Engineering @ SP

ET0053

CIRCUIT THEORY & ANALYSIS

(Version 1.10)

School of Electrical & Electronic Engineering

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For any queries on the notes, please contact:

Name: DANNY QUEK
Room: T15720
Email: Danny_QUEK@sp.edu.sg
Tel: 67721257

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T18A

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T12/T12A level 2

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Industry Co-location Lab : Infineon, LG, SSIA

T12 level 6

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Facilities : Biomedical Engineering Lab
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T8 level 2 & T12 level 1

Smart Urban Transportation Hub

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Industries : Smart Transport, Land Transport

Industry Partners : ST Engineering Land Systems, SMRT, SIIX, SUTD, GreenPower

Industry Co-location Lab : Land Transport Authority, SIEMENS Ingenuity for life

CONTENTS

MODULE OVERVIEW

1 Introduction

This is a second year module for Diploma in Aerospace Electronics, Diploma in Electrical & Electronic Engineering, Diploma in Engineering Systems and Diploma in Energy Systems and Management courses. It is also offered as a third year module for Diploma in Engineering with Business course. It is a continuation of the first year module on Principles of Electrical and Electronic Engineering.

2 Module Aims

This module will provide students with an understanding of circuit theory, which includes mesh analysis, nodal analysis, circuit theorems and applications. The student will also be introduced to three-phase circuits, covering three-phase supply and loads.

CIRCUIT ANALYSIS

Keywords: Voltage Source, Current Source, DC, AC, Current, Voltage, Resistance, Reactance, Impedance, Admittance, Ohm's law, Kirchoff's Current Law (KCL), Kirchoff's Voltage Law (KVL), Star Connection, Delta Connection, Star-Delta Transformation, Delta-Star Transformation, Thevenin's Theorem, Thevenin Equivalent Circuit, Norton's theorem, Norton Equivalent Circuit.

Objective: Student should be able to analyse any given DC and AC electrical circuitry. They should be able to:

1.1 Explain the operating characteristics of a voltage source.

1.2 Explain the operating characteristics of a current source.

1.3 Explain the conversion of voltage and current sources.

1.4 Describe series connection of voltage sources and apply this principle to find the equivalent circuit.

1.5 Describe parallel connection of current sources and apply this principle to find the equivalent circuit.

1.6 Analyse the given circuitry using mesh analysis method by way of writing matrix equation by inspection and solving.

1.7 Analyse the given circuitry using nodal analysis method by way of writing matrix equation by inspection and solving.

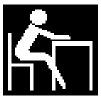
1.8 Apply star to delta and delta to star transformation to simplify the given circuitry.

1.9 State and apply Thevenin's theorem to solve any given DC and AC circuits by finding the equivalent circuit.

1.10 State and apply Norton's theorem to solve any given DC and AC circuits by finding the equivalent circuit.

1.11 Explain the relationship between Thevenin And Norton equivalent circuits.

Number of lecture hours required for completing this topic: 12 ***hours*.**



Follow up activity to be completed by students: ***Tutorial 1, 2, 3 and 4.***

Practical exercise to be completed by students: ***Experiment 3,4,5 and 6.***

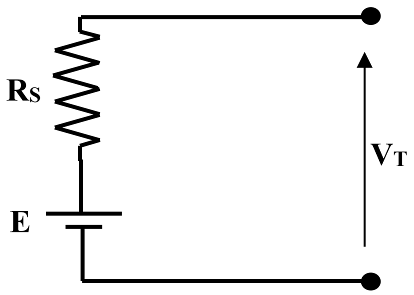
1.0 Introduction

Circuit analysis deals with four basic manifestations of electricity, namely electric charge, magnetic flux, electric potential (or voltage) and electric current. Here we assume that the reader has some familiarity with these concepts and merely state the basic relationship among them. The relationship between the voltage and the current of circuit elements depends entirely on the nature of the element. The circuit elements namely resistors, capacitors, inductors, voltage and current sources are primarily encountered in electrical network. In this chapter, we mainly study the behaviour of these elements when connected into various configurations to form what is called a network or a circuit. Further we also analyse these circuits by finding the voltages across or the current through any circuit element by applying Mesh, Nodal analysis and network theorems, namely, Thevenin's theorem and Norton's theorem.

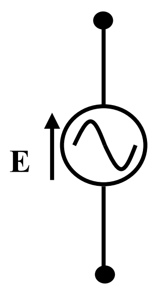
1.1 Constant Voltage Source

A constant voltage source is an ideal source element capable of supplying any current at a given voltage.

The diagram in Figure 1.1 shows a DC voltage source with an emf of E volts and an internal resistance of R_S ohms. E denotes a constant perfect source, independent of the current taken. The terminal voltage V_T depends on the current drawn.

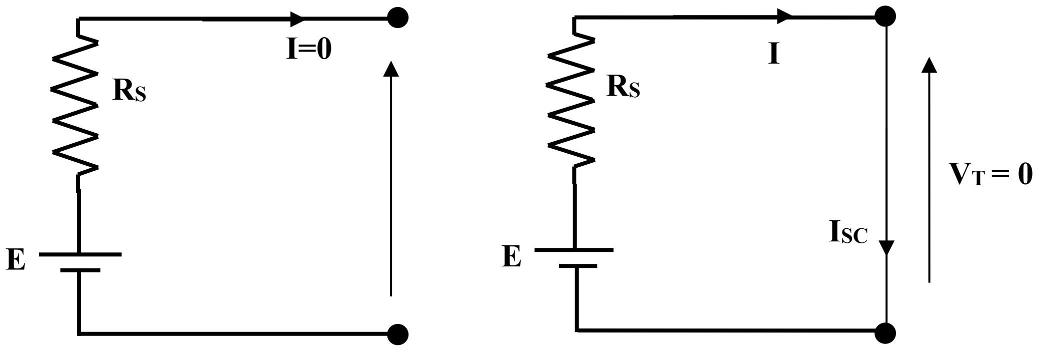


(a) Voltage source



(b) Other symbol

Figure 1.1: Representation of voltage source



(a) Terminal open circuit **(b) Terminal short circuit**

Figure 1.2: Voltage source

When the terminal is open-circuited:

$I = 0$ Amps, $V_T = V_{o/c} = E$ volts

When the terminal is short-circuited:

$V_T = 0$ Volts, $I = I_{S/C} = E/R_S$ Amps

Hence plotting V_T against I gives a straight line as shown in Figure 1.3.

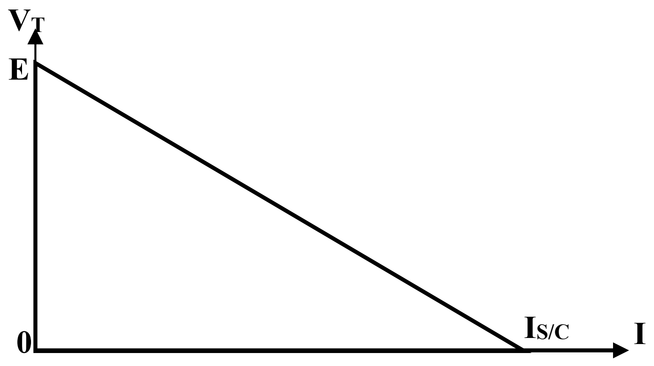


Figure 1.3: Plot of V_T versus I

1.1.1 Analysis Of Voltage Source

Figure 1.4 shows a voltage source connected to a resistive load R_L.

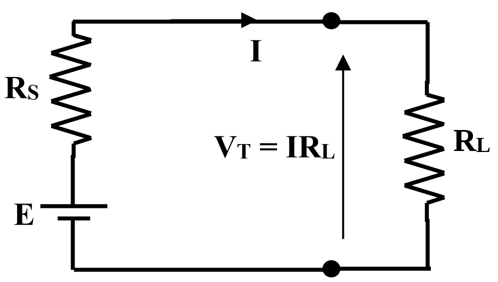


Figure 1.4: Voltage source with load R_L.

Applying KVL:

$$I\,R_S + I\,R_L = E$$

$$I = \frac{E}{R_S + R_L}\ , \quad V_T = I\,R_L$$

$$V_T = E - I\,R_S = \frac{E\,R_L}{R_S + R_L}$$

1.2 Constant Current Source

A constant current source is an ideal source element capable of supplying any voltage at a given current. Figure 1.5 shows a DC current source with current of I_S amperes and an internal resistance of R_S ohms. I_S denotes a constant perfect current source, independent of the voltage drop across the terminal. The current drawn depends on the load.

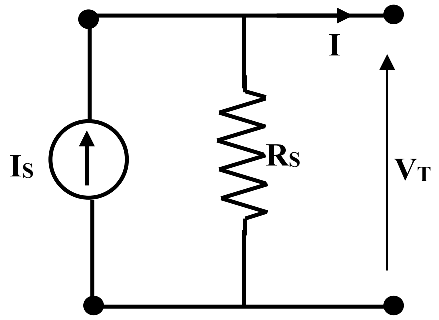


Figure 1.5: Representation of current source

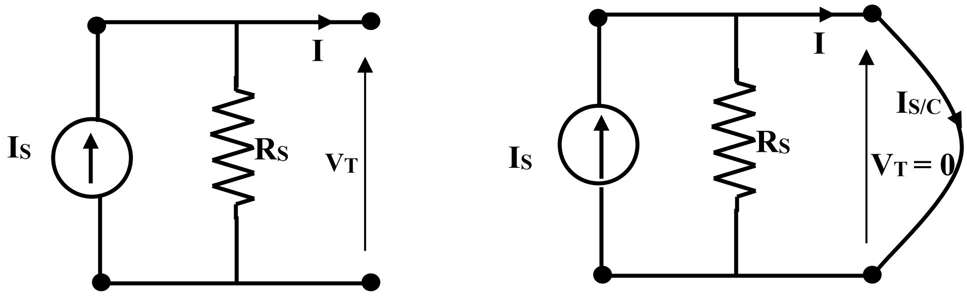


Figure 1.6: Current source under open and short circuit condition

When terminal is open-circuited:

$I = 0$ amperes; $V_T = V_{O/C} = I_S\,R_S$ volts

If $I_S\,R_S = E$, then $V_T = E$ volts

When terminal is short-circuited:

$V_T = 0$ volts, $I = I_{S/C} = I_S$ amperes

Hence plotting I against V_T will be a straight line as shown in Figure 1.7.

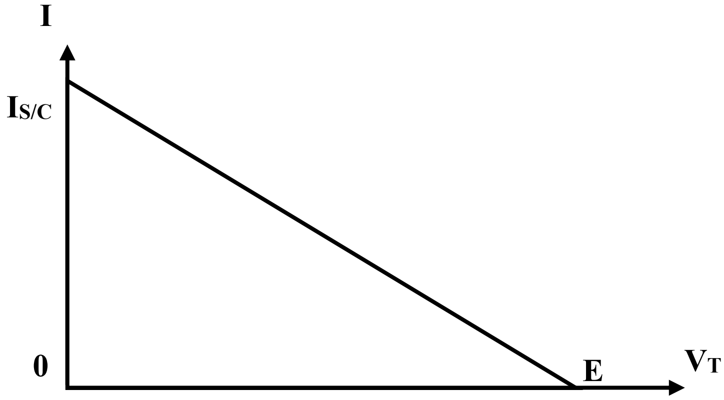


Figure 1.7: Plot of I versus V_T

1.2.1 Analysis Of Current Source

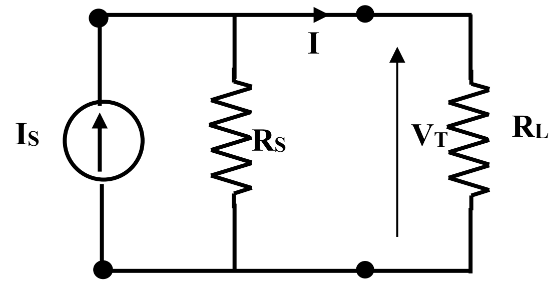


Figure 1.8: Current source with load

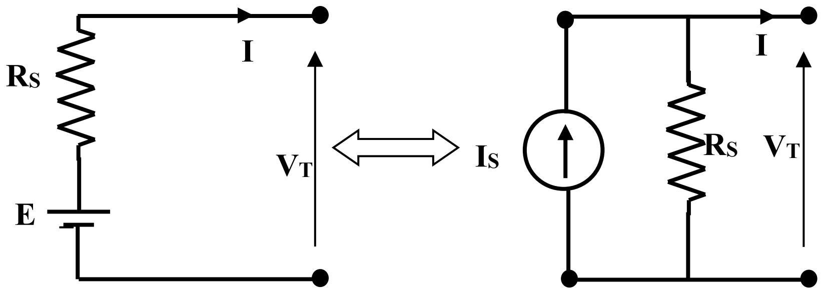
For the circuit shown in Figure 1.8,

$$I = I_S\left(\frac{R_S}{R_S + R_L}\right)$$

$$V_T = I\,R_L = I_S\left(\frac{R_S\,R_L}{R_S + R_L}\right)$$

1.3 Source Conversion (Helmholtz Circuits)

Some circuit networks are so complex that they cannot be analysed by simple series-parallel techniques. Also some circuits have current sources as well as voltage sources. An understanding of the principle of source conversion will allow us to simplify networks before solving.



(a) Voltage source ⟺ **(b) Current source**

Figure 1.9: Source conversion

$E = I_S R_S$ $\qquad$ $I_S = \dfrac{E}{R_S}$

<u>Example 1.1</u>

Convert the voltage source given in Figure 1.10 to its equivalent current source and calculate the current to the load.

<u>Solution 1.1</u>

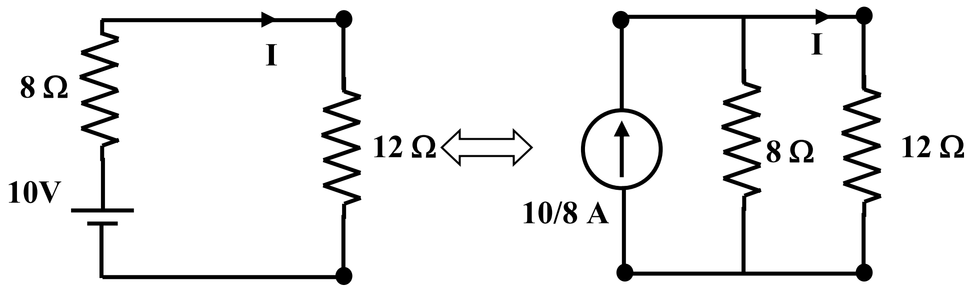
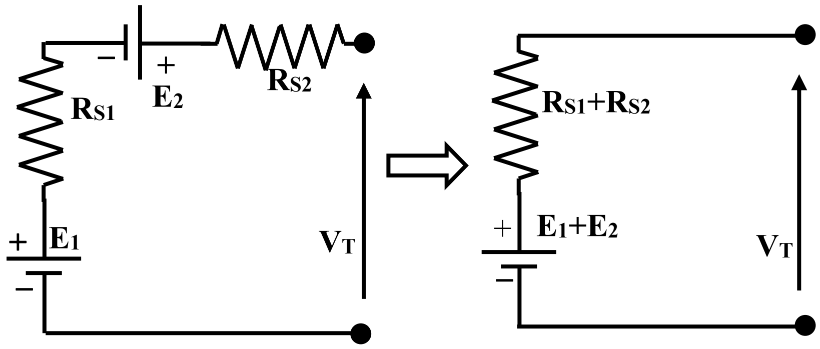


Figure 1.10: Conversion of voltage source to current source

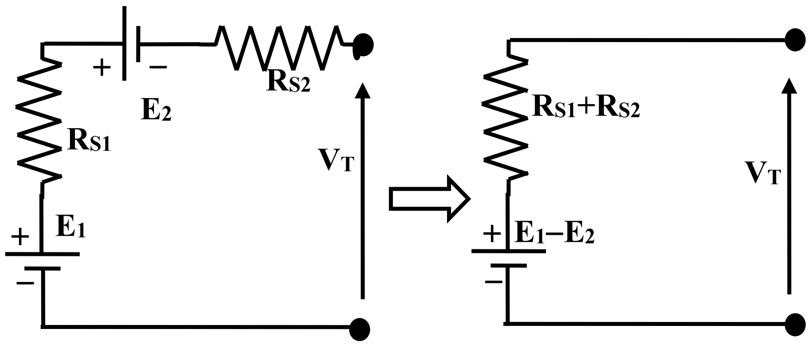
Current $I = \dfrac{10}{8+12} = 0.5$ A $\qquad$ $I = \dfrac{10}{8} \times \dfrac{8}{8+12} = 0.5$ A

1.4 Series Connection of Voltage Sources

Voltage sources can be operated in series aiding to give a total output voltage of (E_1+E_2) as shown in Figure 1.11 (a). It is wasteful to operate voltage sources in series opposing as in Figure 1.11 (b).



(a) Series adding voltage sources



(b) Series opposing voltage sources

Figure 1.11: Series connection of voltage sources

1.5 Parallel Connection of Current Sources

Current sources operated in parallel aiding will give a total output current of I_{S1}+I_{S2} as shown in Figure 1.12(a). It is wasteful to operate current sources in parallel opposing as shown in Figure 1.12(b) as the total output current is (I_{S1}-I_{S2}).

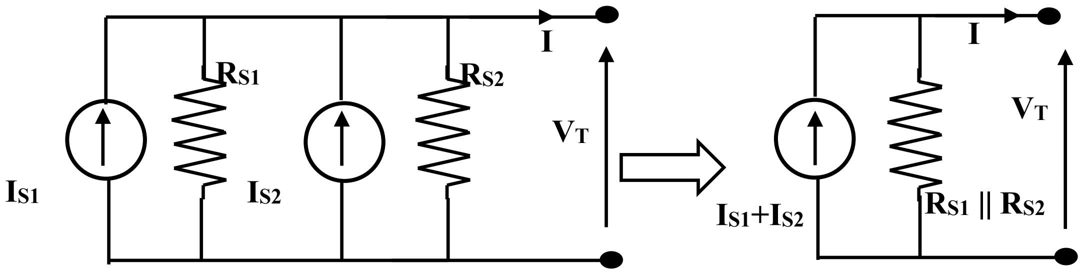
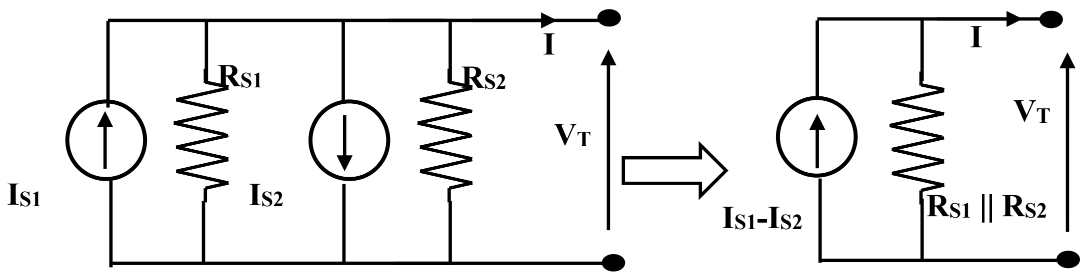


Figure 1.12(a) Parallel - aiding current sources



(b) Parallel - opposing current sources

Figure 1.12: Parallel connection of current sources

Procedure for finding the single equivalent current source or voltage source:

1. *When two voltage sources are in parallel or when a voltage source is in parallel with a current source.*
   - *(a) Always first convert the voltage source to its equivalent current sources with appropriate current direction.*
   - *(b) Identify the type of parallel connection as either:*
     - (i) *Parallel aiding current sources or.*
     - (ii) *Parallel opposing current sources.*
   - *(c) Simplify the circuit to find the single equivalent current source with appropriate current direction.*

2. *When two current sources are connected in series or when a current source is connected in series with a voltage source:*
   - *(a) Always first convert the current source to its equivalent voltage source with appropriate polarity.*
   - *(b) Identify the type of series connection as either:*
     - (i) *Series aiding voltage sources.*
     - (ii) *Series opposing voltage sources.*
   - *(c) Simplify the circuit to find the single equivalent voltage source with appropriate polarity.*

Example 1.2

Simplify the given circuit in Figure 1.13 into an equivalent circuit consisting of a constant current source and a resistor in parallel at the terminals AB.

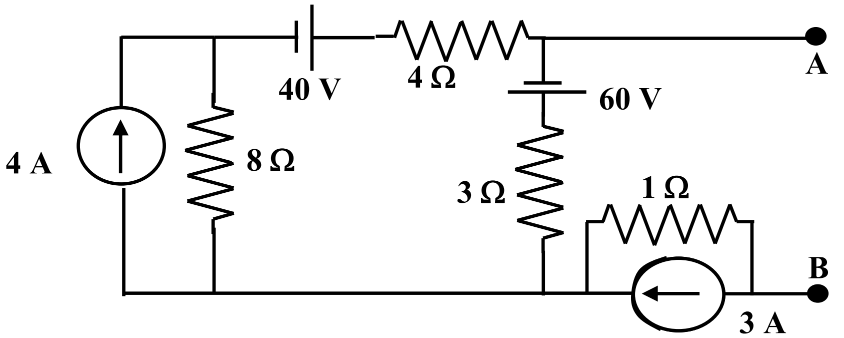
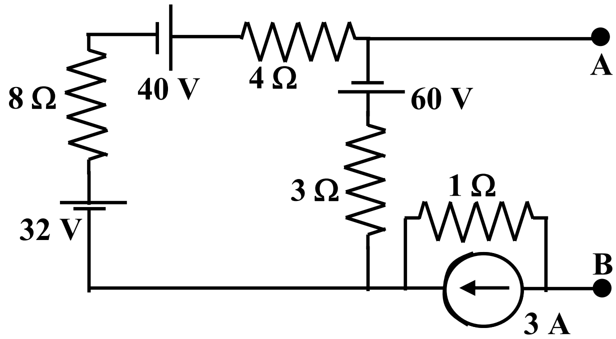


Figure 1.13: Parallel connection of current sources

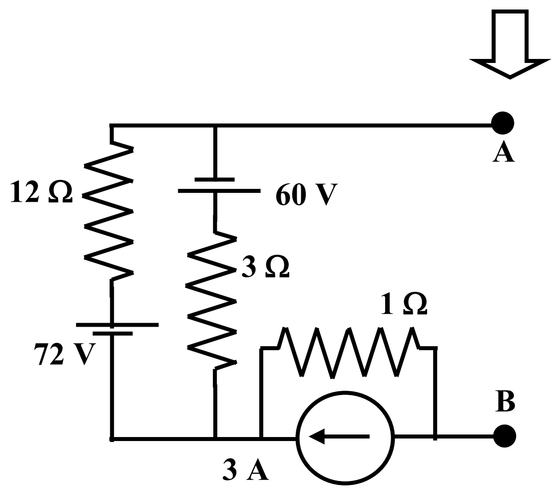
Solution 1.2

Figures 1.14(a) to (g) shows the conversion process to equivalent current source.



(a)

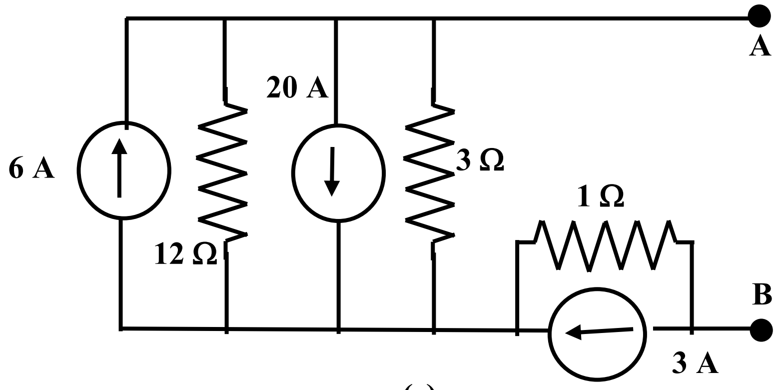
$E = I_S R_S = 4 x 8 = 32$ V



(b)

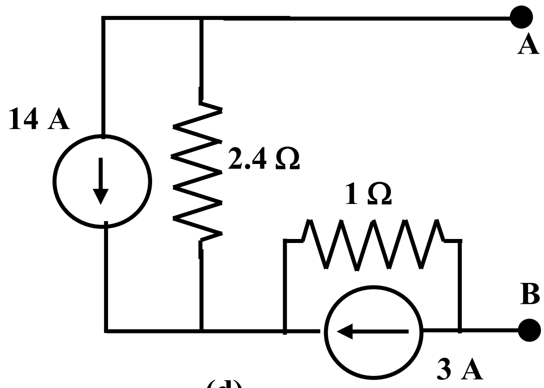
Series aiding voltage sources
$E = E_1 + E_2 = 40 + 32 = 72V$
$R = R_1 + R_2 = 4 + 8 = 12\ \Omega$

Figure 1.14: Conversion of voltage ←→ current source (continued)



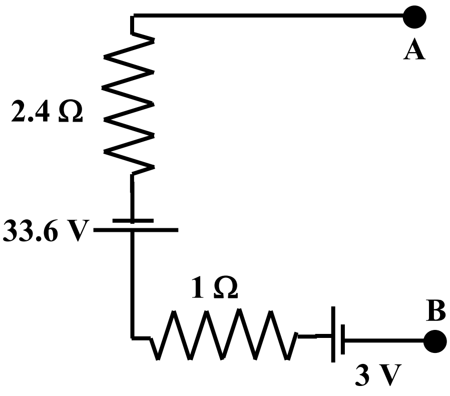
$I_{S1} = 72/12 = 6$ Amps
$I_{S2} = 60/3 = 20$ Amps

(c)



Parallel opposing sources
$I_S = 20 - 6 = 14$ A
R= 12x3/15=2.4 Ω

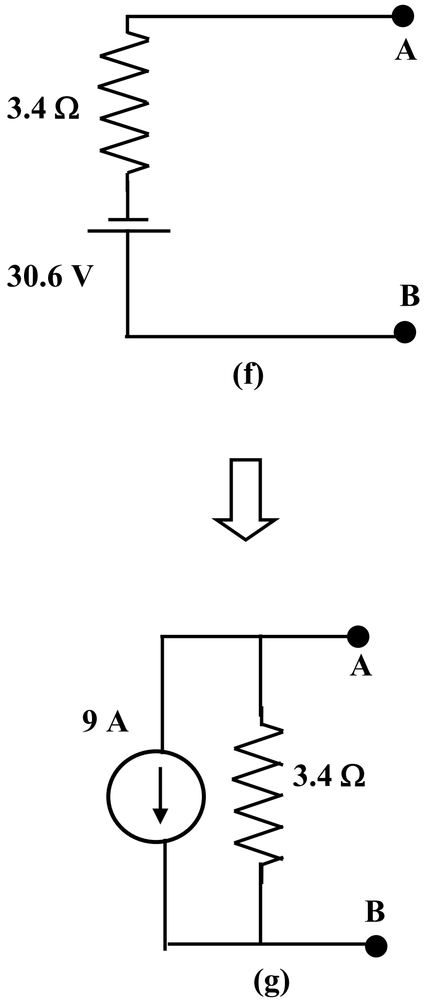
(d)



E =14x2.4 =33.6 V
E =1x3 =3 V

(e)

Figure 1.14: Conversion of voltage ←→ current source (continued)



E =33.6-3=30.6 V
R=2.4+1=3.4 Ω

I_S = 30.6/3.4= 9 A

Figure 1.14: Conversion of voltage ←→ current source

1.6 Mesh –Current Analysis

Mesh current analysis is a standard format procedure which is designed to simplify and speed up the task of writing the set of simultaneous equations for various circuit networks that we need to solve.

1.6.1 Mesh-Current Equations By Analysis

By the use of mesh equations, complex circuits can be solved to find the unknown branch currents. The analysis procedure is presented in this section.

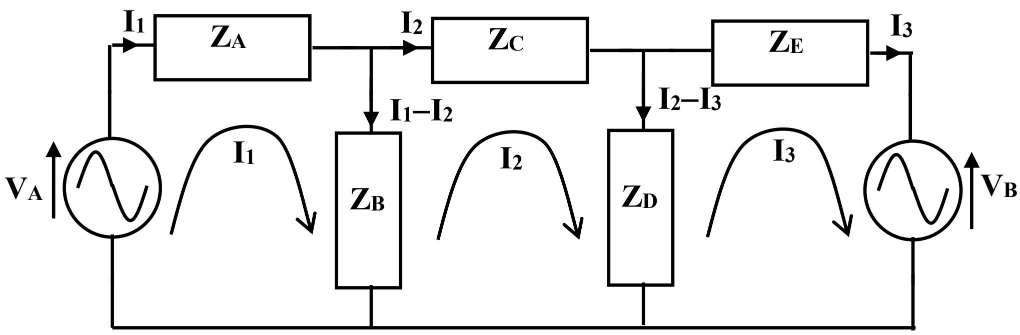


Figure 1.15: Circuit analysis by using mesh current equations

Procedure for writing the mesh current equations:

1. *Determine the number of loops required, and hence find the number of equations to solve. For a network/circuit with N nodes (including the reference node) and B branches (or number of circuit elements both passive and active) in the circuit there are exactly B-N+1 linearly independent KVL (or loop) equations.*
2. *Select loops. (Choose loop for easy solution)*
3. *Label each loop currents.*
4. *Apply Kirchoff's Voltage Law (KVL) to form the loop equations and solve for unknown branch currents.*

Loop equations:

Loop 1: $\mathbf{V_A = I_1 Z_A + (I_1 - I_2) Z_B}$

Loop 2: $\mathbf{0 = I_2 Z_C - (I_1 - I_2) Z_B + (I_2 - I_3) Z_D}$

Loop 3: $\mathbf{-V_B = I_3 Z_E - (I_2 - I_3) Z_D}$

Re-arranging the terms,

$$\mathbf{(Z_A + Z_B)\,I_1 - Z_B\,I_2 + 0I_3 = V_A} \quad \text{.......................}\mathbf{(1)}$$

$$\mathbf{-Z_B\,I_1 + (Z_B + Z_C + Z_D)\,I_2 - Z_D\,I_3 = 0} \quad \text{.......................}\mathbf{(2)}$$

$$\mathbf{0I_1 - Z_D\,I_2 + (Z_D + Z_E)\,I_3 = -V_B} \quad \text{.......................}\mathbf{(3)}$$

$$\begin{bmatrix} (\mathbf{Z_A} + \mathbf{Z_B}) & -\mathbf{Z_B} & \mathbf{0} \\ -\mathbf{Z_B} & (\mathbf{Z_B} + \mathbf{Z_C} + \mathbf{Z_D}) & -\mathbf{Z_D} \\ \mathbf{0} & -\mathbf{Z_D} & (\mathbf{Z_D} + \mathbf{Z_E}) \end{bmatrix} \begin{bmatrix} \mathbf{I_1} \\ \mathbf{I_2} \\ \mathbf{I_3} \end{bmatrix} = \begin{bmatrix} \mathbf{V_A} \\ \mathbf{0} \\ -\mathbf{V_B} \end{bmatrix}$$

[Z] [I] = [V]

The **[Z],** impedance matrix, can be obtained by inspection. The impedance matrix can be expressed as:

$$\begin{bmatrix} \mathbf{Z_{11}} & \mathbf{Z_{12}} & \mathbf{Z_{13}} \\ \mathbf{Z_{21}} & \mathbf{Z_{22}} & \mathbf{Z_{23}} \\ \mathbf{Z_{31}} & \mathbf{Z_{32}} & \mathbf{Z_{33}} \end{bmatrix} \begin{bmatrix} \mathbf{I_1} \\ \mathbf{I_2} \\ \mathbf{I_3} \end{bmatrix} = \begin{bmatrix} \mathbf{V_1} \\ \mathbf{V_2} \\ \mathbf{V_3} \end{bmatrix}$$

Where, for impedance matrix:

Z_{11} = Self-impedance of loop I_1 i.e. sum of all impedance ‘seen’ by I_1.

Similarly,

Z_{22} & Z_{33} = Self-impedance of loop I_2 and I_3 respectively.

$Z_{12} = Z_{21}$ = ($\pm$) Sum of impedance common to loop I_1 and loop I_2.

Use **(+Ve**) if both currents passing through the common impedance are in the same direction and (**– Ve**) if opposite direction.

Similarly,

$Z_{13} = Z_{31}$ = ($\pm$) Sum of impedance common to loop I_1 and loop I_3.

$Z_{23} = Z_{32}$ = ($\pm$) Sum of impedance common to loop I_2 and loop I_3.

The impedance matrix is symmetrical.

For voltage matrix:

V_1 = Sum of ($\pm$) driving voltages for loop 1.

(**+Ve**) if driving voltage is sending a current in the direction of the loop 1,
(**-Ve**) if driving voltage is sending a current in the opposite direction of the loop 1.

Similarly,

V_2 & V_3 = Sum of ($\pm$) driving voltages for loop 2 and loop 3 respectively.

1.6.2 Mesh-Current Equations by Inspection

Example 1.3

Write the mesh current equations by inspection for the circuit shown in Figure 1.16.

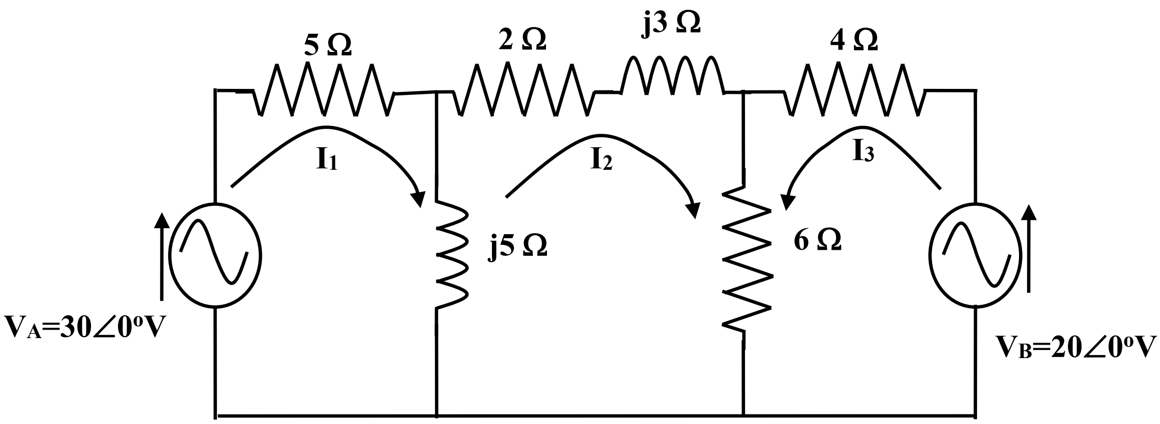


Figure 1.16: Three-loop circuit

Solution 1.3

Form the impedance matrix and the voltage matrix by inspection:

$$\begin{bmatrix} (5+j5) & -j5 & 0 \\ -j5 & (8+j8) & 6 \\ 0 & 6 & 10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30\angle 0^\circ \\ 0 \\ 20\angle 0^\circ \end{bmatrix}$$

Rewriting into equation form

$(5 + j5)\, I_1 - j5\, I_2 + 0\, I_3 = 30\angle 0^\circ$ ....(4)

$-j5\, I_1 + (8 + j8)\, I_2 + 6\, I_3 = 0$ ....(5)

$0\, I_1 + 6\, I_2 + 10\, I_3 = 20\angle 0^\circ$ ....(6)

Using the same circuit but choosing different loop currents as shown in Figure 1.17, the Kirchoff's voltage equation will be as follows:

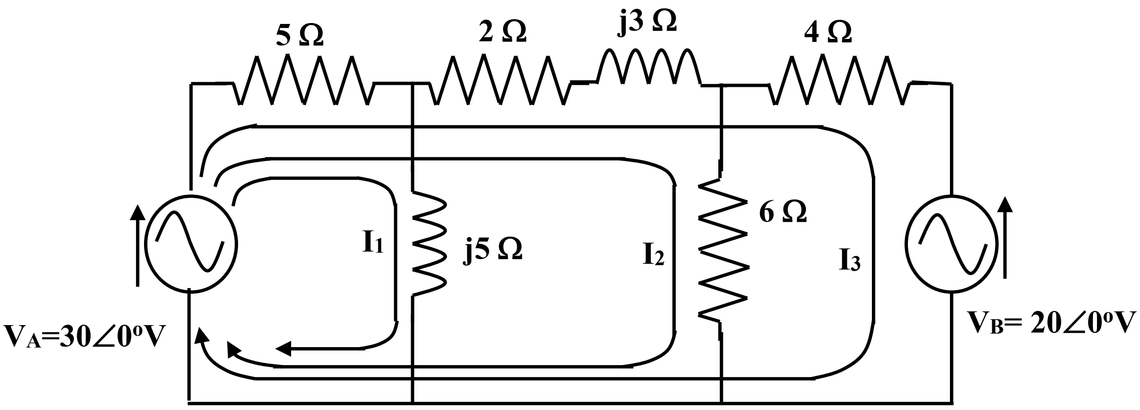


Figure 1.17: Alternate choice of loop currents.

$$\begin{bmatrix} (5+j5) & 5 & 5 \\ 5 & (13+j3) & (7+j3) \\ 5 & (7+j3) & (11+j3) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30\angle 0^\circ \\ 30\angle 0^\circ \\ 30\angle 0^\circ - 20\angle 0^\circ \end{bmatrix}$$

Rewrite into equation form:

$$(5 + j5)\, I_1 + 5\, I_2 + 5\, I_3 = 30\angle 0^\circ \quad \text{...(7)}$$

$$5\, I_1 + (13 + j3)\, I_2 + (7 + j3)\, I_3 = 30\angle 0^\circ \quad \text{...(8)}$$

$$5\, I_1 + (7 + j3)\, I_2 + (11 + j3)\, I_3 = 30\angle 0^\circ - 20\angle 0^\circ \quad \text{......................................(9)}$$

i **Remarks:**

It does not matter which of the loop currents are used in the formation of equations as shown in the two circuits as the final results of the currents flowing in each branch will be the same.

Example 1.4

Determine the current in the 5 Ω branch, for the circuit shown in Figure 1.18 using loop currents analysis method.

Solution 1.4

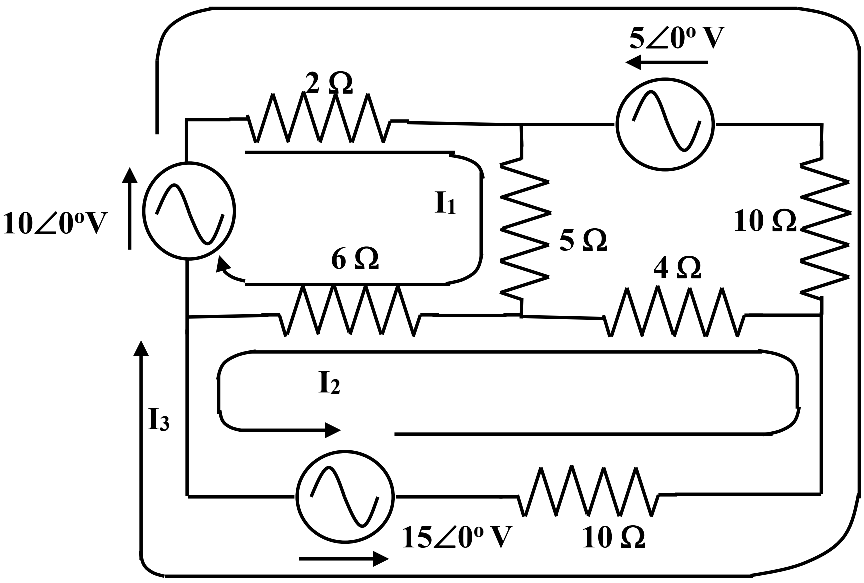


Figure 1.18

The matrix equation is as follow:

$$\begin{bmatrix} 13 & 6 & 2 \\ 6 & 20 & -10 \\ 2 & -10 & 22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 15\angle 0^\circ \\ 10\angle 0^\circ - 5\angle 0^\circ - 15\angle 0^\circ \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 10\angle 0^\circ & 6 & 2 \\ 15\angle 0^\circ & 20 & -10 \\ 10\angle 0^\circ - 5\angle 0^\circ - 15\angle 0^\circ & -10 & 22 \end{vmatrix}}{\begin{vmatrix} 13 & 6 & 2 \\ 6 & 20 & -10 \\ 2 & -10 & 22 \end{vmatrix}} = \frac{\begin{vmatrix} 10 & 6 & 2 \\ 15 & 20 & -10 \\ -10 & -10 & 22 \end{vmatrix}}{\begin{vmatrix} 13 & 6 & 2 \\ 6 & 20 & -10 \\ 2 & -10 & 22 \end{vmatrix}}$$

$$= \frac{10\begin{vmatrix} 20 & -10 \\ -10 & 22 \end{vmatrix} - 6\begin{vmatrix} 15 & -10 \\ -10 & 22 \end{vmatrix} + 2\begin{vmatrix} 15 & 20 \\ -10 & -10 \end{vmatrix}}{13\begin{vmatrix} 20 & -10 \\ -10 & 22 \end{vmatrix} - 6\begin{vmatrix} 6 & -10 \\ 2 & 22 \end{vmatrix} + 2\begin{vmatrix} 6 & 20 \\ 2 & -10 \end{vmatrix}}$$

$$= \frac{10[(20 \text{ x } 22) - (-10 \text{ x } -10)] - 6[(15 \text{ x } 22) - (-10 \text{ x } -10)] + 2[(15 \text{ x } -10) - (20 \text{ x } -10)]}{13[(20 \text{ x } 22) - (-10 \text{ x } -10)] - 6[(6 \text{ x } 22) - (-10 \text{ x} 2)] + 2[(6 \text{ x } -10) - (20 \text{ x } 2)]}$$

$$= \frac{10[440 - 100] - 6[330 - 100] + 2[-150 + 200]}{13[440 - 100] - 6[132 + 20] + 2[-60 - 40]}$$

$$= \frac{10(340) - 6(230) + 2(50)}{13(340) - 6(152) + 2(-100)} = \frac{3400 - 1380 + 100}{4420 - 912 - 200}$$

$$= \frac{2120}{3308} = 0.641\,\text{A}$$

1.7 Nodal-Voltage Analysis

Nodal-voltage analysis is a circuit-analysis format, which allows us to standardise the way we write the Kirchoff's current-law equations. The objective of solving the network by this method is to determine the values of the voltages at different nodes; a voltage node is a junction in an electrical circuit at which a voltage can be measured with respect to another (reference) node.

1.7.1 Nodal-Voltage Equations By Analysis

By the use of node equations, complex circuits can be solved to find the unknown node voltages of circuit with respect to the reference or ground node.

