EM 314 – NUMERICAL METHODS

ASSIGNMENT 01

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SEMESTER 04
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(1) $\sin x \approx x$

Taylor series

$$f(x+h) = f(x) + \frac{f'(x)h}{1} + \frac{f''(x)h^2}{2!} + \frac{f'''(x)h^3}{3!} + \cdots$$

$$\sin(x_0+x) = f(x_0) + \frac{f'(x_0)x}{1} + \frac{f''(x_0)x^2}{2!} + \frac{f'''(x_0)x^3}{3!} + \cdots$$

Then $x_0 = 0$

$$\sin(x) = f(0) + \frac{f'(0)x}{1} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f''''(0)x^4}{4!} + \cdots$$

$$\sin(x) = 0 + x + 0 - \frac{x^3}{3!} + 0 + \frac{x^5}{5!} \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots$$

$$\text{Error} = \frac{x^5}{5!} - \frac{x^3}{3!} \leq 10^{-6}$$

$$\frac{x^5}{120} - \frac{x^3}{6} \leq 10^{-6}$$

~~$x^5 - 20x^3 - 10^{-6} = 0$~~

$$x^5 - 20x^3 - 1.2\times10^{-4} = 0$$

~~$x \approx -0.00368$ / $x \approx 4.47214$ / $x \approx -4.47214$~~

$x \approx -0.01817$ / $x \approx 4.47214$ / $x = -4.72214$

$$-4.47214 \leq x \leq 4.47214$$

02) Any $x_* \in F$ can be represented as

$$x_* = (-1)^s (.a_1 \dots a_t) \beta^m$$

The sign bit can assume 2 values

Each of digits $a_2, a_3, \dots, a_t$ can assume β different values, while a_1 can assume only $\beta - 1$ values.

$\therefore$ The maintissa assumes $(\beta-1)\beta^{t-1}$ different values

The exponent can assume $U - L + 1$ different values.

$\therefore$ The set F contains

$$2(\beta-1)\beta^{t-1}(U-L+1) \text{ elements.}$$

$\vdots$

03) From Taylor series

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(\xi) \quad \text{where } x < \xi < x+h$$

$$\therefore \quad f'(x) = \frac{f(x+h) - f(x)}{h} - \frac{h f''(\xi)}{2}$$

$$= f_h'(x) - \frac{h f''(\xi)}{2}$$

$$\therefore E_h(x) = \left| f'(x) - f_h'(x) \right| = \left| \frac{h f''(\xi)}{2} \right|$$

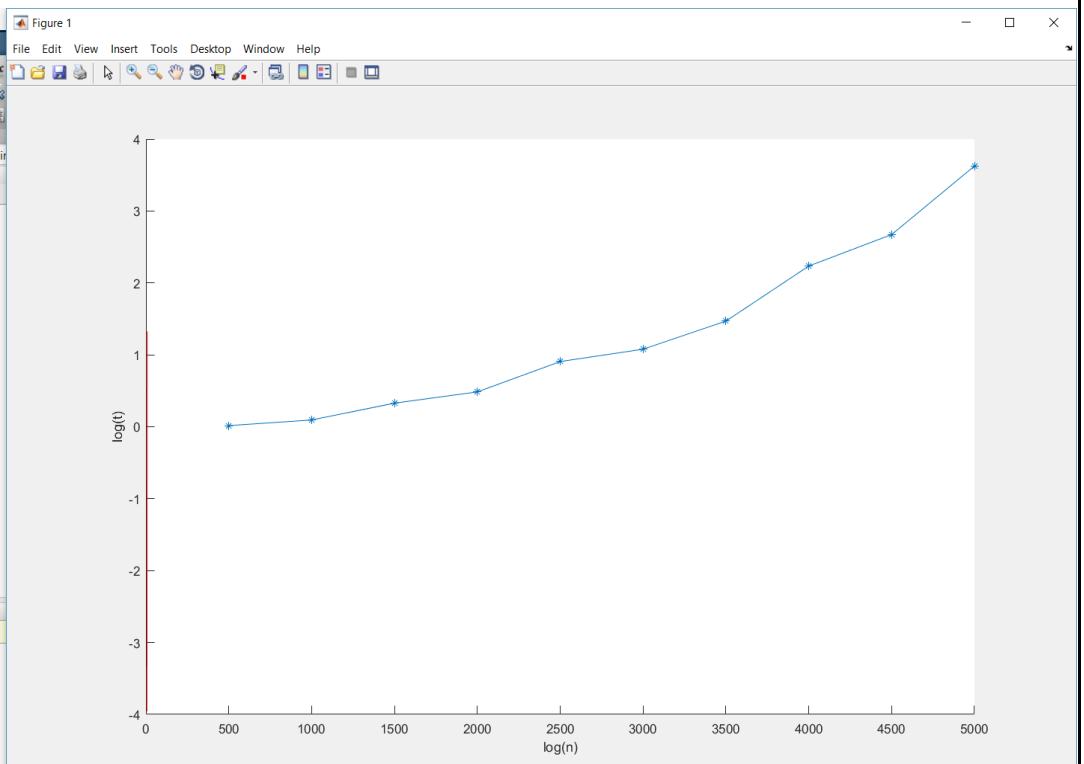
$$\lim_{h \to 0} \frac{|E_h(x)|}{|h|} = \frac{|f''(\xi)|}{2} = \text{constant}$$

