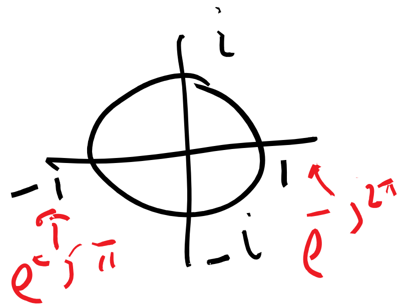
Revision

1. Know your Discrete Fourier Transform well.

A3 Evaluate the $N = 4$-point DFT for $X(0)$ and $X(2)$ if $x(n) = \{0, 2, 0, -2\}$.

A3 When N=4, k= for k = 0, 1,2 and 3



$$X(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi kn}{4}}$$

$$= x(0) + x(1)e^{-j\frac{2\pi k}{4}} + x(2)e^{-j\frac{4\pi k}{4}} + x(3)e^{-j\frac{6\pi k}{4}}.$$

$X(0) = 0 \leftarrow x(0) + x(1) + x(2) + x(3)$

$X(2) = 0 \qquad 0 + 2 + 0 + (-2)$

$$x(0) + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi}$$

$$0 + 2\times -1 + 0 \times 1 + -2 \times -1$$

$$X(2) = 0$$

2. Know your partial fraction

A6 The system function of a digital system is given as:

$$X(z) = \frac{z}{3z^2 - 4z + 1}$$

Using partial fraction, find $x(n)$.

A6 Applying the partial fraction expansion leads to

$$X(z) = \frac{z}{3z^2 - 4z + 1}$$

$$\frac{X(z)}{z} = \frac{1}{(3z-1)(z-1)} = \frac{A}{3z-1} + \frac{B}{z-1}$$

$$A = \frac{X(z)}{z}(3z-1)\Big|\, z = 1/3$$

$$A = -1.5$$

$$B = \frac{X(z)}{z}(z-1)\Big|\, z = 1$$

B=0.5

$$X(z) = \frac{0.5}{1-z^{-1}} - \frac{0.5}{1-\frac{1}{3}z^{-1}}$$

$x(n) = 0.5u(n) - 0.5\,(1/3)^n\,u(n)$ or equivalent

3. Know your Huffman coding

B1 A source is producing sequences of independent symbols A, B, C and D with the following probabilities: A=0.51, B=0.26, C=0.12 and D =0.11.

Compute:

(a) Source entropy;

(b) Design a binary Huffman code such that binary one is sent as often as possible;

(c) The average bit length for the code-word set.

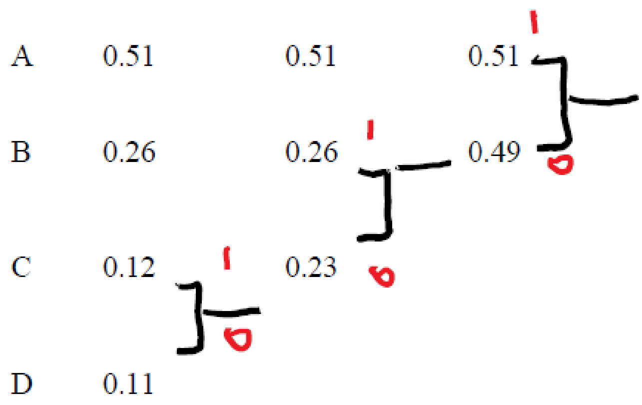
B1 **(a)**

$$H(X)=\sum_{1}^{5} P_i \log_2 \frac{1}{P_i}$$

$= 0.51\log_2(1/0.51) + 0.26\log_2(1/0.26) + 0.12\log_2(1/0.12) + 0.11\log_2(1/0.11)$

$= 1.7181$ bits/symbol

(b)



A = 1, B = 01, C = 001, D = 000

Average bit length = $\bar{n} = \sum_{1}^{4} n_i P(x_i)$ = 1 x 0.51 + 2 x 0.26 + 3 x 0.12 + 3 x 0.11

= <u>1.72 bits/symbol</u>

4. Know your Impulse function

A4 A linear time invariant system's response to a unit step function is given as $y(n) = e^{-n}u(n)$. Determine the impulse response h(n) if this system and calculate the values of h(0), h(1), and h(2).

