2017/2018 SEMESTER TWO EXAMINATION

Diploma in Electrical and Electronic Engineering
3rd Year Full Time

DIGITAL SIGNAL PROCESSING

Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules as set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **TWO** sections:

   Section A - 6 Short Questions, 10 marks each.
   Section B - 2 Long Questions, 20 marks each.

3. **ALL** questions are **COMPULSORY**.

4. **ALL** questions are to be answered in the answer booklet.

5. This paper consists of **6** pages, including 2 pages of mathematical formulae.

SECTION A - SHORT QUESTIONS [10 marks each]

A1.
Using z-transform, determine the impulse response of the digital system shown in Figure A1. Comment on the stability of the system. (10 marks)

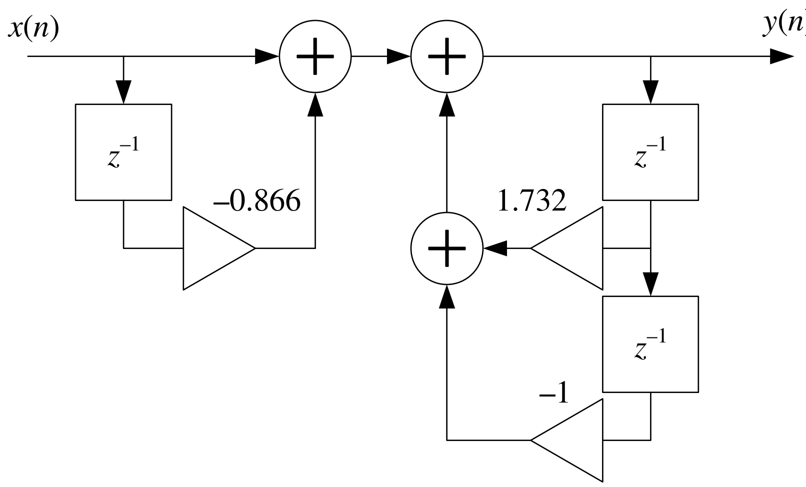


Figure A1

A2 Given y(n)={-1,5,-4,2,1} and impulse response h(n)={1,-1,1}, find the z-transform of y(n) and h(n), hence determine the input x(n) by using the long-division method.

(10 marks)

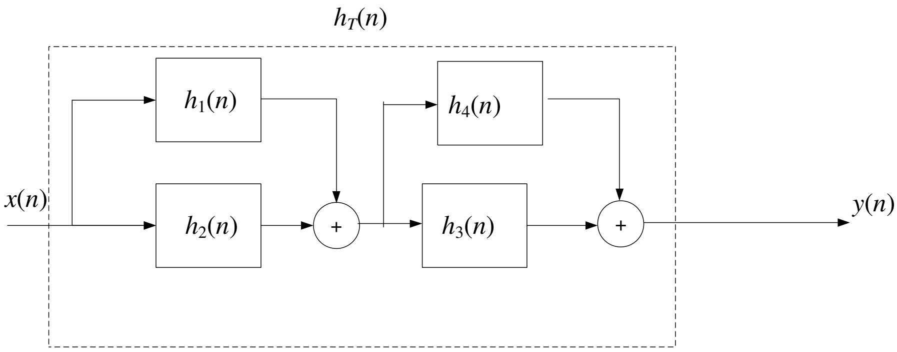
A3 Find the z-transform of $x(n)=20\sin(0.25\pi n)u(n)$ and $y(n)= e^{0.2n}\cos(0.25\pi n)u(n)$.

(10 marks)

A4 Evaluate the $N = 4$-point DFT for $X(0)$ and $X(2)$ if $x(n) = \{0, 2, 0, 2\}$.

(10 marks)

A5 The block diagram of a digital system is given as:



a) Find the overall impulse response of the system, $h_T(n)$ in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$ and $h_4(n)$. Find the z-transform of $h_T(n)$, $H_T(z)$. (6 marks)

b) If $h_1(n) = h_2(n) = h_3(n) = h_4(n)=\{1,1\}$ respectively, find $h_T(n)$. (4 marks)

A6 The system function of a digital system is given as:

$$X(z) = \frac{z}{3z^2 - 4z + 1}$$

Using partial fraction, find $x(n)$. (10 marks)

SECTION B - LONG QUESTIONS [20 marks each]

B1.

A certain digital FIR low pass filter was designed with a Barlett window function using the windowing technique. It was noted that the filter coefficients, $h(20) = h(30)$, and $h(20) = 0$. The sampling frequency used was 10 kHz.

Determine

(a) the peak approximation error of the filter in dB. (3 marks)

(b) the number of tap coefficients that the filter had. [Hint: $h(n) = h(M - n)$] (4 marks)

(c) the width of the transition band in Hz. (4 marks)

(d) the pass band and stop band frequency ranges in Hz. (9 marks)

B2.

Consider a FIR filter with difference equation given by:

$$y(n) = x(n) - x(n-2) - 2x(n-4)$$

(a) Compute and sketch the magnitude response of the filter at $\omega = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$. What type of filter is this? (10 marks)

(b) Find the output of the filter, $y(n)$, when the input $x(n) = \left[\cos\left(\frac{\pi}{2}n\right)\right]u(n)$. Is the steady state output (i.e. when n is large) consistent with the results in part (a) above? Why?

(10 marks)

-End of Paper-

Appendix

The z-transform is defined as $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$

Sequence	Transform
$\delta[n]$	1
$u[n]$	$\dfrac{1}{1-z^{-1}}$
$\delta[n-m]$	z^{-m}
$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$
$na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$
$[\cos\omega_0 n]u[n]$	$\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[\sin\omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[r^n\cos\omega_0 n]u[n]$	$\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$
$[r^n\sin\omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$

Some z-transform properties:	
Sequence	Transform
$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
$x[n-m]$	$z^{-m}X(z)$

Some trigonometric identities:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \;\mu\; \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \;\mu\; \tan A\tan B}$$

Series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

Discrete Fourier Transform:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{N}kn}$$

Complex number theory:

$$z = a + jb = r\angle\theta = re^{j\theta}$$

$$r = \sqrt{a^2+b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

Quadratic equation solution:

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

The characteristics of the different windowing functions:

Window Type	Peak approximation Error $20\log_{10}\delta$ dB	Transition Band $\Delta\omega$
Rectangular: $$w(n)=\begin{cases}1 & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$$	−21	$\dfrac{4\pi}{M+1}$
Bartlett: $$w[n]=\begin{cases}\dfrac{2n}{M} & 0\le n\le \dfrac{M}{2}\\ 2-\dfrac{2n}{M} & \dfrac{M}{2}\le n\le M\\ 0 & \text{otherwise}\end{cases}$$	−25	$\dfrac{8\pi}{M}$
Hanning: $$w(n)=\begin{cases}0.5-0.5\cos\left(\dfrac{2\pi n}{M}\right) & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$$	−44	$\dfrac{8\pi}{M}$
Hamming: $$w(n)=\begin{cases}0.54-0.46\cos\left(\dfrac{2\pi n}{M}\right) & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$$	−53	$\dfrac{8\pi}{M}$
Blackman: $$w(n)=\begin{cases}0.42-0.5\cos\left(\dfrac{2\pi n}{M}\right)+0.08\cos\left(\dfrac{4\pi n}{M}\right) & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$$	−74	$\dfrac{12\pi}{M}$

The impulse response of an ideal low pass filter is: $h_d(n)=\dfrac{\sin\left(\omega_c\left[n-\dfrac{M}{2}\right]\right)}{\pi\left(n-\dfrac{M}{2}\right)}$