Magnetism

EP0605

PRE-CLASS (1 TO 12)

IN-CLASS (14 ONWARDS)

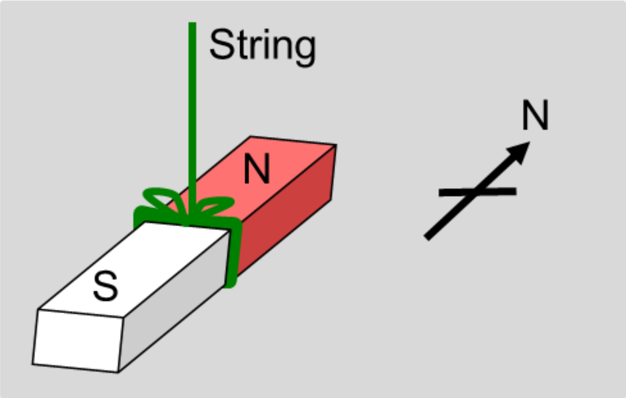
Learning outcomes

At the end of the pre-lecture slides, students should be able to

- define a magnetic field.
- state the formula for magnetic field due to straight wire and solenoid and perform calculations on it.

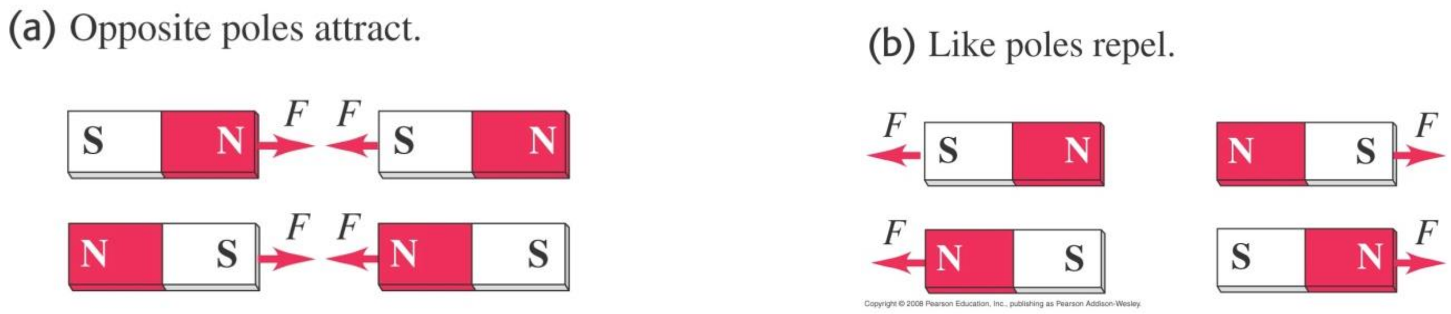
Magnetism

- Magnetic phenomena were first observed more than 2500 years ago in Turkey.
- If a bar magnet is free to rotate about a vertical axis, one end of the magnet points towards the magnetic north and is called the north seeking pole (N).
- The other end point of the magnet points towards the magnetic south and is called the south seeking pole (S).



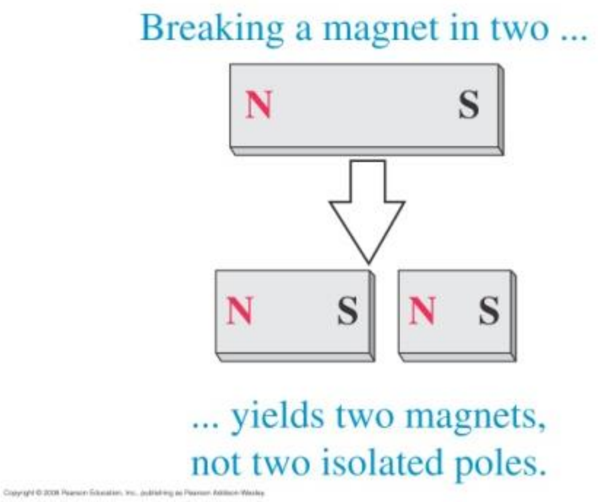
Properties of magnetic poles

- Like poles repel and unlike poles attract.



- If a bar magnet is broken into two parts, each part will have a N pole and a S pole.

- In other words, a magnetic monopole (N or S) does not exist!



Magnetic field

- A magnetic field is a region where a magnetic pole or a moving electric charge experiences a magnetic force.

- A stationary magnetic pole can experience a magnetic force but an electric charge must move in order for it to experience a magnetic force.

- This is because a moving charge is equivalent to an electric current ($I = Q/t$) and an electric current produces a magnetic field around it.

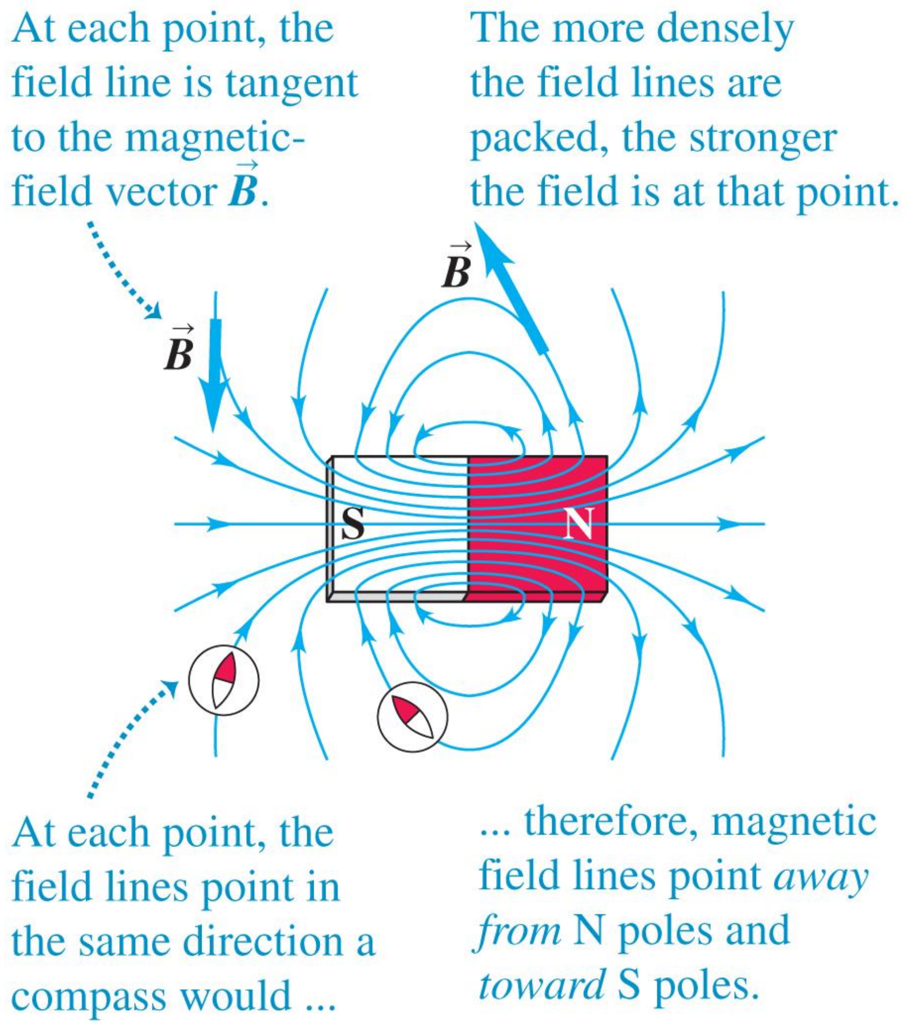
Representing a magnetic field

- A magnetic field is represented by magnetic lines of force or simply magnetic field lines.

