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Stochastic programming of concrete poles

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Synopsis

The optimal design of structures with stochastic constraints is considered. The technique of converting lognormal stochastic constraints to its equivalent deterministic format is presented. The minimum cost design of a reinforced cement concrete pole is considered for illustration. The optimum results obtained from assuming lognormal stochastic constraints. Finally sensitive analysis is carried out to study the effects of various parameters on optimal design of poles.

Key words

Concrete (reinforced); Cost; Design; Optimal; Pole; Reliability; Stochastic programming.

Introduction

Optimization is the process of obtaining best results under given circumstances. Depending on the circumstances and constraints in a particular problem appropriate technique of optimization is chosen. Stochastic programming is one of such techniques and is used whenever parameters are random variables in the optimization problem. The major concern of a structural engineer is the design of safe, functionally suitable and economical structures. But most of the structural engineering problems are stochastic in nature and the designers have to take into account the uncertainties involved. These uncertainties are currently tackled systematically with rational approach of

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reliability theory. In the science of reliability applied to structures, either the reliability (or probability of failure) of the given structure is calculated or the structure is designed for a given probability of failure. Only simple structures or complicated structures with suitable assumptions can be tackled with this theory. Due to this application of stochastic programming (SP) to structural engineering, which is linked to the development of the reliability theory, has been on the limited scale. Moses and Kinser [8] has coupled reliability analysis of frames with structural optimization procedures to produce reliability based optimal design. Loads and strengths have been considered as normal random variables. Rao [10] has presented the optimal cost design of reinforced concrete beams. The limit state of collapse in flexure is the only criterion and the design is taken to be safe if the resisting moment of the beam exceeds the bending moment by a certain number of standard deviations. Murotsu [9] et. al. have optimized a simple portal frame using plastic design based on probabilistic formulation. All the variables have been treated as normal variables and the multidimensional probability distribution has been calculated.

Structural optimization under probabilistic constraints is a time consuming affair. The basic idea of all stochastic programming methods is to convert the probabilistic nature of the problem into an equivalent deterministic problem. The idea of employing deterministic equivalence has been introduced by Charnes and Cooper [3] where the constraints are violated with a prescribed risk. The above technique has been used by Rao [11] for the minimum cost design of a cable stayed box girder. The external loading and stresses have been considered as normal random variables. Hence the formulation of converting probabilistic constraints into its equivalent deterministic model is given considering all variables as normally distributed. In this paper, a technique is developed for converting stochastic nonlinear constraints having lognormal probability distribution to equivalent deterministic constraints. The same technique is applied to the minimum cost design of reinforced cement concrete (RCC) poles at various reliability levels.

Stochastic programming

The standard form of nonlinear SP problem is :

$$\text{Find } \vec{X} \text{ which minimizes } f(\vec{Y})$$

subject to constraints

$$P(g_j(\vec{Y}) \geqslant K) \geqslant R_j \qquad j = 1, 2 \ldots.., m \tag{1}$$

where $\vec{Y}$ is the vector of n random variables $y_1, y_2 \ \ldots, \ y_n$ and it includes the decision variables $x_1, x_2, \ldots\ldots, x_k$; K is a constant. P ( ) is read as probability of ( ). The constraint equations denote that the probability of realizing $g_j(\vec{Y})$ greater than or equal to K must be greater than or equal to the specified reliability level R_j. This type of SP is also called as chance constraint programming technique. Using this technique the stochastic nonlinear programming (NLP) problem is converted to an equivalent deterministic NLP problem. The stochastic objective function $f(\vec{Y})$ is transformed to an equivalent deterministic function $F(\vec{Y})$ given by [11],

$$F(\vec{Y}) = k_1 \bar{\psi} + k_2 \sigma_{\psi} \tag{2}$$

where $k_1 \geqslant 0$ and $k_2 \geqslant 0$,

$$\bar{\psi} = f(\bar{Y}) \tag{3}$$

$$\sigma^2_{\psi} = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial y_i} \Big| \bar{Y} \right)^2 \sigma^2_{y_i} \tag{4}$$

$\bar{\psi}$ and σ_{ψ} are the mean value and standard deviation (SD) of ψ which is the value of the objective function and it is a random variable. Values of k_1 and k_2 indicate the relative importance of $\bar{\psi}$ and σ_{ψ} for minimization. $\partial f/\partial y_i \mid_{\bar{Y}}$ means that the $\partial f/\partial y_i$ is evaluated at $\bar{Y}$ which is the mean of $\vec{Y}$.