A4 A linear time invariant system's response to a unit step function is given as $y(n) = e^{-n}u(n)$. See page 101, Q2-12

x(n) = u(n-1), $y(n-1) = e^{-(n-1)}u(n-1)$

Input, x(n)= u(n) – u(n-1) = δ(n),

Impulse response, y(n)=h(n) = $e^{-n}u(n) - e^{-(n-1)}u(n-1)$

For n= 0, h(0) = $e^{-0}u(0) - e^{-(0-1)}u(0-1)$ = 1 x 1 – 1 x 0 = 1

n= 1, h(1) = $e^{-1}u(1) - e^{-(1-1)}u(1-1)$ = e^{-1}x1 – 1x1

= $e^{-1} - 1$ = - 0.6321

n= 2, h(2) = $e^{-2}u(2) - e^{-(2-1)}u(2-1)$ = e^{-2} x 1 - e^{-1} x 1

= $e^{-2} - e^{-1}$ = - 0.2325

5. Know your z-transform table

A3 Find the z-transform of $x(n)=20\sin(0.25\pi n)u(n)$ and $y(n)= e^{-0.2n}\sin(0.3\pi n)u(n)$.

A3 (a) $x(n)=20\sin(0.25\pi n)u(n)$ see page 126, Table 3.1

$$X(z) = \frac{20\sin(0.25\pi)z^{-1}}{1-2z^{-1}cos(0.25\pi)+z^{-2}}$$ Or equivalent

(b) $y(n)= e^{-0.2n}\sin(0.3\pi n)u(n)$, $Y(z) = \frac{e^{-0.2}\sin(0.3\pi)z^{-1}}{1-2\,e^{-0.2}\cos(0.3\pi)z^{-1}+e^{-0.4}z^{-2}}$

6. Know your sampling theorem

A5 A square wave having period T = 1 ms, is filtered by an ideal low pass filter having cutoff frequency f_c = 2 kHz.

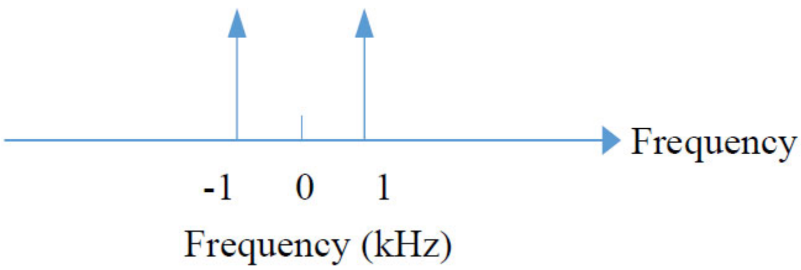
(a) What are the frequency components at the output of the low pass filter?

(b) Sketch the spectrum at the output of the filter from -4 kHz to 4 kHz

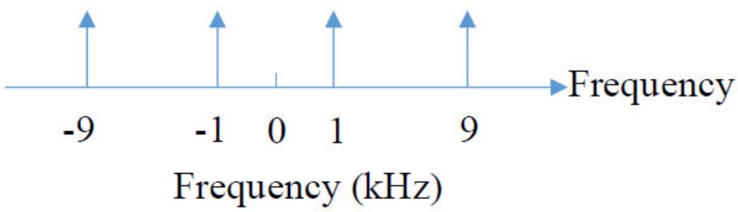
(c) If it is sampled by 10 kHz, sketched the sampled signal spectrum from -10 kHz to 10 kHz.

