Chapter 10

Digital Modulation

(Part 1 of 2)

10.1 Introduction

- Most communication channels have very poor response in the neighbourhood of zero frequency.

passband channels

  - Baseband signal transmission is not suitable.

- To transmit digital signal over passband channels, passband digital signal transmission is required.

- Passband digital signal transmission shifts a baseband digital signal from low frequency to a high frequency band.

  - Impressing a baseband signal upon a high-frequency carrier via digital modulation.

- There are three basic digital modulation techniques:

  Amplitude-shift keying (ASK)
  Frequency-shift keying (FSK)
  Phase-shift keying (PSK)

For binary information: BASK, BFSK, BPSK

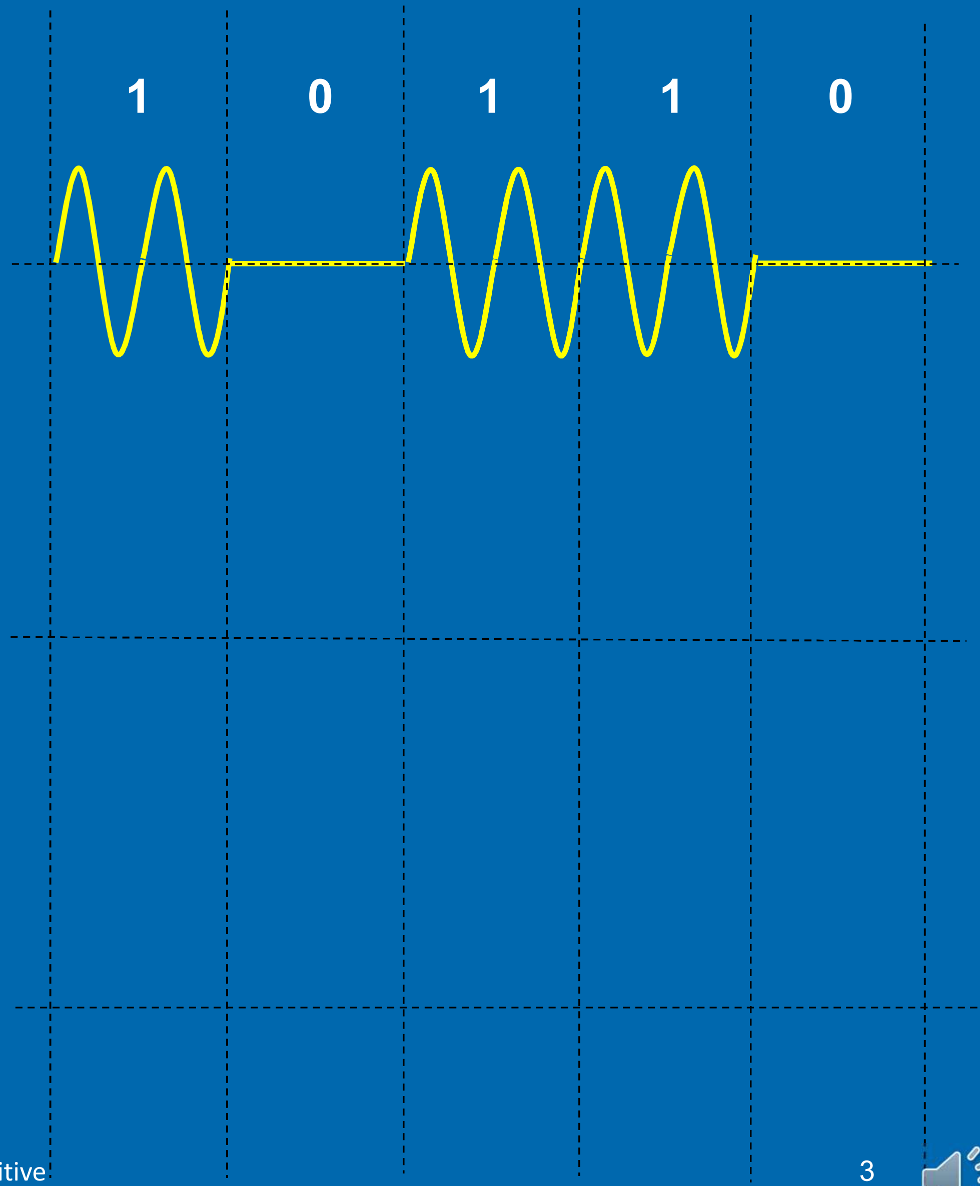
Introduction

BASK

The carrier is switched between two values, also called on and off.

Binary " 1 " $V\cos\omega_c t$

Binary " 0 " 0



Introduction

BASK

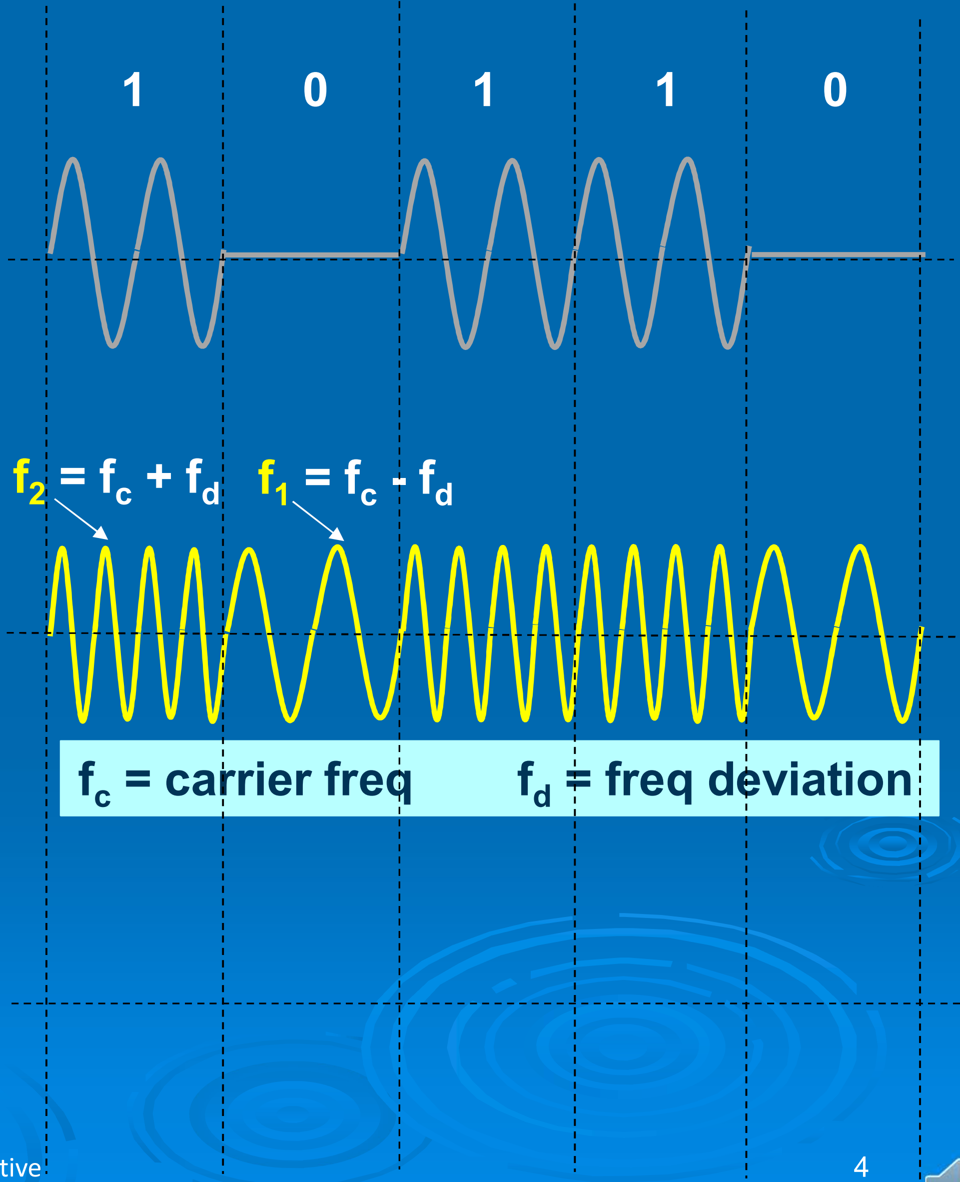
The carrier is switched between two values, also called on and off

Binary " 1 "	$V\cos\omega_c t$
Binary " 0 "	0

BFSK

The frequency of a sinusoidal carrier is switched between two values.

