Chapter 7

Analog to Digital conversion

Part 1 of 4

Introduction

- Digital communication is the dominant form of communication.
- A lot of information in modern communications is in digital form.

e.g. binary data

- However, analog signals like audio and video signals are not compatible with digital processing and signal transmission of digital communication systems

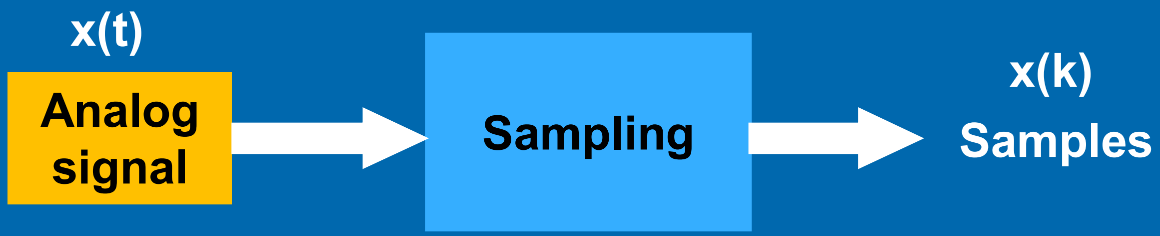
Audio and video signals are two of the most common information signals

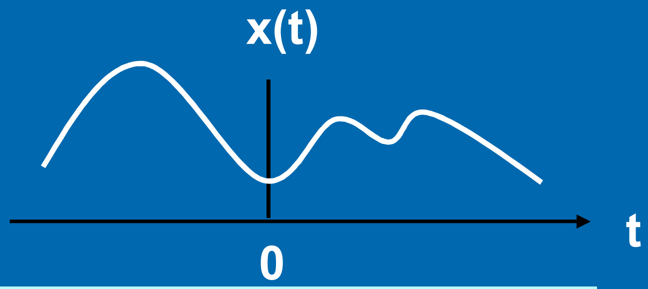
- Analog-to-digital conversion (ADC) is required for transmitting analog signals using digital communication systems.

7.1 Sampling

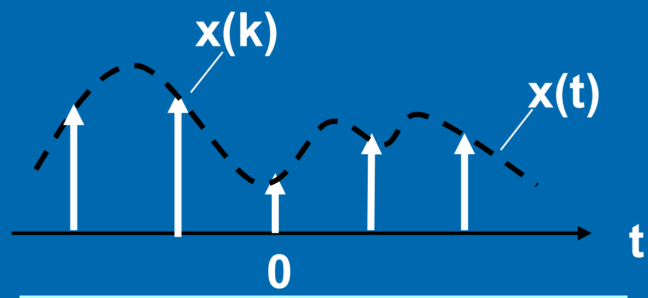
Singapore Polytechnic

- The first step towards the conversion from analog to digital communications. and the bridge between analog and digital signals.
- Converts an analog signal into a corresponding train of samples that are spaced uniformly in time.





continuous time signal x(t)



Discrete-time signal x(k)

7.1 Sampling

Reasons for sampling an analog signal:

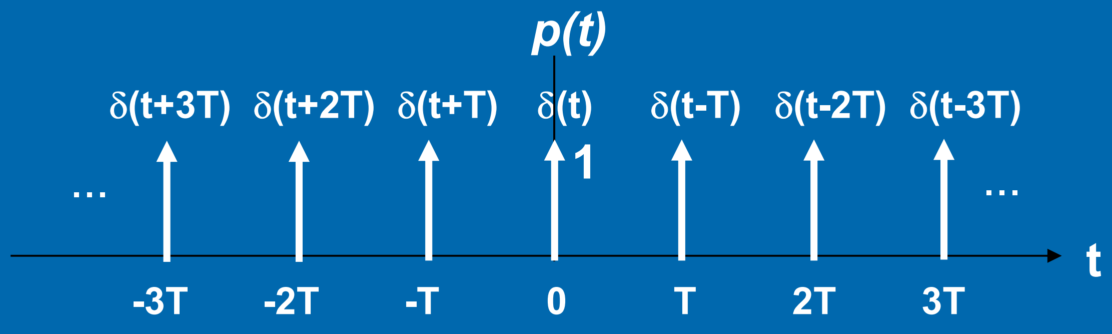
1. To convert an analog signal into a digital signal that is compatible with digital transmission.
   - To be compatible with digital transmission.
   - To allow an analog signal to be digitally processed.
   - To allow TDM (time division multiplexing) which is the simultaneous transmission of several signals over the same channel.
2. Certain signal processing devices (for example high-power microwave tubes; laser) can operate better on a pulse basis.
3. Reduction in power needed to transmit a signal as with sampling, signal is transmitted in burst.

7.1 Sampling

Ideal Sampling / impulse train sampling

- Ideal or impulse-train sampling of is carried as follows :

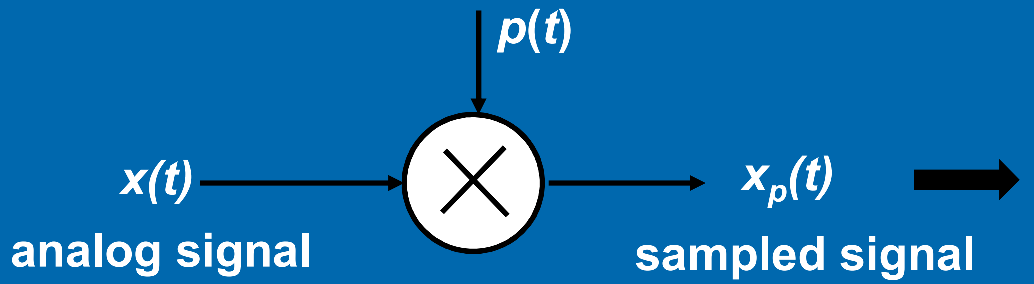
an unit impulse train



sampling frequency

f_0 = fundamental frequency of $p(t)$

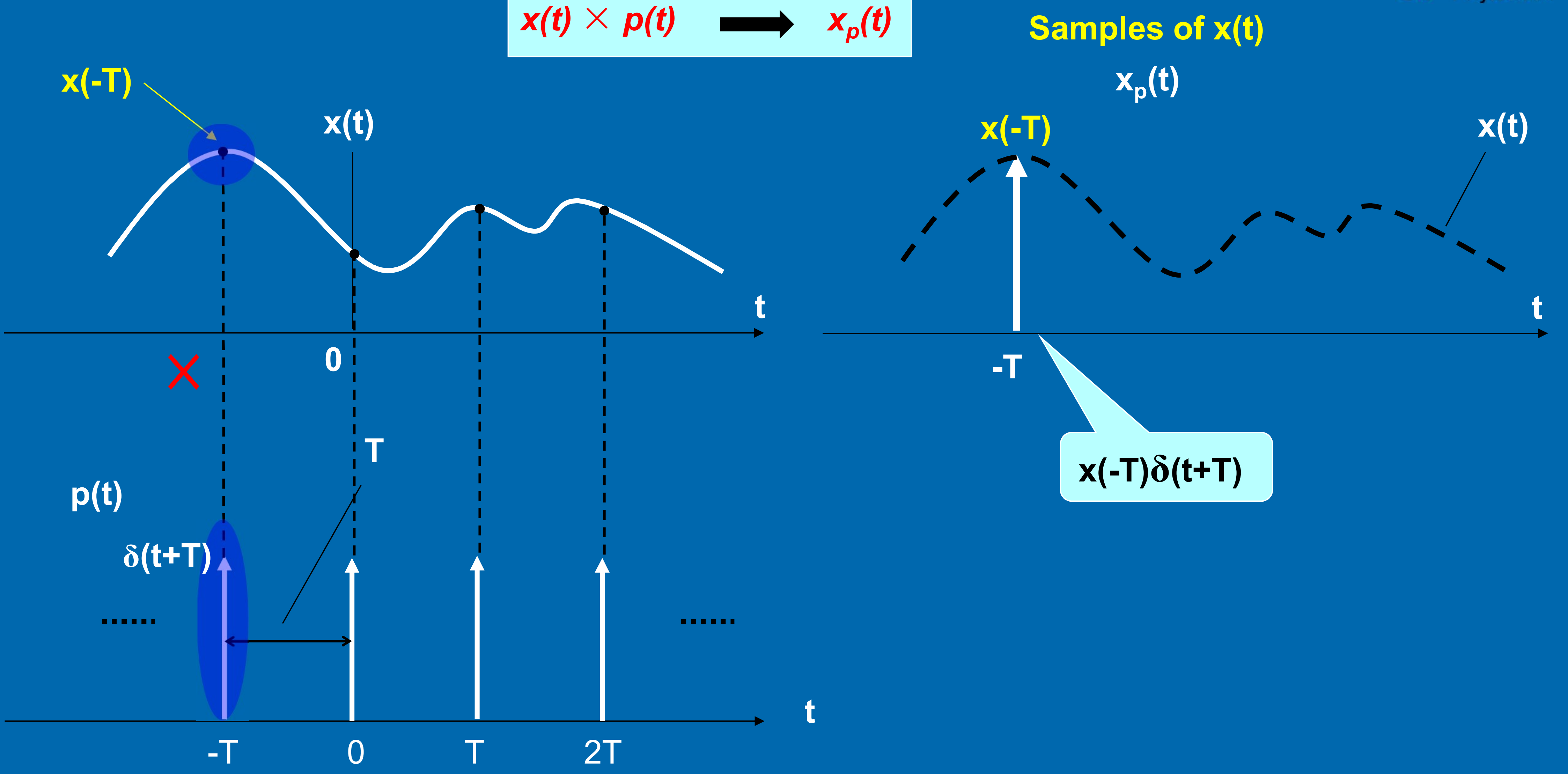
$f_s = f_0 = 1/T$



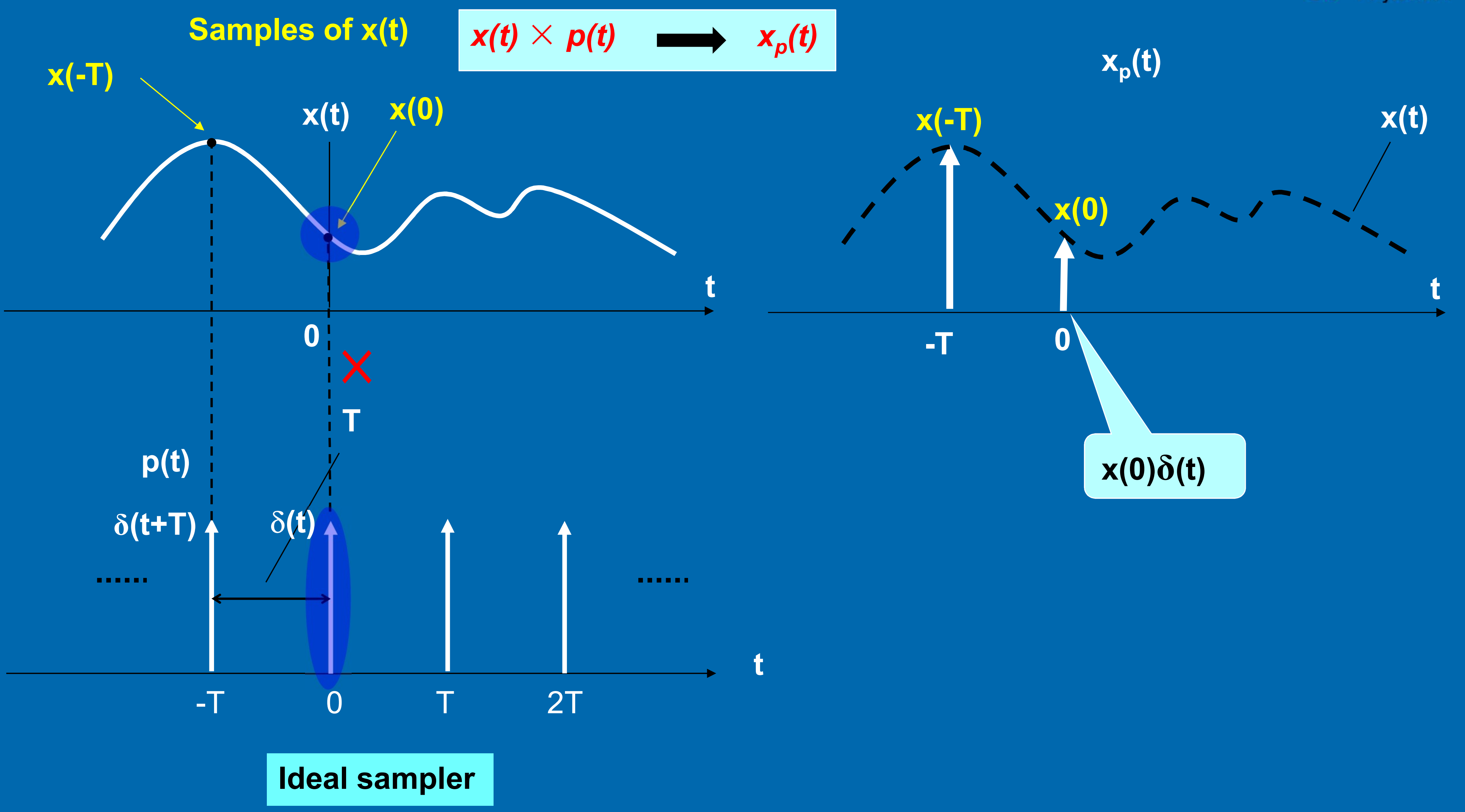
$x(t) \times p(t)$

Ideal sampling takes a sample of a signal without any modification.
Can be done only with an unit impulse train.

