Revision Tutorial
Answers

I. Partial Differentiation

1(a). $5x^4 + y + \dfrac{1}{x+2y}$, $x + \dfrac{2}{x+2y}$

1(b). $2e^{2x}\sin(y)$, $e^{2x}\cos(y)$

1(c). $2x\sin^2 y$, $x^2 \sin 2y$

1(d). $3x^2 + 10xy$, $5x^2 + 6y^2$

1(e). $2xy + 2y^2 - 2$, $x^2 + 4xy$

2(a). $y + 3\sin(3x)$, $9e^{9y} + x$

2(b). $3\sqrt{x^2+5y^2} + \dfrac{3x^2}{\sqrt{x^2+5y^2}}$, $\dfrac{15xy}{\sqrt{x^2+5y^2}}$

2(c). $3y$, $3x + \dfrac{1}{y} + 4y^3$

3. $\dfrac{1}{2}$, $\cos\left(\dfrac{1}{2}\right) \approx 0.878$

4. (a) $\dfrac{3}{5}$ (b) $-\dfrac{1}{3}$

5. $10t^4 - 8t$

6. 11.93cm²/sec

7. 0.04 m^3/s

8(b). $-\dfrac{PQ\sin\theta}{\sqrt{P^2 + 2PQ\cos\theta + Q^2}}$ or $-\dfrac{PQ\sin\theta}{R}$

8(c). 2.04 N/s

II. Integrate functions of linear functions and using trigo identities

1(a) $-\dfrac{1}{6}(1-2x)^3 + C$

1(b) $-\dfrac{2}{9}(4-3x)^{3/2} + C$

1(c) $-\dfrac{1}{8(2x-3)^4} + C$

1(d) $\dfrac{1}{8}\ln|8x+3| + C$

1(e) $-\ln|25-4x| + C$

1(f) $\dfrac{1}{3}\sin\left(3x - \dfrac{\pi}{6}\right) + C$

1(g) $-\dfrac{1}{2}\cos(2x+1) + C$

1(h) $2e^{\frac{x}{2}+5} + C$

2(a) 4

2(b) 0.2882

3(a) $-\dfrac{1}{2}\cos 2x + C$

3(b) $\dfrac{1}{2}\tan 2x + C$

3(c) $\tan 2x - 2x + C$

3(d) $\dfrac{\cos 2x}{2} - \dfrac{\cos 8x}{8} + C$

3(e) $\dfrac{3}{2}(\sin t - \dfrac{1}{4}\sin 4t) + C$

3(f) $\dfrac{\sin 3\theta}{6} - \dfrac{\sin 5\theta}{20} - \dfrac{1}{4}\sin\theta + C$

4(a) 2.41 4(b) 2

5 0.98 amp

III. Integration by substitution

1(a) $\frac{1}{10}\left(x^2-3\right)^5+C$ 1(b) $\frac{1}{2(4-x^2)}+C$

1(c) $\frac{1}{3}\sin^3\theta+C$ 1(d) $\frac{\left(x^3-10\right)^9}{9}+C$

1(e) $-\frac{1}{4}\ln\left|1-2x^2\right|+C$ 1(f) $ln\left|ln\,x\right|+C$

1(g) $-\frac{1}{4}e^{3-2t^2}+C$ 1(h) $\frac{3e^{\frac{y^2}{3}}}{2}+C$

1(i) $-5\sqrt{1-e^{2x}}+C$ 1(j) $\frac{1}{4}\cos t^4+C$

2(a) 1/24 (b) 1.07 (c) 13.33 (d) $\frac{\pi}{8}$

3(a) $2\ln\left|e^x-5\right|+C$ 3(b) $-\sqrt{1-2x^2}+C$

3(c) $-\frac{13+16t}{96\left(4t-5\right)^4}+C$ or $-\frac{1}{24\left(4t-5\right)^3}-\frac{11}{32\left(4t-5\right)^4}+C$

3(d) $-\cos x+\frac{\cos^3 x}{3}+C$

3(e) $\frac{2}{5}(4-x)^{5/2}-\frac{8}{3}(4-x)^{3/2}+C$

3(f) $\frac{1}{40}(1+4e^x)^{5/2}-\frac{1}{24}(1+4e^x)^{3/2}+C$

IV. Integration by partial fraction

1(a). $-\ln|x+3|+\frac{2}{3}\ln|3x-1|+C$

1(b). $-\ln|x+1|+\ln|x-2|+C$

1(c). $-\ln|x|+\frac{3}{2}\ln|x-3|-\frac{1}{2}\ln|x+1|+C$

2(a). $\ln|x+1|-\frac{3}{4}\ln|2x-1|+\frac{1}{4(2x-1)}+C$

2(b). $2\ln|x|+\frac{1}{2}\ln\left|x^2+4\right|-\frac{1}{2}\tan^{-1}\frac{x}{2}+C$