Normal Stochastic Constraint

If all variables in the constraint inequality Eq. 1 are normally distributed and if $g_j(\vec{Y})$ is assumed to be normally distributed, the equivalent deterministic inequality constraint becomes [11],

$$\bar{g}_j - \phi^{-1}(R_j)\, \sigma_{g_j} \geqslant 0, \qquad j = 1, 2, \ldots, m \tag{5}$$

where $\bar{g}_j$ and σ_{g_j} are the mean value and standard deviation of g_j and they are parameters in the case of normal distribution. ϕ^{-1} is the inverse of cumulative probability of a standardized normal random variable.

Lognormal Stochastic Constraint

When the random variable $g_j(\vec{Y})$ in the stochastic constraint Eq. 1 is lognormally distributed the equivalent deterministic constraint is derived as follows.

In the case of lognormally distributed variables, g_j is arranged such that K in Eq. 1 equals 1. Hence the constraint inequality becomes

$$P\,[g_j(\vec{Y}) \geqslant 1] \geqslant R_j, \quad j = 1, 2 \ldots, m \tag{6}$$

Omitting the subscript j for simplicity and if $\tilde{g}$ and $\sigma_{ln\,g}$ are the median and standard deviation of ln g respectively, (the parameters of lognormally distributed g) the left hand side (LHS) of Eq. 6 becomes [2],

$$P\,[g(\vec{Y}) \geq 1] = 1 - F_g(1)$$

$$= \phi\,(ln\,\tilde{g}/\sigma_{ln\,g}) \tag{7}$$

where $F_g(1)$ is the cumulative probability of g, g assuming value ≤ 1.

Hence the inequality condition Eq. 6 can be rewritten as

$$\phi\,(ln\,\tilde{g}/\sigma_{ln\,g}) \geq R \tag{8}$$

Rearranging and simplifying the above equation,

$$ln\,\tilde{g} - \phi^{-1}\,(R_j)\,ln\,g_j \geq 0 \qquad j = 1, 2 \quad , m \tag{9}$$

This is the required equivalent deterministic equation for the lognormal stochastic constraint.

Application

A reinforced cement concrete pole (RCC) used for transmission lines is chosen as the structure for optimization. Reliability theory is the basis of optimal design of the pole for minimum cost. The main load on the structure is due to wind. The actual field data on wind speed observed at Colaba region, Bombay and field data on strengths of concrete and steel are used in the design. The level of reliability or probability of failure is specified before hand. Thus the safety margin is known realistically.

The RCC pole considered for the present problem is shown in Fig. 1. The cross section at the top of the pole is kept to the minimum allowed from constructional aspect and attachment of conductors to the pole. The section is increased linearly towards the bottom. The steel areas in top, middle and bottom tiers and size of the pole at the bottom are to be determined such that the pole is structurally safe at every section and its cost is minimum. Hence the design vector is

$$\vec{x} = (b, D, A_{st1}, A_{st2}, A_{st3})^T \tag{10}$$

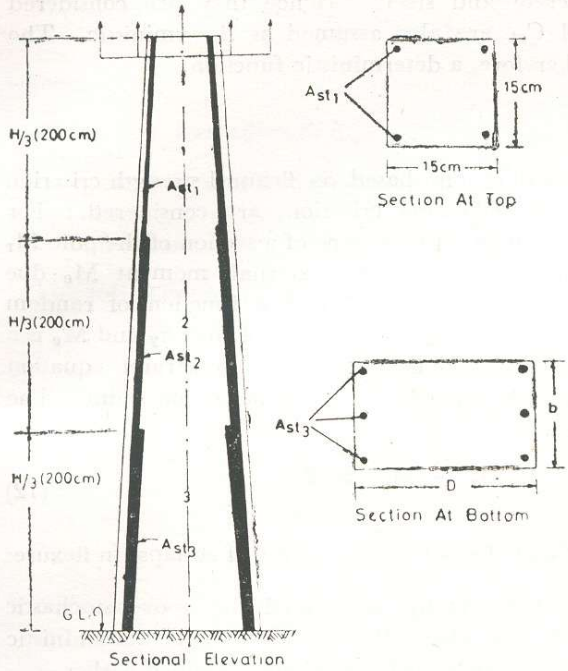

Fig. 1 Details of RCC pole

where

$$b = \text{breadth}$$

$$D = \text{depth of pole at bottom}$$

A_{st1}, A_{st2} and A_{st3} = areas of steel on each face in top, middle and bottom tiers of the pole.