Binary " 1 "	$V\cos\omega_2 t$
Binary " 0 "	$V\cos\omega_1 t$



Introduction

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BASK

The carrier is switched between two values, also called on and off

Binary " 1 " $V\cos\omega_c t$

Binary " 0 " 0

BFSK

The frequency of a carrier is shifted between two values.

Binary " 1 " $V\cos\omega_2 t$

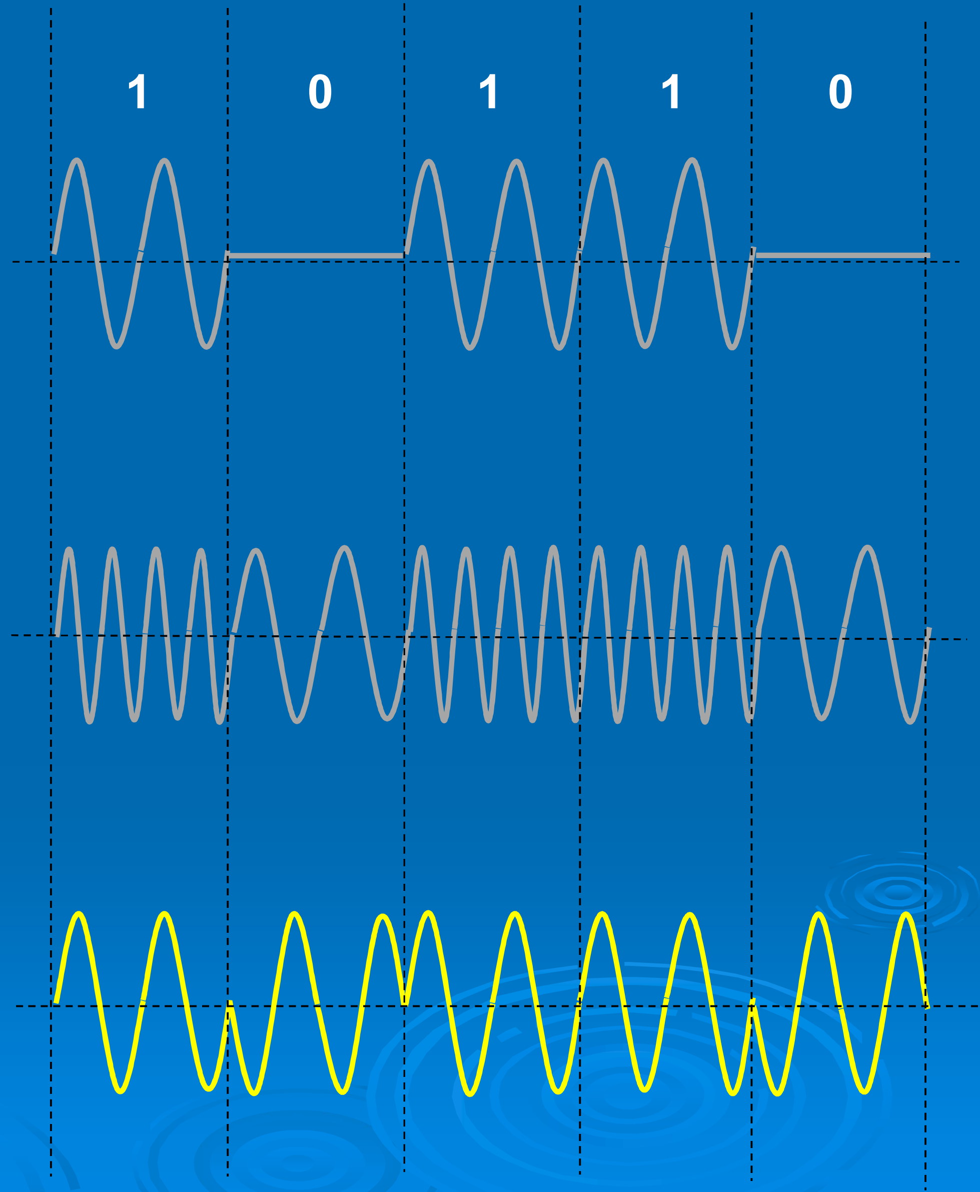
Binary " 0 " $V\cos\omega_1 t$

BPSK

The initial phase-angle of a carrier is shifted between two values.

Binary " 1 " $V\cos\omega_c t$

Binary " 0 " $-V\cos\omega_c t$



10.1 Spectra of BASK, BFSK and BPSK signals

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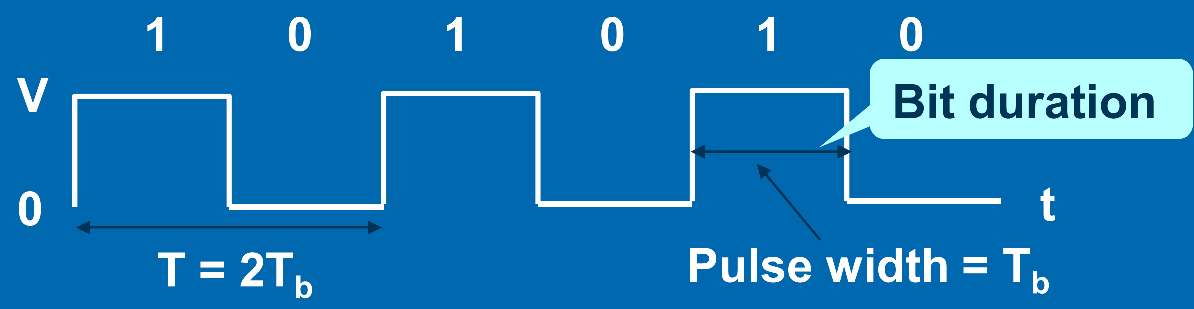
Spectrum of BASK signal

Digital signal, $x(t) \Rightarrow$ ….101010….

Square wave

x(t)