A5 (a) 1 kHz

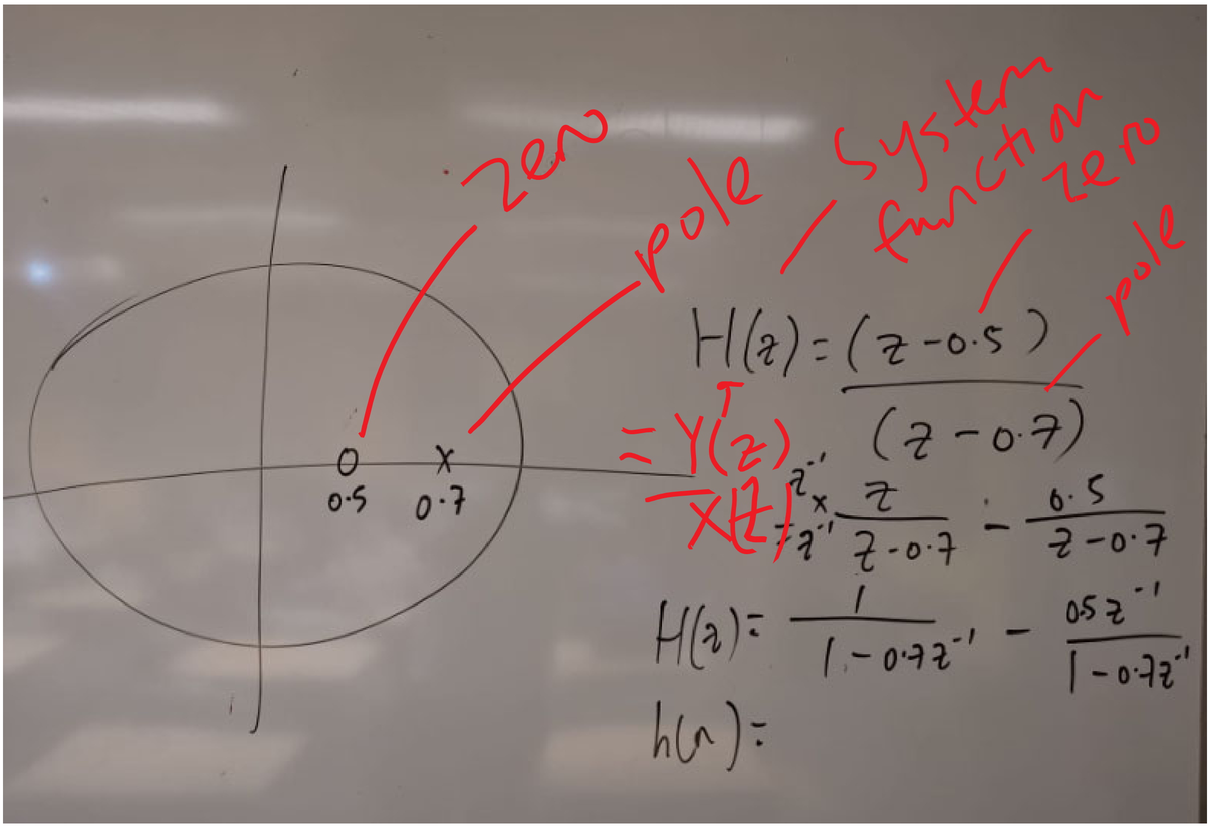
(b)



(c)



7. Know your poles and zeros



8. Know and memorize your MAE, Mean Absolute Error (MAE) formula

1) MAE, mean Absolute Error $= \dfrac{\sum_{i=1}^{n} |y_i - x_i|}{n}$

eminil $= \dfrac{\sum_{i=1}^{n} |e_i|}{n}$

2) MSE, Mean Square Error $= \frac{1}{n} \sum_{i=1}^{n} (y_i - x_i)^2$

3) PSNR, Peak signal-to-noise ratio

$= 20 \log \left(\dfrac{\max}{\sqrt{MSE}} \right)$

or $10 \log \left(\dfrac{\max^2}{MSE} \right)$

Example 9.4

original

255	250
10	15

image 1

249	249
9	14

1) MAE $= \frac{1}{2 \times 2} [1+1+1+1] = 1$

2) MSE $= \frac{1}{2 \times 2} [1^2+1^2+1^2+1^2] = 1$

3) PSNR $= 10 \log \left(\dfrac{255^2}{1} \right) = 48.13$ dB

9. Know your Basics

freq domain

$$H(z) = \frac{Y(z)}{X(z)}$$

System function

$h(n) \rightarrow$ Impulse response

$y(n) = \ldots\ldots$ Difference equation

discrete time

10. Just do your best…do those that you can answer well FIRST,,,,ALL ON THE FRESH PAGE.