x-2}{3-2}\right)\left(\frac{x-4}{3-4}\right)$$

$$= -\tfrac{1}{2}(x-1)(x-3)(x-4) \quad \text{——— (3)}$$

$$l_3(x) = \left(\frac{x-1}{4-1}\right)\left(\frac{x-2}{4-2}\right)\left(\frac{x-3}{4-3}\right)$$

$$= \frac{1}{6}(x-1)(x-2)(x-3) \quad \text{——— (4)}$$

Using,

$$P(x) = \sum_{i=0}^{n} y_i l_i(x)$$

$$p(x) = 0\left\{-\tfrac{1}{6}(x-2)(x-3)(x-4)\right\} + 0.6931\left\{\tfrac{1}{2}(x-1)(x-3)(x-4)\right\} + 1.0986\left\{-\tfrac{1}{2}(x-1)(x-2)(x-4)\right\} + 1.3863\left\{\tfrac{1}{6}(x-1)(x-2)(x-3)\right\}$$

$$p(x) = \frac{283}{10000}x^3 - \frac{196}{625}x^2 + \frac{7179}{5000}x - \frac{2301}{2000}$$

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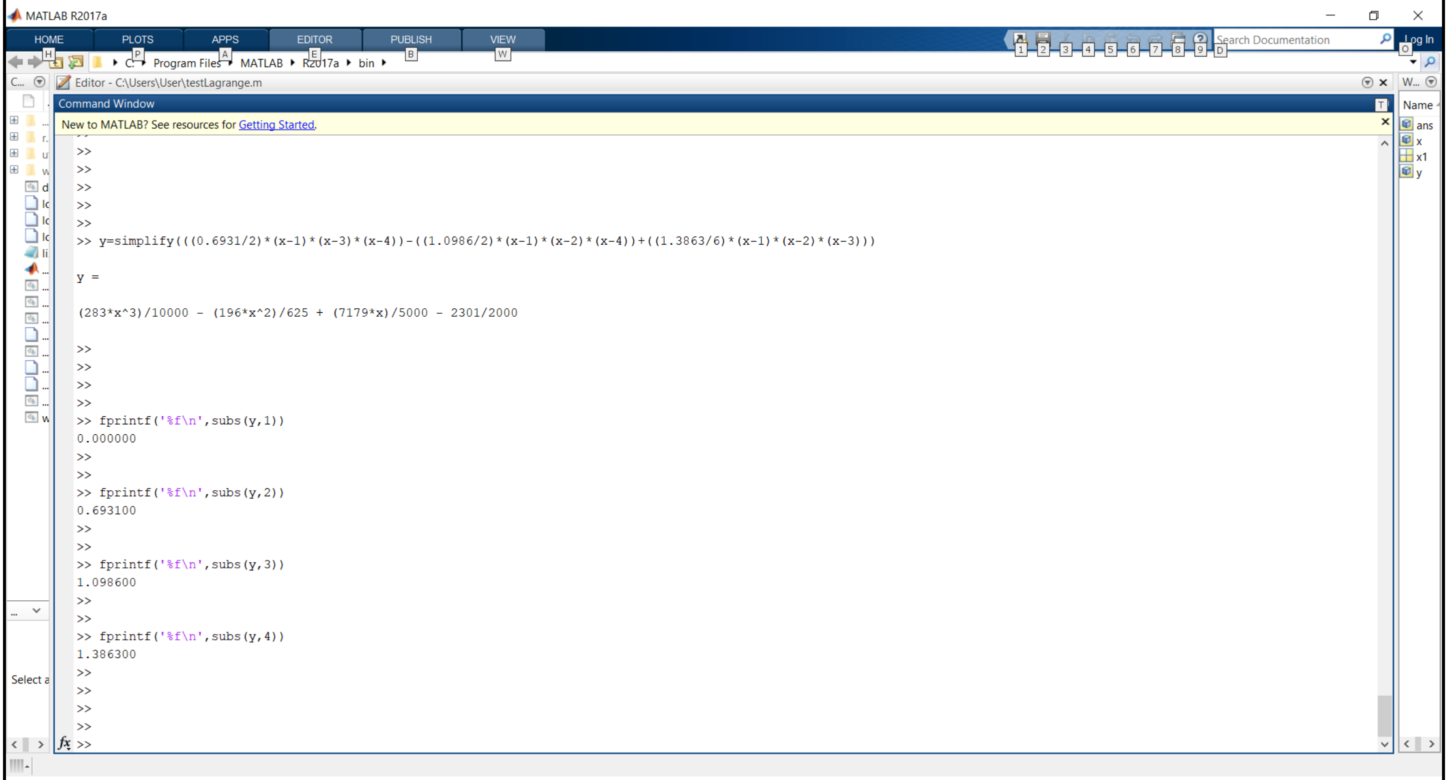
* If we consider,

$$p(x) = \ln(1)\left\{-\tfrac{1}{6}(x-2)(x-3)(x-4)\right\} + \ln(2)\left\{\tfrac{1}{2}(x-1)(x-3)(x-4)\right\} + \ln(3)\left\{-\tfrac{1}{2}(x-1)(x-2)(x-4)\right\} + \ln(4)\left\{\tfrac{1}{6}(x-1)(x-2)(x-3)\right\}$$

