ENGINEERING @ SP

Singapore Polytechnic

ET0930

PRINCIPLES OF COMMUNICATION

(Version 1.2)

School of Electrical & Electronic Engineering

Engineering @ SP

The Singapore Polytechnic's Mission

As a polytechnic for all ages
we prepare our learners to be
life ready, work ready, world ready
for the transformation of Singapore

The Singapore Polytechnic's Vision

Inspired Learner. Serve with Mastery. Caring Community.

A caring community of inspired learners committed to serve with mastery.

The SP CORE Values

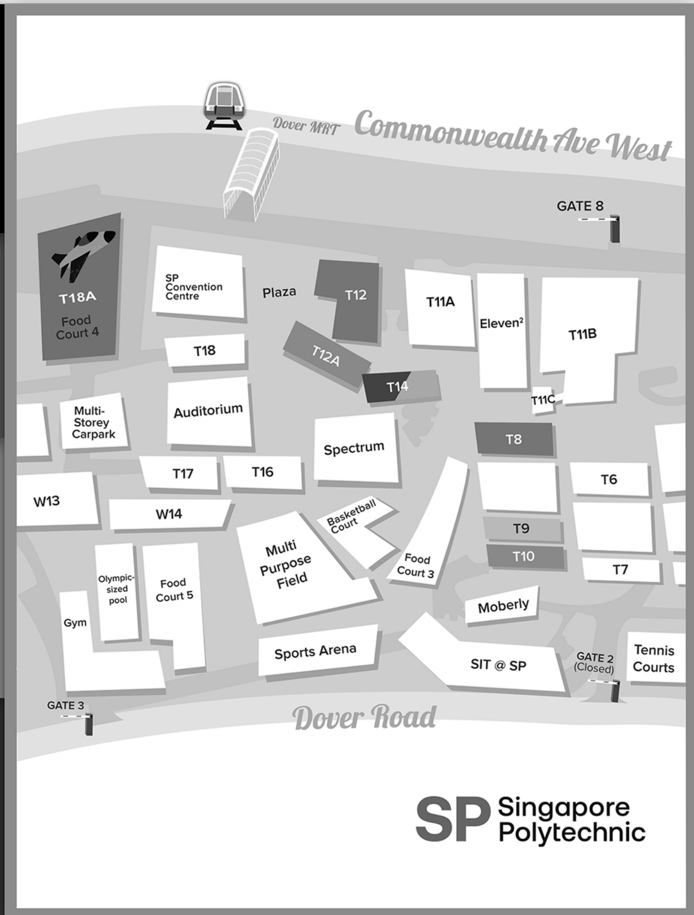
- Self-Discipline
- Personal Integrity
- Care & Concern
- Openness
- Responsibility
- Excellence

For any queries on the notes, please contact:
Name: Yang LiPing
Room: T12515
Email: Yang_Liping@sp.edu.sg
Tel: 68790626

Singapore Polytechnic

SP | SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING

Industry-Collaborated TECH HUB



T18A

Diploma in Aerospace Electronics

Aerospace Engineering Hub

Themed Project : Unmanned Aircraft Applications
Facilities : A320 Flight Simulator
Industries : Aerospace, Air Transport
Industry Partners : AIRBUS, Rolls-Royce

Industry Co-location Lab : ST Engineering Aerospace

T9 level 3 & T14 level 3

Diploma in Computer Engineering

5G & AIoT Technology Hub

Themed Project : Smart Facility Management
Facilities : AIoT Nerve Centre, 5G Garage
SP Connected Solutions Lab
Industries : ICT & Media, Smart City
Industry Partners : ADVANTECH, CONTEC, D'CRYPT, ENVISION, HUAWEI, LINGJACK DIGITAL, unabiz

Industry Co-location Lab : BOSCH, ERICSSON, Singtel

T14 level 3

Diploma in Engineering with Business

Technology to Business Hub

Themed Project : Technology To Business Venture
Facilities : IdeaBox, T2B Hub
Industries : Entrepreneurship & Start-up

Industry Co-location Lab : SPgroup, GEX VENTURES, APOL, BevEat, BitsGreat, MEMORY ARK, OCTOPUS8, Jackdat, REAL IMPACT, SMARTDB, TOP TASTE, Vaster

T10 level 3

Diploma in Electrical & Electronic Engineering

Robotics, Automation & Control Hub

Themed Project : Smart Workflow Solution for I4.0
Autonomous Mobile Robot/Robocup
Facilities : SP-SICK Smart Sensor Lab, SP-Rexroth I4.0 Innovation Lab
Industries : Logistics, Electronics (Manufacturing), Electronics & Chemicals (Manufacturing)
Industry Partners : ABB, BECKHOFF, LKH PRECICON, SEW EURODRIVE, Weidmüller, ZEBRA

Industry Co-location Lab : rexroth A Bosch Company, SICK Sensor Intelligence.

T8 level 2 & T12 level 1

Power & Sustainable Energy Hub

Themed Project : Net-zero Renewable Energy Vertical Farming
Facilities : High Voltage Training, LE & LET Training, Energy Efficiency Lab
Industries : Energy & Power
Industry Partners : CHNT, ebmpapst The engineer's choice, sembcorp, SUNLIGHT A CHNT COMPANY

T12/T12A level 2

Semicon & Electronics Hub

Themed Project : Artificial Intelligence of Things using FPGA
Facilities : Nanofab & IC Design Labs, Infineon Innovation Lab
Industries : Semiconductor & Electronics
Industry Partners : SSIA

Industry Co-location Lab : ADVANTEST, Infineon

T12 level 6

Biomedical Engineering Hub

Themed Project : Engaging Rehabilitation with Data Visualisation & Analytics
Facilities : Biomedical Engineering Lab
Industries : Healthcare
Industry Partners : GE Healthcare, GOLD LITE, Khoo Teck Puat Hospital, NATIONAL INSTRUMENTS

T8 level 2 & T12 level 1

Autonomous & Transportation Systems Hub

Themed Project : 5G-enabled Autonomous Safety Vehicle
Autonomous Underwater Vehicle
Facilities : RSS Lab for RTT Training, Siemens Digital Experience Centre,
SP-TIBCO Adv Mfg Analytics Lab
Industries : Smart Transport, Land Transport
Industry Partners : ST Engineering Land Systems, SMRT, SIIX, SUTD, GreenPower

Industry Co-location Lab : SIEMENS Ingenuity for life, TIBCO

CONTENTS

MODULE OVERVIEW

1. Introduction

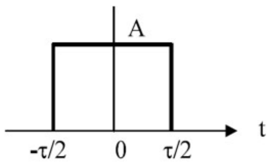
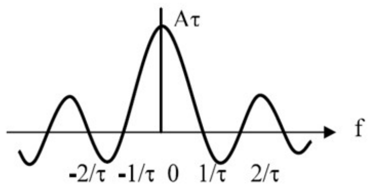
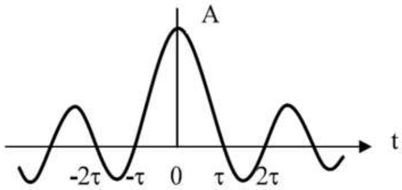
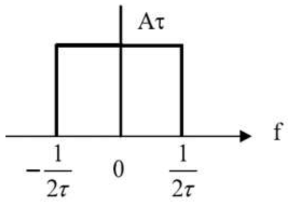
Principles of communication is a third module in the Telecommunications Option.

2. Module Aims

The aim of this module is to introduce the students to the knowledge of analog and digital communication techniques. The module covers principles of analog and digital communication including signals, signal spectra, noise, analog and digital signal transmission and modulation techniques. The module is built on some of the materials covered in Engineering Mathematics IIA & IIB.

Formula List

Analog communications
$P_n = kTB$ $\quad E_n = \sqrt{4kTBR}$ Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J/K   Room Temperature, $T_0 = 290$ K   Velocity of light in free space, $c = 3 \times 10^8$ m/s   Positive envelope = $\left[V_c + v_s(t)\right]$ $\qquad$ Negative envelope = $-\left[V_c + v_s(t)\right]$ $m = \dfrac{\mathrm{Env_{max}} - \mathrm{Env_{min}}}{\mathrm{Env_{max}} + \mathrm{Env_{min}}}$ $B_{FM} = 2(m_f + 1)f_s$, for integer values of m_f. $B_{FM} = 2(m_{f_H} + 1)f_H$

Signal x(t)	Fourier transform X(f)
$\sum_{n=-\infty}^{n=\infty} \delta(t - nT)$	$\frac{1}{T}\sum_{n=-\infty}^{\infty} \delta(f - nf_0)$ where $f_0 = \frac{1}{T}$
$A\mathrm{rect}\frac{t}{\tau}$ 	$A\tau\mathrm{sinc}f\tau$ 
$A\mathrm{sinc}\frac{t}{\tau}$ 	$A\tau\mathrm{rect}f\tau$ 

Quantisation (mid-riser quantiser)		
Step size	Quantisation noise power	Signal-to-noise ratio (dB)
$q = 2\, X_{max} / 2^B$	$N_q = \dfrac{q^2}{12}$	$\left[\dfrac{S}{N_q}\right] = 1.76 + 6B + 20log_{10}\dfrac{V_x}{V}$

PCM TDM
Gross output bit rate, R = commutator speed × no. of inputs × no. of bits per sample

Eye diagram measurements		
Noise margin: $\frac{V_{\min}}{V_{\max}} \times 100\%$	ISI degradation: $20\log_{10}\left(\frac{V_{\max}}{V_{\min}}\right) dB$	Jitter(%): $\frac{\Delta T}{T} \times 100\%$

Baseband transmission of digital signals	
Probability that AWGN exceeds T volts $P(n > T) = \frac{1}{2} erfc\left[\frac{T}{\sqrt{2}\sigma}\right]$	Probability of bit error for a simple comparator receiver $P_e = \frac{1}{2} erfc\left[\frac{R}{\sqrt{2}\sigma}\right]$

Matched filter		
Impulse response, h(t) $s_2(T_b - t) - s_1(T_b - t)$	Probability of bit error $P_e = \frac{1}{2} erfc(\frac{\gamma}{2\sqrt{2}})$ $\gamma^2 = \frac{2}{\eta}\int_0^{T_b} [s_2(t) - s_1(t)]^2 dt$	Probability of bit error with polar NRZ input $P_e = \frac{1}{2} erfc\left[\sqrt{\frac{V^2 T_b}{\eta}}\right]$

Digital Modulation	
Probability of bit error for BPSK $P_e = \frac{1}{2} erfc\left[\sqrt{\frac{V^2 T_b}{2\eta}}\right]$	Probability of bit error for DPSK $P_e = \frac{1}{2} exp\left[\frac{-V^2 T_b}{2\eta}\right] = e^{-\left(\frac{V^2 T_b}{2\eta}\right)}$

Chapter 1

INTRODUCTION

Learning Outcomes

- ❑ **Understand the basics of analog and digital communication systems.**
- ❑ Describe different types of electrical communication systems.
- ❑ Describe and draw the general block diagrams of an analog and digital communication system.
- ❑ Explain briefly the function of each element in an analog and digital communication system.
- ❑ Give examples of analog and digital communication systems.
- ❑ Identify the merits of digital communications by comparing and contrasting with an analog communication system.

INTRODUCTION

Electrical communication is the process of exchanging information such as audio, visual, and computer data in forms of electrical signals from a source to one or more destinations. Electrical signals can be either analog or digital dependent on the type of information source from which a signal is produced. An analog information source produces analog signal which is a smoothly and continuously varying voltage or current. A microphone is an example of an analog source. Its output voltage describing sound information varies continuously. Digital signal is produced by a digital information source which is a set of discrete values. Computer is an example of digital source. The data signal generated by a computer has discrete values.

1.1 TYPES OF ELECTRICAL COMMUNICATION SYSTEMS

Electrical communication systems are divided into analog and digital communication systems base on:

- the nature of the information source
- the type of modulation scheme used

Analog communication systems

A communication system that transfers analog signal from an analog source to the destination (user) using analog modulation techniques is known as analog communication system. A block diagram of an analog communication system is shown in Figure 1.1. Examples of analog communication systems include AM and FM radio broadcasting systems.

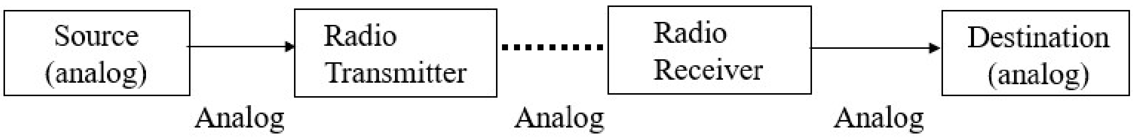


Figure 1.1 analog communication system

Digital communication systems

A communication system that transfers digital signal from a source to a destination (user) is known as digital communication system. Digital communication systems use digital modulation techniques. Computer communication systems and mobile communication systems are examples of digital communication systems.

In digital communication systems, the digital signal may be generated by a digital source or converting an analog signal into digital signal through sampling and quantization. Also, digital signal may be transmitted using an analog waveform by representing different digital signals using high frequency sinusoidal signals of different amplitude, frequency or initial phase.

A communication systems that is part analog and part digital is also known as **hybrid** system. For example, it uses digital modulation schemes for transmitting sampled and quantized values of an analog message signal. Or it transmits digital information as an analog waveform through an analog channel, via a modem. Figure 1.2 shows some examples of digital communication systems.

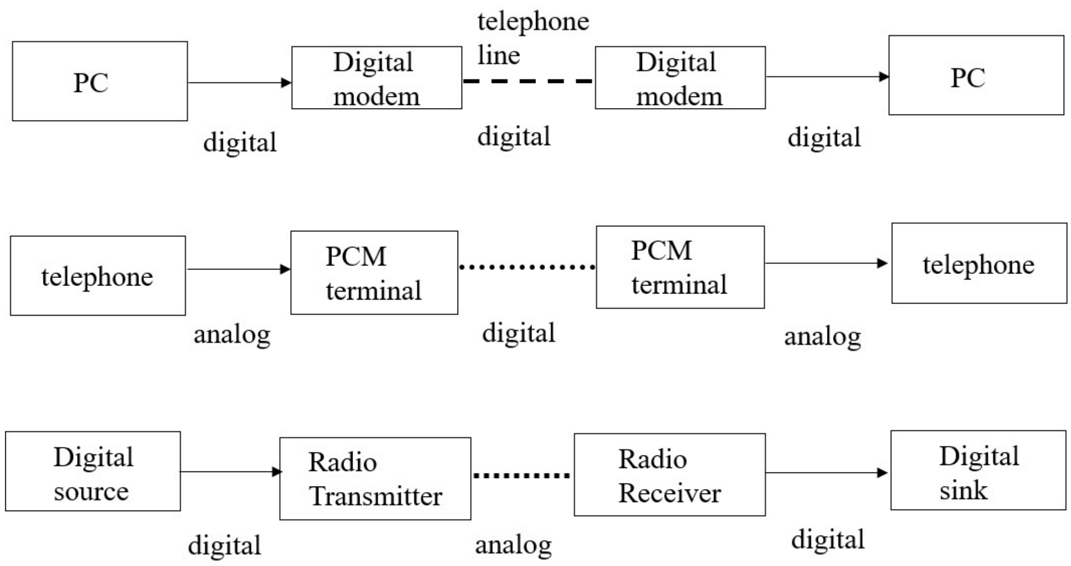


Figure 1.2 Examples of digital communication systems

1.2 ELEMENTS OF ELECTRICAL COMMUNICATION SYSTEMS

An electrical communication system comprises of a transmitter and a receiver on the two ends connected by a communication channel. The transmitter converts the information from a source to a suitable form that can travel through the communication channel. The receiver converts the received signal back to its original information form.

1.2.1 Elements of Analog communication systems

Figure 1.3 shows the basic functional block diagram of an analog communication system.

<u>Input Transducer</u>

The information from a source may be audio, video or other analog type of information, which is generally not electrical signal and cannot be transmitted directly by electrical communication systems. An input transducer is required to convert the information into an electrical signal m(t). When the information is audio, the input transducer is simply a microphone. For video information, the input transducer may be a video camera.

The signal generated by an input transducer or information source is known as **baseband signal**. The frequencies occupied by a baseband signal concentrate about f = 0.

<u>Output transducer</u>

Output transducer at the receiving end converts the received electrical signal to its original information form. Examples of output transducers include loudspeaker which converts audio signal to sound, and LCD or Plasma Screen which converts video signal to visual images.

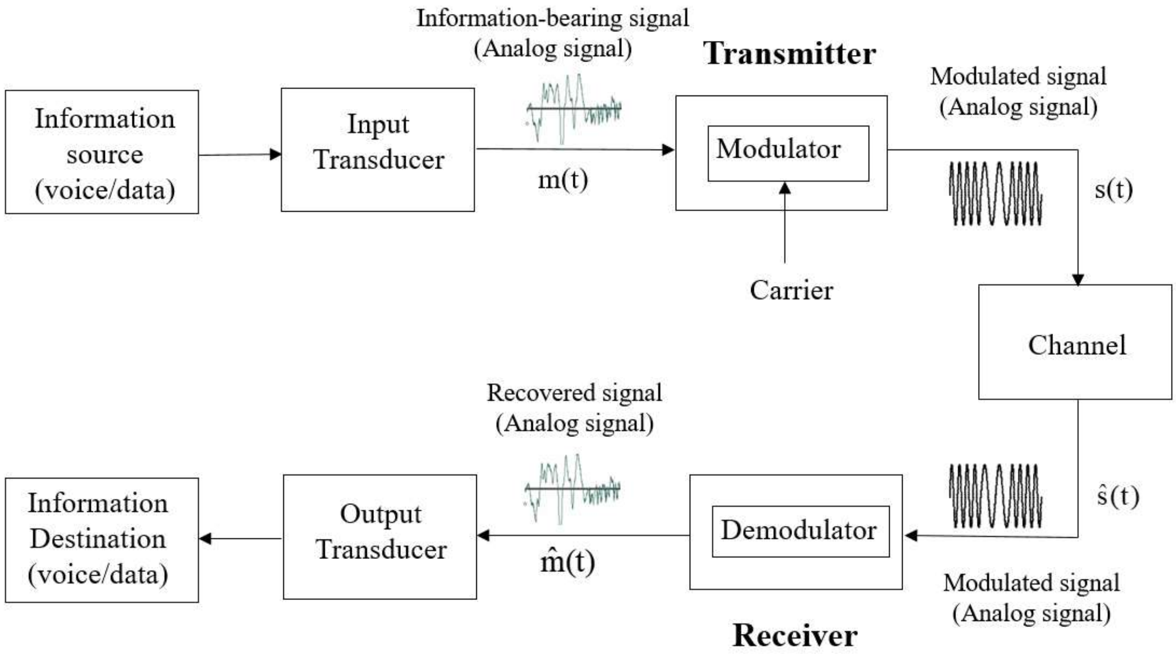


Figure 1.3 A block diagram of analog transmission system

Transmitter

Transmitter processes and converts baseband signal m(t) to a waveform s(t) more suitable for transmission over the communication channel. Signal processing operations performed in the transmitter may include amplification, filtering and modulation.

Baseband signal can be transmitted directly over a dedicated channel such as twisted pair of copper wires or coaxial cable for short distance transmission, e.g. transmitting audio signal over a telephone cable. Other Example of transmitting baseband signal directly includes transmitting video signal from a camera to a TV using a cable. Transmitting baseband signal directly over a communication channel is known as **baseband transmission.**

For long distance transmission where the channel is usually air and baseband transmission is not suitable due to the required impractical antenna size (details covered in Chapter 4), a baseband signal is superimposed on a high frequency sinusoidal signal called **carrier** through a **modulation** process. The output signal of the modulation process, s(t) is known as **passband signal**. Passband signal is designed to have frequencies located in a band about f_c where $f_c >> 0$. Transmitting baseband signal via a passband signal involving modulation is known as **passband transmission.**

In the modulation process, one of the three parameters of a carrier - amplitude, frequency or phase, is made to vary in accordance with the baseband signal. The baseband signal is thus carried by the carrier wave in the form of variation in its amplitude, frequency or phase. If the amplitude of the carrier is varied, the resulting modulation is called **amplitude modulation** (AM). If the frequency of the carrier is varied, the resulting modulation is called **frequency modulation** (FM). If the phase of the carrier is varied, the resulting modulation is called **phase modulation** (PM). Only AM and FM are discussed in this module and the details are given in chapter 5 and 6, respectively.

Modulation makes it possible to match the properties of the passband signal, s(t), to the channel. In addition, modulation is useful for reducing the effect of noise and interference, for simultaneous transmission of several signals over a single channel, and for reducing antenna size (details covered in Chapter 4).

Communication Channel

The information source and the destination are usually separated in space. They are connected by a communication channel. Communication channel can be wireline (twisted-pair wires, coaxial cable or an optical fibre) or wireless (air/vacuum).

Regardless of its form, the channel degrades the transmitted signal in various ways. The degradation is a result of signal distortion due to limited bandwidth of the channel, and the noise present in the channel. Noise and signal distortion are two basic problems of communication channel. The transmitter and receiver in a communication system are carefully designed to avoid signal distortion and minimise the effects of noise at the receiver so that a faithful reproduction of the original message is possible.

Receiver

The receiver takes the corrupted signal $\hat{s}(t)$ at the channel output and converts it to a baseband signal. When $\hat{s}(t)$ is a passband signal, it involves demodulation, the reverse of the transmitter's modulation process. In addition to demodulation, the receiver also provides amplification and filtering.

With the presence of noise and other signal degradations, the receiver cannot recover the information signal perfectly. The aim of the system is to make $\hat{m}(t)$ as close as possible, if not identical, to m(t) of the transmitter.

1.2.2 Elements of digital communication systems

Figure 1.4 shows the functional elements of a digital communication system. The overall purpose of the system is to transmit the information coming out of a source to its destination at as high a rate and accuracy as possible.

Information Source

In digital communication system, the original information source may be discrete or analog. Discrete information source generates a sequence of discrete symbols in contrast to an analog information source which emits continuous amplitude signals. An analog information-signal can be converted to a discrete information signal through a process of sampling and quantisation known as A/D converter (Analog to digital converter). Sampling and quantization is discussed in chapter 7.

Source Encoder/Decoder

The input to the source encoder is a sequence of symbols occurring at a rate of r_S symbols per second. The source encoder converts the symbol sequence into an efficient binary sequence

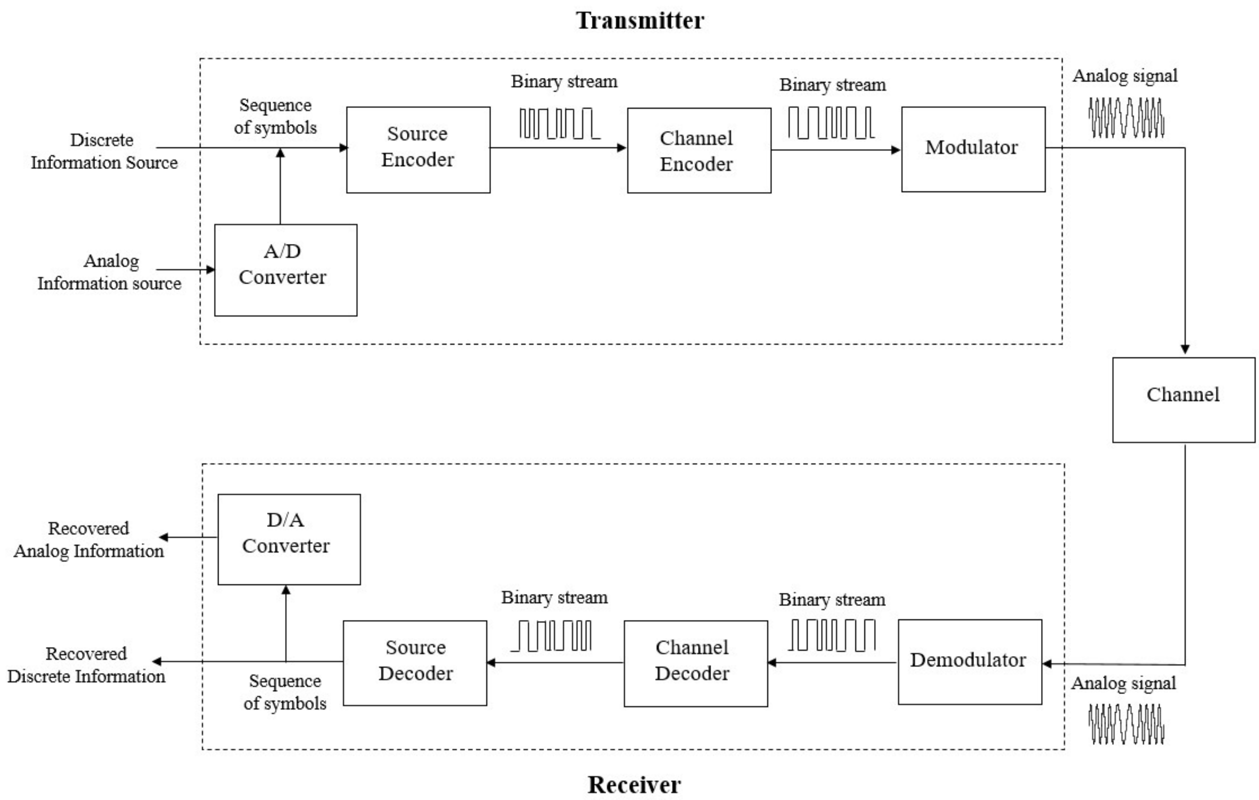


Figure 1.4 Block Diagram of a Digital Communication System

of 0's and 1's by assigning a codeword to each symbol. During the conversion, redundant information is removed.

The simplest way in which a source encoder does coding is to assign a fixed length binary codeword to each symbol in the input sequence. But, a better solution for removing redundancy is to use **variable length coding**.

For example, it can assign 5-bit codeword, 00000 through 11111 for a discrete source that has an alphabet size of 32 symbols. If the symbol rate of the source is 10 symbols per second, the source encoder's output data rate will be 50 (5×10) bits/s.

Fixed length coding of individual symbols is efficient only if the symbols are from a statistically independent sequence with each symbol occurring with equal probabilities. In reality, most information sources do not emits symbols with equal probabilities and the emitted symbols are often dependent of each other. For example, in the English text, 'a' occurs more often than 'z', and given the letter 'q', one can know with high degree of accuracy that the next letter is 'u'. In such a case variable length coding is used, where each codeword has a different word length, i.e. different number of bits. The result is that, on the average, the number of bits to represent a symbol is less than in fixed length coding.

At the receiver, the source decoder converts the binary output of the channel decoder into a symbol sequence.

Common electronic devices using source encoding include digital camera and DVD recorder.

Channel Encoder/Decoder

The channel encoder converts the source encoded binary sequence into a form that will allow the receiver to decode with reduced errors. Digital channel coding results in lower bit errors and thus high transmission reliability.

Error control is accomplished by the channel encoder by systematically adding extra bits to the output of the source encoder. While these extra bits convey no information, they make it possible for the receiver to detect and/or correct some of the errors in the information bearing bits. For example, a simple way of protecting the 7-bit ASCII code is to add a parity check bit to each 7-bit word from the source encoder.

Common electronic devices using channel encoding (and source encoding) include Facsimile and handphone.

Modulator/Demodulator

The digital modulator accepts a bit stream as its input and converts it to a waveform suitable for transmission over the communication channel. Important parameters of the digital modulator are the types of waveforms used, the duration of the waveforms, the power level, and the bandwidth used.

When the channel is a dedicated channel for short distance transmission, a baseband digital signal is directly transmitted over the channel without modulation. Details on baseband transmission and reception of digital signals are covered in Chapter 8 and Chapter 9.

Modulation is required for transmission of digital signals for long distance communication. Commonly used digital modulation schemes are amplitude shift keying (ASK), phase shift keying (PSK) and frequency shift keying (FSK). The details of digital modulation techniques are covered in chapter 10.

At the receiver, the demodulator extracts the message from the information bearing waveform received.

Communication Channel

Distortion and noise in the channel introduce errors in the information being transmitted and limits the rate at which information can be communicated.

Other Functional Blocks

There are a number of other functional blocks, not shown in Figure 1.4, that exist in practical digital communication systems. Examples of such blocks are equalisers, clock recovery networks, scramblers/unscramblers, multiplexers/demultiplexers, encryptors/decryptors, spread spectrum, multiple access and others. These are optional and their implementation depends on the demands imposed on the communication system.

The performance of a digital communication system is often measured based on **error probability**, the probability of incorrectly decoding a message symbol at the receiver.

Digital communication system is aimed to transmit information (or sequences of symbols) at as high a rate and accuracy as possible. It requires that the encoders/decoder and the modulator/demodulator combat jointly the signal degradation caused by a channel and maximise the information rate and accuracy.

1.3 ADVANTAGES AND DISADVANTAGES OF DIGITAL COMMUNICATION

Digital communication has established itself as the dominant form of communication because of its advantages over analog communication.

1.3.1 Advantages of Digital Communication over Analog Communication

- **More robust against channel noise interference**

Binary digital signals operate in two states: 5 V for binary '1'or 0 V for binary '0' normally. The decision at the receiver is the selection between two possible levels and not the shape of the waveform. Thus binary digital signals can be regenerated (or clean up) easily by regenerators placed at some distances apart along a long and noisy transmission channel, as shown in Figure 1.5. In contrast, analog signals can have infinite voltage values making restoration difficult when they are distorted. Thus for digital signals, transmission quality is almost independent of the distance between the transmitter and the receiver. The error rate is virtually unaffected by distance.

- **Ease of integration of various services**

It provides a uniform method for the transmission of all types of signals. A mixture of traffic ranging from telephony and telegraphy to data and video information can be treated as identical signals in transmission and switching – a bit is a bit. With this, a common integrated communication network such as in ISDN (Integrated Service Digital Network) is made possible.

- **Ease of multiplexing and demultiplexing**

Combining digital signals using time-division multiplexing (TDM) is simpler than combining analog signals using frequency-division multiplexing (FDM).

- **Easier to network**

Digital data can be divided into autonomous groups called packets for convenient switching and transmission over digital network, e.g. Internet.

- **Use of better transmission technology**

Newer and better types of transmission media such as optical fibres and circular or helical waveguides have high transmission bandwidth and are more suitable for transmitting digital signals. In optical fibres, information are transmitted by switching the light source on and off at very high rates. All new satellite communication systems use digital communication techniques. Table 1.1 shows a comparison of the various transmission media. Fiber optics has the greatest bandwidth, about 75 THz whereas the traditional twisted-pair has the lowest bandwidth, a maximum of 1MHz.

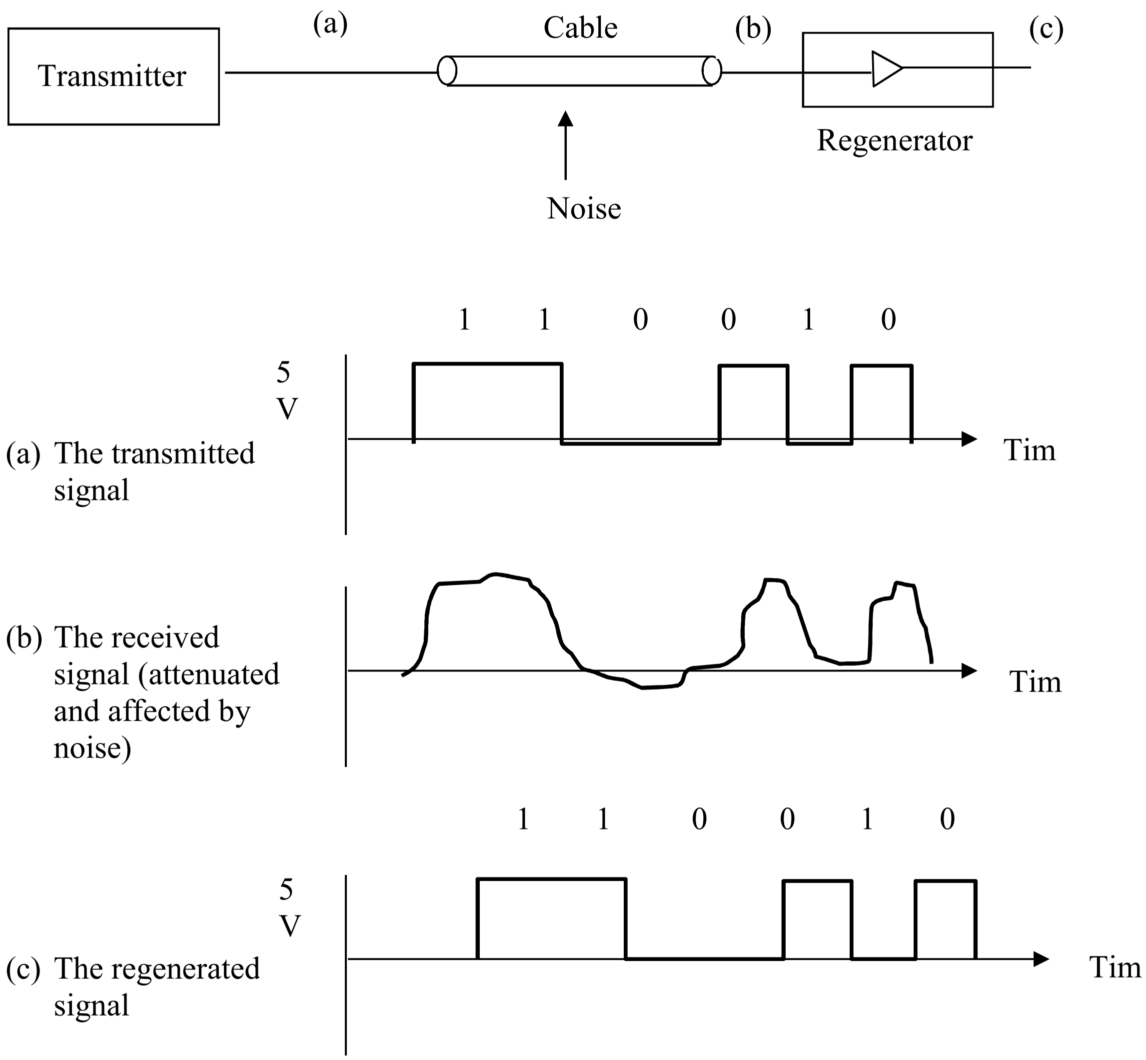


Figure 1.5 The regeneration process

Table 1.1 Comparison of various transmission media

Media Type	**Bandwidth**	**Repeater Distance**
Twisted-pair	1 MHz	Few km
Coaxial cable	1 GHz	Few km
Microwave	100 GHz	Every 10 – 100 km
Satellite	100 GHz	Several thousand km
Fiber	75 THz	Few tens of km

1.3.2 Disadvantages of Digital Communication

- **Higher bandwidth requirement**

The main disadvantage of digital systems is the higher bandwidth required to carry the digital signal as opposed to analog signal. For instance, the bit rate of a standard digital telephone channel is 64 kbits/sec (in a 30-channel PCM system) and the bandwidth required is 32 kHz, while an analog speech channel normally occupies only about 4 kHz bandwidth.

- **Frequent need of repeaters**

Digital repeaters are required at frequent intervals on a line in order to ensure that the digital waveform can be regenerated reliably. This is costly.

- **Overhead on Timing or Synchronisation requirement**

A critical element in a digital communication system is synchronisation. For normal operation of such systems, synchronisation at all levels must be preserved. This includes bit synchronisation, frame synchronisation and network synchronisation. This requires additional processing, bandwidth and transmission time.

Chapter 2

Signals and Spectra

Learning Outcomes

- ❑ **Understand signals and spectra.**
- ❑ Classify signals from different perspectives.
- ❑ Describe analog and digital signals.
- ❑ Determine the period and fundamental frequency of a periodic signal.
- ❑ Explain time domain and frequency domain representations of periodic signals.
- ❑ Identify various representations of the Fourier series.
- ❑ Construct signal spectra from their Fourier Series.
- ❑ Connect the concepts of the Fourier series and Fourier transform.
- ❑ Derive the Fourier Transforms of signals including rectangular pulse, an impulse, a pulse train and an impulse train.
- ❑ Describe briefly the applications of the Fourier transform properties.
- ❑ Explain briefly the concept of convolution as applied to signals.
- ❑ Define bandwidth of signals.
- ❑ Understand basics of signal power measurements.
- ❑ Characterize briefly the impact of linear time invariant (LTI) systems on signals.
- ❑ Understand the effects of various types of communication filters.

INTRODUCTION

In electrical communication systems, information is represented as electrical signal though the original form of information may be in a variety of forms such as human voice, music, video, code output from a computer and etc. Signals include signals with useful information or signals that are unwanted such as noise and interference.

Electrical signal is a variation of electrical voltage/current with respect to time. It is expressed as a single-valued function of time. For any instant of time, there is a unique value of the function. Electrical signal can be analog, discrete (sampled analog), or digital. It can be periodic or non-periodic. It can also be expressed as a function of frequency.

2.1 SIGNAL CLASSIFICATION

2.1.1 Analog and digital signals

The information signals can take analog or digital form. **Analog signals** are signals that can be represented as a continuous function of time and have infinite number of levels. Speech signal is an example of analog signals as shown in Figure 2.1.

Digital signals are signals that have only a finite number of possible levels. For example, binary signals which only have two possible levels as shown in Figure 2.2.

Figure 2.1 An example of analog signals

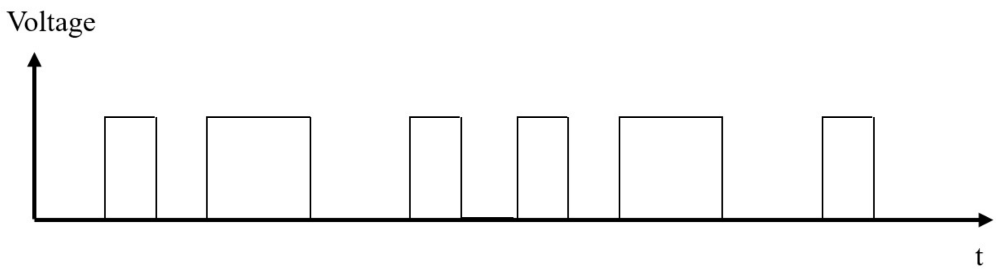


Figure 2.2 An example of a digital signal

2.1.2 Periodic and Non-periodic signals

Both analog and digital signals can be of two forms: Periodic and Non-periodic.

Periodic signals

A signal f(t) is said to be **periodic** if it satisfies the condition in eq. (2.1) for any t and integer n, where T is a positive constant.

$$v(t) = v(t + nT) \tag{2.1}$$

Periodic signal repeats its waveform. The smallest T satisfying Eq. (2.1) is called the **period** of v(t), denoted as T. T is the duration of one cycle. Sine waves and square waves are examples of periodic signals.

Fundamental frequency of periodic signals, f, is defined as the number of cycles per second measured in cycles per second or Hz. It is the rate at which a periodic signal repeats its waveform. f is reciprocal of period T. i.e. f = 1/T.

For example, there are two cycles in one second in the waveform shown in Figure 2.3. The period T is 0.5 sec. Thus, its fundamental frequency f is 2 Hz (or cycles/sec).

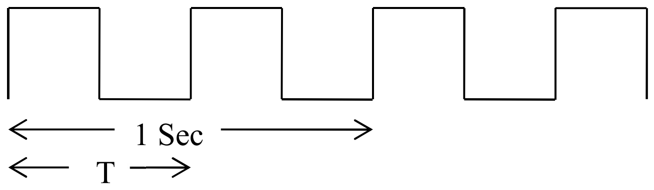


Figure 2.3 Periodic signal with period of 0.5 sec.

Multiples of fundamental frequency are called **harmonics**.

Sometimes angular frequency is used in signal representation. **Angular frequency**, denoted as ω**,** is frequency expressed in radians/sec.:

$\omega = 2\pi \times$ number of cycles/sec $= 2\pi f$ radians/sec.

Non-periodic signals

Signals not satisfying Eq. (2.1) are said to be non-periodic. Non-periodic signal does not repeat itself. Speech and music signals are examples of non-periodic signals. Figure 2.4 shows a waveform of a non-periodic signal.

Figure 2.4 A non-periodic signal

2.1.3 Continuous-time and discrete-time signals

A signal is said to be a **continuous-time signal** if its values are defined at all instants of time. Figure 2.5(a) shows an arbitrary continuous-time signal. A continuous-time signal is naturally produced by a transducer when converting a physical signal into an electrical signal, e.g. a continuous-time electrical signal is produced when a sound wave is converted into electrical signal by a microphone. A continuous-time signal may have zero values at some instants of time or even for certain interval of time. Sine signal, speech signal and music signal are examples of continuous-time signals.

A signal is said to be a **discrete-time signal** if it is defined only at discrete instants of time, k. The time variable on the horizontal time axis takes discrete values only, i.e. it takes values in a set of integers. A discrete-time signal x(k) is usually derived from a continuous time signal x(t) by a sampling process (sampling process is described in chapter 7). Figure 2.5(b) shows a sampled (discrete) signal from arbitrary continuous-time signal given in Figure 2.5(a).

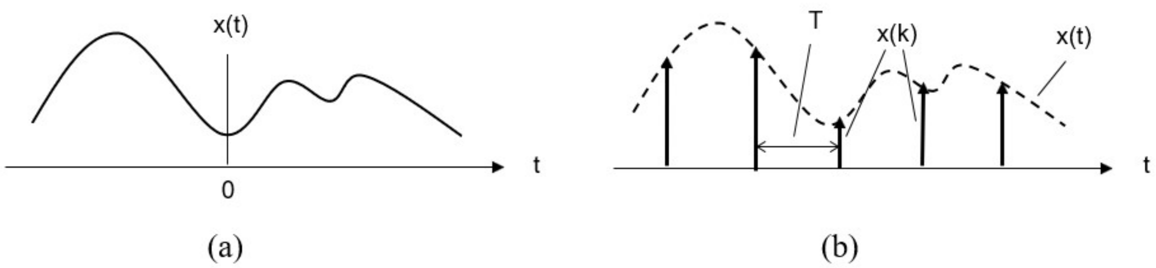


Figure 2.5 (a) an arbitrary continuous-time signal, (b) the sampled (discrete) signal.

2.2 BASIC SIGNALS

Basic signals are ideal signals that can be viewed as building blocks for constructing more complex and practical signals. For instance, basic signals may be used to model other signals that occur in nature. Below are some basic signals and their associated signals which are important to this module.

2.2.1 Sinusoidal signals

Sinusoidal signals are the most fundamental periodic signal. They are the basis for Fourier series and Fourier analysis. Both the continuous and discrete-time versions of a sinusoidal signal x(t), in the form of cosine or sine functions, can be fully described by three characteristics, namely, **peak amplitude (**V_P), **frequency (f)** and **initial phase (**φ) as below:

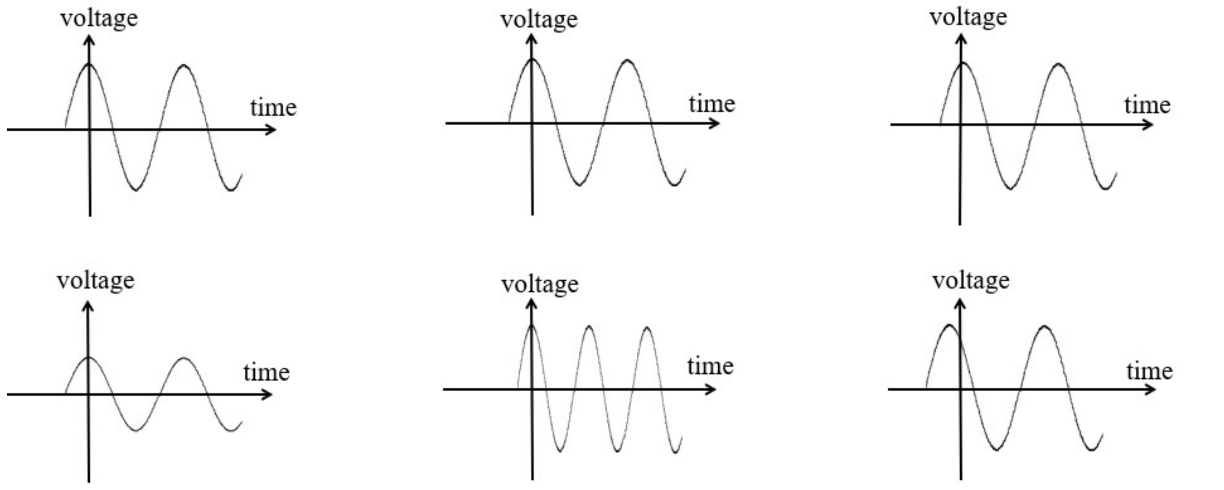
$$\begin{aligned} x(t) &= V_p\cos(2\pi ft + \varphi) = V_P\sin(2\pi ft + \varphi') \\ x(n) &= V_p\cos(2\pi fn + \varphi) = V_P\sin(2\pi fn + \varphi') \end{aligned} \qquad (2.2)$$

- **Peak amplitude (**V_P) is the maximum voltage deviation from zero volt.
- **Frequency** (f) is the rate at which a sine wave repeats its waveform.
- **Initial phase** (φ) describes the position of the waveform at time zero (t=0).

Alternatively, we may use angular frequency ω in instead of frequency f:

$$\begin{aligned} x(t) &= V_P\cos(2\pi ft + \varphi) = V_P\cos(\omega t + \varphi) \\ x(n) &= V_P\cos(2\pi fn + \varphi) = V_P\cos(\omega n + \varphi) \end{aligned} \qquad (2.3)$$

Figure 2.6 shows sinusoidal signals with different amplitude, frequency and initial phase.



(a)Sinusoidal signals with different amplitude

(b) Sinusoidal signals with different frequency

(c) Sinusoidal signals with different initial phase

Figure 2.6 sine waves with different amplitude, fundamental frequency and initial phase.

Example 2.1

Extract the parameters from the following sine waves.

v(t)	Peak Voltage	$\text{freq(Hz)} = \frac{\omega}{2\pi}$	Phase φ
$6\sin(4\pi f_0 + \frac{\pi}{4})$			
$4\sin(6\pi f_0 t + \frac{\pi}{2})$			
8sin5000πt			
2sin3000t			

2.2.2 Rectangular pulse

Rectangular pulse, also named as **rect** function pulse is defined as

$$x(t) = \boldsymbol{A}rect(\frac{t}{\tau}) = \begin{cases} A, & |t| \leq \frac{\tau}{2} \\ 0, & |t| > \frac{\tau}{2} \end{cases} \tag{2.4}$$

Rectangular pulse given by eq. (2.4) is a rectangular-shaped pulse centred at t = 0 with a width of τ and height of A as shown in Figure 2.7. It is commonly used to represent binary "1" in digital communications.

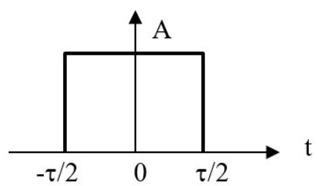


Figure 2.7 Rectangular pulse

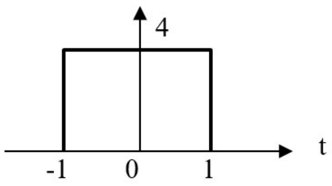
Example 2.2

Sketch x(t) = 4 rect (0.5t)

Solution

$$x(t) = 4\,\text{rect}(0.5\,t) = 4\,\text{rect}\left(\frac{t}{(1/0.5)}\right) = 4\,\text{rect}\left(\frac{t}{2}\right)$$

where A = 4 and τ = 2



2.2.3 Sinc Function

Sinc function, as shown in Figure 2.8, is defined as

$$x(t) = \mathbf{A}\mathrm{sinc}(\frac{t}{\tau}) = \frac{\mathbf{A}\sin(\pi\frac{t}{\tau})}{\pi\frac{t}{\tau}} \tag{2.5}$$

A sinc function is an even function and passes through zero at all positive and negative multiples of τ (i.e., t = ± $\pm\tau,\ \ \pm 2\tau,\ \ldots$). It reaches its maximum of A at t = 0.

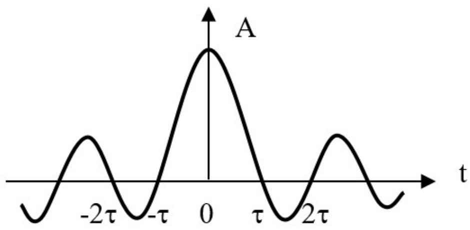


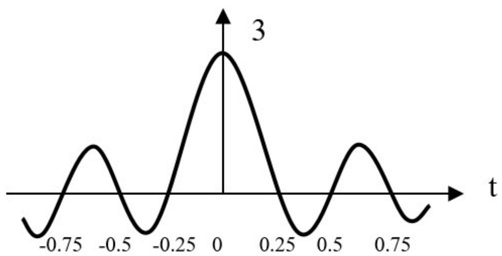
Figure 2.8 A sinc function

Example 2.3

Sketch x(t) = 3 sinc(4t)

Solution

$$x(t) = 3\,\mathrm{sinc}(4t) = 3\,\mathrm{sinc}\left(\frac{t}{1/4}\right) = 3\,\mathrm{sinc}\left(\frac{t}{0.25}\right) \text{ where } A = 3;\ \tau = 0.25$$



2.2.4 Unit Impulse

Unit impulse is a pulse with zero width and yet that the area under the pulse is unity as shown in Figure 2.9. Unit impulse is defined as follows:

$$x(t) = \delta(t) = 0 \quad t \neq 0$$

$$\text{and } \int_{-\infty}^{\infty} \delta(t)dt = 1 \qquad (2.6)$$

The unit impulse is diagrammatically represented by an arrow, whose height denotes its area. It is an idealization of a signal that occurs in an extremely short period of time that is too short to be measured with an extremely large amplitude. It is very useful for simplifying complicated analysis of communication systems.

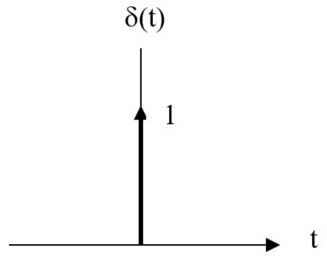


Figure 2.9 Unit impulse signal

2.2.5 Periodic pulse and impulse trains

A periodic **pulse train** with period T consists of rectangular pulses with duration of τ uniformly spaced apart as shown in Figure 2.10. The **duty cycle** of a periodic pulse train is defined as τ/T. A periodic pulse train can be analytically expressed as

$$\boldsymbol{y(t)} = \sum_{\boldsymbol{n}=-\infty}^{\infty} \mathbf{x}(\boldsymbol{t}-\boldsymbol{nT}), \text{ where } x(t) = \begin{cases} A, |t| \leq \frac{\tau}{2} \\ 0, |t| > \frac{\tau}{2} \end{cases} \qquad (2.7)$$

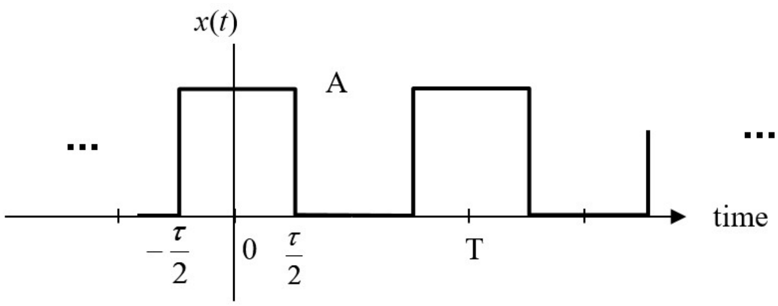


Figure 2.10 A periodic pulse train

A periodic **impulse train** consists of impulses uniformly spaced T seconds apart as shown in Figure 2.11. A periodic impulse train can be analytically expressed as

$$p(t) = \sum_{n=-\infty}^{n=\infty} \delta(t - nT) \tag{2.8}$$

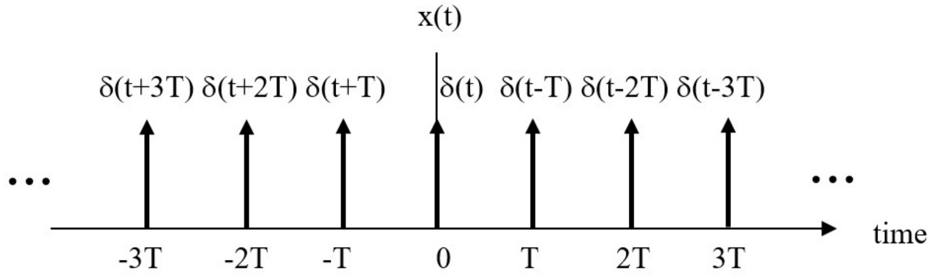


Figure 2.11 Impulse train

2.3 FOURIER SERIES

Signals are described by **time-domain** and **frequency-domain** representations. Description of signal with respect to time, or signal waveform, is known as a time-domain representation. Time domain representation shows the shape and instantaneous magnitude of a signal with respect to time, which can be observed on an oscilloscope. Description of signal with respect to frequency, or signal frequency spectrum, is known as frequency-domain representation. It shows the frequency content of a signal or spectrum of signal frequency components, which can be observed on spectrum analyser.

Signal spectrum consists of amplitude and phase spectrum which indicates amplitude and phase of various frequency components, respectively. In communication applications, it is important to know the frequency content of signals.

Fourier series and Fourier transform are mathematical tools that we can use to determine signal spectrum of periodic and non-periodic signal, respectively.

- **Fourier series** is used for periodic signals. By using a Fourier series expansion, a periodic signal is written as a sum of trigonometric or exponential functions with specific frequencies.

- **Fourier transform** is used for general signals that aren't necessarily periodic. By performing Fourier transform, a signal is written as a continuous integral of exponential functions with a continuum of possible frequencies.

2.3.1 Fourier series of periodic signals

A periodic signal x(t) of period T_0 can be expended into a series of sinusoidal functions known as Fourier series**:**

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t) \tag{2.9}$$

Or

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \varphi_n) \tag{2.10}$$

where $\omega_0=2\pi f_0$ (rad/sec) and $f_0 = 1/T_0$ Hz. f_0 is the repetition frequency of a periodic signal known as **fundamental frequency**. nf_0 is the n^{th} harmonic frequency, and

$$\begin{aligned} a_0 &= \frac{1}{T}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)dt \\ a_n &= \frac{2}{T}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\cos(n\omega_0 t)\,dt \\ b_n &= \frac{2}{T}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)\sin(n\omega_0 t)\,dt \end{aligned} \tag{2.11}$$

and

$$\begin{aligned} A_0 &= a_0, \\ A_n &= \sqrt{a_n^2 + b_n^2} \\ \varphi_n &= -\tan^{-1}\left(\frac{b_n}{a_n}\right) \end{aligned} \tag{2.12}$$

Eq.(2.10) expands a periodic signal into a sum of harmonics of a fundamental frequency f_0. Each harmonic has certain amplitude and phase:

- The term a_0 represents the average value of the periodic signal x(t) known as the **DC component**.
- The summation term consists of an infinite series of harmonic sinusoids known as **AC components**.
- The coefficients A_n and initial phase φ_n are called the **harmonic amplitude and phase** of the n^{th} harmonic, respectively.

The harmonic amplitude indicates how strong a harmonic is in a signal and the phase tells where this harmonic lies in time. Graphical presentation of these two quantities as function of frequency are known as **amplitude spectrum** and **phase spectrum,** respectively, as shown in Figure 2.12 and 2.13. Those spectra are known as **single-sided** amplitude and phase spectrum as the spectrum exists only for DC and positive frequencies. The spectrum of a periodic signal is **discrete** (line spectrum) as it contains only frequency components at multiples of the fundamental frequency f_0.

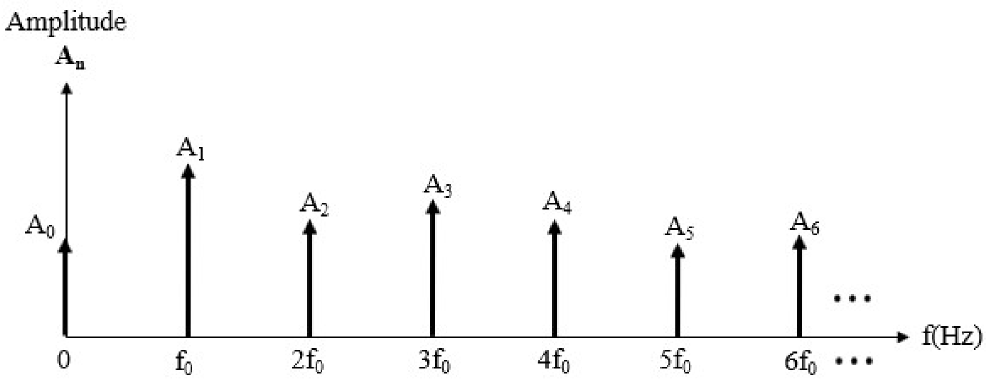


Figure 2.12 Amplitude spectrum of a periodic signal

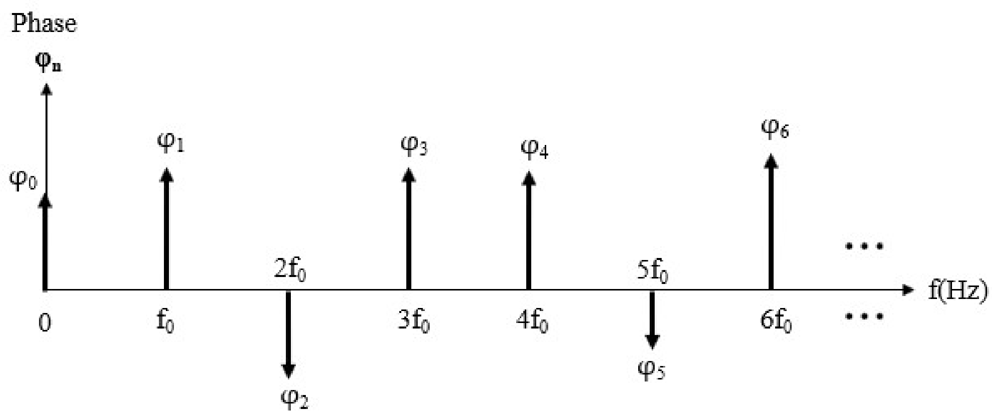


Figure 2.13 Phase spectrum of a periodic signal

The spectrum of a periodic signal may generally consist of the following frequency components:

- a DC component, at frequency 0Hz;
- a fundamental frequency component at the fundamental frequency f_0;
- a series of harmonic frequency components at frequencies nf_0 (n=2,3,4…).

A periodic signal is fully represented by a set of components in **amplitude** and **phase**. Thus, a periodic signal has a dual representation: a representation in time domain and in frequency domain with the following information:

Time Domain Representation	Frequency Domain Representation
Signal waveform	**Amplitude and phase spectrum**
- Signal shape	- Amplitude and phase of various frequency components present
- Signal frequency	
- Type of signal (periodic/non periodic)	

Single-sided amplitude and phase spectrum of sinusoidal signal

A sinusoidal signal, v(t) = $\mathbf{V_P}\cos(2\pi \mathbf{f_v} t + \boldsymbol{\varphi})$, contains only one frequency component of amplitude $\mathbf{V_p}$, frequency $\mathbf{f_v}$ and phase $\boldsymbol{\varphi}$. A sinusoidal signal is known as **single-tone** signal. Its single-sided amplitude and phase spectrum is shown in Figure 2.14.

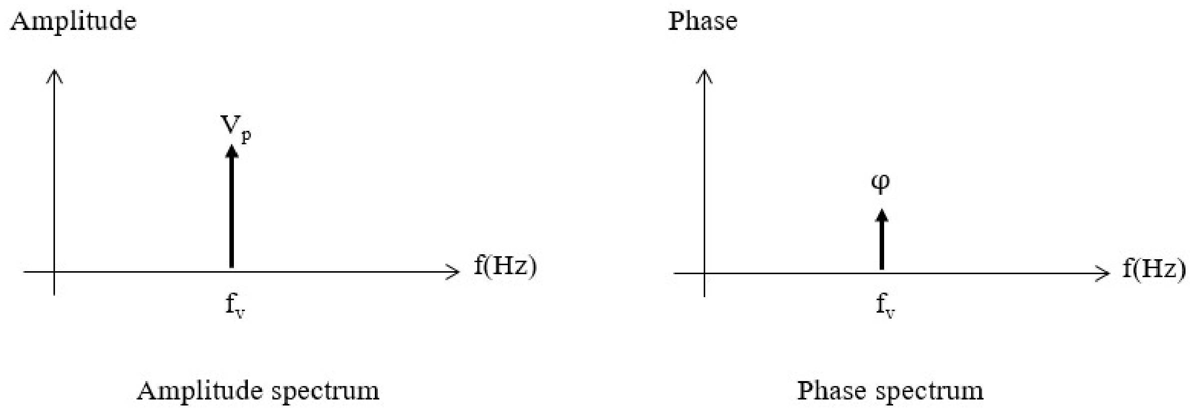


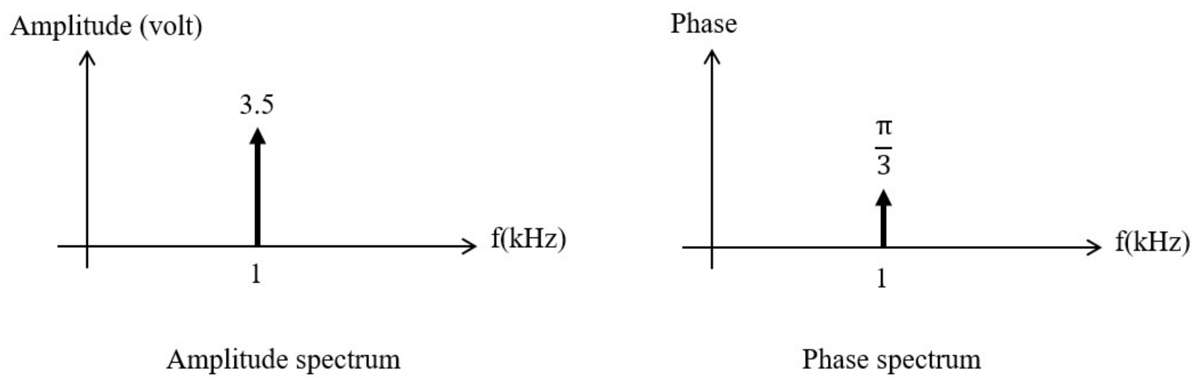
Figure 2.14 Single-sided amplitude and phase spectrum of sinusoidal signal

Example 2.4

Plot the single-sided amplitude and phase spectrum of the following sinusoidal signal:

$$v(t) = \mathbf{3.5}\cos(2000\pi t + \tfrac{\pi}{3})$$

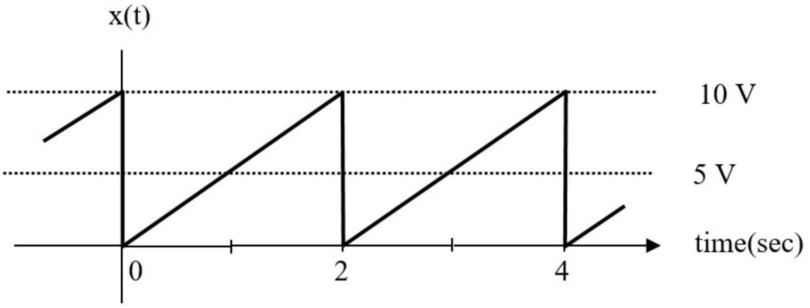
Solution



Single-sided amplitude and phase spectrum of a sawtooth signal

Example 2.5

Plot the single-sided amplitude and phase spectrum of a sawtooth signal.



The Fourier Series of the above waveform is given below:

$$f(t) = 5 + \frac{10}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\cos\left(n\omega_0 t + \frac{\pi}{2}\right)$$

Solution

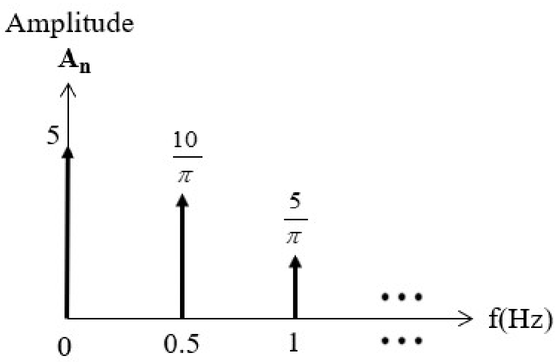
The period of $f(t)$ is 2 s. Therefore, $f_0 = \frac{1}{T} = \frac{1}{2}$ Hz.

Expanding the Fourier series of f(t),

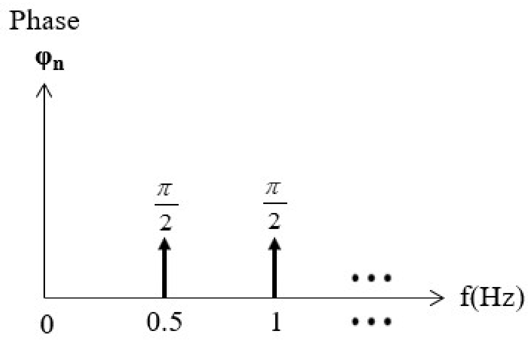
$$\begin{aligned} f(t) &= 5 + \frac{10}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\cos\left(n\omega_0 t + \frac{\pi}{2}\right) \\ &= 5 + \frac{10}{\pi}\cos\left(2\pi f_0 t + \frac{\pi}{2}\right) + \frac{10}{\pi}\cdot\frac{1}{2}\cos\left(4\pi f_0 t + \frac{\pi}{2}\right) + \ldots. \end{aligned}$$

where $\omega_0 = 2\pi f_0$

Hence the amplitude and phase spectrum are:



(a) Amplitude spectrum



(b) Phase spectrum

2.3.2 Exponential form of Fourier Series

It is convenient for many purposes to express a periodic signal x(t) as a complex exponential Fourier series allowing both positive and negative multiples of fundamental frequency as

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$
$$\text{where } C_n = \frac{1}{T}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} x(t)e^{-jn\omega_0 t}dt \tag{2.13}$$

The presence of negative frequencies has no physical meaning but just help to provide a compact mathematical expression.

The Fourier coefficient C_n in Eq. (2.13) may be expressed as

$$C_n = |C_n|e^{j\varphi_n} \tag{2.14}$$

where $| C_n | \geq 0$. $| C_n |$ and φ_n are the amplitude and phase of the n^{th} harmonic component of the periodic signal x(t), respectively. Plotting $| C_n |$ and φ_n versus frequency for both positive and negative frequencies, generates **double-sided** amplitude and phase spectrum, respectively.

The double-sided amplitude spectrum resulted from exponential Fourier series displays half the signal energy at the positive frequencies and the other half at the negative frequencies. The amplitude spectrum of a real-world signal is symmetrical around DC. i.e. the negative half of the spectrum is a mirror image of the positive half of the spectrum. Thus, both the positive spectrum and negative spectrum contain the same information. Most frequency-analysis instruments display only the positive half of the frequency spectrum as the negative frequency information is redundant.

The trigonometric Fourier series coefficients a_n and b_n are related to the complex exponential Fourier series coefficient C_n as following:

$$C_n = \begin{cases} \frac{1}{2}a_n - \frac{1}{2}jb_n & n \geq 1 \\ a_0 & n = 0 \\ \frac{1}{2}a_{|n|} + \frac{1}{2}jb_{|n|} & n \leq -1 \end{cases} \tag{2.15}$$

Thus, a single-sided amplitude spectrum can be converted into a double-sided amplitude spectrum and vice versa using the following equations:

$$|C_0| = a_0 = A_0$$
$$|C_n| = \frac{\sqrt{{a_n}^2 + {b_n}^2}}{2} = \frac{A_n}{2} \quad \text{for n } \neq 0 \tag{2.16}$$

- Double-sided amplitude spectrum can be obtained from a single-sided spectrum by adding negative frequency components which are mirror images of the positive components. The amplitude of each non-DC components of double-sided amplitude spectrum are halved that of single-sided components with DC component remains the same.

- Converting a double-sided spectrum to a single-sided spectrum, the negative half of the frequencies is discarded and every AC coefficient C_n is multiplied by 2. The DC component is unchanged.

2.3.3 Double-sided spectrum of rectangular waveform

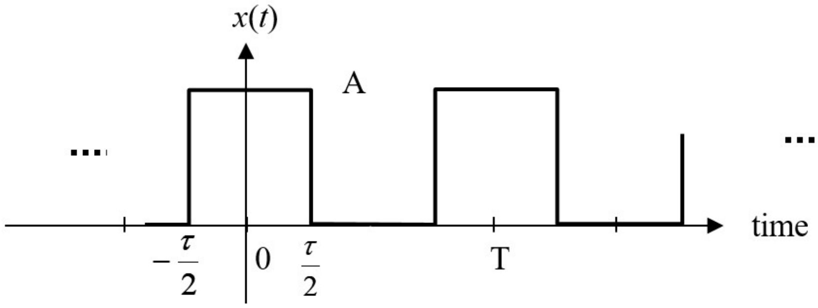
Rectangular waveform and its spectrum are commonly encountered in digital communications. Hence we shall examine its spectrum.

Example 2.6

A signal $x(t)$ has a rectangular waveform as shown in the figure below. Obtain the double-sided frequency spectrum of $x(t)$. Consider two cases :

(a) $\tau = \frac{T}{2}$ (50% duty cycle)

(b) $\tau = \frac{T}{5}$ (20% duty cycle)



Solution

The double-sided spectrum is given by exponential Fourier Series as

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

where

$$C_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-jn\omega_0 t}\,dt$$

Let us now consider the period, $-\frac{\tau}{2} \le t \le T - \frac{\tau}{2}$

$$C_n = \frac{1}{T}\int_{-\frac{\tau}{2}}^{T-\frac{\tau}{2}} x(t)e^{-jn\omega_0 t}\,dt$$

$$C_n = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} Ae^{-jn\omega_0 t}\,dt$$

$$= -\frac{A}{jn\omega_0 T}\left[e^{-jn\omega_0 t}\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}}$$

$$= \frac{2A}{n\omega_0 T}\left[\frac{e^{jn\frac{\omega_0}{2}\tau} - e^{-jn\frac{\omega_0}{2}\tau}}{j2}\right]$$

$$= \frac{2A}{n\omega_0 T}\sin\frac{n\omega_0\tau}{2}$$

$$= \frac{A\tau}{T}\frac{\sin\frac{n\omega_0\tau}{2}}{\frac{n\omega_0\tau}{2}}$$

$$= \frac{A\tau}{T}\frac{\sin\pi\frac{n\tau}{T}}{\pi\frac{n\tau}{T}}$$

$$C_n = \frac{A\tau}{T}\operatorname{sinc}\frac{n\tau}{T}$$

Therefore,

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$= \frac{A\tau}{T}\sum_{n=-\infty}^{\infty}\operatorname{sinc}\frac{n\tau}{T}e^{jn\omega_0 t}$$

To obtain the frequency spectrum of $x(t)$, we simply plot C_n which represent the amplitude of the frequency components of $x(t)$.

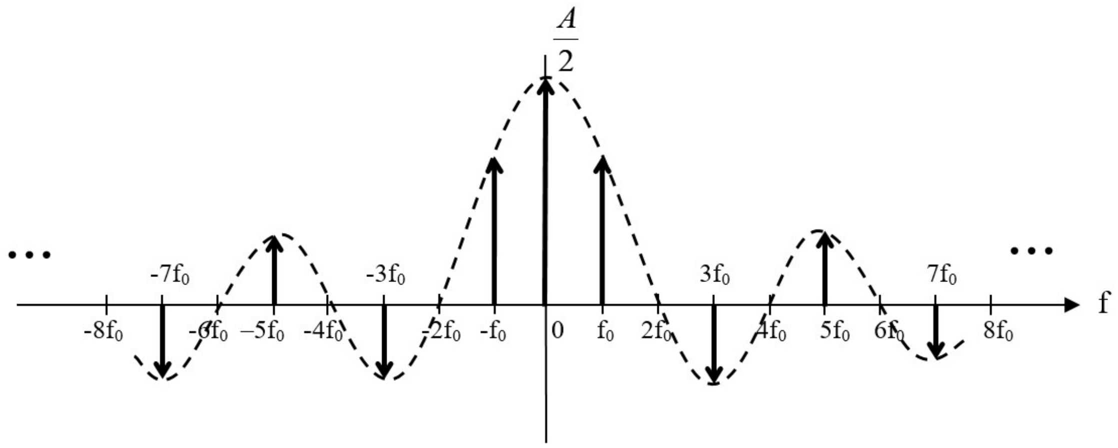
Consider now the first case which represents a square wave where

$$\tau = \frac{T}{2} \quad \text{i.e. } \frac{\tau}{T} = \frac{1}{2}$$

Therefore substituting into the above C_n equation,

$$C_n = \frac{A}{2}\operatorname{sinc}\frac{n}{2}$$

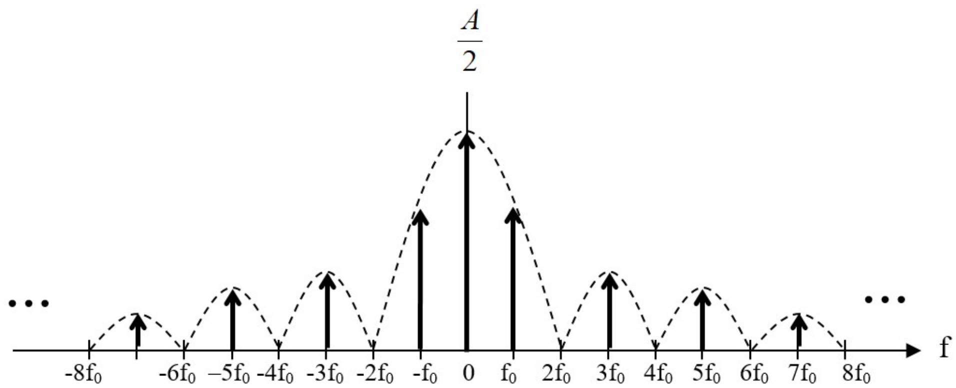
The frequency spectrum is as following:



where $f_0 = \frac{1}{T}$.

Note :

- The zero crossings are found at $n = \pm 2; \pm 4; \pm 6; \ldots$ (ie when $n/2$ is an integer). Even harmonics are suppressed.
- If only the amplitude spectrum is required, the components with negative should be inverted.



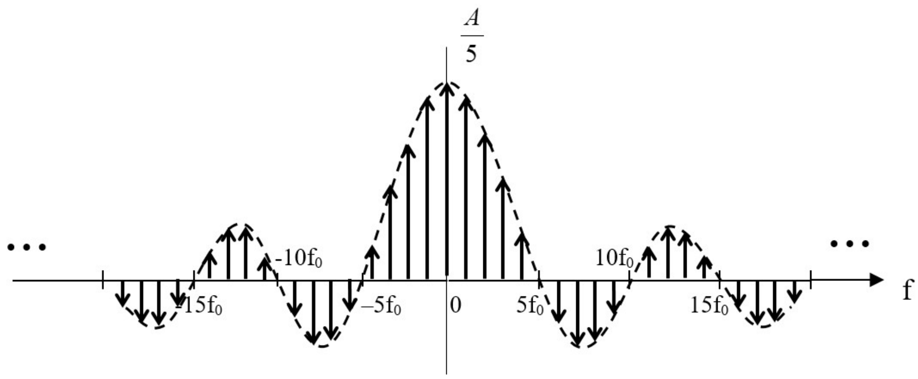
Consider now the second case where

$$\tau = \frac{T}{5} \quad \text{i.e.} \quad \frac{\tau}{T} = \frac{1}{5}$$

Substituting into the above C_n equation,

$$C_n = \frac{A}{5}\,\text{sinc}\,\frac{n}{5}$$

The frequency spectrum is as following:



Note :

- The zero crossings are now at $\pm 5f_o$; $\pm 10f_o$; $\pm 15f_o$; ... (or when n/5 is an integer)
- C_n in this example is real, however in general C_n is complex in which case we need to plot both the amplitude and phase spectra when we want the complete frequency spectrum.

Some observations drawn from this example

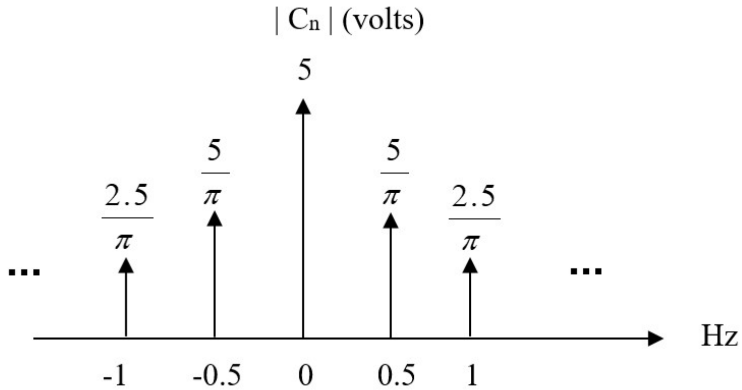
1. Given a rectangular pulse train $x(t)$ having a fundamental frequency f_o, the bandwidth of $x(t)$ increases with decreasing pulse width. That is, with decreasing duty cycle.
2. The envelope of the amplitude spectrum of a rectangular train is always a sinc function with the zero crossings occurring at $f = \pm\frac{k}{\tau}$, where k is a positive integer and τ is the pulse width.
3. A periodic signal has line spectrum. The spacing between the line components is $1/T$ (i.e., 1/period of the signal).

Example 2.7

Plot the double-sided amplitude spectrum of the signal given in example 2.4.

Solution

Applying eq. (2.16), we can derive the following double-sided spectrum.



2.4 FOURIER TRANSFORM

Fourier series is powerful tool for determining the frequency components of a periodic signal from which the spectrum of a periodic signal is derived. But, it does not provide frequency domain representation for non-periodic signal. For non-periodic signal, Fourier transform is employed.

Consider a periodic signal x(t) with a period T. A non-periodic signal can be viewed as a limiting case of a periodic signal x(t) where the period T of the signal approaches infinity as shown in Figure 2.15.

Thus, the Fourier transform of a non-periodic signal can be viewed as a limiting case of the complex exponential Fourier series of a periodic signal whose period is approaching infinity. Since non-periodic signal has a period T that tends to infinity and the fundamental frequency, $1/T$, approaches 0, the interval between two consecutive frequency components approaches 0. The discrete frequency approaches the continuous frequency. The summation becomes an integral. The resultant spectrum consists of a continuous range of frequency which may range from -∞ to ∞.

The Fourier Transform of x(t) is expressed as X(f):

Transform Pair $x(t) \overset{FT}{\Longleftrightarrow} X(f)$

Forward Transform:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt \tag{2.17}$$

Inverse Transform:

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft}\, df \tag{2.18}$$

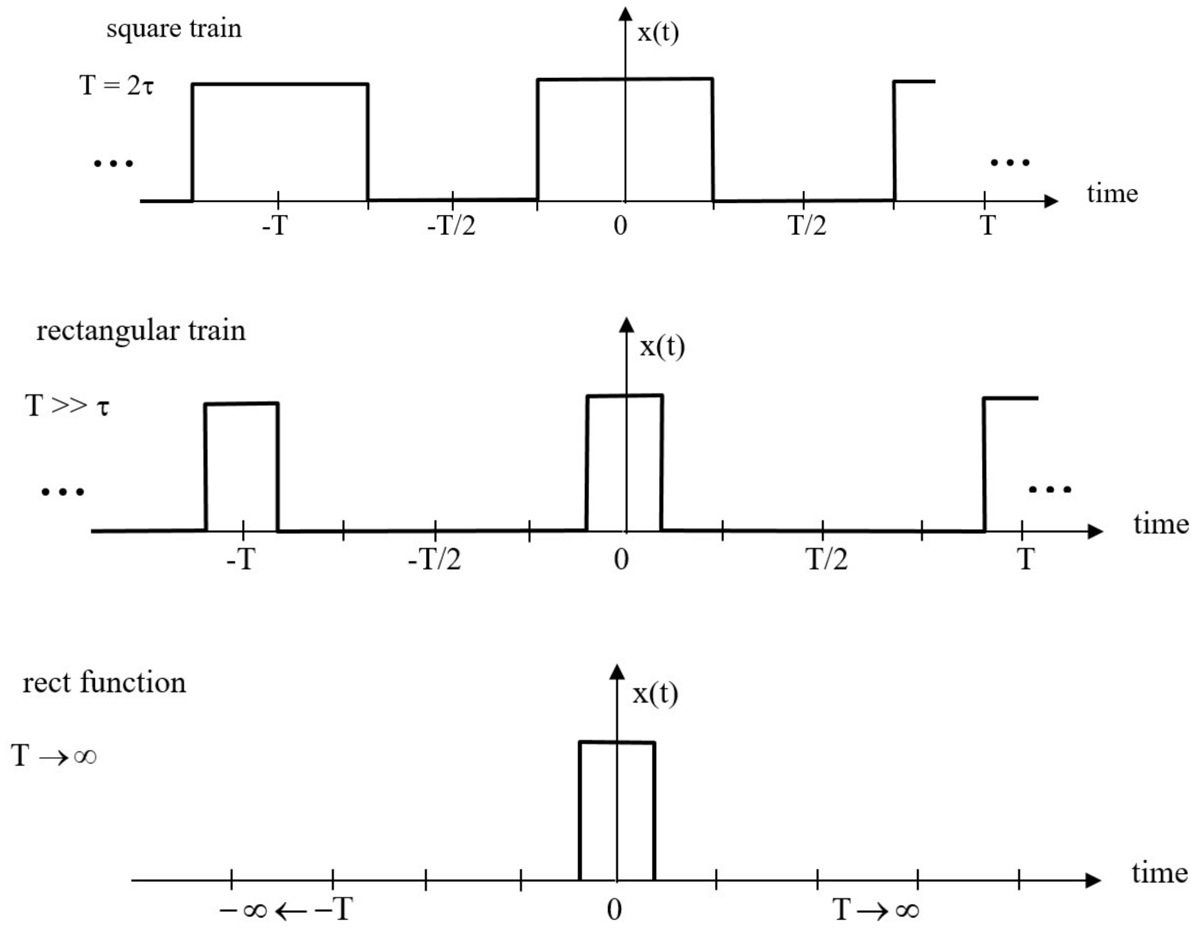


Figure 2.15 Derivation of a non-periodic signal

Eq.(2.17) transforms the time function x(t) into the frequency function X(f). The Fourier transform X(f) is known as the **continuous spectrum** of signal x(t). |X(f)| is known as the amplitude spectrum (even though amplitude density spectrum would be more appropriate) and ∠X(f) is known as the phase spectrum.

