MESH/LOOP ANALYSIS

Objectives

- Analyse a given circuit using mesh/loop analysis method
- Write mesh/loop equations by inspection and solve for the unknown loop currents using Cramer's rule

Mesh Analysis

Mesh analysis is a standard procedure using matrix method in the handling of equations.

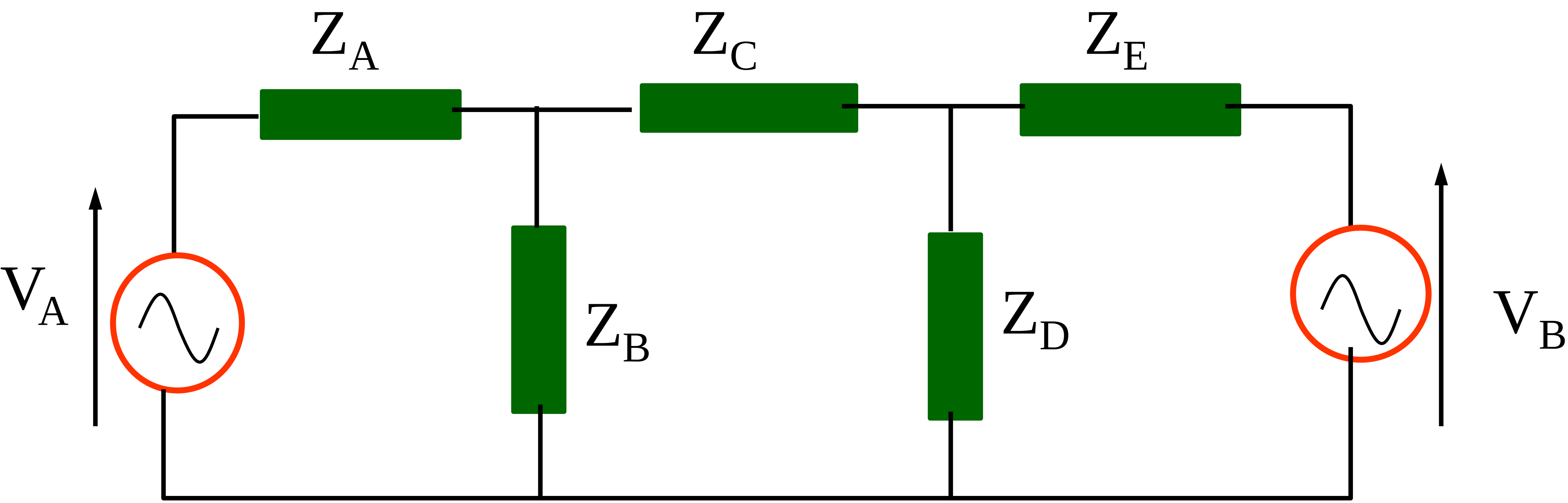
The tool is designed to simplify and speed up the task of writing the set of simultaneous equations required to solve various circuit problems.

Mesh Analysis

In the past, KVL was used to write circuit equations which may be quite tedious and not well organised.

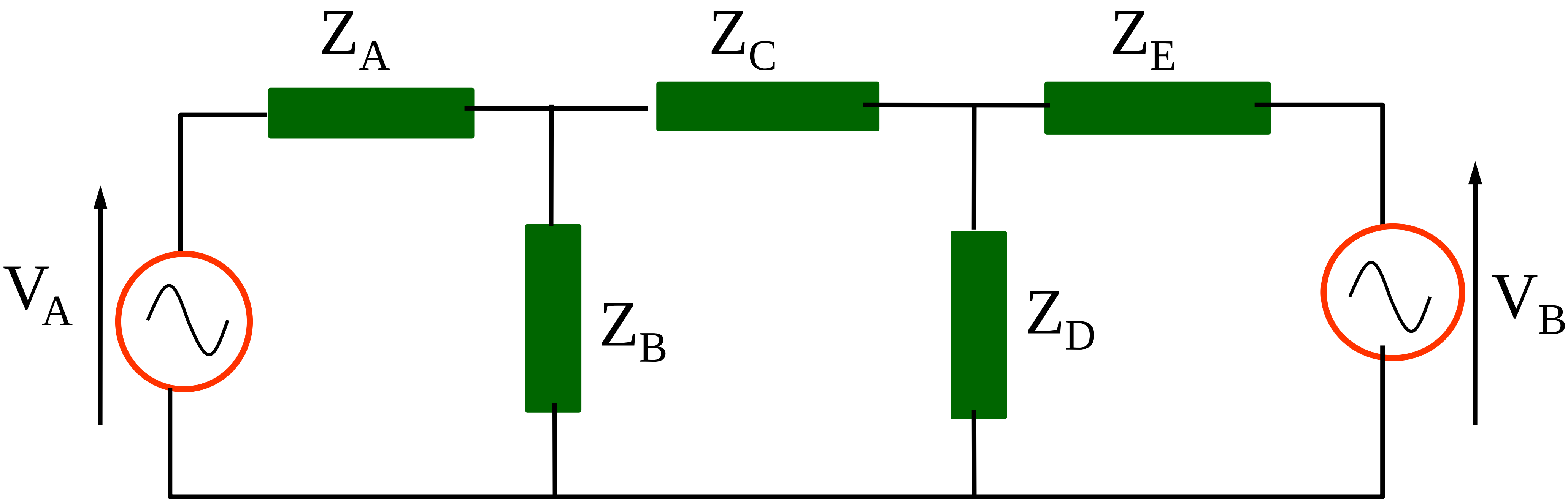
With Mesh/loop analysis, the same equations are written down, according to some established rules and regulations, directly into a matrix equation. From there, the solutions are ***found systematically*** using matrix methods.

Mesh/Loop Analysis



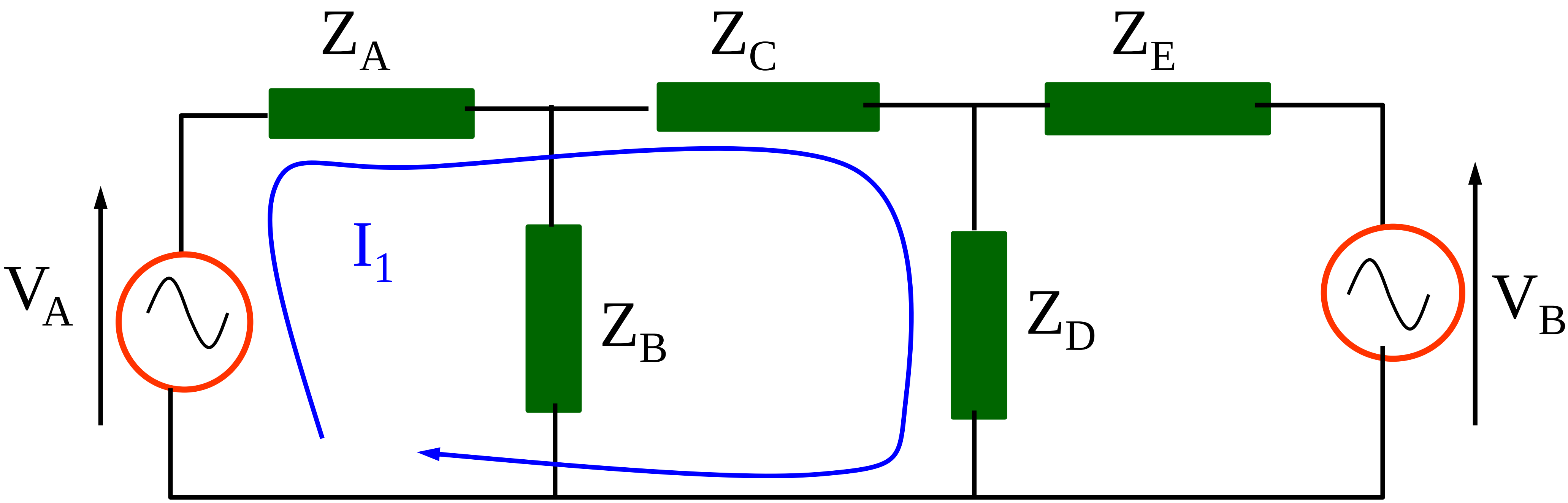
Given a circuit, first determine the number of independent loops, hence the number of independent equations by KVL

Mesh/Loop Analysis



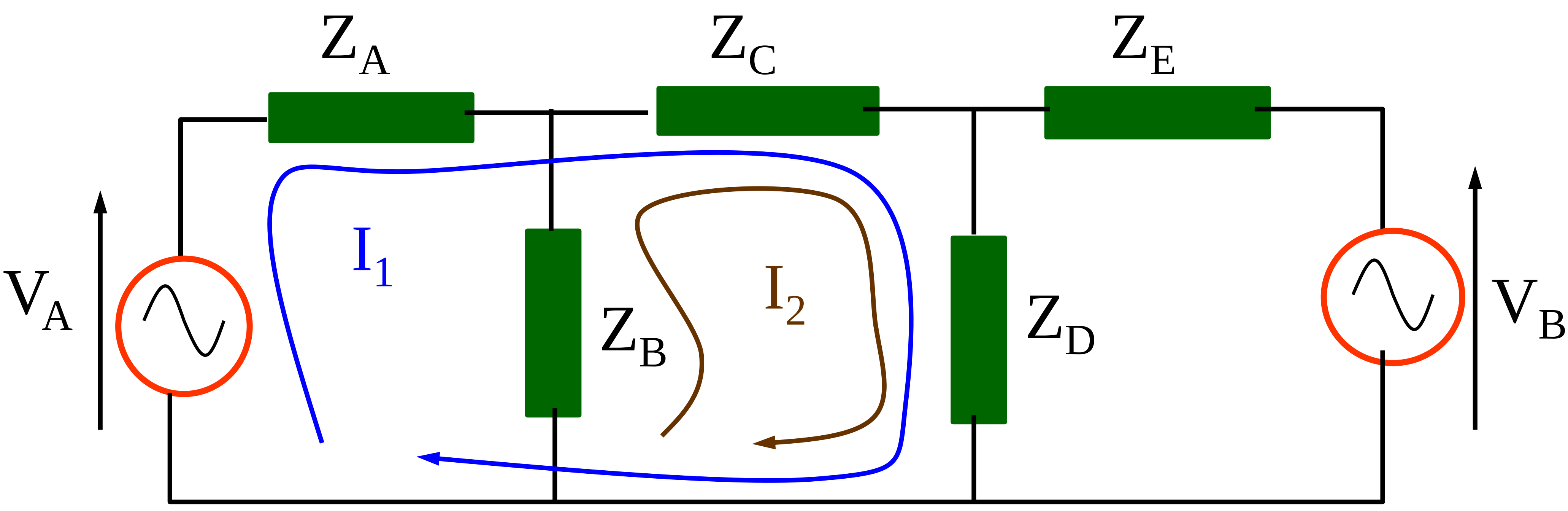
Select the loops. For this circuit, there are 3 loops and hence 3 loop equations are formed.

