SINGAPORE POLYTECHNIC

2017/2018 SEMESTER ONE EXAMINATION

School of Chemical and Life Sciences
DACP, DCHE, DFST, DPCS

School of Architecture & the Built Environment
DCEB

School of Electrical and Electronic Engineering
DASE, DESM, DCPE, DEB, DEEE, DES, DCEP

School of Mechanical and Aeronautical Engineering
DARE, DCEP, DME, DMRO, DBEN

Singapore Maritime Academy (SMA)
DMR

1st Year FT

ENGINEERING MATHEMATICS I Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **THREE** sections:

   Section A: 5 Multiple-Choice Questions (10 marks)
   Answer **ALL** questions.

   Section B: 7 Questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

   Section C: 3 Questions (40 marks)
   Answer **ALL** questions.

3. Unless otherwise stated, leave all answers correct to three significant figures.

4. Except for sketches, graphs and diagrams, no solution or answer is to be written in pencil.

5. This examination paper consists of **7** printed pages.

Section A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks. No mark will be deducted for incorrect answers.

Tick the choice of answer for each question in the box of the **MCQ answer sheet** provided in the answer booklet.

A1. Let $\boldsymbol{A}=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 0 \\ 3 & 0 & 0 \end{pmatrix}$. Which of the following statements is true?

(a) $\boldsymbol{A}$ is both a symmetric matrix and a singular matrix
(b) $\boldsymbol{A}$ is a symmetric matrix, but not a singular matrix
(c) $\boldsymbol{A}$ is not a symmetric matrix, but a singular matrix
(d) $\boldsymbol{A}$ is neither a symmetric matrix nor a singular matrix

A2. The variables y and t are found to follow the equation $y=10+e^{h-kt}$

where h and k are unknown constants.

In order to obtain a straight line to determine h and k, we plot

(a) $\ln y$ against t

(b) $\ln\left(y-10\right)$ against e^t

(c) $\ln y$ against e^t

(d) $\ln\left(y-10\right)$ against t

A3. Given $Z=x+jy=re^{j\theta}$, which of the following is **NOT** true ?

(a) $Z\,\overline{Z}=\left|Z\right|^2$, where $\overline{Z}$ is the conjugate of Z

(b) $Z+\overline{Z}=2r\cos\theta$, where $\overline{Z}$ is the conjugate of Z

(c) The geometrical relationship between Z and jZ is reflection of Z about the Imaginary-axis

(d) The geometrical relationship between Z and $-Z$ is a rotation of Z about the origin through π radian

A4. Suppose $f(x)$ is a continuous function and $f'(x) > 0$ over the interval $a \le x \le b$.

Which of the following statement(s) is/are TRUE over the interval $a \le x \le b$?

(I) $f(x)$ is always positive

(II) $f(x)$ is increasing

(III) There is no stationary point

(IV) There is a point of inflection

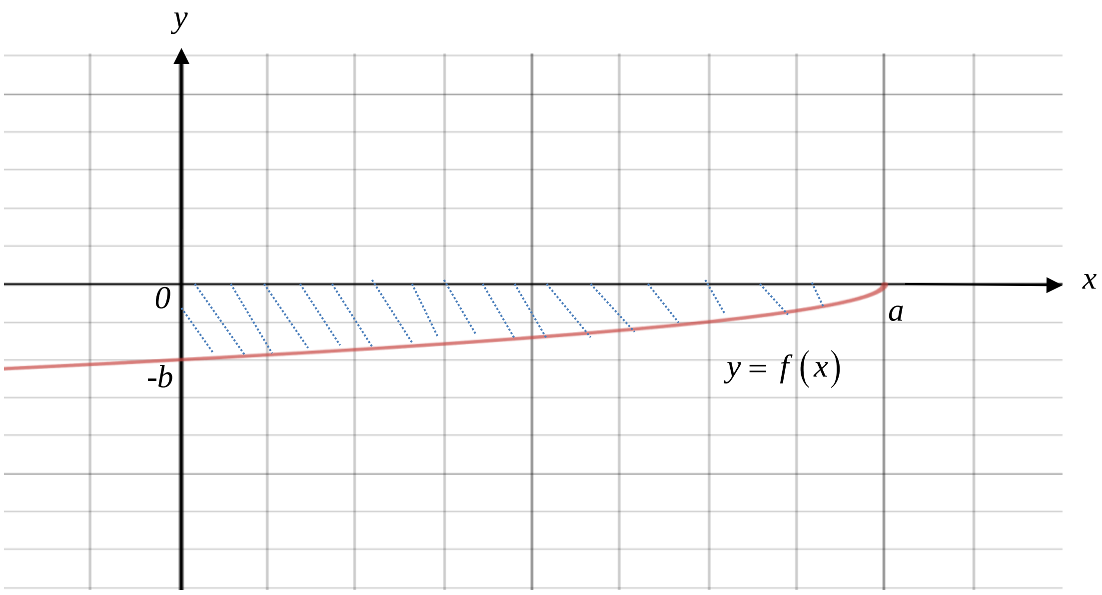
(a) (II) , (III) , (IV)

