SINGAPORE POLYTECHNIC

2018/2019 SEMESTER ONE EXAMINATION

School of Architecture & the Built Environment
DCEB

School of Chemical and Life Sciences
DAPC, DCHE, DFST, DPCS

School of Digital Media & Infocomm Technology
DBIT, DDA, DISM, DIT

School of Electrical and Electronic Engineering
DASE, DESM, DCEP, DCPE, DEB, DEEE, DES

School of Mechanical and Aeronautical Engineering
DARE, DCEP, DME, DMRO, DBEN

Singapore Maritime Academy (SMA)
DMR

1st Year FT

ENGINEERING MATHEMATICS I Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **THREE** sections:

   Section A: 5 Multiple-Choice Questions (10 marks)
   Answer **ALL** questions.

   Section B: 7 Questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

   Section C: 3 Questions (40 marks)
   Answer **ALL** questions.

3. Unless otherwise stated, leave all answers correct to three significant figures.

4. Except for sketches, graphs and diagrams, no solution or answer is to be written in pencil.

5. This examination paper consists of **6** printed pages.

Section A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks. No mark will be deducted for incorrect answers.

Tick the choice of answer for each question in the box of the **MCQ answer sheet** provided in the answer booklet.

A1. Let $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}$ where $a \neq 0$. Which of the following statements is true?

(a) $\mathbf{A}$ is a diagonal matrix

(b) $\mathbf{A}$ is a symmetric matrix

(c) $\mathbf{A}$ is an identity matrix

(d) $\mathbf{A}$ is an invertible matrix

A2. Given the equation $x^2 = a + be^y$, where a and b are constants.

In order to obtain a straight line to determine a and b, we plot

(a) $\ln\left(x^2 - a\right)$ against y

(b) $\ln\left(x^2\right)$ against y

(c) x^2 against e^y

(d) $\ln\left(x^2\right)$ against e^y

A3. Let $Z = r\angle\theta$ be a non-zero complex number and $\overline{Z}$ is the conjugate of Z.

Which of the following is <u>always</u> TRUE ?

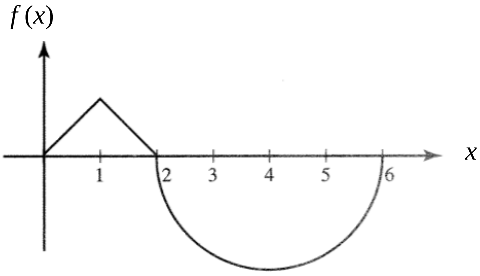
(a) $\left|Z\,\overline{Z}\right| = Z\,\overline{Z}$

(b) $\left|Z + \overline{Z}\right| = 2\left|Z\right|$

(c) $\left|1 + Z\right| = 1 + \left|Z\right|$

(d) $\left|10Z\right| = 10 + \left|Z\right|$

A4.



Refering to the diagram above, $f'(x)$ does not exist for $x =$

(I) 1 (II) 2 (III) 4

(a) (I) only (b) (I) and (II)

(c) (II) only (d) (I) , (II) and (III)

A5. If $\frac{d}{dx}H(x) = h(x)$, then $\int_b^a -h(x)\ dx =$ __________.

(I) $H(a) - H(b)$ (II) $h(a) - h(b)$

(III) $H(b) - H(a)$ (IV) $h(b) - h(a)$

(Note : $H(a) \neq H(b)$ and $h(a) \neq h(b)$)

(a) (I) only (b) (I) and (IV)

(c) (III) only (d) (II) and (III)

Section B (50 marks)

Each question carries 10 marks.

The total mark of the questions in this section is 70 marks.

You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. The currents I_1 , I_2 and I_3 in an electrical circuit are related by the system of equations given as

$$\begin{aligned} I_1 + I_2 + I_3 &= 10 \\ 2I_1 + 3I_2 + kI_3 &= 50 \\ 4I_2 - 3I_3 &= 5 \end{aligned}$$

where k is a constant.
Use Cramer's rule to find the value of I_1 in terms of k .
(Detailed workings of evaluating a determinant must be clearly shown.)

B2. Two matrices $A = \begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 \\ 0 & -2 \\ 8 & 3 \end{pmatrix}$ are given.

(a) Find AB^T.

(b) Given that $C^{-1} = A^2$, find C^{-1}.

(c) Hence or otherwise, find C.

B3. (a) Given $Z_1 = 1 - j5$, $Z_2 = 10\angle 150°$ and $Z_3 = 4\angle 60°$.

Evaluate the following and leave your answers in polar form.

(i) $Z_3 - Z_1$ (ii) $\dfrac{Z_2}{Z_3 - Z_1}$ (iii) $Z_1 Z_2 Z_3$

(Detailed workings must be clearly shown.)