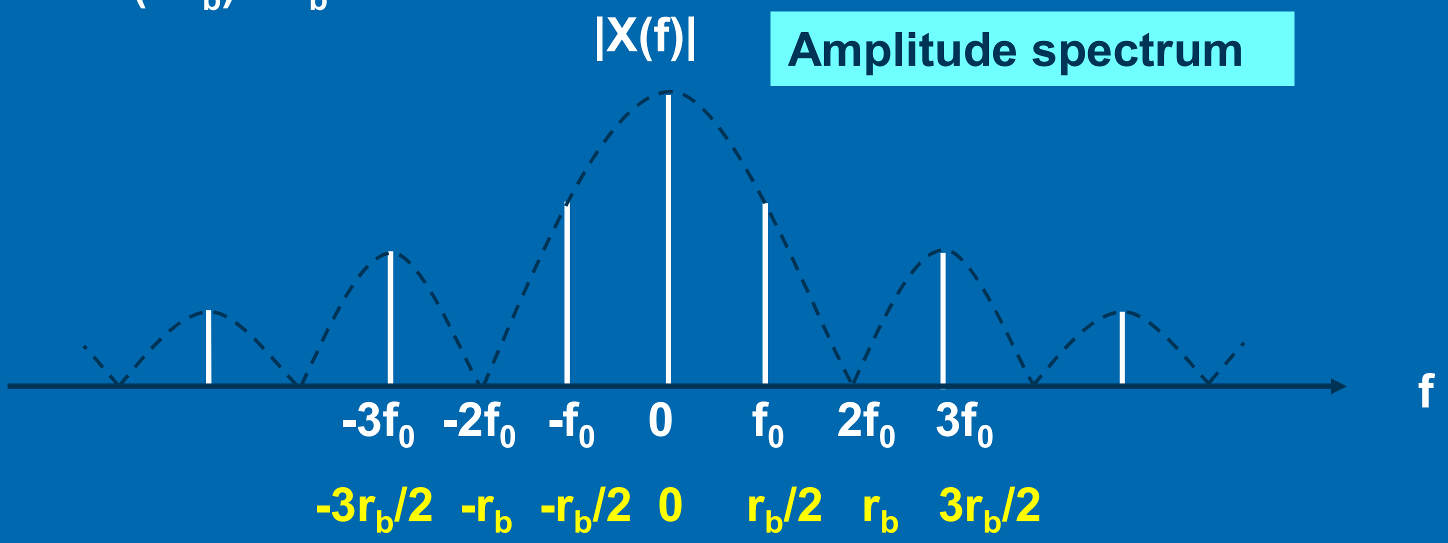
Unipolar NRZ input data



Bit rate $r_b = 1/T_b$

Frequency $f_0 = 1/T = 1/(2T_b) = r_b/2$

Amplitude spectrum



10.1 Spectra of BASK, BFSK and BPSK signals

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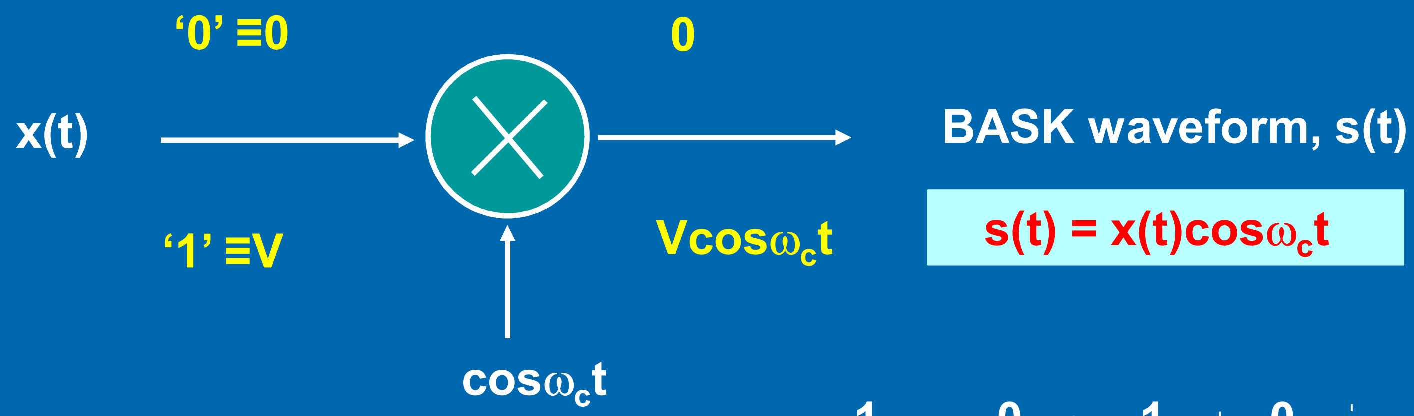
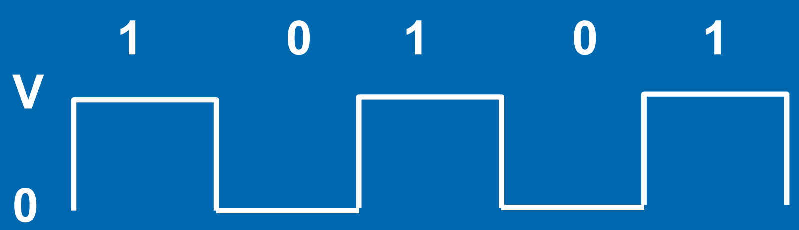
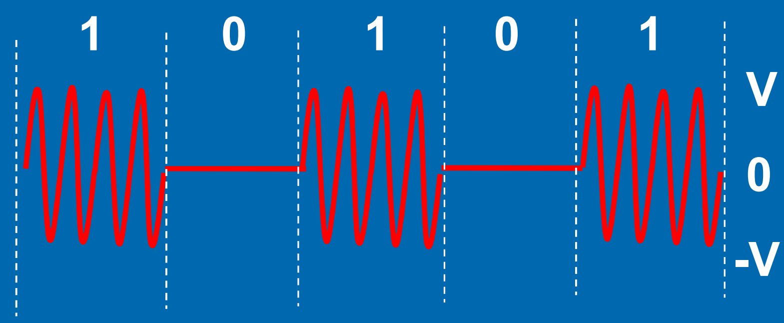
Spectrum of BASK signal



$s(t) = x(t)\cos\omega_c t$





BASK waveform

$\frac{1}{2}[X(f+f_c)+X(f-f_c)]$

Recall

Fourier transform

$x(t) \Leftrightarrow X(f)$

$x(t) \times \cos 2\pi f_c t \Leftrightarrow \frac{1}{2}[X(f+f_c)+X(f-f_c)]$

Shift X(f) left by f_c

Shift X(f) right by f_c

The double-sided spectrum of $x(t)\cos 2\pi f_c t$, consists of two frequency shifted version of X(f).

$x(t) \times \cos 2\pi f_c t \Leftrightarrow X(f-f_c)]$

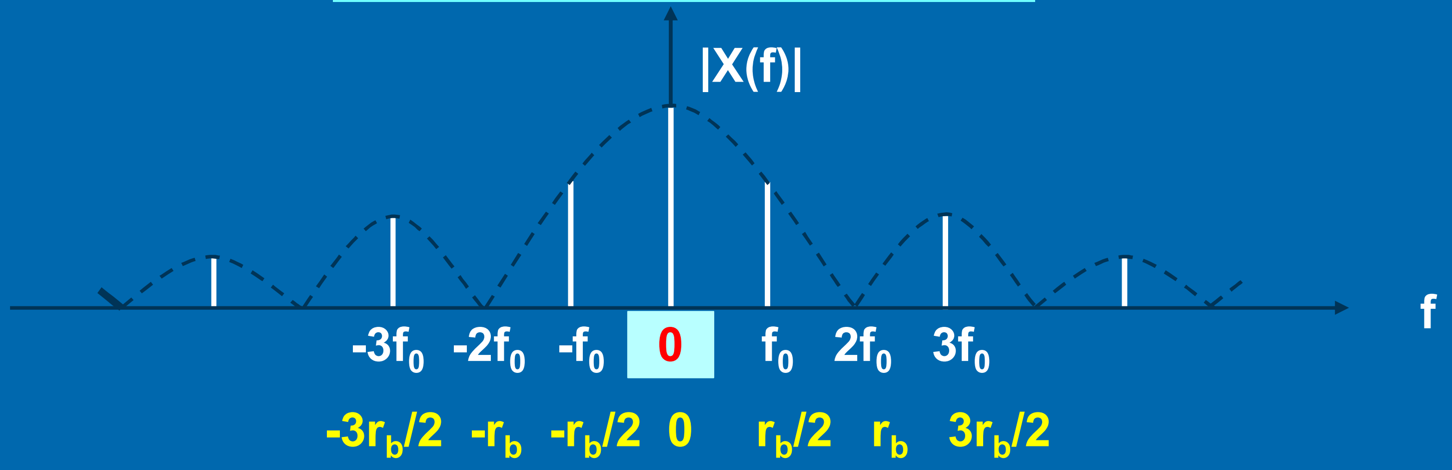
Shift X(f) right by f_c

The single-sided spectrum of $x(t)\cos 2\pi f_c t$, is X(f) shifted right by f_c.

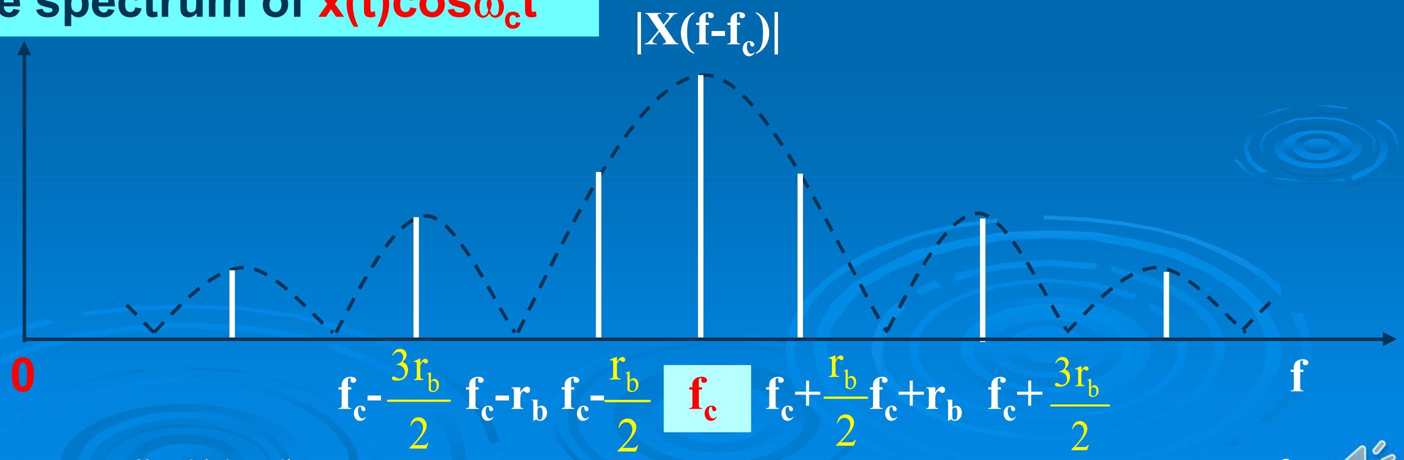
10.1 Spectra of BASK, BFSK and BPSK signals

Spectrum of BASK signal

Amplitude spectrum of x(t)