$$p(x) = \frac{1020209651736479}{36028797018963968}x^3 - \frac{5651838703799463}{18014398509481984}x^2 + \frac{51742823733302861}{36028797018963968}x - \frac{20729677988720207}{18014398509481984}$$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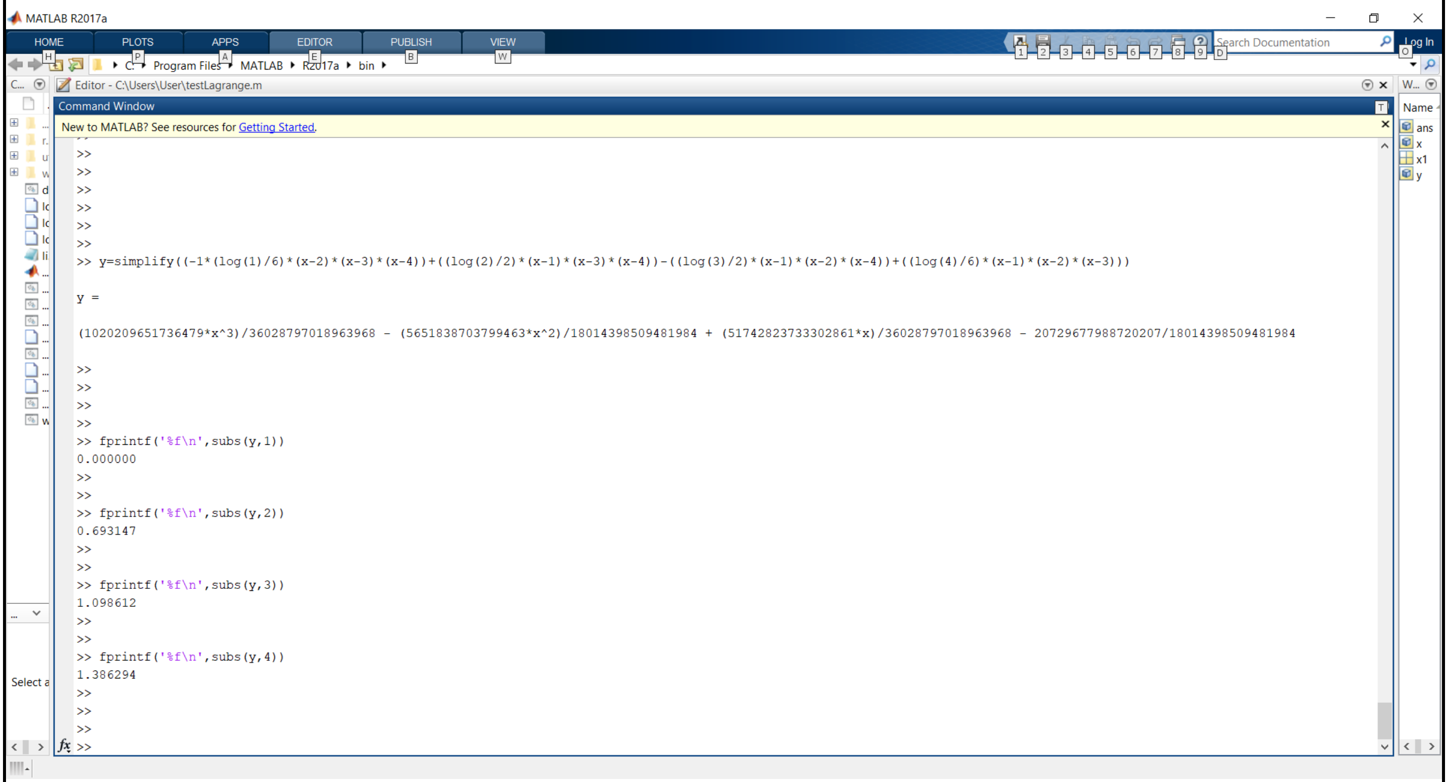
Editor - C:\Users\User\testLagrange.m
Command Window
New to MATLAB? See resources for Getting Started.
>> y=simplify(((0.6931/2)*(x-1)*(x-3)*(x-4))-((1.0986/2)*(x-1)*(x-2)*(x-4))+((1.3863/6)*(x-1)*(x-2)*(x-3)))
y =
(283*x^3)/10000 - (196*x^2)/625 + (7179*x)/5000 - 2301/2000
>> fprintf('%f\n',subs(y,1))
0.000000
>> fprintf('%f\n',subs(y,2))
0.693100
>> fprintf('%f\n',subs(y,3))
1.098600
>> fprintf('%f\n',subs(y,4))
1.386300
Name
ans
x
x1
y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Command Window
New to MATLAB? See resources for Getting Started.
>> y=simplify((-1*(log(1)/6)*(x-2)*(x-3)*(x-4))+((log(2)/2)*(x-1)*(x-3)*(x-4))-((log(3)/2)*(x-1)*(x-2)*(x-4))+((log(4)/6)*(x-1)*(x-2)*(x-3)))
y =
(1020209651736479*x^3)/36028797018963968 - (5651838703799463*x^2)/18014398509481984 + (51742823733302861*x)/36028797018963968 - 20729677988720207/18014398509481984
>> fprintf('%f\n',subs(y,1))
0.000000
>> fprintf('%f\n',subs(y,2))
0.693147
>> fprintf('%f\n',subs(y,3))
1.098612
>> fprintf('%f\n',subs(y,4))
1.386294
Name
ans
x
x1
y

02) $\sum_{i=0}^{n} l_i(x)$

$= l_0(x) + l_1(x) + l_2(x) + \ldots + l_{n-1}(x) + l_n(x)$

$= \prod_{\substack{j=0\\j\neq 0}}^{n} \frac{x-x_j}{x_0-x_j} + \prod_{\substack{j=0\\j\neq 1}}^{n} \frac{x-x_j}{x_1-x_j} + \ldots + \prod_{\substack{j=0\\j\neq n}}^{n} \frac{x-x_j}{x_n-x_j}$

$= \left(\frac{x-x_1}{x_0-x_1}\right)\left(\frac{x-x_2}{x_0-x_2}\right)\cdots\left(\frac{x-x_n}{x_0-x_n}\right) + \left(\frac{x-x_0}{x_1-x_0}\right)\left(\frac{x-x_2}{x_1-x_2}\right)\cdots\left(\frac{x-x_n}{x_1-x_n}\right)$

$+ \ldots + \left(\frac{x-x_1}{x_n-x_1}\right)\left(\frac{x-x_2}{x_n-x_2}\right)\cdots\left(\frac{x-x_{n-1}}{x_n-x_{n-1}}\right)$