Because of that $E_h = O(h)$ //

Q4)

```
n=(500:500:5000);
t=(length(n));
x=(length(n));
y=(length(n));

for i=1:length(n)
    t1=cputime();
    A=rand(n(i));
    det(A);
    t(i)=cputime()-t1;
end

hold on
loglog(n,t,'-*');
xlabel('log(n)');
ylabel('log(t)');
p=polyfit(log(n),log(t),1);
disp(p);
p1=polyval(p,log(n));
plot(log(n),p1,'r');
```

```
>> q4
    2.2946  -18.2192

>>
```

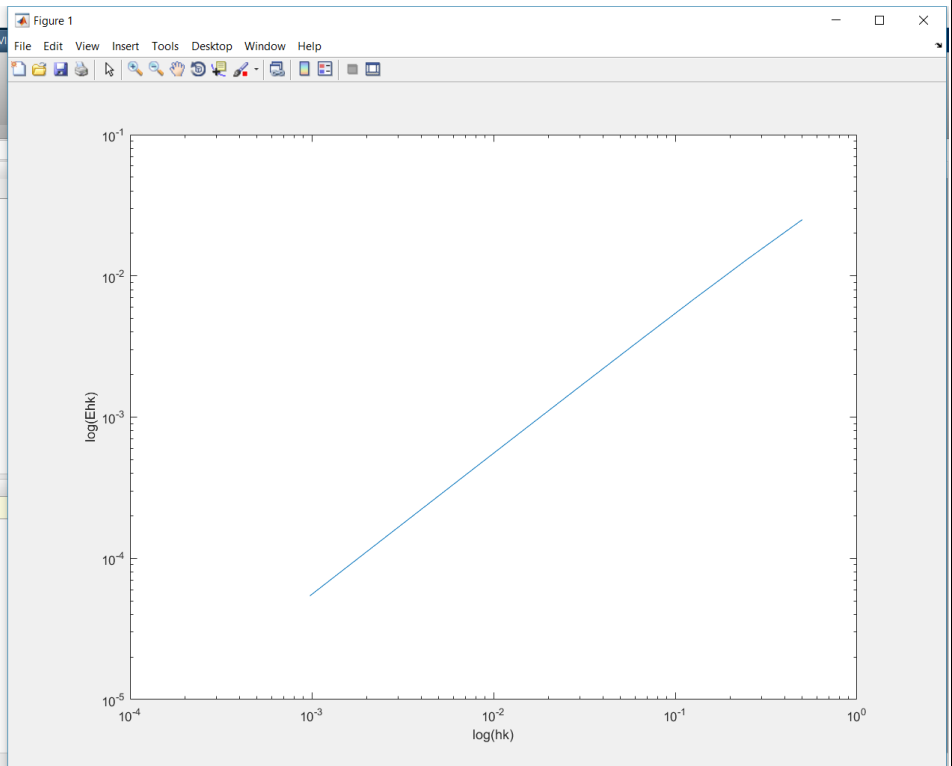


Q5)