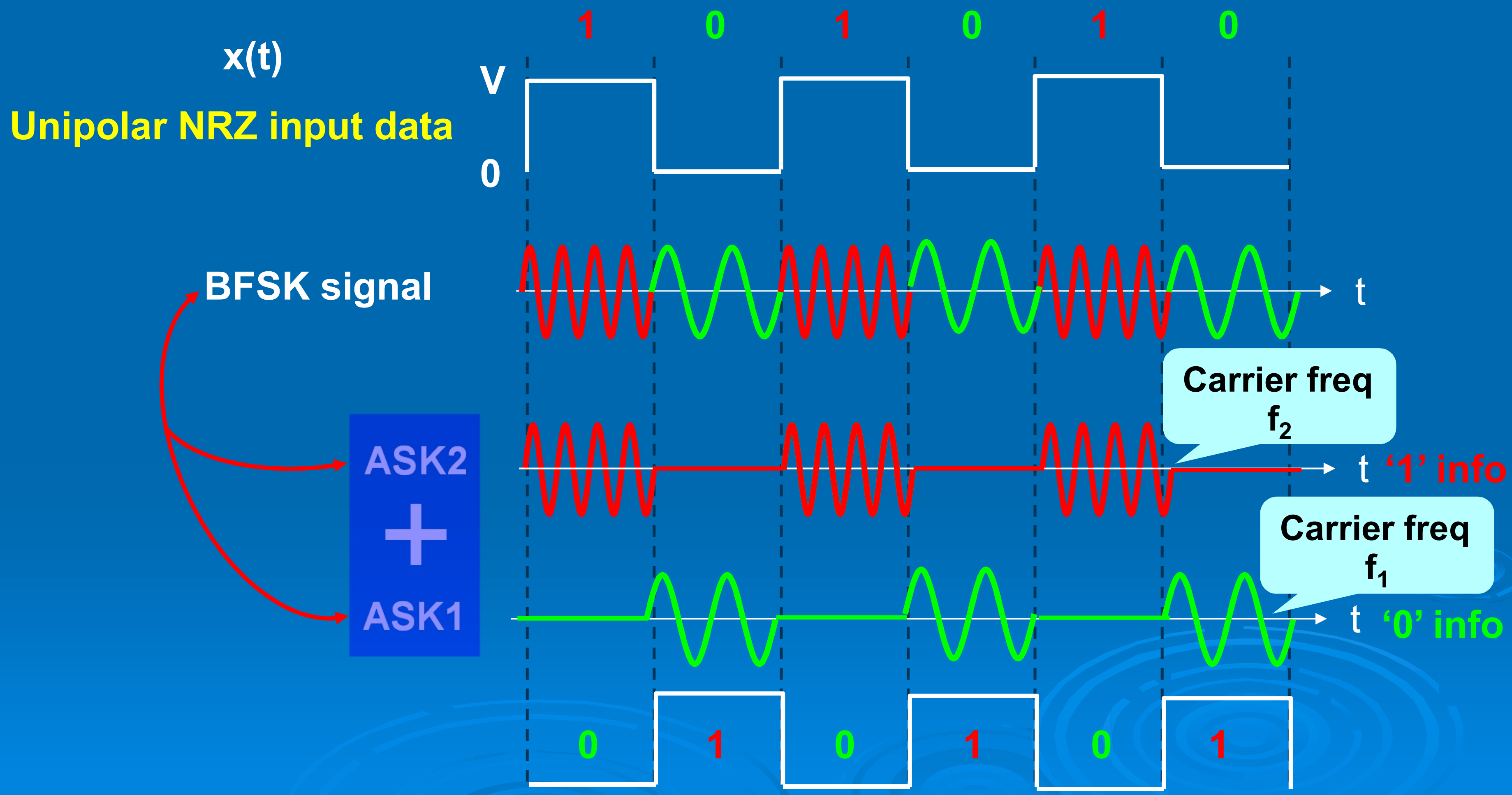
Single-sided Amplitude spectrum of $x(t)\cos\omega_c t$



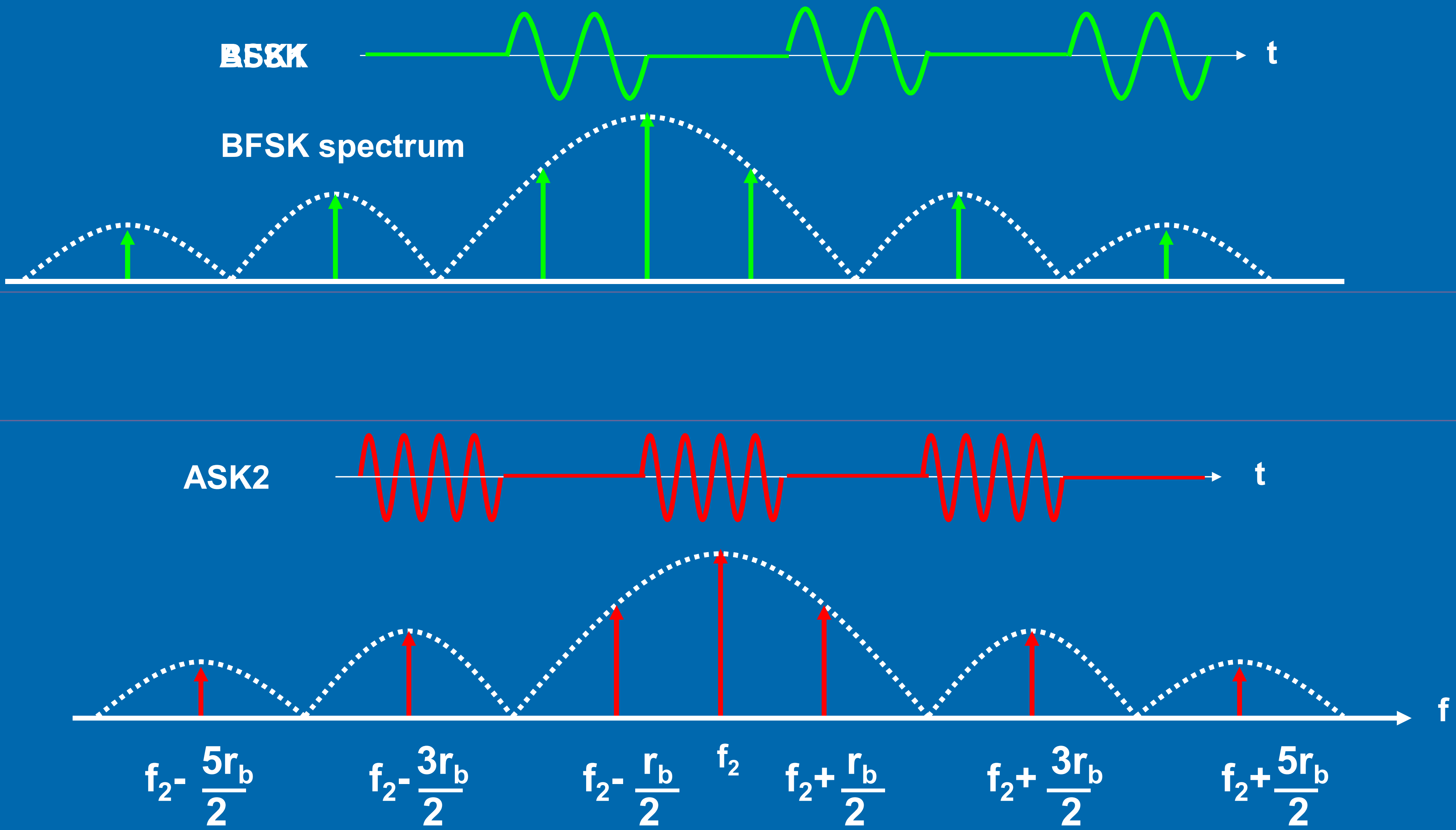
10.1 Spectra of BASK, BFSK and BPSK signals

