2018/2019 SEMESTER ONE EXAMINATION

Diploma in Electrical and Electronic Engineering
3rd Year Full Time

DIGITAL SIGNAL PROCESSING

Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules as set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **TWO** sections:

   Section A - 6 Short Questions, 10 marks each.
   Section B - 2 Long Questions, 20 marks each.

3. **ALL** questions are **COMPULSORY**.

4. **ALL** questions are to be answered in the answer booklet.

5. This paper consists of **6** pages, including 2 pages of mathematical formulae.

SECTION A - SHORT QUESTIONS [10 marks each]

A1. Use partial fraction to find the impulse response of the system described by the following equation,

$$H(z) = \frac{2z^{-2}}{1+6z^{-1}+11z^{-2}+6z^{-3}}$$

Hint:

$(1 + 6z^{-1} + 11z^{-2} + 6z^{-3}) = (1+z^{-1})(1 + 2z^{-1})(1 + 3z^{-1})$ (10 marks)

A2 Using z-transform and long-division method, find the input x(n) given y(n)={2, 3,1,6} and h(n)={1,2} (10 marks)

A3 Find the z-transform of $x_1(n) = e^{-2n}\sin(3n)u(n)$ and $x_2(n)=n\ 5^{n-1}u(n)$. (10 marks)

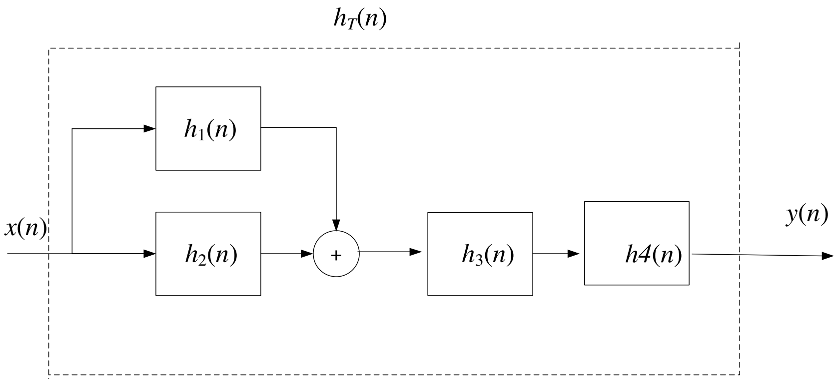
A4 The system function, H(z) of a digital filter is given as

$H(z) \quad = 1 + 2z^{-1} + z^{-2}$

(a) Compute the magnitude of $|H(e^{jw})|$ for $\omega = 0, \pi/2$ and π and sketch the magnitude response. (8 marks)

(b) By observing the magnitude response, comment on the function of this filter. (2 marks)

A5 The block diagram of a digital system is given as:



(a) Find the overall impulse response of the system, $h_T(n)$ in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$ and $h_4(n)$. Find the z-transform of $h_T(n)$, $H_T(z)$. (6 marks)

(b) If $h_1(n)$ ={0,1}, $h_2(n)$ = {1,0}, $h_3(n)$ = $h_4(n)$={1,1} respectively, find $h_T(n)$. (4 marks)

A6 The difference equation of a particular digital network is given as:

$$y(n) = x(n) - x(n-2) - 0.5\, y(n-1)$$

(a) Sketch the digital network. (5 marks)

(b) Find the z-transform of the transfer function, H(z). (3 marks)

(c) Given the input x(n) is a unit step function, find y(n) (2 marks)

SECTION B - LONG QUESTIONS [20 marks each]

B1. A 5000Hz sine wave is sampled at 20000 samples per second. 32-point DFT is applied.

(a) Sketch the spectrum of this 5 kHz sine wave. (2 marks)

(c) Sketch the spectrum of the 5 kHz sine wave sampled at 20 kHz between -20 kHz and 20 kHz. Can the original signal be recovered by a reconstruction filter? Give your reason. (6 marks)

(c) Compute the frequency resolution of DFT. (2 marks)

(d) With reference to B1(b), between 0 and 20 kHz, how many non-zero spectral components? Where are the locations of these non-zero components? (6 marks)

(e) Sketch the 32-point DFT of a 5 kHz sine wave. (4 marks)

B2. A FIR low pass filter is to be designed using the windowing technique. The specification is given below:

Sampling frequency: 4 kHz
Pass band: 0 to 500 Hz
Stop band: 1 kHz to 2 kHz
Peak approximation error: 0.02

(a) Determine the type of Window functions to be used. (4 marks)

(b) Compute the centre of the transition band. (2 marks)

(c) Compute the first two and last two tap coefficients. (6 marks)

(d) Draw the digital network of the FIR filter. Determine if the system is stable or not with justification. (5 marks)

(e) Sketch the diagram for the magnitude response of the filter indicating clearly ω_p, ω_s, ω_c, and attenuation. (3 marks)

-End of Paper-

Appendix

The z-transform is defined as $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$

Sequence	Transform
$\delta[n]$	1
$u[n]$	$\frac{1}{1-z^{-1}}$
$\delta[n-m]$	z^{-m}
$a^n u[n]$	$\frac{1}{1-az^{-1}}$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$
$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$

Some z-transform properties:	
Sequence	Transform
$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
$x[n-m]$	$z^{-m}X(z)$

Some trigonometric identities:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

Series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

Discrete Fourier Transform:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{N}kn}$$

Complex number theory:

$$z = a + jb = r\angle\theta = re^{j\theta}$$

$$r = \sqrt{a^2+b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

Quadratic equation solution:

If $ax^2 + bx + c = 0$

then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

The characteristics of the different windowing functions:

Window Type	Peak approximation Error 20 $\log_{10}\delta$ dB	Transition Band $\Delta\omega$
Rectangular: $w(n)=\begin{cases}1 & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$	-21	$\dfrac{4\pi}{M+1}$
Bartlett: $w[n]=\begin{cases}\dfrac{2n}{M} & 0\le n\le \dfrac{M}{2}\\ 2-\dfrac{2n}{M} & \dfrac{M}{2}\le n\le M\\ 0 & \text{otherwise}\end{cases}$	-25	$\dfrac{8\pi}{M}$
Hanning: $w(n)=\begin{cases}0.5-0.5\cos\left(\dfrac{2\pi n}{M}\right) & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$	-44	$\dfrac{8\pi}{M}$
Hamming: $w(n)=\begin{cases}0.54-0.46\cos\left(\dfrac{2\pi n}{M}\right) & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$	-53	$\dfrac{8\pi}{M}$
Blackman: $w(n)=\begin{cases}0.42-0.5\cos\left(\dfrac{2\pi n}{M}\right)+0.08\cos\left(\dfrac{4\pi n}{M}\right) & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$	-74	$\dfrac{12\pi}{M}$

The impulse response of an ideal low pass filter is: $h_d(n)=\dfrac{\sin\left(\omega_c\left[n-\dfrac{M}{2}\right]\right)}{\pi\left(n-\dfrac{M}{2}\right)}$