- The field lines starts from the N pole and ends at S pole.

- The denser the field lines, the stronger the magnetic field.

- The direction of the magnetic field $\vec{B}$ at any point is tangent to the field line.

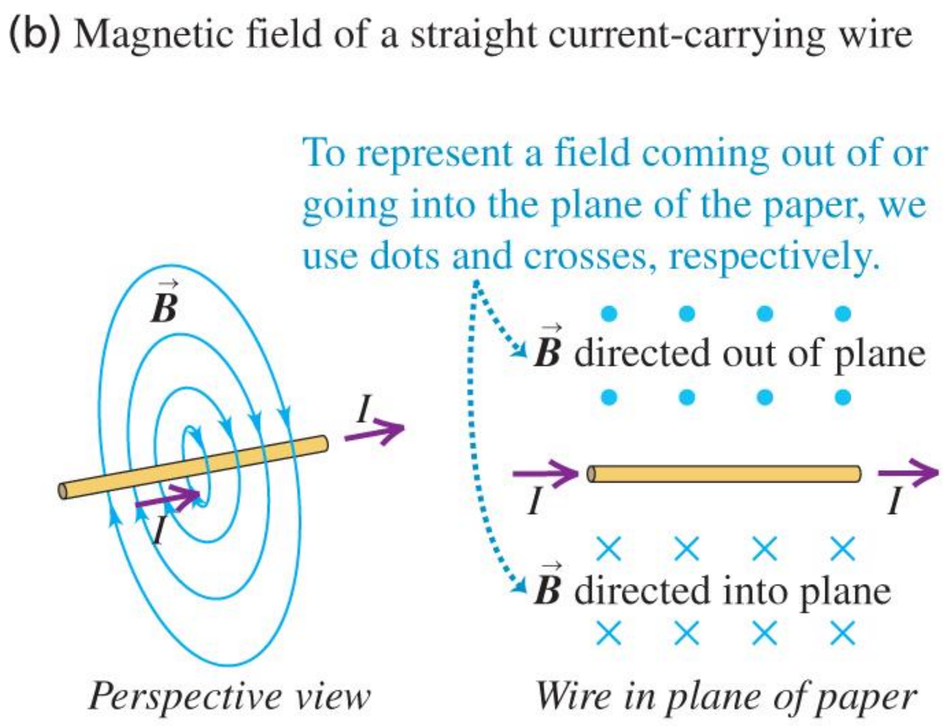


Magnetic field due to electric currents

- In 1820, Hans Christian Oersted discovered that electric currents can affect the movement of a compass.
- This observation shows that there is a relationship between electricity and magnetism.
- We will look at the magnetic field due to straight wire and solenoid.

Representing 3D vectors on a 2D plane

- The study of magnetic field and magnetic effects involves vectors in 3D.
- We can draw 2 vectors in a plane on a piece of paper easily. To draw the 3rd vector, we introduce the **dot** and **cross** notation.
- A **dot** is used to represent a vector pointing **out of the plane of paper**; a **cross** is for pointing **into the plane of paper**.
- The dot and cross notation can be used to represent direction of magnetic field or current.

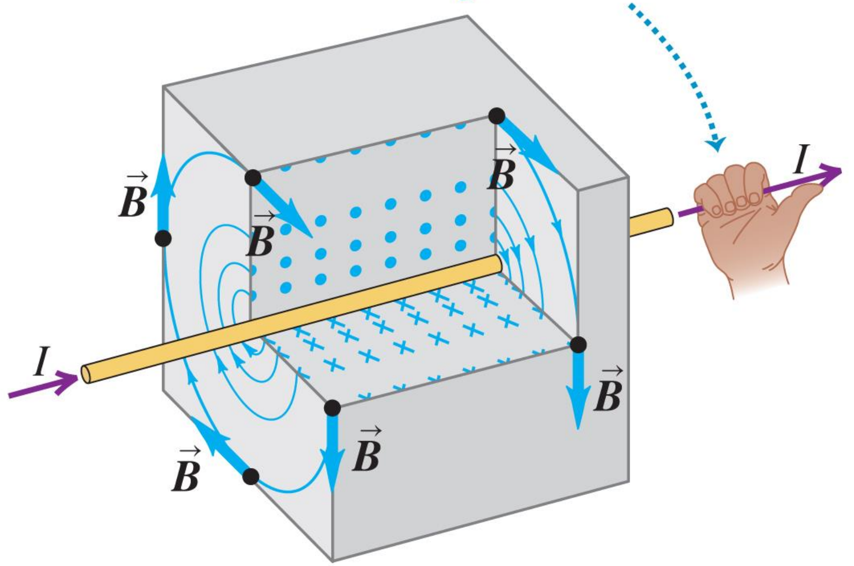


Magnetic field due to current in a long straight wire

- The **magnitude** of the magnetic field due to a current flowing in an infinitely long straight wire at a distance r from the conductor is

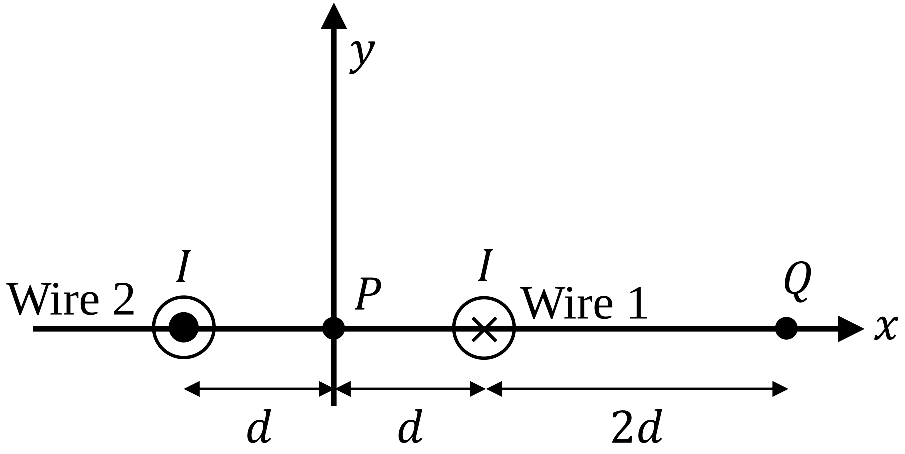
$$B = \frac{\mu_0 I}{2\pi r}$$

- μ_0 = permeability of free space $= 4\pi \times 10^{-7}$ H/m
- The unit of magnetic field is tesla (T).
- The **direction** of the magnetic field at that point can be determined using the **right-hand rule**.



Example 1 – Magnetic field due to 2 wires

Two infinitely long wires carrying current I in opposite directions are placed at distance $2d$ apart. Find the magnetic field due to the two wires at points P and Q. Point P is at the midpoint between the two wires. Point Q is distance $2d$ at the right of wire 1.