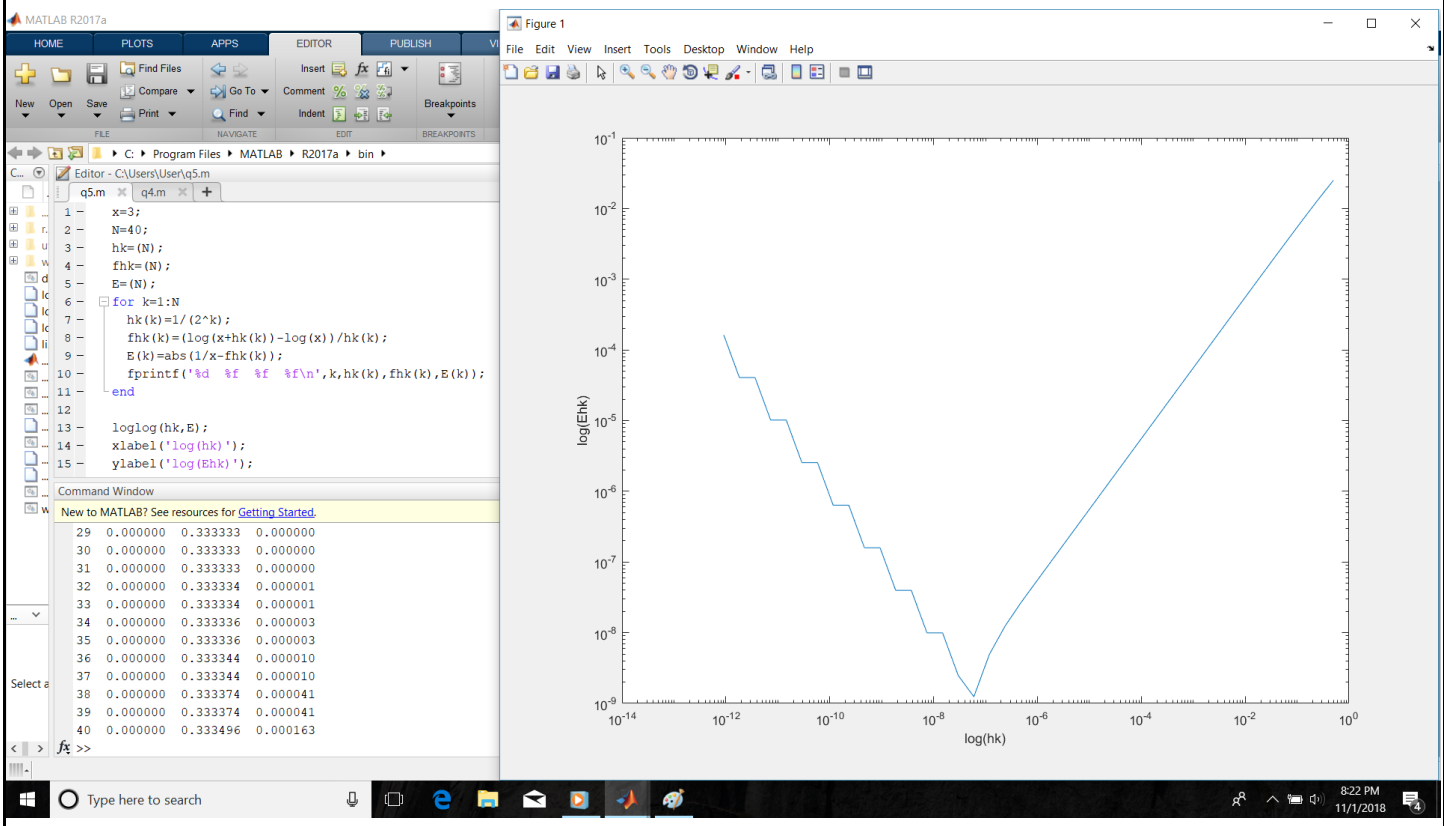
(b)

```
x=3;
N=10;
hk=(N);
fhk=(N);
E=(N);
for k=1:N
   hk(k)=1/(2^k);
   fhk(k)=(log(x+hk(k))-log(x))/hk(k);
   E(k)=abs(1/x-fhk(k));
   fprintf('%d  %f  %f  %f\n',k,hk(k),fhk(k),E(k));
end

loglog(hk,E);
xlabel('log(hk)');
ylabel('log(Ehk)');
```

```
>> q5
 1  0.500000  0.308301  0.025032
 2  0.250000  0.320171  0.013163
 3  0.125000  0.326576  0.006757
 4  0.062500  0.329909  0.003425
 5  0.031250  0.331609  0.001724
 6  0.015625  0.332468  0.000865
 7  0.007813  0.332900  0.000433
 8  0.003906  0.333117  0.000217
 9  0.001953  0.333225  0.000108
10  0.000977  0.333279  0.000054

>>
```



(e)


MATLAB R2017a
HOME
PLOTS
APPS
EDITOR
PUBLISH
C: ▸ Program Files ▸ MATLAB ▸ R2017a ▸ bin ▸
Editor - C:\Users\User\q5.m
q5.m
q4.m
1 – x=3;
2 – N=40;
3 – hk=(N);
4 – fhk=(N);
5 – E=(N);
6 – for k=1:N
7 – hk(k)=1/(2^k);
8 – fhk(k)=(log(x+hk(k))-log(x))/hk(k);
9 – E(k)=abs(1/x-fhk(k));
10 – fprintf('%d %f %f %f\n',k,hk(k),fhk(k),E(k));
11 – end
12
13 – loglog(hk,E);
14 – xlabel('log(hk)');
15 – ylabel('log(Ehk)');
Command Window
New to MATLAB? See resources for Getting Started.
29 0.000000 0.333333 0.000000
30 0.000000 0.333333 0.000000
31 0.000000 0.333333 0.000000
32 0.000000 0.333334 0.000001
33 0.000000 0.333334 0.000001
34 0.000000 0.333336 0.000003
35 0.000000 0.333336 0.000003
36 0.000000 0.333344 0.000010
37 0.000000 0.333344 0.000010
38 0.000000 0.333374 0.000041
39 0.000000 0.333374 0.000041
40 0.000000 0.333496 0.000163
>>
Figure 1
File Edit View Insert Tools Desktop Window Help
log(Ehk)
log(hk)
Type here to search
8:22 PM
11/1/2018