Analysis procedure:

1. *Choose a node as reference (ground). In the circuit shown in Figure 1.19, node 3 is ideal reference node.*
2. *Identify the number of nodes required, hence the number of equations to solve.*
3. *Select nodes. (Choose nodes for easy solution).*
4. *Label each node voltages.*
5. *Apply Kirchoff's Current Law (KCL) to form the node equations and solve for unknown node voltages.*

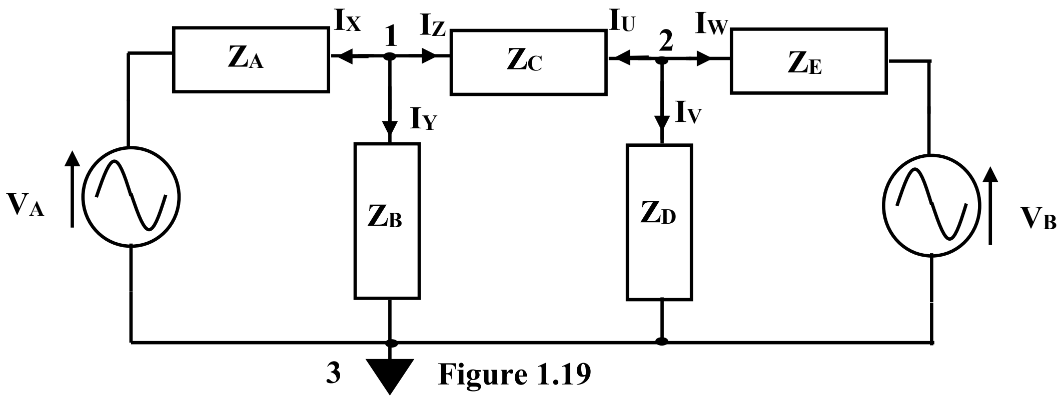


Figure 1.19

$$\text{For Node}:1 \qquad I_X + I_Y + I_Z = 0$$

$$\frac{V_1 - V_A}{Z_A} + \frac{V_1}{Z_B} + \frac{V_1 - V_2}{Z_C} = 0$$

$$\text{Re arranging} \qquad \left(\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C}\right)V_1 - \frac{1}{Z_C}V_2 = \frac{V_A}{Z_A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

$$\text{For Node}: 2 \qquad I_U + I_V + I_W = 0$$

$$\frac{V_2 - V_1}{Z_C} + \frac{V_2}{Z_D} + \frac{V_2 - V_B}{Z_E} = 0$$

$$\text{Rearranging} \quad -\frac{1}{Z_C}V_1 + \left(\frac{1}{Z_C} + \frac{1}{Z_D} + \frac{1}{Z_E}\right)V_2 = \frac{V_B}{Z_E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Equations (1) and (2) can be written in matrix form **[Y] [V] = [I]** i.e.

$$\begin{bmatrix} \left(\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C}\right) & -\left(\frac{1}{Z_C}\right) \\ -\left(\frac{1}{Z_C}\right) & \left(\frac{1}{Z_C} + \frac{1}{Z_D} + \frac{1}{Z_E}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V_A}{Z_A} \\ \frac{V_B}{Z_E} \end{bmatrix}$$

The **[Y]** admittance matrix can be obtained by inspection. The admittance matrix can be expressed as:

$$\begin{bmatrix} +(Y_{11}) & -(Y_{12}) \\ -(Y_{21}) & +(Y_{22}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Where, for admittance matrix:

$\mathbf{Y_{11}}$ = Self-admittance of node 1, given by sum of all admittances connected to node 1.

$\mathbf{Y_{22}}$ = Self-admittance of node 2, given by sum of all admittances connected to node 2,

and $\mathbf{Y_{12} = Y_{21}}$ = Coupling admittance between node 1 and node 2, given by the <u>negative sum</u> of all admittances connected between node 1 and node 2.

The admittance matrix is symmetrical.

For current matrix: $\mathbf{I_1}$ = Sum of ($\pm$) driving currents into node 1.
(**+Ve**) if driving current is flowing into the node,
(**–Ve**) if driving current is flowing out of the node.

Similarly, $\mathbf{I_2}$ = Sum of ($\pm$) driving currents into node 2.
(**+ Ve**) if driving current is flowing into the node,
(**– Ve**) if driving current is flowing out of the node.

1.7.2 Nodal-Voltage Equations by Inspection

Example 1.5

Write the nodal voltage equation by inspection for the circuit shown in Figure 1.20.

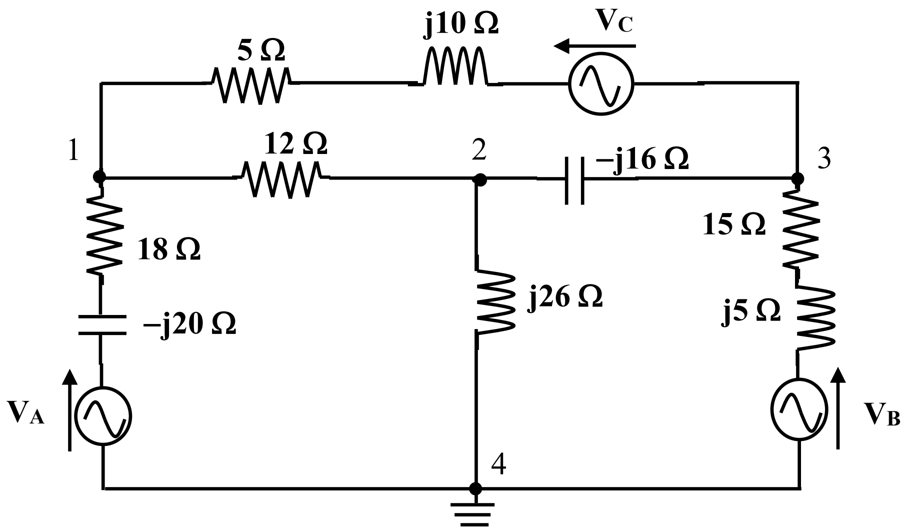


Figure 1.20

Solution 1.5

Writing the matrix equation by inspection as below:

$$\begin{bmatrix} \left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & -\left(\frac{1}{12}\right) & -\left(\frac{1}{5+j10}\right) \\ -\left(\frac{1}{12}\right) & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & -\left(\frac{1}{-j16}\right) \\ -\left(\frac{1}{5+j10}\right) & -\left(\frac{1}{-j16}\right) & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{V_A}{18-j20}+\frac{V_C}{5+j10} \\ 0 \\ \frac{V_B}{15+j5}-\frac{V_C}{5+j10} \end{bmatrix}$$

Example 1.6

Find the current in the 8 Ω resistor using node voltage analysis method for the circuit shown in Figure 1.21.

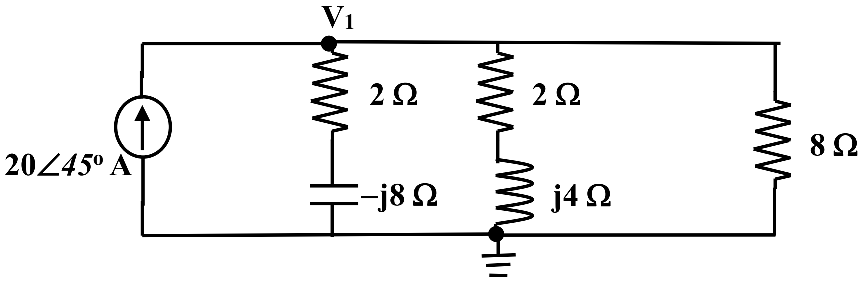


Figure 1.21

Solution 1.6

Writing the matrix equation by inspection as below:

$$\left[\frac{1}{2-j8}+\frac{1}{2+j4}+\frac{1}{8}\right]V_1 = 20\angle 45^\circ$$

$$\left[\frac{1}{8.24\angle -75.96^0}+\frac{1}{4.47\angle 63.43^0}+0.125\right]V_1 = 20\angle 45^\circ$$

$$\left[0.121\angle 75.96^0 + 0.224\angle -63.43^0 + 0.125\right]V_1 = 20\angle 45^\circ$$

$$\left[0.029 + j0.117 + 0.1 - j0.2 + 0.125\right]V_1 = 20\angle 45^\circ$$

$$(0.254 - j0.083)V_1 = 20\angle 45^\circ$$

$$\therefore\ V_1 = \frac{20\angle 45^\circ}{0.254 - j0.083} = \frac{20\angle 45^\circ}{0.267\angle -18.09} = 74.9\angle 63.09^\circ$$

$$I_{8\Omega} = \frac{V_1}{8} = 9.36\angle 63.09^\circ\ \text{A}$$

1.8 Star-Delta and Delta-Star Transformation

Many connections of electrical loads form the shape of a Δ (delta), which is also called a π network. Similarly, another popular connection forms the shape of Y and is called a Y (star) network or referred as T-network.

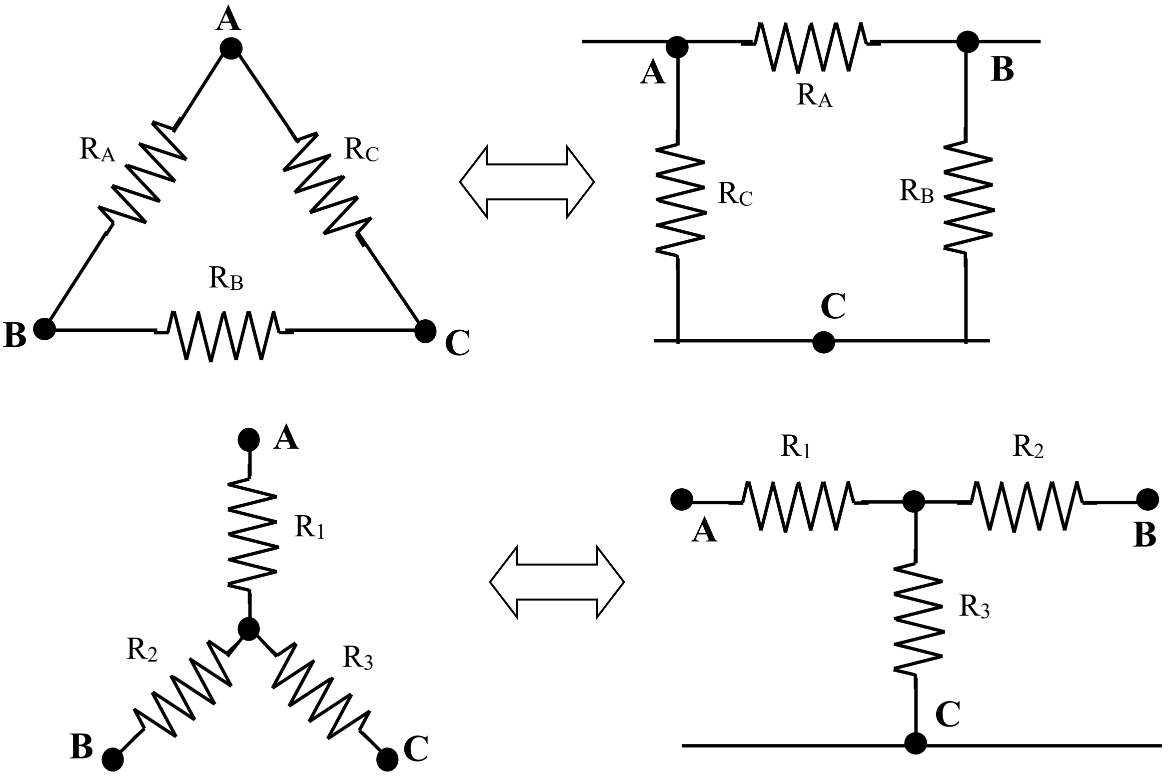


Figure 1.22: Delta and Star networks.

1.8.1 Delta-Star Transformation

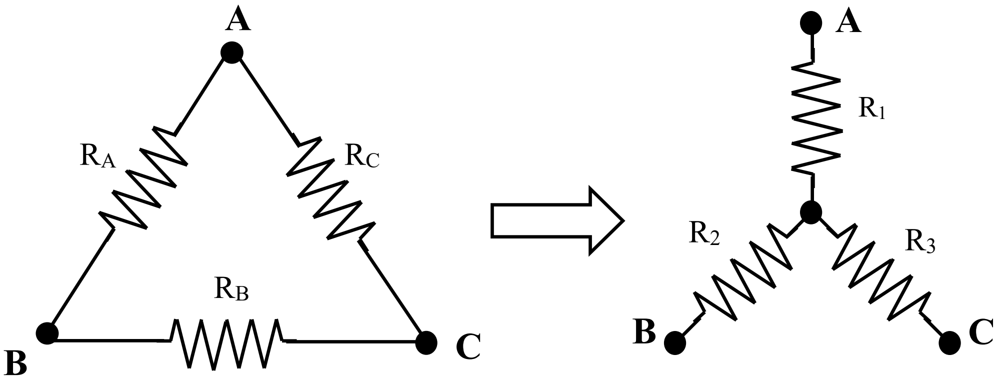


Figure 1.23: Delta to star transformation

Terminals	Star network	Delta network
AB	R_1+R_2	$\dfrac{R_A(R_B+R_C)}{R_A+R_B+R_C}$
BC	R_2+R_3	$\dfrac{R_B(R_A+R_C)}{R_A+R_B+R_C}$
CA	R_3+R_1	$\dfrac{R_C(R_A+R_B)}{R_A+R_B+R_C}$

Simplifying:

$$R_1+R_2=\frac{R_AR_B+R_AR_C}{R_A+R_B+R_C} \quad \dots\dots\dots\dots\dots\dots(12)$$

$$R_2+R_3=\frac{R_AR_B+R_BR_C}{R_A+R_B+R_C} \quad \dots\dots\dots\dots\dots\dots(13)$$

$$R_3+R_1=\frac{R_AR_C+R_BR_C}{R_A+R_B+R_C} \quad \dots\dots\dots\dots\dots\dots(14)$$

Equations $(12)-(13)+(14)$ result in

$$2R_1=\frac{2R_AR_C}{R_A+R_B+R_C} \quad \text{or} \quad R_1=\frac{R_AR_C}{R_A+R_B+R_C} \quad \dots\dots\dots(15)$$

Equations $(13)-(14)+(12)$ result in

$$2R_2=\frac{2R_AR_B}{R_A+R_B+R_C} \quad \text{or} \quad R_2=\frac{R_AR_B}{R_A+R_B+R_C} \quad \dots\dots\dots(16)$$

Equations $(14)-(12)+(13)$ result in

$$2R_3=\frac{2R_BR_C}{R_A+R_B+R_C} \quad \text{or} \quad R_3=\frac{R_BR_C}{R_A+R_B+R_C} \quad \dots\dots\dots(17)$$

Hence the equivalent star resistances may be expressed in words as follows:

The equivalent star resistance connected to a given terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta resistances.

These equations also apply to impedance where impedances are in complex numbers.

1.8.2 Star-Delta Transformation

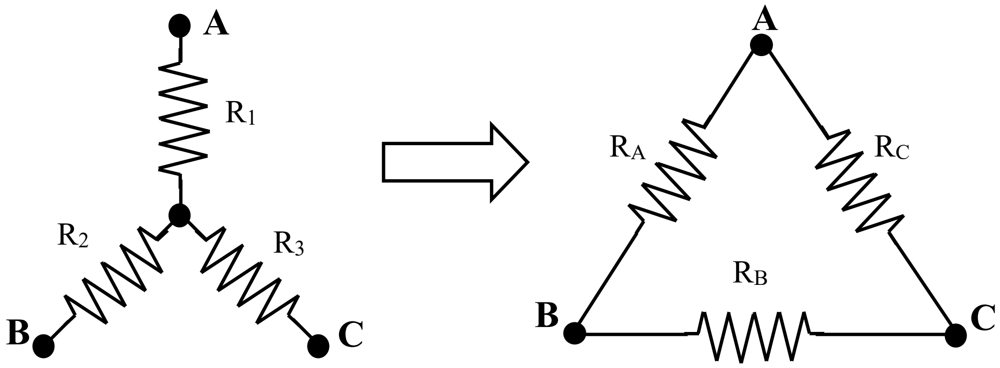


Figure 1.24: Star to delta transformation

Equations (17) divided by (15) will result in

$$R_B = \frac{R_A R_3}{R_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

Equations (17) divided by (16) will result in

$$R_C = \frac{R_A R_3}{R_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

Substituting (18) and (19) into (17) will result in

$$R_3 = \frac{\left(\frac{R_A R_3}{R_1}\right)\left(\frac{R_A R_3}{R_2}\right)}{R_A + \frac{R_A R_3}{R_1} + \frac{R_A R_3}{R_2}} = \frac{R_A{}^2 R_3{}^2}{R_A R_1 R_2 + R_A R_2 R_3 + R_A R_1 R_3}$$

$$1 = \frac{R_A R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$R_A = R_1 + R_2 + \frac{R_1 R_2}{R_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

Similarly with the same approach, we can solve for R_B and R_C. Hence

$$R_B = R_2 + R_3 + \frac{R_2 R_3}{R_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

$$R_C = R_1 + R_3 + \frac{R_1 R_3}{R_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

Similarly, the equivalent delta resistances may be expressed in words as follows:

i ***The equivalent delta resistance between two terminals is the sum of the two star resistances connected to those terminals plus the product of the same two star resistances divided by the third star resistance***.

These equations also apply to impedances where the impedances are in complex numbers.

1.8.3 Summary of Transformation Rules:

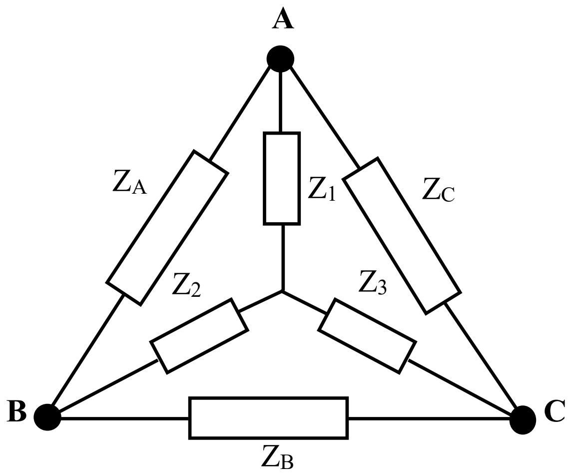


Figure 1.25: Summary of Star to delta and delta to star transformation

Delta to star

$$Z_1 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C}$$

$$Z_2 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C}$$

$$Z_3 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C}$$

Star to delta

$$Z_A = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

$$Z_B = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1}$$

$$Z_C = Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2}$$

1.8.4 Balanced Star Or Delta Impedances

If the network is balanced Star or Delta impedances,

$$Z_D = Z_S + Z_S + \frac{Z_S Z_S}{Z_S} = 3Z_S$$

$\therefore \quad Z_D = 3Z_S$ ....(23)

OR

$Z_S = \frac{Z_D}{3}$ ....(24)

Example 1.7

Find the delta equivalent of the balanced star network shown in Figure 1.26.

Solution 1.7

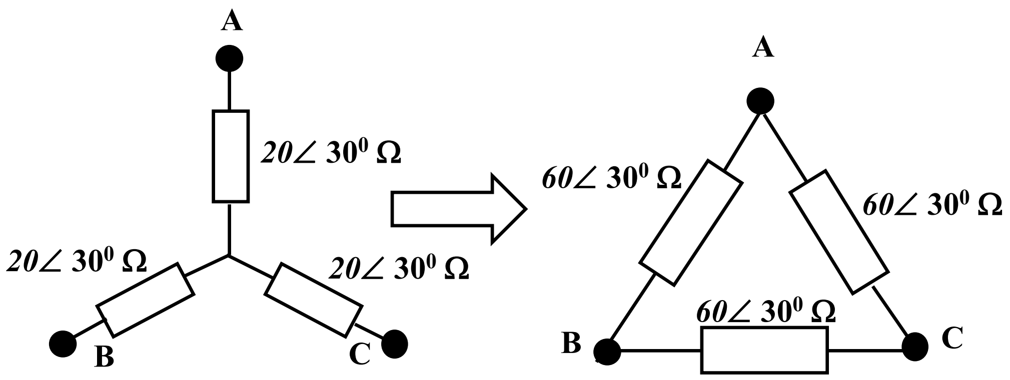


Figure 1.26: Balanced Star to balanced delta transformation

Using the relationship $Z_D = 3Z_S$ so $Z_D = 3 \times 20\angle 30^0\ \Omega = 60\angle 30^0\ \Omega$

Example 1.8

Determine the equivalent resistance across terminals AB for the circuit shown Figure 1.27.

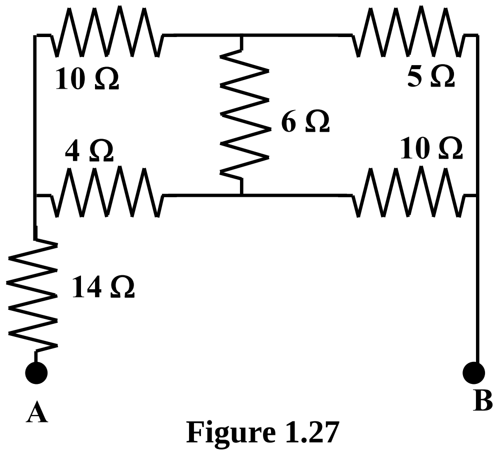


Figure 1.27

Solution 1.8

Using delta to star transformation for the resistors 10 Ω, 6 Ω and 4 Ω, the circuit reduces to Figure 1.28 as below. The circuit is further simplified to obtain R_{AB}.

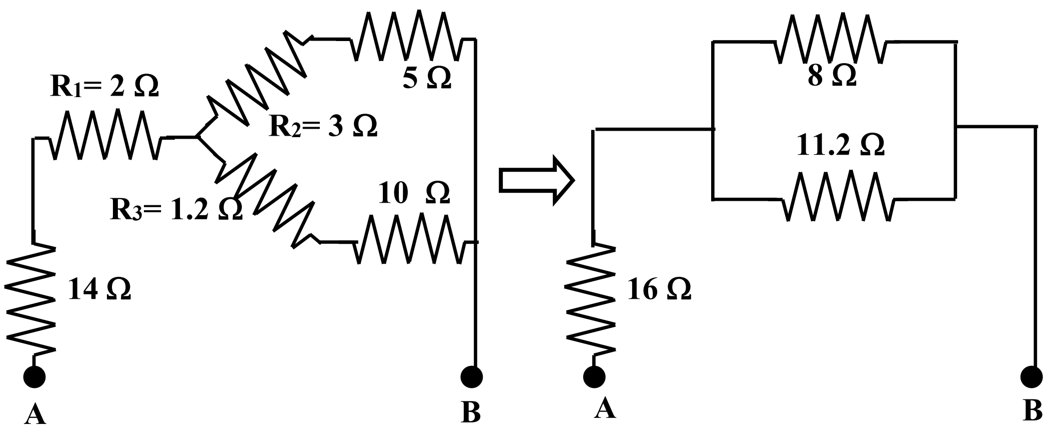


Figure 1.28

$$R_1 = \frac{10 \times 4}{20} = 2\,\Omega, \quad \mathrm{R}_2 = \frac{10 \times 6}{20} = 3\,\Omega, \; \mathrm{R}_3 = \frac{6 \times 4}{20} = 1.2\,\Omega$$

$$R_{AB} = 16 + \frac{8 \times 11.2}{8 + 11.2} = 20.67\Omega$$

1.9 Thevenin's Theorem

The theorem states that an equivalent circuit consisting of one voltage source in series with single impedance can replace any linear circuitry between two points A and B (Figure 1.29).

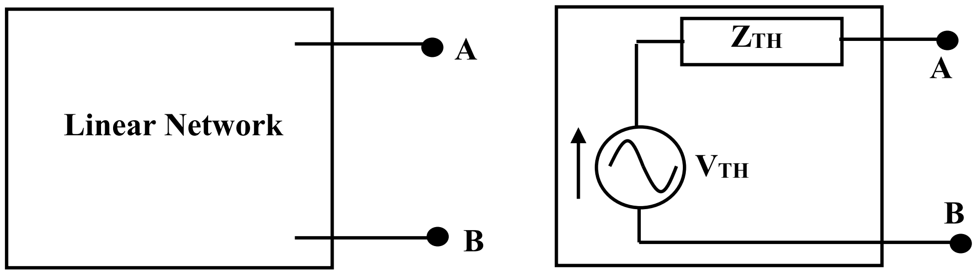


Figure 1.29: Thevenin equivalent circuit for a linear network

Thevenin voltage is the equivalent voltage source and is denoted by $\mathbf{V_{TH}}$. It is defined as the **open circuit** voltage observed across the terminals **AB**.

Thevenin impedance is the equivalent impedance and is denoted by $\mathbf{Z_{TH}}$. It is the impedance measured across terminals **AB** when all circuit sources are set to zero.

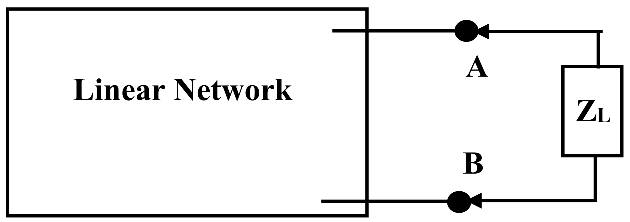
1.9.1 Usage of the Theorem:

1. It is used for solving complicated circuit by replacing it with a simpler equivalent circuit.
2. It can be applied to both DC and AC circuits.

1.9.2 Advantage:

The advantage of Thevenin method is that once the equivalent circuit is formed, it can be re-used for different load conditions connected to the two terminals.

1.9.3 Procedure in applying Thevenin's Theorem to points AB:



Note: The load impedance is usually connected to AB

Figure 1.30

Steps:

1. *Disconnect the load from circuit, i.e. open circuit load terminals AB.*
2. *Calculate or compute or measure the voltage at AB. This is the open-circuit voltage or Thevenin's Voltage, V_{TH}.* ***For computing you can only use either the application of Ohm's Law, Loop or Nodal Analysis method.***
3. *Set all internal circuit supply sources to zero i.e. short circuit all voltage sources; open circuit all the current sources. This is to find Z_{TH}.*
4. *Looking into the 'source free' circuit, calculate the impedance that would exist between the open circuit terminals AB. This gives Z_{TH}.*
5. *Form Thevenin equivalent circuit by connecting V_{TH} in series with Z_{TH}.*
6. *Reconnect the load impedance Z_L or R_L (if any) at the load terminals AB.*
7. *Calculate the load voltage, current and power (if required).*

Note: Source Conversion is not allowed.

<u>Example 1.9</u>

Apply Thevenin's theorem for the network shown in Figure 1.31 and find the Thevenin equivalent circuit across terminals AB.

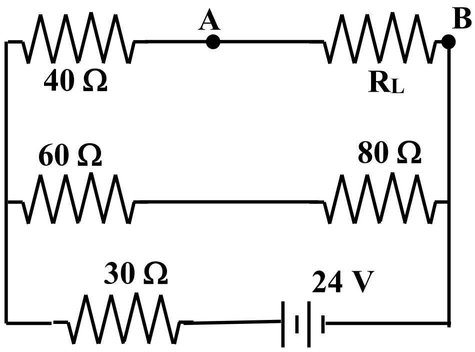


Figure 1.31

<u>Solution 1.9</u>

To find Thevenin's voltage V_{AB} (V_{TH}), remove R_L.

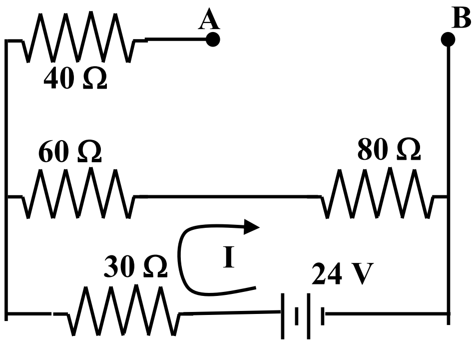


Figure 1.32: To find Thevenin's voltage source, V_{TH}

$$I = \frac{24}{30+60+80} = 0.141\,\text{A}$$

$$V_{60\Omega} = (0.141)\,60 = 8.47\text{ V};\ V_{80\Omega} = (0.141)\,80 = 11.29\text{ V}$$

Therefore $V_{AB} = V_{60\Omega} + V_{80\Omega} = 8.47 + 11.29 = 19.76\text{ V} = V_{TH}$

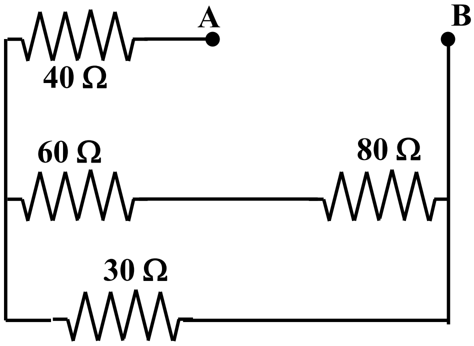


Figure 1.33: To find Thevenin's Resistance, R_{TH}

To find Thevenin's resistance R_{TH} (R_{AB}), short circuit the 24 V supply and find the equivalent resistance between the terminals AB.

$$R_1 = 60 + 80 = 140\,\Omega$$

$$R_{//} = \frac{140 \times 30}{140 + 30} = 24.71\,\Omega$$

$$R_{AB} = 24.71 + 40 = 64.71\,\Omega = R_{TH}$$

<u>Thevenin equivalent circuit</u> with load R_L connected across AB is shown in Figure 1.34 below.

It is now possible to find the current across R_L (with any value) using Ohms Law.

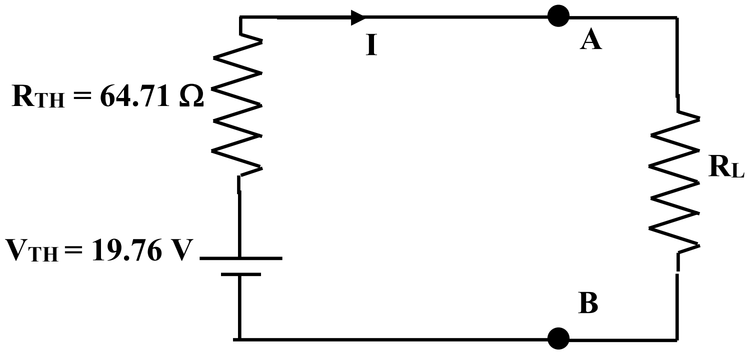


Figure 1.34: Thevenin equivalent circuit.

$$I = \frac{V_{TH}}{R_{TH} + R_L} = \frac{19.76}{64.71 + R_L}\ \text{A}$$

Example 1.10

Apply Thevenin's theorem and calculate the current and the power dissipated in the 10 ohm resistor for the circuit shown in Figure 1.35.

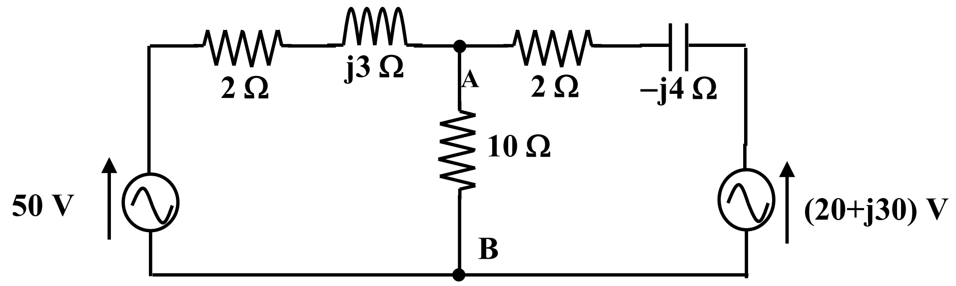


Figure 1.35

Solution 1.10

To find V_{AB} : Remove the 10 Ω load resistor across AB

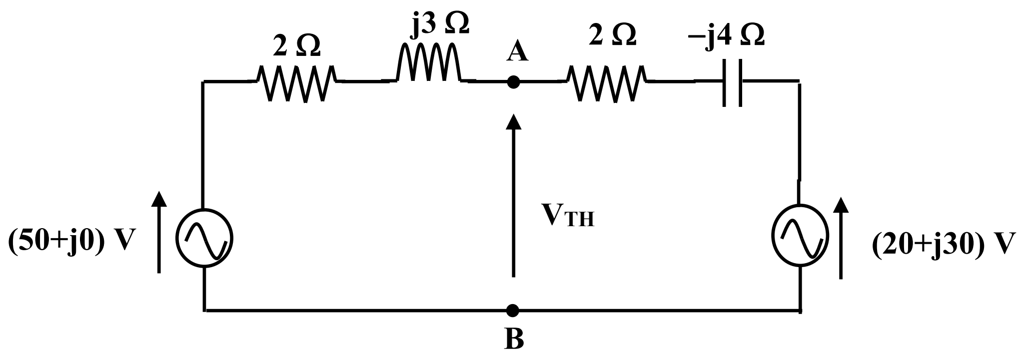


Figure 1.36: To find the Thevenin's equivalent voltage source, V_{TH}

$$I = \frac{50-(20+j30)}{(2+j3)+(2-j4)} = 10.29\angle -30.96^\circ \text{ A}$$

$$\therefore \quad V_{AB} = 50-(10.29\angle -30.96^\circ)(2+j3) = 22.88\angle -43.96^\circ \text{ V}$$

To find Z_{AB} : Short circuit all voltage sources as shown in the Figure 1.36 to find Z_{TH} or Z_{AB}.

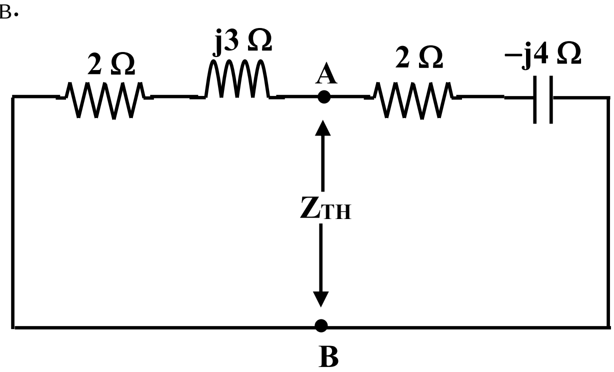


Figure 1.37: To find the Thevenin's equivalent impedance, Z_{TH}

$$Z_{AB} = Z_{TH} = \frac{(2+j3)(2-j4)}{(2+j3)+(2-j4)} = (3.88+j0.47)\Omega$$

Thevenin equivalent circuit:

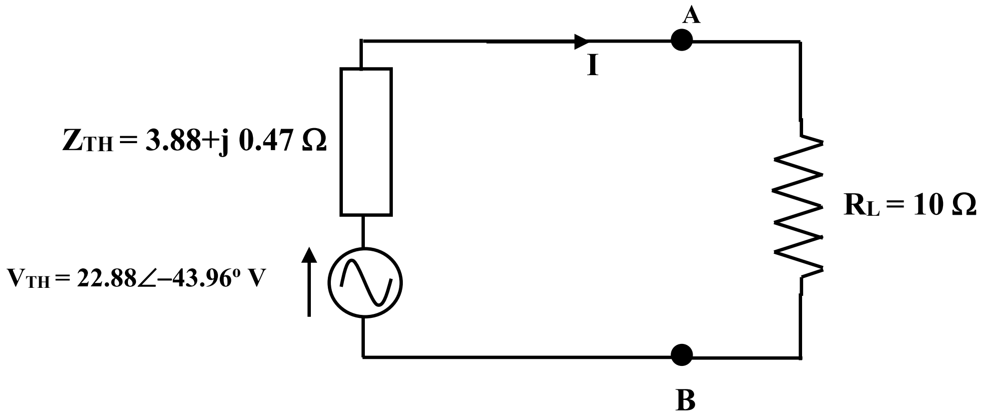


Figure 1.38: Thevenin equivalent circuit

Replace the 10-ohm resistor across AB:

$$\therefore\ I = \frac{V_{TH}}{Z_{TH}+R_L} = \frac{22.88\angle -43.96^\circ}{13.88+j0.47} = 1.65\angle -46^\circ\ \text{A}$$

Power dissipated in the 10-ohm resistor: $P_{10\Omega} = I^2 R_L = 1.65^2 \times 10 = 27.23$ W

1.10 Norton's Theorem

Norton's theorem states that an equivalent circuit consisting of a constant current source paralleled by a shunt resistor (or impedance for AC) can replace any linear circuitry between two points A and B.

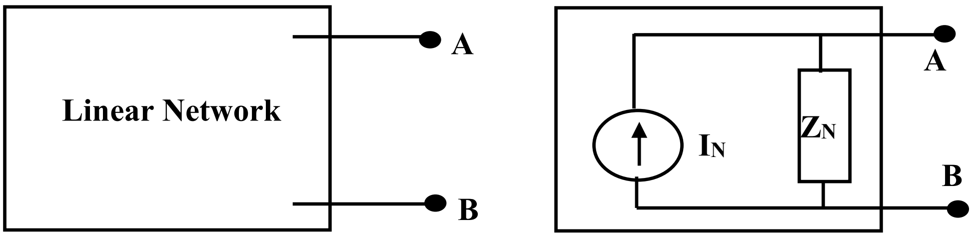
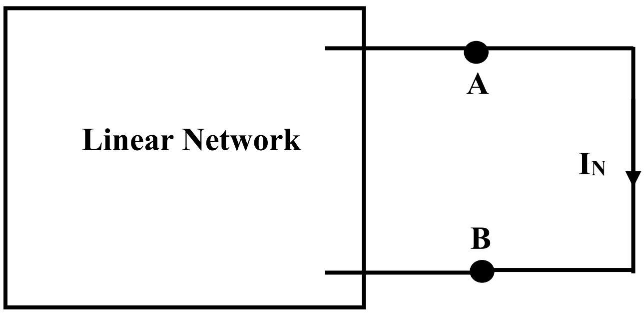


Figure 1.39: Norton equivalent circuit for a linear network

Norton constant current source I_N is the current that would flow at the short-circuited terminal AB.



Note: The load impedance is usually connected to AB

Figure 1.40

Norton shunt impedance Z_N is the impedance 'seen' into the circuit across points AB, with all internal sources set to zero. (Definition is the same as Z_{TH}).

1.10.1 Procedure in applying Norton's Theorem to points AB:

Steps:

1. *Disconnect the load from circuit and short the load terminals AB.*
2. *Calculate or compute or measure the current at AB. This is the short circuit current or Norton current source, I_N.* ***For computing you can only use either the application of Ohm's Law, Loop or Nodal Analysis method.***
3. *Disconnect the load at AB and set all internal circuit supply sources to zero i.e. short circuit all voltage sources; open circuit all the current sources. This is to find Z_N..*
4. *Looking into the 'source free' circuit, calculate the impedance that would exist between the open circuit terminals AB. This gives Z_N.*

5. *Form the Norton equivalent circuit, by connecting the current source I_N, in parallel with Z_N, between the terminals AB.*
6. *Reconstruct the load impedance Z_L or R_L at AB (if any).*
7. *Calculate the load voltage, current and power (if required).*

Note: Source Conversion is not allowed.

Example 1.11

Apply Norton's theorem to find the current in the 4 Ω resistor.

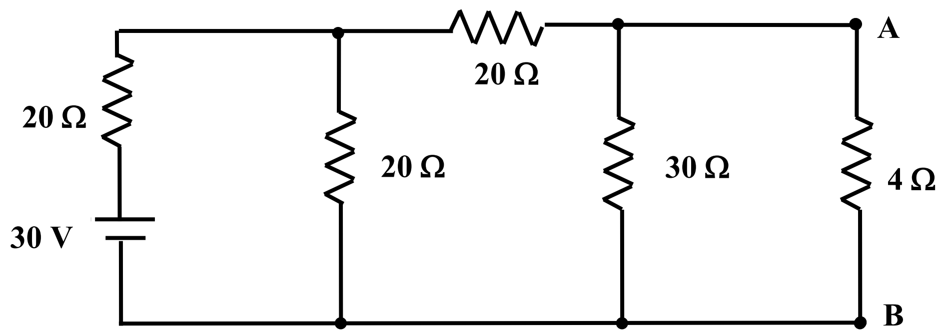


Figure 1.41

Solution 1.11

To find I_N, remove the 4 Ω resistor and s/c AB as shown in Figure 1.42.

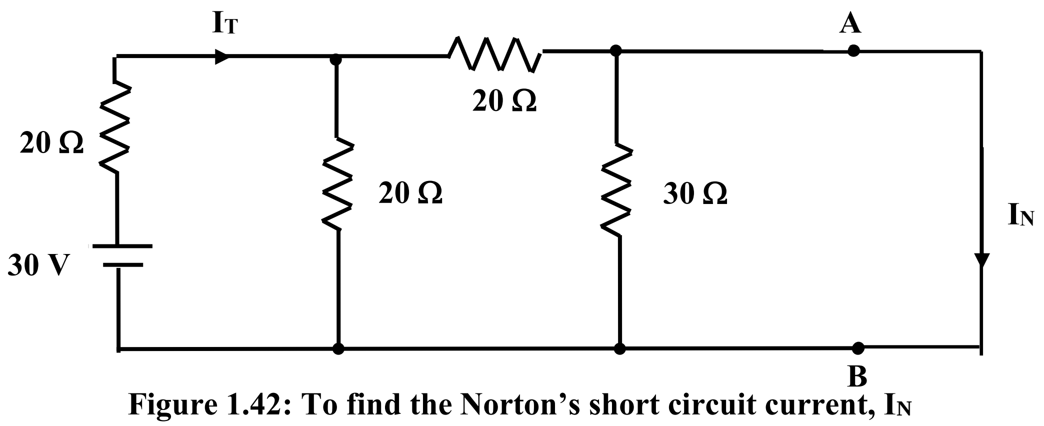


Figure 1.42: To find the Norton's short circuit current, I_N

$$R_{//} = 10\,\Omega$$
$$R_T = 10 + 20 = 30\,\Omega$$
$$I_T = \frac{30}{30} = 1\,\text{A}$$
$$I_N = \frac{1}{2} = 0.5\,\text{A}$$

To find Z_N set all the voltage sources to zero as shown in Figure 1.43.

