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Fantasy Football Case Study

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**Introduction with Problem Statement:**

The purpose of this paper is to develop lineups for a Daily Fantasy Sports (DFS) competition. DFS contests have been exploding in the United States and around the world over the past decade. Even more so with relaxed gambling laws. The presented problem is as follows:

*Consider a fantasy football competition in which each contestant is allowed to participate with at most one fantasy team or entry. Each player has a salary that must be paid to get the player into the entry, a projected score that is an estimation of how many points will the player achieve, and a corresponding position:*

* *Quarterback (QB)*
* *Running Back (RB)*
* *Wide Receiver (WR)*
* *Tight End (TE)*
* *Defense (DST)*

The goal of a DFS contest is to score as many points as possible. There can be various different reward levels for fantasy sports, such as Top 100, Top 500, Top 1000, etc. However, this case study is only concerned with generating the optimal lineup based on scoring the maximal amount of points. For the purpose of this case study the salaries were uniformly distributed to players from $5,000 - $14,000. Additionally, projected scores were uniformly distributed between 1 and 25 points. An example of the data is shown below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **ID** | **Player Name** | **Position** | **Salary** | **Projected Score** |
| 1 | Saints | DST | $5,477 | 18 |
| 2 | Vikings | DST | $12,027 | 12 |
| 4 | Tom Brady | QB | $12,476 | 9 |
| 5 | Cam Newton | QB | $5,158 | 16 |
| 29 | Le’Veon Bell | RB | $12,553 | 7 |
| 30 | David Johnson | RB | $7,731 | 10 |
| 60 | Rob Gronkowski | TE | $12,939 | 21 |
| 61 | Evan Engram | TE | $7,389 | 1 |
| 67 | Antonio Brown | WR | $6,193 | 20 |
| 68 | Deandre Hopkins | WR | $12,513 | 13 |

***\*Table 1: Sample player data***

The following conditions must be enforced on the original model:

* *An entry consists of 6 players*
* *The total combined salary of the selected players is at most $50,000*
* *There must be at least 1 player for each position*
* *The sixth player is a flexible player that can be either a RB, WR, or TE*

**Linear Integer Program Model:**

This model will be a Linear Integer Programing Model meaning that the decisions variables that are being determined will get values of 1 if the model selects that variable or 0 if the model does not select that variable. Note that the player list has already been sorted by position in the data file before we load it into the model. Before discussing the model, here is a quick overview of the sets and the parameters that will be used.

**Sets:**

* Players:
* Defense/ST :
* Quarterbacks:
* Running Backs:
* Tight End:
* Wide Receiver:

The first set is for the players, which corresponds to the Player ID that were assigned. DST, QB, RB, TE, WR are subsets of P for each position.

**Parameters:**

* : the salary (in US$) associated with using player i
* : the projected score of player i

The parameter corresponds to the Salary column of **Table 1**.This will be used in the constraint section to keep the minimum salary below the $50,000 threshold. corresponds to the Projected Score column of **Table 1**. This is what we want to maximize so that our model has the best change of winning a DFS contest.

**Decision variable:**

So, if player i from set P is chosen, this value will become a 1. If player i from set P is not chosen, this value will be a 0.

**Objective function:**

This function states that the projected score for every player i in set P should be multiplied by Xi, if the player is chosen or not, then summed. Thus, giving the lineup that will have the highest score. However, there are constraints that must be applied to the model or the players projected the highest score for each position would be chosen.

**Constraints:**

The first constraint specifies that 6 players must be chosen. The second says that the salary paid to the players must be less than or equal to $50,000. The final constraints specify that one DST, one QB, and either one or 2 RB, TE, WR must be selected.

**Results:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Players** | **Position** | **Projected Score** | **Salary** |
| Saints | DST | 18 | $5,477 |
| Ryan Griffin | QB | 25 | $13,980 |
| Ezekiel Elliott | RB | 24 | $6,519 |
| Rob Kelley | RB | 25 | $5,553 |
| Kyle Rudolph | TE | 23 | $13,330 |
| Kendall Wright | WR | 24 | $5,035 |
| **Total** |  | **139** | **$49,894** |

***Table 2: Dream team***

The model that was run with Xpress IVE resulted in a team with total projected score of 139, with the total salary cap of $50,000 not broken. We can see that the software does a good job helping us choosing players who have very high score with limited budget.

**Linear Integer Program Model for two Entries:**

This portion of the paper focuses on creating a new entry in addition to our first team. The only condition is that the two lineups cannot be exactly the same. The parameters, decision variables, and constraints from the first problem are the still active. The new additions to the model will be shown here.

**Additional decision variable:**

**Objective function:**

We merge the objective function of 2 teams into one, prioritizing team 1’s total score:

We use the number so that the objective function always maximizes the total projected score of team 1 first, only then continue with projected score of team 2.

**Additional constraints:**

The first constraint forces to take a value of 1 when player i appears on both lineups. The second constraint doesn’t allow the lineups to be exactly the same.