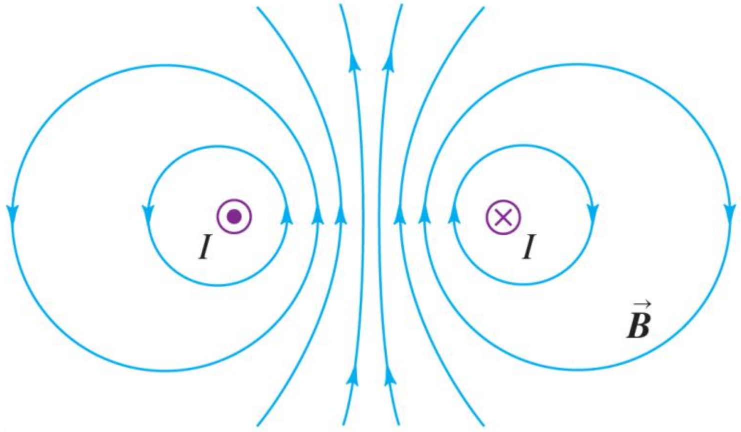
Solution:

At point P, $\vec{B}_1 = \frac{\mu_0 I}{2\pi d}\,\hat{\jmath}$, $\vec{B}_2 = \frac{\mu_0 I}{2\pi d}\,\hat{\jmath}$, so

$$\vec{B}_P = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{2\pi d}\,\hat{\jmath} + \frac{\mu_0 I}{2\pi d}\,\hat{\jmath} = \frac{\mu_0 I}{\pi d}\,\hat{\jmath}$$

At point Q, $\vec{B}_1 = -\frac{\mu_0 I}{2\pi(2d)}\,\hat{\jmath} = -\frac{\mu_0 I}{4\pi d}\,\hat{\jmath}$, $\vec{B}_2 = \frac{\mu_0 I}{8\pi d}\,\hat{\jmath}$, so

$$\vec{B}_Q = \vec{B}_1 + \vec{B}_2 = -\frac{\mu_0 I}{4\pi d}\,\hat{\jmath} + \frac{\mu_0 I}{8\pi d}\,\hat{\jmath} = -\frac{\mu_0 I}{8\pi d}\,\hat{\jmath}$$



Magnetic field due to current in a solenoid

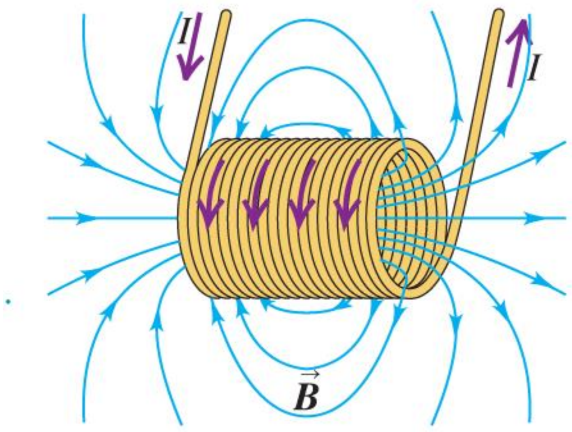
- A solenoid is a helical winding of wire, usually wound around the surface of a cylindrical form.

- The magnitude of the magnetic field at the centre of the solenoid is

$$B = \mu_0 n I$$

- n = number of turns of wire per unit length

- The direction of magnetic field can be determined using the right-hand rule, with the fingers along the current, direction of thumb tells the direction of field.



Similar to a bar magnet

Example 2

A 15.0 cm long solenoid with radius 0.750 cm is closely wound with 600 turns of wire. The current in the windings is 8.00 A. Compute the magnetic field at a point near the centre of the solenoid.

Solution:

Using $B = \mu_0 nI$, $n =$ number of turns per unit length $= \frac{600}{0.15}$

$$B = \mu_0 nI = (4\pi \times 10^{-7})\left(\frac{600}{0.15}\right)(8.00) = 0.0402\ \mathrm{T}$$

End of pre-class slides

Learning outcomes

At the end of the lesson, students should be able to

- solve problems involving magnetic force on a moving charge and magnetic force on current-carrying conductor
- recognize that a moving charge moves in a circular path when it moves in a direction perpendicular to a uniform magnetic field
- describe the applications of electric and magnetic forces on charged particles in the velocity selector

Magnetic force on single moving charge

- From experiments, the magnetic force on a single moving charge is given by

$$\vec{F} = q\vec{v} \times \vec{B}$$

  where q is the quantity of charge in coulombs, $\vec{v}$ is the velocity of the charge in m/s and $\vec{B}$ is the magnetic field in tesla (T).

- Since force is a vector, it has both magnitude and direction.

Magnitude of magnetic force on moving charge

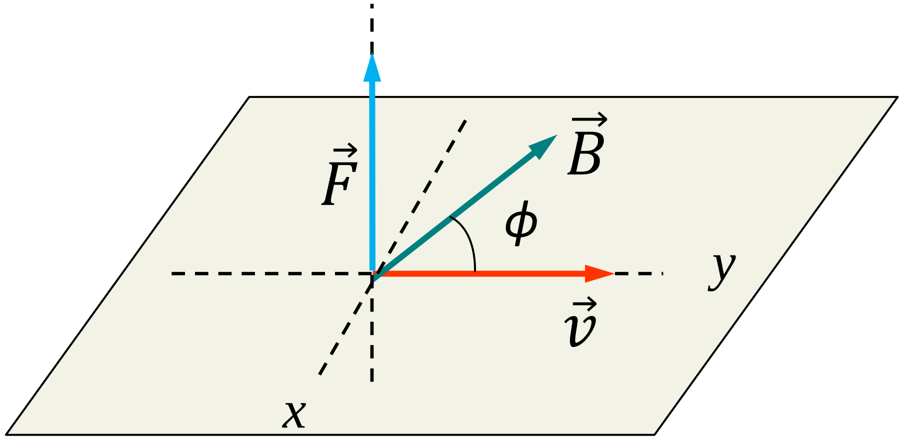
- The magnitude of the magnetic force is given by $F = |q||\vec{v}||\vec{B}| \sin\phi$ or simply $F = qvB \sin\phi$, where ϕ is the angle between $\vec{v}$ and $\vec{B}$.

- By making B the subject, we can see that tesla (T) is the special name for N A^{-1} m^{-1}.

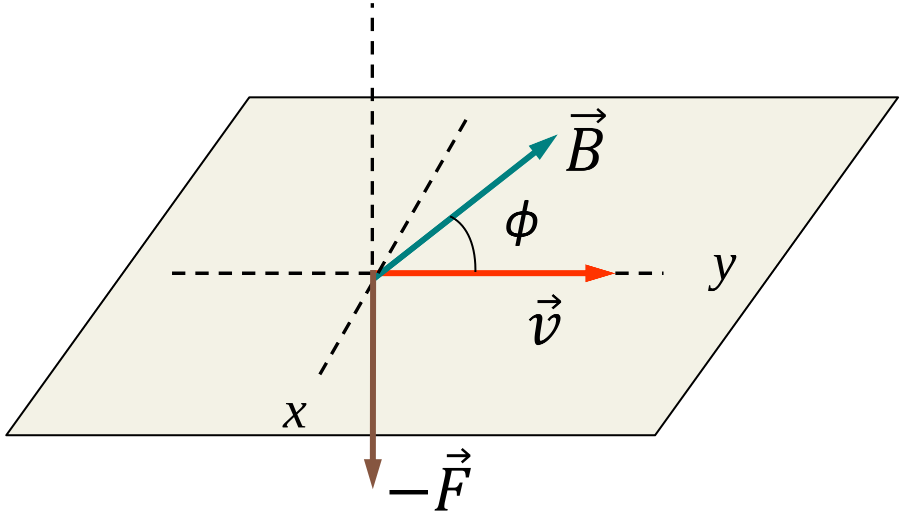
Direction of magnetic force on moving charge

- From the cross product, $\vec{F} = q\vec{v} \times \vec{B}$, it can be seen that $\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.
- Since q can be positive or negative, $\vec{F}$ can point up or down for the same $\vec{v}$ and $\vec{B}$.