Here, if $x = x_0$

$$\prod_{\substack{j=0\\j\neq 0}}^{n} \frac{x-x_j}{x_0-x_j} = \prod_{\substack{j=0\\j\neq 0}}^{n} \frac{x_0-x_j}{x_0-x_j} = 1$$

but, other terms

$$\prod_{\substack{j=0\\j\neq 1}}^{n} \frac{x-x_j}{x_1-x_j} = \prod_{\substack{j=0\\j\neq 1}}^{n} \frac{x_0-x_j}{x_1-x_j}$$

$$= \left(\frac{x_0-x_0}{x_1-x_0}\right)\left(\frac{x_0-x_2}{x_1-x_2}\right)\cdots\left(\frac{x_0-x_n}{x_1-x_n}\right) = 0$$

$\vdots$

$$\prod_{\substack{j=0\\j\neq n}}^{n} \frac{x-x_j}{x_n-x_j} = \prod_{\substack{j=0\\j\neq n}}^{n} \frac{x_0-x_j}{x_n-x_j}$$

$$= \left(\frac{x_0-x_0}{x_n-x_0}\right)\left(\frac{x_0-x_2}{x_n-x_0}\right)\cdots\left(\frac{x_0-x_{n-1}}{x_n-x_{n-1}}\right) = 0$$

$\therefore$ if $\quad x = x_0$

$$\sum_{i=0}^{n} l_i(x) = 1$$

Let consider $x = x_1$ Then,

$$\prod_{\substack{j=0 \\ j\neq 1}}^{n} \frac{x - x_j}{x_1 - x_j} = \prod_{\substack{j=0 \\ j\neq 1}}^{n} \frac{x_1 - x_j}{x_1 - x_j} = 1$$

but, other terms

$$\prod_{\substack{j=0 \\ j\neq 0}}^{n} \frac{x - x_j}{x_0 - x_j} = \prod_{\substack{j=0 \\ j\neq 0}}^{n} \frac{x_1 - x_j}{x_0 - x_j}$$

$$= \left(\frac{x_1 - x_1}{x_0 - x_1}\right)\left(\frac{x_1 - x_2}{x_0 - x_2}\right) \cdots\cdots \left(\frac{x_1 - x_n}{x_0 - x_n}\right) = 0$$

$\vdots$

$$\prod_{\substack{j=0 \\ j\neq n}}^{n} \frac{x - x_j}{x_n - x_j} = \prod_{\substack{j=0 \\ j\neq n}}^{n} \frac{x_1 - x_j}{x_n - x_j}$$

$$= \left(\frac{x_1 - x_1}{x_n - x_1}\right)\left(\frac{x_1 - x_2}{x_n - x_2}\right) \cdots \left(\frac{x_1 - x_{n-1}}{x_n - x_{n-1}}\right) = 0$$

$\therefore$ if $\quad x = x_1$

$$\sum_{i=0}^{n} l_i(x) = 1$$

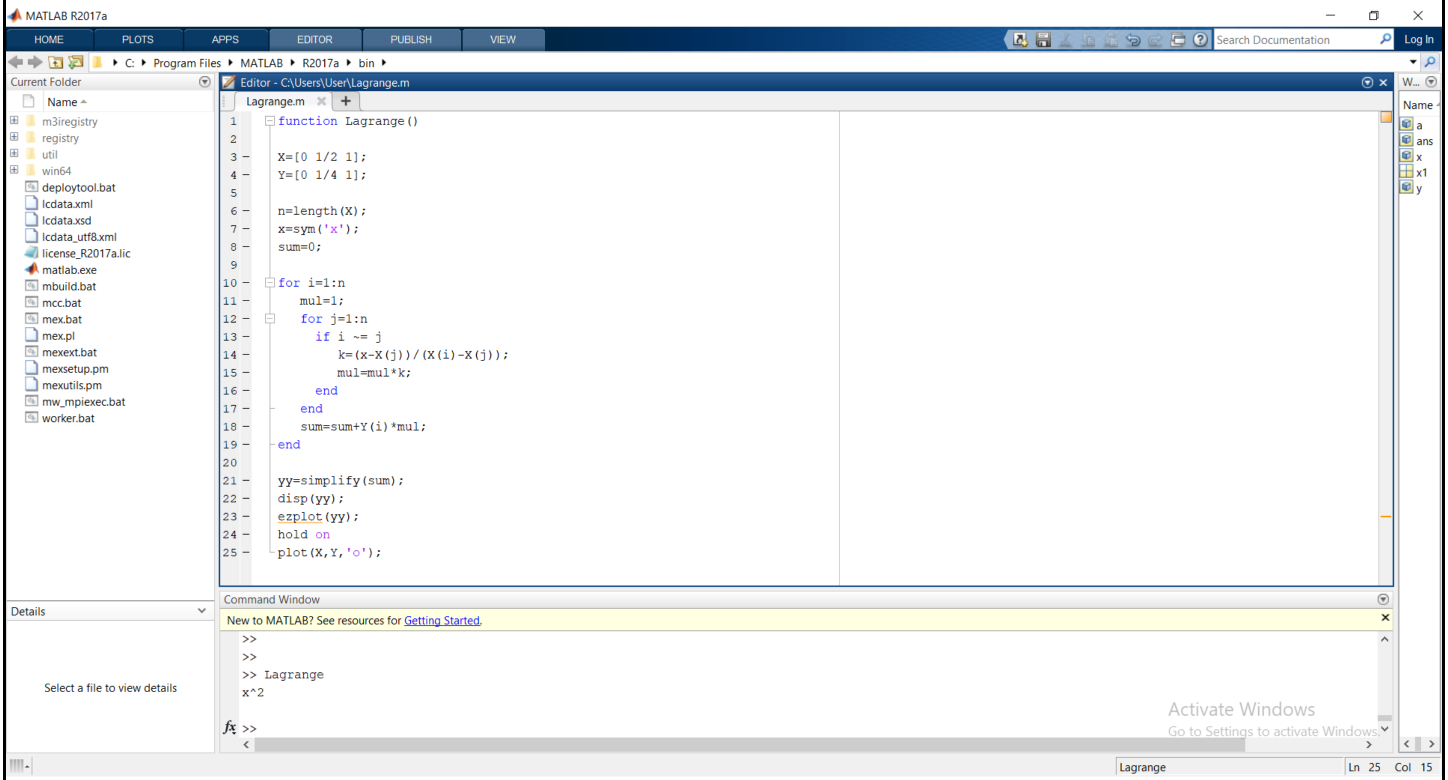
like that we can prove $\forall x = x_i , i = [0, n]$