**Results:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Players** | **Position** | **Projected Score** | **Salary** |
| **Entry 1:** |  |  |  |
| Saints | DST | 18 | $5,477 |
| Ryan Griffin | QB | 25 | $13,980 |
| Ezekiel Elliott | RB | 24 | $6,519 |
| Rob Kelley | RB | 25 | $5,553 |
| Kyle Rudolph | TE | 23 | $13,330 |
| Kendall Wright | WR | 24 | $5,035 |
| **Total** |  | **139** | **$49,894** |
| **Entry 2:** |  |  |  |
| Saints | DST | 18 | $5,477 |
| Lamar Jackson | QB | 24 | $12,624 |
| Ezekiel Elliott | RB | 24 | $6,519 |
| Rob Kelley | RB | 25 | $5,553 |
| Kyle Rudolph | TE | 23 | $13,330 |
| Kendall Wright | WR | 24 | $5,035 |
| **Total** |  | **138** | **$48,538** |

***\*Table 3: 2 entries***

Note that for team 2, the QB position changed from a player with projected score of 25 points to another with 24 points. This is because the model forced at least one of the players to change. Additionally, the salary for both of the entries remains below the $50,000 budget.

**Linear Integer Program Model for two Entries with Diversity Measure:**

To make the entries different the group decided to add a couple of measures. First, no player is allowed to be in both entries. Second, the flex position must be different for both entries. Finally, we do not want to spend that much money, so we decide that the second entry must be $10,000 cheaper than the first entry. The model stays the same for the most part, however, there are new constraints, which can be seen here.

**Additional constraints:**

This constraint does not allow any player to be on both teams:

These constraints do not allow the flex position to be the same for both entries. In this one, we make sure that none of the position has 4 players when counting in both teams:

This final constraint is setting the cost of the second entry to be at least $10,000 less than the first entry:

**Results:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Players** | **Position** | **Projected Score** | **Salary** |
| **Entry 1:** |  |  |  |
| Saints | DST | 18 | $5,477 |
| Lamar Jackson | QB | 24 | $12,624 |
| Ezekiel Elliot | RB | 24 | $6,519 |
| Rob Kelley | RB | 25 | $5,553 |
| Kyle Rudolph | TE | 23 | $13,330 |
| Antonio Brown | WR | 20 | $6,193 |
| **Total** |  | **134** | **$49,696** |
| **Entry 2:** |  |  |  |
| Vikings | DST | 12 | $12,027 |
| Cam Newton | QB | 16 | $5,185 |
| Stevan Ridley | RB | 21 | $6,073 |
| Tyler Eifert | TE | 13 | $5,948 |
| Corey Davis | WR | 17 | $5,113 |
| Kendall Wright | WR | 24 | $5,035 |
| **Total** |  | **103** | **$39,381** |

***\*Table 4: 2 entries with additional constraints***

We can see the model was successful at enforcing all the constraints. No players are the same, the flex position is different between 2 lineups, and the cost for the second entry is at least $10,000 less than the first model.

What surprises us is that even with our Objective function priotizing the projected score of the first entry to force that lineup to be the same as in the result in our first model *(we can see the second model successfully did so)*, the model this time fails to keep the same players for team 1. This means that it is impossible to keep that first lineup AND have the second entry with $10,000 less *(a less than $40,000 total salary for 6 players means less than $6,667 for each, which is dirt cheap considering the salary range is from $5,000 to $14,000)*, and the model has to move Kendall Wright (WR) to the second team, and change QB player to lower the first team cost to less than $50,000.

While we can just drop the constraint about lowering the cost, we decide to make a relaxed model that does all of those objectives *(because why not? we enjoy doing it anyway)*:

* Keep the first lineup
* Make a second optimal lineup that has:
  + Entirely different players
  + Different flex player position
* Lower the second team’s cost as much as possible

With that in mind, our new objective function is:

Again, that number is to ensure our order of priotizing.

**Results:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Players** | **Position** | **Projected Score** | **Salary** |
| **Entry 1:** |  |  |  |
| Saints | DST | 18 | $5,477 |
| Ryan Griffin | QB | 25 | $13,980 |
| Ezekiel Elliott | RB | 24 | $6,519 |
| Rob Kelley | RB | 25 | $5,553 |
| Kyle Rudolph | TE | 23 | $13,330 |
| Kendall Wright | WR | 24 | $5,035 |
| **Total** |  | **139** | **$49,894** |
| **Entry 2:** |  |  |  |
| Vikings | DST | 12 | $12,027 |
| Garrett Gilbert | QB | 20 | $8,442 |
| Stevan Ridley | RB | 21 | $6,073 |
| Tyler Eifert | TE | 13 | $5,948 |
| JuJu Smith-Schuster | WR | 25 | 8170 |
| Allen Hurns | WR | 24 | 9251 |
| **Total** |  | **115** | **$49,911** |

***\*Table 5: 2 entries with*** *(less)* ***additional constraints***

As we can see, now we have 2 teams with scores of 139 and 115, which is better than 134 and 103 in our third model. *(It just seems that there is no way to achieve 115 for team 2 without spending that much money though* ☹*. In fact, if we just drop the lowering-the-cost constraints, the result is still the same which is sad).*

With the results above, we have successfully reach our goals for the project, which are the team lineups in **Table 2**, **Table 3** and **Table 5**. The same models can also be used when player data change in the future.

**Appendix:**

**Data files:**



**Model files:**