Objective Function

The objective function F for the minimum of the pole is given by

$$F = C_c \left[H (b\,D+225)/2 + \gamma\,(A_{st1}\,H_1+A_{st2}\,H_2+A_{st3}\,H_3)\right] \qquad (11)$$

where

C_c = cost of concrete per unit volume

C_{st} = cost of steel per unit volume

γ = ratio of C_{st} to C_c (i.e. $\gamma = C_{st}/C_c$)

H = height of the pole above ground level

$H_1=H_2=H_3=H/3$ = height of each tier of pole.

The statistical variations in the design variables breadth, depth and areas of steel are very small compared to the variations in wind speed and strengths of concrete and steel. Hence they are considered deterministic. C_c and C_{st} are also assumed as deterministic. The objective function is, therefore, a deterministic function.

Stochastic Constraints

Two behaviour constraints, one based on flexural strength criterion and the other based on deflection criterion, are considered. For safety of the pole the moment of resistance of a section of the pole M_r must be greater than or equal to the external moment M_e due to wind at the same section. Since M_r is a function of random variables strength of concrete σ_{cu} and strength of steel σ_y and M_e is a function of random variable wind speed V, the constraint equation based on flexural strength criterion is a stochastic constraint. The same can be expressed as

$$P\,(M_r \geq M_e) \geq R \tag{12}$$

where

R = reliability of the pole at limit state of collapse in flexure.

If M_r and M_e are lognormally distributed, the above stochastic constraint is converted into the following equivalent deterministic constraint by using the stochastic NLP technique explained earlier.

$$ln\,(\widetilde{M}_r/\widetilde{M}_e) - \phi^{-1}\,(R)\;[\sigma^2_{ln\,M_r} + \sigma^2_{ln\,M_e}]^{1/2} \geq 0 \tag{13}$$

where

$\widetilde{M}_r$ and $\widetilde{M}_e$ = medians of M_r and M_e

$\sigma_{ln\,M_r}$ and $\sigma_{ln\,M_e}$ = standard deviations of lognormal variables M_r and M_e

This constraint is to be evaluated at the bottoms of three tiers of the pole, thus resulting three constraints. Similarly the constraint based on the deflection criterion of a RCC pole is also stochastic constraint. The same can be expressed as

$$P\,(\delta \leq \delta_a) \geq R \tag{14}$$

where

δ = deflection of the tip of the pole

δ_a = allowable deflection

R = reliability of the pole at this limit state of deflection

If the allowable short deflection is (1/250) th of height of the pole and if δ is lognormally distributed, the equivalent deterministic constraint for Eq. 14 becomes

$$ln\,(0.004\ H/\tilde{\delta}) - \phi^{-1}(R)\ \sigma_{ln\ \delta} \geq 0 \qquad (15)$$

where

$\tilde{\delta}$ = median of δ

$\sigma_{ln\ \delta}$ = standard deviation of lognormal variate δ.

Side Constraints

The following side constraints are also included in the problem.

$$b - 15 \geq 0 \qquad (16)$$

$$D - 15 \geq 0 \qquad (17)$$

$$A_{st1} - A_{stm} \geq 0 \qquad (18)$$

$$A_{st2} - A_{st1} \geq 0 \qquad (19)$$

$$A_{st3} - A_{st2} \geq 0 \qquad (20)$$

where

A_{stm} = minimum area of the steel to be provided at the top.

IS : 785-1964 [6] specifies that the strength of the poles in the direction of the transmission line shall not be less than one-quarter of the strength required in the transverse direction, and the strength in the transverse direction shall be greater than or equal to the strength in the direction of the line. These conditions lead to

$$2\,b - D \geq 0 \qquad (21)$$

$$D - b \geq 0 \qquad (22)$$

Area of steel at any sectfon is governed by [5] :

$$A_{st} \geq 0.85\ bd/\sigma_y$$

where

d = effective depth.

As actual variations in σ_y and its probability distribution are taken into account in the reliability theory, the above equation is taken as

$$A_{st} \geq bd/\bar{\sigma}_y \qquad (23)$$

where

$\bar{\sigma}_y$ = mean value of σ_y in N/mm^2.

This constraint is checked for bottom sections of all the three tiers.

