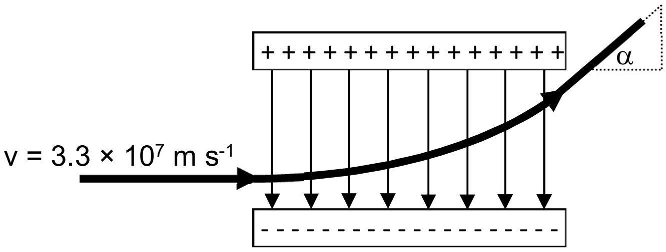
Solutions to Review paper

Q1. An electron gun creates a beam of electrons moving horizontally with a speed of 3.3×10^7 m s^{-1}. The electrons enter two parallel electrodes of length 2.0 cm, where the electric field is E = 5.0 $\times 10^4$ N C^{-1} downwards. In which direction and by what angle is the electron beam deflected by the electrodes?



Solution

When the electron is within the plates, the force on the electron is

$F = qE = 1.6 \times 10^{-19}$ C $\times 5.0 \times 10^4$ N $C^{-1} = 8.0 \times 10^{-15}$ N (directed upwards)

Acceleration of the electron (directed upwards) = F/m = 8.78×10^{15} m/s^2

There is no acceleration in the horizontal direction.

Writing the kinematical equations for both x and y directions as function of time t , using $x = x_o + v_{ox}t + \frac{1}{2} a_x t^2$ and $y = y_o + v_{oy}t + \frac{1}{2} a_y t^2$ we get,

$$x = (3.3 \times 10^7 \text{ m s}^{-1})\, t$$
$$y = \tfrac{1}{2} (8.78 \text{ x } 10^{15} \text{ m/s}^2)\, t^2$$

The time taken by the electrons to travel, t can be found out as

0.02 m = $t = x/v_x = 0.02$ m $/(3.3 \times 10^7$ m $s^{-1}) = 6.6 \times 10^{-10}$ s

The velocity in the x direction is the same as the initial velocity (there is no acceleration in the x-direction). The velocity in the y direction as function of time t can be obtained from $v_y = v_{0y} + a_y t$.

$$v_x = 3.3 \times 10^7 \text{ m s}^{-1}$$
$$v_y = (8.78 \times 10^{15} \text{ m/s}^2)t$$

At the moment when the electrons leave the electrodes, $v_x = 3.3 \times 10^7$ m s^{-1}
and $v_y = (8.78 \times 10^{15}$ m/s^2)(6.6×10^{-10} s) = 5.3×10^6 m s^{-1}

Therefore $\tan \alpha = v_y / v_x = (5.3 \times 10^6 \text{ m s}^{-1} / 3.3 \times 10^7 \text{ m s}^{-1})$

or $\alpha = \tan^{-1}(v_y / v_x) = 9.1^o$

Q2. A straight 2.00-m, 150-g wire carries a current in a region where the earth's magnetic field is horizontal with a magnitude of 0.55 gauss. (a) What is the minimum value of the current in this wire so that its weight is completely supported by the magnetic force due to earth's field (assuming no forces other than gravity acts on it)? Does it seem likely that such a wire could support this kind of current? (b) Show how the wire would have to be oriented relative to the earth's magnetic field to be supported in this way. 1 gauss = 10^{-4} T.

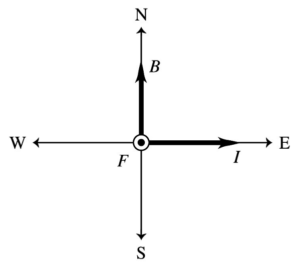
Solution

The magnetic force is $F = IlB\sin\phi$. For the wire to be completely supported by the field requires that $F = mg$ and that $\vec{\mathbf{F}}$ and $\vec{\mathbf{w}}$ are in opposite directions.
The magnetic force is maximum when $\phi = 90°$. The gravity force is downward.

