CSDS 310: Algorithms Fall 2024

Assignment 2: Asymptotic Notation & Divide and Conquer

Instructor: Orhan Özgüner Due: October 1 before 11:59 PM

Problem 1

For each of the following pairs of functions, write down the asymptotic relation between $f(n)$ and $g(n)$; *i.e.*, if $f(n) = x(g(n))$ where $x \in \{o, \Theta, \omega\, O, \Omega\}$. Assume that $k \geq 0, \epsilon > 0$, and $c > 1$. Provide a justification for your answers.

	$f(n)$	$g(n)$
a)	$\log^k n$	n^ϵ
b)	n^k	c^n
c)	$\sqrt{n}$	$n^{\sin(n)}$

Problem 2

Considering functions $f(n) \geq 0$, $g(n) \geq 0$, and constant $c > 0$, indicate whether each of the following statements is true. Prove the statements that are true by providing a formal argument that is based on the definition of asymptotic notation. For statements that are false, provide a counter-example to prove that they are false.

(*a*) If $f(n) \geq 1$, then $f(n) + c = O(f(n))$.

(*b*) $f(2n) = \Theta(f(n))$.

(*c*) If $f(n) = O(n^c)$, then $f(2n) = O(n^c)$.

Problem 3

Let $0 < \lambda < 1 < a < b$ be constants. Solve the following recurrences using Master Method, noting the case that applies.

(a) $T(n) = bT(n/a) + \Theta(n)$.

(b) $T(n) = a^2T(n/a) + \Theta(n^2)$.

(c) $T(n) = T(\lambda n) + n^\lambda$.

Problem 4

You are given an array of k sorted arrays each of which has a length n/k elements. Describe an efficient algorithm to merge these arrays to obtain one sorted array of length n.