Physical quantities

Learning outcomes

At the end of the lesson, students should be able to

- appreciate the importance of SI units and their standards
- perform unit conversion and dimensional analysis to prove homogeneity of a physical equation
- express answers in appropriate significant figures

Physical quantities

- Physics is the most fundamental physical science.
- It is the foundation of engineering and technology.
- It is based on experimental observations and measurements.
- Physical quantities are measurable quantities.
- Examples are the weight and height of a person.

Physical quantities

- If the length of a stick is 2.98 m, we mean it is 2.98 times the standard length defined as 1.00 m.
- Physical quantities must be quoted with values and units.
- E.g. the mass of a stone is 50.0 kg, the distance is 100 m.
- The Systeme International (French) or SI units is used globally by scientists and engineers.

Advantage of the SI system

- Prefixes can be used with any unit, e.g.
  - 1 nanosecond (ns) = 10^{-9} s
  - 1 millimetre (mm) = 10^{-3} m
  - 1 kilometre (km) = 10^{3} m
- Very large or very small numbers can be expressed in scientific notations, e.g.
  - Radius of Earth = 6.4×10^{6} m
  - Radius of hydrogen atom = 5.29×10^{-11} m

Standard of time

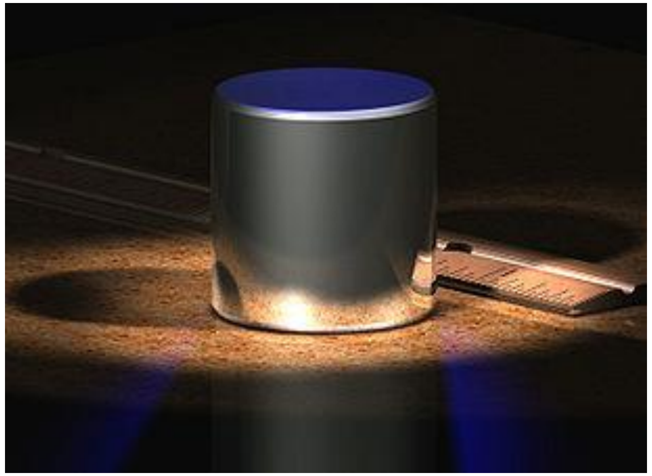
- The SI unit of time is the second (s).
- This is defined as 9,192,631,770 times the period of vibration of the radiation from the cesium-133 atom.
- Based on this standard, one day is 8.6×10^4 s.

Standard of length

- The SI unit of length is the meter (m).
- This is defined as the distance travelled by light in vacuum in 1/299,792,458 s.
- This standard was chosen because the speed of light in vacuum is the same everywhere and every time.
- Based on this standard, the distance from our Sun to the nearest star, Proxima Centauri, is 4×10^{16} m or 4.4 light-years (ly).

Standard of mass

- The SI unit of mass is the kilogram (kg).
- This is defined as the mass of a specific platinum-iridium alloy cylinder kept at the international Bureau of Weights and Measures at Sevres, France.
- Based on this standard, the mass of an electron is 9.11×10^{-31} kg.



Base physical quantities

- There are seven SI physical base quantities, each with a symbol and a base unit.

Base quantity	SI unit	Symbol
mass	kilogram	kg
length	metre	m
time	second	s
current	ampere	A
temperature	Kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Derived quantities and units

- Derived quantities are products or quotients of base quantities, e.g.
  - Area is length multiply with length.
  - Speed (distance/time) is length divide with time.
  - Density (mass/volume) is mass divide with length^3.
- Each derived quantity has a derived unit which is the product or quotient of base units.
- The derived units of the above derived quantities are m^2, m/s and kg/m^3.

Special names of derived units

- Some derived units have special names.
- Special names are in small letters.
- Symbols for units are single capital letter or start with capital letter.

Quantity	SI unit	Special name	Symbol
Force	kg m/s^2	newton	N
Work	kg m^2/s^2	joule	J
Pressure	N/m^2	pascal	Pa
Power	J/s	watt	W

Prefixes and scientific notations

- We can represent big numbers and small numbers using scientific notation.
- For example: Radius of Earth = 6.4×10^6 m
- However, we can also replace the 10^6 with prefixes.
- For example, Radius of Earth = 6.4 Mm (mega-metre)

Prefixes and scientific notations

- Here is a table of commonly used prefixes:

Multiplication factor	Prefix	Abbreviation
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Unit conversions

- It is important to learn how to convert from one unit to another unit in physics calculations.

- It is recommended to use SI units when we perform calculations using physics formula, so that our answers will be in SI units.