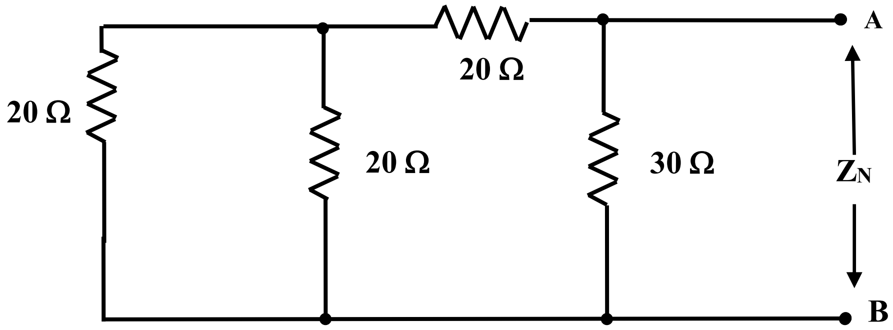


Figure 1.43: To find the Norton's impedance, Z_N

$R_{//1} = 10\,\Omega$

$R_N = R_{AB} = (10 + 20) // 30 = 15\,\Omega$

Norton Equivalent Circuit

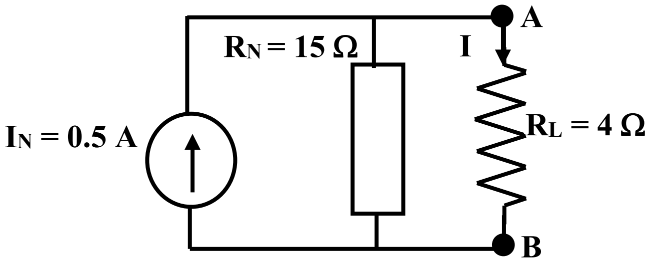


Figure 1.44: Norton equivalent circuit.

$$\text{Load current } I = \frac{I_N R_N}{R_N + R_L} = \frac{0.5 \times 15}{19} = 0.395\,\text{A}$$

Example 1.12

Apply Norton's theorem and calculate the current in the 10 ohm resistor for the circuit shown in Figure 1.45.

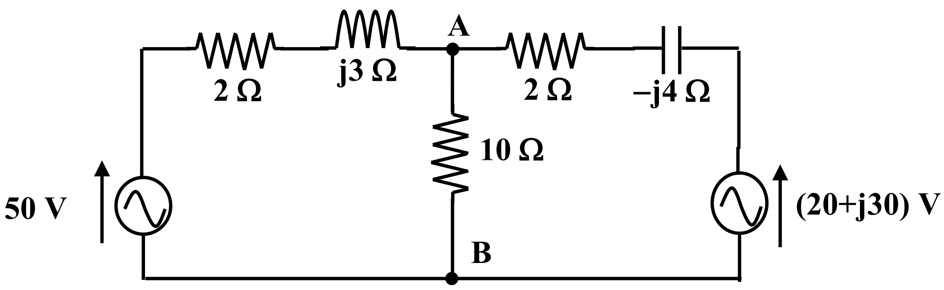


Figure 1.45

Solution 1.12

To find I_N : Remove the 10 Ω load resistor across AB

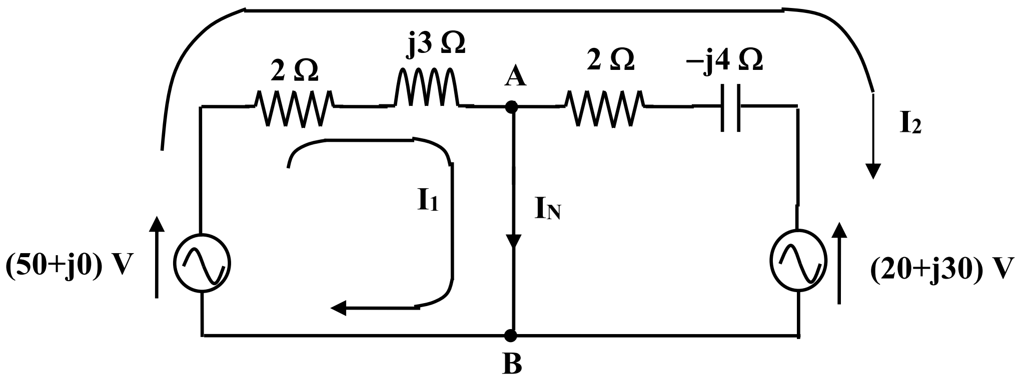


Figure 1.46: To find the Norton's short-circuit current, I_N

$$\begin{bmatrix} 2+j3 & 2+j3 \\ 2+j3 & 4-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50+j0 \\ 50+j0-(20+j30) \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 50 & 2+j3 \\ 30-j30 & 4-j1 \end{vmatrix}}{\begin{vmatrix} 2+j3 & 2+j3 \\ 2+j3 & 4-j1 \end{vmatrix}}$$

$$= \frac{50(4-j1)-(2+j3)(30-j30)}{(2+j3)(4-j1)-(2+j3)(2+j3)}$$

$$= \frac{50(4.123\angle -14^0)-(3.61\angle 56.3^0)(42.43\angle -45^0)}{(3.61\angle 56.3^0)(4.123\angle -14^0)-(3.61\angle 56.3^0)(3.61\angle 56.3^0)}$$

$$= \frac{206.15\angle -14^0 - 153.17\angle 11.3^0}{14.88\angle 42.3^0 - 13.03\angle 112.6^0}$$

$$= \frac{200.03 - j49.87 - 150.21 - j30.01}{11.01 + j10.01 + 5 - j12.03}$$

$$= \frac{49.82 - j79.88}{16.01 - j2.02} = \frac{94.14\angle -58^0}{16.13\angle -7.19^0}$$

$$= 5.84\angle -50.81^0 \text{ A}$$

To find Z_{AB}: Remove the 10 Ω load resistor across AB and short circuit all voltage sources as shown in the Figure 1.47 to find Z_N or Z_{AB}.

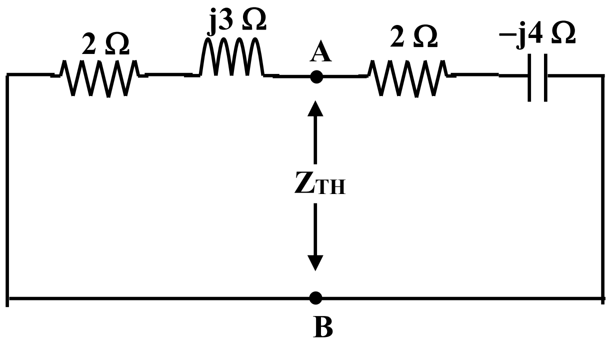


Figure 1.47: To find the Norton's equivalent impedance, Z_N

$$Z_{AB} = Z_N = \frac{(2+j3)(2-j4)}{(2+j3)+(2-j4)} = (3.88 + j0.47)\,\Omega$$

Norton equivalent circuit:

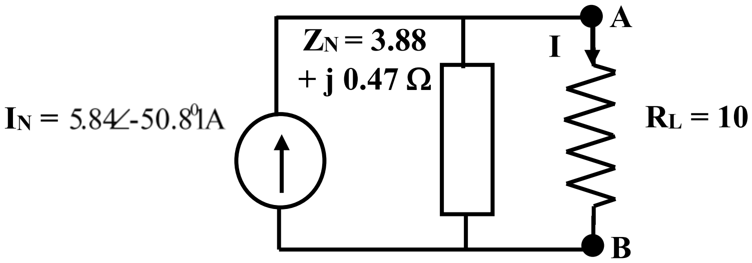


Figure 1.48: Norton equivalent circuit

Replace the 10-ohm resistor across AB:

$$\text{Load current } I = \frac{I_N Z_N}{Z_N + R_L}$$

$$= \frac{5.84\angle -50.81^0 \times (3.88 + j0.47)}{13.88 + j0.47}$$

$$= \frac{5.84\angle -50.81^0 (3.91\angle 6.91^0)}{13.89\angle 1.94^0}$$

$$= \frac{22.83\angle -43.9^0}{13.89\angle 1.94^0} = 1.64\angle -45.84^0\,\text{A}$$

1.11 Relationship Between Thevenin And Norton Equivalent Circuits

The same linear circuit, therefore, can be represented as Thevenin or Norton equivalent circuit.

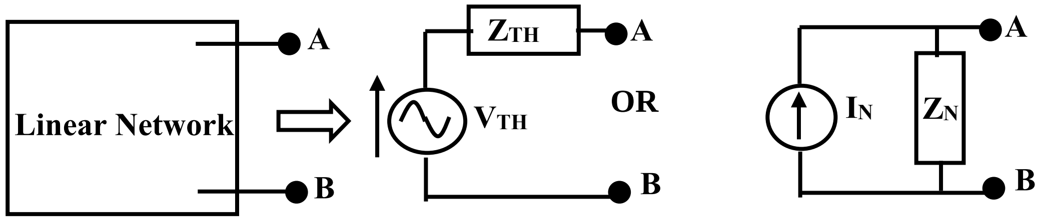


Figure 1.49: Relationship between Thevenin and Norton equivalent circuit

Conversion of Thevenin to Norton equivalent and vice versa

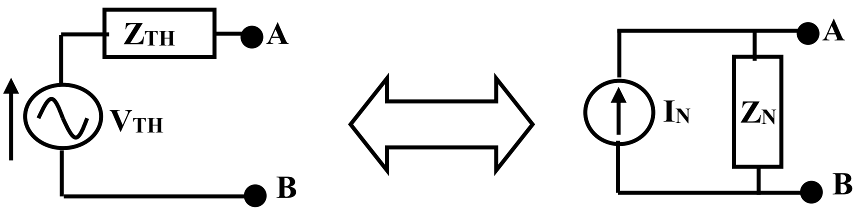


Figure 1.50: Conversion from Thevenin and Norton equivalent circuit

$$V_{TH} = I_N Z_N \qquad I_N = \frac{V_{TH}}{Z_{TH}}$$

$$Z_{TH} = Z_N \qquad Z_N = Z_{TH}$$

$$Z_{TH} = Z_N = \frac{V_{TH}}{I_N}$$

THREE PHASE CIRCUITS

Keywords: AC voltage, AC current, Frequency, Three Phase Voltage, Phase Sequence, Phase Voltage, Phase Current, Line Voltage, Line Current, Neutral Current, Phase Impedance, 3-wire, 4-wire, Balanced Load, Unbalanced Load, Star-Connection, Delta Connection, Power Factor, Lagging Power Factor, Leading Power Factor, Real Power, Reactive Power, Apparent Power, Power Factor Correction, Wattmeter, Two-Wattmeter Method

Objectives: Student should be able to analyse three phase circuits.

They should be able to:

2.1 Explain the principle of generating single and three phase AC Emf's.

2.2 Define phase sequence and state its importance.

2.3 Describe the usage of double subscript notation for three phase voltages and currents.

2.4 Construct circuit diagrams for three phase windings, star and delta.

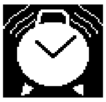
2.5 Draw three-phase star (Y or T) connected supply system and state the relationship between phase and line voltages.

2.6 Draw three phase star (Y or T) and delta (π) connected balanced load and with the aid of phasor diagram, derive the relationships for line voltages, phase voltages, line currents, power and power factor for the respective loads.

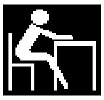
2.7 Explain power factor correction with the aid of power triangle.

2.8 Describe the need and importance of three-phase four-wire star connected system. Solve problems based on unbalanced star connected load.

2.9 Describe the various methods of measuring three phase power for star and delta loads.



Number of lecture hours required for completing this topic: 14 ***hours.***



Follow up activity to be completed by students: ***Tutorial 5, 6, 7 and 8.***

Practical exercise to be completed by students: ***Experiment 7,8,9 and 10.***

2.0 Introduction

This chapter mainly describes three phase supply system and three phase loads, because it is commonly used in every power and industrial applications and has several advantages over its single counter part. Here in this chapter it is assumed that the reader has some familiarity with the concepts of generating electricity and hence it is merely reviewed. The phase relationship between the voltages and the currents in a three phase circuit depends entirely on the type of load (balanced or unbalanced) and the connection (star or delta). With the aid of circuit and phasor diagram, the relationships between the line voltage, phase voltage, line current and phase current, for balanced star, delta connected supply, load and unbalanced star connected load are derived. Furthermore the problems associated with poor power factor, its effect on the three phase supply system and the method of improving it are identified. Finally, the chapter is concluded by describing the various methods that could be used for measuring three phase power in the case of star and delta connected load.

2.1 Generation of AC Emf's

Generation of AC voltage is based on the principle of electromagnetic induction. Alternating voltage may be generated by rotating a coil in a magnetic field (as in DC machine) or by rotating a magnetic field within a stationary coil (as in synchronous generator). According to Faraday's laws of electromagnetic induction, the value of the voltage generated (i.e. emf induced) depends upon the number of turns in the coil, strength of the field and the speed at which the coil or magnetic field rotates. The direction of the induced emf is given by Fleming's right hand rule.

2.1.1 Generation of single phase emf

In Figure 2.1, AA_1 represents a conductor loop (rectangular coil) driven anti-clockwise in the magnetic field between the poles NS. For the position shown, the emf generated in the loop is a maximum. The direction of the emf is indicated by the arrowheads and is from the 'start' terminal towards the 'finish' terminal. Let's regard this direction as positive. Consequently, the emf generated in the coil AA_1 can be represented by the curve shown in Figure 2.2.

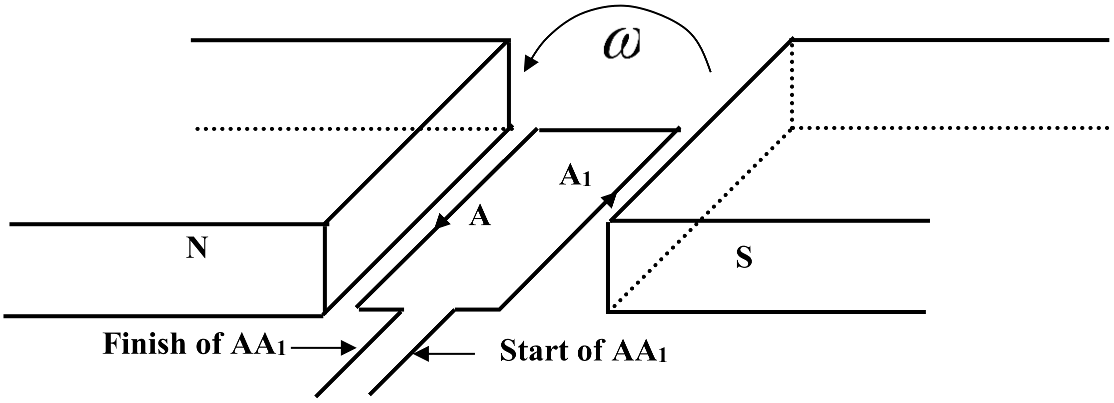


Figure 2.1: Generation of single phase emf

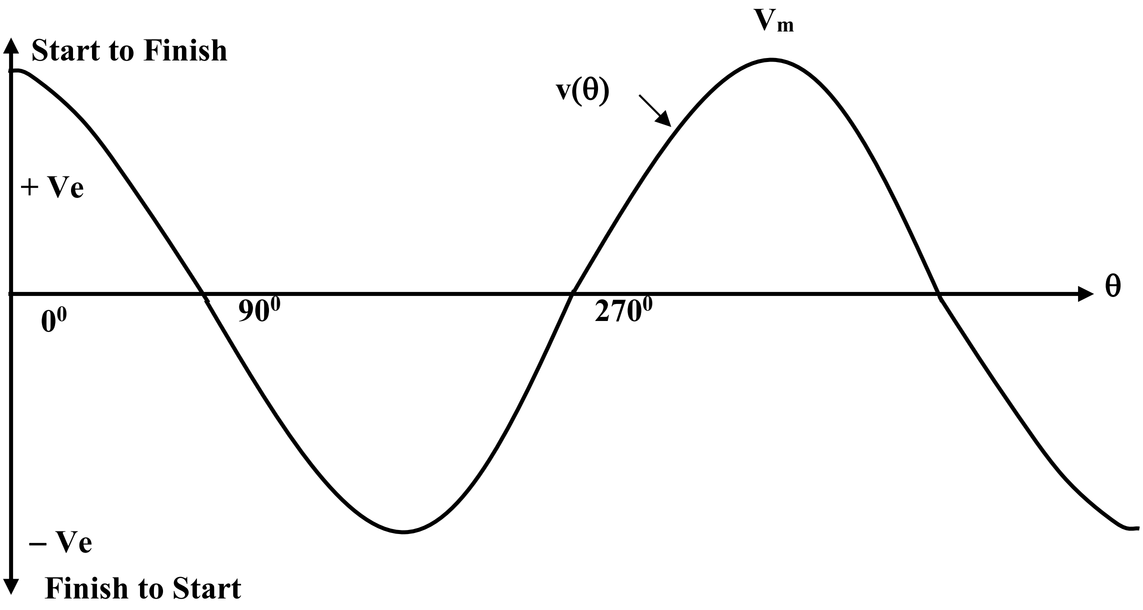


Figure 2.2: Waveforms of single-phase emf

2.1.2 Generation of three-phase emf

In Figure 2.3, AA_1, BB_1 and CC_1 represent three identical loops fixed to one another at angles of 120°, each loop terminating in a pair of slip-rings on the shaft as indicated in Figure 2.4. Those slip rings connected to the sides A, B and C are referred to as the 'finishes' of the respective phases and those connected to A_1, B_1 and C_1 as the 'starts'.

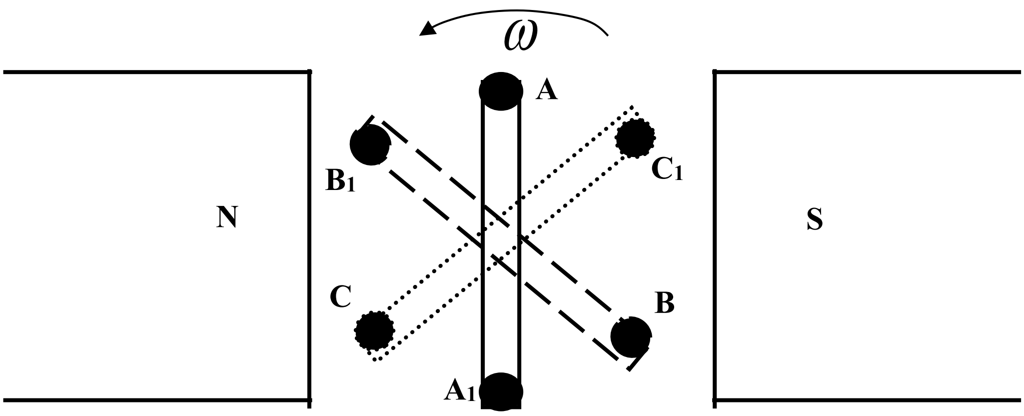


Figure 2.3: Generation of three-phase emf

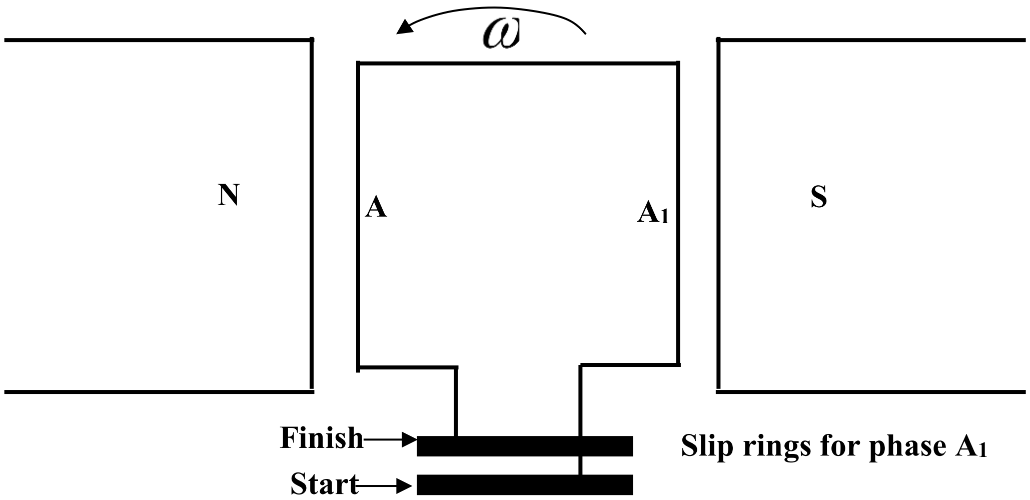


Figure 2.4: Loop AA_1 at instant of maximum emf

Suppose the three coils are rotating anti-clockwise at a uniform angular speed of ω rad/s in the magnetic field due to poles NS. The emf generated in loop AA_1 is zero for the position shown in Figure 2.3. When this loop has moved through 90° to the position shown in Figure 2.4, the generated emf is at its maximum value, with its direction round the loop being from the 'start' slip-ring towards the 'finish' slip-ring. Again taking this direction as positive, the emf induced in AA_1 can therefore be represented by the curve v_A in Figure 2.5. Since the loops are being rotated anti-clockwise, it is evident from Figure 2.3 that the emf generated in side B has exactly the same magnitude as that generated in side A, but lags by 120^0 (or 1/3 cycle). Similarly, the emf generated in side C lags that of side B by another 120° Consequently, the emf's generated in the three loops are represented by the three equally spaced curves v_A, v_B and v_C shown in Figure 2.5.

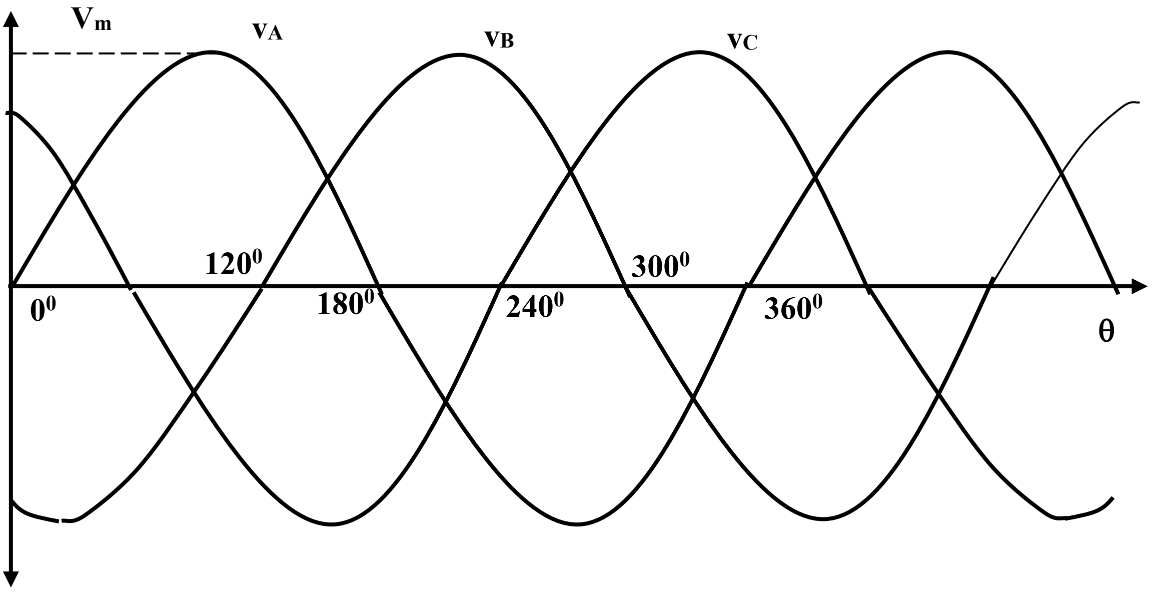


Figure 2.5: Waveforms of three phase emf

If the instantaneous value of the emf generated in phase AA_1 is written as

$$v_A = V_m \sin\theta,$$

Then instantaneous emf of $BB_1 = v_B = V_m \sin(\theta - 120^0)$

and instantaneous emf of $CC_1 = v_C = V_m \sin(\theta - 240^0) = V_m \sin(\theta + 120^0)$

These three emf's are termed the phase voltages. In phasor form, and taking, say, V_A as the reference, we have

$$v_A = V\angle 0^0$$

$$v_B = V\angle -120^0 = V\angle +240^0$$

$$\text{and} \quad v_C = V\angle -240^0 = V\angle +120^0$$

$$\text{where } V = 0.707\, V_m$$

2.2 Phase Sequence

Phase sequence is the order in which the voltages of the three phases attain their maximum values. In the configuration of Figure 2.3 therefore, the maximum of emf occur in the sequence A - B - C. The phase sequence is thus ABC which is known as the ***POSITIVE PHASE SEQUENCE*** as shown in Figure 2.6.

If the maximum of emf occur in the sequence A – C – B, the phase sequence will then be ACB which is known as ***NEGATIVE PHASE SEQUENCE*** as in Figure 2.7. In power supply systems, positive phase sequence is usually assumed.

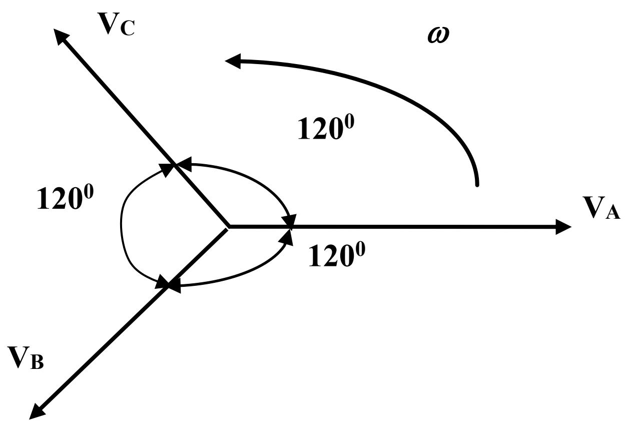


Figure 2.6: Positive phase sequence

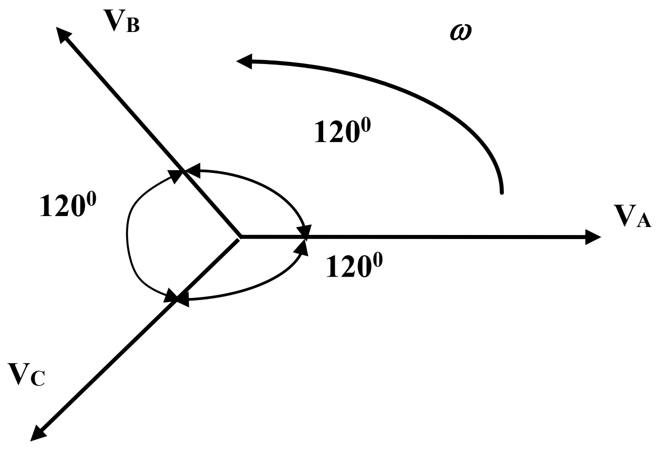


Figure 2.7: Negative phase sequence

Note: the direction of rotation of the phasors is always anti-clockwise.

Importance of phase sequence:

(i) *If the sequence is wrong, the required direction of rotation of machines will not be obtained.*

(ii) *All generators connected to the same system must have the same phase sequence.*

Example 2.1

Express the 3-phase balanced voltages V_A, V_B, V_C, taking $V_A = |V|\angle 30^o$ with respect to the reference. Draw the corresponding phasor diagram, for (a) Phase sequence ABC and (b) Phase sequence ACB.

Solution 2.1

Given $|V_A| = |V_B| = |V_C| = |V|$

(a) For phase sequence ABC

$\mathbf{V_A = |V|\angle 30^o = |V|\angle 30^o}$ **Volts,** $\mathbf{V_B = |V|\angle(30^o - 120^o) = |V|\angle -90^o}$ **Volts,**

$\mathbf{V_C = |V|\angle (30^o - 240^o) = |V|\angle -210^o}$ **Volts** $\mathbf{= |V|\angle +150^o}$ **Volts.**

(b) For phase sequence ACB

$\mathbf{V_A = |V|\angle 30^o = |V|\angle 30^o}$ **Volts,** $\mathbf{V_C = |V| \angle(30^o - 120^o) = |V|\angle -90^o}$ **Volts,**
$\mathbf{V_B = |V|\angle(30^o - 240^o) = |V|\angle -210^o}$ **Volts** $\mathbf{= |V|\angle +150^o}$ **Volts.**

The phasor diagram is shown in Figure2.8 for positive phase sequence ABC and negative phase sequence ACB.

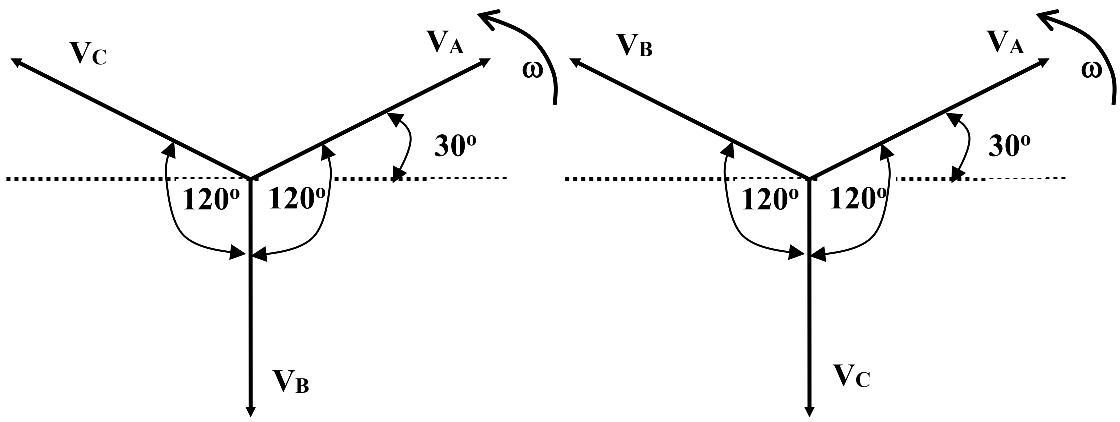


Figure 2.8: Phasor diagram for ABC and ACB phase sequence

2.3 Double Subscript Notation

Voltages always reflect potential differences between any two points in an electric circuit. They can be denoted according to the name of the element between the two points; for example, V_C is the voltage across an element C. They can also be denoted according to the symbols given to the two points; for example, V_{AB} is the voltage of point A with respect to point B. This is called double-subscripted voltage.

In three-phase circuits, it is important to maintain a conventional positive direction in which voltages are measured. Thus, $\mathbf{V_{AB}}$ is used to denote the voltage of the **A** line with respect to the **B** line. $\mathbf{V_{BC}}$ is used to denote the voltage of the **B** line with respect to the **C** line. $\mathbf{V_{CA}}$ is used to denote the voltage of the **C** line with respect to the **A** line. The voltages $\mathbf{V_{AN}}$, $\mathbf{V_{BN}}$, $\mathbf{V_{CN}}$ is used to denote the voltage of the **A, B, C** line with respect to the **NEUTRAL** point, respectively. When this double subscript notation is extended to currents, then, for example, I_{AB} denotes a current, which flows from point A to point B. Similarly, I_{BC} and I_{CA} denotes a current, which flows from point B to point C, and from point C to point A, respectively.

2.4 Star and Delta Connections of Three Phase Windings

The output voltage levels produced by a three-phase generator depend on the connection arrangement of the three individual generator coils. The two possible connections are knows as *wye* (Y) and *delta* (Δ), because of circuit resemblances to those symbols.

2.4.1 Delta (Δ) Connection

The three phases (or windings) of Figure 2.3 can be represented in Figure 2.9 where the phases are shown isolated from each other. This arrangement requires six line conductors and is therefore expensive. Simplification can be made by joining A_1 and B together, thus enabling conductors 2 and 3 to be replaced by one single conductor. Similarly, joining conductors between 4 and 5, 6 and 1 can also be made. However, before joining conductors 6 and 1, it should be realized that the resultant voltage between conductors 6 and 1 is zero at every instant so that joining conductors 6 and 1 will not cause any circulating current around the circuit.

Instantaneous value of total emf acting from 6 and 1 is

$$\begin{aligned} v_A + v_B + v_C &= E_m \sin\theta + E_m \sin(\theta - 120^0) + E_m \sin(\theta - 240^0) \\ &= 0 \end{aligned}$$

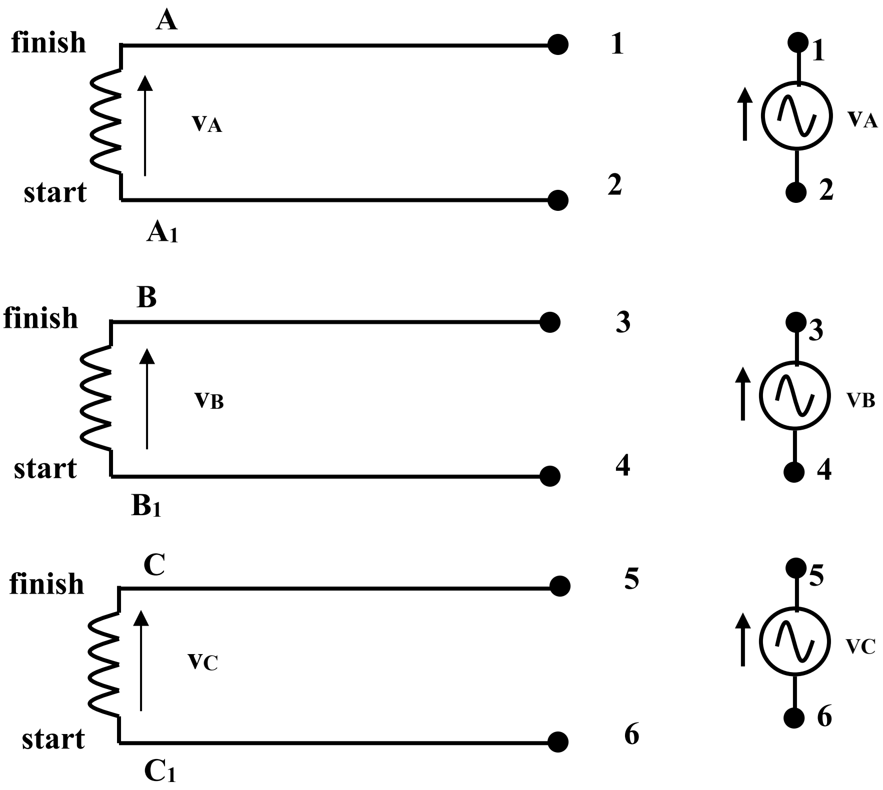


Figure 2.9: Three phase supply windings with six conductors

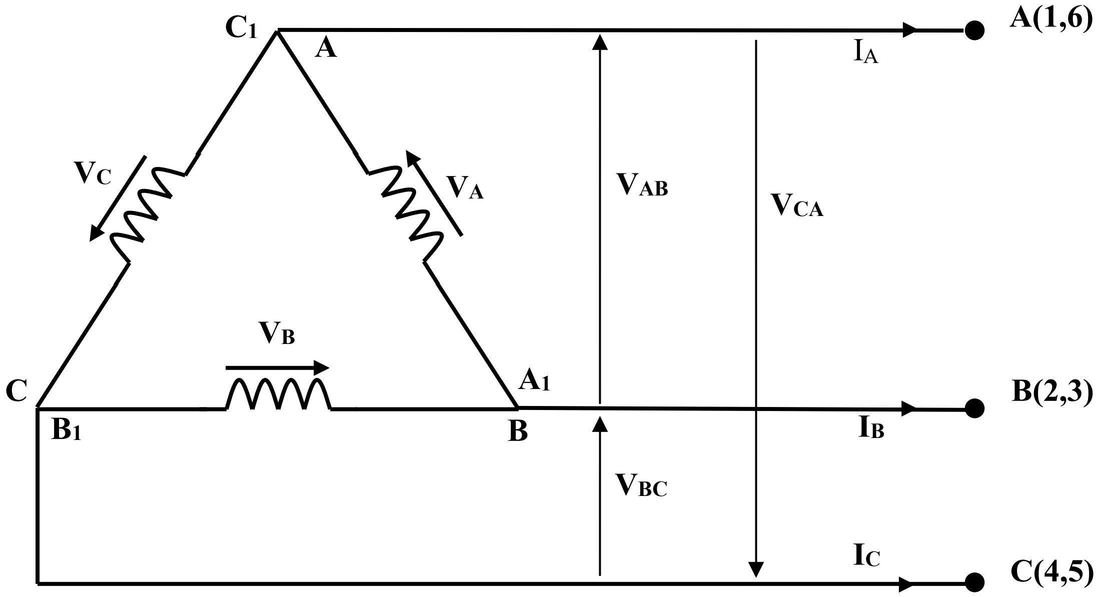


Figure 2.10: Shows the mesh (or delta) connection of a 3-phase, 3-wire supply system, 'start' of one phase must be connected to the 'finish' of another phase

$|\mathbf{V_{AB}}| = |\mathbf{V_{BC}}| = |\mathbf{V_{CA}}| = \mathbf{V_L}$ ($\mathbf{V_L}$ ---- the **magnitude** of the line voltage)

And $|\mathbf{V_A}| = |\mathbf{V_B}| = |\mathbf{V_C}| = \mathbf{V_{PH}}$ ($\mathbf{V_{PH}}$ ---- the **magnitude** of the phase voltage)

so that $\mathbf{V_L} = \mathbf{V_{PH}}$

but the line current magnitude $\mathbf{I_L} \neq$ phase current magnitude $\mathbf{I_{PH}}$

If a balanced 3-phase load is connected to this balanced delta connected supply then the magnitude of line currents $|\mathbf{I_A}| = |\mathbf{I_B}| = |\mathbf{I_C}| = \mathbf{I_L}$

2.4.2 Star (Y or T) Connection

If all the 'starts' (or 'finishes') of the windings in Figure 2.9 are joined together to a single neutral terminal (N), we have the star or Y connection as shown in Figure 2.11. There are four output terminals: A, B, C and N. This is known as three-phase four-wire system.

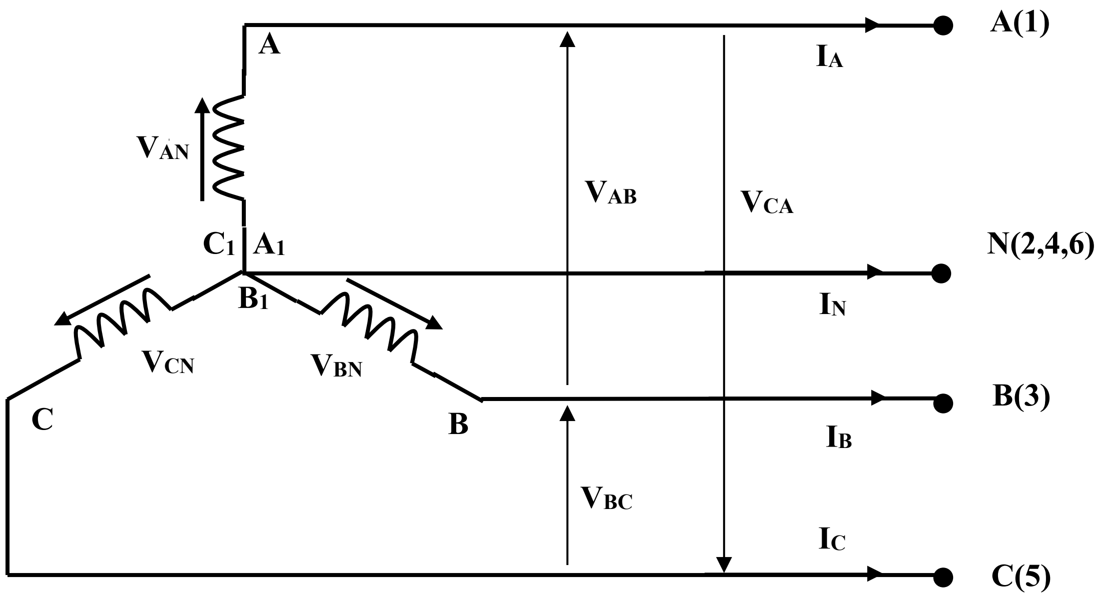


Figure 2.11: Star-connected 3-phase, 4-wire supply system

2.5 Voltages and Currents in a Star - connected System

For balanced supply, we have the

Line voltage magnitude $|\mathbf{V_{AB}}| = |\mathbf{V_{BC}}| = |\mathbf{V_{CA}}| = \mathbf{V_L}$, and

Phase voltage magnitude $|\mathbf{V_{AN}}| = |\mathbf{V_{BN}}| = |\mathbf{V_{CN}}| = \mathbf{V_{PH}}$

where N is the star or neutral point of the 3-phase star connected supply.

It is obvious that

$$\mathbf{I_L = I_{PH}} \quad \textbf{and} \quad \mathbf{V_L \neq V_{PH}}$$

From Figure 2.11 and by Kirchhoff's voltage law, it can be seen that

$$\mathbf{V_{AB} = V_{AN} + V_{NB} = V_{AN} + (-V_{BN})}$$
$$\mathbf{V_{BC} = V_{BN} + V_{NC} = V_{BN} + (-V_{CN})}$$
$$\mathbf{V_{CA} = V_{CN} + V_{NA} = V_{CN} + (-V_{AN})}$$

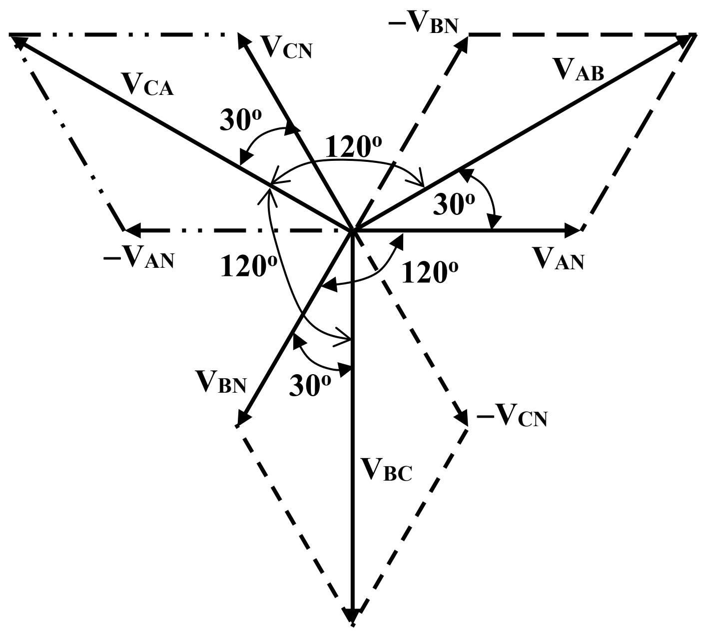


Figure 2. 12: Relationship between line and phase voltages in a 3-phase, balanced star-connected 4-wire supply system

The line and phase voltages are plotted in the phasor diagram of Figure 2.12, taking V_{AN} as the reference (in fact you can use any one of the above voltages as the reference). It can be seen that the three **LINE** voltages are all equal in magnitude **(=V_L)** and displaced from one another by **120^0**, and that the three **PHASE** voltages are also equal in magnitude **(=V_{PH})** and displaced from one another by 120^0. Also

$$\mathbf{V_L = 2\ V_{PH} \cos 30^o}$$
$$\mathbf{V_L = \sqrt{3}\ V_{PH}}$$

The line voltages is each of magnitude $\sqrt{3}$ V_{PH} and leading the phase voltages by 30^o.

2.6 Three-Phase Balanced Supply with Star and Delta Connected Balanced Loads

2.6.1 Three Phase 4-Wire Star Connected Load

The 3-phase load of Figure 2.13 is supplied from a balanced star connected supply. The supply neutral point N is connected to the load neutral point N by the neutral conductor.