An example of non-periodic signal and its continuous amplitude spectrum is shown in Figure 2.16 and Figure 2.17, respectively.

Figure 2.16 Time domain representation of a non-periodic signal

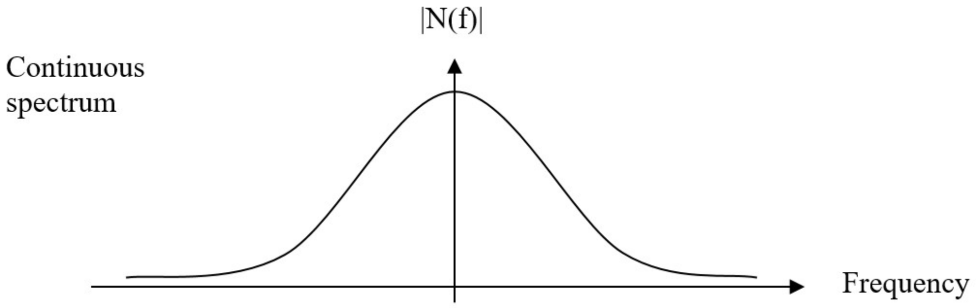


Figure 2.17 Frequency domain representation of a non-periodic signal

2.4.1 Some Useful Properties of Fourier Transform

- **Addition Property**

$$\text{If} \quad x_1(t) \overset{FT}{\Longleftrightarrow} X_1(f)\,; \qquad x_2(t) \overset{FT}{\Longleftrightarrow} X_2(f)$$
$$\text{then} \quad x_1(t) + x_2(t) \overset{FT}{\Longleftrightarrow} X_1(f) + X_2(f) \tag{2.19}$$

- **Time-shift Property**

$$\text{If} \quad x(t) \overset{FT}{\Longleftrightarrow} X(f)$$
$$\text{then} \quad x(t-\tau) \overset{FT}{\Longleftrightarrow} e^{-j\omega\tau}X(f) \tag{2.20}$$

where $e^{-j\omega\tau}$ is the phase term i.e. only the phase spectrum will changed while the amplitude spectrum remains unchanged when a waveform is shifted in time.

- **Frequency-shift Property** (see Appendix 2.1)

$$\text{If} \quad x(t) \overset{FT}{\Longleftrightarrow} X(f)$$
$$\text{then} \quad x(t)e^{j2\pi f_c} \overset{FT}{\Longleftrightarrow} X(f-f_c) \tag{2.21}$$
$$x(t)\cos 2\pi f_c t \overset{FT}{\Longleftrightarrow} \frac{1}{2}[X(f+f_c) + X(f-f_c)] \tag{2.22}$$

Eq. (2.22) is extremely important in communications. It states that multiplying x(t) with a sinusoidal signal cos $2\pi f_c t$ shifts the spectrum of x(t) to $\pm f_c$. In practice, f_c is much larger than the frequencies of x(t) for the purpose of modulation.

- **Convolution Property**

The convolution of two signals, $x_1(t)$ and $x_2(t)$, is defined by the following integral operation:

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)\mathrm{d}\tau = \int_{-\infty}^{\infty} x_2(\tau)x_1(t-\tau)\mathrm{d}\tau \qquad (2.23)$$

Where τ is a dummy variable. The integral of eq. (2.23) is generally very difficult to evaluate in closed form. We will only consider convolution of a function with a unit impulse function.

Note: Convolution is a mathematical operation and it is an important analytical tool used in communication engineering. It is a method of determining the response of a linear system to a specific input signal, or the result of the interaction between two signals.

- Time Convolution Theorem

$$x(t) \overset{FT}{\Longleftrightarrow} X(f)$$

$$y(t) \overset{FT}{\Longleftrightarrow} Y(f)$$

$$x(t) * y(t) \overset{FT}{\Longleftrightarrow} X(f)\cdot Y(f) \qquad (2.24)$$

- Frequency Convolution Theorem

$$x(t) \overset{FT}{\Longleftrightarrow} X(f)$$

$$y(t) \overset{FT}{\Longleftrightarrow} Y(f)$$

$$x(t)\cdot y(t) \overset{FT}{\Longleftrightarrow} X(f) * Y(f) \qquad (2.25)$$

2.4.2 Fourier transform of an impulse

Unit impulse, often denoted as $\delta(t)$, can be seen as the limiting case of the rectangular pulse of height A and width τ. This concept is used to derive the Fourier transform of an impulse, as shown in section 2.7 of the Appendix 2.

The Fourier Transform of a unit impulse, $\delta(t)$, can be shown to be equal to a constant of value 1 at all frequency i.e.

$$\delta(t) \overset{FT}{\Longleftrightarrow} 1 \qquad (2.26)$$

and graphically shown in Figure 2.18.

- **The spectrum of a unit impulse contains components at all frequencies, with equal amplitude of 1 and has zero phase.**

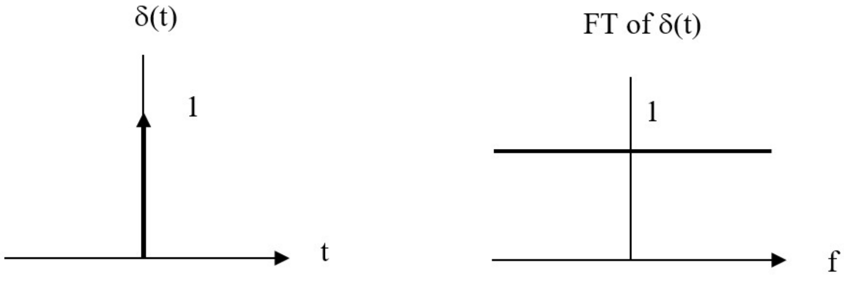


Figure 2.18 Unit impulse and its Fourier transform

For an impulse of strength A i.e. $A\delta(t)$, the Fourier Transform is:

$$A\delta(t) \overset{FT}{\Longleftrightarrow} A \tag{2.27}$$

It can be shown (refer to Appendix 2.2), the Fourier transform of a DC signal V_0 is

$$x(t)=V_o \quad \overset{FT}{\Longleftrightarrow} \quad X(f) = V_o\,\delta(f) \tag{2.28}$$

- **Convolution of any function $f(t)$ with a unit impulse function $\delta(t)$ gives the function $f(t)$ itself - Replication property of $\delta(t)$**

$$f(t) * \delta(t) = f(t) \text{ - the function f(t) itself} \tag{2.29}$$

and

$$f(t) * \delta(t-t_0) = f(t-t_0) \text{ - the time-shifted version of f(t)} \tag{2.30}$$

Proof :

Since $\int_{-\infty}^{\infty} \delta(t)dt = 1$ and $\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$

it follows, $\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)$

Then,

$$f(t) * \delta(t) = \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)d\tau = f(t)$$

Similar result follows in the frequency domain:

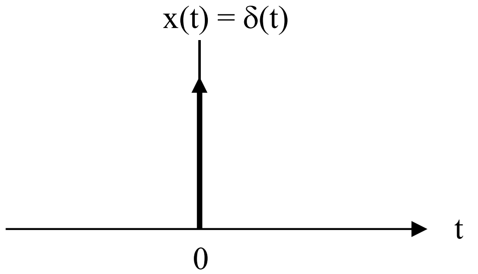
- **Convolution of any function $X(f)$ with a unit impulse function $\delta(f)$ gives the function $X(f)$ itself.**

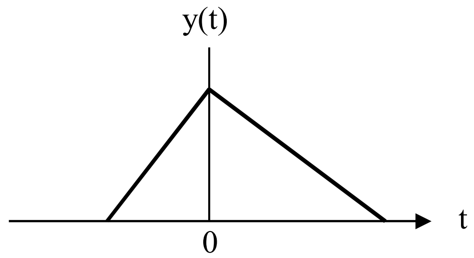
$$X(f) * \delta(f) = X(f) \tag{2.31}$$

$$X(f) * \delta(f-f_0) = X(f-f_0) \tag{2.32}$$

Example 2.8

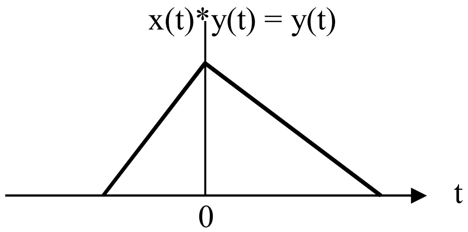
Two signals $x(t)$ and $y(t)$ are shown below. Obtain the graphical convolution of $x(t)$ and $y(t)$.





Solution

According to eq. (2.29), $y(t)*x(t) = y(t)*\delta(t) = y(t)$



2.4.3 Fourier transform of a sinusoid signal

A sinusoidal signal of a frequency f_0 and an amplitude V_p can be expressed as a complex exponential function applying Euler's identity (refer to Appendix 2.4):

$$x(t) = V_p\cos(2\pi f_0 t)$$

$$= V_P\left(\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right)$$

Applying the frequency-shift property in eq. (2.21), the Fourier transform of sinusoidal x(t) is

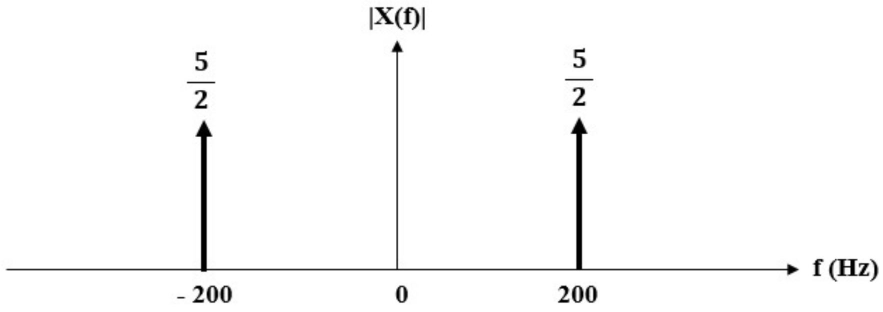
$$x(t) = V_p\cos(2\pi f_0 t) \overset{FT}{\Longleftrightarrow} X(f) = \frac{V_p}{2}\left[\delta(f + f_0) + \delta(f - f_0)\right] \qquad (2.33)$$

Example 2.9

Plot the amplitude spectrum of signal $x(t) = 5\cos 400\pi t$

Solution

$$x(t) = 5\cos(400\pi t) \quad \overset{FT}{\Longleftrightarrow} \quad X(f) = \frac{5}{2}\,[\delta(f + 200) + \delta(f - 200)]$$



2.4.4 Fourier transform of an amplitude modulated signal

As mentioned in Chapter 1, for long distance transmission, a baseband signal x(t) is superimposed on a high frequency sinusoidal signal through a **modulation** process, which shifts x(t) to a high frequency band. One type of modulation is achieved by multiplying x(t) with a high frequency sinusoidal signal which is $x(t)\cos 2\pi f_c t$.

According to Frequency-shift property in eq. (2.21) and (2.22), the spectrum of the modulated signal $x(t)\cos 2\pi f_c t$ is

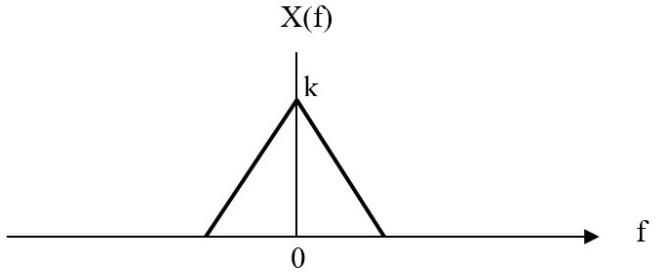
$$\text{If } x(t) \overset{FT}{\Longleftrightarrow} X(f)$$

$$x(t)\cos 2\pi f_c t \overset{FT}{\Longleftrightarrow} \frac{1}{2}\,[X(f+f_c) + X(f-f_c)]$$

- **The spectrum of an modulated signal, $x(t)\cos 2\pi f_c t$, consists of two frequency shifted version of the spectrum of x(t), one version shifted left by f_c and another version shifted right by f_c. The amplitude of the two frequency shifted versions is half of X(f)**.

Example 2.10

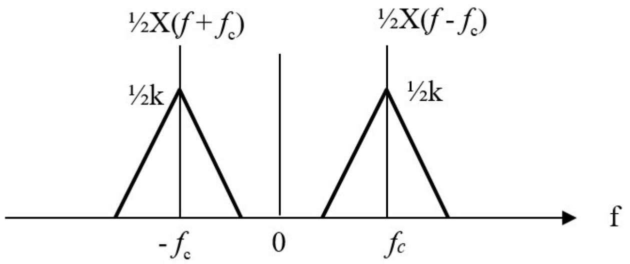
The spectrum of x(t) is given below. Plot the spectrum of $x(t)$ cos $2\pi f_c t$.



Solution

$$x(t)\cos 2\pi f_c t \quad \overset{FT}{\Longleftrightarrow} \quad \frac{1}{2}\left[X(f+f_c)+X(f-f_c)\right]$$

The above equation tells us that the spectrum x(t)cos $2\pi f_c t$ consists of two frequency shifted version of the spectrum of x(t) as shown below:



2.4.5 Two common transform pairs

Fourier transform pairs shown in Figure 2.19 and Figure 2.20 are commonly used in the study of communication systems:

$$\boldsymbol{A}rect\frac{t}{\tau} \quad \overset{FT}{\Longleftrightarrow} \quad \boldsymbol{A}\tau sinc f\tau \qquad (2.34)$$

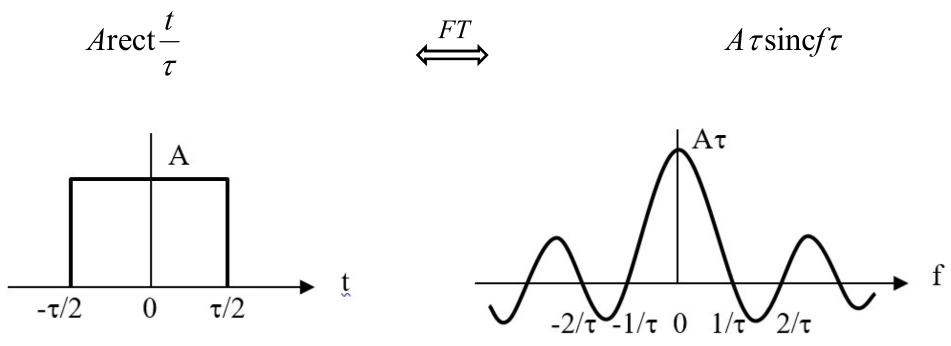


Figure 2.19 Rect function and its Fourier transform

$$\boldsymbol{A}\mathrm{sinc}\frac{t}{\tau} \quad \overset{FT}{\Longleftrightarrow} \quad \boldsymbol{A}\tau\mathrm{rect}f\tau \tag{2.35}$$

$$A\mathrm{sinc}\frac{t}{\tau} \quad \overset{FT}{\Longleftrightarrow} \quad A\tau\mathrm{rect}f\tau$$

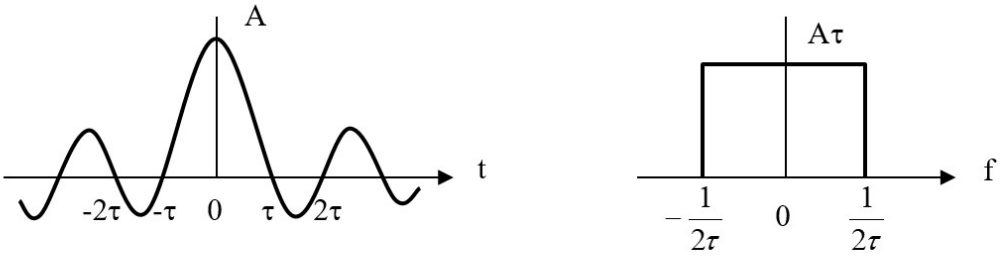


Figure 2.20 Sinc function and its Fourier transform

Example 2.11

Sketch the amplitude spectrum of $x(t) = 3\mathrm{rect}\,20t$

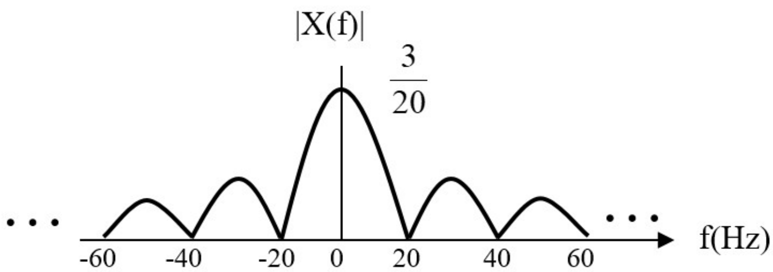
Solution

Comparing with the standard transform in the FT pair in (2.34),

$x(t) = 3\mathrm{rect}20t \equiv Arect\frac{t}{\tau}$

Hence A = 3, and $\tau = \frac{1}{20}$ s

The amplitude spectrum is



2.4.6 Fourier transform of a unit impulse train

Consider a train of impulse mathematically expressed as

$$x(t) = \sum_{n=-\infty}^{n=\infty} \delta(t - nT) \qquad (2.36)$$

The Fourier transform of a train of impulses given by eq. (2.36) is shown in Figure 2.21 (refer to Appendix 2.6)

$$\therefore \qquad x(t) \Leftrightarrow X(f) = \frac{1}{T}\sum_{n=-\infty}^{\infty} \delta(f - nf_0) \quad \text{where } f_0 = \frac{1}{T} \qquad (2.37)$$

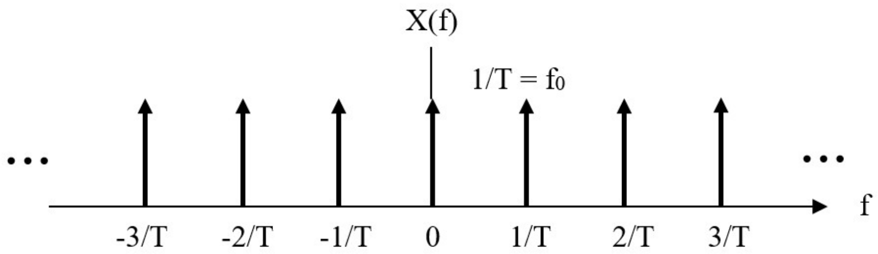


Figure 2.21 unit impulse train and its Fourier transform

Note:

- The spectral lines are all of the same amplitude and the frequency separation between the spectral lines is 1/T. The bandwidth is infinity.
- This is a very important result which is used in analysing the sampling process.

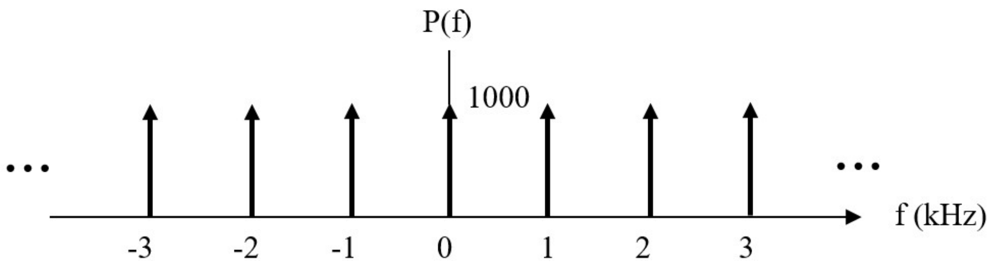
Example 2.12

Sketch the amplitude spectrum of a unit impulse train $p(t)$ of frequency 1 kHz.

Solution

Given $f_0 = \frac{1}{T} = 1$ kHz.

According to the discussion above, the amplitude spectrum is an impulse train with f_0 = 1kHz:



2.5 SIGNAL BANDWIDTH

Signal bandwidth, B, is defined as the width of **positive frequencies** contained in a signal. For a frequency-limited signal with positive frequencies ranging from f_L to f_H, the signal bandwidth is the difference between the highest frequency, f_H, and lowest frequencies, f_L:

$$B = f_H - f_L \quad (\text{Hz}) \tag{2.38}$$

where to $0 \le f_L \le f_H$.

However, many natural signals are not bandlimited and may have frequency components extending from zero to infinity. It is therefore not possible to measure the bandwidth of those type of signals theoretically as the bandwidth is infinite. Practically, only the range of positive frequencies with significant spectrum is considered. For signal with infinite bandwidth, there usually exists some upper frequency, above which the energy level is negligible. Similarly, there usually exists some lower frequency below which the energy level is insignificant. In practice, we need to send only the significant spectrum of speech and digital signals. We can recover the speech and digital signals with reasonable accuracy at the receiver with minimum distortion. Thus the bandwidth is taken to be the bandwidth of the **significant spectrum** of the infinite spectrum. For example, the natural speech has a significant spectrum from 100 Hz to 10 kHz. Thus, the bandwidth of natural speech is approximately 10 kHz.

Example 2.13

The significant frequency range of a telephone signal is between 300 Hz and 3.4 kHz. What is the bandwidth of the signal?

Solution

Bandwidth, B: $B = f_H - f_L = 3.4 - 0.3 = 3.1$ (kHz)

2.6 SIGNAL POWER MEASUREMENTS

Signal power is often measured relative to a reference level in communication systems which is expressed in decibels (dB). dB is a relative measure of two different power level in base 10 logarithmic measure. The dB makes it convenient to represent very large or small numbers. A dB is considered as a dimensionless unit as it is a ratio of two similar quantities with the same units.

In electrical communications, the gain/loss of a system, attenuation of signal power, and signal to noise ratios are often expressed in dB.

The ratio of the power level at the output of a circuit, P_o, compared to that at the input, P_i, is often expressed by the decibel gain instead of the actual ratio, denoted as G (dB):

$$G(dB) = 10log\frac{P_o}{P_i} \tag{2.39}$$

where P_o and P_i are measured in same units (watts or milliwatts).

Eq. (2.39) gives a number that indicates the relative value of output power with respect to the input power. It does not indicate the actual magnitude of the power levels involved.

If the dB value is known, the power ratio can be obtained by inversion of eq. (2.39):

$$\frac{P_o}{P_i} = 10^{\frac{G(dB)}{10}} \tag{2.40}$$

Example 2.14

Express the power gain/loss in dB in the table.

P_1 (Watts)	P_2 (Watts)	$\frac{P_2}{P_1}$	dB $(10\log\frac{P_2}{P_1})$	Remarks
2	4	2	3	Power gain is 3 dB
2	20	10	10	Power gain is 10 dB
2	2	1	0	Power gain is 0 dB
4	1	0.25	-6	Power gain is -6 dB/ Power loss is 6 dB

Decibel measures of absolute power level with respect to some reference level are also commonly used. One frequently used measurement is **dBm**. **dBm** indicates power measurement relative to **1 mW** (milliwatt / one thousandth Watt) giving by eq.(2.39). The actual value of the power in milliwatts can be obtained:

$$10\log\frac{\mathrm{P(mW)}}{1\ \mathrm{mW}} = \mathrm{X\ dBm} \tag{2.41}$$

$$P = 10^{\frac{x}{10}}\ mW \tag{2.42}$$

For example, 0 dBm equals one milliwatt, and 1 dBm is about 1.259 mW.

When power is measured relative to 1 Watt, the decible level is denoted dBW:

$$10log\frac{P\ (Watt)}{1\ Watt} = X\ dBW \tag{2.43}$$

Example 2.15

Express the power in table in terms of dBW, dBm.

Power	dBW	dBm
30 W		
3 μW		

Solution

P = 30 W
P (dBW) = $10log\frac{P\ (Watt)}{1\ Watt}$ =10log(30) = 14.77 dBW
P (dBm) = $10\log\frac{\mathrm{P(mW)}}{1\ \mathrm{mW}}$ = 10log(30×10^3) = 14.77 + 30 = 34.77 dBm

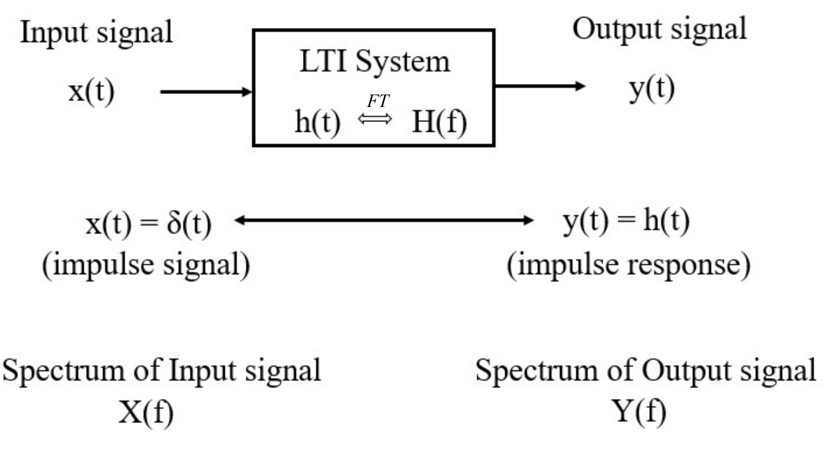
P = 3 μW
P (dBW) = $10log\frac{P\ (Watt)}{1\ Watt}$ =10log(3×10^{-6}) = -55.23 dBW
P (dBm) = $10\log\frac{\mathrm{P(mW)}}{1\ \mathrm{mW}}$ = 10log(3×10^{-3}) = -25.23 dBm

2.7 SIGNAL TRANSMISSION THROUGH LINEAR TIME INVARIANT (LTI) SYSTEMS

A system is said to be **linear** if the output due to a sum of different inputs is the sum of the corresponding individual outputs. If the input-output relationship of a system does not change with time, it is said to be **time invariant**.

A LTI system is described by its **impulse response**, h(t), i.e. the response of the system when its input is a unit impulse, δ(t) in time domain. It can be shown that the system's impulse response can be used to obtain its output when the input is not an impulse using the following convolution integral:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\mathrm{d\tau} = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\mathrm{d\tau} = x(t) * h(t) \qquad (2.44)$$



The output signal of a LTI system can be obtained by convolving the input signal with the impulse response of the system. The spectrum of the output signal is obtained by taking the Fourier transform of both sides of eq. (2.44) applying Time Convolution Theorem in eq. (2.24).

$$Y(f) = X(f) \cdot H(f) \tag{2.45}$$

or

$$H(f) = \frac{Y(f)}{X(f)} \tag{2.46}$$

where H(f) is the Fourier transform of h(t) known as **frequency response of the system**. The impulse response and frequency response are a Fourier transform pair:

$$h(t) \overset{FT}{\Longleftrightarrow} H(f)$$

The frequency response, H(f), consists of amplitude and phase response:

$$H(f) = \frac{Y(f)}{X(f)} = |H(f)| \angle H(f) \text{ and}$$

$$|H(f)| = \frac{|Y(f)|}{|X(f)|} \tag{2.47}$$

where |H(f)| is the amplitude response and
∠H(f) is the phase response

LTI systems play significant role in communication system analysis and design as LTI system can be easily characterised either in time domain using the system impulse response h(t) or in frequency domain using the system frequency response H(f).

2.8 FILTERS

A filter is a frequency-selective LTI system that can shape and limit the spectrum of a signal. Filters are widely used to process signals in communication systems. It passes signals of certain frequencies and rejects or attenuates signals of other frequencies. As such, filters are used to filter out (or eliminate) the signals in an unwanted range such as interference, noise and distortion products.

The characteristic of a filter is determined by its frequency response H(f):

$$H(f) = \frac{Y(f)}{X(f)} = |H(f)| \angle H(f) \text{ and}$$

$$|H(f)| = \frac{|Y(f)|}{|X(f)|} \tag{2.48}$$

where |H(f)| is the amplitude response and
∠H(f) is the phase response of a filter

The amplitude response |H(f)| and phase response ∠H(f) specifies the **voltage gain** and **phase shift** applied by the filter at each frequency, respectively.

2.8.1 Ideal Filters

An ideal filter passes signals at certain sets of frequencies exactly and completely reject the rest. The following are common types of ideal filters:

- Low pass filter (LPF)
- High pass filter (HPF)
- Band pass filter (BPF)

Figure 2.22 shows the amplitude response (voltage gain) of ideal LPF, HPF and BPF. The following are observed:

(a) LPF passes low frequency components from $-f_c$ to $+f_c$ and rejects high frequencies beyond $\pm f_c$.

(b) HPF passes high frequency components beyond $\pm f_c$ but rejects frequencies from $-f_c$ to $+f_c$ (opposite of LPF).

(c) BPF passes frequencies in the range of $-f_U$ to $-f_L$ and f_L to f_U, and rejects all frequencies outside this range.

The parameters of the ideal filters are defined below:

- **Passband**: the range(s) of frequency that is passed by a filter.

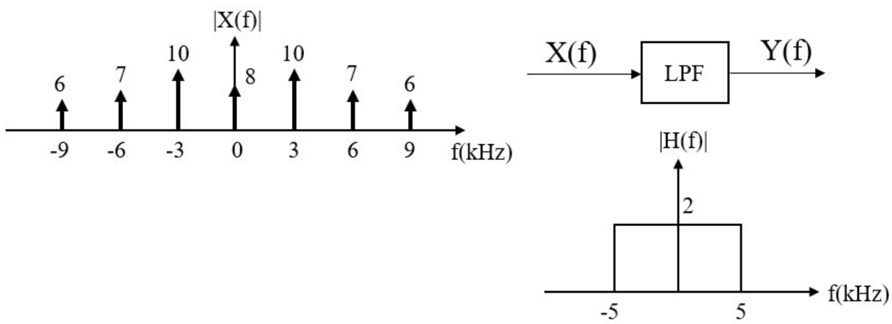
- **Stopband**: the range(s) frequencies that is rejected by a filter is called.

- **Cut-off frequencies**: the frequencies mark the transition from passband to stopband, or vice versa.

For BPF there are two cut-off frequencies, denoted as f_L and f_U. The transition from stopband to passband and vice versa is instantaneous.

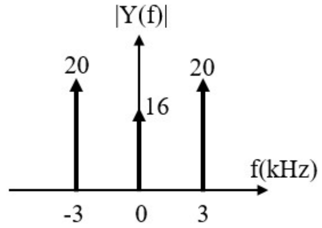
<u>Example</u> <u>2.16</u>

The amplitude spectrum of the input signal of an ideal LPF is shown below. The LPF has a cut-off frequency of 5 kHz. Determine the amplitude spectrum of the filter output signal.



Solution

The filter passes frequencies from -5 kHz to +5 kHz with a voltage gain of 2 and rejects high frequencies beyond ±5 kHz. Thus, the amplitude spectrum of the filter output signal only has frequency components from -3 kHz to +3 kHz.



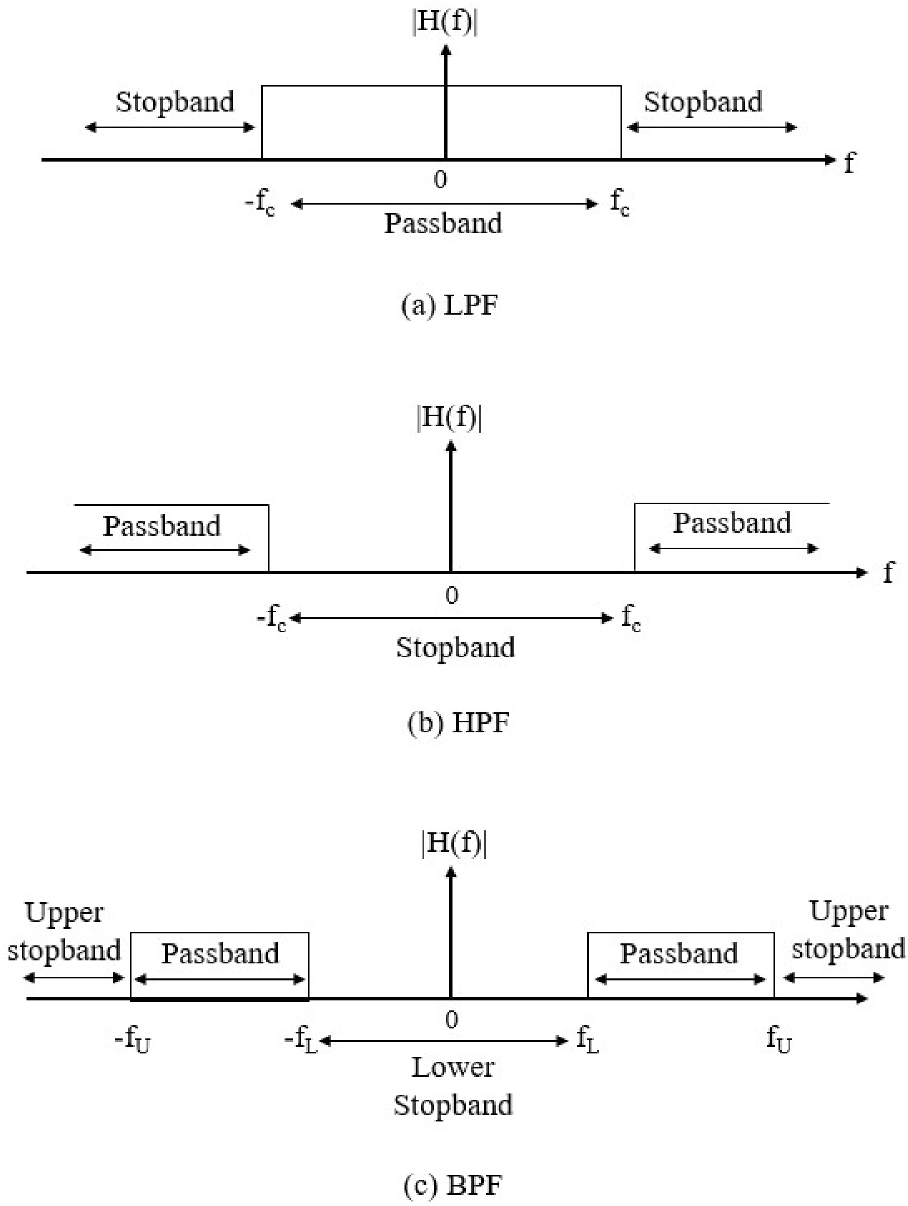


Figure 2.22 Amplitude response (voltage gain) of ideal filters

2.8.2 Practical Filters

In practice, it is impossible to achieve the ideal frequency response because of instantaneously transition from stopband to passband or vice versa. Practical filters have a gradual transition from passband to stop band and vice-versa as shown in Figure 2.23.

Terms used in Practical Filters

Below are some technical terms that are commonly used when describing frequency response of practical filters:

(i) **Cut-off frequency:**

In a practical filter, the change in gain is gradual rather than abrupt. Thus, the cut-off frequency of practical filters is defined as the frequency at which the filter voltage gain has drop to $1/\sqrt{2}$ or 0.707 of its maximum voltage gain, A_v.

$$\begin{aligned} |H(f)|_{(max)} &= A_v \\ |H(f)|_{(3dB)} &= A_{v(3dB)} = \frac{Av}{\sqrt{2}} \end{aligned} \tag{2.49}$$

This change in voltage gain is equivalent to a 3 dB drop in power gain. Hence, the cut-off frequency of practical filters is also known as the **3 dB cut-off frequency**.

For LPF and HPF, there is only one cut-off frequency and this is designated f_{3dB} in Figure 2.23 (a) and (b).

However for the BPF, there are two cut-off frequencies and they are designated as f_L (lower cut-off frequency) and f_U (upper cut-off frequency).

(ii) **Passband:**

This is the range of frequency in which the filter will pass within a specified gain. This specified gain is traditionally taken to be $|H(f)| = A_v/\sqrt{2}$, i.e. $0.707A_v$.
Therefore, for LPF, its passband is the frequency range from $-f_{3dB}$ to f_{3dB}. For BPF, the passband is from f_L to f_U. For HPF, the passband is defined as all frequencies above f_{3dB}. For BSF, there are 2 passbands, from 0 to f_L and above f_U.

(iii) **Stop band:**

This is the band of frequencies in which the voltage gain |H(f)| is below a value **y**, as specified by the user. Frequency components within the stop band will be amplified much less than those frequencies in the passband.

(iv) **Transition band:**

This is the band of frequencies between the passband and the stop band and vice-versa.

(v) **Filter Bandwidth:**

This is defined as the width of the passband of the positive frequency.

Hence for LPF, the bandwidth is (f_{3dB} - 0) = f_{3dB}.

For BPF, its bandwidth is equal to (f_U - f_L).

In the case of a HPF, it's bandwidth is (∞ - f_{3dB}). However, this has no practical meaning and therefore the term "bandwidth of HPF is never used.

The filters used in communication systems are mostly LPF and BPF.

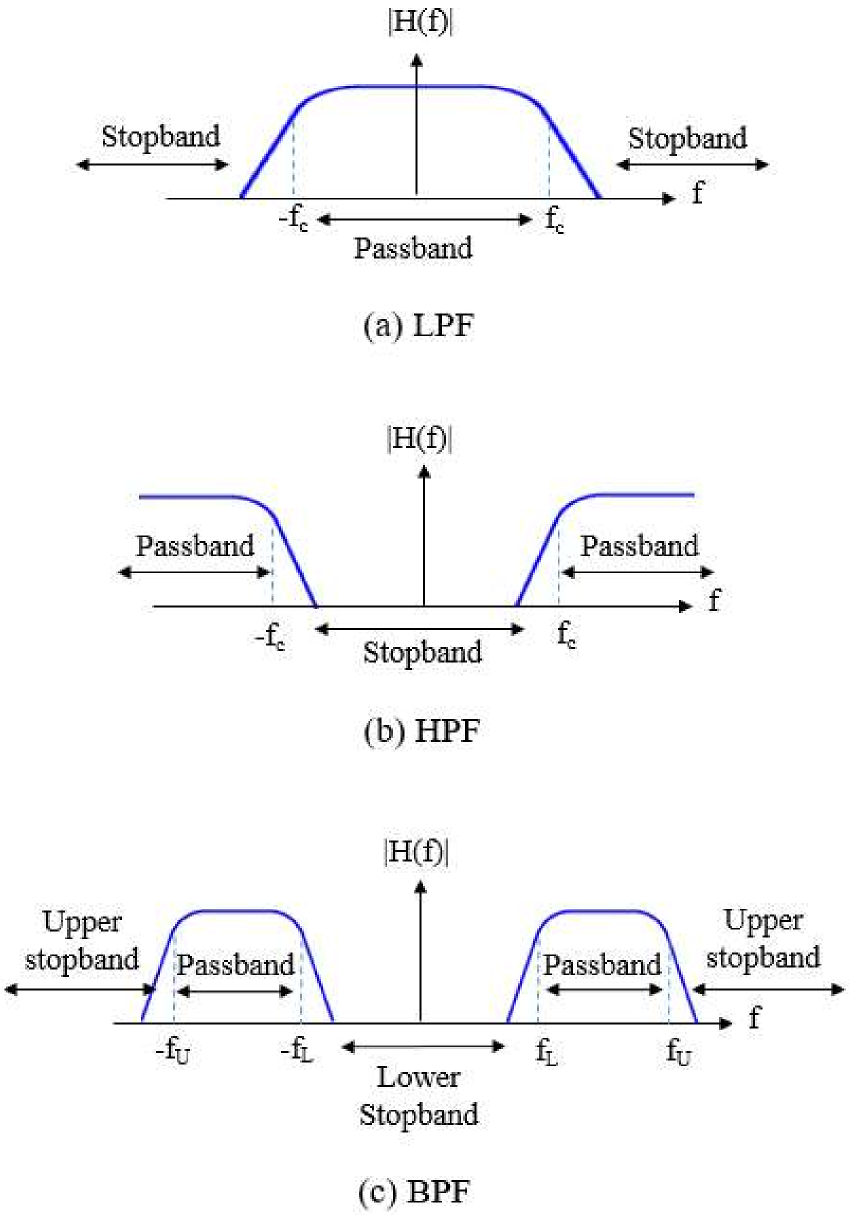
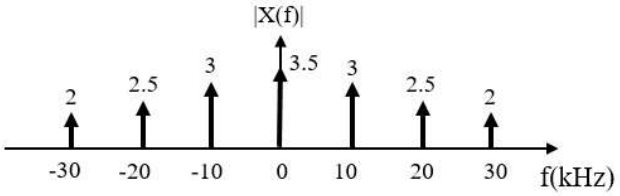
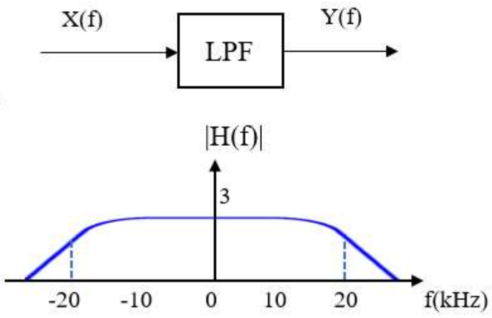


Figure 2.23 Amplitude response of practical filters

Example 2.17

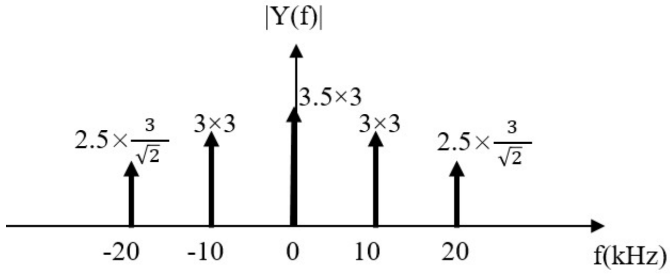
The amplitude spectrum of the input signal of a LPF is shown below. The LPF has a 3dB cut-off frequency of 20 kHz. Sketch the amplitude spectrum of the filter output signal.





Solution

|Y(f)| = |H(f)| × |X(f)|



APPENDIX

2.1 FREQUENCY-SHIFT PROPERTY

If $x(t) \overset{FT}{\Leftrightarrow} X(f)$

then $x(t)e^{j2\pi f_c t} \overset{FT}{\Leftrightarrow} X(f-f_c)$

$x(t)\cos 2\pi f_c t \overset{FT}{\Leftrightarrow} \frac{1}{2}[X(f+f_c) + X(f-f_c)]$

To prove this property, we take the Fourier transform of $x(t)e^{j2\pi f_c t}$ as follows:

$$\Im\left[x(t)e^{j2\pi f_c t}\right] = \int_{-\infty}^{\infty} e^{j2\pi f_c t}\, x(t)e^{-j2\pi f t} dt$$
$$= \int_{-\infty}^{\infty} x(t)e^{-j2\pi(f-f_c)t}\, dt = X(f - f_c)$$

2.2 FOURIER TRANSFORM OF DC SIGNAL

$x(t)=V_o \quad \overset{FT}{\Leftrightarrow} \quad X(f) = V_o\, \delta(f)$

To prove this, we check the inverse FT of X(f):

$$\int_{-\infty}^{\infty} V_o \delta(t) e^{j2\pi f t}\, df = V_o$$

2. 3 A USEFUL FOURIER TRANSFORM RESULT

Prove the relation :

$Ae^{jn\omega_0 t} \overset{FT}{\Leftrightarrow} A\delta(f - nf_0)$

where $\omega_o = 2\pi f_o$

We use the inverse FT,

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j\omega t} df$$
$$= \int_{-\infty}^{\infty} A\delta(f - nf_0)e^{j2\pi f t} df$$
$$= Ae^{j2\pi n f_0 t} \int_{-\infty}^{\infty} \delta(f - nf_0) df = Ae^{jn\omega_0 t}$$

If A = 1, then $e^{jn\omega_0 t} \overset{FT}{\Leftrightarrow} \delta(f - nf_0)$

2.4 Fourier transform of Sinusoid

$$x(t) = V_p\cos(2\pi f_0 t) = V_p\left(\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right)$$

According to 2.3 above,

$$\Im[x(t)] = \frac{V_p}{2}\,[\delta(f + f_0) + \delta(f - f_0)]$$

2.5 FOURIER TRANSFORM OF A PERIODIC FUNCTION

Strictly, the Fourier Transform of a periodic function does not exist, since for a periodic function, $x(t)$,

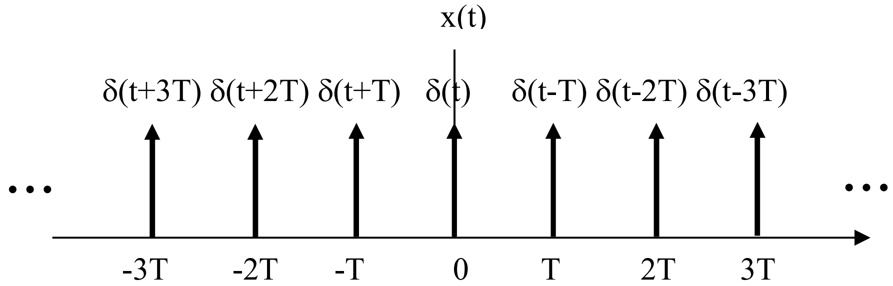
$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt \qquad \text{cannot be found.}$$

However, using the Fourier Transform pair below whose proof is shown in 2.3 above:

$$\Im\left[e^{jn\omega_0 t}\right] = \delta(f - nf_0)$$

then a periodic function can have a Fourier Transform.

2.6 FOURIER TRANSFORM OF A TRAIN OF IMPULSES



The train of impulses can be expressed as,

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

We first need to find its complex exponential fourier series,

$$C_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)e^{-jn\omega_0 t}\,dt$$

$$C_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t)e^{-jn\omega_0 t}\,dt$$

Since $\int_{-\infty}^{\infty} \delta(t) e^{-jn\omega_0 t} dt = \int_{-\infty}^{\infty} \delta(t) e^{-jn\omega_0 0} dt = 1$

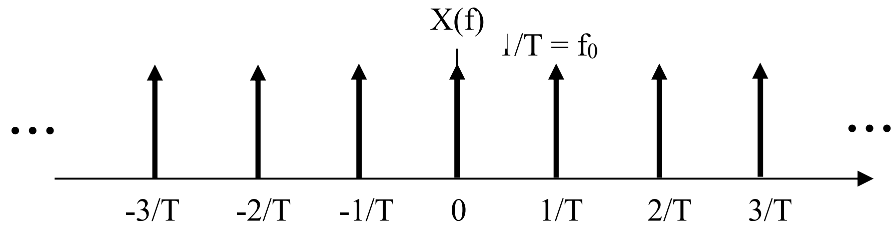
$$\therefore \quad C_n = \frac{1}{T}$$

Hence its complex exponential fourier series is

$$\therefore \quad x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{jn\omega_0 t}$$

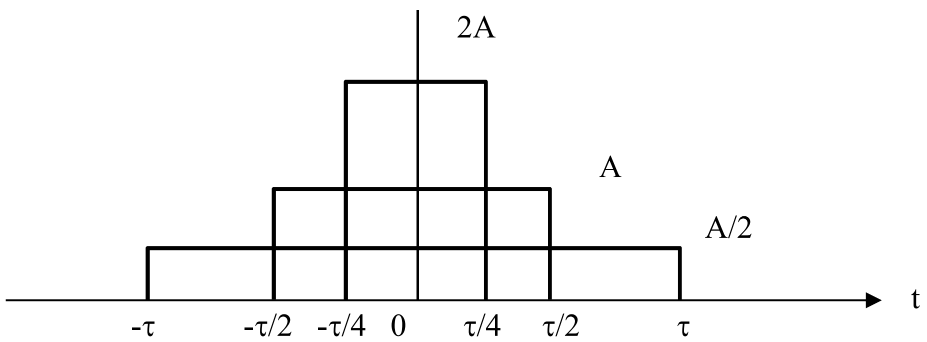
Taking Fourier transform,

$$\therefore \quad x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{jn\omega_0 t} \overset{FT}{\Leftrightarrow} X(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta(f - nf_0) \quad \text{where } f_0 = \frac{1}{T}$$



2.7 THE SPECTRUM OF AN IMPULSE

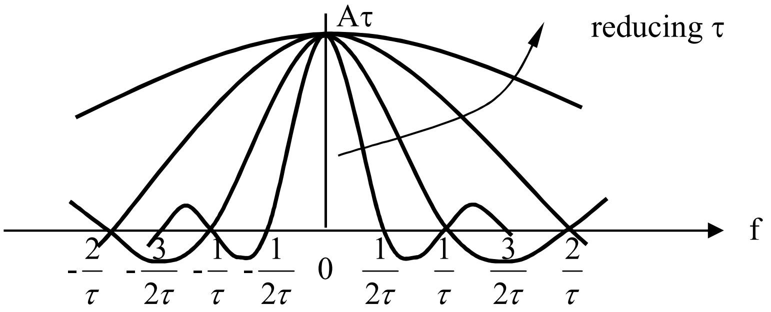
Consider a pulse of width 2τ and amplitude A/2. If the pulse width is reduced but the pulse area is held constant [Aτ] then the corresponding spectrum will be widen, that is 1/τ becomes bigger as τ becomes smaller.



Note that the area under the pulse remained the same, that is :

2A x τ/2 = A x τ = A/2 x 2τ = Aτ

Therefore, $X(f)$ at $f = 0$ remains constant and is equal to the area under the pulse, that is Aτ.



In the limit, as τ becomes very small, the pulse becomes an impulse of strength 'Aτ' and its associated spectrum becomes a <u>constant</u> of value 'Aτ' <u>at all frequency</u>.

Chapter 3

Noise

Learning Outcomes

- ❑ **Understand the characteristics of noise in communication systems.**
- ❑ Define electrical noise and describe the most common types of noise
- ❑ Identify noise sources.
- ❑ Describe the characteristics of thermal noise and its power spectrum density.
- ❑ Calculate noise voltage and noise power for thermal noise.
- ❑ Define the term SNR and identify the uses of SNR
- ❑ Define the term noise factor, F and identify the uses of noise factor.
- ❑ Compare noise performances of systems using SNR, F and Friiss' formulas.

INTRODUCTION

Noise is any unwanted electrical signal interfering with or distorting signal being communicated in a communication system. Noise corrupts a (wanted) signal during the process of transmission and reception and thus deteriorates the performance of the systems. Noise is usually random in nature and gets to add on a signal at a transmitter, in the communication channel and at a receiver. But, the noise present in the communication channel and at the input of a receiver is of major concern as it interferes with the proper reception or reproduction of the wanted signal at the receiver. Noise may be produced internally by sources within a communication system or it may be picked up by a communication system from external sources. It is important to study the characteristics of noise and take possible measure in the design of a communication system to reduce its effect.

3.1 NOISE SOURCES AND CLASSIFICATION

Noise may be broadly classified into two types: **internal noise** and **external noise** based on it source of generation as shown in Figure 3.1. The terms, external and internal are used with reference to the receiver. External noise produced by external sources like lightning discharge, fluorescent lights and electric motors may be further divided into **atmospheric noise**, **space noise** and **man-made noise**. Internal noise produced by noisy components within a system like transistors and resistors is divided into two types: **conductor noise** and **semiconductor noise**.

Atmospheric noise, space noise and some man-made noise travel as radio wave and are picked up by communication systems through electromagnetic induction inducing undesired current to interfere with the signals in communication systems. Noise originated from electromagnetic radiations is known as **radiated interference**.

Internal noise and some man-made noise are conducted into a communication system causing undesired voltage or current to interfere with a system. Noise of this origin is known as **conducted interference**.

3.1.1 External Noise

Atmospheric noise (Radiated interference)

Such noise is produced by lightning and other natural electrical disturbances in the atmosphere. They take the form of electromagnetic energy and spreads over a wide range of frequencies. However at frequencies of 30 MHz and above, the energy produced tends to be small.

When lightning strikes, a single current pulse I, flows through the air and produces an **electromagnetic (EM)** wave as shown in Figure 3.2 (a) & (b). The EM wave which originates from the current pulse has the same spectrum as that of a pulse (Figure 3.2 (c)).

When the EM wave cuts a conductor in an equipment, it induces a current pulse that flows in that conductor and this may interfere with the operation of that equipment. For example, lightning may induce a current pulse in telephone lines and damage modems connected to the lines.

Some of the frequency components in the EM wave may interfere with radio reception, causing a cracking sound on an AM radio, or jagged horizontal lines to appear on a TV screen. It may also trigger a car alarm system.

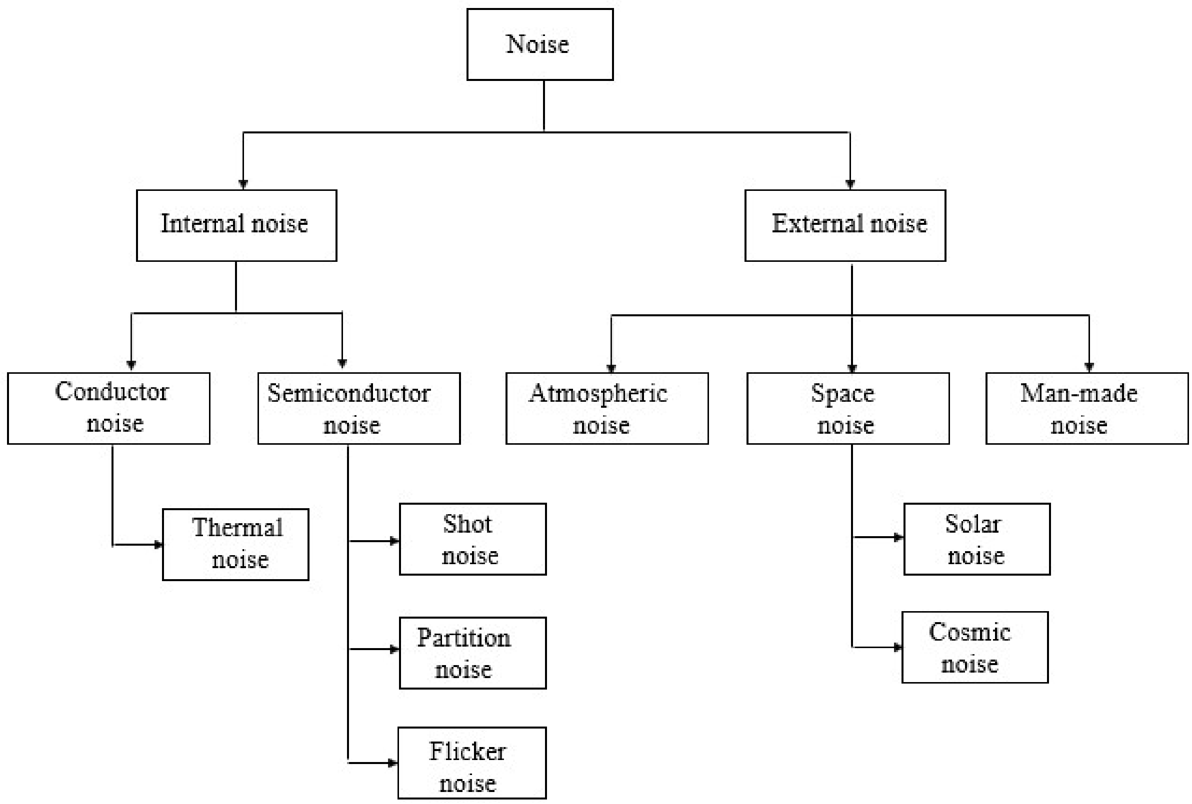


Figure 3.1 noise sources

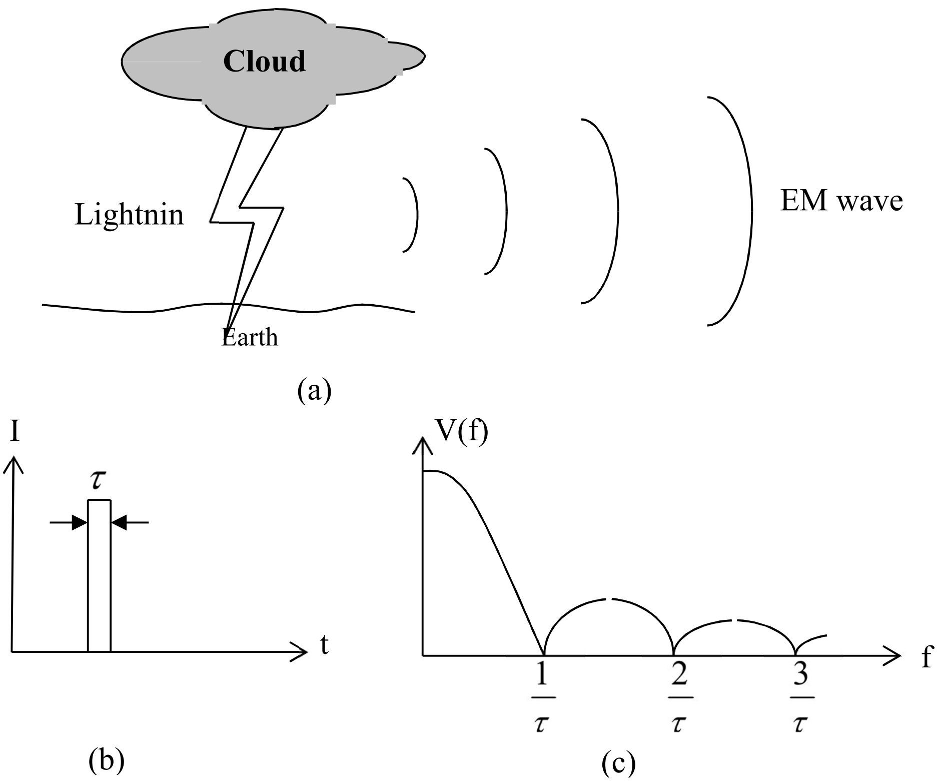


Figure 3.2 Radiated interference caused by lightning

Space noise (Radiated interference)

Such noise is produced by the sun (solar noise) and the stars (cosmic noise).

(i) Solar noise

Solar noise is in the form of electromagnetic energy which is radiated by the sun. It spreads over a very wide spectrum of frequencies. During intense solar activities like solar flares and sunspots, its energy increases and may be strong enough to jam the reception of some communication equipment.

(ii) Cosmic noise

Stars also radiate a wide spectrum of electromagnetic energy which interfere with the proper reception of radio signals. Although they are very remote, the noise energy is in no way small since there is a very large number of them.

Man-made noise (Conducted or radiated interference)

Man made noise is noise from electrical machines or equipment made by man. It may be conducted or radiated interference.

Radiated Man-made Interference

(i) Clock signals in Digital equipment

All digital devices like computers, contain clock signals which are rectangular or square waves. These clock signals consist of fundamental and harmonic components that generate electromagnetic wave when they flow through the conductors in circuit boards as shown in Figure 3.3. Some of these frequency components may interfere with the operation of communication equipment. This is the reason why digital devices like notebook, gameboy, and CD players must be switched off on board aeroplanes. Emission from the clock signals inside these devices may cause erroneous operation in the plane navigation system.

(ii) Interruption of current in electrical machines

Radiated Man–made noise is also caused by current pulses in electrical machines like car ignition systems, electric motors, air conditioners, fluorescent lights, hair dryers and refrigerators. These pulses are caused by interruptions of current in them. Rapid and large changes in voltages or currents tend to produce a large number of frequency components. These frequency components will radiate out from the electrical machine as an EM wave and affect radio reception.

Conducted Man-made Interference

(i) Noise generated by one equipment may flow through the mains wire to the mains socket of another equipment as shown in Figure 3.4. Noise spread in this way is known as conducted interference.

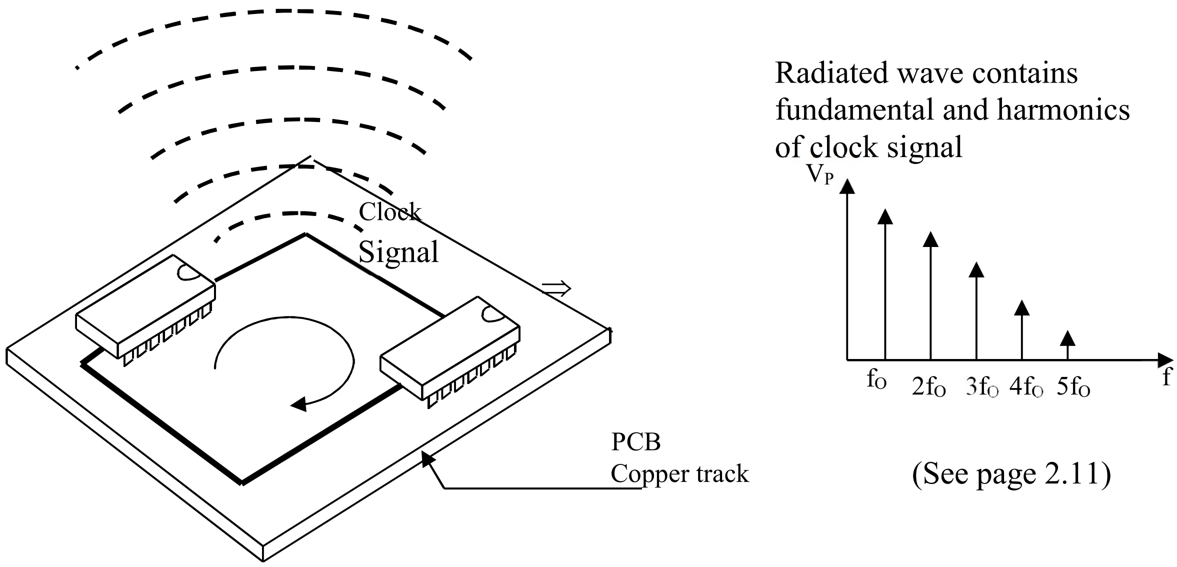
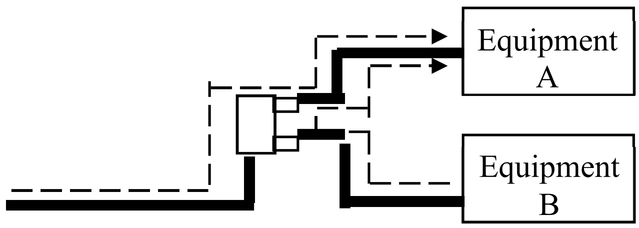
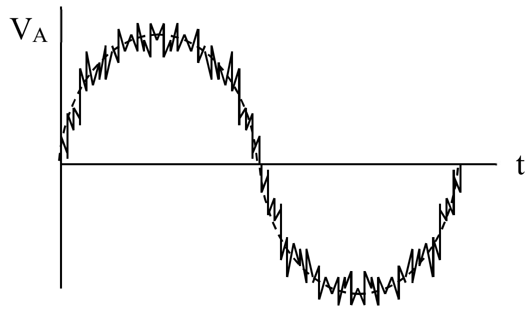


Figure 3.3 Radiated interference from a digital circuit.



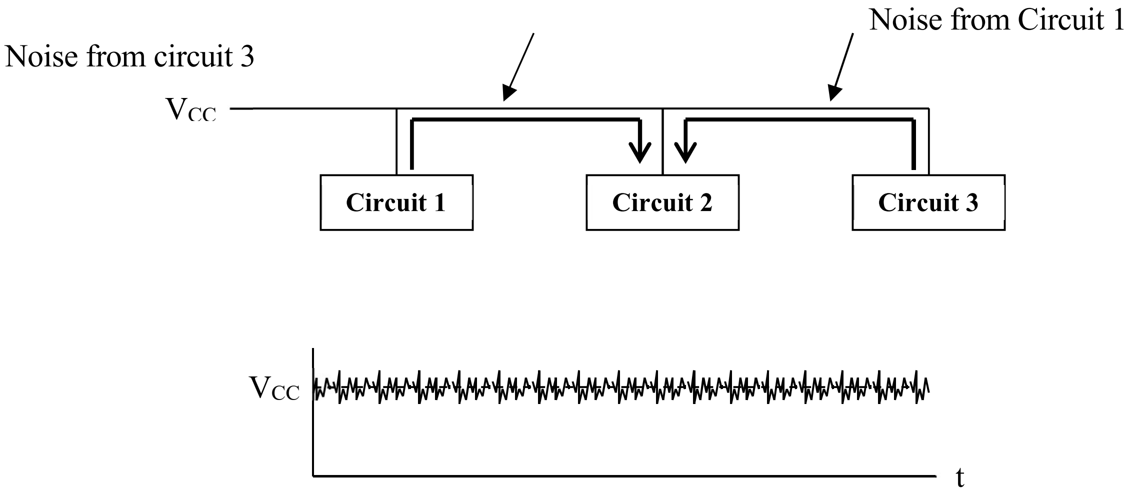
AC power line



Noise riding on 50 Hz mains supply.

Figure 3.4 Conducted interference

(ii) Similarly, noise generated by one circuit may flow through the dc power supply line to another circuit in a printed circuit board as shown in Figure 3.5.



Noise riding on DC supply voltage at circuit 2

Figure 3.5 Conducted interference

3.1.2 Internal noise

These are noise generated by components or devices present within the communication systems.

Noise in conductors: Thermal noise (Conducted interference)

This type of noise is generated in any conductor and is due to the random motion of free electrons within the conductor. As these electrons move randomly across the finite resistance of the conductor, a small (noise) voltage is generated (see Figure 3.6(a)). Although the average or dc value of such voltages is zero, they do have a finite rms value.

The random motion of free electrons increases with temperature and thus does the noise voltage.

As thermal noise is due to random motion of electrons, thermal noise is non-periodic (see Figure 3.6(a)) and therefore, the noise power, P_n has a continuous spectrum.

Studies have shown that the power spectral density of thermal noise, P(f), defined as noise power per unit bandwidth, has a uniform spectrum as shown in Figure 3.6(b). P(f) is proportional to temperature, T, and independent of frequency. Thermal noise is often referred to as '**white noise**' because of its uniform 'spectral density'. And, the power spectral density of thermal noise is given by:

$$P(f) = \frac{\eta}{2} = \frac{kT}{2} \qquad \text{Watts/Hz} \tag{3.1}$$

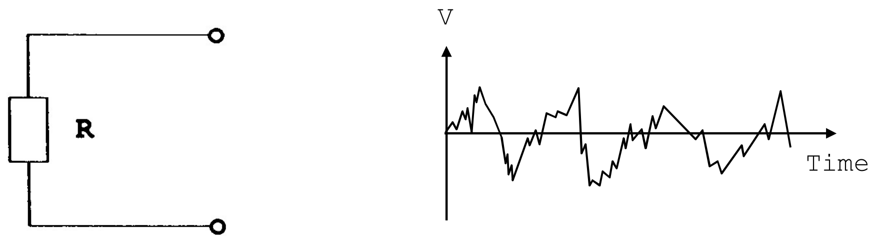
where k = Boltzmann's constant = 1.38×10^{-23} J/K
and T is the temperature of the conductor in Kelvin.

Note: The factor 2 in the denominator on the right hand side of eq. (3.1) is for double-sided power spectral density.

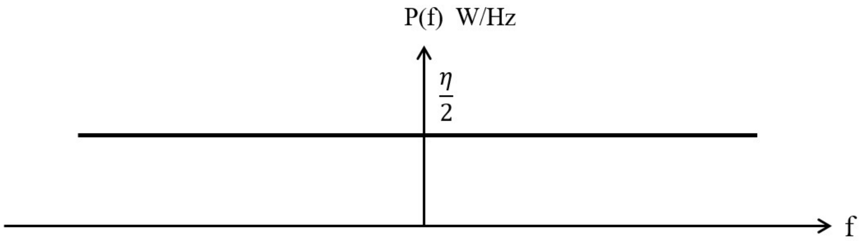
The thermal noise power, P_n over a bandwidth, B is given by

$$P_n = kTB \text{ Watts} \tag{3.2}$$

Where k = Boltzmann's constant (1.38×10^{-23} J/K)
T = temperature (K) $\Rightarrow y°c = (273 + y)K$
B = bandwidth (Hz)



(a)



(b)

Figure 3.6 (a) Thermal noise waveform
(b) Power Spectral density of Thermal noise

It is interesting to note that the noise power is not a function of the resistance of the conductor. Thus all conductors, regardless of the resistance, generate the same amount of thermal noise power over the same bandwidth and temperature.

The rms noise voltage across a resistor of R ohms at a temperature of T K over a bandwidth of B is given by

$$E_n = \sqrt{4kTBR} \text{ volts} \tag{3.3}$$

The proof for eq. (3.3) is contained in Appendix 3.1

While it was noted in Eq. (3.1) that thermal noise power is independent of resistance, it should be borne in mind that thermal noise voltage increases with resistance as evident from eq. (3.3).

Therefore, for voltage circuits like operational amplifiers and voltage comparators, it is preferable to use low value resistors.

Noise in semiconductors (Conducted interference)

Examples of semiconductor noise are:

(i) Shot noise. Shot noise also has a uniform power spectral density.

(ii) Partition noise.

(iii) Low frequency or flicker noise.
The low frequency components in the spectrum of this type of noise have high amplitudes while the high frequency components have low amplitudes. Hence the name "low frequency" noise.

Example 3.1

The temperature of a 10kΩ resistor is 20°C.
(a) Draw the spectrum of the noise produced by the resistor.
(b) Calculate the noise power and rms noise voltage over a 15kHz bandwidth.

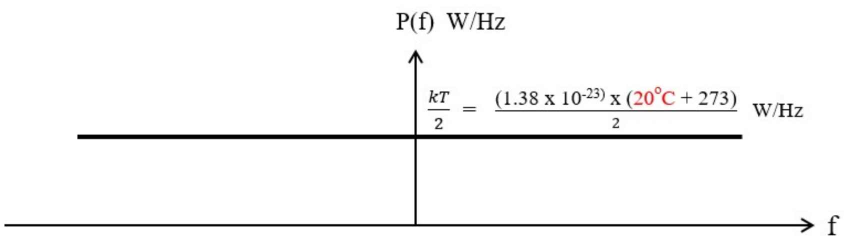
Solution

(a)

$$P(f) = \frac{\eta}{2} = \frac{kT}{2} \quad \text{Watts/Hz}$$

$$= \frac{(1.38 \times 10^{-23}) \times (20^{\circ}C + 273)}{2}$$

$$= 2.02 \times 10^{-21} \text{ Watts/Hz}$$



(b)

$$P_n = kTB \text{ Watts}$$

$$= (1.38 \times 10^{-23}) \times (20^{\circ}C + 273) \times (15 \times 10^{3}) \text{ W}$$

$$= 6.06 \times 10^{-17} \text{ W}$$

$$E_n = \sqrt{4kTBR} = \sqrt{4P_nR}$$

$$= \sqrt{4 \times (6.06 \times 10^{-17}) \times (10 \times 10^3)}$$

$$= 1.56\ \mu V$$

Although P_n and E_n are small, they may be of the same order as the signal power or voltage at the input of a radio receiver. So P_n and E_n cannot be ignored.

3.2 NOISE EXPRESSION

Noise expressions are used to quantify or express the noise performance of a device, circuit or communication system. Common noise expressions used are listed below:

1. Signal to Noise ratio, SNR
2. Noise factor, F and noise figure
3. Noise Temperature (not in syllabus)

3.2.1 Signal to noise ratio (SNR)

Signal to noise ratio (SNR) defines the ratio of signal power to the noise power at a specific point in a communication system. Thus,

$$SNR = \frac{\text{Signal Power}, P_S \text{ at a point in a communication system}}{\text{Noise Power}, P_n \text{ at the same point}} = \frac{P_S}{P_n} \qquad (3.4)$$

SNR is more often defined in decibels, i.e.

$$SNR(\text{in dB}) = 10 \log SNR \qquad (3.5)$$

The noisiness of a signal is not determined by how much noise it contains but rather the amplitude of this noise <u>compared</u> with the signal amplitude. This is illustrated in Figure 3.7, where the noise amplitude is the same in both (c) and (f) but its effect is more serious in (f) because of the lower signal amplitude in (d) compared to that in (a). Hence, SNR and not noise power is the correct indicator of signal noisiness.

<u>Use of SNR</u>

SNR tells us how noisy a signal is. The noisier the signal, the lower the SNR (See example 3.2(a)).

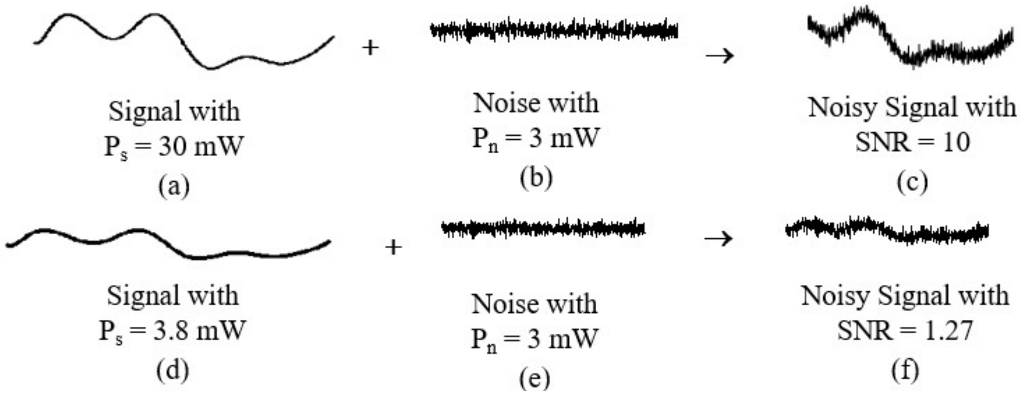


Figure 3.7 If the signal is weak (d), the noise effect will be serious (f); if the signal is stronger (a), the noise effect will not be as serious (c). A high SNR means $P_S >> P_n$. Hence signal (c) is less noisy than (f).

3.3.2 Noise Factor and noise figure

The **Noise Factor** F of an amplifier (or any circuit) is defined as follows:

$$F = \frac{\text{SNR at input of amplifier or any circuit}}{\text{SNR at output of the same amplifier or circuit}} \qquad (3.6)$$

$$= \frac{\mathbf{SNR_i}}{\mathbf{SNR_O}} \quad \text{(with } P_{ni} \text{ set at } kT_oB \text{ watts, where } T_o = 290\text{K)}$$

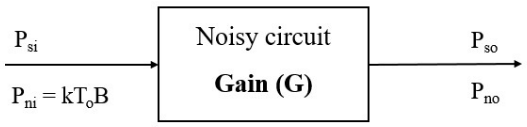


Figure 3.8 A noisy circuit

When measuring noise factor, the input noise power, P_{ni} must be set to a standard value equal to kT_0B watts (See Appendix 3.2 to find out why this must be set to this standard value).

The SNR at the output will always be less than the SNR at the input since any amplifier or circuit will add noise to the incoming signal. Consequently,

- amplifying a signal does not make the signal less noisy, and
- $F > 1$, in practice

  (See Appendix 3.2 for a full explanation of why $F > 1$)

When noise factor is expressed in decibels, it is referred as **Noise Figure**:

$$\text{Noise Figure (NF)} = 10 \log_{10} F \text{ dB} \qquad (3.7)$$

Use of Noise Factor

Tells us how noisy a circuit is. The noisier the circuit, the larger the noise factor. (See example 3.2 (d)).

Example 3.2

With the available noise power at input standardised at kT_0B, The measurements performed on Amplifiers A and B produce the following results:

Amplifier A	-	SNR at input = 40 SNR at output = 10
Amplifier B	-	SNR at input = 75 SNR at output = 15

(a) Which output signal is noisier?
(b) Why does SNR reduces as the signal travels from the input to the output of each amplifier?
(c) Does amplifying a signal make the signal less noisy?
(d) If all the SNR values were measured with $P_{ni} = kT_oB$ which amplifier is noisier?

Solution

(a) Output from amplifier A is noisier because SNR_o is lower.
(b) The signal picks up noise from the amplifier as it travels through it.
(c) No. In fact amplifying a signal makes it noisier (due to part b).
(d) Since $P_{ni} = kT_oB$, the formula $F = \dfrac{SNR_i}{SNR_O}$ can be used.

$$\therefore \; F_A = \frac{40}{10} = 4$$

$$F_B = \frac{75}{15} = 5$$

$$\because \; F_B > F_A \text{ Amplifier B is noisier.}$$

Note:

- Output signal being more noisy does not always mean that the amplifier is more noisy (Compare the answers in part a and part d). The output signal could be noisy because the input signal was noisy.

- High F means $SNR_o << SNR_i$

  i.e. output is noisier than the input
  i.e. Signal picks up more noise from the amplifier as it travels through it.
  i.e. Amplifier is noisy.

3.3 TOTAL NOISE FACTOR OF CASCADED CIRCUITS

For n-cascaded circuits shown in Figure 3.9, the total noise factor of the cascaded circuit can be determined by using Friiss' formula:

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \ldots + \frac{F_n - 1}{G_1 G_2 G_3 \ldots G_{(n-1)}} \qquad (3.8)$$

When using the Friiss' formula, all power gains must be expressed in linear units and noise factor (not noise figure) must be used. The resultant F_t is the noise factor, not the noise figure.

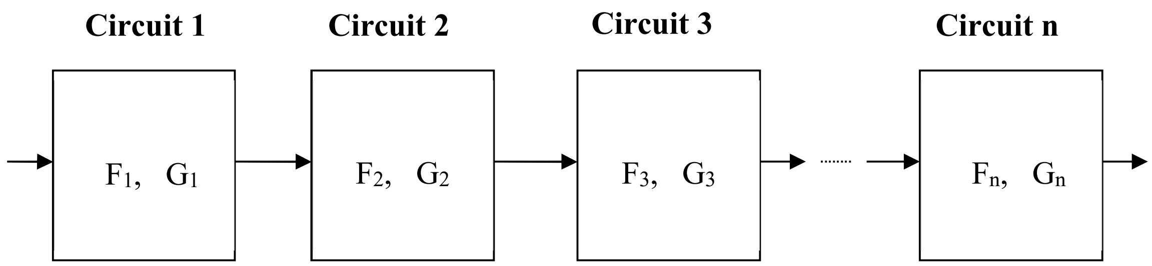


Figure 3.9 n Circuits in cascade

Where G is the Power Gain. i.e.

$$G_1 = \frac{\text{Signal power at the output of Circuit1}}{\text{Signal power at the input of Circuit1}}$$

$$G_2 = \frac{\text{Signal power at the output of Circuit2}}{\text{Signal power at the input of Circuit2}}$$

etc.

3.4 IMPROVEMENT OF OVERALL NOISE FACTOR

Amplifiers are used in communication systems to amplify weak signals. However these amplifiers also amplify the noise, as well as add noise to the system. Thus the SNR at the amplifier output is always less than the SNR at its input. This results in an overall degradation in the noise performance of a communication system. Such degradation can be minimised by choose an appropriate amplifier in the first stage.

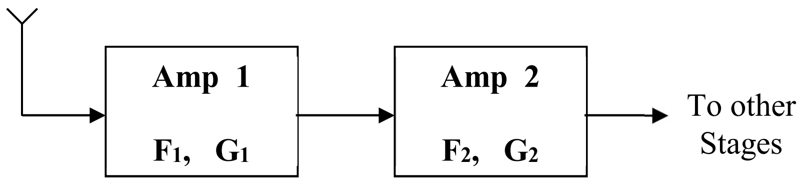
(i) As can be seen from eq. (3.8), the noise contribution of the second and subsequent stages are reduced by the power gain G_1 of first stage. If G_1 is quite large, the overall noise factor F of a cascade connection will be approximately equal to the noise factor of the first stage F_1 in a cascade connection.

(ii) To achieve a low overall noise factor of a cascade connection, it is necessary to choose a first stage with high power gain G_1 and low noise factor F_1.

Thus, a low noise factor and high power gain amplifier should be used as first stage in order to minimise the degradation of noise performance in communication systems.

Example 3.3

Figure below is the front end of a radio receiver consisting 2 amplifiers to amplify the weak signal received by the antenna. Show that an amplifier 1 with high power gain and low noise factor is necessary in order to achieve a low overall noise factor F of the two cascade amplifier connection.



Solution

The overall noise factor Ft of the two amplifiers is given by the Friiss' formula,

$$F_t = F_1 + \frac{F_2 - 1}{G_1}$$

$$F_t > F_1$$

$$F_{t(min)} = F_1$$

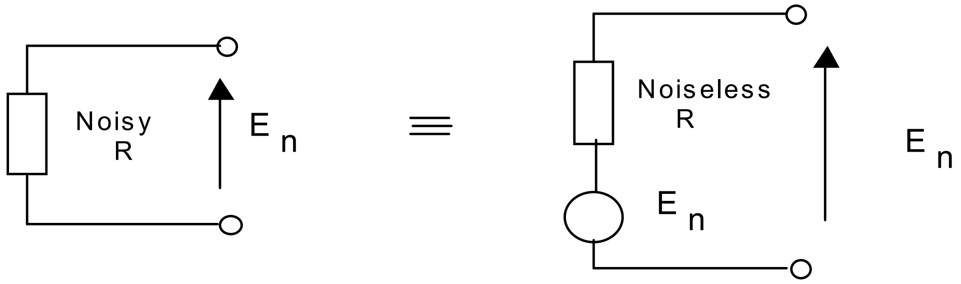
If $G_1 >> (F_2 - 1)$ then, $F_t \approx F_1 = F_t(min)$

If F_i is low, then F_t will also be low.