Positive q



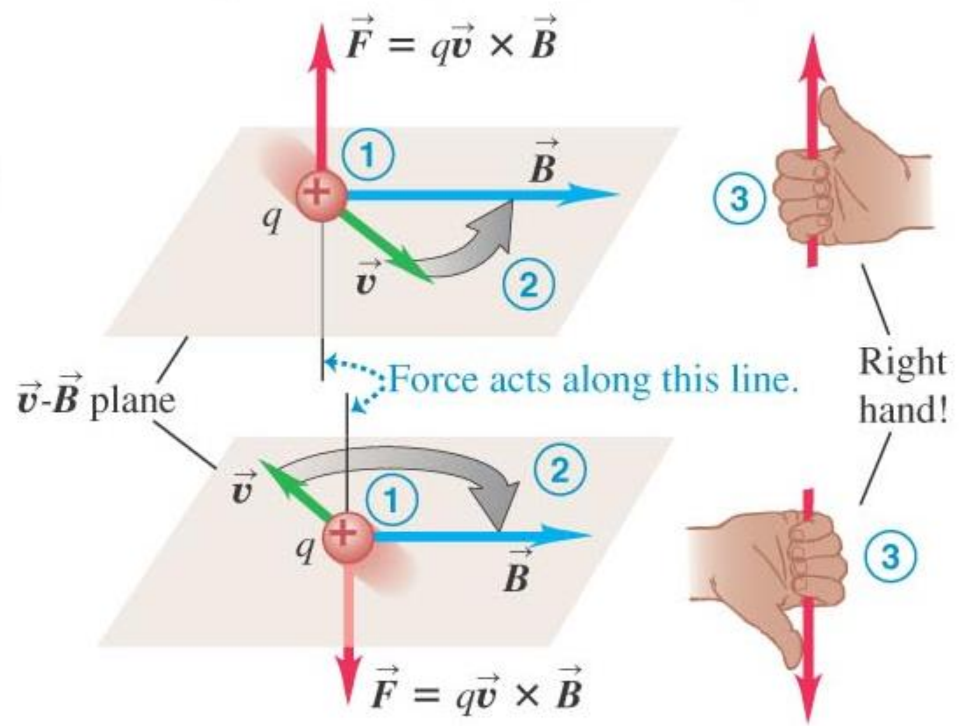
Negative q



OFFICIAL (CLOSED), NON-SENSITIVE

Right-hand grip rule for direction of magnetic force

- A quick way to find the direction of the magnetic force on a moving charge is to use the right hand grip rule.
- We point our fingers along the velocity vector $\vec{v}$ and move them towards the magnetic field vector $\vec{B}$.
- The thumb points in the direction of the magnetic force if q is positive.
- If q is negative, the direction of the force is opposite to the direction of the thumb.



Right-hand grip rule for direction of magnetic force

- The cross product, $\vec{F} = q\vec{v} \times \vec{B}$, gives both the magnitude and direction of the magnetic force on a moving charge.

- The right hand grip rule only tells us the direction of the magnetic force relative to the charge's velocity and the magnetic field.

Example 3a

A proton moves along the positive y-axis at 1.0×10^5 m/s through a uniform magnetic field of magnitude 1.2 T directed along the negative x-axis as shown. What is the direction and magnitude of the magnetic force on the proton? Charge of a proton is 1.6×10^{-19} C.

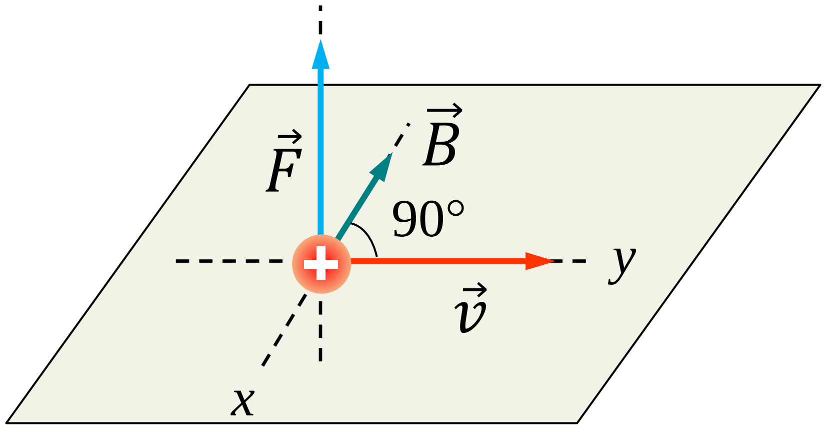
Solution:

Using the vector cross product,

$\vec{v} = 1.0 \times 10^5\ \hat{j}$ m/s, $\vec{B} = -1.2\ \hat{\imath}$ T

$$\vec{F} = q\vec{v} \times \vec{B} = 1.6 \times 10^{-19} \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ 0 & 1.0 \times 10^5 & 0 \\ -1.2 & 0 & 0 \end{vmatrix}$$

$\vec{F} = 1.92 \times 10^{-14}\ \hat{k}$ N $= 1.9 \times 10^{-14}\ \hat{k}$ N (2 s.f.)



Example 3b

An electron moves along the positive y- axis at 1.0×10^5 m/s through a uniform magnetic field of magnitude 1.2 T directed along the negative x-axis as shown. What is the direction and magnitude of the magnetic force on the electron? Charge of an electron is -1.6×10^{-19} C.

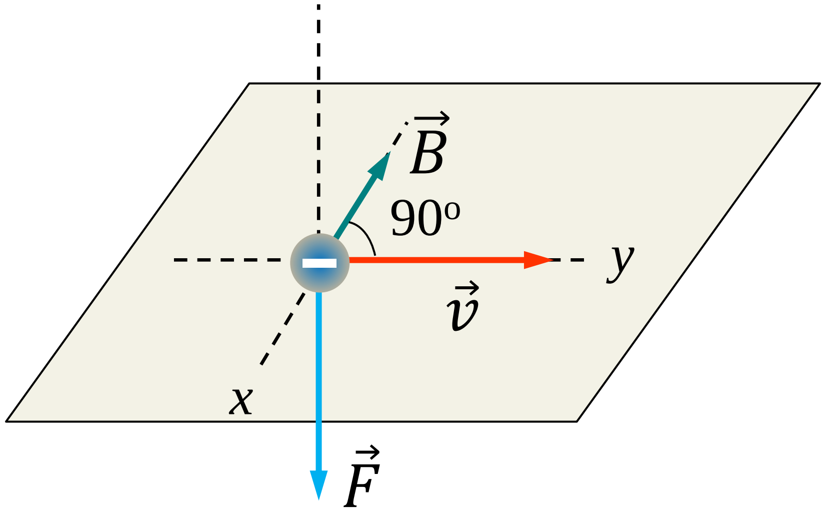
Solution:

Using the vector cross product,

$\vec{v} = 1.0 \times 10^5\ \hat{\jmath}$ m/s, $\vec{B} = -1.2\ \hat{\imath}$ T

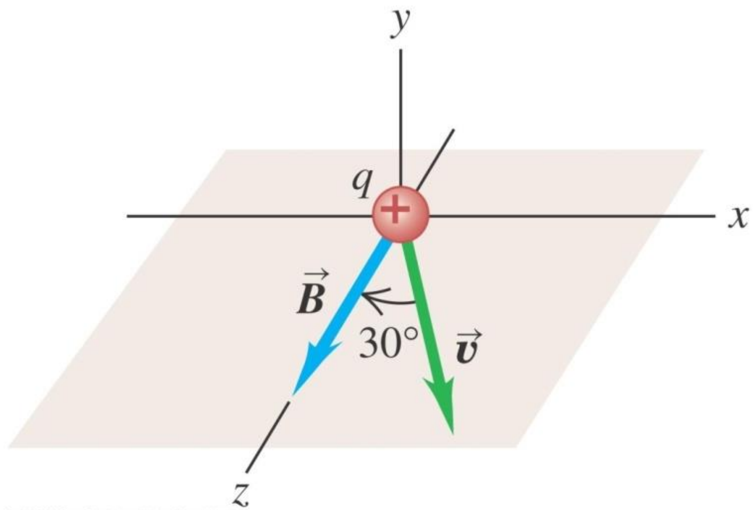
$$\vec{F} = q\vec{v} \times \vec{B} = -1.6 \times 10^{-19} \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1.0 \times 10^5 & 0 \\ -1.2 & 0 & 0 \end{vmatrix}$$

$$\vec{F} = -1.92 \times 10^{-14}\ \hat{k}\ \text{N} = -1.9 \times 10^{-14}\ \hat{k}\ \text{N (2 s.f.)}$$



Example 4

A beam of protons moves at 3.0×10^5 m/s through a uniform magnetic field with a magnitude 2.0 T. The magnetic field is directed along the positive *z*-axis as shown in the below figure. The velocity of each proton lies in the *x*-*z* plane at an angle of 30° to the positive *z*-axis. Find the force on a proton. Charge of a proton is 1.6×10^{-19} C.



Charge moving perpendicular to uniform ***B*** field

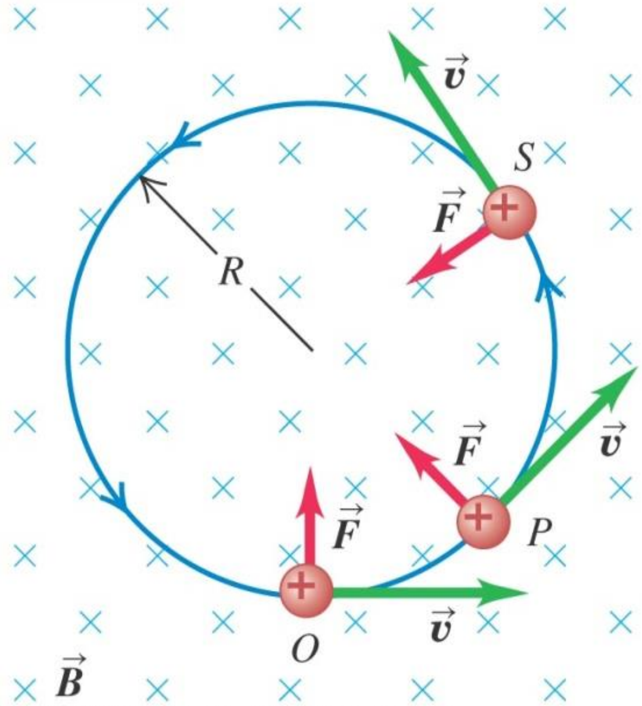
- The magnetic force is always perpendicular to the velocity and the magnetic field vectors.

- A particle projected into a uniform magnetic field perpendicular to its trajectory will perform uniform circular motion.

- For motion in a circle

$$F = qvB = \frac{mv^2}{R}$$

$$R = \frac{mv}{qB}$$



Cyclotron frequency

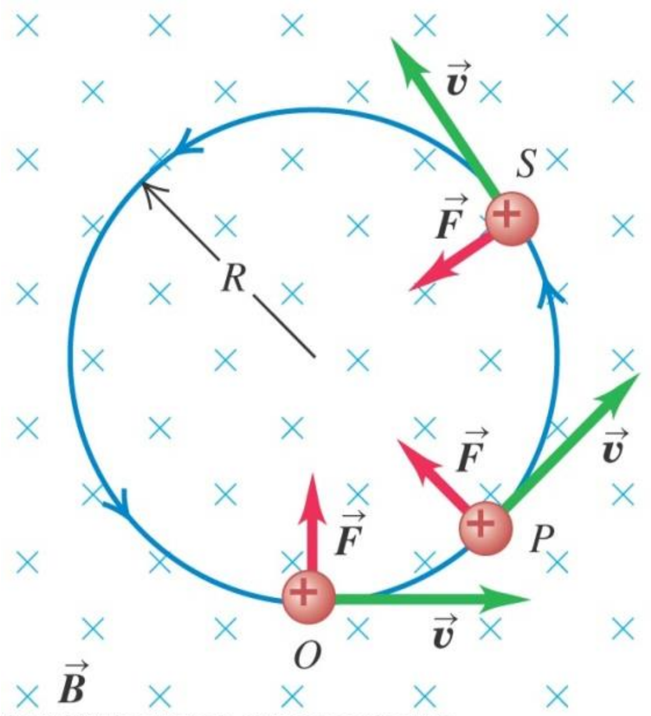
- The number of cycles per second also known as the cyclotron frequency is

$$f = \frac{qB}{2\pi m}$$

- The period is

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

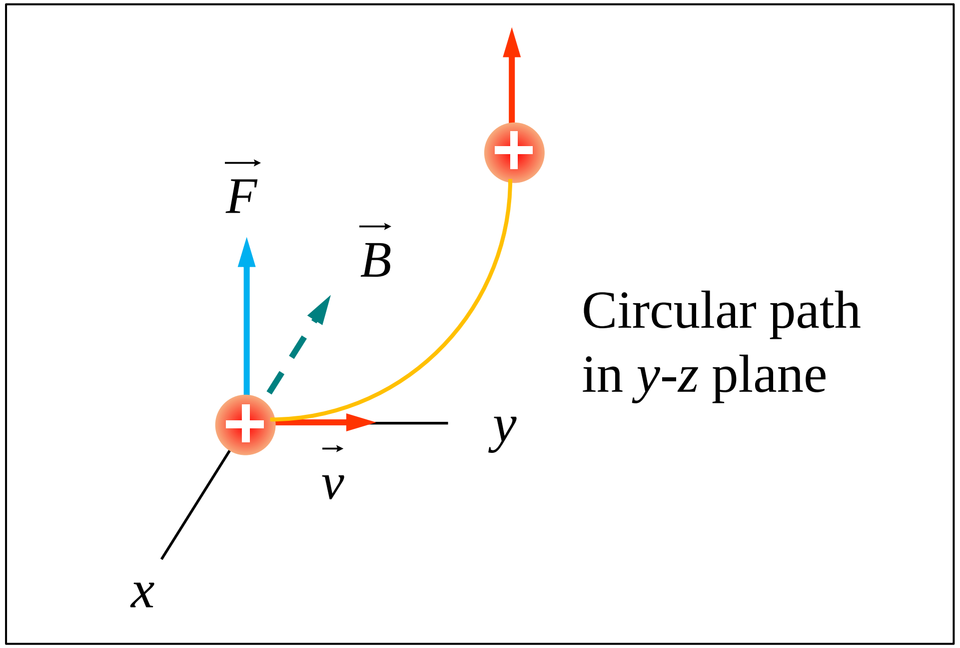
- Both frequency and period are independent of R.



Example 5

A proton moving at 1.00×10^5 m/s along y-axis enters a uniform magnetic field with a magnitude 2.00 T. If the field points into the page ($+x$), determine the centripetal force on the proton and its period of rotation. Charge of a proton is 1.60×10^{-19} C. Mass of proton is 1.67×10^{-27} kg.