$$\sum_{i=0}^{n} l_i(x) = 1$$

$$\begin{bmatrix} l_0(x_0) & l_1(x_0) & \cdots & l_n(x_0) \\ l_0(x_1) & l_1(x_1) & \cdots & l_n(x_1) \\ l_0(x_2) & l_0(x_2) & \cdots & l_n(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ l_0(x_n) & l_0(x_n) & \cdots & l_n(x_n) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}}_{\text{Identity matrix}}$$

$$\therefore \sum_{i=0}^{n} l_i(x) = 1 \text{ for all } x$$

3)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mbuild.bat
mcc.bat
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mex.pl
mexext.bat
mexsetup.pm
mexutils.pm
mw_mpiexec.bat
worker.bat
Details
Select a file to view details
Editor - C:\Users\User\Lagrange.m
Lagrange.m
function Lagrange()
X=[0 1/2 1];
Y=[0 1/4 1];
n=length(X);
x=sym('x');
sum=0;
for i=1:n
mul=1;
for j=1:n
if i ~= j
k=(x-X(j))/(X(i)-X(j));
mul=mul*k;
end
end
sum=sum+Y(i)*mul;
end
yy=simplify(sum);
disp(yy);
ezplot(yy);
hold on
plot(X,Y,'o');
Command Window
New to MATLAB? See resources for Getting Started.
>>
>>
>> Lagrange
x^2
>>
Activate Windows
Go to Settings to activate Windows.
W...
Name
a
ans
x
x1
y
Lagrange
Ln 25 Col 15

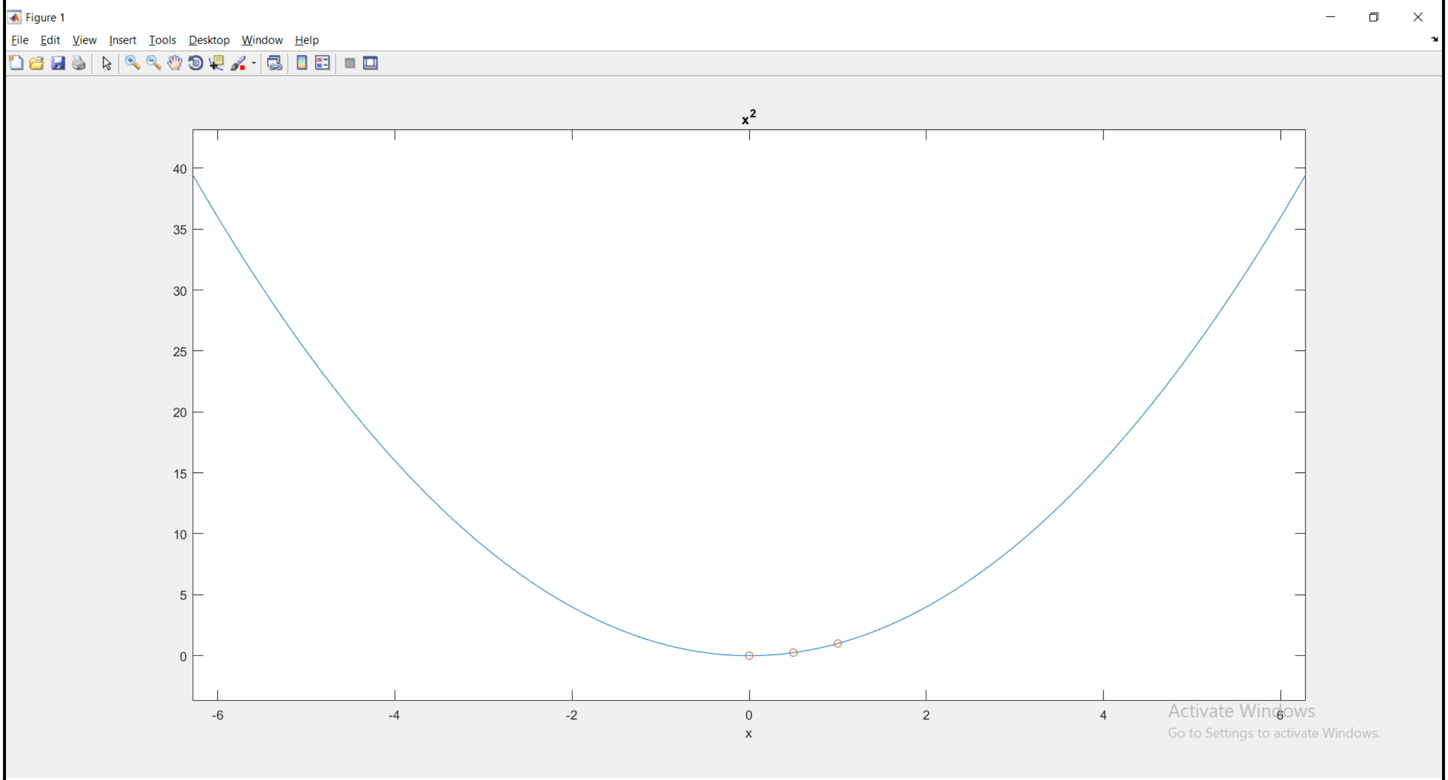
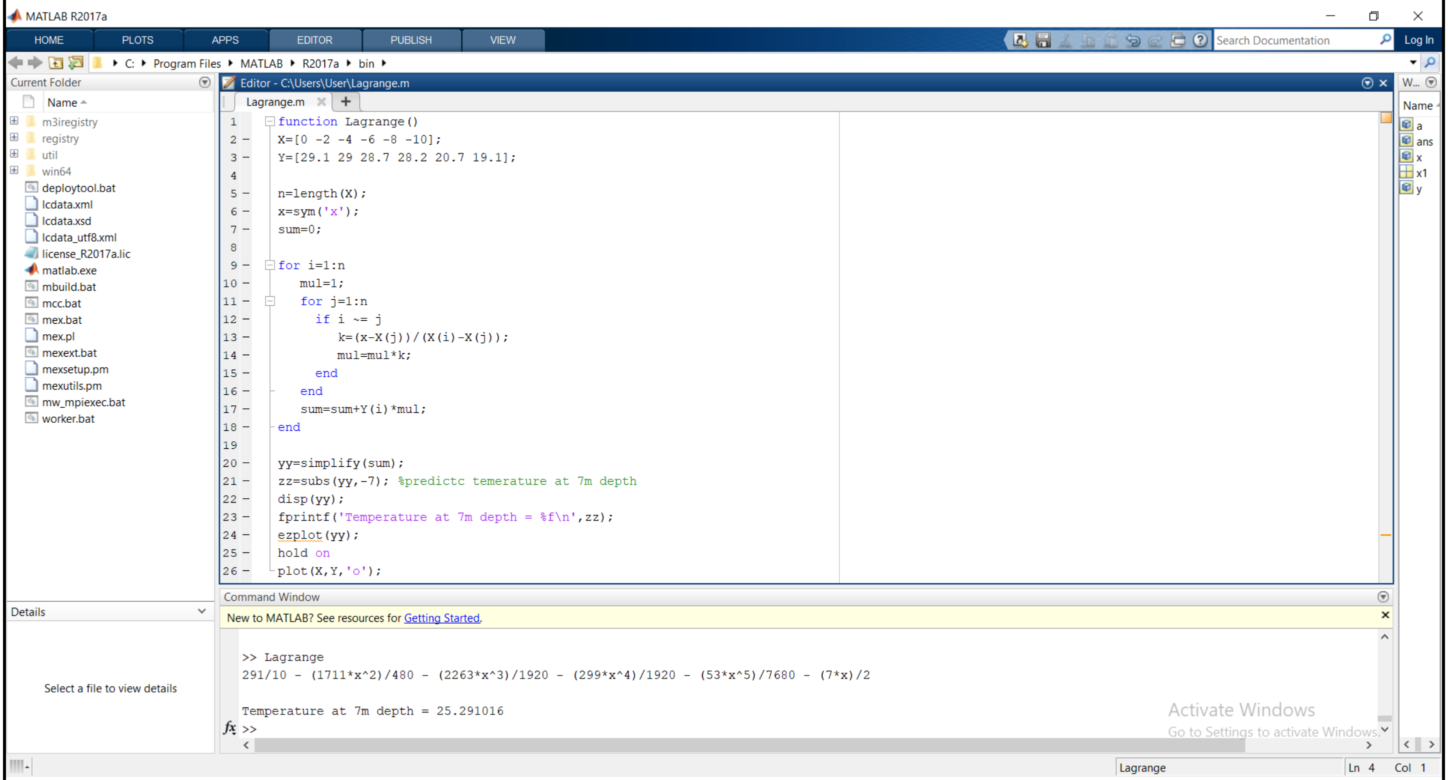


