SINGAPORE POLYTECHNIC

2019/2020 SEMESTER TWO EXAM

ENGINEERING MATHEMATICS II Time allowed: 2 hrs

2nd Year Full-Time

School of Chemical and Life Sciences
DCHE

School of Electrical and Electronic Engineering
DASE, DCPE, DEB, DEEE, DES, DESM

School of Mechanical and Aeronautical Engineering
DARE, DBEN, DCEP, DME, DMRO

Instructions to Candidates:

1. The examination rules set out on the last page of the answer booklet are to be complied with.
2. **The questions are printed on Pages 2 – 5**.
3. This paper consists of THREE (3) sections.

   Section A: 5 Multiple Choice Questions (10 marks)
   Answer **ALL** questions.

   Section B: 7 Structured Questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all the questions you answered will be added, but the maximum mark you can obtain from this section is 50 marks.

   Section C: 3 Structured Questions (40 marks)
   Answer **ALL** questions.

4. Unless otherwise stated, leave your decimal answers correct to **two** decimal places.
5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply will result in loss of marks.

<u>Section A</u> (10 marks):

Answer <u>ALL</u> multiple choice questions on the MCQ answer sheet of the answer booklet.

A1. Which of the following methods can be used to find the integral $\int \ln(2x)\,dx$?

I. Integration by substitution and let $u = \ln(2x)$.

II. Integration by parts with $u = \ln(2x)$ and $dv = dx$.

III. Integration by parts with $u = 1$ and $dv = \ln(2x)\,dx$.

(a) I only
(b) II only
(c) III only
(d) I, II & III

A2. Which of the following statements is **false** about the Simpson's rule?

(a) It is a numerical method of integration to calculate definite integrals.

(b) The required number of ordinates (y values) is even.

(c) The exact solution of a definite integral cannot be calculated using the Simpson's rule.

(d) The width of each strip is $h = \dfrac{b-a}{n}$, where a and b are the lower and upper limits of the integral, n is the number of strips.

A3. The Fourier series of a periodic function $f(t)$ of period 2π is given by

$$f(t) = \frac{1}{\pi}\left(1 - \cos t + 9\cos 3t + 25\cos 5t + \cdots\right).$$

Then the value of $\int_0^{2\pi} f(t)\cos 3t\,dt$ is ______________

(a) 1
(b) $\dfrac{1}{\pi}$
(c) 9
(d) $\dfrac{9}{\pi}$

A4. If $\mathcal{L}\{f(t)\} = F(s)$, then $F(s-7) =$ ________

(a) $e^{7s}\mathcal{L}\{f(t)\}$
(b) $\mathcal{L}\left\{e^{t} f(t-7)\right\}$
(c) $e^{s-7}\mathcal{L}\{f(t)\}$
(d) $\mathcal{L}\left\{e^{7t} f(t)\right\}$

A5. Which of the following differential equations is linear and homogeneous?

(a) $\frac{d^2y}{dx^2}+\frac{dy}{dx}-4y=0$ (b) $\frac{d^2y}{dx^2}+y\frac{dy}{dx}+8y=0$

(c) $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y+x=0$ (d) $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-15=0$

Section B (50 marks):

Each question carries 10 mark. The total marks of the questions in this section is 70. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you can obtain from this section is 50 marks.

B1. (a) Find $\int x\sin(2x^2)\,dx$, by using the substitution $u=2x^2$.

(b) Use integration by parts to evaluate $\int_0^{\pi} 2t\cos(t)\,dt$.

B2. Using Simpson's rule with $n=6$ strips, find the approximation for the definite integral $\int_0^3 \sqrt{1+x^2}\,dx$.

B3. A periodic function $f(t)$ of period 2π is defined as

$$f(t)=\begin{cases} 0, & -\pi \le t < -\dfrac{\pi}{2} \\ 4, & -\dfrac{\pi}{2} \le t < \dfrac{\pi}{2} \\ 0, & \dfrac{\pi}{2} \le t < \pi \end{cases}, \qquad f(t+2\pi)=f(t).$$

(a) Find the d.c. component (i.e. a_0).

