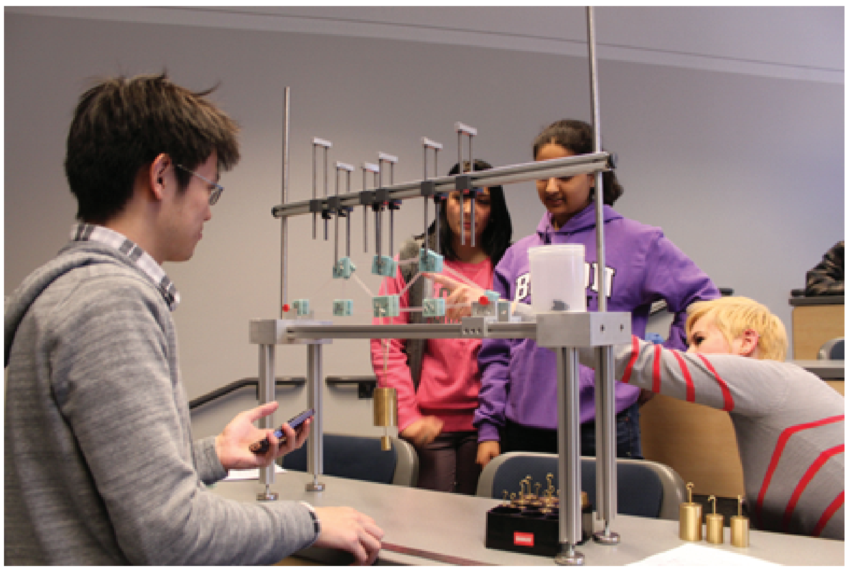
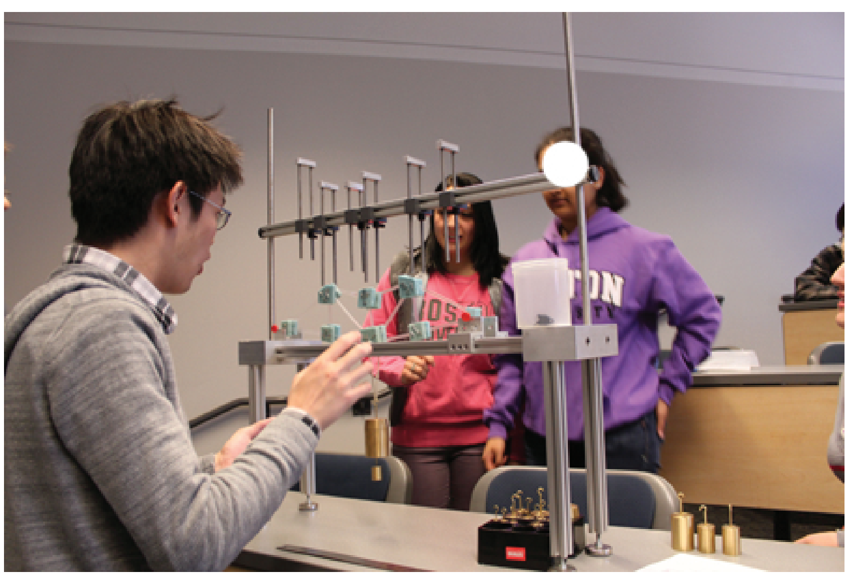
EK 301: Truss Design Project Description

Fall 2017





1 Overview

Memo from Washington Roebling, Chief Engineer of design and construction of the Brooklyn Bridge, to his assistant:

"There are so many points to be considered, so many conflicting interests to be reconciled on the parts of the truss that it is perfectly bewildering to pick out the best thing. For example, I want to reduce the aggregate weight so as to keep down the pressure on the masonry. I want to simplify the superstructure so as to make work in the shop easy and erection easy and safe and I also want to keep down the wind surface as much as possible. On the other hand I want the truss sufficiently strong to resist a reasonable amount of bending, and this goes against the other points. But the only possible way in which I can reduce pressure on masonry and wind surface is by reducing the height and weight of the trusses and increasing the strain per square inch on the iron. I do not see that any reduction of weight is possible in any other parts of the structure. By making the truss rods of steel we make some reduction in weight but it is only in the low truss that the rod section is great enough to enable us to attain any appreciable advantage by the substitution of steel for iron. In the high truss with rods through two panels the section is hardly sufficient to make it worth while to change. This therefore would be one argument in favor again of reducing the weight of the intermediate truss and leaving the rods in all the trusses within one panel. This includes the two central trusses even if they are arranged with a square bar in the middle of two flat ones outside."

Source: McCullough, David G. The Great Bridge: The Epic Story of the Building of the Brooklyn Bridge. New York: Simon and Schuster, 1972. Print.

Luckily for you, the challenging design and construction of the Brooklyn Bridge is already completed! Instead, your task is to work in teams of three students within your section to design, construct, and test a simple planar truss made from soda straws (structural members), foam core gusset plates, and T-pins (pin connectors). Like all bridges, your truss must satisfy a given set of specifications, such as span, minimum load, and cost (see Sec. 2 below). You will go through several design stages and your final design and build will be evaluated based on its theoretical maximum achievable load, the accuracy of your theoretical load with respect to the actual maximum load, and the cost of the truss.

This effort will require material testing to determine the strength of the straws, a failure analysis, and a final physical test of the maximum load bearing capability of your final design. The project will not only test your understanding of trusses, but will also require you to interpret and apply experimental data and to create and judge the merit of alternative designs, all while exposing you to the benefits of teamwork.

The project consists of four main components:

1. **Materials analysis:** In this element you will measure, analyze, and report on the relevant physical properties of your primary material – soda straws.
2. **Preliminary design:** Everything has to start somewhere. Here you will use both your creative and analytical skills to design and analyze two trusses that meet the specifications.
3. **Redesign and build:** Having never built a truss before, it is unlikely your first attempt will be the best you can come up with. Thus, you will come up with at least two more alternative designs. After picking your dream truss, you will build it using materials provided by us.
4. **Testing:** Every good thing must come to and end and so it is with your truss. You will load up your truss with increasing weight until it fails, hopefully in a crash of glory.

The project deliverables are a report on your straw testing, a preliminary design report, and a final report. Oh, and your completed truss of course.

Important dates:

Week of October TBD (tentative): Straw testing (in B01, 110 Cummington Mall)
October 25th/26th: Straw testing report due (in class)
November 17th by 4 p.m.: Preliminary design report due (in the ME Department office)
December 8th by 4 p.m.: Final report due (in the ME Department office)
December 9th: Truss testing, approximately 30-45 minute slot (note that is a Saturday!) (PHO 210, 211)

2 Specifications

The truss you design and built must conform to the following specifications.

1. **The truss must be a single, planar, simple truss. No truss member may be designed to extend above the line connecting the two end joints.**

   A simple truss starts with three members and three joints configured in a triangle and then is built up by adding two members and one joint each time to complete an additional triangle. Members may not cross each other or be doubled-up, and can only be joined to other members at their ends.

   Make sure your truss is a simple truss by checking first that it is comprised only of triangles. If any squares are formed, your truss is incorrect! Second, the number of joints J and members M must be related by

$$M = 2J - 3.$$

2. **You must use only the materials provided to you.**

   A week before the straw lab you will be provided a bag with soda straws and pins, and later in the semester you will be given a bag of foam gusset plates. This collection is meant to serve all aspects of the project (straw testing and final build) so hold onto it until you complete your testing.

3. **Members must be joined concurrently at joints using the provided pins and double pinned to the guest plates as described below. There must be a gusset plate on each side of the joint.**

   Each straw should be surrounded between two gusset plates and double pinned in their center, with the outer pin 5 mm from the straw end and the inner pin another 5 mm from the outer pin, as illustrated in Fig. 2.1. The center axes of the straws should intersect at a single point to prevent the generation of a moment; such a moment can cause premature buckling. The ends of the straws should be as close as possible without actually touching. Pins should have their points on the same side of the truss.