3(a). $\frac{4}{3\left(1-x\right)}-\frac{17x+20}{3\left(x^2+2\right)}$

3(b). $\frac{1}{6}\left[-20\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-8\ln|1-x|-17\ln\left|2+x^2\right|\right]+C$

4(a). $\frac{1}{x-1}+\frac{2}{\left(x+2\right)^2}-\frac{1}{x+2}$

4(b). $\ln|x-1|-\frac{2}{x+2}-\ln|x+2|+C$

5(a). $2\ln|x|-\frac{1}{2}\ln|x^2+3|-\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}+C$

5(b). 0.349

V. Integration by completing the square

1(a). $\sqrt{3}\tan^{-1}\left(\frac{x+3}{\sqrt{3}}\right)+C$

1(b). $\frac{1}{2}\ln\left([x-5]^2+25\right)+C$

2(a). $a=3, b=4$

2(b). 0.1865

3. $\frac{1}{\sqrt{3}}\tan^{-1}\frac{(2x-1)}{\sqrt{3}}-\frac{1}{2}\ln|x^2-x+1|+C$

4. $\frac{1}{8}\tan^{-1}\left(\frac{x-2}{8}\right)+C$, $\frac{1}{16}\tan^{-1}\left(\frac{x-2}{8}\right)+C$

5(a). $\frac{1}{\sqrt{3}}tan^{-1}\left(\frac{x-3}{\sqrt{3}}\right)+C$

5(b). $x+\frac{1}{\sqrt{3}}tan^{-1}\left(\frac{x-3}{\sqrt{3}}\right)+C$

VI. Integration by parts

1(a) $-\frac{x^2}{3}\cos 3x+\frac{2}{9}x\sin 3x+\frac{2}{27}\cos 3x+C$ 1(b) 0.0384 1(c) 4.575

1(d) $\frac{1}{29}e^{5x}(2\sin 2x+5\cos 2x)+C$ 1(e) $x\,\ell n(1-4x)-x-\frac{1}{4}\ell n(1-4x)+C$

2(a) $-\frac{\ln|x|}{4(2x+1)^2}+\frac{1}{4}\left[\ln|x|-\ln|2x+1|+\frac{1}{2x+1}\right]+C_1$

2(b) $\frac{1}{4}\left[-\sin^{-1}(2x)\sqrt{1-4x^2}+2x\right]+C$

3. $\frac{1}{3}\left(2\sqrt{3}\,\pi-\frac{\pi}{2}-2+\ln 2\right)$ or 2.668

VII. Simpson's Rules

1(a). 1.3001

1(b). Simpson's Rule cannot be calculated using 7 strips since n = 7 is odd.

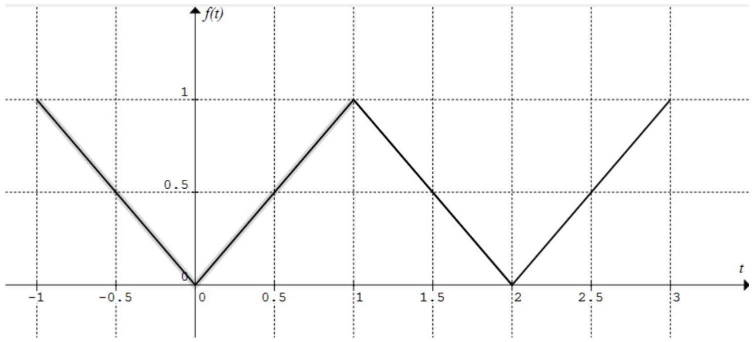
2. 0.993

3. 0.984

4. 0.3697 or 0.370 (3 dp)

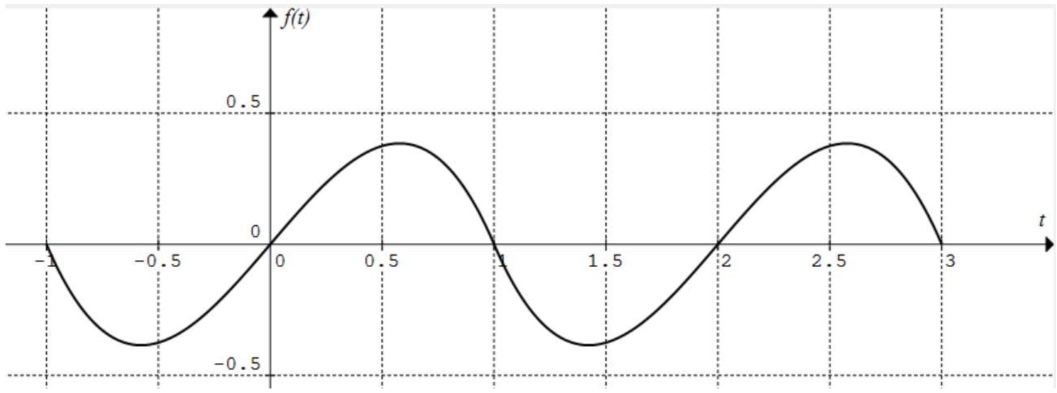
VIII. Fourier Series

1(a).



