

Disturbance Estimation
And Cancellation
for
Linear Uncertain Systems

H. J. Kim

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DISTURBANCE ESTIMATION AND CANCELLATION FOR LINEAR UNCERTAIN SYSTEMS

Hwi Jun KIM

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Summary

This thesis was born on the boundary of the theory of control and a particular application, namely a smart structural system. More specifically, the primary motivation of this study comes from the following questions. Is it really possible to use smart structural system in a practical situation? What is the problem from the view point of the theory of control? In this work, it is assumed that one of the most important problems is that of robustness of the controlled system, and it is supposed that one of the solution would be to estimate uncertainty and disturbance without *a priori* knowledge in order to cancel their effect on system behaviour. This study provides a method, including its theoretical foundation, to estimate and cancel out any bounded disturbance and/or uncertainty.

One of the most important problems in control systems is the robustness of the controlled system. It is known that almost all physical systems, such as mechanical or structural system, contain some form of uncertainty. Even smart structural systems cannot escape from this problem. Such systems consist of host materials, sensing and actuating layers, which are attached or embedded to the host materials. The modelling of smart structural systems gives rise to infinite dimensional models if it were modelled by ordinary differential equations. In practice, however, a model is obtained by using the Finite Element Method, and, hence, a high order finite dimensional model is obtained. Such approximate models will inevitably generate uncertainty, representing unmodelled dynamics. In addition, such system models would suffer from parametric uncertainty and external disturbances. Thus, this type of system model would include many types of uncertainties.

In past decades, much research has been done using a deterministic approach for the robust control problem. The majority of this work assumes a known upper bound to uncertainty and disturbance, and robust controllers are determined deterministically. However, for smart structural systems, it may be difficult, or even impossible, to obtain such *a priori* knowledge of any disturbance. If this is the case, what can be said about the robustness of controlled system without *a priori* knowledge of any disturbance? Part of the answer to this question can be found in this thesis.

This thesis considers a linear uncertain system in which the uncertainty and/or disturbance is known to be bounded, but its bound is unknown. The main contribution is that an adaptive feedback control law is designed to estimate the bounded disturbance. The design of the adaptive control algorithm is novel and the adaptive control algorithm is easy to implement. This information can then be used to cancel the effect of the disturbance in the system. This has the advantage that, if further design objectives are to be realized, the controls can be designed based on the information from the known nominal model only

and not on the model with uncertainty.

This thesis is organized as follows. In Chapter 1, firstly, concept of stability of systems and deterministic approach of robust control are recalled. At the end of that chapter, it is implied that there is some limitation of that approach of robust control. In Chapter 2, motivated by the limitation discussed at Chapter 1, the method of disturbance estimation is introduced. Firstly, the statement of the problem is provided. It is followed by some preliminary works which are required for analysis. Then, for each class of systems, an adaptive algorithm, lemmas, theorem, and simulation examples are provided. The class of systems examined are second-order single-input linear systems, n^{th} order single-input linear systems, and multi-input linear systems. In Chapter 3, based on the works of Chapter 2, some applications of the method of disturbance estimation are presented. In Section 3.2, an adaptive algorithm which guarantees robustness of a controlled system is presented. In Section 3.3, treatment of input uncertainty and unmodelled dynamics is discussed. In the following section, Section 3.4, it is shown that under appropriate assumptions, it is possible to treat residual disturbance by the method proposed. In Section 3.5, the method is extended so that the method can be used only by outputs of a system. At last, in Section 3.6, it is demonstrated that parameter variations can be extracted from estimated disturbances. In Chapter 4, conclusion remarks and suggestions for future works are provided.

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Chapter 4

Conclusions and further research

4.1 Concluding remarks

As it is shown in previous chapters, in this study, the following topics are investigated.

1. For both single-input and multi-input systems, estimation and cancellation of bounded disturbance/uncertainty can be performed without *a priori* knowledge of bounded disturbance/uncertainty (see Theorem 3, 4, 5, and related remarks).
2. Estimation and cancellation can be achieved, even in the presence of residual uncertainty/disturbance under appropriate conditions (see Section 3.4).
3. Using the disturbance estimation method, a tracking controller can be designed with respect to the nominal model (see Theorem 7).

4. Estimation and cancellation of unknown bounded disturbance/uncertainty can, also, be achieved by using only output measurement, even in the presence of sensor noise (see Section 3.5).
5. For both the stabilization and tracking problems, the system to be controlled is robust against parametric uncertainty, input uncertainty, unmodelled dynamics, external disturbance and/or sensor noise using the disturbance estimation/cancellation method (see Theorem 7, 8, and Section 3.5.3).

