SINGAPORE POLYTECHNIC
2020/2021 SEMESTER ONE END OF SEMESTER TEST

EP0604/MS837M FURTHER MATHEMATICS
Time Allowed: 1 hour 30 min + 10 min reading time

Instructions to Candidates
1. The Singapore Polytechnic examination rules are to be complied with.
2. This examination paper consists of FOUR printed pages.
3. Answer **ALL** the questions.
4. Give all non-exact answers to 3 significant figures.
5. A mathematical formulae and tables card is provided for reference.

Additional Formulae

Absolute value Inequalities: (i) $|x-a|<k$ is equivalent to $-k<x-a<k$

(ii) $|x-a|>k$ is equivalent to $x-a>k$ or $x-a<-k$

VECTOR EQUATION OF A LINE

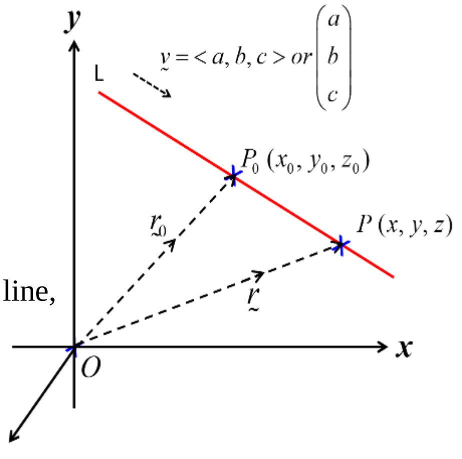
$$\underset{\sim}{r}=\underset{\sim}{r}_0+\lambda\underset{\sim}{v} \quad , \ \lambda\in\mathbb{R}$$

where

$\underset{\sim}{r} = \ <x,y,z>$ is the position vector of any point on the line,

$\underset{\sim}{r}_0 = \ <x_0,y_0,z_0>$ is the position vector of a known point on the line,

$\underset{\sim}{v} = \ <a,b,c>$ is a non-zero vector parallel to the line.



PARAMETRIC EQUATIONS OF A LINE

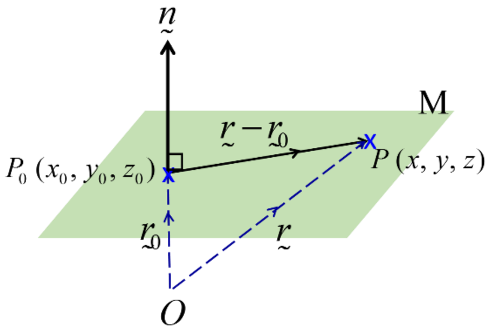
$$x=x_0+\lambda a, \quad y=y_0+\lambda b, \quad z=z_0+\lambda c \qquad \text{where } \lambda\in\mathbb{R}$$

EQUATION OF A PLANE

The plane in $\mathbb{R}^3$ that passes through the point $P_0(x_0,y_0,z_0)$ and is normal to the non-zero vector $\underset{\sim}{n}=\langle a,b,c\rangle=a\underset{\sim}{i}+b\underset{\sim}{j}+c\underset{\sim}{k}$ has equations:

In vector form: $\underset{\sim}{n}\bullet\overrightarrow{P_0P}=0$ or $\underset{\sim}{r}\bullet\underset{\sim}{n}=\underset{\sim}{r}_0\bullet\underset{\sim}{n}$

In point-normal form: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$



1. Solve the following.

   (a) $\dfrac{1}{x-2} \geq \dfrac{2x-3}{(x-2)(x-3)}$ (7 marks)

   (b) $\left|\dfrac{x+1}{x-1}\right| \leq 2$ (8 marks)

2. (a) Find the equation of the curve which passes through the point $(1,\ 0)$ and for which

   $\dfrac{dy}{dx} = x\ln x$. (8 marks)

   (b) Find $\int \sin(x)\left[\cos(x)\right]^2\, dx$. (7 marks)

3. The graphs of $y=\sqrt{2x-1}$ and $y=x-0.5$ are shown in Figure 1.

   (a) Find the x-coordinates of the points of intersection of the two curves. (2 marks)

   (b) Find the area enclosed by the curve $y=\sqrt{2x-1}$ and the line $y=x-0.5$.