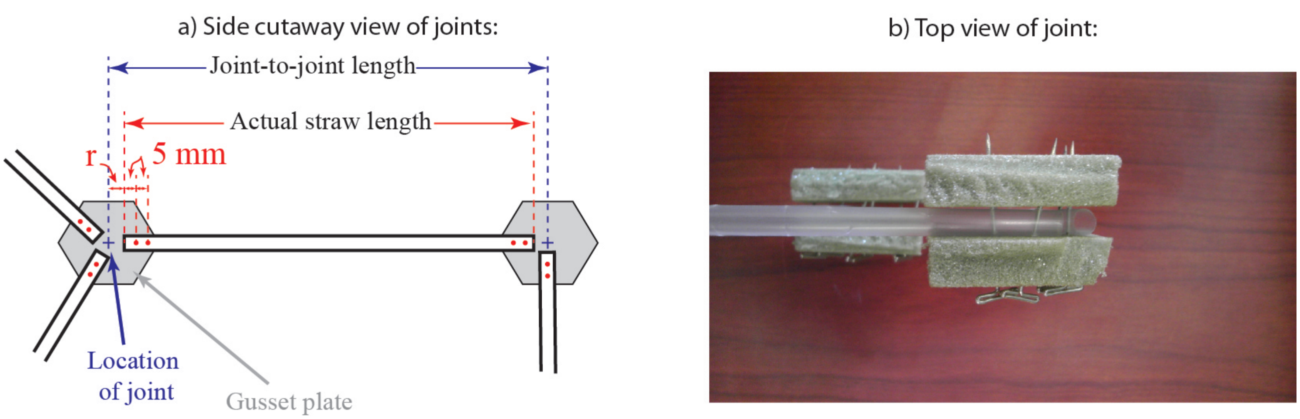


Figure 2.1: a.) A cutaway schematic of 2 connected joints. ***NOTE:*** *The distance from the end of the straw to the true center of the joint should correspond to the straw radius r, or the EQUIVALENT MINIMUM distance based on the specific joint geometry that prevents the multiple straws from physically intersecting. The hexagonal grey shape represents the 'back' gusset plate, and a fully-formed joint will also have an opposing gusset plate that will cover the 'top' of the joint exposed in the figure, surrounding (but not squeezing) the straw. b.) The top view of a joint; note the placement of a gusset plate on both sides of the straw, connected by 2 pins per straw. See Sec. 8.1 for more detail.*

4. **Gusset plates may be any geometry but must have an area of less than 16 cm^2 with no straw extending along the gusset plate by more than 2.5 cm from the point of concurrency.**

   The gusset plates provided are initially cut from foam core material into approximately square shapes. Truss performance is improved by using the smallest gusset plates possible so you are encouraged to shape the gusset plates for your particular truss design.

5. **All *joint-to-joint* distances (as defined in Fig. 2.1) must be at least 10 cm and no longer than 15 cm.**

   Since the straws don't quite go joint to joint, the shortest straw length allowed is approximately 9.5 cm.

6. **The truss must span a distance of 54.5 cm $\pm$ 1 cm.**

   This must be true not only of your designs but also of the constructed truss you bring to testing.

7. **The truss should be designed such that it is supported and loaded in an "inverted" arrangement (see Fig. 2.2 below), where the truss is supported by two joints along the top horizontal line and the load is placed at a joint positioned *below* the joints.**

8. **The truss must support a minimum load of 4.91 N (500 g mass) for 60 seconds. The load must be placed on a joint located at a horizontal distance of 25 cm $\pm$ 0.5 cm away from the pin joint.**

   This implies that there must be a joint at that location along the bottom line. You are free to have any other number of joints along that line but you must at least have that one. Feel free to deviate from the classic 'train bridge' design where all the bottom-most joints are horizontal and in-line with each other (e.g. it doesn't have to *look* like the truss shown in Fig. 2.2, only be supported and loaded as shown).

9. **The unloaded truss must not sag more than 2 cm below a line connecting the end joints.**

10. **The loaded truss must not have any member sag more than 6 cm below the end joint.**

    If that happens prior to the truss failing in a spectacular buckling event, it will be deemed to have failed.

11. **The total (virtual) cost of the truss must be less than \$350** , where the cost of the truss is defined as

$$\text{Cost} = \$10J + \$1L$$

and J = number of joints and L = all straw lengths summed together in cm.

Summary of distance and cost specifications

Joint-to-joint span L_{jj}	10 cm $\leqslant L_{jj} \leqslant$ 15 cm
Truss span	54.5 cm $\pm$ 1 cm
Load to pin joint span	25 cm $\pm$ 0.5 cm
Total virtual cost	$<$ \$350

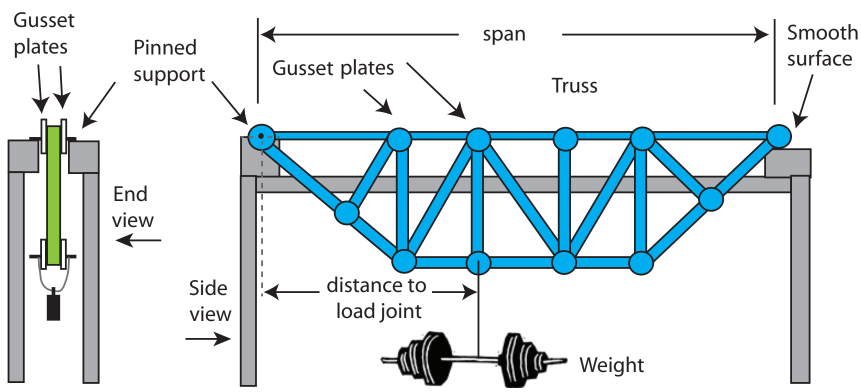


Figure 2.2: Cartoon of the testing apparatus

3 Group work

Working in groups is a vital component of most engineering projects. You will encounter group projects that involve scenarios ranging from working with close friends to complete strangers, trustworthy and seasoned engineers who speak your language & have a similar work approach to non-engineers who have no knowledge of the technical jargon and analysis (I know, it's sad but true). Regardless of the setting, your engineering profession demands a responsibility to the challenge presented to you, and to conduct yourself in a professional manner. This responsibility includes integrating yourself with your team.

The EK301 Truss Design Project is no different. You may find yourself working with peers who you've never met prior to EK301, both within and outside of your major, and future engineering classes are likely to have significant group components as well. To assist with achieving optimal group dynamics, your **straw lab report** should include the two deliverables described below regarding your group effort:

1. Effort-based contract: You should read through the entirety of this project manual to better understand what is required in completing all aspects of the project. Based on this project, your group should write a contract wherein you will briefly describe the following issues:
   - Strengths/weaknesses: Your particular strengths and weaknesses that pertain to accomplishing a group-based engineering design project. For example, if you feel especially confident in your Matlab/coding skills but less confident in technical writing, write these thoughts down and discuss them with your group.
   - Scheduling issues: You should discuss hard deadlines or extenuating non-academic circumstances that your group should be aware of, for scheduling purposes.
   - Group contract: As a group, you should collectively write a short document that describes what measures you feel comfortable with if someone misses a deadline, and the parameters involved with keeping everyone updated on individual deliverables that you're each responsible for. Note this is something **YOU** write; we do not distribute a downloadable form for this exercise.
2. Case study: You will need to individually read through a short case study on group dynamics ('It Takes Two to Tango'[1]), and discuss the case study with your group setting. You should take meeting minutes and especially focus your discussion on the group dynamics discussed in the case study. The case study is posted in the project folder on the course website.
   - The date, time, and place of the meeting should be recorded along with the names of the participants. The (acting) chair of the meeting and the recorder (minute taker) should be identified.
   - The planned agenda should be attached.
   - The important points should be summarized, along with who suggested each of them.
   - The conclusions should be recorded and any action items listed, with the responsible person labeled.

The goal of this effort is have clear communication and a healthy working relationship with each other. If problems arise throughout the course of the project timeline, try to resolve them amongst yourselves and by referencing your group contract. You will individually fill out a peer evaluation for yourself and your group members at the end of the project to let us know things went, and negative feedback may result in a grade adjustment. If problems crop up before then or if you need more guidance, don't hesitate to approach your section instructor.

[1] Oakley B., J. Student Centered Learning, Vol 1 (1), 19-27 (2002)

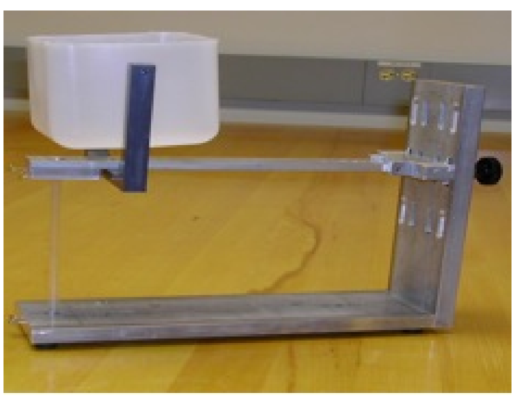
4 Straw testing lab and analysis

In order to determine the failure load for your truss, you need to know the strength of the materials you are working with. In a properly designed and built truss, all the load is carried by the straws. The purpose of the straw testing is to determine the buckling strength as a function of the length of the straw. In this section, we detail the straw testing lab, the analysis to be performed, and the report that you need to write.

4.1 Straw testing lab

Straw testing will take place in a lab during Week of October TBD (tentative) at specific times to be arranged. The lab will take place in room B01 in 110 Cummington Mall and should take approximately 60-90 minutes to complete. Sign up will be done using a sheet posted on the door of B01 with times taken on a first-come, first sign up basis. **All the members of your team must be present at testing.** If any of your team is missing, you will not be allowed to test. If you do not test, or if you show up late for your test or are not being prepared for testing, there will be a significant reduction in your grade.