(b) (I) , (II) , (III) , (IV)

(c) (II) , (III)

(d) (I) , (II) , (IV)

A5. The diagram below shows the graph of $y = f(x)$.



Which of the following expression(s) give(s) the area of the shaded region?

I. $-\int_0^a y\,dx$

II. $\int_{-b}^0 x\,dy$

III. $-\int_{-b}^0 x\,dy$

IV. $-\int_{-b}^0 y\,dx$

(a) I only

(b) I and II only

(c) I and III only

(d) II and IV only

Section B (50 marks)

Each question carries 10 marks.

The total mark of the questions in this section is 70 marks.

You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. The currents I_1, I_2 and I_3 in a certain network are related by the following equations.

$$\begin{array}{ccccccc} I_1 & + & 3I_2 & + & 2I_3 & = & 19 \\ 2I_1 & + & I_2 & + & I_3 & = & 13 \\ 4I_1 & + & 2I_2 & + & 3I_3 & = & 31 \end{array}$$

By using Cramer's Rule, find the value of I_3 .

(Detailed workings of evaluating a determinant must be clearly shown.)

B2. Given $\boldsymbol{A}=\begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{pmatrix}$ and $\boldsymbol{C}=\begin{pmatrix} 2 & 0 & 3 \\ p & 4 & r \\ q & -8 & 5 \end{pmatrix}$.

(i) Find $\boldsymbol{B}^{\boldsymbol{T}}$.

(ii) Find $\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{A}$.

(iii) Find $\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{A}-\boldsymbol{C}$

(iv) If $\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{A}-\boldsymbol{C}$ is a symmetric matrix, determine the values of the real numbers p , q and r .

B3. The total impedance of a circuit with series and parallel components is given by

$$Z_T = Z_1 + \frac{Z_2}{Z_3+Z_4}$$

where $Z_1 = 3 + j2$, $Z_2 = 2\angle -45^o$, $Z_3 = 1 + j$ and $Z_4 = 6\angle 40^o$.

Find the following and express all answers in polar forms.

(i) $Z_3 + Z_4$ (ii) $\dfrac{Z_2}{Z_3+Z_4}$ (iii) Z_T

(Detailed workings must be clearly shown.)

B4. The velocity (in cm/s) of a ball oscillating at the end of a vertical spring is given by

$$v^2 = a - bx$$

where x is the displacement (in cm) ; a and b are constants.
The readings collected are shown in the table below.

x	0	5	10	15	20
v	20	17.89	15.49	12.65	8.944

(i) State the variables that should be plotted on the vertical and horizontal axes of a graph so that a best fit straight line can be drawn to show that v and x obey the above equation.

(ii) Hence compute in a table the values of the variables to be plotted on the horizontal and vertical axes, correct to 4 significant figures. *Do not plot the values*.

(iii) Suppose the best fit straight line passes through the second and the fourth points of the new values, use these two points to estimate the gradient and the vertical intercept. Hence, determine the values of a and b.

B5. Find $\frac{dy}{dx}$ for each of the following.

(a) $y = 2\ln\left(1+x^2\right) - \tan^{-1}(2x)$

(b) $3y - 2\sin x = 3y^2 + x$

B6. (a) The total amount of a bacteria in a laboratory sample, N, as a function of time t (seconds) is given as

$$N = 10000e^{-0.001t}$$

(i) What is the total amount of bacteria initially?

(ii) Find the rate at which the amount of bacteria is changing with time when t = 1000 seconds.

(iii) Is the amount of bacteria increasing or decreasing with time?