Mesh/Loop Analysis



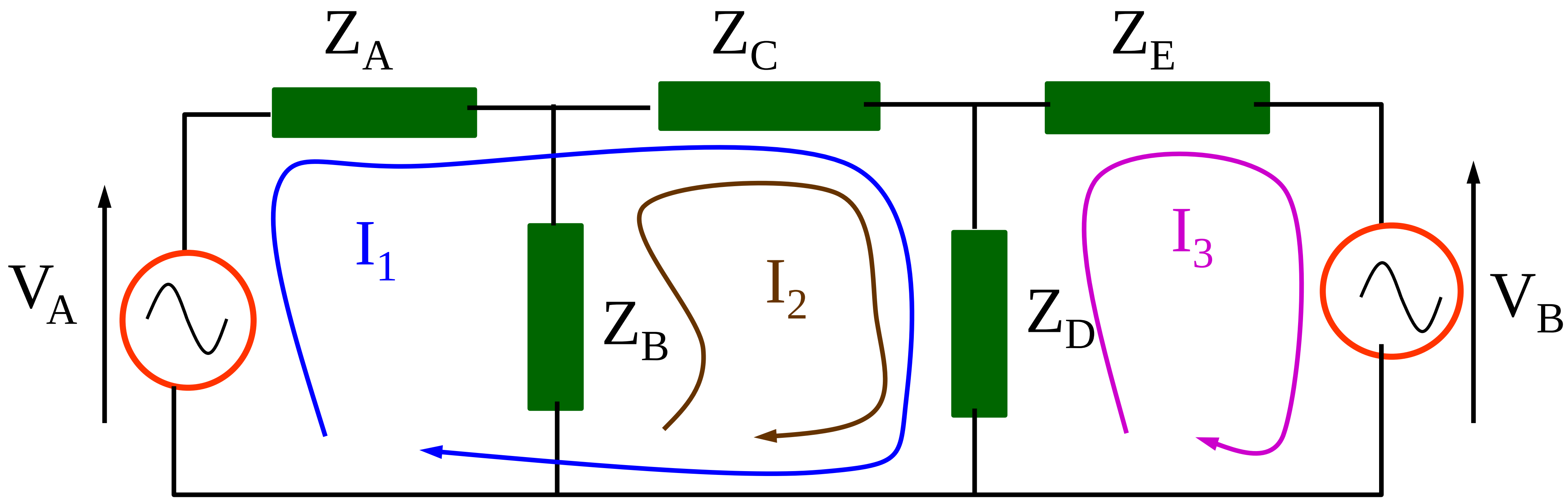
Select the loops. For this circuit, there are 3 loops and hence 3 loop currents I_1…..

Mesh/Loop Analysis



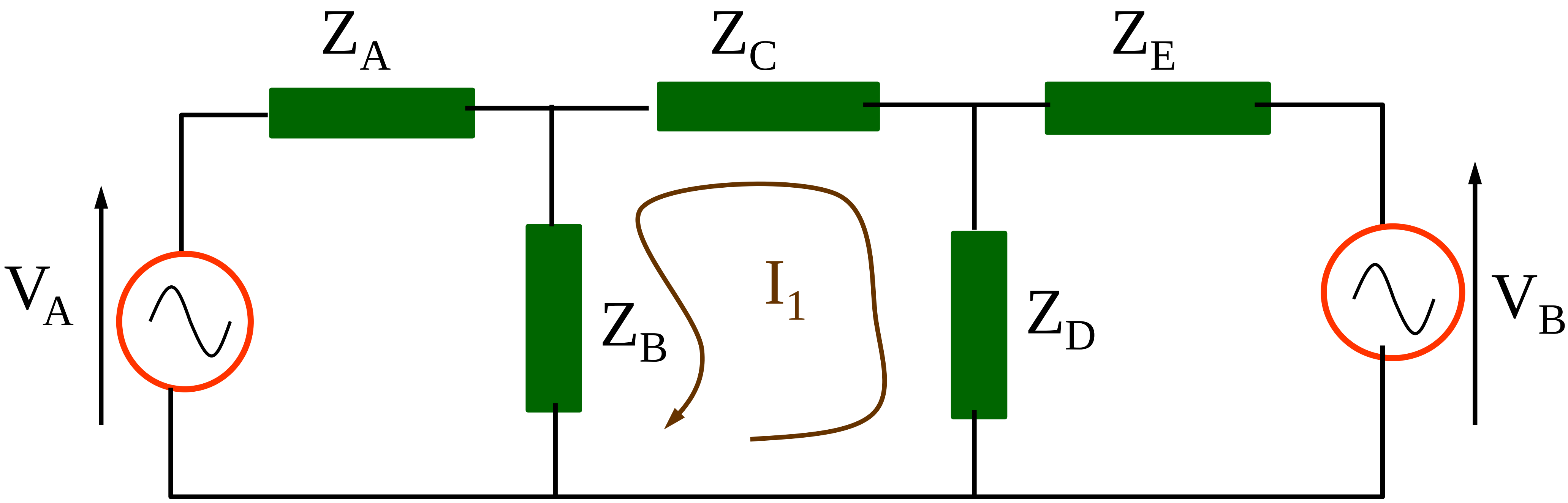
Select the loops. For this circuit, there are 3 loops and hence 3 loop currents I_1, I_2…

Mesh/Loop Analysis



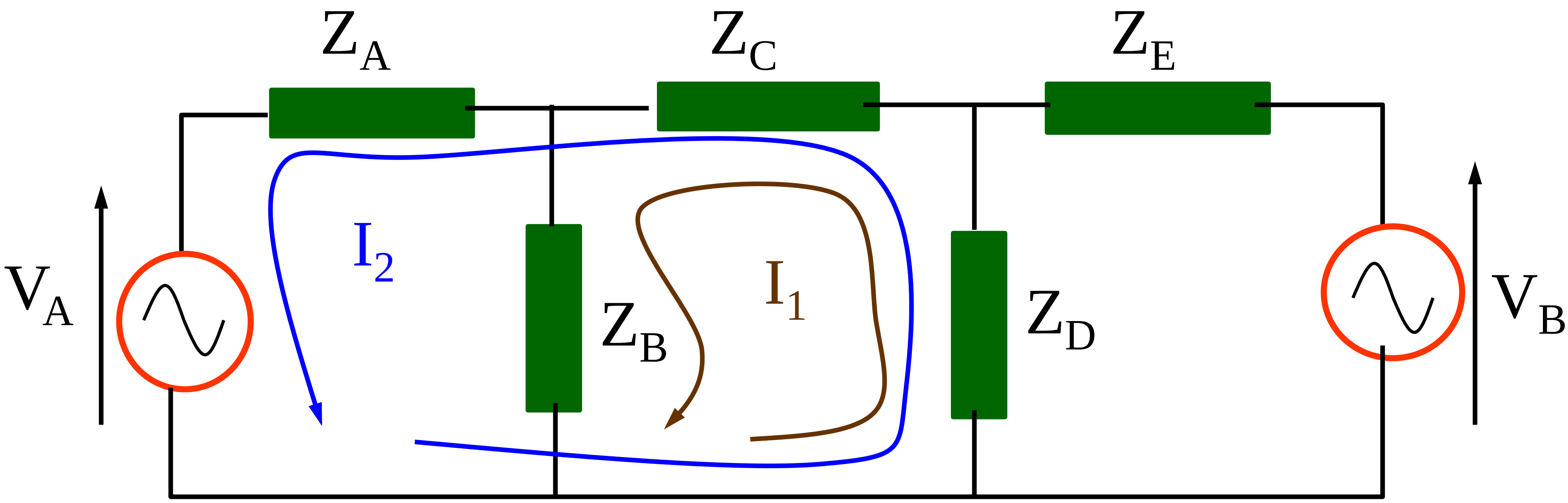
Select the loops. For this circuit, there are 3 loops and hence 3 loop currents I_1, I_2 and I_3.