6. The parameter variations for the nominal model can be estimated from the estimated disturbance under certain assumptions (see Section 3.6).

In addition, numerical simulations are presented to demonstrate the methods developed.

The author believes that theory is enhanced as a result of interaction with various applications. Although the primary motivation of this study comes from one specific area of application and the assumptions developed are constructed so that these assumptions are realistic in the specific practical situation, it is uncertain what kind of problems exist when the theory is implemented for applications. The author believes that almost exact estimation of such uncertainty can be achieved but perfect estimation of such uncertainty is not possible. Thus, to make clear and resolve such problems, it is recommended that the methods be implemented in a number of applications.

In relation to *Treatment of input constraint*, improvements of the adaptive algorithms and a better understanding of the nominal model and modelling is encouraged. The method proposed gives an engineering solution for the worst case situation regarding uncertainty and disturbance, as well as control of a nonlinear and time-varying system, by cancelling out their effect. However, due to input and uncertainty that can be tolerated as modelling errors. Thus, improvements of the adaptive algorithms and a better understanding of modelling and the characteristics of the nominal model, which are fundamental characteristics of nonlinear system and time-varying system, are required.

Since the methods developed do not require any *a priori* knowledge of disturbance and the methods can be applied using only output measurement, the methods will be relatively easy to implement in a practical situation. The author believes that the method proposed will contribute to the development of high performance/reliable systems.

2. Treatment of the case when there are input constraints.
1. Implementation of the methods in some practical application.

Topics which are not studied, but are suggested for further work, are listed as follows.

4.2 Recommendations for further work

Bibliography

- [1] A. Ichmann and E. P. Ryan, Universal λ -Tracking for Nonlinearly-Perturbed Systems in the Presence of Noise, *Automatica*, vol. 30, 1994, pp.337-346.
- [2] B. Mårtensson, The order of any stabilizing regulator is sufficient a priori information for adaptive stabilization, *Systems & Control Letters*, vol. 6, 1985, pp.87-91.
- [3] B. R. Barmish and G. Leitmann, On Ultimate Boundedness Control of Uncertain Systems in the Absence of Matching Assumptions, *IEEE Transactions On Automatic Control*, vol. AC-27, 1982, pp.153-158.
- [4] C. K. Lee, Theory of laminated piezoelectric plates for the design of distributed sensors/actuators. Part I: Governing equations and reciprocal relationships, *Journal of Acoustical Society of America*, vol. 87, 1990, pp.1144-1158.
- [5] D. E. Miller and E. J. Davison, An Adaptive Controller Which Provides Lyapunov Stability, *IEEE Transactions On Automatic Control*, vol. 34, 1989, pp.599-609.
- [6] D. E. Miller and E. J. Davison, An Adaptive Controller Which Provides an Arbitrarily Good Transient and Steady-State Response, *IEEE Transactions on Automatic Control*, vol. 36, 1991, pp.68-81.
- [7] D. G. Luenberger, Canonical Forms for Linear Multivariable Systems, *IEEE Transactions On Automatic Control*, vol. AC-12, 1967, pp.290-293.
- [8] G. Feng, New robust model reference adaptive control algorithm, *IEEE Proceedings-Control Theory and Applications*, vol. 141, 1994, pp.177-180.
- [9] G. Leitmann, On the Efficacy of Nonlinear Control in Uncertain Linear Systems, *Journal of Dynamic Systems, Measurement, and Control*, vol. 102, 1981, pp.95-102.
- [10] H. S. Tzou and C. I. Tseng, Distributed Modal Identification and Vibration Control of Continua: Piezoelectric Finite Element Formulation and Analysis, *Transactions of ASME*, vol. 113, 1991, pp.500-505.
- [11] I. Mareels, A simple selftuning controller for stably invertible systems, *Systems & Control Letters*, vol. 4, 1984, pp.5-16.
- [12] K. Ogata, *Modern Control Engineering*, Prentice-Hall International, 1997.