Figure 1
File Edit View Insert Tools Desktop Window Help
x^2
40
35
30
25
20
15
10
5
0
-6
-4
-2
0
2
4
6
x
Activate Windows
Go to Settings to activate Windows.

04)

a)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lcdata_utf8.xml
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matlab.exe
mbuild.bat
mcc.bat
mex.bat
mex.pl
mexext.bat
mexsetup.pm
mexutils.pm
mw_mpiexec.bat
worker.bat
Editor - C:\Users\User\Lagrange.m
Lagrange.m
function Lagrange()
X=[0 -2 -4 -6 -8 -10];
Y=[29.1 29 28.7 28.2 20.7 19.1];
n=length(X);
x=sym('x');
sum=0;
for i=1:n
mul=1;
for j=1:n
if i ~= j
k=(x-X(j))/(X(i)-X(j));
mul=mul*k;
end
end
sum=sum+Y(i)*mul;
end
yy=simplify(sum);
zz=subs(yy,-7); %predictc temerature at 7m depth
disp(yy);
fprintf('Temperature at 7m depth = %f\n',zz);
ezplot(yy);
hold on
plot(X,Y,'o');
Details
Select a file to view details
Command Window
New to MATLAB? See resources for Getting Started.
>> Lagrange
291/10 - (1711*x^2)/480 - (2263*x^3)/1920 - (299*x^4)/1920 - (53*x^5)/7680 - (7*x)/2
Temperature at 7m depth = 25.291016
>>
Activate Windows
Go to Settings to activate Windows.
W...
Name
a
ans
x
x1
y
Lagrange
Ln 4
Col 1