Using the right hand grip rule, the initial force on the proton is in the positive z-axis.

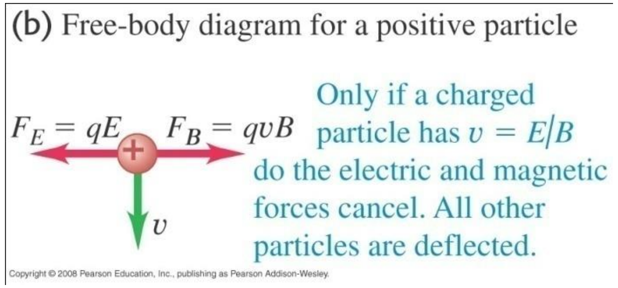


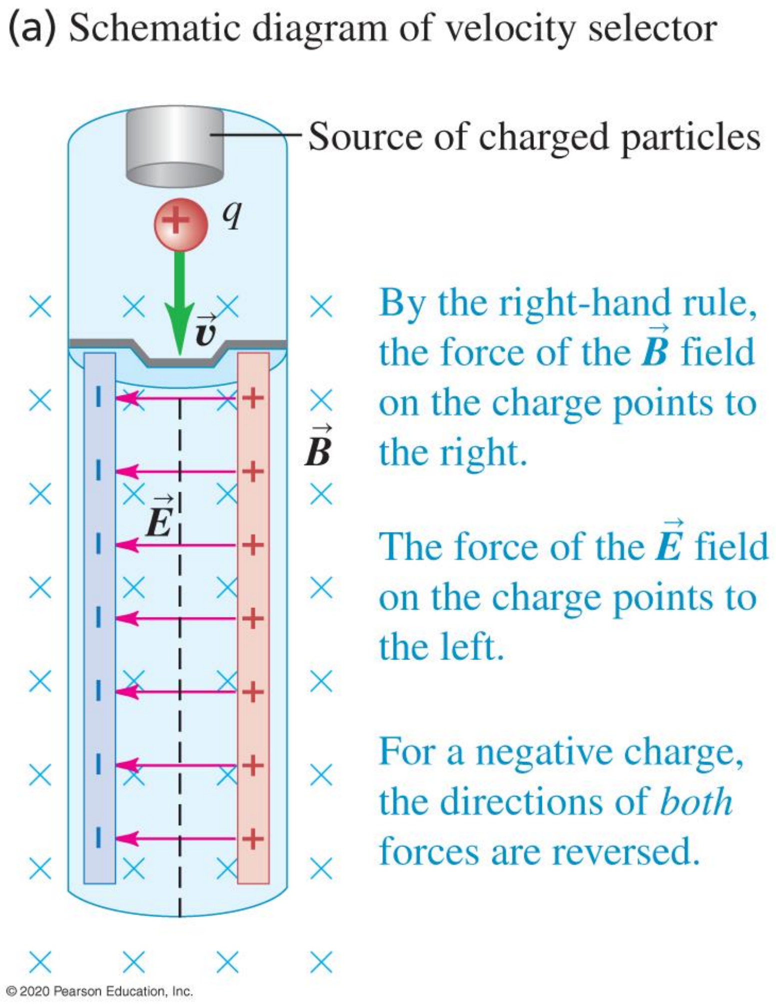
Velocity selector

- Velocity selector is used to select positive and negative charges having a particular speed.
- Inside the region, the electric field and magnetic field are perpendicular to each other.
- The directions of the fields are chosen such that the net force on the charged particle is **zero**.