Mesh/Loop Analysis



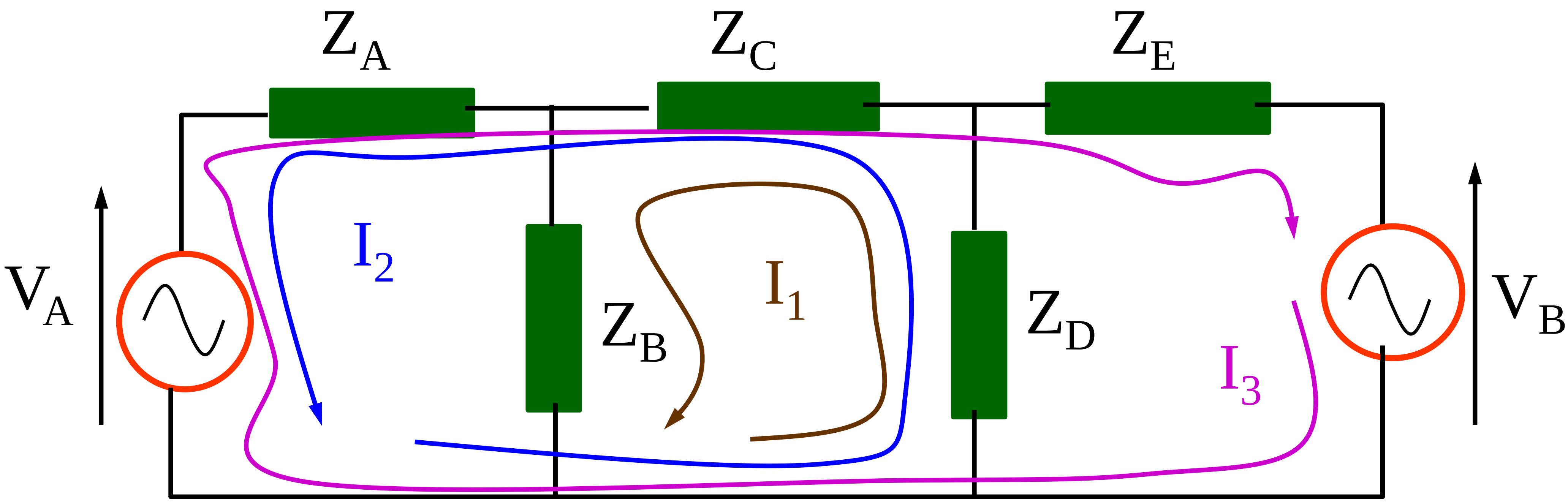
OR, you can select the loop currents I_1…

Mesh/Loop Analysis



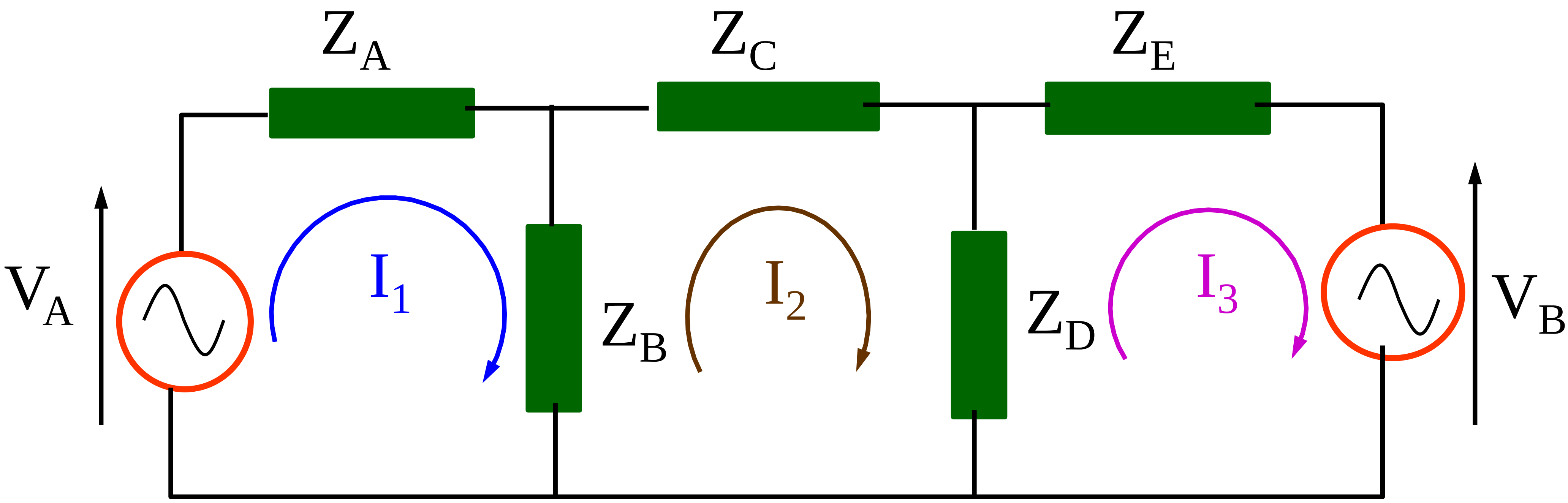
OR, you can select the loop currents I_1, I_2…

Mesh/Loop Analysis



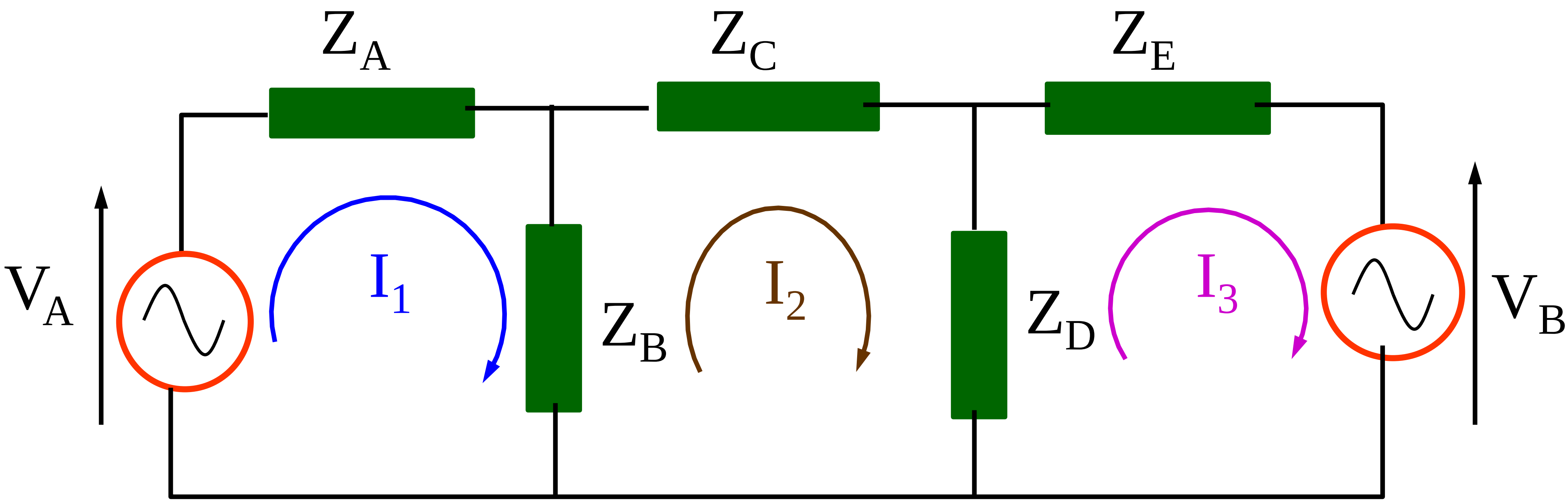
OR, you can select the loop currents I_1, I_2 and I_3

Mesh/Loop Analysis



Or you can select the mesh currents as above.

Mesh/Loop Analysis



The rules on the selection of mesh currents are:

* No. of mesh currents = No. of independent loops

* Each impedance must have at least one mesh current

Mesh/Loop Analysis

- **Mesh/loop analysis of an electric circuit is based on Kirchhoff's Voltage Law (KVL).**
- **KVL : Phasor sum of all the Voltages in a closed loop is zero. OR**
- **Phasor sum of voltage rises must be equal to the phasor sum of all the voltage drops in a closed loop.**

Mesh/Loop Analysis

Loop 1: $V_A = I_1 Z_A + (I_1 - I_2) Z_B$

Loop 2: $(I_2 - I_1) Z_B + I_2 Z_C + (I_2 - I_3) Z_D = 0$

Loop 3: $I_3 Z_E + (I_3 - I_2) Z_D + V_B = 0$

Rearranging the equations gives:-

Loop 1: $(Z_A + Z_B)\, I_1 - Z_B\, I_2 + 0\, I_3 = V_A$

Loop 2: $- Z_B\, I_1 + (Z_B + Z_C + Z_D)\, I_2 - Z_D\, I_3 = 0$

Loop 3: $0\, I_1 - Z_D\, I_2 + (Z_D + Z_E)\, I_3 = - V_B$

Mesh/Loop Analysis

Loop 1: $(Z_A + Z_B)\, I_1 - Z_B\, I_2 + 0\, I_3 = V_A$

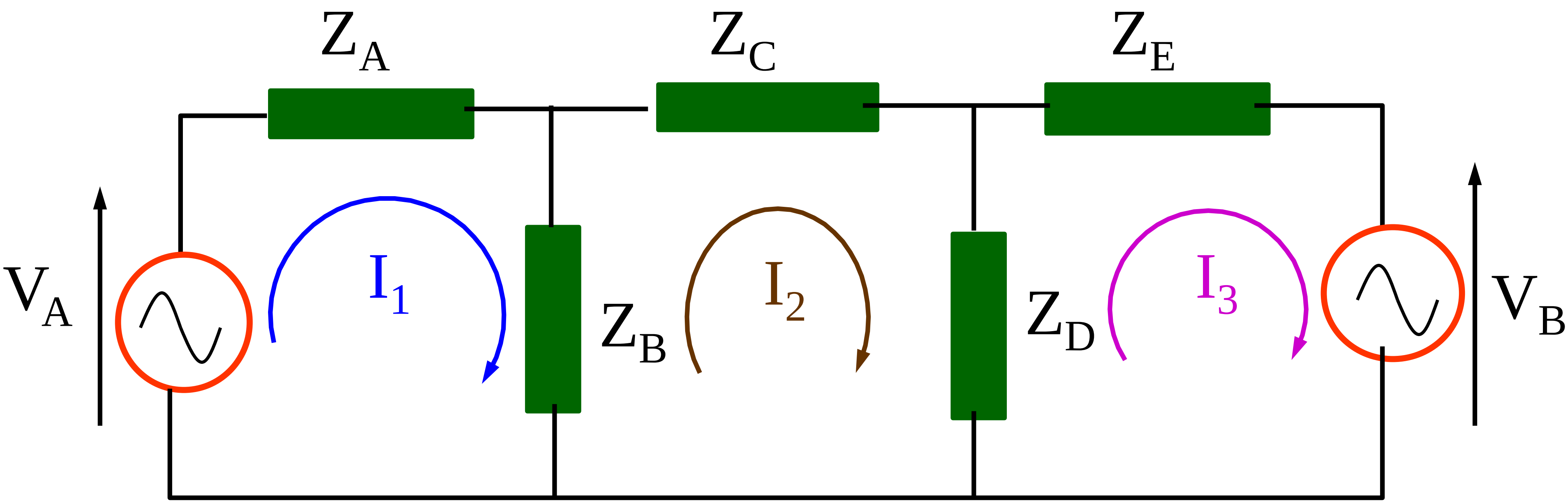
Loop 2: $- Z_B\, I_1 + (Z_B + Z_C + Z_D)\, I_2 - Z_D\, I_3 = 0$