$x(t) \times p(t) \longrightarrow x_p(t)$

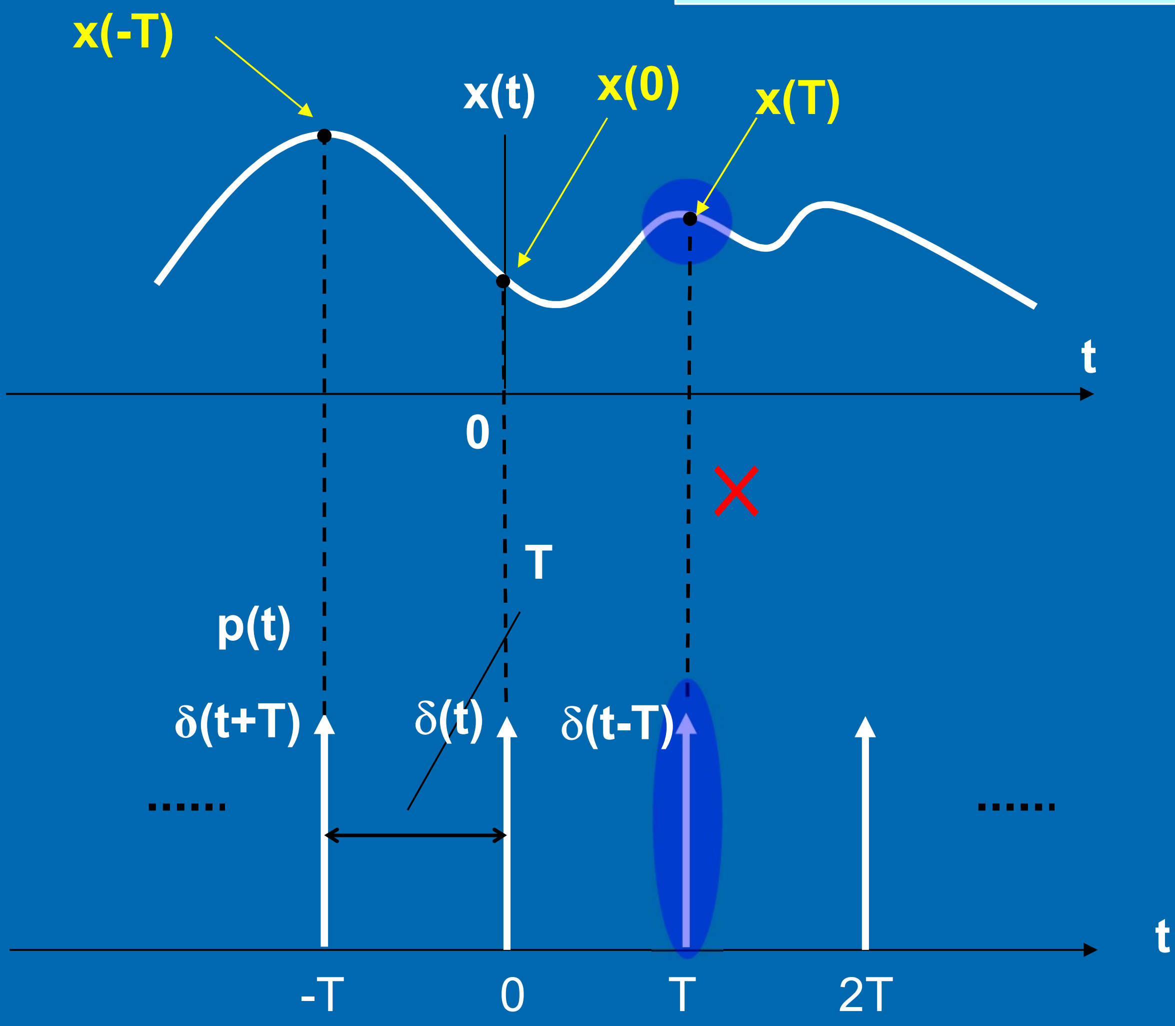


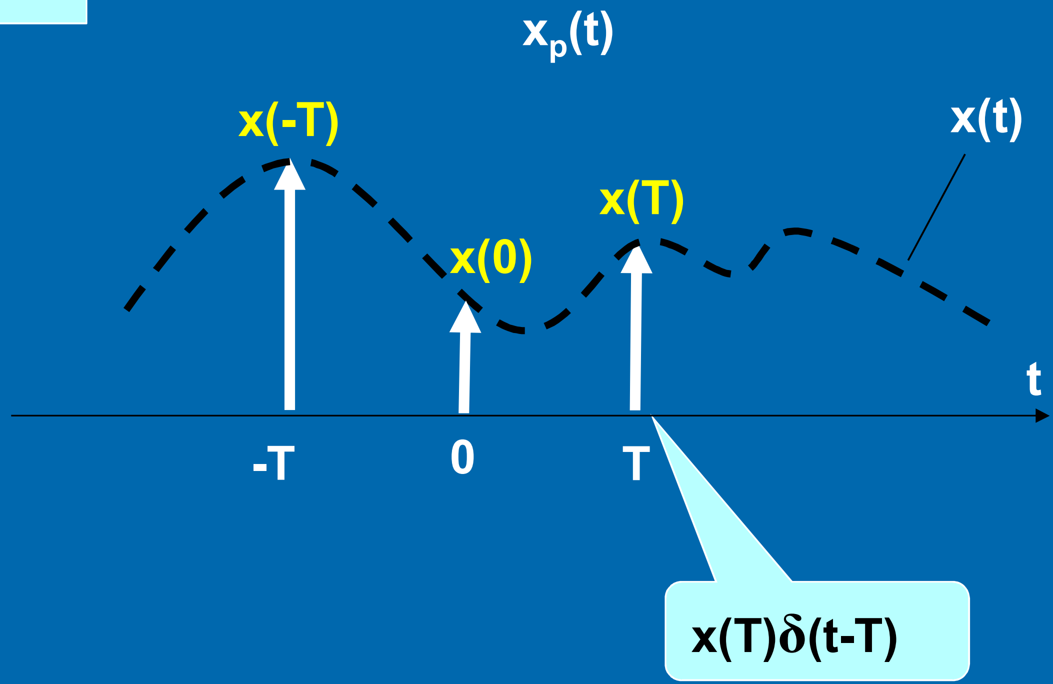
Ideal sampler


Samples of x(t)
x(t) × p(t) ➡ xp(t)
x(-T)
x(t)
x(0)
t
0
T
p(t)
δ(t+T)
δ(t)
......
......
t
-T
0
T
2T
xp(t)
x(-T)
x(t)
x(0)
t
-T
0
x(0)δ(t)
Ideal sampler

$x(t) \times p(t) \longrightarrow x_p(t)$

Samples of x(t)

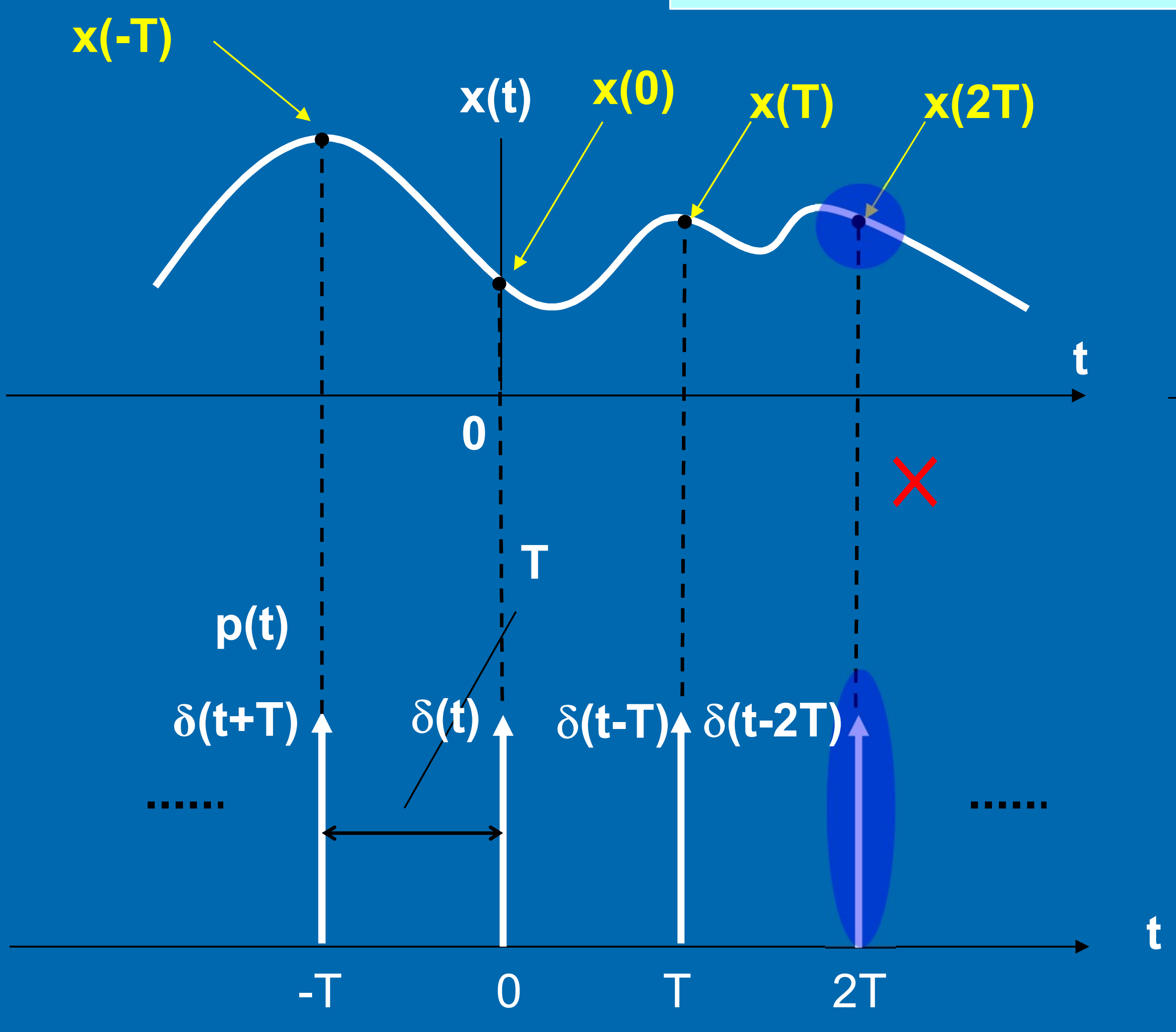


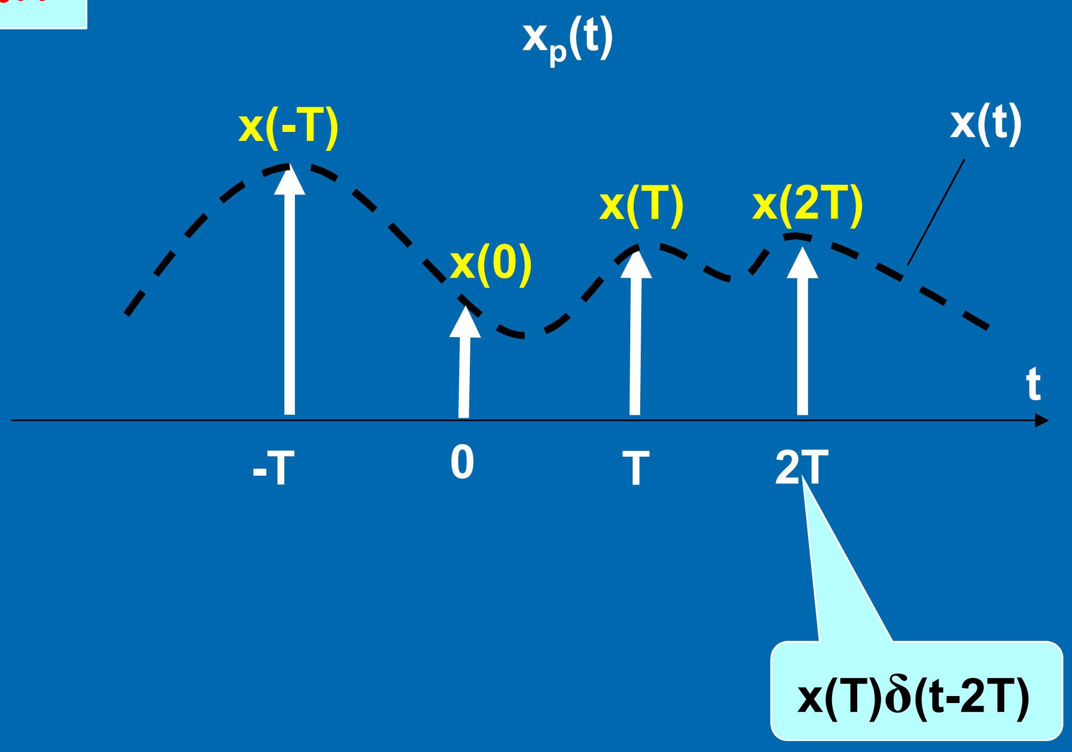


Ideal sampler

Samples of x(t)

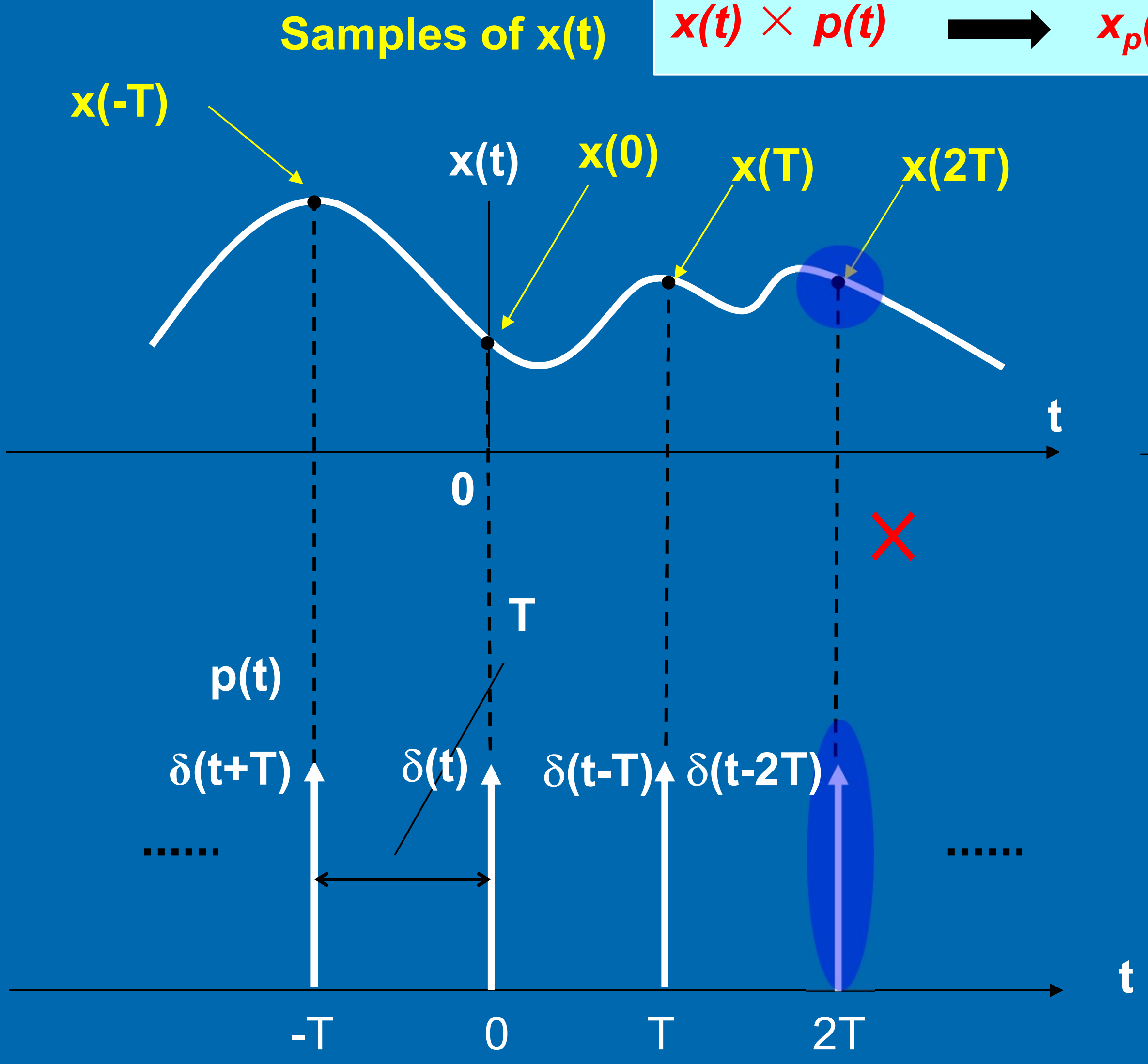
$x(t) \times p(t) \longrightarrow x_p(t)$



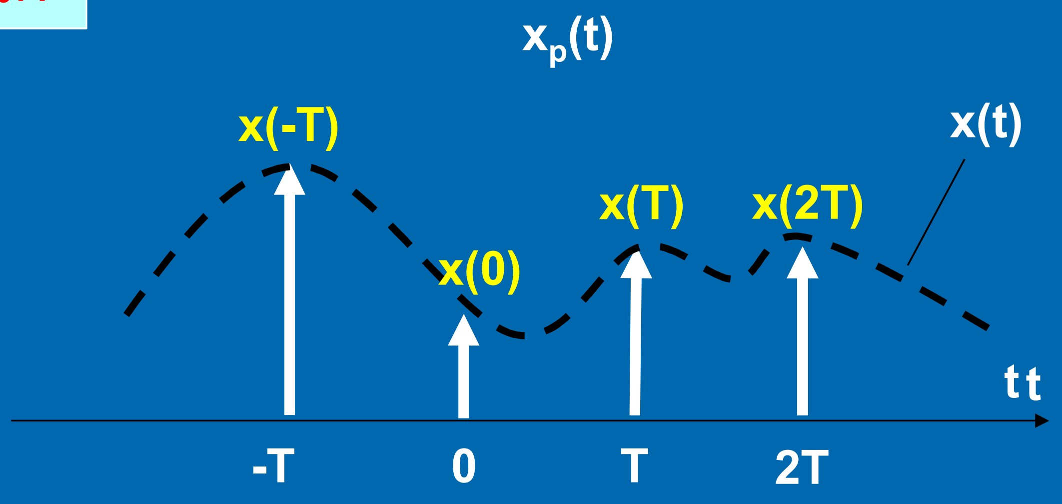


Ideal sampler

$x(t) \times p(t) \longrightarrow x_p(t)$



Ideal sampler



An impulse train whose amplitudes are enveloped by the original signal.