- Examples of unit conversions:
  - 1 km = 1000 m
  - 2 ms = 0.002 s
  - 1 g = 0.001 kg
  - 16 MB = 16×10^6 bytes

More unit conversions

- 1 minute in s =
- 1 hour in minutes =
- 1 kg in g =
- 1 day in hours =
- 1 mg in kg =
- 100 ns in s =
- 50 hours in days =

More unit conversions

- Complex examples on unit conversions
  - Convert 1 m^2 into cm^2
  - Convert 1 m^3 to cm^3
  - Convert 36 km/h into m/s
  - Convert 1 g/cm^3 into kg/m^3

Dimensions of base quantities

- Each base quantity has an associated dimension.
- Dimensions of a unit is independent of the system of units used.
- E.g. the dimension of length is [L], whether it is in inches or metres.

Base Quantity	SI Base Unit	Dimension
mass	kg	[M]
length	m	[L]
time	s	[T]
current	A	[I]
temperature	K	[θ]
amount of substance	mol	[N]
luminous intensity	cd	[J]

Dimensions of derived quantities

- The dimension of a derived quantity is the product or quotient of base quantities dimensions.

Derived quantity	Derived SI unit	Derived Dimension
volume	m^3	$[L]^3$
velocity	m/s	[L]/[T]
force	kg m/s^2	$[M][L]/[T]^2$
work	kg m^2/s^2	$[M][L]^2/[T]^2$
specific heat capacity	m^2/s^2 K	$[L]^2/[T]^2[\theta]$

Dimensionless quantities

- Dimensionless quantities are ratios of physical quantities with the same dimensions.
- Examples are:
  - relative density
  - relative atomic mass
  - efficiency
  - angle in radians
  - relative humidity

Homogeneous vs physically correct equations

- When an equation is homogeneous, the left side and right side of the equation have the same dimension.
- A homogeneous (or dimensionally correct) equation may not be physically correct.
- For example, the kinematic equation $v = u + \frac{1}{2}at$ is homogeneous because both sides of the equal sign have the same dimension but it is physically wrong.

Uses of dimension analysis

- Dimension analysis is used to
  - determine the unit of a physical quantity.
  - check if a relationship or equation is incorrect.
  - predict relationships between physical quantities.

Example 1

If k is dimensionless, m is mass, l is length, g is acceleration due to gravity and T is period, is the equation $T = k\sqrt{\frac{ml}{g}}$ correct?

The dimension of period T is [T].

The dimension of $k\sqrt{\frac{ml}{g}}$ is $\sqrt{\frac{[M][L]}{[L]/[T]^2}} = [T][M]^{1/2}$.

Hence the equation is not homogeneous or dimensionally inconsistent.

Example 2

Prove whether $v^2 - u^2 = 2as$ is homogeneous, where u and v are velocities, a is acceleration and s is displacement.

The dimension of $v^2 - u^2$ is

$$\frac{[L]^2}{[T]^2} - \frac{[L]^2}{[T]^2} = \frac{[L]^2}{[T]^2}$$

The dimension of $2as$ is

$$\frac{[L]}{[T]^2} \times [L] = \frac{[L]^2}{[T]^2}$$

Both sides have the same dimension and the equation is homogeneous.

Note that the number 2 does **not** contribute any dimension to the term $2as$.

If this number is missing, the equation is dimensionally correct but physically wrong.

Example 3

Show that k is dimensionless in the equation $F = k\rho r^2 v^2$ where F is force, v is velocity, r is radius and ρ is density.