Loop 3: $0\, I_1 - Z_D\, I_2 + (Z_D + Z_E)\, I_3 = - V_B$

Putting the equations in matrix form:

$$\begin{bmatrix} (Z_A + Z_B) & -Z_B & 0 \\ -Z_B & (Z_B + Z_C + Z_D) & -Z_D \\ 0 & -Z_D & (Z_D + Z_E) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_A \\ 0 \\ -V_B \end{bmatrix}$$

Mesh/Loop Analysis

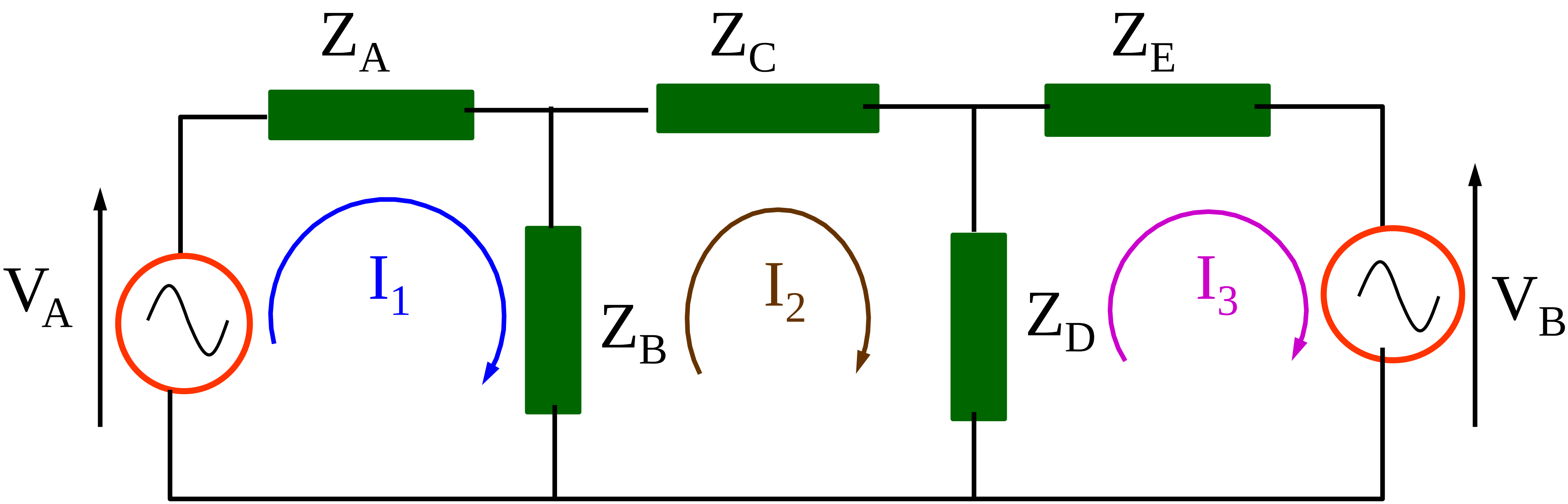


The general mesh matrix equation is therefore:

$$Z \times I = V$$

where Z is a 3 x 3 impedance matrix (for a 3 loops circuit), and I & V are 3 x 1 vectors

Mesh Analysis by Inspection



The purpose of mesh analysis is to be able to write this matrix equation Z x I = V by INSPECTION on the circuit without using KVL.

Mesh Analysis by Inspection

Let's look at the matrix again: Z x I = V

$$\begin{bmatrix} (Z_A + Z_B) & -Z_B & 0 \\ -Z_B & (Z_B + Z_C + Z_D) & -Z_D \\ 0 & -Z_D & (Z_D + Z_E) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_A \\ 0 \\ -V_B \end{bmatrix}$$

In general, the matrix Z x I = V can be expressed as:

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Mesh Analysis by Inspection

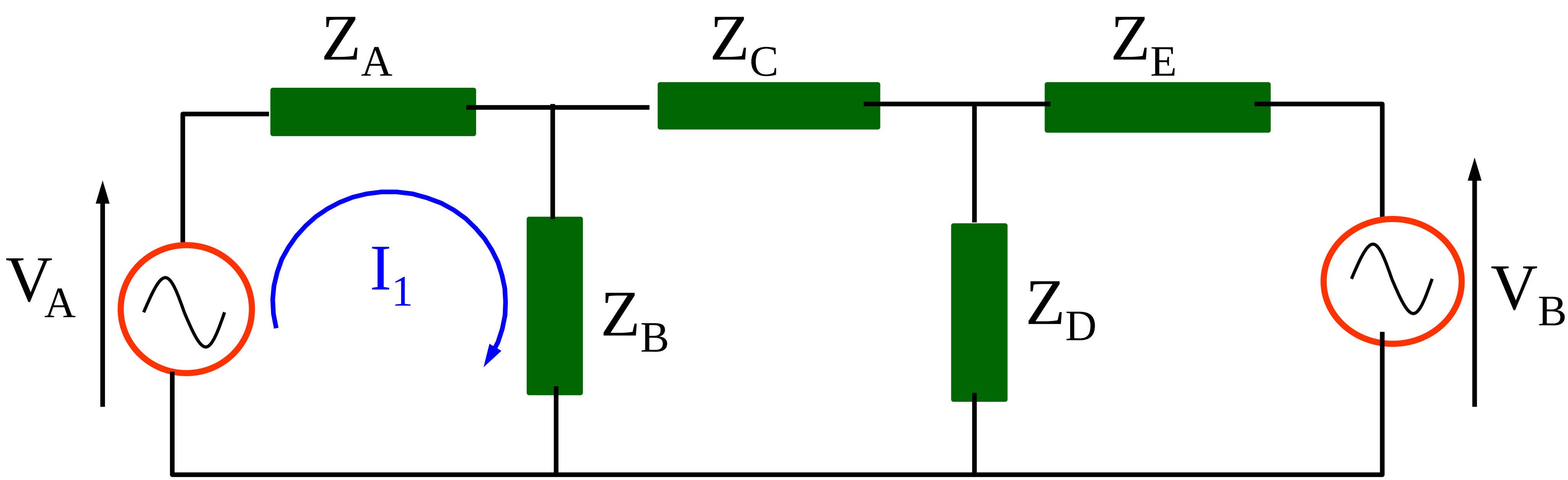
Here comes the regulations to follow in order to write the mesh matrix equation by inspection ***without using Kirchhoff's Voltage Law***

This is then followed by the use of Cramer's Rule to solve for the unknown mesh currents I_1, I_2 etc.

Mesh Analysis by Inspection

Self-impedances (those in the diagonal)

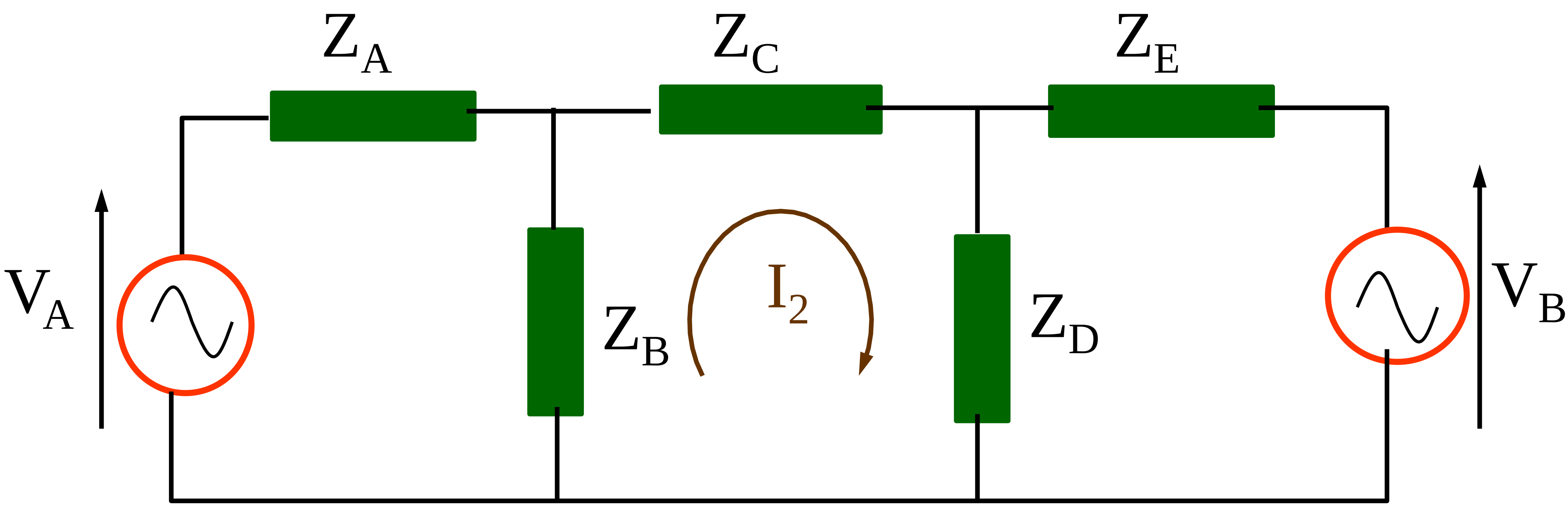
Z_{11} = sum of impedances flowed through by current I_1 = $(Z_A + Z_B)$



Mesh Analysis by Inspection

Self-impedances (those in the diagonal)