The straw properties will be tested using a simple lever/hinge mechanism shown in Fig. 4.1. This tester applies an axial load force to the straw. This force is generated by masses you will add to the high-tech bucket on the top of the tester. Note that the weight you add is **not** equal to the load experienced by the straw. You will need to analyze the tester to determine the relationship between the added weight and the applied load by drawing a free body diagram of the testing apparatus.



(a) Side view

(b) Front view

Figure 4.1: Straw testing apparatus in (a) side and (b) front views.

You will test at least three straw lengths in the range 9.5 cm± 0.1 cm to 15 cm± 0.1 cm. You are free to choose the lengths as you see fit. For accurate modeling of the straws, we need as many different lengths as possible so please try to choose differently than your colleagues on other teams. For each straw length you must run a minimum of 5 tests for a total of 15 tests. To ensure accurate results, please make sure that

1. All straws used are straight.
2. All straws are double-pinned at both ends with the first pin 5 mm from the end and the second 5 mm in from that (see Fig. 2.1).

3. All pins (at both ends) are aligned perpendicular to the straw axis and parallel to each other.
4. The straws are oriented vertically in the tester.
5. The straws are supported by the pins sitting squarely in the slots, not by the straw ends.
6. The hinged lever arm is horizontal and perpendicular to the straw in all directions.

4.1.1 PRELAB ASSIGNMENT: Prior to coming to lab

There are a few items you need to complete before coming into your testing time.

1. Read this document carefully (hey, you're already doing that one! Good work.).
2. Sign up for a testing time.
3. Prepare a procedure checklist for the steps described in Sections 4.1.2 - 4.1.4 below; add a check box next to each step to help you track your progress. Leave a space to write down the information that you'll need to gather for these steps. Show this procedure to the GST once you arrive, since you won't be allowed to start testing otherwise.
4. Decide upon the three lengths of straws you will test and then carefully cut at least six straws in each length. A pair of scissors is sufficient for this.

4.1.2 Prior to commencing data collection

Once you arrive in the lab but before beginning your data collection, you need to do a few things.

1. Pin all your straws carefully. To ensure your pinning is consistent, it is highly recommended that you take advantage of the pinning jigs.
2. Record necessary experimental information. There are a few data points you will need in your analysis and report. Among the items you should record are
   (a) the number of the tester,
   (b) the mass of the straw tester arm (engraved on the arm),
   (c) the distance from the center of the arm's pin to its center of gravity as noted by a line engraved on the arm.
   (d) Note that if your group uses more than 1 testing apparatus, you'll need to obtain this information for each tester and carefully note which of your data points correspond to each apparatus.
3. Draw a FBD of your apparatus. This needs to be checked off by the Graduate Student Teacher (GST) and included in your report. Note that the GST is *only* verifying that you did a FBD, not checking its accuracy. Be sure to double-check (and maybe even triple-check) your FBD to ensure it accurately describes the testing scenario.
4. Consider the rigid body static equilibrium analysis of the straw tester to be sure you have recorded all the information you need.

4.1.3 Data collection

It is important to have accurate results. This means you will need to load the straws in small increments. As buckling nears (indicated by a straw that is bending), you should go in sizes of no more than 20 g. Of course, you have no idea where to start or what the onset of buckling looks like. That's why you cut *six* straws at each length, even though only five straws are needed. This first straw is a sacrificial straw. This one should be quickly tested first to get a sense of where buckling will occur and what buckling will look like. This will then guide your initial choice of loading and your initial step sizes.

For each straw, testing should proceed as follows.

1. Place the next mass in the bucket. Ensure the increment is less than 20 g if buckling is imminent.
2. Record the mass increment.
3. Wait for 30 seconds or until buckling occurs.
4. Repeat until buckling occurs.

Note that the max load is the last one to successfully hold for 30 seconds, not the one at which buckling occurred.

4.1.4 Before leaving the lab

There are a few things you need to do before leaving the lab.

1. Report your results to the GST.
2. Obtain data from your colleagues for three other lengths, as well as the experimental specifications for the tester(s) that they used.
3. Clean up your area.
4. Check one more time that you've done everything you needed to.

4.1.5 After leaving the lab but before doing your report

Your recordings of the load were in terms of the mass placed on the tester. The relevant number, of course, is the axial force applied to the straw. Using your FBD and the physical measurements of your apparatus, do a static analysis of your tester and convert the mass load into the actual force load (in N).

You should then organize your results. For each straw length, create a table that reports the final load (in N) for each of the five straw trials. Arrange these in descending magnitude. Report also the average magnitude and the standard deviation. An example is shown below.

Straw length: xx cm

Trial 1	147 N
Trial 2	146 N
Trial 3	146 N
Trial 4	140 N
Trial 5	120 N
Average	140 N
Std. dev.	11.4 N

4.2 Straw analysis

The load that the truss can hold is related to the strength of the straws from which it is made. Hopefully this comes as no surprise to you! The goal of this straw analysis is therefore to obtain the best estimate of the buckling strength W for a given length l of straw from that data you have obtained. It is of course impossible to test all possible lengths of straw. Therefore, the best estimate of buckling strength will be obtained from testing a finite number of lengths, and then fitting a curve (a functional relationship between the strength and the length) to that data to obtain a functional relationship between buckling strength and straw length.

4.2.1 What YOU need to do

You have in your hands measurements from three lengths of straws (plus another three you so cleverly obtained from your colleagues – more on that in a bit). You will fit these three mean load values with a linear curve. Note that while in reality, the use of software is common place for doing curve fits, all your calculations here must be done by hand. (Why? Because we believe it is important to see and experience the details to better understand the technique.)

As your multiple trials revealed, your experimental data is uncertain due to a variety of reasons. In order to decrease this uncertainty, one can increase the number of different lengths tested, as well as the number of tests per straw length. This larger data set would allow you to obtain a better idea of the functional relationship between buckling strength and length. Of course, these two approaches are not equal in dealing with experimental uncertainty. **In your straw testing report, discuss which measurement (increasing number of tests per length, or increasing number of lengths tested) will result in a more certain estimate of buckling load versus length.**

Procedure for fitting and analyzing your data

1. Calculate the sample mean of the buckling load for each straw length according to

$$\text{Mean}_j = \overline{W}_j = \frac{1}{N}\sum_{i=1}^{N} W_i$$

   where j is an index running from 1 to 3, denoting the straw length, N is the total number of trials you performed at the length (which will be at least, and probably exactly, five), and W_i is the maximum load at that length in trial i.

2. Calculate the sample standard deviation for each mean according to

$$\sigma_j = \sqrt{\frac{\sum_{i=1}^{N}\left(W_i - \overline{W}_j\right)^2}{N-1}}$$

   where again j is an index running from 1 to 3 and denoting which straw length the data refer to.

3. Plot your sample means for each of the straw lengths you measured on a linear graph of buckling weight ($y-$axis) versus straw length ($x-$axis). This plot should be either drawn by hand to appropriate scale (graph paper is available on the course website) or using computer software (such as Excel or Matlab). Be sure to include appropriate axis labels, a legend, a figure number, and a descriptive caption. Furthermore, be sure to minimize the data range for the weights and lengths. In other words, do not draw a vertical axis spanning 200 N if all of the averages for your weight fall within a 20 N range. The smaller the range, the more accurate your fits for the graph will be. Fig. 4.2 shows an example of an acceptable plot, albeit from a completely different application.

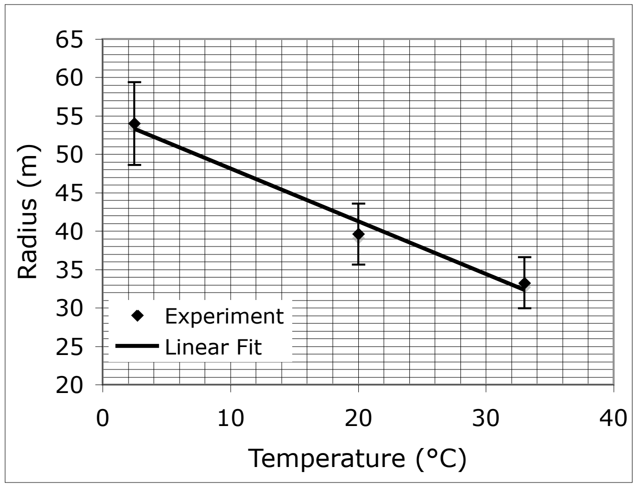


Figure 4.2: Example of an acceptable plot: Maximum expansion radius vs. temperature for a single bubble sonoluminescence experiment at 23.1 kHz.