   (8 marks)

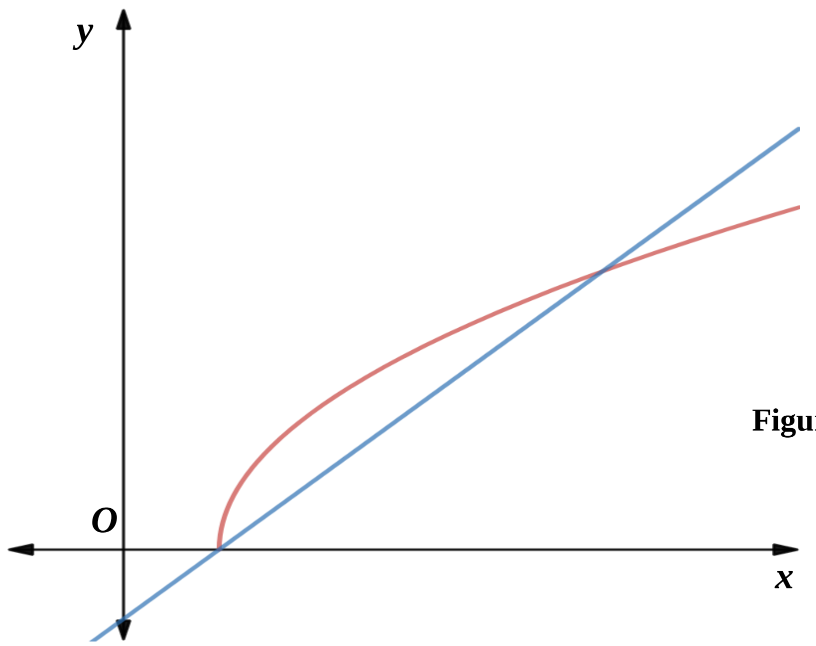


Figure 1

4. The diagram in Figure 2 shows the graphs of $y=|2x-3|+2$ and $4x^2+4y=41$. Given that the shaded region R is the area bounded by the graph $y=|2x-3|+2$, the curve $4x^2+4y=41$ and the y-axis. Find the volume of solid formed when the bounded region R is rotated about the x-axis. Leave your answer in term of π.

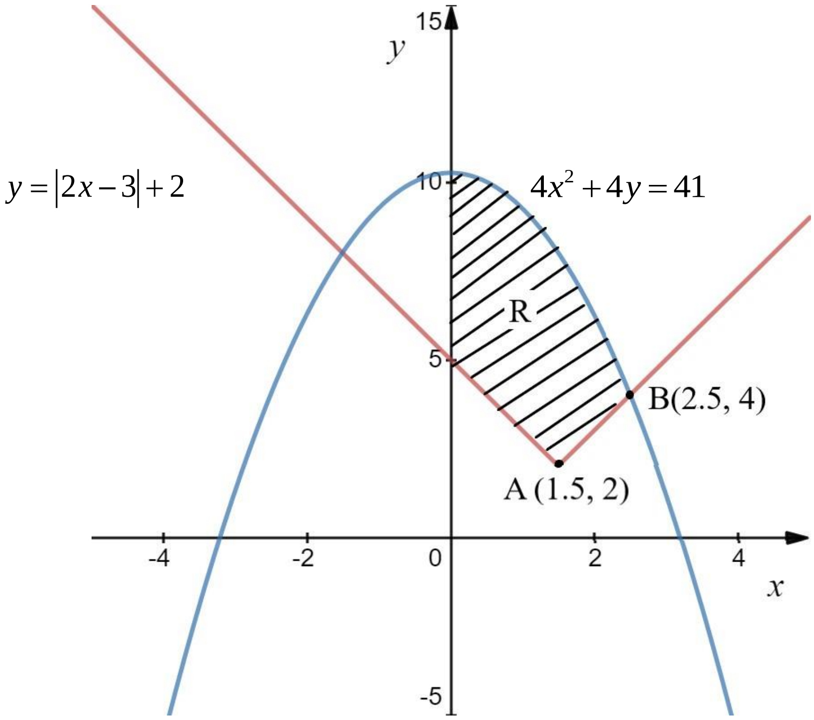


Figure 2

(15 marks)