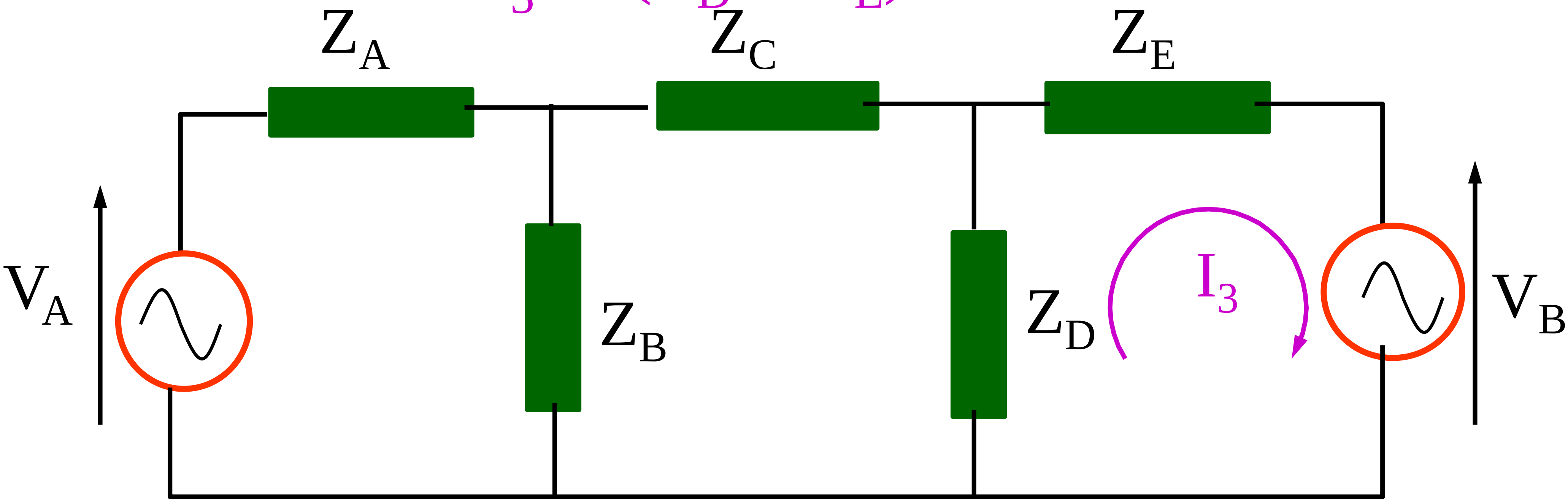
Z_{22} = sum of impedances flowed through by current I_2 = $(Z_B + Z_C + Z_D)$



Mesh Analysis by Inspection

Self-impedances (those in the diagonal)

Z_{33} = sum of impedances flowed through by current I_3 = $(Z_D + Z_E)$



Mesh Analysis by Inspection

Self-impedances (those in the diagonal)

$Z_{11} = (Z_A + Z_B)$ = sum of impedances flowed through by current I_1

$Z_{22} = (Z_B + Z_C + Z_D)$ = sum of impedances flowed through by current I_2

$Z_{33} = (Z_D + Z_E)$ = sum of impedances flowed through by current I_3

Mesh Analysis by Inspection

Mutual impedances (those off the diagonal)

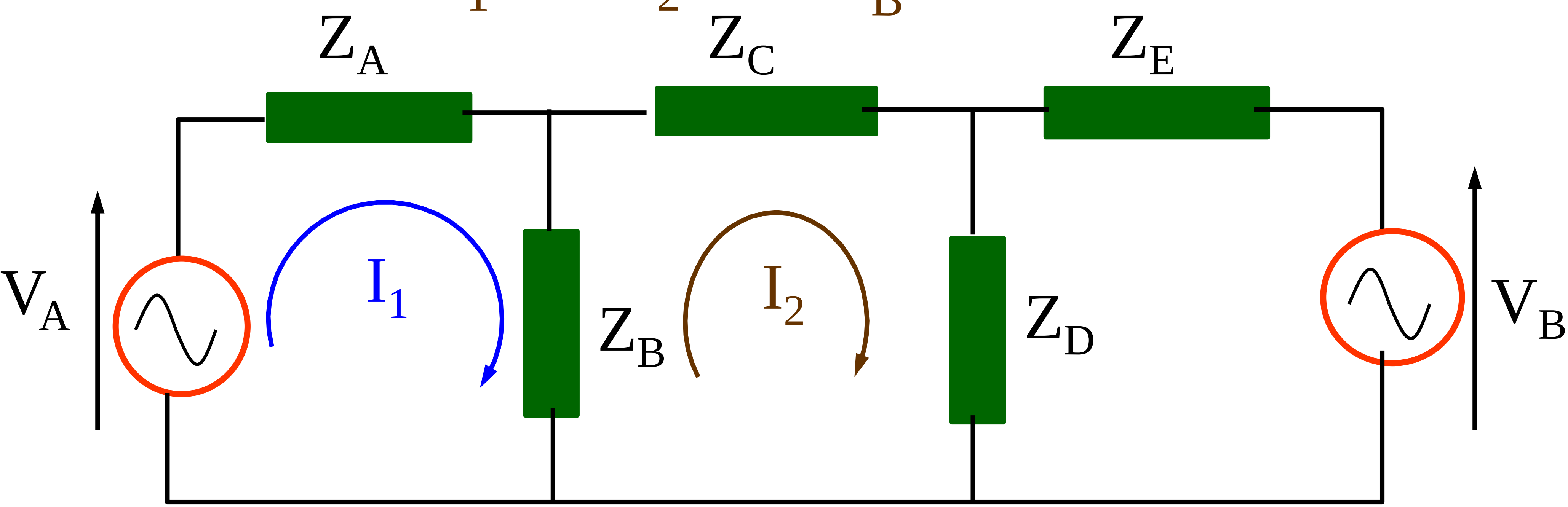
$Z_{12} = Z_{21}$ = sum of impedances flowed through by I_1 and I_2

Positive (+) sum when both currents flowed in the same direction through the impedances, otherwise negative (-).

Mesh Analysis by Inspection

Mutual impedances (those off the diagonal)

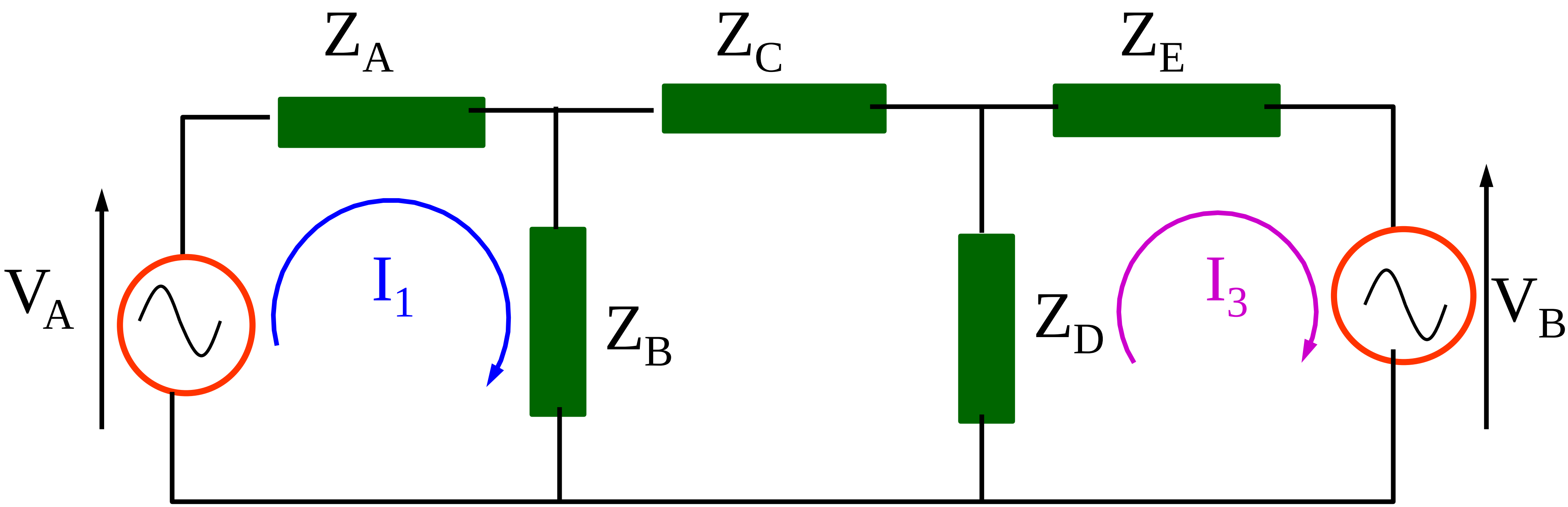
$Z_{12} = Z_{21}$ = sum of impedances flowed through by I_1 and I_2 = $-Z_B$



Mesh Analysis by Inspection

Mutual impedances (those off the diagonal)