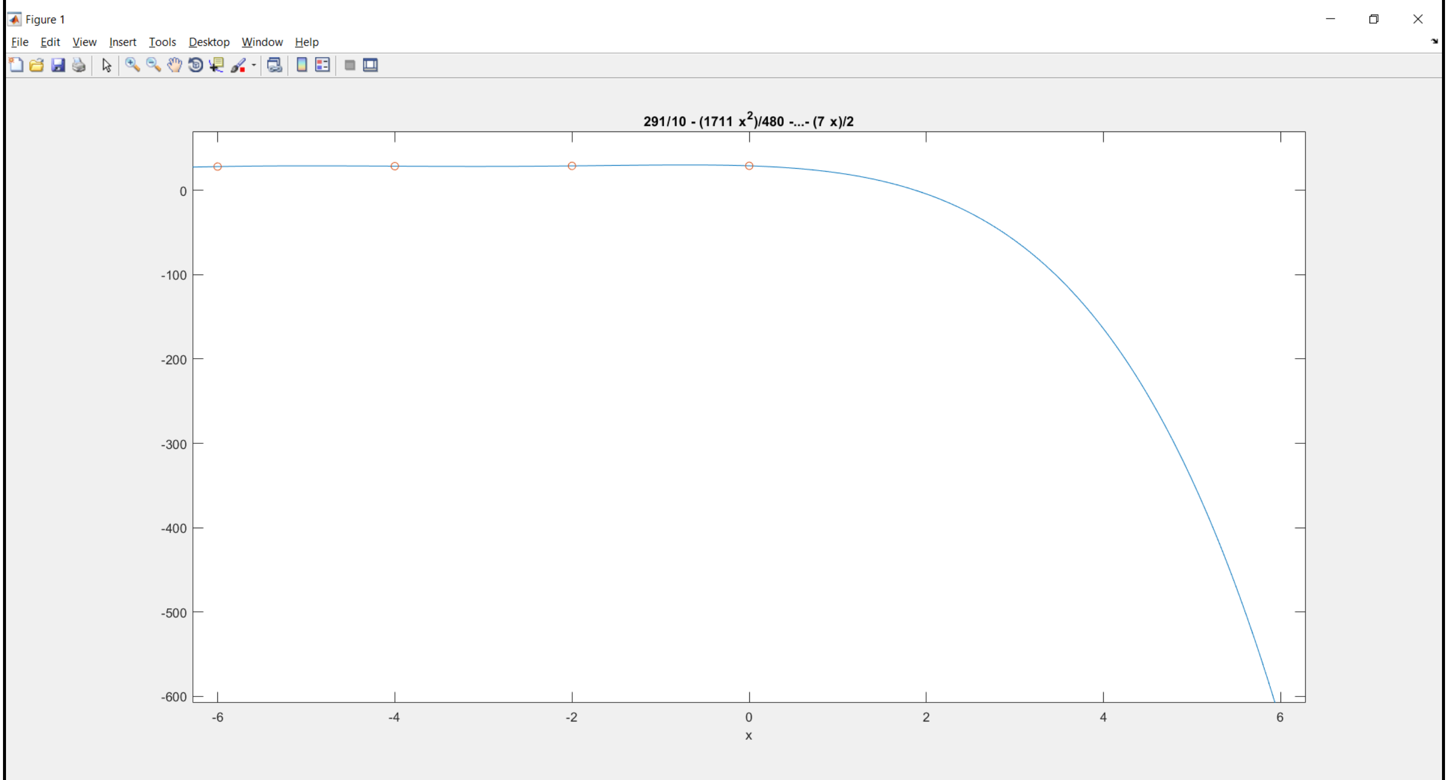


Figure 1
File
Edit
View
Insert
Tools
Desktop
Window
Help
291/10 - (1711 x^2)/480 -...- (7 x)/2
0
-100
-200
-300
-400
-500
-600
-6
-4
-2
0
2
4
6
x

b)

Temperature at 7m depth = 25.291016

By observing the table, this is can be a valid answer.

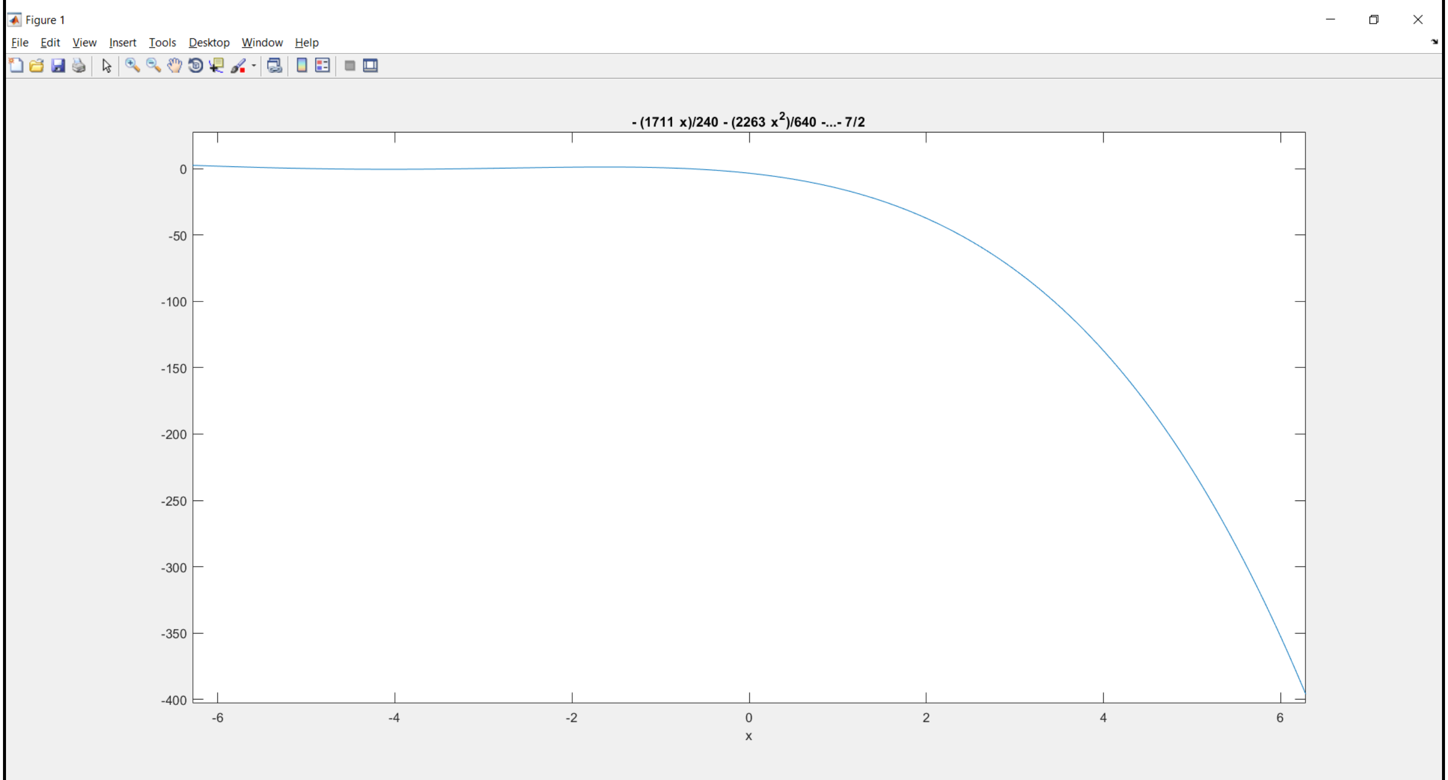
Because, at 6m depth temperature = 28.2 and