In general, by Kirchhoff's Current Law,

$$\mathbf{I_A + I_B + I_C + I_N = 0} \text{ (vector addition) } \textbf{……………….(1)}$$

and the supply phase voltages are also the load phase voltages (provided that the impedance's of the line conductors are neglected) so that

$$I_A = \frac{V_{AN}}{|Z_A|\angle\phi_A} \qquad I_B = \frac{V_{BN}}{|Z_B|\angle\phi_B} \qquad I_C = \frac{V_{CN}}{|Z_C|\angle\phi_C}$$

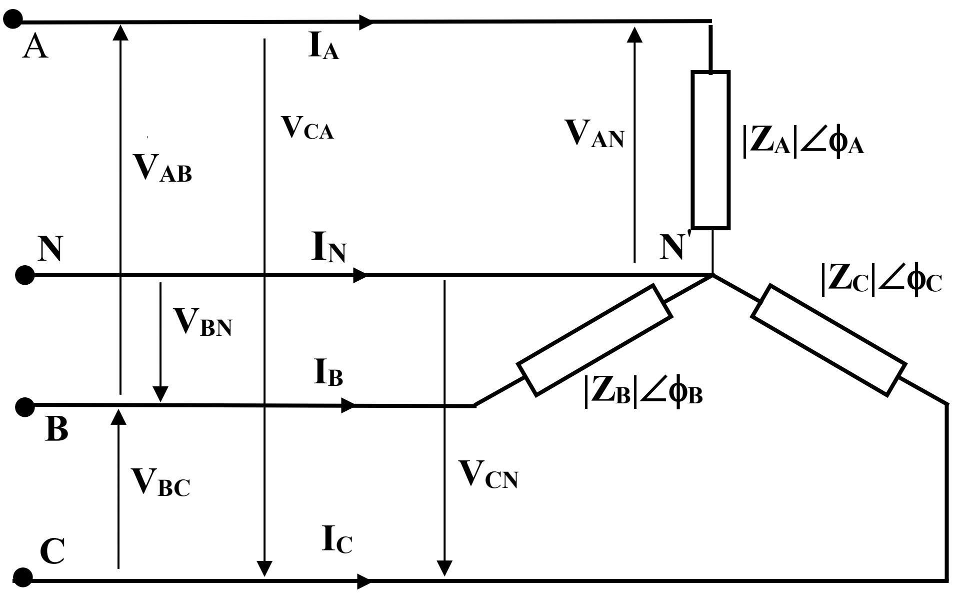


Figure 2.13: Three phase, 4-wire, star (or Y)-connected load

The total power consumption of the 3 phase load is equal to the sum of the three powers consumed in each phase, i.e.

$$\begin{aligned} P_T = {} & |V_{AN}|\,|I_A|\cos(\text{angle between } V_{AN} \text{ \& } I_A) + \\ & |V_{BN}|\,|I_B|\cos(\text{angle between } V_{BN} \text{ \& } I_B) + \\ & |V_{CN}|\,|I_C|\cos(\text{angle between } V_{CN} \text{ \& } I_C) \end{aligned}$$

When the three phases of the load are identical (3 phase balanced load) i.e.

$$Z_A = Z_B = Z_C = Z, \qquad \phi_A = \phi_B = \phi_C = \phi$$

Then $|I_A| = |I_B| = |I_C| = I_L = I_{PH}$

and remembering $V_L = \sqrt{3}\ V_{PH}$ or $V_{PH} = V_L / \sqrt{3}$

so that $\mathbf{P = 3\ V_{PH}\ I_{PH} \cos\phi}$

$\mathbf{= 3\ (V_L / \sqrt{3})\ I_L \cos\phi}$

$\mathbf{= \sqrt{3}\ V_L\ I_L \cos\phi}$

Also, taking I_A as reference and substituting for I_A, I_B and I_C in equation (1), we note that $I_A + I_B + I_C = I_L \angle 0^o + I_L \angle{-120^o} + I_L \angle{-240^o}$

$= I_L\{(1-0.5-0.5) + j\,(0 - 0.8600 + 0.8600)\} = 0$

Since $\mathbf{I_A + I_B + I_C = 0}$ (see the phasor diagram of Figure 2.14), therefore $I_N = 0$ i.e. **the 4th wire (the neutral wire) is not required for a balanced Y-connected load connected to a balanced Y-connected supply.**

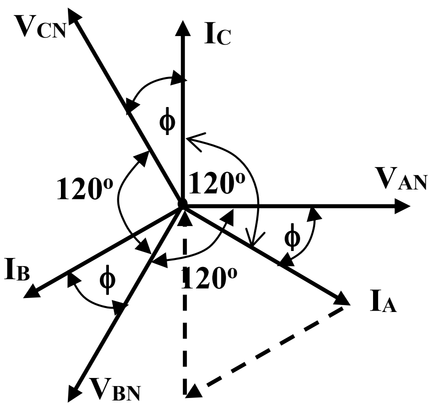


Figure 2.14: Phasor diagram for a balanced star-connected net inductive load

Example 2.2

A three-phase three-wire **199 V** ABC supply system feeds a balanced Y-connected load, as shown in Figure 2.15, with a phase impedance of $Z = 23 \angle 30^o\ \Omega$. Taking $\mathbf{V_{AN}}$ as the reference voltage, find the line currents and draw the phasor diagram.

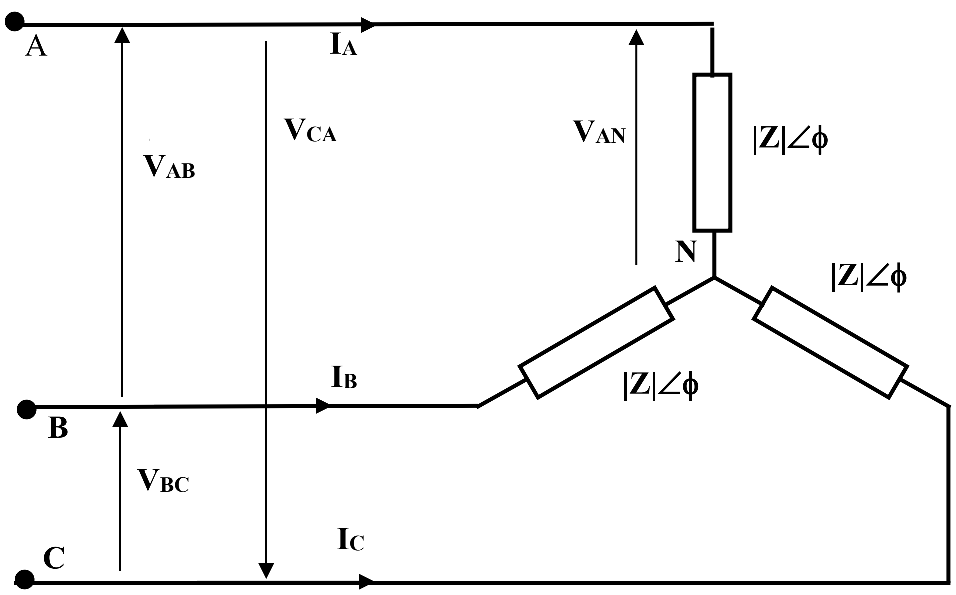


Figure 2.15: Balanced star connected load

Solution 2.2

Here the line-to-line voltage magnitude, V_L, in rms value, is **199 V**. As the supply and the load are both balanced, the load phase voltages must also be balanced with

$$V_{PH} = V_L / \sqrt{3} = 199 / \sqrt{3} = 114.89 \text{ V}$$

Taking $\mathbf{V_{AN}}$ as the reference, so $V_{AN} = 114.9 \angle 0^0$ V, then

$V_{BN} = 114.89 \angle{-120^0}$ V and

$V_{CN} = 114.89 \angle{-240^0}$ V or $114.9 \angle 120^0$ V

$I_A = V_{AN} / Z_A = 114.89 \angle 0^o \ / \ 23 \angle 30^o = 5 \angle{-30^o}$ A

$I_B = V_{BN} / Z_B = 114.89 \angle{-120^o} \ / \ 23 \angle 30^o = 5 \angle{-150^o}$ A

$I_C = V_{CN} / Z_C = 114.89 \angle{-240^o} \ / \ 23 \angle 30^o = 5 \angle{-270^o}$ A $= 5 \angle 90^o$ A

OR

$I_A = V_{AN} / Z_A = 114.89 \angle 0^o \ / \ 23 \angle 30^o = 5 \angle{-30^o}$ A

Since this is a ABC system and that the load and supply are balanced, all line currents are equal in magnitude and lagging by 120^o apart. Thus

$I_B = 5 \angle{-30^o} - 120^o = 5 \angle{-150^o}$ A and

$I_C = 5 \angle{-150^o} - 120^o = 5 \angle{-270^o} = 5 \angle 90^o$ A

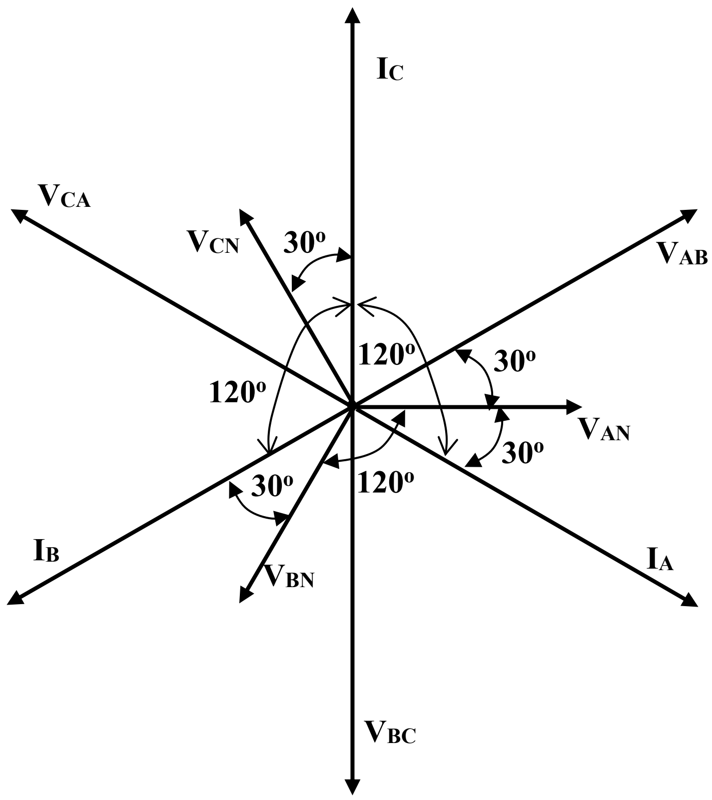


Figure 2.16: Phasor diagram for balanced star connected load

Example 2.3

Each phase of a star-connected load consists of a resistance of 100 Ω in parallel with a capacitor of capacitance 31.8 μF. Calculate the line currents, the power absorbed and the power factor when the load is connected to a 416 V, 3-phase, 50 Hz supply. Take V_{BC} as reference.

Solution 2.3

Given: $V_L = 416$ V, $V_{BC} = 416\angle 0^0$ V, f = 50 Hz, $R_L = 100\ \Omega$; C = 31.8 μF,

$$V_{PH} = \frac{V_L}{\sqrt{3}} = \frac{416}{\sqrt{3}} = 240.18\text{V}\ ; X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 31.8 \times 10^{-6}} = 100\ \Omega$$

$$Z_{PH} = \frac{100 \times -j100}{100 - j100} = (50 - j50) = 70.71\angle -45^\circ\ \Omega$$

$$Since\ \ V_{BC} = 416\angle 0^\circ V\ \ , V_{BN} = 240.18\angle -30^0 V$$

$$I_B = \frac{V_{BN}}{Z_{PH}} = \frac{240.18\angle -30^\circ}{70.71\angle -45^\circ} = 3.397\angle 15^\circ A,$$

then $I_C = 3.397\angle -105^\circ A$ and $I_A = 3.397\angle -225^\circ A$

$\cos\phi = \cos(45^0) = 0.707$ leading

$P_T = 3V_{PH} I_{PH} \cos\phi = 3 \times 240.18 \times 3.397 \times 0.707 = 1.731$ kW

2.6.2 Three Phase 3-Wire Delta Connected Load

The 3-phase delta connected load of Figure 2.17 can be supplied from a star or delta connected supply. Only three line conductors are required. For this arrangement,

supply line voltages = load phase voltages

The phase currents are:

$$I_{AB} = \frac{V_{AB}}{|Z_A|\angle\phi_A} \qquad I_{BC} = \frac{V_{BC}}{|Z_B|\angle\phi_B} \qquad I_{CA} = \frac{V_{CA}}{|Z_C|\angle\phi_C}$$

When the supply and the load are both balanced, then

$$\mathbf{Z_A = Z_B = Z_C = Z}, \quad \phi_A = \phi_B = \phi_C = \phi$$

so that the magnitude of load phase current $|\mathbf{I_{AB}}| = |\mathbf{I_{BC}}| = |\mathbf{I_{CA}}| = \mathbf{I_{PH}}$, and

magnitude of line current $|\mathbf{I_A}| = |\mathbf{I_B}| = |\mathbf{I_C}| = \mathbf{I_L}$

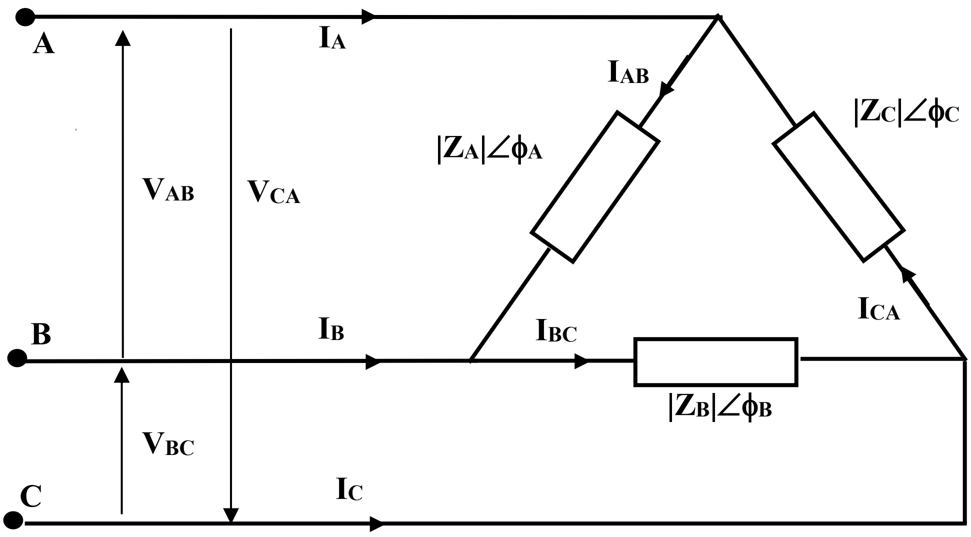


Figure 2.17: Three phase, 3-wire delta connected load

From Figure 2.17 and apply Kirchhoff's current law at each junction A, B and C of the load, we have

$I_A = I_{AB} - I_{CA}$, $I_B = I_{BC} - I_{AB}$, and $I_C = I_{CA} - I_{BC}$

so that from the current phasor diagram of Figure 2.18 (**for balanced supply and load**), it is obvious that

$$\mathbf{I_L = 2I_{PH}\cos 30^o = \sqrt{3}\, I_{PH}}$$

The line currents is each of magnitude $\sqrt{3}\, I_{PH}$ and lagging the phase currents by 30^o.

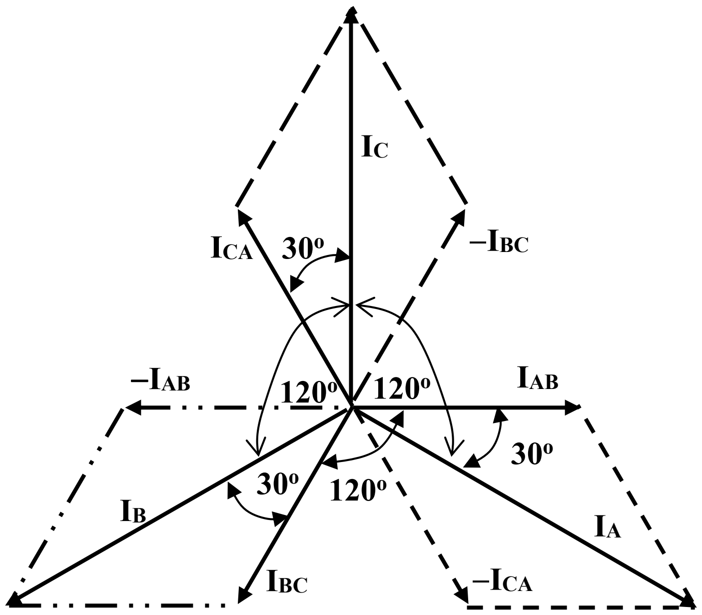


Figure 2.18: Current relationship in balanced delta connected load

The total power consumption of the 3 phase load is equal to the sum of the three powers consumed in each phase, i.e.

$P = \ |V_{AB}|\ |I_{AB}| \cos$ (angle between V_{AB} & I_{AB}) +

$|V_{BC}|\ |I_{BC}| \cos$ (angle between V_{BC} & I_{BC}) +

$|V_{CA}|\ |I_{CA}| \cos$ (angle between V_{CA} & I_{CA})

so that when the load and supply are both balanced, then

$P = 3\ V_{PH}\ I_{PH} \cos \phi = 3\ V_L\ (\ I_L / \sqrt{3}\ ) \cos \phi = \sqrt{3}\ V_L\ I_L \cos \phi$

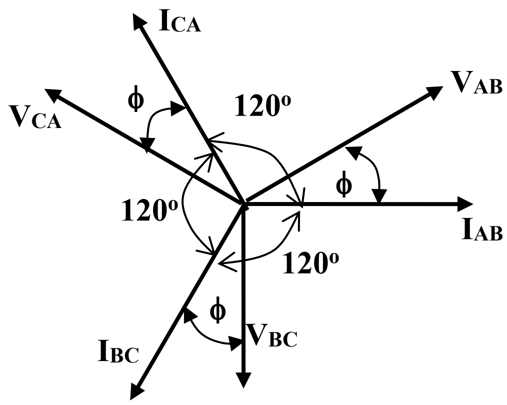


Figure 2.19: Phasor diagram for balanced delta connected net inductive load

Example 2.4

A 3-ϕ, 3−wire 199 V, ABC supply system feeds a balanced delta connected load, as shown below with a phase impedance of **Z = 23∠30°**. Taking phase voltage **V_{AB}** as reference voltage, find the line currents and draw the phasor diagram.

Solution 2.4

Given $V_L = 199$ V; $Z = 23\angle 30^\circ$

Taking phase voltage **V_{AB}** as reference $V_{AB} = 199\angle 0^\circ$ V,

$V_{BC} = 199\angle -120^\circ$ V,

$V_{CA} = 199\angle -240^\circ = 199\angle 120^\circ$ V

Phase currents $I_{AB} = \dfrac{V_{AB}}{Z} = \dfrac{199\angle 0^\circ}{23\angle 30^\circ} = 8.652\angle -30^\circ$ A

$I_{BC} = \dfrac{V_{BC}}{Z} = \dfrac{199\angle -120^\circ}{23\angle 30^\circ} = 8.652\angle -150^\circ$ A

$I_{CA} = \dfrac{V_{CA}}{Z} = \dfrac{199\angle 120^\circ}{23\angle 30^\circ} = 8.652\angle 90^\circ$ A

Line current $I_L = \sqrt{3}I_{PH} = \sqrt{3}\times 8.652 = 14.985A$

Line current lags the respective phase current by 30^0.

Therefore the line currents $I_A = 14.985\angle -30^0 - 30^0 = 14.985\angle -60^0$ A

$I_B = 14.985\angle -150^0 - 30^0 = 14.985\angle -180^0$ A

$I_C = 14.985\angle 90^0 - 30^0 = 14.985\angle 60^0$ A

OR

Phase currents $I_{AB} = \dfrac{V_{AB}}{Z} = \dfrac{199\angle 0^\circ}{23\angle 30^\circ} = 8.652\angle -30^\circ$ A

Because this is a ABC system and that load and supply are balanced, all phase currents are equal in magnitude and lagging by 120° apart. Thus

$I_{BC} = 8.652\angle -30^\circ - 120^\circ = 5\angle -150^\circ$ A and

$I_{CA} = 8.652\angle -150^\circ - 120^\circ = 5\angle -270^\circ = 5\angle 90^\circ$ A

Line current $I_L = \sqrt{3}I_{PH} = \sqrt{3}\times 8.652 = 14.985A$

Line current lags the respective phase current by 30^0.

Therefore the line currents $I_A = 14.985\angle -30^0 - 30^0 = 14.985\angle -60^0$ A

Because this is a ABC system and that load and supply are balanced, all line currents are equal in magnitude and lagging by 120° apart. Thus

$I_B = 14.985\angle{-60^0-120^0} = 14.985\angle{-180^0}$ A

$I_C = 14.985\angle{-180^0-120^0} = 14.985\angle{-300^0}$ A or $14.985\angle{60^0}$ A

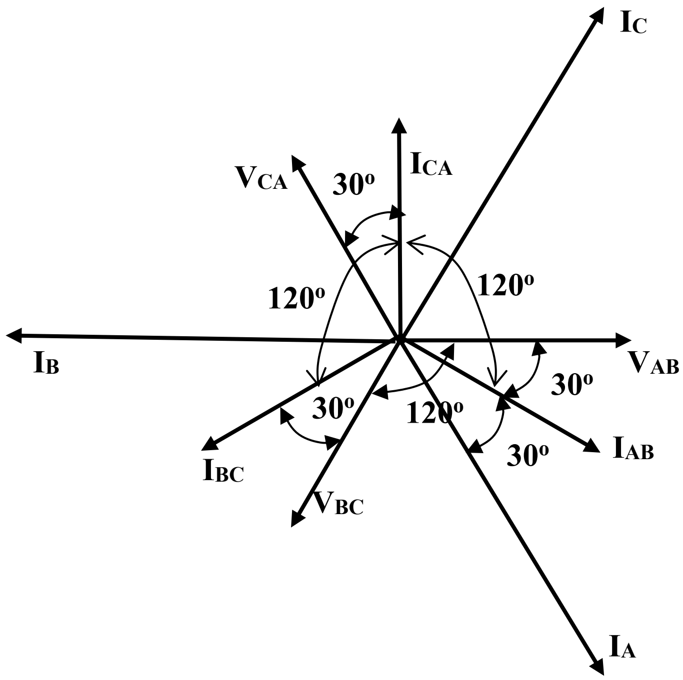


Figure 2.20: Phasor diagram for balanced delta connected load

Example 2.5

A 3-ϕ balanced delta-connected load is connected to a 3-ϕ balanced, 50 Hz, 400 V supply. The load is consuming a power of 100 kW at a power factor of 0.8 lagging.

Find: (a) The line current.

(b) The values of the passive elements which make up the load in each phase.

Solution 2.5

(a) Power = $\sqrt{3}$ x V_L x I_L x power factor

$100 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.8$

$I_L = 180.4$ A

(b) $I_{PH} = I_L / \sqrt{3} = 180.4 / \sqrt{3} = 104.2$ A

$V_{PH} = V_L = 400$ V

Therefore, magnitude of the impedance/phase $Z = V_{PH} / I_{PH}$

$= 400 / 104.2 = 3.84\ \Omega$

so that impedance (in complex form)/phase $Z = 3.84 \angle\cos^{-1} 0.8$

$$= 3.84 \angle 36.7^{\circ} = (3.08 + j2.29)\ \Omega$$

i.e. the resistance/phase $R = 3.08\ \Omega$

and $X_L = \omega L = 2.29\ \Omega$

$$L = 2.29 / \omega = 2.29 / (2 \times \pi \times 50) = 7.29 \text{ mH}$$

i.e. the inductance/phase $L = 7.29$ mH

2.7 Power Triangle and Power Factor Correction For Balanced Loads

2.7.1 Reasons for power factor correction

Most industrial loads consist of electric motors. Thus they are inductive and they have lagging power factors. The power factor becomes increasingly important since

the load current, $I_L = P/(\sqrt{3}V_L\cos\phi)$.

For the same amount of true power, a low power factor requires high load current and is costly to the customer. For this reason, the power factor should be corrected (i.e. improved) to approach unity.

Power factor correction for an inductive load consists of connecting capacitance in parallel with the load.

2.7.2 Power Triangles

TRUE or ACTIVE POWER:- The power dissipation across each phase of a 3-phase load is given by:

$$\mathbf{P_{PH} = V_{PH}\, I_{PH} \cos\phi \quad (watts, W\ or\ kW)}$$

Where V_{PH} is the rms value of the voltage across one phase,

I_{PH} the rms value of the current flowing through the **SAME** phase, and

cos ϕ the power factor, with angle ϕ being the phase angle between the phase voltage across and the phase current through the same phase.

For a balanced load, the power dissipation is the same in all three phases. Thus the equation for the total power consumption is:

$$\mathbf{P_T = 3 \times P_{PH} = 3\, V_{PH}\, I_{PH} \cos\phi}$$

or

$$\mathbf{= \sqrt{3}\, V_L\, I_L \cos\phi}$$

REACTIVE POWER:- The reactive power across each phase is given by:

$$\mathbf{Q_{PH} = V_{PH}\, I_{PH} \sin \phi \quad (VAR\ or\ kVAR)}$$

Where V_{PH}, I_{PH} and angle ϕ have the same definition stated earlier under *TRUE POWER*.

When the load is balanced, the total reactive power is then

$$\mathbf{Q_T = 3 \times Q_{PH} = 3\, V_{PH}\, I_{PH} \sin \phi}$$

or

$$\mathbf{= \sqrt{3}\, V_L\, I_L \sin \phi}$$

APPARENT POWER:- The apparent power per phase is calculated as:

$$\mathbf{S_{PH} = V_{PH}\, I_{PH}\ (Volt\text{-}Ampere,\ VA\ or\ kVA)}$$

For a balanced load, the total apparent power is

$$\mathbf{S_T = 3 \times S_{PH} = 3\, V_{PH}\, I_{PH}}$$

or

$$\mathbf{= \sqrt{3}\, V_L\, I_L}$$

For a balanced load, star or delta, the power factor is defined as $\mathbf{\cos \phi = P / S}$.

Thus, P, Q and S can be represented by a right-angled triangle called the power triangle as shown in Figures 2.21 and 2.22.

Also $\mathbf{P^2 + Q^2 = S^2}$

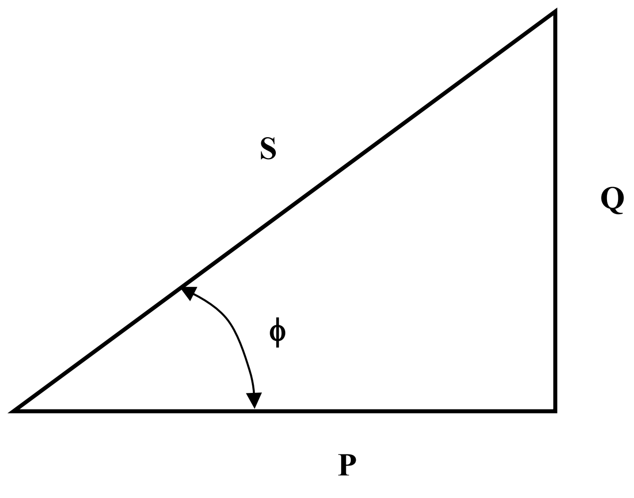


Figure 2.21: Power triangle for net inductive load

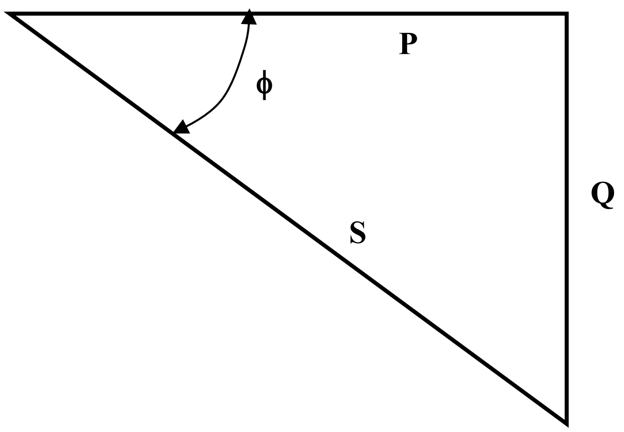


Figure 2.22: Power triangle for net capacitive load.

From Figures 2.21 and 2.22, it can be seen that the apparent power S can also take on a ***COMPLEX FORM*** given by:

S = P + jQ	for net inductive load.
= jQ	for purely inductive load.
S = P − jQ	for net capacitive load.
= − jQ	for purely capacitive load.

2.7.3 Solve problems on power factor correction

Example 2.6

A 3-phase, 55.2 kW, 400 V, 50 Hz induction motor operates on full load with an efficiency of 92% and at a power factor of 0.8 lagging. Calculate the total kVAR rating of capacitors required for raising the full-load power factor to (a) 0.95 lagging and (b) unity.

Solution 2.6

Motor power input **P = 55.2k / 0.92 = 60 kW**

Given that the p.f. is 0.8 lagging, then

$\cos\phi_1 = 0.8$, $\phi_1 = 36.87^o$ **and** $\tan\phi_1 = 0.75$

and the motor reactive power input, $\mathbf{Q = P \times \tan\phi_1 = 60\ k \times 0.75 = 45\ kVAR}$
(Triangle ABC below is the power triangle for the motor alone)

(a) For a new pf. of 0.95 lagging after adding the capacitor, we have, by referring to the Figure 2.23,

$$\mathbf{\cos\phi_2 = 0.95, \quad \phi_2 = 18.19^o \quad and \quad \tan\phi_2 = 0.328}$$

Thus, the net reactive power consumption for both the motor and capacitors is given by $\mathbf{Q - Q_C = P \times \tan\phi_2 = 60 \times 0.328 = 19.68\ kVAR}$
(Triangle ADC below is the power triangle for the motor and the capacitors together)
Where Q_C is the reactive power consumption (or rating) of the capacitors.
Therefore,
$\mathbf{Q_C = Q - 19.68k = 45k - 19.68k = 25.32\ kVAR = S_C}$ **[for pure capacitors]**
Therefore, $\mathbf{S_C = 25.32\ kVA}$

(b) Now, for a new pf. of unity after adding the capacitor,

$$\mathbf{\cos\phi_2 = 1, \quad \phi_2 = 0^o}$$

the net reactive power consumption for both the motor and capacitors is 0.
Thus $\mathbf{Q_C = Q = 45\ kVAR = S_C}$ **[for pure capacitors]**

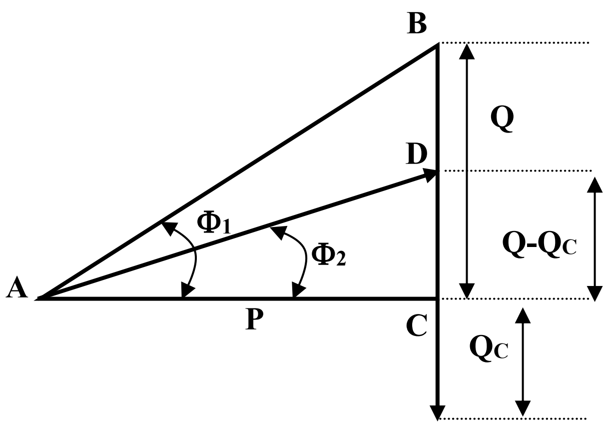


Figure 2.23: Power triangle before and after power factor correction

Example 2.7

A 3-ϕ load takes 150 kW at 400 V, 50 Hz and a power factor of 0.8 lagging.

Determine the kVA rating of a capacitor bank to improve the power factor to 0.88 lagging.

hat capacitance per phase is required if the capacitor bank is connected (a) in delta and (b) in star?

Solution 2.7

At original power factor,

Power, P = 150 kW, $\cos\phi_1 = 0.8$ so $\phi_1 = 36.87^0$

Reactive power, Q = P x $\tan\phi_1$ = 150 k x 0.75 = 112.5 kVAR

At the new power factor,

Power, P = 150 kW (unchanged), $\cos\phi_2 = 0.88$ so $\phi_2 = 28.36^0$

Reactive power, $Q - Q_C$ = P x $\tan\phi_2$ = 150 k x 0.539 = 80.8 kVAR

Therefore the total kVA of the capacitor bank $S_C = Q_C$ = 112.5 k – 80.8 k

= 31.7 kVA (for 3 phases)

$S_{C(PH)}$ = 10.6 kVA (per phase)

(a) ***Capacitor bank in delta:***

Phase voltage of capacitor bank, V_{PH} = line voltage $\mathbf{V_L = 400\ V}$

Therefore, phase current, $\mathbf{I_C = (10.6 \times 10^3) / 400 = 26.5\ A}$

Since $\mathbf{V_{PH} / X_C = I_C}$ **and** $\mathbf{X_C = 1/\omega C}$**,** we have

$X_C = \mathbf{V_{PH} / I_C = 400 / 26.5 = 15.09\ \Omega}$

$\mathbf{C = 1 / (\omega X_C) = 1 / (2\pi \times 50 \times 15.09) = 210.94\ \mu F /phase}$

(b) ***Capacitor bank in star:***

Phase voltage $\mathbf{V_{PH} = V_L / \sqrt{3} = 400 / \sqrt{3} = 230.94\ V}$

Phase current in capacitor bank $\mathbf{I_C = (10.6 \times 10^3) / 230.94 = 45.9\ A}$

$X_C = \mathbf{V_{PH} / I_C = 230.94 / 45.9 = 5.03\ \Omega}$

So that $\mathbf{C = 1 / (2\pi \times 50 \times 5.03) = 632.82\ \mu F/phase}$

2.8 Three-phase four-wire unbalanced star-connected system

The three-phase four-wire system is used extensively in industrial and commercial occupancies where both three-phase power and one-phase power are required. Figure 2.24 is the 400/230 V, three-phase, four-wire system. This system provides the dual advantages of furnishing 230 V one-phase loads (for lighting, single phase motors and small appliances) and balanced 400 V three-phase loads (for large electric motors and ovens). In practice, an attempt is made to balance the 230 V one-phase loads so that there are limited (but equal) numbers of branch circuits on each phase. But depending on the circuits energized, there is always the possibility of some imbalance.

The presence of the neutral wire ensures that the load phase voltages are always equal in magnitude and displaced by 120° regardless of load imbalance.

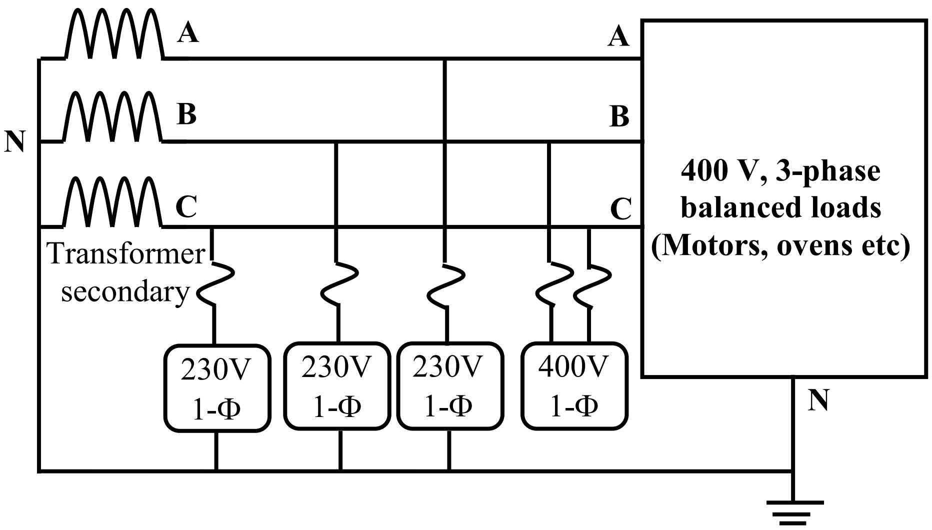


Figure 2.24: A 400/230 V, three-phase, four-wire system

2.8.1 Effect of the neutral wire on unbalance three-phase loads

(a) The phase voltages of a three-phase four-wire system remain undisturbed regardless of load imbalance, whereas the phase voltages of a three-phase three-wire unbalanced load will increase or decrease alarmingly and possibly, dangerously. This will cause damage to the electrical appliances.

(b) The line currents of a three-phase four-wire system tend to remain fairly balanced despite the severely unbalanced load. A three-phase three-wire unbalanced load (lacking the neutral) will result in greater line current imbalance.

2.8.2 Solve problems on three-phase 4-wire system supplying unbalanced star-connected loads

For the unbalanced star load of Figure 2.25,

$$\mathbf{I_A + I_B + I_C + I_N = 0} \quad \textbf{(by KCL)}$$

And $I_A = \dfrac{V_{AN}}{|Z_A|\angle\phi_A}$, $I_B = \dfrac{V_{BN}}{|Z_B|\angle\phi_B}$, and $I_C = \dfrac{V_{CN}}{|Z_C|\angle\phi_C}$,

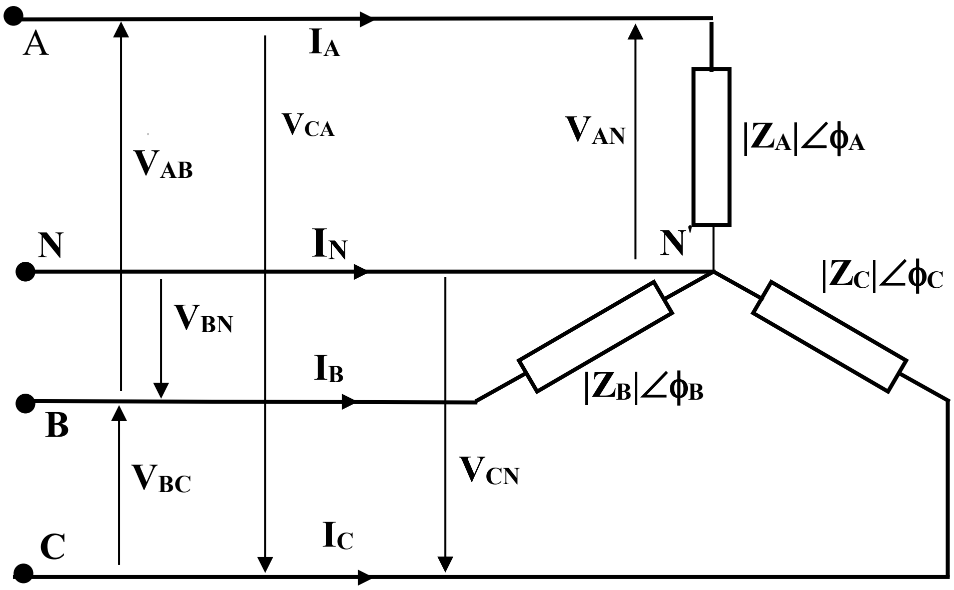


Figure 2.25: 3-phase, 4-wire, Y-connected unbalanced load

Total power consumption of an unbalanced load star is equal to the sum of the power consumption across individual impedances.

Example 2.8

The branch impedances of a 3-phase Y-connected load are $Z_A = (5 - j10)\ \Omega$, $Z_B = (6 + j5)\ \Omega$, and $Z_C = (3 + j15)\ \Omega$. The load is connected to a 3-phase, 440 V, 50 Hz balanced Y-connected supply system. Calculate the line currents, each phase power, reactive power, the overall system power factor and neutral current. Take V_{AN} as reference.

Solution 2.8

$$\text{For a balanced Y-connected supply system } \mathrm{V_{PH}} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254V$$

$$\therefore\ \mathrm{V_{AN}} = 254\angle 0^O; \quad \mathrm{V_{BN}} = 254\angle -120^O; \quad \mathrm{V_{CN}} = 254\angle -240^O;$$

$$\text{In a star-connected system, } \mathrm{I_L} = \mathrm{I_{PH}}$$

$$\therefore\ \mathrm{I_A} = \frac{254\angle 0^O}{5 - j10} = \frac{254\angle 0^O}{11.18\angle -63.43^O} = 22.72\angle 63.43^O\,A$$

$$\mathrm{I_B} = \frac{254\angle -120^O}{6 + j5} = \frac{254\angle -120^O}{7.81\angle 39.80^O} = 32.52\angle -159.8^O\,A$$

$$\mathrm{I_C} = \frac{254\angle -240^O}{3 + j15} = \frac{254\angle -240^O}{15.3\angle 78.7^O} = 16.6\angle -318.7^O\,A$$

$$\text{Power in each phase} = \mathrm{V_{PH}} I_{PH} \cos\Phi$$

$$\therefore \quad \mathrm{P_A} = |V_{AN}||I_A|\cos\Phi_A = 254\times 22.72\times\cos(63.43^O) = 2581.26\,W$$

$$\mathrm{P_B} = |V_{BN}||I_B|\cos\Phi_B = 254\times 32.52\times\cos(39.80^O) = 6346.08\,W$$

$$\mathrm{P_C} = |V_{CN}||I_C|\cos\Phi_C = 254\times 16.6\times\cos(78.7^O) = 826.18\,W$$

$$\text{Total power } \mathrm{P_T} = P_A + P_B + P_C = 9753.52\,W$$

$$\text{Reactive power in each phase} = \mathrm{V_{PH}} I_{PH} \sin\Phi$$

$$\therefore \quad \mathrm{Q_A} = |V_{AN}||I_A|\sin\Phi_A = 254\times 22.72\times\sin(63.43^O) = 5161.40\,VAR \ \ (leading)$$

$$\mathrm{Q_B} = |V_{BN}||I_B|\sin\Phi_B = 254\times 32.52\times\sin(39.80^O) = 5287.36\,VAR \ \ (lagging)$$

$$\mathrm{Q_C} = |V_{CN}||I_C|\sin\Phi_C = 254\times 16.6\times\sin(78.7^O) = 4134.66\,VAR \ \ (lagging)$$

$$\text{Total reactive power } \mathrm{Q_T} = -\mathrm{Q_A} + \mathrm{Q_B} + \mathrm{Q_C}$$

$$= +4260.62\,VAR \text{ (lagging)}$$

$$\text{Power factor angle } \ \Phi_\mathrm{S} = \tan^{-1}\frac{Q}{P} = \tan^{-1}\left(\frac{4260.62}{9753.52}\right) = 23.59^o$$

and overall system power factor $\cos\Phi_\mathrm{S} = 0.916$ lagging

Applying KCL at the neutral point N, we have $I_N + I_A + I_B + I_C = 0$

$$\therefore \ \mathrm{I_N} = -(I_A + I_B + I_C) = -(22.72\angle 63.43^\circ + 32.52\angle -159.8^\circ + 16.6\angle 41.3^\circ)A$$

$$I_N = (7.886 - j20.04) = 21.535\angle -68.51^\circ\,A \ \ \text{(flowing towards the load)}$$

2.9 Measurement of Three-Phase Power Using Wattmeters

2.9.1 One Wattmeter Method For Three Phase Balanced Star and Delta Loads

If a star or delta three-phase load is balanced, then the power consumption across each phase is the same, and the total power is therefore:

Total power = 3 x power in each phase

A single wattmeter can be used and the connections for balanced star and delta loads are as shown in Figures 2.26 and 2.27 respectively.