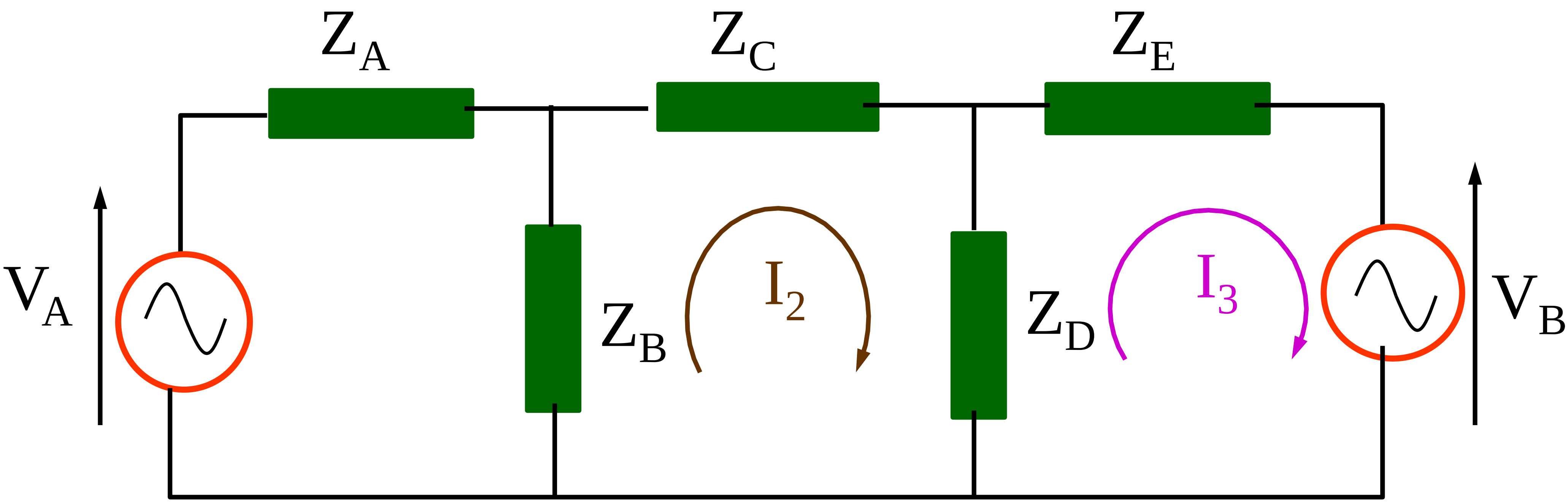
$Z_{13} = Z_{31}$ = sum of impedances flowed through by I_1 and $I_3 = 0$



Mesh Analysis by Inspection

Mutual impedances (those off the diagonal)

Z_{23} = Z_{32} = sum of impedances flowed through by I_2 and I_3



Mesh Analysis by Inspection

Mutual impedances (those off the diagonal)

$Z_{23} = Z_{32}$ = sum of impedances flowed through by I_2 and I_3 $= -Z_D$

Positive (+) sum when both currents flowed in the same direction through the impedances, otherwise negative (-).

Mesh Analysis by Inspection

Mutual impedances (those off the diagonal)

$Z_{12} = Z_{21} = -Z_B$ = sum of impedances flowed through by I_1 and I_2

$Z_{13} = Z_{31} = 0$ = sum of impedances flowed through by I_1 and I_3

$Z_{23} = Z_{32} = -Z_D$ = sum of impedances flowed through by I_2 and I_3

Positive (+) sum when both currents flowed in the same direction through the impedances, otherwise negative (-).

Mesh Analysis by Inspection

Voltage vector V

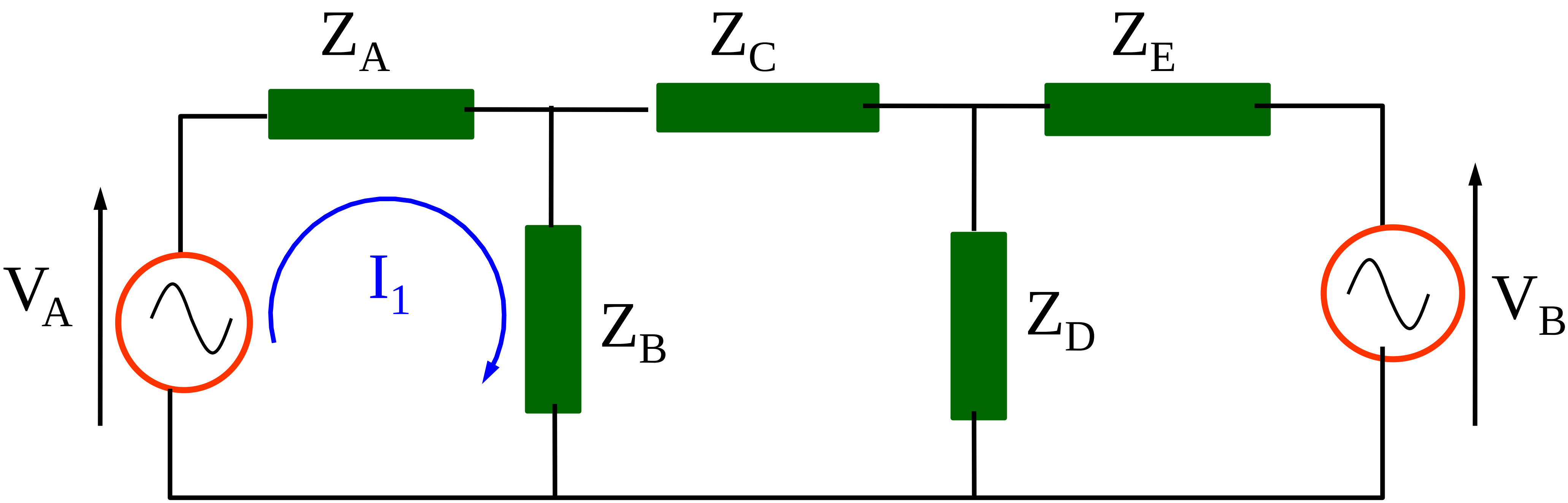
V_1 = sum of voltage sources in loop 1

A positive (+) sign should be applied on a voltage source in the same direction of the mesh current, and negative (-) sign for a voltage source in the opposite direction of the mesh current.

Mesh Analysis by Inspection

Voltage vector V

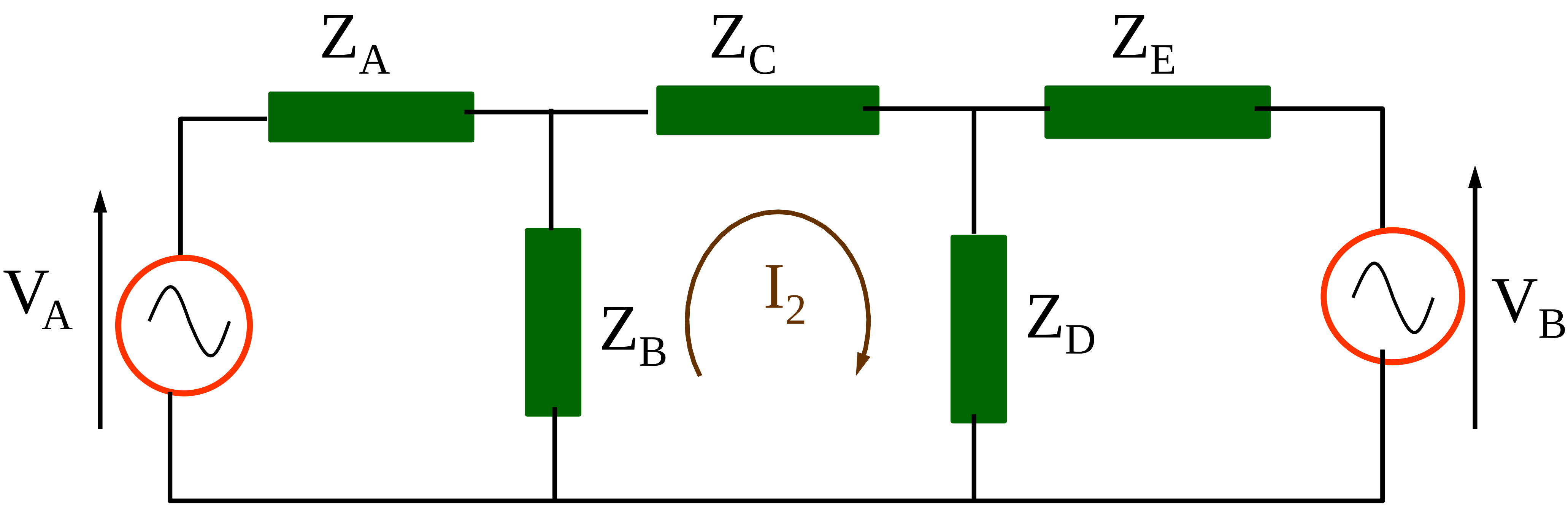
V_1 = sum of voltage sources in loop 1 = V_A



Mesh Analysis by Inspection

Voltage vector V

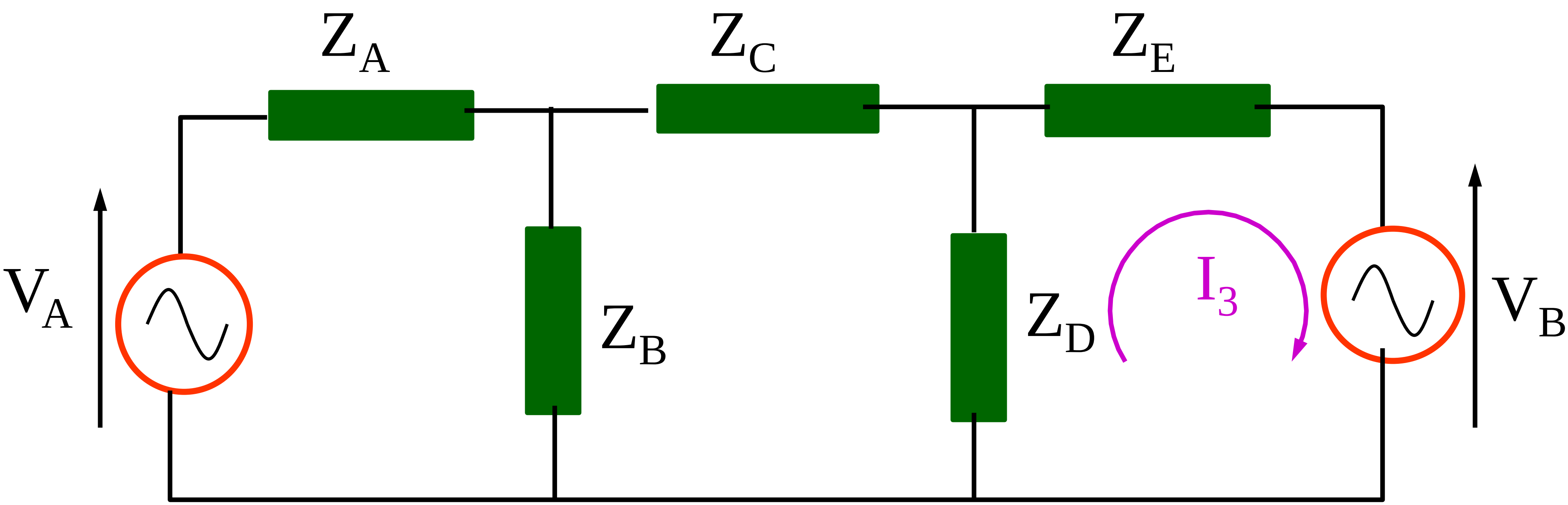
V_2 = sum of voltage sources in loop 2 = 0



Mesh Analysis by Inspection

Voltage vector V

V_3 = sum of voltage sources in loop 3



Mesh Analysis by Inspection

Voltage vector V

V_3 = sum of voltage sources in loop 3 $= - V_B$

A positive (+) sign should be applied on a voltage source in the same direction of the mesh current, and negative (-) sign for a voltage source in the opposite direction of the mesh current.