(b) The charge q (coulombs) in a certain capacitor at time t (seconds) follows the equation

$$q = e^{-0.1t}\sin\left(120\pi t\right)$$

Find the expression for i (amperes) in terms of t, given that $i = \frac{dq}{dt}$.

Simplify the answer.

B7. (a) Find the following integrals:

(i) $\int \left(\frac{2}{\sqrt{x}} + 3\sin(4x) + \pi \right) dx$

(ii) $\int \frac{2}{16 + x^2} \, dx$

(b) An object moves horizontally through a medium with an initial velocity of 15m/s. The acceleration of its horizontal motion is $a = -10e^{-t}$, where a is in m/s^2 and t is the time in seconds. Find the velocity of the object after 2 seconds. [Hint: $v = \int a \, dt$].

Section C (40 marks)

Answer ALL **THREE** questions.

C1. (a) Given $f(x) = \ln(9 + x^2)$.

(i) Find $f'(x)$.

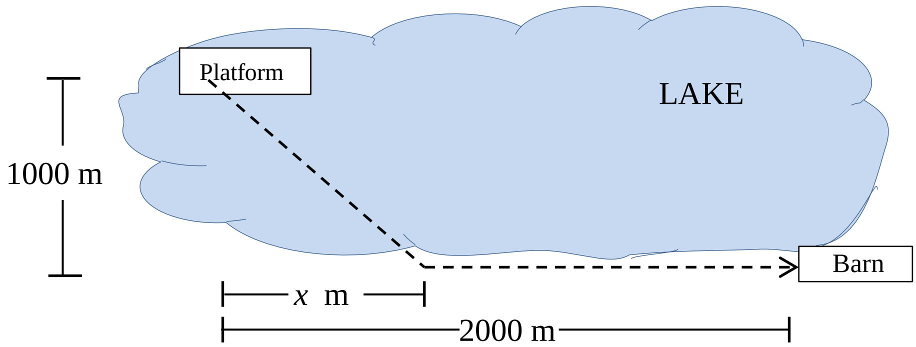
(ii) Hence or otherwise, find $\int \frac{1+3x}{9+x^2} \, dx$. (6 marks)

(b) The heat capacity, C, of a substance varies with temperature, T, measured in kelvins (K), as

$$C = \alpha + \beta T - \sigma T^2$$

where α , β and σ are positive constants. If the temperature is decreasing at the rate of 2 K/s, what is the rate of change of C at T = 300 K? Leave your answer in terms of β and σ . (6 marks)

C2. In an annual pigeon racing event, pigeons are released from a platform on a lake where they would fly to a barn located on the other side of the lake. The path taken by the pigeons is represented by the dotted line.



Assuming that each pigeon uses 30 joules of energy for every metre it flies over water and 10 joules for every metre it flies over land,

(i) show that the total amount of energy E used by a pigeon during the race is

$$E = 30\sqrt{x^2 + 1000000} + 20000 - 10x$$

(ii) find the value of x such that the amount of energy used by a pigeon during the race is the minimum. (14 marks)

C3. (a) If $z = -1 + j\sqrt{3}$, find the real number a such that $\arg(z(z+a)) = \dfrac{5\pi}{6}$.

(b) A complex number is given as $z = e^{j\theta}$.

(i) Express $z^n + \dfrac{1}{z^n}$, where n is a positive integer, in terms of cosine only.