To assist in the correct connection of the wattmeter, two terminals of the wattmeter are marked with " ± " identification. As shown in Figures 2.26 and 2.27, the identified terminal of the current coil is connected to the source, and the identified terminal of the voltage coil is connected to the line containing the current coil.

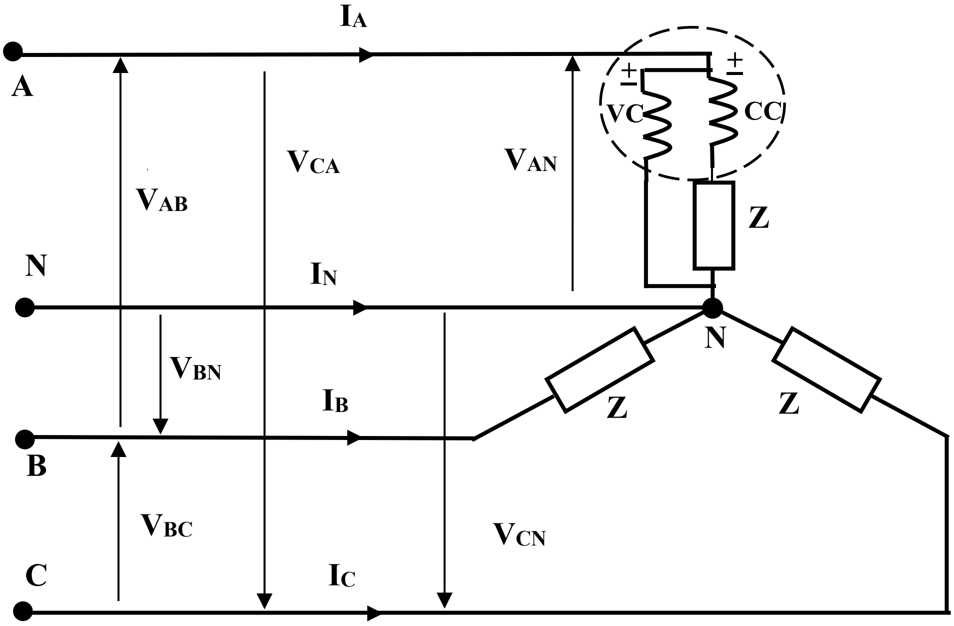


Figure 2.26: One wattmeter method for balanced star load power measurement

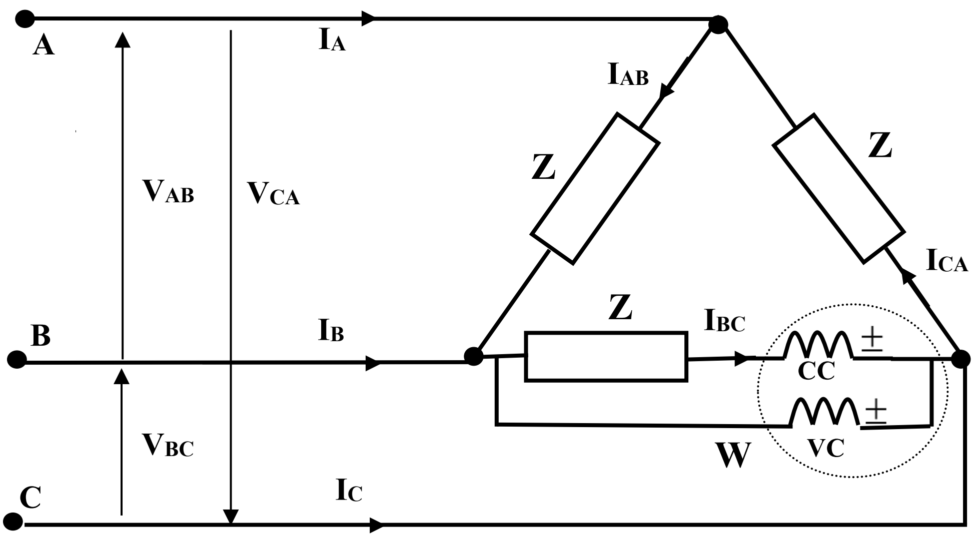


Figure 2.27: One-wattmeter method for balanced delta load power measurement

2.9.2 Two Wattmeter Method For Measuring Three Phase Power

The most widely used method for measuring three-phase power is as shown in Figures 2.28 and 2.29. Its advantages are:

(a) The same connections apply to 3 wire both star and delta loads.

(b) It shows the total power for both 3 wire balanced and unbalanced loads, and

(c) For balanced loads, the power factor of the load can be determined from the two-wattmeter readings.

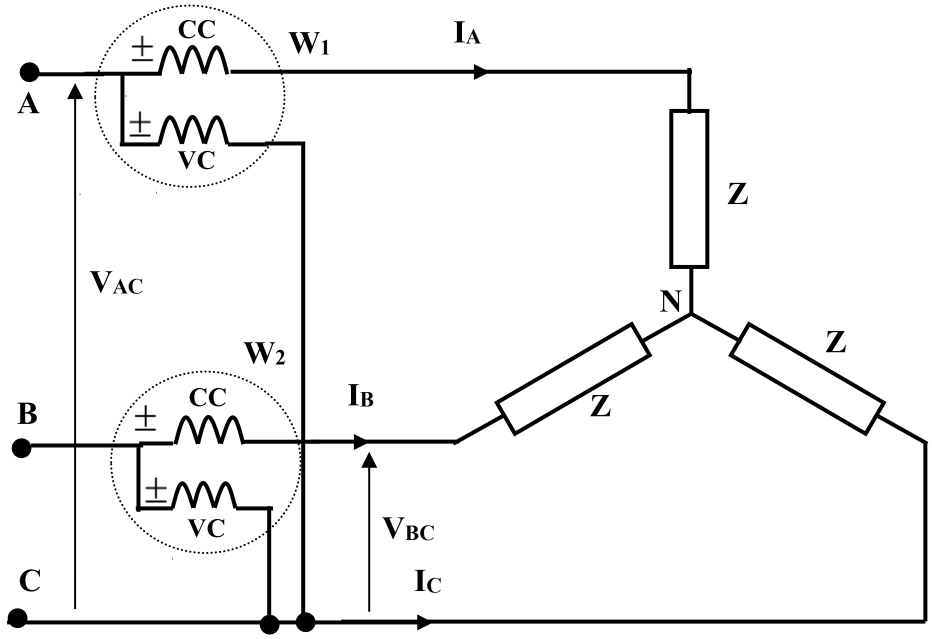


Figure 2.28: Two-wattmeter method for balanced star load power measurement

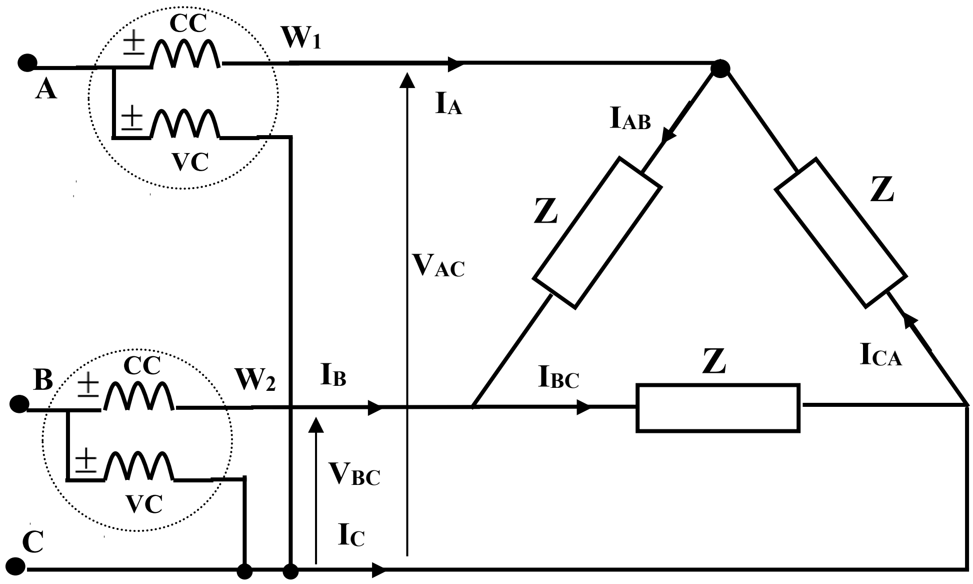


Figure 2.29: Two-wattmeter method for balanced delta load power measurement

2.9.3 Two-Wattmeter Method Measures The Total Power Of Balanced Three Phase Loads

Proof: Consider the star load of Figure 2.28 having a lagging power factor of cos θ. Taking V_{AN} as the reference for the star connected load, a phasor diagram of all voltages and currents is as shown in Figure 2.30. Similarly, taking V_{AB} as reference for the delta connected load, a phasor diagram of all voltages and currents is as shown in Figure 2.31.

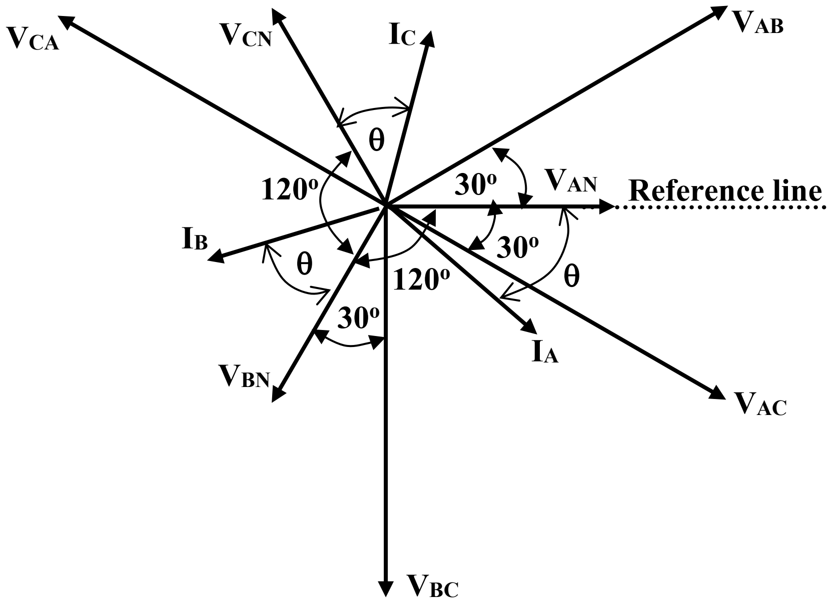


Figure 2.30: Phasor diagram for balanced star-load with lagging power factor

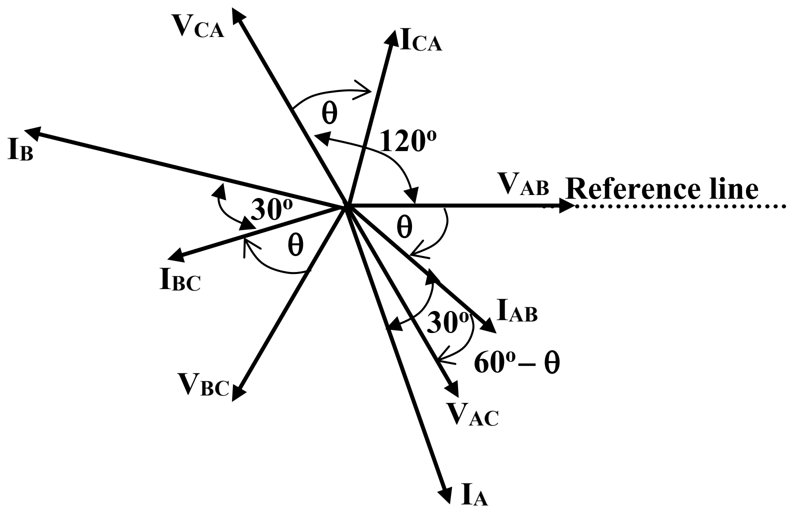


Figure 2.31: Phasor diagram for balanced delta-load with lagging power factor

Reading of wattmeter W_1 = | I_A | x | V_{AC} | x cos (phase angle between I_A and V_{AC})

Reading of wattmeter W_2 = | I_B | x | V_{BC} | x cos (phase angle between I_B and V_{BC})

From the phasor diagram of Figures 2.30 and 2.31, and with/

$| I_A | = | I_B | = I_L$,

and $| V_{AC} | = | V_{BC} | = V_L$,

The above two equations for wattmeters W_1 and W_2 can be rewritten as:

$W_1 = I_L \times V_L \times \cos(\theta - 30^o)$ ....**(1)**

and $W_2 = I_L \times V_L \times \cos(\theta + 30^0)$ ....**(2)**

Giving $\mathbf{W_1 + W_2 = \sqrt{3} \times V_L \times I_L \times \cos\theta}$ ....**(3)**

= **total power of the 3-phase balanced load**

If the star load of Figure 2.28 has a leading power factor of cos θ, then the two equations for wattmeters W_1 and W_2 will be rewritten as:

$W_1 = I_L \times V_L \times \cos(\theta + 30^o)$

and $W_2 = I_L \times V_L \times \cos(\theta - 30^0)$

In general, the wattmeter readings can be obtained from the expression given below:

$\mathbf{W_1 \text{ or } W_2 = I_L \times V_L \cos(\theta \pm 30^o)}$ ....**(4)**

Equation (4) can be used only when the current and voltage coil connection details for the two wattmeters are not specified.

Similarly by drawing the phasor diagram of a balanced delta load, it can be shown that the summation of the two-wattmeter readings again gives the total power consumption of the balanced delta load.

The effect of power factor angle on the wattmeter readings has been tabulated in the table below. For power factor angle $\theta = 60^0$, W_2 will read zero, and W_1 will read the full three phase power. For angles of $\theta > 60^0$, one of the wattmeters will read negative and hence in order to get positive deflection, one has to change the connection of the current or voltage coil.

Power factor angle θ	**Type of balanced load**	**Wattmeter readings**	
		W_1	**W_2**
0^0	**Resistive load**	**$W_1 = W_2 = +Ve$**	**$W_1 = W_2 = +Ve$**
30^0	**Net inductive load**	**$W_1 > W_2 = +Ve$**	**$W_2 < W_1 = +Ve$**
60^0	**Net inductive load**	**$W_1 = +Ve$**	**$W_2 = 0$**
$> 60^0 < 90^0$	**Net inductive load**	**$W_1 = +Ve$**	**$W_2 = -Ve$**

2.9.4 Power Factor Calculation For Balanced Loads

From equations 1 & 2 above, it can be shown that

$W_1 - W_2 = V_L$ x I_L x $\sin\theta$ ....(5)

The power factor of the three phase loads can be determined by the readings of W_1 and W_2 using equation (3) and (5).

Thus:

$$\tan\theta = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) \quad \text{and power factor} = \cos\theta = \cos\left(\tan^{-1}\left[\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right]\right)$$

The above equation is only applicable for the balanced loads.

2.9.5 Readings of wattmeters using complex conjugate

W_1 = Real part of [V_{AC} x I_A*]

W_2 = Real part of [V_{BC} x I_B*]

Where **I_A * and I_B *** are the complex conjugates of **I_A** and **I_B** respectively. The complex conjugate method described above is usually used to determine the respective wattmeter readings when reference vector is given or when the voltage and current coil connection details are given.

Example 2.9

A 3-ϕ star connected load takes 86.5 kW at 440 V, 50 Hz and a power factor of 0.87 lagging. Determine the line current and the readings on each of the two wattmeters connected between A and C and B and C lines, respectively to measure the input power.

Solution 2.9

$$P = \sqrt{3}V_L I_L \cos\Phi\,; \quad \text{or } 86500 = \sqrt{3}\times 440 \times I_L \times 0.87$$

$$\therefore\ \mathrm{I_L} = 130.46A$$

$$\Phi = \cos^{-1}(0.87) = 29.54^0;$$

$$W_1 \ or\ W_2 = 440\times 130.46\times\cos(29.54-30) = 57.40kW$$

$$W_1 \ or\ W_2 = 440\times 130.46\times\cos(29.54+30) = 29.09kW$$

OR

$$\tan\Phi = \frac{\sqrt{3}\times(W_1 - W_2)}{W_1 + W_2} \qquad \text{Or} \quad 0.5667 = \frac{\sqrt{3}(W_1 - W_2)}{86500}$$

$$(W_1 - W_2) = 28301.45$$

$$(W_1 + W_2) = 86500.00$$

$$\therefore \quad \mathrm{W}_1 = 57400.72W = 57.40kW$$

$$\mathrm{W}_2 = 29099W = 29.099kW$$

****

Tutorial 1

Conversion of Voltage and Current Sources

1. The current source in Figure 1 has I = 12 A, R_S = 2 Ω and R_L = 6 Ω. Find the values of V_L and I_L for this circuit. Convert the source into an equivalent voltage source and recalculate V_L and I_L.
   Ans: V_L = 18 V, I_L = 3 A

2. Find the equivalent voltage source between terminals AB for the circuit shown in Figure 2 using source conversion.
   V_{AB} = 9 V, R = 9 Ω

3. Using source conversion method, simplify the given circuit in Figure 3 to its equivalent current source between the terminals AB.
   Ans: I = 4 A (A +ve), R = 2.5 Ω

4. Using source conversion method, simplify the given circuit in Figure 4 to its equivalent current source between the terminals AB.
   Ans: I = 1 A (B +ve), R = 2.5 Ω

5. Using source conversion method, simplify the given circuit in Figure 5 to its equivalent current source between the terminals AB and calculate the current in the 4 Ω resistor.
   Ans: 10 A, 1 Ω, $I_{4\Omega}$ = 2A

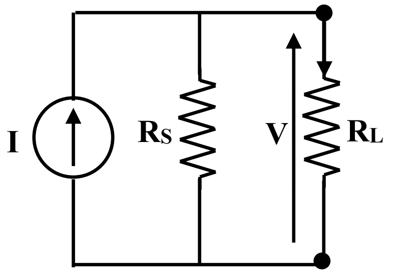


Figure 1

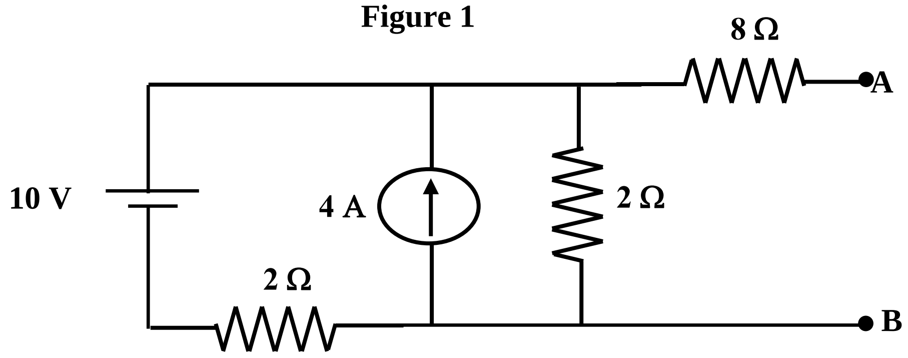


Figure 2

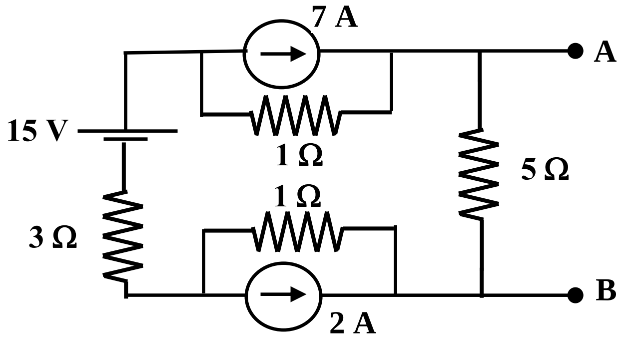


Figure 3

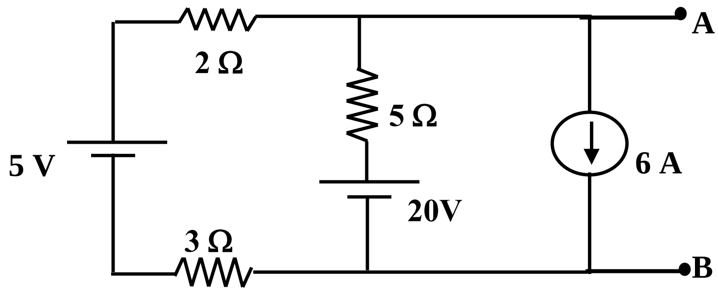


Figure 4

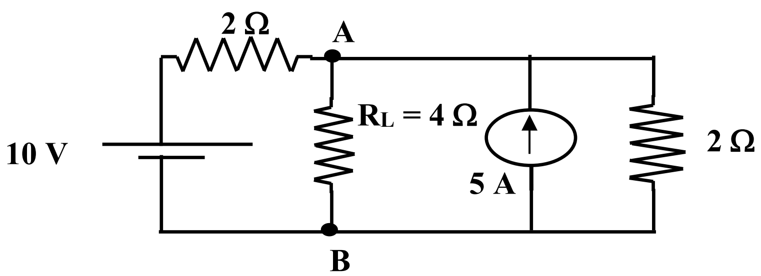


Figure: 5

Tutoria1 2

Mesh Current and Nodal Voltage Analysis

1. Write the mesh current equations for I_1, I_2 and I_3 in matrix form by inspection for the network shown in Figure 1.

   Ans: $\begin{bmatrix} 5 - j5 & j5 & 4 \\ j5 & 5 & j5 \\ 4 & j5 & 5 + j5 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0^0 \\ 0 \\ 20\angle 90^0 \end{bmatrix}$

2. Loop currents are shown in the network of Figure 2. Write the matrix equation by inspection and solve for I_1, I_2 and I_3.

   Ans: I_1 = 3.55 A, I_2 = –1.98 A, I_3 = –2.98 A

3. Find the node voltages V_1 and V_2 in the network of Figure 2 and with them verify the three currents obtained in Problem 2.

   Ans: V_1 = 7.11 V, V_2 = –3.96 V

4. Obtain the node voltage V_1 in the network shown in Figure 3.

   Ans: $74.9\angle 62.84^\circ$ V

5. Write the nodal voltage equations for V_1, V_2 and V_3 in matrix form by inspection for the network shown in Figure 4.

   Ans:

$$\begin{bmatrix} \frac{1}{2-j2}+\frac{1}{5}+\frac{1}{10} & -\left(\frac{1}{2-j2}\right) & -\left(\frac{1}{10}\right) \\ -\left(\frac{1}{2-j2}\right) & \frac{1}{2-j2}+\frac{1}{j5}+\frac{1}{8} & -\left(\frac{1}{8}\right) \\ -\left(\frac{1}{10}\right) & -\left(\frac{1}{8}\right) & \frac{1}{2-j4}+\frac{1}{8}+\frac{1}{10} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -\left(\frac{10\angle 0^0}{2-j2}\right) \\ \frac{10\angle 0^0}{2-j2}+\frac{5\angle 30^0}{8} \\ -\left(\frac{5\angle 30^0}{8}\right) \end{bmatrix}$$

6. For the circuit shown in Figure 5
   (a) Write down in matrix form, by inspection, the node voltage equations required to calculate the voltages at nodes A and B, with respect to the ground node C.
   (b) By solving the matrix equation in (a), calculate the current flowing in the 1000 Ω resistor.
   (c) Write down in matrix form, by inspection, the mesh current equations required to calculate the currents I_1, I_2, and I_3.
   (d) By solving the matrix equation in (c), show that current I_2 is the same as obtained in (b).

   Ans:

$$\begin{bmatrix} 0.103 & -0.001 \\ -0.001 & 0.008 \end{bmatrix}\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 510 & 10 & 0 \\ 10 & 1510 & -500 \\ 0 & -500 & 700 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 0 \end{bmatrix}$$

$V_A = 9.72V, V_B = 1.215V$

$I_{1000\,\Omega} = I_2 = 8.5mA$

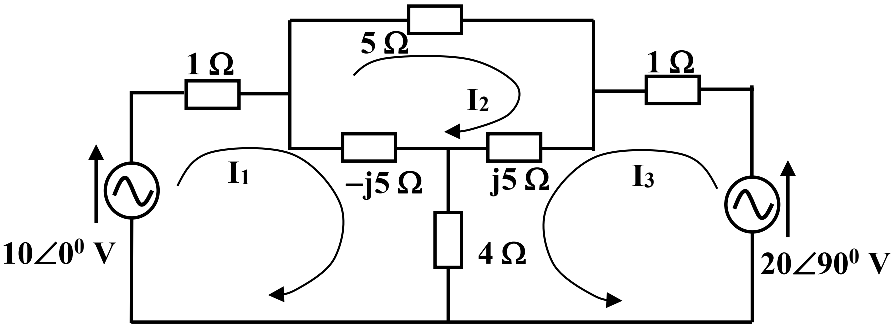


Figure 1

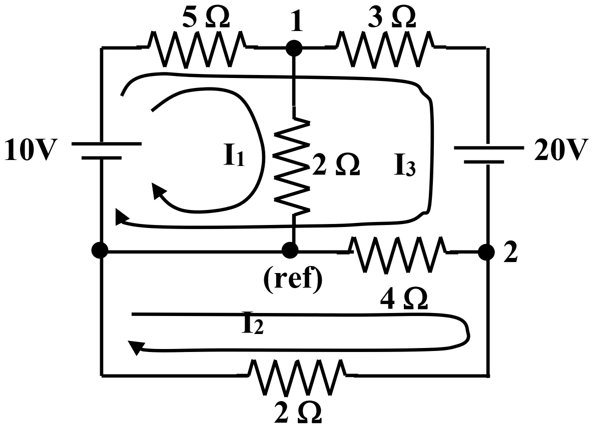


Figure 2

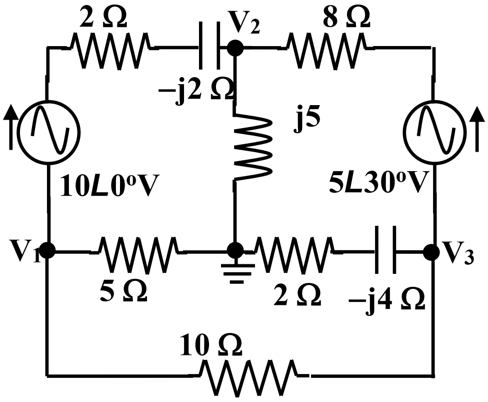


Figure 4

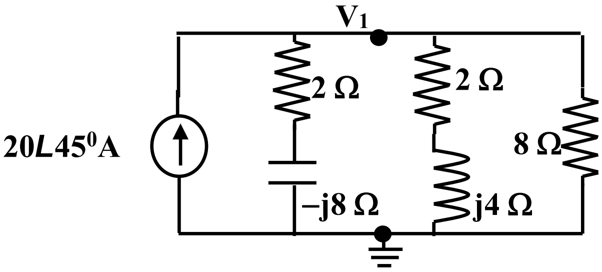


Figure 3

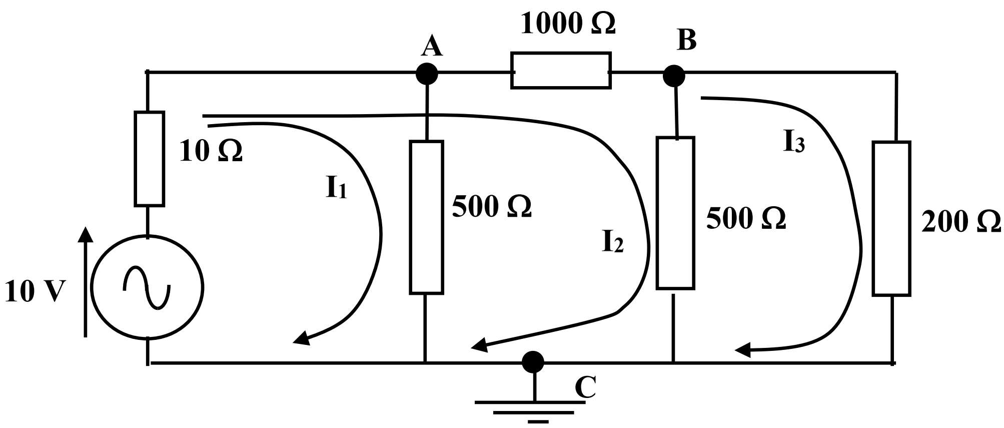


Figure 5

Star-Delta and Delta-Star Transformation

1. A network is arranged as shown in Figure 1. Using star-delta transformation , calculate the equivalent resistance between A and C.
   Ans: 2.86 Ω

2. Referring to the circuit shown in Figure 2, perform the delta-star conversion on the 500 Ω, 2500 Ω, 100 Ω branches and hence, find the current I in the 50 ohm resistor.
   Ans: 1.32 mA

3. Convert the three impedances forming a Δ in Figure 3 into an equivalent Y. Hence find an overall Y-equivalent.
   Ans:(11 + j2) Ω; (4 – j1) Ω; (1 + j2) Ω

4. The three-terminal network shown in Figure 4 contains a balanced delta in parallel with a balanced Y. Obtain the Y-connected equivalent.
   Ans: $\mathbf{2.28\angle{-3.54^0}}$ Ω **(balanced)**

5. By converting the star-connected resistors of 2 Ω, 4 Ω and 6 Ω (shown in the dotted box) into an equivalent delta connection, calculate the effective resistance between the terminals A and B of the circuit shown in Figure 5.
   Ans: 22 Ω; 11 Ω; 7.33 Ω; $\mathbf{R_{AB}}$ **= 4.09 Ω.**

6. Simplify the circuit shown in Figure 6 to prove that the equivalent resistance at the terminals AB is R_{AB} = 4.87 Ω using star to delta transformation for the 6 Ω star connected resistances connected to the terminals ABC.
   Ans: $\mathbf{R_{\Delta}}$ **=18 Ω;** $\mathbf{R_{AB}}$ **= 4.87 Ω.**

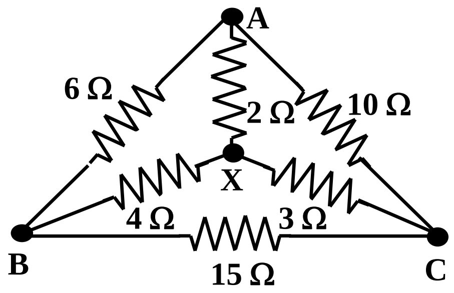


Figure 1

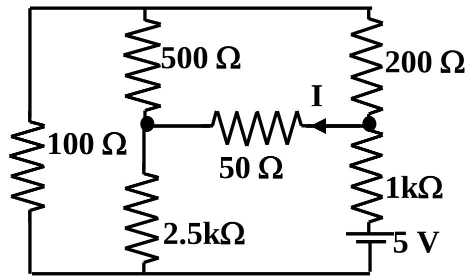


Figure 2

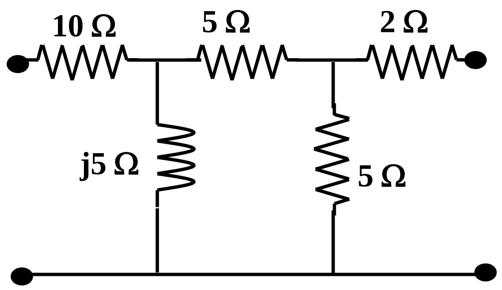


Figure 3

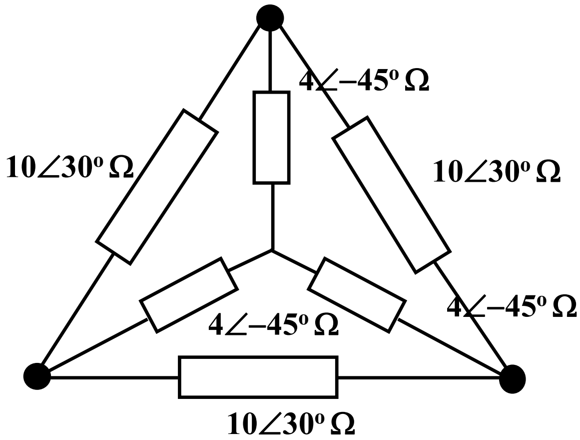


Figure 4

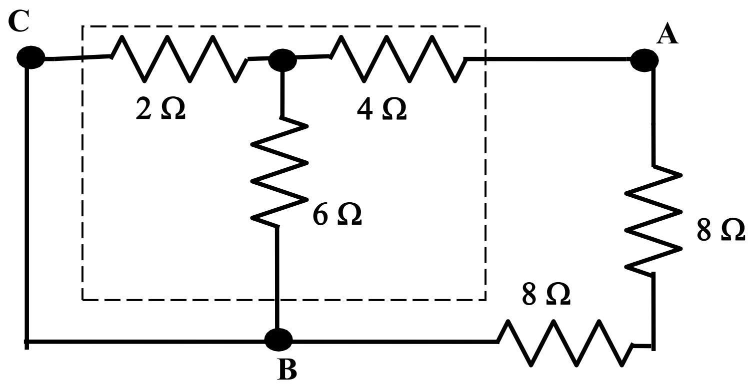


Figure 5

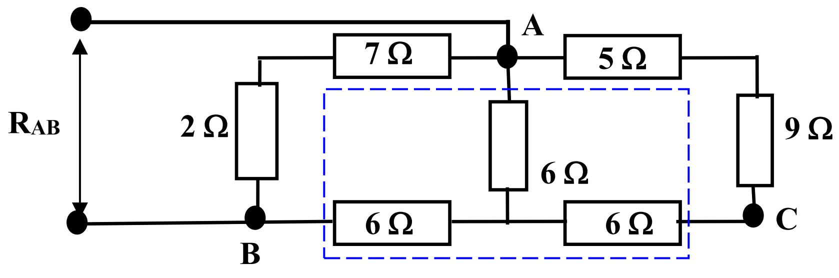


Figure 6

Tutorial 4

Thevenin's and Norton's Theorems

1. Obtain the Thevenin and Norton equivalent circuits between terminals AB for the active network shown in Figure 1.
Ans: V_{TH} = 6.29 V with B +Ve, R_{TH} = 9.43 Ω, I_N = - 0.67 A, R_N = 9.43 Ω

2. For the circuit shown in Figure 2, find the current through the 8 Ω using the Thevenin and Norton equivalent circuits.
Ans: V_{TH} = 4.27 V, R_{TH} = 2.69 Ω, I_N = 1.59 A, R_N = 2.69 Ω, $I_{8\Omega}$ = 0.4 A

3. Obtain the Thevenin equivalent and Norton equivalent by applying the respective theorems as seen across terminals AB of the circuit in Figure 3.
**Ans: V_{TH} = 11.17 $\angle -63.4°$ V, Z_{TH} = 10.6 $\angle 45°$ Ω, I_N = 1.06 $\angle -108.4°$ A,
Z_N = 10.6 $\angle 45°$ Ω**

4. For the circuit shown in Figure 4, apply Thevenin's and Norton's theorems to
   (a) Find the equivalent circuit parameters between the terminals A and B.
   (b) Calculate the voltage across the load R_L.
Ans: V_{AB} = V_{TH} = –5 V, R_{TH} = 15 Ω, I_{AB} = I_N = –0.33 A, R_N = 15 Ω, V_{RL} = 2.22 V

5. For the circuit shown in Figure 5,
   (a) Applying Norton's theorem, find the Norton equivalent circuit parameters I_N and R_N at the terminals AB.
   (b) Using the Norton equivalent circuit obtained above calculate the current flowing and voltage across the load R_L.
   (c) Convert the Norton equivalent circuit to find its Thevenin equivalent circuit.
Ans: I_N = 1.818 A, R_N = 5.5 Ω, I_{RL} = 1.33A, V_{RL} = 2.66 V, V_{TH} = 10 V, R_{TH} = 5.5 Ω

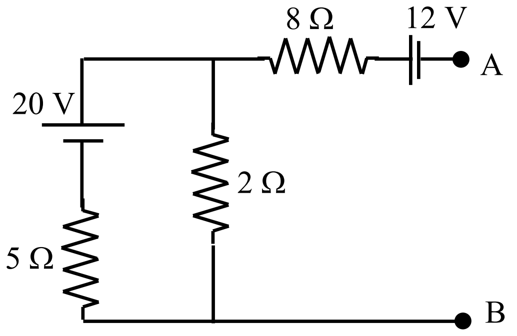


Figure 1

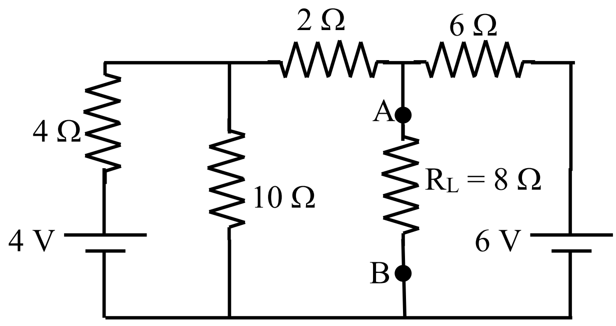


Figure 2

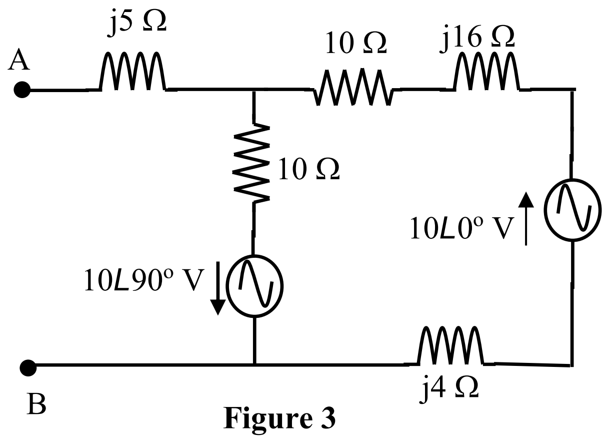


Figure 3

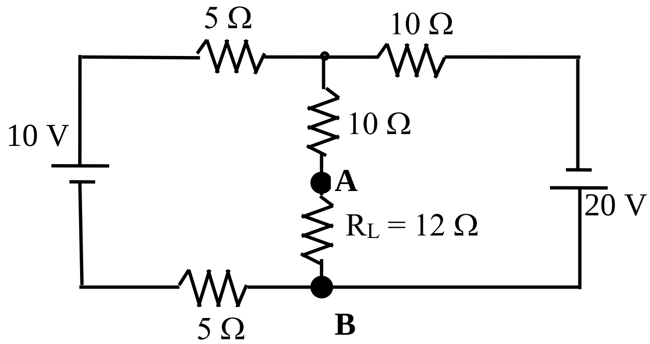


Figure 4

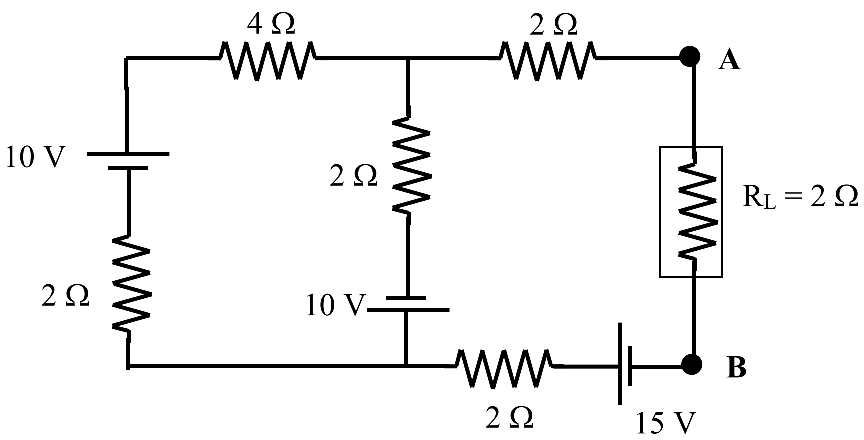


Figure 5

Tutoria1 5

Three-Phase Circuits: Balanced Star Connected Load

1. A three-phase, four-wire, 208 V system serves a balanced star-connected load with impedances of $20\angle-30^o\ \Omega$ each. Find the line currents and draw the phasor diagram, taking $V_{AN} = 120.09\ \angle-90^o$ V.
 Ans: $I_A = 6\angle300^o$ A or $6\angle-60^o$ A, $I_B = 6\angle180^o$ A or $6\angle-180^o$ A, $I_C = 6\angle60^o$ A or $6\angle-300^o$ A

2. An ABC three-phase, 4-wire system serves a star-connected load with impedances of $10\angle30^0\ \Omega$ each. The line current I_A drawn is found to be $12\angle-30^0$ A. Find all the line voltages, the phase voltages and the remaining line currents.
 Ans: $V_{AN} = 120\angle0^o$ V, $V_{BN} = 120\angle-120^o$ V, $V_{CN} = 120\angle-240^o$ V,
 $V_{AB} = 208\angle30^o$ V, $V_{BC} = 208\angle-90^o$ V, $V_{CA} = 208\angle-210^o$ V,
 $I_A = 12\angle-30^o$ A, $I_B = 12\angle-150^o$ A, $I_C = 12\angle-270^o$ A

3. A three-phase, four wire, 208 V system serves a balance star-connected load drawing a line current $I_A = 10\angle145°$A. Find the line current I_C and the load impedance in each phase, taking V_{BC} as reference.
 Ans: $I_C = 10\ \angle-95^o$ A, $Z = 12.01\ \angle-55^o\ \Omega$

4. Given a balanced three-phase, three-wire system with a Y-connected load for which the line voltage is 230 V and the impedance of each phase is $(6 + j\ 8)\ \Omega$. Taking V_{AB} as reference, find the line current I_A and power absorbed by each phase.
 Ans: $I_A = 13.28\ \angle-83.13^o$ A, $P_{PH} = 1058$ W

5. A balanced star load, having a power factor of 0.8 lagging, is connected to a 400 V, 3-phase, 3-wire balanced star supply. The total power consumption of the load is 3 kW. Find the impedances of the load in polar form.
 Ans: $Z = 42.67\ \angle36.87^o\ \Omega$

6. A small plant requires the installation of a new three-phase device. The device is to be connected using a 4-wire system with a line voltage of 208 V. Each of the 3-phases has impedance of $10\angle60^\circ\ \Omega$. Calculate the line currents for each phase and the overall power dissipated if the impedances are connected in star. Take V_{AN} as the reference voltage and phase sequence as ACB (negative sequence).
 Ans: $I_A = 12\angle-60^o$ A, $I_B = 12\angle-300^o$ A, $I_C = 12\angle-180^o$ A, P = 2161.59 W

Tutoria1 6

Three-Phase Circuits: Balanced Delta Connected Load

1. A three-phase, three-wire, 110 V system supplies a delta connection of three equal impedances of $5\angle 45^{o}$ Ω. Determine the line currents I_A, I_B and I_C and draw the phasor diagram, taking V_{AB} as the reference.
   Ans: I_A = 38.1∠–75º A, I_B = 38.1∠–195º A, I_C = 38.1∠45º or 38.1∠–315º A

2. A balanced delta connected load with impedances each of $5\angle -30^{o}$ Ω is connected to a three phase, three wire, 208 V, 50 Hz supply system. Taking V_{BC} as reference, calculate the phase current I_{BC} and the line currents I_A, I_B and I_C in polar form.
   Ans: I_{BC} = 41.6∠30º A, I_B = 72.05∠0º A, I_C = 72.05∠–120º A, I_A = 72.05∠–240º A

3. A three-phase, 3-wire, 220V system supplies a balanced delta connected load with a phase impedance of $10\angle 40°$ Ω. Taking V_{CN} as the reference, determine:
   (a) the phase current I_{CA} in polar form.
   (b) the total power consumed.
   Ans: I_{CA} = 22∠–10º A, P_T = 11.12 kW

4. A 220 V, three-phase voltage is applied to a balanced delta-connected three-phase load of phase impedance (15 + j20) Ω. Taking V_{BC} as the reference, determine:
   (a) the line current I_C , and
   (b) the power consumption per phase.
   Ans: I_C = 15.242∠–203.13º A or 15.242∠156.87º A, P_{PH} = 1162 W

5. A star-connected, three-phase load consists of three similar impedances. When the load is connected to a three-phase, 500 V, 50 Hz supply, the line current is 28.85 A and the power factor is 0.8 lagging.
   (a) Calculate
      (i) the total power taken by the load, and
      (ii) the resistance of each phase of the load.
   (b) If the phase loads were now re-connected in delta and supplied from the same three-phase system, determine the magnitude of line current.
   Ans: P_T = 20 kW, R_{PH} = 8 Ω, $I_{L(\Delta)}$ = 86.6 A

6. A balanced delta-load with impedances each of $27\angle -25^{o}$ Ω and a balanced Y-load with impedances each of $10\angle -30^{o}$ Ω are both connected to a three-phase, three-wire, 208 V, ABC system. Find the total line currents in phasor form and the power in each load. Take V_{AB} as the reference.
   Ans: I_A = 25.3∠–2.62ºA, I_B = 25.3∠–122.62ºA , I_C =25.3∠117.38º A or

   25.3∠–242.62º A, P_Δ = 4355.6 W, P_Y = 3744 W

Tutoria1 7

Real, reactive, apparent power, power factor and power factor correction

1. A 440 V, 50 Hz, star-connected induction motor takes a line current of 50 A at a power factor of 0.8 lagging. A three-phase capacitor bank, connected in star, is to be used to improve the power factor to 0.95 lagging. Determine the capacitance of each capacitor. **Ans: C = 211 μF**

2. A 415 V, 50 Hz, three-phase distribution system supplies a 20 kVA, three-phase induction motor load at a power factor of 0.8 lagging, and a star-connected set of impedances, each having a resistance of 10 Ω in series with and an inductive reactance of 8 Ω. Calculate the phase capacitance of delta-connected capacitors required to improve the overall power factor to 0.95 lagging.
Ans: C = 72.1 μF

3. A three-phase, 50 hp (1 hp = 746 W), 440 V, 50 Hz induction motor operates on full load with an efficiency of 89% and at a p.f. of 0.85 lagging. Calculate the total kVA rating of capacitors required, to raise the full load p.f. to 0.95 lagging. What will be the capacitance per phase if the capacitors are
(a) delta-connected, and
(b) star-connected.
Ans: S_C = 12.193 kVA, C_Δ = 66.77 μF, C_Y = 200.27 μF

4. A 3-phase, 50 Hz induction motor is connected to a 3-phase 450V, 50 Hz, power supply. The motor has 4000 W output power on full-load operation. If the efficiency of the motor running on full load is 89%, calculate the input power of the motor.
(a) If the power factor of the motor running on full load is 0.8 lagging, calculate its apparent input power at full load.
(b) Calculate the reactive input power supplied to the motor on full load.
(c) Calculate the total kVA rating of the capacitor bank required, to raise the full load power factor to 0.95 leading.
(d) Calculate the capacitance per phase if the 3 balanced capacitors are connected in delta.
Ans: S = 5.618 kVA, Q = 3.373 kVAR, S_C = Q_C =4.85 kVA, C = 25.39 μF

5. A 400 V, 60 Hz distribution system supplies a 15 kVA three-phase motor at a power factor of 0.7 lagging. A three-phase capacitor bank of 6 kVAR is connected across the motor terminals to improve the power factor. Determine the new power factor.
Ans: 0.912 (lagging)

Tutoria1 8

Unbalanced star connected load and measurement of three phase power

1. A star load with $Z_A = (2 - j1)\ \Omega$, $Z_B = (2 + j3)\ \Omega$ and $Z_C = (3 + j0)\ \Omega$ is connected to a 3-phase, 4-wire, 100 volts, ABC system. Find the line currents including the neutral assuming the positive direction is towards the load. Take V_{AB} as the reference voltage.