Mesh Analysis by Inspection

Voltage vector V

$V_1 = \ V_A \ =$ sum of voltage sources in loop 1

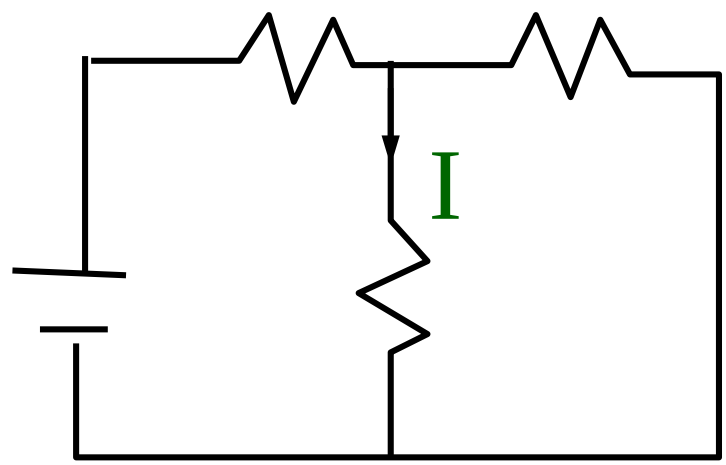
$V_2 = \quad 0 \quad =$ sum of voltage sources in loop 2

$V_3 = -\ V_B =$ sum of voltage sources in loop 3

A positive (+) sign should be applied on a voltage source in the same direction of the mesh current, and negative (-) sign for a voltage source in the opposite direction of the mesh current.

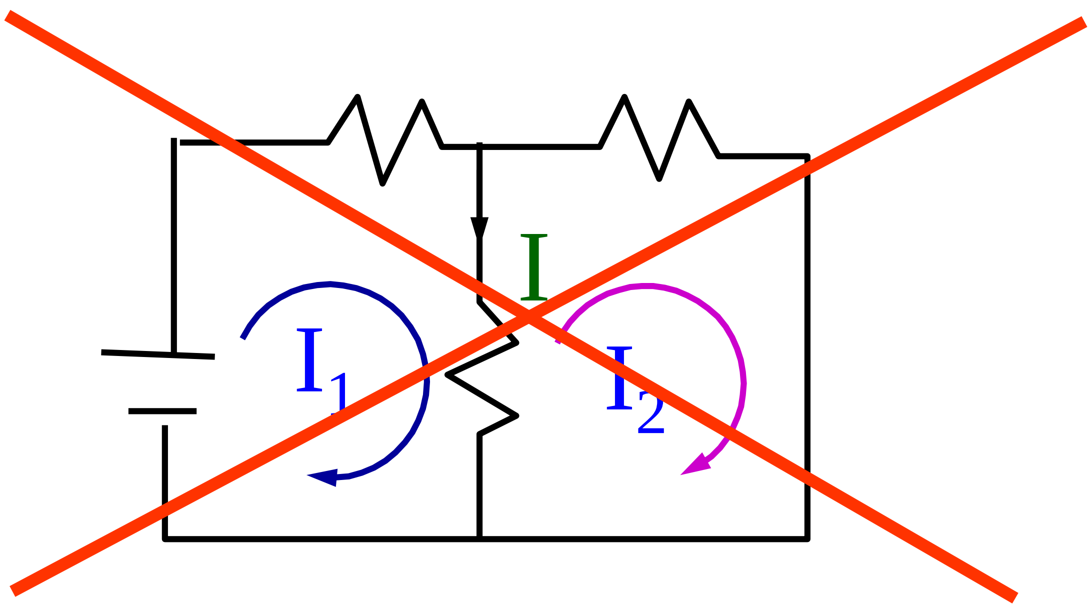
Wise choice of loop currents

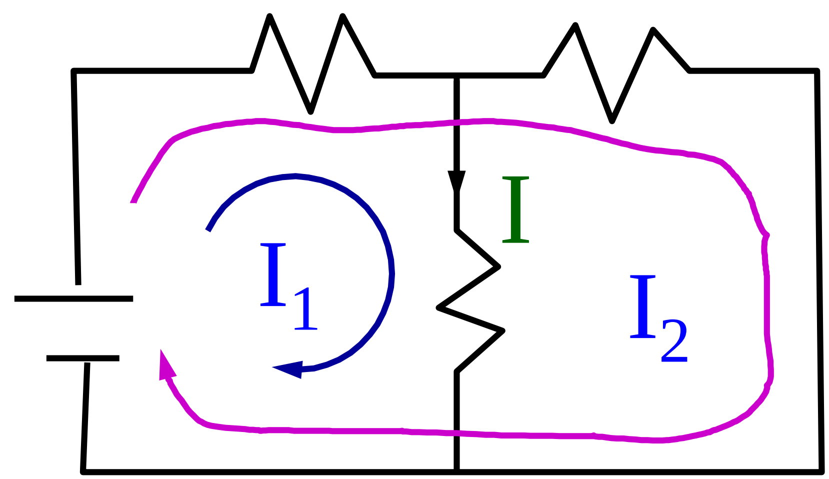
e.g. Find I in the following circuit using loop analysis.



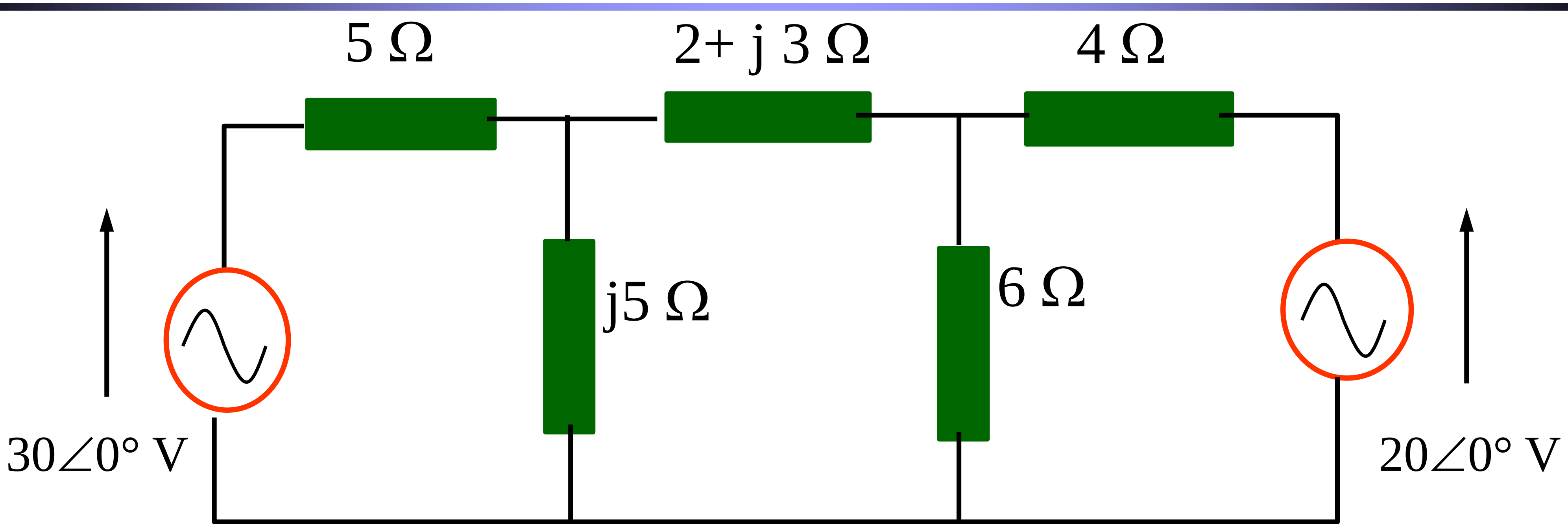
Wise choice of loop currents

Which one is the wiser choice of loop currents?





