1. **IDENTIFY** and **SET UP:** For parts (a) through (d), identify the appropriate value of ϕ and use the relation $W = F_P s = (F\cos\phi)s$. In part (e), apply the relation $W_{net} = W_{student} + W_{grav} + W_n + W_f$.

EXECUTE: (a) Since you are applying a horizontal force, $\phi = 0°$. Thus,
$W_{student} = (2.40\text{ N})(\cos 0°)(1.50\text{ m}) = 3.60\text{ J}$
(b) The friction force acts in the horizontal direction, opposite to the motion, so $\phi = 180°$.
$W_f = (F_f\cos\phi)s = (0.600\text{ N})(\cos 180°)(1.50\text{ m}) = -0.900\text{ J}.$
(c) Since the normal force acts upward and perpendicular to the tabletop, $\phi = 90°$.
$W_n = (n\cos\phi)s = (ns)(\cos 90°) = 0.0\text{ J}$
(d) Since gravity acts downward and perpendicular to the tabletop, $\phi = 270°$.
$W_{grav} = (mg\cos\phi)s = (mgs)(\cos 270°) = 0.0\text{ J}.$
(e) $W_{net} = W_{student} + W_{grav} + W_n + W_f = 3.60\text{ J} + 0.0\text{ J} + 0.0\text{ J} - 0.900\text{ J} = 2.70\text{ J}.$

EVALUATE: Whenever a force acts perpendicular to the direction of motion, its contribution to the net work is zero.

2. **IDENTIFY:** The gravity force is constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

SET UP: The displacement is upward along the ladder and the gravity force is downward, so $\phi = 180.0° - 30.0° = 150.0°$. $w = mg = 735\text{ N}$.

EXECUTE: (a) $W = (735\text{ N})(2.75\text{ m})\cos 150.0° = -1750\text{ J}.$
(b) No, the gravity force is independent of the motion of the painter.

EVALUATE: Gravity is downward and the vertical component of the displacement is upward, so the gravity force does negative work.

3. **IDENTIFY:** We want to find the work done by a known force acting through a known displacement.

SET UP: $W = \vec{F}\cdot\vec{s} = F_x s_x + F_y s_y$. We know the components of $\vec{F}$ but need to find the components of the displacement $\vec{s}$.

EXECUTE: Using the magnitude and direction of $\vec{s}$, its components are $x = (48.0\text{ m})\cos 240.0^{o} = -24.0\text{ m}$ and $y = (48.0\text{ m})\sin 240.0^{o} = -41.57\text{ m}$. Therefore, $\vec{s} = (-24.0\text{ m})\hat{\imath} + (-41.57\text{ m})\hat{\jmath}$. The definition of work gives $W = \vec{F}\cdot\vec{s} = (-68.0\text{ N})(-24.0\text{ m}) + (36.0\text{ N})(-41.57\text{ m}) = +1632\text{ J} - 1497\text{ J} = +135\text{ J}$

Evaluate: The mass of the car is not needed since it is the given force that is doing the work.

4. **IDENTIFY** and **SET UP:** Let point 1 be at the bottom of the incline and let point 2 be at the skier. Work is done by gravity and by friction. Solve for K_1 and from that obtain the required initial speed.

EXECUTE: $W_{tot} = K_2 - K_1$
$K_1 = \frac{1}{2}mv_0^2$, $K_2 = 0$
Work is done by gravity and friction, so $W_{tot} = W_{mg} + W_f$.
$W_{mg} = -mg(y_2 - y_1) = -mgh$
$W_f = -fs$. The normal force is $n = mg\cos\alpha$ and $s = h/\sin\alpha$, where s is the distance the box travels along the incline.
$W_f = -(\mu_k mg\cos\alpha)(h/\sin\alpha) = -\mu_k mgh/\tan\alpha$
Substituting these expressions into the work-energy theorem gives
$-mgh - \mu_k mgh/\tan\alpha = -\frac{1}{2}mv_0^2$.
Solving for v_0 then gives $v_0 = \sqrt{2gh(1 + \mu_k/\tan\alpha)}$.