Appendix 3.1

RMS noise voltage

We may use Thevenin's theorem to represent the noise voltage produced by a noisy resistor as shown below.



E_n = rms noise voltage produced across its terminals.

Fig: Thevenin's equivalent of a noisy resistor.

The value of E_n is derived assuming matched condition, i.e. when all the noise power from the resistor is transferred to its load (maximum power transfer). This is usually the case in most communication systems where the output of one stage is usually matched to the input of the next stage so that signal power is effectively transferred through the system.

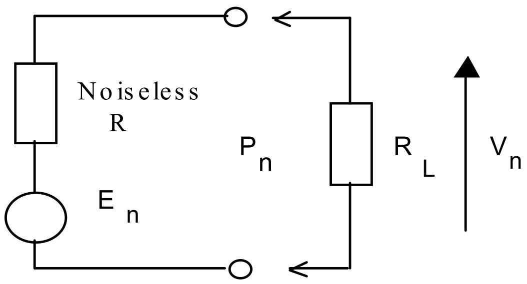


Fig: Calculation of E_n under matched condition

Under matched condition, $R_L = R$. Hence all the noise power, Pn is transferred to R_L.

From the above figure $P_n = \frac{V_n^2}{R_L} = \frac{V_n^2}{R}$

Where V_n = rms noise voltage across R_L.

Now, $V_n = \frac{1}{2} E_n$

Therefore $P_n = \frac{E_n^{\ 2}}{4R}$ i.e. $E_n^{\ 2} = 4RP_n$

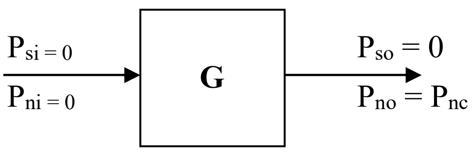
From equation 4.1 , $P_n = kTB$

$\therefore E_n^{\ 2} = 4kTBR$

i.e. $E_n = \sqrt{4kTBR}$

Appendix 3.2

Noise Factor

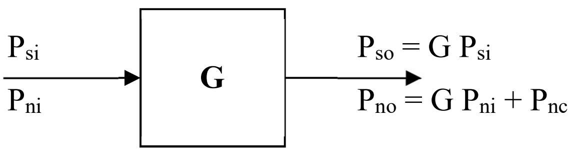


P_{nc} is the noise at the output of the circuit when input = 0.

This noise is produced by the circuit.

Amplifier will amplify both signal and noise

Power gain, G = $\frac{P_{SO}}{P_{Si}}$



Noise factor, $F = \frac{SNR_i}{SNR_o} = \frac{P_{si}}{P_{ni}} \times \frac{P_{no}}{P_{so}} = \frac{P_{si}}{P_{so}} \times \frac{P_{no}}{P_{ni}} = \frac{GP_{ni} + P_{nc}}{GP_{ni}} = 1 + \frac{P_{nc}}{GP_{ni}}$

F will have different values for different values of P_{ni}.

Hence P_{ni} is standardised at KT_oB where T_o = 290K.

Hence the formula, $F = \frac{SNR_i}{SNR_o}$ cannot be used unless $P_{ni} = kT_oB$

$$F = 1 + \frac{P_{nc}}{GP_{ni}}$$

If the amplifier circuit is noiseless, then $P_{nc} = 0$ and hence $F = 1$ (ideal case).

i.e. $SNR_i = SNR_o$ and the output is as noisy as the input.

However, in practice $P_{nc} \neq 0$ and hence $F > 1$.

i.e. $SNR_o < SNR_i$ and the output is noisier than the input.

Chapter 4

Signal Transmission

Learning Outcomes

- ❑ Describe the electromagnetic frequency spectrum.
- ❑ Describe classification of radio frequency bands and state the main users of each RF band.
- ❑ Describe briefly characteristics of transmission channels and transmission impairments
- ❑ Distinguish between baseband and passband transmission
- ❑ Define modulation and understand need of modulation.
- ❑ Calculate the minimum antenna length for effective transmission.
- ❑ Describe the types of modulation, viz. analog sinusoidal and digital modulations.

INTRODUCTION

There are many aspects involved in signal transmission which decide the characteristics and performance of a communication system. They mainly include the type of signals employed - analog or digital; the transmission channel signal propagates -wired or wireless; signal transmission method used - baseband or passband transmission. Some of those aspects are discussed in this chapter.

4.1 ELECTROMAGNETIC FREQUENCY SPECTRUM

Communication systems transmit information from one point to another in the form of electromagnetic energy. Electromagnetic energy includes voice/power, radio waves, infrared light, visible light, ultraviolet light, and X, gamma, and cosmic rays. All these constitute the electromagnetic spectrum (see Figure 4.1). Not all portions of the electromagnetic spectrum are currently usable for communications. The portions that are used for communications includes voice-band frequencies, radio frequencies, infrared light and visible light. Voice-band frequency signals are generally transmitted over metal cables, such as twisted-pair or coaxial cable. Radio frequency signals can travel through air or vaccum. Visible light is used for communications using fiber-optic cable.

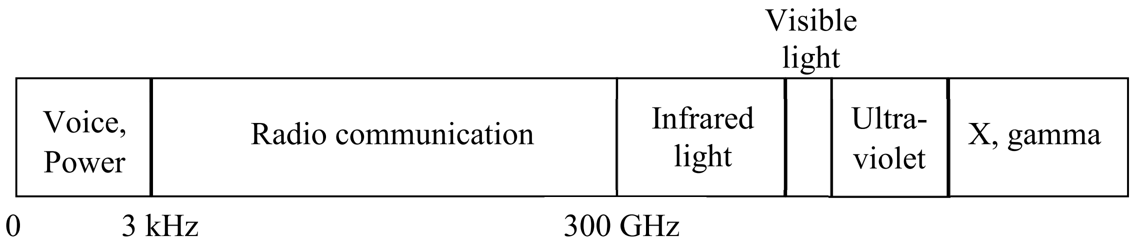


Figure 4.1 Electromagnetic spectrum

4.1.1 Radio frequency bands.

The portion of the electromagnetic spectrum used for radio transmission is divided into eight ranges, known as **radio frequency** (RF) bands. The RF bands are regulated by government authorities. They are rated from very low frequency (VLF) to extremely high frequency (EHF). Figure 4.2 shows all eight bands and their acronyms. The frequency range of the RF bands are given in Table 4.1.

Figure 4.3 shows the allocation of the radio communication services in the RF bands. These frequencies are allocated by the International Telecommunication Union (ITU).

VLF - Very Low Frequency
LF - Low Frequency
MF - Medium Frequency
HF - High Frequency
VHF - Very High Frequency
UHF - Ultra High Frequency
SHF - Super High Frequency
EHF - Extremely High Frequency

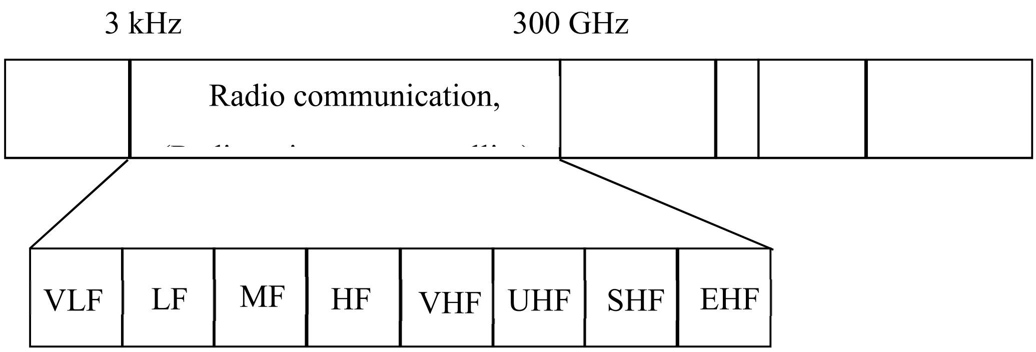


Figure 4.2 Radio frequency bands

Frequency Range	Name of Band	Other Names
30 kHz - 300 kHz	Low Frequency (LF)	Long Wave (LW)
300 kHz - 3 MHz	Medium Frequency (MF)	Medium Wave (MW)
3 MHz - 30 MHz	High Frequency (HF)	Short Wave (SW)
30 MHz - 300 MHz	Very High Frequency (VHF)	-
300 MHz - 3 GHz	Ultra High Frequency (UHF)	-
3 GHz - 30 GHz	Super High Frequency (SHF)	Microwave
30 GHz - 300 GHz	Extremely High Frequency (EHF)	Millimetre Wave
300 GHz - 3 THz	-	Infrared

Table 4.1 RF Bands

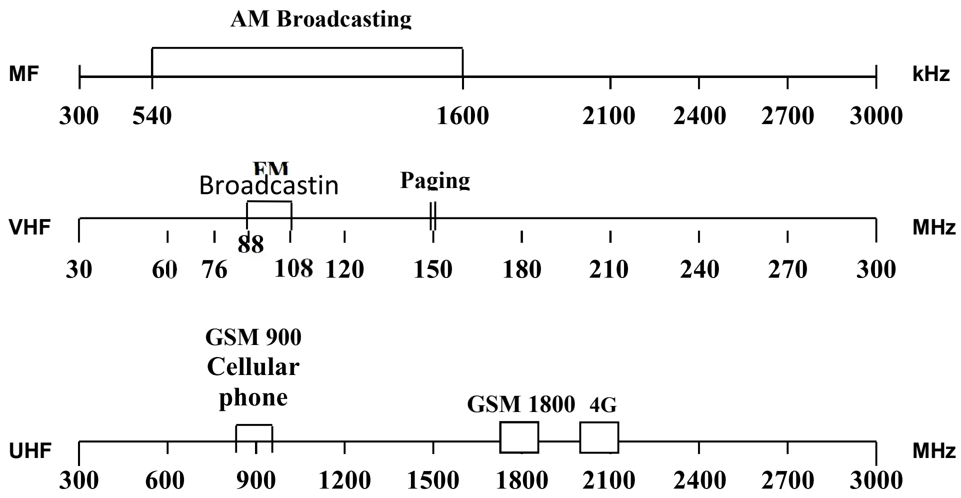


Figure 4.3 Common Users of Radio Frequencies

4.1.2 Relationship between frequency and wavelength

It is common to use the term wavelength rather than frequency when dealing with radio waves. Wavelength, **λ** is defined as the distance travelled by radio wave in **T** seconds where **T = 1/f**, is the period of the signal of frequency **f**. Thus, wavelength is determine by

$$\lambda = V_{(Prop)}/f \qquad (4.1)$$

where $\mathbf{V_{(Prop)}}$ is the velocity of radio wave.

When radio wave is sent down a transmission channel, it will travel at a certain velocity determined by the characteristic of the transmission medium. For example, radio wave in free space (Vacuum) travels at the speed of light, $\mathbf{c = 3 \times 10^8}$ **m/s** while signals in air and cables tend to travel slower.

Example 4.1

(a) A radio station is transmitting at 20 MHz. What is the wavelength of this transmission?

(b) A 20 MHz signal travels along a certain length of co-axial cable at 2×10^8 m/s. What is the wavelength of this transmission?

Solution

(a) Assume, $V_{(Prop)} = c = 3 \times 10^8$ m/s, for simplicity

$$\therefore \quad \lambda = V_{(Prop)}/f = c/f$$

$$= \frac{3 \times 10^8}{20 \times 10^6}$$

$= 15$ m

(b) $V_{(Prop)} = 2 \times 10^8$ m/s,

$\therefore \lambda = V_{(Prop)} / f$

$$= \frac{2 \times 10^8}{20 \times 10^6}$$

$= 10$ m

4.2 TRANSMISSION CHANNELS

The transmission channels connect the transmitters and receivers and may take a variety of forms. In some communication systems, there exist a physical connection between the transmitters and receivers. In some other systems, no physical connectivity is present. The most commonly employed channels can be divided into two broad categories: wired and wireless channels.

4.2.1 Wired channels

Wired channel includes twisted-pair, coaxial cable, fiber-optic cable and a combination of them.

- Twisted-pair wires are used for speech transmission such as voice transmission in telephone networks, where twisted-pair wires are used to connect subscribers and the local exchange. Digital data from a computer can also be transmitted over such channel. Twisted-pair wires can provide a bandwidth of a few 100 kHz. But they suffer from cross-talk and their attenuation sharply increases with frequencies above 100 kHz.

- Coaxial cable is used for sending signals of frequency ranging from a few 100 kHz to about 1 GHz. Some popular use of coaxial cable includes distributing television signals in cable TV systems, data transmission in LANs.

- An optical fiber is a dielectric waveguide that transports light signals. Information is transmitted by varying the intensity of the light source with the message signal. Optical fibers provides very large bandwidth, very little attenuation and immunity to interferences and induced noise. Optical fiber transmission systems are widely deployed in backbone networks.

4.2.2 Wireless channels

Wireless or radio transmission channels transport electromagnetic waves without using a physical conductor. Information bearing radio wave propagates through the air or vacuum. Depending on frequency, radio wave propagates differently through the air. It may travel as a surface wave (below 3 MHz), sky wave (3MHz to 30MHz) or space wave (above 30MHz).

Wireless channel has both advantage and disadvantage. The most important advantage is that it is suitable for signal broadcasting. Mobile communication becomes possible using wireless communication channel. The main disadvantages of wireless channel includes channel characteristics that are highly dependent on transmission frequency, channel noise and limited allocation of available bandwidth.

4.2.3 Transmission impairments

Transmission channels are not perfect and they cause impairment to the signal sent through them. This means that the signal at the sending end and receiving end of the channel are not the same. There are mainly four types of transmission impairments, namely attenuation, distortion, noise and interference.

- Attenuation is the loss of transmission signal strength as the signal travels through the channel. The longer the transmission distance, the more the amount of attenuation. For wired channels, attenuation has an exponential dependence on distance, i.e. the attenuation in dB increases linearly with the distance, whereas for wireless channels, the attenuation in dB increases logarithmically with the distance.

- Distortion takes place when signal changes its form or shape after going through a transmission channel. Distortion may be linear or nonlinear.

  There are two types of linear distortion: amplitude distortion and phase distortion. When the amplitude response of the channel is not constant (flat), the result is amplitude distortion. When the phase response of the channel is not linear (i.e. different frequency components suffer different amount of delay), the result is phase (delay) distortion.

  Nonlinear distortion occurs when the relationship between the transmitted signal and received signal is not linear.

- Noise is the unwanted and unavoidable random waves that tend to disturb signal transmission which can be added by the channel or by the receiver. Sources of noise include the thermal noise present in any conductors, noise from semiconductor devices, noise from lighting and from the sun (refer to Chapter 3). Noise is usually assumed to have a Gaussian distribution and uniform spectral density, and add to signals, which is known as additive white Gaussian noise (AWGN).

  Noise limits the performance of communication system. It causes errors in digital signals or degrade the quality of analog signals.

- Interference refers to random man-made signals that appear at the receiver from sources other than its own transmitter. It may be caused by other communication systems or other electrical devices. Interference alters, modifies or disrupts a signal as it travels along a channel.

  Wireless communication systems are particularly vulnerable to interference because of the wide difference in the transmitted and received signal levels.

4.2.4 Channel models

The impairments of communication channel cause amplitude and phase variations in a signal that travels through. For analysis and design of communication systems, channels are represented by system models into which the most important characteristics of channel is incorporated. Generally, communication channels are modelled as filters as shown in Figure 4.4, with additive noise, n(t). Their parameters may be deterministic or random, time-invariant or variant, linear or nonlinear. In this module, we assume the communication channel is linear AWGN channel.

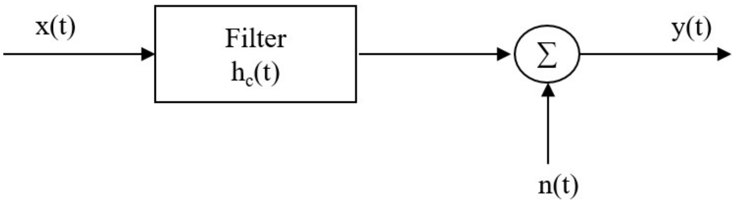


Figure 4.4 Communication channel model

4.3 BASEBAND AND PASSBAND SIGNAL TRANSMISSION

Signals generated by the information sources/the input transducers are known as **baseband signal**. Baseband signals have frequencies relatively close to zero. For example, the frequencies of natural speech range from about 100 Hz to 5 kHz and video signal from a TV camera has a frequency range of 0 Hz to 5MHz.

When baseband signals are directly transmitted over transmission channels as shown in Figure 4.4, it is known as **baseband signal transmission**. It may be used for transmitting either analog signals (e.g, voice or music signals) or digital signals (e.g, data signals generated by a computer). It is preferred for low frequency and short distance dedicated communication usually over wired channels like twisted pair wires, coaxial cable or other wirelines.

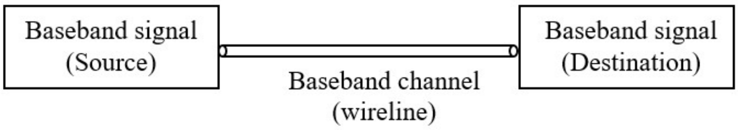


Figure 4.5 Baseband signal transmission

Baseband signal transmission is not suitable for wireless communication (due to impractical antenna size explained in section 4.5), long-distance communications or shared transmission channel by multiple users. For those cases, passband signal transmission is used. Passband signal transmission shifts the baseband signal to a radio frequency (RF), prior transmission. It is done by impressing a baseband signal upon a high-frequency sinusoidal signal by a modulator as shown in Figure 4.6. The resultant signal is known passband signal. At the receiver, the Passband signal is shifted back to baseband (original frequency) by a demodulator to recover the information.

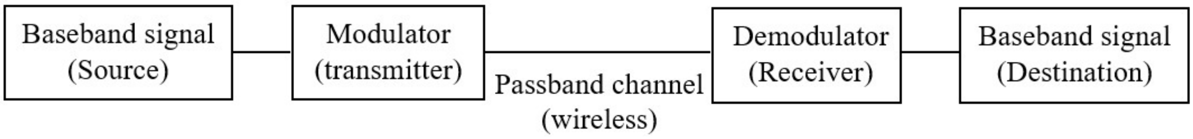


Figure 4.6 Passband transmission

Thus, baseband signal transmission sends the information signal as it is without modulation (frequency shifting) while passband signal transmission shifts a baseband signal to a higher frequency through modulation.

4.4 MODULATION

The process of impressing a baseband signal (information signal) onto a high frequency sinusoid is known as **modulation**. It is achieved by varying the characteristics, i.e. amplitude, phase or frequency, of a high frequency sinusoid in accordance with a baseband signal.

Modulation is implemented using a circuit called modulator. As shown in Figure 4.7, a modulator combines the low frequency baseband signal with a high frequency sinusoid to produce a high frequency modulated signal known as passband signal. The high frequency sinusoid is known as **carrier signal** or **carrier**. The baseband signal (information signal) that we want to send at the input of the modulator is known as **modulating signal**. The high frequency passband signal at the output of the modulator is known as **modulated signal** or modulated carrier.

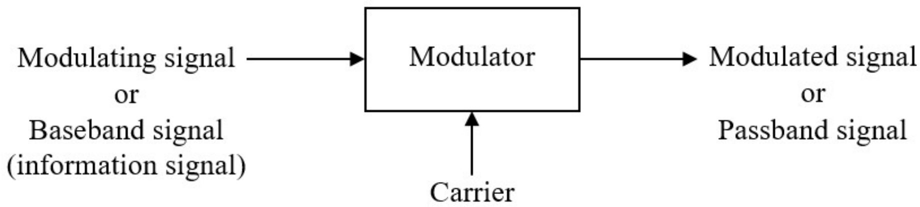


Figure 4.7 The Modulation Process

The value of the carrier frequency will determine the frequency band the modulating signal will be shifted to. For example, in "Class 95 FM", a 95.0 MHz carrier is used to shift the frequency of the modulating signal to a higher frequency band centred around 95.0 MHz.

4.5 THE NEED FOR MODULATION

4.5.1 Modulation for Practical Antenna Length

Efficient transmission refers to the ability of the waves to travel very far without the need for high transmission power. Generally, efficient radio transmission requires antenna length to be at least one-tenth of the signal wavelength, i.e. antenna length $\geq 0.1\lambda$. Thus, the antenna length is inversely proportional to the frequency of the transmitted signal. The higher the signal frequency, the shorter is the antenna required for efficient transmission.

The **minimum antenna length l_{min}** for efficient transmission and good reception is

$$l_{min} = 0.1\lambda \qquad (4.2)$$

If a signal of frequency 100 Hz were to be propagated, then the minimum antenna length required will be

$$\begin{aligned} \text{minimum antenna length} = 0.1\,\lambda &= 0.1 \text{ x } c/f \\ \text{for 100 Hz} \quad &= 0.1 \text{ x } 3 \text{ x } 10^8/100 \text{ m} \\ &= 300 \text{ km} \end{aligned}$$

Obviously, a 300 km antenna length is impractical. If the 100 Hz signal is shifted to a 100 MHz, then

$$\begin{aligned} \text{minimum antenna length} \quad &= 0.1 \text{ x } 3 \text{ x } 10^8/100 \text{ x } 10^6 \text{ m} \\ \text{for 100 MHz} \quad &= 0.3 \text{ m} \end{aligned}$$

The 0.3 m antenna length is a huge reduction and is practical to construct and use.

4.5.2 Modulation for Simultaneous Transmission without Interference

When two or more stations broadcast, say, speech at the same time within hearing distance without modulation, it impossible for one to listen to one station without hearing the other stations. This is because all the speech signals occupy the same frequency band ranging from 100 Hz to 10 kHz. Hearing multiple stations at the same time is known as **co-channel interference**, or cross talk.

However, if each station is modulated onto a different carrier frequency, the modulated signal can then be broadcast simultaneously without interference with one another. The listener can then, by band selection, tune to the station of his choice without hearing the other stations.

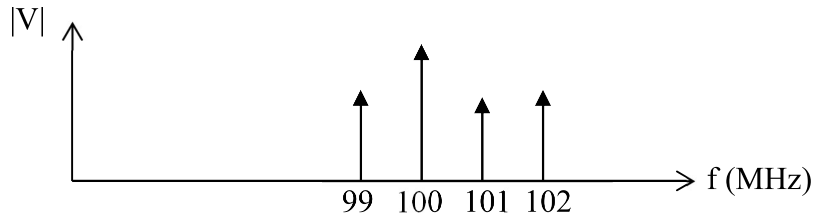
Modulation therefore allows simultaneous transmission of music, video or data signals without interference by the use of different carrier frequencies.

4.5.3 Modulation to Reduce the Effects of Noise

By using certain types of modulation (such as FM) it is possible to minimise the effects of noise. This will be covered in greater details in Chapter 6.

Example 4.2

1. A signal with the following amplitude spectrum is to be transmitted. What is the minimum antenna length required for efficient transmission?



Solution

For 99MHz, min. length required = L_1 = 0.1c/99MHz = 0.303m

For 102MHz, min. length required = L_2 = 0.1c/102MHz = 0.294m

For 100 & 101MHz, min. length required will be between L_1 and L_2.

L_1 should be chosen because it will be long enough for ALL the 4 frequency components.

L_2 will be too short for the 99, 100 & 101MHz component.

4.6 TYPES OF MODULATION

There are many ways to achieve modulation. The modulation techniques can be grouped into two categories

(a) Analog Modulation.
(b) Digital Modulation.

4.6.1 Analog Modulation

In analog modulation, the modulating signal is analog and the carrier is sinusoid. There are three basic analog modulation techniques used in analog communication systems:

- Amplitude modulation (**AM**)
- Phase modulation (**PM**)
- Frequency modulation (**FM**)

In AM, the peak amplitude of the carrier is varied in accordance with the instantaneous amplitude of the modulating signal as shown in Figure 4.8(a).

In FM, the frequency of the carrier is varied in accordance with the instantaneous amplitude of the modulating signal as shown in Figure 4.8(b).

In PM, the relative phase of the carrier is varied in accordance with the instantaneous amplitude of the modulating signal as shown in Figure 4.8(c).

AM is mainly used for radio broadcasting in the medium and shortwave band. AM is covered in details in Chapter 5. FM is mainly used for radio broadcasting in the VHF band as well as in cordless phones and walkie-talkies. The details of FM is covered in Chapter 6. PM is not covered in this module.

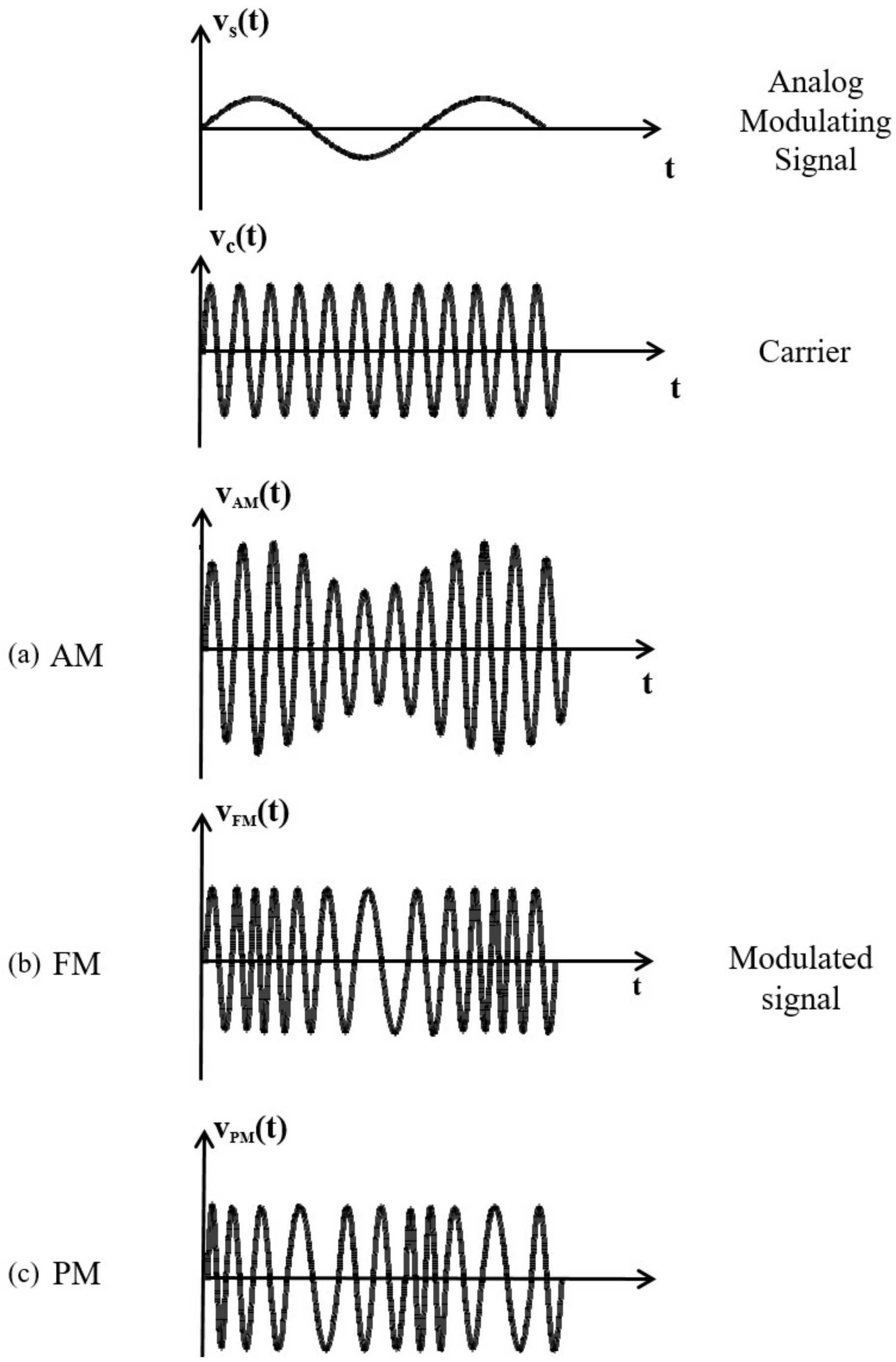


Figure 4.8 Basic analog modulation techniques

4.6.2 Digital Modulation

In digital modulation, the modulating signal is digital (having two or more discrete levels) and the carrier is sinusoid. The amplitude, frequency or phase of the sinusoidal carrier is varied in accordance with the logic states of the modulating signal. Three are three basic digital modulation techniques used in digital communication systems:

- Amplitude-shift keying (ASK)
- Frequency-shift keying (FSK)
- Phase-shift keying (ASK)

For binary digital information, the three basic digital modulation techniques are as follows:

ASK - the carrier is turned on and off so that logic 1 may be represented by the presence of carrier while logic 0 is represented by the absence of carrier shown in Figure 4.9(a).

FSK - the two logic states are represented by two different carrier frequencies shown in Figure 4.9(b).

PSK - one phase of the carrier is used to represent one binary state, and a second phase (usually 180^o apart) is used for the second state shown in Figure 4.9(c).

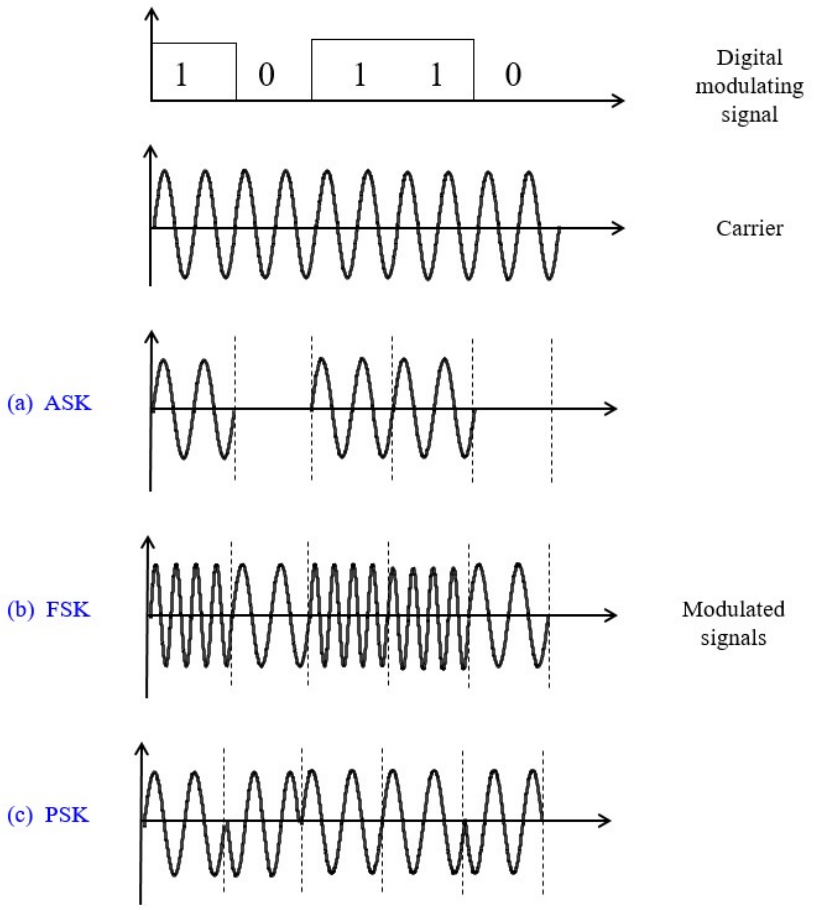


Figure 4.9 Basic digital Modulation techniques

Chapter 5

Amplitude Modulation

Learning Outcomes

- **Apply the time domain and frequency domain knowledge about AM.**
- Describe how information is carried in AM signals and write the mathematical expression for AM signals.
- Draw the block diagram of an AM modulator.
- Calculate modulation index, m and explain its significance.
- Explain AM demodulation using envelope detector.
- Explain the meaning and effect of over-modulation.
- Sketch the frequency spectrum of a single-tone AM signal.
- Sketch the frequency spectrum of multi-tone AM signal.
- Determine the bandwidth of an AM signal.
- Explain the evolution of DSBSC from AM and write the equation for a DSBSC signal.
- Explain the evolution of SSB from DSBSC modulation.
- Compare AM, DSBSC and SSB in terms of power saving, bandwidth saving and circuitry complexity.
- List the applications of AM, DSBSC and SSB.

INTRODUCTION

Amplitude modulation (AM) is the process of varying the amplitude of a high frequency carrier in accordance with the modulating signal (information signal). In AM, the amplitude of the carrier changes directly proportional to the instantaneous value of a modulating signal. There are two things happened in the process:

- The modulating signal is impressed onto the carrier in such a way that it can be recovered at the receiver.

- The modulating signal is shifted to a higher frequency band.

5.1 PRINCIPLES OF AM

Let the high frequency carrier be expressed as

$$v_c(t) = V_c \cos(2\pi f_c t + \theta) = V_c \cos(\omega_c t + \theta) \qquad (5.1)$$

where V_c is its maximum amplitude in volts, f_c is its frequency in Hz and θ is its initial phase which we assume to be zero, without losing generality.

To make the peak amplitude of the carrier (the swing from zero to positive peak or negative peak in a cycle) vary in accordance with the modulating signal $v_s(t)$, the carrier amplitude V_c, is replaced by $[V_c + v_s(t)]$. Thus, the amplitude of the carrier varies with the instantaneous amplitude of a modulating signal $v_s(t)$. The modulated signal is thus given by

$$\begin{aligned} v_{AM}(t) &= [V_c + v_s(t)]\cos(\omega_c t) \\ &= V_c \cos(\omega_c t) + v_s(t)\cos(\omega_c t) \end{aligned} \qquad (5.2)$$

5.1.1 Time domain description of single-tone AM signals

A modulating signal is usually an arbitrary complex signal, such as audio or music signal. For easy understanding and analysis, we start with a modulating signal that is a simple sinusoidal signal expressed as

$$v_s(t) = V_s \cos 2\pi f_s t = V_s \cos\omega_s t \qquad (5.3)$$

Sinusoidal modulating signal is known as **single-tone modulating signal**.

Substituting eq. (5.3) into eq. (5.2), we obtain an AM signal modulated by a single-tone modulating signal known as **single-Tone AM signal**.

The time domain description of single-tone AM signal is thus as below:

$$v_{AM}(t) = [V_c + V_s\cos2\pi f_s t]\cos2\pi f_c t \qquad (5.4)$$

Figure 5.1 illustrates a single-tone modulating signal $v_s(t)$, carrier signal $v_c(t)$ and the modulated signal $v_{AM}(t)$. From Figure 5.1, the following observation can be made:

Before modulation: $v_{AM}(t) = v_c(t) = V_c\cos\omega_c t$
After modulation: $v_{AM}(t) = [V_c + V_s\cos\omega_s t]\cos\omega_c t$
AM instantaneous peak amplitude = $[V_c + V_s\cos\omega_s t]$
Carrier frequency: $f_c = 1/T_c$
modulating signal frequency: $f_s = 1/T_s$

After modulation, the carrier amplitude is no longer constant but follows the instantaneous changes in the amplitude of the modulating signal. The time-varying shape of the AM waveform is called the **envelope** of the AM signal. The envelope of the AM signal has the same shape as the modulating signal and thus carries the information of modulating signal. And, the following are observed on the AM envelope:

Positive envelope = $[V_c + V_s\cos2\pi f_s t]$
Negative envelope = $-[V_c + V_s\cos2\pi f_s t]$
Maximum envelope $Env_{max} = V_c + V_s$
Minimum envelope $Env_{min} = V_c - V_s$

A functional block diagram for generating AM signal is shown in Figure 5.2.

Example 5.1

A carrier signal $v_c(t) = 10\cos(2\pi\times10^5 t)$ is amplitude modulated by a modulating signal $v_s(t)=2\cos(2\pi\times10^3 t)$. Determine:

(a) The values of V_c, V_s, T_c, T_s, Env_{max} and Env_{min}.
(b) The expression of the AM signal.

Solution

(a) $V_c = 10$ volt, $V_s = 2$ volt
$T_c = 1/f_c = 1/10^5 = 10\ \mu s$, $T_s = 1/f_s = 1/10^3 = 1$ ms
$Env_{max} = V_c + V_s = 12$ volt, $Env_{min} = V_c - V_s = 8$ volt

(b) $v_{AM}(t) = [V_c + V_s\cos2\pi f_s t]\cos2\pi f_c t = [10 + 2\cos(2\pi\times10^3 t)]\cos2\pi\times10^5 t$

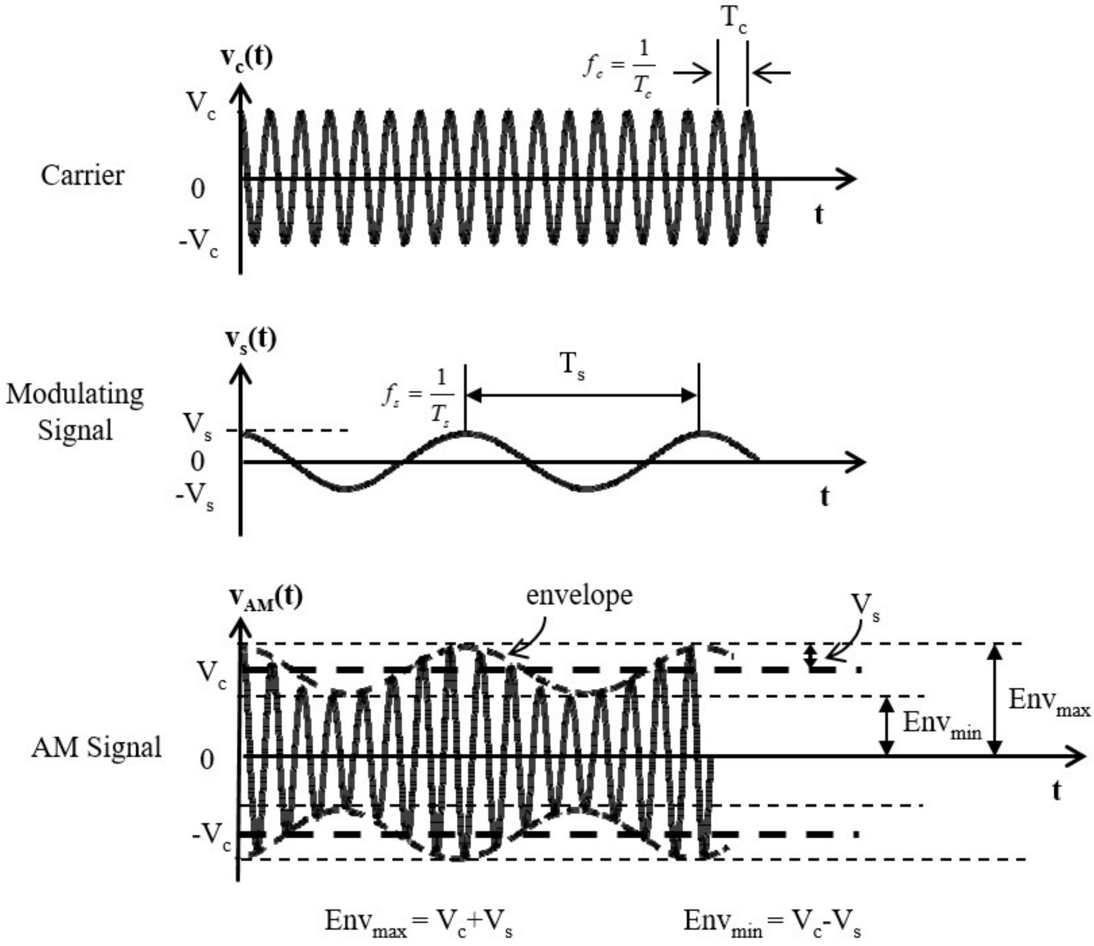


Figure 5.1 Sinusoidal modulating signal, carrier and modulated signal.

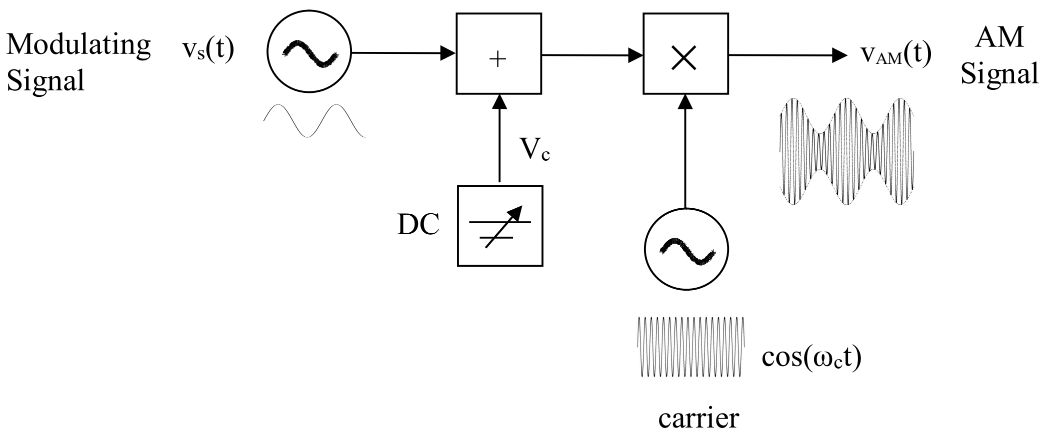


Figure 5.2 Generation of AM signal

5.1.2 Modulation index

Modulation index m is defined as the ratio of the peak amplitudes of modulating signal to the peak amplitude of unmodulated carrier:

$$m = \frac{V_s}{V_c} \tag{5.5}$$

m is a constant, determined by the peak amplitudes of the carrier signal and modulating signal, and **$0 \le m \le 1$**. It is also expressed in percentage, known as percentage modulation.

The carrier amplitude is usually fixed in practice. Thus, the modulation index is controlled by varying the peak amplitude of the modulating signal. When the amplitude of the modulating signal changes while the carrier amplitude is kept constant, the envelope of the AM signal will also change. The **modulation index** m, is used to describe the size of the envelope.

Figure 5.3 shows the AM signals with different modulation index. It can be observed that the size of the envelope of AM signal varies directly proportional to modulation index, m. A larger modulating signal, thus large modulation index, will result in a larger envelope. This applies for any arbitrary modulating signals as well.

At the receiver, the modulating signal is recovered by extracting the envelope (details are given in Section 5.3). Thus, a larger envelope will result in a larger receiver's output and hence a high SNR. Large envelope implies large m. In AM transmission, m is usually kept as high as possible.

Example 5.2

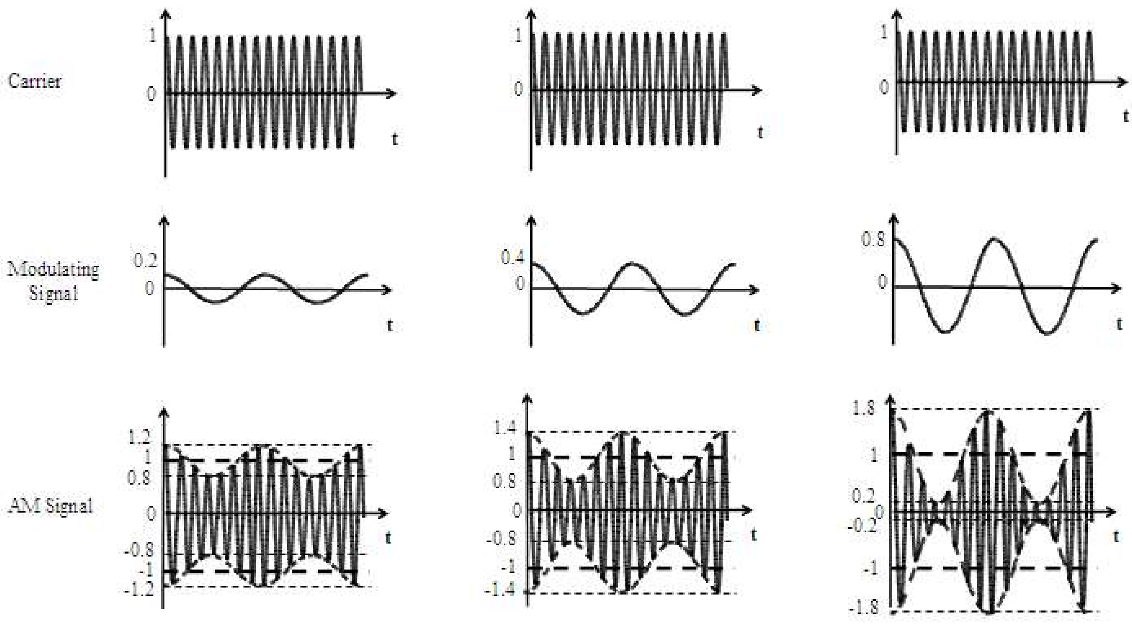
A carrier signal with amplitude of 4 Volt and frequency of 500 kHz is amplitude modulated by a sinusoidal modulating signal with frequency of 3 kHz and amplitude of 2.4 Volt.

(a) Write the expression for the resulting AM signal.
(b) Determine the modulation index.

Solution

(a) $V_c = 4$ volt, $V_s = 2.4$ volt
$f_c = 500$ kHz, $f_s = 3$ kHz
$v_{AM}(t) = [V_c + V_s\cos 2\pi f_s t]\cos 2\pi f_c t$
$= [4 + 2.4\cos(6\pi\times10^3 t)]\cos 10\pi\times10^5 t$

(b) $m = V_s / V_c = 2.6 / 4 = 0.6$



(a) Vs = 0.2 V
Vc = 1 V
m = 0.2 (20%)

(b) Vs = 0.4 V
Vc = 1 V
m = 0.4 (40%)

(c) Vs = 0.8 V
Vc = 1 V
m = 0.8 (80%)

Figure 5.3 AM waveforms for different values of m.

5.1.3 Over-modulation

While it is important to use as high a modulation index as possible, it should not exceed 1. When AM modulation index is great than one, the AM signal is said to be **over-modulated**. This situation occurs when $V_s > V_c$. Figure 5.4 shows an example of over-modulation with m=1.5. It can be observed that the envelope of the overmodulated AM signal is no longer a faithful reproduction of the shape of the modulating signal. A distorted envelope will result in a distorted receiver's output during demodulation.

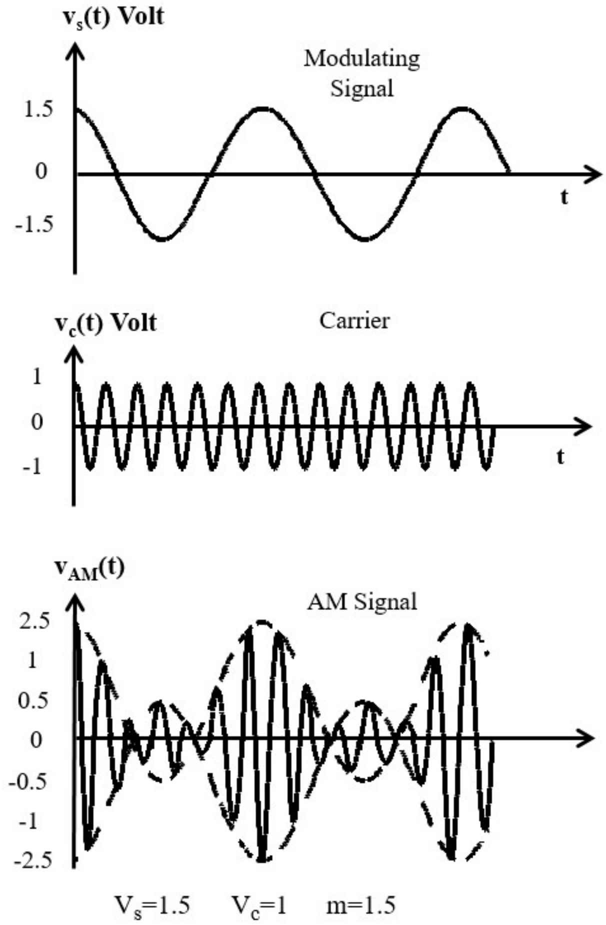


Figure 5.4 An example of over-modulation

5.1.4 Measurement of modulation index

The modulation index m can also be determined by measuring the maximum and minimum envelope of the AM signal using an oscilloscope. We can compute the modulation index from the measured maximum and minimum envelope values as following (refer to Figure 5.1):

$$V_s = \frac{Env_{max} - Env_{min}}{2}$$
$$V_c = \frac{Env_{max} + Env_{min}}{2} \tag{5.6}$$

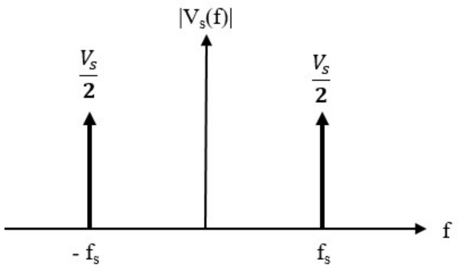
$$m = \frac{Env_{max} - Env_{min}}{Env_{max} + Env_{min}} \tag{5.7}$$

5.1.5 Frequency domain description of single-tone AM signal

When a sinusoidal carrier signal is modulated by a sinusoidal signal, the resultant AM signal is **NOT** a sinusoidal signal. To find out more about the AM signal, it is essential to analysis the AM signal in frequency domain.

Recall the Fourier transform of a sinusoid (refer to chapter 2) and its double-side amplitude spectrum shown below.

$$v_s(t) = V_s \cos 2\pi f_s t \xLeftrightarrow{FT} V_s(f) = \frac{V_s}{2}\delta(f - f_s) + \frac{V_s}{2}\delta(f + f_s)$$



Applying Fourier transform's modulation theorem, the Fourier transform of a single-tone AM signal, $V_{AM}(f)$, can be obtained as following:

$$v_{AM}(t) = [V_c + v_s(t)]\cos\pi f_c t \xLeftrightarrow{FT} V_{AM}(f) = \frac{V_c}{2}\delta(f + f_c) + \frac{V_c}{2}\delta(f - f_c)$$

$$= V_c \cos 2\pi f_c t \qquad + \frac{1}{2}[V_s(f + f_c) + V_s(f - f_c)]$$

$$+ v_s(t) \times \cos 2\pi f_c t$$

$V_s(f+f_c)$ and $V_s(f-f_c)$ are frequency-shifted versions of the modulating signal spectrum $V_s(f)$. $V_s(f-f_c)$ is $V_s(f)$ shifted to the right by f_c and $V_s(f+f_c)$ is $V_s(f)$ shifted to the left by f_c. Thus, modulation of carrier with $v_s(t)$ has shifted the frequency of $v_s(t)$ by the carrier frequency f_c. Figure 5.5 plots the double-sided amplitude spectrum of the single-tone AM signal.

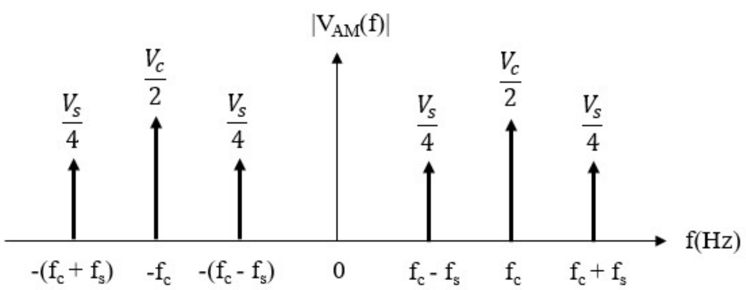


Figure 5.5 double-sided amplitude spectrum of single-tone AM signal

Figure 5.6 indicates that the AM signal, $v_{AM}(t)$, contains three frequency components:

- A carrier frequency (CF) component with frequency of f_c,
- An **upper side-frequency** (USF) component with frequencies of $f_c + f_s$, and
- A **lower side-frequency** (LSF) component with frequency of f_c - f_s.

The upper side-frequency and lower side-frequency components space symmetrically about the carrier frequency. They are called upper and lower side-frequency components because they are on either side of the carrier frequency component.

5.1.6 Single-tone AM signal bandwidth

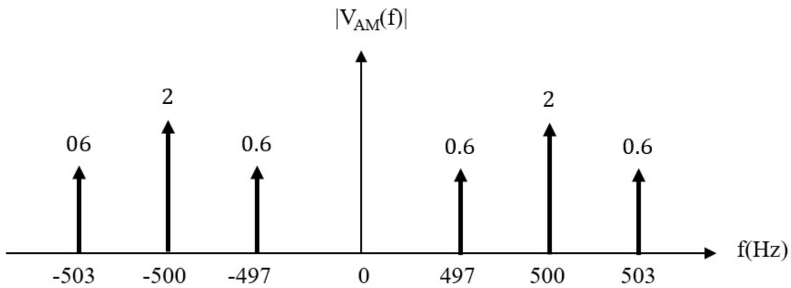
The bandwidth of single-tone AM signal is equal to the difference between the upper side-frequency and lower side-frequency.

$$B_{AM} = (f_c + f_s) \ - (f_c - f_s) = 2f_s \tag{5.8}$$

Example 5.3

A carrier signal with amplitude of 4 Volt and frequency of 500 kHz is amplitude modulated by a sinusoidal signal with frequency of 3 kHz and amplitude of 2.4 Volt. Draw the double-sided amplitude spectrum of the AM signal.

Solution



The single-sided spectrum of the AM signal can be obtained by combining the negative and positive frequency components as shown in Figure 5.6.

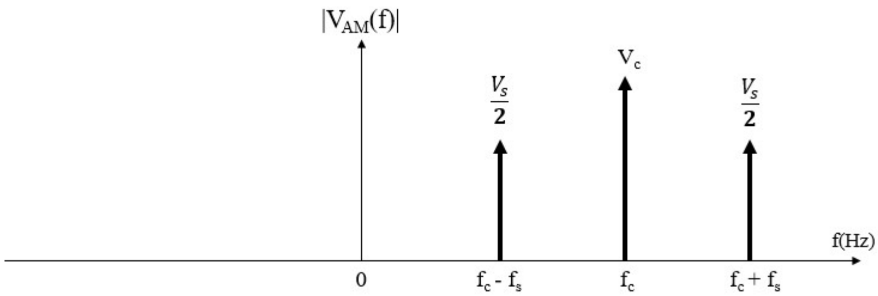


Figure 5.6 Single-sided amplitude spectrum of single-tone AM signal

Alternatively, the single-sided spectrum of the AM signal can be obtained by expanding the terms on the right-hand side of eq. (5.4) and using trigonometric identity, cosAcosB=1/2[cos(A-B)+cos(A+B)]. We obtain the expression of AM signal in frequency domain:

$$\begin{aligned} v_{AM}(t) &= [V_c + V_s\cos\omega_s t]\cos\omega_c t \\ &= V_c\cos\omega_c t + V_s\cos\omega_s t \times \cos\omega_c t \\ &= V_c\cos\omega_c t + \frac{V_s}{2}\cos(\omega_c - \omega_s)t + \frac{V_s}{2}\cos(\omega_c + \omega_s)t \end{aligned} \tag{5.9}$$

Eq. (5.9) represent a single-tone AM signal consisting of three frequency components with frequency of f_c-f_s, f_c and f_c+f_s, respectively. Based on Eq. (5.9), a single-side spectrum can be plotted as shown in Figure 5.7.

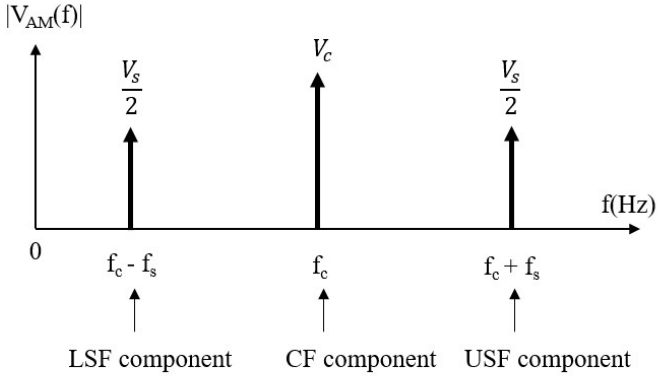


Figure 5.7 single-sided amplitude spectrum of sinusoidal modulated AM signal

5.2 MULTI-TONE AM SIGNALS

As mentioned earlier, modulating signal is usually not a simple sinusoid but a complex signal consisting of more than one frequency components known as **multi-tone modulating signal**. An AM signal modulated by a multi-tone modulating signal is known as **multi-tone AM signal**.

Consider a multi-tone modulating signal consisting of two frequency components given below:

$$v_s(t) = V_1\cos w_1 t + V_2\cos w_2 t \qquad \text{where } V_1 > V_2 \text{ and } f_2 > f_1$$

The amplitude spectrum of $v_s(t)$ shown in Figure 5.8 is obtained by perform the following Fourier transform.

$$v_s(t) = V_1\cos 2\pi f_1 t + V_2\cos 2\pi f_2 t \overset{FT}{\Longleftrightarrow} V_s(f) = \frac{V_1}{2}\delta(f - f_1) + \frac{V_1}{2}\delta(f + f_1) + \frac{V_2}{2}\delta(f - f_2) + \frac{V_2}{2}\delta(f + f_2)$$

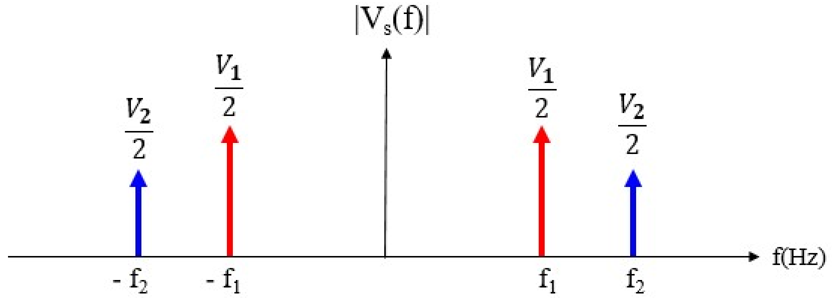


Figure 5.8 double-sided spectrum of a multi-tone modulating signal

When $v_s(t)$ modulates the amplitude of a carrier, the modulated signal is

$$v_{AM}(t) = [\ V_c + V_1\cos w_1 t + V_2\cos w_2 t]\cos\omega_c t \qquad (5.10)$$

Perform Fourier transform on eq. (5.10) givens the double-sided spectrum of the mulit-tone AM signal as shown in Figure 5.9.

$$v_{AM}(t) = [V_c + v_s(t)\ ]\cos\pi f_c t \quad \overset{FT}{\Longleftrightarrow} \quad V_{AM}(f) = \frac{V_c}{2}\delta(f + f_c) + \frac{V_c}{2}\delta(f - f_c)$$

$$= V_c\cos 2\pi f_c t \qquad\qquad + \frac{1}{2}[\ V_s(f + f_c) + V_s(f - f_c)\ ]$$

$$+\ v_s(t) \times \cos 2\pi f_c t$$

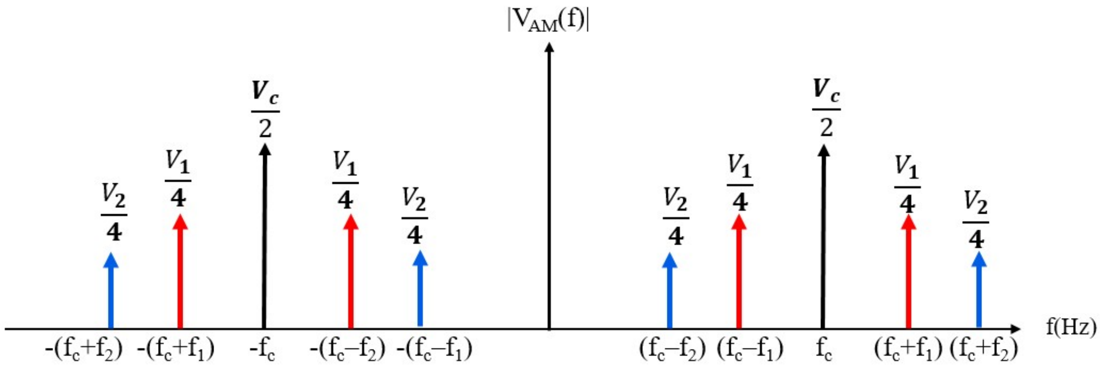


Figure 5.9 Double-sided spectrum of a multi-tone AM signal

As shown in Figure 5.9, the multi-tone AM signal contains the following frequency components:

- A carrier frequency component of frequency of f_c
- A group of 2 upper side-frequency components of frequencies $f_c + f_1$ and $f_c + f_2$

- A group of 2 lower side-frequency components of frequencies f_c - f_1 and f_c - f_2

Again, we can combine the negative and positive frequency components to obtain the single-sided spectrum of the multi-tone AM signal as shown in Figure 5.10.

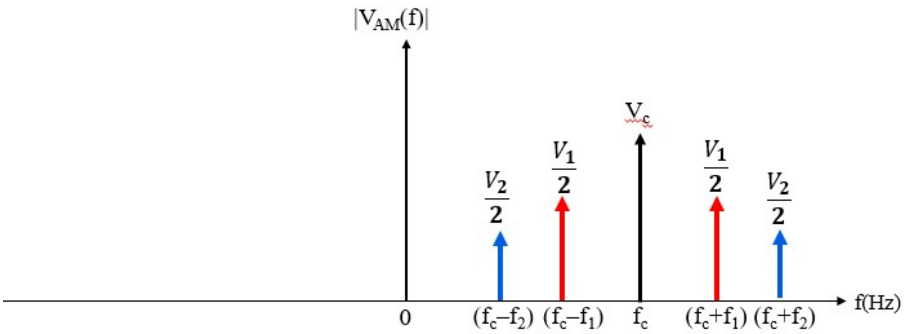


Figure 5.10 Single-sided spectrum of a multi-tone AM signal

The above analysis can be applied to multi-tone modulating signal consisting of more than two frequency components. When a multi-tone modulating signal consists of several frequency components, each one of these components will produce a corresponding upper side-frequency component and lower side-frequency component in AM signal. Thus, the AM process shifts each and every frequency component in the spectrum of the modulating signal to a pair of side-frequency components about the carrier frequency f_c.

Similar results can be obtained for multi-tone modulating signal $v_s(t)$ with a continuous spectrum $V_s(f)$ ranging from f_L to f_H Hz given by Figure 5.11. Note that the shape of $V_s(f)$ has no particular significance.

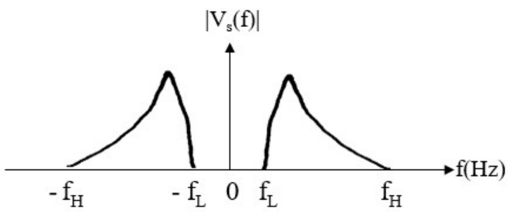


Figure 5.11 Double-sided amplitude spectrum of a multi-tone modulating signal

The spectrum of the multi-tone AM signal is obtained by performing the following Fourier transform.

$$v_s(t) \quad \overset{FT}{\Longleftrightarrow} \quad V_s(f)$$

$$v_{AM}(t) = [V_c + v_s(t)]\cos\omega_c t \quad \overset{FT}{\Longleftrightarrow} \quad V_{AM}(f) = \frac{V_c}{2}\delta(f - f_c) + \frac{V_c}{2}\delta(f + f_c)$$

$$= V_c\cos\omega_c t + V_s\cos\omega_c t \qquad\qquad + \frac{1}{2}[\,Vs(f - f_c) + V_s(f + f_c)\,]$$

$V_s(f - f_c)$ is $V_s(f)$ shifted to the right by f_c and $V_s(f + f_c)$ is $V_s(f)$ shifted to the left by f_c. Figure 5.12 shows the amplitude (magnitude) spectrum of the multi-tone AM signal described above. For multi-tone AM signal with continuous spectrum, we need to consider sidebands instead of side-frequencies. The multi-tone AM signal contains the following frequency components:

- An upper sideband (USB) ranging from $(f_c + f_L)$ to $(f_c + f_H)$ Hz
- An lower sideband (LSB) ranging from $(f_c - f_H)$ to $(f_c - f_L)$ Hz
- The upper sideband and lower sideband space symmetrically about the carrier frequency. They are mirror image of each another.

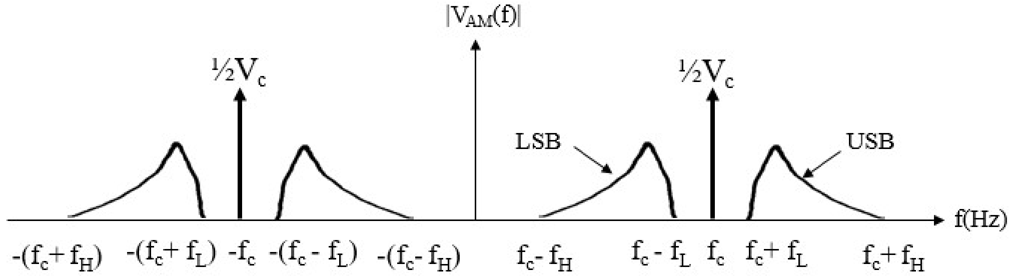


Figure 5.12 Double-sided magnitude spectrum of a multi-tone AM signal

Combining the negative and positive frequency components, we can obtain the single-sided amplitude spectrum of the multi-tone AM signal shown in Figure 5.13.

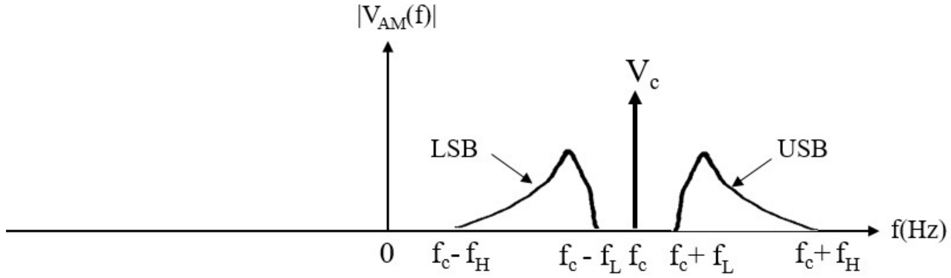


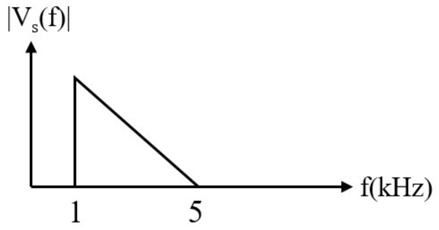
Figure 5.13 Single-sided amplitude spectrum of the multi-tone AM signal

The **bandwidth (B_{AM}) of a multi-tone AM signal** will be twice the maximum frequency of the modulating signal, f_H, given by

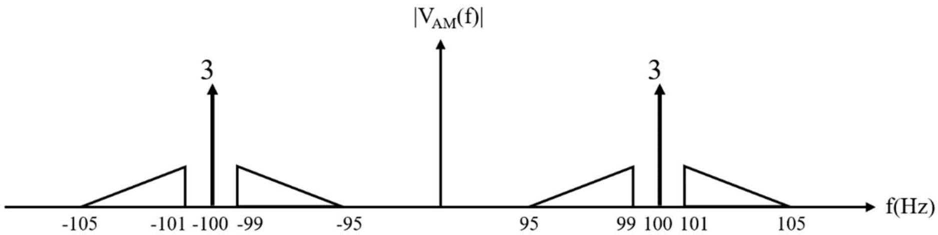
$$B_{AM} = (f_c + f_H) - (f_c - f_H) = 2f_H \quad (5.11)$$

Example 5.4

A carrier signal with amplitude of 6 volt and frequency of 100 kHz is amplitude modulated by a modulating signal that has an amplitude spectrum shown below. Plot the double-sided amplitude spectrum of the AM signal.



Solution



5.3 DEMODULATION OF AM SIGNALS

Demodulation is the process of recovering the original modulating signal from a modulated carrier at the receiver. There are several techniques available for demodulation of AM signal: Coherent/synchronous detection, Square-law detection and Envelope detection. Envelope detection technique is the simplest and most widely used technique. We shall only consider the envelope detection technique in this module.

Since the modulating signal is contained in the envelope of an AM signal, it can be recovered by detecting the voltage variations in the envelope. A circuit that can perform this function is shown in Figure 5.14. It is known as Envelope Detector. Because it uses a diode, this circuit is also sometimes referred to as a diode detector. The envelope detector is a combination of half-wave rectifier and low-pass filter.

During the positive half cycle of the AM waveform $v_{AM}(t)$, the diode conducts and the capacitor is charged with voltage v(t). The capacitor voltage v(t) increases as $v_{AM}(t)$. v(t) reaches the peak voltage value when $v_{AM}(t)$ reaches its peak voltage. And, v(t) increases very fast as the capacitor is selected in such a way that it charges very quickly and discharges very slowly. When $v_{AM}(t)$ decreases, v(t) also decreases but at much slower speed. $v_{AM}(t)$ shortly becomes less than v(t) and the diode is reverse biased. The capacitor will thus discharge through resistor R till the next positive half cycle of AM. At the next positive half cycle of AM, the diode starts conducting and the capacitor is charged again. This cycle of events repeats following the cycle of the AM waveform envelope, which results in v(t) following the variation of the envelope of the AM waveform. Next, the dc component of the detected envelope is blocked using a coupling capacitor and the modulating signal is recovered.

If the AM signal is over-modulated as shown in Figure 5.15, the output of the envelope detector before the DC block will be a distorted signal as shown in Figure 5.16. Thus, m should not be allowed to exceed 1 in AM transmission.

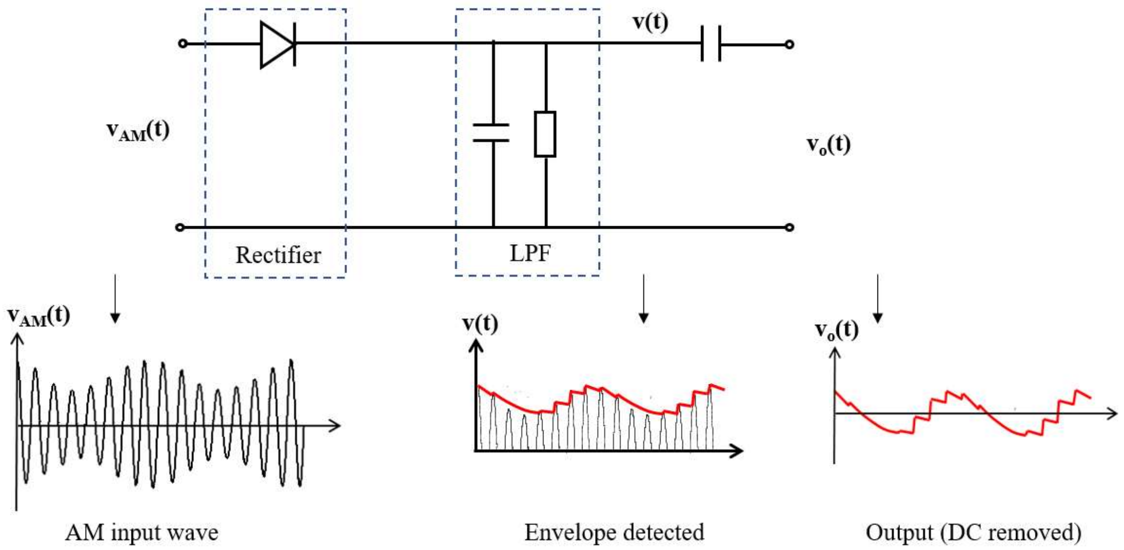


Figure 5.14 Working of envelope detector

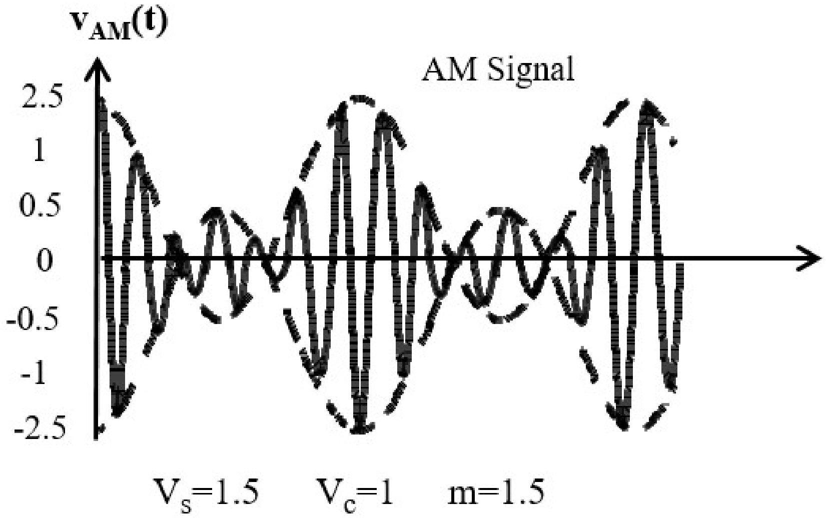


Figure 5.15 Over-modulated AM signal

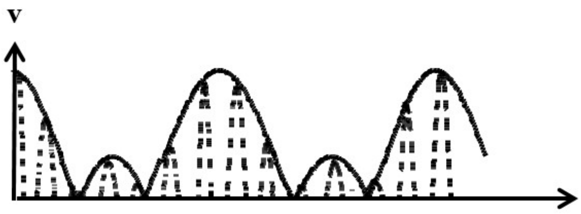


Figure 5.16 Distorted output of envelope detector due to over-modulation

5.4 DOUBLE SIDEBANDS SUPPRESSED CARRIER (DSBSC) AND SINGLE-SIDEBAND (SSB) MODULATION

AM signal consists of a carrier component and two sidebands. A large portion of total transmitted power of the AM signal lies in the carrier component. The useful power that lies in the sidebands containing modulating signal is very small. For example, for single-tone AM signal, the carrier component takes up to 67% of the total transmitted power (see Appendix). Since the information of the modulating signal is contained only in the sidebands and not in the carrier, the carrier component power is a waste.

The modified AM process in which the modulated signal contains no carrier component but only two sidebands is known as **Double Sideband Suppressed Carrier** (DSBSC) modulation. In this modulation process, the carrier is suppressed, leaving only the two sidebands to be transmitted.

Further more, the information contained in the lower-sideband is identical to the information contained in the upper-sideband. It is not essential to transmit both sidebands at the same time. In fact, no information will be lost when the carrier and one of the sidebands are suppressed. This leads to the next logical step of suppressing not only the carrier, but also one of the sidebands. The modified AM process that transmits one sideband only is known as **Single Sideband (SSB) modulation**.

5.4.1 Time domain representation of DSBSC signal

Since DSBSC signal contains only two sidebands and no carrier component, we can generate DSBSC signal by removing the carrier component of AM signal (eq. (5.2)) as

$$v_{DSBSC}(t) = v_s(t)\cos\omega_c t \tag{5.12}$$

Thus, a DSBSC signal can be generated easily by taking the product of the carrier and the modulating signal.

A block diagram of a DSBSC modulator is shown in Figure 5.17. A DSBSC modulator is also known as a **Balanced Modulator**.

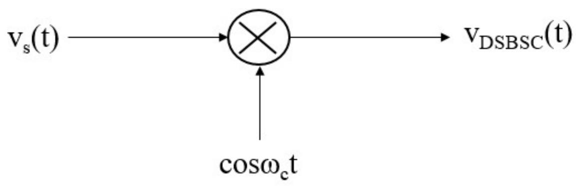


Figure 5.17 DSBSC Modulator

Figure 5.18 shows the waveform of a DSBSC signal modulated by a sinusoidal signal. It can be observed that whenever modulating signal $v_s(t)$ changes sign, there is a 180^o carrier phase reversal on the DSBSC signal since $v_s(t)$ multiplies the carrier $\cos\omega_c t$, which does not happen in AM unless it is over-modulation. And, the envelope of the DSBSC signal is not a faithful representation of the modulating signal. A simple envelope detector using a diode cannot recover the modulating signal. Recovering modulating signal from DSBSC signal requires a complex demodulator.

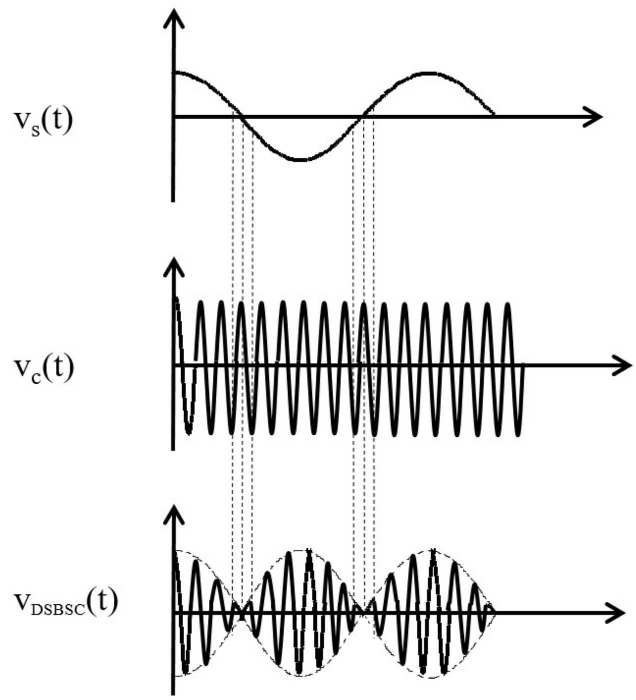


Figure 5.18 The DSBSC waveform for a sinusoidal modulating signal

5.4.2 Frequency domain representation of DSBSC signals

We consider a multi-tone modulating signal with the amplitude spectrum shown in Figure 5.19. Taking Fourier transform on both side of eq. (5.12), we have

$$v_s(t) \overset{FT}{\Longleftrightarrow} V_s(f)$$

$$v_{DSBSC}(t) = v_s(t)\cos\omega_c t \overset{FT}{\Longleftrightarrow} V_{DSBSC}(f) = \frac{1}{2}[\,Vs(f - f_c) + V_s(f + f_c)\,]$$

Figure 5.20 plots the amplitude spectrum of the DSBSC signal. Comparing the spectrum shown in Figure 5.12 and Figure 5.20, it is clear that a DSBSC signal contains two sidebands without carrier component. Because of no carrier, all the average power of the DSBSC signal resides in the two sidebands. As both sidebands are present, DSBSC signal has a bandwidth of $2f_H$ just like AM signal.

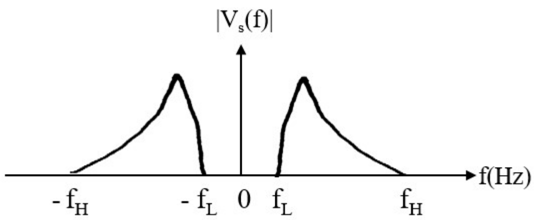


Figure 5.19 Magnitude spectrum of complex modulating signal

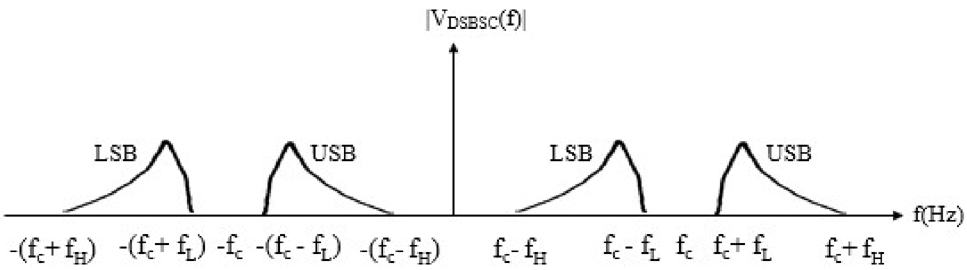
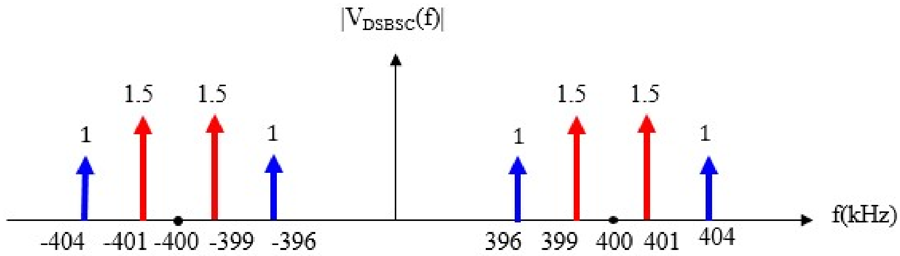


Figure 5.20 Spectrums of AM, DSBSC and SSB Signals

Example 5.5

The modulating signal in a DSBSC modulator is $v_s(t) = 6\cos 2000\pi t + 4\cos 8000\pi t$. The carrier signal is $v_c(t) = \cos 8\times10^5\pi t$. Plot the double-sided spectrum of modulated signal and determine the bandwidth of the AM signal.

Solution



The bandwidth of the AM signal: $B_{AM} = 404 - 396 = 8$ kHz

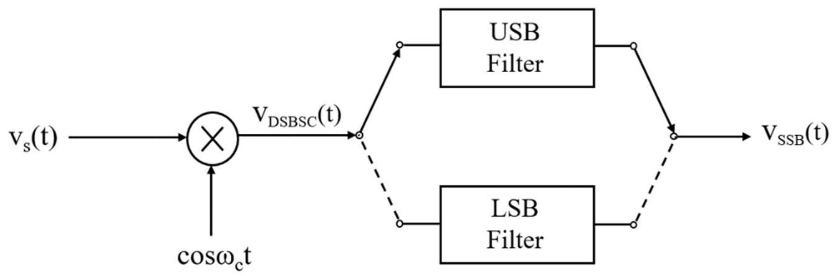
5.4.3 Single-Sideband (SSB) modulation

There are three methods to produce SSB signals:

1. Filter method
2. Phase shift method
3. Weaver or third method

All the three method employ a two-step operation to generate a SSB signal. In the first step, they all use a balanced modulator to obtain a DSBSC signal. In the second step, the filter method uses a filter to extract one sideband from the DSBSC signal. In the phase shift method, the unwanted sideband is attenuated by cancelling it with an identical but anti-phase signal. The weaver or third method uses a combination of filtering and cancellation techniques. In this module, only the filter method is considered.