Spectrum of BFSK signal

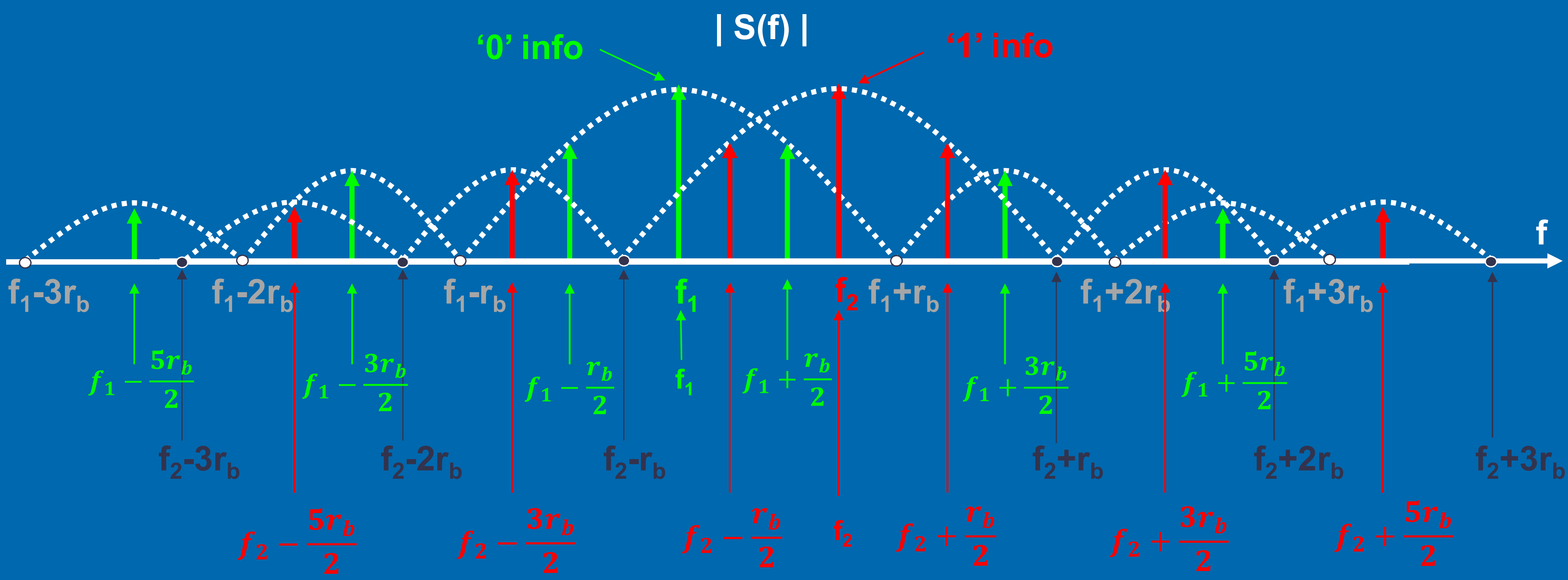
Digital signal, x(t)⇒ ….101010….



Spectrum of BFSK signal



Spectrum of BFSK signal



- f_1 and f_2 are two transmitting frequencies (two frequencies of the carrier)
- r_b is bit rate

10.1 Spectra of BASK, BFSK and BPSK signals

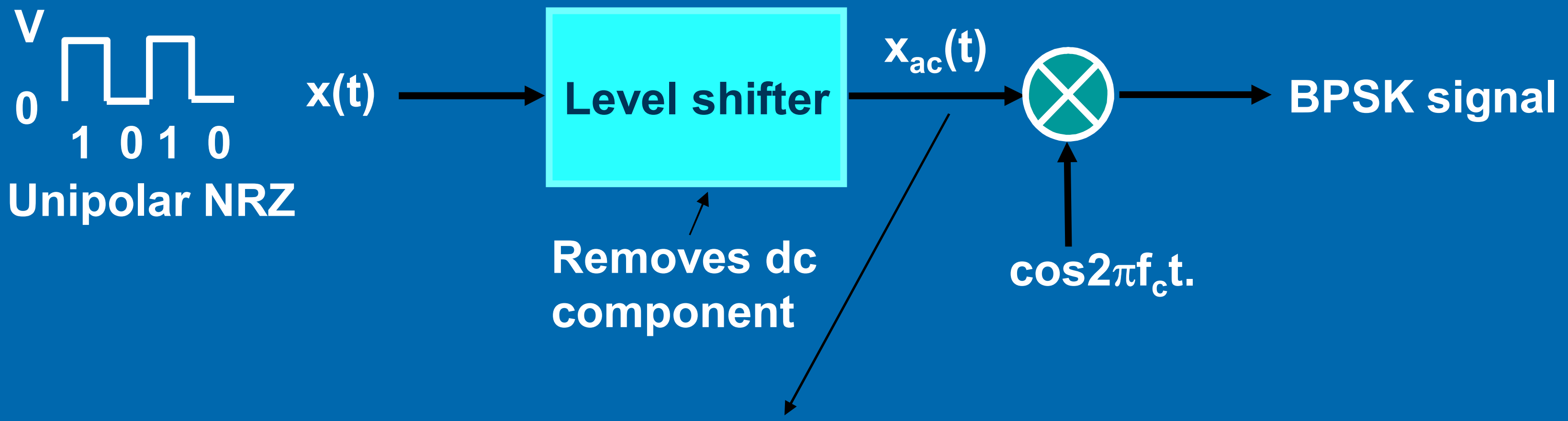
Spectrum of BPSK signal

'1' ≡ V/2

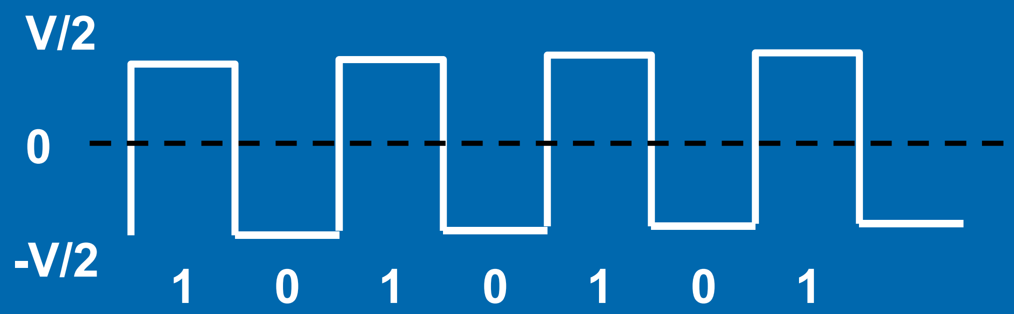
'0' ≡ -V/2

'1' ⇨ $V/2 \cos 2\pi f_c t$

'0' ⇨ $-V/2 \cos 2\pi f_c t = V/2 \cos(2\pi f_c t + \pi)$



Polar version of x(t) = $x_{ac}(t)$



10.1 Spectra of BASK, BFSK and BPSK signals

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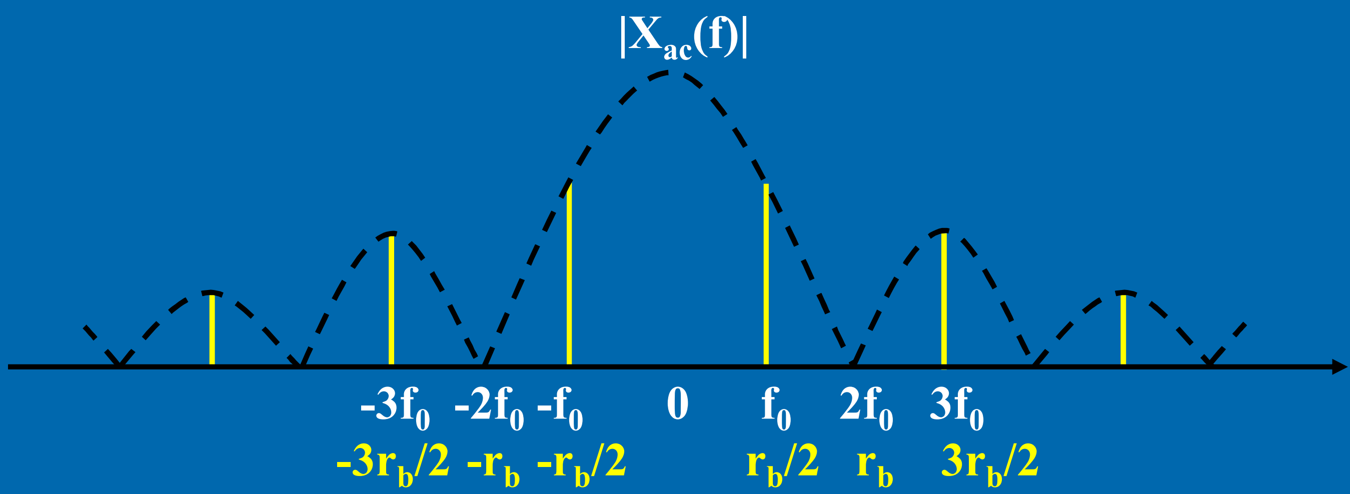
Spectrum of BPSK signal

- $x_{ac}(t)$ has no dc component.

