CX 4220/CSE 6220 High Performance Computing
Spring 2023
Homework 5
Due Wednesday, April 19

Adhere to the following guidelines while working on and submitting the homework

- You are strongly encouraged to be concise in answering homework problems, and type up your solutions (preferably using LaTeX).
- It is your responsibility to ensure that your solutions are legible. You will risk losing points if your solutions are illegible to the TAs.
- Submissions are due 11:59PM EST. The deadline for distance learning students is one week after the date on the homework. Late homeworks are not accepted.
- Your submission MUST be made in PDF format. Specify your name and GT username at the top. Do not put your GTID.

1. (5 points) Consider matrix-vector multiplication $y = Ax$ where A is an $n \times n$ matrix and x and y are $n \times 1$ vectors. Output y vector elements are computed as $y[i] = \sum_{j=0}^{n} A[i,j] \cdot x[j]$. Consider matrix is partitioned with 1D column-wise partitioning, such that each processor stores $\frac{n}{p}$ complete columns of the matrix A, and has *matching* $\frac{n}{p}$ elements of the vectors x and y, i.e., if processor has column j (i.e., $A[*,j]$), it also owns $x[j]$ and $y[j]$.

   Design an efficient algorithm to compute matrix-vector multiplication in parallel. Analyze the run-time of this algorithm and specify the computation and communication time.

2. (5 points) Given two vectors $\vec{X} = [x_0, x_1, \ldots, x_{n-1}]$ and $\vec{Y} = [y_0, y_1, \ldots, y_{n-1}]$, their convolution is defined to be the $n \times n$ matrix M whose ij^{th} entry is $M[i,j] = x_i + y_j$.

   (a) Give an algorithm to compute the convolution of X and Y on an n^2-processor machine. Use a logical $n \times n$ mesh topology, and assume that the vectors are initially stored in the first column of processors (one element per processor). Ignore communication constants etc. in your run-time analysis. Your algorithm should run in $O(\log n)$ time.

   (b) Determine the computation time and communication time when using p processors by now taking the communications constants into account. What is the maximum value of p that still results in optimum efficiency? (You may assume p is a perfect square; $\sqrt{p}$ divides n etc.)

3. (5 points) Design a parallel prefix algorithm for the CRCW PRAM Sum Model, i.e., if multiple processors attempt to write to the same location, the sum of the values gets deposited there.

Present an algorithm that uses $\Theta(n^2)$ processors and runs in $O(1)$ time. Argue why this algorithm is of no practical value.

4. (5 points) In the mid 1980s, a team consisting of a computer scientist and a physicist at CalTech set out to build the first parallel computer. They decided to build a 64-processor configuration and use it to solve problems in physics. The physicist argued for using the three dimensional torus interconnect as it would naturally correspond to the physics problems. The computer scientist argued for using the hypercube interconnection topology. Which choice is superior and why?

5. (5 points) A pancake network of order n, denoted S_n, is defined as follows: The network has $n!$ processors. Each processor corresponds to a distinct permutation of the first n positive integers and the id of a processor is the permutation to which it corresponds. Two processors are connected iff one id can be obtained from the other by reversing the first i $(2 \leq i \leq n)$ integers of its permutation. For example, 2413 is the id of a processor in S_4 and is connected to processors with id's 4213, 1423 and 3142. Show that the diameter of S_n is $O(n)$.