The block diagram shown in Figure 5.21 illustrates the generation of a SSB signal using filter method. After generating a DSBSC signal by a balance modulator, a sideband filter is used to extract out the required sideband (USB or LSB) signal for transmission. Figure 5.22 shows the single-sided amplitude spectrum of the modulating signal $v_s(t)$, DSBSC signal $v_{DSBSC}(t)$ and SSB signal $v_{SSB}(t)$.



Note: LSB filter refers to a BPF that passes the LSB only.
USB filter refers to a BPF that passes the USB only.

Figure 5.21 Generating SSB Signals using Filter Method

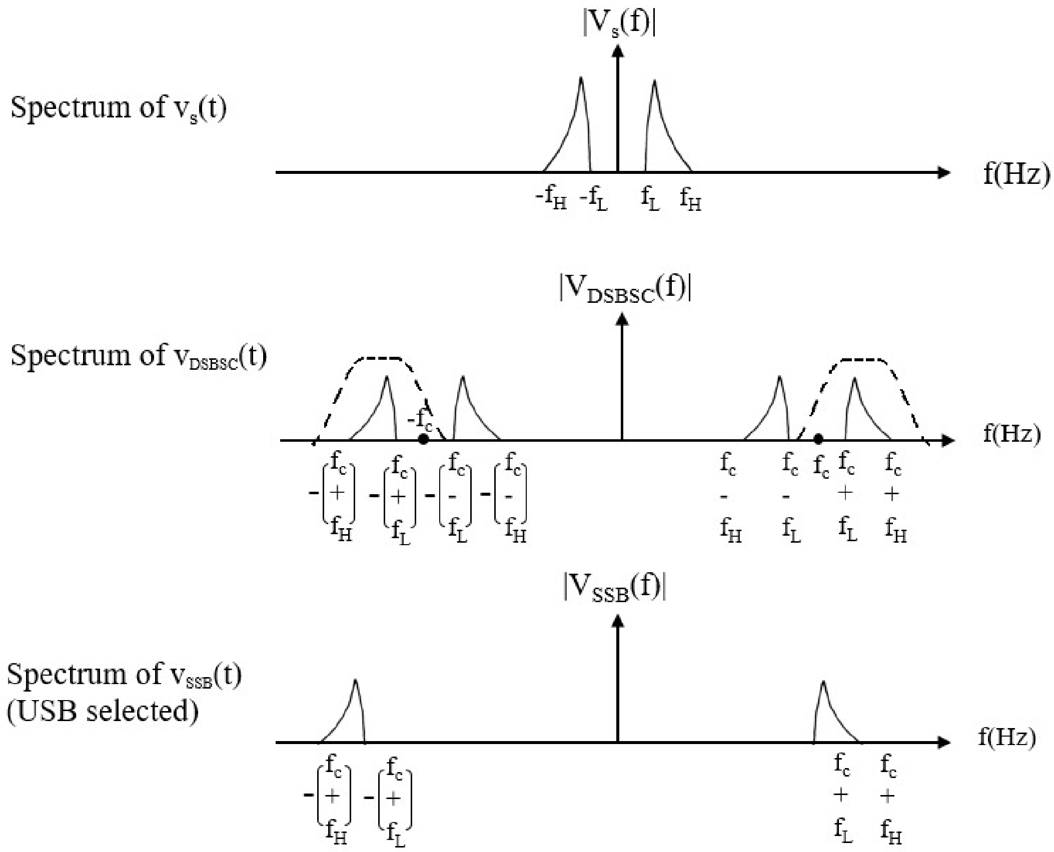


Figure 5.22 SSB signal spectrum

Example 5.6

Figure E5.6(a) shows the frequency response of a BPF given by Figure E5.6(b).

(a) Sketch the double-sided amplitude spectrum at point A and B.
(b) What is the function of the block diagram in E5.6(b) ?

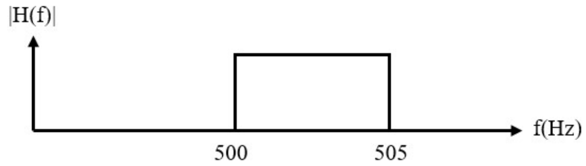


Figure E5.6(a)

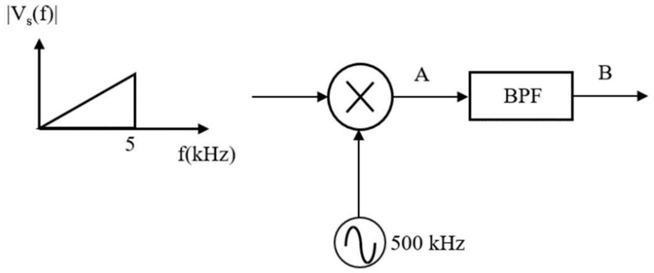
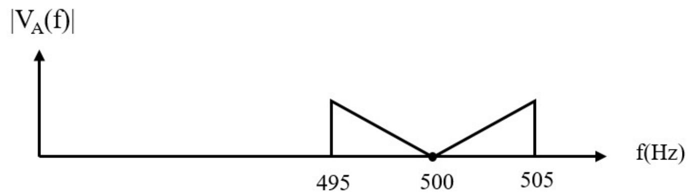


Figure E5.6(b)

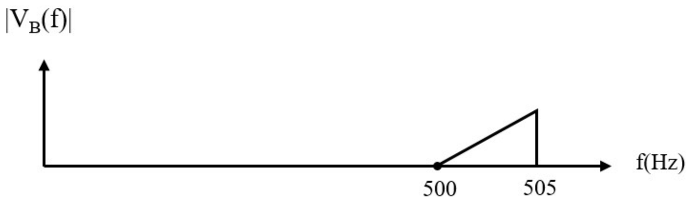
Solution

(a)

Spectrum at A



Spectrum at B



(b) SSB modulator

5.4.4 DSBSC and SSB demodulators

A block diagram of a DSBSC and SSB demodulator is shown in Figure 5.23. The operation of DSBSC and SSB demodulator is illustrated by Figure 5.24. Note that the illustration uses a DSBSC signal as the received signal shown in Figure 5.24(a).

The output of the multiplier in Figure 5.23 is given by

$$v_x(t) = v_{DSBSC}(t) \times \cos\omega_c t$$

The spectrum of $v_x(t)$ is obtain by performing the following Fourier transform:

$$v_{DSBSC}(t) \quad \overset{FT}{\Longleftrightarrow} \quad V_{DSBSC}(f)$$

$$v_x(t) = v_{DSBSC}(t) \times \cos 2\pi f_c t \quad \overset{FT}{\Longleftrightarrow} \quad V_X(f) = \frac{1}{2}\left[V_{DSBSC}(f + f_c) + V_{DSBSC}(f - f_c) \right]$$

The spectrum of $v_x(t)$ consists of two frequency-shifted versions of the spectrum of DSBSC signal $v_{DSBSC}(t)$, one version shifted right by f_c, another shifted left by f_c, as shown in Figure 5.24(b). The low pass filter stops the high frequency component around $2f_c$ and passes the low frequency modulating signal shown in Figure 5.24(c). Thus, the low frequency modulating signal is recovered. Similar result can be obtained for SSB. Note that the modulating signal is recovered without error only if the receiver can generate $\cos\omega_c t$ without frequency and phase errors.

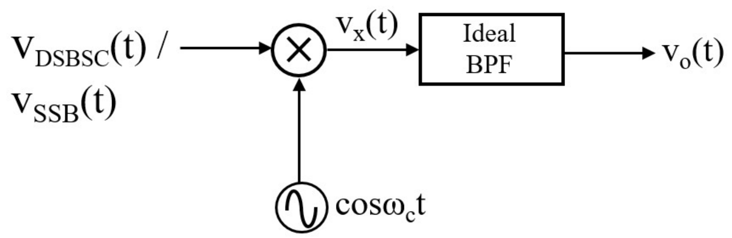


Figure 5.23 DSBSC and SSB demodulator

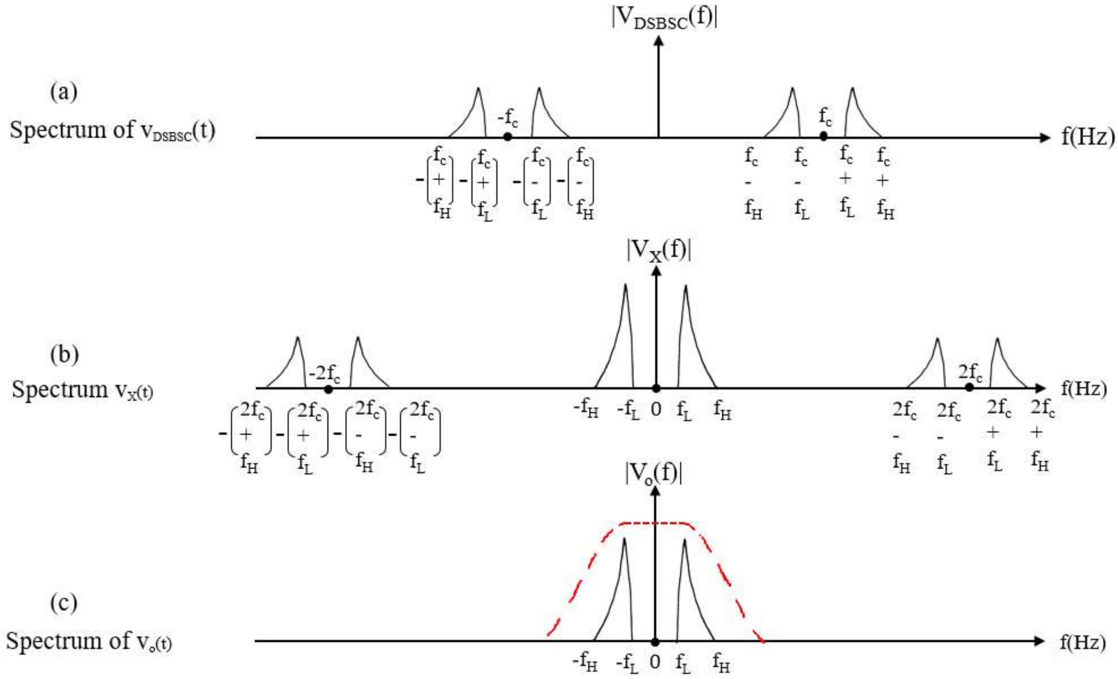


Figure 5.24 DSBSC signal demodulation

5.4.5 Advantages and disadvantages of DSBSC and SSB

The advantages of DSBSC and SSB modulation compared with AM are as follows:

- **Power saving** (Advantage of DSBSC and SSB): By not transmitting the carrier (i.e. DSBSC), there will be a total power saving of more than 67%. If only one sideband (i.e. SSB) is transmitted, the total power saving will be more than (67% + 17%) = 84% (see Appendix)

- **Bandwidth saving** (Advantage of SSB but not DSBSC): Comparing Figure 8.1c with Figures 8.1a and 8.1b, it can be observed that the bandwidth of SSB transmission is half that of AM and DSBSC systems, i.e. a bandwidth reduction of 50%. Since SSB transmission can occupy one-half the frequency space required for two sidebands, it allows for better utilisation of the channel bandwidth. More transmission can fit into a given frequency range than would be possible with double sideband signals.

The main disadvantage of DSBSC and SSB modulation systems is the high cost of the hardware. DSBSC and SSB signals cannot be demodulated using the simple envelope detector. A much more sophisticated method, synchronous detection, has to be used which increases hardware costs.

Appendix Power of AM signal

From equation (5.9), the sinusoidally modulated AM signal can be expressed as

$$v_{AM}(t) = V_C\cos\omega_C t + ½V_S\cos\omega_C + \omega_S)t + ½V_S\cos\omega_C - \omega_S)t$$

carrier component + USF component + LSF component

The total transmitted power, P_{AM} is given by

$$P_{AM} = P_C + P_{USF} + P_{LSF}$$

Where P_C = power of the carrier component

P_{USF} = power of the USF component

P_{LSF} = power of the LSF component

Since these three components are sinusoidal,

$$P_C = \frac{(V_{rms})^2}{R_L} = \left[\frac{V_C}{\sqrt{2}}\right]^2 \times \frac{1}{R_L} = \frac{V_C^{\;2}}{2R_L}$$

$$P_{USF} = \left[\frac{\frac{1}{2}V_S}{\sqrt{2}}\right]^2 \times \frac{1}{R_L} = \frac{V_S^{\;2}}{8R_L}$$

$$P_{LSF} = \left[\frac{\frac{1}{2}V_S}{\sqrt{2}}\right]^2 \times \frac{1}{R_L} = \frac{V_S^{\;2}}{8R_L} = P_{USF}$$

Where R_L = load in which P_{AM} is dissipated.

We know $m = V_S/V_C$ or $V_S = mV_C$

$$P_{USF} = P_{LSF} = \frac{m^2V_C^{\;2}}{8R_L} = \frac{1}{4}m^2\left[\frac{V_C^{\;2}}{2R_L}\right]$$

$$\therefore P_{USF} = P_{LSF} = \frac{1}{4}m^2P_C$$

$$P_{AM} = P_C + \tfrac{1}{4}m^2P_C + \tfrac{1}{4}m^2P_C$$

$$\therefore P_{AM} = \left(1 + \tfrac{1}{2}m^2\right)P_C$$

$P_{sidebands}$ = power of both the sidebands

$= P_{USF} + P_{LSF}$

$$\therefore P_{sidebands} = \tfrac{1}{2}m^2P_C$$

Using the equations for P_{AM}, $P_{sidebands}$ and P_{USF} the following relations can be derived.

$$\frac{P_C}{P_{AM}} = \frac{1}{1 + \tfrac{1}{2}m^2} = \frac{2}{2 + m^2}$$

$$\frac{P_{sidebands}}{P_{AM}} = \frac{\tfrac{1}{2}m^2P_C}{(1 + \tfrac{1}{2}m^2)\,P_C}$$

$$\therefore \frac{P_{sidebands}}{P_{AM}} = \frac{m^2}{2 + m^2}$$

$$\frac{P_{USF}}{P_{AM}} = \frac{P_{LSF}}{P_{AM}} = \frac{m^2}{2(2 + m^2)}$$

For $m = 0$

$P_{USF} = P_{LSF} = 0 \qquad \Rightarrow \dfrac{P_C}{P_{AM}} = 100\%$

For $m = 1$

$$\frac{P_{USF}}{P_{AM}} = \frac{P_{LSF}}{P_{AM}} = \frac{1^2}{2(2 + 1^2)} = \frac{1}{6}$$

$$P_{USF} = P_{LSF} = \frac{P_{AM}}{6}$$

$$P_{USF} = P_{LSF} = 16.67\% \text{ of } P_{AM} \Rightarrow \frac{P_C}{P_{AM}} = 66.67\%$$

Chapter 6

Frequency Modulation

Learning Outcomes

- ❑ Describe how information is conveyed in FM.
- ❑ Manipulate calculations involving the instantaneous frequency, frequency deviation, peak frequency deviation, conversion gains and demodulated output voltage.
- ❑ Write the mathematical expression for the single-tone FM signal in non-series form.
- ❑ Determine the frequencies of the frequency components of a single-tone FM signal.
- ❑ Calculate the FM modulation index, m_f.
- ❑ Describe the effect of m_f on the FM spectrum.
- ❑ Distinguish between wideband and narrowband FM.
- ❑ Calculate the FM signal bandwidth by using Carson's rule.
- ❑ Calculate the power of an FM signal.
- ❑ Describe the advantages and disadvantages of WBFM and NBFM.

INTRODUCTION

In AM, information is conveyed by varying the peak amplitude of the carrier in proportion to the instantaneous amplitude of the modulating signal as shown in Figure 6.1.

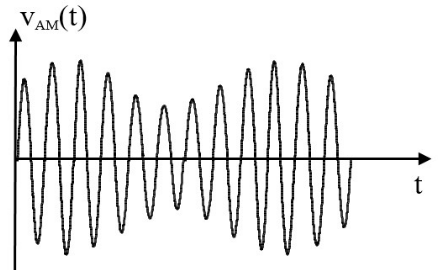


Figure 6.1 AM signal: Frequency constant, Amplitude varies.

In FM, the carrier amplitude is kept constant. Instead, information is conveyed by varying the frequency of the carrier signal in proportion to the instantaneous amplitude of the modulating signal as shown in Figure 6.2.

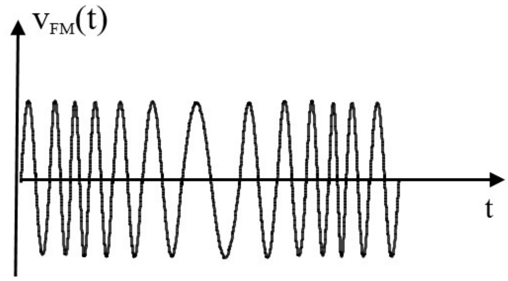


Figure 6.2 FM signal: Amplitude constant, Frequency varies.

6.1 BASIC CONCEPTS OF FM

FM signal is given by (refer to Appendix)

$$v_{FM}(t) = V_c \cos \theta(t) = V_c \cos 2\pi\left(f_c t + k_f \int_0^t v_s(\tau)d\tau\right) \quad (6.1)$$

where $V_c\cos(2\pi f_c t)$ is the unmodulated carrier signal and
$v_s(t)$ is the modulating signal.

The frequency of FM signal does not remain constant but varies in time. It is made to deviate from carrier $\mathbf{f_c}$ in proportion to the instantaneous value of the modulating signal $v_s(t)$. FM frequency at any instant in time is known as the **instantaneous frequency**, denoted as fi(t):

$$f_i(t) = f_c + k_f v_s(t) \quad (6.2)$$

k_f represents the amount of frequency change for a unit of amplitude of modulating signal with units of Hz/Volt. k_f relates frequency changes to instantaneous values of modulating signal $v_s(t)$. k_f is known as **conversion gain** of FM modulator. The instantaneous frequency of the FM signal varies from $f_c + k_f$[min $v_s(t)$] to $f_c + k_f$[max $v_s(t)$] ((min/max $v_s(t)$ –

minimum/maximum value of $v_s(t)$). When there is no modulating signal, $v_s(t)=0$ and the FM waveform becomes a pure unmodulated carrier.

FM signal is generated by a frequency modulator which essentially converts the voltage variation in modulating signal to frequency variation in the carrier signal as shown in Figure 6.3.

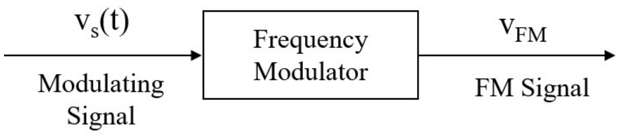


Figure 6.3 Frequency modulatior

6.2 SINGLE-TONE FM

A single-tone modulating signal (Figure 6.4(a)) is given by

$$v_s(t) = V_s \cos 2\pi f t = V s \cos \omega_s t \quad (6.3)$$

Substituting eq.(6.3) into eq.(6.1), we get the expression for single-tone FM signal shown in Figure 6.4(b) (refer to Appendix):

$$v_{FM}(t) = V_c \cos(\omega_c t + m_f \sin \omega_s t) \quad (6.4)$$

where m_f is the modulation index of the FM signal (defined in section 6.2.2)

The instantaneous frequency of single-tone FM (Figure 6.4(c)) is given by

$$f_i(t) = f_c + k_f v_s(t) = f_c + k_f V_s \cos \omega_s t \quad (6.5)$$

The following can be observed from Figure 6.4(c):

- From 0 to t_1, the frequency of FM signal = f_c as $v_s(t) = 0V$.
- From t_1 to t_2, the frequency of FM signal increases gradually from f_c to $f_c+k_fV_s$ following the changes in modulating signal $v_s(t)$.
- From t_2 to t_3, the frequency of FM signal decreases gradually from $f_c+k_fV_s$ to f_c as the modulating signal decreases.
- From t_3 to t_4, the frequency of FM signal further decreases from f_c to $f_c-k_fV_s$ as the modulating signal decreases to its minimum.
- From t_4 to t_5, the frequency of FM signal increases gradually from $f_c-k_fV_s$ to f_c.

The rate at which $f_i(t)$ changes is determined by the rate at which $v_S(t)$ changes. And, $f_i(t)$ changes in the same manner as $v_S(t)$ and thus has the same shape as $v_S(t)$. If $v_S(t)$ is sinusoidal, $f_i(t)$ is also sinusoidal.

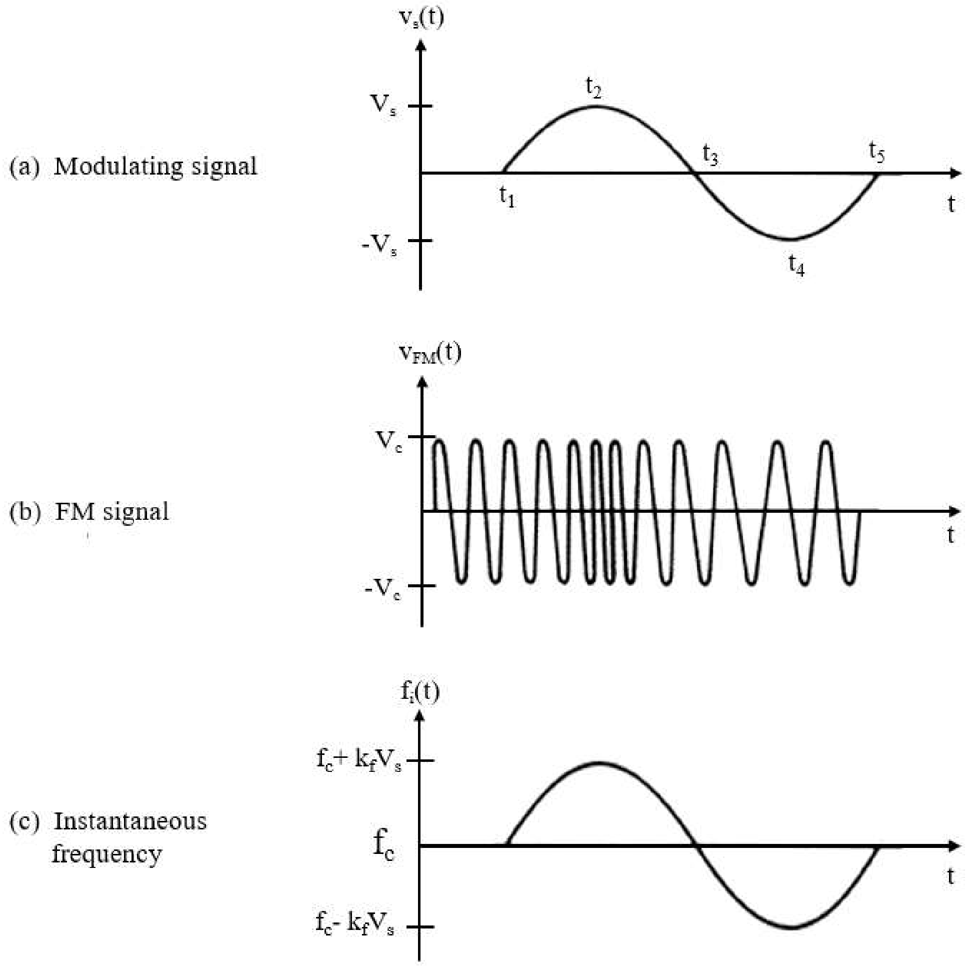


Figure 6.4 FM waveform

6.2.1 Frequency deviation

The amount of frequency change away from f_c at any instant in time is known as the **frequency deviation** and is proportional to the modulating voltage at that instant in time.

$$\textbf{frequency deviation} = k_f \times v_s(t) \tag{6.6}$$

The **peak frequency deviation** (the maximum frequency change on either side of carrier frequency f_c), denoted as Δ_f, is given by

$$\Delta_f = k_f \text{ x peak modulating voltage } = k_f V_S \tag{6.7}$$

Therefore,

$$\begin{aligned} f_{i(max)}(t) &= f_C + \Delta_f \\ f_{i(min)}(t) &= f_C - \Delta_f \end{aligned} \tag{6.8}$$

6.2.2 Frequency modulation index

For a single-tone modulating signal $v_S(t) = V_S\sin\omega_S t$, the **modulation index** of the FM signal is defined as:

$$m_f = \frac{\Delta_f}{f_S} \qquad (6.9)$$

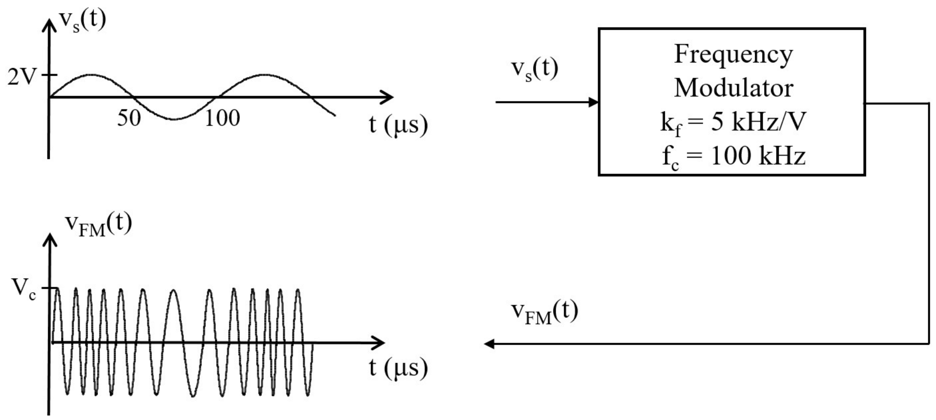
Note:

- The modulation index in AM, $m = V_S/V_C$ expresses the size of the envelope. However, in FM, the envelope is constant. m_f expresses the amount (Δ_f) and speed (f_S) of frequency change.
- A larger m_f means the amount of frequency change (Δ_f) is large or the frequency of the FM signal is changing slowly (f_S).
- In AM, m cannot be greater than 1 or else the envelope will overlap. In FM, Δ_f can be set independent of f_S and therefore m_f can be higher than 1.
- The only limitation for Δ_f is that it must not be larger than f_C or else $f_{i(min)} = (f_C - \Delta_f)$ will be a negative frequency.
- $f_S << f_C$ as f_C cannot change faster than itself.

<u>**Example** 6.1</u>

For the FM waveform shown below

(i) calculate the instantaneous frequency at t = 50 μs.
(ii) determine Δ_f.
(iii) determine $f_{i(max)}$ and specify when it occurs.
(iv) determine $f_{i(min)}$ and specify when it occurs.
(v) show how $f_i(t)$ changes with time.



Solution

(i) calculate the instantaneous frequency at t = 50 μs.

At t = 50 μs, $v_S(t) = 0$ V

$\therefore\ f_i = f_C = \underline{\underline{100 \text{ kHz}}}$

(ii) determine Δ_f.

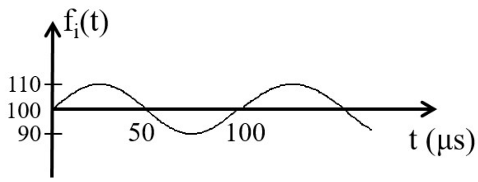
$\Delta_f = k_f V_S = 5 \times 2 = \underline{\underline{10 \text{ kHz}}}$

(iii) determine $f_{i(max)}$ and specify when it occurs.

$f_{i(max)} = f_C + \Delta_f = 100 + 10 = 110$ kHz
occurring at t = 25 μs and 125 μs

(iv) determine $f_{i(min)}$ and specify when it occurs.
$f_{i(min)} = f_C - \Delta_f = 90$ kHz
occurring at t = 75 μs

(v) show how $f_i(t)$ changes with time.



Note: Change in $f_i(t)$ has the same shape as $v_S(t)$. i.e. frequency modulation converts voltage change to frequency change.

6.2.3 Frequency spectrum of single-tone FM signal

When $v_S(t)$ is sinusoidal, the FM signal is written as

$$v_{FM}(t) = V_c \cos(\omega_c t + m_f \sin\omega_s t)$$

This expression can be expanded in Fourier series to obtain the single-sided spectrum of FM signal: (**Note**: only single-sided FM spectrum is considered in this Module)

$$
\begin{aligned}
v_{FM}(t) = V_c\{&J_0(m_f)\cos\omega_c t \\
&+ J_1(m_f)[\cos(\omega_c + \omega_s)t - \cos(\omega_c - \omega_s)t] \\
&+ J_2(m_f)[\cos(\omega_c + 2\omega_s)t + \cos(\omega_c - 2\omega_s)t] \\
&+ J_3(m_f)[\cos(\omega_c + 3\omega_s)t - \cos(\omega_c - 3\omega_s)t] \\
&+ J_4(m_f)[\cos(\omega_c + 4\omega_s)t + \cos(\omega_c - 4\omega_s)t] \\
&+ \ldots\ldots\}
\end{aligned} \tag{6.10}
$$

*Equation (6.10) is non-examinable.

where $J_n(m_f)$ are Bessel coefficients whose numerical values can be obtained from Bessel function table, given the modulation index. The value of J_n will be different for different values of m_f. Use of Bessel function table will not be covered in this module. Eq. (6.10) shows that a single-tone FM signal is composed of a carrier with an amplitude $J_0(m_f)V_C$ and a set of side-frequencies spaced symmetrically on both sides of the carrier at frequency separation of $\pm nf_S$ where n = 1, 2, 3, etc. So the side-frequency components consists of

- Infinite number of lower side frequencies of peak amplitudes, $J_n(m_f)V_C$ and frequencies, $(f_C - nf_S)$ where n = 1, 2, 3 to ∞.

- Infinite number of upper side frequencies of peak amplitudes, $J_n(m_f)V_C$ and frequencies, $(f_C + nf_S)$ where n = 1, 2, 3 to ∞.

Figure 6.5 plots the single-sided spectra of single-tone FM signals for m_f=0.2, 1 and 13 respectively. The vertical lines represent the magnitudes only of the frequency components of significant terms. The following observations can be made from Figure 6.5:

- The number of side frequencies increases with m_f, but not proportionally, i.e. if m_f increases 13 times, the number of side frequencies does not increase by 13 times.

- The amplitude levels of side-frequency components tend to decrease as these are away from the carrier frequency component.

- Bandwidth of FM signal is an EVEN multiple of f_S and thus increase as f_S.

- Theoretically, FM consists of infinitive number of side-frequency components and the bandwidth required for transmitting FM signal is infinite.

As higher order side-frequencies tend to have very small amplitudes (see Figures 6.5), they can be ignored in practice. So the **significant bandwidth** of an FM signal is finite. The significant bandwidth can be defined as the spacing between the two frequencies beyond which none of the side-frequency components has significant amplitude levels.

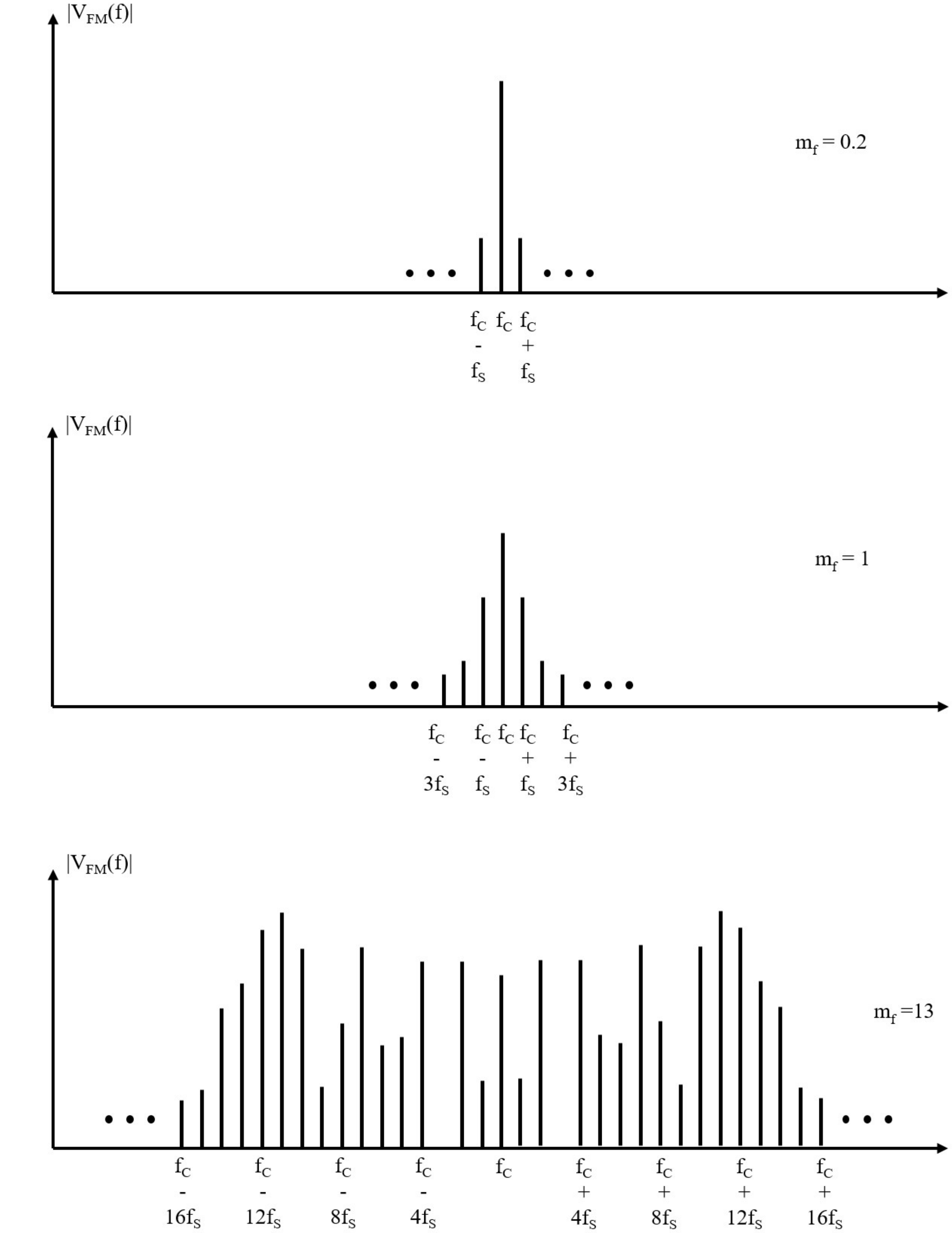


Figure 6.5 Single-sided Spectra of FM signal for $m_f = 0.2, 1, 13$

6.2.4 Carson's Rule for FM bandwidth

Carson's rule approximates the bandwidth required to transmit FM signal. It estimates the required bandwidth as twice the sum of the peak frequency deviation and the frequency of modulating signal.

$$\text{Estimated } B_{FM} \approx 2(m_f + 1)f_s \approx 2(\Delta_f + f_s) \quad (6.11)$$

Eq.(6.11) is valid for integer values of m_f only. This is because B_{FM} is an even multiple of f_S.

6.3 WIDEBAND AND NARROWBAND FM

Eq. (6.11) shows that B_{FM} changes with m_f. If m_f decreases, B_{FM} decrease as well. However B_{FM} will only decrease up to a certain minimum value as the simplest FM spectrum consists of one carrier component and two side-frequency components. The bandwidth of the simplest FM signal is thus $(f_C+f_S) - (f_C-f_S) = 2f_S$, which is the minimum FM bandwidth (see Figure 6.6).

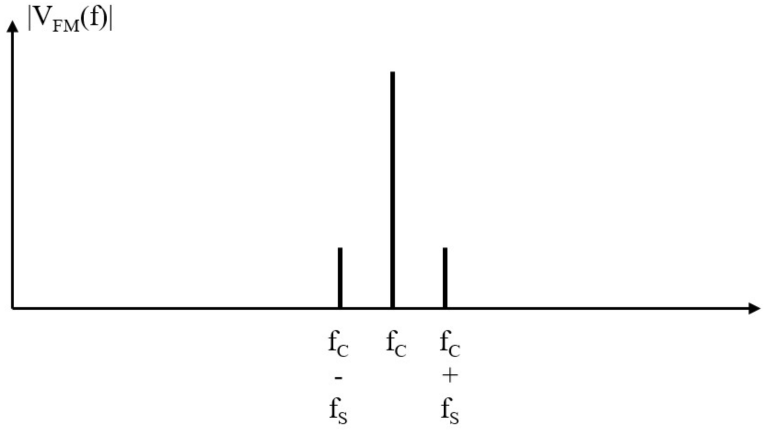


Figure 6.6 Frequency Spectrum of a Narrowband FM Signal($m_f < 0.5$)

It can be proved that when m_f decreases to less than 0.5, $B_{FM} = B_{FM(min)} = 2f_S$.

FM systems with **m_f < 0.5** are referred to as **Narrowband FM (NBFM)** systems.

Conversely, FM systems with **$m_f \geq 0.5$** are known as **wideband FM (WBFM)** systems because of their larger bandwidths.

For $m_f < 0.5$, i.e. narrowband FM, $B_{FM} = B_{FM(min)} = 2f_S = B_{AM}$

For $m_f \geq 0.5$, i.e. wideband FM, $B_{FM} > B_{AM}$

Note:

- The bandwidth of an FM signal increases with m_f but the bandwidth of an AM signal is not dependent on modulation index.

	NBFM	WBFM
m_f	<0.5	≥ 0.5
B_{FM}	$2f_S$	$>2f_S$

NBFM is widely used because it has the first three advantages of WBFM (covered in section 6.9) but does not have the large bandwidth of WBFM.

6.4 FM DEMODULATOR

A frequency demodulator recovers the information contained in the FM signal by converting the frequency variation in the FM signal to a voltage variation with a certain conversion gain k_d. It works as following:

When the instantaneous frequency $f_i(t)$ equals to f_C, the output voltage, $v_o(t)$, is 0 V.
When $f_i(t)$ increases above f_C, $v_o(t)$ will increase above 0 V.
When $f_i(t)$ decreases below f_C, $v_o(t)$ will decrease below 0 V. And,

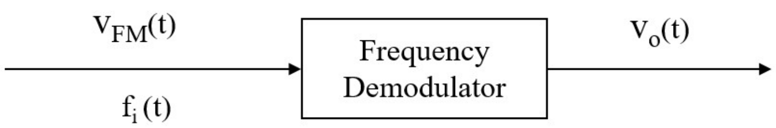
$$v_o(t) = k_d[f_i(t) - f_c] \tag{6.12}$$

$$f_i(t) = f_C + k_f v_S(t)$$

i.e. $$f_i(t) - f_C = k_f v_S(t)$$

Therefore, $$v_O(t) = k_d k_f v_S(t)$$

i.e. $$v_O(t) \propto v_S(t) \tag{6.13}$$



Convert frequency change
to voltage change
Conversion gain k_d (V/Hz)

The conversion gain of a frequency demodulator, k_d (V/Hz) tells us how much the output voltage changes for a given change in input frequency. If $k_d = 0.1$ V/kHz, it means the output voltage will change by 0.1 V for every 1 kHz change in $f_i(t)$.

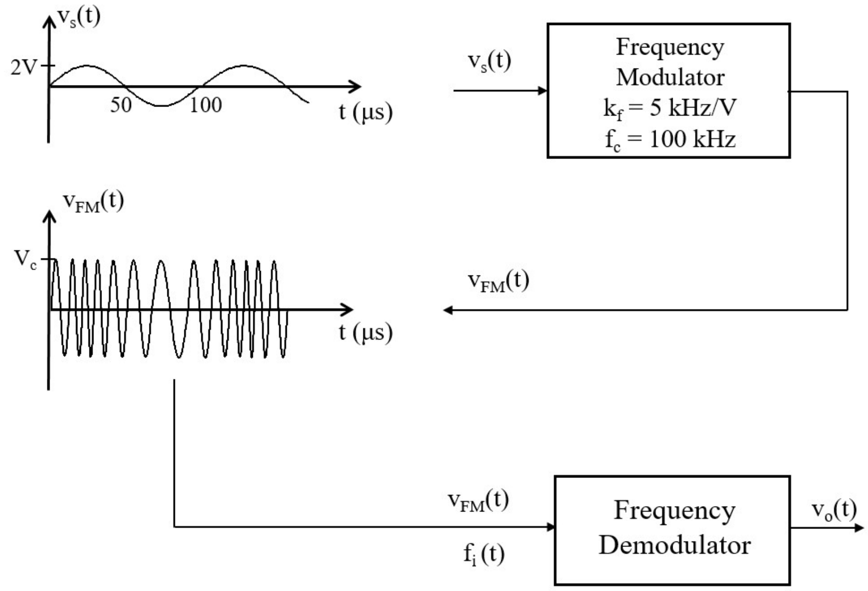
Eq.(6.13) shows that the output of the demodulator $v_O(t)$ has the same shape as the modulating signal. Thus, the modulating signal is successfully recovered. Note that it is not necessary that $v_O(t)$ be exactly the same size as $v_S(t)$. As long as $v_O(t)$ has the same shape as $v_S(t)$ (i.e. $v_O(t) \propto v_S(t)$), it is sufficient for anolog signals. This is because the size of the analog output can be adjusted by using a volume control. And,

$$v_{O(max)} = k_d \Delta_f \tag{6.14}$$

$$v_{O(min)} = - k_d \Delta_f \tag{6.15}$$

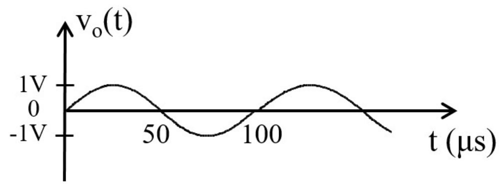
Example 6.2

Sketch the demodulated output waveform if $k_d = 0.1$ V/kHz for the FM system shown below.



Solution

$v_O(t) \propto v_S(t)$, i.e. $v_O(t)$ has the same shape as $v_S(t)$ as below:



From Equation (6.14), $v_{O(max)} = k_d \Delta_f$

From Example 6.1, $\Delta_f = 10$ kHz

Therefore,

$v_{O(max)} = 0.1$ V/kHz x 10 kHz = $\underline{\underline{1\text{ V}}}$

From Equation (6.15), $v_{O(min)} = -\underline{\underline{1\text{ V}}}$

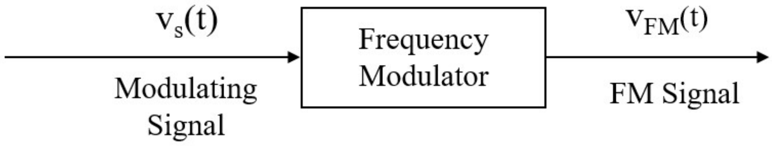
6.5 MULTI-TONE FM SIGNAL

Modulating signal is usually a complex signal. When a carrier is frequency modulated by a complex modulating signal, it is known as **multi-tone FM signal**. We use examples to demonstrate modulation and demodulation of multi-tone FM in the following sections.

6.5.1 Modulation of multi-tone FM signal

Example 6.3

A carrier of frequency of 3 MHz is frequency modulated by a modulating signal shown in Figure 6.7(a). The conversion gain, k_f=1 MHz/V. Plot the waveform of FM signal $v_{FM}(t)$ and the instantaneous frequency $f_i(t)$.



Solution

The FM modulator converts voltage change to frequency change:

$$f_i(t) = f_c + k_f\, v_s(t) \text{ where conversion gain, } k_f = 1 \text{ MHz/V;}$$

When $v_s(t) = 0$, output frequency, $f_i = f_c = 3$MHz.

Thus, between 0 to 1 μs, $v_s(t) = 0$V, the frequency of the FM output, $f_i(t) = f_c$ which is 3 MHz.

After 1 μs, $f_i(t)$ increases as the modulating signal, $v_s(t)$. It reaches its maximum, $f_{i(max)}$ between 2 to 3 μs:

$$f_{i(max)} = f_c + \Delta_f = f_c + k_f V_s = 3 + 1\text{MHz/V x 2V} = 5\text{MHz}$$

After 3 μs, $f_i(t)$ decreases as $v_s(t)$ and reaches its minimum, $f_{i(max)}$ between 5 to 6 μs:

$$f_{i(max)} = f_c - \Delta_f = f_c - k_f V_s = 3 - 1\text{MHz/V x 2V} = 1\text{MHz}$$

After 7 μs, $f_i(t)$ is 3 MHz as $v_s(t) = 0$V.

Between 1 to 2 μs and 3 to 4 μs, $v_s(t)$ is 1V and $f_i(t)$ becomes 4 MHz:

$$f_i(t) = f_c + k_f v_s(t) = 3 + 1\text{MHz/V x } 1\text{V} = 4\text{MHz}$$

Between 4 to 5 μs and 6 to 7 μs, $v_s(t)$ is -1V and $f_i(t)$ becomes 2 MHz:

$$f_i(t) = f_c + k_f v_s(t) = 3 + 1\text{MHz/V x}(-1)\text{V} = 2\text{MHz}$$

The waveform of the FM signal $v_{FM}(t)$ is shown in Figure 6.7(b). The instantaneous frequency is shown in Figure 6.7(c). The instantaneous frequency has the same shape as that of Figure 6.7(a) as expected. This is because in FM, the voltage variation in the modulating signal is converted to a frequency variation in the FM signal. Thus, unlike AM where the information is contained in the amplitude variation of the AM signal, in FM, the information is contained in the frequency variation of the FM signal. The frequency of FM signal is changing as modulating signal but its amplitude is constant.

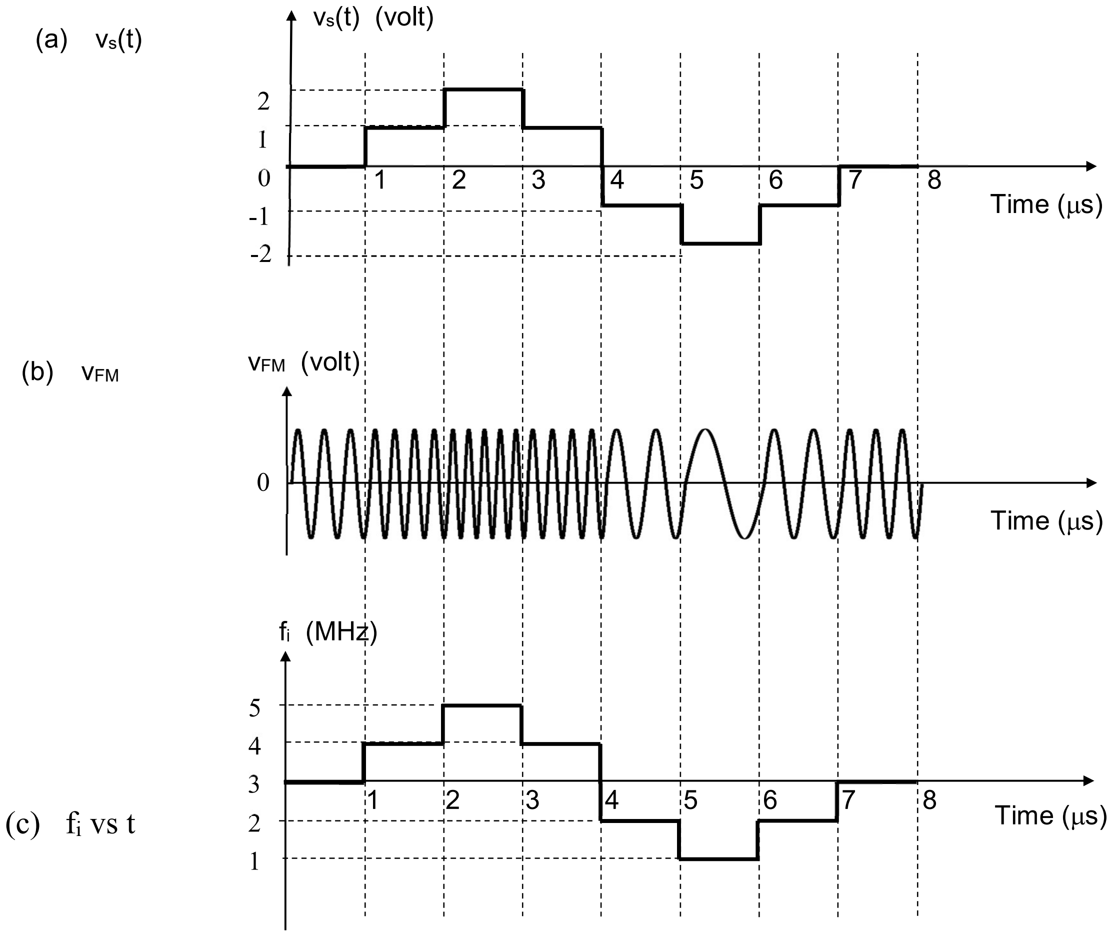
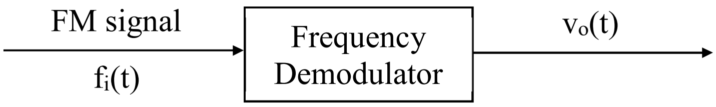


Figure 6.7 Frequency modulation with complex modulating signal

6.5.2 Demodulation of Multi-tone FM signal

Example 6.4

At FM demodulator shown in Figure 6.8, the modulating signal is recovered by converting the frequency variation to a corresponding voltage variation. The conversion gain, k_d = 1 V/MHz. Plot the recovered modulating signal if the input signal of FM demodulator is the FM signal from example 6.3.



Converts Frequency change to Voltage change.

When $f_i(t) = f_C$, $V_o(t) = 0V$

Figure 6.8 Frequency Demodulation

Solution

The FM demodulator converts frequency variation to voltage variation:

$v_o(t) = k_d[f_i(t) – f_c]$ where conversion gain, k_d = 1 MHz/V;

The recovered modulating signal is shown in Figure 6.9 which is obtained as following:

When $f_i(t) = f_c$, output voltage, $v_o(t) = 0V$.

Thus, between 0 to 1 μs, $v_o(t) = 0V$, as $f_i(t) = f_c = 3$ MHz.

Between 1 and 2 μs, $v_o(t)$ increases as $f_i(t)$:

$$v_o(t) = k_d[f_i(t) – f_c] = 1V/MHz \times (4 - 3) MHz = 1V$$

$v_o(t)$ reaches its maximum, $v_{o(max)}$ between 2 to 3 μs:

$$v_{o(max)} = k_d(f_{i(max)} – f_c) = 1V/MHz \times (5 - 3) MHz = 2V$$

Between 3 and 4 μs, $v_o(t)$ decreases as $f_i(t)$:

$$v_o(t) = k_d[f_i(t) – f_c] = 1V/MHz \times (4 - 3) MHz = 1V$$

Between 4 and 5 μs, $v_o(t)$ decreases further as $f_i(t)$:

$$v_o(t) = k_d(f_i(t) – f_c) = 1V/MHz \times (2 - 3) MHz = -1V$$

$v_o(t)$ reaches its minimum, $v_{o(min)}$ between 5 to 6 μs:

$$v_{o(min)} = k_d(f_{i(min)} - f_c) = 1V/MHz \times (1 - 3)\ MHz = -2V$$

After 7 μs, $v_o(t) = 0V$ as $f_i(t) = 3$ MHz.

The recovered signal is an exact replica of the modulating signal sent at the transmitter as expected.

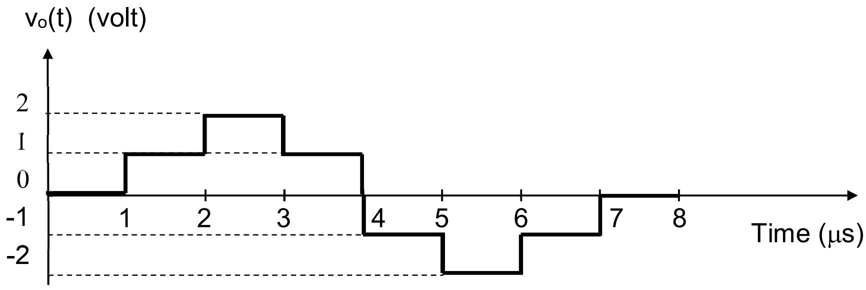


Figure 6.9 Recovered in FM

6.5.3 Bandwidth of multi-tone FM

Multi-tone modulating signal contain a number of sinusoidal components. The frequency of the highest frequency component is denoted as f_H. If Δf is the same for all the frequency components, the bandwidth of multi-tone FM may be <u>estimated</u> using f_H as

$$B_{FM} \approx 2(m_{f_H} + 1)f_H \tag{6.16}$$

$$\approx 2(\Delta f + f_H)$$

where $m_{f_H} = \dfrac{\Delta f}{f_H}$ rounded to the next highest integer.

6.6 TRANSMITTED POWER IN FM

The peak amplitude of FM signal is constant and equals to V_C (see Figure 6.10). The FM waveform is said to have a constant envelope.

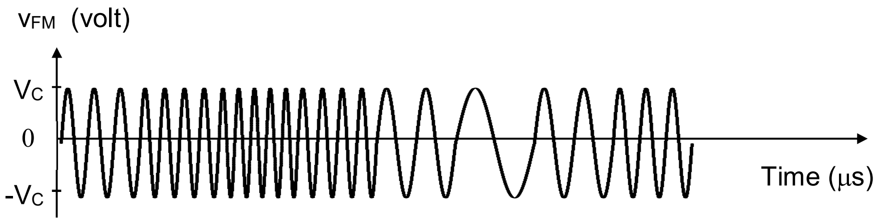


Figure 6.10 FM Signal

FM waveform is a sinusoidal signal whose frequency is changing. Frequency does not affect the power of a sine wave. Hence, the transmitted power is simply given by

P_{FM} = Power of a sine wave = $(V_{rms})^2/R_L$

$$P_{FM} = \frac{V_c^{\ 2}}{2R_L} \tag{6.17}$$

where V_C is the peak value and R_L is the load in which the FM signal is dissipated.

As it can be seen from Eq.(6.17), the power of an FM signal is constant.

In Singapore, the transmitted power for FM broadcasting is 10 kW. In other countries, it generally does not exceed 50 kW.

6.7 ADVANTAGES & APPLICATIONS OF FM

- Demodulated o/p level is independent of FM level (NBFM and WBFM)

In FM, the information is contained in the frequency variation of the FM signal. Fluctuations in the received FM level will not affect the demodulated output level, which is illustrated in Figure 6.11. This makes FM suitable for mobile applications, where mobility results in variations in received signal strength. For AM, fluctuations in the received signal level will cause variations in the audio output volume. But this will not happen in FM.

- No need to transmit at high power (NBFM and WBFM)

Since the demodulated output level is not dependent on the received FM level, there is no need to ensure a high level FM signal at the receiver. Hence transmission power can be lower than the AM to cover the same area. This makes FM suitable for battery operated transmitted like cordless phones, wireless microphone and Walkie-Talkies.

- Good noise immunity (WBFM only)

At the transmitter, the FM modulator converts the voltage variation in the modulating signal to a frequency variation. The maximum frequency variation is denoted as Δ_f and occurs at the peak voltage of the modulating signal.

At the receiver, this frequency variation is converted back to a voltage variation. If the frequency variation is large, then the output voltage will also be large.

Thus the use of a large Δ_f in FM transmission produces a large demodulated output, resulting in a high SNR at the receiver's output. Conversely, a low Δ_f will result in a smaller and hence noisier output.

Since NBFM has lower Δ_f, the noise immunity of NBFM is not as good as the WBFM.

Increasing Δ_f in FM does not increase V_C. This means, higher SNR at the receiver output can be achieved without increasing transmission power! This is unlike other modulation systems like AM, DSBSC and SSB where higher SNR can only be obtained by increasing transmission power.

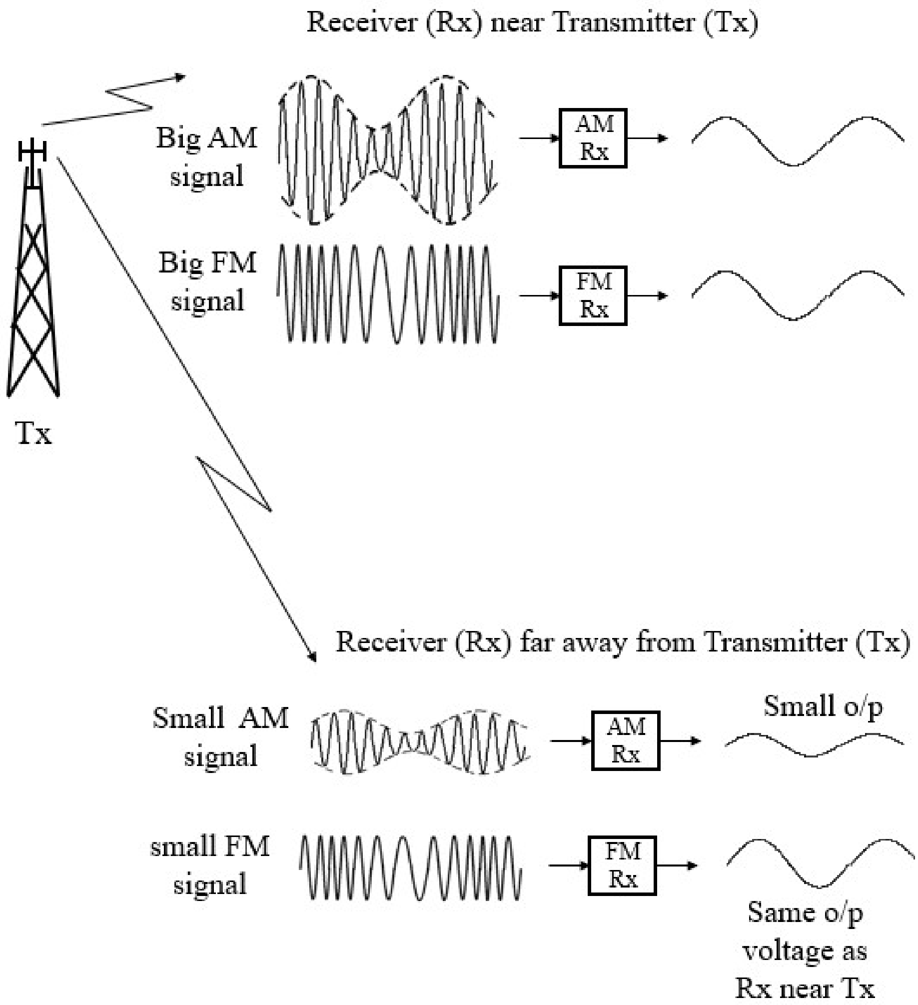


Fig. 6.11 Demodulated O/P level is independent of FM level**.**

For this reason, FM has its use in applications where there is a requirement for very good noise immunity without the need to transmit at high power. Examples of such applications are cordless phones, wireless microphones and walkie-talkies where high transmission power is not possible because of battery operation. Other applications are radio-cab, police and military radio where a high immunity to noise from the vehicle's ignition system is required.

FM is also used in these systems because any fluctuation in the received FM level as the receiver moves about does not affect the demodulated output level.

Although it is true that in AM, higher SNR can also be obtained by increasing m, there is a limit on how high m can be. In any case, increasing m will also increase transmission power.

6.8 Disadvantage of FM (WBFM only)

The noise immunity of FM however comes with a disadvantage. That is, increasing Δ_f will increase the bandwidth of the FM signal since $B_{FM} = 2(\Delta f + f_H)$. Thus a higher transmission bandwidth is incurred when FM is used. This increase the cost of transmission.

Appendix

Frequency modulation (FM) is a modulation process in which the instantaneous frequency of the carrier signal varies in proportion to the instantaneous values of the modulating signal. FM waveform is generally expressed as

$$v_{FM}(t) = V_c \cos \theta(t) \qquad \text{(A6.1)}$$

where V_c is a constant.

To understand FM, it is very important to understand what is meant by frequency when the frequency is varying with time. The **instantaneous frequency** $f_i(t)$ of $v_{FM}(t)$ is defined as the rate of change of its phase angle:

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta}{dt} \quad \text{(Hz)} \qquad \text{(A6.2)}$$

And

$$\theta(t) = 2\pi \int_0^t f_i(\tau)d\tau \qquad \text{(A6.3)}$$

In FM, the instantaneous frequency of the carrier signal is made to deviate from carrier f_c in proportion to the Instantaneous value of the modulating signal $v_s(t)$. That is

$$f_i(t) = f_c + k_f v_s(t) \qquad \text{(A6.4)}$$

k_f represents the amount of frequency change for a unit of amplitude of modulating signal with units of Hertz/Volt.

So the phase θ(t) of FM signal is

$$\theta(t) = 2\pi \int_0^t f_i(\tau)d\tau = 2\pi \int_0^t [f_c + k_f v_s(\tau)]d\tau = 2\pi\left(f_c t + k_f \int_0^t v_s(\tau)d\tau\right) \qquad \text{(A6.5)}$$

The FM signal given by eq.(A.1) is thus expressed as

$$v_{FM}(t) = V_c \cos\theta(t) = V_c \cos 2\pi\left(f_c t + k_f \int_0^t v_s(\tau)d\tau\right) \qquad \text{(A6.6)}$$

For single-tone FM, where the modulating signal is a cosine wave, $v_s(t)=V_s\cos 2\pi f_s t$, we have

$$f_i(t) = f_c + k_f V_s \cos 2\pi f_s t \qquad \text{(A6.7)}$$

Thus, FM signal modulated by a sinusoidal modulating signal is given by

$$
\begin{aligned}
v_{FM}(t) &= V_c\cos 2\pi \left(f_c t + k_f \int_0^t V_s \cos 2\pi f_s \tau \, d\tau \right) \\
&= V_c \cos 2\pi \left(f_c t + \frac{k_f V_s}{2\pi f_s} sin 2\pi f_s t \right) \\
&= V_c \cos (2\pi f_c t + \frac{k_f V_s}{f_s} sin 2\pi f_s t \\
&= V_c \cos (2\pi f_c t + m_f sin 2\pi f_s t \qquad \text{(A6.8)}
\end{aligned}
$$

where $m_f = \frac{k_f V_s}{f_s}$ is called FM modulation index.

Chapter 7

Analog to Digital Conversion

Learning Outcomes

- ❑ Explain the need for sampling.
- ❑ State the sampling theorem.
- ❑ Determine the Nyquist sampling rate and sampling intervals.
- ❑ Describe the aliasing effect and the means to mitigate it.
- ❑ Differentiate among the ideal, natural and flat-top sampling techniques.
- ❑ Understand the effect of oversampling and undersampling.
- ❑ Appreciate the role of quantization.
- ❑ Outline the basics of uniform quantization.
- ❑ Determine the quantization noise power for a uniform quantizer.
- ❑ Determine the signal-to-quantization noise ratio for sinusoidal signal for a uniform quantizer.
- ❑ Describe the need for nonuniform quantization techniques.
- ❑ Comprehend all elements involved in the pulse-code modulation (PCM).
- ❑ Explain the operation of a PCM-TDM system.
- ❑ Design PCM-TDM systems.
- ❑ Describe the operation of a DPCM system.
- ❑ Understand the operation of PCM systems

INTRODUCTION

We have studied mainly analog communications in the previous chapters. For the rest of the chapters, we cover signal transition using digital communication systems. Audio and video signals, among the most common information signals, are analog and not compatible with digital processing and signal transmission of digital communication systems, though a lot of information in modern communications is in digital form such as binary data in computer programs and computer communication systems. In this chapter, we focus on analog-to-digital conversion (ADC).

7.1 SAMPLING

The transition from analog to digital communication requires conversion of analog signals to digital signals. Sampling is the first step towards the conversion from analog to digtal communications and the bridge between analog and digital signals.

Sampling process converts an analog signal into a corresponding train of samples that are spaced uniformly in time. Under certain circumstances, it is possible for an analog signal to be completely represented by its instantaneous samples (values) taken at equal and sufficiently short intervals in time. The original continuous signal can then be fully recovered from the train of samples by processing them through an appropriate filter.

Reasons for sampling an analog signal

There are a number of reasons why we sample analog signals:

- To convert an analog signal into a digital signal that is compatible with digital transmission. For example, PCM transmission.

- To allow an analog signal to be digitally processed.

- To allow TDM (time division multiplexing) which is the simultaneous transmission of several signals over the same channel.

- Certain signal processing devices (for example high-power microwave tubes; laser) can operate better on a pulse basis.

- Reduction in power needed to transmit a signal as with sampling, signal is transmitted in burst.

7.1.1 Ideal sampling

Ideally, the sampling of a signal x(t) is considered as the multiplication of x(t) with a periodic unit impulse train p(t), which takes instantaneously the signal value without any modification. This sampling process is refered as impulse sampling, or ideal sampling.

The unit impulse train $p(t)$ has a period, T which is the sampling interval. The fundamental frequency of $p(t)$, $f_s = 1/T$ is referred as the sampling frequency.

In time domain, ideal/impusle sampling can be modelled as in Figure 7.1 and expressed as

$$x_p(t) = x(t).p(t) \tag{7.1}$$

$$\text{where } p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

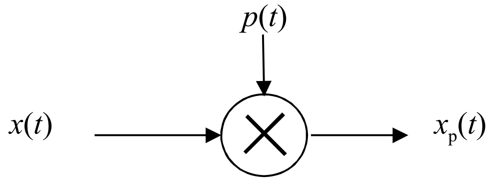


Figure 7.1 Model of Impulse sampling

$x_p(t)$ is an impulse train with the strength of the impulses equal to the values of $x(t)$ at the sampling instants, at intervals of T, as shown in Figure 7.2 (c) and expressed as.

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT) \tag{7.2}$$

From the frequency convolation property of th Fourier transform mentioned in chapter 2, the time domain product of $x_p(t)$ can be transformed to the freqeuncy domain convolution as

$$X_p(f) = X(f) * P(f)$$

where $*$ denotes convolution

The Fourier transform of the impulse train is given as

$$P(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta(f - kf_s)$$

The convolution of any function with an impulse function simply shifts that function, that is

$$X_p(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(f - kf_s) \tag{7.3}$$

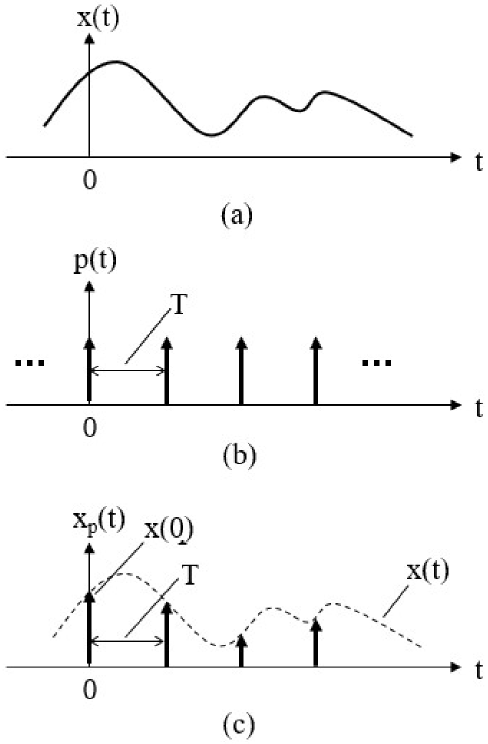


Figure 7.2 Time domain representation of impulse sampling: (a) analog signal x(t), (b) impluse train, (c) sampled version of x(t)

Hence, $X_p(f)$ is a periodic function of frequency consisting of a sum of **shifted replicas** of $X(f)$, scaled by $1/T$. Assume x(t) is bandlimited siganal with spectrum given as in Figure 7.3(a), where f_m is the highest frequency of the signal $x(t)$. The frequency domain representation of impulse sampling under different f_m is shown in Figure 7.3 (c) and (d).

The following can be observed from Figure 7.3 (c) and (d):

- When $f_s > 2f_m$, that is $(f_s - f_m) > f_m$ there is no overlapping between the shifted replicas of $X(f)$. $X(f)$ is reproduced at integer multiples of the sampling frequency. Consequently, the original signal $x(t)$ can be recovered exactly from the sampled signal, $x_p(t)$ by passing it through an ideal LPF with constant gain of T and cut-off frequency f_c in the range, $f_m < f_c < (f_s - f_m)$. The output signal will exactly be equal to $x(t)$.

- When $f_s < 2f_m$, that is $(f_s - f_m) < f_m$ there is overlap and the original signal cannot be recovered from the sampled signal. Such overlap of replicas of $X(f)$ is known as **aliasing**.

The above observations form the basis of sampling theorem.

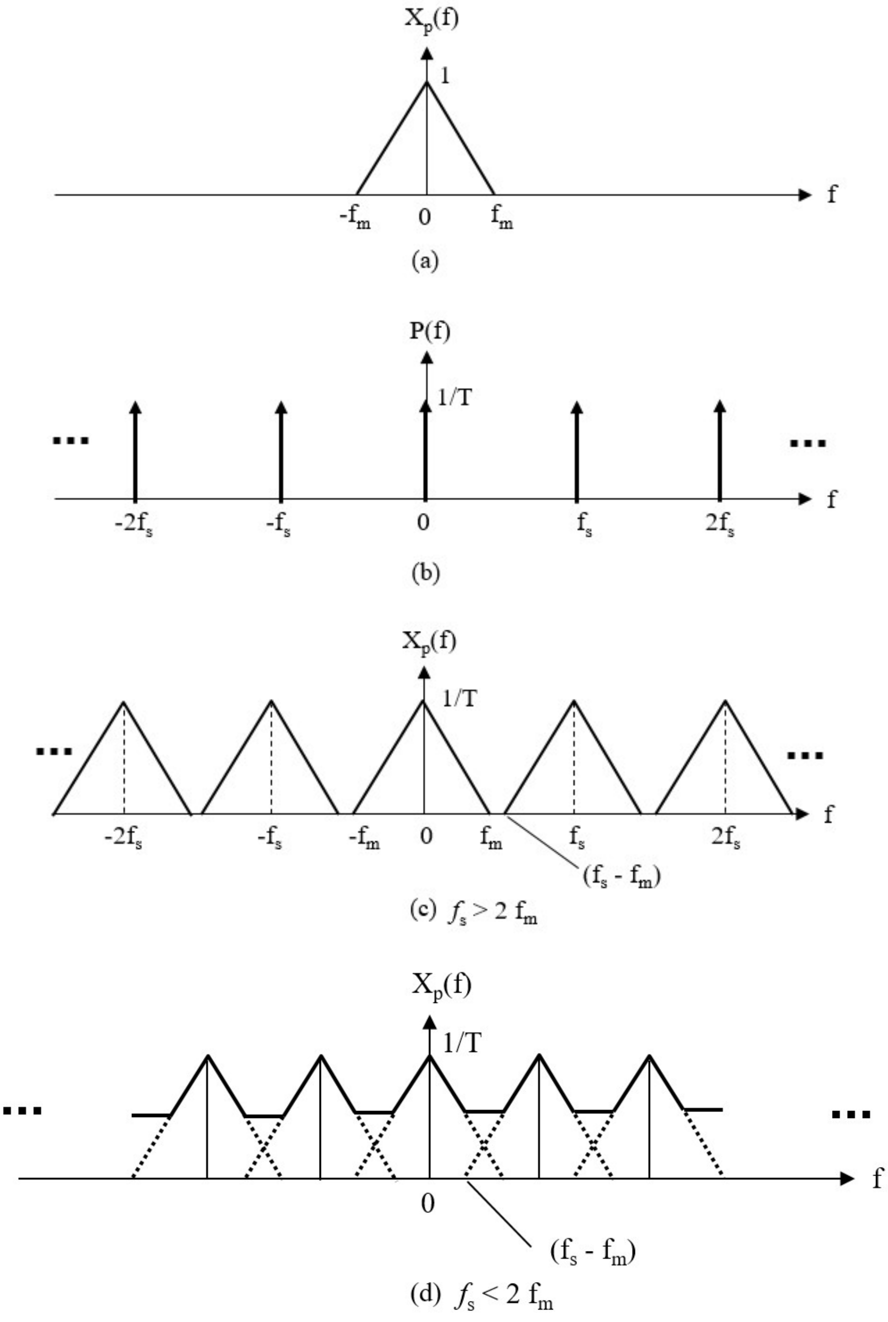


Figure 7.3 Frequency Domain representation of impulse sampling:
(a) spectrum of signal x(t),
(b) spectrum of impulse train,
(c) spectrum of sampled signal when $f_s > 2f_m$,
(d) spectrum of sampled signal when $f_s < 2f_m$.

Uniform Sampling Theorem I

For bandlimited baseband signals

If a baseband signal $x(t)$ is **bandlimited** to f_m, it must be sampled at a sampling frequency $f_s \geq 2f_m$, for it to be completely recovered from its samples.

Important note

f_m must be well defined to avoid the occurrence of aliasing. For this reason, it is common in communication system design, to perform an anti-aliasing filtering before any sampling operation. For example in telephony, speech is bandlimited to 3.4kHz using a LPF with cut-off frequency of 3.4 kHz, even though there can be frequency components in the 7kHz range or higher.

Nyquist frequency and Nyquist rate

The minimum sampling frequency, $f_s = 2f_m$ is referred to as **Nyquist Frequency** or **Nyquist Rate** and minimum sampling interval, $T_s = 1/2f_m$ as **Nyquist Interval**.

The effect of using Non-ideal reconstruction filter

In practice, it is very difficult to build filters with a very sharp roll-off at the cut-off frequency. As shown in Figure 7.4, some spurious frequency components are allowed by the reconstruction filter to reach the output. Note that these components are considerably attenuated compared to the baseband signal. To minimise these spurious frequencies, a higher sampling frequency should be used.

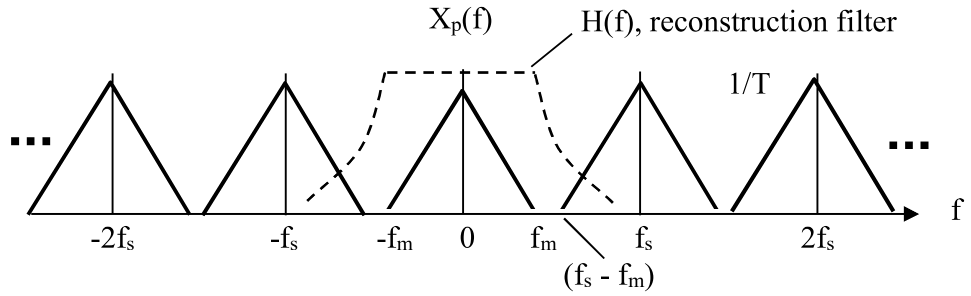


Figure 7.4 Effect of using non-ideal reconstruction filter

Example 7.1

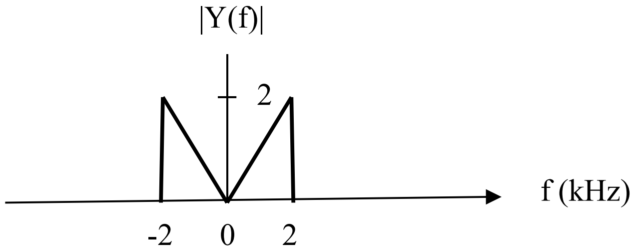
A baseband signal has frequency components up to 3.6 kHz. What is the minimum sampling frequency to ensure that the original signal can be recovered from its samples?

Solution

Using sampling theorem I, min. sampling frequency = $2f_m$ = 2 x 3.6 kHz = 7.2 kHz.

Example 7.2

A signal y(t) with a amplitude spectrum shown below is sampled at 5 kHz by an ideal unit impulse train, p(t). Sketch the amplitude spectrum of the sampled signal.



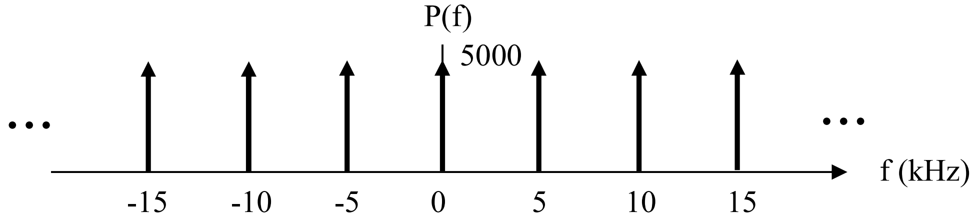
Solution

Sampled signal, $y_p(t) = y(t) \times p(t)$

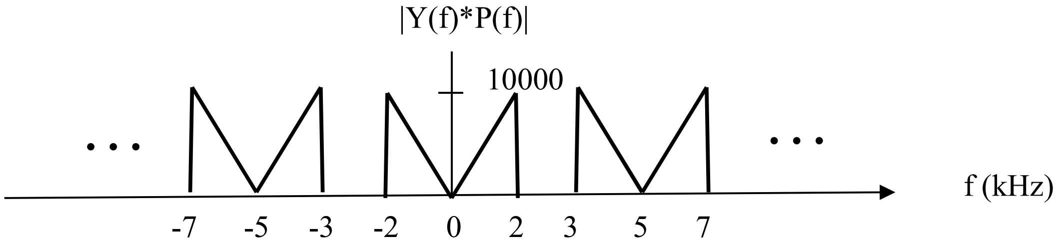
From the frequency convolution theorem, amplitude spectrum of sampled signal

$$Y_p(f) = Y(f)*P(f).$$

The amplitude spectrum of P(f) is



Hence amplitude spectrum of sampled signal is given by

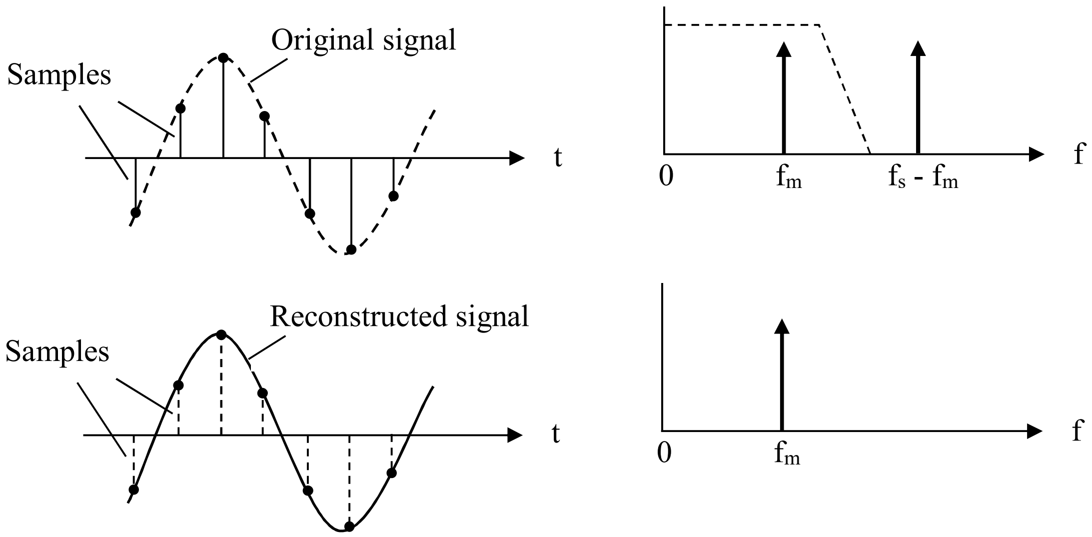


7.1.2 The effects of oversampling and undersampling

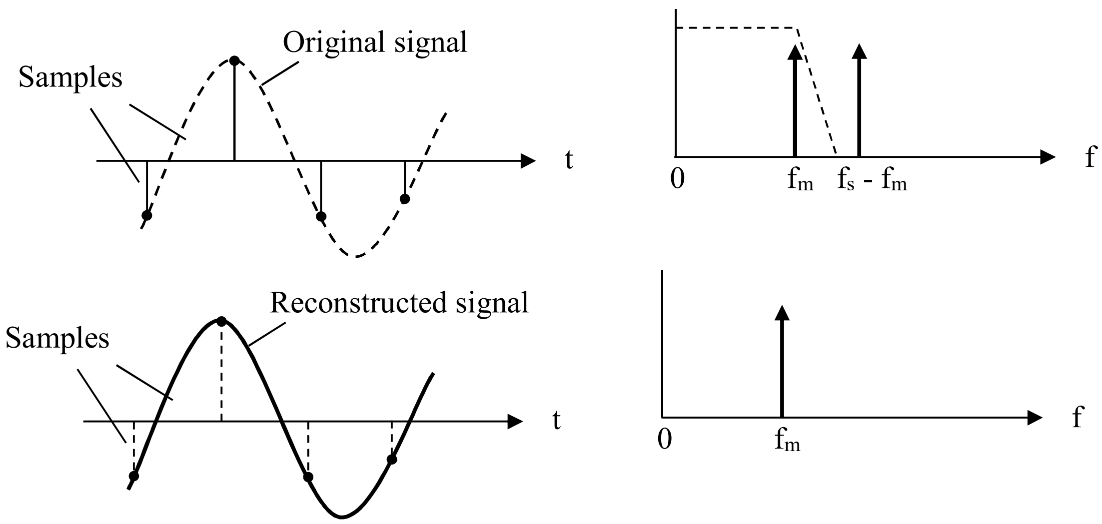
Consider $x(t)$ as a pure sinusoidal signal of frequency f_m. Assume also an ideal reconstruction LPF with cut-off frequency, f_c between f_m and (f_s - f_m). We consider two cases of oversampling as shown in Figure 7.5 (a) and (b).

From Figure 7.5, in either case the reconstructed waveform is identical to the original. It can be seen that the original signal is fully recovered whether the sampling frequency $f_s >> 2f_m$ or $f_s > 2f_m$.