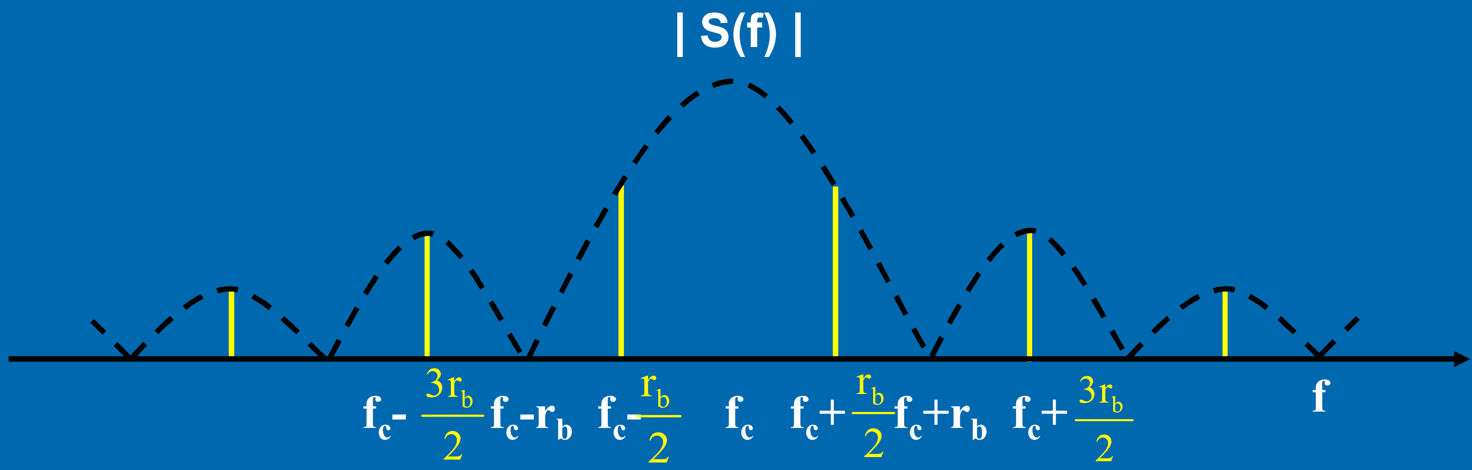
Amplitude spectrum



10.1 Spectra of ASK, FSK and PSK signals

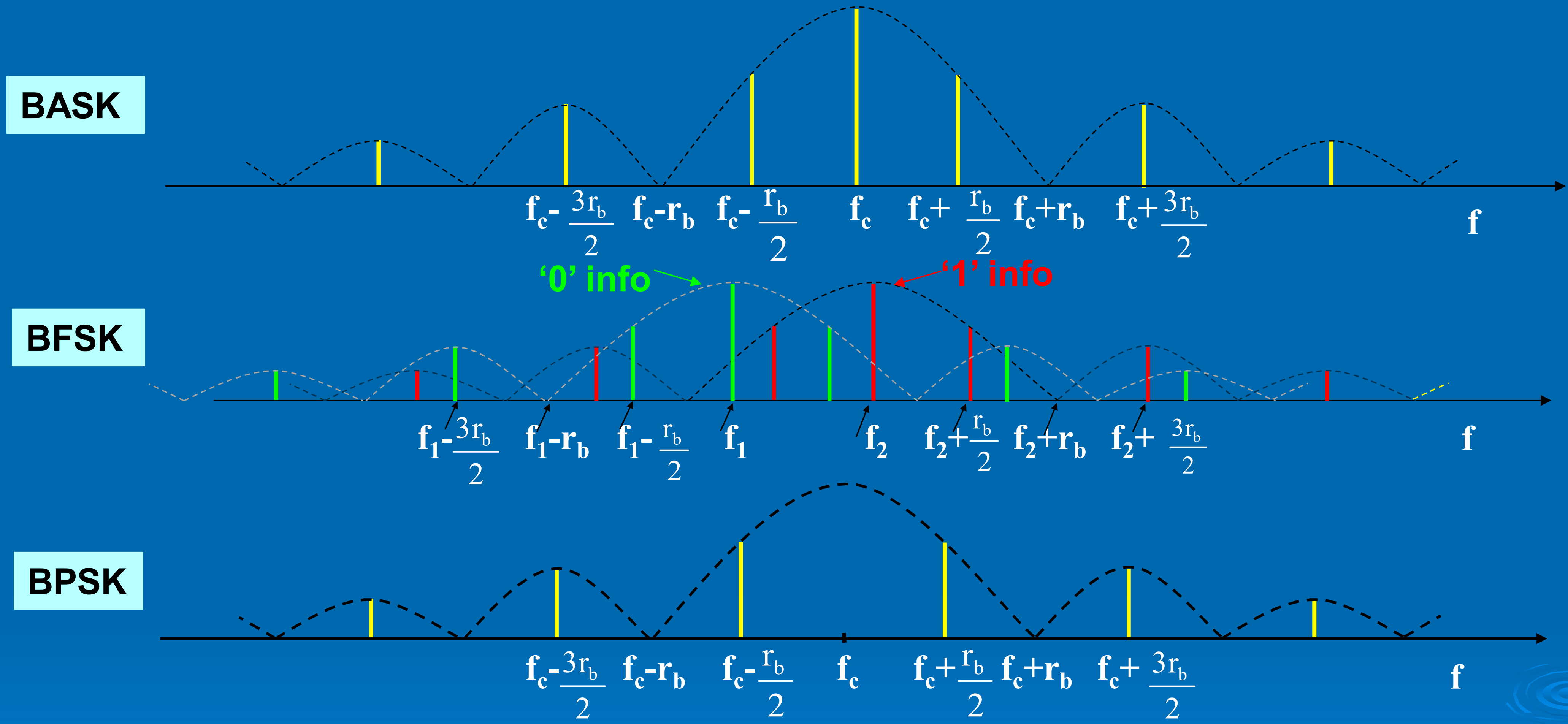
Spectrum of BPSK signal

- **BPSK waveform, $s(t) = x_{ac}(t) \cos 2\pi f_c t$**
- **The single-sided Amplitude spectrum, $S(f) = X_{ac}(f-f_c)$, frequency shifted by f_c**



Spectra of ASK and PSK are almost the same except PSK has no component at f_c.

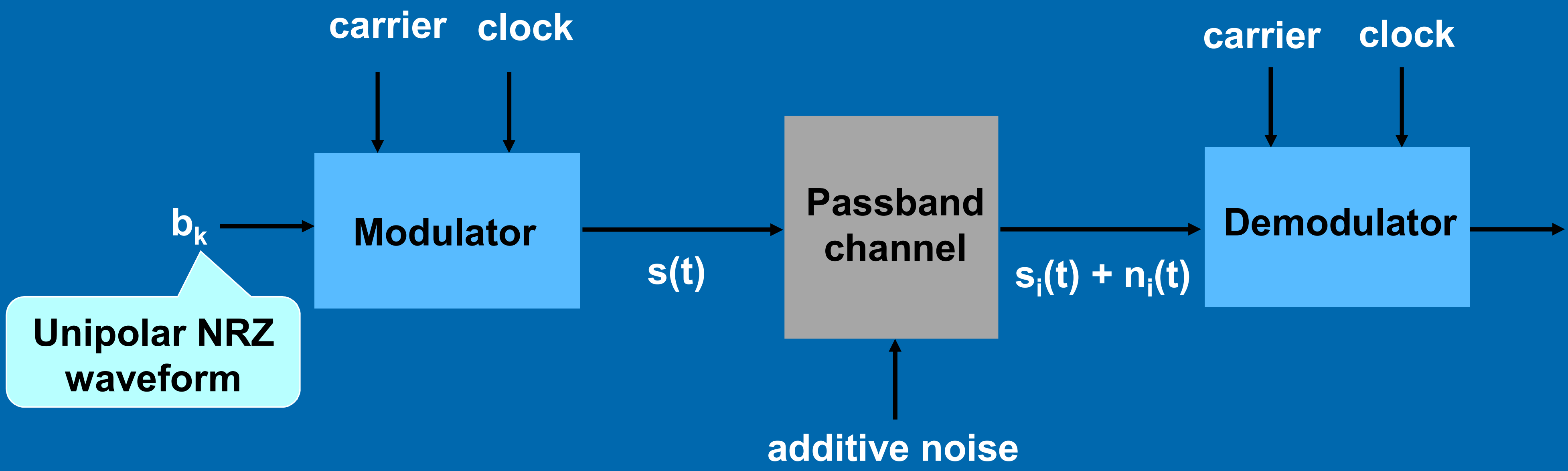
10.1 Spectra of BASK, BFSK and BPSK signals



- Input data is ….101010….
- r_b is the data bit rate.
- f_c the carrier frequency, where $f_c >> r_b$.
- For FSK, $f_1 = f_c – f_d$; $f_2 = f_c + f_d$ where f_d = frequency deviation.

