SINGAPORE POLYTECHNIC

2019/2020 SEMESTER TWO EXAMINATION

School of Architecture & the Built Environment
DCEB

School of Chemical and Life Sciences
DAPC, DCHE, DFST, DPCS

School of Electrical and Electronic Engineering
DASE, DCEP, DCPE, DEB, DEEE

School of Mechanical and Aeronautical Engineering
DARE, DCEP, DME, DMRO, DBEN

Singapore Maritime Academy (SMA)
DMR

1st Year FT

ENGINEERING MATHEMATICS I Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **THREE** sections:

   Section A: 5 Multiple-Choice Questions (10 marks)
   Answer **ALL** questions.

   Section B: 7 Questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

   Section C: 3 Questions (40 marks)
   Answer **ALL** questions.

3. Unless otherwise stated, all non-exact answers should be corrected to three significant figures.

4. Except for sketches, graphs and diagrams, no solution or answer is to be written in pencil.

5. This examination paper consists of **6** printed pages.

Section A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks. No mark will be deducted for incorrect answers.

Tick the choice of answer for each question in the box of the **MCQ answer sheet** provided in the answer booklet.

A1. If $\boldsymbol{A}$ is a $\mathrm{m} \times \mathrm{n}$ matrix and $\boldsymbol{B}$ is a $\mathrm{n} \times \mathrm{n}$ matrix, where m ≠ n, which of the following products will be a square matrix?

(a) $\boldsymbol{AB}$ (b) $\boldsymbol{BA}^{\mathrm{T}}$ (c) $\boldsymbol{AA}^{\mathrm{T}}$ (d) $\boldsymbol{AB}^2$

A2. Variables x and y are related by the equation $y = ae^{-x} + b$, where a and b are constants. In order to reduce the equation to linear form and have a best-fit straight line drawn, we plot

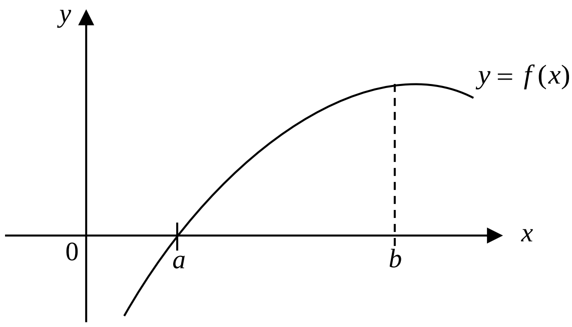
(a) y against e^{-x} (b) $\ln(y)$ against $\ln(x)$

(c) $\ln(y)$ against x (d) y against x

A3. Given the complex number $z = je^{j\theta}$, where $0 < \theta < \dfrac{\pi}{2}$. Then argument of z is

(a) $\theta - \dfrac{\pi}{2}$ (b) $\theta + \pi$ (c) $-\theta$ (d) $\theta + \dfrac{\pi}{2}$

A4. The diagram below shows the graph of the function $y = f(x)$.



Given that $\dfrac{dx}{dt}$ is negative over the interval $a < x < b$, then $\dfrac{dy}{dt}$ over the same interval

(a) is zero (b) is positive

(c) is negative (d) can be postive or negative

A5. If $\dfrac{d}{dx}F(x) = f(x) + 1$, then $\int_a^b f(x)\,dx =$ __________.

(a) $F(b) - F(a) - 1$

(b) $F(b) - F(a) - b + a$

(c) $F(b) - F(a)$

(d) $F(b) - F(a) + b - a$

Section B (50 marks)

Each question carries 10 marks.

The total mark of the questions in this section is 70 marks.

You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. The forces X, Y and Z exerted by three components of a machine are represented by the following system of equations :

$$\begin{aligned} -2X + 3Y + 2Z &= -13 \\ 3X - 8Y + \ \ Z &= 51 \\ 5X \qquad\quad - kZ &= 11 \end{aligned}$$

Use Cramer's rule to find the value of Z. Leave the answer in terms of the constant k.
[Note : *Detailed workings of evaluating a determinant must be clearly shown*.]

B2. Let $\boldsymbol{A} = \begin{pmatrix} 2 & 0 \\ 9 & 1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$.

(a) Evaluate $\boldsymbol{A} + 3\boldsymbol{B}$.

(b) Evaluate the product $\boldsymbol{AB}$.

(c) Find $\boldsymbol{A}^{-1}$.

(d) Find matrix $\boldsymbol{C}$, given that $\boldsymbol{CA} = \boldsymbol{B}$.

B3. Given $z_1 = 1 - j$ and $z_2 = 4\angle -120^\circ$.

(a) Express z_1 in polar form and z_2 in rectangular form.
[Note : *Using calculator to do the conversion is allowed*.]