It is therefore wasteful to sample a signal at too high rate, since it increases the bandwidth requirement for the same task without any gain as most of the samples transmitted are redundant and do not carry any extra information about the signal.



(a) $f_s >> 2f_m$



(b) $f_s > 2f_m$

Figure 7.5 Effect of oversampling: (a) $f_s >> 2f_m$, (b) $f_s > 2f_m$

If $f_s < 2f_m$, the signal is undersampled and an effect known as **aliasing** occurs. As shown in Figure 7.6, the reconstructed signal takes on an identity of a lower frequency signal, which is the difference between f_s and f_m, ie (f_s - f_m). The original signal cannot be recovered from its samples.

In the case when $f_s = f_m$, the reconstructed signal is a constant (i.e. dc). This is consistent with the fact that when sampling once per cycle of the original signal, the samples are all equal. The result is as though a dc signal was being sampled.

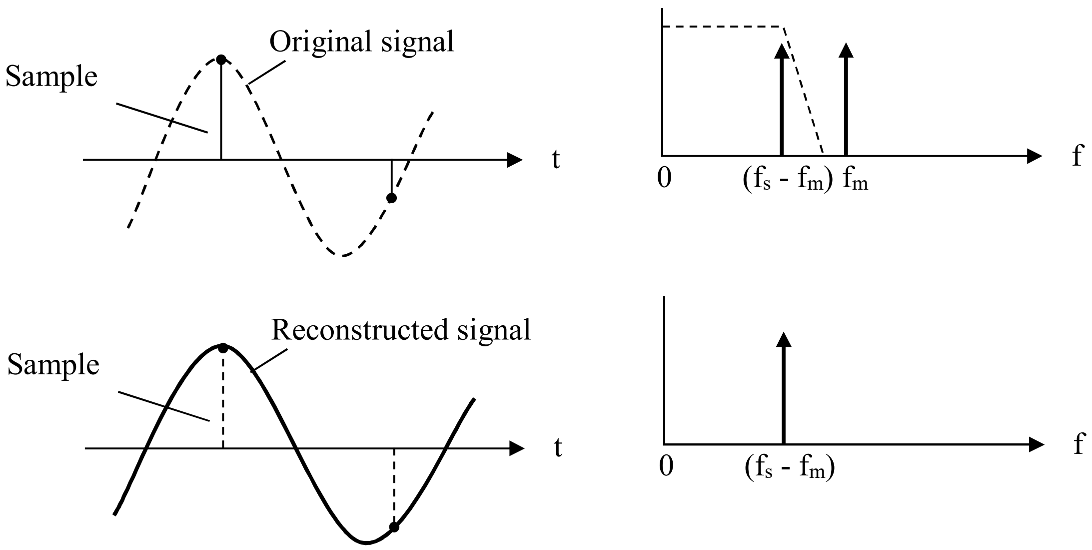


Figure 7.6 Undersampling, $f_s < 2f_m$,

Aliasing

Aliasing (or sometimes referred to as spectral folding) causes higher frequencies to be translated as lower frequencies in the recovered signal. In voice transmission, aliasing will cause serious degradation in the intelligibility.

Aliasing can occur under two conditions:

1. When the signal to be sampled is not bandlimited as shown in Figure 7.7.

2. When the bandlimited signal is sampled at a frequency less than twice the highest frequency present (Undersampling) as shown in Figure 7.8.

To remedy the problem in case 1, anti-aliasing filter is used to bandlimit the signal before sampling.

Based on above discussion, we may conclude that the uniform sampling theorem allows us to completely reconstruct a bandlimited signal from instantaneous samples taken at a rate of at least twice the highest frequency present in the signal.

The signal can be totally recovered by processing the samples with an ideal LPF having a bandwidth equal to the highest frequency present in the signal.

On the other hand, if the sampling theorem conditions are not satisfied, aliasing will occur.

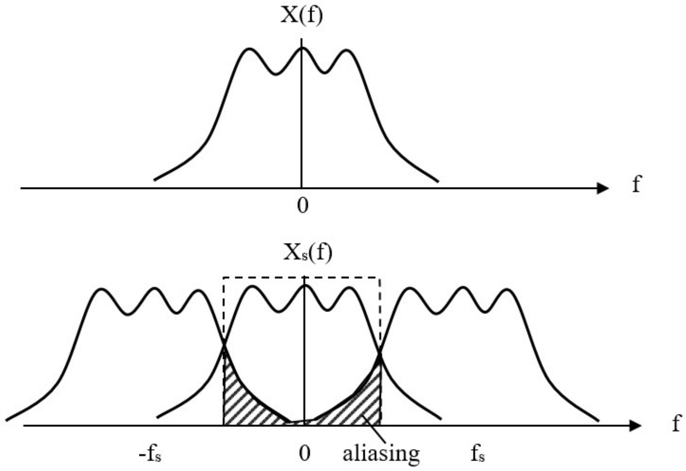


Figure 7.7 Aliasing occurs when sampling non-bandlimited signal.

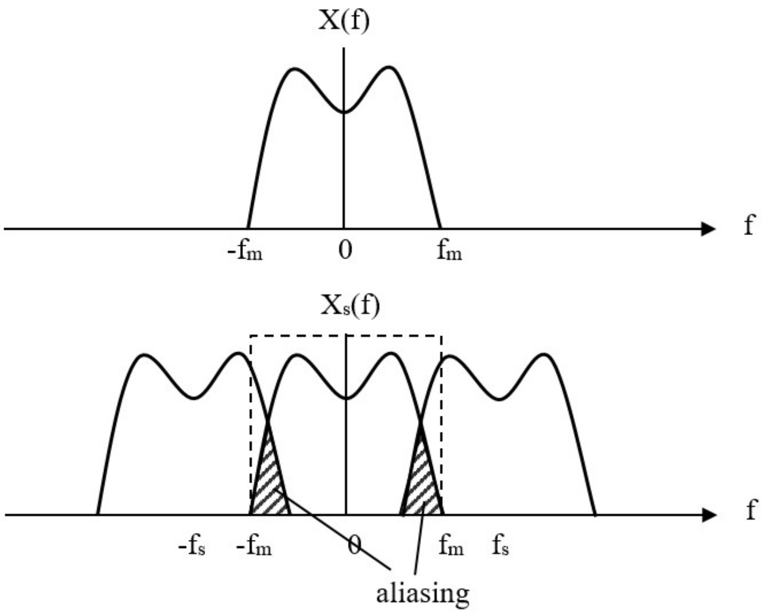


Figure 8.8 Aliasing occurs when signal is under-sampled.

7.1.3 Practical sampling

The ideal sampling techniques discussed earlier, used impulse sampling train which do not exist in practice. Besides, practical filters cannot be perfectly rectangular in frequency response. Signal recovery is still possible from a sampled signal where the sampling signal is a pulse train.

Therefore in practical sampling :

- The sampling waveform consists of pulses of finite amplitude and duration.
- Practical reconstruction filters do not possess ideal characteristics.

There are two forms of practical sampling,

1. Natural sampling
2. Flat-top sampling

Natural Sampling

In natural sampling the signal to be sampled is multiplied by the sampling **pulse** train as shown in Figure 7.9.

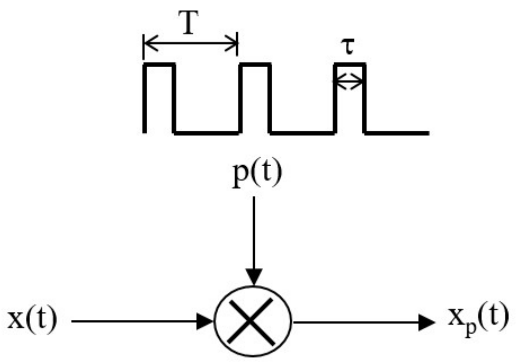


Figure 7.9 Model of natural samping

The expression of sampled waveform using natural sampling is the same as that in ideal sampling, which is $x_p(t)=x(t)\times p(t)$. Figure 7.10(c) shown the sampled waveform of signal x(t). We can make the following observation on natural sampling:

1. In time domain, the sampled signal is a pulse train whose pulse amplitude follows the message signal over the duration of the pulse width, τ.

2. $x(t)$ can be reconstructed from $x_p(t)$ by processing it through an ideal LPF having a system transfer function:

$$H(f) = \begin{cases} T/\tau & ; |f| \le |fc| \\ 0 & ; |f| > |fc| \end{cases} \tag{7.4}$$

and $|f_m| \le |f_c| \le |f_s - f_m|$

3. We have assumed that the sampling frequency $f_s \ge 2f_m$ and hence no aliasing occurs.

Flat-top sampling

In Flat-top sampling, the amplitude of each pulse in the sampled pulse train is constant during the complete duration of the pulse as shown in Figure 7.11. The amplitude of the pulse is determined by the instantaneous sample of the analog signal $x(t)$. It is generated by a Sample-and- Hold circuit given in Figure 7.12.

Unlike natural sampling, a low pass filter operating on the flat-top sampled signal will not give a distortion-free output. However, this distortion may be corrected by adding a second filter - the equalising filter, in cascade to the reconstruction filter. This distortion is called the **Aperture Distortion/Effect**.

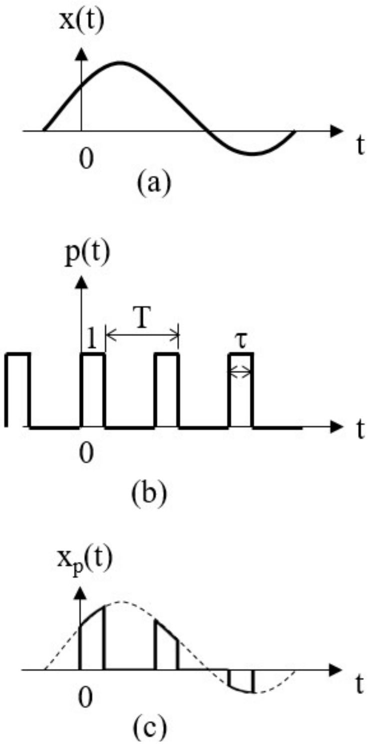


Figure 7.10 Waveforms of natural sampling: (a) message signal (b) sampling train (c) Naturally sampled signal

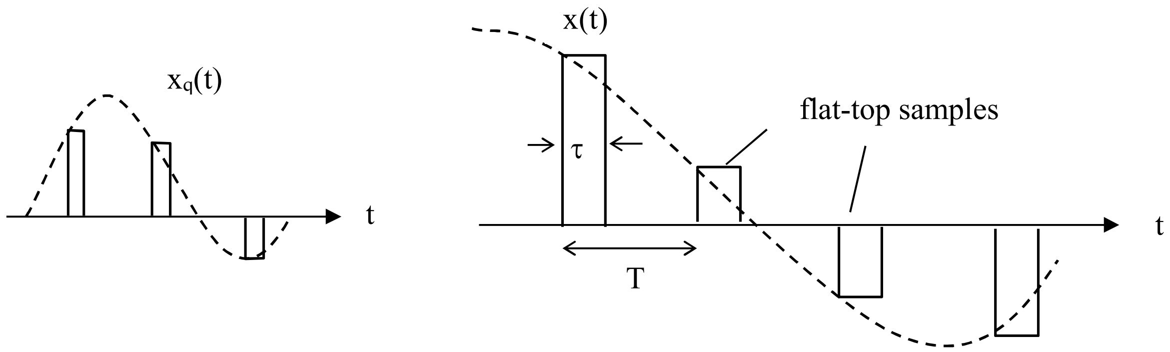


Figure 7.11 Examples of flat-top sampled signals

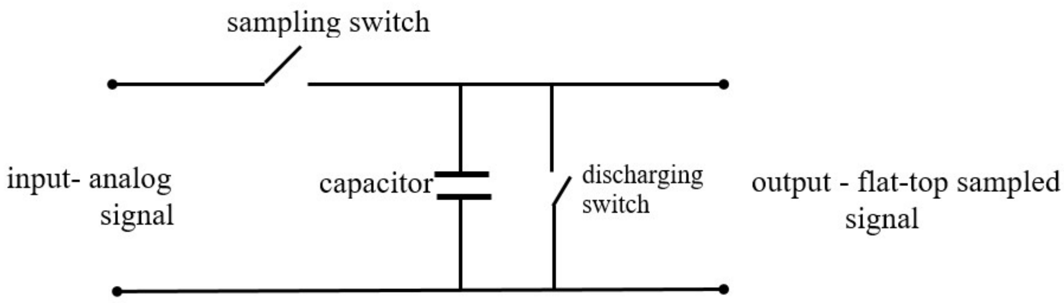
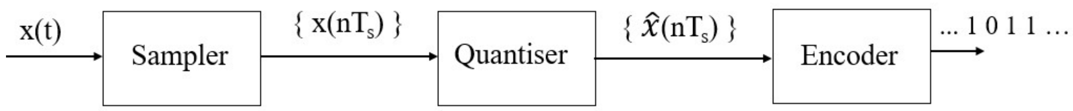


Figure 7.12 Sample-and-Hold circuit

7.2 QUANTIZATION

The sampled analog signal from a sampling process is still an analog signal as its amplitude is continuous though it is discrete in time. To obtain a digital representation of such analog signal, two additional operations are required: **quantization** of the amplitude values of the sampled signal and **encoding** of the quantised values as shown below:



A quantiser converts the analog samples of an information signal into discrete levels. Thus, an infinite number of continuous values are converted to a finite number of discrete levels. An encoder translates each quantised sample to a unique digital codeword via mapping of codewords to quantised values.

The concept of quantisation is represented in Figure 7.13. Assume the amplitude of the signal m(t) is confined to the range V_L to V_H. We divide this range into M equal intervals each of size q, denoted as L_{01}, L_{12} L_{23}… and so on. Accordingly q, called the <u>step-size</u>, is

$$q = (V_H - V_L) / M \tag{7.5}$$

As an example, we assume M = 8. In the centre of each of the M intervals we locate <u>quantisation levels</u> $m_0, m_1, ..., m_7$. The quantised signal $m_q(t)$ is generated in the following way:

$$\mathbf{m_q(t)} \in \{ m_0, m_1, ..., m_7 \}$$

At any time, $m_q(t)$ has the value of quantisation level to which m(t) is closest.

Thus the signal $m_q(t)$ will at all times be found at one of the quantisation levels $m_o, m_1, ..., m_7$. The transition in $m_q(t)$ from $m_q(t) = m_o$ to $m_q(t) = m_1$ is made abruptly when m(t) passes the transition level L_{01} which is midway between m_o and m_1 and so on. Thus the signal $m_q(t)$ does not change at all with time or it makes a "quantum" jump of step size q.

The **quantisation error**, defined as $m_q(t)$-m(t), has a magnitude which is equal or less than q/2.The quantisation error is also termed as **quantisation noise**.

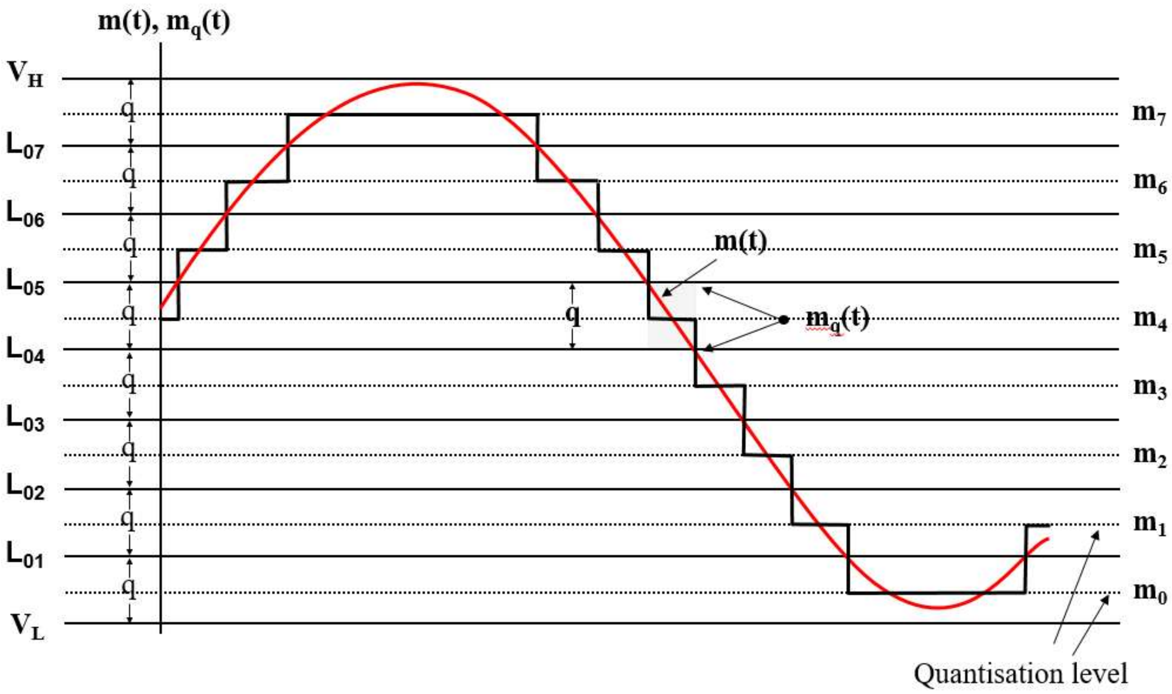


Figure 7.13 Concept of quantisation

We see therefore, that the quantised signal is an approximation to the original signal. The quality of the approximation may be improved by reducing the size of the steps, thereby increasing the number of allowable levels. With small enough steps, the human ear or eye will not be able to distinguish the original from the quantised signal.

7.2.1 Uniform quantisation

The quantisation ranges and levels may be chosen in a variety of ways depending on the intended application of the digital representation, that is whether the source signal to be quantised is voice, video, music or any other type of analog source, each of which possesses unique statistical characteristics. When the digital representation is to be processed by a digital system, the quantisation levels and ranges are generally distributed uniformly. Thus to define a uniform quantiser using the example of Figure 7.14, we set

$$x_i - x_{i-1} = q$$
$$\hat{x}_i - \hat{x}_{i-1} = q$$

where q is the quantisation step size. A common uniform quantiser characteristic is shown in Figure 7.15 for the case of eight quantisation levels. Here the origin appears in the middle of a rising part of the staircase-like function. This class of quantisers is called the "**mid-riser**" class. It can be seen that the mid-riser quantiser has the same number of positive and negative levels, and these are symmetrically positioned about the origin.

For a uniform quantiser (as shown in Figure 7.15) there is only two parameters: the number of levels and the quantisation step size, as we discussed above. The number of levels is generally chosen to be of the form 2^B so as to make the most efficient use of B-bit binary code words. Together, q and B must be chosen so as to cover the range of input samples. Then we should set the peak-to-peak signal amplitude to be equal to the quantiser input range i.e.

$$2\,X_{max} = q2^B \qquad (7.6)$$

where X_{max} is the peak signal amplitude.

That is, $$q = 2\,X_{max} / 2^B \quad \text{mid riser} \qquad (7.7)$$

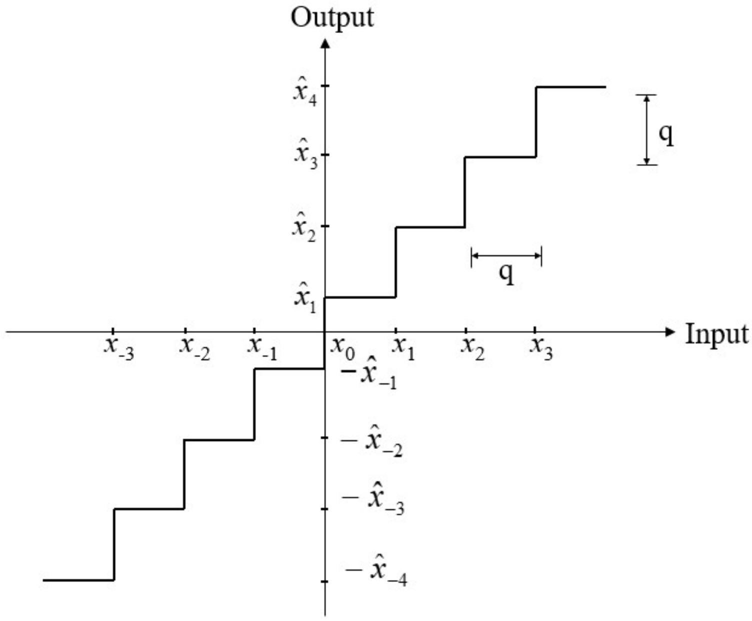


Figure 7.14 Input-output characteristics of a 3-bit quantiser

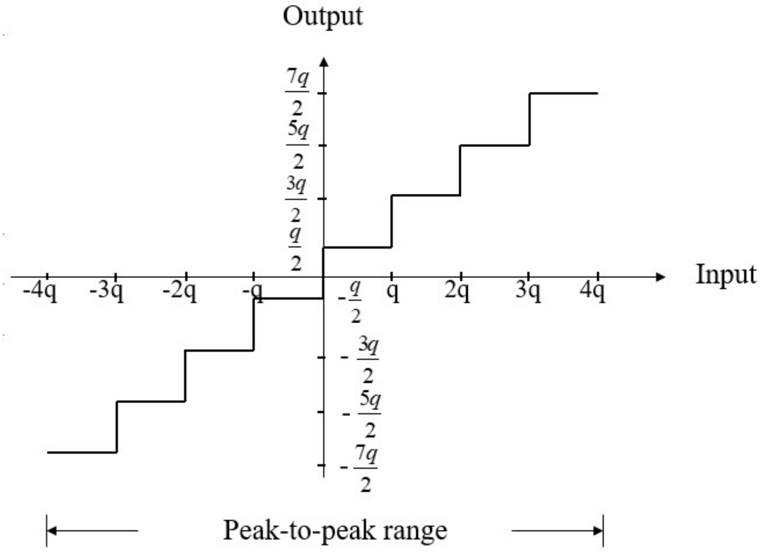


Figure 7.15 Uniform mid-riser 3-bit quantiser

7.2.2 Overload error

In addition to quantisation error (or quantisation noise), the quantisation process also introduces overload error (or clipping), as illustrated in Figure 7.16(b). As long as the input signal lies within the quantiser permitted input range, the only form of error introduced is quantisation error, limited to a maximum value equal to q/2 for the linear quantiser. If the input signal exceeds the allowed range, the quantiser output will remain in the maximum allowed level, resulting in the input signal being clipped. Careful choice of overload thresholds x_o and x_N controls a trade-off between the relative amounts of quantisation and overload distortion.

7.2.3 Quantisation noise power

In Figure 7.16(a) a linearly increasing signal is fed into the uniform quantiser and the result is a staircase-like waveform (the quantised signal) while the difference between the output and input waveforms is a sawtooth waveform with a peak-to-peak magnitude equal to q. The error waveform can be described mathematically as

$$e = -x + q/2 \tag{7.8}$$

The rms value of the error signal is then given by

$$e_{rms} = \sqrt{\frac{1}{q}\int_0^q \left(-x + \frac{q}{2}\right)^2 dx}$$

$$= \sqrt{\frac{q^2}{12}} \tag{7.9}$$

The quantisation noise power is then obtained as the rms value squared, that is,

$$N_q = \frac{q^2}{12} \tag{7.10}$$

It should be pointed out that this result (quantisation noise power) is not limited to the case of a ramp input to an uniform quantiser. The same result can be obtained if we make the assumption that the quantisation noise is a random signal with a uniform distribution in the interval -q/2 to +q/2, that is

$$p_e(e) = \frac{1}{q}, \qquad -\frac{q}{2} \le e \le +\frac{q}{2} \tag{7.11}$$
$$= 0\,, \qquad \text{otherwise}$$

7.2.4 Signal-to-Quantisation noise ratio

Performance of a quantiser can be described by a signal-to-noise ratio that takes both quantisation error and overload error into account. In this module we neglect overload error to simplify our analysis. To compute the ratio of signal to quantisation noise S/N_q, the input signal characteristics must also be specified. Quite often performance for a quantiser is based on sinusoidal inputs, because S/N_q for speech and sinusoidal inputs compare favourably and use of sinusoids facilitates measurements and calculations of S/N_q. For the case of a full range (peak voltage = V) sinusoidal input that has zero overload error, the average signal power is

$$
\begin{aligned}
S &= \frac{{V_{rms}}^2}{R} = V_{rms}^2 \\
&= \frac{V^2}{2} \quad where\ R = 1\Omega\ and\ V_{rms} = \frac{V}{2}
\end{aligned}
\tag{7.12}
$$

Equation 8 thus gives the normalised average signal power where R is normalised to 1 Ω to simplify the mathematics. Now, the peak value of the sinusoid can be expressed in terms of step size (q) and number of levels in the quantiser (M), as follows

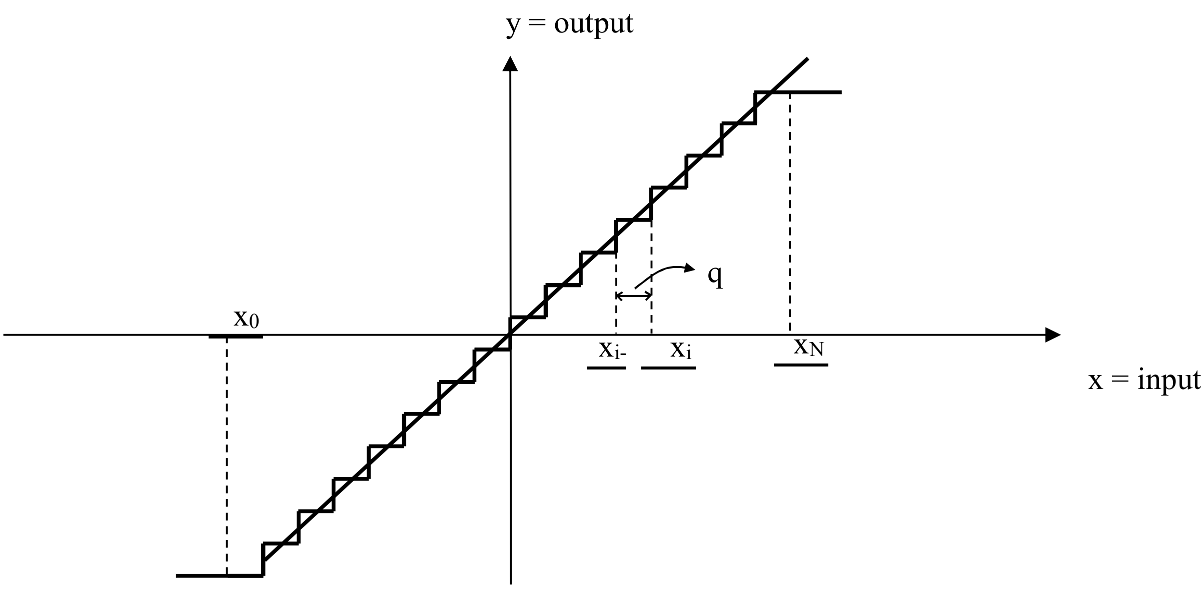
$$V = (q \text{ x } M)/2 \tag{7.13}$$

The rms value is therefore,

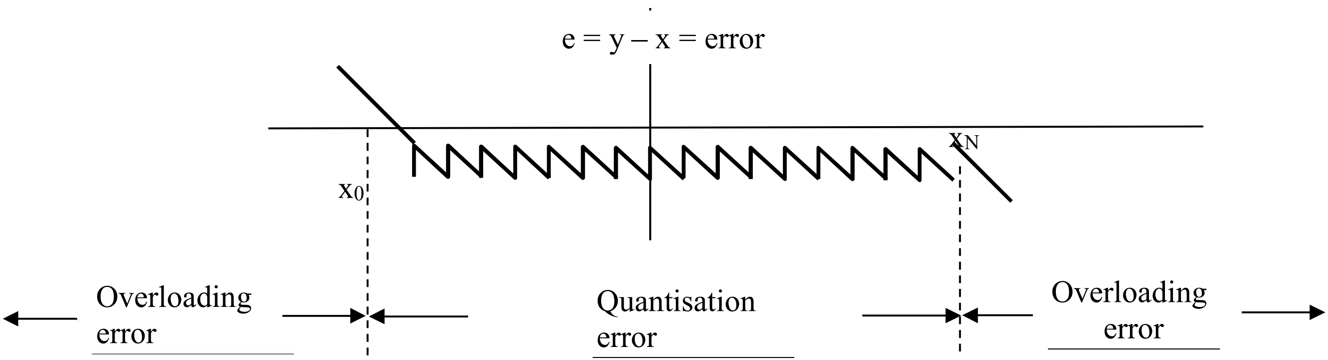
$$V_{rms} = \frac{q \text{ x } M}{2\sqrt{2}} \tag{7.14}$$

Hence, the average signal power is

$$S = \frac{q^2 M^2}{8} \tag{7.15}$$



(a) Linear quantiser characteristic



(b) Error characteristic

Figure 7.16 Linear Quantisation

Combing eqs.(7.6) and (7.11), the signal to quantization noise ratio is

$$\frac{S}{N_q} = 1.5M^2 \tag{7.16}$$

Expressing the ratio in dB's, we have

$$\left[\frac{S}{N_q}\right]_{dB} = 10\log_{10}\left(1.5M^2\right) \tag{7.17}$$

$$= 10\log_{10} 1.5 + 10\log_{10} M^2 = 1.76 + 20\log_{10} M$$

Since M = 2^B,

$$\left[\frac{S}{N_q}\right]_{dB} = 1.76 + 20\log_{10} 2^B = 1.76 + B.20\log_{10} 2$$

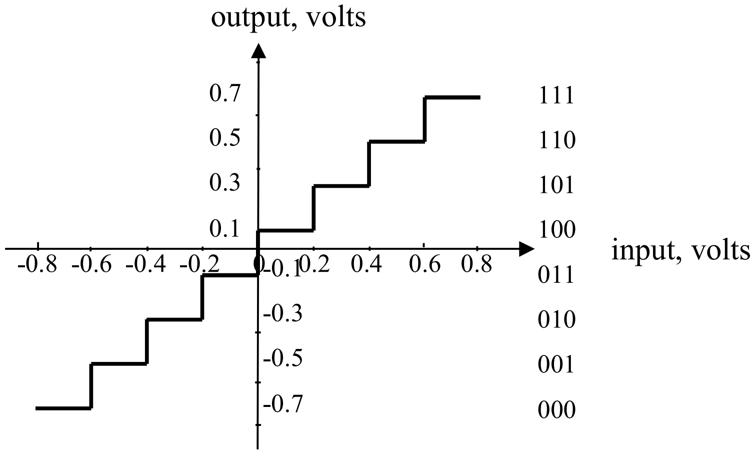
$$\left[\frac{S}{N_q}\right]_{dB} = 1.76 + 6B \qquad \text{dB} \tag{7.18}$$

Note that eq (9.14) is the signal-to-noise ratio for a sinusoid whose amplitude range coincides with the range of the quantiser. For sinusoidal inputs whose amplitude, V_x, is less than the full range of the quantiser, V_q, then

$$\left[\frac{S}{N_q}\right]_{dB} = 1.76 + 6B + 20log_{10}\frac{V_x}{V_q} \tag{7.19}$$

To compute the mean square error due to overload distortion, the input signal probability density function must be specified. The mathematical treatment of this topic is very involved and it is beyond the scope of this course.

Example 7.5



The input-output characteristic of a 3-bit quantiser is shown in the above figure.

(a) Determine its step size.
(b) Calculate its quantisation noise power.
(c) Determine the maximum quantisation error.
(d) Determine the quantised output voltage and its codeword when an input of 0.58 V dc is applied to the quantiser.

Solution

(a) From the input-output characteristic we note that the step size, q = 0.2 V.
(b) Quantisation noise power,

$$N_q = \frac{q^2}{12} = \frac{0.2^2}{12} = 3.33mW$$

(c) Maximum quantisation error $= \frac{q}{2}$ = 0.1 V
(d) Quantised voltage = 0.5 V and codeword is 110

7.2.5 Non-uniform qunatization (logarithmic quantization)

One of the problems with the uniform quantiser is that the speech signal changes with time and the variance can be quite different from one speech segment to another. If the quantiser is designed to accommodate strong signals with large variance (like vowels), the quantisation step size will be large and weaker signals (like consonants) will be subjected to a larger quantisation error which in turn will have an effect on the speech quality. To solve this problem, we abandon the uniform quantiser and design a quantiser whose step size increases with the signal amplitude. This technique is called *non-uniform quantisation*, since a variable step size is used. The effect of non-uniform quantisation can be obtained by first passing the analog signal through a compression (non-linear) amplifier and then into a uniform quantiser. For speech signal, the uniform quantiser will usually operate on the logarithm of the speech signal, which is a compressed version of the original signal.

The signal is reconstructed at the receiver by expanding it. This process of compression-expansion is called COMPANDING (COMpressing-exPANDING).

An example of a compression characteristic is shown in Figure 7.17. Note that at low amplitudes the slope is larger than at large amplitudes. A signal transmitted through such a network will have the extremities of its waveform compressed. The peak signal which the system is intended to accommodate will, as before, range through all available quantisation regions. But now, a small amplitude signal will range through more quantisation regions than would be the case in the absence of compression. Figure 7.18 gives a comparison of the variation of output signal-to-noise ratio as a function of the input signal power when companding is used, to the case of an uncompanded system. Note that the companded system has a far greater dynamic range than the uncompanded system and that theoretically the companded system has an output signal-to-noise ratio which exceeds 30 dB over a dynamic range of input signal power of 48 dB, while the uncompanded system has a dynamic range of 18 dB for the same conditions.

The ITU-T standard for digital coding of voice signals with companding is 8000 samples/s, 8-bit/sample giving a PCM bit rate of 64 kbits/s. The companding characteristic is either A-law (used in Europe) or μ-law (used in America).

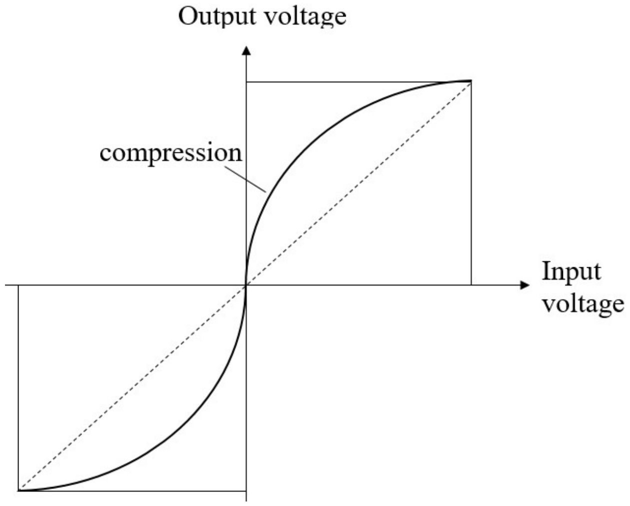


Figure 7.17 Input-output characteristic for compression

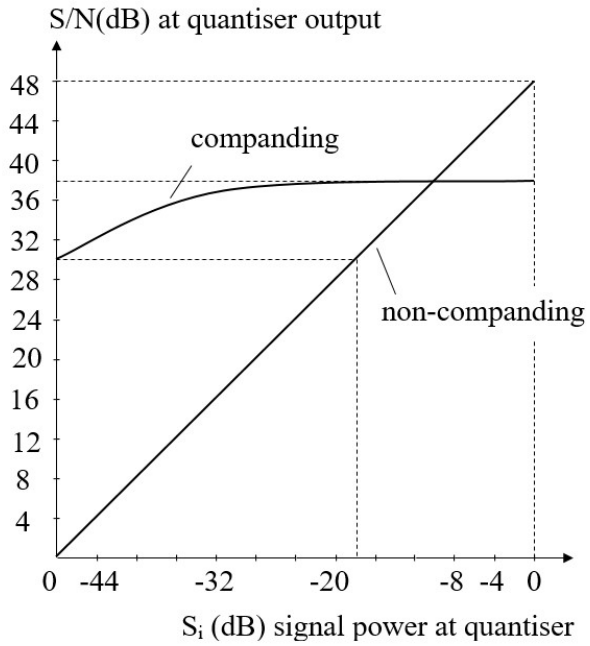


Figure 7.18 Comparison of companded and uncompanded systems

7.3 PULSE CODE MODULATION (PCM)

Pulse code modulation (PCM) is the most common techniques used today for analog to digital conversion where the information contained in the samples of an analog siganl is represented by digital words in a serial bit stream. PCM is used in many applications, such as telephone system, compact disc (CD) recording, PC audio – wav format, voice mail, and many other applications.

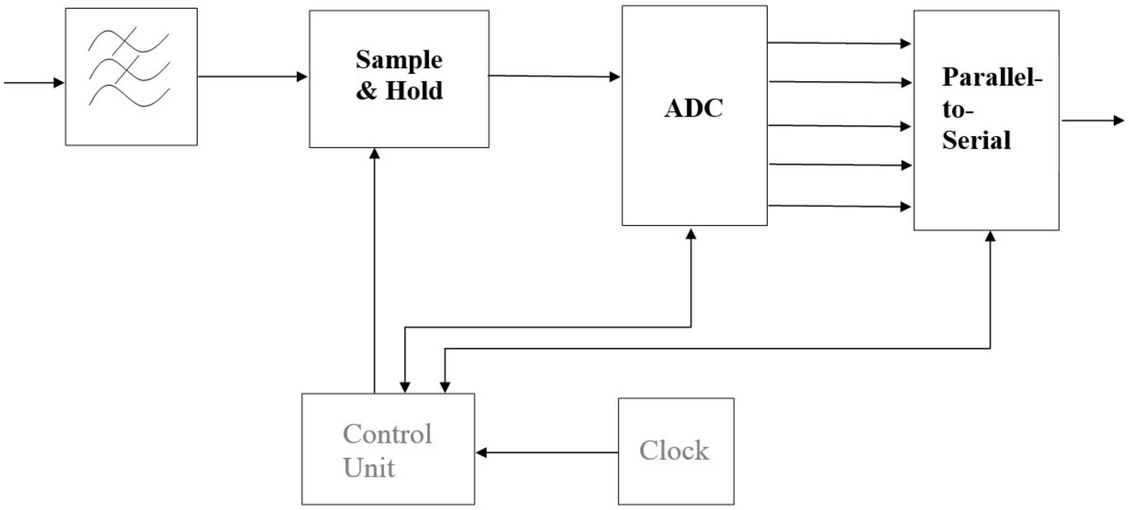
7.3.1 PCM system

Figure 7.19 shows a block diagram of a PCM transmission system. The operation of PCM transmitter is briefly described below (refer to Figure 7.19(a)).

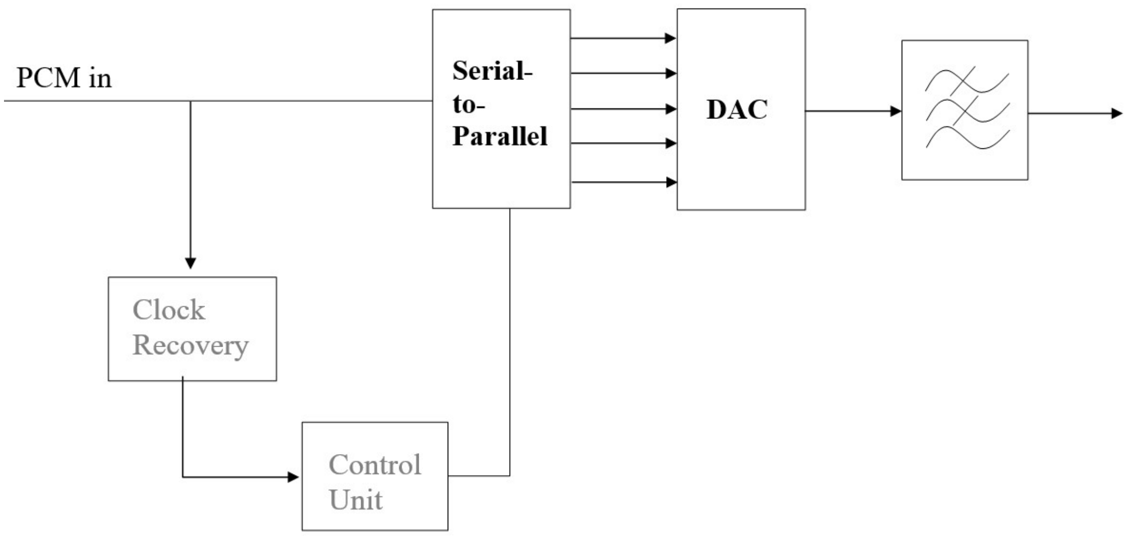
- **Anti-aliasing filter**: Analog signal is bandlimited by attenuating high-frequency components of the original analog signal that are not significant so as to avoid aliasing effect.

- **Sampling and Hold**: The bandlimited analog signal is sampled and held constant to allow time for the conversion to take place from a sampled analogue value to a digital code word.

- **Analog to Digital Converter (ADC)**: Inside the ADC unit there are a quantiser and an encoder. The quantiser converts the analogue sample to the nearest allowed output discrete level, while the encoder generates a unique binary pulse pattern (5-bit parallel PCM codes) corresponding to a given quantisation level.



(a) PCM transmitter



(b) PCM Receiver

Figure 7.19 PCM transmission system: (a) PCM transmitter, and (b) PCM receiver.

- **Parallel to Serial Converter**: PCM codes are converted to serial binary data and presented to the transmission line as serial digital pulses.

The quantising and encoding operations are usually performed in the same circuit, which is called an analog-to-digital converter.

The operation of the PCM receiver is essentially the reverse of the transmitter (refer to Figure 7.19(b)). Digitally encoded signals arrive at the receiver in serial bit-stream. If necessary, the received pulses are first reshaped before entering the serial to parallel converter. Once in parallel format, the data is fed into the DAC (digital to analogue converter) to generate sequene of qantized multi-level sampled pulses. In order to recover the original transmitted signa, this qantized multi-level sampled pulses is passed through a reconstruction filter to recover the original analog signal by filtering out unwanted frequency components.

7.3.2 Minimum PCM transmission bandwidth

The minimum bandwidth occupied by a PCM bit-stream, assuming NRZ (non-return-to-zero) signalling format, is related to its bit rate in the following way,

$$BW = R/2 \tag{7.20}$$

where R is the bit rate. To understand the above relationship, let us consider a bit pattern 101010... which has the highest fundamental frequency of any given pattern. Note that in this pattern two bits are transmitted per cycle, hence the fundamental frequency is the line bit rate divided by two.

Example 7.6

We want to transmit serially a 4 kHz audio using 8-bit PCM.
Output bit rate = sampling frequency x no. of bits per sample = 8 kHz x 8 bits = 64 kbits/sec, assuming the sampling frequency used is 8 kHz. So a transmission bandwidth of at least 32 kHz is required, exceeding the bandwidth of a telephone voice channel.

7.3.3 Time division Multiplexing of PCM signals

When a large number of PCM signals are to be transmitted over a common channel, time division multiplexing (TDM) of these PCM signals is required. Such a system is known as a Pulse Amplitude Modulation (PAM)-TDM system. TDM is the time interleaving of samples from several sources so that the information from these sources can be transmitted serially over a single communication channel.

Figure 7.20 illustrates the TDM concept applied to three analog signals. By sampling the three signals at the same rate and interleaving the pulses into separate time slots (Figure 7.20(b)), the three signals share the same channel simultaneously. Samples of the same baseband signal are separated by T. For convenience, all samples belonging to the same signal are of equal amplitude in the illustration.

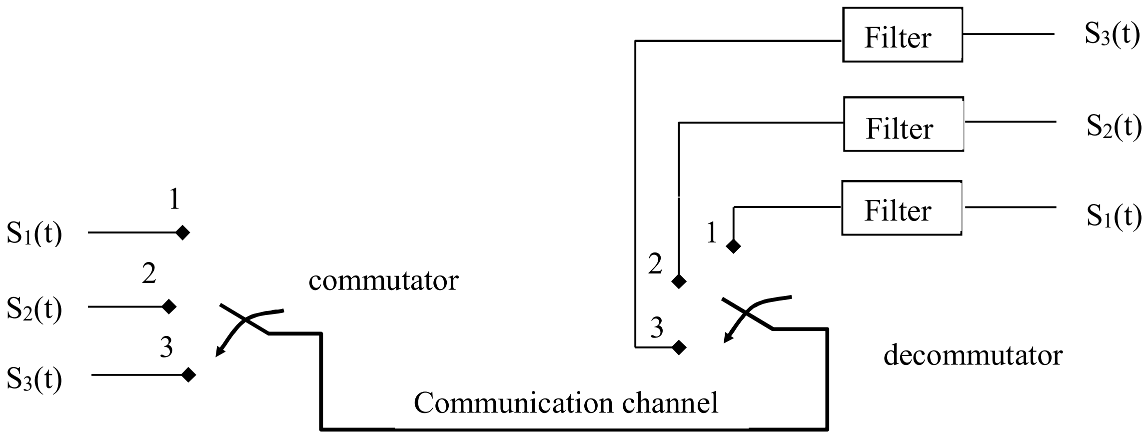
The commutator in Figure 7.20(a) performs both the sampling and multiplexing. The commutator must operate at rate that satisfies the sampling theorem for each signal. Consequently, the signal with highest frequency determines the commutator speed.

For example, suppose the maximum frequencies for the three input signals are f_1 = 4 kHz, f_2 = 12 kHz, and f_3 = 4 kHz. Then the commutator speed must be 24 kHz to satisfy the worst-case condition. We can calculate the communication channel pulse rate as follows: The commutator completes one cycle, called a frame, every 1/24 kHz = 41.67 μs. Each time around, the commutator picks up a pulse from each of the three signals. Hence, there are 3 pulses/cycle x 24 k cycles/s = 72 k pulses/s.

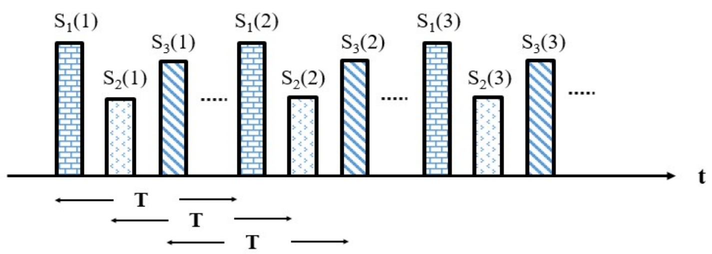
In general, the gross channel output pulse rate is given by

Gross channel output pulse rate = number of signal inputs × commutator speed (7.21)

Multiplexing of many signals will require relatively high pulse rate/bitrate transmission systems and thus high bandwidth. To minimize the transmission bandwidth required, an optimum commutator structure which generates the lowest pulse rate/bitrate is necessary.



(a) Time-division multiplexing of three signals



(b) Time interlacing of three baseband signals

Figure 7.20 TDM concept

It is possible to modify the above commutator structure to minimize channel pulse rate, thus, the bandwidth required for transimission. Figure 7.21 shows a modified structure with insertion of signals $S_1(t)$, $S_3(t)$, and a dummy input (unconnected input) in between 3 inputs of $S_2(t)$. Now, with uniform sampling the commutator speed can be 8 kHz (i.e. (2 pulses/41.67 μs)/(5 + 1 pulses/cycle) = 8 kHz), and the channel pulse rate will be reduced to 8 kHz x (5+1) pulses/cycles = 48 k pulses/s.

In the modified structure, the sampling frequency for $S_2(t)$ is given by:

$$f_{s2} = \text{commutator speed x no.of inputs for } S_2(t)$$
$$= 8 \text{ kHz x } 3 = 24 \text{ kHz}$$

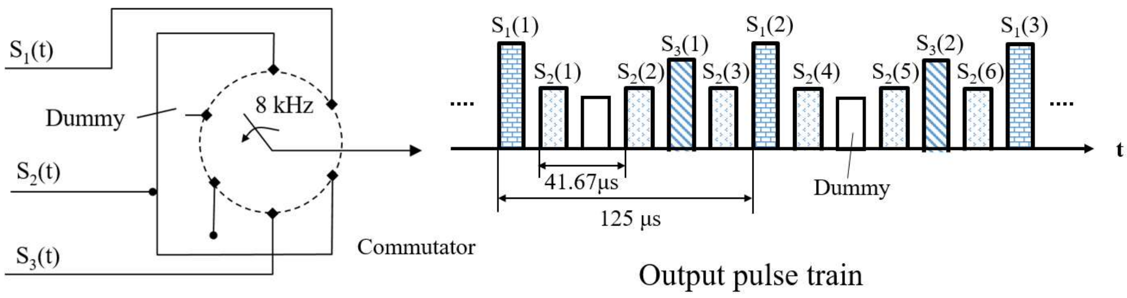


Figure 7.21 TDM for minimum channel output pulse rate

In general the sampling frequency for any signal in a commutator structure is given by,

f_s = commutator speed x number of inputs for that signal

A commutator structure with minimise channel pulse rate is called **optimum commutator structure**. To optimize a commutator structure, the following steps are involved:

- Choosing a commutator speed (CS)/rotation rate that equals to the sampling rate of the signal with minimum bandwidth.
- For signals with larger bandwidth, multiple-equally-spaced inputs are allocated and sampled in one cycle of commutator rotation determined by sampling theorem. Thus, the sampling rates of all the signals will be multiples of CS.

It should be realised that decreasing the commutator speed will result in a more complex commutator structure as more inputs are taken in one cycle of commutator rotation. Sometime it is necessary to trade-off between the channel pulse rate and the complexity of commutator structure.

Example 7.7

A PAM-TDM system is used to multplex four signals $m_1(t) = \cos \omega_0 t$, $m_2(t) = 0.5\cos \omega_0 t$, $m_3(t) = 2\cos 2\omega_0 t$, and $m_4(t) = \cos 4\omega_0 t$ where $\omega_0 = 2000\pi$ radians/s.

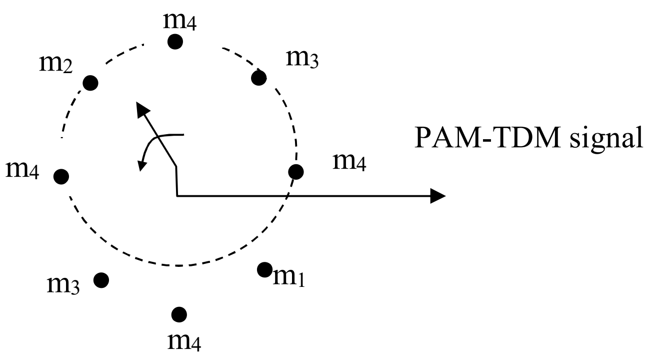
(b) What is the commutator speed for minimum channel pulse rate assuming uniform sampling?

(b) Sketch the optimum commutator structure.

Solution

(a)

Signal	f_m (kHz)	f_S(kHz)	No. of inputs
m_1	1	2	1
m_2	1	2	1
m_3	2	4	2
m_4	4	8	4



Note that to get uniform sampling:

- all the signal inputs should be evenly spaced
- for the same signal all inputs must be evenly spaced, e.g. all 4 m_4's are evenly spaced.

(b) The rule for determining the commutator speed is as follows: the lowest value of f_s in column 3 is chosen to be the commutator speed i.e. commutator speed = 2 k cycles/s

The **gross channel output bit rate** is:

Gross channel output bit rate, R = commutator speed x no. of inputs x no. of bits per sample (7.22)

The same general concepts discussed above on PAM-TDM can apply as well to the TDM of PCM signals as just an additional encoder is needed as shown in Figure 7.22.

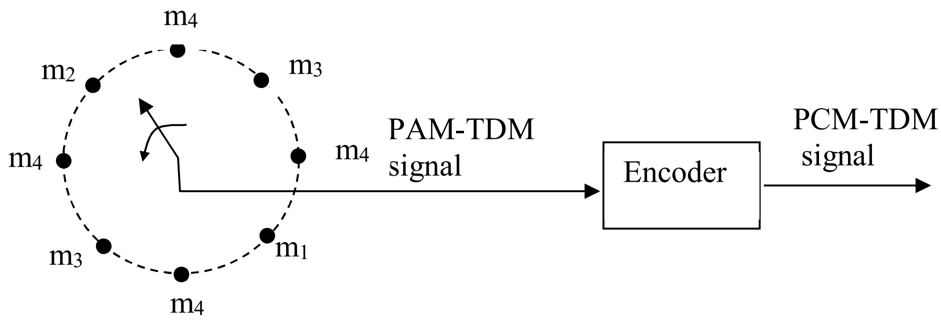


Figure 7.22 Commutator structure for PCM-TDM signal

Example 7.8

A 4 kHz signal is transmitted through a PCM system. The system employs a 8-bit quantiser which has a step size of 5 mV.

(a) Calculate the quantisation noise power and the maximum output signal-to-quantisation noise ratio (in dBs) of the system.

(b) If five similar systems are time-multiplexed, what is the minimum transmission bandwidth required by the multiplexed system?

Solution

(a) Quantisation noise power, $N_q = \frac{q^2}{12} = \frac{(5\times10^{-3})^2}{12}$ = 2.08 x 10^{-6} W

Signal-to-quantisation noise = 1.76 + 6B = 1.76 + 6 x 8 dB
= 49.76 dB

(b) f_m = 4 kHz, $f_s = 2f_m$ = 8 kHz

Gross output bit rate, R = commutator speed x no. of inputs x no. of bits per sample
= 8000 x 5 x 8 = 320 kbps

Note: Here commutator speed equals f_s as the five signals are similar.

Hence minimum transmission bandwidth = R/2 = 320/2 = 160 kHz.

7.4 DIFFERENTIAL PULSE-CODE MODULATION (DPCM)

DPCM is a variant of PCM and designed specifically to take advantage of the sample-to-sample redundancies in a typical speech waveform. It is capable of providing better speech quality than the straight PCM for the same bit rate. A closer examination of the speech signal, especially during voiced periods, reveals that there is a relatively smooth change from one speech sample to the next. In other words, there is considerable **correlation between adjacent samples**. As a result, the difference between adjacent samples will have a smaller variance and dynamic range than the speech samples themselves. Since the range of sample differences is less than the range of individual amplitude samples, fewer bits are needed to encode difference samples. The sampling rate is often the same as for comparable PCM system.

Figure 7.23 depicts an analog signal being sampled to produce S_1, S_2 and S_3. In DPCM, the differences in the samples (e.g. d_2 and d_3) are quantised instead of the samples (e.g. S_1, S_2 and S_3).

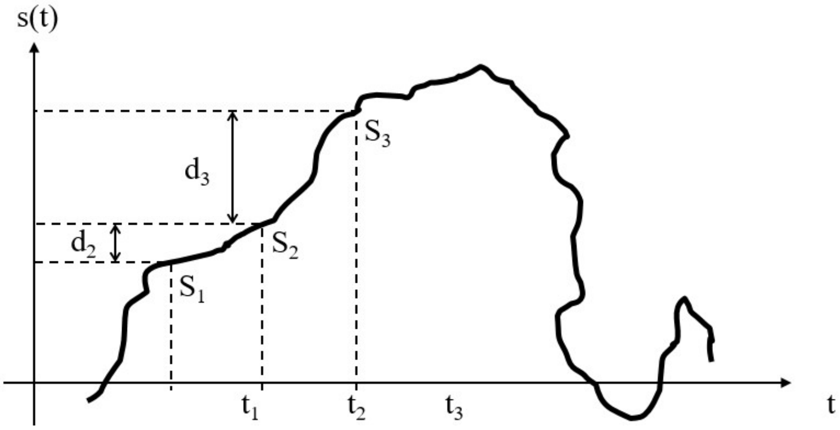


Figure 7.23 DPCM encoding

The simplest means of generating the difference samples for a DPCM coder is to store the previous input sample directly in a sample-and-hold circuit and use an analogue subtractor to measure the change between the two adjacent samples. The change in the signal is then quantised and encoded for transmission. The DPCM structure shown in Figure 7.24 is more complicated than PCM, because the previous input value is reconstructed by a feedback loop that integrates the encoded sample differences. In essence, the feedback signal is an estimate of the input signal as obtained by integrating the encoded sample differences.

The advantage of the feedback implementation is that quantisation errors do not accumulate indefinitely. If the feedback signal drifts from the input signal, as a result of an accumulation of quantisation errors, the next encoding of the difference signal automatically compensates for the drift. In a system without feedback the output produced by a decoder at the other end of the connection might accumulate quantisation errors without bound.

As in PCM systems, the quantiser used can be uniform or companded. Further improvements can be achieved by using certain adaptive techniques to adjust the the quantisation step size in accordance with the average power level of the signal. The resultant system is known as ADPCM (Adaptive Differential Pulse Code Modulation).

DPCM encoders and decoders can be implemented in a variety of ways depending on how the signal processing functions are partitioned between analogue and digital circuitry. At one extreme the differencing and integration functions can be implemented with analogue circuitry, while at the other extreme all signal processing can be implemented digitally using conventional PCM samples as input.

ITU-T has adopted the 32 kbits/s ADPCM encoder/decoder as a standard under the G.721 specification. The G. 721 encoder uses a 4-bit quantiser to transmit the quantised difference signal.

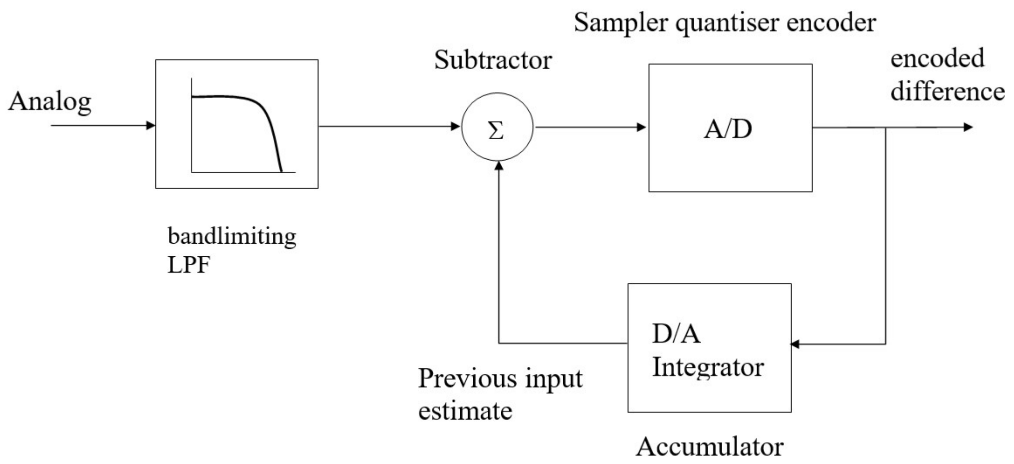


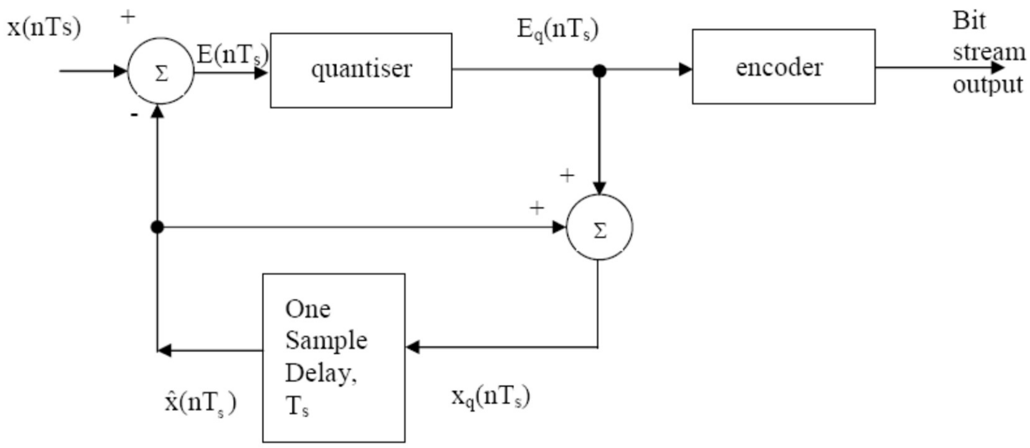
Figure 7.24 Functional Block diagram of DPCM

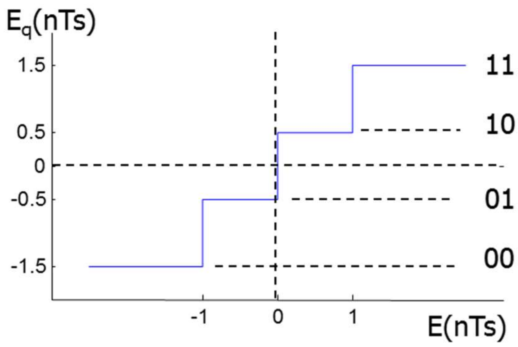
Example 7.9

A DPCM modulator is shown in the figure below. Assume that the input $x(nT_s)$ is given in Table 7.1.

(a) Complete the remaining boxes in the table

(b) What is the output bit stream?





Quantiser rule:

$V_o = 1.5\,V: \quad 1\,V < V_i \le \infty$

$0.5\,V: \quad 0\,V < V_i \le 1\,V$

$-0.5\,V: \quad -1\,V < V_i \le 0\,V$

$-1.5\,V: \quad -\infty < V_i \le -1\,V$

Time	$x(nT_s)$ (V)	$\hat{x}(nT_s)$ (V)	$E(nT_s)$ (V)	$Eq(nT_s)$ (V)	$Xq(nT_s)$ (V)	Output bit stream
0	1.3	0.5				
T_s	2.2					
$2T_s$	1.8					

Table 7.1

Solution

The left adder gives: $E(nT_s) = x(nT_s) - \hat{x}(nT_s)$

The right adder gives: $X_q(nT_s) = E_q(nT_s) + \hat{x}(nT_s)$

We then complete the table using the two equations. For example for the first sampling instant, time = 0,

$E(nT_s) = 1.3 - 0.5 = 0.8$ V. This voltage 0.8 V then goes into the quantiser and is quantised to 0.5 V which is equivalent to binary 10.

Using the second equation, $X_q(nT_s) = 0.5 + 0.5 = 1$ V. This completes the first row.

For the second row, we first note that $\hat{x}(nT_s) = X_q(nT_s)$ but delayed by one sample.

Hence, $\hat{x}(nT_s) = 1$ V i.e. value of $X_q(nT_s)$ in first row.

We can then complete the rest of the table using the method just outline above.

Time	**$x(nT_s)$ (V)**	**$\hat{x}(nT_s)$ (V)**	**$E(nT_s)$ (V)**	**$Eq(nT_s)$ (V)**	**$Xq(nT_s)$ (V)**	**Output bit stream**
0	1.3	0.5	0.8	0.5	1	10
T_s	2.2	1	1.2	1.5	2.5	11
$2T_s$	1.8	2.5	-0.7	-0.5	2	01

7.5 APPLICATIONS

7.5.1 30-Channel PCM-TDM system

In this section, we describe the 30-channel PCM-TDM system used in public telephone networks e.g. the Singapore telephone network. The standard for converting analog voice lines into digital ones is standardized in ITU-T G.711.

In the 30-channel PCM system, there are 30 incoming speech channels. In addition, there are two extra channels. These 2 channels are used for signalling and synchronization. Thus there are 32 channels altogether.

Basic parameters of 30-channel PCM system

Three important parameters of a 30-channel PCM system are highlighted:

- Sampling rate
- Companding and encoding schemes
- Total output bit rate

<u>Sampling rate</u>

The rate at which the sampling is carried out depends on the highest frequency to be sampled in the signal. Here the highest frequency will be 3.4 kHz (voice signal over telephone varies from 0.3-3.4 kHz). As 3.4 kHz is the highest frequency to be sampled, it follows that the sampling rate should be at least 6800 times/sec.

In practice because of imperfect conditions it is better to sample at a frequency in excess of 6800 times per second The 30-channel PCM system uses the sampling frequency of 8 kHz i.e. 8000 samples/sec. The sampling interval is 125 μs (= 1/8000).

Companding and encoding schemes

The companding scheme used is A-law and each quantised sample is encoded into 8 bits. The encoding technique allocates 7 bits to represent the amplitude of the sample and 1 bit to represent the sign. The number of quantisation levels is thus 256 (128 positive and 128 negative).

Total output bit rate

In a 125 μs time frame there are 32 channels. Therefore, each channel occupies 3.9 μs. Each signal sample is represented by 8 bits. Therefore, each bit occupies 0.488 μs. So the output bit rate is 2.048 Mbits/S.

The 30-channel PCM system is often referred to as 2 Mbits PCM system.

Higher order pulse code modulation (HOPCM)

In order to fully utilize the high bandwidth available on many trunk links it is necessary to transmit at a high bit rate. To achieve this, several 30-channel PCM systems can be multiplexed together to form a higher speed group. There are several ways of multiplexing PCM to form higher order systems but in Singapore a 2-8-34-140 Mbit/s hierarchy is used. In Figure 7.25, 4 PCM groups, each consisting of one 30- channel system and having the usual bit rate of 2.048 Mbit/s (referred to as 2Mbit/s), are multiplexed together. This forms a single high speed link having a bit rate which in practice is slightly higher than 4x2.048 = 8.192 Mbits/s. This is referred to as an 8Mbit/s group. Since several inputs combine to form a single high speed bit stream each input is known as a tributary.

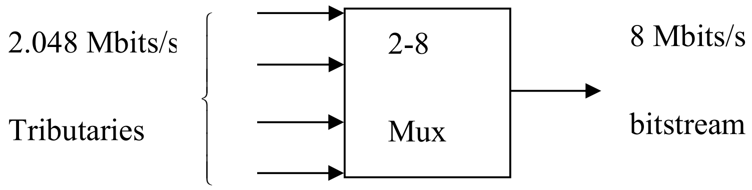


Figure 7.25 2-8 multiplexer

The 8 Mbit/s group is then multiplexed with other 8 Mbit/s groups to form a 34 Mbit/s group as shown in Figure 7.26. Note this is not the expected 32 Mbit/s because of the overhead involved in synchronisation and framing.

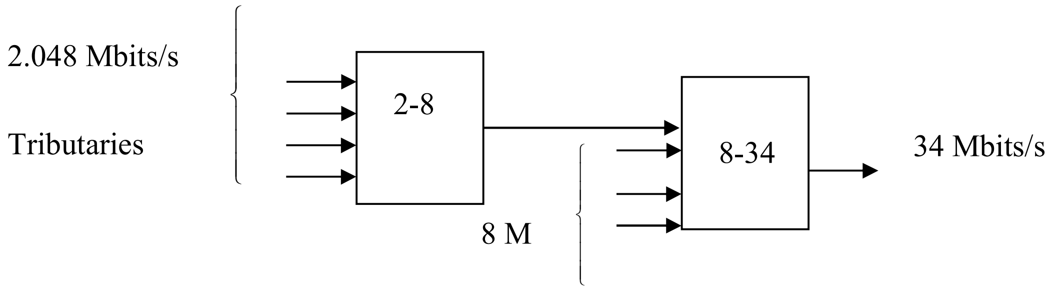


Figure 7.26 8-34 multiplexer

Figure 7.27 shows the final stage in the process, four 34Mbit/s groups are multiplexed to form a 140Mbit/s bit stream which is ideally suited for transmission using optical fibre.

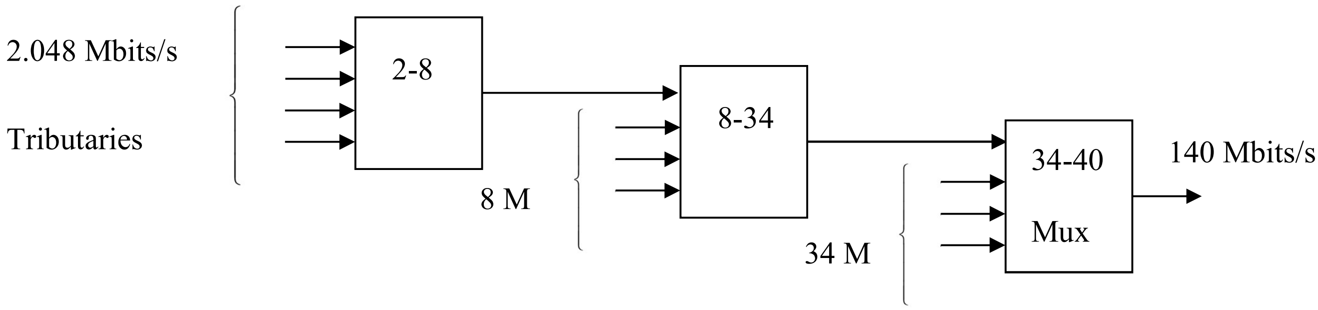


Figure 7.27 34-140 multiplexer

The 140Mbit/s system carries 30x4x4x4 = 1920 telephone channels and forms the main or "backbone" network between major exchanges in Singapore.

7.5.2 CD Recording system

The compact disc (CD) is another application of PCM. Music recording in CD requires the signal bandwidth to be 15 kHz. Although the Nyquist sampling rate is 30 kHz, the actual sampling rate is 44.1 kHz to take into account non-ideal filters. Each sample is quantised and converted into 16 bits. This means that the number of levels is 2^{16} = 65,536 to reduce the quantisation error. There are 2 channels, left and right, for stereo recording. The total bit rate is therefore,

$$44.1 \text{ kHz} \times 16 \text{ bits} \times 2 = 1.41 \text{ Mbps}$$

Chapter 8

Baseband Transmission of Digital Signals

Learning Outcomes

- ❑ Describe the various PCM waveforms: unipolar/polar NRZ, unipolar/polar RZ, and Manchester formats.
- ❑ Evaluate the characteristics of each PCM waveform to identify its suitability for a particular application.
- ❑ Understand the two main problems in digital transmission.
- ❑ Identify the undesirable effect of pulse spreading which is intersymbol interference (ISI).
- ❑ Describe how thermal noise in the transmission medium can affect the correct detection of digital data.
- ❑ Analyse whether ISI will occur when transmission rate varies (binary impulsive channel inputs) and understand how ISI in digital transmission can be minimised.
- ❑ Evaluate the effect of additive white Gaussian noise (AWGN) on bit error rate (BER) in digital transmission and calculate the probability of an AWGN peak crossing a threshold voltage.
- ❑ Explain what an eye diagram is.
- ❑ Determine the various performance parameters of a digital transmission system from an eye diagram: Noise margin, ISI degradation and Jitter.

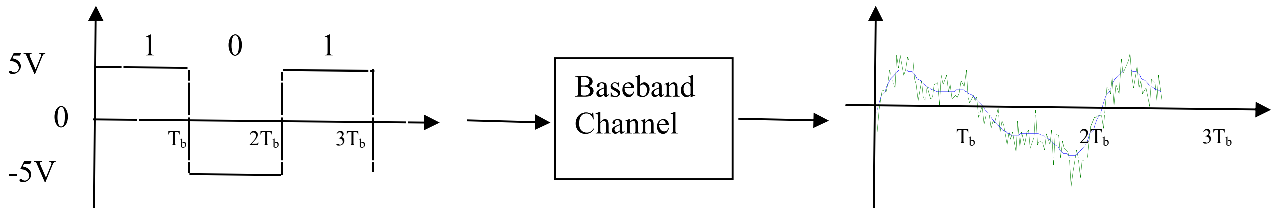
INTRODUCTION

The objective of a digital communication system is to transmit a message in a prescribed amount of time with a minimum number of errors.

When transmitting digital message, it is not necessary to preserve the signal waveform to ensure perfect message reproduction; it is sufficient to prevent each binary signalling waveform from becoming so distorted that it can be mistaken for the other.

The problems that prevent a binary data from being faithfully received occur mainly at the transmission channel. For example, what causes a transmitted message 0110110 to be received as 0010110 (second bit is wrongly received) are mainly due to:

- channel noise, which usually is added to the signal. The resultant signal can confuse the receiver into making a wrong decision (i.e., decoding a 1 as 0, or vice versa).
- bandwidth limitation of the channel in relation to transmission bit rate, resulting in intersymbol interference (ISI).



Before we look into details of these problems, we first

- consider the difference between the analysis of baseband signals and Passband signals.

- consider how binary digits are represented by electrical pulses.

In the analysis of transmission problems (ISI and noise) caused by the transmission channel, we need only to concentrate on baseband signals and baseband channels. The same analysis can be extended to passband signals and channels, since in this case, the spectral analysis is just the shifted version of the baseband spectrum. In the case of time domain analysis, the constraint of the passband channel bandwidth will affect the envelope of the passband signal, which has the shape of the baseband signal.

8.1 LINE CODING

Binary data can be transmitted by various pulse types. The choice of a particular pair of pulses to represent binary 1 and 0 is called line coding. Figure 8.1 illustrates a number of commonly used line code waveforms for transmission of binary data 1011001. The various line code waveforms can be classified into the following groups:

1. Nonreturn-to-zero (NRZ)
2. Return-to-zero (RZ)
3. Phase-encoded
4. Multilevel binary

1. NRZ group

- This group is probably the most commonly used line code. NRZ can be partitioned into the following subgroups: unipolar NRZ and polar NRZ.

- Unipolar NRZ - Binary 0 is represented as zero level. Binary 1 will be represented by a +V pulse of constant amplitude for the entire duration of the bit interval. Unipolar NRZ waveforms are used in digital logics, but are seldom used for direct transmission because the dc component in the waveform carries no information and waste transmission power.

- Polar NRZ - Binary 1 and 0 are represented by pulses of equal positive and negative amplitudes i.e. binary 1 will be represented by a +V pulse and binary 0 will be represented by a -V pulse. In either case, the assigned pulse amplitude is constant throughout the bit interval. Polar NRZ waveforms are used in RS232C interface, digital logic and magnetic tape recording.

2. RZ group

- RZ can be partitioned into the following subgroups: unipolar RZ, polar RZ, and RZ-AMI. These line codes find application in baseband data transmission and in magnetic recording.

- With unipolar RZ, a 1 is represented by a half-width pulse that returns to zero level before the end of the bit interval, and a 0 is represented by the absence of a pulse.

- With polar RZ, the 1s and 0s are represented by pulses of opposite polarities respectively that are one-half bit wide. In either case, the pulse returns to zero level before the end of the bit interval.

- RZ-AMI (AMI for "alternate mark inversion") is the line coding scheme most often used in telemetry systems. The 1s are represented by equal amplitude alternating pulses of opposite polarities. The 0s are represented by the absence of pulses.

3. Phase-encoded group

- This group consists of four subgroups. The phase-encoding schemes are used in magnetic recording systems and optical communications and in some satellite telemetry links.

- The better known subgroup is Manchester coding. In Manchester coding, a half-bit-wide pulse positioned during the first half of the bit interval represents a 1; a half-bit-wide pulse positioned during the second half of the bit interval represents a 0. This is used in Ethernet LAN networks.

4. Multilevel group

- Many binary waveforms use three levels, instead of two, to encode the binary data. Polar RZ and RZ-AMI belong to this group.

One might ask why there are so many line codes. Are there really so many unique applications necessitating such a variety of waveforms to represent binary digits? The reason for the large selection relates to the differences in performance that characterises each waveform. In choosing a line coding scheme for a particular application, some of the parameters worth examining are the following:

- **Transmission bandwidth.** This should be as small as possible.

- **DC component.** Eliminating the dc energy from the signal's power spectrum enables the system to be coupled. Magnetic recording systems, or systems using transformer coupling, have little sensitivity to very low frequency signal components. Thus low-frequency information could be lost.

- **Self-Clocking.** Symbol or bit synchronization is required for any digital communication system. Some line coding schemes have inherent synchronizing or clocking features that aid in the recovery of the clock signal. For example, the Manchester code has a transition in the middle of every bit interval whether a one or a zero is being sent. This guaranteed transition provides a clocking signal.

- **Noise immunity.** The various line code waveforms can be further characterised by probability of bit error versus signal-to-noise ratio. Some of the schemes are more immune to noise than others. For example, the NRZ waveforms have better error performance than does the unipolar RZ waveform.

Having discussed the various ways of representing binary data in electrical waveforms, we shall now go into more details of the transmission problems of these waveforms.

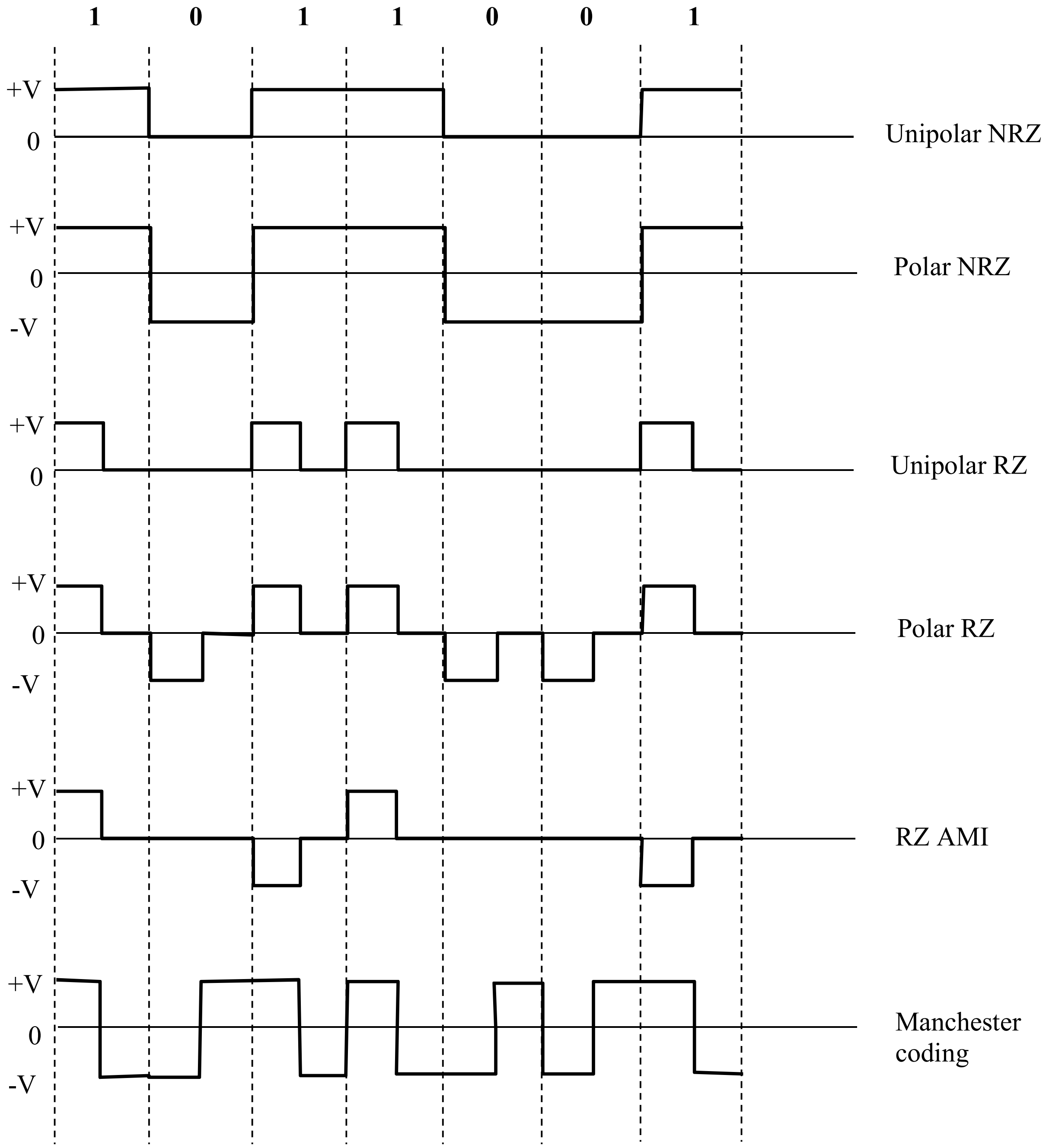


Figure 8.1 Various line code waveforms for binary data transmission.

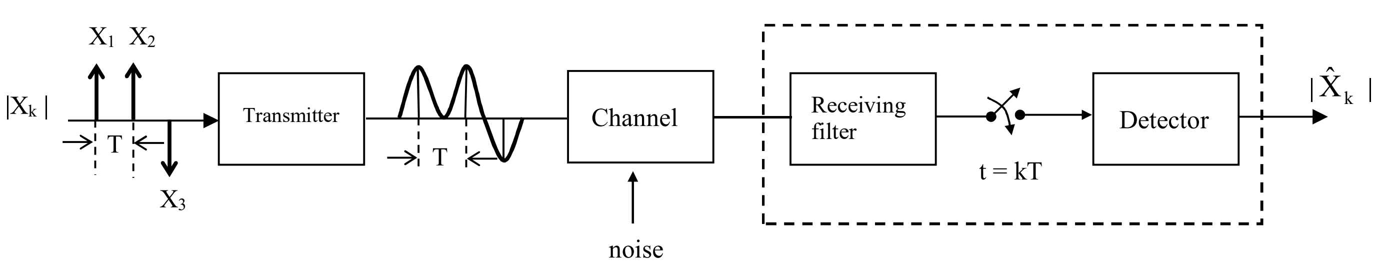
8.2 INTERSYMBOL INTERFERENCE (ISI)

Figure 8.2(a) highlights the major filtering aspects of a typical baseband digital system; there are circuit reactances throughout the system - in the transmitter, in the receiver, and in the channel. The pulses at the input might be impulse-like samples, or flat-top samples. In either case, they are low-pass filtered at the transmitter to confine them to some desired bandwidth. Channel reactances can cause amplitude and phase variations that distort the pulses. The receiving filter, called the equalizing filter, should be configured to compensate for the distortion caused by the transmitter and the channel. Let us assume that the equalizer filter does a perfect job, such that the received pulses are similar to the transmitted pulses (refer to Figure 8.2(a)). The function of the detector is to sample the received pulses at regular intervals of T_b seconds, at the instant when the amplitude of the pulse is a maximum, and compare each received pulse to a threshold. For example, the detector decides that a binary one was sent if the received pulse is positive, and that a binary zero was sent if the received pulse is negative.

