CX 4220/CSE 6220 High Performance Computing
Spring 2023
Homework 4
Due Monday, March 27

Adhere to the following guidelines while working on and submitting the homework

- You are strongly encouraged to be concise in answering homework problems, and type up your solutions (preferably using LaTeX).
- It is your responsibility to ensure that your solutions are legible. You will risk losing points if your solutions are illegible to the TAs.
- Submissions are due 11:59PM EST. The deadline for distance learning students is one week after the date on the homework. Late homeworks are not accepted.
- Your submission MUST be made in PDF format. Specify your name and GT username at the top. Do not put your GTID.

1. (5 points) Consider the following modification to the *Sample Sort* algorithm described in class. Suppose instead of choosing $p-1$ local splitters per processor we choose $kp-1$ local splitters per processor. Prove that this modification guarantees the number of elements received by any processor is bounded by $\frac{n}{p}\left(1+\frac{1}{k}\right)$.

2. (5 points) Show that a p-processor ring can be embedded into a p-processor array with load $= 1$ and dilation $= 2$. Specify the mapping function from ring to array that works for arbitrary values of p.

3. (5 points) A three dimensional torus of size $16 \times 16 \times 8$ is embedded in a $2,048$-processor parallel computer that can route hypercubic permutations.

   (a) Determine rank of the processor to which $(9, 13, 6)$ is mapped.

   (b) What is the torus rank of processor 532?

4. (5 points) Consider the embedding of a 16-leaf complete binary tree into a 16-node hypercube assuming only one tree level at a time is involved computationally. Consider processor with rank 12 in the hypercube.

   (a) Which levels of the tree does this processor participate in?

   (b) For each level above, compute the tree rank of the node, and the hypercube rank of its parent, left child, and right child.

5. (5 points) A Binomial tree of height d, termed $B(d)$ is defined as follows: If $d = 0$, then $B(0)$ is a single node. Otherwise, $B(d)$ is constructed by taking two binomial trees of height $d-1$

and making the root of one tree the child of the root of the other tree. Show that $B(d)$ can be embedded in a d-dimensional hypercube. (**Hint:** Think recursion).