

Initialization.

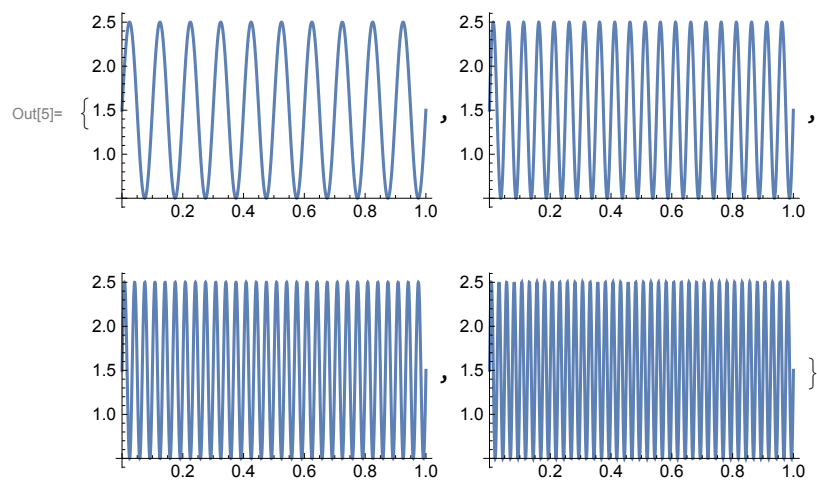
```
In[10]:= fs = 18;  
In[11]:= ff = Times;  
In[12]:= mystyle[text_] := Style[text, FontSize → fs, FontFamily → ff]
```

First-taste example.

Defining boundary value problem.

```
In[1]:= L = 1;  
In[2]:= c[k_, x_] :=  $\frac{3}{2} + \sin[2 \pi k x]$ ;  
In[3]:= f[x_] = x;  
In[5]:= Table[Plot[c[10 k, x], {x, 0, L}], {k, 1, 4}]
```

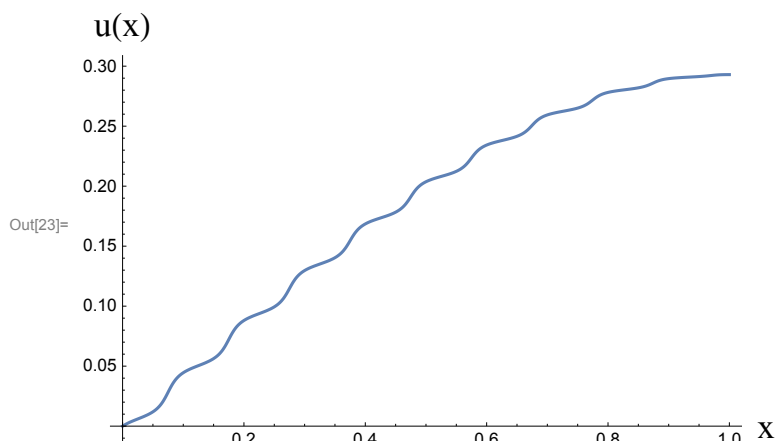
Out[5]= {



, }

```
In[62]:= nsol = NDSolve[{D[c[10, x] u'[x], x] + f[x] == 0, u[0] == 0, u'[L] == 0},  
u[x], {x, 0, L}, AccuracyGoal → 20];
```