7.1 Sampling

Singapore Polytechnic

Ideal Sampling

- **In time domain,**

$x_p(t) = x(t) \times p(t)$ **where** $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

$x_p(t)$ is an impulse train with the strength of the impulses equal to the values of $x(t)$ at the sampling instants, at intervals of T.

7.1 Sampling

Singapore Polytechnic

Ideal Sampling

- In frequency domain (from frequency convolution theorem)

$$X_p(f) = X(f) * P(f)$$

where $*$ denotes convolution and $P(f) = \frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f - kf_s)$

$\because \quad X(f) * \delta(f - kf_s) = X(f - kf_s)$ Chapter 2 eq (2.32)

Hence,

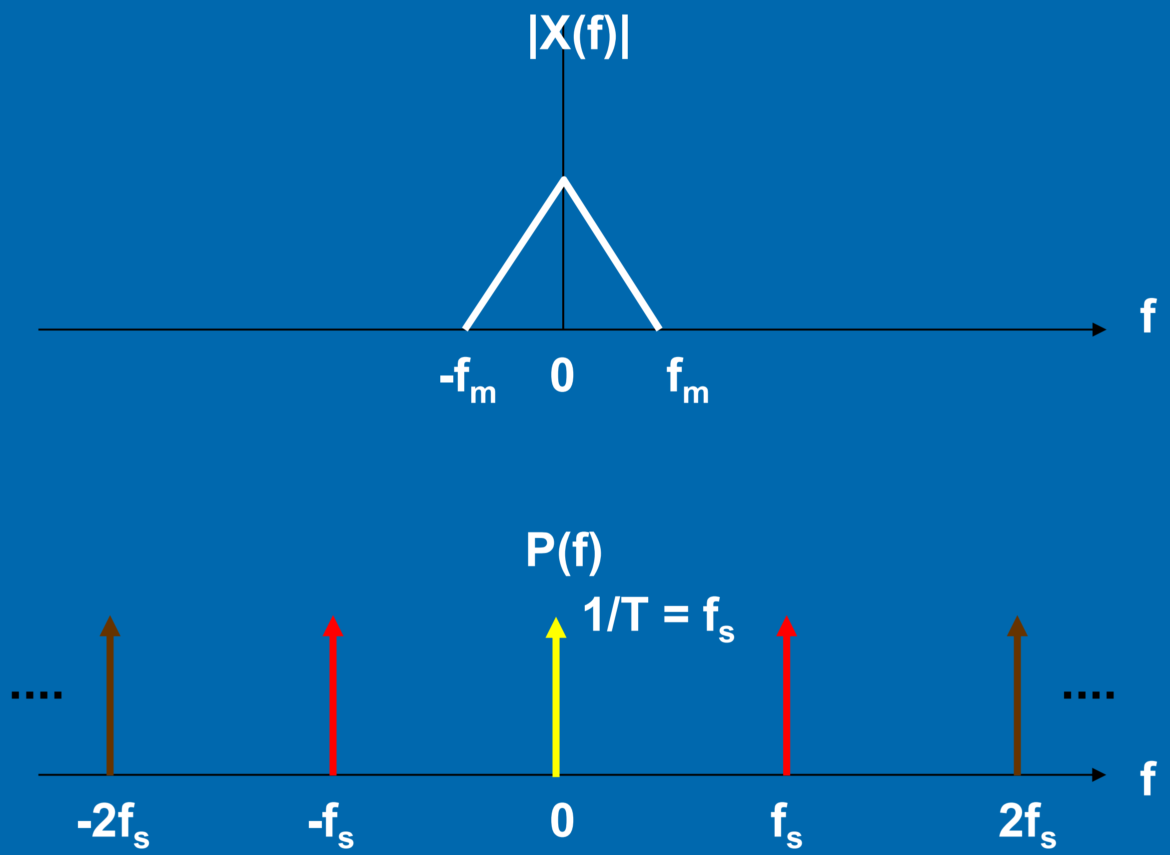
$$X_p(f) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(f - kf_s)$$

$X_p(f)$ is a periodic function of f consisting of sum of shifted replicas of $X(f)$, scaled by $1/T$.

7.1 Sampling

<u>**Ideal Sampling**</u>

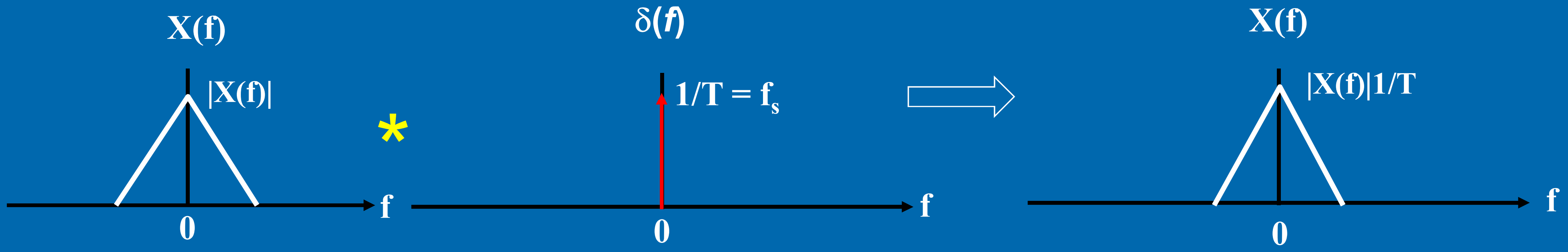
Let X(f) be the amplitude spectrum of x(t) as shown below with highest frequency component, f_m and P(f) be the spectrum of p(t).



Recall (Chapter 2)

Convolution of any function *X*(*f*) with a unit impulse function δ(*f*) gives the function *X*(*f*) itself.

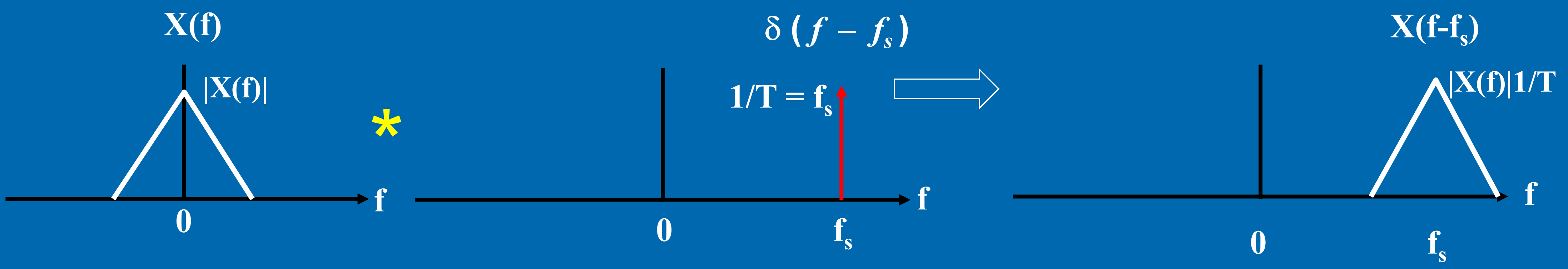
$$X(f) * \delta(f) = X(f)$$



Recall (Chapter 2)

Convolution of any function *X*(*f*) with a unit impulse function $\delta(f-f_s)$ gives the frequency shifted version: $X(f-f_s)$

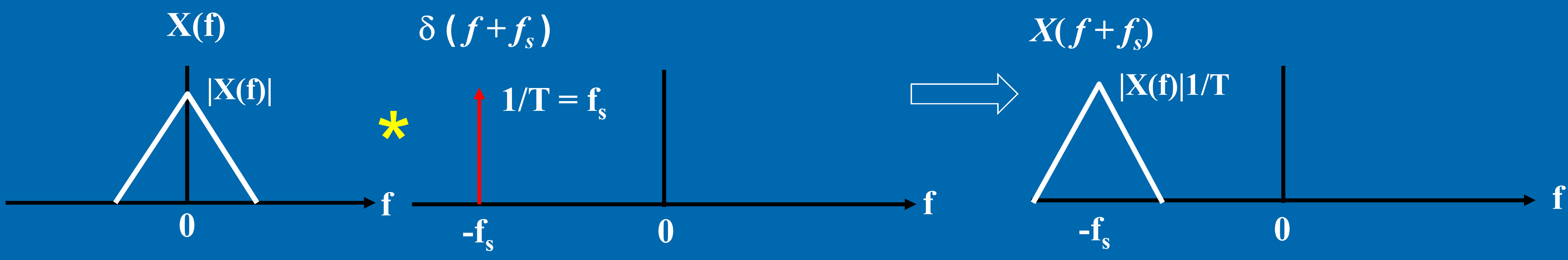
$$X(f) * \delta(f-f_s) = X(f-f_s)$$



Recall (Chapter 2)

Convolution of any function $X(f)$ with a unit impulse function $\delta(f+f_s)$ gives the frequency shifted version: $X(f+f_s)$