10.2 Passband Binary Data Transmission System

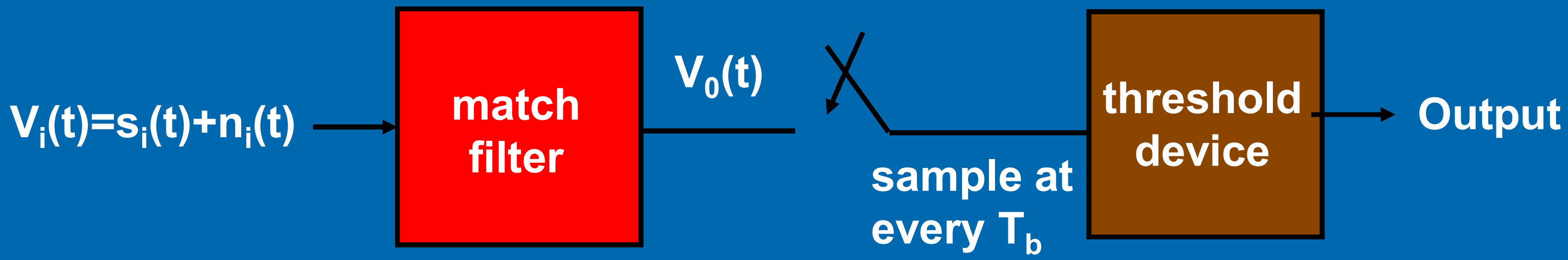
Passband binary data transmission system



10.3 Optimum Receiver for Binary Digital Modulation Systems

- Optimum receiver minimises the probability of error.
- For digital modulation systems, optimum receiver = matched filter receiver.

Receiver using a matched filter to minimize the probability of error.

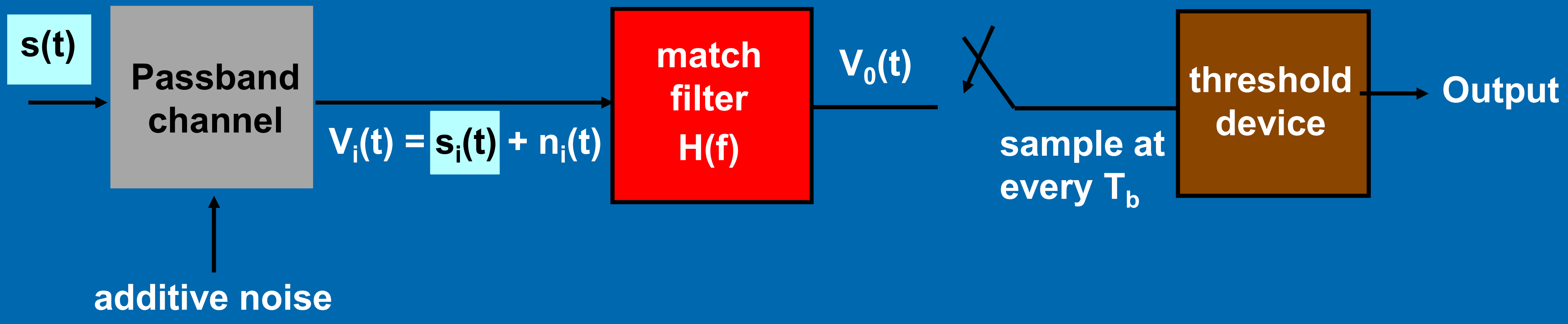


- Assumptions
  - AWGN
  - ‘1’ and ‘0’ are equiprobable and independent.
  - ISI-free channel.
  - Zero propagation delay

10.3 Optimum Receiver for Binary Digital Modulation Systems

$$s(t) = \begin{cases} s_2(t); & 0 < t < T_b; \quad \text{binary 1;} \\ s_1(t); & 0 < t < T_b; \quad \text{binary 0;} \end{cases}$$

binary 1 and binary 0 is are equal-probable and statistically independent.



- **Matched filter H(f) has impulse response:**

  $$h(t) = s_2(T_b - t) - s_1(T_b - t)$$

- **Implemented by integrate-and-dump correlation receiver.**

10.4 Coherent BPSK System

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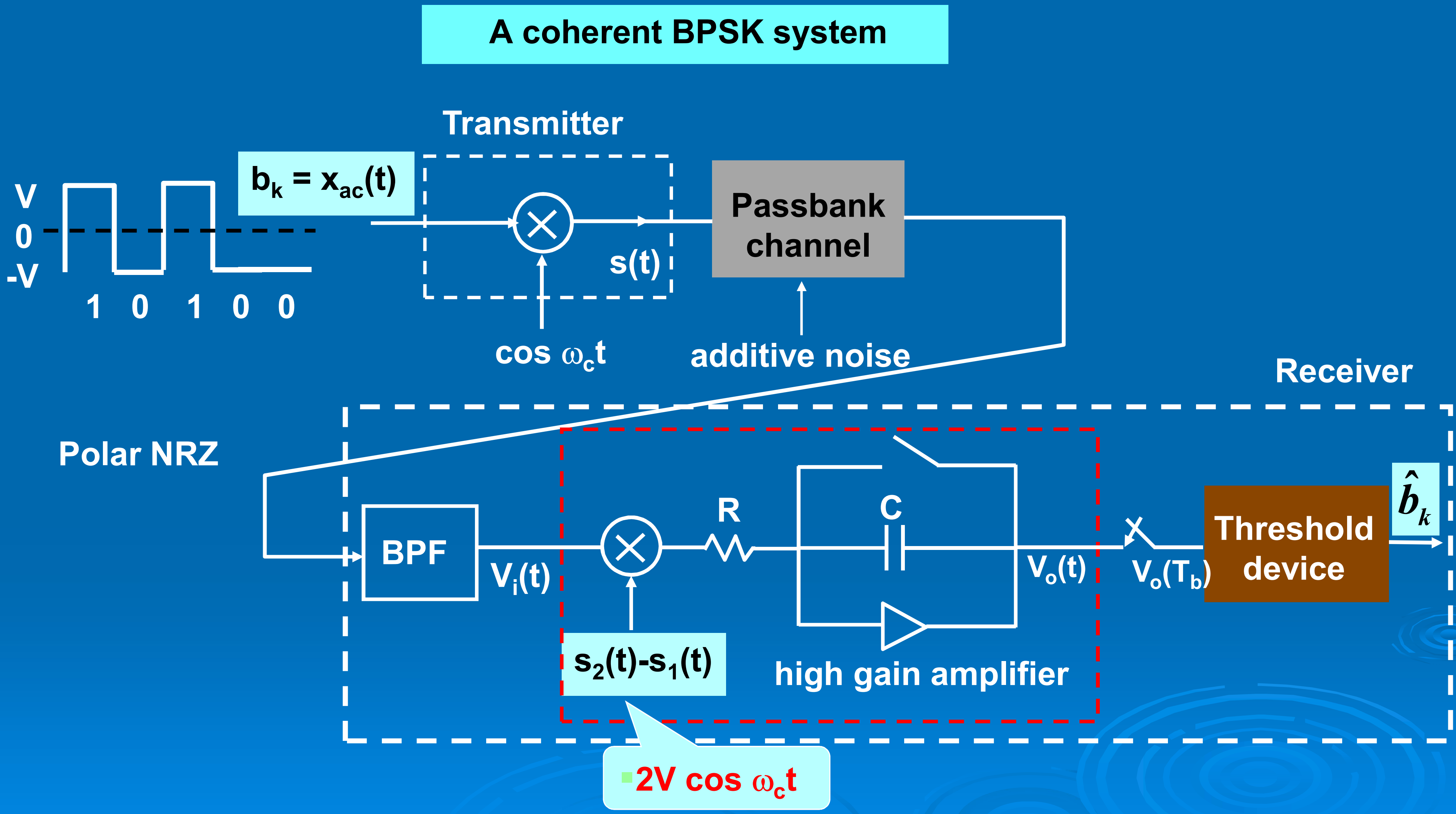
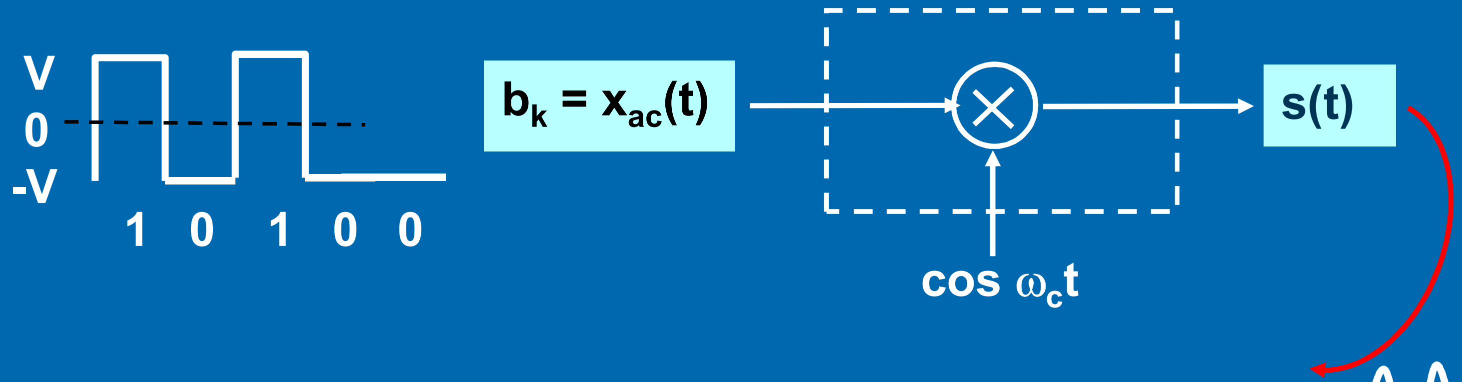
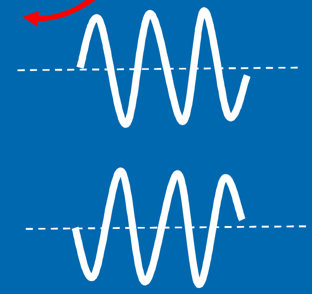