(b) Evaluate the following and leave your answers in polar form.

(i) $z_1 + z_2$ (ii) $z_1\overline{z_2}$ (iii) $\dfrac{z_1}{z_2}$ (iv) $(z_1)^3$

[Note : *Detailed workings must be clearly shown*.]

B4. A hypothetical study of a phenomenon shows that the values of W and S follow a certain behavioural trend which can be modelled by the equation $WS = a + Wb$, where a and b are constants. The measured values of W and S are tabulated as shown below.

S	5	10	15	20	25
W	7.20	2.40	1.44	1.03	0.80

(a) State the variables that should be plotted on the vertical and horizontal axes such that a best-fit straight line can be drawn.

(b) Hence compute in a table, the values of the variables to be plotted on the horizontal and vertical axes. *<u>Do not plot the values</u>*.

(c) Suppose the best-fit straight line passes through the second and the fifth points of the new values, use these two points to estimate the gradient and the vertical intercept. Hence, determine the values of a and b.

B5. (a) Given $f(x) = \ln(4x-1) + \tan^{-1}(2x)$, find the value of the derivative $f'(0)$.

(b) Given $y = 3e^{2x}\sin(x)$, find $\frac{dy}{dx}$ and simplify the answer.

B6. (a) When a viscous liquid is poured onto a flat surface, it forms a circular patch of area A (cm^2) and radius r (cm).

(i) Find the rate of change of A with respect to r when r = 3 cm.

(ii) If r is increasing at a steady rate of 0.5 cm/s, find the rate of change of A.

[Note : area of circle = πr^2]

(b) When an object speeds upward, its equation of motion v (m/s) is given by

$$v = 1100 - 110t - 4.9t^2$$

If the force (N) on the object is given by $F = 10\frac{dv}{dt}$, find the force F at t = 10 s.

B7. (a) Find the following integrals:

(i) $$\int\left(\frac{2}{x}-e^{6x}+9\sin(3x)\right)dx$$

(ii) $$\int\left(\frac{1}{\sqrt{100-x^2}}\right)dx$$

(b) An object fired from ground level rises upwards. Its velocity v (m/s) at t (s) is given by $v=100-25t$.

(i) Find an expression for the position of the object h (m) from ground level in terms of t.

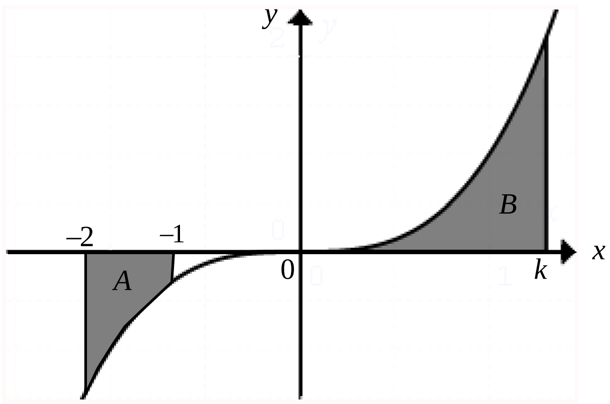
(ii) Find the value of h after 3 s.

$\left[\text{Note: } h=\int v\ dt\right]$

Section C (40 marks)
Answer ALL **THREE** questions.

C1. The diagram below shows the curve $y=x^3$.

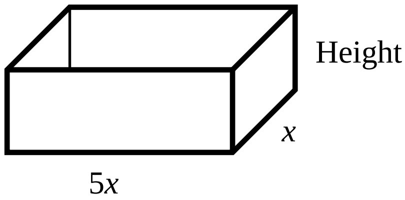
Region A is bounded by the curve, the lines $x = -2$, $x = -1$ and the x-*axis*.
Region B is bounded by the curve, the x-*axis* and the line $x = k$.
If the area of region B is four times that of region A, find the value of k.



(13 marks)

C2. (a) Given $y + x^2 = 2y^3 - \ln x$, find $\dfrac{dy}{dx}$. (6 marks)

(b) An <u>open</u> rectangular container of volume 300 cm^3 has a width of x cm and a length of $5x$ cm. The bottom of the container is made of a material that costs 10 cents per square centimeter while the sides are made of another material that costs 5 cents per square centimeter.



(i) Show that the material cost C (<u>cents</u>) of the container is given by

$$C = 50x^2 + \frac{3600}{x}$$

(ii) Find the value of x such that the material cost is minimum. (11 marks)

C3. The equation $Z^2 + mZ + 2 = 0$ has two distinct complex roots Z_1 and Z_2. If $|Z_1 - Z_2| = 2$, find the value(s) of the real number m. (10 marks)

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