$$X(f) * \delta(f+f_s) = X(f+f_s)$$

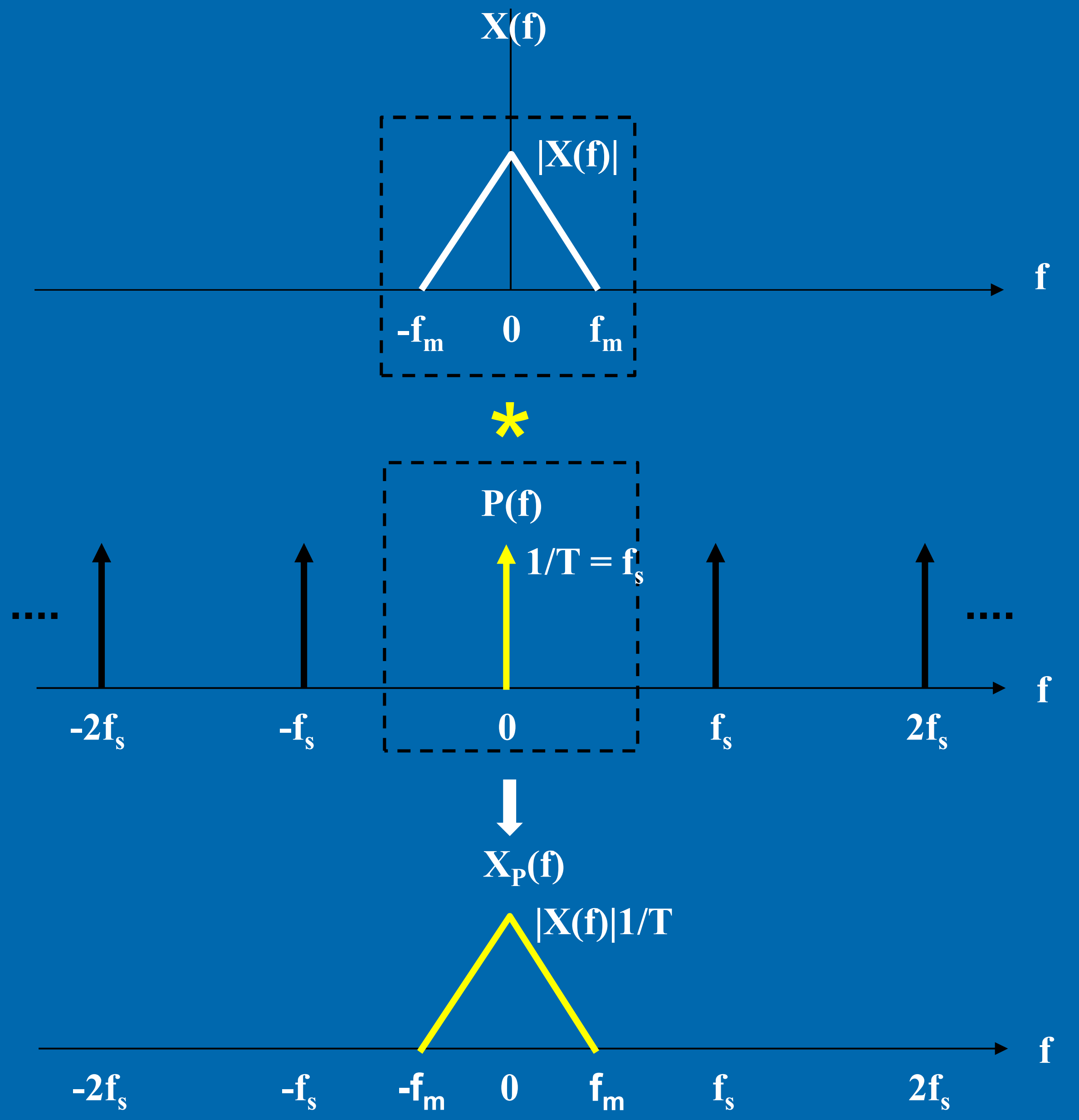


7.1 Sampling

Ideal Sampling

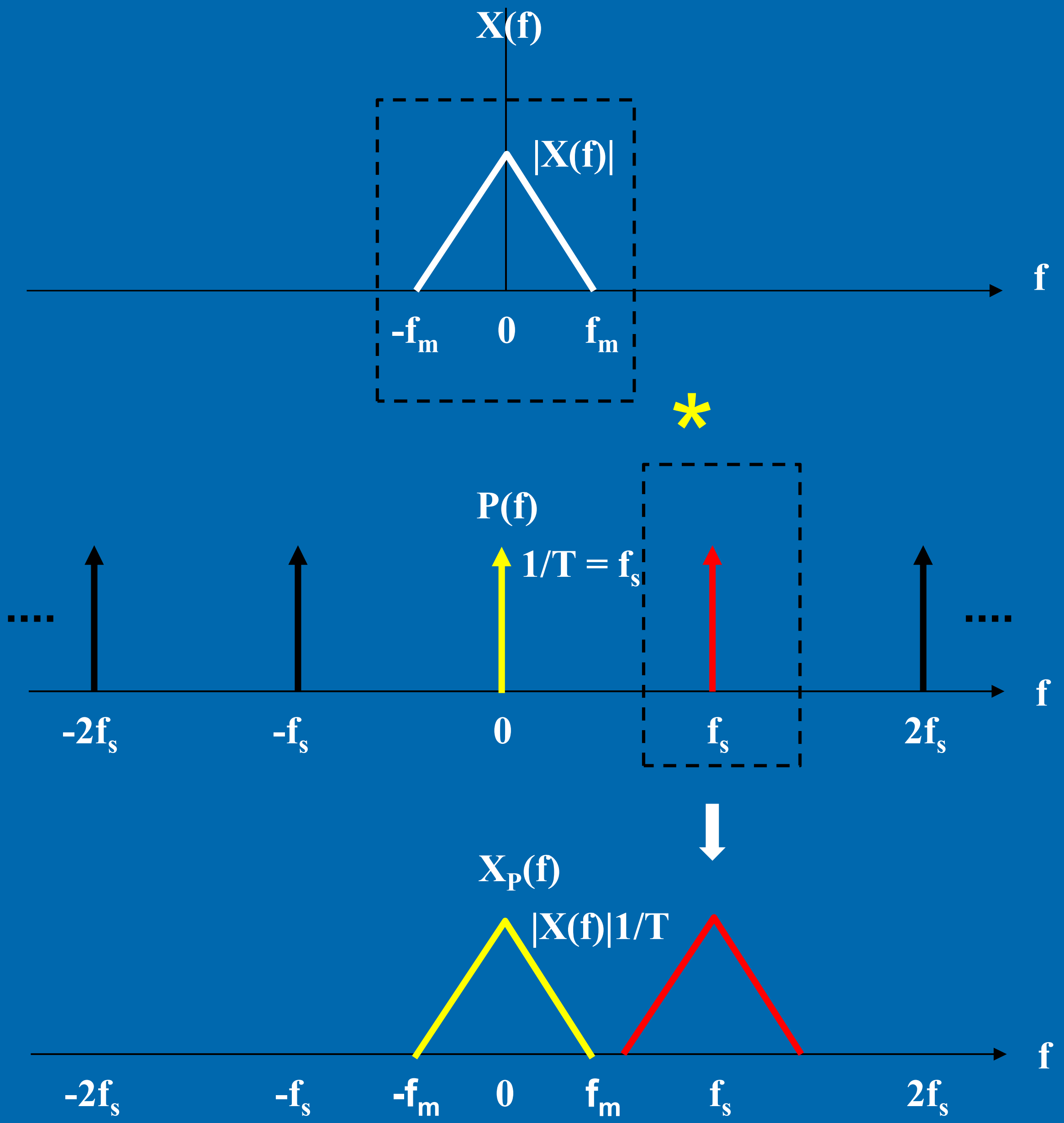
$$P(f)=\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f-kf_s)$$

$$X_p(f)=X(f)*P(f)$$



7.1 Sampling

Ideal Sampling

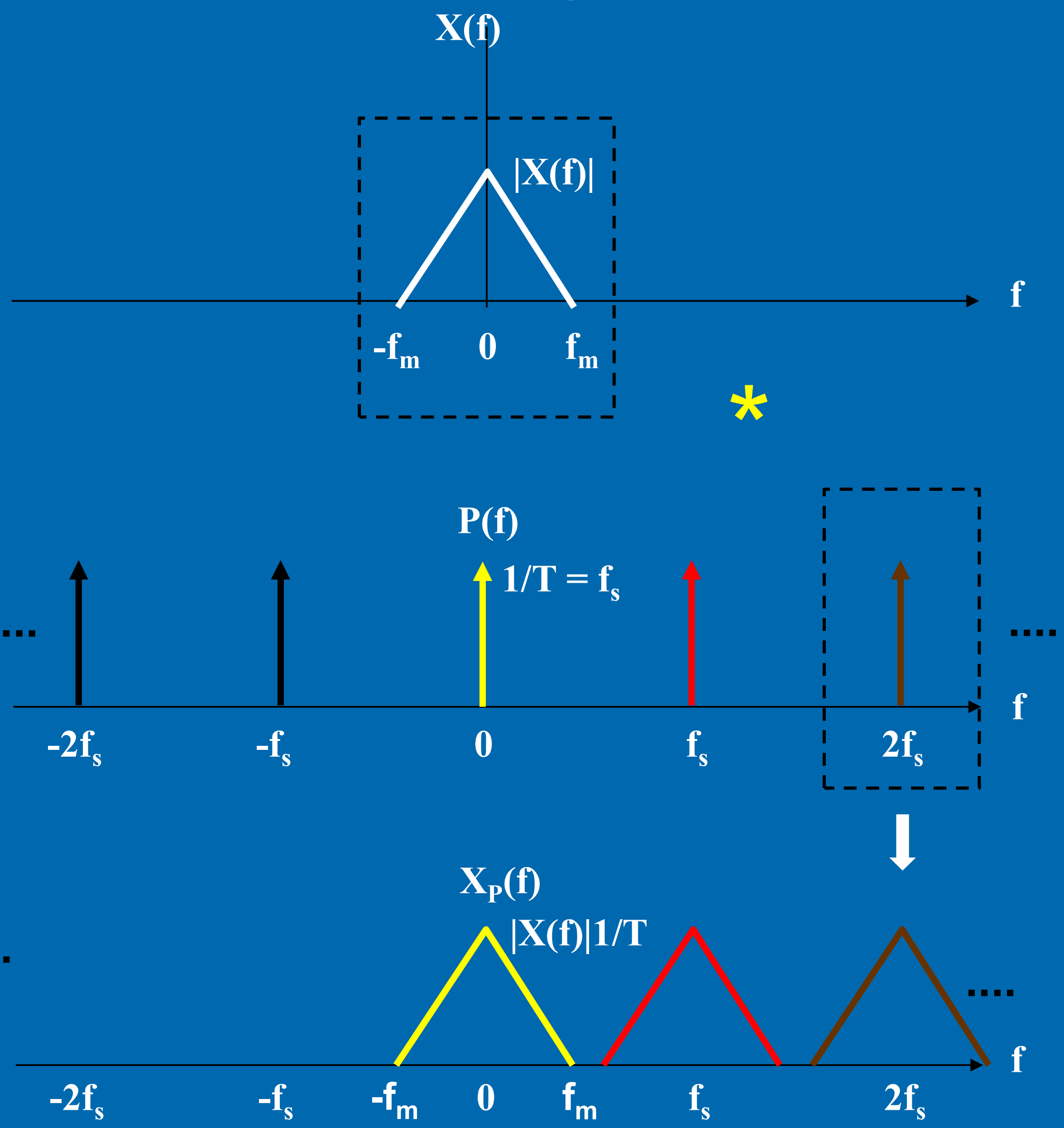


$$P(f) = \frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f - kf_s)$$

$$X_p(f) = X(f) * P(f)$$

7.1 Sampling

Ideal Sampling



$$P(f)=\frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f-kf_s)$$ ....

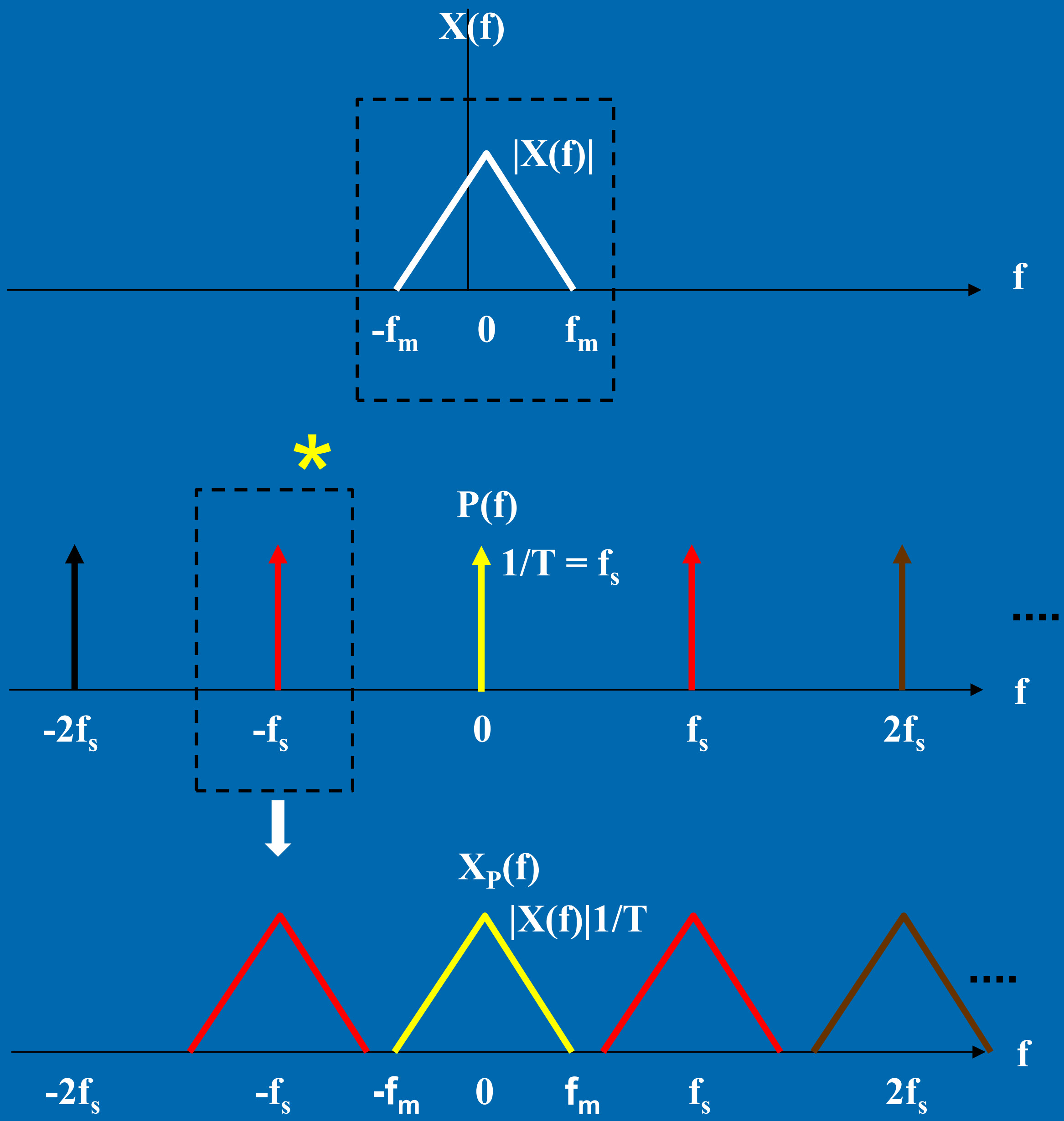
$$X_p(f)=X(f)*P(f)$$ ....

7.1 Sampling

Ideal Sampling

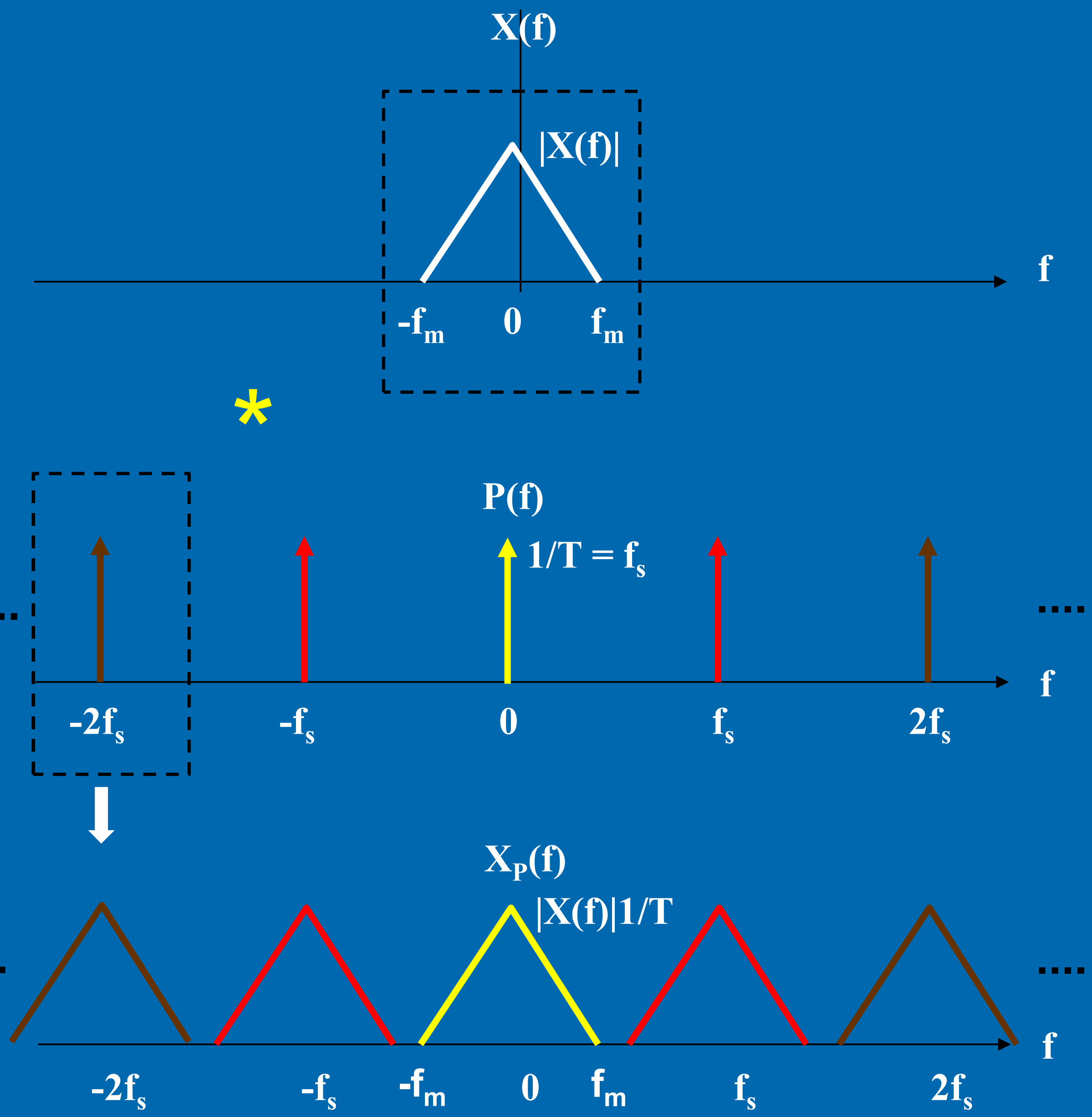
$$P(f) = \frac{1}{T}\sum_{k=-\infty}^{\infty}\delta(f - kf_s)$$