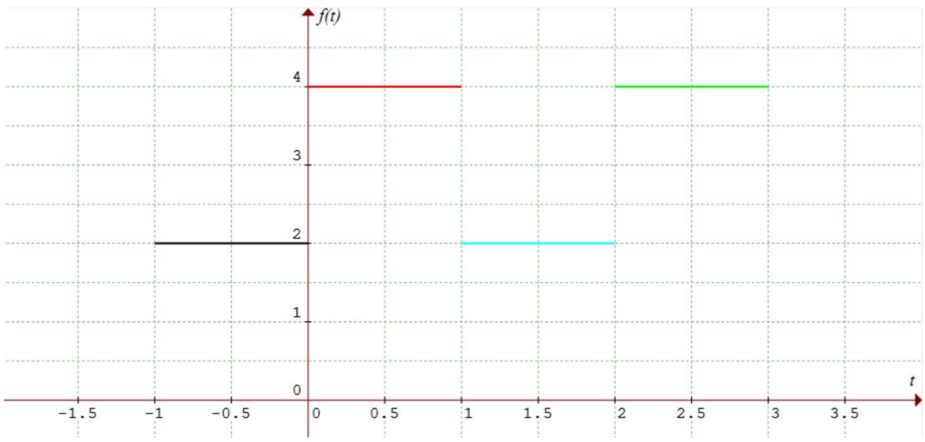
it is an even function

1(b).



it is an odd function

1(c).



it is neither even nor odd.

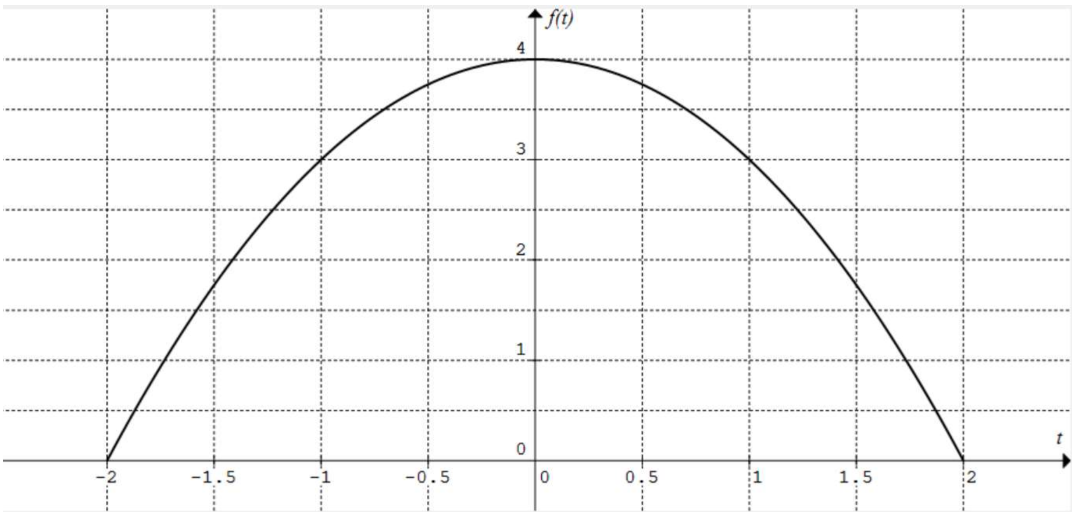
2(a). $T = 4,\ f(t) = \begin{cases} 1, & -1 \le t < 1 \\ 0, & 1 \le t < 3 \end{cases}$ $\qquad f(t+4) = f(t)$, it is an even function

2(b). $T = 2,\ f(t) = \begin{cases} 0, & -1 \le t < 0 \\ t, & 0 \le t < 1 \end{cases} \qquad f(t+2) = f(t)$, it is neither even nor odd

3(a). $f(t) = \frac{1}{2} - \frac{4\cos(\pi t)}{\pi^2} - \frac{4\cos(3\pi t)}{9\pi^2} + \ldots$

3(b). $f(t) = \frac{12\sin(\pi t)}{\pi^3} - \frac{3\sin(2\pi t)}{2\pi^3} + \frac{4\sin(3\pi t)}{9\pi^3} + \ldots$

4(i).



5. Assume that the function is continuous with period $T = 2$.

$$i(t) = \begin{cases} -20t + 10, & 0 \le t < 1 \\ 20t - 30, & 1 \le t < 2 \end{cases} \qquad i(t+2) = i(t)$$

$i(t) = \frac{80\cos(\pi t)}{\pi^2} + \ldots$, and $V(t) = -\frac{8\sin(\pi t)}{\pi} + \ldots$