EVALUATE: The result is independent of the mass of the box. As $\alpha \to 90°$, $h = s$ and $v_0 = \sqrt{2gh}$, the same as throwing the box straight up into the air. For $\alpha = 90°$ the normal force is zero so there is no friction.

5. **IDENTIFY:** We know (or can calculate) the change in the kinetic energy of the crate and want to find the work needed to cause this change, so the work-energy theorem applies.

SET UP: $W_{\text{tot}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2$.

EXECUTE: $W_{\text{tot}} = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}(30.0\text{ kg})(5.62\text{ m/s})^2 - \frac{1}{2}(30.0\text{ kg})(3.90\text{ m/s})^2$.

$W_{\text{tot}} = 473.8\text{ J} - 228.2\text{ J} = 246\text{ J}$.

EVALUATE: Kinetic energy is a scalar and does not depend on direction, so only the initial and final speeds are relevant.

6. **IDENTIFY:** The force does work on the box, which gives it kinetic energy, so the work-energy theorem applies. The force is variable so we must integrate to calculate the work it does on the box.

SET UP: $W_{\text{tot}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2$ and $W_{\text{tot}} = \int_{x_1}^{x_2} F(x)dx$.

EXECUTE: $W_{\text{tot}} = \int_{x_1}^{x_2} F(x)dx = \int_0^{14.0\text{m}} [18.0\text{ N} - (0.530\text{ N/m})x]dx$

$W_{\text{tot}} = (18.0\text{ N})(14.0\text{ m}) - (0.265\text{ N/m})(14.0\text{ m})^2 = 252.0\text{ J} - 51.94\text{ J} = 200.1\text{ J}$. The initial kinetic energy is zero, so $W_{\text{tot}} = \Delta K = K_{\text{f}} - K_{\text{i}} = \frac{1}{2}mv_{\text{f}}^2$. Solving for v_{f} gives $v_{\text{f}} = \sqrt{\frac{2W_{\text{tot}}}{m}} = \sqrt{\frac{2(200.1\text{ J})}{6.00\text{ kg}}} = 8.17\text{ m/s}$.

Evaluate: We could not readily do this problem by integrating the acceleration over time because we know the force as a function of x, not of t. The work-energy theorem provides a much simpler method.

7. **IDENTIFY** and **SET UP:** Use Eq. to relate the power provided and the amount of work done against gravity in 16.0 s. The work done against gravity depends on the total weight which depends on the number of passengers.

EXECUTE: Find the total mass that can be lifted:

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} = \frac{mgh}{t}, \text{ so } m = \frac{P_{\text{av}}t}{gh}$$

$$P_{\text{av}} = (40\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 2.984\times10^4\text{ W}$$

$$m = \frac{P_{\text{av}}t}{gh} = \frac{(2.984\times10^4\text{ W})(16.0\text{ s})}{(9.80\text{ m/s}^2)(20.0\text{ m})} = 2.436\times10^3\text{ kg}$$

This is the total mass of elevator plus passengers. The mass of the passengers is

$2.436\times10^3\text{ kg} - 600\text{ kg} = 1.836\times10^3\text{ kg}$. The number of passengers is $\frac{1.836\times10^3\text{ kg}}{65.0\text{ kg}} = 28.2$.

28 passengers can ride.

EVALUATE: Typical elevator capacities are about half this, in order to have a margin of safety.