Evaluation of Stochastic Constraints

The moment of resistance of a section of the pole is a function of σ_{cu}, σ_y and geometric properties of the section.

$$M_r = f\,(\sigma_{cu}, \sigma_y, b, d, A_{st}) \tag{24}$$

Here σ_{cu} and σ_y are considered as random variables. Using partial derivative method [4], the approximate moments of M_r are calculated. They are given by

$$\overline{M}_r = f\,(\bar{\sigma}_{cu}, \bar{\sigma}_y, b, d\ A_{st}) \tag{25}$$

$$\sigma^2_{M_r} = \left(\frac{\partial M_r}{\sigma\sigma_{cu}}\right)\Bigg|^2_{\overline{M}_r} \left(\sigma^2_{\sigma_{cu}}\right) + \left(\frac{\partial M_r}{\partial\sigma_y}\right)\Bigg|^2_{\overline{M}_r} \left(\sigma^2_{\sigma_y}\right) \tag{26}$$

If M_r follows normal distribution, $\overline{M}_r$ and σ_{M_r} become parameters of M_r. The results of the statistical analysis of the field data on the strengths of M15 and M20 concretes, and the strength of steel show that σ_{cu} and σ_y follow lognormal distribution and when they are so, the resulting M_r also tends to follow lognormal distribution [12]. The lognormal distribution is a better model for M_r than normal distribution. When M_r follows lognormal distribution, the parameters $\tilde{M}_r$ and $\sigma_{ln\,M_r}$ of M_r can be evaluated from the calculated values $\overline{M}_r$ and σ_{M_r}.

$$\tilde{M} = \overline{M}_r \exp\left(-\sigma^2_{ln\,M_r}/2\right) \tag{27}$$

$$\sigma^2_{ln\,M_r} = ln\,[(\sigma_{M_r}/\overline{M}_r)^2 + 1] \tag{28}$$

The external bending moment due to wind is given by

$$M_e = K_1 V^2 \tag{29}$$

where K_1 is a constant depending on the drag coefficient, exposed area of the pole, height of the pole etc. The approximate moments of M_e are

$$\overline{M}_e = K_1\,(\overline{V}^2 + \sigma^2_V) \tag{30}$$

$$\sigma_{M_e} = K_1\,[4\,(\overline{V})^2\,\sigma^2_V + \sigma^4_V]^{1/2} \tag{31}$$

If the wind speed is assumed to follow lognormal distribution M_e is also lognormally distributed. Hence $\tilde{M}_e$ and $\sigma_{\ln M_e}$ can be obtained for lognormal V using similar Eqs. 27 and 28. Hence the stochastic constraint Eq. 13 can be evaluated.

In the case of RCC pole with a tapered section the calculation of deflection becomes difficult and closed form solution for deflection is not possible. So element method is used to calculate the deflection at the tip of the pole. Using the virtual work theorem, the expression for the deflection at the tip of the pole is

$$\delta = \sum_{i=1}^{n} (m_i \, M_i \, \Delta x)/(E_c \, I_i); \quad i = 1, 2 \ldots, n \qquad (32)$$

where

m_i = bending moment on the element i due to horizontal unit load at the tip of the pole

M_i = bending moment on the element i due to horizontal wind speed

Δx = height of the element

E_c = modulus of elasticity of concrete

I_i = moment of inertia of the average section of the element i

and, n = number of elements.

Using codal provisions, E_c and I are calculated and hence the mean value of deflection can be evaluated. Calculation of the partial derivatives of the deflection with respect to σ_{cu} and σ_y, to determine the variation of δ due to variations in σ_y and σ_{cu}, is quite complicated as there are number of steps involved in the calculation of deflection and there is no closed form solution for deflection. However, it is possible to find the partial derivative of δ with respect to V. To overcome this difficulty, the variation of deflection of a doubly reinforced concrete beam given by ACI Committee [1] is directly taken. The committee suggests lognormal distribution for deflection and the variation for deflection for a given static load is about 20 percent in the case of simply supported beams prepared and tested under laboratory conditions. Since the pole is a cantilever and field variations are likely to be more than that observed in laboratory conditions, a value of 25 percent is taken for the coefficient of variation of deflection due to strength of concrete and steel only.

From Eq. 32 it is seen that M_i is a function of random variable V only and hence partial derivative of δ with respect to V can be evaluated and hence the standard deviation of ∂. Thus the stochastic constraint can be evaluated after evaluating $\bar{\delta}$ and σ_δ.