at 8m depth temperature = 20.7 so at 7m depth

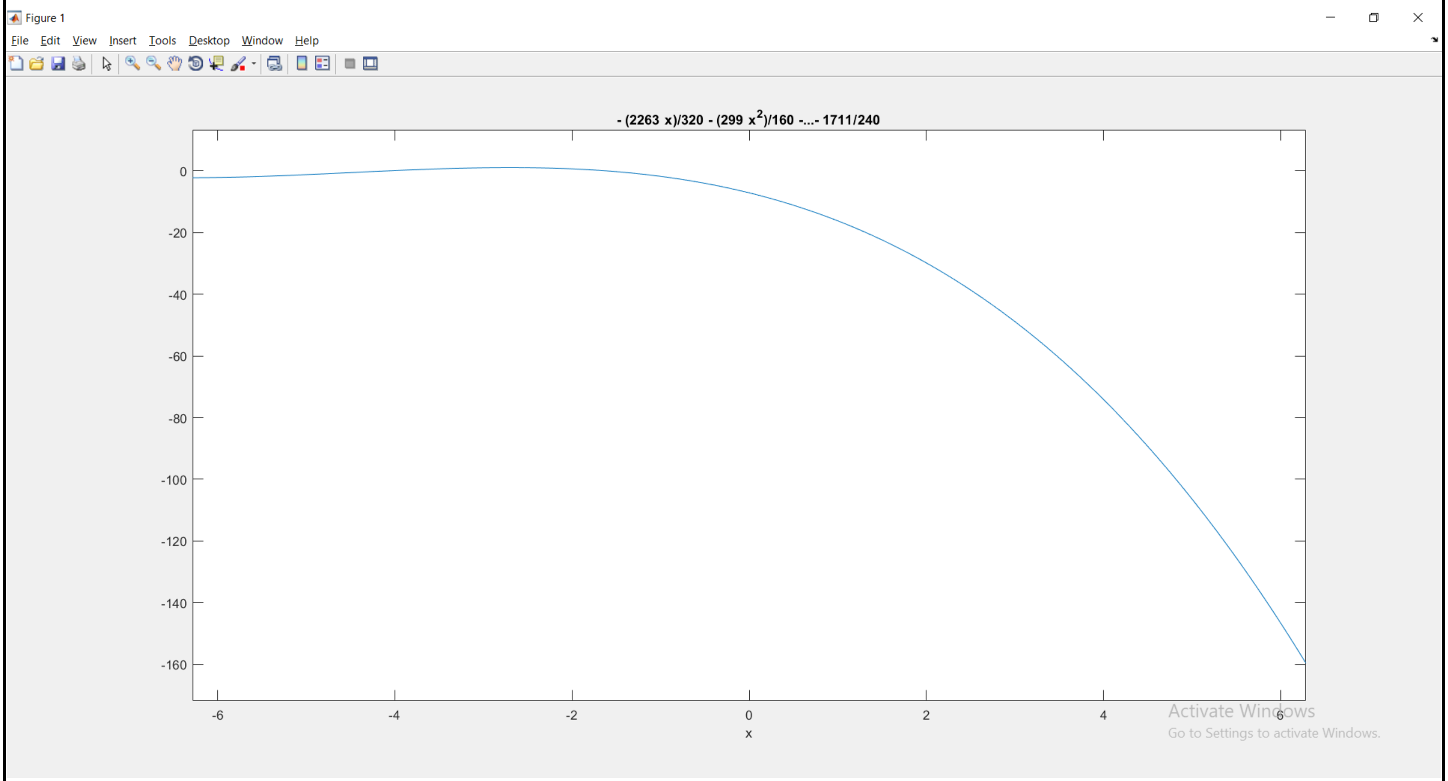
temperature would between these two values

c)