(a) $IlB = mg$. $I = \dfrac{mg}{lB} = \dfrac{(0.150\text{ kg})(9.80\text{ m/s}^2)}{(2.00\text{ m})(0.55\times10^{-4}\text{ T})} = 1.34\times10^4$ A. This is a very large current and ohmic heating due to the resistance of the wire would be severe; such a current isn't feasible.

Another way to think about it is this. Suppose the resistance of the wire is 0.1 ohms. Then with the current calculated, the power dissipated is 18 MW (using $P = I^2R$). A typical power station can provide 250 MW – it would mean fourteen wires could replace an entire power station!

(b) The magnetic force must be upward. The directions of I, $\vec{\mathbf{B}}$ and $\vec{\mathbf{F}}$ are shown in the figure below, where we have assumed that $\vec{\mathbf{B}}$ is south to north. To produce an upward magnetic force, the current must be to the east. The wire must be horizontal and perpendicular to the earth's magnetic field.
The magnetic force is perpendicular to both the direction of I and the direction of $\vec{\mathbf{B}}$

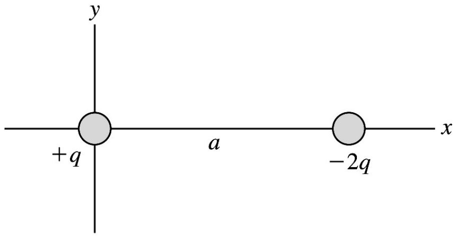


Q3. A positive charge q is fixed at the point x = 0, y = 0. A negative charge -2q is fixed at the point x = a and y = 0. (a) Show the positions of the charges in a diagram. (b) Derive an expression for the potential V at points on the x-axis as a function of the coordinate x (take V to be zero at an infinite distance from the charges).(c) At which positions on the x-axis is V = 0?

Solution

For a point charge, $V = \frac{kq}{r}$. The total potential at any point is the algebraic sum of the potential of the two charges.

(a) The positions of the two charges are shown in the below figure.



(b) We have to consider 3 scenarios:

$$x > a : V = \frac{kq}{x} - \frac{2kq}{x-a} = \frac{-kq(x+a)}{x(x-a)}.$$

$$0 < x < a : V = \frac{kq}{x} - \frac{2kq}{a-x} = \frac{kq(3x-a)}{x(x-a)}.$$

$$x < 0 : V = \frac{-kq}{x} + \frac{2kq}{x-a} = \frac{kq(x+a)}{x(x-a)}.$$

A general expression valid for any y is $V = k\left(\frac{q}{|x|} - \frac{2q}{|x-a|}\right)$.

(c) The potential is zero at $x = -a$ and $a/3$.

Q4. Three point charges are arranged along the x-axis. Charge $q_1 = +3.00\ \mu C$ is at the origin and charge $q_2 = -5.00\ \mu C$ is at $x = 0.200$ m. There is a third charge $q_3 = -8.00\ \mu C$ located on the x-axis. Where is q_3 located so that the net force on q_1 is 7.00 N in the negative x direction?

Solution

Apply Coulomb's law to calculate the force exerted by q_2 and q_3 on q_1. Add these forces as vectors to get the net force. The target variable is the x-coordinate of q_3.

$\mathbf{F}_2$ is in the x-direction.

$$F_2 = k\frac{|q_1 q_2|}{r_{12}^2} = 3.37 \text{ N, so } F_{2x} = +3.37 \text{ N}$$

$$F_x = F_{2x} + F_{3x} \text{ and } F_x = -7.00 \text{ N}$$

$$F_{3x} = F_x - F_{2x} = -7.00 \text{ N} - 3.37 \text{ N} = -10.37 \text{ N}$$

For F_{3x} to be negative, q_3 must be on the negative x axis.

$$F_3 = k\frac{|q_1 q_3|}{x^2}, \text{ so } |x| = \sqrt{\frac{k|q_1 q_3|}{F_3}} = 0.144 \text{ m, so } x = -0.144 \text{ m}$$

Note that q_2 attracts q_1 in the + x-direction so q_3 must attract q_1 in the –x direction, and q_3 is at negative x.