(b) Given that $x + jy = (4 + j3)(6 + j7)$, find the real numbers x and y.

B4. A virus culture is exposed to an experimental vaccine and it was observed that the number of living cells N in the culture and the time t hours are related according to the law

$$t = \frac{a}{N^2} - b$$

where a and b are constants.
The readings collected are shown in the table below.

N	100	70.71	57.74	50.00	44.72
t	0	1	2	3	4

(a) State the variables that should be plotted on the vertical and horizontal axes of a graph so that a best fit straight line can be drawn to show that the readings obey the above law.

(b) Hence compute in a table, the values of the variables to be plotted on the horizontal and vertical axes, correct to 4 significant figures. *Do not plot the values*.

(c) Suppose the best fit straight line passes through the third and the fifth points of the new values, use these two points to estimate the gradient and the vertical intercept. Hence, determine the values of a and b.

B5. Find $\dfrac{dy}{dx}$ for each of the following.

(a) $y = 2e^{3x} + 3\sin^{-1}(4x)$

(b) $8y^2 + xy = \sin(3x) + 5$

B6. (a) Given the curve $y = \ln\sqrt{2x+1}$.

(i) Find the slope of the tangent line to the curve at $x = 0$.

(ii) Find $\dfrac{d^2y}{dx^2}$ at $x = 0$.

(b) In an electrical circuit, the voltage v (volts) is given by

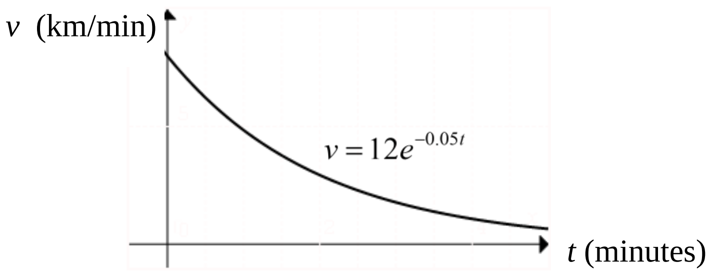
$$v = 250t + 5\cos(2t)$$

Find the current i (amperes) at time $t = 0.5$ second, given the capacitance, $C = 30\mu\text{F}$. (Note : $i = C\dfrac{dv}{dt}$)

B7. (a) Find the following integrals:

(i) $\displaystyle\int \left(\frac{3}{x} - 2\sec^2(4x)\right) dx$ (ii) $\displaystyle\int \frac{1}{x^2+5}\, dx$

(b) The vertical velocity of a satellite that is launched from the earth to the space is shown by the graph below .



(i) Determine the acceleration a as a function of time t .

Is the satellite accelerating or decelerating ? Explain your answer.

(ii) Find the area under the curve from $t=0$ to $t=10$ minutes.

What does the area represent ?

[Hint: $v = \dfrac{ds}{dt}$, $a = \dfrac{dv}{dt}$]

Section C (40 marks)
Answer ALL **THREE** questions.

C1. At time t seconds, a cylindrical container is expanding in such a way that the area of the cross-section is

$$A = \frac{3}{4}h^2 \quad \text{cm}^2$$

where h (centimeters) is the height of the cylinder.

(a) When $h = 4$ centimeters, the height is changing at the rate of 0.5 cm/s. At this instant, find the rate of change of

(i) the area of the cross-section of the cylindrical container,

(ii) the volume of the cylinder.

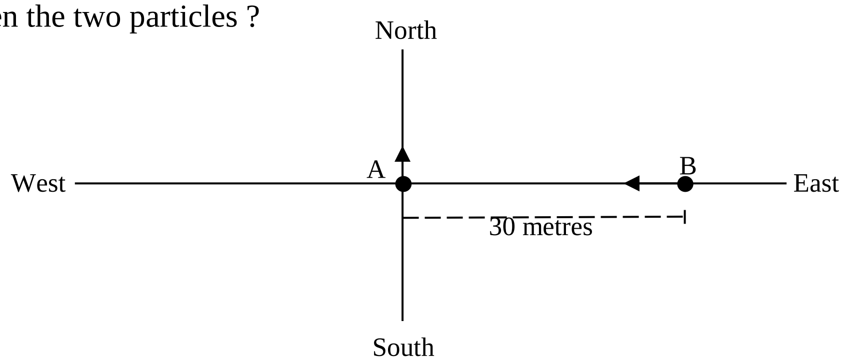
(b) What will be the rate of change of the height if A is increasing at $1\ \text{cm}^2/\text{s}$ at the instant when $h = 3$ centimeters? (15 marks)

C2. Given $\left|\frac{z+j}{z}\right| = 4$ and $\arg\left(\frac{z+j}{z}\right) = \frac{\pi}{3}$.

(a) Express $\frac{z+j}{z}$ in polar form.