   Ans: $I_A = 25.77\angle{-3.44^o}$ A, $I_B = 16.04\angle 153.7^o$ A, $I_C = 19.24\angle 90^o$ A,
   $I_N = -27.3\angle 65.42^o$ A $= 27.3\angle{-114.58^o}$ A

2. A 3-phase, 4-wire, 440 V system is loaded as follows:
   Resistance loads of 150 kW, 250 kW and 400 kW connected between neutral and the A,B and C lines respectively. Calculate the:
   (a) line currents,
   (b) neutral current (flowing towards the supply) and
   (c) total power of the system.
   Phase sequence, ABC. Take V_{AN} as reference.

   Ans: $I_A = 590.48\angle 0^o$ A, $I_B = 984.14\angle{-120^o}$ A, $I_C = 1574.62\angle{-240^o}$ A
   $I_N = 857.95\angle 143.41^o$ A, Total power = 800 kW

3. Three equal resistances of 20 Ω each are connected in delta, and a star load with phase impedances $Z_A = (3 + j4)\ \Omega$, $Z_B = (6 - j8)\ \Omega$ and $Z_C = (9 + j12)\ \Omega$ are connected to a three phase, 4-wire, 440 V, ABC system. Taking the phase voltage V_{BN} as the reference, calculate the:
   (a) line currents and three phase power for the balanced delta connected resistive load.
   (b) line currents and each phase power for the star connected unbalanced load.
   (c) total power for the combined loads.
   (d) neutral current I_N flowing towards the loads.

   Ans: (a) Delta load: $I_B = 38.1\angle 0^o$ A, $I_C = 38.1\angle{-120^o}$ A, $I_A = 38.1\angle{-240^o}$ A,
   P = 29.04 kW
   (b) Star load: $I_B = 25.40\angle 53.13^o$ A, $I_C = 16.93\angle{-173.13^o}$ A, $I_A = 50.8\angle{-293.13^o}$ A,
   $P_B = 3.871$ kW, $P_C = 2.58$ kW, $P_A = 7.74$ kW
   (c) $P_T = 43.23$ kW
   (d) $I_N = 67.56\angle{-105.79^o}$ A (flowing towards the load)

4. The power input to a 2 kV, 50 Hz 3-phase motor running on full load at an efficiency of 90 per cent is measured by the two wattmeters which indicate as 300 kW and 100 kW.
   Calculate the:
   (a) input power,
   (b) output power and
   (c) line current if the power factor is 0.756 lagging.

   Ans: $P_{in} = 400$ kW, $P_{out} = 360$ kW, $I_L = 152.74$ A

5. A balanced star-connected load, with each phase having a resistance of 10 Ω in series with an inductive reactance of 30 Ω is connected to a 400 V 50 Hz supply. The phase rotation is ABC. Using two-wattmeter method the two wattmeters are connected to read the total power with their respective current coils connected in the A and C lines respectively. Calculate the reading of each wattmeter.

Ans: W_1 or W_2 = 2184.59 W, W_2 or W_1 = –585.65 W

6. For a 3-phase 3-wire ABC system, determine the apparent power, reactive power, and true power for a Y-connected load consisting of $Z_A = Z_B = Z_C = 47\angle 45^0$. The line voltage is 122 V. If two wattmeter method were used for power measurement, with the current coils of the two wattmeters connected to the A and C lines respectively, determine the power indicated by each wattmeter.

Ans: S = 316.54 VA, Q = 223.8 VAR, P = 223.8 W, W_1 or W_2 = 47.3 W, W_2 or W_1 = 176.53 W

7. (a) Draw a labelled circuit diagram showing the connections of the wattmeter voltage and current coils in the two-wattmeter method, with the current coils of the two wattmeters connected in the A and B lines for measuring the total power consumption in a balanced three phase

(i) star-connected load and
(ii) delta-connected load.

(b) A balanced delta-connected load of impedance (15 + j10) Ω per phase is connected to a 400 V, 50 Hz, three-phase supply. The phase sequence is ABC. The total power consumption of the load is measured by the two-wattmeter method, with the current coils of the two wattmeters connected in the A and B lines. Calculate the reading of each wattmeter.

Ans: I_L = 38.43 A, W_1 or W_2 = 15.34 kW, W_2 or W_1 = 6.81 kW

Name: ____________________________ Admission no: ____________________

Class: _______________ Date: ____________________

LABORATORY EXPERIMENT 1

Series RLC Resonant Circuit

E1.1 Objective

I. To determine the effect of impedance and current with changes in frequency of an AC source.

II. To determine the resonant frequency and bandwidth of a series RLC circuit.

III. To measure the effect of circuit Q factor on frequency response and on bandwidth at half power points.

E1.2 Equipment

1 Signal generator/function generator

1 Decade resistance box

1 Decade inductance box

1 Decade capacitance box

2 Digital multimeters

E1.3 Information

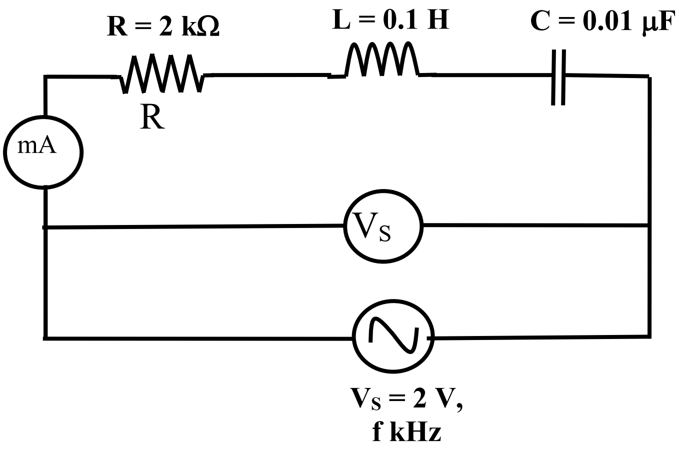


Figure: 1 Series RLC resonant

In a series RLC resonant circuit as in Figure 1, there is frequency f_o of the voltage source V at which $\mathbf{X_L = X_C}$. Therefore

$$\mathbf{f_O = \frac{1}{2\pi\sqrt{LC}}\ Hz}$$

At this frequency the impedance of the RLC circuit is minimum and Z=R, the resistance in the circuit. The current in the circuit is then maximum and

$$\mathbf{I_O = \frac{V}{R}}$$

The quality factor Q of the circuit is defined as $\mathbf{Q = \frac{X_L}{R}}$.

In a series resonant circuit the voltages across L and C are equal and are affected by the Q of the circuit.

$$\mathbf{V_L = V_C = QV}$$

Figure 2 shows the frequency response of a series resonant circuit. The two frequencies f_1 and f_2, which are called the half power points, are located at 70.7 % of the maximum current. Bandwidth BW of the response curve is defined as the difference between the two frequencies f_1 and f_2.

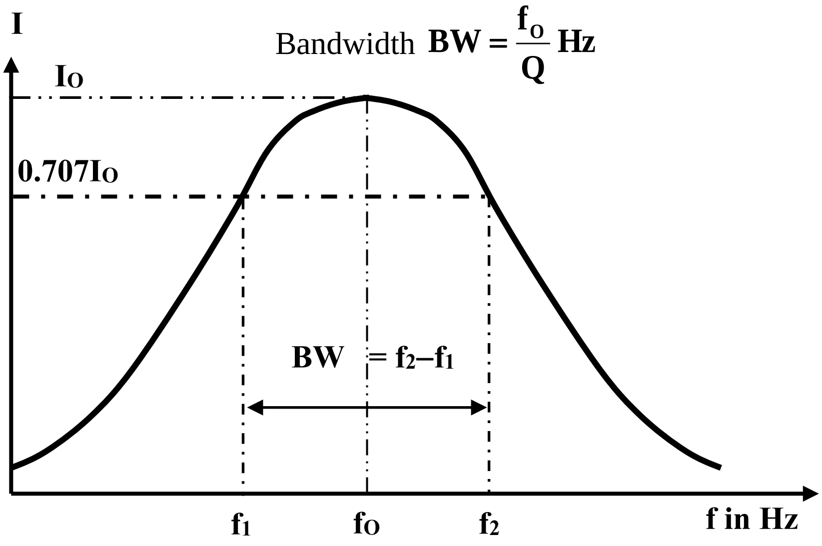


Figure:2 Frequency Response curve for series RLC resonant circuit

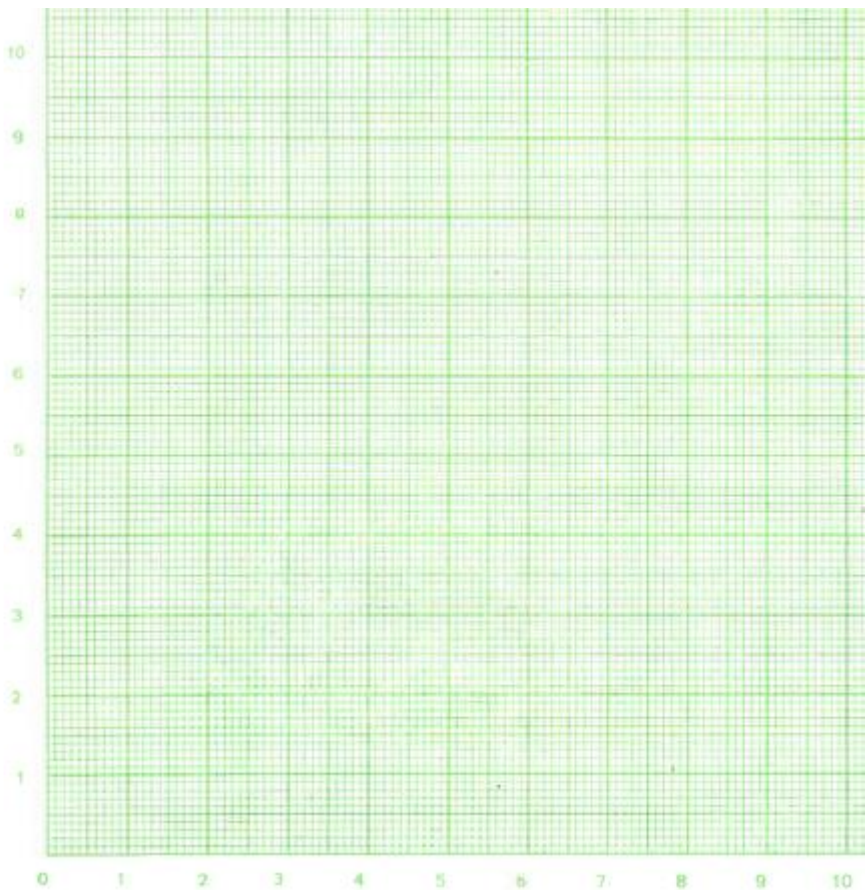
E1.4 Procedure

1. Connect the circuit as shown in Figure 1.
2. Set the voltage on the function generator to ***2 V*** using the amplitude knob by observing the multimeter V_S and maintain this supply voltage throughout the experiment.
3. Vary the frequency of the oscillator in accordance with the values set in Table 1 and record the multiammeter reading.

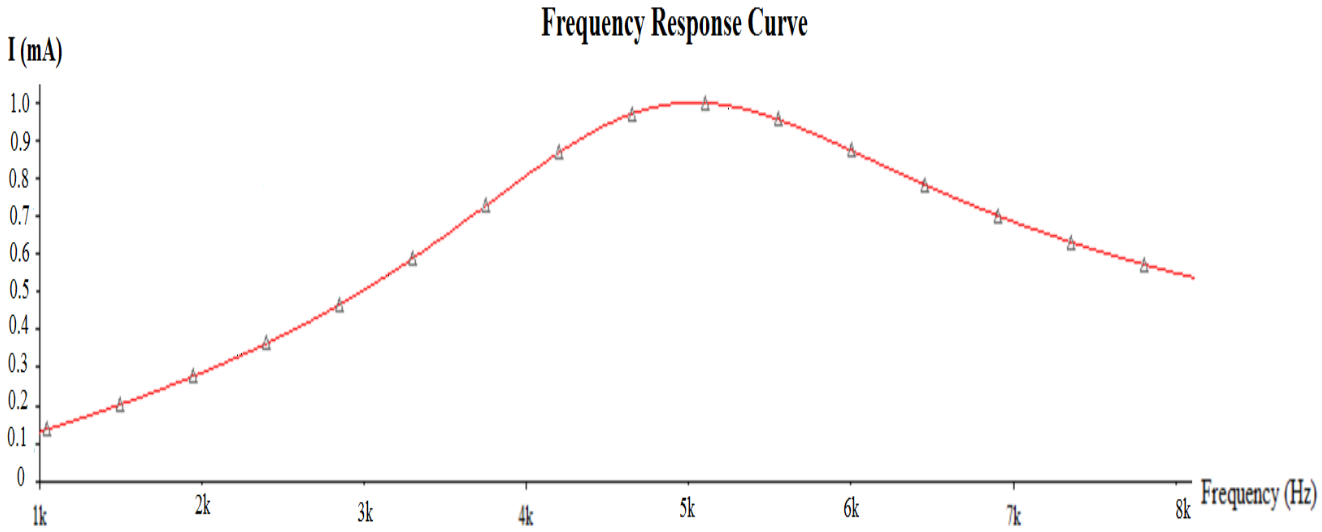
E1.5 Results

Table 1

f (kHz)	**V_S = 2 V and R = 2 kΩ**
	I (mA)
1	
2	
3	
3.5	
4	
4.5	
5	
5.5	
6	
7	
8	



If there is insufficient time, you may make use of the frequency response curve below to collect the necessary data.



E1.6 Discussion

1. The theoretical calculations are as shown.

Resonant Frequency $\mathbf{f_o} = \dfrac{\mathbf{1}}{\mathbf{2\pi\sqrt{LC}}}$ **Hz** $= \dfrac{1}{2\pi\sqrt{(0.1)(0.01\text{x}10^{-6})}} =$

Quality Factor $\mathbf{Q} = \dfrac{\mathbf{X_L}}{\mathbf{R}} = \dfrac{2\pi f_0 L}{R} = \dfrac{2\pi(\qquad\qquad)(0.1)}{2\text{x}10^3} =$

Bandwidth $\mathbf{BW} = \dfrac{\mathbf{f_o}}{\mathbf{Q}}$ **Hz** $=$

Lower half-power or cut-off frequency $\mathbf{f_1} = \mathbf{f_0} - \dfrac{\mathbf{BW}}{\mathbf{2}} =$

Upper half-power or cut-off frequency $\mathbf{f_2} = \mathbf{f_0} + \dfrac{\mathbf{BW}}{\mathbf{2}} =$

2. Compare the theoretical readings with your experimental results.

1.

	Experimental Readings	**Theoretical Readings**
$\mathbf{f_1}$		
$\mathbf{f_0}$		
$\mathbf{f_2}$		
BW		
Q		

3. How does the value of ***R*** affect the resonant frequency f_o, bandwidth ***BW*** and the ***Q*** factor of the series RLC circuit

Increase in R ___________ (increase, decrease, does not affect) the resonant frequency, f_o.

Increase in R ___________ (increase, decrease, does not affect) the bandwidth, ***BW.***

Increase in R ___________ (increase, decrease, does not affect) the quality factor, **Q**.

LABORATORY EXPERIMENT 2

Parallel RLC Resonant Circuit

E2.1 Objective

I. To draw a schematic diagram of a parallel RLC circuit using NI Multisim software.
II. To perform simulations on the parallel RLC circuit using NI Multisim software.

E2.2 Equipment

Notebook installed with NI Multisim

E2.3 Information

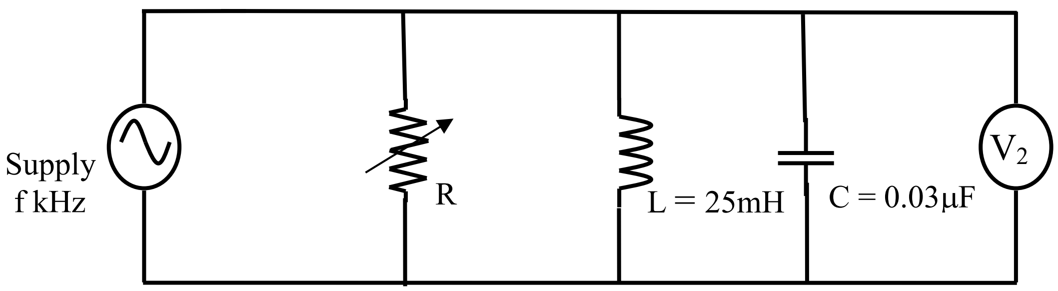


Figure 1

In a three branch parallel RLC resonant circuit as in Figure 1, there is frequency f_o of the voltage source V at which $\mathbf{X_L = X_C}$. Therefore

$$\mathbf{f_O = \frac{1}{2\pi\sqrt{LC}}\ Hz}$$

At this frequency the impedance of the RLC circuit is maximum and $Z_O = R$, the resistance in the circuit. The current in the circuit is then minimum and

$$I_O = \frac{V}{Z_O} = \frac{V}{R}$$

The quality factor Q of the circuit is defined as $Q = \frac{R}{X_L} = \frac{R}{X_C}$.

n a parallel resonant circuit the current through L and C are equal and are affected by the Q of the circuit. $I_L = I_C = QI_O$

Figure 2 shows the frequency response of a parallel resonant circuit. The two frequencies f_1 and f_2 which are called the half power points are located at 70.7 % of the maximum impedance curve. Bandwidth BW of the response curve is defined as the difference between the two frequencies f_1 and f_2.

$$\text{Bandwidth, } \mathbf{BW} = \frac{\mathbf{f_O}}{\mathbf{Q}}\ \mathbf{Hz}$$

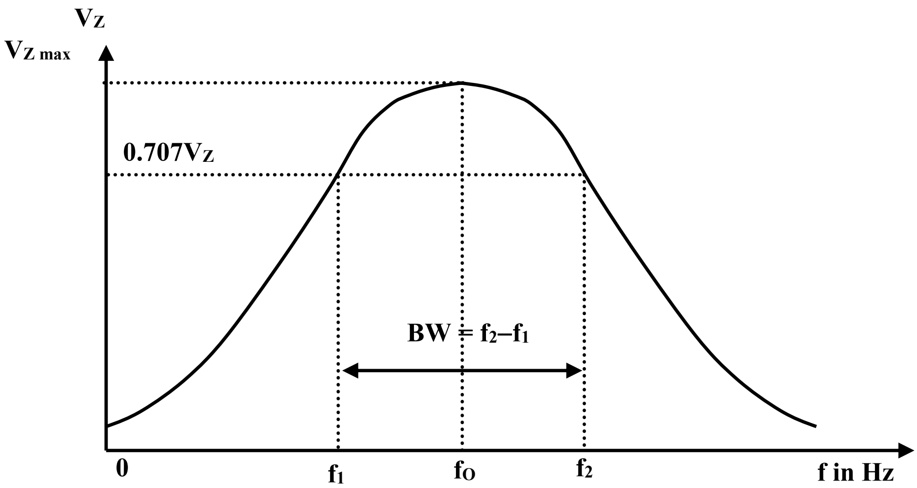
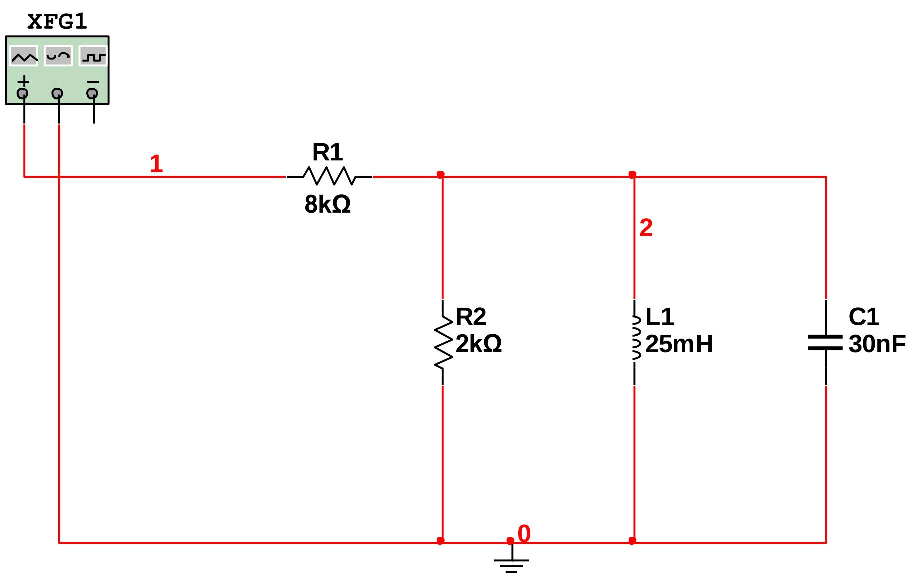


Figure 2: Frequency response curve for parallel RLC resonant

E2.3 Procedure

1. Construct the circuit below.



This schematic needs the following components from the Master Database.

(a) **Resistors, inductor** & **capacitor** from the **Basic** Group.

(b) Ground from the **Sources** Group > **POWER_SOURCES** > **GROUND**

The function generator can be obtained from **Simulate** > **Instruments** > **Function Generator**.

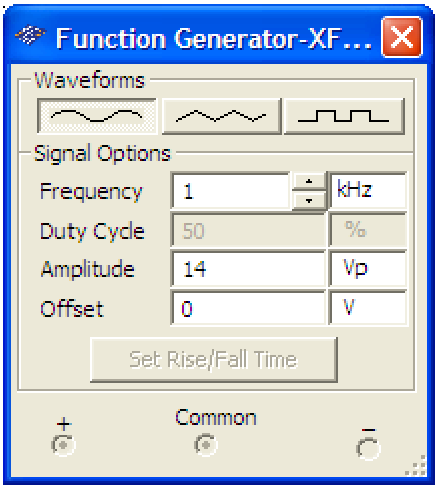
The Component Browser is used to select components for placement onto the schematic. To access the Component Browser, click on any icon in the parts bin, or select **Place/Component**.

You can change the property of the component by selecting the component (left click the mouse) and then right click the mouse and select Properties.

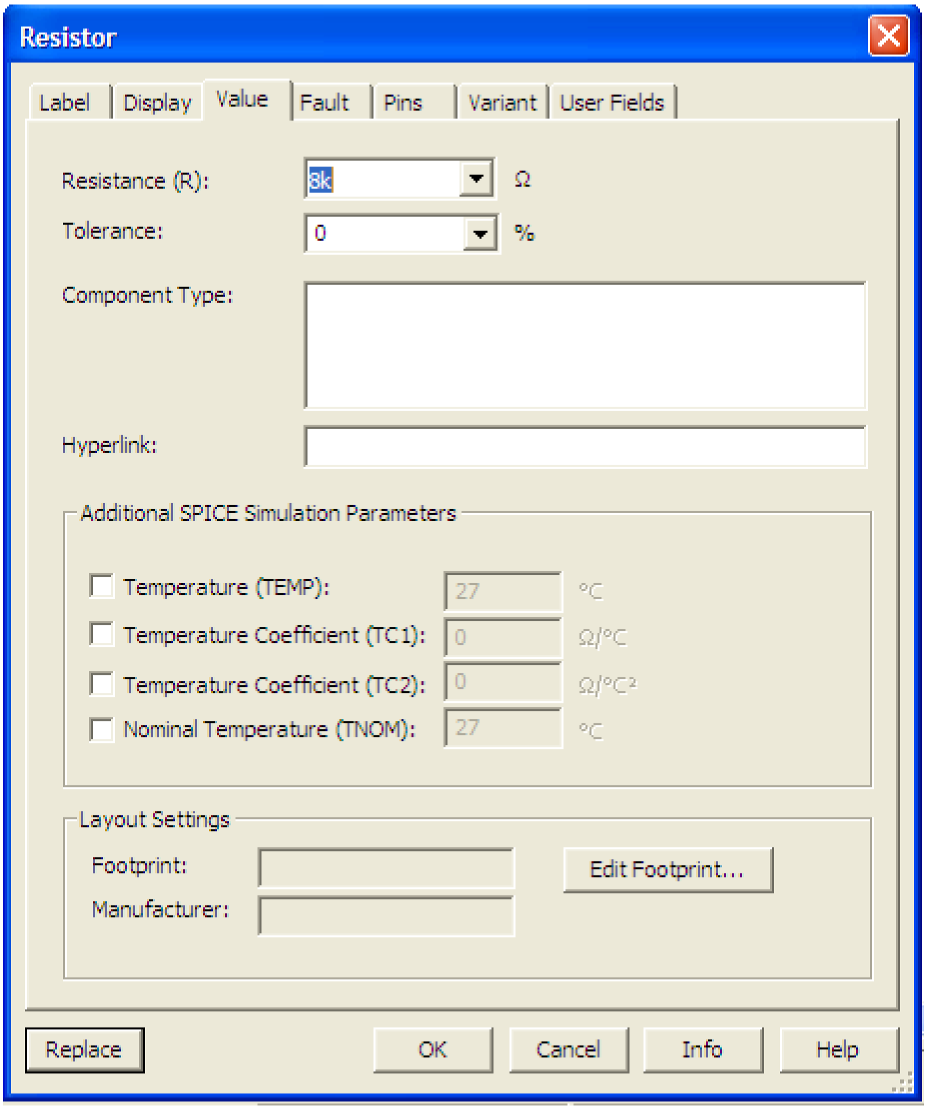
You may need to rotate some of the components for a prefer placement position. This can be done by selecting the component and then right click on the mouse.

Now, you need to connect the components together to form the circuit. Place the cursor at the pin of the components and the cursor will turn into a cross and drag it to the other end of the component to be connected.

2. Double click on the Function Generator and edit the **Frequency** and **Amplitude**.

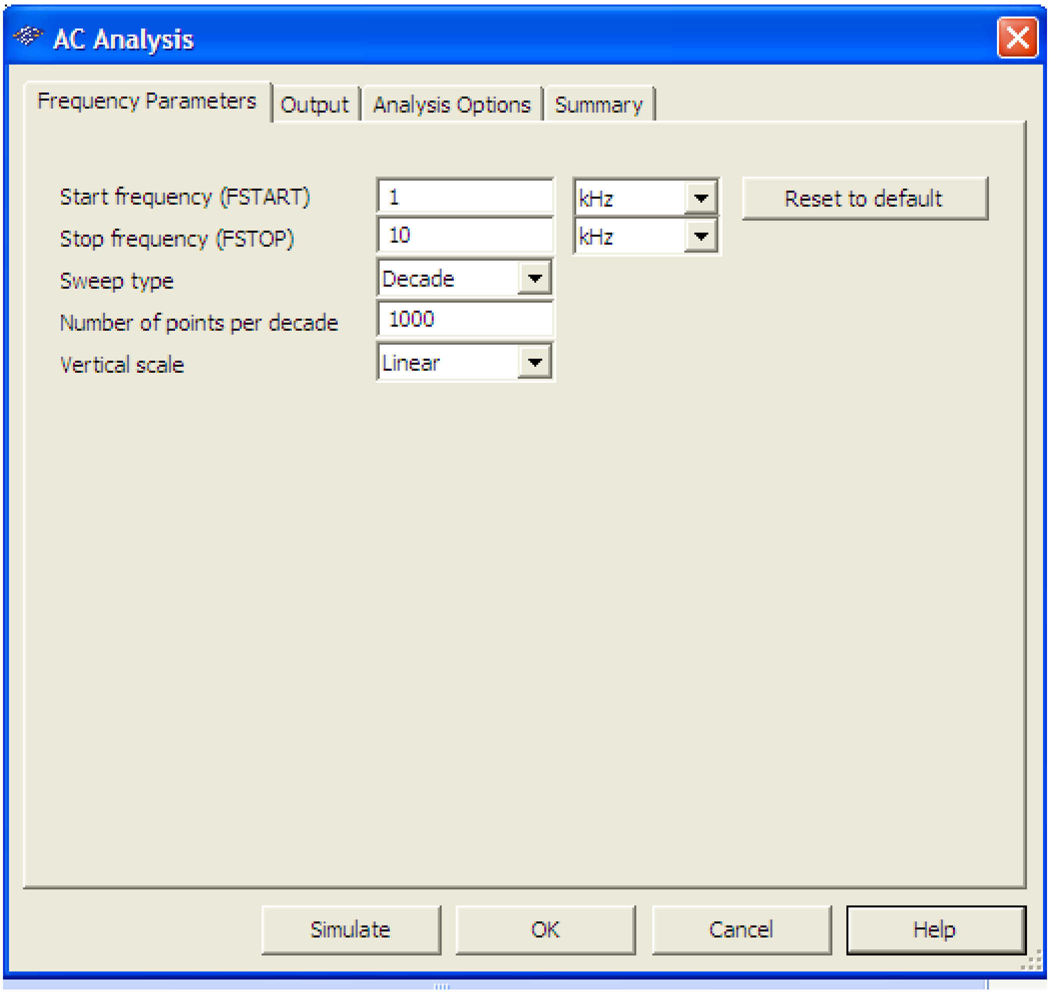


3. Double click on **Resistor** to change the resistor value.



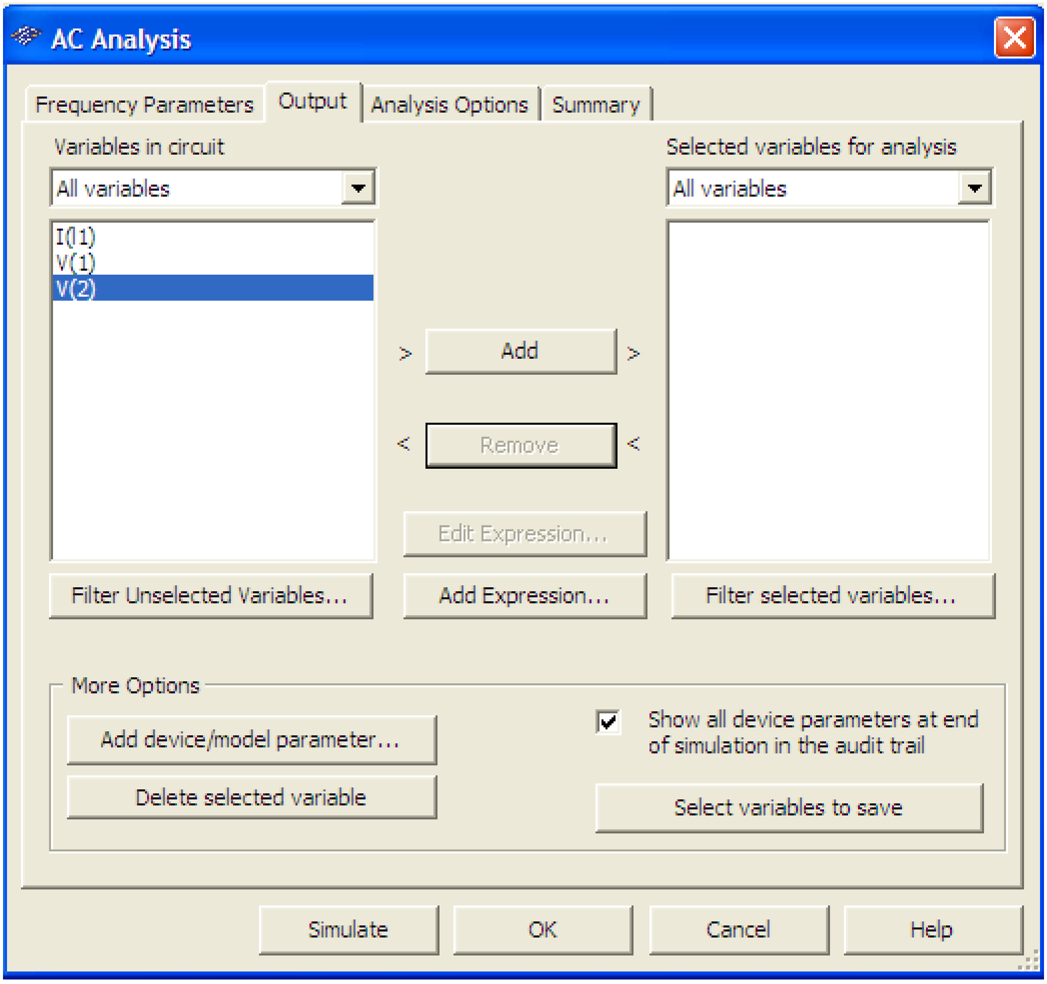
4. Repeat the above for the **Capacitor** and **Inductor**.
5. After finished, save your schematic.
6. To configure and begin an analysis, go to **Simulate - Analyses a**nd select **AC Analysis.**

7. At the **AC Analysis Setup Window**, edit **Frequency Parameters** as below.



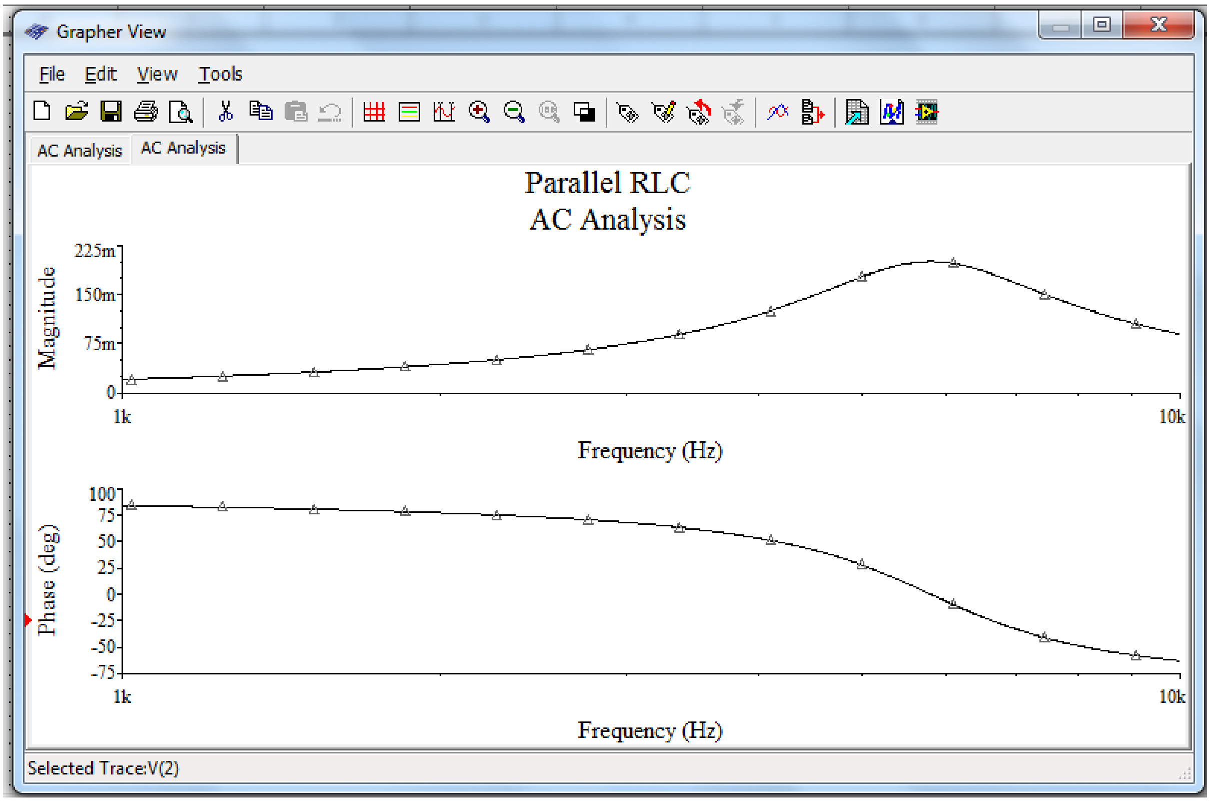
8. Click **Output**, **V(2)** and **Add**.

V(2) refers to the parallel part of the schematic circuit and hence it is added in for analysis.