$$X_p(f) = X(f) * P(f)$$



7.1 Sampling

Ideal Sampling



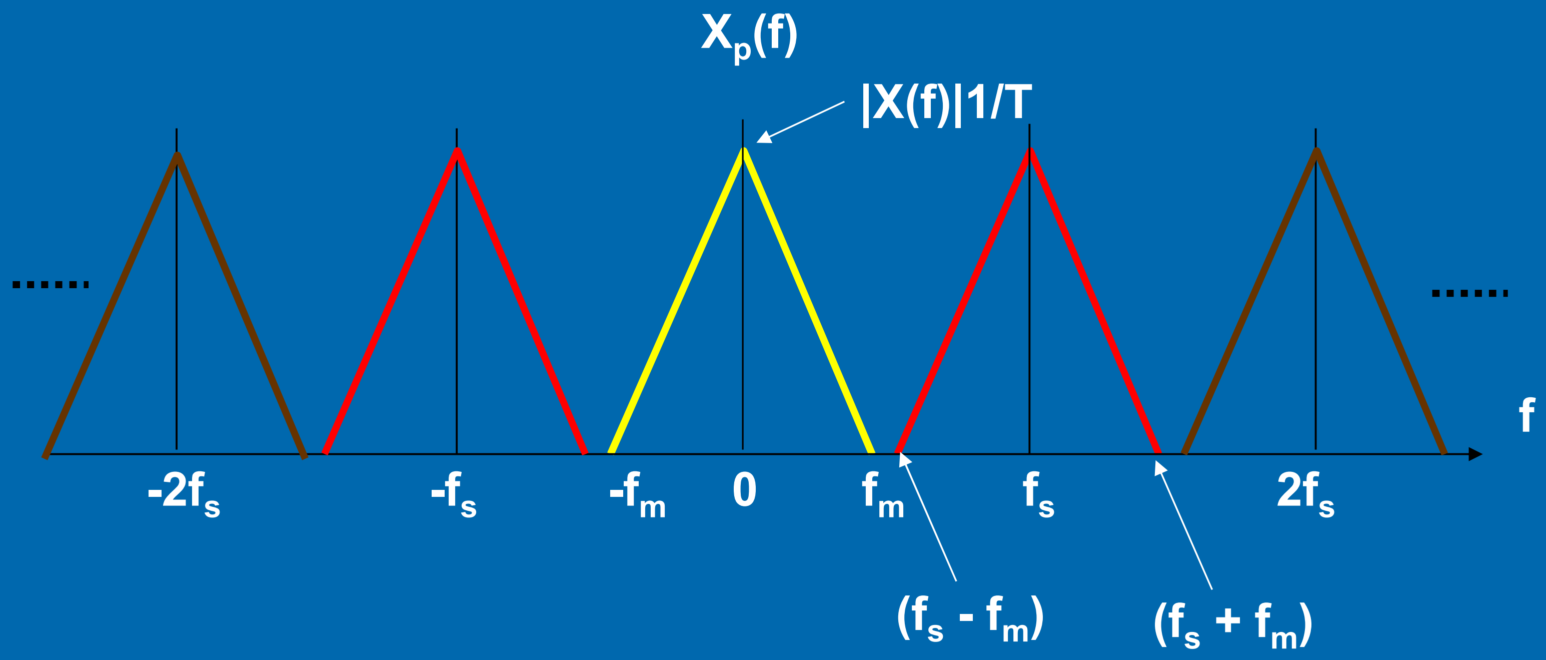
$$P(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta(f - kf_s)$$

$$X_p(f) = X(f) * P(f)$$

7.1 Sampling

Ideal Sampling

Amplitude spectrum of the sampled signal



7.1 Sampling

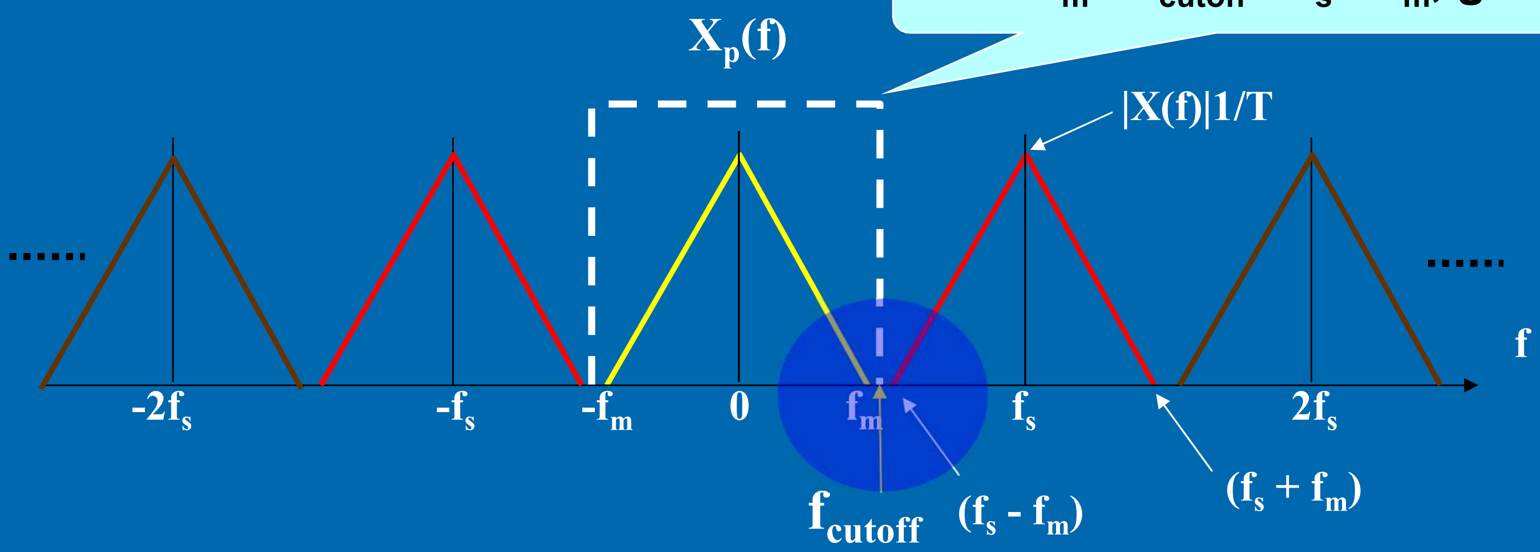
Ideal Sampling

- If $f_s > 2\,f_m$, $f_s > f_m + f_m \Rightarrow f_s - f_m > f_m$

When $f_s > 2f_m$, i.e. $(f_s - f_m) > f_m$ there is no overlap and $x(t)$ can be recovered exactly, by processing $x_p(t)$ through an ideal LPF of gain T and $f_m < f_c < (f_s - f_m)$.

Observation 1 Exact recovery of original signal from sampled signal is possible

LPF : $f_m < f_{cutoff} < f_s - f_m$, gain = T



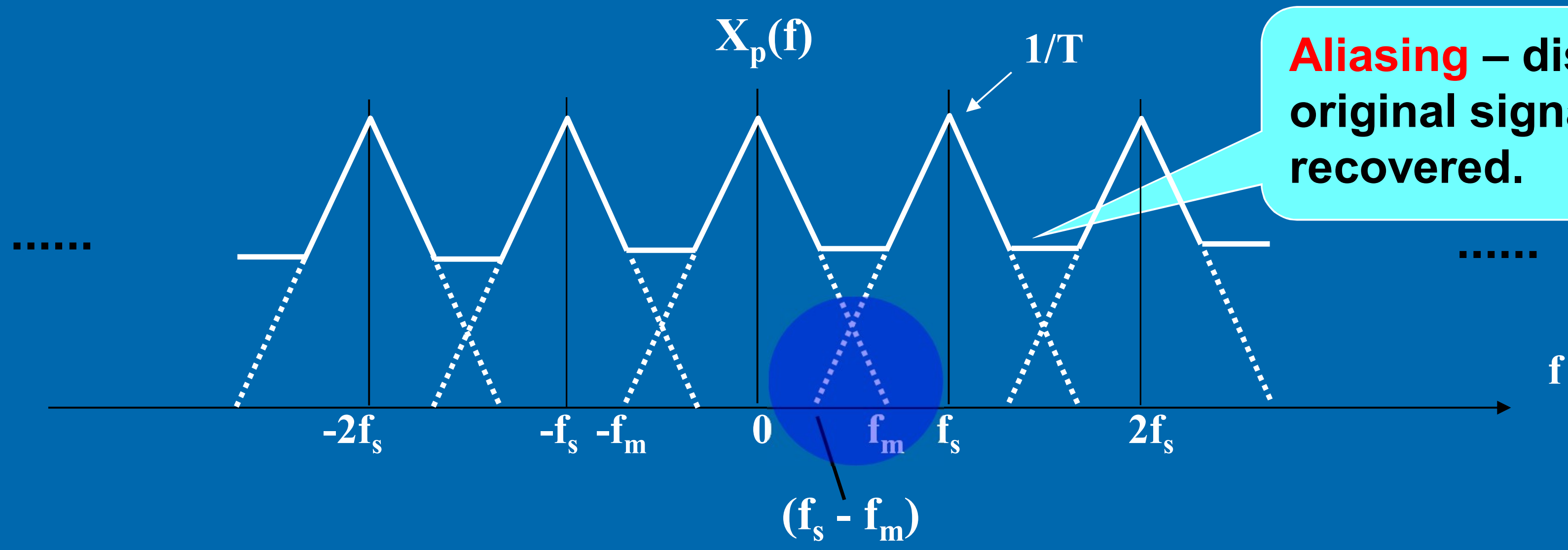
7.1 Sampling

Ideal Sampling

- If $f_s < 2 f_m$, $f_s < f_m + f_m \Rightarrow f_s - f_m < f_m$

When $f_s < 2f_m$, i.e. $(f_s - f_m) < f_m$ there is overlap and $x(t)$ cannot be recovered from the sampled signal. Such overlap of replicas of $X(f)$ is known as **aliasing.**

Observation 2: Aliasing



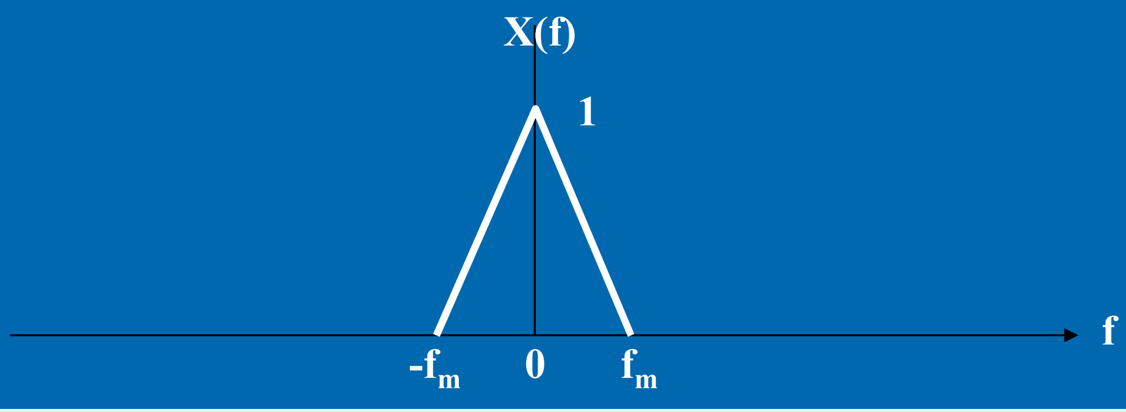
Aliasing – distortion so original signal cannot be recovered.

7.1 Sampling

Ideal Sampling

Uniform Sampling Theorem I

If $x(t)$ is bandlimited to f_m, it must be sampled at $f_s \geq 2f_m$, in order for it to be recovered completely from its samples.



Bandlimited baseband signal

i.e. signals having components from dc to about 1 MHz, e.g.

- **It is common in communication system design, to perform an anti-aliasing filtering before any sampling operation to avoid the occurrence of aliasing.**

7.1 Sampling

Ideal Sampling

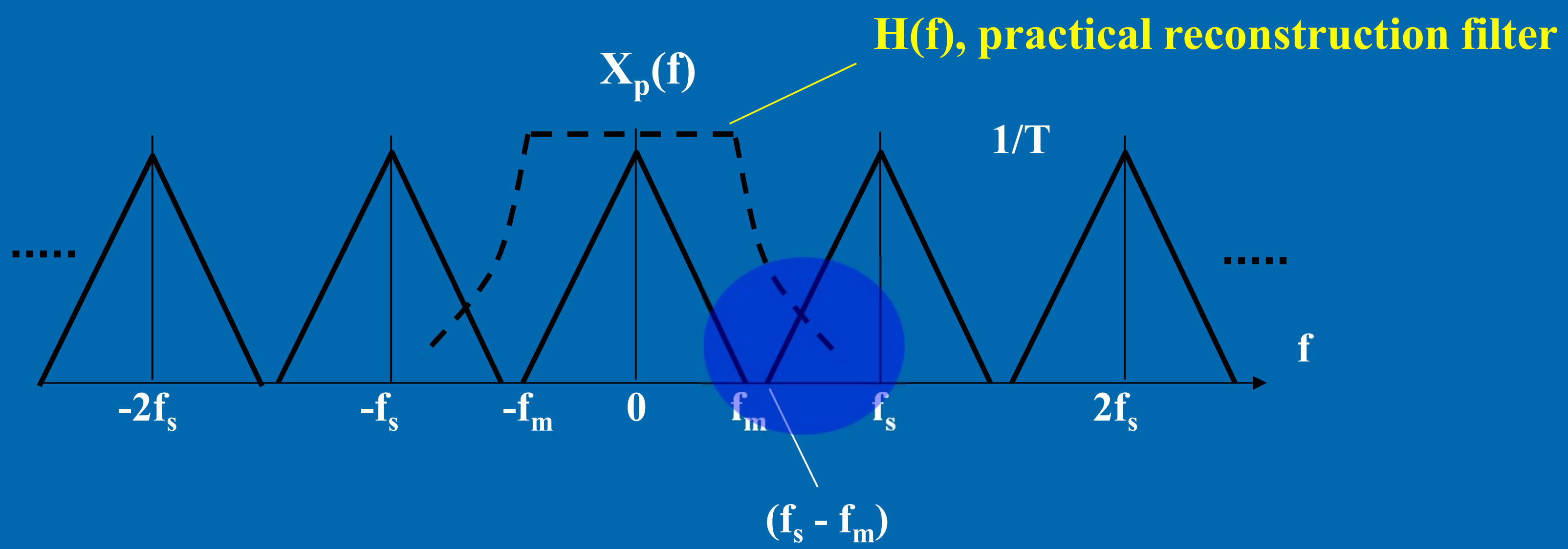
- **Nyquist frequency** and **Nyquist rate**

  - The minimum sampling frequency, $f_s = 2f_m$ is referred to as Nyquist Frequency or Nyquist Rate and

  - Minimum sampling interval, $T_s = 1/f_s = 1/(2f_m)$ as Nyquist Interval.