The optimization of RCC poles with lognormal stochastic constraints have been converted into deterministic format. The problem now reduces to finding of minimum of a multivariable nonlinear function subject to nonlinear inequality constraints. Sequential unconstrained minimization technique [7] (interior penalty function method) is used to find out the minimum cost of the pole subject to the constraints for given value of R.

Results and discussion

Details of the pole other than given in Fig. 1 are listed below ;

Spacing of poles = 50 m

Types and number of conductors

Type	Diameter mm	No.	Height above ground level m
Copper conductor	7.1	3	5.80
Copper neutral wire	5.0	1	5.80
Street lighting wire	4.0	1	6.15

Details of the random variables considered and other numerical data that are necessary for the solution of the stochastic programming problem of RCC poles are given in Table 1. For parametric study

Table 1 Numerical data for the case study

Random variable	Mean	Standard deviation	Probability distribution
σ_{cu} (N/mm^2) (M15 concrete)	22.67	5.01	lognormal
σ_y (N/mm^2) (Mild steel)	295.3	5.51	lognormal
Wind speed (kmph) (Colaba, Bombay)	86.38	9.92	lognormal

Cost of concrete = Rs. 1000/cu.m.

Cost ratio γ = 70

Number of elements for deflection calculation = 3 some of the parameters involved in the problem are changed in the examples solved. The results of the specific cases solved are presented.

Using the data given in Table 1 and the method developed earlier to transform the stochastic constraints to equivalent deterministic constraints, the optimal design of RCC pole shown in Fig. 1 is obtained for various reliability levels and the results are given in Table 2. It is seen from Table 2 that the cost of the pole increases with reliability as expected. To present the picture numerically, it is noted that for increase in reliability from $(1 - 10^{-2})$ to $(1-10^{-7})$, the increase in the cost is 41.38 percent.

It is also observed during the optimization that the constraint specifying the minimum steel is the most active of all the constraints at optimum. This implies that the optimum steel is not decided by the strength or deflection criteria but merely by a specification governing the minimum steel. It is also found that optimal size of pole is always dictated by deflection constraint and the required area of steel is governed by minimum requirement.

To make a comparative study, the same problem has been solved assuming all the random variables σ_{cu}, σ_y and V as normally distributed and assuming the strength and deflection constraints as normal stochastic constraints. The results of optimization are given in Table 3. Comparing the results in Tables 2 and 3, it is found that with assumption of normal distribution for random variables the optimal size of the pole is less and hence the cost is less than that of the cost of the pole with lognormal distribution for random variables. Moreover, the difference in costs for the two cases increases with increase in reliability level. Hence the assumption of normal variables and normal stochastic constraints when they are not so as in this case will lead to the solution on the unsafer side.

For parametric study, the values of γ, the coefficient of variation of σ_y, the grade of concrete, the grade of steel and number of elements for deflection calculation are changed and the results of the optimization are given in Table 4.

Comparing Tables 3 and 4, it is observed that change in the cost ratio in the range of 50 to 80 and obviously above does not affect the optimum design. This is because that the optimum steel area is not decided by the cost ratio but by the specification for minimum steel area.

The change in either mean value or the coefficient of variation of strength of steel does not affect the optimal design indicating again

Table 2 Specified reliability versus optimum design (lognormal variables)

Specified reliability (Probability of failure)	Optimum design					Optimum cost
	Breadth (cm)	Depth (cm)	A_{st1} (sq. cm)	A_{st2} (sq. cm)	A_{st3} (sq. cm)	Rs.
0.99 (10^{-2})	15.337	30.673	1.570	1.570	1.570	340.51
0.999 (10^{-3})	16.527	33.054	1.570	1.570	1.626	364.84
0.9999 (10^{-4})	17.598	35.142	1.570	1.570	1.856	392.92
0.99999 (10^{-5})	18.553	37.106	1.570	1.570	2.081	420.19
0.999999 (10^{-6})	19.978	38.667	1.570	1.661	2.349	455.50
0.9999999 (10^{-7})	20.344	40.688	1.585	1.768	2.561	481.43

Note : Minimum stipulated steel area = 1.57 cm^2 (2-10 mm bars on each face)

Table 3 Specified reliability versus optimum design (normal variables)