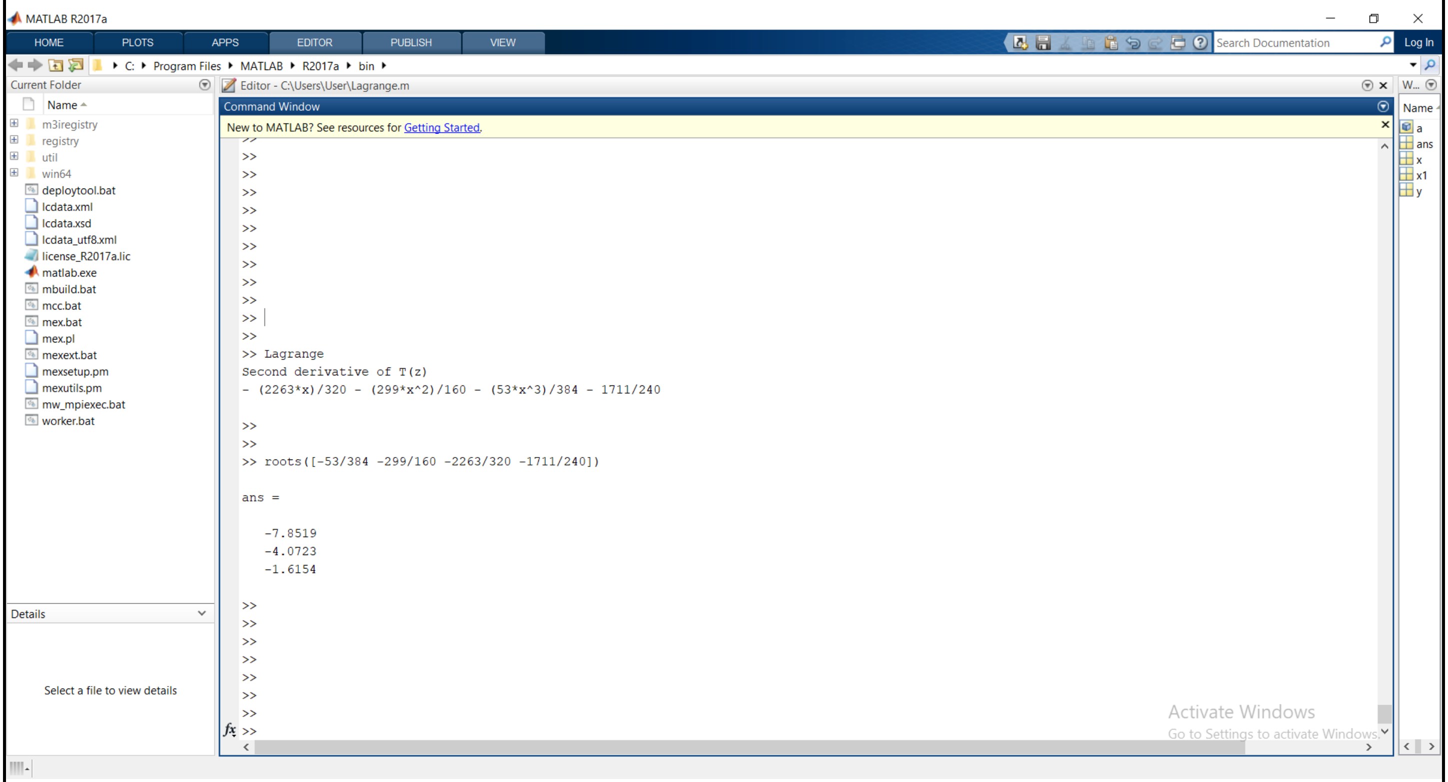
Graph of first derivative (dT/dz)



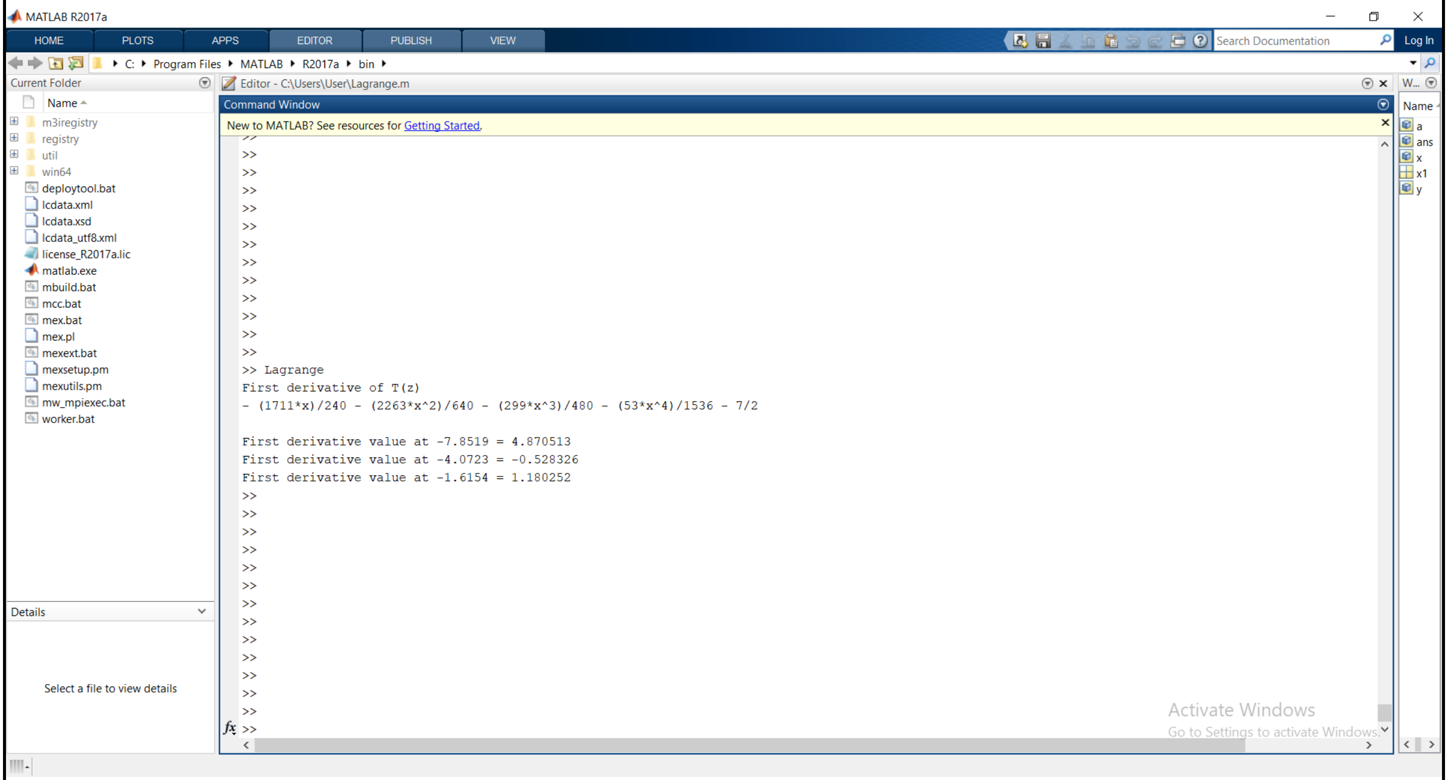
Graph of second derivative (d^2T/dz^2)



Roots of second derivative



Roots of second derivative function substitute to first derivative function to find maximum



Maximum value of first derivative = 4.870513

At 7.8519m depth