(b) Find the complex number z in rectangular form. (10 marks)

C3. As shown in the diagram, particle B is initially 30 metres to the east of particle A. Both particles start moving at the same time, B moves to the west while A moves to the north as indicated by the arrows. At any instant, the ratio of the distance travelled by particle A to that travelled by particle B is 3:5. What is the minimum distance between the two particles ? (15 marks)



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No.	SOLUTION
A	A1) **d** A2) **c** A3) **a** A4) **b** A5) **c**
B1	$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & k \\ 0 & 4 & -3 \end{vmatrix} = -9+0+8-0-4k-(-6) = 5-4k$ $\Delta_{I_1} = \begin{vmatrix} 10 & 1 & 1 \\ 50 & 3 & k \\ 5 & 4 & -3 \end{vmatrix} = -90+5k+200-15-40k-(-150) = 245-35k$ $I_1 = \dfrac{245-35k}{5-4k}$
B2 (a)	$B^T = \begin{pmatrix} 4 & 0 & 8 \\ 1 & -2 & 3 \end{pmatrix}$ $AB^T = \begin{pmatrix} 11 & -6 & 25 \\ 5 & -10 & 15 \end{pmatrix}$
(b)	$C^{-1} = \begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 21 \\ 0 & 25 \end{pmatrix}$
(c)	$C = (C^{-1})^{-1}$ $= \dfrac{1}{100}\begin{pmatrix} 25 & -21 \\ 0 & 4 \end{pmatrix}$
B3 (a) (i)	$Z_3 - Z_1 = (2 + j3.464) - (1 - j5)$ $= 1 + j8.464$ $= 8.523\angle 83.26^o$
(ii)	$\dfrac{Z_2}{(Z_3 - Z_1)} = \dfrac{10\angle 150^o}{8.523\angle 83.26^\circ}$ $= 1.173\angle 66.74^o$
(iii)	$Z_1 Z_2 Z_3 = (5.099\angle -78.69)\ (10\angle 150^\circ)\ (4\angle 60^\circ)$ $= 203.96\angle 131.31^o$
B3 (b)	$x + jy = 24 + j28 + j18 + j^2 21$ $= 3 + j46$ $x = 3$, $y = 46$

<table>
<tr><th>No.</th><th>SOLUTION</th></tr>
<tr><td>B4</td><td>

(i) *Vertical axis is* t

Horizontal axis is $\dfrac{1}{N^2}$

(ii)

1/N^2	1.000(10^{-4})	2.000(10^{-4})	2.999(10^{-4})	4.000(10^{-4})	5.000(10^{-4})
t	0	1	2	3	4

(iii) $a = gradient = \dfrac{4-2}{5\left(10^{-4}\right)-2.999\left(10^{-4}\right)} = 9995$

Take point $(5.000\left(10^{-4}\right)$, 4) and substitute into $t = \dfrac{10000}{N^2} - b$