$$qE = qvB$$

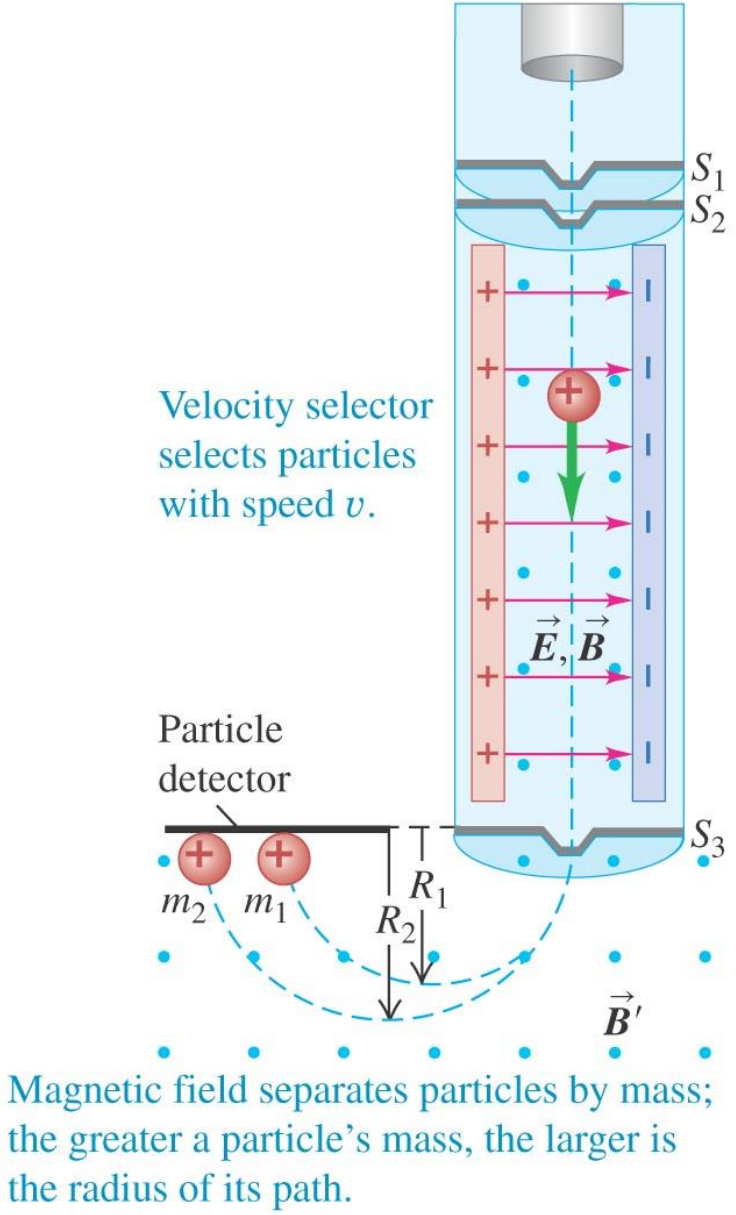
$$v = \frac{E}{B}$$





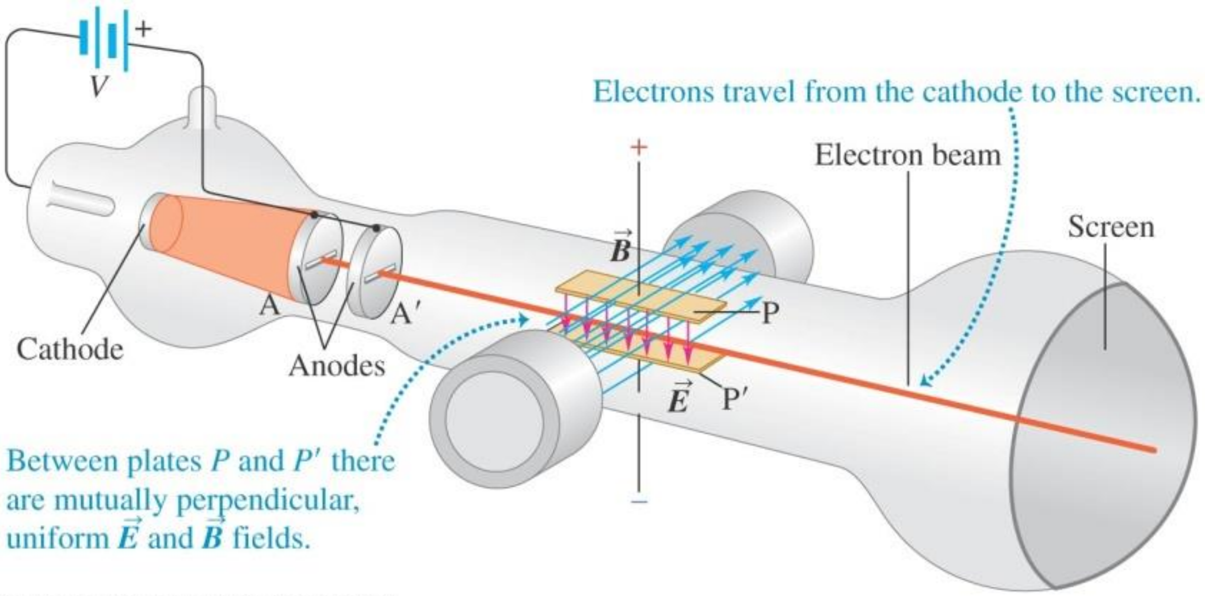
Mass spectrometer

- In a mass spectrometer, singly charged positive ions with velocity $v = E/B$ pass through S_3.
- These ions then move into a region with a different magnetic field B' that is perpendicular to the page.
- They move in circular arcs with radius $R = \frac{mv}{qB'}$.
- Isotopes of the same element (with different m) will move with different radii.



Thomson's e/m experiment

- Electrons in a glass container are accelerated by a p.d. V between anodes A and A' and enter a magnetic field.
- The work done by the electric field on the electrons is eV, where e is the electronic charge. The electrons gain kinetic energy.



Thomson's e/m experiment - cont

- From conservation of energy,

$$\frac{1}{2}mv^2 = eV \quad or \quad v = \sqrt{\frac{2eV}{m}}$$

- Only electrons whose magnetic force equals its electric force will pass through and strike the screen, i.e.,

$$eE = Bev$$

$$v = \frac{E}{B} = \sqrt{\frac{2eV}{m}} \quad \Rightarrow \quad \frac{e}{m} = \frac{E^2}{2VB^2}$$

Thomson's e/m experiment - cont

- There is only a single value of e/m = 1.76 x 10^{11} C kg^{-1}.
- Since this value did not depend on the cathode material, the electrons in the beam is a common constituent of all matters.
- After some years Millikan succeeded in measuring the charge of the electron.
- Combining with Thomson's result and Millikan's experiments, the mass of the electron was determined to be about 9.11 x 10^{-31} kg.

Example 6

In a Thomson's e/m experiment, the accelerating potential is 150 V with a deflecting electric field of magnitude 6.0×10^{6} N/C. Speed of light is 3×10^{8} m/s. Mass of the electron = 9.11×10^{-31} kg.

a) At what fraction of the speed of light do the electrons move?

b) What should be the magnetic field?

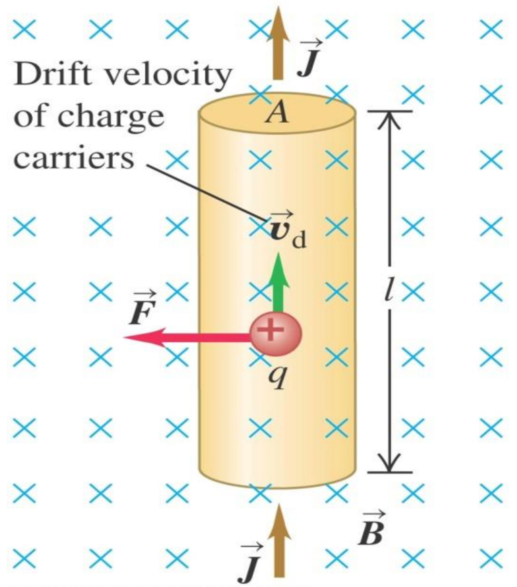
c) With this magnetic field, what will happen to the electron beam if the accelerating potential is increased above 150 V?

Magnetic force on a current-carrying conductor

- Suppose positive charges flow with drift velocity v_d in a wire of length l and area A and perpendicular to a uniform field $\vec{B}$.
- The total number of charges is nAl, where $n =$ number of charges per unit volume.
- The total force on all the charges is

$$F = (nAl)(qv_dB) = nqAv_dlB = ilB$$

where $i = nqAv_d$



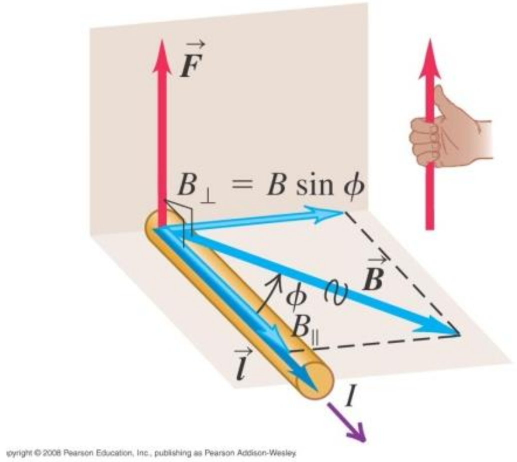
Magnetic force on a current-carrying conductor

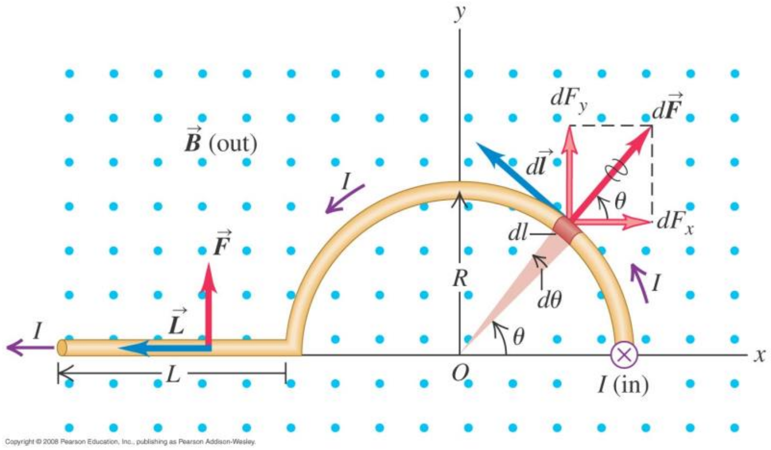
- If $\vec{B}$ makes an angle ϕ with the conductor, the force becomes $F = ilB \sin\phi$.

- In vector form $\vec{F} = i\vec{l} \times \vec{B}$.

- If the conductor is not straight, the force on each segment is $d\vec{F} = id\vec{l} \times \vec{B}$

- The total force is obtained by integrating over the whole length of the conductor.