7.1 Sampling

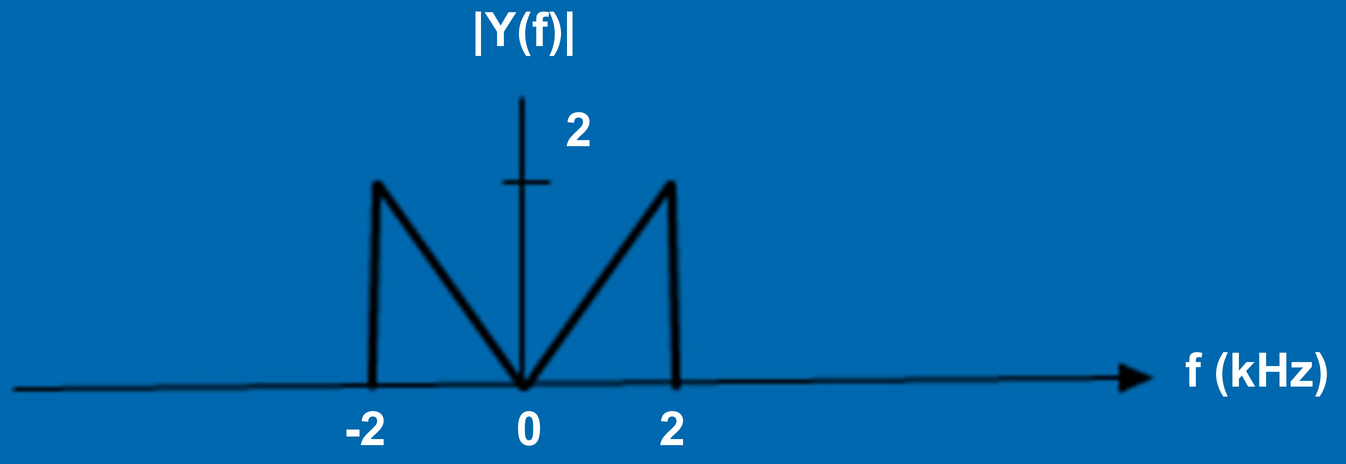
The effects of using Non-ideal reconstruction filter



- In practice, difficult to build filters with a very sharp roll-off at f_c. Some spurious frequency components are allowed by the reconstruction filter to reach the output.
- To minimise these spurious frequencies, a higher sampling frequency should be used.

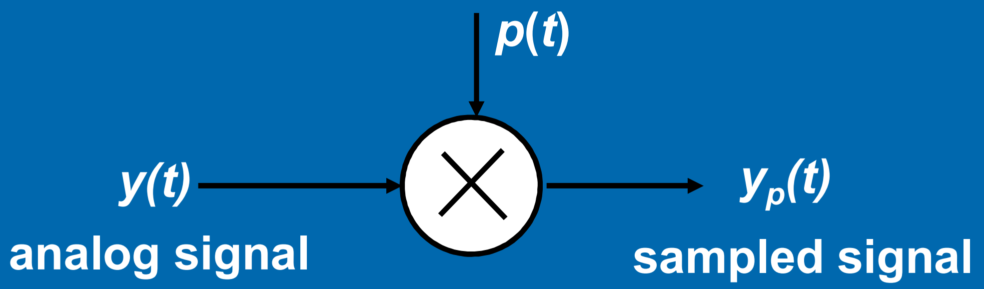
Worked Example 7.1

A signal y(t) with an amplitude spectrum shown below is sampled at 5 kHz by an ideal unit impulse train, p(t). Sketch the amplitude spectrum of the sampled signal.



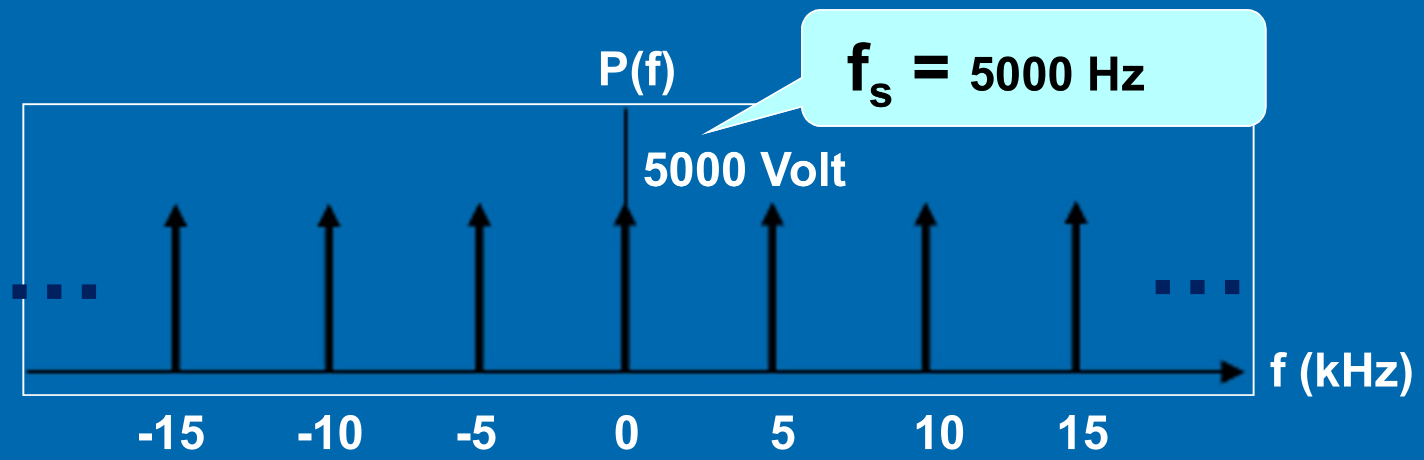
Solution

Sampled signal = y(t) x p(t)

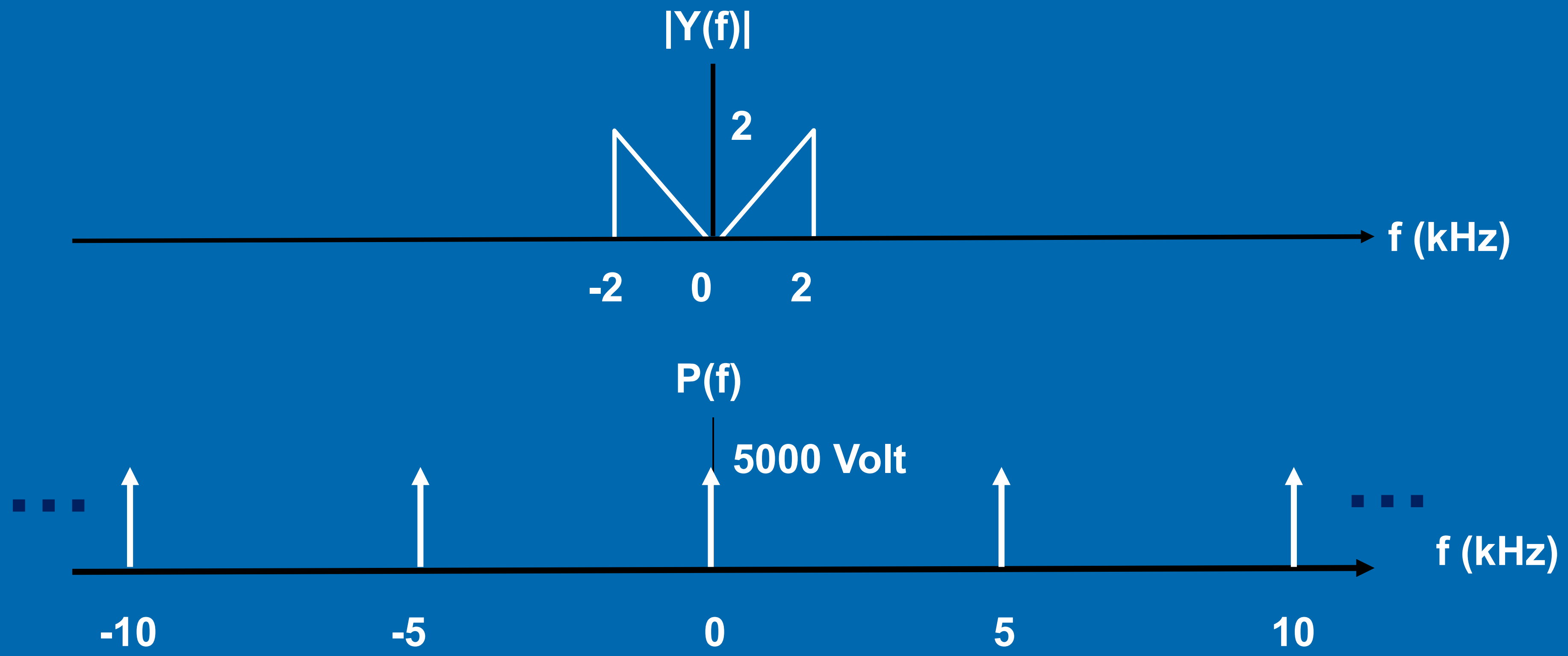


The amplitude spectrum of sampled signal = Y(f)*P(f)

The amplitude spectrum of P(f) is

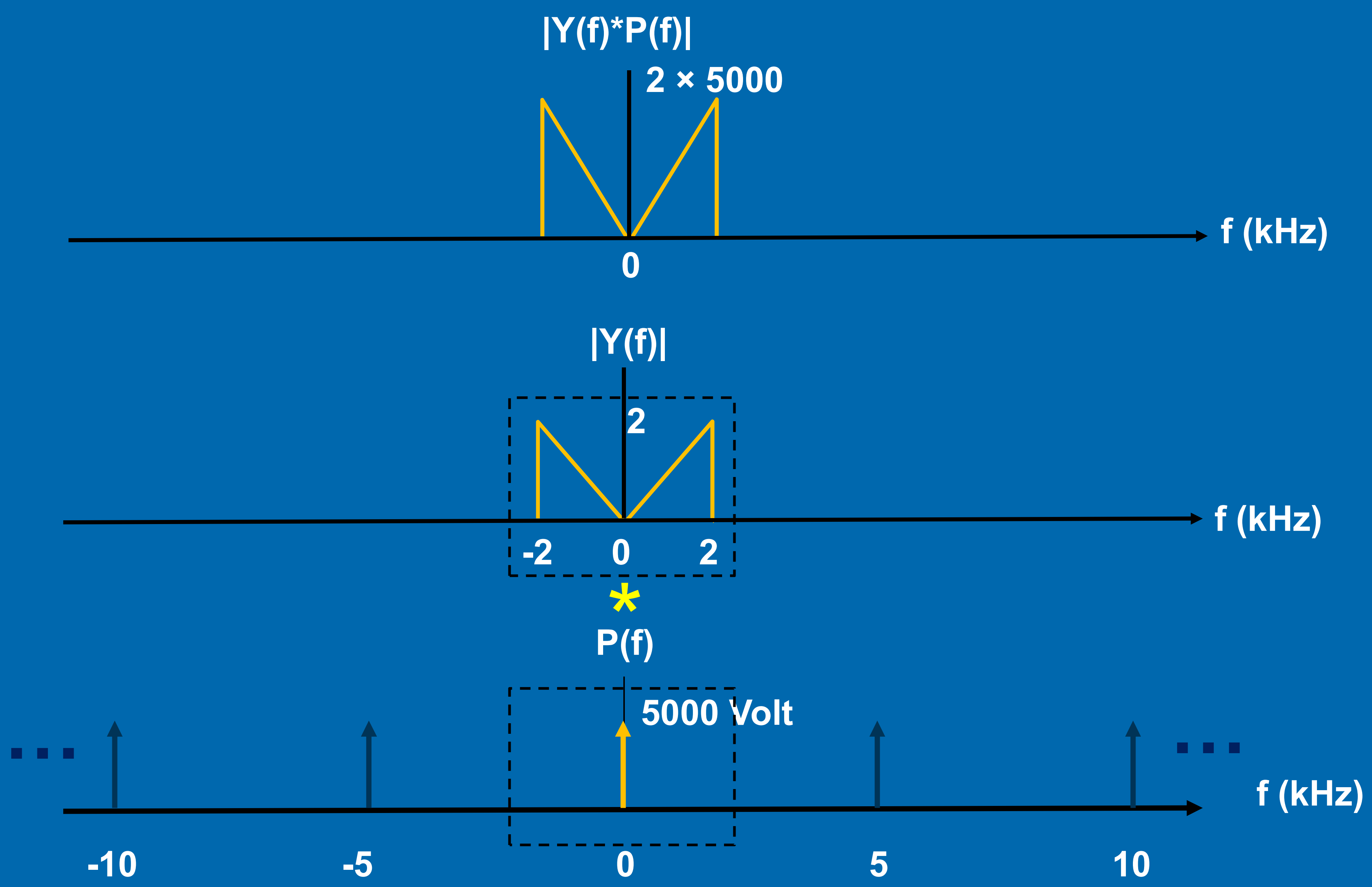


Solution The amplitude spectrum of sampled signal = Y(f)*P(f)



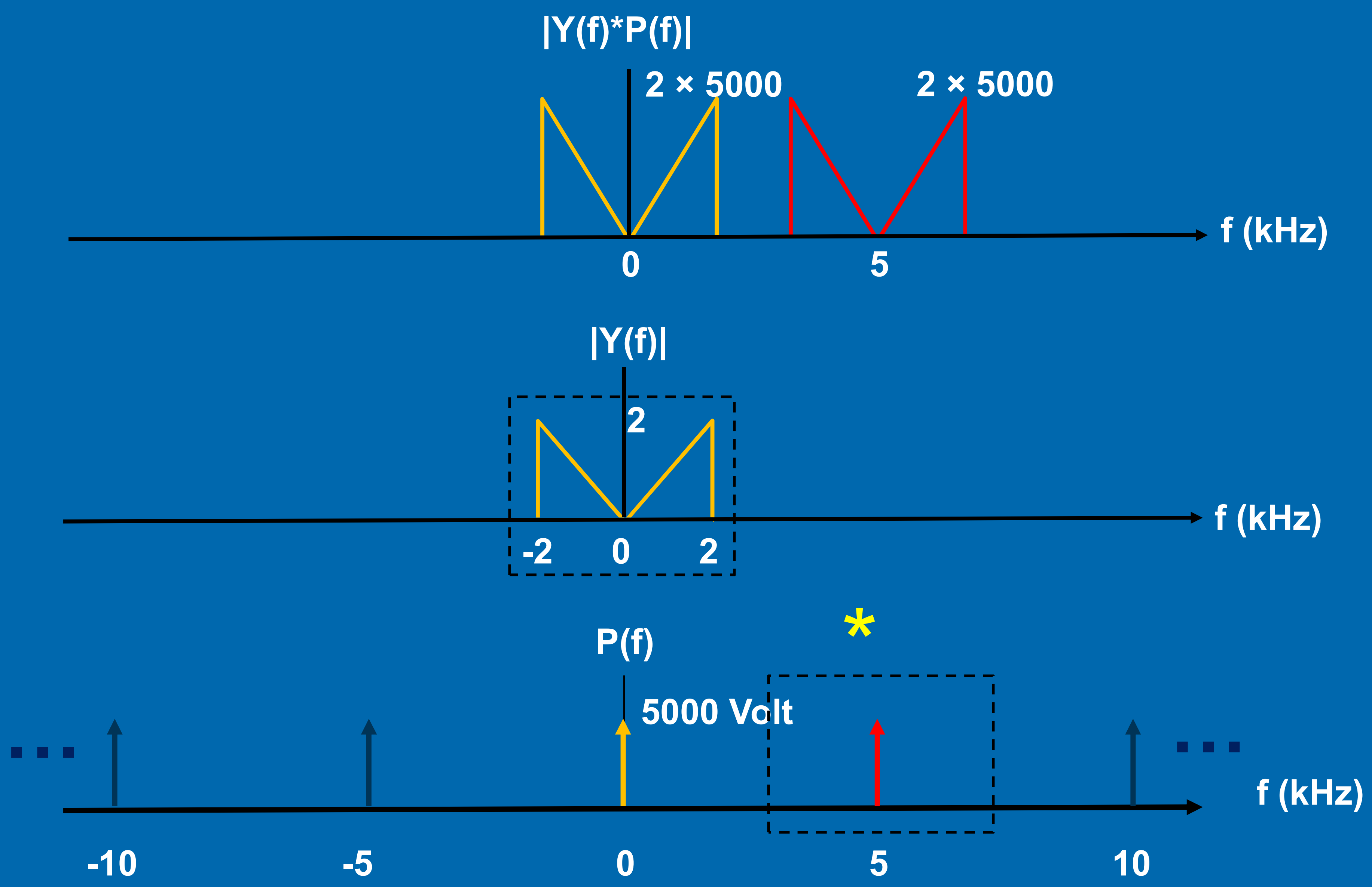
Solution

The amplitude spectrum of sampled signal = Y(f)*P(f)