4. Draw vertical error bars symmetrically about each data point. The error bar should extend above and below the data point by an amount equal to twice the sample standard deviation for that data point. (Recall that you calculated this above as σ_j.) This range corresponds to 95% of randomly distributed data falling within the limits defined by the error bars (known as 95% confidence intervals). You can see such error bars on the example plot in Fig. 4.2.

5. Fit a line to the data by drawing the line that visually minimizes the sum of the vertical distances between the points and the line. Determine the equation of this line by finding the slope and intercept. Report these values. Note that this line represents the best estimate of the buckling strength of any length of straw, given the data that you collected.

6. Estimate the error of the fit by determining the average vertical distance from your data points to the line. This error will have units of Newtons. In your report, discuss whether the error of the fit is small or large relative to the error bars. Discuss in your report whether it is more appropriate to use the error of the fit or the sample deviation of any single measurement when determining the failure load of your truss.

7. Plot at least three more load points for different lengths using the data you acquired from your colleagues (I did say we'd talk about these later. Later is now.) Plot these data points using a different symbol. In your report, discuss how well your fit describes their data and any reasons for the discrepancies you may find. Then, determine a new best-fit line using all six data points. In your report, discuss also the benefits of using all of the class data to determine a fit (using the results of your six-point fit as a guide).

4.2.2 FYI: What the Graduate Student Teachers (GSTs) (not you!) will do

Data analysis

For your reference, a detailed discussion on a curve fit analysis for the data and buckling strength is included in Appendix A. The GSTs will conduct an analysis of the full data set that was generated by the entire class based off this comprehensive curve fit analysis. They will use this large data set to fit the data to both the theoretical model (allowing the Young's modulus to be varied to find the best fit) and the semi-empirical model (allowing both the constant A and the exponent α to be varied to find the best fit). This information will be posted to Blackboard, with an indication of the best fit and its values.

Note that the GSTs' result will be much more accurate than yours since it is based on a much larger data set and a better model. Therefore, despite all your hard work in doing your own analysis and generating a linear model, for your truss design and failure analysis, **you must use the curve fit produced by the GSTs in all of the analysis of your trusses**.

Uncertainty analysis

Experimental expectation and historical data both agree that the curve fit will not accurately describe the data. The inherent nature of data, especially when working with a mass-produced product such as a plastic drinking straw, results in a deviation of results about a mean value. The GSTs will apply an uncertainty analysis that will account for the scatter in the data, resulting in a length-dependent uncertainty in the buckling force. This uncertainty is important to recognize and incorporate in your ultimate design analysis, since it will allow you to determine range of loads that your truss can reasonably be expected to hold before collapse. Pay close attention to the uncertainty results that the GSTs report after they analyze the data. You'll need these numbers when you design your truss.

4.3 Report on straw testing and analysis

Due in class on October 25th/26th.

The goal of this report is to describe your straw buckling experiment, present your data reduction and analysis, and give a discussion and interpretation of those results. For general guidelines on report writing, please see Appendix B.

The specific elements that need to be included in this report are as follows:

1. Introduction

   Describe the goal of the experiments and why you are doing them (which is not because you have to do it as part of this course...).

2. Procedure

   Describe the experimental setup and give the details on your tester, including the dimensions you measured and the number of the tester. Describe how your experiments were carried out.

3. Analysis

   Describe your method of analysis. Provide a FBD of a straw in the tester unit. Present a static equilibrium analysis of the testing machine based on that FBD. Note that it is OK to submit a revised FBD that differs from the one you showed to the GSTs during the testing. Describe your method of curve fitting.

4. Data

   You have data. Present it in a clear format. That almost certainly means a well thought-out table that includes the mass (in grams) on the tester just before buckling (not the failure mass), the size of the increment that caused buckling, the corresponding load applied to the straw (as derived from your statics analysis), and the average and standard deviation for each straw length. Organize your results for each straw in descending magnitude.

5. Results

   This is basically the plot described above.

6. Discussion

   Describe your results, the errors in your results, and how your results will impact your truss design. Answer all questions asked in this handout related to the straw testing and analysis. Provide a short commentary on any points of interest such as methods you used to cut the straws, methods for pinning, and other items that may be particularly useful to you when you design and build your truss.

7. Appendix

   The appendix should include the minutes of your case study discussion and group contract, and whatever additional information you feel is pertinent (additional sample calculations, etc).

This report will be graded according to the rubric given in Appendix C.

5 Computational analysis

The main thrust of this project is to design the best truss you can that meets or exceeds the specifications laid out in Section 2. This effort implies that you will need to analyze your truss designs to determine their maximum failure load. You have learned (or will learn soon) how to do this in the course of the course. You will also have learned that figuring everything out for even a small truss is no small task, as determining the forces in each of the members with a known load and known support reactions leads to m coupled linear algebraic equations, where m is the number of members in the truss. The trusses you design will almost certainly have more than ten members leading to essentially a big pain in the brain – to say nothing of design optimization and the large possibility of human error if the calculations are done by hand. Thankfully, they do not need to be done by hand. Instead, you will leverage your knowledge and experience in EK 127/8 to write a computational analysis code to do this for you. Matlab is highly recommended, but an alternative language, such as Python, is ok to use as well. Keep in mind you will be asked to submit a Matlab-formatted input file, as described below.

Why use Matlab? Matlab excels at solving systems of equations, particularly linear systems as in this scenario. The use of Matlab here will refresh your coding skills and is good practice for future courses and indeed your future career. Matlab may be found on the computers in the ME CAD lab (room 302 in 110 Cummington Mall), in the Ingalls Engineering Center, or online through your personal computer (access instructions can be found at `http://collaborate.bu.edu/engit/MatlabRemoteAccess`).

When building any piece of software, it is essential that you verify its correctness. This is especially true in this project since your design results rely entirely on your code. Therefore, you will need to verify the results of your program against a problem you solve by hand. A small problem will be assigned on the main class website. You will need to solve this problem by hand (on paper) and using your computational analysis code (again, Matlab, Python, or something similar). The results must, of course, agree. You will need to report both results in your Preliminary design report (see Section 6).

5.1 Program description

The purpose of the truss analysis is to determine the internal tensile force in each member and the support reaction forces. Here we assume that all applied loads are known and that the structure is a simple truss. Start off by numbering each of the J joints and each of the M members, in no particular order, along with the reaction forces S, as illustrated in Fig. 5.1. where $S_{x,1}, S_{y,1}$ are the unknown pin support reactions at joint 1, $S_{y,2}$ is

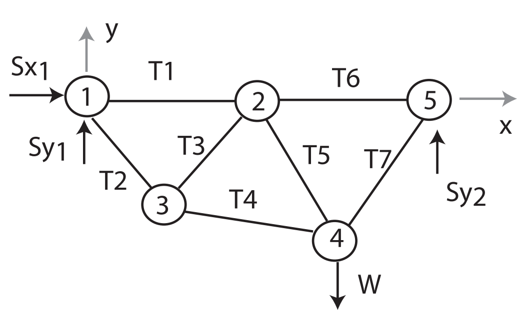


Figure 5.1: Illustration of joint and member numbering for a 5-joint/7-member truss

the unknown roller support reaction at joint 5, and W is an applied load (i.e., the weight on the relevant joint in your design).

There will be M+3 unknowns, M coming from the unknown forces in the members and three coming from the unknown reactions since there is one pin joint and one rocker joint. By doing an equilibrium analysis at all the joints (i.e. using sum of forces in x equals zero and sum of forces in y equal zero), we can obtain $2J$ equations. In order to solve for the member tensions in a simple truss, the number of equations should be equal to the number of unknowns. Of course, you are asked to build a simple truss in which the number of joints and the number of members is related by $M = 2J - 3$. How fortuitous.

5.2 Algorithm

The following algorithm consists of two stages:

1. A definition of the truss parameters: the joint locations, the member-joint connections, the reaction forces locations, and the load magnitude.
2. A construction of the system of equilibrium equations and solution for the unknown forces.