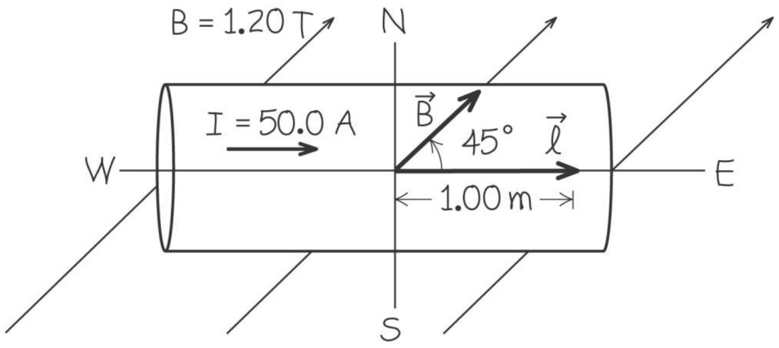
Example 7

A straight horizontal copper rod carries a current of 50.0 A from west to east in a region between the poles of a large electromagnet. In this region there is a horizontal magnetic field toward the north east (45° north of east) with magnitude 1.20 T.

a) Find the magnitude and direction of the force on a 1.00-m section of rod.

b) What should be done to the rod to maximize the magnitude of force?

c) What is the magnitude of the force in the above case in b)?



Magnetic force between two parallel conductors

- When two parallel conductors are placed near each other, they will experience magnetic force.

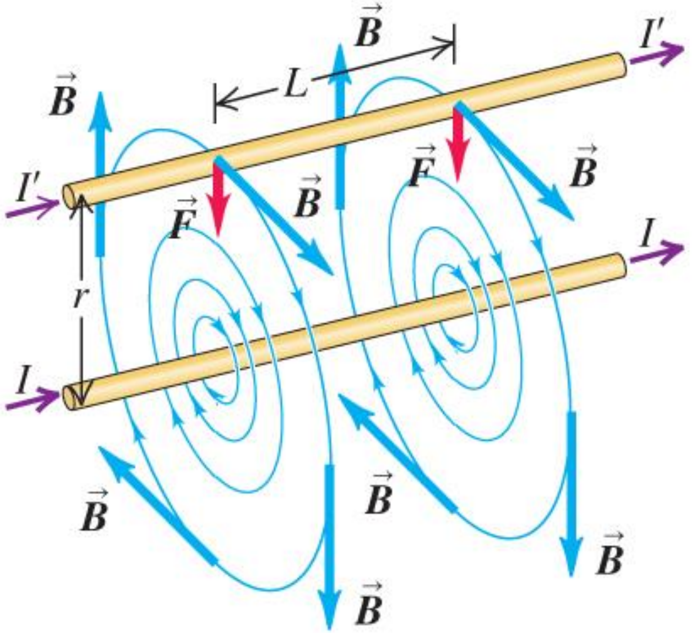
- Suppose two parallel wires carrying currents I and I' are at distance d from each other, the **magnitude** of the magnetic force is

$$F = I'LB = \frac{\mu_0 I I' L}{2\pi d}$$

- The **direction** of the magnetic force can be determined by **right-hand-grip-rule**.

The magnetic field of the lower wire exerts an attractive force on the upper wire. By the same token, the upper wire attracts the lower one.

If the wires had currents in *opposite* directions, they would *repel* each other.

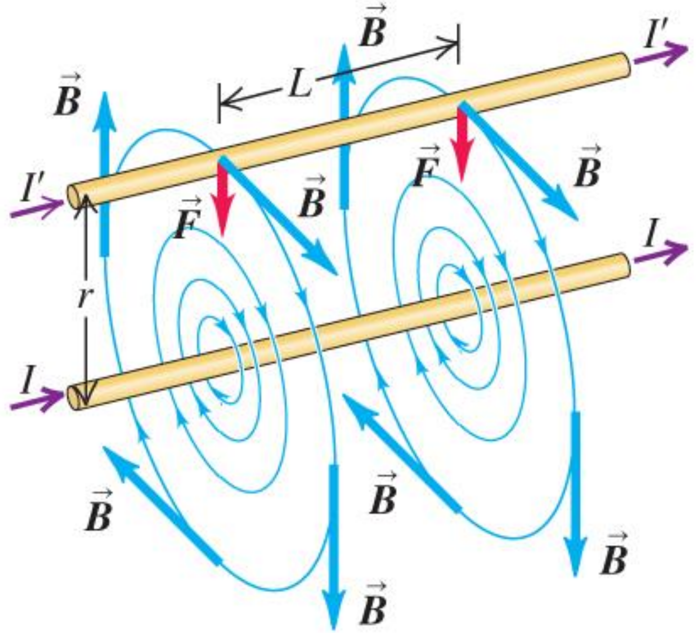


Magnetic force between two parallel conductors

- When the currents in the parallel conductors are in the **same** direction, they **attract** each other.
- When the currents in the parallel conductors are in **opposite** directions, they **repel** each other.
- Video: [MIT Physics Demo -- Forces on a Current-Carrying Wire - YouTube]

The magnetic field of the lower wire exerts an attractive force on the upper wire. By the same token, the upper wire attracts the lower one.

If the wires had currents in *opposite* directions, they would *repel* each other.



Magnetic force between two parallel conductors

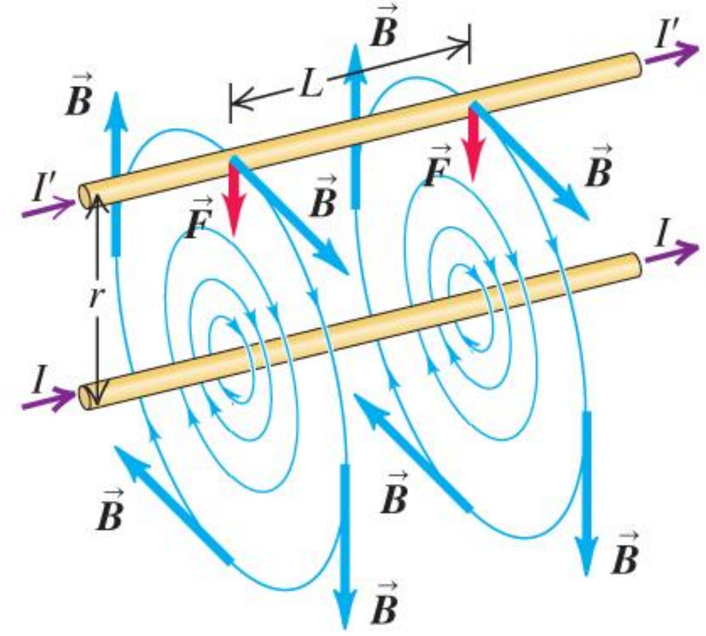
- Force per unit length is

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi d}$$

- We use this formula to define 1 ampere:
  - One ampere is that unvarying current that, if present in each of the two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience a force of exactly 2×10^{-7} N/m.

The magnetic field of the lower wire exerts an attractive force on the upper wire. By the same token, the upper wire attracts the lower one.

If the wires had currents in *opposite* directions, they would *repel* each other.



Example 8

Two long, parallel wires are separated by a distance of 3.50 cm. The force per unit length that each wire exerts on the other is 3.80×10^{-5} N/m, and the wires repel each other. The current in one wire is 0.580 A.

(a) What is the current in the second wire?

(b) Are the two currents in the same direction or opposite direction?

Chapter summary

- Magnetic field due to a long straight wire: $B = \frac{\mu_0 I}{2\pi r}$
- Magnetic field at the centre of a solenoid: $B = \mu_0 n I$
- Magnetic force on a moving charge: $\vec{F} = q\vec{v} \times \vec{B}$
- Velocity selector: $v = \frac{E}{B}$
- Magnetic force on a current-carrying conductor: $\vec{F} = I\vec{l} \times \vec{B}$
- Force per unit length in parallel conductors: $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

End of chapter