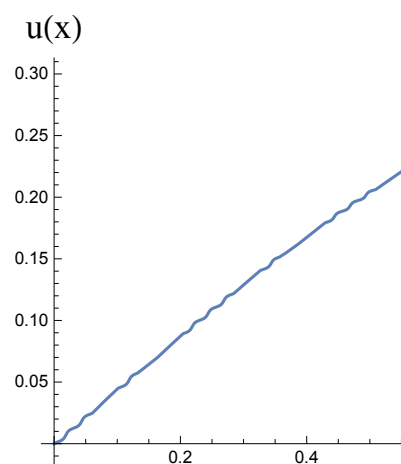
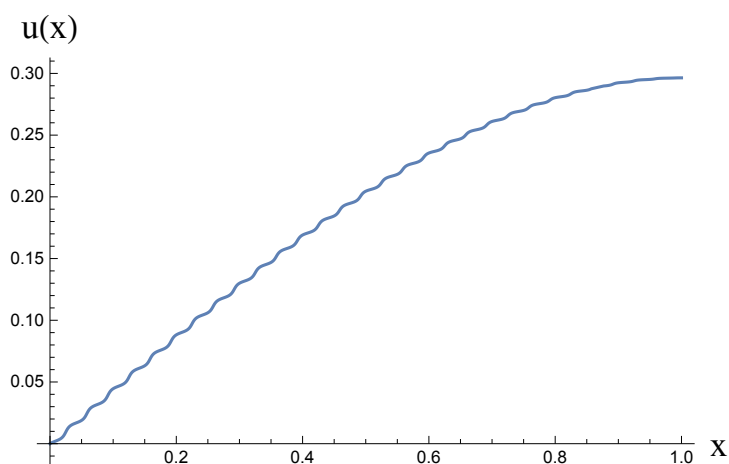
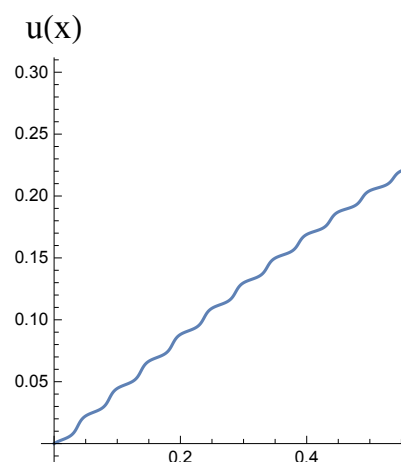
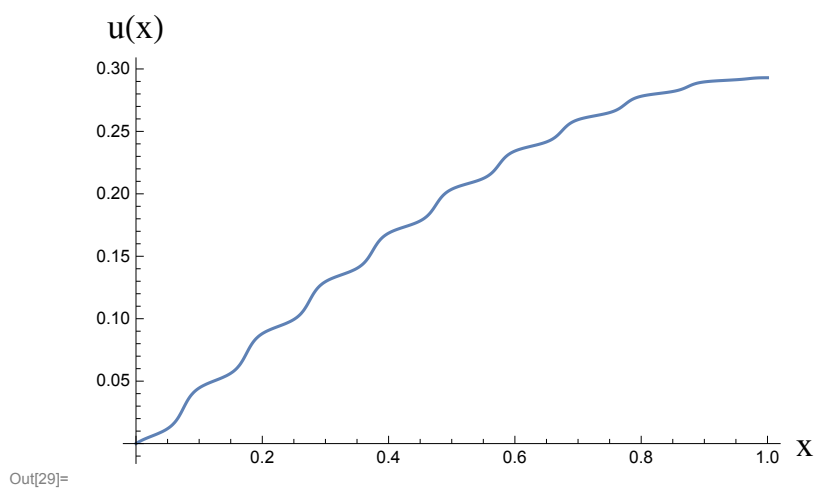
```
In[23]:= Plot[u[x] /. nsol[[1]], {x, 0, L}, ImageSize → Medium,
  AxesLabel → {mystyle["x"], mystyle["u(x)"]}]
```




List of equations with small wavelengths of material parameters.

```
In[28]:= nsolList = Table[NDSolve[
  {D[c[10 k, x] u'[x], x] + f[x] == 0, u[0] == 0, u'[L] == 0}, u[x], {x, 0, L}], {k, 1, 4}];
```

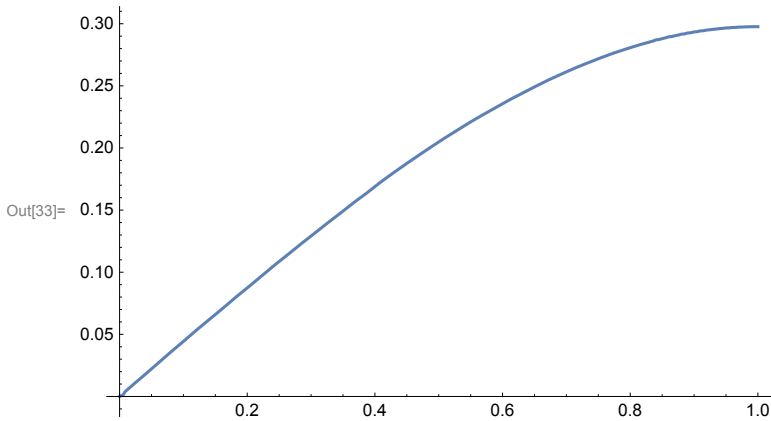
```
In[29]:= GraphicsGrid[Partition[Table[
  Plot[u[x] /. nsolList[[j, 1]], {x, 0, 1}, AxesLabel → {mystyle["x"], mystyle["u(x)"]}],
  {j, Length[nsolList]}], 2], ImageSize → 800]
```



```
In[63]:= nsol2 =
NDSolve[{D[c[10 x 10, x] u'[x], x] + f[x] == 0, u[0] == 0, u'[L] == 0}, u[x], {x, 0, L}];
```

 **NDSolve**: The scaled boundary value residual error of 11.878190169874852` indicates that the boundary values are not satisfied to specified tolerances. Returning the best solution found.

```
In[33]:= Plot[u[x] /. nsol2[[1]], {x, 0, L}]
```



```
In[26]:= Clear[u, k]
```