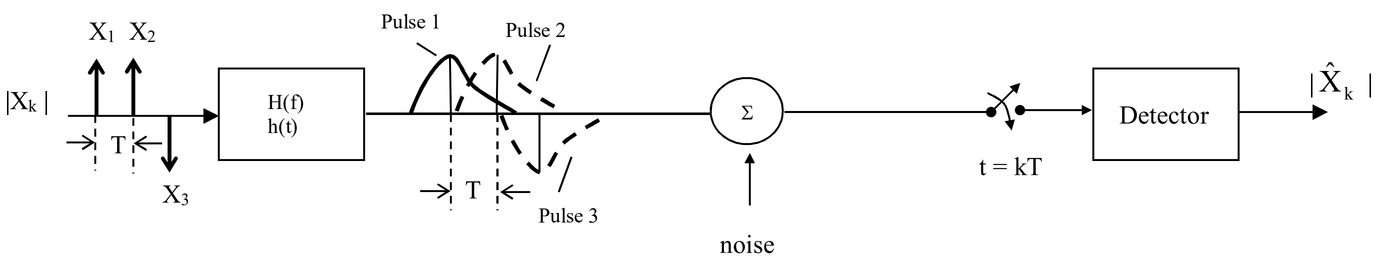
Figure 8.2(b) illustrates a convenient model for the system, lumping all the filtering effects into one overall equivalent system transfer function, H(f):

$$H(f) = H_t(f)H_c(f)H_r(f) \tag{8.1}$$

where $H_t(f)$ characterizes the transmitting filter, $H_c(f)$ the filtering within the channel, and $H_r(f)$ the receiving or equalizer filtering. The characteristic H(f), then, represents the composite system



(a)



(b)

Figure 8.2 Intersymbol interference in the detection process.
(a) Typical baseband digital system.
(b) Equivalent model.

transfer function due to all of the filtering at the various locations throughout the transmitter/channel/receiver chain. Due to the effects of system filtering, the received pulses overlap one another as shown in Figure 8.2(b); the tail on one pulse "smear" into adjacent symbol intervals so as to interfere with the detection process; such interference is termed intersymbol interference (ISI). Even in the absence of noise, imperfect filtering and system bandwidth constraint lead to ISI. In practice, $H_c(f)$ is usually specified, and the problem remains to determine $H_t(f)$ and $H_r(f)$ such that the ISI of the pulses are minimized at the output of $H_r(f)$.

Nyquist investigated the problem of specifying a received pulse shape so that no ISI occurs at the detector. **He showed that the theoretical minimum system bandwidth needed to transmit R_b bits/s, without ISI, is $R_b/2$ hertz.** This occurs when the system transfer function, H(f), is an ideal low-pass filter, as shown in Figure 8.3.

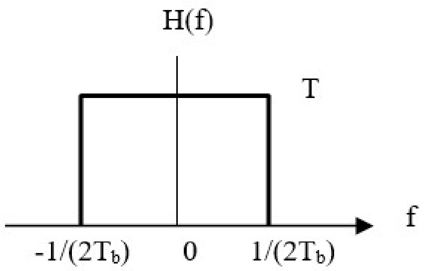


Figure 8.3 Rectangular system transfer function H(f).

For most communication systems, our goal is to reduce the required system bandwidth as much as possible. Nyquist has provided us with a basic limitation to such bandwidth reduction. What would happen if we tried to force a system to operate at smaller bandwidth than the constraint dictates? We would find that restricting the bandwidth would spread the pulses in time. This would degrade the system's error performance, due to the increase in ISI.

Example 8.1

A binary source is transmitting information at a rate of 1 kb/s. A binary 1 is transmitted as a unit impulse and a binary 0 as no pulse. The channel is an ideal, rectangular low pass filter with zero-phase shift. What is the minimum cutoff frequency of the channel to allow ISI-free transmission?

Solution

Given R_b = 1 kb/s.
From Nyquist's Theorem, the theoretical system bandwidth (in this case, the channel bandwith) to detect R_b bits without ISI is $R_b/2$ hertz, if the system (or channel) has an ideal low pass filter characteristic.

Hence the channel cutoff frequency is $f_c = R_b/2 = 500$ Hz.

8.3 CHANNEL NOISE

We have identified ISI as a possible source of transmission errors. Another possible cause of transmission errors is due to the noise in the communication system. "Noise" refers to unwanted electrical signals that accompany the message signals. Sources of these noises: electromagnetic pickup of other radiating signals, 50-cycle hum due to inadequate power supply filtering, atmospheric disturbances, etc. These noises can be eliminated or reduced by careful engineering design for example by using shielded cables.

However one unavoidable noise is the **thermal noise** caused by the random motion of electrons in conducting media - wires, resistors, and so forth. The main source of this noise is from the communication channel. Characteristics of thermal noise include:

- zero-mean voltage
- Gaussian probability density function (pdf)
- corrupts the desired signal in an additive fashion
- Power spectral density (psd) of thermal noise voltage is essentially flat over the range of frequencies used in conventional communication systems.

Hence a channel affected by thermal noise is usually called an additive, white, Gaussian noise (AWGN) channel.

8.3.1 Probability Density Function and Probability of Gaussian Noise

The probability density function of thermal noise is Gaussian, and is given by

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left[\frac{n}{\sigma}\right]^2 \right] \tag{8.2}$$

where σ = the standard deviation of the noise signal or the rms noise voltage
n = noise voltage

This distribution is shown in Figure 8.4. It can be seen the pdf is symmetrical about n = 0.

The probability of noise voltage n being in the range (n_1,n_2) is given by the area of p(n) under (n_1,n_2):

$$\text{Probability } (n_1 < n < n_2) = \int_{n_1}^{n_2} p(n)dn \tag{8.3}$$

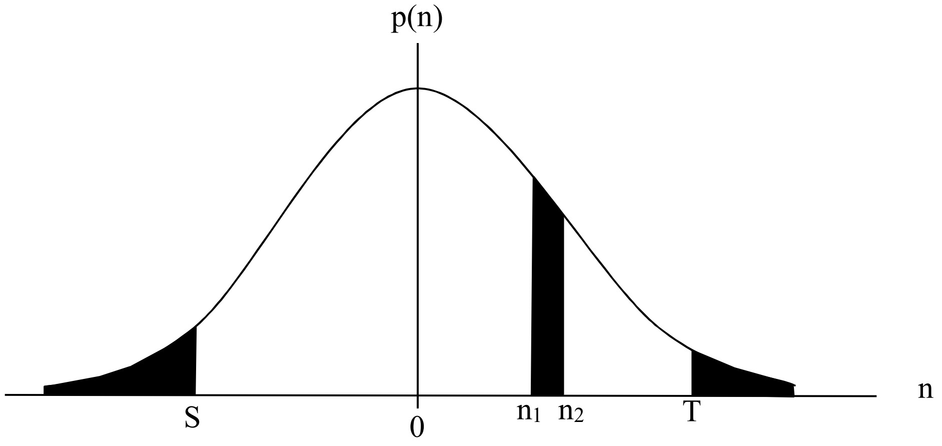


Figure 8.4 Probability density function of Gaussian Noise.

Similarly, the probability of observing n > T volts is given by:

$$\text{Probability } (n > T) = \int_T^{\infty} p(n)dn \tag{8.4}$$

and

$$\text{Probability } (n < S) = \int_{-\infty}^{S} p(n)dn \tag{8.5}$$

The probability of n > T volts is evaluated using the complementary error function, *erfc(z)*:

$$\text{Probability}(n > T) = \int_T^{\infty} p(n)dn = \frac{1}{2} erfc\left[\frac{T}{\sqrt{2}\sigma}\right] \tag{8.6}$$

$\int_{\alpha}^{T} p(n)dn$ cannot be determined in closed form. Instead, *erfc(z)* is used because its values have been extensively worked out and tabulated. The *erfc(z)* table is in Appendix I at the end of chapter 8.

Example 8.2

What is the probability of a zero-mean white Gaussian noise:

(i) exceeding 8 mV, if it has an rms value of 2 mV ?
(ii) exceeding a magnitude of 5 mV, if it has an rms value of 3 mV ?
(iii) equal or less than a magnitude of 5 mV, if it has an rms value of 3 mV ?

<u>Solution</u>

(i) The probability that the noise exceeds T volt is given by

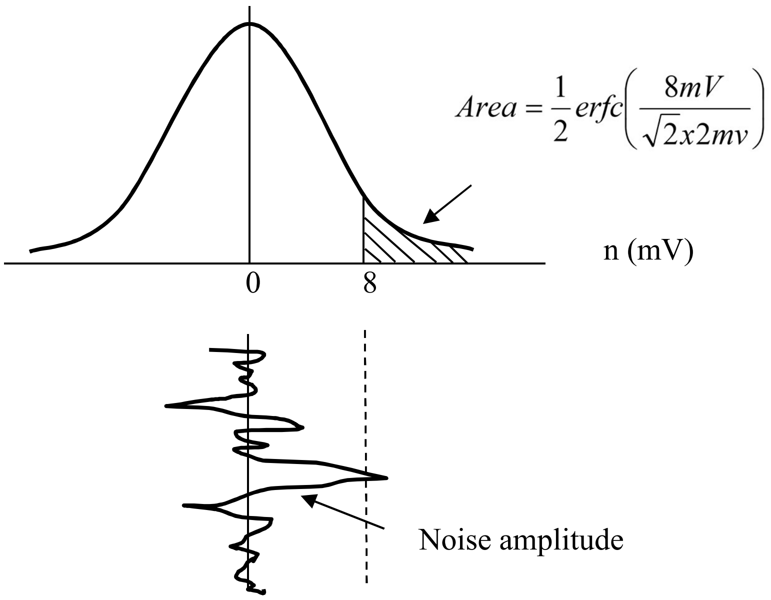
$$P(n > T) = \frac{1}{2} erfc\left[\frac{T}{\sqrt{2}\sigma}\right]$$

where σ = rms value of noise signal.

Therefore P(n > 8 mV) = 1/2 erfc [8mV/($\sqrt{2}$ x 2 mV)]
= 1/2 erfc [2.828]

Referring to the Probability and Statistic table:
For z = 2.82; erfc(z) = 0.666 x 10^{-4} (Note: z is round down for worse case bit error)

Therefore P(n > 8 mV) = 1/2 x 0.666 x 10^{-4} = 3.33 x 10^{-5}



(ii) The probability that the noise exceeds a magnitude of T volt is given by

P(|n| > T) = erfc [T/($\sqrt{2}\sigma$)]

as

$$P(|n| > T) = P(n > T) + P(n < -T)$$

$$= 2x1/2 \text{ erfc}[T/(\sqrt{2}\sigma)]$$

(since $P(n > T) = P(n < -T)$ – symmetrical about n=0 axis)

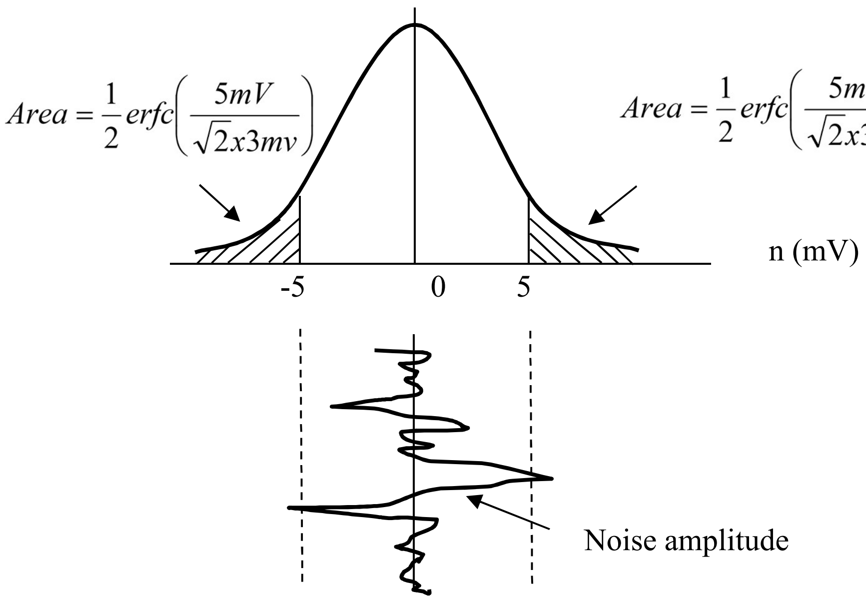
$$= \text{erfc}[T/(\sqrt{2}\sigma)]$$

Therefore

$$P(|n| > 5 \text{ mV}) = \text{Total shaded area}$$

$$= \text{erfc}[5 \text{ mV}/(\sqrt{2} \text{ x } 3 \text{ mV})]$$

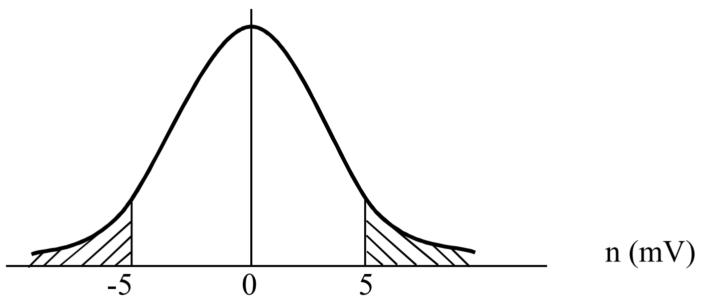
$$= \text{erfc}(1.179)$$



From the Prob & Stat table:
$z = 1.17$; $\text{erfc}(z) = 0.98 \times 10^{-1}$

Therefore $P(|n| > 5 \text{ mV}) = 0.98 \times 10^{-1}$

(iii)



Unshaded area = 1 - 0.98 x 10^{-1} = 0.902

Hence P(|n| < 5 mV) = 0.902

8.3.2 Probability of Error

In an analog communication system, the extent to which noise affects the performance of the system is measured by the signal-to-noise ratio at the output of the system. For a digital communication system, the probability of error is used. Figure 8.5 shows the relationship between probability of error and signal-to-noise ratio. As the signal to noise ratio increases, the probability of error reduces.

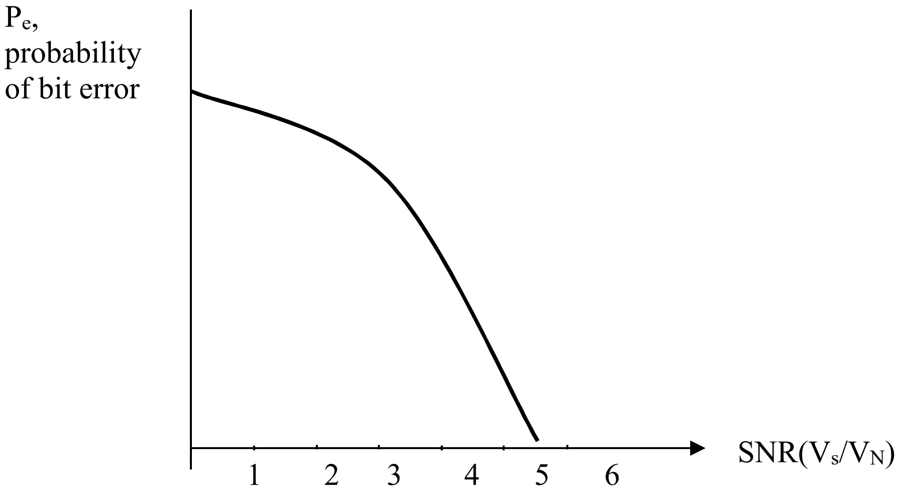


Figure 8.5 Graph of bit error probability vs. signal- to-noise ratio for polar NRZ baseband transmission.

Consider a digital communication system whose input is a sequence of bits $\{S_k\}$. For example, $\{S_k\}$ may take the form: ...101101110010... . The output of the system will be a sequence $\{\hat{S}_k\}$. In an ideal system, $\{\hat{S}_k\}$ will be the same as $\{S_k\}$. However, in practical systems, the input and the output sequences will differ occasionally, due to errors caused by the channel noise. The overall performance of a digital communication system is measured in terms of the probability of bit error P_e, defined as

$$P_e = \text{Probability}(\hat{S}_k \neq S_k)$$

Practical systems have P_e ranging from 10^{-4} to 10^{-7}. Bit error rate (BER) is the practical measurement of P_e. The BER can be obtained in practice by:

$BER = N_e/N_t$
where N_e = total number of error bits over time interval T
N_t = total number of bits transmitted over time interval T.

N_t should be very large. For example, to measure the BER of a transmission system with possible P_e of 10^{-6} and a bit rate of 1 kb/s would take at least $T = 10^6/10^3 = 10^3$ seconds or about 17 minutes. Usually T must be much longer to give a better approximation of P_e.

8.3.3 Operation of a Baseband Binary Receiver

Figure 8.6 shows a baseband binary receiver. The received signal plus noise is first applied to a lowpass filter which has been designed to remove excess noise without introducing ISI. A sample-and-hold (S/H) device, triggered at the optimum times, extracts the sample values from y(t):

$$y(t_k) = a_k + n(t_k)$$

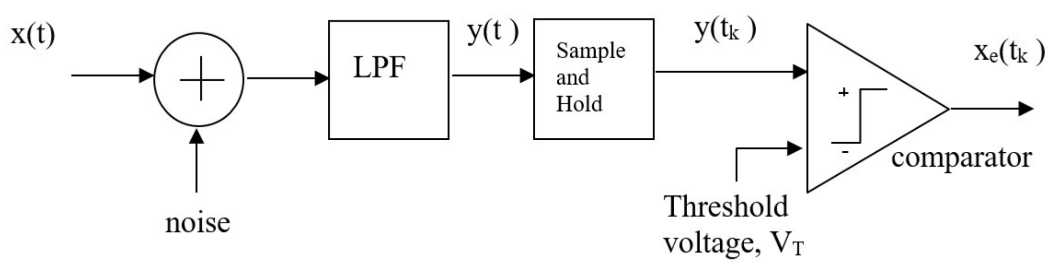


Figure 8.6. Baseband binary receiver

The comparator compares values of $y(t_k)$ with a fixed threshold level V_T. If $y(t_k) > V_T$, the comparator goes HIGH to indicate a 1; if $y(t_k) < V_T$, the comparator goes LOW to indicate 0. The regenerator i.e. sample-and-hold circuit and the comparator thereby acts as an analog-to-digital converter, converting the noisy analog waveform y(t) into a noiseless digital signal $x_e(t)$ with occasional errors. Thus the original transmitted data is decoded or recovered.

Assume x(t) to be a unipolar signal in which $a_k = A$ represents bit '1' and $a_k = 0$ represents the bit '0'. The threshold voltage should be set at some intermediate level, $0 < V_T < A$. *For minimum bit error, the threshold is set at A/2.* The regeneration process is illustrated by the waveforms in Figure 8.7. Notice that noise has caused two error bits in the regenerated signal.

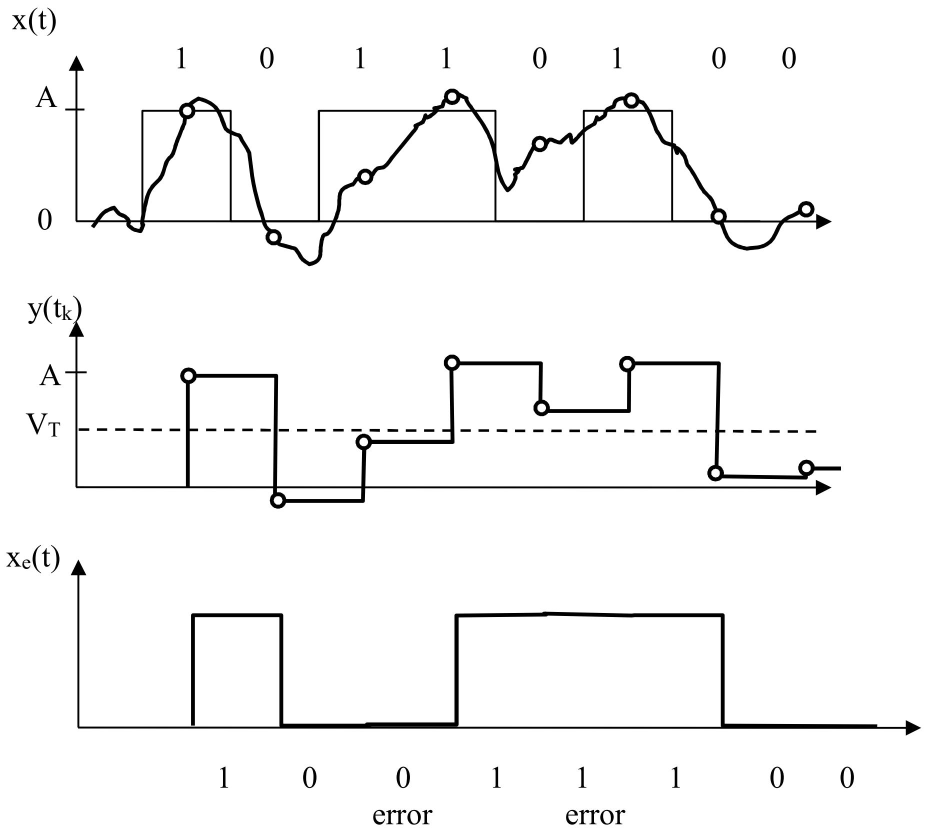


Figure 8.7 Regeneration of a unipolar NRZ signal.
(a) Signal plus noise;
(b) S/H output;
(c) Decoded output (comparator output).

We shall now calculate the probability of error in a baseband binary receiver. The probability of error is determined as follows:

Probability of error $P_e = P(1)P([V_1 + n] < V_T) + P(0)\ P([V_0 + n] > V_T)$
where $P(1)$ = probability of transmitting binary '1'
$P(0)$ = probability of transmitting binary '0'
V_1 = voltage level for binary '1'
V_0 = voltage level for binary '0'
n = noise

If $P(1) = P(0) = 0.5$, then $P_e = 0.5P([V_1 + n]<V_T) + 0.5P([V_0 + n] >V_T)$.

$$= 0.5P(n <[V_T - V_1]) + 0.5P(n >[V_T - V_0]).$$

Since V_T is at the mid-point of V_1 and V_0, $V_T = 0.5(V_1 + V_0)$.
Therefore $P_e = 0.5P(n <[0.5(V_1 + V_0) - V_1]) + 0.5P(n >[0.5(V_1 + V_0) - V_0])$

$$= 0.5P(n <[-0.5(V_1 - V_0)]) + 0.5P(n >[0.5(V_1 - V_0)])$$

Let R be the half signal excursion i.e. $R = 0.5(V_1 - V_0)$.

Hence $P_e = 0.5P(n <-R) + 0.5P(n>R)$
$= P(n>R)$

(since $P(n <-R) = P(n >R)$, symmetrical about n=0)

Therefore, $P_e = P(n > R) = \frac{1}{2} erfc\left[\frac{R}{\sqrt{2}\sigma}\right]$

Example 8.3

A discrete data source is transmitting a random binary signal such that the probability of transmitting 1 or 0 is equiprobable. The signal input to the comparator at the receiver is 0.5 volt for binary 1 and -0.5 volt for binary 0. The channel noise is AWGN with an rms value of 0.2 volt.

(i) What is the bit error rate?
(ii) If a million bit is transmitted for each block of message, on the average, how many bits are received incorrectly per block?

Solution

(i) $P_e = 1/2\ erfc\ (R/(\sqrt{2}\sigma))$

where R is half the signal excursion range.

Signal excursion is from -0.5 V to +0.5 V.
Therefore signal excursion range is 1 V and R = 0.5 V
Thus P_e = 1/2 erfc (0.5/($\sqrt{2}$ x 0.2) = 1/2 erfc (1.76)

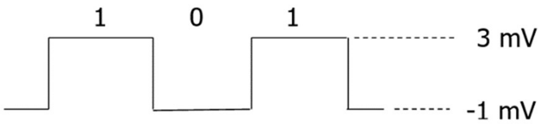
From the Prob & Stat Table : if z = 1.76, erfc(z) = 0.128 x 10^{-1}

Therefore P_e = 1/2 erfc (1.76) = 1/2 x 0.128 x 10^{-1} = 6.4 x 10^{-3}

(ii) On the average, 10^6 x 6.4 x 10^{-3} = 6400 bits are received incorrectly, for every million bits transmitted.

Example 8.4

The signal component to the receiver of a baseband transmission system is of the form:



The signal is corrupted by additive white Gaussian noise (AWGN) which has an rms value of 0.8 mV. Assume equal probability of transmitting binary 1 or 0 and independent bit transmission.

(i) Calculate the threshold voltage V_T of the receiver comparator for minimum bit error.

(ii) Calculate the probability of bit error P_e at the receiver.

(iii) If the transmission bit rate is 1 Mb/s, what is the average duration between bit errors?

Solution

(i) $V_H = 3$ mV; $V_L = -1$ mV; $\quad V_T = \dfrac{V_H + V_L}{2} = 1\,\text{mV}$

(ii)

$$R = \frac{V_H - V_L}{2} = 2mV$$

$$P_e = \frac{1}{2}\text{erfc}\left(\frac{R}{\sqrt{2}\,\sigma}\right) = \frac{1}{2}\text{erfc}\left(\frac{2\,\text{mV}}{\sqrt{2}\;0.8\,\text{mV}}\right) = \frac{1}{2}\text{erfc}(1.768) = \frac{1}{2}\text{erfc}(1.76)$$

$$= \frac{1}{2}\text{ x } 0.128 \text{ x } 10^{-1} = 6.4 \text{ x } 10^{-3}$$

Z	erfc(Z)
1.72	0.149972D-01
1.73	0.144215D-01
1.74	0.138654D-01
1.75	0.133283D-01
1.76	0.128097D-01

(iii) In one second, there are 10^6 x 6.4 x 10^{-3}= 6400 error bits. Therefore average duration between error bits = 1/6400 =1.563 x 10^{-4} = 0.156 ms.

8.4 JITTERS

The baseband binary receiver explained earlier is reproduced again in Figure 8.8.

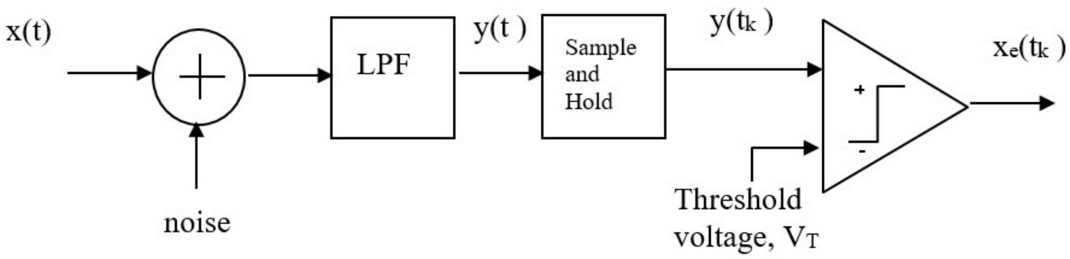


Figure 8.8 Baseband binary receiver

For a self-synchronization system, the "sync", which is the bit timing clock, is derived from the received signal itself, usually from the zero crossings of the incoming pulse stream. Due to the imperfect channel characteristics, these pulses will be affected by ISI and noise. Hence the zero crossings will not be regular, as it was at the transmitting end if the appropriate line code waveform types (e.g. bipolar RZ) are used. This variations in the zero crossing which results in the variations of clock rate and phase, is called jitters. The ITU has defined jitters as the short time variations of the significant instants (sampling instants) of a digital signal from their ideal positions in time.

8.5 EYE DIAGRAM

The eye diagram technique is a useful experimental method for assessing the data handling ability of a digital transmission system. The eye diagram test allows us to present a visual display, on an oscilloscope, of the effects of signal processing and/or transmission path on a digital waveform.

Figure 8.9 illustrates a typical set-up of the eye diagram test system. The signal $v_o(t)$ is fed to channel 1 of an oscilloscope. The bit clock is fed to the external trigger of the oscilloscope. The horizontal time base is set approximately equal to the bit duration. The inherent persistence of the cathode ray tube displays the superimposed segments of the $v_o(t)$ signal resulting in the eye diagram.

Figure 8.10 shows some examples of eye diagrams measured using the technique described.

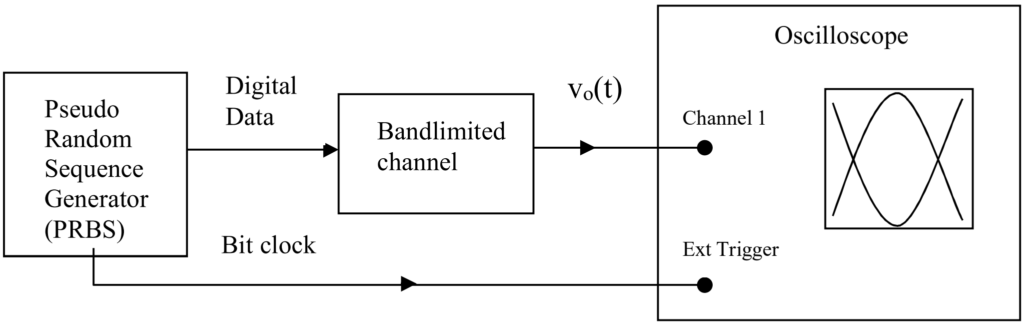
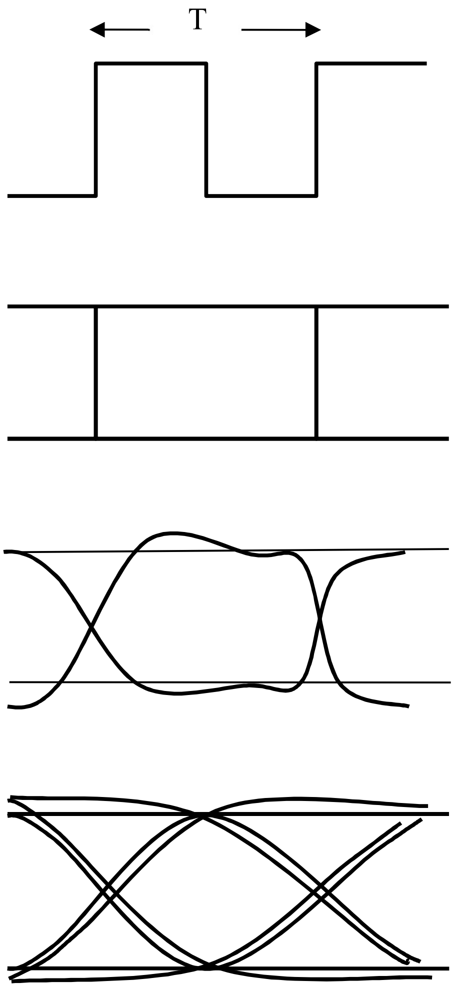


Figure 8.9 Eye diagram measurement set-up.



(a) Clock.

(b) Eye pattern of PRBS connected directly to CRO (bypassing system under test).

(c) Eye pattern of a channel that is free from ISI and noise.

(d) Eye pattern of a channel with ISI (but free from noise) due to insufficient bandwidth.

Figure 8.10 Examples of eye diagrams

8.5.1 Eye Diagram Measurement

For systems with high signal-noise-ratio we can simplify the eye diagram as shown in Figure 8.11.

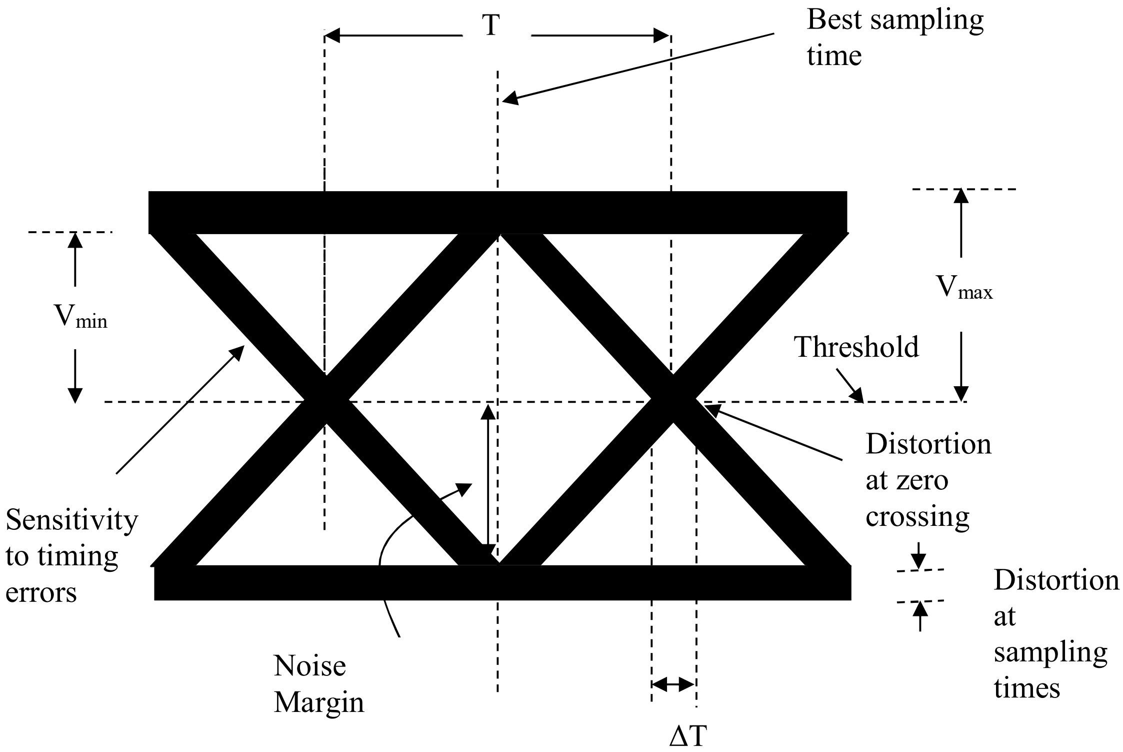


Fig 8.11 Characteristics of an eye diagram.

The following observations can be made.

- The best time to sample the received waveform is when the eye opening is largest.

- Amplitude variations at the sampling instant due to ISI can lower the noise margin, where noise margin (or noise immunity) is proportional to the width of the eye opening. We can define the percentage of noise margin as

  Noise Margin (%) = V_{min}/V_{max} x 100 %

  Also, ISI degradation may be defined as

  ISI Degradation = 20 $\log_{10}$ (V_{max}/V_{min}) dB
- The sampling time is midway between zero crossings in this case. If the clock

information is derived from the zero crossings, then the amount of distortion of zero crossings indicates the amount of jitter which can be measured by defining

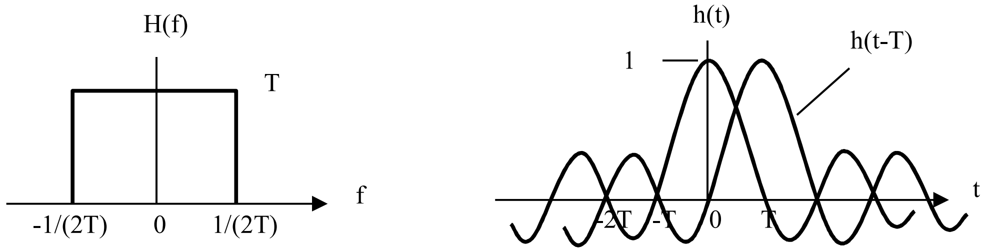
Jitter (%) = $\Delta T/T \times 100$ % where T = one bit interval.

- Asymmetry in the eye diagram indicates non-linearities in the system under test.

APPENDIX

FURTHER ANALYSIS OF ISI PROBLEM

Nyquist investigated the problem of specifying a received pulse shape so that no ISI occurs at the detector. He showed that the theoretical minimum system bandwidth needed to transmit R_b bits/s, without ISI, is $R_b/2$ hertz. This occurs when the system transfer function, H(f), is made rectangular, as shown in Figure A8.1(a). When H(f) is such an ideal low pass filter with bandwidth 1/(2T) or $R_b/2$, its impulse response, the inverse Fourier transform of H(f) is h(t) = sinc t/T, as shown in Figure A8.1(b).



(a) (b)

Figure A8.1 (a) Rectangular system transfer function H(f),
(b) Received pulse shape h(t) = sinc (t/T).

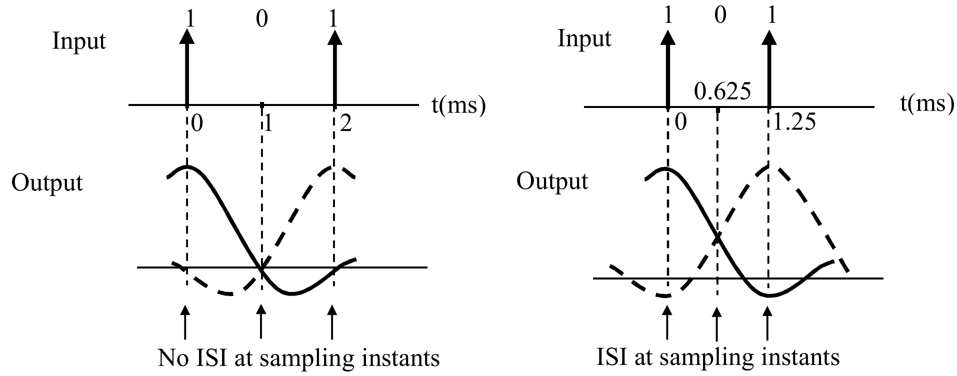
Thus h(t) is the received pulse shape resulting from the application of an impulse at the input of such an ideal system. Nyquist established that if each pulse from a received sequence is of the form h(t), the pulses can be detected without ISI. Figure A8.1(b) illustrates how ISI is avoided. The figure shows two successive received pulses, h(t) and h(t - T). Even though h(t) has a long tail, it passes through zero at the instant that h(t - T) is sampled (at t = T) and therefore causes no interference to the detection process.

Example A.1

A binary source is transmitting information at a rate of 1 kb/s. A binary 1 is transmitted as a unit impulse and a binary 0 as no pulse. The channel is an ideal, rectangular low-pass filter with zero-phase shift and a cutoff frequency of 500 Hz.
Assume a 101 sequence is sent. Show that when the transmission rate is increased from 1 kb/s to 1.6 kb/s, ISI occurs at the output of the channel.

Solution

Transmission rate at 1 kb/s: Transmission rate at 1.6 kb/s



PULSE SHAPING TO REDUCE ISI

The Nyquist requirement for a sinc(t/T) received pulse shape is not physically realizable since the system characteristic have to be an ideal low pass filter. Also, with such a characteristic, the detection process would be very sensitive to small timing errors. In Figure A8.1(b), the pulse h(t) has zero value in adjacent pulse times only when the sampling is performed at exactly the correct sampling time; timing errors will produce ISI. Therefore we cannot implement such systems in practice; we need to provide some "excess bandwidth" beyond the theoretical minimum.

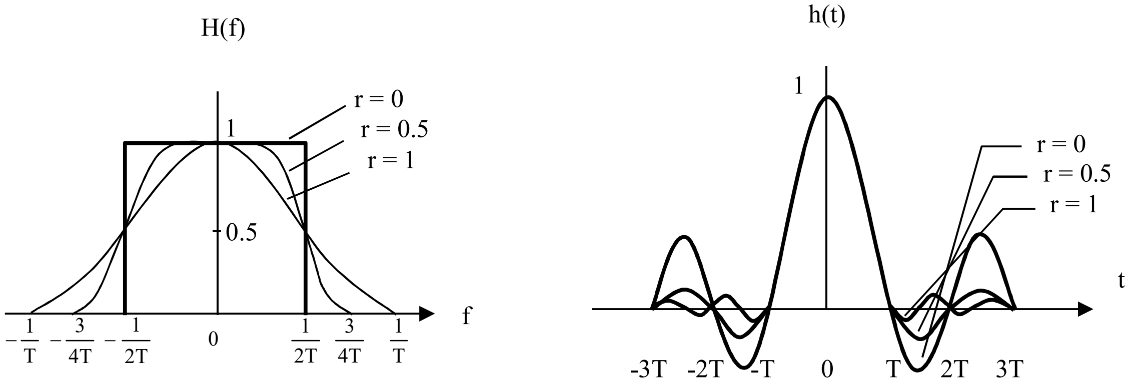
One frequently used system transfer function, H(f), is called the raised cosine filter. It can be expressed as

$$H(f)=\begin{cases}1 & for\,|f|<2W_o-W\\ \cos^2\left[\dfrac{\pi(|f|+W-2W_o)}{4(W-W_o)}\right] & for\ 2W_o-W<|f|<W\\ 0 & for\ |f|>W\end{cases} \qquad (A8.1)$$

where W is the absolute bandwidth, and $W_0 = 1/2T$ represents the minimum Nyquist bandwidth for the rectangular LPF spectrum. The difference $(W - W_0)$ is termed the excess bandwidth; notice that $W = W_0$ for the rectangular spectrum. The **roll-off factor** is defined to be $r = (W - W_0)/W_0$. For a given W_0, r characterized the steepness of the filter roll-off. The raised cosine characteristic is illustrated in Figure A8.2(a) for roll-off values of r = 0, r = 0.5, and r = 1.0. The r = 0 roll-off is the Nyquist minimum-bandwidth case. Notice that when r = 1.0, the required excess bandwidth is 100 % (twice the Nyquist bandwidth).

The corresponding impulse response for the H(f) of eq.(A8.1) is shown in Figure A8.2 (b) for r =

0, r = 0.5, and r = 1.0. Notice that in Figure A2(b) that timing errors will still result in some ISI degradation for r = 1. However, the problem is not as serious as it is for r = 0, because the tails of the h(t) waveforms are of much smaller amplitude for r = 1 than they are for r = 0.



(a) (b)

Figure A2 Raised cosine filter characteristics.
(a) System transfer function.
(b) System impulse response.

Chapter 9

Optimum Baseband Receiver

Learning Outcomes

- ❑ Understand the principles of baseband receivers.
- ❑ Identify the matched filter as a means of reducing the probability of bit error
- ❑ in baseband detection.
- ❑ Describe and derive graphically the impulse response of a matched filter.
- ❑ Determine the probability of bit error for matched filter receivers.
- ❑ Verify that an integrate-and-dump receiver is an example of a matched filter
- ❑ implementation.

INTRODUCTION

Consider the transmission of a random, binary digital signal over a distortionless channel of infinite bandwidth. The input and output waveforms of the channel may appear as shown in Figure 9.1(b) and 9.1(c).

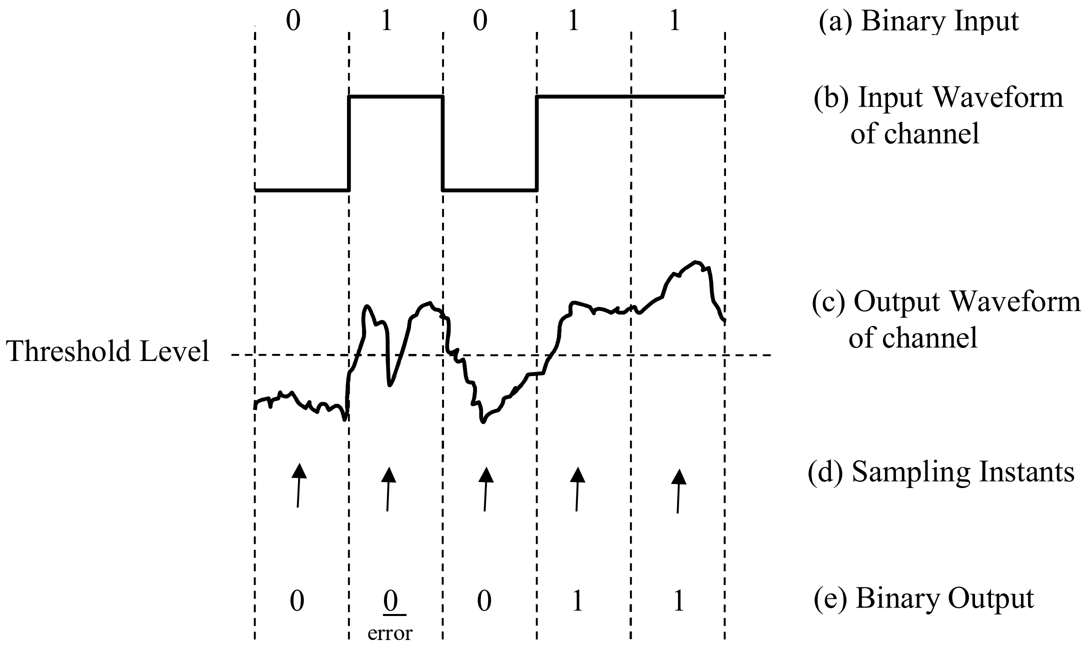


Figure 9.1 Showing the process of detection by a binary receiver

Figure 9.1(c) shows the digital signal being corrupted by channel noise. Let us assume the noise is additive, white and Gausian, with zero mean.

The receiver at the end of the channel samples the received waveform at regular bit intervals and decide whether a +V volt or -V volt was sent. Due to noise in the channel, it is possible for the receiver to decode wrongly.

The **optimum receiver** is one which distinguishes between +V and -V from their noisy versions with minimum probability of error.

9.1 OPTIMUM RECEIVER FOR BINARY TRANSMISSION

The optimum receiver consists of a filter, a sampler and a threshold device as shown in Figure 9.2.

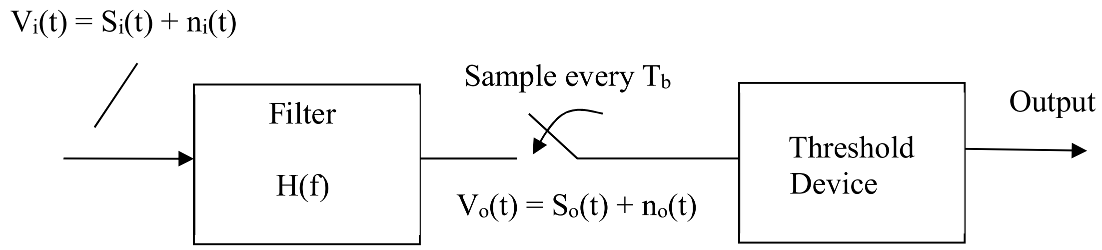


Figure 9.2 Optimum Receiver Structure

Let the input to the optimum receiver be a signal as following:

$$V_i(t) = s_i(t) + n_i(t)$$

where $s_i(t)$ and $n_i(t)$ are the signal and noise components, respectively. For simplicity, let the signal component be of the form as shown in Figure 9.3(a).

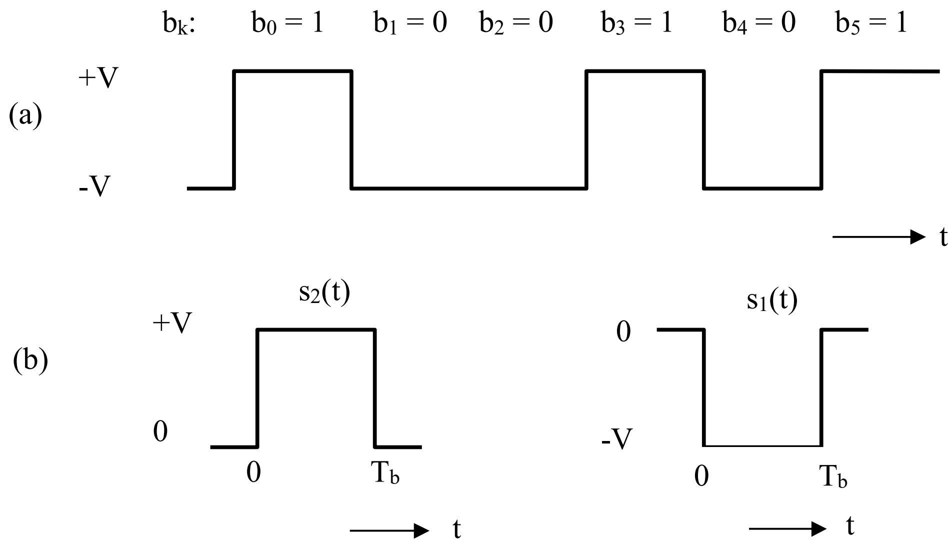


Figure 9.3 (a) Input signal waveform to optimum receiver
(b) Basic pulses that made up the input signal waveform
- polar NRZ inputs

The pulse sequence can be considered to be made up of basic pulses, $s_2(t)$ and $s_1(t)$, where

$$s_i(t) = \begin{cases} s_2(t - kT_b) = +V, & \text{if } b_k = 1 \\ s_1(t - kT_b) = -V, & \text{if } b_k = 0 \end{cases}$$

for $kT_b \le t \le (k+1)T_b$

The input at the receiver also has a noise component, $n_i(t)$, which is assumed to be zero-mean Gaussian. The output of the filter, H(f), is

$$V_0(t) = \int_{-\infty}^{\infty} V_i(\tau)h(t - \tau)d\tau \tag{9.1}$$

where $V_i(t) = s_i(t) + n_i(t)$

Note that $V_o(t)$ has a signal component $s_o(t)$ due to $s_i(t)$, and a noise component $n_o(t)$ due to $n_i(t)$ and

$V_o(t) = s_o(t) + n_o(t)$

The signal component at the output of the filter, $s_o(t)$, can be considered to comprise two basic pulses, $s_{o2}(t)$ and $s_{o1}(t)$, where

$$s_o(t) = \begin{cases} s_{o2}(t) & \text{for } +V, (\text{or } s_2(t)) \text{ input} \\ s_{o1}(t) & \text{for -} V, (\text{or } s_1(t)) \text{ input} \end{cases}$$

For an optimum receiver, the output $V_o(t)$ is sampled at the end of every bit interval, T_b. Further, the output should have zero ISI. To satisfy the zero ISI requirement, we can make $V_o(t)$ to be zero outside the current bit frame. Thus, for the current bit frame, equation 5.1 can be written as:

$$V_0(T_b) = \int_0^{T_b} V_i(\tau)h(T_b - \tau)d\tau$$

where $V_o(t) = 0$ for *t* not in the range of the current bit frame.

The signal component at the output of the filter is

$$s_o(T_b) = \begin{cases} \int_0^{T_b} s_1(\tau)h(T_b - \tau)d\tau = s_{o1}(T_b) \text{ for binary 0} \\ \int_0^{T_b} s_2(\tau)h(T_b - \tau)d\tau = s_{o2}(T_b) \text{ for binary 1} \end{cases}$$

The noise component at the output of the filter is

$$n_0(T_b) = \int_0^{T_b} n_i(\tau)h(T_b - \tau)d\tau$$

9.1.1 Matched Filter

It can be shown that to minimise the probability of error, the filter should take on the form of a **matched filter**, whose impulse response, h(t), is related to $s_1(t)$ and $s_2(t)$ by

$$h(t) = s_2(T_b - t) - s_1(T_b - t)$$

The process of obtaining h(t) from $s_2(t)$ and $s_1(t)$ is shown in an example in Figure 9.4. It involves a few steps.

For polar NRZ inputs, where $s_2(t) = +V$ and $s_1(t) = -V$,

$$h(t) = \begin{cases} \text{2V for } 0 \le t \le T_b \\ \\ \text{0V for other t} \end{cases}$$

9.1.2 Probability of bit error

The probability of bit error for the matched filter is given by

$$P_e = \frac{1}{2} erfc\left(\frac{\gamma}{2\sqrt{2}}\right)$$

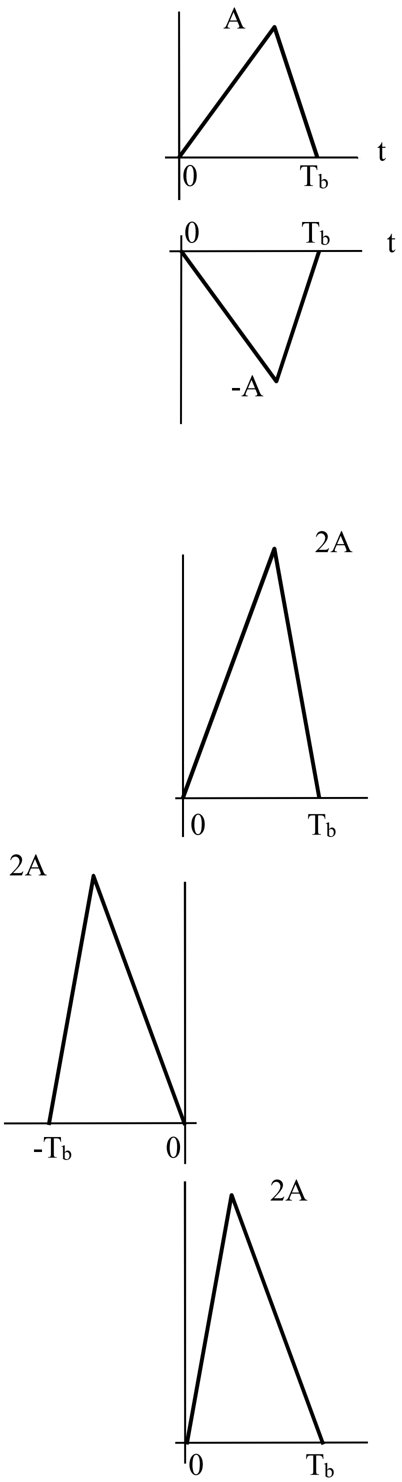
where $\gamma^2 = \frac{2}{\eta}\int_0^{T_b} [s_2(t) - s_1(t)]^2 dt$ and $\frac{\eta}{2}$ is the double sided power spectral density of the white channel noise.

Taking the example of polar inputs: $s_2(t) = +V$ and $s_1(t) = -V$,

$$\gamma^2 = \frac{2}{\eta}\int_0^{T_b} 4V^2 dt = \frac{8}{\eta}V^2 T_b$$

$$\gamma = V\sqrt{\frac{8T_b}{\eta}}$$

Therefore $P_e = \frac{1}{2} erfc\left\{V\sqrt{\frac{T_b}{\eta}}\right\} = \frac{1}{2} erfc\left\{\sqrt{\frac{V^2 T_b}{\eta}}\right\}$



(a) $s_2(t)$

(b) $s_1(t)$

(c) $s_2(t) – s_1(t)$

(d) $s_2(-t) – s_1(-t)$

Fold about the
vertical axis

(e) $h(t) = s_2(T_b-t) – s_1(T_b-t)$

Shift to the right by T_b

Figure 9.4 Showing the process of obtaining h(t) from $s_2(t)$ and $s_1(t)$

9.1.3 Implementation of the matched filter

The matched filter receiver can be implemented using an Integrate-and-Dump Correlation Receiver. From Figure 9.3, the output of the filter, H(f), at the end of each bit frame is

$$V_0(T_b) = \int_0^{T_b} V_i(\tau)h(T_b - \tau)d\tau \tag{9.2}$$

We have said that for a matched filter,

$$h(t) = s_2(T_b - t) - s_1(T_b - t)$$

If we let $t = T_b - \tau$, we have

$$h(T_b - \tau) = s_2(\tau) - s_1(\tau) \tag{9.3}$$

Substituting eq.(9.3) into eq.(9.2) above, we have

$$V_0(T_b) = \int_0^{T_b} V_i(\tau)[s_2(\tau) - s_1(\tau)]d\tau$$

$$= \int_0^{T_b} V_i(t)[s_2(t) - s_1(t)]dt \tag{9.4}$$

Eq.(9.4) can be implemented by the Integrate-and-Dump Correlation receiver shown in Figure 9.5.

In Figure 9.5, SW1 and SW2 are closed (and opened) at the end of each bit interval, T_b. Actually , the action of SW1 occurs just before SW2. SW1 is used to sample $V_o(T_b)$ and SW2 is closed to reset (dump) the integrator to zero initial condition before the occurrence of the next bit frame.

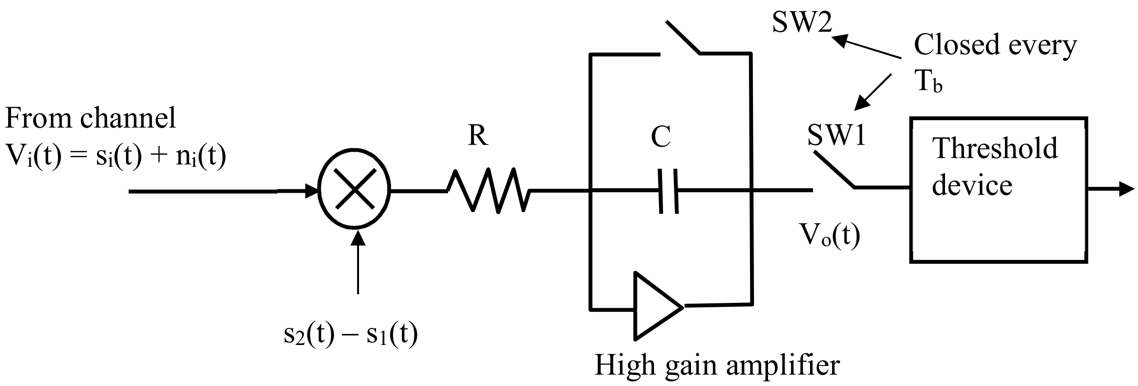


Figure 9.5 Integrate-and-Dump Correlation Receiver

Using the example of polar NRZ inputs,

$$s_i(t) = \begin{cases} s_2(t - kT_b) = +V, & \text{if } b_k = 1 \\ s_1(t - kT_b) = -V, & \text{if } b_k = 0 \end{cases}$$

for $kT_b \le t \le (k+1)T_b$

The bandwidth of the filter preceding the integrator is assumed to be wide enough to pass $s_i(t)$ (signal component) without distortion.

$$V_o(T_b) = \begin{cases} k\int_0^{T_b} 2V^2 dt = 2kV^2 T_b \text{ for + V input} \\ k\int_0^{T_b} -2V^2 dt = -2kV^2 T_b \text{ for - V input} \end{cases}$$

where k is the circuit constant

The above result is obtained as follows. Assume that binary 1 was transmitted. Then assuming ideal channel,

$$\begin{aligned} \boldsymbol{V_0\left(T_b\right) = k\int_0^{T_b} V_i(t)[s_2(t) - s_1(t)]dt} \\ \boldsymbol{= k\int_0^{T_b} V[V - \left(-V\right)]dt} \\ \boldsymbol{= k\int_0^{T_b} 2V^2 dt = 2kV^2\int_0^{T_b} dt = 2kV^2\left[t\right]_0^{T_b} = 2kV^2 T_b} \end{aligned}$$

Similar analysis can be done for binary 0.

The output $V_o(t)$ for a 101 input is shown in Figure 9.6, assuming no channel noise.

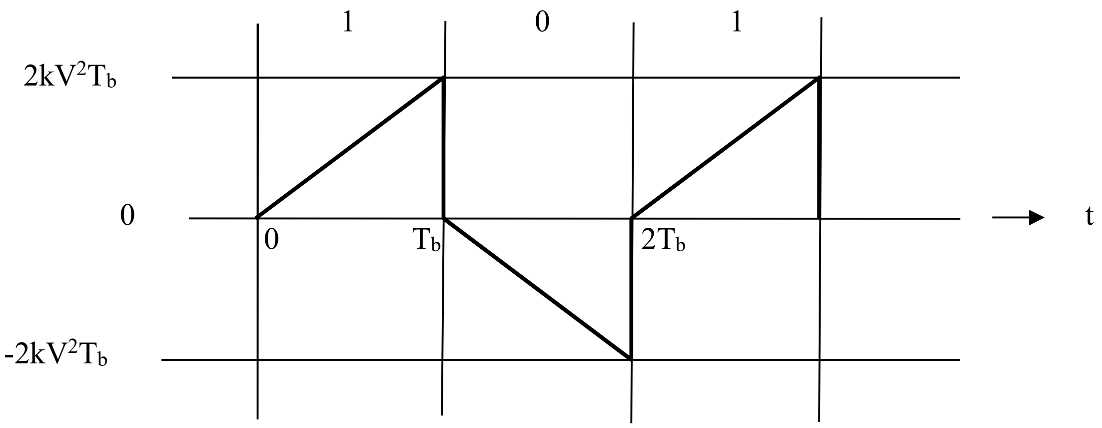


Figure 9.6 Output of Integrate-and-Dump receiver for a binary 101 sequence. The noise component is not shown.

Example 9.1

A polar NRZ binary signal, s(t), is a +1 V or –1 V pulse during the interval (0, T_b). The transmission rate of the signal is 100 bps. AWGN noise having two-sided power spectral density of 10^{-3} W/Hz is added to the signal. If the received signal is detected with a matched filter, calculated the bit error probability.

Solution

Given: r_b = 100 bps $\eta/2 = 10^{-3}$

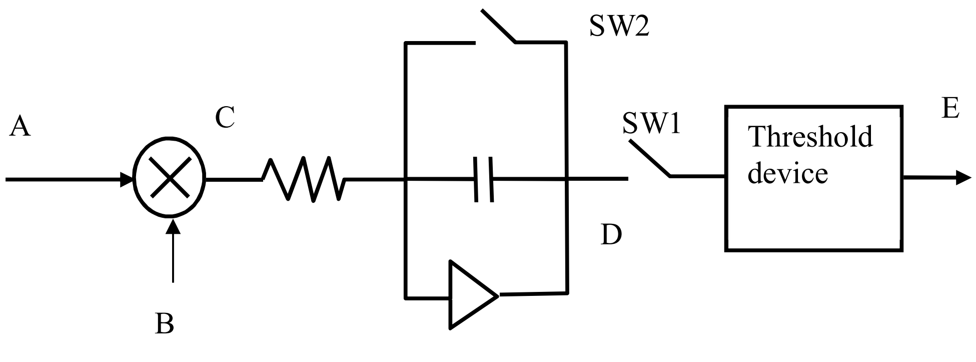
therefore $T_b = 1/r_b = 1/100$ and $\eta = 2 \times 10^{-3}$ W/Hz

For matched filter with polar NRZ inputs,

$$
\begin{aligned}
P_e &= \frac{1}{2} erfc\left\{\sqrt{\frac{V^2 T_b}{\eta}}\right\} \\
&= \frac{1}{2} erfc\left\{\sqrt{\frac{1^2 x 0.01}{2x10^{-3}}}\right\} \\
&= \frac{1}{2} erfc\{2.236\} \\
&= 8.1x10^{-4}
\end{aligned}
$$

Example 9.2

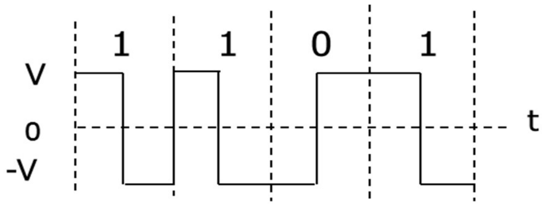
An integrate and dump correlation receiver is shown in Figure E9.2. If its input is a Manchester code waveform of amplitude V volt, sketch the waveforms at A to E for a 1101 sequence.



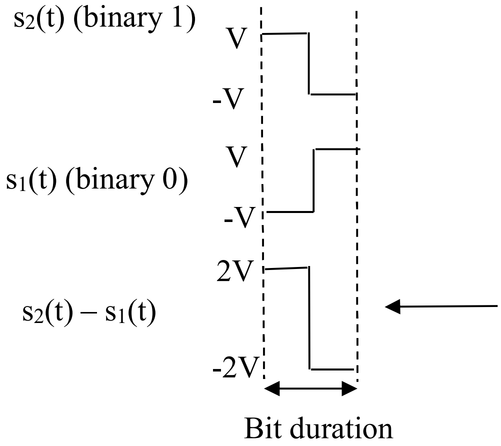
FigureE9.2

Solution

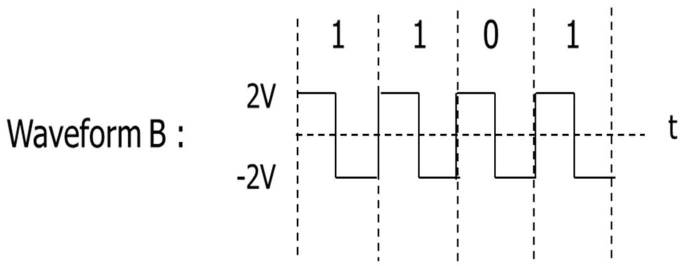
For binary sequence { 1 1 0 1}, Manchester code waveform at A is:



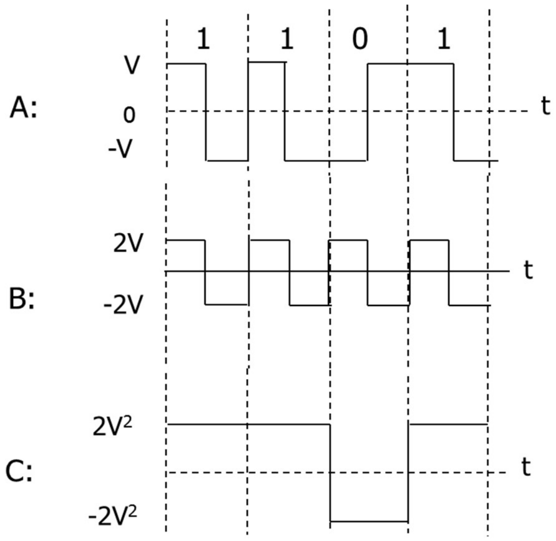
Waveform at B is based on the equation: $s_2(t) - s_1(t)$ for each bit frame:



To sketch waveform B, repeat $s_2(t) - s_1(t)$ waveform pattern for every bit frame.



Waveform C is the multiplication of Waveform A and Waveform B



Waveform D is the integration of Waveform C.
Consider an integrator circuit as shown in Fig (a) below:

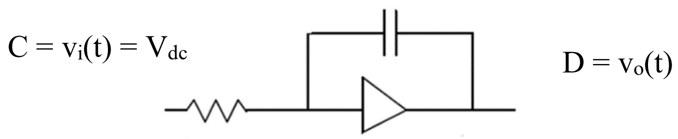


Fig (a)

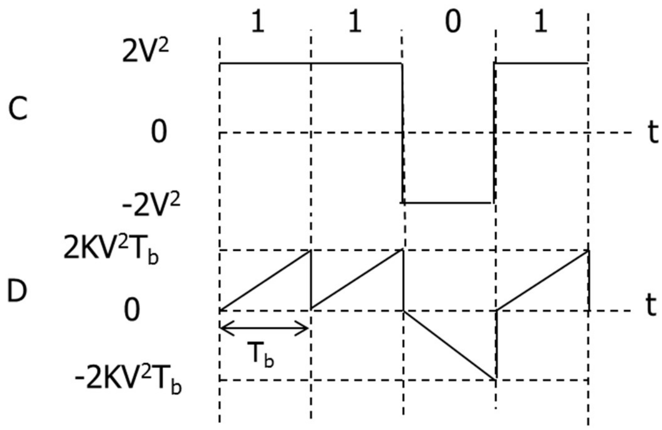
$$v_o(t) = K\int_0^{T_b} v_i(t)dt = K\int_0^{T_b} V_{dc}dt = KV_{dc}\int_0^{T_b} dt = KV_{dc}\left[t\right]_0^{T_b}$$

where K = circuit constant

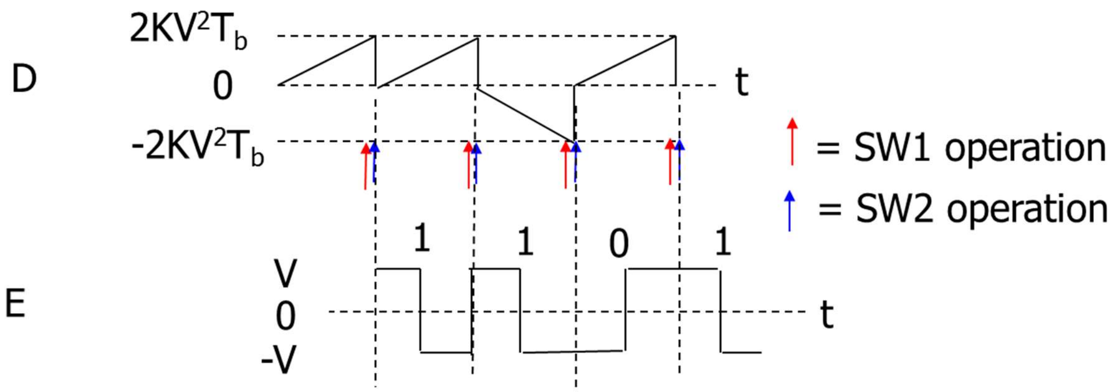
If C = $v_i(t) = 2V^2$ (a constant), D = $v_o(t) = K2V^2\left[t\right]_0^{T_b} = 2KV^2T_b$

A ramp

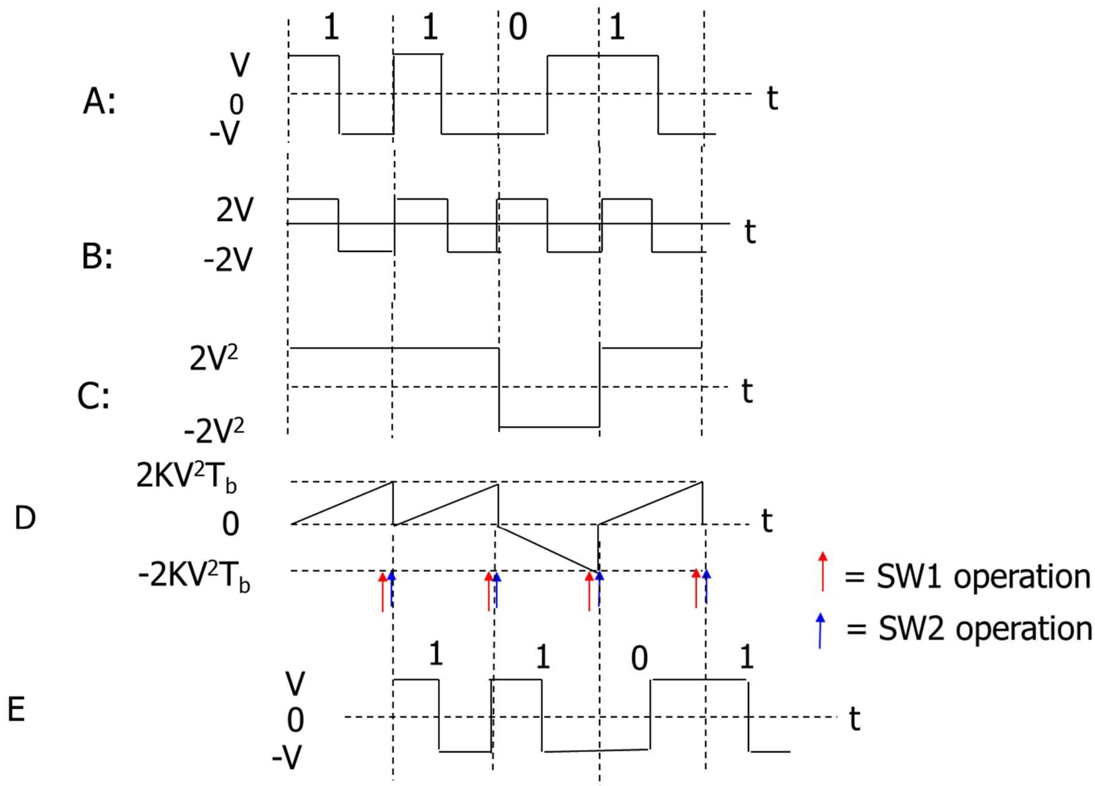
Value of D at $t = T_b$



SW1 is closed at the end of each bit duration for a very short duration to sample the D waveform. After sampling D, SW1 is opened again followed by the short closure of SW2 to discharge the capacitor so that D waveform drops to zero to initialize the start of the next bit-frame waveform of D.



The complete waveforms:



Chapter 10

Digital Modulation

Learning Outcomes

- Derive the frequency spectra of ASK, FSK and PSK systems of various binary input sequences.
- Identify differences in the modulated waveforms of ASK, FSK and PSK systems and their frequency spectra.
- Explain the modulation techniques QPSK, MPSK and QAM.
- Understand the implementation of the 3 basic keying techniques in digital modulation systems.
- Draw the simplified block diagrams of the modulator/demodulator for BPSK and DPSK systems.
- Determine the probabilities of bit error for each of the above systems.
- Evaluate the systems in terms of probabilities of bit error, bandwidth, transmission speed and system complexity.

INTRODUCTION

Most real communication channels have very poor response in the neighbourhood of zero frequency and hence are regarded as passband channels. In order to transmit digital information over passband channels, we have to transfer the information to a carrier wave of appropriate frequency. Digital information can be encoded upon a carrier in many different ways. There are three basic digital modulation techniques:

- Amplitude-shift keying (ASK)
- Frequency-shift keying (FSK)
- Phase-shift keying (PSK)

ASK - The amplitude of the carrier is switched between two values (on and off) (see Figure 10.1(a)). For example, binary 1 is represented by a sinusoidal carrier and binary 0 a zero volt signal.

FSK - This is generated by switching the frequency of the carrier between two values corresponding to the binary information to be transmitted.(see Figure 10.1(b)) For example, binary 1 is transmitted as $\sin(2\pi(f_c + f_d)t)$, and binary 0 as $\sin(2\pi(f_c - f_d)t)$, where f_c is the carrier frequency and f_d the frequency deviation.

PSK - The carrier phase is shifted between two values. (See Figure 10.1(c)). For example, for binary 1, the waveform transmitted is $\sin 2\pi f_c t$, and for binary 0, $\sin(2\pi f_c t + \pi)$.

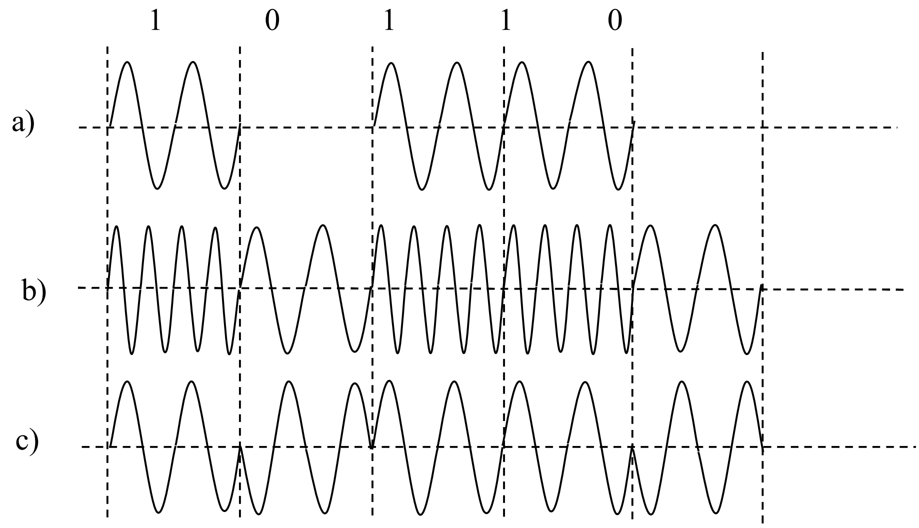


Figure 10.1 Binary digital carrier modulation waveforms
(a) Amplitude-shift keying (ASK)
(b) Frequency-shift keying (FSK)
(c) Phase-shift keying (PSK)

10.1 SPECTRA OF ASK, FSK AND PSK SIGNALS

We use a long sequence of ...101010...., to get an idea how spectra of ASK, FSK and PSK look like. Presuming the information to be transmitted is a long sequence of ...101010...., the frequency spectra of ASK, FSK and PSK can appear as shown in Figure 10.2. In Figure 10.2, the data has a bit rate of r_b bits/s. The information is encoded onto the carrier of frequency f_c.

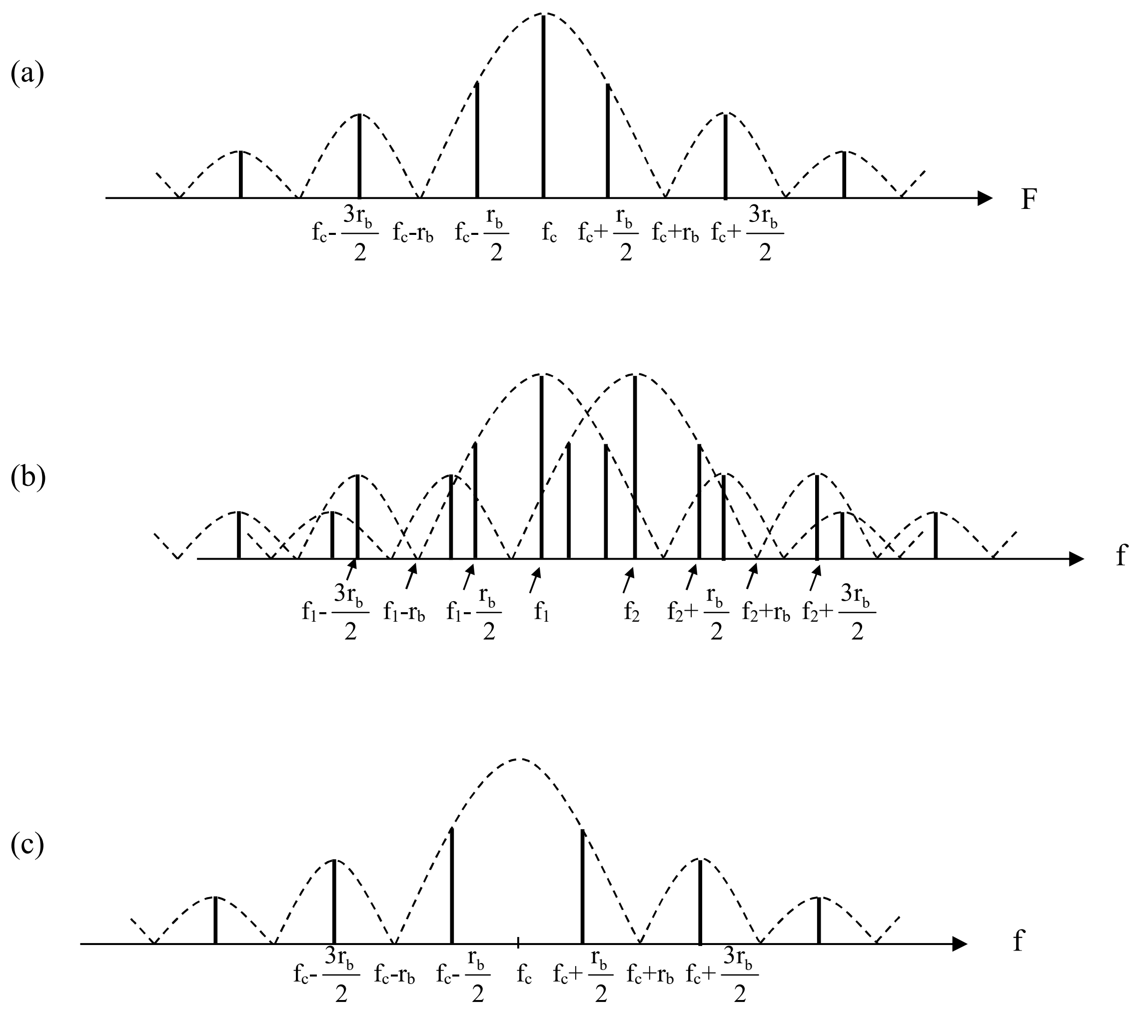


Figure 10.2 Single-sided amplitude spectrum for (a) ASK (b) FSK (c) PSK.
r_b is the data bit rate; f_c the carrier frequency, where $f_c >> r_b$.
For FSK, $f_1 = f_c – f_d$; $f_2 = f_c + f_d$.

The steps to obtain the spectra appear is show below.

Let the data be denoted by x(t) and the carrier be cos $2\pi f_c t$ where x(t) is as shown in Figure 10.3.

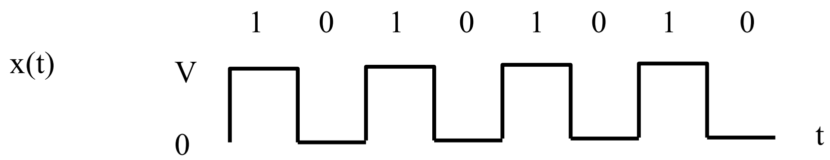


Figure 10.3 A series of ...101010... data to be transmitted.
This is represented by the unipolar NRZ waveform.

The Fourier Transform of x(t) has been previously found to be

$$X(f) = \frac{V}{2}\sum_{n=-\infty}^{\infty} \sin c\frac{n}{2}\delta(f - nf_0) \qquad (10.1)$$

where $f_0 = r_b/2$.

This has the spectrum shown in Figure 10.4:

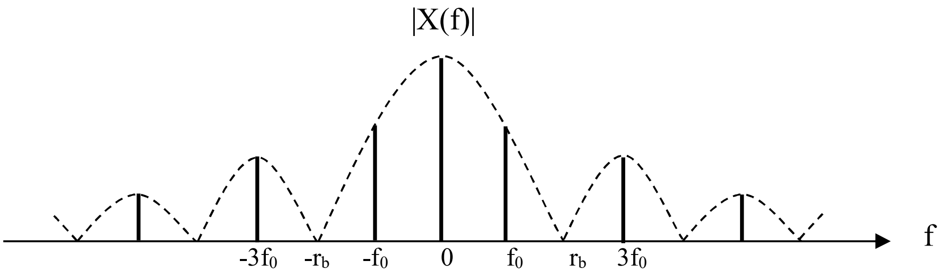


Figure 10.4 Frequency spectrum of the periodic NRZ signal in Figure 10.3.