- [13] K. Yamada, S. Komada, M. Ishida, and T. Hori, Characteristics of servo system using high order disturbance observer, *IEEE Proceedings of 35th Conference of Decision and Control*, 1996 pp.3252-3257.
- [14] L. Mirsky, *An Introduction To Linear Algebra*, Oxford University Press, 1955.
- [15] M. Corless, Simple Adaptive Controllers for Systems which are Stabilizable via High Gain Feedback, *IMA Journal of Mathematical Control & Information*, vol. 8, 1991, pp.379-387.
- [16] M. Corless and G. Leitmann, Continuous State Feedback Guaranteeing Uniform Ultimate Boundedness for Uncertain Dynamic Systems, *IEEE Transactions on Automatic Control*, vol. AC-26, 1981, pp.1139-1144.
- [17] M. Corless and G. Leitmann, Adaptive Control of Systems Containing Uncertain Functions and Unknown Functions with Uncertain Bounds, *Journal of Optimization Theory and Applications*, vol. 41, 1983, pp.155-168.
- [18] M. Fu and R. Barmish, Adaptive Stabilization of Linear Systems Via Switching Control, *IEEE Transactions On Automatic Control*, vol. AC-31, 1986, pp.1097-1103.
- [19] M. Nakao, K. Ohnishi, and K. Miyachi, A Robust Decentralized Joint Control Based on Interference Estimation, *Proceedings of IEEE International Conference of Robotics and Automation*, 1987 pp.326-331.
- [20] O. C. Zienkiewicz and R. L. Taylor, *Finite Element Method - The Basis, Butterworth Heinemann*, 2000.
- [21] R. E. Kalman and J. E. Bertram, Control System Analysis and Design Via the "Second Method" of Lyapunov I Continuous-Time Systems, *Journal of Basic Engineering*, vol. 82, 1960, pp.371-393.
- [22] Robert M. Jones, *Mechanics of Composite Materials*, Taylor and Francis, 1999.
- [23] S. Gutman, Uncertain Dynamical Systems - A Lyapunov Min-Max Approach, *IEEE Transactions on Automatic Control*, vol. AC-24, 1979, pp.437-443.
- [24] S. Gutman and Z. Palmor, Properties of Min-Max Controllers in Uncertain Dynamical Systems, *SIAM Journal of Control and Optimization*, vol. 20, 1982, pp.850-861.
- [25] S. Komada, K. Ohnishi, and T. Hori, Hybrid Position/Force Control of Robot Manipulators Based on Acceleration Controller, *Proceedings of the 1991 IEEE International Conference on Robotics and Automation Sacramento, California*, April 1991, 1991 pp.48-55.
- [26] S. L. Shah, D. G. Fisher, and D. E. Seborg, Eigenvalue/Eigenvector Assignment For Multivariable Systems And Further Results For Output Feedback Control, *Electronics Letters*, vol. 11, 1975, pp.388-389.

- [27] S. N. Singh, Adaptive Model Following Control of Nonlinear Robotic Systems, *IEEE Transactions on Automatic Control*, vol. AC-30, 1985, pp.1099-1100.
- [28] S. Srinathkumar, and R. P. Rhoten, Eigenvalue/Eigenvector Assignment For Multivariable Systems, *Electronics Letters*, vol. 11, 1975, pp.124-125.
- [29] Shankar Sastry, Nonlinear Systems Analysis, Stability, and Control, Springer-Verlag New York, 1999.
- [30] Vivtor-Emil Neagoe, Inversion of the Vander Monde Matrix, *IEEE Signal Processing Letters*, vol. 3, 1996, pp.119-120.
- [31] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, Numerical Recipes in C, Cambridge University Press, 1988.
- [32] W. Yim, and S. N. Singh, Adaptive Output Feedback Force Control of a Cantilever Beam Using a Piezoelectric Actuator, *Journal of Vibration and Control*, vol. 9, 2003, pp.567-581.
- [33] Walter Gautschi, On inverses of Vandermonde and confluent Vandermonde matrices, *Numerische Mathematik*, vol. 4, 1962, pp.117-123.
- [34] Walter Gautschi, Norm Estimates for Inverse of Vandermonde Matrices, *Numerische Mathematik*, vol. 23, 1975, pp.337-347.
- [35] X. Chen, and Chun-Yi Su, Robust output tracking control for the systems with uncertainties, *International Journal of Systems Science*, vol. 33, 2002, pp.247-257.
- [36] X. Chen, S. Komada, and T. Fukuda, Design of a Nonlinear Disturbance Observer, *IEEE Transactions on Industrial Electronics*, vol. 47, 2000, pp.429-437.
- [37] Y. H. Chen and G. Leitmann, Robustness of uncertain systems in the absence of matching assumptions, *International Journal of Control*, vol. 45, 1987, pp.1527-1542.
- [38] Yu-Sheng Lu and Jian-Shiang Chen, Design of a Perturbation Estimator Using the Theory of Variable-Structure Systems and Its Application to Magnetic Levitation Systems, *IEEE Transactions On Industrial Electronics*, vol. 42, 1995, pp.281-289.