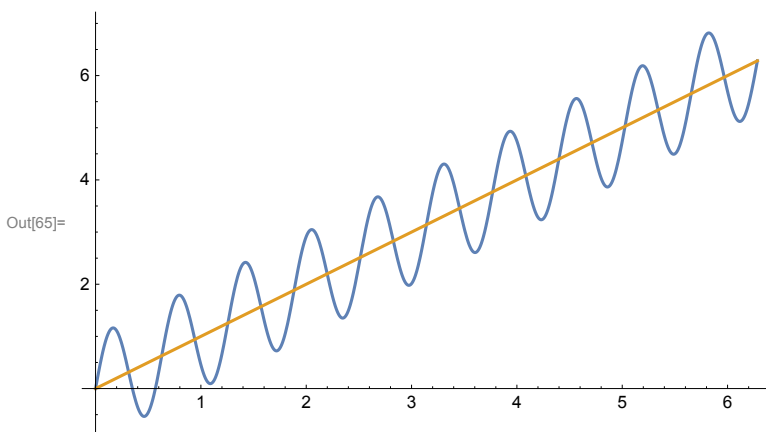
```
In[60]:= Re[ComplexExpand[
DSolve[{D[c[k, x] u'[x], x] + f[x] == 0, u[0] == 0, u'[L] == 0}, u[x], {x, 0, L}]]];
```

```
In[61]:= g[x_] = Sin[10 x] + x;
```

```
In[35]:= λ = 2 π / 10;
```

```
In[64]:= fbar[x_] = 1/λ ∫_{x-λ/2}^{λ/2+x} g[τ] dτ;
```

```
In[65]:= Plot[{g[x], fbar[x]}, {x, 0, 2 π}]
```



Solve the microscale periodic boundary value problem.

$$k = 1/2 \pi;$$

$$\lambda = 1/k;$$

$$c[k_, x_] := 3/2 + \text{Sin}[2 \pi k x];$$

```
strain = 0.025;
```

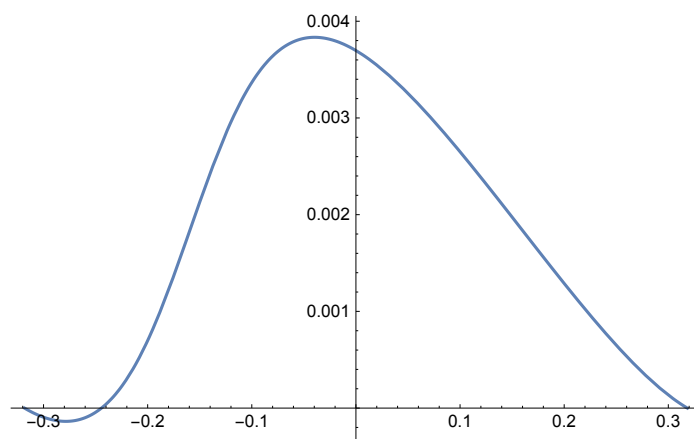
```
nsol = NDSolve[
  {D[c[k, x] (v'[x] + strain), x] == 0, v[-λ/2] == 0, v[λ/2] == 0}, v[x], {x, -λ/2, λ/2}]
```

```
{ {v[x] → InterpolatingFunction[ Domain: {{-0.318, 0.318}} Output: scalar] [x] } }
```

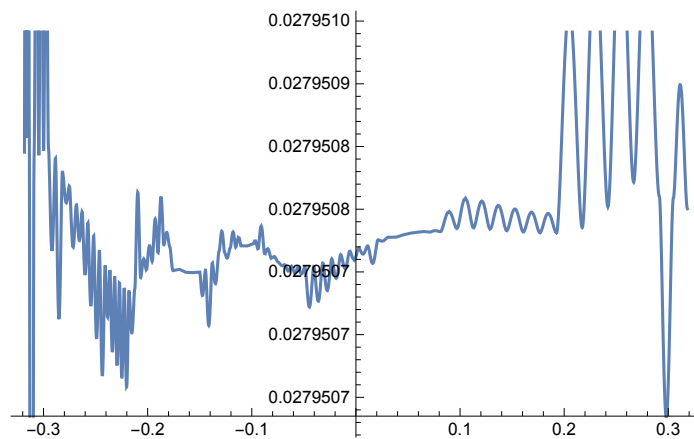
```
u[x_] = v[x] /. nsol[[1]];
```

```
σ[x_] = c[k, x] (D[u[x], x] + strain);
```

```
Plot[u[x], {x, -λ/2, λ/2}]
```



```
Plot[σ[x], {x, -λ/2, λ/2}]
```



```
 $\frac{1}{\lambda}$  NIntegrate[σ[x], {x, -λ/2, λ/2}]
```

```
0.0279508
```

```
 $\frac{1}{1}$  (1 - r) /. {r → 0}
```