

## ASSIGNMENT 5: GRAPH ALGORITHMS

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Due: December 8 before 11:59 PM

**Problem 1**

There are  $n$  basketball teams in the world. The ranking of these teams from the previous year is available. This year, some of these  $n$  teams played against each other and the winner of each game was determined. There were  $m$  games in total. The International Basketball Association wants to introduce a new performance criterion, called “domination factor”, defined as follows: Team  $i$  is said to “dominate” team  $j$  if we can find a chain of games such that  $j$  was beaten by a team that was beaten by a team that was beaten by a team ... that was beaten by  $i$  (observe that, according to this definition, domination can be bi-directional, i.e.,  $i$  and  $j$  can dominate each other). Then, for each team  $i$ , the domination factor  $z_i$  is defined as the rank of the best team (that is, the highest ranked team according to last year’s rankings) that is dominated by team  $i$ . Describe an efficient algorithm to compute the domination factor for all the  $n$  teams.

**Problem 2**

Prove or disprove the following statements:

- (a) Let  $G = (V, E)$  be a directed graph. For any  $uv \in E$ , if some run of Depth-First-Search (DFS) on  $G$  results in  $v.f > u.f$ , then  $uv$  must be on a cycle.
- (b) Consider any run of DFS on a directed graph  $G = (V, E)$ . For any edge  $uv \in E$ , if there is a path from  $v$  to  $u$  in  $G$ , then  $uv$  cannot be a cross edge.

**Problem 3**

There are two types of professional wrestlers: “babyfaces” (“good guys”) and “heels” (“bad guys”). Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have  $n$  professional wrestlers and we have a list of  $r$  pairs of wrestlers for which there are rivalries. Give an efficient algorithm that determines whether it is possible to designate some of the wrestlers as babyfaces and the remainder as heels such that each rivalry is between a babyface and a heel. If it is possible to perform such a designation, your algorithm should produce it.

**Problem 4 (Bonus: 10 pts)**

We had a great semester full of algorithm problems. I enjoy teaching this class and I am trying to improve my class. Therefore your feedback on this class is important. CSDS310 is an UGER “communication in the discipline” course. Please include in your evaluation, feedback on those particular aspects of the class: the “job interview” type presentations, recitations, and the written assignments.

Please attach evidence of completion of course evaluation (a screenshot of the confirmation page will suffice, please do not include your actual evaluations). This bonus will help you to avoid any borderline issues. Here is the link for the course evaluations: [webapps.case.edu/courseevals](http://webapps.case.edu/courseevals)