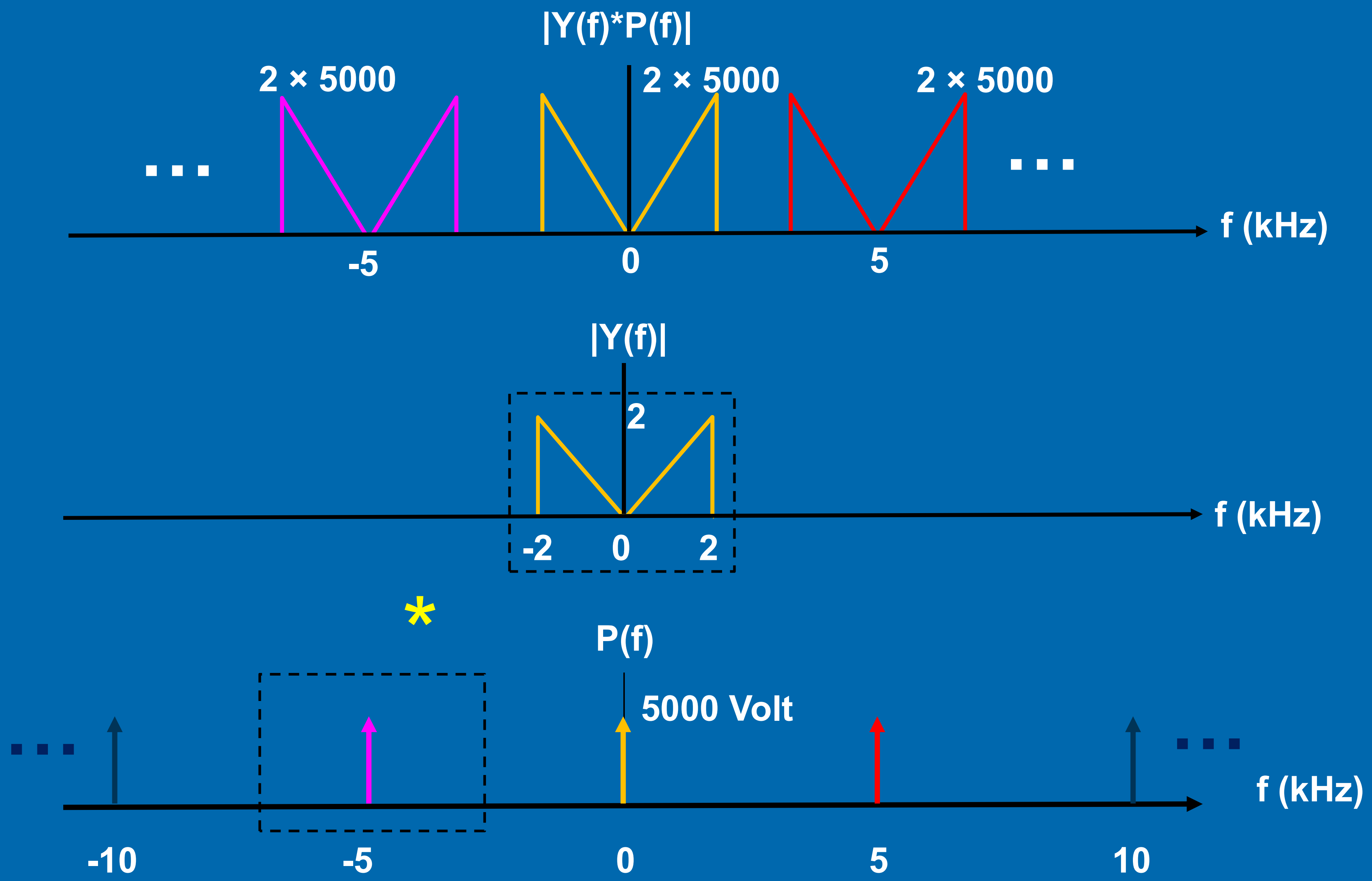
Solution

The amplitude spectrum of sampled signal = Y(f)*P(f)



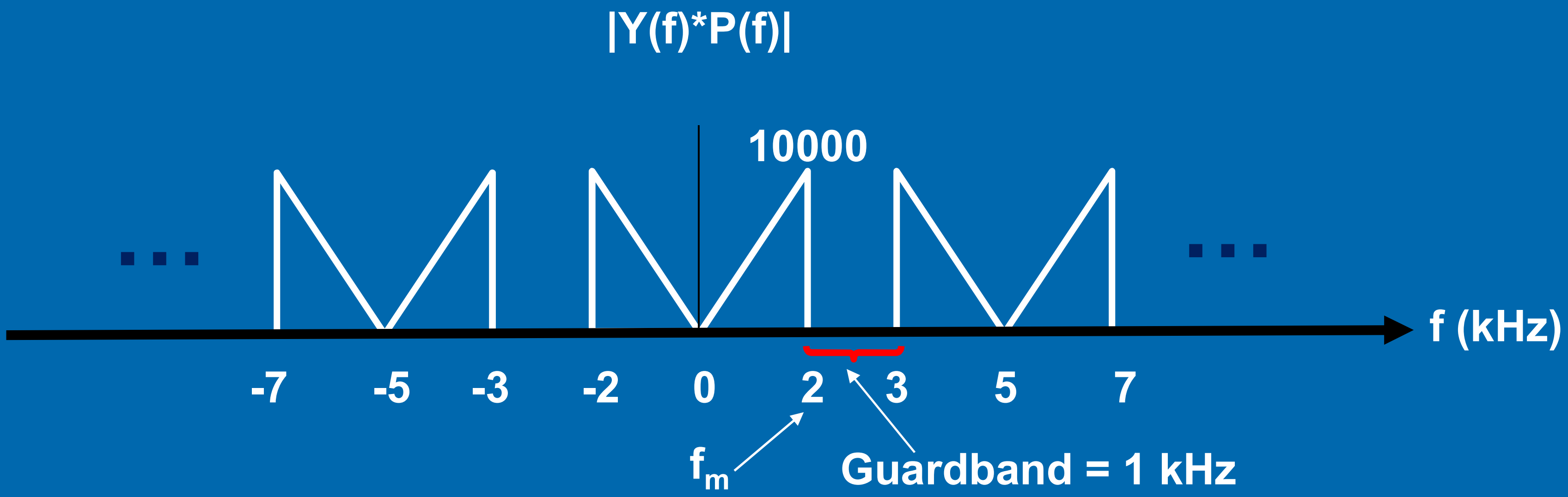
Solution

The amplitude spectrum of sampled signal = Y(f)*P(f)



Solution

Hence, amplitude spectrum of sampled signal is given by



The sample frequency $f_s = 2\ f_m$ + guardband = 5 kHz

7.1 Sampling

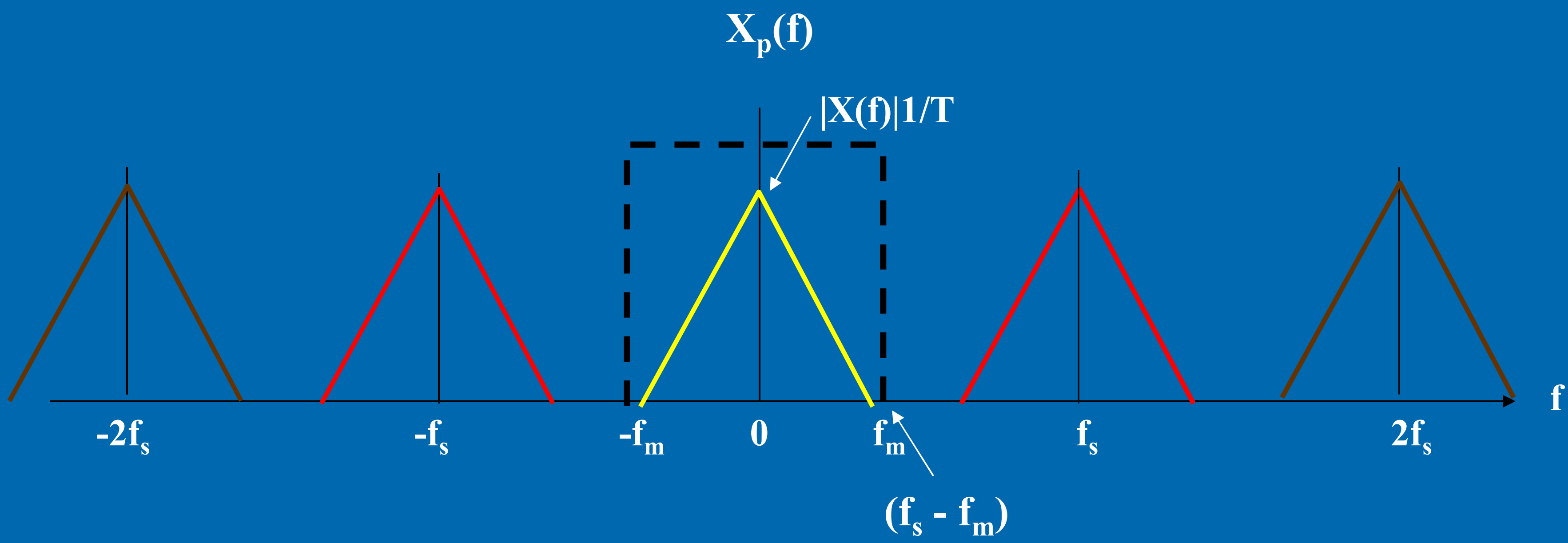
The effects of oversampling and undersampling

Oversampling

$f_s >> f_m + f_m \Rightarrow f_s - f_m >> f_m$

Recovery of original signal is easy but at expense of higher transmission bandwidth.

- For higher sampling freq, $f_s >> 2 f_m$, it is wasteful to sample the signal at so high rate as it increases the bandwidth without any gain (most samples are redundant).



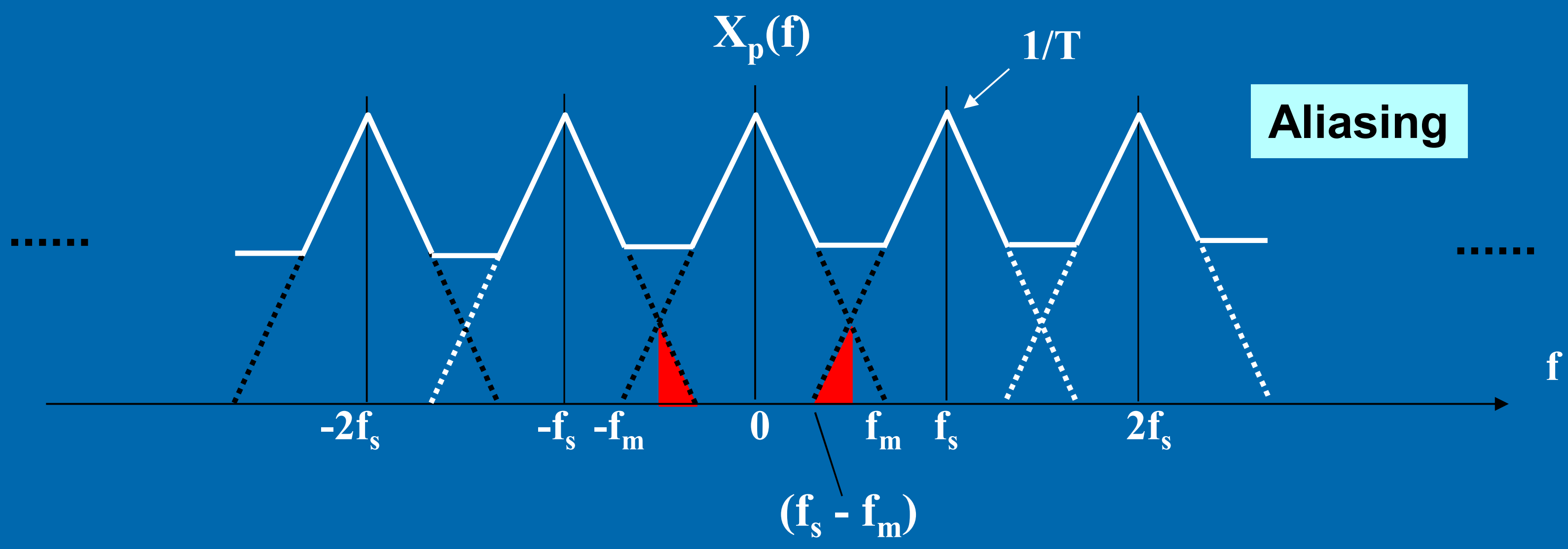
7.1 Sampling

The effects of oversampling and undersampling

Undersampling **Aliasing**

$f_s < f_m + f_m \Rightarrow f_s - f_m < f_m$

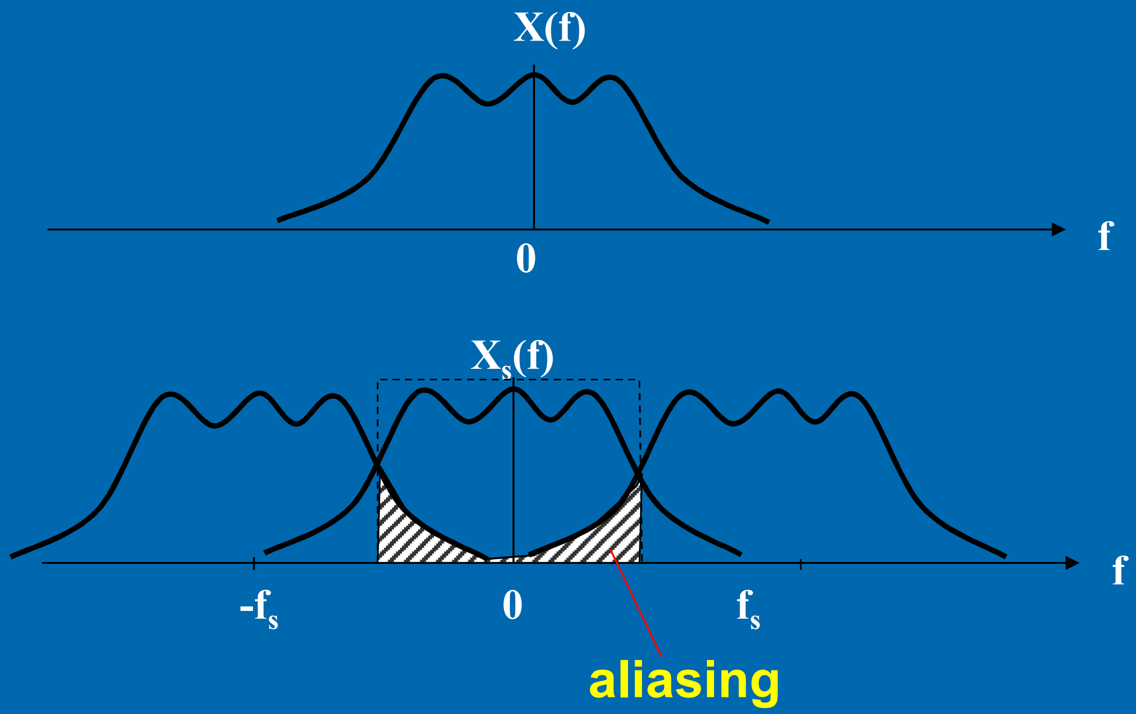
Recovery of original signal is not possible.