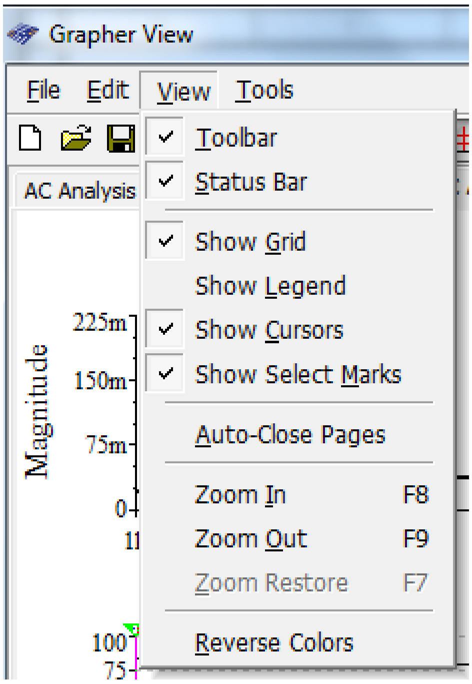


9. Click **Simulate**.

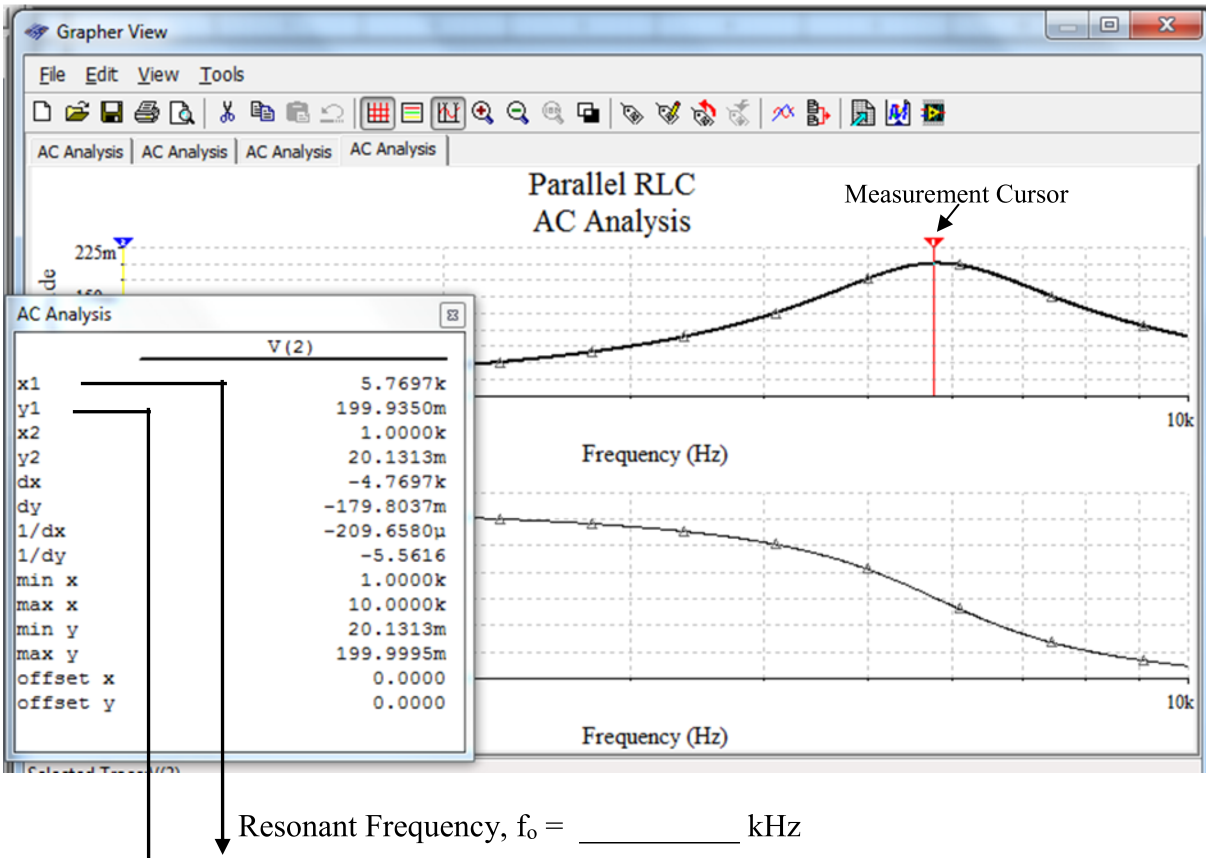
10. The results are displayed on a plot in Multisim's **Grapher**.



11. At the **Grapher View**, click **View.** Check **Show/Hide Grid** and **Show/Hide Cursors** in order to measure the values of the waveform.



12. Drag the measurement cursor from the y-axis to the peak of the frequency response curve to find out the resonant frequency.



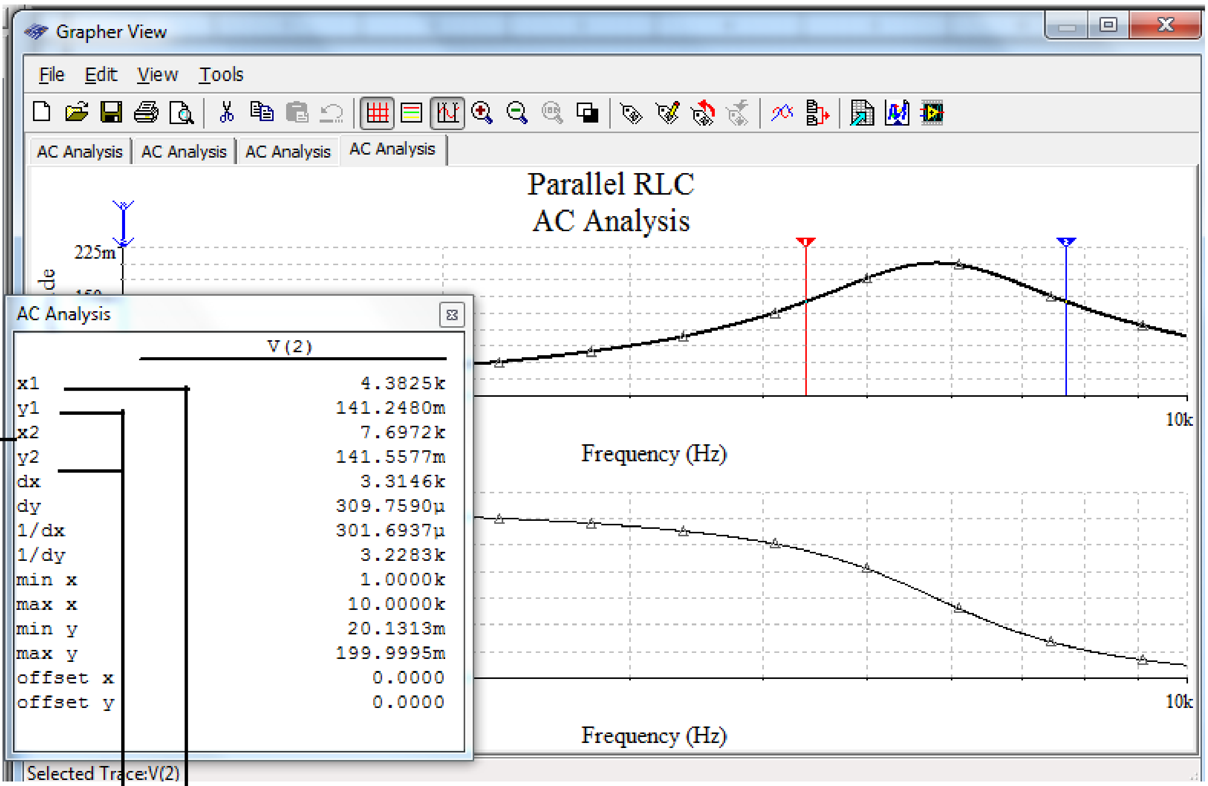
Resonant Frequency, f_o = __________ kHz

Peak value, V_2 / V_1 = __________

13. You can make use of the measurement cursors to obtain the half power frequencies f_1 and f_2. Hence, calculate the bandwidth BW and Q factor.

Hint: (a) Calculate the voltage (V_{12}) at half power frequencies. (0.707 x Peak value)

(b) Drag the two measurement cursors to find out the values on either side of the response curve.



f_1 = __________ kHz

0.707 x peak value = __________

f_2 = __________ kHz

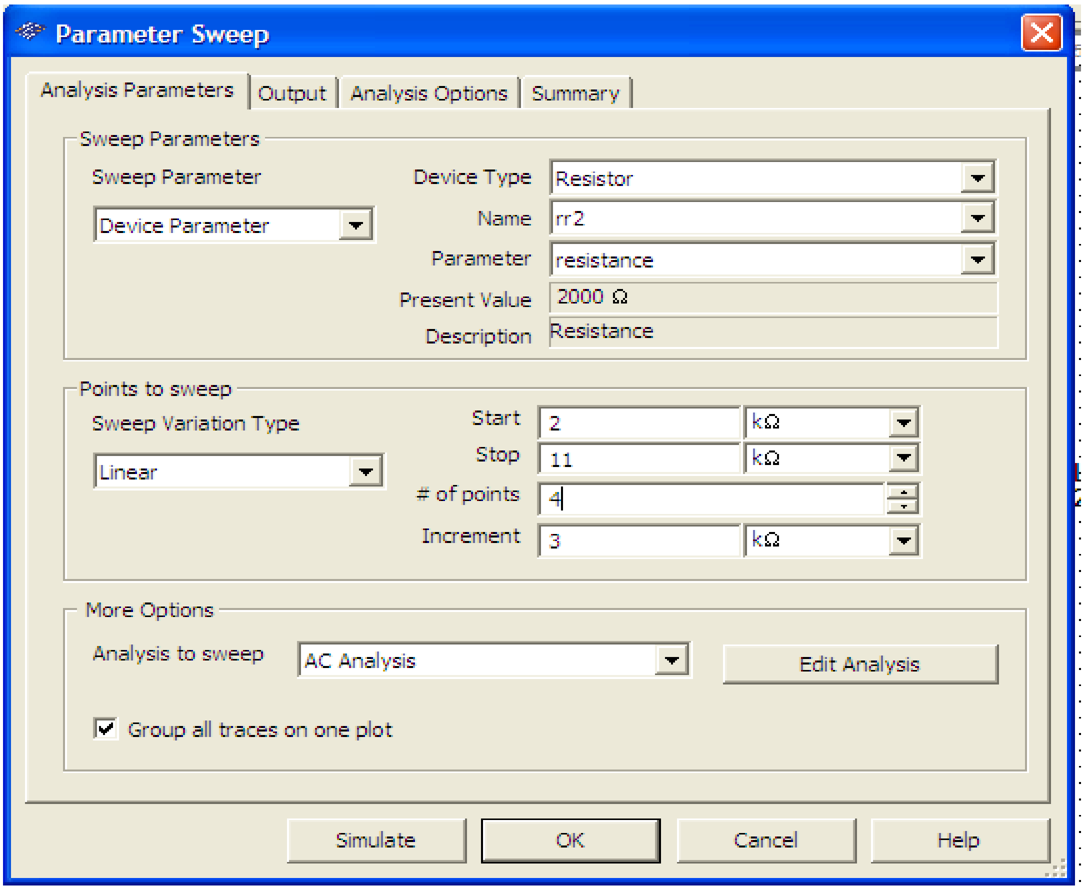
Hence, calculate:

Bandwidth BW = f_2 - f_1 = __________ kHz

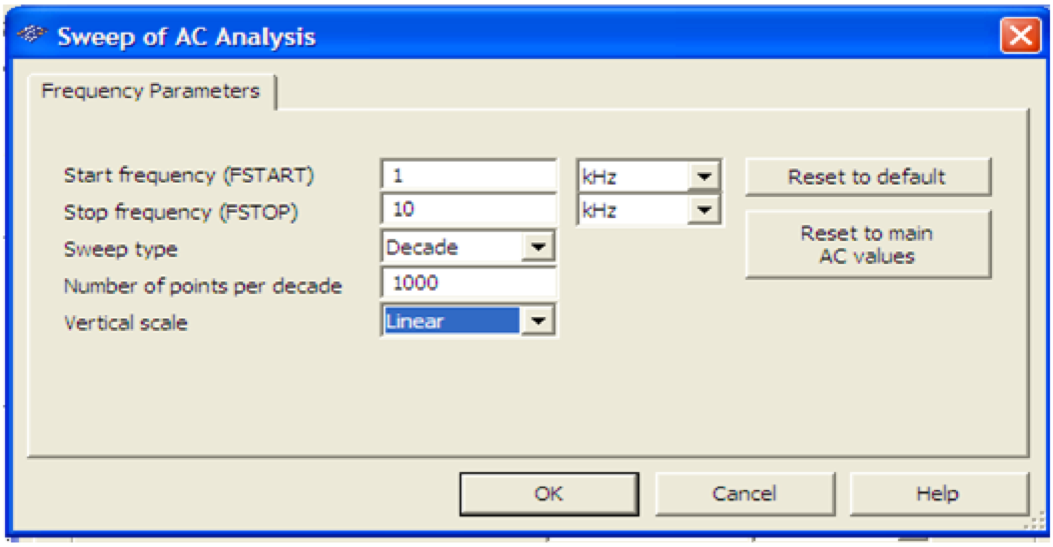
$$Q = \frac{f_o}{BW} = \frac{\qquad}{\qquad} = \_\_\_\_\_\_\_\_$$

To observe how the value of R_2 (the parallel resistor) affect the resonant frequency, f_o and bandwidth BW of the parallel RLC circuit.

14. Go to **Simulate - Analyses - Parameter Sweep** to set multiple AC Analysis simulations.

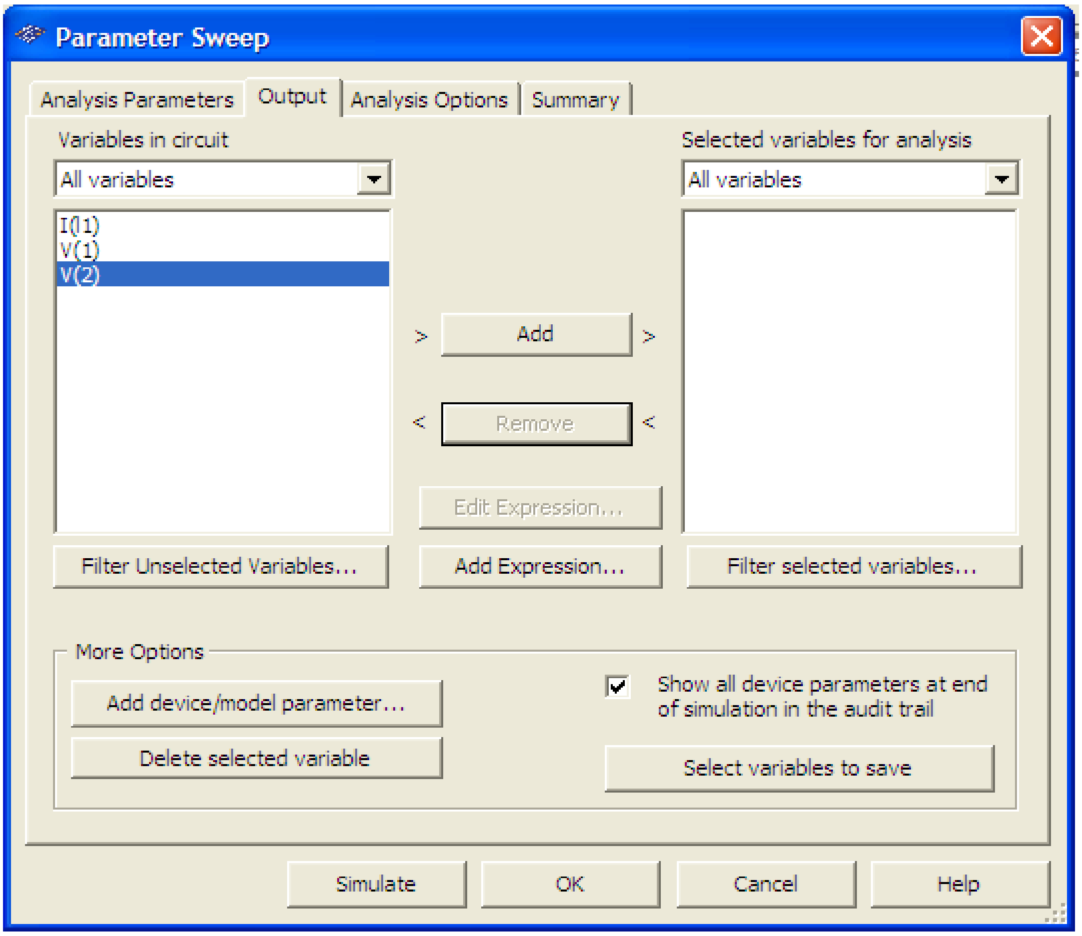


15. Click **Edit Analysis** to set the **Frequency Parameters.**

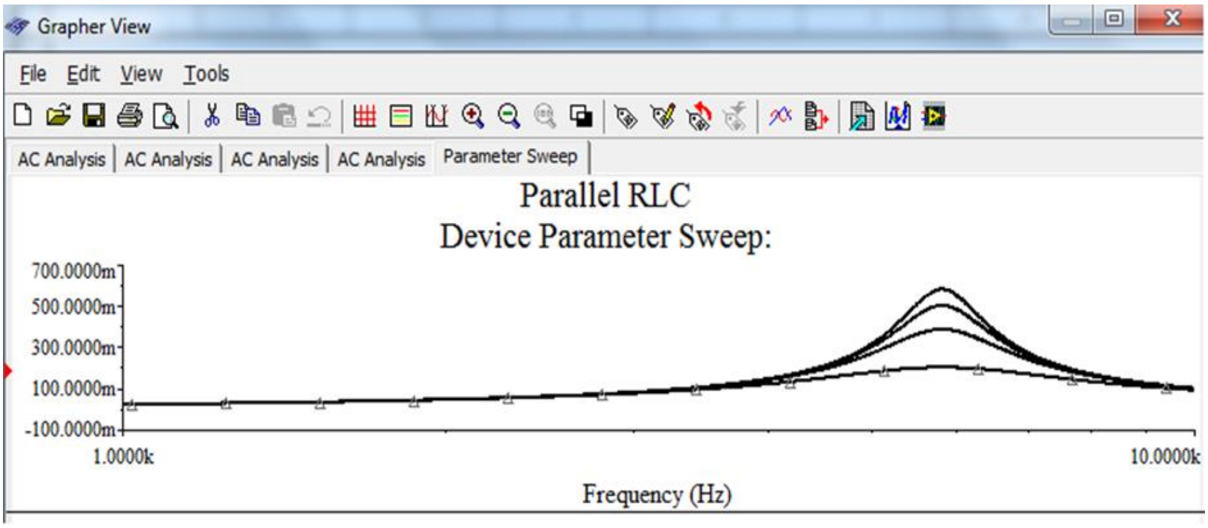


16. Click **OK.**

17. At the **Parameter Sweep** view, click **Output**, **V(2)** and **Add**.



18. Click **Simulate**.



19. From the response curve above, how does R_2 affect the resonant frequency $\boldsymbol{f_o}$ and bandwidth $\boldsymbol{BW}$.

Increase in R_2 ___________ (increase, decrease, does not affect) the resonant frequency, $\boldsymbol{f_o}$.

Increase in R_2 ___________ (increase, decrease, does not affect) the bandwidth, $\boldsymbol{BW}$.

E2.4 Discussion

1. The theoretical calculations are as shown.

Resonant Frequency $\mathbf{f_o} = \dfrac{\mathbf{1}}{\mathbf{2\pi\sqrt{LC}}}$ **Hz** $= \dfrac{1}{2\pi\sqrt{(25x10^{-3})(30x10^{-9})}} =$

Quality Factor $\mathbf{Q} = \dfrac{R}{X_L} = \dfrac{R}{2\pi f_0 L} = \dfrac{2x10^3}{2\pi(\qquad\qquad)(25x10^{-3})} =$

Bandwidth $\mathbf{BW} = \dfrac{\mathbf{f_o}}{\mathbf{Q}}$ **Hz** =

Lower half-power or cut-off frequency $\mathbf{f_1} = \mathbf{f_0} - \dfrac{BW}{2} =$

Upper half-power or cut-off frequency $\mathbf{f_2} = \mathbf{f_0} + \dfrac{BW}{2} =$

2. Compare the theoretical readings with the experimental results.

	Experimental Readings	**Theoretical Readings**
$\mathbf{f_1}$		
$\mathbf{f_0}$		
$\mathbf{f_2}$		
BW		
Q		

Name: ______________________________ Admission no: ________________

Class: _______________ Date: ________________

LABORATORY EXPERIMENT 3

Mesh and Nodal Analysis in a DC Network

E3.1 Objective

I. To analyze a DC network using the mesh and nodal analysis.

II. Perform simulations on the network given to verify experimental results.

E3.2 Equipment

1 Network board

1 DC power supply 0 – 30 V

1 Digital multimeter

E3.3 Information

E3.3.1 Mesh analysis method

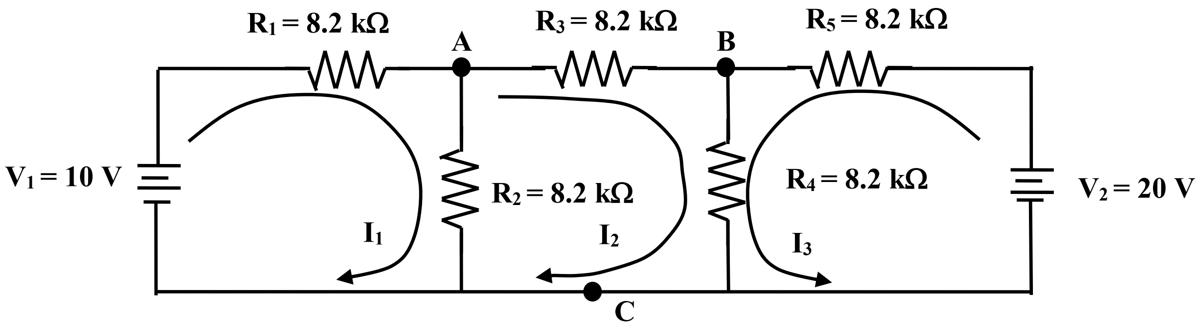


Figure 1

According to Kirchoff's voltage law, the sum of voltages around each loop must be zero. For example, the voltage equation for the loop current I_1 in Figure 1 is:

$$I_1R_1 + (I_1 - I_2)R_2 = V_1 \; ; \qquad \text{or } (R_1 + R_2)I_1 + (-R_2)I_2 + (0)I_3 = V_1$$

By considering the sum of the voltages within each loop in turn, one should be able to write down a matrix equation of the form

$$[\,\boldsymbol{R}\,]\,[\,\boldsymbol{I}\,] = [\,\boldsymbol{V}\,] \quad \text{...(1)}$$

Where,

[***R***] is an known **n x n** matrix (assuming there are n loops in the network), and [***I***] and [***V***] are the column vectors.

By using Cramers rule one should be able to evaluate the unknown column vectors [***I***].

E3.3.2 Nodal Analysis

According to the Kirchoff's current law, the sum of currents flowing out of a node must be zero. For example, by choosing the node **C** in Figure 1, as the reference node, we have for node **A.**

$$\frac{V_A - V_1}{R_1} + \frac{V_A}{R_2} + \frac{V_A - V_B}{R_3} = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_A + \left(-\frac{1}{R_3}\right)V_B = \left(\frac{1}{R_1}\right)V_1$$

$$(G_1 + G_2 + G_3)V_A + (-G_3)V_B = G_1V_1$$

Similarly, the nodal equations for any circuit can be written down in matrix form as

$$[\boldsymbol{G}]\ [\boldsymbol{V}] = [\boldsymbol{I}]$$

Where,

[***G***] is the conductance matrix (n x n matrix assuming n unknown nodes)

[***I***] is current source column matrices and

[***V***] is the unknown node voltage vector.

E3.4 Procedure

1. Redrawn circuit diagram given in Figure 1 showing meter connections in the respective positions to measure currents in R_1, R_2, R_3, R_4, R_5 and voltages at nodes A and B with respect to reference node C .

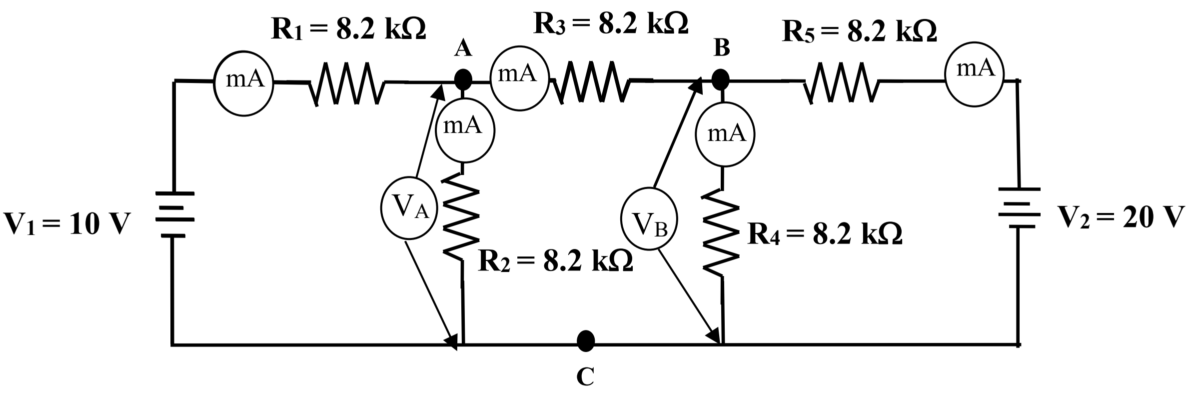


Figure 2: Redrawn circuit diagram of Figure 1 showing all meter connections

2. Connect the circuit drawn (Figure 2) in step 1. Adjust the power supplies so that $V_1 = 10$ V and $V_2 = 20$ V. Use the multimeter to check these values.
3. Use the multimeter as an ammeter to measure the current through each resistor ***one at a time***. Indicate their polarities (direction of current flow) in Figure 2 and tabulate the results in Table1.

4. Use the multimeter as a voltmeter to measure the voltages at nodes A and B with respect to reference node C. Note their polarities in Figure 2 and record the results in Table 2.

E3.5 Results

Table 1: Results for Mesh Analysis

I_{R1} (mA)	I_{R2} (mA)	I_{R3} (mA)	I_{R4} (mA)	I_{R5} (mA)

Table 2: Results for Nodal Analysis

V_A (V)	V_B (V)

E3.6 Discussion

1. How would you choose the mesh currents if you are asked to find the currents only through the resistors R_2 and R_4 (show your choice of loop currents by redrawing Figure 1)? Give reasons for your choice.

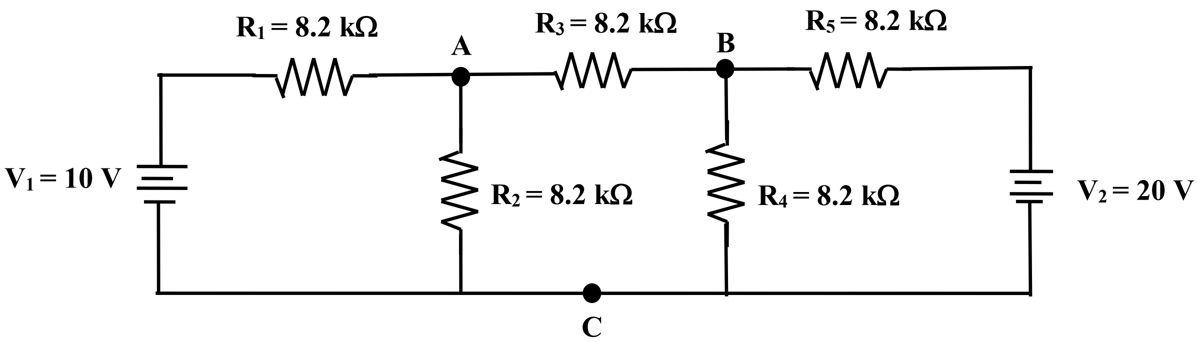


Figure 1

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LABORATORY EXPERIMENT 4
Star-Delta Transformation

E4.1 Objective

At the end of the experiment the student should be able:

I. To transform the star-connected network to an equivalent delta-connected network.

II. To transform the delta-connected network to an equivalent star-connected network.

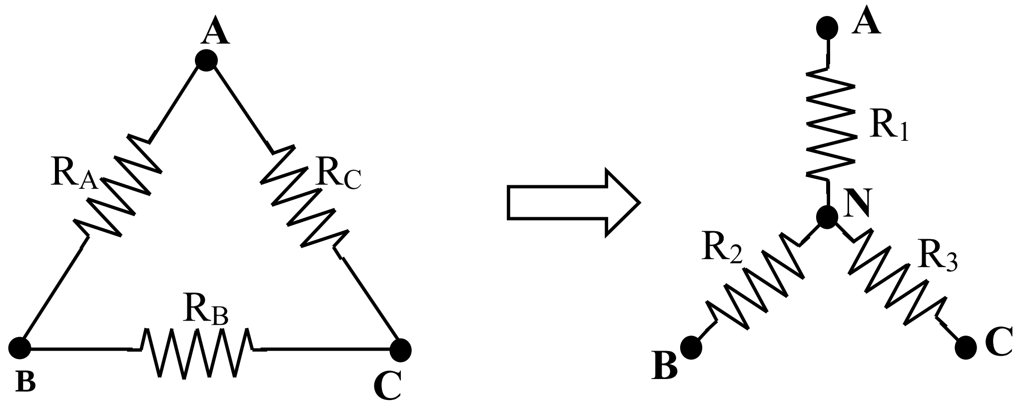
E4.2 Equipment

1 Digital multimeters

3 Decade resistance boxes 0-10,000 Ω

E4.3 Information

E4.3.1 Delta to star equivalent



(a) (b)

Figure 1

Figure 1(a) shows three resistors R_A, R_B, and R_C connected in a closed mesh or delta to three terminals A, B and C. It is possible to replace these delta connected resistors by three resistors R_1, R_2 and R_3 connected respectively between the same terminals A, B,C and a common point N as shown in Figure 1(b). Such an arrangement is said to be star-connected. If the star-connected network is to be equivalent to the delta-connected network, the resistance between any two terminals in Figure 1(b) must be the same as that between the same two terminals in Figure 1(a).

The formula to convert the delta network into a star equivalent is as follows:

$$\mathbf{R_1 = \frac{R_A R_C}{R_A + R_B + R_C}} \quad \text{................(1)}$$

$$\mathbf{R_2 = \frac{R_A R_B}{R_A + R_B + R_C}} \quad \text{................(2)}$$

$$\mathbf{R_3 = \frac{R_B R_C}{R_A + R_B + R_C}} \quad \text{................(3)}$$

Hence the above equations may expressed as: the equivalent star resistance connected to a given terminal is equal to the product of the two delta resistance connected to the same terminal divided by the sum of the delta resistances.

E4.3.2 Star to delta equivalent

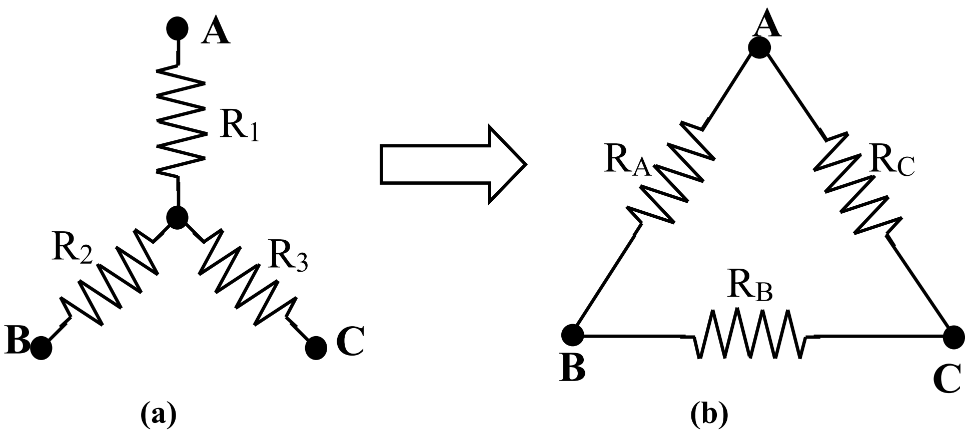


Figure 2

Figure 2(a) shows three resistors R_1, R_2 and R_3 connected to three terminals A, B, C and a common point N in star. Again it is possible to replace these star-connected resistors by three resistors R_A, R_B and R_C connected in mesh or delta between the same terminals A, B and C as in Figure 2(b). For the two networks to be equivalent, the resistance between any two terminals in Figure 2(a) must be the same as that between the same two terminals in Figure 2(b). The formula to convert the star network into a delta equivalent is as follows:

$$\mathbf{R_A = R_1 + R_2 + \frac{R_1 R_2}{R_3}} \quad \text{........................(4)}$$

$$\mathbf{R_B = R_2 + R_3 + \frac{R_2 R_3}{R_1}} \quad \text{........................(5)}$$

$$\mathbf{R_C = R_1 + R_3 + \frac{R_1 R_3}{R_2}} \quad \text{........................(6)}$$

Hence the above equations may be expressed as: the equivalent delta resistance between two terminals is the sum of the two star resistance connected to these terminals plus the product of the same two star resistance divided by the third star resistance.
The star-delta transformation and vice versa is also applicable to AC circuits by replacing the resistance to impedance.

E4.4 Procedure:

E4.4.1 Delta to star equivalent

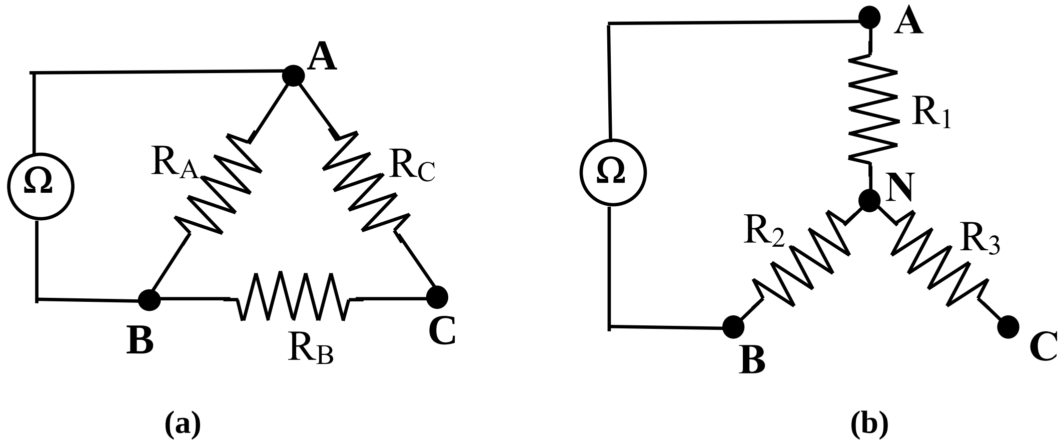


Figure 3

1. Set the three decade resistance boxes such that $\mathbf{R_A = 700\ \Omega}$, $\mathbf{R_B = 400\ \Omega}$ and $\mathbf{R_C = 900\ \Omega}$.
2. Connect the resistances R_A, R_B and R_C in delta as in Figure 3(a).
3. Use the multimeter as an ohmmeter and measure the resistance across terminals AB, BC, and CA ***one at a time.***
4. Calculate the equivalent star connected resistance R_1, R_2 and R_3.

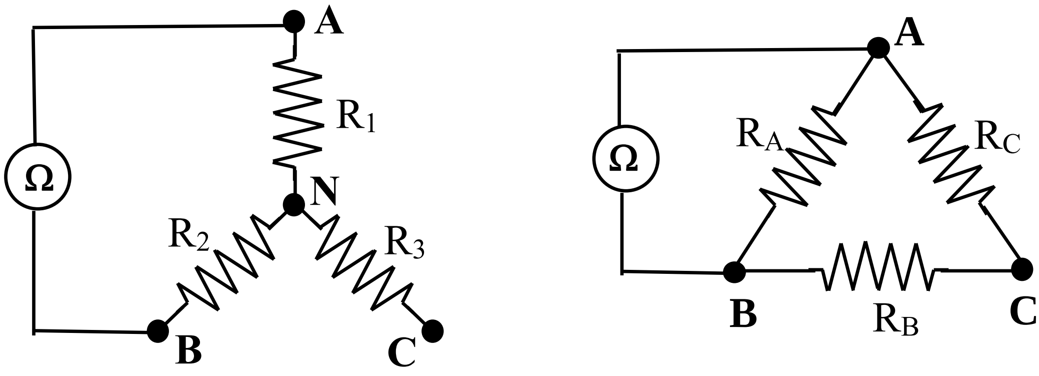
$$R_1 = \frac{R_A R_C}{R_A + R_B + R_C} =$$

$$R_2 = \frac{R_A R_B}{R_A + R_B + R_C} =$$

$$R_3 = \frac{R_B R_C}{R_A + R_B + R_C} =$$

5. Re-adjust the three resistance boxes to the calculated values in step 4 for R_1, R_2 and R_3 and connect them in star as in Figure 3(b).
6. Repeat the procedure step 3 to the star connected network.
7. Tabulate the results in Table1.

E4.4.2 Star to Delta equivalent



(a) (b)

Figure 4

1. Set the three decade resistance boxes such that **R_1 = 100 Ω, R_2 = 400 Ω** and **R_3 = 200 Ω.**
2. Connect the resistances R_1, R_2 and R_3 in star as in Figure 4(a).
3. Use the multimeter as an ohmmeter and measure the resistance across terminals AB, BC, and CA ***<u>one at a time.</u>***
4. Calculate the equivalent delta connected resistance R_A, R_B and R_C.

$$\mathrm{R}_{\mathrm{A}} = R_1 + R_2 + \frac{R_1 R_2}{R_3} \quad =$$

$$\mathrm{R}_{\mathrm{B}} = R_2 + R_3 + \frac{R_2 R_3}{R_1} \quad =$$

$$\mathrm{R}_{\mathrm{C}} = R_1 + R_3 + \frac{R_1 R_3}{R_2} \quad =$$

5. Re-adjust the three resistance boxes to the calculated values in step 4 for R_A, R_B and R_C and connect them in delta as in Figure 4(b).
6. Repeat the procedure step 3 to the delta-connected network.
7. Tabulate the results in Table 2.

E4.5 Results

Table 1: Delta to Star equivalent

Terminals	DELTA CONNECTION	STAR CONNECTION
R_{AB}		
R_{BC}		
R_{CA}		

Table 2: Star to Delta equivalent

Terminals	STAR CONNECTION	DELTA CONNECTION
R_{AB}		
R_{BC}		
R_{CA}		

E4.6 Discussion

1. Compare and comment on the readings obtained in Table 1 and 2 respectively.

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LABORATORY EXPERIMENT 5
Thevenin's and Norton's Theorems

E5.1 Objective

To verify and examine the working principles of Thevenin's and Norton's theorems.

E5.2 Equipment

1	Network board
1	DC power supply 0–30 V
2	Digital multimeters
2	Variable resistors

E5.3 Information

E5.3.1 Thevenin's Theorem

An active network having two terminals A and B as shown in Figure 1(a) can be replaced by a constant voltage source V_{TH} in series with an internal resistance R_{TH} or impedance Z_{TH} as shown in Figure 1(b). The value of V_{TH} (Thevenin voltage) is equal to the open circuit voltage across the two terminals A and B. The internal resistance R_{TH} or impedance Z_{TH} (Thevenin resistance or impedance) is the value of resistance or impedance measured between terminals A and B with the load disconnected and as viewed back into the network with all its voltage sources shorted.

E5.3.2 Norton's Theorem

Norton's theorem states that an active network having two terminals A and B as shown in Figure 1(a) can be replaced by a constant current source I_N in parallel with an internal resistance R_N or impedance Z_N as shown in Figure 1(c). The value of I_N (Norton current) is equal to the short circuit current between the two terminals A and B if the load resistance is replaced by a short circuit. The internal resistance R_N or impedance Z_N (Norton resistance or impedance) is the value of resistance or impedance measured between terminals A and B with the load disconnected and as viewed back into the network with all its voltage sources shorted. Thus R_N is defined exactly the same manner as R_N in Thevenin's theorem.

E5.4 Circuit diagram

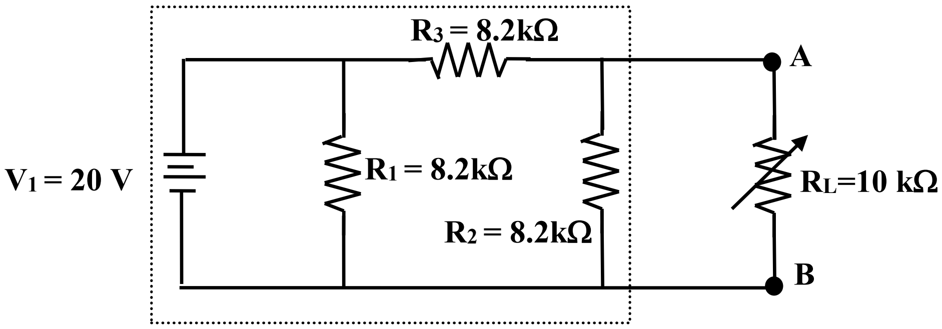
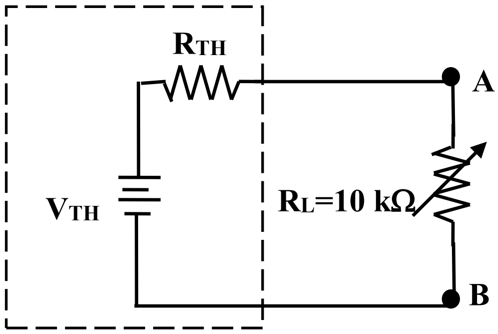


Figure 1(a)



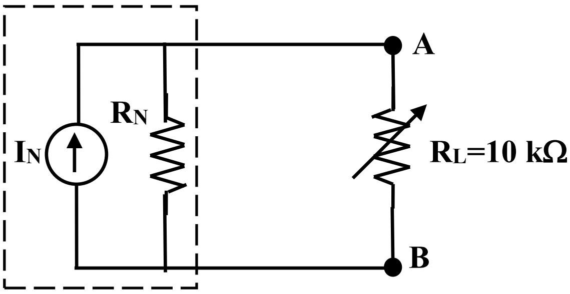


Figure 1(b) **Figure 1(c)**

E5.5 Procedure

E5.5.1 Thevenin's Theorem

1. Redrawn circuit diagram given in Figure 1(a) showing meter connections with polarity marked in the respective positions to measure the voltage (V_{RL}) across and current (I_{RL}) flowing in R_L.

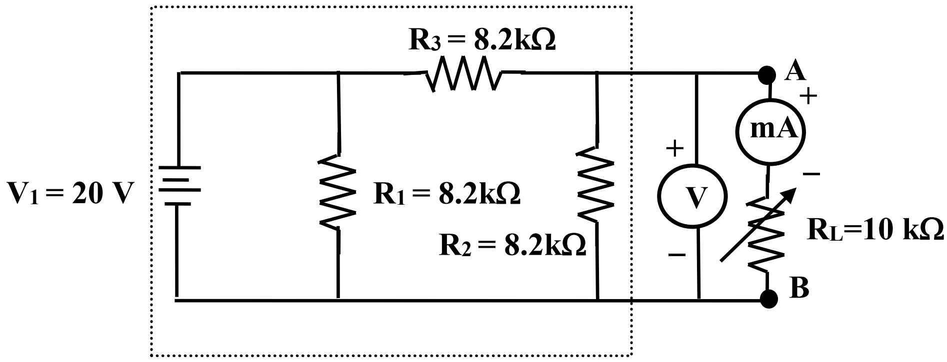


Figure 2: Measurement of V_{RL} and I_{RL}

2. Connect the circuit drawn (Figure 2) in step1.
3. Measure the voltage across (V_{RL}) and current flowing (I_{RL}) in R_L and tabulate the results in Table1.
4. To determine the Thevenin's voltage V_{TH}, remove the load R_L from the circuit connected in step 2, and use the multimeter as a voltmeter and measure the voltage across the open terminals A and B. Tabulate the results in Table 1. Draw the corresponding connection diagram (Figure 3a) in your laboratory sheet.
5. To determine the Thevenin's resistance R_{TH}, switch off the power supply and short its terminals. Use the multimeter as an ohmmeter and measure the resistance at the open load terminals A and B (***Note: Do not switch on the supply***). Draw the corresponding connection diagram (Figure 3b) in your laboratory sheet. Tabulate the measured result in Table 1.