5.2.1 Defining the truss parameters

First we define a *connection matrix* **C**. This matrix has j rows and m columns, where the row represents the joint number and the column represents the member number (thus, the first column corresponds to member 1 and the first row corresponds to joint 1). We indicate the connection of member 1 to joints 1 and 2 by placing a '1' in column 1 of rows 1 and 2. If a given joint is not connected to a given member, we place a '0' in the matrix location that corresponds to the joint and member numbers. This can be summarized as:

$$\mathbf{C}_{j,m} = \begin{cases} 1, & \text{if member } m \text{ is connected to joint } j \\ 0, & \text{elsewhere} \end{cases}$$

The end result should work out such that the sum of each $column$ is 2, since each member is only connected to two joints. The sum of each row should equal the number of members attached to the corresponding joint. For the arrangement shown in Fig. 5.1 (ignoring member 6), we would have the following connection matrix:

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

Next we construct a connection matrix for the support forces along each axis, where $\mathbf{S_x}$ and $\mathbf{S_y}$ are matrices with j rows and 3 columns. Note that for our statically-determinate truss, supported by one pin and one roller joint, we will have a total of three unknown reactions. In each matrix, for each unknown reaction force, put a '1' in the column that corresponds to the joint j (there should be only a single entry of '1'; note that this is true in both matrices even though we know that for the loading conditions in this project, there are no support forces in the x direction) (and, another parenthetical note: be sure you understand and agree with the previous parenthetical remark!). For the example in Fig. 5.1 (where a pin and roller support is modeled), we get

$$\mathbf{S_x} = \begin{array}{c} \begin{array}{ccc} S_{x1} & S_{y1} & S_{y2} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}, \qquad \mathbf{S_y} = \begin{array}{c} \begin{array}{ccc} S_{x1} & S_{y1} & S_{y2} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

The truss is also defined by the location of the joints; to capture these we construct two location vectors $\mathbf{X}$ and $\mathbf{Y}$. Each has j elements corresponding to the relevant location of the jth joint; that is

$$\mathbf{X} = [x_1, x_2, x_3, x_4, ...],\ \mathbf{Y} = [y_1, y_2, y_3, y_4, ...];$$

Choose your reference frame carefully and then locate the position for the joints accordingly.

Finally, we define the load vector $\mathbf{L}$ which represents the known forces that act on each joint. This vector has $2j$ elements; the first j elements correspond to loads in the x direction for each of the j joints, and the last j elements represent loads along the y direction. In our example in Fig. 5.1, the vertical force $\mathbf{W} = -mg\hat{\mathbf{j}}$ for the load is placed on joint 2. In a standard force equilibrium equation, the force $\mathbf{W} = -mg\hat{\mathbf{j}}$ would be combined with the other vertical forces to equal zero. Once this known force is moved into the $\mathbf{L}$ matrix (see Eqn. 5.1 below), its sign is flipped and becomes positive. Therefore

$$\mathbf{L} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ mg \\ 0 \end{bmatrix} \begin{array}{l} \text{no horizontal load at J1} \\ \text{no horizontal load at J2} \\ ... \\ ... \\ ... \\ \text{no vertical load at J1} \\ ... \\ ... \\ \text{load } mg \text{ at J4} \\ ... \end{array}$$

5.2.2 Constructing the equilibrium equations

We would like to construct the equilibrium equations using only the matrices and vectors described above. We can achieve this by using the method of joints of determine the forces at each joint j. To describe the method, we start with joint 1 in Fig. 5.1. Its FBD is detailed in Fig. 5.2.

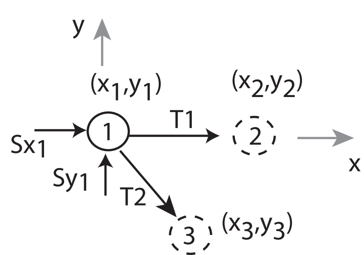


Figure 5.2: Illustration of the method of joints at joint 1.

Summing the forces in the x–direction, we get

$$\begin{aligned}\sum F_{x,1} \quad : \quad & \left(\frac{x_2 - x_1}{r_{1,2}}\right) T_1 + \left(\frac{x_3 - x_1}{r_{1,3}}\right) T_2 + 0 \cdot T_3 + 0 \cdot T_4 \\ & +0 \cdot T_5 + 0 \cdot T_6 + 0 \cdot T_7 + 1 \cdot S_{x,1} + 0 \cdot S_{y,1} + 0 \cdot S_{y,2} = 0\end{aligned}$$

where $r_{1,2}$ is the distance between joint 1 and 2 and the overall fraction is the x-axis unit vector for that force. Pay close attention to the direction along which you are defining the tension force! The rest of the joints would be considered by working out the equilibrium equations along the x axis, resulting in a total of j equations along the x–axis. For the arrangement in Fig. 5.1, the second equation would represent $\sum F_x$ for joint 2, and so on.

After considering the x–axis, we move on to the y–axis. The sum of the y–forces at the first joint is given by

$$\begin{aligned}\sum F_{y,1} \quad : \quad & \left(\frac{y_2 - y_1}{r_{1,2}}\right) T_1 + \left(\frac{y_3 - y_1}{r_{1,3}}\right) T_2 + 0 \cdot T_3 + 0 \cdot T_4 \\ & +0 \cdot T_5 + 0 \cdot T_6 + 0 \cdot T_7 + 0 \cdot S_{x,1} + 1 \cdot S_{y,1} + 0 \cdot S_{y,2} = 0\end{aligned}$$

Note that this is written in a general form given the position of the joints. For example, the first term in the equation $(y_2 - y_1)$ is clearly zero since those two joints are on the same horizontal line.

The goal now is to use linear algebra to solve of the unknown forces by separating the system of linear force equations into three matrices, $\mathbf{A}$, $\mathbf{L}$, and $\mathbf{T}$ where

$$\mathbf{A} \times \mathbf{T} = \mathbf{L} \tag{5.1}$$

and $\mathbf{A}$ is a matrix that should be populated by the coefficients of the force for the respective member tension at each joint, starting with the forces along the x axis for rows 1 to j, and finishing with the forces along the y axis for rows $j + 1$ to $2j$. It should be $2j$ rows and m+3 columns, where the last three columns are the $\mathbf{S}_x$ and $\mathbf{S}_y$ matrices. That is,

$$\mathbf{A} = \begin{array}{c} \begin{array}{|ccccccc|ccc|} m_1 & m_2 & m_3 & m_4 & m_5 & m_6 & m_7 & & \mathbf{S} & \end{array} \\ \left[\begin{array}{cccccccccc} \frac{x_2-x_1}{r_{1,2}} & \frac{x_3-x_1}{r_{1,3}} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{x_1-x_2}{r_{1,2}} & 0 & \frac{x_3-x_2}{r_{2,3}} & 0 & \frac{x_4-x_2}{r_{2,4}} & \frac{x_5-x_2}{r_{2,5}} & 0 & 0 & 0 & 0 \\ 0 & \frac{x_1-x_3}{r_{1,3}} & \frac{x_2-x_3}{r_{2,3}} & \frac{x_4-x_3}{r_{3,4}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{x_3-x_4}{r_{3,4}} & \frac{x_2-x_4}{r_{2,4}} & 0 & \frac{x_5-x_4}{r_{5,4}} & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \\ - & - & - & - & - & - & - & - & - & - \\ \frac{y_2-y_1}{r_{1,2}} & \frac{y_3-y_1}{r_{1,3}} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \frac{y_1-y_2}{r_{1,2}} & 0 & \frac{y_3-y_2}{r_{2,3}} & 0 & \frac{y_4-y_2}{r_{2,4}} & \frac{y_5-y_2}{r_{2,5}} & 0 & 0 & 0 & 0 \\ 0 & \frac{y_1-y_3}{r_{1,3}} & \frac{y_2-y_3}{r_{2,3}} & \frac{y_4-y_3}{r_{3,4}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{y_3-y_4}{r_{3,4}} & \frac{y_2-y_4}{r_{2,4}} & 0 & \frac{y_5-y_4}{r_{5,4}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{y_2-y_5}{r_{2,5}} & \frac{y_4-y_5}{r_{5,4}} & 0 & 0 & 1 \end{array}\right] \end{array} \begin{array}{l} \text{x components} \\ - \\ \text{y components} \end{array}$$

The magnitudes of the unknown forces, $\mathbf{T}$, are defined in a matrix consisting of one column and m+3 rows:

$$\mathbf{T} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ S_{x,1} \\ S_{y,1} \\ S_{y,2} \end{bmatrix}$$

The $\mathbf{L}$ matrix was defined above. Since we are interested in solving for the unknown forces in $\mathbf{T}$, but know the values that populate $\mathbf{A}$ and $\mathbf{L}$, we can invert Eq. 5.1 to solve for $\mathbf{T}$ to get

$$\mathbf{T} = \mathbf{A}^{-1}(\mathbf{L}). \tag{5.2}$$

Important note: Even if there is no loading on the truss along the x-axis, as will be the case for your straw truss load, you must make sure to still represent the $S_{x,1}$ constraint in the matrix calculation. Failure to do so will prevent your matrix from being invertible. Also, an analysis and subsequent inversion based solely on the truss in Fig. 5.1 will fail because it's not a complete and fully-supported truss (only one support was modeled).

5.2.3 Maximum theoretical load

One aspect of the analysis that you'll have to grapple with is how to accurately determine the failure load of your designs. One method could be to ramp up the test load until one of the straws is at its maximum load (as determined by the straw testing analysis and the joint-to-joint lengths). Adjusting that load up by hand is an inefficient and inaccurate way to find that max load – – that's just the concept behind it.