10.1.1 ASK spectrum

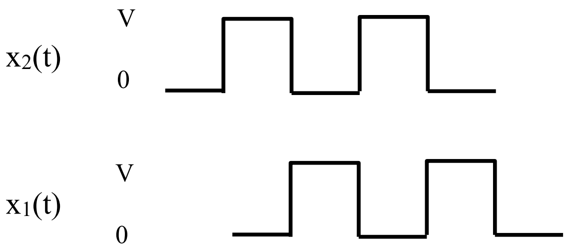
The ASK waveform can be expressed by

$$s(t) = x(t)\cos \omega_c t, \text{ where } \omega_c = 2\pi f_c. \qquad (10.2)$$

It has a frequency spectrum $S(f) = X(f)*[1/2\delta(f + fc) + 1/2\ \delta(f - f_c)] = \frac{1}{2}\,[X(f+f_c) + X(f-f_c)]$ which consists of two frequency shifted version of the spectrum of x(t), one version shifted left by fc and another version shifted right by fc. The single-sided spectrum of the ASK signal is shown in Figure 10.2(a).

10.1.2 FSK spectrum

To obtain the frequency spectrum of FSK, we can consider x(t) to comprise of two components:



where the FSK waveform is

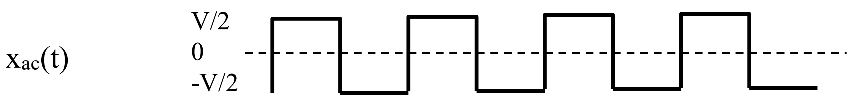
$$s(t) = x_2(t)\cos 2\pi f_2 t + x_1(t)\cos 2\pi f_1 t \tag{10.3}$$

where $f_1 = f_c - f_d$; $f_2 = f_c + f_d$;

Hence the single-sided FSK spectrum is the summation of two ASK spectra; one centering at f_2 and the other at f_1 (see Figure 10.2(b)).

10.1.3 BPSK spectrum

The binary phase-shift keying (BPSK) waveform is obtained by multiplying the polar version of x(t) in Figure 12.3 with $\cos 2\pi f_c t$



Thus, $$s(t) = x_{ac}(t)\cos 2\pi f_c t. \tag{10.4}$$

The spectrum of BPSK is almost similar to ASK, except for the absence of the line spectrum at f_c (see Figure 10.2(c)).

In summary,

For ASK:

$$s(t) = \begin{cases} V\cos\omega_c t = s_2(t); \quad 0 < t < T_b; \text{ binary } 1 \\ \\ 0 = s_1(t); \quad 0 < t < T_b; \text{ binary } 0 \end{cases} \tag{10.5}$$

For FSK:

$$s(t) = \begin{cases} V\cos\omega_2 t = s_2(t); \quad 0 < t < T_b; \text{ binary } 1 \\ \\ V\cos\omega_1 t = s_1(t); \quad 0 < t < T_b; \text{ binary } 0 \end{cases} \tag{10.6}$$

For PSK:

$$s(t) = \begin{cases} V\cos\omega_c t = s_2(t); \quad 0 < t < T_b; \text{ binary } 1 \\ \\ -V\cos\omega_c t = s_1(t); \quad 0 < t < T_b; \text{ binary } 0 \end{cases} \tag{10.7}$$

Note that for PSK, $x_{ac}(t)$ now has +V amplitude instead of +V/2 so that all three systems have a carrier of similar amplitude.

10.2 PASSBAND BINARY DATA TRANSMISSION SYSTEM

Figure 10.5 shows the overall diagram of binary data transmission over a passband channel.

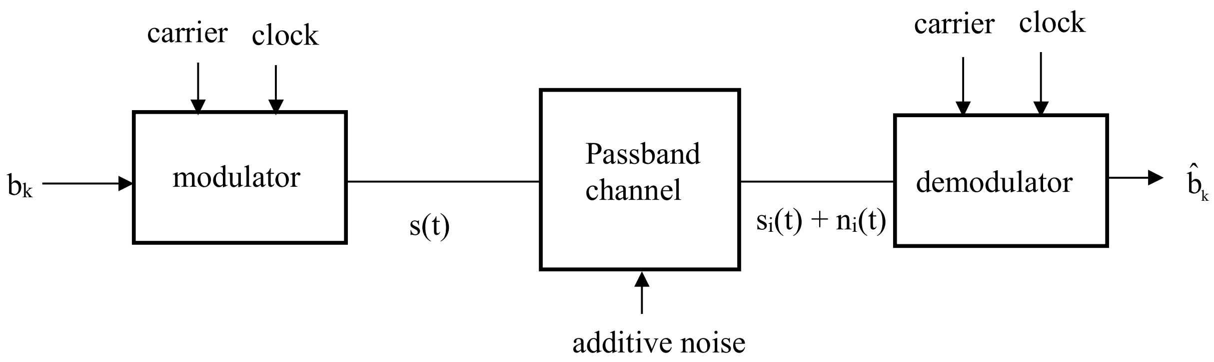


Figure 10.5 Passband binary data transmission system

10.3 OPTIMUM RECEIVER FOR BINARY DIGITAL MODULATION SYSTEMS

The optimum receiver for binary digital carrier modulation systems is a matched filter receiver as shown in Figure 10.6 if the channel noise is presumed AWGN. This is similar in form to the one discussed earlier in chapter 8 for the baseband system.

For simplicity in analysis, let us assume a passband channel which is ideal with adequate bandwidth so that signal s(t) passes through it without suffering any distortion. Let us also neglect the propagation delay. Hence $s(t) = s_i(t)$ (refer to Figure 10.5. The channel noise is assumed to be AWGN. We use the same assumptions as in the baseband system, i.e. $s_1(t)$, $s_2(t)$ sre equiprobable and statistically independent, and transmission is ISI-free.

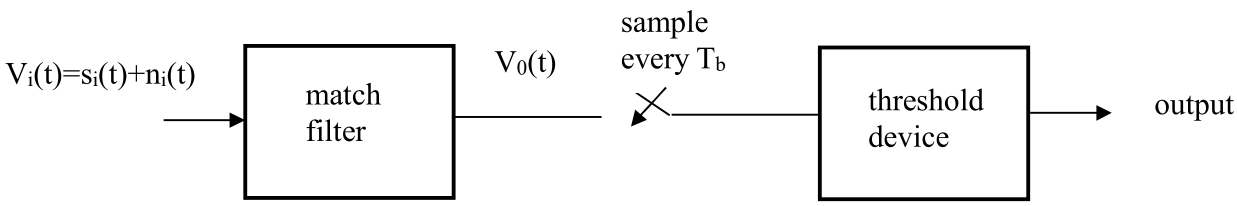


Figure 10.6 Demodulator (Receiver) of Figure 10.5, using a matched filter to minimize the probability of error.

The matched filter H(f) should have an impulse response $h(t) = s_2(T_b - t) - s_1(T_b - t)$. This filter can be implemented using integrate and dump correlation receiver.

10.4 COHERENT BPSK SYSTEM

The binary input to the BPSK transmitter is in the form of a polar NRZ waveform. The transmitted wave is

$$s(t) = \begin{cases} s_2(t) = V\cos\omega_c t = s_2(t); & 0 < t < T_b;\ \text{binary 1} \\ s_1(t) = -V\cos\omega_c t = s_1(t); & 0 < t < T_b;\ \text{binary 0} \end{cases}$$

Consider a coherent BPSK system shown in Figure 10.7. Let us assume the passband channel and the passband filter (used to limit the noise power) have negligible effect on the signal waveform. Hence $V_i(t) = s_i(t) + n_i(t)$ where $s_i(t) = s(t)$ and $n_i(t)$ is the bandlimited AWGN from the channel. The waveform $V_i(t)$ is the input to the integrate and dump receiver. Hence the output of the integrate and dump receiver at the end of each bit interval is

$$V_o(T_b) = k\int_0^{T_b} V_i(t)[s_2(t) - s_1(t)]dt \tag{10.8}$$

where $s_2(t) - s_1(t) = 2V\cos\omega_c t$ and k is a constant depending on circuit parameter.

If we consider only the signal component then for $s_i(t) = s_2(t) = V\cos\omega_c t$

$$V_o(T_b) = \int_0^{T_b} 2kV^2\cos^2\omega_c t dt$$

$$= 2kV^2\int_0^{T_b}\frac{(1+\cos 2\omega_c t)}{2}dt$$

$$= kV^2 T_b$$

Similarly, if $s_i(t) = s_1(t) = -V\cos\omega_c t$

$V_o(T_b) = -kV^2T_b$

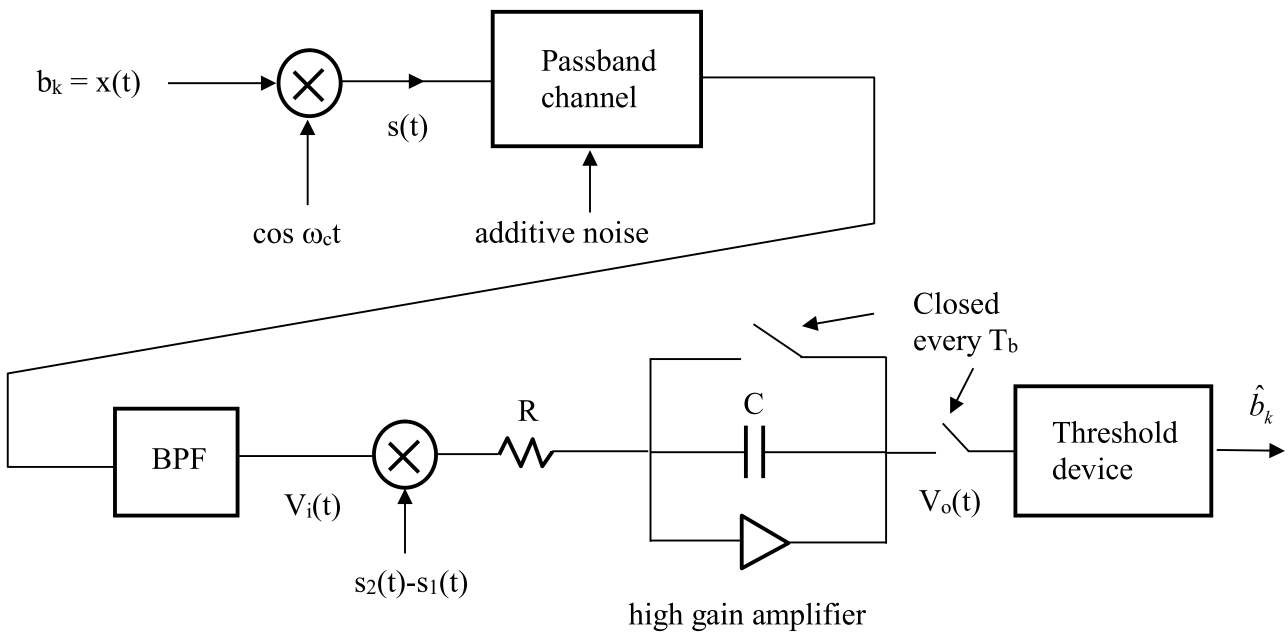


Figure 10.7 A coherent BPSK system

Hence the threshold level for the threshold device is at zero.

To obtain the probability of bit error for BPSK, we note previously that for a matched filter,

$$P_e = \frac{1}{2} erfc\left(\frac{\gamma}{2\sqrt{2}}\right)$$

where $\gamma^2 = \frac{2}{\eta}\int_0^{T_b} [s_2(t) - s_1(t)]^2 \, dt$

and η is the single-sided power spectral density of the white noise, $n_i(t)$.

so $\gamma^2 = \frac{2}{\eta}\int_0^{T_b} 4V^2 \cos^2 \omega_c dt$

$$= \frac{4V^2 T_b}{\eta}$$

or $\gamma = \sqrt{\frac{4V^2 T_b}{\eta}}$

Therefore

$$P_e = \frac{1}{2} erfc\left[\sqrt{\frac{V^2 T_b}{2\eta}}\right] \qquad (10.9)$$

10.5 DIFFERENTIAL PHASE-SHIFT KEYING (DPSK)

In the BPSK system of Figure 10.7, we presume that the local reference signal, $s_2(t) - s_1(t) = 2V\cos \omega_c t$ is available for the integrate and dump receiver. This local carrier must be of the same phase and frequency with the carrier of the received BPSK waveform. The generation of a local carrier would require complex hardware. A DPSK system get around the need for a coherent reference signal.

Figure 10.8 shows a DPSK system, the binary input sequence $\{b_k\}$ is first differentially encoded to produce $\{d_k\}$using the rule

$$d_k = \overline{b_k \oplus d_{k-1}}$$

where d_{k-1} is d_k one bit interval earlier. Figure 10.9 shows the encoding process, presuming an arbitrary starting bit of $d_k = 1$ at bit time 0. Hence at bit time 1, the starting bit becomes

$$d_k = \overline{b_k \oplus d_{k-1}}$$

$$= \overline{1 \oplus 1} = 1$$

The level shifter changes the unipolar NRZ digital waveform to a polar NRZ waveform. The latter is multiplied by the carrier V $\cos \omega_c t$ to produce the BPSK waveform which is then transmitted across the passband channel. Figure 10.9 shows that if $d_k = 1$, transmit wave is V$\cos\ \omega_c t$ (zero phase shift); if $d_k = 0$ the transmit wave is -V$\cos \omega_c t$ (or half-cycle phase shift).

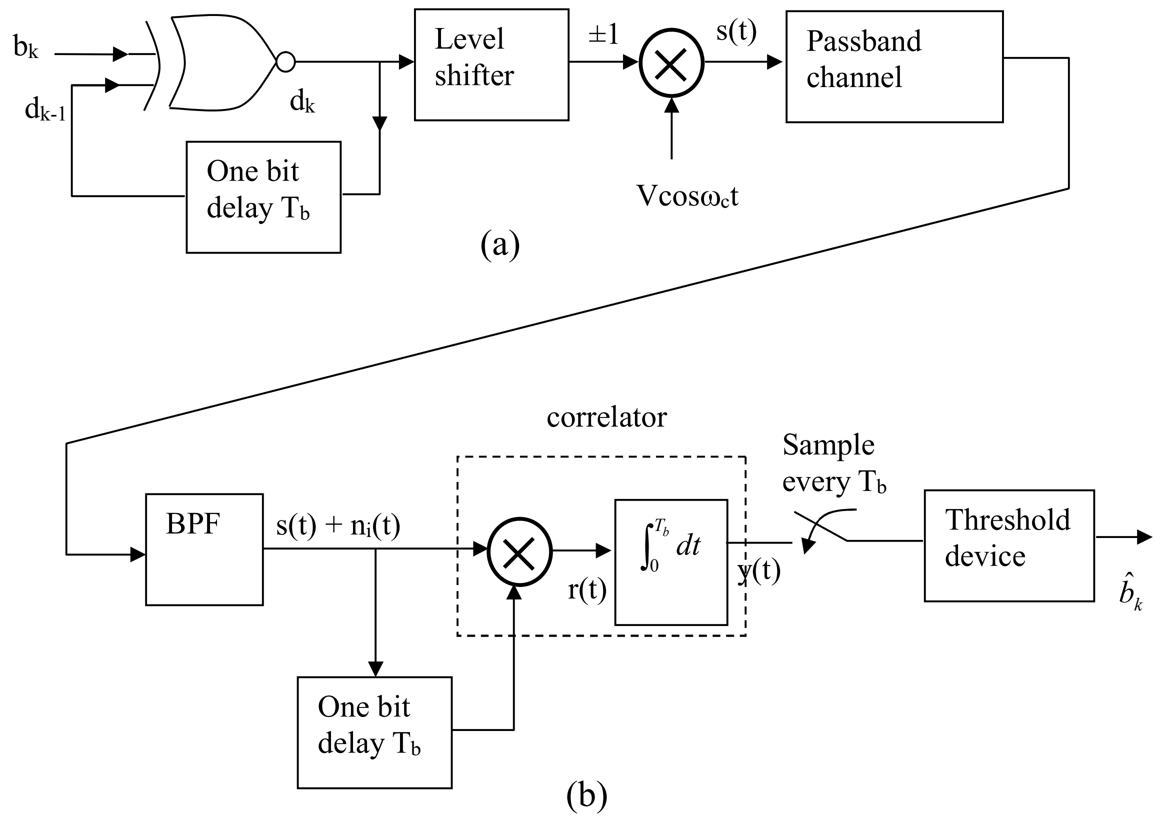


Figure 10.8 (a) DPSK modulator (b) DPSK demodulator. The channel and BPF are presumed to cause negligible distortion to s(t).

Bit Time	0	1	2	3	4	5	6	7
Input sequence, b_k		1	1	0	1	0	0	0
Encoded sequence, d_k	1*	1	1	0	0	1	0	1
Transmitted phase	0	0	0	π	π	0	π	0
Correlator output		+	+	-	+	-	-	-
Output bit sequence		1	1	0	1	0	0	0

*Arbitrary starting reference bit

Figure 10.9 Differential encoding and decoding

At the receiver, the passband filter is used to reduce the effect of noise that is superimposed on the signal without altering the shape of the signal waveform appreciably. To recover the bit sequence $\{b_k\}$, the present BPSK wave over one bit interval is multiplied with the previous BPSK wave, over the same duration. For example, let us presume the passband channel and BPF cause minimal distortion to the the transmitted BPSK wave, where

$$s(t) = \begin{cases} s_2(t) = V\cos\omega_c t = s_2(t); \quad 0 < t < T_b;\ d_k = 1 \\ s_1(t) = -V\cos\omega_c t = s_1(t); \quad 0 < t < T_b;\ d_k = 0 \end{cases}$$

Hence the correlator input is also s(t). If the present BPSK waveform s(t) is similar to the previous waveform s(t - T_b), then, considering only the signal component,

$$r(t) = V^2\cos^2\omega_c t = \frac{V^2}{2} + \frac{V^2}{2}\cos 2\omega_c t$$

The integrator is used to eliminate the high frequency component and after sampling the output is $KT_b(V^2/2)$ where K is a constant.

If s(t) = -s(t – T_b), then

$$r(t) = -V^2\cos^2\omega_c t = -\frac{V^2}{2} - \frac{V^2}{2}\cos 2\omega_c t$$

so that after integrating and sampling, we have $-KT_b(V^2/2)$.

Hence the threshold device is set at zero volt. Sampling is done at every T_b at the instant when y(t) is a maximum signal. If the correlator output is positive, then $b_k = 1$ is presumed sent, if the correlator output is negative, then $b_k = 0$ is presumed sent. Figure 10.9 shows the demodulaltion process to obtain the source bit sequence.

The probability of bit error for a DPSK system is found to be

$$P_e = \frac{1}{2}\exp(\frac{-V^2T_b}{2\eta}) = \frac{1}{2}e^{(\frac{-V^2T_b}{2\eta})}$$

where η is the single-sided power spectral density of the white channel noise. For a given transmitted power, the DPSK system will give a higher error rate when compared with the coherent BPSK system. Other disadvantages of DPSK are:

- Asynchronous transmission is not possible. This is because the correlator requires the multiplication of the present waveform of the same duration. Thus the system is locked on a specific signalling speed.

- Another minor problem is that an error will propagate to the adjacent bit.

Example 10.1

Figure E10.1 shows the block diagram of a DPSK transmitter. The binary input is in unipolar NRZ format of amplitude 2V volts, at a bit rate of 1200 b/s. The carrier is $\sin\omega_c t$ where $\omega_c = 4800\pi$ rad/s. Assume that the input is a long series of …10101010… . Sketch the waveforms at points A to D as indicated in Fig E10.1 for a 1010 frame. Assume distortionless transmission path. Also assume that the encoder output is binary 1 prior to the 1010 frame

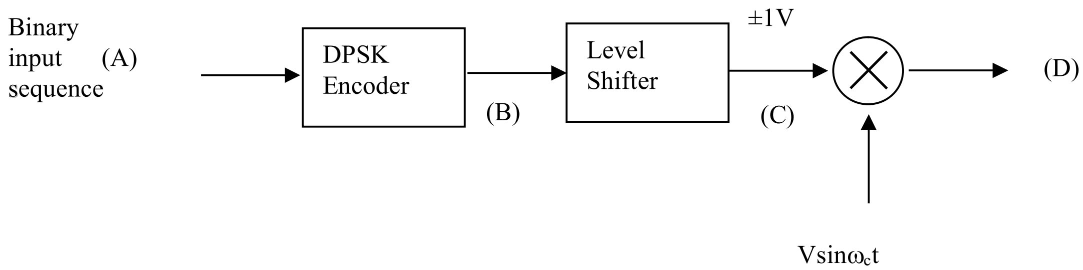


Figure E10.1 DPSK Transmitter

Solution

$$\omega_c = 2\pi f_c = 4800\pi \text{ rad/s} \qquad T_b = \frac{1}{1200} \text{ sec}$$

$$2f_c = 4800 \rightarrow f_c = 2400 \text{ Hz} \rightarrow \text{Carrier Period } T_c = \frac{1}{2400} \text{ sec}$$

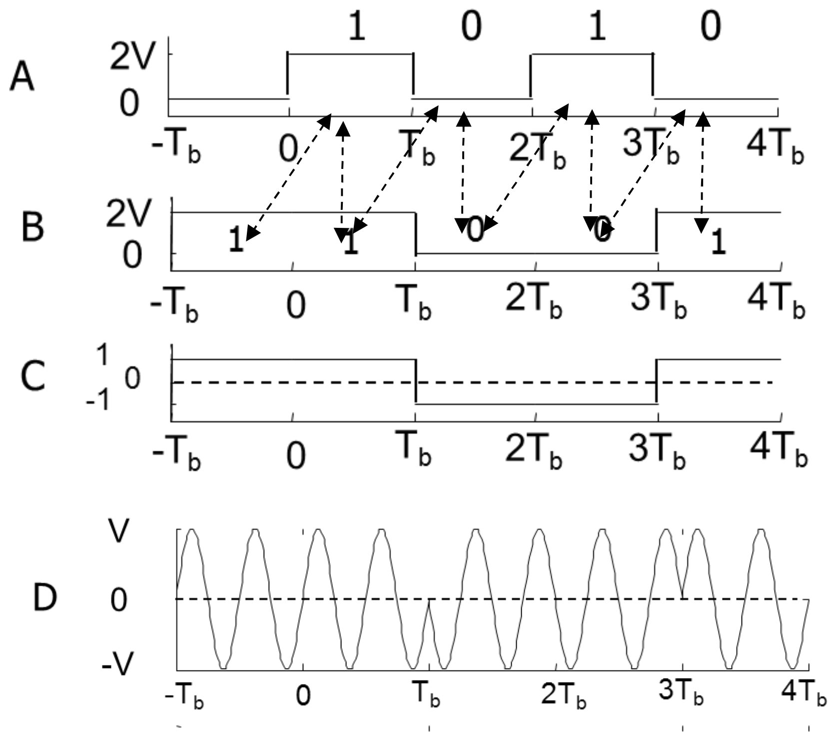
Waveform at A has unipolar format; bit rate r_b = 1200 b/s

One bit duration $T_b = \frac{1}{r_b} = \frac{1}{1200} = 2 \text{ x } \frac{1}{2400} = 2T_c$ = two carrier periods

X-NOR Truth Table

X	Y	Z(o/p)
0	0	1
0	1	0
1	0	0
1	1	1

Waveforms at various points



10.6 QUADRATURE PHASE SHIFT KEYING(QPSK)

A limitation of binary modulated signals is that they do not make the most efficient use of channel bandwidth. One way of improving bandwidth utilisation is to use quadrature multiplexing. One important example of such an approach is *quadrature phase shift keying* (QPSK).

In BPSK we transmitted each bit individually. In QPSK we lump two bits together to form a *symbol*. The symbol can have any one of four possible values corresponding to the two-bit sequences 00, 01, 10 and 11. In QPSK we therefore transmit four distinct symbols.

In QPSK, the phase of the carrier takes on one of four equally spaced values to represent the four permutations of two bits, for example

$$s_i(t) = \begin{cases} A\cos(2\pi f_c t + \phi_i) \quad 0 \le t \le T_s \\ \phi_i = \begin{cases} 0 & for \text{"00"} \\ 90^\circ & for \text{"01"} \\ 180^\circ & for \text{"10"} \\ 270^\circ & for \text{"11"} \end{cases} \end{cases}$$

where T_s is the symbol duration.

Figure 10.10 shows how the bit stream 01111000 (LSB at the right most) would be modulated by a QPSK modulator. Note that the LSB is transmitted first.

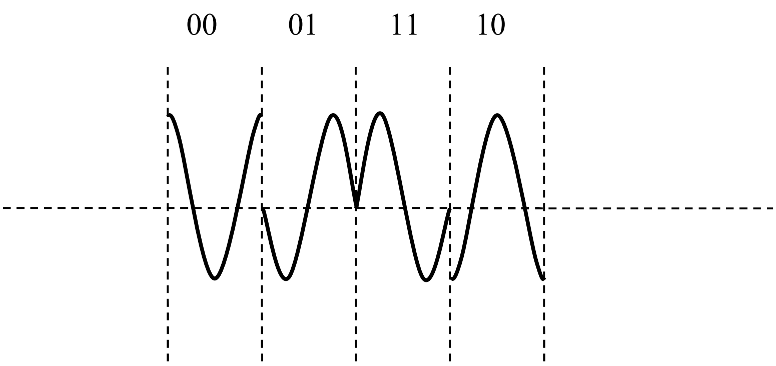


Figure 10.10 QPSK waveform

The probability of bit error for a QPSK system is given by

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{V^2 T_b}{2\eta}}\right)$$

This is the same probability of bit error as BPSK. Hence we have the important result that, in spite of the reduction (by a factor of two) in the bandwidth required by QPSK compared to BPSK, both have the same probability of bit error. Thus QPSK is the preferred system if smaller channel bandwidth is desired.

10.7 M-ARY PSK (MPSK)

In MPSK, the phase of the carrier takes on one of M possible values, namely, ϕ_i = 360i/M, where i = 0, 1, …, M-1. Accordingly, during each symbol interval of duration T_s, one of M possible symbols

$$s_i(t) = A\cos\left(2\pi f_c t + \frac{360i}{M}\right) \qquad i = 0,1,\ldots,M-1$$

is sent.

The QPSK system in section 10.7 is an example of MPSK with M = 4. In MPSK, as the number of phase levels M is increased, the bandwidth efficiency is improved but at the expense of an increase in transmitted power or an increase in P_e. Among the family of MPSK, QPSK offers the best trade-off between power and bandwidth requirements. For this reason, QPSK is the first to be widely used in practice. For M > 8, power requirements is excessive. Thus MPSK systems with M > 8 are not widely used in practice. Also MPSK requires more complex equipment as M increases.

10.8 QUADRATURE AMPLITUDE MODULATION (QAM)

The channel bandwidth utilisation can be improved further by the hybrid use of amplitude and phase modulation. A common form of this hybrid scheme is QAM. The principle of QAM is illustrated using 16-QAM.

In 16-QAM there are 16 possible symbols, each representing a 4-bit data. The way to represent these 16 symbols is by a constellation diagram. A possible constellation diagram for 16-QAM is shown in Figure 10.11. This constellation shown uses three amplitudes, two with four phases and one with eight phases. This constellation is used in 9600 bps modems.

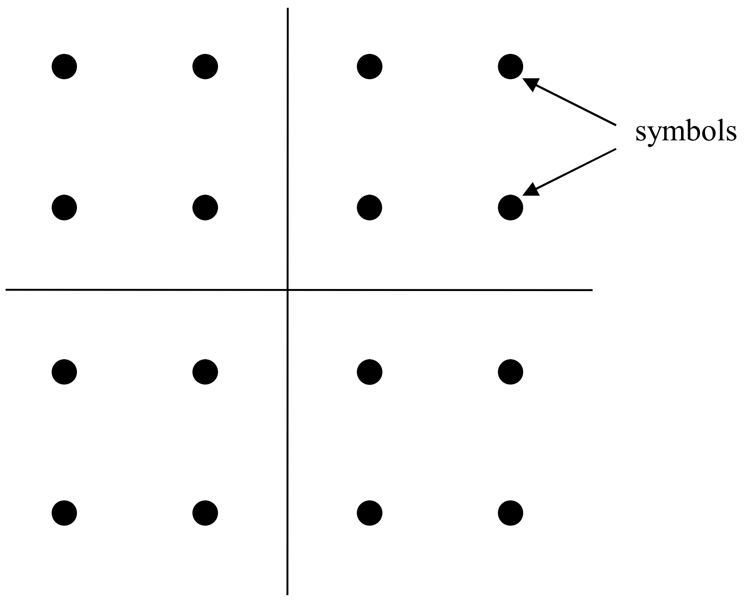
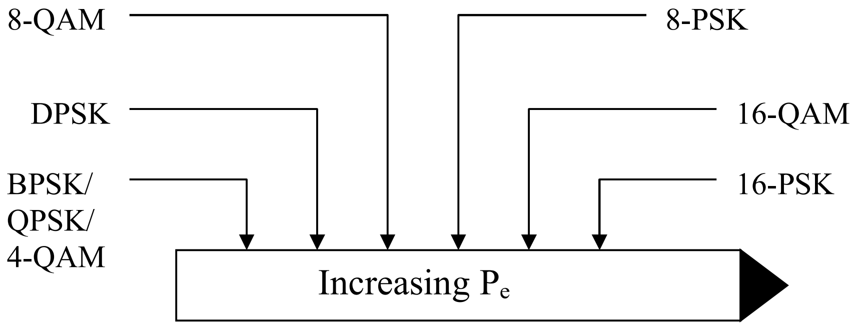


Figure 10.11 Constellation diagram for 16 QAM

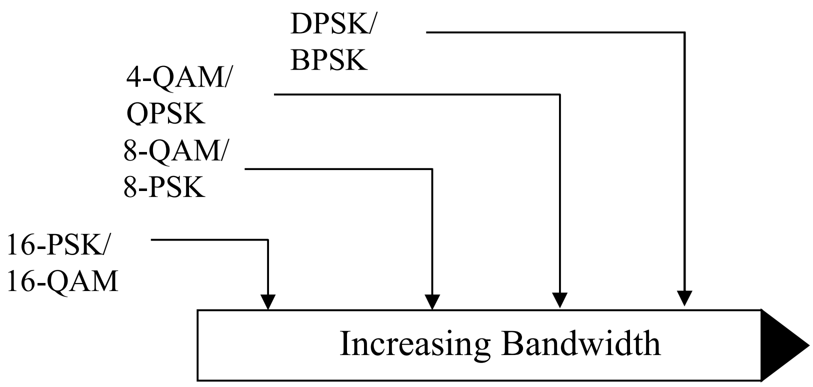
10.9 COMPARISON OF DIGITAL MODULATION SYSTEMS

Choice of digital modulation methods is dependent mainly on error performance, bandwidth efficiency (in bps/Hz) and equipment complexity.

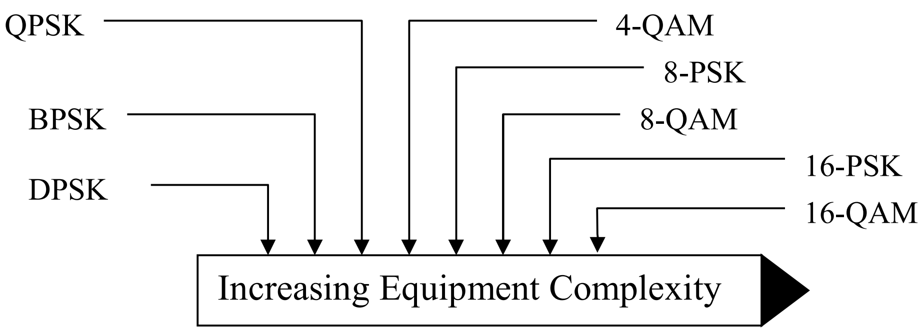
Error Performance Comparison



Bandwidth Comparison



Equipqment Complexity Comparison



10.10 APPLICATIONS

10.10.1. Cable Data Modems

Modern CATV systems that provide a high-speed internet connection (via a cable modem) use fibre to distribute the TV and data signals from the CATV head end to the neighbourhood of the customer. In each neighbourhood the signals are converted from light to RF and transmitted to the home via coaxial cable. This allows the neighbourhood coaxial cable network to operate up to 800 MHz as the distances are relatively short.

The user connects the CATV cable that enters his home to the cable modem which demodulates the downstream data and modulates the upstream data. The data modem is usually connected to a PC or in-house data network via an ethernet line. The downlink is usually around 3 Mb/s and the uplink speed is around 500 kb/s.

A single downlink 6 MHz-wide channel can support a combined downstream data rate of 27 Mb/s (shared among subscribers in the neighbourbood) if 64 QAM is used. Up to 36

Mb/s if 256 QAM is used. One upstream channel supports a combined data rate of 10 Mb/s using QPSK or 30 Mb/s if 16 Qam is used.

Cable Modem Standards

Component	**Downstream**	**Upstream**
Carrier frequency range	50 - 750 MHz	5 – 42 MHz
Channel bandwidth	6 MHz	6 MHz or 2 MHz
Modulation	64 QAM or 256 QAM	QPSK or 16 QAM
Composite data rate	27 Mb/s or 36 Mb/s	10 – 30 Mb/s
Subscriber data rate	1.5 – 6 Mb/s	256 kb/s – 1.5 Mb/s
Coding	Block code (Reed Solomon)	Block code (RS)
Encryption	DES	DES

DES – Data Encryption Standard

10.10.2 Digital Radio

In digital radio, information originating from a source is transmitted to its destination by means of digital modulation techniques (usually QAM e.g. 64 QAM) over an appropriate number of microwave radio links, with each link offering a line-of-sight propagation path.

10.10.3 Digital Communications by Satellite

In digital communications by satellite, time-division multiple access (TDMA) is used. Each ground station transmits a burst of RF signal. These are time-division multiplexed and transmitted to the satellite. The satellite acts as a relay station; it retransmits the information downward using TDMA to the ground receiving station.

The satellite's RF power amplifier at the output operates at maximum constant amplitude to maximize the downlink carrier-to-noise ratio. This feature excludes digital modualtion schemes where carrier amplitude is not constant (for example, ASK, QAM). The TDMA frame consists of many RF signal time slots, one for each station, plus a reference used for frame supervisory (from control station). Typically a burst consists of an initial preamble which is followed by a message portion.

The preamble, among other things, consists of a part for carrier recovery and a part for symbol-timing recovery. This feature makes the use of coherent PSK attractive. Hence the digital modulation technique in used in digital satellite
communication is coherent BPSK or more popularly QPSK because of its bandwidth-saving capability.

10.10.4 Software-defined radio

What is Software-defined radio (SDR)?

Software-defined radio (SDR) is a radio communication system where components that have been typically implemented in hardware (e.g. mixers, filters, amplifiers, modulators/demodulators, detectors, etc.) are instead implemented by means of software on a personal computer or embedded system. While the concept of SDR is not new, the new capabilities in digital electronics render practical many processes which used to be only theoretically possible.

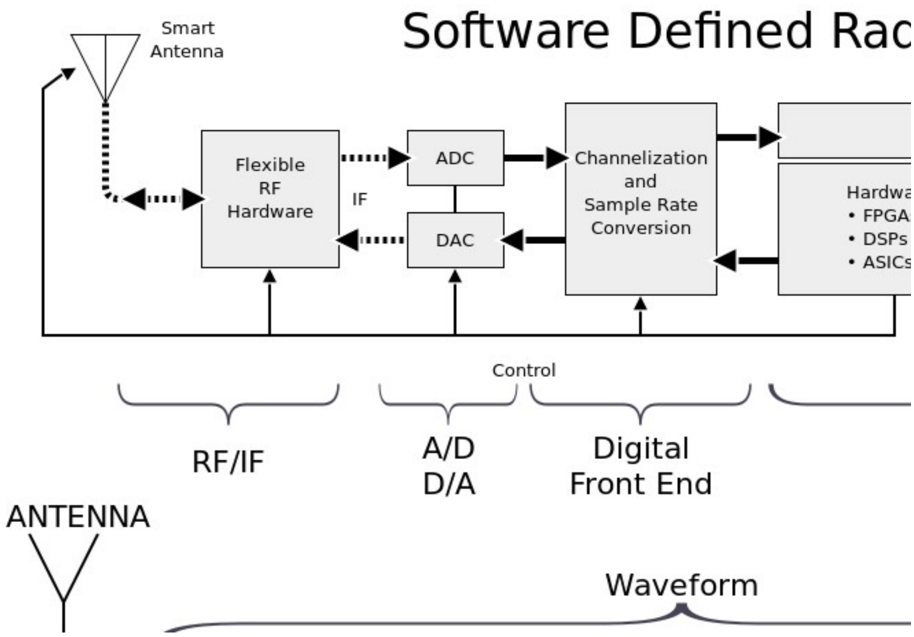
A basic SDR system may consist of a personal computer equipped with a sound card, or other analog-to-digital converter, preceded by some form of RF front end. Significant amounts of signal processing are handed over to the general-purpose processor, rather than being done in special-purpose hardware (electronic circuits). Such a design produces a radio which can receive and transmit widely different radio protocols (sometimes referred to as waveforms) based solely on the software used.

The ideal receiver scheme would be to attach an analog-to-digital converter to an antenna. A digital signal processor would read the converter, and then its software would transform the stream of data from the converter to any other form the application requires. An ideal transmitter would be similar. A digital signal processor would generate a stream of numbers. These would be sent to a digital-to-analog converter connected to a radio antenna.
The ideal scheme is not completely realizable due to the actual limits of the technology. The main problem in both directions is the difficulty of conversion between the digital and the analog domains at a high enough rate and a high enough accuracy at the same time, and without relying upon physical processes like interference and electromagnetic resonance for assistance.

Receiver architecture

Most receivers use a variable-frequency oscillator, mixer, and filter to tune the desired signal to a common intermediate frequency or baseband, where it is then sampled by the analog-to-digital converter. However, in some applications it is not necessary to tune the signal to an intermediate frequency and the radio frequency signal is directly sampled by the analog-to-digital converter (after amplification).
Real analog-to-digital converters lack the dynamic range to pick up sub-microvolt, nanowatt-power radio signals. Therefore, a low-noise amplifier must precede the conversion step and this device introduces its own problems. For example, if spurious signals are present (which is typical), these compete with the desired signals within the amplifier's dynamic range. They may introduce distortion in the desired signals, or may block them completely. The standard solution is to put band-pass filters between the antenna and the amplifier, but these reduce the radio's flexibility. Real software radios

often have two or three analog channel filters with different bandwidths that are switched in and out.



COMPLIMENTARY ERROR FUNCTION TABLE (ERFC)

Table Complementary Error Function

Z	erfc(Z)	Z	erfc(Z)	Z	erfc(Z)	Z	erfc(Z)	Z	erfc(Z)
0.00	1.000000	0.40	0.571608	0.80	0.257899	1.20	0.896860D-01	1.60	0.236516D-01
0.01	0.988717	0.41	0.562031	0.81	0.251997	1.21	0.870445D-01	1.61	0.227932D-01
0.02	0.977435	0.42	0.552532	0.82	0.246189	1.22	0.844661D-01	1.62	0.219619D-01
0.03	0.966159	0.43	0.543113	0.83	0.240476	1.23	0.819499D-01	1.63	0.211572D-01
0.04	0.954889	0.44	0.533775	0.84	0.234857	1.24	0.794948D-01	1.64	0.203782D-01
0.05	0.943628	0.45	0.524518	0.85	0.229332	1.25	0.770999D-01	1.65	0.196244D-01
0.06	0.932378	0.46	0.515345	0.86	0.223900	1.26	0.747540D-01	1.66	0.188951D-01
0.07	0.921142	0.47	0.506255	0.87	0.218560	1.27	0.724864D-01	1.67	0.181896D-01
0.08	0.909922	0.48	0.497250	0.88	0.213313	1.28	0.702658D-01	1.68	0.175072D-01
0.09	0.898719	0.49	0.488332	0.89	0.208157	1.29	0.681014D-01	1.69	0.168474D-01
0.10	0.887537	0.50	0.479500	0.90	0.203092	1.30	0.659920D-01	1.70	0.162095D-01
0.11	0.876377	0.51	0.470756	0.91	0.198117	1.31	0.639369D-01	1.71	0.155930D-01
0.12	0.865242	0.52	0.462101	0.92	0.193232	1.32	0.619348D-01	1.72	0.149972D-01
0.13	0.854133	0.53	0.453536	0.93	0.188436	1.33	0.599850D-01	1.73	0.144215D-01
0.14	0.843053	0.54	0.445061	0.94	0.183729	1.34	0.580863D-01	1.74	0.138654D-01
0.15	0.832004	0.55	0.436677	0.95	0.179109	1.35	0.562378D-01	1.75	0.133283D-01
0.16	0.820988	0.56	0.428384	0.96	0.174576	1.36	0.544386D-01	1.76	0.128097D-01
0.17	0.810008	0.57	0.420184	0.97	0.170130	1.37	0.526876D-01	1.77	0.123091D-01
0.18	0.799064	0.58	0.412077	0.98	0.165768	1.38	0.509840D-01	1.78	0.118258D-01
0.19	0.788160	0.59	0.404063	0.99	0.161492	1.39	0.493267D-01	1.79	0.113594D-01
0.20	0.777297	0.60	0.396144	1.00	0.157299	1.40	0.477149D-01	1.80	0.109095D-01
0.21	0.766478	0.61	0.388319	1.01	0.153190	1.41	0.461476D-01	1.81	0.104755D-01
0.22	0.755704	0.62	0.380589	1.02	0.149162	1.42	0.446238D-01	1.82	0.100568D-01
0.23	0.744977	0.63	0.372954	1.03	0.145216	1.43	0.431427D-01	1.83	0,965319D-02
0.24	0.734300	0.64	0.365414	1.04	0.141350	1.44	0.417034D-01	1.84	0.926405D-02
0.25	0.723674	0.65	0.357971	1.05	0.137564	1.45	0.403050D-01	1.85	0.888897D-02
0.26	0.713100	0.66	0.350623	1.06	0.133856	1.46	0.389465D-01	1.86	0.852751D-02
0.27	0.702582	0.67	0.343372	1.07	0.130227	1.47	0.376271D-01	1.87	0.817925D-02
0.28	0.692120	0.68	0.336218	1.08	0.126674	1.48	0.363459D-01	1.88	0.784378D-02
0.29	0.681716	0.69	0.329160	1.09	0.123197	1.49	0.351021D-01	1.89	0.752068D-02
0.30	0.671373	0.70	0.322199	1.10	0.119795	1.50	0.338949D-01	1.90	0.720957D-02
0.31	0.661092	0.71	0.315334	1.11	0.116467	1.51	0.327233D-01	1.91	0.691006D-02
0.32	0.650874	0.72	0.308567	1.12	0.113212	1.52	0.315865D-01	1.92	0.662177D-02
0.33	0.640721	0.73	0.301896	1.13	0.110029	1.53	0.304838D-01	1.93	0.634435D-02
0.34	0.630635	0.74	0.295322	1.14	0.106918	1.54	0.294143D-01	1.94	0.607743D-02
0.35	0.620618	0.75	0.288844	1.15	0.103876	1.55	0.283773D-01	1.95	0.582066D-02
0.36	0.610670	0.76	0.282463	1.16	0.100904	1.56	0.273719D-01	1.96	0.557372D-02
0.37	0.600794	0.77	0.276178	1.17	0.979996D-01	1.57	0.263974D-01	1.97	0.533627D-02
0.38	0.590990	0.78	0.269990	1.18	0.951626D-01	1.58	0.254530D-01	1.98	0.510800D-02
0.39	0.581261	0.79	0.263897	1.19	0.923917D-01	1.59	0.245380D-01	1.99	0.488859D-02

(cont'd)

Table Complementary Error Function

Z	erfc(Z)	Z	erfc(Z)	Z	erfc(Z)	Z	erfc(Z)	Z	erfc(Z)
2.00	0.467773D-02	2.40	0.688514D-03	2.80	0.750132D-04	3.20	0.602576D-05	3.60	0.355863D-06
2.01	0.447515D-02	2.41	0.653798D-03	2.81	0.706933D-04	3.21	0.563542D-05	3.61	0.330251D-06
2.02	0.428055D-02	2.42	0.620716D-03	2.82	0.666096D-04	3.22	0.526935D-05	3.62	0.306423D-06
2.03	0.400365D-02	2.43	0.589197D-03	2.83	0.627497D-04	3.23	0.492612D-05	3.63	0.284259D-06
2.04	0.391419D-02	2.44	0.559174D-03	2.84	0.591023D-04	3.24	0.460435D-05	3.64	0.263647D-06
2.05	0.374190D-02	2.45	0.530580D-03	2.85	0.556563D-04	3.25	0.430278D-05	3.65	0.244483D-06
2.06	0.357654D-02	2.46	0.503353D-03	2.86	0.524012D-04	3.26	0.402018D-05	3.66	0.226667D-06
2.07	0.341785D-02	2.47	0.477434D-03	2.87	0.493270D-04	3.27	0.375542D-05	3.67	0.210109D-06
2.08	0.326559D-02	2.48	0.452764D-03	2.88	0.464244D-04	3.28	0.350742D-05	3.68	0.194723D-06
2.09	0.311954D-02	2.49	0.429288D-03	2.89	0.436842D-04	3.29	0.327517D-05	3.69	0.180429D-06
2.10	0.297947D-02	2.50	0.406952D-03	2.90	0.410979D-04	3.30	0.305771D-05	3.70	0.167151D-06
2.11	0.284515D-02	2.51	0.365705D-03	2.91	0.386573D-04	3.31	0.285414D-05	3.71	0.154821D-06
2.12	0.271639D-02	2.52	0.365499D-03	2.92	0.363547D-04	3.32	0.266360D-05	3.72	0.143372D-06
2.13	0.259298D-02	2.53	0.346286D-03	2.93	0.341828D-04	3.33	0.248531D-05	3.73	0.132744D-06
2.14	0.247471D-02	2.54	0.328021D-03	2.94	0.321344D-04	3.34	0.231850D-05	3.74	0.122880D-06
2.15	0.236139D-02	2.55	0.310660D-03	2.95	0.302030D-04	3.35	0.216248D-05	3.75	0.113727D-06
2.16	0.225285D-02	2.56	0.294163D-03	2.96	0.283823D-04	3.36	0.201656D-05	3.76	0.105236D-06
2.17	0.214889D-02	2.57	0.278489D-03	2.97	0.266662D-04	3.37	0.188013D-05	3.77	0.973591D-07
2.18	0.204935D-02	2.58	0.263600D-03	2.98	0.250491D-04	3.38	0.175259D-05	3.78	0.900547D-07
2.19	0.195406D-02	2.59	0.249461D-03	2.99	0.235256D-04	3.39	0.163338D-05	3.79	0.832821D-07
2.20	0.186285D-02	2.60	0.236034D-03	3.00	0.220905D-04	3.40	0.152199D-05	3.80	0.770039D-07
2.21	0.177556D-02	2.61	0.223289D-03	3.01	0.207390D-04	3.41	0.141793D-05	3.81	0.711851D-07
2.22	0.169205D-02	2.62	0.211191D-03	3.02	0.194664D-04	3.42	0.132072D-05	3.82	0.657933D-07
2.23	0.161217D-02	2.63	0.199711D-03	3.03	0.182684D-04	3.43	0.122994D-05	3.83	0.607981D-07
2.24	0.153577D-02	2.64	0.188819D-03	3.04	0.171400D-04	3.44	0.114518D-05	3.84	0.561711D-07
2.25	0.146272D-02	2.65	0.178488D-03	3.05	0.160798D-04	3.45	0.106605D-05	3.85	0.518863D-07
2.26	0.139288D-02	2.66	0.168689D-03	3.06	0.150816D-04	3.46	0.992201D-06	3.86	0.479189D-07
2.27	0.132613D-02	2.67	0.159399D-03	3.07	0.141426D-04	3.47	0.923288D-06	3.87	0.442464D-07
2.28	0.126234D-02	2.68	0.150591D-03	3.08	0.132595D-04	3.48	0.858995D-06	3.88	0.408473D-07
2.29	0.120139D-02	2.69	0.142243D-03	3.09	0.124292D-04	3.49	0.799025D-06	3.89	0.377021D-07
2.30	0.114318D-02	2.70	0.134333D-03	3.10	0.116487D-04	3.50	0.743098D-06	3.90	0.347922D-07
2.31	0.108758D-02	2.71	0.126838D-03	3.11	0.109150D-04	3.51	0.690952D-06	3.91	0.321007D-07
2.32	0.102449D-02	2.72	0.119738D-03	3.12	0.102256D-04	3.52	0.642341D-06	3.92	0.296117D-07
2.33	0.983805D-03	2.73	0.113015D-03	3.13	0.957795D-05	3.53	0.597035D-06	3.93	0.273103D-07
2.34	0.935430D-03	2.74	0.106649D-03	3.14	0.896956D-05	3.54	0.554816D-06	3.94	0.251829D-07
2.35	0.889267D-03	2.75	0.100622D-03	3.15	0.839821D-05	3.55	0.515484D-06	3.95	0.232167D-07
2.36	0.845223D-03	2.76	0.949176D-04	3.16	0.786174D-05	3.56	0.478847D-06	3.96	0.213999D-07
2.37	0.803210D-03	2.77	0.895197D-04	3.17	0.735813D-05	3.57	0.444728D-06	3.97	0.197214D-07
2.38	0.763142D-03	2.78	0.844127D-04	3.18	0.688545D-05	3.58	0.412960D-06	3.98	0.181710D-07
2.39	0.724936D-03	2.79	0.795818D-04	3.19	0.644190D-05	3.59	0.383387D-06	3.99	0.167392D-07

(cont'd)

Table Complementary Error Function

Z	erfc(Z)	Z	erfc(Z)	Z	erfc(Z)	Z	erfc(Z)	Z	erfc(Z)
4.00	0.154173D-07	4.40	0.489171D-09	4.80	0.113521D-10	5.20	0.192491D-12	5.60	0.238284D-14
4.01	0.141969D-07	4.41	0.446950D-09	4.81	0.102914D-10	5.21	0.173138D-12	5.61	0.212646D-14
4.02	0.130707D-07	4.42	0.408293D-09	4.82	0.932791D-11	5.22	0.155701D-12	5.62	0.189730D-14
4.03	0.120314D-07	4.43	0.372906D-09	4.83	0.845298D-11	5.23	0.139992D-12	5.63	0.169250D-14
4.04	0.110726D-07	4.44	0.340520D-09	4.84	0.765861D-11	5.24	0.125844D-12	5.64	0.150951D-14
4.05	0.101882D-07	4.45	0.310886D-09	4.85	0.693754D-11	5.25	0.113103D-12	5.65	0.134604D-14
4.06	0.937269D-08	4.46	0.283775D-09	4.86	0.628312D-11	5.26	0.101632D-12	5.66	0.120003D-14
4.07	0.862073D-08	4.47	0.258978D-09	4.87	0.568932D-11	5.27	0.913067D-13	5.67	0.106965D-14
4.08	0.792756D-08	4.48	0.236302D-09	4.88	0.515062D-11	5.28	0.820141D-13	5.68	0.953249D-15
4.09	0.728870D-08	4.49	0.215568D-09	4.89	0.466202D-11	5.29	0.736527D-13	5.69	0.849347D-15
4.10	0.670003D-08	4.50	0.196616D-09	4.90	0.421893D-11	5.30	0.661308D-13	5.70	0.756621D-15
4.11	0.615769D-08	4.51	0.179295D-09	4.91	0.381721D-11	5.31	0.593654D-13	5.71	0.673885D-15
4.12	0.565816D-08	4.52	0.163467D-09	4.92	0.345307D-11	5.32	0.532816D-13	5.72	0.600078D-15
4.13	0.519813D-08	4.53	0.149008D-09	4.93	0.312304D-11	5.33	0.478119D-13	5.73	0.534249D-15
4.14	0.477457D-08	4.54	0.135801D-09	4.94	0.282401D-11	5.34	0.428952D-13	5.74	0.475548D-15
4.15	0.438468D-08	4.55	0.123740D-09	4.95	0.255311D-11	5.35	0.384766D-13	5.75	0.423213D-15
4.16	0.402583D-08	4.56	0.112729D-09	4.96	0.230774D-11	5.36	0.345063D-13	5.76	0.376564D-15
4.17	0.369564D-08	4.57	0.102677D-09	4.97	0.208554D-11	5.37	0.309396D-13	5.77	0.334990D-15
4.18	0.339186D-08	4.58	0.935034D-10	4.98	0.188437D-11	5.38	0.277362D-13	5.78	0.297948D-15
4.19	0.311245D-08	4.59	0.851326D-10	4.99	0.170226D-11	5.39	0.248595D-13	5.79	0.264949D-15
4.20	0.285549D-08	4.60	0.774960D-10	5.00	0.153746D-11	5.40	0.222766D-13	5.80	0.235559D-15
4.21	0.261924D-08	4.61	0.705306D-10	5.01	0.138834D-11	5.41	0.199585D-13	5.81	0.209387D-15
4.22	0.240207D-08	4.62	0.641787D-10	5.02	0.125343D-11	5.42	0.178779D-13	5.82	0.186087D-15
4.23	0.220247D-08	4.63	0.583874D-10	5.03	0.113141D-11	5.43	0.160110D-13	5.83	0.165347D-15
4.24	0.201907D-08	4.64	0.531083D-10	5.04	0.102107D-11	5.44	0.143363D-13	5.84	0.146889D-15
4.25	0.185057D-08	4.65	0.482970D-10	5.05	0.921310D-12	5.45	0.128342D-13	5.85	0.130466D-15
4.26	0.169581D-08	4.66	0.439130D-10	5.06	0.831132D-12	5.46	0.114873D-13	5.86	0.115856D-15
4.27	0.155369D-08	4.67	0.399191D-10	5.07	0.749634D-12	5.47	0.102797D-13	5.87	0.102862D-15
4.28	0.142319D-08	4.68	0.362814D-10	5.08	0.675994D-12	5.48	0.919719D-14	5.88	0.913078D-16
4.29	0.130341D-08	4.69	0.329687D-10	5.09	0.609469D-12	5.49	0.822708D-14	5.89	0.810352D-16
4.30	0.119347D-08	4.70	0.299526D-10	5.10	0.549382D-12	5.50	0.735785D-14	5.90	0.719040D-16
4.31	0.109259D-08	4.71	0.272071D-10	5.11	0.495122D-12	5.51	0.657916D-14	5.91	0.637892D-16
4.32	0.100005D-08	4.72	0.247084D-10	5.12	0.446133D-12	5.52	0.588172D-14	5.92	0.565791D-16
4.33	0.915161D-09	4.73	0.224348D-10	5.13	0.401912D-12	5.53	0.525717D-14	5.93	0.501740D-16
4.34	0.837317D-09	4.74	0.203664D-10	5.14	0.362004D-12	5.54	0.469802D-14	5.94	0.444852D-16
4.35	0.765944D-09	4.75	0.184850D-10	5.15	0.325994D-12	5.55	0.419751D-14	5.95	0.394336D-16
4.36	0.700518D-09	4.76	0.167742D-10	5.16	0.293508D-12	5.56	0.374959D-14	5.96	0.349488D-16
4.37	0.640556D-09	4.77	0.152187D-10	5.17	0.264208D-12	5.57	0.334880D-14	5.97	0.309679D-16
4.38	0.585612D-09	4.78	0.138048D-10	5.18	0.237786D-12	5.58	0.299027D-14	5.98	0.274350D-16
4.39	0.535276D-09	4.79	0.125198D-10	5.19	0.213964D-12	5.59	0.266959D-14	5.99	0.243004D-16

TUTORIALS

Tutorial 1 - Introduction

1. Draw the block diagram of a radio broadcasting system and describe the functions of each block.

2. Draw the block diagram of a digital communication system and describe the function of each block.

3. Explain the difference between analog and digital communication systems.

4. What are the two main causes of signal degradation in the communication channel of a digital communication system? How does this signal degradation affect the recovery the original data in the digital receiver?

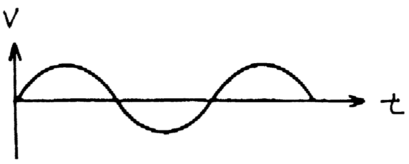
5. Explain why digital communications is preferred over analog communications.

6. Name some common devices that use source coding and channel coding techniques.

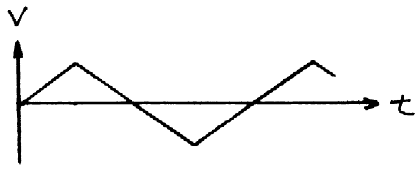
Tutorial 2 - Signal and Spectra

1. State, with reasons, whether the following are analog or digital signals.

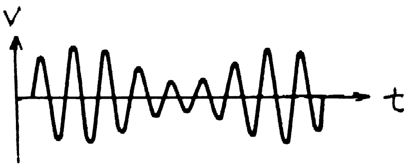
(a)



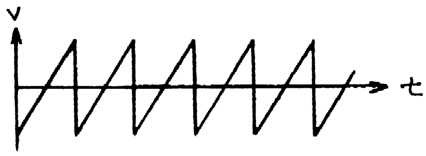
(e)



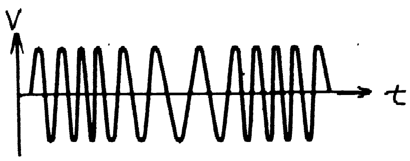
(b)



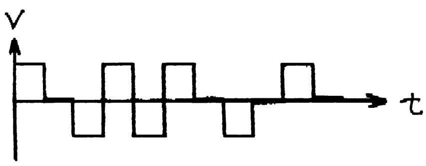
(f)



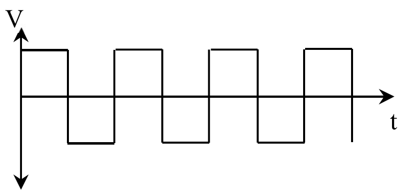
(c)



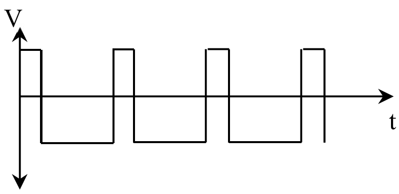
(g)



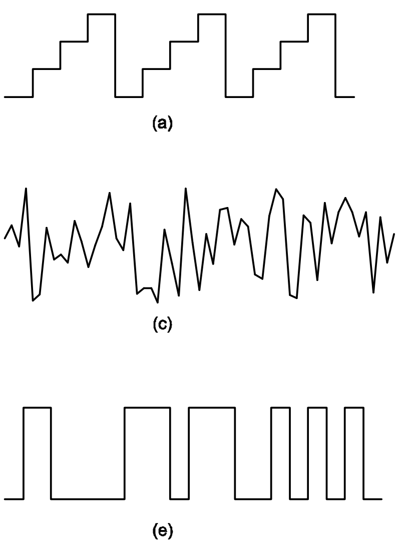
(d)



(h)



2. State with reason whether the following signals are periodic or non-periodic. Indicate the period, T for the periodic signals.



3. Sketch the single-sided and double-sided amplitude spectrum for the sinusoidal waveform in Figure T2.3.

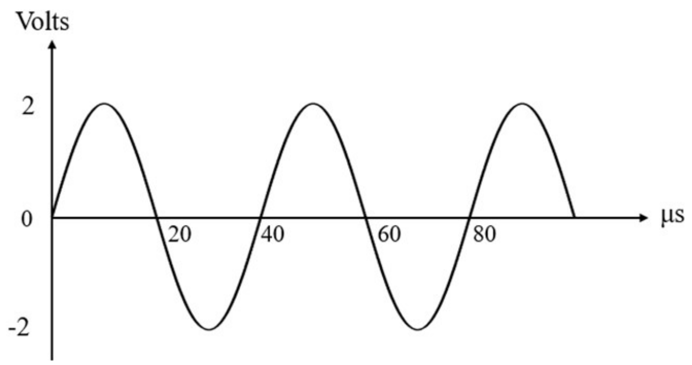


Figure T2.3

4. Sketch the single-sided and double-sided spectrum for signal v(t) given below:

$$v(t) = V_1\cos\omega_1 t + V_2\cos\omega_2 t + V_3\cos\omega_3 t \quad \text{where } V_1 > V_2 > V_3 \text{ and } f_3 > f_2 > f_1$$

5. The trigonometric Fourier series of a waveform which repeats itself every 125 μs is given by

$$v(t) = 0.4 + \frac{0.8\sin 0.2\pi \cos\omega_0 t}{0.2\pi} + \frac{0.8\sin 0.4\pi \cos 2\omega_0 t}{0.4\pi} + \frac{0.8\sin 0.6\pi \cos 3\omega_0 t}{0.6\pi} + \frac{0.8\sin 0.8\pi \cos 4\omega_0 t}{0.8\pi} + \ldots$$

Sketch the single-sided and double-sided amplitude spectrum of the signal up to the 4th harmonic, showing the amplitude and frequency of each component.

6. Sketch the amplitude spectrum of rect (t).

7. Sketch x(t) whose frequency spectrum is given below in Figure. T2.7.

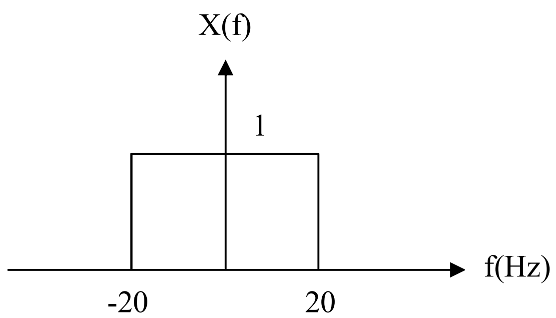
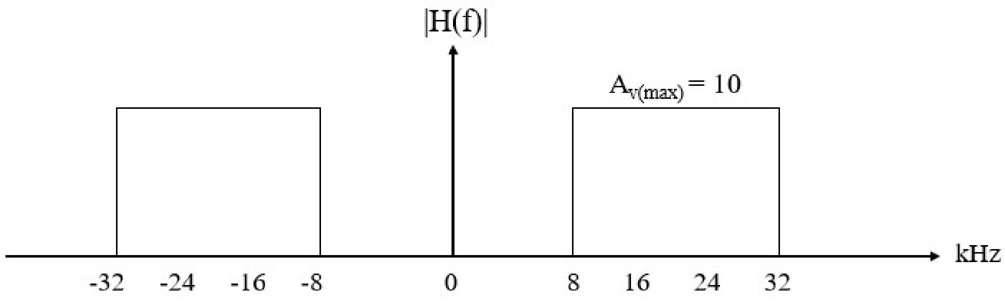


Figure T2.7

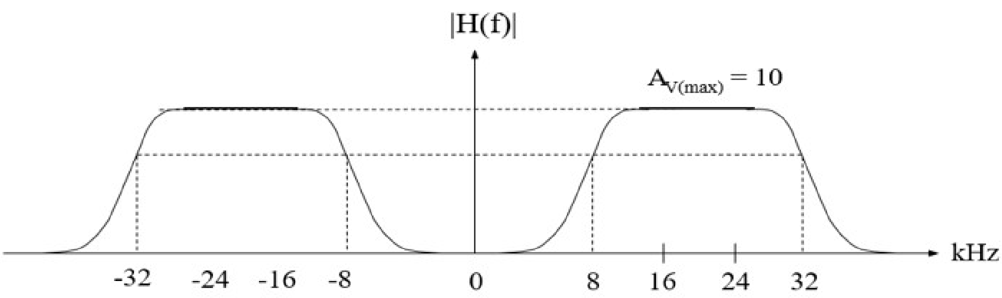
8. State the main difference between the amplitude spectrums of periodic and non periodic signals.

9. (a) The signal in question 5 is input to the ideal BPF shown in Figure T2.9 (a). Draw the double-sided amplitude spectrum of the filter output signal.

   (b) The same signal is input to the practical BPF shown in Figure T2.9 (b). The BPF has a passband from 8 kHz to 32 kHz. Draw the amplitude spectrum of the filter output signal.



(a) Ideal BPF



(b) Practical BPF

Figure T2.9

Tutorial 3 - Noise

1. The temperature of a 12 kΩ resistor is 30^{O}C.
   (a) What is the thermal noise power and rms noise voltage over a 10 kHz bandwidth? Sketch the noise spectrum.
   (b) For the same bandwidth, what is the rms noise voltage if the temperature of the resistor is increased by 25^{O}C?

2. (a) If the signal and noise power at the output of an amplifier is 100 mW and 4mW respectively, what is the SNR at the output?
   (b) Another amplifier has signal and noise power of 1000 mW and 10mW respectively at its output. Which output signal is noisier?
   (c) Why low SNR means signal is noisy?

3. The noise factor of a noiseless amplifier which generates no noise would be:
   (a) less than one. (b) one.
   (c) greater than one. (d) zero.

4. Explain why high noise factor means the circuit is noisy?

5. Identify the main sources of noise in a battery-operated amplifier. The amplifier is enclosed in a metallic chassis. Give reasons for your answers.

Tutorial 4 - Signal Transmission

1. A radio transmitter is connected to an antenna by a length of cable. If the wavelength of the signal on the cable is 0.8 times the wavelength in free space, determine the velocity of the signal along the cable.

2. GSM 900 uses carrier frequency from 890 MHz to 915 MHz to send information from the mobile phone to the base station (uplink). Calculate the minimum antenna length for efficient transmission.

3. (a) Define modulating signal and modulation.
   (b) Name the input and output signals of a modulator.
   (c) Give 3 reasons for the need of modulation.
   (d) Name three examples of analog modulation.
   Name three examples of digital modulation.

4. (a) Give the frequency range for frequency bands MF, VHF, and UHF..
   (b) Name one application for each of these frequency bands.

5. Describe briefly transmission impairments.

Tutorial 5 – Amplitude Modulation

1. For the AM signal, $v_{AM}(t)$, shown in Figure T5.1
   (a) Determine V_S, V_C, f_S, f_C.
   (b) Calculate the modulation index, m
   (c) Write the equation for the modulating signal, $v_S(t)$
   (d) Write the equation for the AM signal, $v_{AM}(t)$
   (e) Sketch the double-sided amplitude spectrum of $v_{AM}(t)$
   (f) Calculate the bandwidth of $v_{AM}(t)$

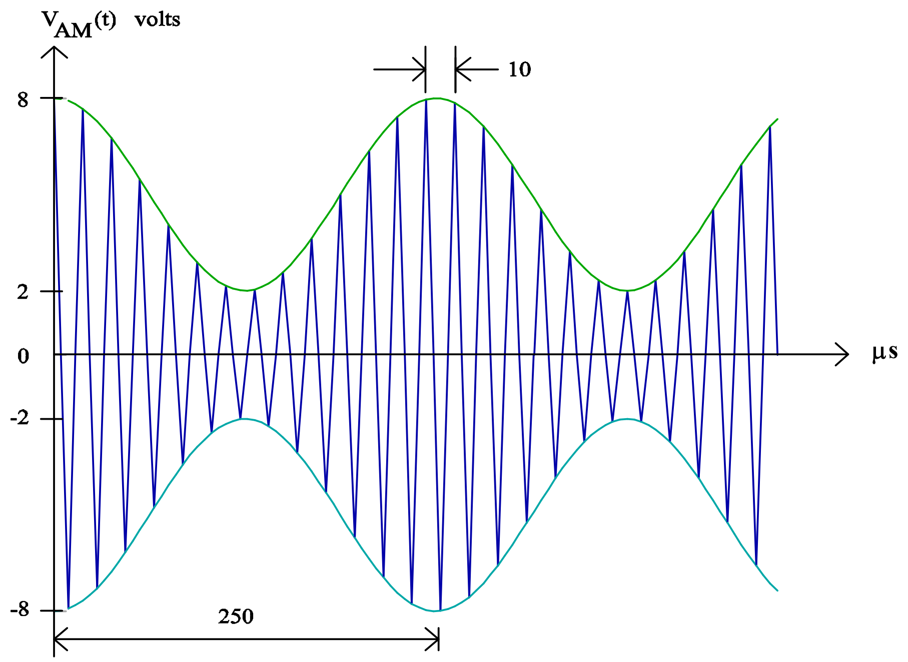


Figure T5.1 An AM signal

2. For the amplitude spectrum shown in Figure T5.2, what is the modulation index of the AM signal?
   (a) 0.20 (b) 0.33
   (c) 0.40 (d) 0.67

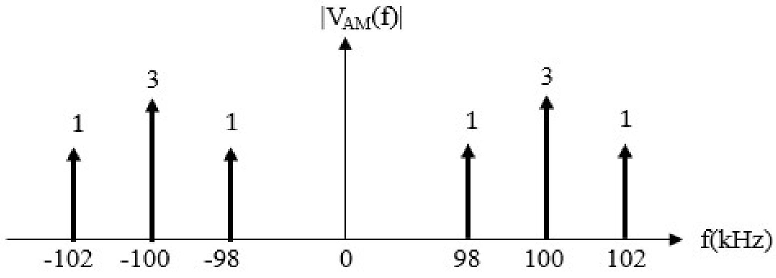


Figure T5.2

3. The AM signal spectrum is shown in Figure T 5.3.
   (a) Determine the frequency values of A and B.
   (b) Sketch the spectrum of the modulating signal and state its bandwidth.
   (c) Write the equation for the modulating signal.

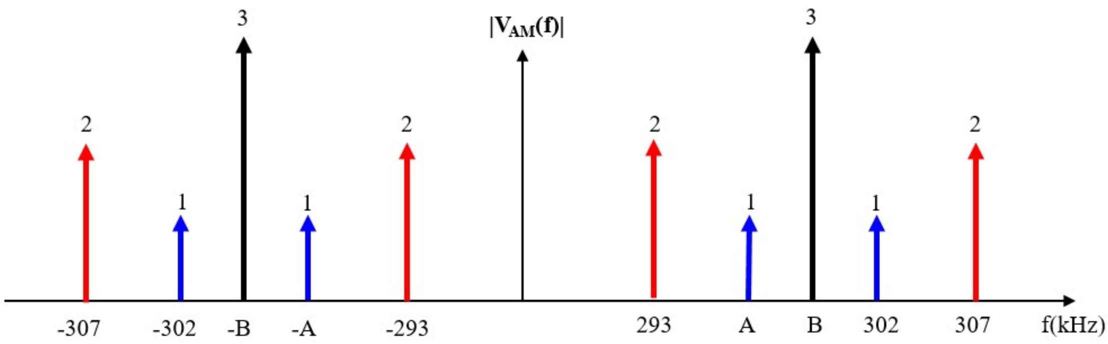


Figure T5.3

4. When a carrier, $8\cos 1200\times10^{3}\pi t$ is amplitude modulated by a modulating signal, $v_s(t)$. The spectrum of $v_s(t)$ is given in Figure T5.4.
   (a) Sketch the block diagram to produce the AM signal.
   (b) Sketch the double-sided amplitude spectrum of the AM signal.

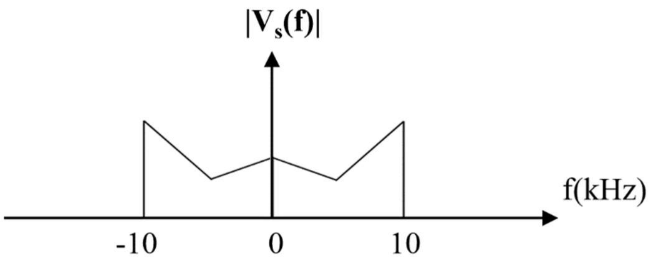


Figure T5.4

Tutorial 6 – Frequency Modulation

1. An FM modulator has a conversion gain of 10 kHz/V. Its carrier frequency is set to 200 kHz.

   Plot a graph showing how the output frequency changes when the modulating signal in Figure T6.1 is applied. Indicate the frequency at t_1, t_2, t_3 and t_4.

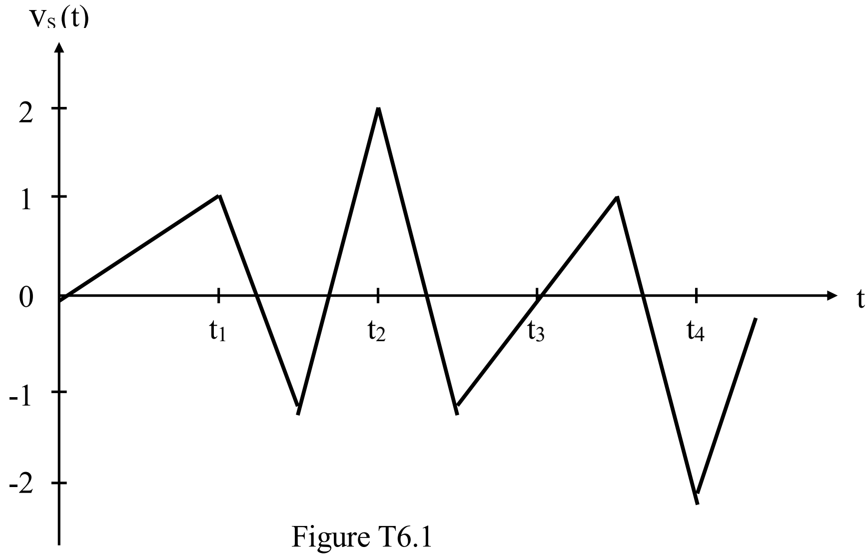


Figure T6.1

2. A 2.4 V_{peak}, 500 Hz sinusoidal modulating signal when fed to a frequency modulator results in a peak frequency deviation of 4.8 kHz.
   (a) Calculate the conversion gain of the modulator.
   (b) What is the peak frequency deviation when the peak amplitude of the modulating signal is increased to 7.2 V?
   (c) What is the modulation index in each case?

3. A 100 MHz carrier is frequency modulated by a 5 kHz sine wave to a modulation index of 4. Given that the conversion gain of the demodulator is 6 mV/kH. Determine the peak output voltage of the demodulator.

4. A 2 V_{peak}, 100 MHz sinusoidal carrier is frequency modulated by a 4 V_{peak}, 2 kHz sinusoidal signal. The modulator has a conversion gain of 3 kHz/V.
   (a) State, with reason, whether it is narrowband or wideband transmission.
   (b) Calculate the bandwidth of the FM signal.
   (c) Calculate the total FM power if dissipated over a 50Ω load.

5. When the modulation index of an FM signal increases from 2 to 6, the transmitted power
   (a) increases by 3 times
   (b) increases by 9 times
   (c) remains the same
   (d) reduces by 9 time

6. (a) List 4 advantages and 1 disadvantage of wideband FM.
   (b) List 4 advantages and 1 disadvantage of narrowband FM.
   (c) The transmission power of FM can be lower than that of AM to cover the same area. Why?

Tutorial 7 – Analog and digital conversion

1. (a) State sampling theorem I.
   (b) A signal, v(t), bandlimited to 3kHz, has a amplitude spectrum shown in Figure T7.1. The signal is sampled by an ideal unit impulse train such that the guardband of the sampled output is 1 kHz.
   - i) Determine the sampling frequency and sketch the amplitude spectrum of the sampled signal.
   - ii) State how v(t) can be recovered from the samples.
   - iii) If the sampling rate is 5000 samples per second, comment on the recovery of v(t).

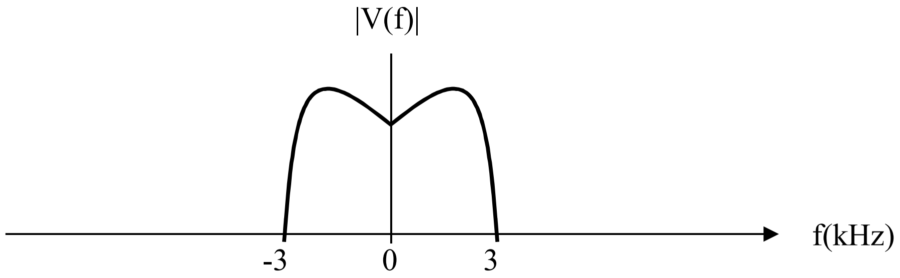


Figure T7.1

2. The bandlimited signal f(t) is ideally sampled at 80 samples per second as shown in Figure T7.2, where f(t) = 100sinc 100t. Draw the amplitude spectrum of the output signal g(t).

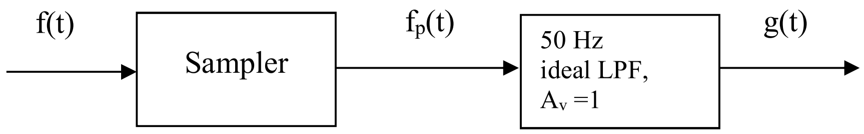


Figure T7.2

3. (a) Sketch the amplitude spectra of the following signals:
   (i) m(t) = 2rect1000t
   (ii) p(t) = $\sum_{n=-\infty}^{\infty} \delta(t - nT_s)$ where p(t) is unit impulse train and T_s = 0.2ms

   (b) The signal m(t) is filtered and then sampled by signal p(t) as shown in Figure T7.3. Sketch the amplitude spectra at points X and Y over a range of ± 8kHz.

   (c) Explain how to recover signal at point X from the signal y(t) at point Y.

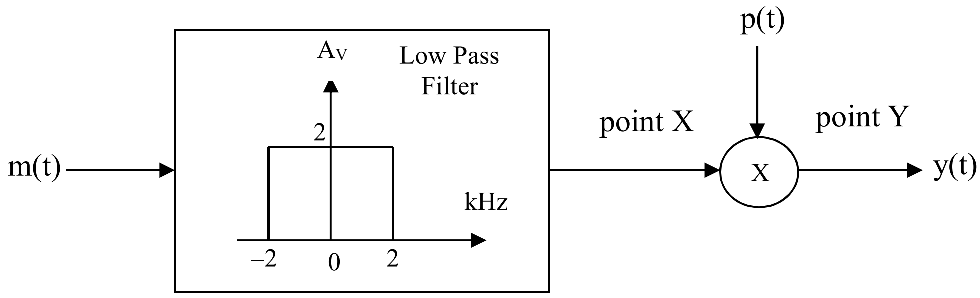


Figure T7.3

4. A signal, x(t) given below is low pass filtered and then sampled by a unit impulse train, p(t), as shown in Figure 7.4.

$$x(t) = \frac{1}{2} + \sin 2000\pi t + \frac{1}{2}\sin 4000\pi t + \frac{1}{3}\sin 6000\pi t + \frac{1}{4}\sin 8000\pi t + \cdots$$

   (a) Sketch the double-sided amplitude spectrum of x(t) up to the 3rd harmonic and the double-sided amplitude spectrum of y(t).
   (b) Using uniform sampling theorem I, determine the minimum sampling frequency for y(t).
   (c) If y(t) is sampled at a sampling frequency of 4.5 kHz, sketch the amplitude spectrum of the sampled signal, z(t), for a frequency range of ±7 kHz.

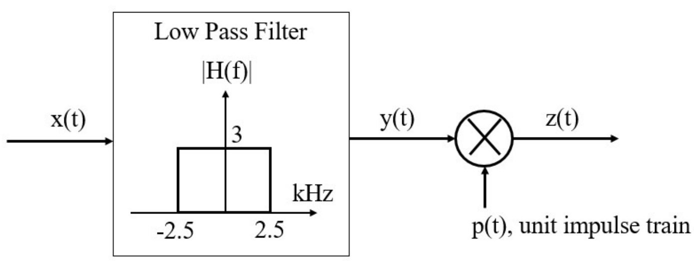


Figure T7.4

5. State two main differences between natural and flat top sampling.

6. Sketch the i/p-o/p characteristics of a 3-bit mid-riser linear quantiser of a PCM system used to digitise analogue signals whose range of amplitudes vary between ± 3.5 volts.

7. A PCM system employs a uniform 5-bit quantiser/encoder. The <u>maximum permissible</u> input voltage to the quantiser is 15 volts peak-to-peak. Calculate the signal-to-quantisation noise ratio (in dB) for the following input signals:
   (a) 7.5 Sin ωt
   (b) 5 Sin ωt

8. Four voice channels plus one music channel are sampled and transmitted through a PCM-TDM system in which 8-bit uniform quantisers are employed. The music signal is bandlimited to 10 kHz and each voice channel is bandlimited to 4 kHz.
   (a) sketch the PCM-TDM commutator system capable of handling both the voice and music signals. Ensure that uniform sampling is achieved. Synchronisation information is needed.
   (b) calculate the gross bit rate of the system

9. A commutator shown in Figure T7.8 is connected to a communication channel with bandwidth of 300 kHz.

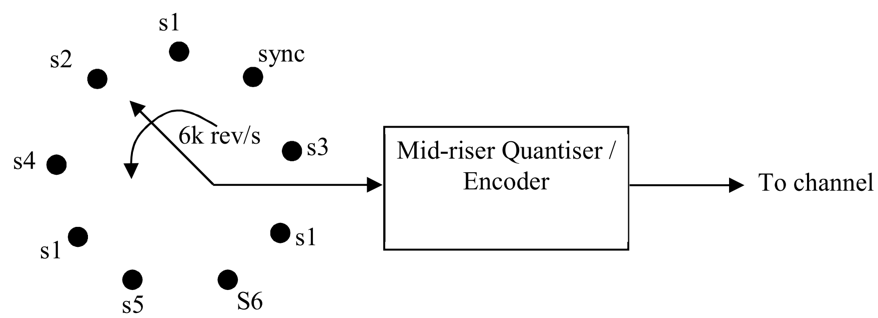


Figure T7.8

(a) Determine the maximum signal frequency of each input signal s_1 to s_6.
(b) Show that the maximum number of bits per sample for each signal is 11 bits assuming each sample of the signals is encoded using unipolar NRZ format.