Figure 3(a) Measurement of V_{TH} Figure 3(b) Measurement of R_{TH}

Figure 3: Measurement of V_{TH} and R_{TH} for the Thevenin's equivalent circuit

6. Redrawn a fully labeled circuit diagram (Figure 4) of the circuit given in Figure 1(b) in your laboratory sheet showing meter connections in the respective positions to measure the voltage (V_{RL}) across and current (I_{RL}) flowing in R_L. Mark the values of V_{TH} and R_{TH} obtained in step 4 and 5, respectively in the diagram.

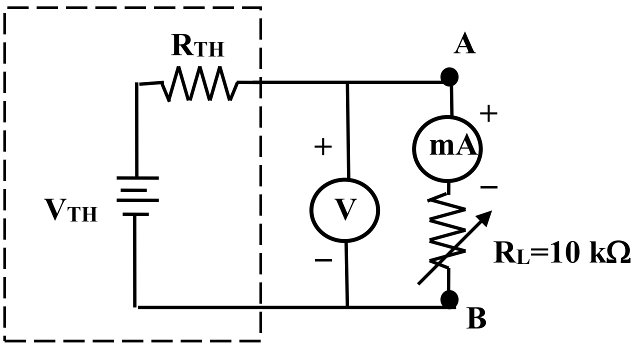


Figure 4: Measurement of V_{RL} and I_{RL} using Thevenin's equivalent circuit

7. Connect up the circuit drawn in step 6.

8. Measure the voltage (V_{RL}) across and current (I_{RL}) flowing in R_L and tabulate in Table 1.

E5.5.2 Norton's Theorem

1. Redrawn a fully labeled circuit diagram of the circuit given in Figure 1(a) in your lab-sheet, showing meter connection with polarity marked to measure the short circuit current (I_N) flowing at the shorted load terminals AB.

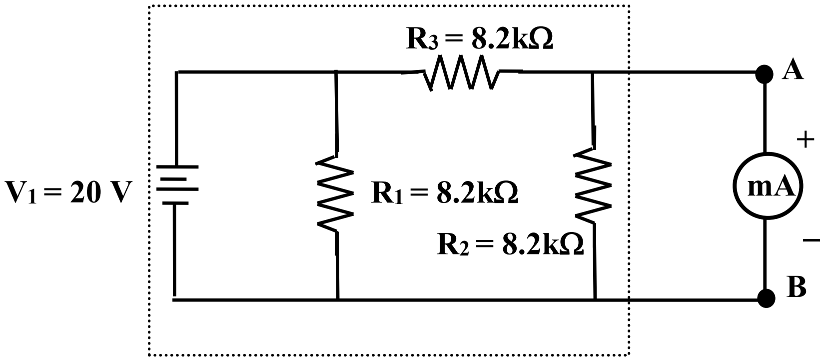


Figure 5: Measurement of I_N (short circuit current at AB)

2. Connect the circuit drawn (Figure 5) in step 1 and measure I_N (short circuit current) and tabulate in Table 1.

3. To determine the Norton's resistance R_N, off the power supply and short its (power supply) terminals. Use the multimeter as an ohmmeter and measure the resistance at the open load terminals A and B (***<u>Note: Do not switch on the supply</u>***. Connection is same as Figure 3(b). Hence $R_{TH} = R_N$.

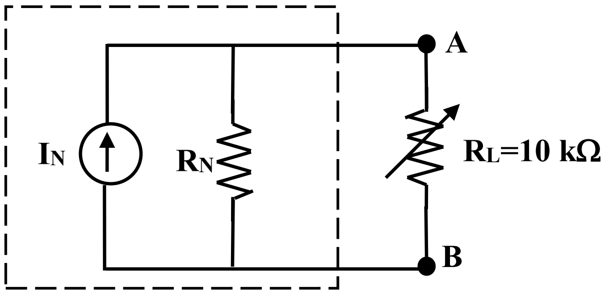
E5.6 Results

Table: 1

V_{RL} Figure 2	I_{RL} Figure 2	V_{TH} Figure 3(a)	$R_{TH} = R_N$ Figure 3(b)	V_{RL} Figure 4	I_{RL} Figure 4	I_N Figure 5

E5.7 Discussion

1. Making use of the readings obtained in steps 2 and 3 for the Norton's equivalent circuit, calculate the voltage across and current flowing in R_L. Compare your results with the experimental results and explain for any discrepancies.



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LABORATORY EXPERIMENT 6

GUIDANCE FOR LAB ASSESSMENT

E6.1 Objective

Student will be given details of the assessment requirements, including the description of task, organization, presentation of work and assessment scheme. Upon completion of the assignment, student should be able to apply the knowledge gained in the module to obtain the Thevenin equivalent circuit for the given network and carry out investigations to justify.

E6.2 Equipment

1 Network board

1 DC power supply 0-30 V

2 Digital multimeters

2 Variable resistors

E6.3 Scenario

For this assessment, you will work to apply the fundamental knowledge of Electrical Engineering circuits, theorem and the usage of basic measuring instruments such as the voltmeter and ammeter to:

- Analyse the task.
- Conduct voltage and current measurements for the load connected to terminals A and B.
- Design an equivalent circuit to replace the given network and determine its parameters.
- Connect up the designed equivalent circuit and carry out investigations to justify.
- Record all the results in the investigations.

E6.4 Assessment

The lab assignment is to be carry out individually during the lab session and will comprise 10 % of the total marks for this module.

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LABORATORY EXPERIMENT 7

Three-Phase Star-Delta Connections

E7.1 Objective

I. To connect up a balanced delta and star network and to verify the relationships between line and phase currents as well as voltages.

II. To examine the neutral current for an unbalanced star network.

E7.2 Equipment

1 x 3 resistive load bank (Lab Volt)

1 x 3 AC voltmeter 0 – 250 V (Lab Volt)

1 AC Clamp Meter

E7.3 Information

Three alternating voltages of the same frequency energise a three-phase electric circuit. These three voltages have equal magnitudes and are displaced by 120^0 from each other. The three-phase circuit is merely a combination of three single-phase circuits. In a balanced star-connected three phase load, the line to line (V_L) voltage is $\sqrt{3}$ times each line to neutral (V_{PH}) voltage. Each line current (I_L) is the same as each load current (I_{PH}).

However in a balanced delta-connected three-phase load, the voltage and current characteristics are opposite to those of a star connection. The line to line (V_L) voltage is the same as each load voltage (V_{PH}) but the line current (I_L) is $\sqrt{3}$ times each load current (I_{PH}).

E7.4 Procedure

E7.4.1 Balanced Delta Connections

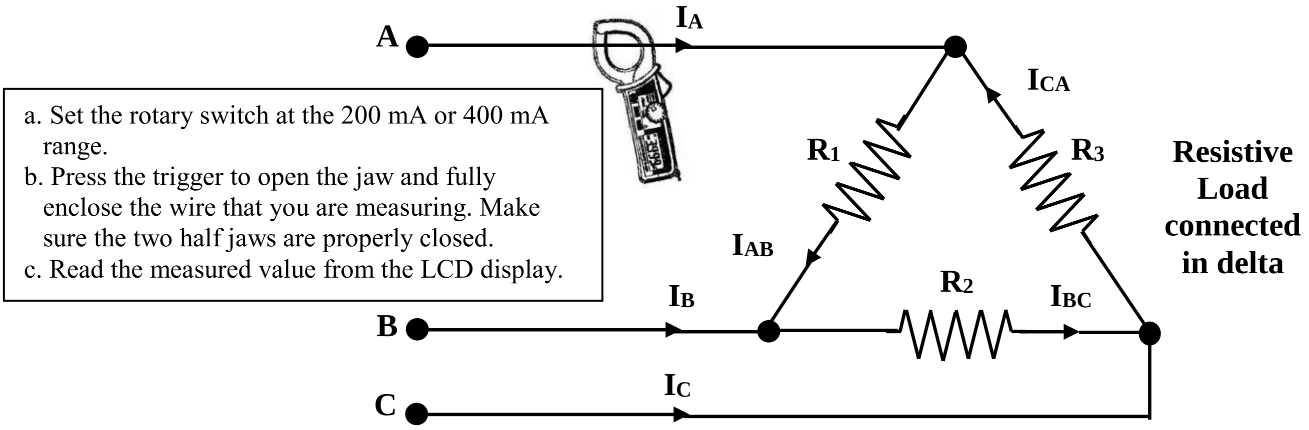

Figure 1

1. Put all the resistive bank switches in the '0' position.
2. Connect up the circuit in Figure 1.
3. Set R_1, R_2 and R_3 to 2400 Ω by putting the respective switches in the '1' position.
4. Switch on the 3-phase supply to the loads.
5. Use the clamp meter to measure all the currents one at a time and record them in Table 1.
6. Switch off the supply to the loads.

E7.4.2 Balanced and Unbalanced Star-connections

1. Put all the resistive bank switches in the '0' position.
2. Connect up the circuit in Figure 2.
3. Set R_1, R_2 and R_3 to 1200 Ω by putting the respective switches in the '1' position.
4. Switch on the 3-phase supply to the loads.
5. Use the clamp meter to measure all the currents one at a time and tabulate your results in Table 2.
6. Switch off the supply to load.
7. Repeat the experiment (step 4 to step 6) for unbalanced loads with R_3 set to 2400 Ω. Tabulate your results in Table 3.

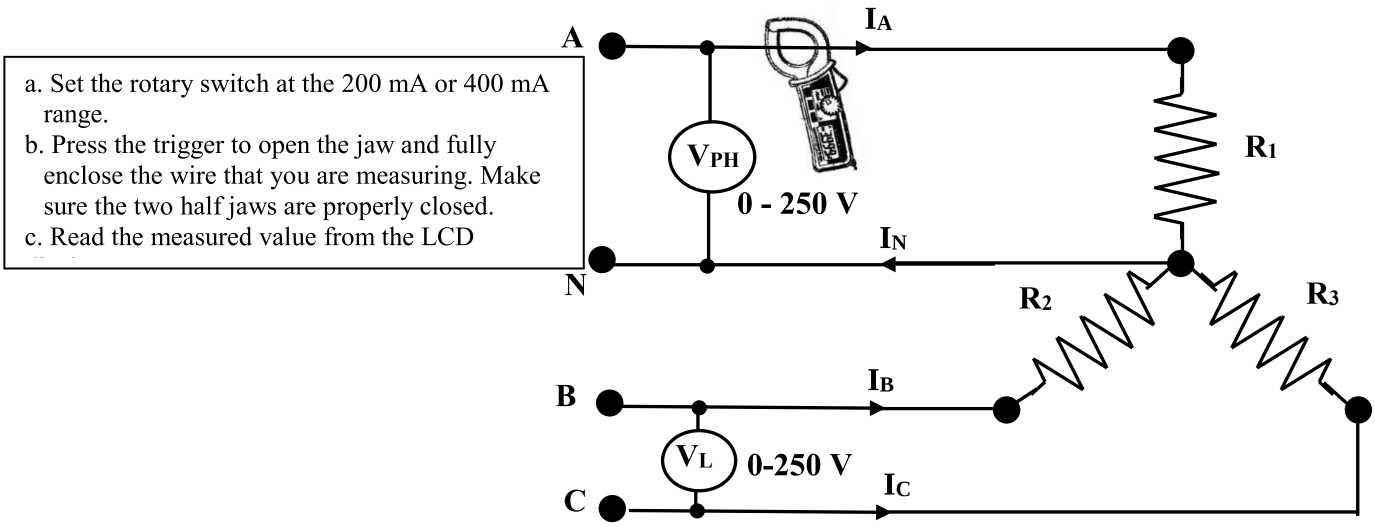

Figure 2

E7.5 Results

Table 1: Balanced Delta Connections

I_A (A)	I_B (A)	I_C (A)	I_{AB} (A)	I_{BC} (A)	I_{CA} (A)

Table 2: Balanced Star Connections

V_L (V)	V_{PH} (V)	I_A (A)	I_B (A)	I_C (A)	I_N (A)

Table 3: Unbalanced Star Connections

V_L (V)	V_{PH} (V)	I_A (A)	I_B (A)	I_C (A)	I_N (A)

E7.6 Discussion

1. For Table 1, draw to scale the current phasor diagrams.

2. Determine the ratio of $\frac{\mathbf{I_L}}{\mathbf{I_{PH}}}$ of Table 1.

3. Calculate the ratio of $\frac{\mathbf{V_L}}{\mathbf{V_{PH}}}$ for Tables 2 and 3.

4. Draw the line currents, neutral current, line voltages and phase voltages phasor diagrams for Table 2 and 3. Compare the neutral current with that obtained experimentally.

V_{AN}

Balanced Star-Connected Load

V_{AN}

Unbalanced Star-Connected Load

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LABORATORY EXPERIMENT 8

Three Phase Power Measurement

E8.1 Objective

I. To determine the apparent, real and reactive power in a three phase circuit.

II. To calculate the power factor in three phase loads.

E8.2 Equipment

1 x 3 resistive load bank (Lab Volt)

1 x 3 inductive load bank (Lab Volt)

1 x 3 AC voltmeter 0 – 250 V (Lab Volt)

1 AC voltmeter 0 - 300 V

1 AC Clamp Meter

E8.3 Information

Power developed in each phase (P_{PH}) of either a delta or star connection is

$$P_{PH} = V_{PH} I_{PH} \cos \phi \quad \ldots\ldots (1)$$

Where ϕ is the angle between the phase current and the phase voltage. The total power in all three phase of a balanced three phase load is then

$$P_{Total} = 3 \times P_{PH} = 3 V_{PH} I_{PH} \cos \phi \quad \ldots\ldots (2)$$

In a star connection: $I_{PH} = I_L,\ V_{PH} = \dfrac{V_L}{\sqrt{3}}$

Thus, the three phase power in a balanced star connected system in terms of line voltage and current is

$$P_{Total} = 3 \times P_{PH} = 3 V_{PH} I_{PH} \cos \phi$$

$$P_{Total} = 3 \left(\frac{V_L}{\sqrt{3}}\right) I_L \cos \phi = \sqrt{3}\, V_L I_L \cos\phi \quad \ldots\ldots (3)$$

In a delta connection: $V_{PH} = V_L,\ I_{PH} = \dfrac{I_L}{\sqrt{3}}$

Thus, the three phase power in a balanced star connected system in terms of line voltage and current is

$$P_{Total} = 3 \times P_{PH} = 3 V_{PH} I_{PH} \cos \phi$$

$$\mathbf{P_{Total} = 3\, V_L \left(\frac{I_L}{\sqrt{3}}\right) \cos\phi = \sqrt{3}\, V_L I_L \cos\phi} \text{.......................................} \mathbf{(4)}$$

Hence the expression for three phase power in a balanced system of either star or delta connected is equal to

$$\mathbf{P_T = \sqrt{3}\, V_L I_L \cos\phi\ (W)} \text{.......................................} \mathbf{(5)}$$

So the three phase apparent power in balanced system is

$$\mathbf{S_T = \sqrt{3}\, V_L I_L\ (VA)} \text{.......................................} \mathbf{(6)}$$

Three phase reactive power is equal to the sum of the three reactive powers per phase

$$\mathbf{Q_T = \sqrt{3}\, V_L I_L \sin\phi\ (VAR)} \text{.......................................} \mathbf{(7)}$$

The power factor of balanced three-phase connection is equal to

$$\textbf{Power factor } = \cos\phi = \frac{P_T}{S_T} \text{.......................................} \mathbf{(8)}$$

E8.4 Procedure

E8.4.1 Three phase resistive load

a. Set the rotary switch at the 200 mA or 400 mA range.
b. Press the trigger to open the jaw and fully enclose the wire that you are measuring. Make sure the two half jaws are properly closed.
c. Read the measured value from the LCD display.

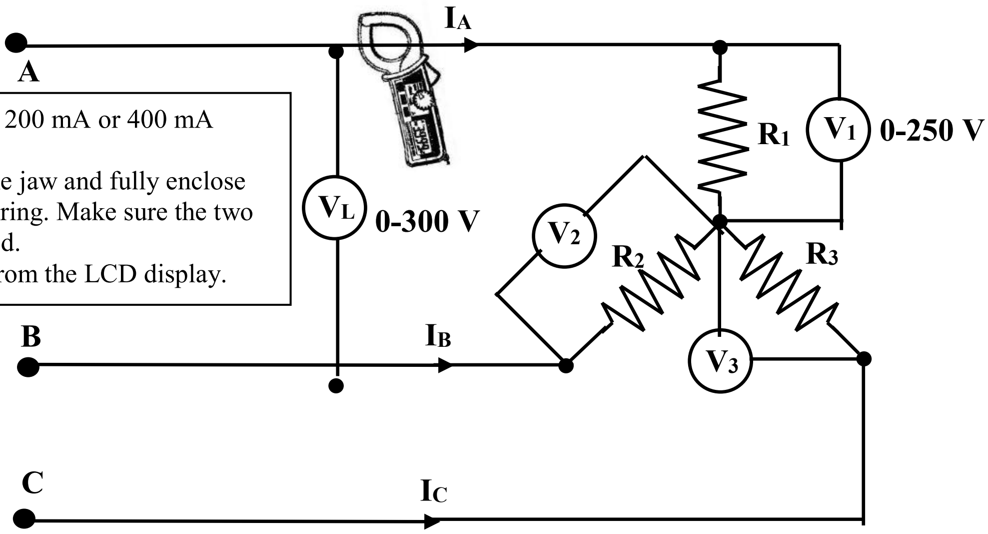

Figure 1

1. Put all the resistive bank switches in the ‘0’ position.
2. Connect up the circuit in Figure 1.
3. Set R_1, R_2 and R_3 to 1200 Ω by putting the respective switches in the ‘1’ position.
4. Switch on the 3-phase supply to the loads.
5. Use the clamp meter to measure all the currents one at a time and record them in Table 1.
6. Switch off the supply to the loads.

E8.4.2 Three phase resistive and inductive loads in series

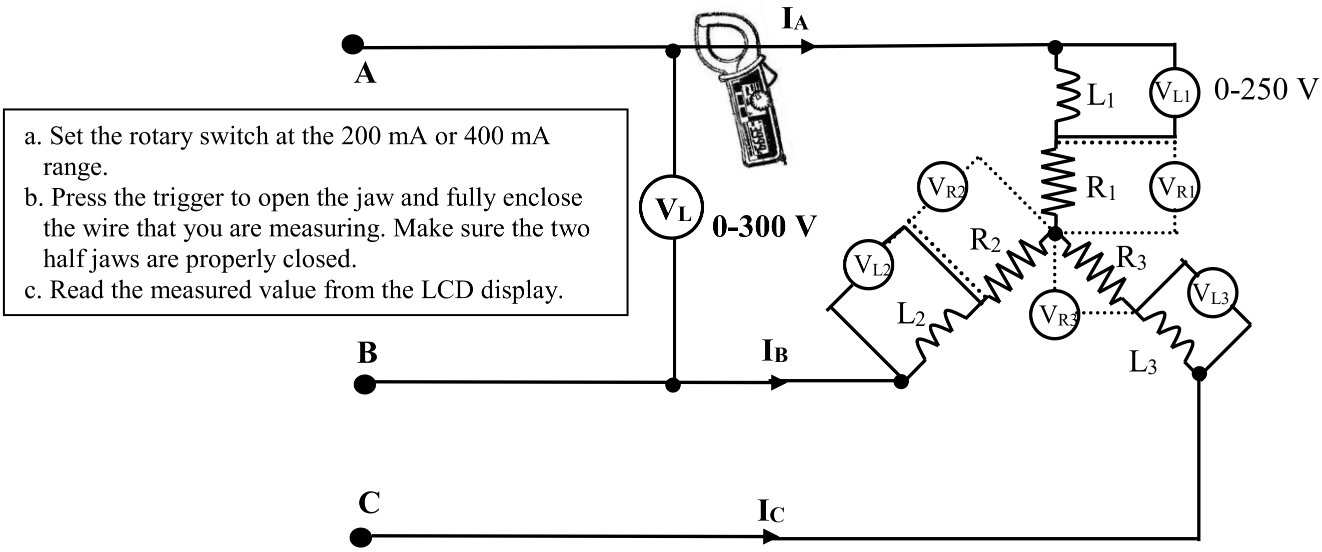

Figure 2

1. Put all the resistive bank switches in the ‘0’ position.
2. Connect up the circuit as shown in Figure 2 to measure the voltage across the inductor.
3. Set R_1, R_2 and R_3 to 1200 Ω by putting the respective switches in the ‘1’ position.
4. Set L_1, L_2 and L_3 to 7.6 H by putting the respective switches in the ‘1’ position.
5. Switch on the three phase supply to the loads.
6. Use the clamp meter to measure all the currents one at a time and tabulate in Table 2.
7. Switch off the supply
8. Reconnect each of the voltmeters to measure the voltage across the resistor as shown in the dotted lines in Figure 2. (**Note: connect all the three voltmeters to measure the voltages across all the three resistors simultaneously**).
9. Switch on the three phase supply and record the voltmeter readings in Table 2.
10. Switch off the supply.

E8.5 Results

Table 1: Resistive Loads only

Line voltage V_L=_______V

SET	I_A (A)	I_B (A)	I_C (A)	V_1 (V)	V_2 (V)	V_3 (V)	P_1 (W)	P_2 (W)	P_3 (W)
$R_1 = R_2 = R_3 = 1200\ \Omega$									

Table 2: Resistive and Inductive Loads in series

Line voltage V_L=_______________V

SET	I_A (A)	I_B (A)	I_C (A)	V_{R1} (V)	V_{R2} (V)	V_{R3} (V)	V_{L1} (V)	V_{L2} (V)	V_{L3} (V)
$R_1 = R_2 = R_3 = 1200\ \Omega$ $L_1 = L_2 = L_3 = 7.6$ H									

From Table 2, calculate the real power per phase for the resistive loads and the reactive power per phase for the inductive loads and tabulate in Table 3.

Table 3: Resistive and Inductive Loads in series

SET	P_1 (W)	P_2 (W)	P_3 (W)	Q_1 (VAR)	Q_2 (VAR)	Q_3 (VAR)
$R_1 = R_2 = R_3 = 1200\ \Omega$ $L_1 = L_2 = L_3 = 7.6$ H						

E8.6 Discussion

1. Calculate the total three-phase power using the sum of the phase power from Table 1 and compare with the calculated value of equation (3).

2. What is the value of the total apparent power, reactive power and the power factor of Figure 1.

3. Calculate the total three phase real power and reactive power using the sum of the real power and reactive phase power respectively from Table 3.

4. Calculate the total apparent power from step 3 and compare with the calculated value of equation (6).

5. Determine the power factor of Figure 2.

6. Would only one wattmeter be needed to measure the total three-phase power on a balanced load, three-phase four wire system? Give your reasons.

Name: ________________________________ Admission no: ____________________

Class: ______________ Date: ______________________

LABORATORY EXPERIMENT 9
Power Factor Improvement

E9.1 Objective

To learn how to use a capacitor to improve power factor.

E9.2 Equipment

1 Variac, 0 – 260 V, 2.5 A

1 AC voltmeter, 0 – 75 V

3 AC ammeters 0 – 1 A

1 Wattmeter, 0 – 120 V, 1 A

1 Power factor meter

1 RL load

1 Capacitor load

E9.3 Information

Most residential and industrial loads are inductive and have a lagging power factor. A low lagging power factor leads to a high current flow. If a load takes a current of I ampere with a lagging power factor, then the current I can be resolved into two components, as shown in Figure 1.

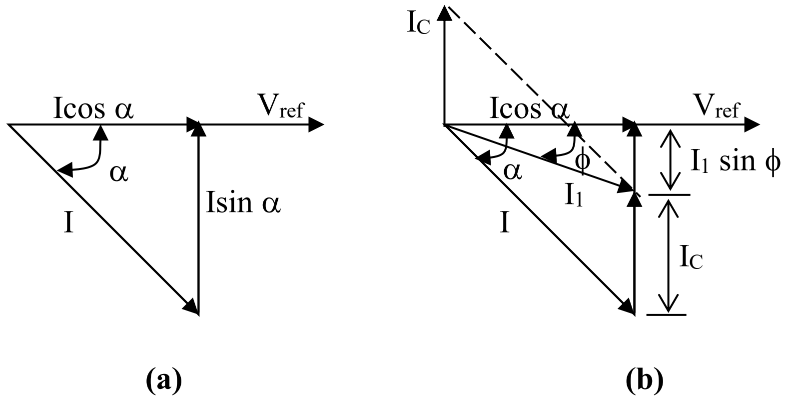

Figure 1

$I\cos\alpha$ is the active component of the current and is in phase with the voltage. $I\sin\alpha$ is the reactive component of the current and is 90^0 out of phase with the voltage. In order to reduce the current I, a capacitor is connected in parallel with the load. As the capacitor takes a current I_C, which leads the voltage by 90^0, the resultant reactive current is reduced so that the overall power factor of the new supply current I_1 is raised, i.e. the angle between I_1 and the voltage is reduced.

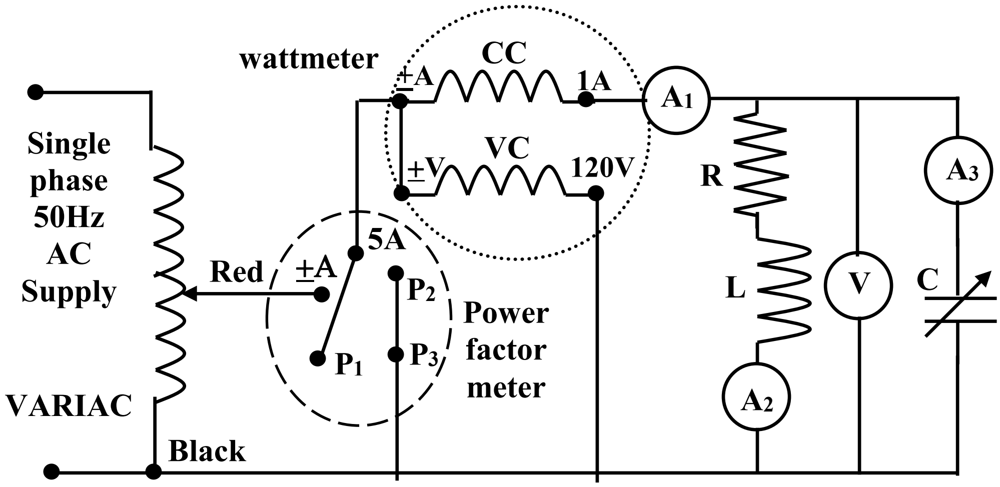

Figure 1

E9.4 Procedure

1. Connect the circuit shown above.
2. With all the capacitance set to zero, vary the variac slowly until the voltmeter reads 30 V. Maintain this voltage throughout the experiment.
3. With the capacitor bank set to the values indicated in Table 1, take the readings of all the meters and complete Table 1.
4. Slowly bring the variac voltage to 0 V and switch off the supply.

E9.5 Results

Table 1

C in μF	Cos α measured	Power W	I_1 (A)	I_2 (A)	I_3 (A)
0					
10					
20					
30					
40					

E9.6 Discussion

1. Calculate the values of R and L of the load impedance.

2. Calculate the capacitance value required to give unity power factor. Compare this value with the experimental one.

Name: ______________________________________ Admission no: ________________

Class: _______________ Date:_____________

LABORATORY EXPERIMENT 10

Two Wattmeter Method of Power Measurement

E10.1 Objectives

I. To learn how to connect two wattmeters to three-phase loads for power measurements.

II. To measure the power of various three-phase loads using two wattmeter method and to determine the power factor of balanced-loads.

E10.2 Equipment

1 x 3 resistive load bank (Lab Volt)

1 x 3 inductive load bank (Lab Volt)

2 Wattmeters

1 AC Voltmeter 0 – 250 V (Lab Volt)

1 AC Clamp Meter

E10.3 Information

Power in a three phase circuit can be measured by using two wattmeters as shown in Figure 1 for a star-connected circuit.

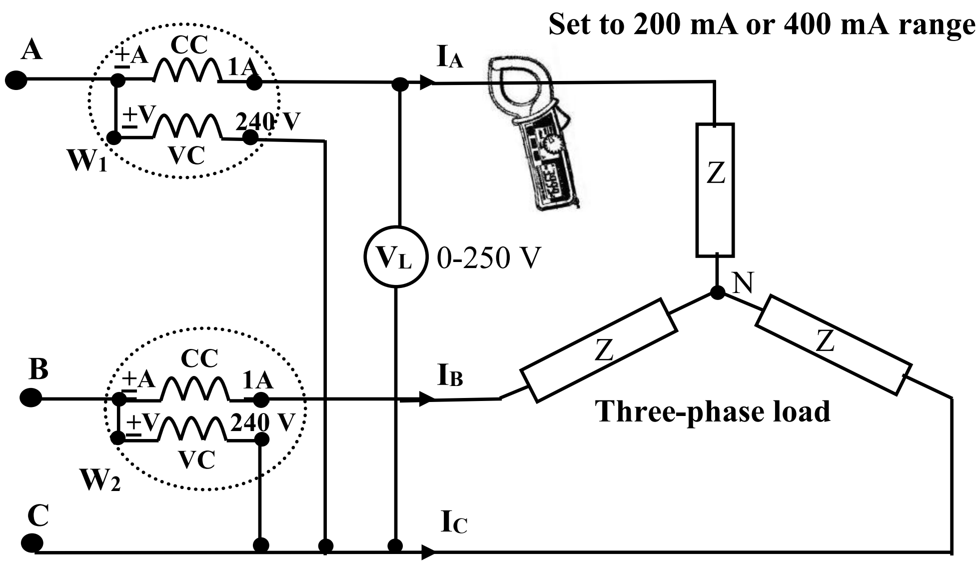

Figure 1

The phasor diagram of the current taken by the current coils and voltage across the voltage coils of W_1 and W_2 are as shown:

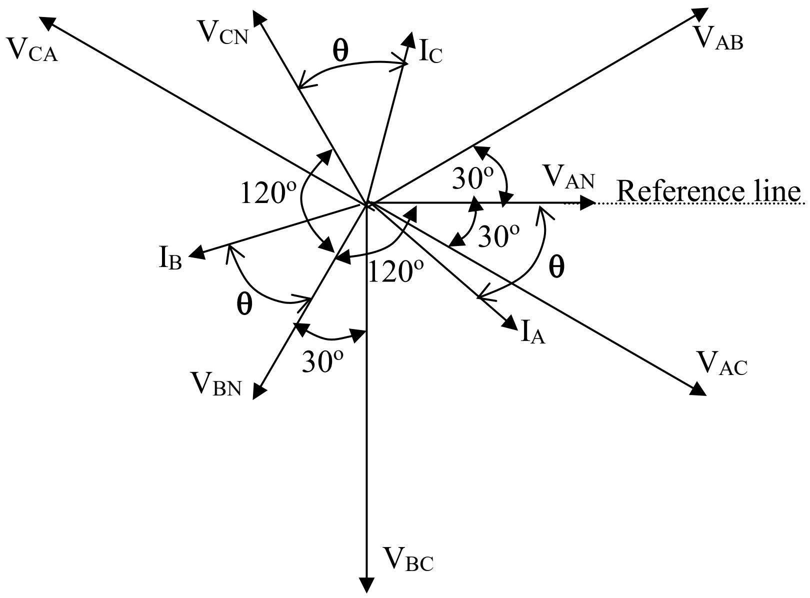

Figure 2

From the phasor diagram above, the readings of W_1 and W_2 are:

$$\mathbf{W_1 = V_{AC}\, I_A \cos(\theta - 30^o)\ watts}$$
$$\mathbf{= V_L\, I_L \cos(\theta - 30^o)\ watts}$$
$$\mathbf{W_2 = V_{BC}\, I_B \cos(\theta + 30^o)\ watts}$$
$$\mathbf{= V_L\, I_L \cos(\theta + 30^o)\ watts}$$

The sum of W_1 and W_2 should give correctly the power measured in a balanced three-phase system of any power factor. Thus:

$$\mathbf{P_{Total} = W_1 + W_2 = V_L\, I_L \cos(\theta - 30^o) + V_L\, I_L \cos(\theta + 30^o)}$$
$$\mathbf{= \sqrt{3}\ V_L\, I_L \cos\theta\ \ watts}$$

The same approach can be used to derive the readings of W_1 and W_2 if the balanced three phase loads are connected in delta.

The power factor of the three phase loads can be determined by the readings of W_1 and W_2. This is only applicable to the balanced loads.

Thus:

$$\tan\theta = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) \text{ and power factor} = \cos\theta = \cos\left(\tan^{-1}\left[\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right]\right)$$

When the power factor of the load is 0.5 lagging, θ is 60^{o}, then the readings of W_2 is zero and W_1 is maximum.

When the power factor is less than 0.5 lagging, θ is greater than 60^{o}, the reading on W_2 is **negative**. To measure this power, it is necessary to **reverse** the connections to either the current or the voltage coil of the wattmeter and thus the reading obtained must be taken as negative when the total power is considered.

E10.4 Procedure

1. Put all the resistive bank switches in the '0' position.
2. Connect up the circuit as shown in Figure 1 using resistive loads.
3. Set R_1, R_2 and R_3 to 1200 Ω by putting the respective switches in the '1' position.
4. Switch on the three-phase supply.
5. Use the clamp meter to measure any one of the line currents for values of R set in Table 1.
6. Switch off the supply to the loads.
7. Repeat the experiment using inductive loads only and record all readings in accordance to the settings in Table 2.

   Note: One of the wattmeters will read negative. To measure this power, it is necessary to **reverse** the connections of either the current or voltage coil of that wattmeter. The reading shown on that wattmeter will then be taken as negative. **Remember to switch off the supply when doing the reversal.**

8. Repeat the experiment for resistive-inductive loads in parallel and record all readings in accordance to the settings in Table 3.

   Note: Connect the current or voltage coil of the wattmeter back to its original position.

Redrawn the connection circuit diagram for the resistive-inductive loads in parallel in Figure 3 along with meter connections.

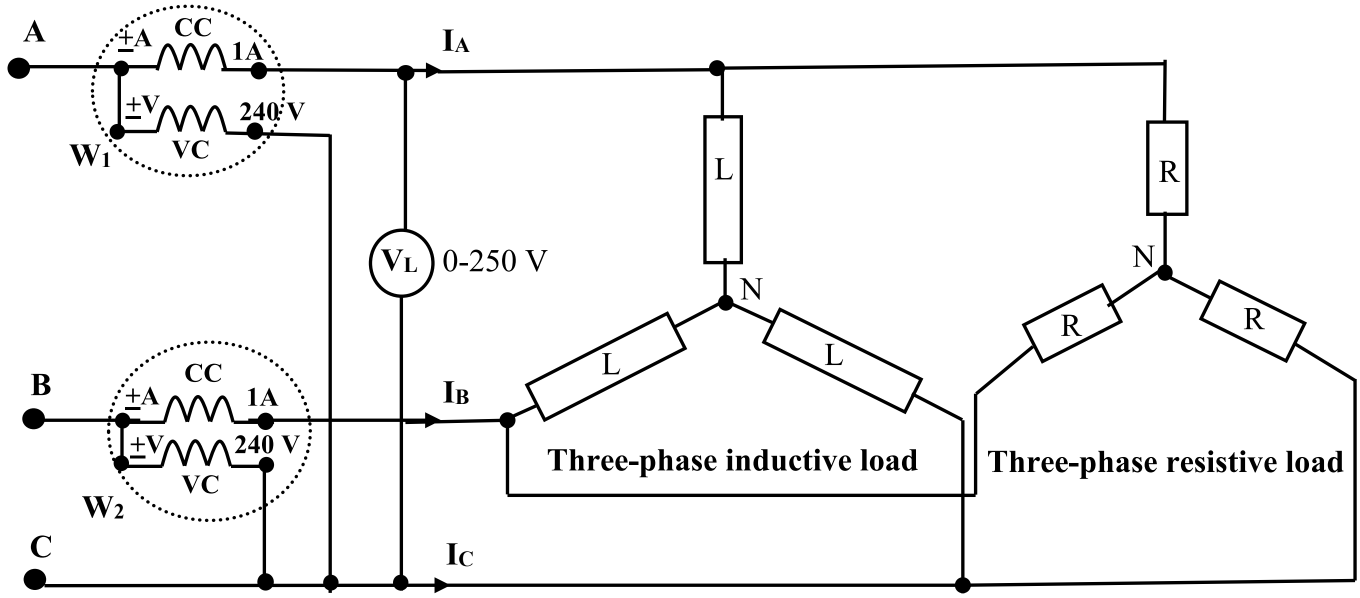

Figure 3: Circuit diagram for resistive and inductive load in parallel.

E10.5 Results

Table 1 : Resistive Loads Only

SET R (for all 3 phases)	W_1 (W)	W_2 (W)	V_L (V)	I_L (A)	P_{Total} W_1+W_2	p.f. cos θ	θ (deg)
1200 Ω to '1'							
1200 Ω & 2400 Ω to '1'							

Table 2 : Inductive Loads Only

SET L (for all 3 phases)	W_1 (W)	W_2 (W)	V_L (V)	I_L (A)	P_{Total} W_1+W_2	p.f. cos θ	θ (deg)
3.8 H to '1'							
7.6 H to '1'							

Table 3: Resistive and Inductive Loads in parallel

SET	W_1 (W)	W_2 (W)	V_L (V)	I_L (A)	P_{Total} W_1+W_2	p.f. cos θ	θ (deg)
$R_1 = R_2 = R_3 = 1200\ \Omega$ $L_1 = L_2 = L_3 = 3.8$ H							
$R_1 = R_2 = R_3 = 1200\ \Omega$ $L_1 = L_2 = L_3 = 7.6$ H							

E10.6 Discussion

1. Write your comments on the experimental results:-

   (a) Resistive Loads Only

   (b) Inductively Loads Only

   (c) Resistive-Inductive Loads in Parallel

2. If two wattmeters are used to measure total power in a three-phase three-wire system, does each wattmeter measure single phase power? Explain.

3. When do you use two wattmeters to measure the total three-phase power of a three-phase system ?

4. What is the significance of a negative indication on a wattmeter?

Engineering @ SP

The School of Electrical & Electronic Engineering at Singapore Polytechnic offers the following full-time courses.

1. Diploma in Aerospace Electronics (DASE)

The Diploma in Aerospace Electronics course aims to provide students with a broad-based engineering curriculum to effectively support a wide spectrum of aircraft maintenance repair and overhaul work in the aerospace industry and also to prepare them for further studies with advanced standing in local and overseas universities.

2. Diploma in Computer Engineering (DCPE)

This diploma aims to train technologists who can design, develop, setup and maintain computer systems; and develop software solutions. Students can choose to specialise in two areas of Computer Engineering & Infocomm Technology, which include Computer Applications, Smart City Technologies (IoT, Data Analytics), Cyber Security, and Cloud Computing.

3. Diploma in Electrical & Electronic Engineering (DEEE)

This diploma offers a full range of modules in the electrical and electronic engineering spectrum. Students from 2019/20 Year 1 intake can choose one of the six available specialisations (Biomedical, Communication, Microelectronics, Power, Rapid Transit Technology and Robotics & Control) for their final year. Students from earlier intakes and direct-entry 2nd year students can choose one of the seven available double-specialisation tracks (Aerospace + Communication, Biomedical + Robotics & Control, Computer + Communication, Microelectronics + Nanoelectronics, Microelectronics + Robotics & Control, Power + Control and Rapid Transit Technology + Communication) for their final year.

4. Diploma in Energy Systems & Management (DESM)*

The Diploma in Energy Systems & Management course aims to equip students with the knowledge and expertise in three specialisations: clean energy, power engineering and energy management, so as to design clean and energy efficient systems that will contribute to an economically and environmentally sustainable future.

5. Diploma in Engineering Systems (DES)*

The Diploma in Engineering Systems course aims to provide students with a broad-based engineering education to support activities and future challenges requiring interdisciplinary engineering systems capabilities. The course leverages on the experience and expertise of two schools, namely the School of Electrical & Electronic Engineering and the School of Mechanical & Aeronautical Engineering.

6. Diploma in Engineering with Business (DEB)

Diploma in Engineering with Business provides students with the requisite knowledge and skills in engineering principles, technologies, and business fundamentals, supported by a strong grounding in mathematics and communication skills, which is greatly valued in the rapidly changing industrial and commercial environment.

7. Common Engineering Program (DCEP)

In Common Engineering Program, students will get a flavour of electrical, electronics and mechanical engineering in the first semester of their study. They will then choose one of the 7 engineering courses specially selected from the Schools of Electrical & Electronic Engineering and Mechanical & Aeronautical Engineering.

*Course is applicable only for AY2018 intake and earlier

School of Electrical & Electronic Engineering

More than 60 Years of solid foundation	8 Tech Hubs	Unique PTN Scheme	More than 35,000+ Alumni

SINGAPORE POLYTECHNIC SP

Electives offered by SP SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING

All SP students, including EEE students are free to choose electives offered by ANY SP schools, subject to meeting the eligibility criteria.

Like all schools, School of Electrical and Electronic Engineering offers electives for:

- EEE students only
- *and* for all SP students

EEE students are required to complete 3 electives, starting from Year 2 to Year 3 (one elective per semester).

Electives Choices for All SP students

Mod Code	Module Title
EP0400	Unmanned Aircraft Flying and Drone Technologies
EP0401	Python Programming for IoT*
EP0402	Fundamentals of IoT*
EP0403	Creating an IoT Project*
EP0404	AWS Cloud Foundations
EP0405	AWS Cloud Computing Architecture

Certificate in IoT (Internet of Things)

* A certificate in IoT would be awarded if a student completes the 3 modules: EP0401, EP0402 and EP0403

Electives Choices for EEE students

Mod Code	Module Title
EM0400	Commercial Pilot Theory
EM0401	Autonomous Electric Vehicle Design
EM0402	Artificial Intelligence for Driverless Cars
EM0403	Autonomous Mobile Robots
EM0404	Smart Sensors and Actuators
EM0405	Digital Manufacturing Technology
EM0406	Linux Essential
EM0407	Advanced Linux
EM0408	Linux System Administration
EM0409	Rapid Transit System
EM0410	Rapid Transit Signalling System
EM0411	Smart City Systems Design
EM0412	Data Analytics
EM0413	Mobile App Development
EM0414	Client-Server App Development
EM0415	Machine Learning & Artificial Intelligence
EM0416	Solar Photovoltaic System Design
EM0417	Energy Management and Auditing
EM0418	Integrated Building Energy Management System
EM0419	Digital Solutioning Skills