A better way would be to use the fact that the truss internal forces are all linear. If you apply a load of 1 N and analyze all the internal forces and then double the applied load to 2 N, all the internal forces will also double! We can use this to our advantage by applying the 1 N load and then calculate a scaling ratio SR of compression force to buckling load for each of the members in compression:

$$SR_i = F_{internal,i}/F_{buckling,i}$$

The member with the largest ratio SR is the one that will fail first. Furthermore, this ratio can then be used to calculate the maximum theoretical load for the truss, since

$$F_{failure} = \frac{\text{1 N load}}{SR_{max}}$$

5.2.4 Verification of computational analysis

The most efficient way to analyze (and optimize) multiple designs is to write a central analysis code and set it up to accept an input file that contains the details for the truss design (e.g., the C, Sx, Sy, X, Y, and L matrix values) and output the results to the command line or a separate file. Additionally, it's in your best interest to know whether your code is valid: the basis of your eventual test results depend on this code. To this end, the GSTs will verify whether your results are correct, and will provide feedback on your preliminary report to alert you of any mistakes. In order to do this efficiently, it is a requirement that you email the details for one of your preliminary truss designs to your GST, with these details described in a Matlab input file.

Input file

With all this in mind, set up a general analysis code that will accept an input file with the following input parameters and their values:

1. The connection matrix **C**; (i.e.: C = [1 1 0 0 ... 0; 1 0 0 0 ... 0; ...];)
2. The S_x matrix of reaction forces in the x-direction;
3. The S_y matrix of reaction forces in the y-direction;
4. The joint location vectors X and Y;
5. The vector of applied external loads **L**.

Save these parameters to a .mat file, using the following syntax in Matlab:

```
save('TrussDesign1_MaryJoeBob_A1.mat','C','Sx','Sy','X','Y','L')
```

NOTE: Your input file MUST be in the .mat format (NOT PDF, Word, .dat, .txt, etc!). Failure to format this correctly will result in a loss of points due to the difficulty in checking your file to give you the necessary feedback. See your instructor or any of the GSTs or LAs for assistance if necessary.

Output format

The output from your code should display your group member names and course section identifier, along with the applied load, truss cost, and load/cost ratio for the truss. Your code should print out the identifier for each member, and the magnitude of the member's load with a (T) if in tension or a (C) if the member is in compression. It should also list the label for the reaction force, followed its value. For example, if the load you apply is 1 N, and the members have been designated as m1, m2, ... m15, the output should appear as:

```
\% EK301, Section A1, Group 1: Billy B, Suzy Q, 2/15/20xx.
Load: 1 N
Member forces in Newtons
m1: 0.991 (C)
m2: 0.273 (T)
. . .
m15: 0.827 (T)
Reaction forces in Newtons:
Sx1: 0.0
Sy1: 0.75
Sy2: 0.25
Cost of truss: $319
Theoretical max load/cost ratio in N/$: 0.0031
```

5.2.5 Matlab script

E-mail your Matlab .mat input file (one per group) to the course gmail account (ek301docs@gmail.com) by the preliminary report deadline so that your program and its output can be verified using our analysis code. The subject title (and body) of your email should include your: course section identifier ('Section A1') and your group member's names. You only need to send this information for the **preliminary report**, not the final report.

6 Preliminary designs

This project specifies a set of somewhat conflicting requirements for the truss. In general, stronger trusses are more expensive. There is not really a "best" solution and thus you will need to balance strength, cost, and load-to-cost ratio. The primary goal of the preliminary design phase is to explore (at least) two different options to get a sense of how the different design decisions affect the results. All results in this preliminary design report should be based on the joint-to-joint length (you cannot use the actual straw lengths because you are not building the truss... yet.). Each truss should be fully analyzed, a process that includes the following:

1. **Prediction of the force in each member for a given vertical load given at the specified load joint.**

   This prediction should be computed using your analysis program. It should include not only the magnitude of the force in each member but also whether that member is in tension or compression and the uncertainty of the force (to 2 significant figures).

2. **Determination of the member to buckle first.**

   This should be calculated based on the class average best-fit nonlinear relationship between buckling load and straw length. This should be based on the *joint-to-joint* length. (Keep in mind, however, that the *actual load* the straw can support is determined by its actual length.) Report the member, its length, its predicted buckling strength, and the uncertainty in that strength.

3. **Specification of the maximum load that the physical truss could support.**

   Report the maximum load and the uncertainty in that load. To calculate this last item, use the uncertainty in the strength of the member that will buckle first, expressed as a percentage uncertainty. Then multiply your maximum load by this percentage to get the uncertainty.

4. **Calculation of cost and theoretical load-to-cost ratio.**

   The cost of the truss should be calculated based on the joint-to-joint length. Please note that you are not actually paying any money; this is an artificial cost from an artificial formula. The formula (given in the Specifications section), however, captures some real engineering considerations. For example, the number of joints is weighted heavily, partly because joints are expensive to fabricate but also because it is a reflection of the number of shorter straws that must be used. The formula also depends on the total strength length; this of course represents the total amount of material needed.

   The load-to-cost ratio is exactly that: the ratio of the maximum load to the cost for the truss.

It is strongly suggested that you develop an automated means for calculating the member to fail, the cost, and load-to-cost values using Matlab, Excel, or similar. This will allow you to easily compare different designs. To encourage design optimization, the load and load-to-cost ratio will be graded relative to the performance of the final design for each group in your section.

6.1 Preliminary design report

Due to ME front office on November 17th by 4 p.m.

The goal of this report is to describe and discuss your two preliminary designs. Follow the general guidelines on report writing described Appendix B but be sure to address the specific elements described below. The grading rubric can be found in Appendix D.

1. Introduction

   Describe the motivation for your preliminary designs, the rationale for using a computer program to analyze a truss, and how you plan on using the results of the analysis for the remainder of the project.

2. Procedure

   Describe also any procedures you used for coming up with your designs.

3. Analysis

   Describe your computational approach. Demonstrate that you can use a computer to correctly analyze a truss by working the problem assigned on the web site in two ways,

   (a) by hand (Note that it is OK to submit a handwritten solution so long as it is neatly presented.)

   (b) by your computational approach. Include a print-out of the code output and of the final result

   The results from the two methods must agree. Resolve any discrepancies before submitting your report.

   Describe your uncertainty analysis and any other relevant methods you used.

4. Data

   Data in this portion of the project includes the diagram of each design with all members and joints clearly labeled in a fashion consistent with the output of your software. Highlight the member that will buckle first. It also includes the output of the code with the load in each member. This is probably best reported using a table that includes, for each member:

   (a) member number (consistent with the design drawing)

   (b) Joint-to-joint theoretical member length

   (c) Whether in tension or compression

   (d) Buckling strength and uncertainty based on the joint-to-joint length and the class average buckling strength fit and uncertainty of the buckling strength. These parameters should only be stated for members that are in compression.

   (e) Magnitude of the force at theoretical maximum truss load

5. Results

   The results include the critical member, its length and buckling strength and uncertainty, the maximum theoretical load and uncertainty, truss cost, and load-to-cost ratio for each design.

6. Discussion

   Discuss your results, comparing the two designs. Draw some conclusions about which is the better design and why as well as how you might further improve your designs.

7. Separately, make sure to email your Matlab input file to ek301docs@gmail.com by the due date for this report, specifying your section number in the email Subject heading.

7 Final design

The focus of this effort is to show the full details of your final truss design. This design can be an entirely new plan or an optimization of one of your earlier designs. After completing your preliminary design report, we will report to you the top designs in your section in terms of load and load-to-cost, and you should use this information to optimize your truss. Since your grade is based in part on how your design stacks up against other designs in your section, it is worthwhile to spend some time trying to improve your design. Of course, your design will not likely be the best in all categories and you'll need to consider various design tradeoffs. Eventually you need to settle on a final design and provide a full analysis as in the preliminary design phase.

7.1 Modeling details

Maximum actual load

Another critical aspect to address in the computational analysis is best illustrated by referring back to Figure 2.1. Notice that there are *two* relevant straw lengths: the joint-to-joint length and the actual straw length. Think for just a minute: which one of these lengths determines the **load** in the members?

Did you think about it and try to answer this? Please do so! (Really, it does help with understanding.) We'll wait...

The answer is that it depends on the context. If you're only simulating a truss and not building it, then only the **joint-to-joint** length ($l_{joint-to-joint}$) matters in determining the load; this is the length we've used in our class analysis up to this point. When it comes to building a truss, the strength of the straws, however, is determined entirely by their actual length (l_{actual}). Here we need to be careful with our terminology. For the rest of the manual, $F_{theoretical}$ will refer to the failure load when the analysis only accounts for the joint-to-joint lengths. $F_{theoretical}$ under-predicts the actual failure load. Why under predict? The reason is that shorter straws are stronger than longer straws and the actual straws are all shorter than the joint-to-joint lengths. You therefore need to adjust your theoretical failure load based on the ratio of the joint-to-joint length and the actual length of the straw that's going to fail to find the actual failure load F_{actual}.