10.4 Coherent BPSK System

Transmitter



$$s(t) = \begin{cases} s_2(t) = V\cos\omega_c t & 0 < t < T_b; \quad \text{binary 1;} \\ s_1(t) = -V\cos\omega_c t & 0 < t < T_b; \quad \text{binary 0;} \end{cases}$$



10.4 Coherent BPSK System

Integrate and dump correlation receiver

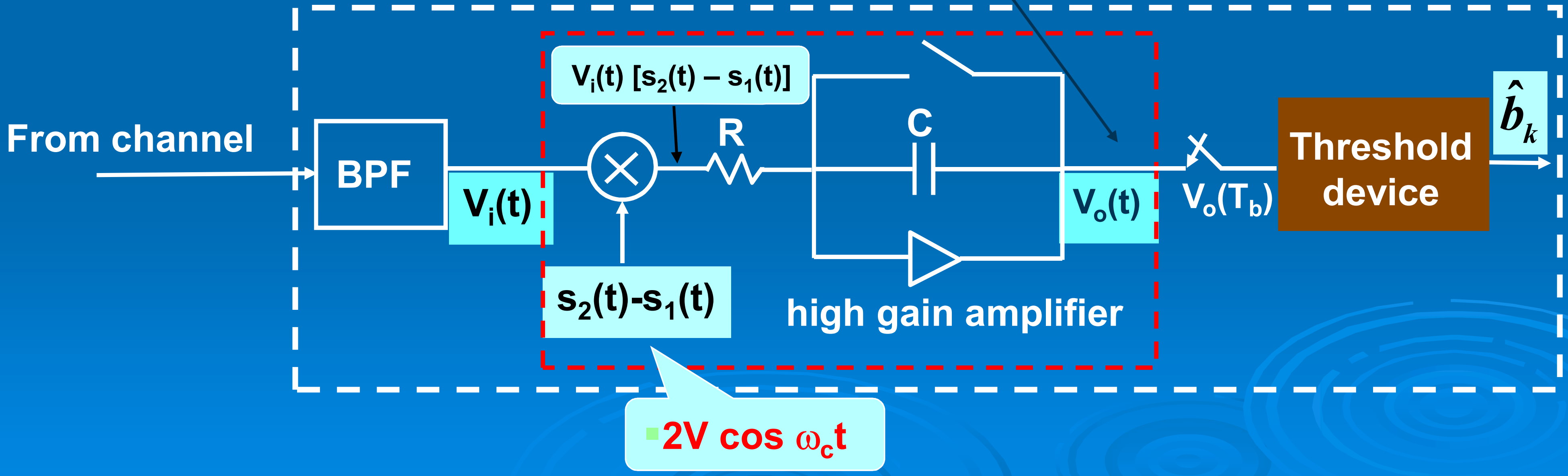
$V_i(t) = s_i(t) + n_i(t)$

- $s_i(t) = s(t)$ – output of transmitter
- $n_I(t) =$ AWGN from the channel.

$$V_o(T_b) = K\int_0^{T_b} V_i(t)[s_2(t) - s_1(t)]dt$$

where $s_2(t) - s_1(t) = 2V\cos\omega_c t$

k is a circuit constant.



10.4 Coherent BPSK System

- **Consider only the signal component for simplicity i.e. no channel noise**

Binary '1' **for noise - free channel**

$V_i(t) = s_2(t) = V\cos\omega_c t$

$$V_o(T_b) = k\int_0^{T_b} V\cos\omega_c t(2V\cos\omega_c t)dt = k\int_0^{T_b} 2V^2\cos^2\omega_c t dt$$

using $\cos 2\theta = 2\cos^2\theta - 1$

$$\Rightarrow \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= 2kV^2\int_0^{T_b}\frac{(1+\cos 2\omega_c t)}{2}dt = 2kV^2\int_0^{T_b}\frac{1}{2}dt + 2kV^2\int_0^{T_b}\frac{\cancel{\cos 2\omega_c t}}{2}dt$$

0

Assume ideal case: whole cycles within 1 bit.

$$= kV^2\int_0^{T_b}dt = kV^2[t]_0^{T_b} = kV^2 T_b$$

10.4 Coherent BPSK System

- **Consider only the signal component for simplicity i.e. no channel noise**

Binary '0' **for noise - free channel**

using $\cos 2\theta = 2\cos^2\theta - 1$

$\Rightarrow \cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$V_i(t) = s_1(t) = -V\cos\omega_c t$

$$V_o(T_b) = k\int_0^{T_b} -V\cos\omega_c t\,(2V\cos\omega_c t)\,dt = k\int_0^{T_b} -2V^2\cos^2\omega_c t\,dt$$

$$= -2kV^2\int_0^{T_b}\frac{(1+\cos 2\omega_c t)}{2}dt = -2kV^2\int_0^{T_b}\frac{1}{2}dt - 2kV^2\int_0^{T_b}\frac{\cos 2\omega_c t}{2}dt$$

0

$$= -kV^2\int_0^{T_b}dt = -kV^2T_b$$

Assume ideal case: whole cycles within 1 bit.

10.4 Coherent BPSK System

Probability of bit error for BPSK signals

- **The probability of matched filter receiver:**

$$P_e = \frac{1}{2} erfc\left(\frac{\gamma}{2\sqrt{2}} \right)$$

where $\gamma^2 = \frac{2}{\eta} \int_0^{T_b} [s_2(t) - s_1(t)]^2 dt$

η **is the single-sided power spectral density of the white noise, $n_i(t)$.**

10.4 Coherent BPSK System

Probability of bit error for BPSK

For BPSK system

$s_2(t)-s_1(t) = 2V \cos \omega_c t$

$$\gamma^2 = \frac{2}{\eta}\int_0^{T_b}(2V\cos\omega_c t)^2 dt = \frac{2}{\eta}\int_0^{T_b} 4V^2\cos^2\omega_c t dt = \frac{8V^2}{\eta}\int_0^{T_b}\frac{(1+\cos 2\omega_c t)}{2}dt$$

$$\gamma^2 = \frac{8V^2}{\eta}\int_0^{T_b}\frac{1}{2}dt + \frac{8V^2}{\eta}\int_0^{T_b}\frac{\cancel{\cos 2\omega_c t}}{2}dt$$

0

Assume ideal case: whole cycles within 1 bit.

$$= \frac{4V^2}{\eta}[t]_0^{T_b} = \frac{4V^2 T_b}{\eta}$$

or $$\gamma = \sqrt{\frac{4V^2 T_b}{\eta}}$$

10.4 Coherent BPSK System

Probability of bit error for BPSK

Therefore

$$P_e = \frac{1}{2} erfc\left[\frac{\gamma}{2\sqrt{2}}\right] = \frac{1}{2} erfc\left[\frac{\sqrt{\frac{4V^2T_b}{\eta}}}{2\sqrt{2}}\right] = \frac{1}{2} erfc\left[\sqrt{\frac{V^2T_b}{2\eta}}\right]$$

End

CHAPTER 10

(Part 1 of 2)