5. Given two vectors $\underset{\sim}{u}=3\underset{\sim}{i}+4\underset{\sim}{j}$ and $\underset{\sim}{v}=5\underset{\sim}{i}-12\underset{\sim}{j}$.

(a) Sketch $\underset{\sim}{v}$. (2 marks)

(b) Find $\|\ 2\underset{\sim}{u}+\underset{\sim}{v}\ \|$. (2 marks)

(c) Find the unit vector of $\underset{\sim}{u}$. (2 marks)

(d) Find the angle between the vector $\underset{\sim}{u}$ and $\underset{\sim}{v}$. (4 marks)

6 (a) Plane Π has equation $x-2y+z=20$ and the line l is perpendicular to Π.

(i) Write down the vector equation of the line l given that l passes through the coordinates $(4,-1,2)$. (4 marks)

(ii) Find the coordinates of the point of intersection of line l and plane Π. (4 marks)

(b) In Figure 4, the pyramid has vertices $B(2,2,0)$, $C(0,2,0)$ and $V(0,0,4.5)$.

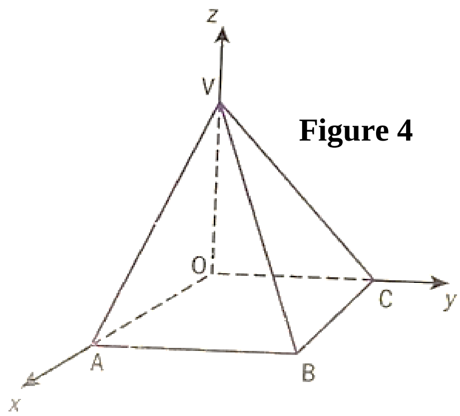


Figure 4

(i) Find the vector which is orthogonal to both the vectors $\overrightarrow{BC}$ and $\overrightarrow{BV}$. (7 marks)

(ii) Find the Cartesian equation of the plane BCV. Express your answer in the form of $ax+by+cz=k$. (5 marks)

7. Given $f(x)=2x+1$ and the composite function $f^2(x)=(f\circ f)(x)$.

(a) Show that $f^2(x)=4x+3$. (1 mark)

(b) Prove by mathematical induction $f^n(x)=2^n x+2^n-1$ for every positive integer *n*. (14 marks)

~END OF PAPER~

Answers

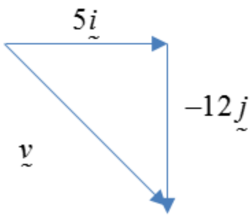
1 (a) $x \leq 0 \ \ or \ \ 2 < x < 3$ (b) $x \leq \frac{1}{3} \ or \ x \geq 3$

2 (a) $y = \ln x \frac{x^2}{2} - \frac{x^2}{4} + \frac{1}{4}$ (b) $-\frac{\cos^3 x}{3} + C$

3 (a) $x = 0.5, x = 2.5$ (b) $\frac{2}{3}$ units^2

4 $146\frac{7}{12}\pi \ unit^3$

5 (a)



5 (b) $\sqrt{137}$ (c) $\frac{1}{5}\left(3\underset{\sim}{i} + 4\underset{\sim}{j}\right)$ (d) $\theta = 120.51°$

6 (a)(i) $\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ (ii) $\lambda = 2, (6, -5, 4)$

(b)(i) $9\underset{\sim}{j} + 4\underset{\sim}{k}$ (ii) $9y + 4z = 18$

7 (b) Step 3 need to prove: $f^{n+1}(x) = 2^{n+1}x + 2^{n+1} - 1$