That adjustment will likely differ for each member, since it's dependent on its individual length and therefore will depend on the actual straw fit. The straw buckling strength fit will be posted on Blackboard (so make sure to use this value once it's posted!), but will take the form of

$$F_{fit}(l) = Cl^{-\alpha} \tag{7.1}$$

where C will be a fit coefficient, l is the straw length, and α will be a power law coefficient. The correction factor that you should make will depend on the value for α:

$$F_{actual} = \left(\frac{l_{jj}}{l_{actual}}\right)^{\alpha} F_{theoretical} \tag{7.2}$$

Note that the failure mode of this scenario is for buckling, which only occurs for members that are in a state of compression. Thus, you should only conduct this analysis for members that are under compression.

7.2 Final design report

Due to the ME Front office (Room 101, 110 Cummington Mall) on December 8th by 4 p.m.

The report should be in the style of a technical report. The grading rubric can be found in Appendix E. It should include (only) the following specific elements:

1. Introduction

   Describe your motivation and approach for your final design; did you focus on cost? on max load? on load-to-cost ratio? on a combination?

2. Procedure

   Describe any changes you made in the design procedure you came up with for the preliminary designs. (If you have not made any changes, you can simply note that.) Describe your plans for construction.

3. Analysis

   Note any changes from your analysis used in the preliminary designs. As before, if you have not made any changes then you can simply note that. Be sure to factor in Equation 7.2 where appropriate!

4. Data

   Include your final design drawing with all members and joints clearly labeled in a fashion consistent with the output of your software. Highlight the member that will buckle first. Include the same type of table as you did in your preliminary design report listing the output of your analysis code.

   In addition, add columns to that table for the following parameters:

   (a) Actual straw length for each member
   (b) Maximum supported load and uncertainty, using the actual straw length
   (c) An ”engineering intuition” adjustment to that buckling strength. This adjustment is intended to account for the quality of your construction, the quality of your particular materials, your past experience with the straws, previous lives as a builder of trusses, effects due to phase of the moon and the color of your best friend’s pet fish, and so on. Keep in mind that this adjustment is purely optional and could be up or down, but should definitely be *small*.
   (d) Force in the member for the predicted actual maximum truss load for the constructed truss.

   Of course, to get some of those numbers you will need to at the very least have cut your actual straws and, preferably, have constructed your final truss.

5. Results

   Report the following:

   (a) The internal force and length of each member
   (b) The buckling load and uncertainty of all compressive members
   (c) Clearly identify the critical member
   (d) The maximum load, truss cost, and load-to-cost ratio for both the theoretical and actual analyses
   (e) The adjusted maximum load and its uncertainty (but don’t worry about factoring in a range for the load-to-cost ratios based on the uncertainty).

6. Discussion

   Discuss the rationale for your design, including how you optimized it. Describe how your design evolved and what design decisions you made along the way.

   Discuss your (intended) building technique.

   If you decided to adjust your load, provide a discussion of the adjusted estimate and how you chose this adjustment. What is it capturing?

7. Appendix: Meeting minutes of the Hartford roof collapse discussion

   On the course web site you will find assigned reading on the Hartford roof collapse. Your team should have a meeting to discuss these readings. In your discussion, pay special attention to the use of computer programs in analysis and design. Discuss also an appropriate safety factor to be used on your own truss if it were to be used in a case where human life is at risk and report what maximum load you would claim for your design. Consider variability in materials and construction and any other factors you deem relevant. Be sure to keep in mind the Fundamental Canon No. 1 from the Code of Ethics of Engineers, National Society of Professional Engineers, 1997: "Engineers shall hold paramount the safety, health, and welfare of the public in performance of their professional duties."

   During this meeting you should assign someone to take minutes; those minutes should be submitted in an appendix to the final report. The minutes should adhere to the following guidelines.

   (a) The date, time, and place of the meeting should be recorded along with the names of the participants. The (acting) chair of the meeting and the recorder (minute taker) should be identified.
   (b) The planned agenda should be attached.
   (c) The important points should be summarized, along with who suggested each of them.
   (d) The conclusions should be recorded and any action items listed, with the responsible person identified.
   (e) The date, time, and place of the next meeting should be given, if schedule, or the disposition of the group require. For example, a date for when a decision as to a follow up meeting will be made or a decision that no further meetings of this group are needed. (Note that one meeting is sufficient for this exercise!)

8 Construction and testing

This is arguably the most fun element of the project. I mean, analysis is great, design is interesting, but building something and then breaking it? Now that's fun!

The testing apparatus is shown again in Fig. 8.1. The truss will be placed between the two horizontal bars of the apparatus (only the rear bar is shown in the figure). A pin will be inserted into the center of the joints at each end of your truss. One of these pins will be placed in a 'v' groove, creating a pin joint. The other will rest on a smooth surface, creating a rocker joint. Adjustable vertically oriented support rods are provided to keep the truss from tipping over. Pins will be inserted through the center of the load joint and a loop of string will be attached. The weights will be hung on this string.

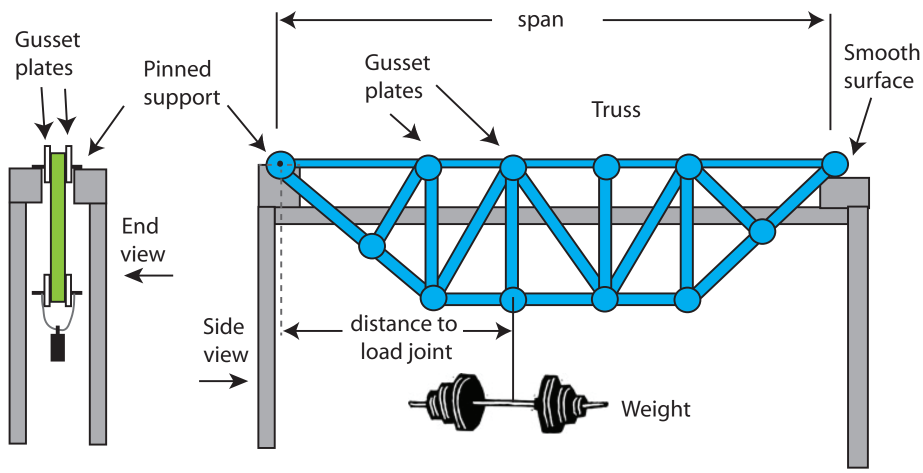


Figure 8.1: Cartoon of the testing apparatus

8.1 Construction

The accuracy of your prediction and the performance of your truss is highly dependent on the quality of its construction. Key points to keep in mind:

- all straws at a joint should intersect at a common point (a moment will result, otherwise)
- the entire construction should be planar (the truss will twist, otherwise)
- the truss should conform to all specifications (locations of pins, lengths, cost, etc.)
- Trim the gusset plates so that the pins have enough physical material to act upon but that there is a minimal amount of overlap with the foam material and the straw. When you surround the straw with the 2 plates, allow for a millimeter-size gap between the foam and the straw.
- You will have 3-4 foam pieces that are thicker than the standard pieces. Use these thicker pieces for one (only!) of the gusset plates on the support and load joints to keep the pins from ripping through the foam.

Finally, once it's built, **measure** your truss to make sure it is within the specifications for the support joint span and the load joint location. If it's too long, check your straw lengths to make sure they're shorter than the joint-joint distance and also check to make sure your straws aren't spaced too far apart at each joint. Deviation from your design will result in an inaccurate test result, relative to your design analysis.

8.2 Testing

Testing will occur on December 9th. Your entire team must be present for the actual testing. If you have a legitimate conflict on this day, see your instructor as soon as possible to make alternative arrangements.

When you show up for testing, your truss will be inspected by a GST or instructor. The design requirements (overall length, minimum straw length, etc.) will all be verified. Note that if these requirements are not met you will not be allowed to test your truss (and will thus receive a zero on this portion of the project). The overall quality of the build will be evaluated. The truss will then be loaded into the testing apparatus.

You will begin by adding the minimum weight. This weight must be successfully supported for a full minute. At that point, you may add additional weight in whatever increment you like. Each new weight must be held for 5-10 seconds. Failure is defined as either a member (or members) buckling or a sag of a portion of the truss by more than 6 cm below the end joint line. The buckling load is the last successfully supported load, not the one that actually causes failure. Since your final grade depends in part on how close your actual load is to your prediction (within the uncertainty bounds – this is why the uncertainty is so important!), you will want to go in small increments as you get close to your theoretical limit.