Example 1.3 (Different Loops)

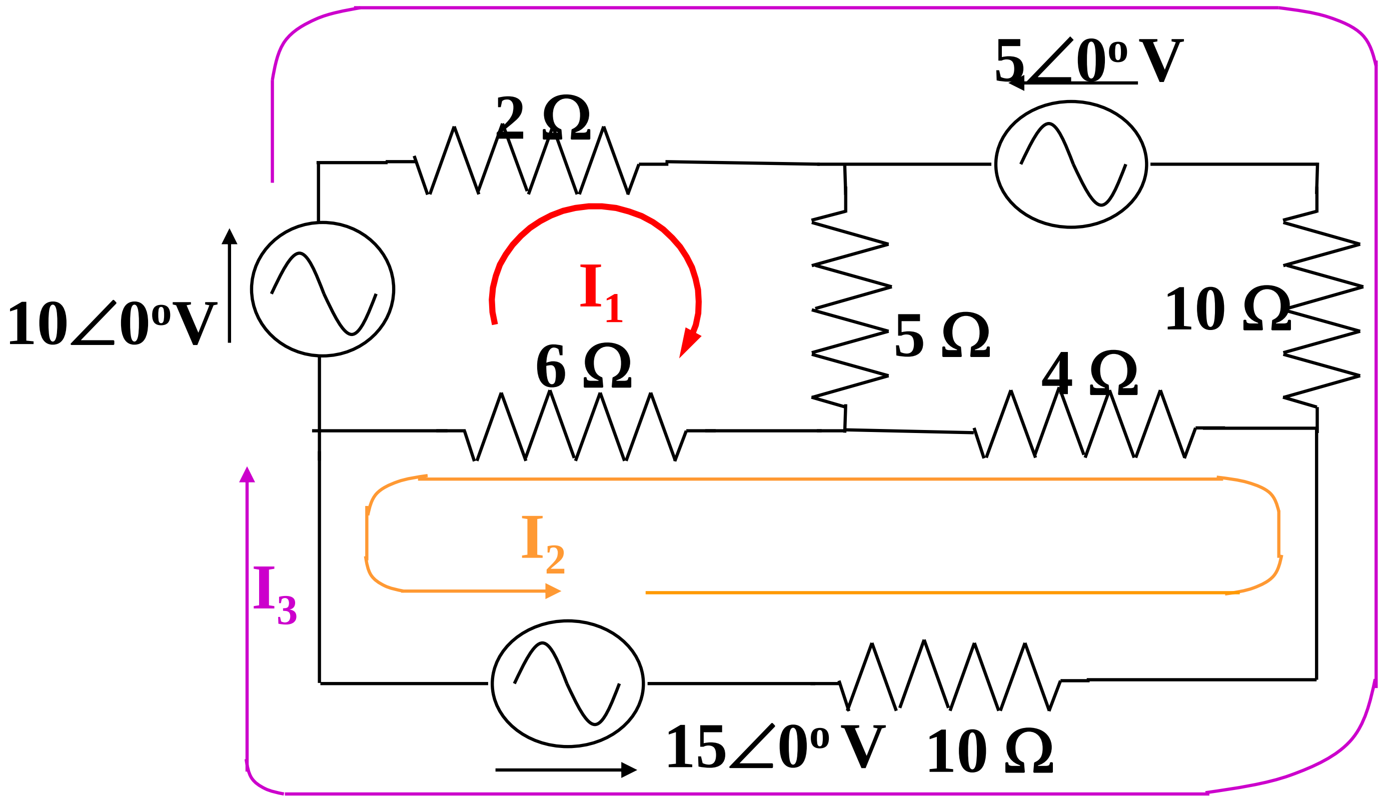


Example 1.3

Remarks:

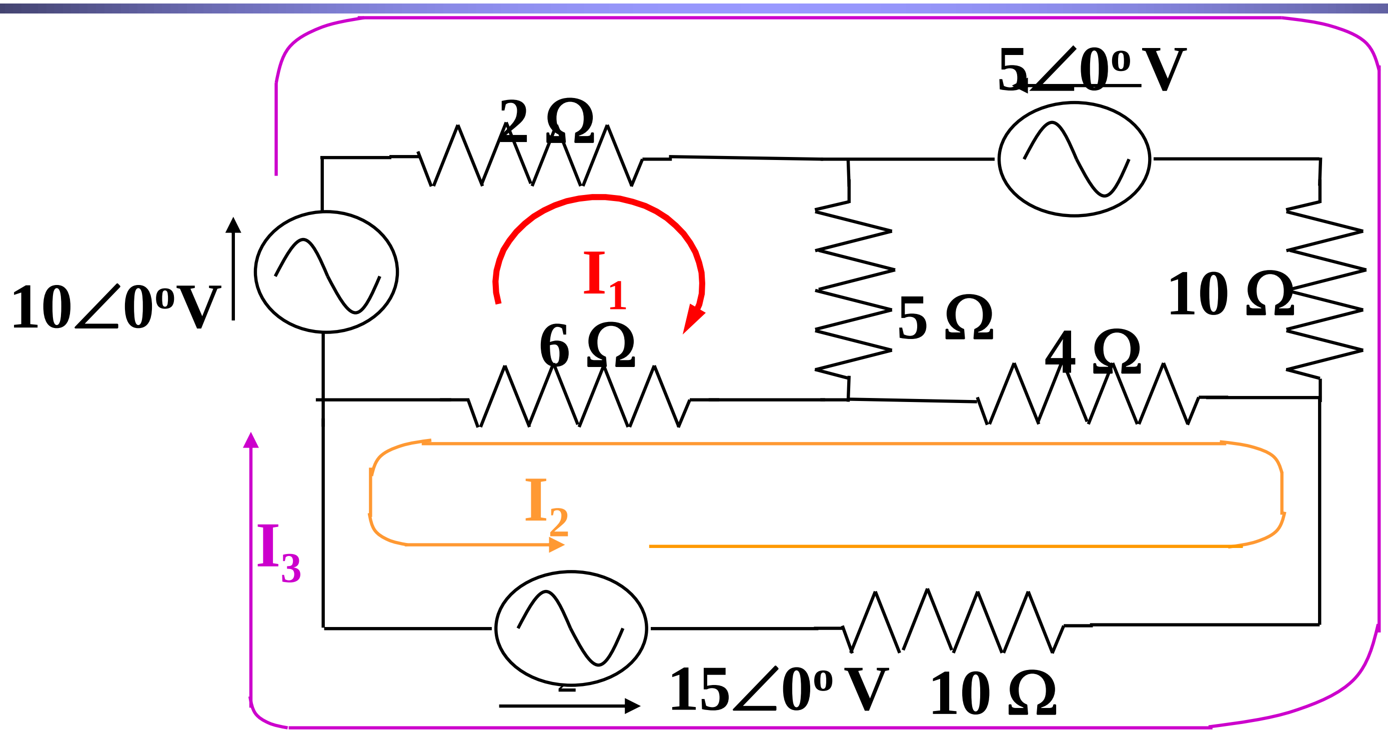
It does not matter which of the loop currents are used in the formation of equations as shown in the two circuits as the final results of the currents flowing in each branch will be the same.

Example 1.4



Determine the current in the 5 Ω branch, for the circuit shown using loop current analysis method.

Example 1.4



$$\begin{bmatrix} 13 & 6 & 2 \\ 6 & 20 & -10 \\ 2 & -10 & 22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 15\angle 0^\circ \\ 10\angle 0^\circ - 5\angle 0^\circ - 15\angle 0^\circ \end{bmatrix}$$

Example 1.4

$$\begin{bmatrix} 13 & 6 & 2 \\ 6 & 20 & -10 \\ 2 & -10 & 22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 15\angle 0^\circ \\ 10\angle 0^\circ - 5\angle 0^\circ - 15\angle 0^\circ \end{bmatrix}$$

Using Cramer's Rule, solve only I_1.
Simplify the above matrix.

$$\begin{bmatrix} 13 & 6 & 2 \\ 6 & 20 & -10 \\ 2 & -10 & 22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ -10 \end{bmatrix}$$

Example 1.4

$$I_1 = \frac{\Delta I_1}{\Delta} = \frac{\begin{vmatrix} 10 & 6 & 2 \\ 15 & 20 & -10 \\ -10 & -10 & 22 \end{vmatrix}}{\begin{vmatrix} 13 & 6 & 2 \\ 6 & 20 & -10 \\ 2 & -10 & 22 \end{vmatrix}}$$

Example 1.4

$$\begin{array}{cccc} & + & - & + \\ \Delta I_1 = & \multicolumn{3}{c}{\begin{vmatrix} 10 & 6 & 2 \\ 15 & 20 & -10 \\ -10 & -10 & 22 \end{vmatrix}} \end{array}$$

$$\Delta I_1 = 10\begin{vmatrix} 20 & -10 \\ -10 & 22 \end{vmatrix} - 6\begin{vmatrix} 15 & -10 \\ -10 & 22 \end{vmatrix} + 2\begin{vmatrix} 15 & 20 \\ -10 & -10 \end{vmatrix}$$

$$\begin{aligned} &= 10\ [(20 \times 22) - (-10 \times -10)] \\ &\quad - 6\ [(15 \times 22) - (-10 \times -10)] \\ &\quad + 2\ [(15 \times -10) - (20 \times -10)] \end{aligned}$$

Example 1.4

$$= 10[440 - 100] - 6[330 - 100] + 2[-150 + 200]$$

$$= 10(340) - 6(230) + 2(50)$$

$$= 3400 - 1380 + 100$$

$$= 2120$$

Example 1.4

$$\Delta = \begin{array}{c} \begin{matrix} + & \quad - \quad & + \end{matrix} \\ \begin{vmatrix} 13 & 6 & 2 \\ 6 & 20 & -10 \\ 2 & -10 & 22 \end{vmatrix} \end{array}$$

$$\Delta = 13\begin{vmatrix} 20 & -10 \\ -10 & 22 \end{vmatrix} - 6\begin{vmatrix} 6 & -10 \\ 2 & 22 \end{vmatrix} + 2\begin{vmatrix} 6 & 20 \\ 2 & -10 \end{vmatrix}$$

$$\begin{aligned} &= 13\,[(20 \times 22) - (-10 \times -10)] \\ &\quad - 6\,[(6 \times 22) - (-10 \times 2)] \\ &\quad + 2\,[(6 \times -10) - (20 \times 2)] \end{aligned}$$

Example 1.4

$$= 13[440 - 100] - 6[132 + 20] + 2[-60 - 40]$$

$$= 13(340) - 6(152) + 2(-100)$$

$$= 4420 - 912 - 200 = 3308$$

$$\boxed{\text{Hence } I_1 = \frac{2120}{3308} = 0.641\text{ A}}$$

Similarly using Cramer's Rule you can solve for I_2 and I_3.

Example 1.4

Similarly using Cramer's Rule you can solve for I_2 and I_3.

$$I_2 = \frac{\Delta I_2}{\Delta}$$

$$I_3 = \frac{\Delta I_3}{\Delta}$$

ΔI_2 and ΔI_3 and be found by replacing the second and third column in matrix [Δ] by the [V] column matrix respectively.

…next topic

Nodal Analysis

Nurturing Curious Minds, Producing Passionate Engineers

Lee M L
Office: T12A522
School of EEE
Tel: 6879-0657