(b) If $a_n=\frac{8}{n\pi}\sin\left(\frac{n\pi}{2}\right)$, find a_1, a_2 and a_3.

(c) Given that $f(t)$ is an even function, use the results from parts (a) and (b), write down the Fourier series of $f(t)$ as far as the 3rd harmonic.

B4. (a) Find the general solution of the differential equation $\frac{dy}{dx} = 2xy$ by separating the variables.

(b) (i) Find $\int_4^{12} (1+2x)^2\,dx$.

(ii) The average value of a function $y = f(x)$ in the interval from $x = a$ to $x = b$ is given by $y_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$. Use this definition and the result from part (i), find the average value of the function $y = (1+2x)^2$ in the interval from $x = 4$ to $x = 12$.

B5. (a) Find $\mathcal{L}\left\{t^3 - 5\cos 3t\right\}$.

(b) Find $\mathcal{L}\left\{4t\cos 2t + 3e^{2t}\right\}$.

(c) Find $\mathcal{L}\left\{\sin \pi t\right\}$. Hence, use first shift theorem to find $\mathcal{L}\left\{e^{-t}\sin \pi t\right\}$.

B6. (a) Find $\mathcal{L}^{-1}\left\{\frac{3}{4s} + \frac{5}{s^4} - \frac{5s}{s^2+4}\right\}$.

(b) By partial fraction, find $\mathcal{L}^{-1}\left\{\frac{s-3}{(s-1)(s-2)}\right\}$.

B7. Given the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0$, find

(a) the general solution;

(b) the particular solution if $y(0) = 2$ and $y'(0) = -4$.

Section C (40 marks):

Answer all THREE questions below.

C1. The length, width and height of a rectangular box are measured to be 1, 2 and 3 metres respectively. If each measurement is estimated to be in error by 1% too large, use partial differentiation to estimate the percentage error in the measurement of the volume of the box. (11 marks)

C2. An object cools in an air-conditioned room maintained at a constant temperature of 20°C. Assume the object cools down according to Newton's law of cooling which states that the rate of change of temperature is proportional to the temperature difference between the object and the surrounding environment. Ten minutes after the object began to cool, its temperature was observed to be 75°C. Another 10 minutes later its temperature was 50°C.

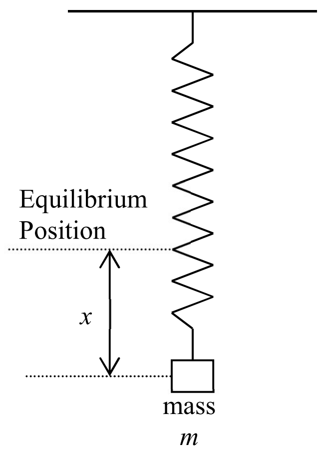
(a) Set up the differential equation that models the cooling process of the object.

(b) Find the initial temperature of the object.

(c) When the temperature of the object is 50°C, a cooling fan is set up to cool down the object faster. It is assumed that this fan further reduced the temperature at a constant rate of 5°C/min. Write down the differential equation that models this new setup.

(14 marks)

C3. In a spring–mass system, a 1-kg mass is attached to a spring of stiffness 4 (N/m). Assume the mass is suspended in a medium that offers no resistance. At any time t (s), there is an external force of $f(t) = 3\cos 2t$ (N) applied to the mass. Let the displacement of the mass from the equilibrium position be x (m). The mass was pulled 0.5 m below its equilibrium position and released from rest.



(a) Write down the second order differential equation and the initial conditions governing the motion of the system.

(b) Hence solve the differential equation using Laplace Transform.

(c) Determine the displacement x (m) after one second.

(15 marks)

- End of Paper -