(ii) Hence show that $\cos^3\theta = \dfrac{1}{8}\left(z^3 + 3z + \dfrac{3}{z} + \dfrac{1}{z^3}\right)$. (14 marks)

~~~ END OF PAPER ~~~

	Solution
A	A1) **b** A2) **d** A3) **c** A4) **c** A5) **b**
B1	$\Delta = \begin{vmatrix} 1 & 3 & 2 \\ 2 & 1 & 1 \\ 4 & 2 & 3 \end{vmatrix}$ $= 1\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} - (3)\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} + (2)\begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix}$ or $3+12+8-8-2-18$ $= -5$ $\Delta I_3 = \begin{vmatrix} 1 & 3 & 19 \\ 2 & 1 & 13 \\ 4 & 2 & 31 \end{vmatrix}$ $= 1\begin{vmatrix} 1 & 13 \\ 2 & 31 \end{vmatrix} - (3)\begin{vmatrix} 2 & 13 \\ 4 & 31 \end{vmatrix} + (19)\begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix}$ or $31+156+76-76-26-186$ $= 5-30+0 \quad = -25$ $i_3 = \frac{\Delta i_3}{\Delta} = \frac{-25}{-5} = 5$
B2 (i)	$B^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix}$
B2 (ii)	$B^T A = \begin{pmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{pmatrix}$
B2 (iii)	$B^T A - C = \begin{pmatrix} -13 & 6 & -4 \\ -22-p & 8 & -2-r \\ -11-q & 14 & -6 \end{pmatrix}$
B2 (iv)	For Symmetric matrix, *-22-p = 6* *-11-q= – 4* *-2-r=14* *p=-28 , q = - 7 , r = - 16*

	Solution
B3 (i)	$Z_3 + Z_4 = 1 + j \quad + 6\angle 40^o$ $= 1 + j \quad + (4.596 + j3.857)$ $= 5.596 + j4.857$ $= 7.41\angle 40.96^o$
(ii)	$\dfrac{Z_2}{Z_3 + Z_4} = \dfrac{2\angle -45^o}{7.41\angle 40.96^o}$ $= 0.270\angle -85.96^o$
(iii)	$Z_T = 3 + j2 + 0.270\angle -85.96^o$ $= 3 + j2 + 0.019 - j0.269$ $= 3.02 + j1.731$ $= 3.481\angle 29.8^o$
B4	Vertical axis : v^2 Horizontal axis : x *x* *0* *5* *10* *15* *20* v^2 *400* *320.1* *239.9* *160.0* *80.00* *a = intercept on vertical-axis* $-b = gradient$ $= \dfrac{160 - 320.1}{15 - 5} \quad = -16.01$ $b = 16.01$ *Substitute (5,320.1) into* $v^2 = a - bx$ 320.1=*a -16.01(5)* *a=400.2*
B5 (a)	$\dfrac{dy}{dx} = 2\left(\dfrac{1}{1+x^2}\right)(2x) - \left(\dfrac{1}{1+4x^2}\right)(2)$
B5 (b)	$3\dfrac{dy}{dx} - 2\cos x = 3(2y)\dfrac{dy}{dx} + 1$ $3\dfrac{dy}{dx} - 6y\dfrac{dy}{dx} = 2\cos x + 1$ $\dfrac{dy}{dx}(3 - 6y) = 2\cos x + 1$ $\dfrac{dy}{dx} = \dfrac{2\cos x + 1}{(3 - 6y)}$
B6 (a)	i) 10000 ii) $\dfrac{dN}{dt} = 10000e^{\left(\frac{-t}{1000}\right)}\left(\dfrac{-1}{1000}\right)$ at t = 1000 s, $\dfrac{dN}{dt} = -10e^{\left(\frac{-1000}{1000}\right)} \quad = \dfrac{-10}{e} = -3.7/\text{s}$ iii) Decreasing

	Solution
B6 (b)	$i=\frac{dq}{dt}$ $=e^{-0.1t}\cos(120\pi t)(120\pi)+\sin(120\pi t)e^{-0.1t}(-0.1)=e^{-0.1t}\left(120\pi\cos(120\pi t)-0.1\sin(120\pi t)\right)$
B7 (a)i ii	$\int\left(\frac{2}{\sqrt{x}}+3\sin(4x)+\pi\right)dx=2\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+3\left(\frac{1}{4}\right)\left(-\cos(4x)\right)+\pi x+C$ $\int\frac{2}{16+x^2}\,dx=2\left(\frac{1}{4}\right)\tan^{-1}\left(\frac{x}{4}\right)+C$