Specified reliability (Probability of failure)	Optimum design					Optimum cost
	Breadth (cm)	Depth (cm)	A_{st1} (sq. cm)	A_{st2} (sq. cm)	A_{st3} (sq. cm)	Rs.
0.99 (10^{-2})	15.000	29.409	1.57	1.57	1.570	331.72
0.999 (10^{-3})	15.368	30.713	1.57	1.57	1.570	340.98
0.9999 (10^{-4})	15.819	31.628	1.57	1.57	1.570	349.48
0.99999 (10^{-5})	16.188	32.376	1.57	1.57	1.570	356.60
0.999999 (10^{-6})	16.511	32.989	1.57	1.57	1.624	364.55
0.9999999 (10^{-7})	16.875	33.542	1.57	1.57	1.688	372.53

Note : Minimum stipulated area = 1.57 sq. cm. (2-10 mm dia. bars on each face)

Table 4 Sensitivity of optimal design to change in various parameters (normal variables)

Probability of failure	Parameters changed and their values	Optimal design					Optimum cost	Remarks
		b (cm)	D (cm)	A_{st1} (cm^2)	A_{st2} (cm^2)	A_{st3} (cm^2)	Rs.	
10^{-5}	$\gamma = 50$	16.188	32.376	1.57	1.57	1.57	318.93	
10^{-5}	$\gamma = 80$	16.188	32.376	1.57	1.57	1.57	375.45	
10^{-5}	$n = 6$	16.186	32.371	1.57	1.57	1.57	356.57	
10^{-7}	$n = 6$	16.784	33.528	1.57	1.57	1.678	371.23	
10^{-5}	$\sigma_{\sigma_y} = 22.15$ N/mm^2	16.188	32.376	1.57	1.57	1.57	356.61	Other data are same as in Table 1 for each case
10^{-7}	$\sigma_{\sigma_y} = 29.53$ N/mm^2	16.807	33.613	1.57	1.57	1.69	372.09	
10^{-5}	$\bar{\sigma}_y = 441.7$; $\sigma_{\sigma_y} = 24.3$ N/mm (Torsteel)	16.188	32.376	1.57	1.57	1.57	356.61	
10^{-5}	$\bar{\sigma}_{cu} = 29.16$; $\sigma_{\sigma_{cu}} = 5.49$ N/mm^2 (Mix M20)	15.601	31.202	1.57	1.57	1.57	345.41	

that the steel does not have any influence in the optimization process in the present case study. The increase in the grade of the concrete decreases the size of the pole and steel area as expected.

The increase in the number of elements for deflection calculation from 3 to 6 produces a reduced deflection and a slight reduction in size of the pole and hence less cost. However, this difference in design is very small and negligible. For all practical purposes the calculation of deflection based on three elements is enough.

Conclusion

The equivalent deterministic formulation has been presented when the constraint equation involves lognormal random variables and the resulting stochastic constraint also is a lognormal variate. The application of stochastic programming to RCC poles with lognormal constraint is illustrated. The suggested approach can be applied to the optimal design of structures under different limit states and different failure modes for specified reliability of the structure.

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Notations

The following symbols are used in this paper :

A_{st} = area of steel on each face

A_{st1} = area of steel on each face in tier 1; similarly A_{st2} and A_{st3}

b = breadth of pole

C_c = cost of concrete per unit-volume

C_{st} = cost of steel per unit volume

D = depth of pole

d = effective depth of section

E_c = modulus of elasticity of concrete

H = height of pole

H_1 = height of tier 1; similarly H_2 and H_3

I = moment of inertia

M_e = external bending moment

M_r = moment of resistance of section

$\overline{M}_r$ = mean value of M_r; similarly $\overline{M}_e$, $\bar{\delta}$, $\bar{\sigma}_y$, $\bar{V}$

$\tilde{M}_r$ = median of M_r; similarly $\tilde{M}_e$, $\tilde{\delta}$, $\tilde{V}$

n = number of elements for deflection calculation

P ( ) = probability of ( )

R = reliability

V = wind speed

X = vector of decision variables

Y = vector of random variables

δ = deflection

δ_a = allowable deflection

γ = ratio of cost of steel to cost of concrete

ϕ = cumulative probability of standardized normal variable

σ = standard deviation or stress

σ_{cu} = cube strength of concrete at 28 days

σ_y = yield strength of steel

σ_{M_r} = standard deviation of M_r ; similarly σ_{M_e}, σ_δ and σ_v

$\sigma_{ln M_r}$ = standard deviation of $ln\ M_r$; similarly $\sigma_{ln\ M_e}$, $\sigma_{ln\delta}$ and σ_{lnv}.