Q5. A ice block of mass 10 kg floating in a river is pushed through a displacement **d** given by **d** = (20 m) **i** – (16 m) **j** along a straight embankment by rushing water, which exerts a force **F** = (210 N) **i** – (150 N) **j** on the block.
(a) How much work does the force do on the block during the displacement?
(b) What is the magnitude of the final velocity of the block if it was initially at rest?

Solution

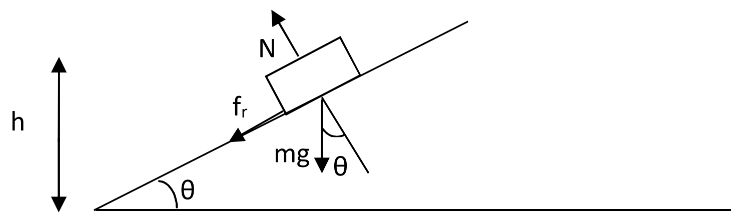
(a) The force is constant. Therefore, W = F•d = 210 × 20 + 16 × 150 = 6600 J
(b) From the work-energy principle we can calculate the final velocity of the block.

$$W = KE_{final} - KE_{initial} = \frac{1}{2}mv^2 - 0$$

$$v = \sqrt{\frac{2W}{m}} = \sqrt{1320} = 36.3 \text{ m/s}$$

Q6. An object of mass 2.4 kg moves up a plane that is inclined at 37.0^0 to the ground. If the initial speed of the object is 3.8 m/s and the coefficient of kinetic friction between the object and the plane is 3/10, how far does the object travel before it stops?

Solution



Let the system consist of earth, object and the inclined plane. There are no external forces doing work to change the energy of the system. Let v_0 and v be the initial and final velocities of the object respectively, N be the normal reaction on the object due to the inclined plane, f_r be the force of friction and h be the vertical height at which the object stops. Applying the work-energy theorem with friction to the system,

$$W_{\text{ext}} = \Delta E_{\text{mech}} + \Delta E_{\text{therm}}$$
$$= \Delta K + \Delta U + \Delta E_{\text{therm}}$$
$$0 = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\,2} + mgh + \mu_k Nd$$
But $N = mg\cos(\theta)$

and $h = d\sin(\theta)$

Therefore

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\,2} + mgd\sin(\theta) + \mu_k mg\cos(\theta)d = 0$$
$v = 0$ as the object comes to rest.

This implies $d = \dfrac{u^2}{2g\left(\mu_k\cos(\theta) + \sin(\theta)\right)} = 0.87\,\text{m}$

Alternate method

$$v^{\,2} - v_0^{\,2} = 2a_x(x - x_0) = 2a_x d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$
$$F_{\text{net}} = ma_x = -mg\sin(\theta) - \mu_k N$$
$$= -mg\sin(\theta) - \mu_k mg\cos(\theta)$$
or $a_x = -g\sin(\theta) - \mu_k g\cos(\theta)$

The final velocity $v = 0$

Substituting in (1)

$$-v_0^{\,2} = -2gd(d\sin(\theta) + \mu_k\cos(\theta))$$
This implies $d = \dfrac{u^2}{2g\left(\mu_k\cos(\theta) + \sin(\theta)\right)} = 0.87\,\text{m}$

Q7. A force of 30 N is applied to a body of mass 10 kg which was originally at rest. If the object moved 24 m in the same direction as the force, calculate. i) the work done on the body and its final kinetic energy, ii) the final velocity of the body, iii) the rate of work done by the force.