Trusses can hold either less or more weight than predicted (or, rarely, exactly as much as predicted). Once you reach your predicted load, the natural inclination is then to throw on as much additional weight as possible to force a failure. Unfortunately, this does not give a true representation of the strength of your truss. Thus, each additional increment after the first can be no larger than the previous one.

Items to remember for testing day:

1. Make sure you have your group members' phone numbers! Important in case they're still asleep during your testing timeslot...
2. Bring your completed truss and extra straws and pins. If your truss is incorrectly built you may need to perform an on-the-spot adjustment!
3. Make sure your gusset plate geometry and joint locations are correct (see Fig. 2.1). Remember that the joint is measured from the point in space where the straws would intersect, and has nothing to do with the edge locations of the gusset plates.
4. Trim your gusset plates to be as small as possible. Large gusset plates may induce out-of-plane contact forces on the straw that may cause your truss to fail at a lower load! Just make sure that the pins have enough foam material to provide sufficient constraint on the pin.
5. Fill out a truss test sheet (one per group) and your peer evaluation (one per person). These will be available for download off the course website and the non-grey boxes should be filled out in advance.
6. Determine your failure load and uncertainty in terms of equivalent mass, for straightforward reference during testing.

9 Grading

A numerical grade with a maximum of 100 points will be assigned via the following proportions.

1. Straw testing and report: 20%
2. Preliminary design report: 30%
3. Final design report: 20%
4. Calculated theoretical load and load-to-cost ratio: 15%
5. Accuracy of maximum load prediction and failure member: 15%

Your grade on the theoretical load, cost, load-to-cost ratio, and accuracy of your maximum load prediction will be determined **in comparison to the other teams in your section**. In each category, the team with the best result will get maximum credit with the other teams getting lower credit based on how close they are to that maximum (and yes, if you are very close to the maximum you will also get full credit. A one gram difference out of a 1 kg load, for example, is not significant). We will report the maximums in each category after the preliminary design phase so that you may choose to redesign your truss.

Note that your final design must meet all the specifications. Failure to come up with a design to meet those specifications will lead to a maximum grade of 60% on the project.

The different grading rubrics are given in the appendices.

Acknowledgements

This project was originally conceived and developed by M. Isaacson and S. Grace in 1996. Major revisions over the years have been done by, among others, S.B. Andersson, C. Farny, S. Grace, G. Holt, M. Isaacson, T. Melamed, T. Murray, J. Sullivan, A. Tomboulides, and D. Wroblewski. There have probably been others as well.

Date of this (the latest and greatest (?) revision): September 27, 2017

A About fitting a curve to data

There are several different techniques for fitting a curve to a data set, depending on how much information and knowledge you have about the system you were testing. Below we discuss three approaches. The first, an empirical fit, is the one **you** will use in your analysis. The second and third fit types will be investigated by GSTs, where the second is a fit to a functional form that is based on a theoretical model of the straw. The final is a semi-empirical fit, that relaxes the theoretical relationship to allow for a better fit.

Empirical fitting

This method is perhaps the most straight-forward to apply. It is not, however, based on a physical theory that relates buckling strength and straw length. This method relies on the belief that a physical relationship does exist and that the data will reveal this fit. One method to display this relationship is to use a polynomial function given by

$$W(l) = a_0 + a_1 l + a_2 l^2 + a_3 l^3 + \cdots + a_m l^m$$

where m must be smaller than the number of data points in your data set.

Since you have only three data points to work with, it is most appropriate to select a *linear* fit, so ignore the nonlinear terms and fit the data with the linear fit equation.

Fit to a theoretical curve

Since the GSTs will have the entire set of class data to analyze, they have a basis for investigating a fit that better matches buckling theory. A treatment of the straw as a long, thin, and massless rod yields the relationship for the buckling strength W as a function of the straw length l to be

$$W(l) = \frac{\pi^2 EI}{l^2}.$$

Here E is a material property known as Young's modulus, I is the moment of inertia, and l is the length. The moment of inertia and the length are easily determined from the straw geometry but E is not easily found. It can therefore serve as an adjustable parameter to find the best fit.

Semi-empirical fit

The equation relating the buckling strength to the straw length given above is theoretically justified. However, the straws may not be kind enough to obey our theoretical derivation (perhaps due to material imperfections, experimental error, etc). A better fit may be achieved by selecting a function motivated by the theory but which can be adjusted based on the measured data. For example, recognizing that the theory predicts a buckling strength inversely proportional to some power of the length, we could define

$$W(L) = \frac{A}{L^\alpha}$$

where both A and α are to be chosen to get the best fit.

B Guidelines on writing reports

For all reports, proper format and proper English usage are required. All reports must be clear and user friendly. Each deliverable should be interpreted as one per design group. Be sure to include your section number on the title page, along with your team and member names. All of your reports for this course must be at least two pages long, though you should find that they will be longer than that. Note that the final design report only needs to include the sections that are specified in Sec. 7.

General guidelines to follow include the following.

1. Do not write in the first person.
2. Include all experimental data you collect.
3. Show all equations used and define all variables.
4. Typeset your equations (rather than writing them by hand).
5. In fact, typeset just about everything. The only exceptions are handwritten meeting notes or hand-drawn schematics.
6. Be consistent with your you name and number your plots, figures, and equations throughout the text.
7. Cite all references
8. Proofread for typos, spelling, grammar, formatting, and general writing flow. This is especially important when multiple authors are involved.
9. Don't bother with using a special plastic binder for your completed report. A single staple is easier to handle.

A comprehensive report should include the following:

1. Cover page

   This should contain the title of the report, the name of the group, the names of the members of the group, the section, and the name of the professor of the section.

2. Table of contents

   A listing of the sections of the report and the pages on which those sections begin. (You have opened your textbooks, right? They have tables of contents that you can use as examples.)

3. Introduction

   This explains the purpose of the report/lab/testing that was done. Discuss what you intended to learn from the experiment/test/analysis/design and how you plan on using what you have learned.

4. Procedure (if applicable)

   This describes what you did to gather your data. You should describe any experimental apparatus in detail.

5. Data

   Present the data in tabular form. Do not forget to include units! Be sure to include a brief description that orients the reader to the source of the data.

6. Analysis

   Describe the analysis that you performed on the data. Show all equations with all symbols defined. Show an example for each unique calculation (e.g. sum of moments, standard deviation, mean buckling load, etc.). References any figures (such as FBDs), data tables, or tabular presentations of your results as needed.

7. Results

   Present your results in tabular and/or graphical form in a way that is easy for the reader to follow and interpret. Label and reference each table and graph.

8. Discussion

   Answer any questions that you have been asked to answer in the lab/project manual. Discuss your results. Do they make physical sense? Do they help you accomplish the goals you set out in your introduction? Discuss the uncertainty in your data and the sources of error.

9. Conclusion

   What did you learn? How will the results be used? What might you suggest doing differently next time?

10. Appendices

    Attach any hand calculations, computational code, extra data tables, alternative designs you considered but didn't use, and other relevant information that are important but did not make their way into the main report. Be sure to reference your appendices at some relevant point in your report. (Notice, for example, that this appendix was referenced earlier in this document.)

C Straw report grading rubric

Category	**Percentage**
Writing quality and overall presentation	15
Data presented in a clear manner (such as a table)	10
All units specified	5
FBD drawing provided	10
Correct FBD analysis	15
Uncertainty estimate calculated	10
Graph has axis labels, legend, etc.	5
All data points (your three, plus three others) drawn	5
Error bars drawn	5
Line fit drawn	5
Dominant error sources identified	5
General discussion error given	5
Discussion of data and fitting given	5
Group member contract & case study minutes attached	5

D Preliminary design report grading rubric

Category	Percentage
Writing quality and overall presentation	15
Rationale for using a computer program	10
Solution of assigned problem (by hand and via computer)	15
Design 1 diagram (consistent labels, legible diagram)	5
Design 1 max load calculation and accuracy	5
Design 1 truss cost and load-to-cost ratio and accuracy	5
Design 1 critical member correctly identified with length and buckling strength	2
Design 1 error analysis	8
Design 2 diagram (consistent labels, legible diagram)	5
Design 2 max load calculation and accuracy	5
Design 2 truss cost and load-to-cost ratio and accuracy	5
Design 2 critical member correctly identified with length and buckling strength	2
Design 2 error analysis	8
Discussion of the best truss	10

E Final report grading rubric

Category	Percentage
Writing quality and overall presentation	10
Cover page exists, lists team members with ID numbers, course section, and date	2
Introduction: explanation of motivation and approach	10
Final design drawing given with all members and joints labeled and all lengths given	10
Table of results with accurate theoretical and actual calculations	15
All calculations and analysis of critical member correct	21
Uncertainty analysis given	5
Discussion of experience-adjusted estimate	2
Discussion of design rationale and optimization approach	15
Minutes of Hartford meeting (date, time, attendees, agenda, summary, conclusions)	10