10. A PCM-TDM system is used to multiplex three similar sinusoidal signals, each uniformly quantised and represented by a binary code. The bit duration is 10 μsec. If the maximum signal-to-quantisation noise ratio is to be maintained greater than 1000, determine
    (a) the number of bits required
    (b) the sampling frequency (uniform sampling and `sync' info are required)
    (c) the highest analogue signal frequency allowed
    (d) the minimum transmission bandwidth required

Tutorial 8 – Baseband transmission of digital signal

1. Encode the data 101101 using
   (a) Polar NRZ
   (b) Manchester coding
   (c) Unipolar RZ

2. A binary source transmits 1's using unit impulses and 0's using no pulses as shown in Figure T8.1 for the data sequence 110. The bit rate is 500 b/s. If the transmission channel is an ideal low-pass filter having minimum cut-off frequency determine the minimum cut-off frequency for zero ISI transmission.

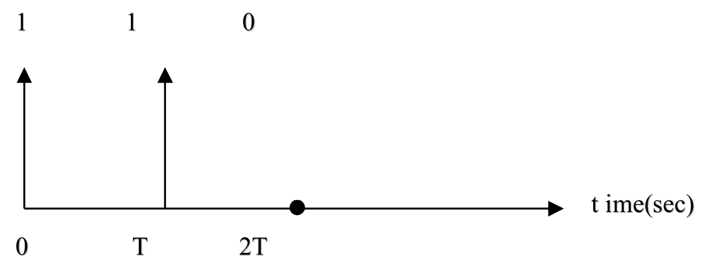


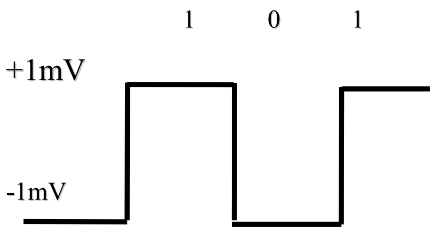
Figure T8.1

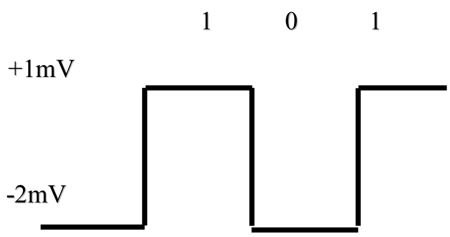
3. What is the probability that a 0.1 volt RMS Gaussian noise signal exceeds a magnitude of 0.5 volt?

4. What is the probability that a 0.5 volt RMS Gaussian noise signal exceeds +1 volt?

5. In a digital communication system, it is equally likely to send a '1' (5 volts) or a '0' (0 volt). If the rms Gaussian noise is 0.4 volt, calculate the error probability.

6. The signal component of the input to the receiver of a baseband transmission system is of the form :





This signal is corrupted by additive, white Gaussian noise with an rms value of 0.5 mV. The threshold device at the receiver is a comparator. If the received voltage is above V_T (threshold voltage) at the sampling instant, the signal is decoded as binary 1; if it is below V_T then it is decoded as binary 0. For both cases above, determine the value of V_T to obtain a minimum probability of bit error (P_e), and hence find P_e. The probability of occurrences of binary 1 and 0 are assumed equal and independent.

Tutorial 9 – Optimum baseband transmission

1. What is the impulse response of a matched filter for the Manchester coding signal given below in Figure T9.1:

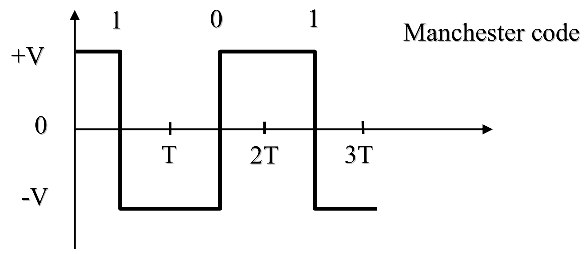


Figure T9.1

2. A polar binary signal, s(t), is a +1 V or -1 V pulse during the spectral interval (0, T_b). Additive white Gaussian noise having two-sided power density of 10^{-3} W/Hz is added to the signal. If the received signal is detected with a matched filter, determine the maximum bit rate that can be sent with a bit error probability of $P_e \leq 10^{-3}$.

3. An integrate and dump correlation receiver is shown in Figure T9.2. For the following input :
(a) a polar NRZ waveform of amplitude V volt,
(b) a Manchester code waveform of amplitude V volt,

Sketch the waveforms at A to E for a 1011 sequence. Explain the operations of SW1 and SW2.

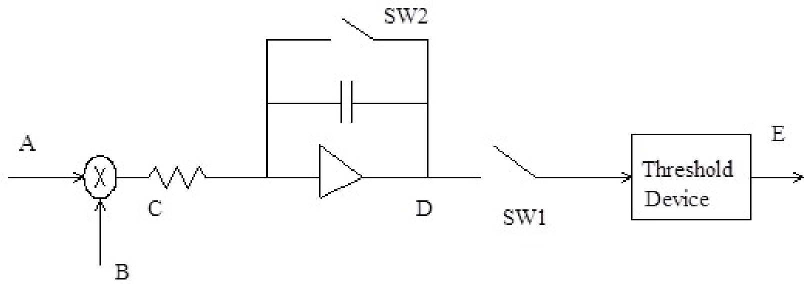


Figure T9.2

4. The input to a matched filter receiver is a baseband signal of data rate 1200 b/s. The signal is in the form of :
   (a) a polar NRZ waveform of amplitude 5 mV.
   (b) a unipolar NRZ waveform of amplitude 5 mV

   For both cases, find the probability of bit error, if the single-sided power spectral density of the channel AWGN is 2 nanowatt/HZ. Assume that the bits of the source binary sequence are independent and equiprobable.

Tutorial 10 – Digital Modulation

1. A BPSK system with a carrier frequency of 16 kHz is used to transmit binary data of bit rate of 8 kbps.
   (a) Sketch the block diagram of the BPSK transmitter.
   (b) If the input bit stream to the BPSK transmitter is 11010010, sketch the polar NRZ waveform of the bit stream.
   (c) Sketch the BPSK waveform for the input bit stream in (b).

2. A binary data at rate 3 kb/s is transmitted over a passband channel using BPSK. The carrier amplitude at the receiver is 10 mV, and the single-sided power spectral density of the channel AWGN is 2 nanowatt/Hz. Calculate the bit error rate, assuming that an integrate-and-dump correlation receiver is used.

3. Figure T10.3 shows the block diagram of a DPSK transmitter. The binary input is in unipolar NRZ format, at a bit rate of 2400 b/s. The carrier is $A\sin\omega_c t$ where $\omega_c = 4800\pi$ rad/s. Assume that input is a long sequence of …101010… Sketch the waveforms at points A to C as indicated in Figure T10.1 for a 1010 frame. Assume a distortionless transmission path. Also assume that the encoder's output is binary 0 prior to the 1010 frame.

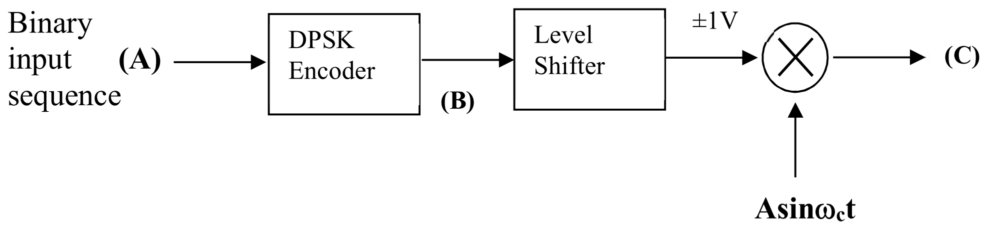


Figure T10.3 DPSK Transmitter

4. Rank, from low to high, the following modulation schemes in terms of BER, bandwidth efficiency and equipment complexity: BPSK, DPSK, QPSK, 4- QAM, 8-PSK and 8-QAM.

LAB SHEETS

Experiment 1
Signal Measurement

Objectives

1. To know how to use the Digital Oscilloscope.
2. To know how to generate signals using FUNCTION GENERATOR.

Equipment

Oscilloscope
Function Generator

Procedure

TIME DOMAIN SIGNAL MEASUREMENT USING DIGITAL OSCILLOSCOPE

The Oscilloscope can be set to display 1 signal or 2 signals (refer the front panel of TDS1012B Digital Oscilloscope in Figure E1.1):

- Press CH 1 to display signal at CH 1.
- Press CH 2 to display signal at CH 2.
- You can select both CH1 and CH2 to display 2 signals simultaneously.

Note: You can switch off CH 1/CH 2 display by press it again after it is on.

A. Display the test signal from PROBE COMP

Input the test signal from PROBE COMP to CH1 (or CH2) of the oscilloscope using a BNC-to-Crocodile Co-axial cable (see Figure E1.2).

1. Press **AUTOSET** button to obtain a stable waveform of the signal.

2. Set the **Attenuation** factor of CH 1/CH2:
   Press CH 1/CH2 MENU ➔ select Probe ➔ Voltage ➔ Attenuation ➔ 1X.

B. Measure the test signal using MEASURE method

You can access automatic measurements by pressing **MEASURE** button. Five types of automatic measurements can be observed simultaneously out of 11 types of automatic measurements given below:

Freq, **Period**, **Pk-Pk** (Peak-to-Peak), **Min**, **Max**, **Mean**,
Pos Width, **Neg Width, Rise Time**, **Fall Time**, and **None**.

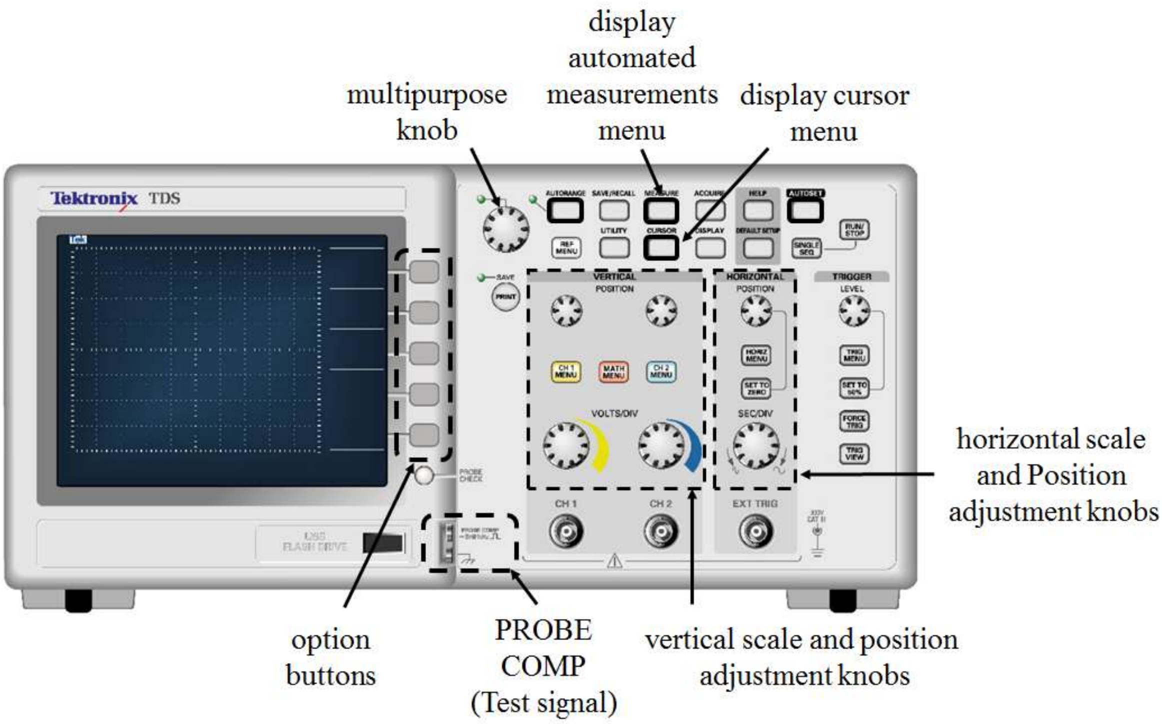


Figure E1.1 Front panel of the TDS1012B Digital Oscilloscope

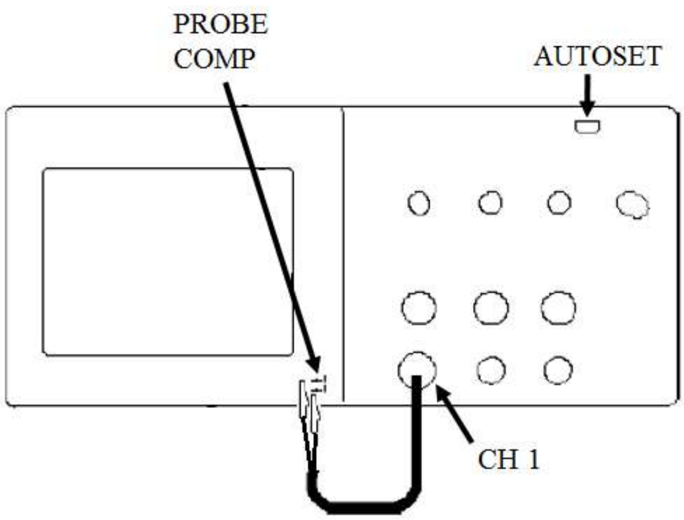


Figure E1.2 Display the signal from PROBE COMP

Set the oscilloscope to automatically measure the <u>Frequency</u>, <u>Period</u>, and <u>Peak-to-Peak Amplitude</u>, <u>Rise Time</u>, and <u>Positive Width</u> of the test signal as shown in Figure E1.3 following the steps below:

1. Press **Measure** button to access the Measure Menu if it is not yet done.

2. Press the first option button (refer to Figure E1.1) from the top to active Measure 1 menu. Select Type **Frequency**. Press the Back option button.

3. Press the second option button from the top to active Measure 2 menu. Select Type **Period** and Press the Back option button.

4. Press the remaining three option buttons one by one to set the measurements to **Pk-Pk**, **Rise Time** and **Pos Width** respectively**.**

   Note: Make sure **Source** is set to the same channel (CH1/CH2) that the input signal is connected to.

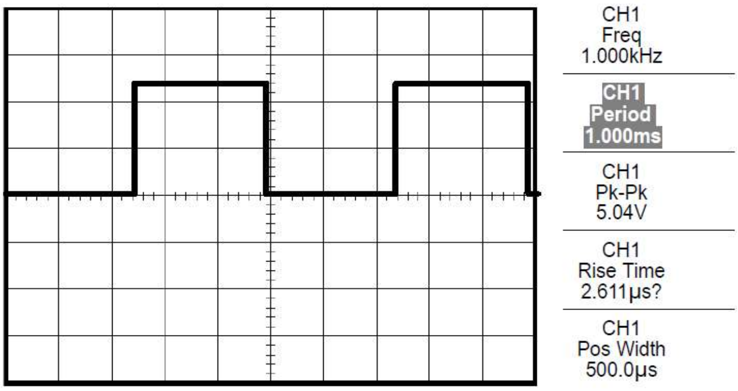


Figure E1.3 Automatic measurements

C. Measure the test signal using CURSOR method

Cursor method allows you to take measurements by moving the cursors, which always appear in pairs, and reading their numeric values from the display readouts. There are two types of cursors:

- **Amplitude Cursors** - appear as horizontal lines on the display.
- **Time Cursors** - appear as vertical lines on the display.

When you use cursors, be sure to set the **Source** to the waveform on the display that you want to measure. For instance, you need to press CH1 if the waveform to be measured is displayed at CH1.

Measure the Peak-to-Peak amplitude of the test signal using Amplitude Cursors following the steps below (see Figure E1.4):

1. Press the CURSOR button to access the Cursor Menu.
2. Select Type **Amplitude** and Source **CH1/CH2**.
3. Select the **Cursor 1** option button. Turn the multipurpose knob to place cursor 1 on the peak of the signal.
4. Select the **Cursor 2** option button. Turn the multipurpose knob to place Cursor 2 on the lowest part of the signal.
5. You can obtain the measured peak-to-peak amplitude, **ΔV**, of the signal in the Cursor Menu (refer to Figure E1.4). Record your measured Peak-to-Peak amplitude below:

   peak-to-peak amplitude $V_{P\text{-}P}$ = ______________________.

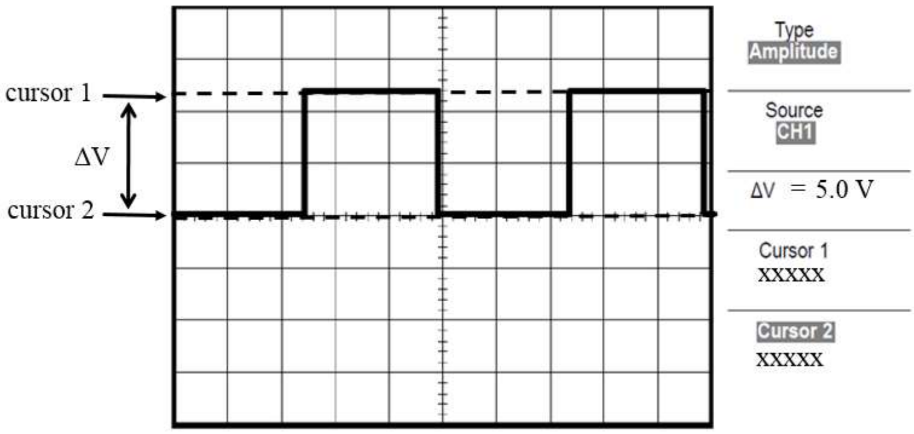


Figure E1.4 Measuring Amplitude

Measure the Frequency and Period of the test signal using Time Cursor following the steps below (see Figure E1.5):

1. Press the **CURSOR** button.
2. Select Type **Time** and Source **CH1/CH1**.
3. Select the **Cursor 1** option button. Turn the multipurpose knob to place a cursor on the start of a cycle of the signal.
4. Select the **Cursor 2** option button. Turn the multipurpose knob to place a cursor on the end of a cycle of the signal.

5. You can obtain the measured period, Δt and frequency, 1/Δt in the Cursor Menu (refer to Figure E1.5). Record your measured frequency and period below:

   Period = ______________________.

   Frequency = ______________________.

6. Measure the **Pos Width** (Pulse Width) of the signal based on the steps described above and record your measured **Pos Width** below:

   Pos Width = ______________________.

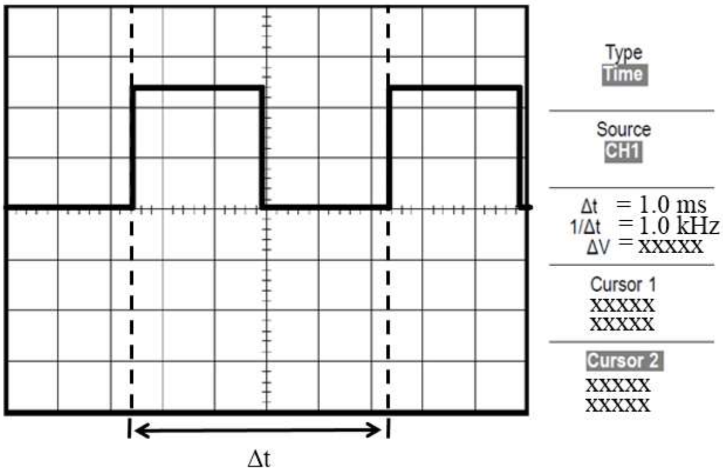


Figure E1.5 Measuring Period and Frequency

D. Adjust the vertical and horizontal scale of the test signal.

The vertical scale and position of the waveform can be adjusted by turning the **Volts/Div** and **POSITION** knobs. The vertical scale can be adjusted using **Coarse** or **Fine** adjustment. The coarse adjustment changes the Volts/Div in steps of 1-2-5 volts (or mVolts). The fine adjustment changes the Volts/Div by either 10 or 40 mVolts.

1. Set the vertical adjustment to coarse (or fine) by pressing CH 1 MENU ➔ select Vots/Div ➔ select Coarse (or fine).

2. Adjust the **vertical** scale and position to allow the waveform to occupy 6 divisions vertically (see Figure E1.6) and record the VOLT/DIV setting:

   VOLTS/DIV = ______________________.

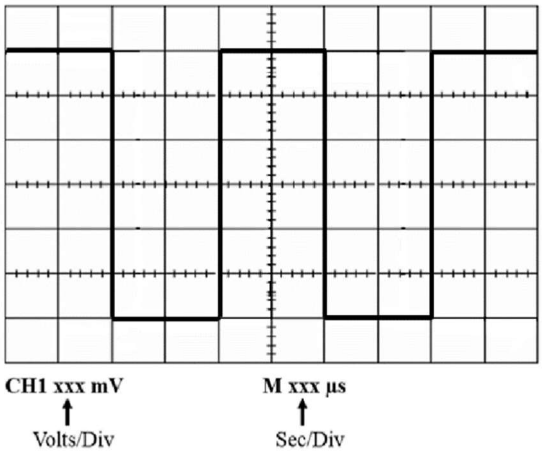


Figure E1.6 Vertical and horizontal scale

3. Adjust the **horizonta**l scale of the waveform by turning the **SEC/DIV** and **POSITION** knob to display 2.5 cycles of the waveform (see Figure E1.6). Record the SEC/DIV setting:

   SEC/DIV =______________________

SIGNAL GENERATION OF FUNCTION GENERATOR

The function generator,TG1010 FUNCTION GENERATOR, can produce different types of waveforms up to a maximum frequency of 10 MHz. Figure E1.7 shows the front panel of the function generator.

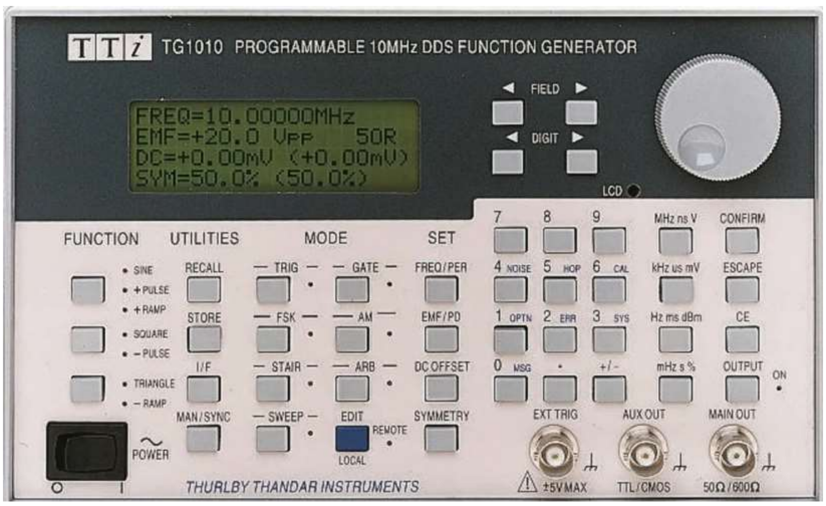


Figure E1.7 Front panel of TG1010

A. Generate and display a sine wave with frequency of 50 kHz and amplitude of $4V_{pp}$

1. Under the "FUNCTION" choose "SINE".

2. Under the "FIELD" use the right arrow button move the cursor to the 1st line of the display panel "FREQ=xxxx kHz" of the numerical part xxxx, key in 50 then press "kHz µs mV". (50 kHz).

3. Under the "FIELD" use the right arrow button move the cursor to the 2nd line of the display panel "EMF = +20.0 V" of the +20.0 part, key in 4 then press "MHz ns V". (4 volts peak-to-peak).

4. Make sure the 3rd line of the display panel is "DC = +0.00mV" and the last line of the display panel is "SYM = 50.0%".

5. Press "CONFIRM" then "OUTPUT", the "ON" will light.

   Note "CE" is back space, can be used to correct error key.

FREQ = 50 . 0 0 0 0 0 KHZ
EMF = + 2 . 0 Vpp 50Ω
DC = + 0 . 0 0 mV (+0.00mV)
SYM = 5 0 . 0 % (5 0 . 0 %)

6. Display the sine waveform on the oscilloscope.

B. Generate and display a rectangular wave shown in Figure E1.8.

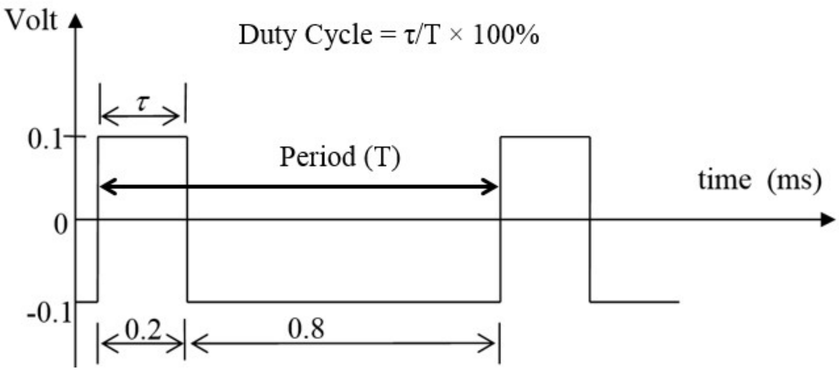


Figure E1.8 Rrectangular waveform

Determine the following **before** you start generating and displaying the waveform:

Frequency : ____________________

Amplitude : ____________________

SYMMETRY (duty cycle) : ____________________

Set the function generator to generate the waveform and display about 4 cycles of the waveform on the oscilloscope. **Show your result to your Lecturer.**

C. Generate and display a sawtooth wave shown in Figure E1.9.

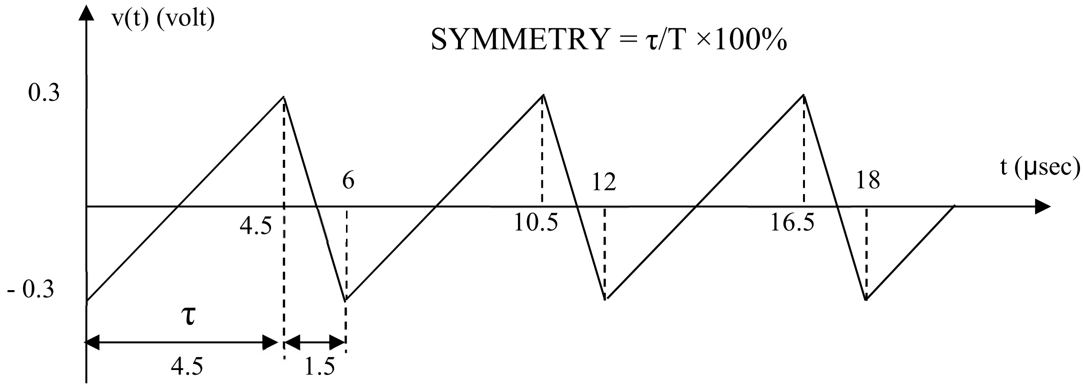


Figure E1.9 Sawtooth waveform

Determine the following **before** you start generating and displaying the waveform:

Frequency : ________________________________

Amplitude : ________________________________

SYMMETRY : ________________________________

Set the function generator to generate the waveform and display about 4 cycles of the waveform on the oscilloscope. **Show your result to your Lecturer.**

D. Generate a 1.5 V DC voltage

Generate a DC voltages from the function generator.

1. Enter the 1.5 V DC value on the third line in the display (leave the other lines as they are).

2. Press the "**GATE**" button.

   Note: Remember to deactivate the "gate" button when other waveforms e.g. sine waves are required.

FREQ =xx. xxxx KHZ
EMF = + 1 . 0 Vpp 50Ω
DC = **+ 1 . 5 0 V** (+0.00mV)
SYM = 5 0 . 0 % (5 0 . 0 %)

Experiment 2
Signal Spectrum Measurement

Objectives

1. To get familiar with the user interface of spectrum analyser

2. To get familiar with important spectrum analyzer parameters.

3. To perform frequency domain signal measurements of periodic signal.

Equipment

R&S FSV Spectrum Analyser
Tektronix TDS1012B Digital Oscilloscope
TG1010 Function Generator

Procedure

Caution

The Spectrum Analyser has a **maximum allowable input power** of 1 W (30 dBm).

To avoid potential damage, please make sure that **you have set the voltage level of the input signal to the Spectrum Analyser as instructed.**

To display signal spectrum using spectrum analyser, we need to specify the frequency range to be displayed on the spectrum analyser by setting the **Start** and **Stop** frequency or **Centre frequency** and **Span** on the Spectrum Analyser as shown in Figure E2.1.

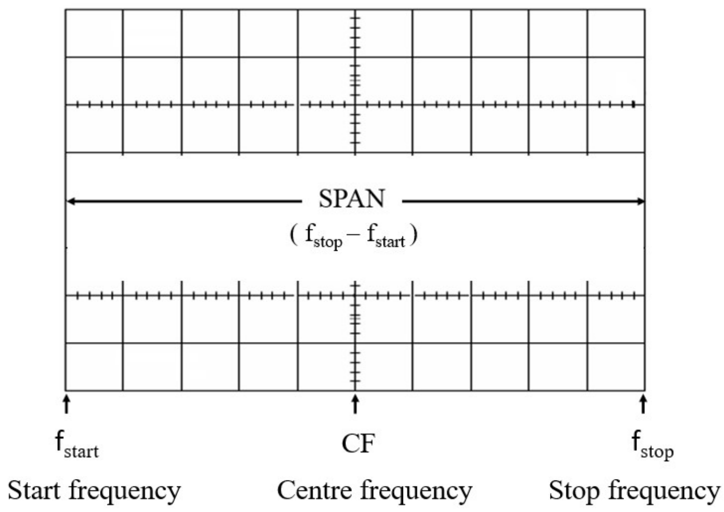


Figure E2.1 Spectrum Analyser CRT screen

You will use the **two** methods to display the spectrum of a signal:

- Method 1 - set **Start and Stop** frequency
- Method 2 - set **Centre** frequency and **Span**

A. Measure frequency and power of a 100 kHz and 600 mV_{pp} sine wave signal

1. Input a sine wave signal of 100 kHz and amplitude of 600 mV_{pp} to the spectrum analyser through a RF cable as shown in Figure E2.2.

Important
Make sure that you have set the input signal voltage level as instructed before connect the signal to the spectrum analyser.

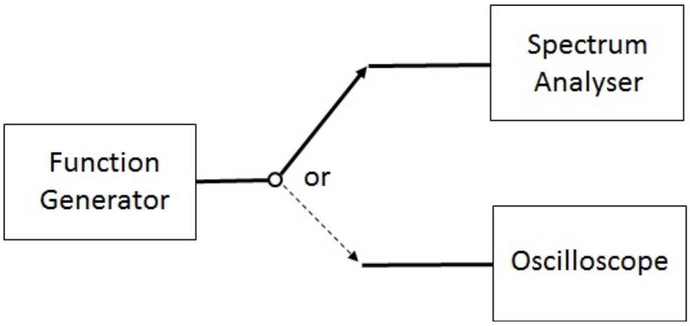


Figure E2.2 Connection diagram

2. Measure the frequency and power of the sine wave:

   FREQUENCY ➔ CENTER ➔ enter 100 kHz ➔ SPAN ➔ enter 200 kHz

3. Adjust the reference level (refer to appendix):

   AMPLITUDE ➔ Ref Level ➔ enter 0 dBm

4. Press PEAK SEARCH to place a marker (**M**1) on the frequency component. Record down the frequency and amplitude of the sine signal below:

   Frequency: ____________________.

   Amplitude: ____________________.

Question 1: How many frequency components does a sine signal have? ____

Question 2: What is the expected frequency of the component?

Expected frequency: ______________

B. Display and measure the spectrum of a rectangular wave

1. Input a 200mV$_{p-p}$ rectangular wave with a period of 5μs and a pulse width of 1μs to the spectrum analyser (the timings must be exact).

2. Based on what you have learnt in Chapter 2, sketch the **single-sided** spectrum of the rectangular wave up to the 7th harmonic below. Indicate the frequency of each component but not their amplitudes.

3. Display the spectrum of the rectangular wave up to the 7th harmonic (from fundamental frequency to 7th harmonic) on the spectrum analyser using **Method 1.**

a) Determine **Start** frequency, $\mathbf{f_{start}}$:

Start frequency**,** Calculated f_{start} = 200 kHz (to display the fundamental frequency component)

Note: The actual Start frequency, f_{start} should be chosen slightly smaller than 200 kHz, usually the difference is $\boldsymbol{\Delta F}$, so that the fundamental component will be fully included within the display window:

$\boldsymbol{\Delta F}$ **= 10% × $\mathbf{f_0}$ (fundamental frequency)**

Chosen $\mathbf{f_{start}}$ = Calculated $\mathbf{f_{start}}$ - $\boldsymbol{\Delta F}$ = 180 kHz

b) Determine **Stop** frequency, $\mathbf{f_{stop}}$:

Stop frequency, f_{stop} =1400 kHz (to display the 7th harmonic component).

Note: The actual **Stop** frequency, $\mathbf{f_{stop}}$ should be chosen slightly larger than 1400 kHz so that the 7th harmonic component will be fully included within the display window.

Chosen $\mathbf{f_{stop}}$ = Calculated $\mathbf{f_{stop}}$ + $\boldsymbol{\Delta F}$ = 1420 kHx

Now you can display the spectrum using the **chosen Start** and **Stop** frequency:

FREQUENCY ➔ Start ➔ enter **180** kHz ➔ Stop ➔ enter **1420** kHz.

4. Display the spectrum using **Method 2**

a) Determine **Center** Frequency:

CF = (Calculated f_{start} + Calculated f_{stop}) / 2 = 800

b) Calculate Span:

Span = Calculated f_{stop} – Calculated f_{start} = 1400 – 200 = 1200 kHz.

Note: The actual Span should be chosen slightly larger than the calculated Span so than the first and the last components will be fully included within the display window.

Chosen Span = Calculated **Span** + 2$\boldsymbol{\Delta F}$ = 1240 kHz

Set the spectrum analyser to display the spectrum:

FREQUENCY ➔ Center ➔ enter 800 kHz ➔ SPAN ➔ enter **1240** kHz.

5. Measure the frequency and amplitude of each frequency component

a) Record down the reading of the amplitude and frequency of the frequency components in the table below.

MRK-> ➔ Peak ➔ Next Peak, and so on

Note: You can also turn the knob manually to measure component by component using PEAK SEARCH.

	Fundamental	2nd Harmonic	3rd Harmonic	4th Harmonic	5th Harmonic	6th Harmonic	7th Harmonic
Frequency (kHz)							
Amplitude (dBm)							

Question 3. While maintain the frequency of the rectangular wave constant, slowly increase the SYMMETRY until it reaches 50%. Describe and explain the change in the spectrum.

Observation: ____

Explanation: ____

____

C. Display and measure the spectrum of a Sawtooth wave

1. Change the signal to a sawtooth wave while mantaining all other values unchanged.

2. Record down your reading on frequency and amplitude of the frequency components in the table below.

	Fundamental	2nd Harmonic	3rd Harmonic	4th Harmonic	5th Harmonic	6th Harmonic	7th Harmonic
Frequency (kHz)							
Amplitude (dBm)							

Question 4: Compare the spectrum of the rectangular wave and the sawtooth wave, describe and explain the difference.

Observation: ____

Explanation: ____

____

D. Practice

1. Practice 1

Display the spectrum of a 400mV$_{p-p}$ rectangular wave with a period of 10μs and a pulse width of 3μs up to the 9th harmonic. Show how you calculate and choose Start and Stop frequency, SPAN and Centre frequency. **Show the spectrum to your lecturer.**

Method 1

Start frequency = __________

Stop frequency = __________

Chosen Start frequency = __________

Chosen Stop frequency = __________

Method 2

Center frequency = ______________

Calculated **Span** = ______________

Chosen Span = ______________

2. PRACTICE 2

Display the spectrum of a 500 mV_{p-p}, 250 kHz square wave, showing the **3rd and 5th harmonics only**. Show your working below. Measure the frequency and power of each component. Show how you calculate and choose Start and Stop frequency, or SPAN and Centre frequency. **Show the spectrum and measurements to your lecturer.**

Method 1

Start frequency = __________

Stop frequency = __________

Chosen Stop frequency = __________

Chosen Stop frequency = __________

Method 2

Center frequency = ______________

Calculated **Span** = ______________

Chosen Span = ______________

E. TEST

Without referring to the lab sheets, set the functional generator to generate the waveform shown in Figure E2.3. Display the spectrum up to 5th components on the spectrum analyser. **Show your results to your lecturer.**

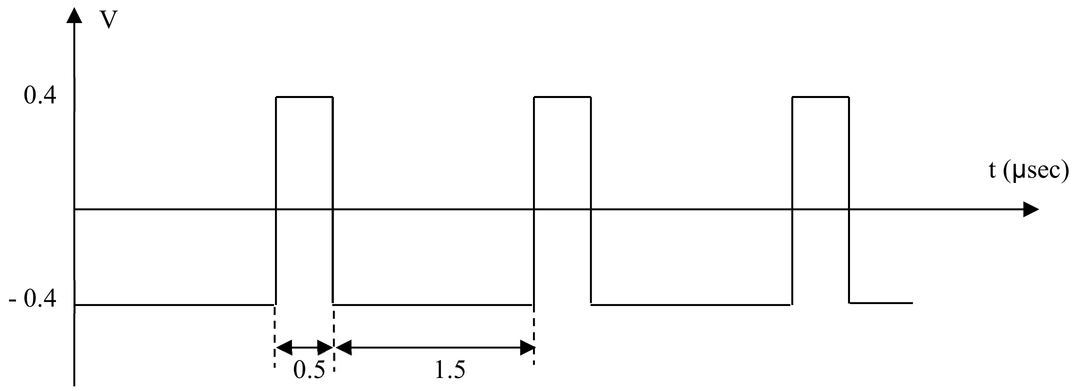


Figure E2.3 A rectangular waveform

Determine the following **before** you start:

Frequency :________________________

Amplitude :________________________

SYMMETRY (Duty cycle) :________________________

Appendix

1. REFERENCE LEVEL (Ref LVL)

The REFERENCE LEVEL (REF LVL) must be set to an appropriate value. Look at the spectrum displayed on the CRT screen. Can you see the peaks of all the frequency components of the input signal? If yes, then you don't need to change the REF LVL setting. If the displayed spectrum does not appear correctly on the screen, you need to adjust REFERENCE LEVEL .

Follow this sequence to adjust the REFERENCE LEVEL (RFF LVL): AMPLITUDE ➔ Ref Level ➔ Rotate the "SPINNER" to move the displayed spectrum ↑ or ↓ until the peaks of all the frequency components are clearly displayed.

> **Note**: *You will notice that each vertical line representing a frequency component is a bit rounded. This is normal and is due to the way the spectrum analyzer works.*

Five settings of reference level are available i.e. 20 dBm, 10 dBm, 0 dBm, -10 dBm and -20dBm. The Ref LVL should be set to a value slightly higher than the power of the input signal expressed in dBm. Since we do not know the input power, we have been setting the Ref LVL by trial and error.

If you know the input power in dBm, then you should set the Ref LVL to the next higher level. For example if the input power is +15 dBm, set the REF LVL to +20 dBm. If the input power is -15 dBm set the Ref LVL to -10 dBm.

2. DC COMPONENT

Spectrum analysers cannot display DC component

3. OTHER SETTINGS

a) The resolution Bandwidth (RBW) will be set to the optimum every time when the span selection is changed. But, it can be set manually to see a fine spectrum by pressing the following buttons in sequence: BW ➔ RBW ➔ enter XXX Hz/ kHz/MHz.

b) You can place a triangle-shaped marker on the peak of a frequency component to find the frequency and amplitude (power) of that component. Follow the sequence blow to get the reading of all frequency components. The Marker Readout is the frequency and amplitude of the component (Marker Readouts of amplitude and frequency appear in the upper-right corner of the display).

MKR-> ➔ Peak ➔ Next Peak

Experiment 3
Band Pass Filter Measurement

Objective

1. Plot the Frequency Response of a BPF.
2. Determine the lower and upper 3dB cut-off frequencies of a BPF.

II THEORY

Question 1: On the frequency response of a practical BPF shown in Figure E3.1, identify the axes, maximum voltage gain, lower and upper 3dB cut-off frequencies, the voltage gain at the upper and lower 3dB cut-off frequencies, passband, stopband, transition band, and the centre frequency.

Note: The centre frequency of a BPF is at the centre of the passband where the voltage gain is maximum.

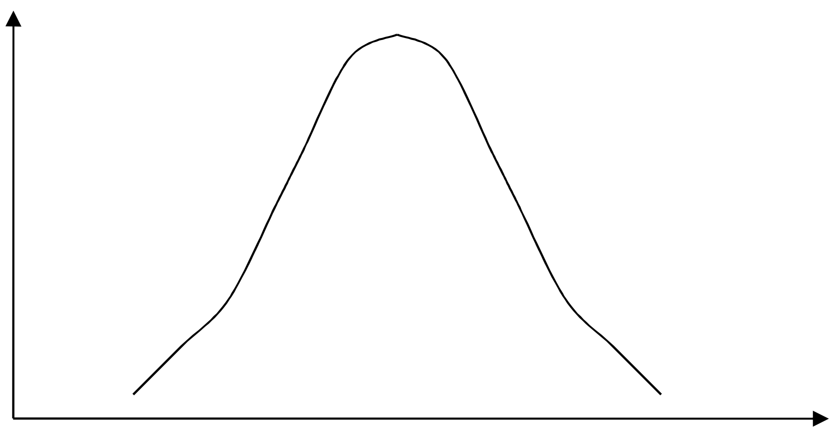


Figure E3.1 BPF Frequency Response

Question 2: When measuring the frequency response, what type of input signal is used?

____

Question 3: A sinewave is fed to a practical BPF. Sketch the output waveform for each of the 3 cases below. You need not indicate the actual output voltage. The peak-to-peak voltage of the input sinewave is the same for all the 3 cases.

Frequency of input sinewave	A_V	Output waveform
a) at the centre frequency		
b) in the transition band		
c) in the stopband		

Question 4: Referring to your answer in Q3, what happens to the output voltage when the input signal frequency is equal to the centre frequency of the BPF?

____

Question 5: Referring to your answer in Q3, how would the amplitude of the output waveform changes as the input signal frequency shifts from centre frequency, to transition band and then to stopband?

____

Procedure

A. Plot the frequency response of bandpass filter (BPF)

1. Set the Function Generator to produce a $4V_{p-p}$, 855 kHz sine wave. Press the output button until it lights up. Connect the signal to the BPF input, TP1 and monitor this signal on CH1 of the oscilloscope.

2. Monitor the output signal of BPF, TP2 on CH2 (Set TRIG MENU>SOURCE>CH1). Press AUTOSET on the oscilloscope so that the displayed waveforms are ‘tall’ enough for accurate voltage measurements.

3. Sketch a **block diagram** to show how you would connect up the equipment to perform measurements on the BPF. Indicate TP1, TP2 and GND terminals on your block diagram.

4. Fill in the values for 855 kHz in Table E3.1.

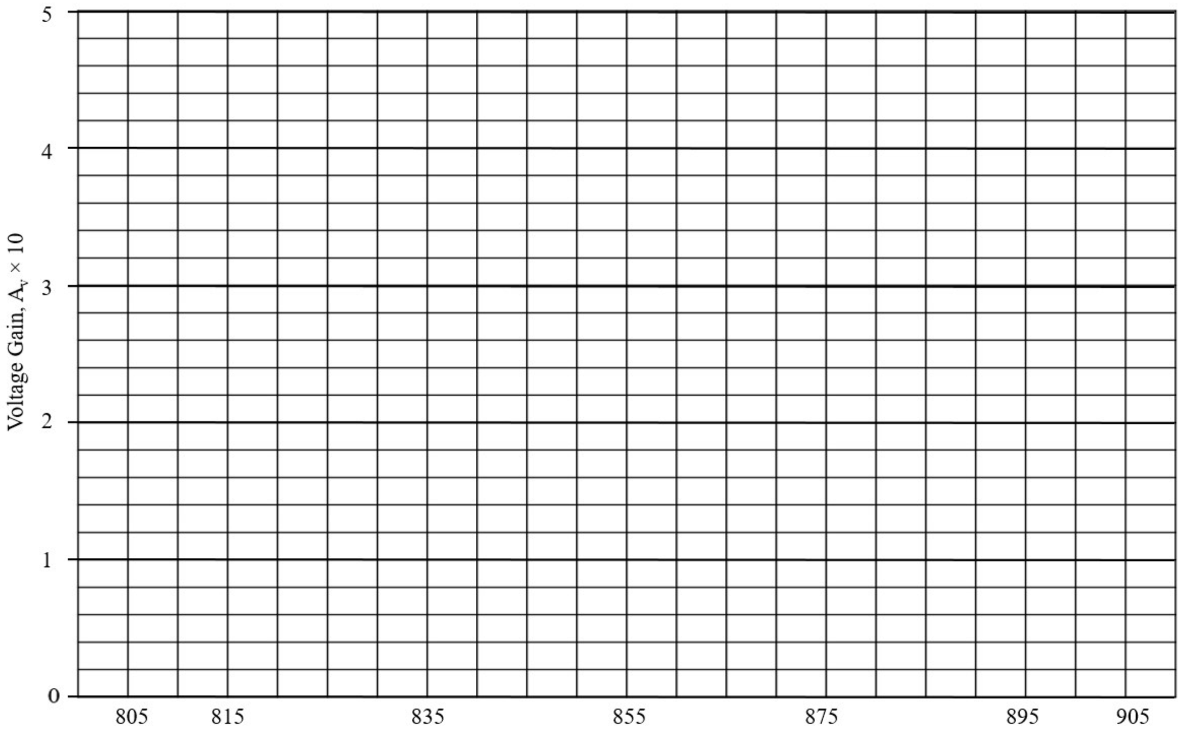
   Note: The voltage gain is scaled by a factor of 10 for easy plotting.

5. Repeat the measurements for each frequency in Table E3.1.

Table 3.1

Input frequency (kHz)	Input voltage $V_{i\,(p\text{-}p)}$	output voltage $V_{o\,(p\text{-}p)}$	Voltage Gain A_v**×10**
805 kHz			
810 kHz			
815 kHz			
820 kHz			
825 kHz			
830 kHz			
835 kHz			
841 kHz			
845 kHz			
850 kHz			
855 kHz			
860 kHz			
865 kHz			
870 kHz			
875 kHz			
880 kHz			
885 kHz			
890 kHz			
895 kHz			
900 kHz			
905 kHz			

6. Sketch the frequency response of the BPF from the results obtained in Table E3.1.



Frequency of input sinusoidal signal (kHz)

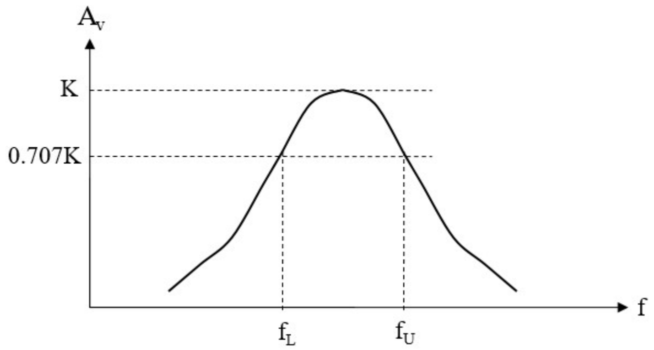
B. Determine the following parameters of the BPF based on the frequency response drawn.

K =

f_L =

f_U =

Bandwidth =



Experiment 4
Digital Communication Link

Objectives

1. To show how data represented by data words can be sent as a digit stream, one bit at a time and reconstructed at a distant receiver.

2. To show that analogue signals can be converted to data words and sent by this process.

3. To show that communication can be accomplished in this way.

Equipment

Feedback Digital Communications Systems modules: 297A, 297H, 297K, 297M.
Function Generator
Oscilloscope

Procedure

Important
Keep the Spectrum Analyser **unconnected** unless you are told otherwise.

A. Establishing the data source

1. With power switched on, connect the Data Source Module to oscilloscope as shown in Figure E4.1.

2. On the Data Source Module, set the "Format" switch to "8-bit" and the "Data Source" to "Push buttons". This sets the module to output 8 data bits for each word, and the word is determined by the setting of the eight push buttons.

3. Put a jumper between "160 kHz" and "clock-in" on the Data Source Module. The 160 kHz clock determines the bit rate at the output of the Data Source Module. Without this connection, no binary data will be produced at the output.

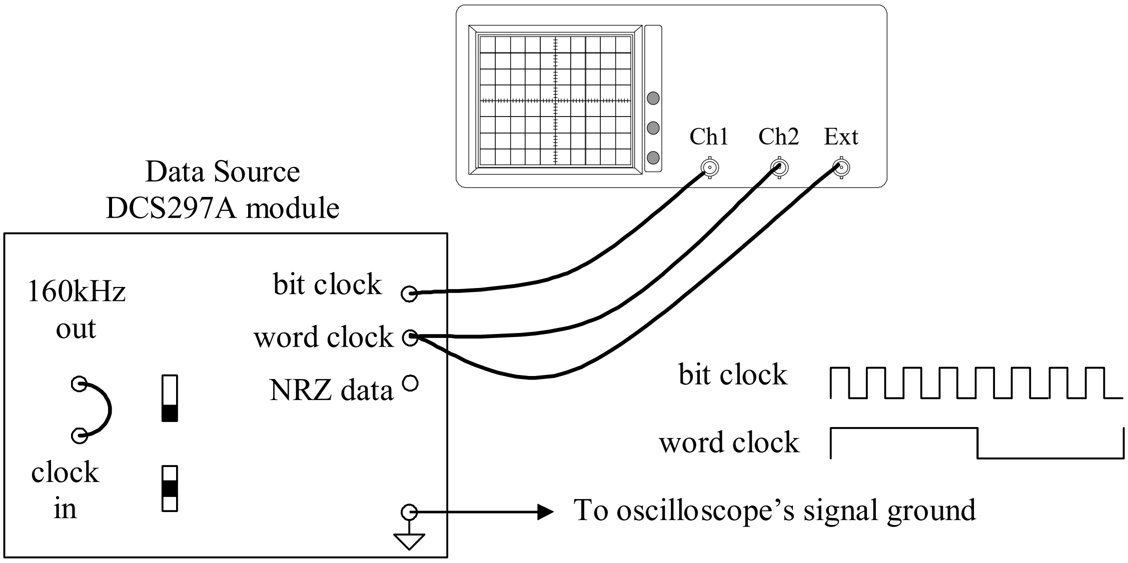


Figure E4.1 Observation of the bit clock and word clock

4. To view the signals on the scope, depress the "autoset" button or manually set the oscilloscope as follows:
   (i) CH1 and CH2 DC-coupled
   (ii) VOLTS/DIV = 5 V/Div
   (iii) TIME/DIV = 10 us/Div
   (iv) TRIGGER SOURCE = EXT, positive going edge

   On the oscilloscope, adjust the time/div knob and the "horizontal position" knob until you see the 8 cycles of the bit clock and 1 cycle of the word clock as shown in Figure E4.1.

   CH1 shows the BIT CLOCK which determines the rate at which bit signals will be generated.

   CH2 is the WORD CLOCK which marks the start of a WORD (eight bits).

   Question 1: What is the clock frequency of the bit clock and word clock?

   Frequency of bit clock: ____________________________

   Frequency of word clock: ____________________________

5. Refer to Figure E4.1 and connect the "NRZ data" to CH2. Also connect the word clock to "Ext" on the oscilloscope. Using the row of black push-buttons on the Data Source Module, set up the binary pattern **01001100**.

   Observe the NRZ data waveform in CH2. To observe the NRZ data waveform correctly ensure that the following steps are carried out:

(i) Ensure that you have connect the word clock to “Ext” on the oscilloscope.
(ii) Press “autoset”.
(iii) On the oscilloscope, press the “trig menu” button and set the trigger source to “Ext”.
(iv) Zoom in, using the sec/div knob, to view only 8 bits for both the NRZ data and bit clock waveforms.
(v) Using the “horizontal position” knob move the “trigger arrow” at the centre of the oscilloscope’s display until it is at the extreme left of the display where it will become horizontal.
(vi) Use the “measure” button to measure values in the waveforms like period, frequency, peak-to-peak voltage if required.

The CH2 waveform is now a simple form of digital signal in which a stream of "bits" is sent, one after the other. The word set up on the push buttons presents all eight bits of the word at once, "in parallel". They are applied to a "parallel to serial converter". This sends each of the input bits in turn to the "NRZ data" output, timed by the bit clock.

6. Set the word using the push buttons to **01110100**. Record the waveform of CH1 and CH2 accurately.

B. Sending an analogue signal digitally

1. In order to send an analogue signal, it must first be converted into digital form. This is called analogue-to-digital Conversion (ADC).

   Set the <u>data source switch to ADC now</u> so that the bits of the signal are now determined not by the push buttons but by a voltage, at the "analogue input" socket.

   Prior to connecting the analogue input, use the small "**zero**" knob to adjust the data word to **10000000** while shorting the “analogue input” to ground. Remove the shorting wire.

2. Set the function generator to produce a 2 V DC signal.

3. Connect the function generator output to the "analogue input" and earth terminals of the DCS297A. Observe the waveform of the 8 bits NRZ data output on oscilloscope CH1.

4. Sketch the NRZ data waveforms.

5. Reset the function generator to produce a 1.5 V_{pp} **sine** waveform at 0.01 Hz with DC = 0 V.

6. Observe the NRZ data output.

   Note: The NRZ data output is changing continuously as the ADC encodes different samples of the input and each sample is converted to a different digital output.

C. Receiving digital words

1. Add the data **receiver** module **DCS297H**, by placing it to the right of data source. Connect the two together and set the two switches on the data receiver as shown in Figure E4.2.

   Note that the "received data" lamps in the receiver module now show the same patterns as are being sent from the source.

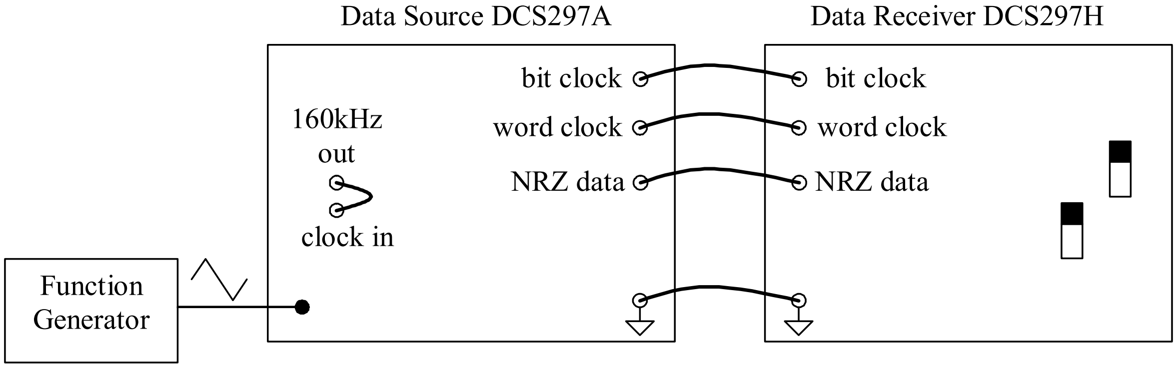


Figure E4.2 Receiving digital words.

D. Obtaining an analogue output

1. If the output is to be presented to an analogue device, the received word has to be processed by a digital-to-analogue converter (DAC).

2. Change the oscilloscope connections:

(i) CH1 to analogue input of Data Source
(ii) CH2 to analogue output of Data Receiver

Increase the frequency of the input signal (Function Generator) to 100 Hz.

Press "autoset" and you should see an analogue output which is a reasonable copy of the original input. What you have is a simple digital communication system.

3. Increase frequency of the input signal (Function generator) to 2 kHz.

4. Using the spectrum analyser, observe and sketch the spectrum of the reproduced signal, i.e. the analogue output.

Experiment 5
Signal Sampling

Objective

1. To demonstrate the operation of taking and holding samples of signals.

2. To study the effects of changing the sampling frequency.

Equipment

Sample-and-hold module 296E
Function Generator
Spectrum Analyser
Power supply

Important Notice

Ensure that the 296E module is connected to the power supply as follows:
Red to +15 V
Green to 0 V (ground)
Black to -15V

Introduction

In digital communications, the first step in processing a continuous analogue waveform, is to take its samples. "Samples" are particular values of the signal at regular intervals. From this point on, the "samples" will represent the signal, which could be digitised, or be subjected to various processing techniques.

The samples must be taken frequently enough, so that reconstruction of the original waveform from its discrete samples can be possible.

According to the Sampling Theorem, at least two samples per cycle should be taken so that a faithful reconstruction of the signal can be made. Theoretically, as shown in the waveforms in Figure E5.1, each sample is an instantaneous measurement, but in real life, taking a sample measurement takes time. Furthermore, an instantaneous sample would contain no energy and it would be difficult to transmit or process it. In practice, it is necessary to stretch the sample out in time, or to hold on to it long enough to use it. That is where the name "Sample-and-Hold" comes from. This is also known as flat-top sampling.

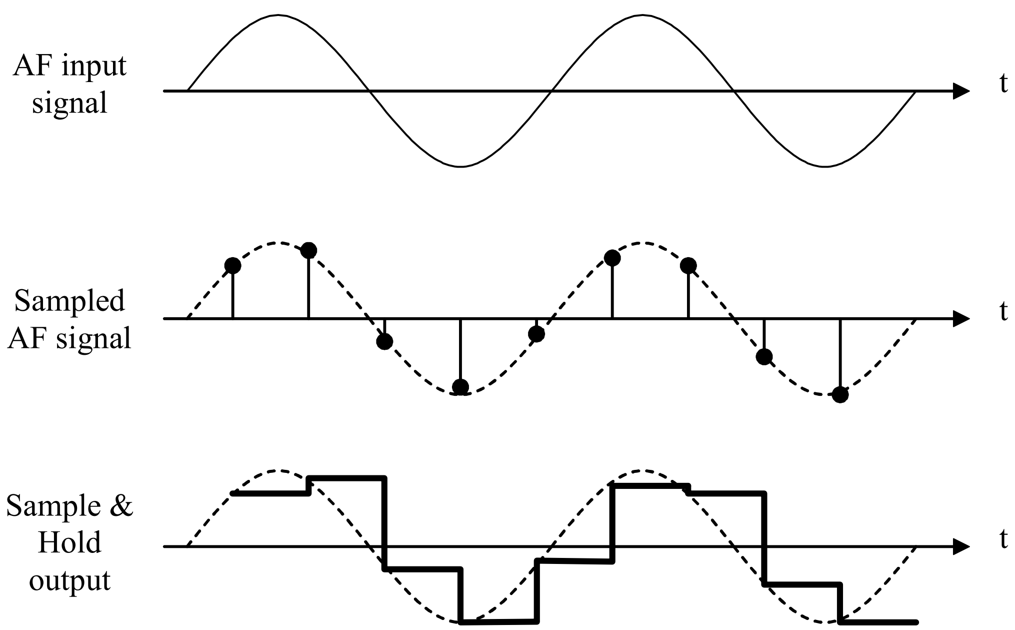


Figure E5.1 Waveform of AF input signal, sampled AF signal, and Sample & Hold output.

Figure E5.2 shows the Sample & Hold module (Module 296E) to be used in this experiment. In Module 296E, there are two sampling circuits. Looking at "Input 1", we can see that it is first buffered and then sampled. The switch at the output of the buffer is usually open, but when the sampling signal (pulse 1) goes "high", the switch is closed and the capacitor is quickly charged (according to the time constant of the circuit). Moments later, when the "pulse 1" goes "low", the switch is opened and the capacitor holds the sample value.

Procedure

Important
Keep the Spectrum Analyser **unconnected** unless you are told otherwise.

A. Sampling and Recovery of Sampled Signal

1. Ensure that the module is correctly connected to the power supply and switch on the latter.

2. Connect the "internal clock output" on the 296E module to "clock input" of the logic control unit.

3. Observe "pulse 1" on oscilloscope. Adjust the period of pulse 1(using clock frequency knob) to 200µs (frequency = 5 kHz) and pulse width (using the sampling pulse knob) to 5µs.

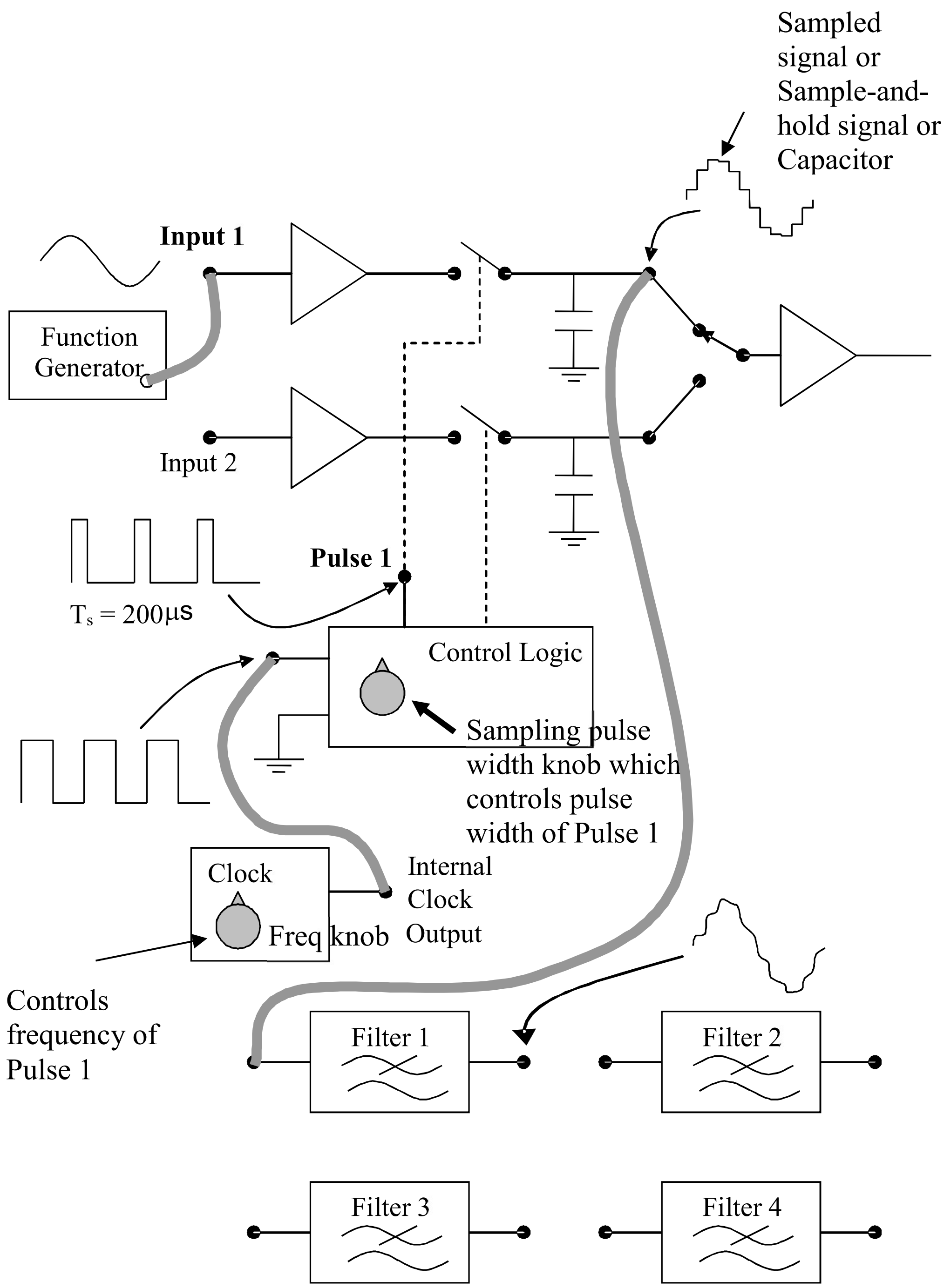


Figure E5.2 Module 296E (Sample-&-Hold)

Question1: what is the sampling frequency ? ________________________

4. Generate and connect a 500 Hz sine wave with 3 V_{pp} amplitude to "input 1" of the 296E module. Observe the "input 1" on CH1 of the oscilloscope. The sampled signal (or sample-and-hold signal) is the voltage across the capacitor. Monitor this voltage on CH2. Sketch both waveforms in the space provided below.

Signal at Input 1

Sampled signal

5. The original signal can be recovered from the sampled signal (or sample-and-hold signal) although it has many harmonics or spurious components. Recovery is done using low-pass filtering which removed or reduced the spurious components. There are four low-pass filters on the 296E module.

6. Connect the sampled signal to the input of filter 1. Observe both input and output of the filter on the oscilloscope; hence verify the effect of low-pass filtering.

7. Now observe the filter output together with the sinusoidal input waveform on the oscilloscope and hence verify their similarity. Filter 1 (a LPF) has recovered the original sinusoidal input although with some distortion (See Figure E5.3).

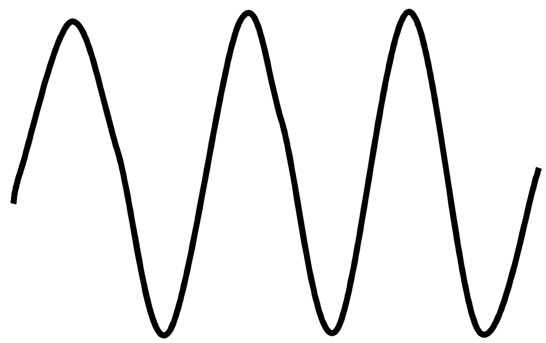


Figure E5.3 Recovered signal at the LPF output

B. Relation Between Sampling And Signal Frequencies

1. With the same sinusoidal input signal connected to "input 1" and the **sampling frequency** remaining at 5 kHz, set the pulse width to maximum.

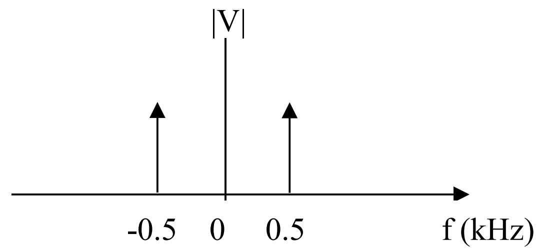
3. Monitor the spectrum of the sampled signal (or sample-and-hold signal) using the spectrum analyser up to 15 kHz. Sketch the spectrum.

Important

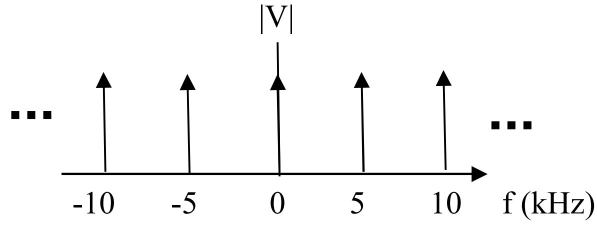
Make sure that you have set the input signal voltage level as instructed before connect the signal to the spectrum analyser.

3. Apply the concepts learnt in the chapter of sampling i.e. verify by deriving the spectrum of the sampled signal theoretically (assume ideal sampling): you need to find the convolution of the sine spectrum and the unit impulse train spectrum. Leave out amplitude values.

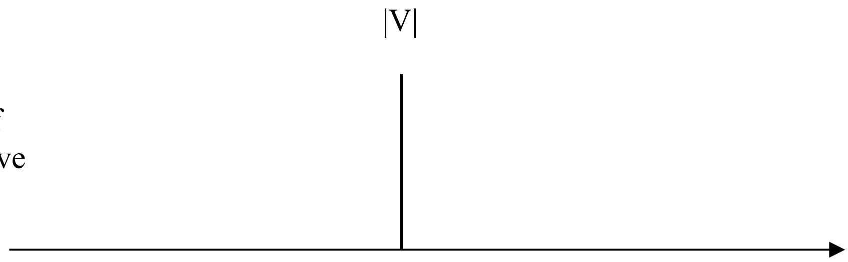
Spectrum of 500 Hz sine wave



Spectrum of unit impulse train of frequency 5 kHz



Spectrum of the sampled signal = Convolution of the 2 spectra above



Question 2: Is the observed spectrum on the spectrum analyser as expected?

____

____

4. Continue to raise the frequency of the function generator, and observe that at a particular frequency, aliasing just occurs in spectrum of the sampled signal i.e. the second copy of the signal spectrum just coincides with the first. Note down the input signal frequency at this point (This is the maximum signal frequency for the sampling frequency of 5 kHz):

$f_{m\ (max)}$: ____________________________

Question 3: How does your observation relate to Sampling Theorem I?

____

____

Experiment 6
Pulse Code Modulation (PCM)

Objective

To examine the operation of a PCM encoder and decoder.

Equipment

1 PCM Module 296F
2 Power Supplies
Oscilloscope
Function Generator

Important Notice

The PCM Module 296F MUST be connected as follows:

Red to +15 volts
Green to 0 volts (GND)
Black to -15 volts

Introduction

PCM is a very well known type of digital pulse modulation system which provides the basis for voice coding in Integrated Services Digital Network (ISDN). The PCM Module 296F is a useful learning module. It contains both a PCM encoder and a PCM decoder. Figure E6.1 shows the Module 296F.

Procedure

Important

Keep the Spectrum Analyser **unconnected** unless you are told otherwise.

PCM Encoding and Decoding of the 296F Module

1. Referring to Figure E6.1, set the 3 bit/4 bit switch to the 4 bit position. Set encoder's clock to SLOW. Plug the 296F Module to the power supply.

2. Observe that the output of the latch stops at a value for a long interval. Record this value.

   Value of the latch's output = ____________ .

What is the analogue voltage level represented by this latch output?

Analogue voltage level = ____________ .

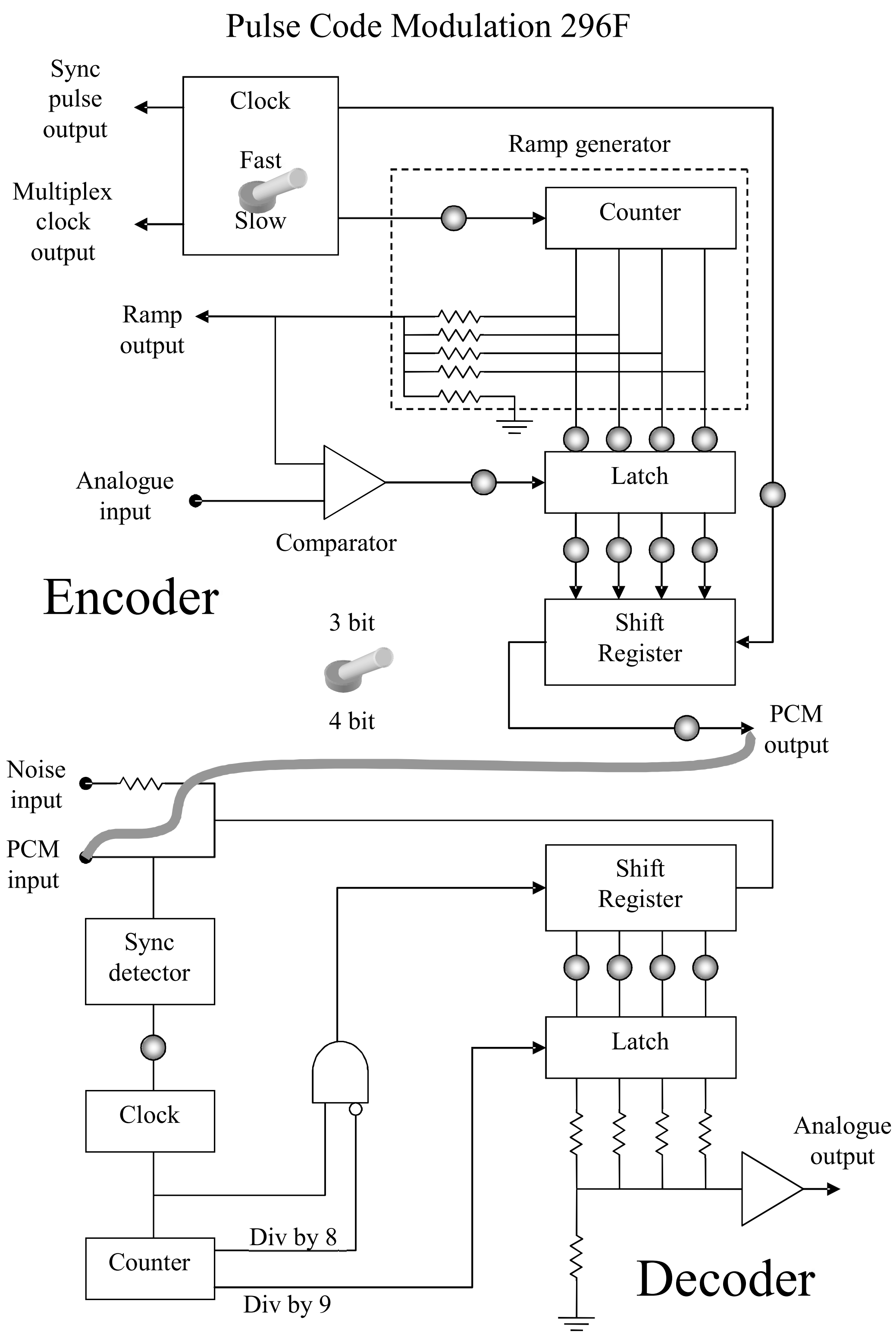


Figure E6.1 PCM Encoder and Decoder module (296F)

Note: The counter counts in binary numbers, and the four bits, which are displayed on the lamps above, on which, at successive counts, we can see in turn all the binary numbers from 0000 to 1111. The indicator lamps indicate a logic `1' state when lighted. The most significant bit is at the left of the counter.

3. Connect the `ramp output' to oscillosope CH1. Set encoder's clock to **FAST**. Press Autoset and use the time/div knob to observe 3 or 4 cycles of the ramp output signal. Sketch the ramp output signal as displayed on the oscilloscope, labelling its minimum and maximum voltage levels and the step size

4. Determine the following values for the 'ramp output'' signal.

   a) the sampling interval: ________________

   b) the sampling frequency: _________________

   c) the maximum input frequency that can be handled by the system: _________________

   d) Explain your answer in part c) above.

5. Note: A comparator is used to make a continuous comparison between the changing ramp voltage and that of the analogue signal. The latter is attenuated by a factor of 0.35 before it is input to the comparator i.e. 1 volt input appears as 0.35 volt to the input of comparator. When the comparator operates, it delivers a `1' output if the attenuated analogue input voltage is more positive than the ramp, and a `0' if the attenuated analogue signal is lower that the ramp. A `1' signal from the comparator causes the 'latch' circuit to transfer the data at its input to its output. Thus if the attenuated analogue signal exceeds the ramp signal each count of the counter appears at the output of the latch. If on the other hand the ramp signal exceeds the attenuated analogue signal, the counter outputs, although presented to the latch are not accepted by it and the latch output remains steady, until the count of zero is reached, when it is reset.

6. Set encoder clock to SLOW. Connect the function generator output to the analogue input of the DCS 296F module.

7. Generate the DC voltage shown in the table, observe and record the digital output at the latch for the DC input voltage below:

DC input voltage	Digital Output of latch
-1.4 V	
+1.6 V	

8. The binary number, represented by the four bits held in the latch output when the counter reaches the count of 15, is a digitally encoded equivalent of the analogue signal at some time during the current count.

9. The encoded number must be sent into the transmission system with a particular bit pattern so that it can be sorted out at the far end. The 296F Module uses the following bit pattern:

   0 0 **A B C D** 0 0 1 1 1 1 1 1 1 1

   where the four bits A, B, C, D, represent the encoded number being sent.

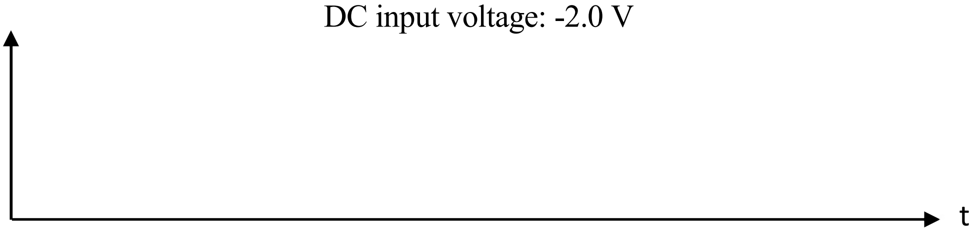
10. Set encoder's clock to **FAST**.

11. With the 'ramp output' signal connected to oscilloscope CH1, connect the PCM output to CH2. Connect the **`SYNC' pulse output** at the top left corner of the Module to Oscilloscope EXT input. Set the trigger source to EXT using the TRIG Menu.

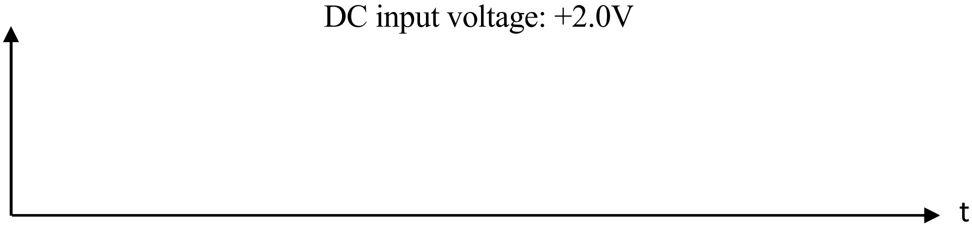
12. Adjust the DC voltage output from the function generator to –2 V and +2 V. For each case, observe the PCM output signal, then record the PCM transmitted bit and draw the signal displayed on the oscilloscope.

DC input voltage	PCM transmitted bit pattern
-2.0 V	
+2.0 V	

DC input voltage: -2.0 V



DC input voltage: +2.0V



13. **Reset the DC value to 0 V first and disable the "gate" button.** Change the function generator output to a 2 kHz sine wave with amplitude of 1.5 V_{pp} .

14. Connect the analogue input of the encoder to oscilloscope CH1.

15. Connect the `PCM output' of the encoder to the PCM input of the decoder.

16. Connect the analogue output of the decoder to oscilloscope CH2.

17. Press Autoset on the oscilloscope. Compare the original signal (CH1) and the reproduced signal at the end of the whole PCM communication chain (CH2).

    Are they similar? ________________

18. Using the spectrum analyser, observe and sketch the spectrum of the reproduced signal (up to a frequency range of 50 kHz), i.e. the analogue output of the **decoder**.

Important
Make sure that you have set the input signal voltage level as instructed before connect the signal to the spectrum analyser.

Engineering @ SP

The School of Electrical & Electronic Engineering at Singapore Polytechnic offers the following full-time courses.

1. Diploma in Aerospace Electronics (DASE)

The Diploma in Aerospace Electronics course aims to provide students with a broad-based engineering curriculum to effectively support a wide spectrum of aircraft maintenance repair and overhaul work in the aerospace industry and also to prepare them for further studies with advanced standing in local and overseas universities.

2. Diploma in Computer Engineering (DCPE)

This diploma aims to train technologists who can design, develop, setup and maintain computer systems; and develop software solutions. Students can choose to specialise in two areas of Computer Engineering & Infocomm Technology, which include Computer Applications, Smart City Technologies (IoT, Data Analytics), Cyber Security, and Cloud Computing.

3. Diploma in Electrical & Electronic Engineering (DEEE)

This diploma offers a full range of modules in the electrical and electronic engineering spectrum. Students can choose one of the six available specialisations (Biomedical, Communication, Microelectronics, Power, Rapid Transit Technology and Robotics & Control) for their final year.

4. Diploma in Engineering with Business (DEB)

Diploma in Engineering with Business provides students with the requisite knowledge and skills in engineering principles, technologies, and business fundamentals, supported by a strong grounding in mathematics and communication skills, which is greatly valued in the rapidly changing industrial and commercial environment.

5. Common Engineering Program (DCEP)

In Common Engineering Program, students will get a flavour of electrical, electronic and mechanical engineering in the first semester of their study. They will then choose one of the 7 engineering courses specially selected from the Schools of Electrical & Electronic Engineering and Mechanical & Aeronautical Engineering.

School of Electrical & Electronic Engineering

More than 60 Years of solid foundation	8 Tech Hubs	Unique PTN Scheme	SP-NUS SP-SUTD Programmes	More than 35,000+ Alumni

SP Singapore Polytechnic

Electives offered by SP | SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING

All SP students, including EEE students are free to choose electives offered by ANY SP schools, subject to meeting the eligibility criteria.

Like all schools, School of Electrical and Electronic Engineering offers electives for:

- EEE students only
- *and* for all SP students

EEE students are required to complete 3 electives, starting from Year 2 to Year 3 (one elective per semester).

Electives Choices for All SP students

Mod Code	Module Title
EP0400	Unmanned Aircraft Flying and Drone Technologies
EP0401	Python Programming for IoT*
EP0402	Fundamentals of IoT*
EP0403	Creating an IoT Project*
EP0404	AWS Academy Cloud Foundations
EP0405	AWS Academy Cloud Architecting
EP0406	Fundamentals of Intelligent Digital Solutions
EP0407	Technology to Business
EP0408	Cybersecurity Essentials
EP0409	Low Code 5G & AIoT

Certificate in IoT (Internet of Things)

* A certificate in IoT would be awarded if a student completes the 3 modules: EP0401, EP0402 and EP0403

Electives Choices for EEE students

Mod Code	Module Title
EM0400	Commercial Pilot Theory
EM0401	Autonomous Electric Vehicle Design
EM0402	Artificial Intelligence for Autonomous Vehicle
EM0403	Autonomous Mobile Robots
EM0404	Smart Sensors and Actuators
EM0405	Digital Manufacturing Technology
EM0406	Linux Essential
EM0407	Advanced Linux
EM0408	Linux System Administration
EM0409	Rapid Transit System
EM0410	Rapid Transit Signalling System
EM0412	Data Analytics
EM0413	Mobile App Development
EM0414	Client-Server App Development
EM0415	Machine Learning & Artificial Intelligence
EM0416	Solar Photovoltaic System Design
EM0418	Integrated Building Energy Management System
EM0419	Digital Solutioning Skills