8. (a) The SI unit of α is N/m^2.The force is not constant. The properties of work done by a conservative force are described

$$W = \int_1^2 \vec{F} \cdot d\vec{l},$$

$$\vec{F} = -\alpha x^2 \hat{i}$$

(b) $d\vec{l} = dx\hat{i}$

$$\vec{F} \cdot d\vec{l} = (-\alpha x^2 \hat{i}) \cdot (dx\hat{i}) = -\alpha x^2 \ dx$$

$$W = \int_{x_1}^{x_2} (-\alpha x^2) dx = -\tfrac{1}{3}\alpha x^3 \Big|_{x_1}^{x_2} = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = -\frac{12 \text{ N/m}^2}{3}((0.300 \text{ m})^3 - (0.10 \text{ m})^3) = -0.10 \text{ J}$$

now $x_1 = 0.30$ m and $x_2 = 0.10$ m

$$W = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = +0.10 \text{ J}$$

(c) The total work for the displacement along the x-axis from 0.10 m to 0.30 m and then back to 0.10 m is zero. The total work is zero when the starting and ending points are the same, so the force is conservative.

$$W_{x_1 \to x_2} = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = \tfrac{1}{3}\alpha x_1^3 - \tfrac{1}{3}\alpha x_2^3$$

The definition of the potential energy function is

$$W_{x_1 \to x_2} = U_1 - U_2.$$

Comparison of the two expressions for W gives $U = 1/3\ \alpha x^3$. This does correspond to U = 0 when x = 0.

9. **IDENTIFY:** The normal force does no work, so only gravity does work and Eq. (7.4) applies.

SET UP: $K_1 = 0$. The crate's initial point is at a vertical height of $d\sin\alpha$ above the bottom of the ramp.

EXECUTE: **(a)** $y_2 = 0$, $y_1 = d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $U_{\text{grav},1} = K_2$. $mgd\sin\alpha = \tfrac{1}{2}mv_2^2$ and $v_2 = \sqrt{2gd\sin\alpha}$.

(b) $y_1 = 0$, $y_2 = -d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $0 = K_2 + U_{\text{grav},2}$. $0 = \tfrac{1}{2}mv_2^2 + (-mgd\sin\alpha)$ and $v_2 = \sqrt{2gd\sin\alpha}$, the same as in part (a).

(c) The normal force is perpendicular to the displacement and does no work.

EVALUATE: When we use $U_{\text{grav}} = mgy$ we can take any point as $y = 0$ but we must take $+y$ to be upward.

10. **IDENTIFY:** Only the spring does work and Eq. (7.11) applies. $a = \frac{F}{m} = \frac{-kx}{m}$, where F is the force the spring exerts on the mass.

SET UP: Let point 1 be the initial position of the mass against the compressed spring, so $K_1 = 0$ and $U_1 = 11.5$ J. Let point 2 be where the mass leaves the spring, so $U_{\text{el},2} = 0$.

EXECUTE: (a) $K_1 + U_{\text{el},1} = K_2 + U_{\text{el},2}$ gives $U_{\text{el},1} = K_2$. $\tfrac{1}{2}mv_2^2 = U_{\text{el},1}$ and

$$v_2 = \sqrt{\frac{2U_{\text{el},1}}{m}} = \sqrt{\frac{2(11.5 \text{ J})}{2.50 \text{ kg}}} = 3.03 \text{ m/s}.$$

K is largest when U_{el} is least and this is when the mass leaves the spring. The mass achieves its maximum speed of 3.03 m/s as it leaves the spring and then slides along the surface with constant speed.

(b) The acceleration is greatest when the force on the mass is the greatest, and this is when the spring has its maximum compression. $U_{\text{el}} = \tfrac{1}{2}kx^2$ so $x = -\sqrt{\frac{2U_{\text{el}}}{k}} = 2\sqrt{\frac{2(11.5 \text{ J})}{2500 \text{ N/m}}} = -0.0959$ m. The minus sign indicates compression. $F = -kx = ma_x$ and $a_x = -\frac{kx}{m} = -\frac{(2500 \text{ N/m})(-0.0959 \text{ m})}{2.50 \text{ kg}} = 95.9 \text{ m/s}^2$.

EVALUATE: If the end of the spring is displaced to the left when the spring is compressed, then a_x in part (b) is to the right, and vice versa.