The effects of oversampling and undersampling

Undersampling **Aliasing**

- **Besides undersampling, aliasing can also occur when the signal to be sampled is not bandlimited.**



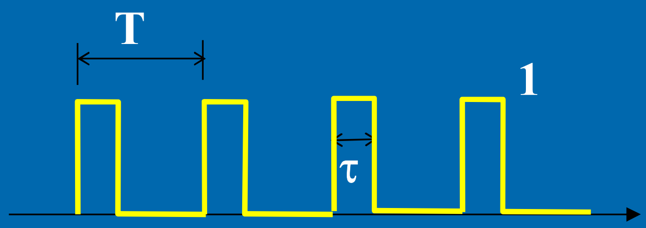
- **Hence, anti-aliasing filter is required to bandlimit the signal before sampling.**

7.1 Sampling

Practical sampling

- **Practical sampling :**

  a) Sampling waveform consists of pulses of finite amplitude and duration.



  b) Practical reconstruction filters do not possess ideal characteristics.

Signal recovery is possible using higher sampling frequency (guardband ↑)

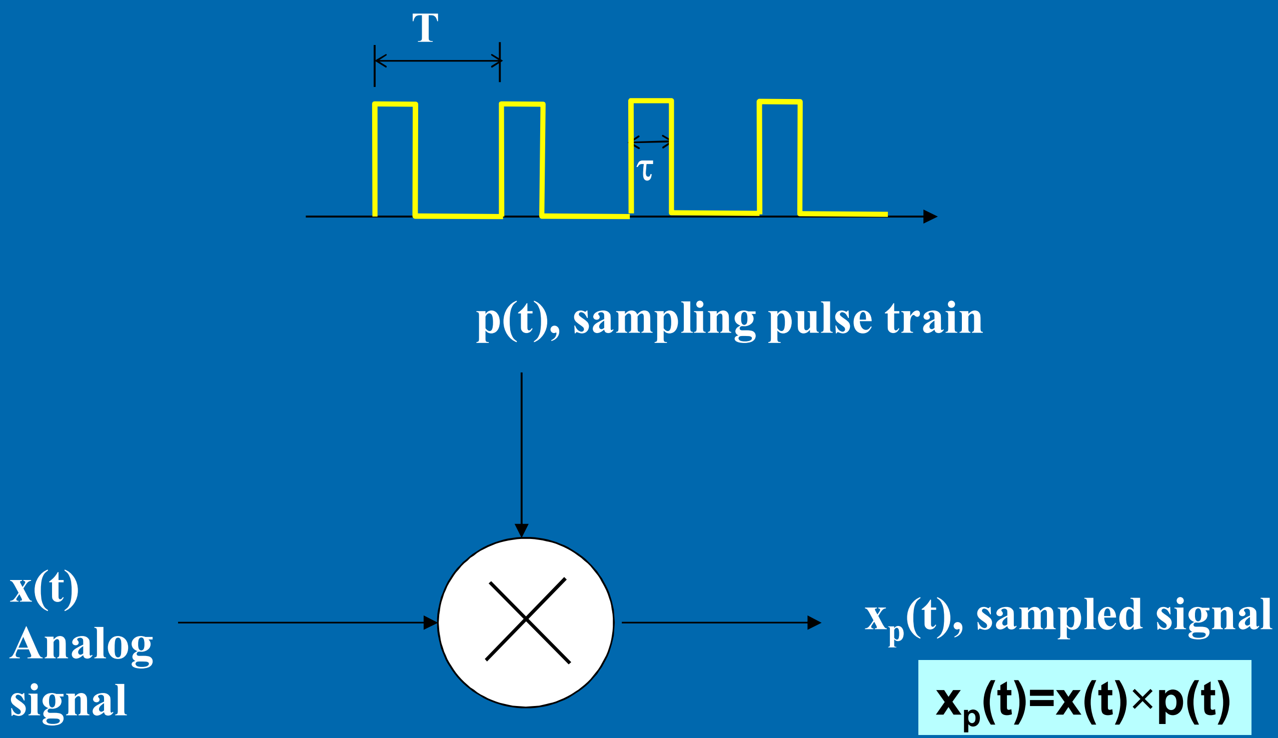
- **There are two forms of practical sampling:**

  1. Natural sampling
  2. Flat-top sampling

Practical sampling

Natural Sampling

- In natural sampling the signal to be sampled is multiplied by a sampling pulse train.

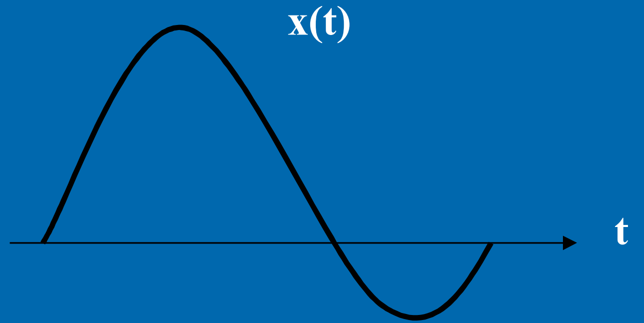


7.1 Sampling

<u>Practical sampling</u>

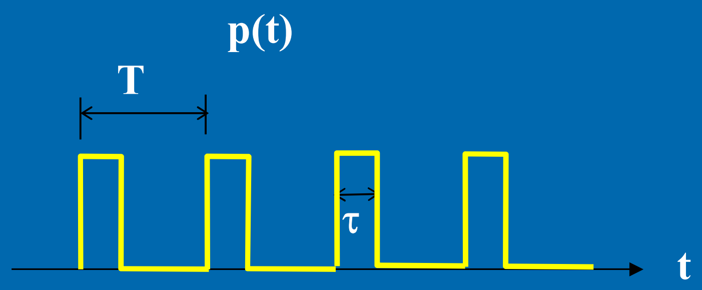
Natural Sampling

a) message signal

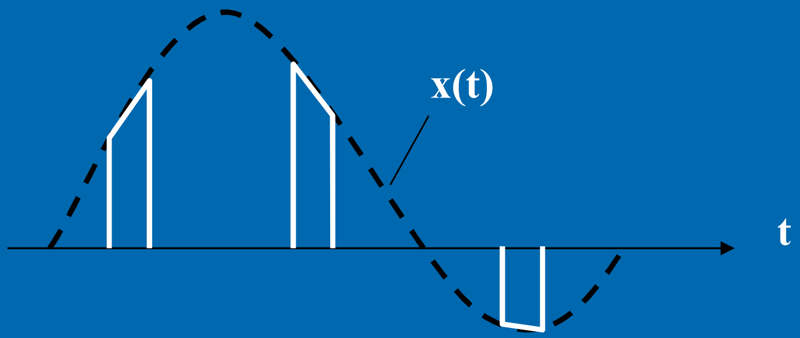


X

b) sampling train



c) Naturally sampled signal $x_p(t)=x(t)\times p(t)$



7.1 Sampling

Singapore Polytechnic

Practical sampling

Natural Sampling

Observation on natural sampling based on the above :

1. **The sampled signal is a pulse train whose amplitude follows the signal over duration τ.**

2. **$f_s \geq 2f_m$ and hence, no aliasing occurs.**

3. ***$x(t)$* can be reconstructed from *$x_p(t)$* by processing it through an ideal LPF:**

$$H(f) = \begin{cases} T/\tau \, ; & |f| \leq |f_c| \\ 0 \, ; & |f| > |f_c| \end{cases} \quad \text{and} \quad |f_m| \leq |f_c| \leq |f_s - f_m|$$

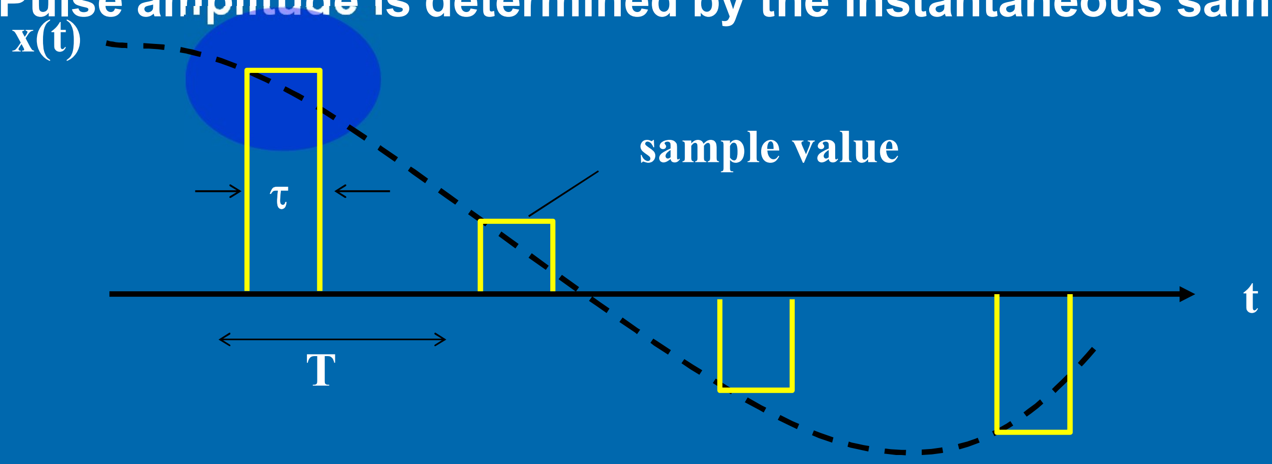
7.1 Sampling

Practical sampling

Flat-top Sampling

- Also known as 'instantaneous sampling'.

- Amplitude of each pulse in the sampled pulse train is constant over duration τ. Pulse amplitude is determined by the instantaneous sample of $x(t)$.

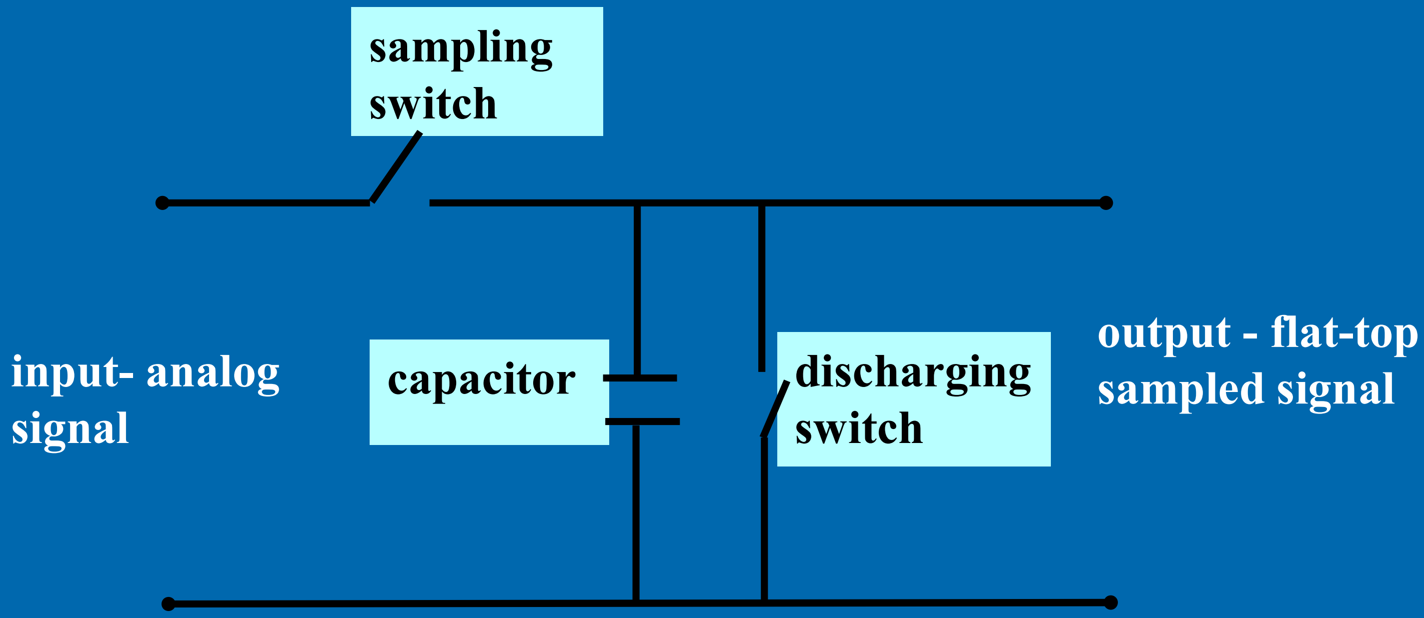


- Generated by a Sample-and-Hold circuit.

7.1 Sampling

Practical sampling

Flat-top Sampling



Sample-and-Hold circuit

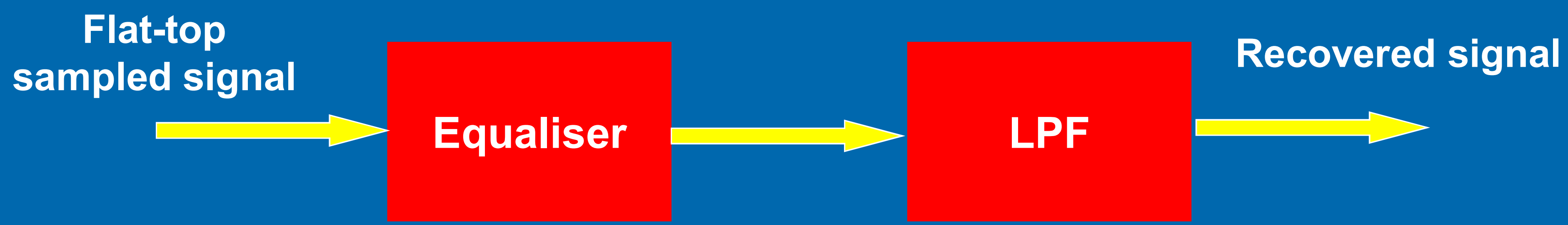
SP Singapore Polytechnic

Practical sampling

Flat-top Sampling

- Unlike Natural sampling, Flat-top sampling does not produce a distortion-free signal. This distortion is called <u>Aperture Distortion.</u>

- This distortion may be corrected by adding a second filter - the equalising filter, in cascade to the reconstruction filter.



END

CHAPTER 7

(Part 1 of 4)