The dimension of F is

$$\frac{[M][L]}{[T]^2}$$

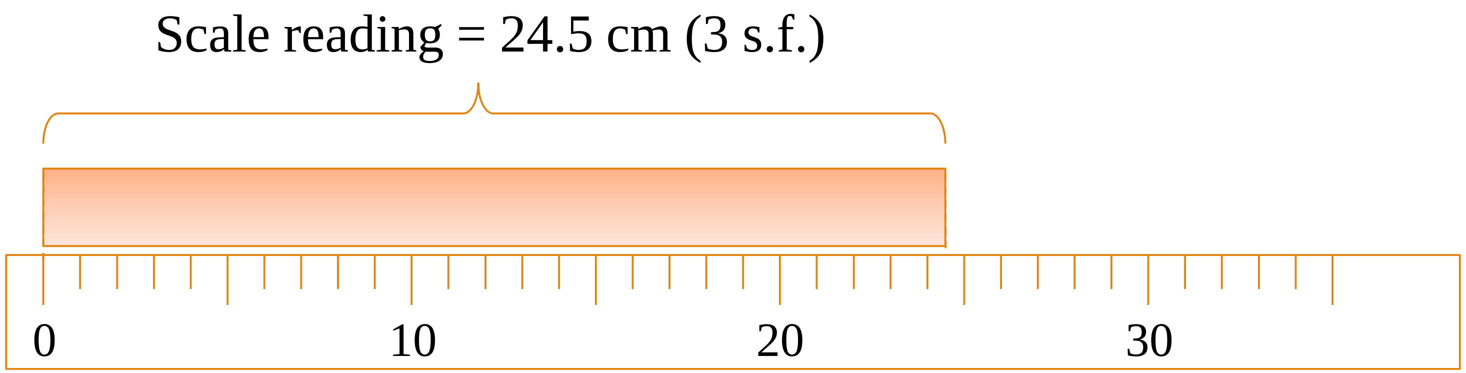
The dimension of $\rho r^2 v^2$ is

$$\frac{[M]}{[L]^3} \times [L]^2 \times \frac{[L]^2}{[T]^2} = \frac{[M][L]}{[T]^2}$$

Therefore k must be dimensionless.

Scale reading

- A scale reading is the process of comparing a physical quantity against a scale.
- We usually quote readings up to half the smallest division in the scale.
- In the example below, the length of the rod is read as 24.5 cm because the smallest scale is 1.00 cm.



OFFICIAL (CLOSED), NON-SENSITIVE

Accuracy vs precision

- Precision of an instrument refers to the limit of its sensitivity.
- E.g.
  - meter rule : precision is up to 0.1 cm
  - micrometer screw-gauge : precision is up to 0.001 cm.
- Accuracy refers to how close the measured quantity is to the “true” value.

Uncertainty

- Measurements always have uncertainties because of
  - limitations of the person taking the measurement.
  - the instruments used.
  - the methods used.
- Example:
  - The smallest measurement a ruler can read is 1 mm while the smallest reading for a Vernier calliper is 0.1 mm. So, these two instruments have different uncertainties.
- Another meaning of uncertainty is the maximum likely difference between the measured value and the true value.

Expressing uncertainty

- Absolute uncertainty of R is $\pm\Delta R$.
- Fractional uncertainty of R is $\pm\frac{\Delta R}{R}$.
- Percentage uncertainty of R is $\pm\frac{\Delta R}{R}\times 100\%$.
- We should always measure small quantities with high precision.

Significant figures (s.f.)

- The uncertainty in a measurement is indicated by its number of significant figures.
- E.g. if we measure the thickness of a book as 2.91 mm, the first two digits are certain while the third digit has an uncertainty of about 0.01 mm depending on the instrument we use.
- The result of calculations must also be written with an appropriate number of significant figures.
- E.g. $2.0 \times 11.0 = 22$ (2 s.f.) and not 22.0 (3 s.f)

Identifying significant figures

- In whole numbers with trailing zeroes, the zeroes may or may not be significant.
  - The number 500 may have 1, 2 or 3 significant figures depending on the context
- In such cases, use the scientific notations.
  - 500 written as 5×10^2 implies 1 s.f.
  - 500 written as 5.0×10^2 implies 2 s.f.
  - 500 written as 5.00×10^2 implies 3 s.f.

Identifying significant figures

- Zeroes before first non zero digit are not significant.
  - 0.0023 has 2 s.f.
- Zeroes within numbers are significant.
  - 0.0203 has 3 s.f.
- Zeroes after the decimal point are significant.
  - 2705.40 has 6 s.f.
  - 1.00 has 3 s.f.

Significant figures in answers

- The answer obtained from mathematical operations on numbers cannot be more precise than the numbers used in the operation.
- There are rules on the number of s.f. in the answer arising from
  - multiplication
  - division
  - addition
  - subtraction

Rules on multiplication and division

- The number of significant figures in the answer follows the quantity with the least significant figures.

- Examples:
  - Multiplication: 2.4 (2s.f.) $\times$ 3.65 (3 s.f.) $= 8.76 \approx 8.8$ (to 2 s.f.)
  - Division: 725.0 (4 s.f.) $\div$ 0.125 (3 s.f.) $= 5800 = 5.80 \times 10^3$ (to 3 s.f.)

Rules on addition and subtraction

- The answer has the same number of decimal places as the quantity with the **least** number of decimal places.
- Examples:
  - Addition: 23.1 (1 d.p.) + 0.546 (3 d.p.) + 1.45 (2 d.p.) = 25.096 ≈ 25.1 (to 1 d.p.)
  - Subtraction: 1.002 (3 d.p.) – 0.998 (3 d.p.) = 0.004 (to 3 d.p.)

End of chapter