B7 (b)	$a=\frac{dv}{dt}=-10e^{-t}$ $v=\int-10e^{-t}\,dt=10e^{-t}+C$ When $t=0,\quad v=15\ ;\quad 15=10+C$ $\therefore C=5$ Hence, $v=10e^{-t}+5$ When $t=2,\quad v=10e^{-2}+5\quad=6.353$ The velocity of the object is $6.353\ \ m/s$
C1 (a)(i) (ii)	$f^{'}(x)=\frac{2x}{9+x^2}$ $\int\frac{1+3x}{9+x^2}\,dx=\int\frac{1}{9+x^2}\,dx+\int\frac{3x}{9+x^2}\,dx$ $\int\frac{1}{9+x^2}\,dx=\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C_1$ $\int\frac{3x}{9+x^2}\,dx=3\int\frac{x}{9+x^2}\,dx$ $=3\int\frac{1}{2}f'(x)\,dx$ $=\frac{3}{2}\ln\left(9+x^2\right)+C_2$ $\therefore\int\frac{1+3x}{9+x^2}\,dx=\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+\frac{3}{2}\ln\left(9+x^2\right)+C$

	Solution
C1 (b)	$\frac{dC}{dT} = \beta - 2\sigma T$ $\frac{dT}{dt} = -2K/s$ $\frac{dC}{dt} = \frac{dC}{dT}\frac{dT}{dt} \quad = (\beta - 2\sigma T)(-2)$ $\left.\frac{dC}{dt}\right\|_{T=300} = -2(\beta - 600\sigma)$ or $2(600\sigma - \beta)$
C2 (i)	Distance of flight over water = $\sqrt{x^2 + 1000^2}$ Energy spent over water = $30\sqrt{x^2 + 1000^2}$ Distance of flight over land = $2000 - x$ Energy spent over land = $10(2000 - x)$ Total energy spent, E = $30\sqrt{x^2 + 1000^2} + 10(2000 - x)$
(ii)	$\frac{dE}{dx} = 30 \cdot \frac{1}{2}(x^2 + 1000^2)^{-\frac{1}{2}} \cdot 2x - 10 = 30x(x^2 + 1000^2)^{-\frac{1}{2}} - 10 = \frac{30x}{\sqrt{x^2 + 1000^2}} - 10$ $\frac{dE}{dx} = \frac{30x}{\sqrt{x^2 + 1000^2}} - 10 = 0$ $30x = 10\sqrt{x^2 + 1000^2}$ $3x = \sqrt{x^2 + 1000^2}$ $9x^2 = x^2 + 1000^2$ $8x^2 = 1000^2$ $x^2 = \frac{1000^2}{8}$ $x = 353.6 \ or - 353.6 \ \ (NA)$ To verify: $\frac{d^2E}{dx^2} = 30x \cdot -\frac{1}{2}(x^2 + 1000^2)^{-\frac{3}{2}} \cdot 2x + 30(x^2 + 1000^2)^{-\frac{1}{2}}$ $= -30x^2(x^2 + 1000^2)^{-\frac{3}{2}} + 30(x^2 + 1000^2)^{-\frac{1}{2}}$ $= 30(x^2 + 1000^2)^{-\frac{3}{2}}[1000^2]$ At x = 353.6 , $\frac{d^2E}{dx^2} > 0$ => Minimum point The least energy is spent when *x* is 353.6 m.

No.	SOLUTION
C3 (a)	$\arg(z)=\frac{2\pi}{3}$
	$\arg(z+a)=\tan^{-1}\left(\frac{\sqrt{3}}{a-1}\right)$
	$\arg(z(z+a))=\arg(z)+\arg(z+a)$
	$\frac{2\pi}{3}+\tan^{-1}\left(\frac{\sqrt{3}}{a-1}\right)=\frac{5\pi}{6}$
	$\tan^{-1}\left(\frac{\sqrt{3}}{a-1}\right)=\frac{5\pi}{6}-\frac{2\pi}{3}$
	$\frac{\sqrt{3}}{a-1}=\tan\left(\frac{\pi}{6}\right)$
	$\frac{\sqrt{3}}{a-1}=\frac{1}{\sqrt{3}}$
	$a-1=3$
	$a=4$
C3 (b)i	$z^n=e^{jn\theta}=\cos(\mathrm{n}\theta)+j\sin(\mathrm{n}\theta)$, $\frac{1}{z^n}=e^{-jn\theta}=\cos(\mathrm{n}\theta)-j\sin(\mathrm{n}\theta)$
	$z^n+\frac{1}{z^n}=2\cos(n\theta)$
(ii)	Let $n=1$, $2\cos(\theta)=z+\frac{1}{z}$
	$\left[2\cos(\theta)\right]^3=\left(z+\frac{1}{z}\right)^3$
	$8\cos^3(\theta)=z^3+3z^2\left(\frac{1}{z}\right)+3z\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3$
	$\cos^3(\theta)=\frac{1}{8}\left[z^3+3z+\frac{3}{z}+\frac{1}{z^3}\right]$ ➔ Shown