$4 = 9995(5.000)\left(10^{-4}\right) - b$

$b = 1$

</td></tr>
<tr><td>B5 (a)</td><td>

$\dfrac{dy}{dx} = 2e^{3x}(3) + 3\left(\dfrac{1}{\sqrt{1-16x^2}}\right)(4)$

</td></tr>
<tr><td>B5 (b)</td><td>

$8(2y)\dfrac{dy}{dx} + \left(y + x\dfrac{dy}{dx}\right) = 3\cos(3x) + 0$

$\dfrac{dy}{dx}[16y + x] = 3\cos(3x) - y$

$\dfrac{dy}{dx} = \dfrac{3\cos(3x) - y}{16y + x}$

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<tr><td>B6 (a)</td><td>

$y = \dfrac{1}{2}\ln(2x+1)$

$\dfrac{dy}{dx} = \dfrac{1}{2}\left[\dfrac{2}{2x+1}\right]$

$\left.\dfrac{dy}{dx}\right|_{x=0} = \dfrac{1}{2}[2] = 1$

$\dfrac{d^2y}{dx^2} = (-1)(2x+1)^{-2}(2)$

$\left.\dfrac{d^2y}{dx^2}\right|_{x=0} = -2[1]^{-2} = -2$

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<tr><td>(b)</td><td>

$v = 250t + 5\cos(2t)$

$\dfrac{dv}{dt} = 250 - 10\sin(2t)$

$i = C\dfrac{dv}{dt}$

$i(0.5) = 30(10^{-6})\left[250 - 10\sin(1)\right]$

$= 7.25\ mA$

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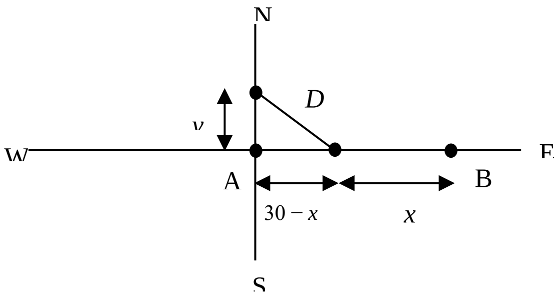
No.	SOLUTION
B7 (a)i	$\int \left(\frac{3}{x} - 2\sec^2(4x)\right) dx = 3\ln\lvert x\rvert - 2\left(\frac{1}{4}\tan(4x)\right) + C$
B7 (a)ii	$\int \frac{1}{5+x^2}\, dx = \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$
B7 (b)i	$a = \frac{d}{dt}\left(12e^{-0.05t}\right) = 12e^{-0.05t}(-0.05) \quad = -0.6e^{-0.05t}$ <u>Decelerating</u> because the value of a is always <u>negative</u>.
(ii)	$s = \int_0^{10} 12e^{-0.05t}\, dt = \left[\frac{12}{-0.05}e^{-0.05t}\right]_0^{10} = -240\left[e^{-0.05(10)} - e^0\right]$ $= 94.43$ The area represents the <u>vertical displacement</u> after 10 minutes.
C1 a(i)	$\frac{dA}{dh} = \frac{3}{4}(2h)$ $\frac{dA}{dt} = \left(\frac{dA}{dh}\right)\left(\frac{dh}{dt}\right) \quad = \frac{3}{4}(2h)0.5$ $\left.\frac{dA}{dt}\right\rvert_{h=4\partial} \quad = \frac{3}{4}(4) \quad = 3 \quad \text{cm}^2/\text{s}$
a(ii)	Volume, $V = A(h) \quad = \frac{3}{4}h^3$ $\frac{dV}{dh} = \frac{3}{4}\left[3h^2\right]$ $\frac{dV}{dt} = \left(\frac{dV}{dh}\right)\left(\frac{dh}{dt}\right) \quad = \frac{9}{4}\left(h^2\right)0.5$ $\left.\frac{dV}{dt}\right\rvert_{h=4cm} \quad = \frac{9}{8}\left(4^2\right) \quad = 18 \quad \text{cm}^3/\text{s}$
b	$\frac{dA}{dt} = \left(\frac{dA}{dh}\right)\left(\frac{dh}{dt}\right)$ $1 \quad = \frac{3}{2}(h)\left(\frac{dh}{dt}\right)$ $\frac{dh}{dt} \quad = \frac{2}{3h}$ $\left.\frac{dh}{dt}\right\rvert_{h=3cm} \quad = \frac{2}{9}\text{cm/s}$

No.	SOLUTION
C2	Since $r = \left\|\frac{z+j}{z}\right\| = 4$ and $\arg\left(\frac{z+j}{z}\right) = \theta = \frac{\pi}{3}$ Therefore : $\frac{z+j}{z} = 4\angle\frac{\pi}{3}$ $\frac{z+j}{z} = 4\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$ $z + j = 4z\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$ $z + j = z\left(2 + j2\sqrt{3}\right)$ $z - z\left(2 + j2\sqrt{3}\right) = -j$ $z\left(1 - 2 - j2\sqrt{3}\right) = -j$ $z = \frac{-j}{-(1 + j2\sqrt{3})}$ $z = \frac{j}{(1 + j2\sqrt{3})} \cdot \frac{\left(1 - j2\sqrt{3}\right)}{\left(1 - j2\sqrt{3}\right)}$ $z = \frac{2\sqrt{3}}{13} + j\frac{1}{13}$ or $z = \frac{1}{13}\left(2\sqrt{3} + j)\right)$

No.	SOLUTION
C3	 Let D be the distance between the two particles . Let the distance travelled by particle A and B be x and y respectively. $\frac{y}{x}=\frac{3}{5}\Rightarrow y=\frac{3}{5}x$ From the figure we can see that $D=\sqrt{y^2+(30-x)^2}$ $D=\sqrt{(\frac{3}{5}x)^2+(30-x)^2}$ $\frac{dD}{dx}=\frac{1}{2}\left(\frac{9}{25}x^2+(30-x)^2\right)^{-\frac{1}{2}}\left(\frac{68}{25}x-60\right)$ For maxima or minima, $\frac{dD}{dx}=0$ $\frac{68}{25}x-60=0$ $x=22\frac{1}{17}$ or 22.06 $\left.\frac{dD}{dx}\right\|_{x<22\frac{1}{17}}<0$ and $\left.\frac{dD}{dx}\right\|_{x>22\frac{1}{17}}>0$ Therefore, D is mimium when $x=22\frac{1}{17}$ m D = $\sqrt{\left(\frac{3}{5}(22\frac{1}{17})\right)^2+\left(30-22\frac{1}{17}\right)^2}$ =15.4 m