Solution

i) W = F × d = 30 × 24 = 720 J
Work done on the object goes on to increase the KE from 0.
Therefore W = KE (final) – KE (initial)
But KE (initial) is 0 (since initially the body was at rest) which means
W = KE (final) = 720 J

ii) KE (final) = $1/2mv^2$ = 720 J
m = 10 kg
Therefore from the above equation, v (final) = 12 m/s

iii) rate of work done is the power = Work/time = (F× d)/t

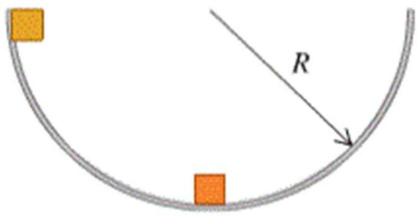
Average speed = (initial velocity + final velocity)/2

= (0 + 12) /2 = 6 m/s

Time taken = Distance/ Average velocity = 24/6 = 4 s,

Therefore Power = (30 x 24)/4 = 180 W

Q8. Two identical masses m_1 and m_2 are placed in a smooth hemispherical bowl of radius R at the top and bottom, respectively, as shown. The mass m_1 is released from rest and collides with m_2 at a certain velocity. The masses stick together when they collide and move together at half the original speed that m_1 had just before the collision. How high above the bottom of the bowl will the masses go after colliding? You can ignore friction between the masses and the surface of the bowl.



Solution

Apply conservation of energy to the motion before and after the collision Let v be the speed of the mass m_1 just before it strikes m_2. We know that m_1 and m_2 are identical. Therefore let each object have mass m.

From conservation of energy, the loss in GPE for m_1 at the top will result in KE at the bottom.

$\frac{1}{2}mv^2 = mgR;\ \ v = \sqrt{2gR}.$

Let y be the final height above the bottom of the bowl. Apply conservation of energy to the motion of the combined object after the collision that moves at $v/2$.

$$\frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = (2m)gy.$$

$$y = \frac{v^2}{8g} = \frac{2gR}{8g} = R/4.$$

Q9. A singly charged Lithium ion has a mass 1.16×10^{-26} kg. It is accelerated through a potential difference of 220 V and then enters a magnetic field with magnitude 0.723 T perpendicular to the path of the ion. What is the radius of the ion's path in the magnetic field?

Solution
After being accelerated through a potential difference V the ion has kinetic energy qV. The acceleration in the circular path is
$v^2/R.$.
The ion has charge
$q = +e.$
$K = qV = +eV.$

$$\tfrac{1}{2}mv^2 = eV$$

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\ \text{C})(220\ \text{V})}{1.16\times10^{-26}\ \text{kg}}} = 7.79\times10^4\ \text{m/s}.$$

$F_B = |q|vB\sin\phi.$ $\phi = 90°.$

$\vec{F} = m\vec{a}$ gives $|q|vB = m\dfrac{v^2}{R}.$

$$R = \frac{mv}{|q|B} = \frac{(1.16\times10^{-26}\ \text{kg})(7.79\times10^4\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.723\ \text{T})} = 7.81\times10^{-3}\ \text{m} = 7.81\ \text{mm}.$$

The larger the accelerating voltage, the larger the speed of the particle and the larger the radius of its path in the magnetic field.

Q10. Singly ionized atoms are accelerated and then passed through a velocity selector consisting of perpendicular electric and magnetic fields. The electric field is 155 V/m and the magnetic field is 0.0315 T. The ions next enter a uniform magnetic field of magnitude 0.0175 T that is oriented perpendicular to their velocity. (a) How fast are the ions moving when they emerge from the velocity selector? (b) If the radius of the path of the ions in the second magnetic field is 17.5 cm, what is their mass?

Solution
The velocity selector eliminates all ions not having the desired velocity. Then the magnetic field bends the ions in a circular arc.
In a velocity selector, $E = vB.$ For motion in a circular arc in a magnetic field of magnitude B,
$R = \dfrac{mv}{|q|B}.$ The ion has charge $+e.$

(a) $v = \dfrac{E}{B} = \dfrac{155\ \text{V/m}}{0.0315\ \text{T}} = 4.92\times10^3\ \text{m/s}.$

(b) $m = \dfrac{R|q|B}{v} = \dfrac{(0.175\ \text{m})(1.60\times10^{-19}\ \text{C})(0.0175\ \text{T})}{4.92\times10^3\ \text{m/s}} = 9.96\times10^{-26}\ \text{kg}.$

Ions with larger ratio $\dfrac{m}{|q|}$ will move in a path of larger radius.