7. **IDENTIFY:** $\Delta x = v_{\text{av-}x}\Delta t$

SET UP: We know the average velocity is 6.25 m/s.

EXECUTE: $\Delta x = v_{\text{av-}x}\Delta t = 25.0\text{ m}$

EVALUATE: In round numbers, 6 m/s × 4 s = 24 m ≈ 25 m, so the answer is reasonable.

8. (a) **IDENTIFY:** Calculate the average velocity.

SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so use $x(t)$ to find the displacement Δx for this time interval.

EXECUTE: $t = 0$: $x = 0$

$t = 10.0\text{ s}$: $x = (2.40\text{ m/s}^2)(10.0\text{ s})^2 - (0.120\text{ m/s}^3)(10.0\text{ s})^3 = 240\text{ m} - 120\text{ m} = 120\text{ m}.$

Then $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{120\text{ m}}{10.0\text{ s}} = 12.0\text{ m/s}.$

(b) **IDENTIFY:** calculate $v_x(t)$ and evaluate this expression at each specified t.

SET UP: $v_x = \dfrac{dx}{dt} = 2bt - 3ct^2.$

EXECUTE: (i) $t = 0$: $v_x = 0$

(ii) $t = 5.0\text{ s}$: $v_x = 2(2.40\text{ m/s}^2)(5.0\text{ s}) - 3(0.120\text{ m/s}^3)(5.0\text{ s})^2 = 24.0\text{ m/s} - 9.0\text{ m/s} = 15.0\text{ m/s}.$

(iii) $t = 10.0\text{ s}$: $v_x = 2(2.40\text{ m/s}^2)(10.0\text{ s}) - 3(0.120\text{ m/s}^3)(10.0\text{ s})^2 = 48.0\text{ m/s} - 36.0\text{ m/s} = 12.0\text{ m/s}.$

(c) IDENTIFY: Find the value of t when $v_x(t)$ from part (b) is zero.

SET UP: $v_x = 2bt - 3ct^2$

$v_x = 0$ at $t = 0.$

$v_x = 0$ next when $2bt - 3ct^2 = 0$

EXECUTE: $2b = 3ct$ so $t = \dfrac{2b}{3c} = \dfrac{2(2.40\text{ m/s}^2)}{3(0.120\text{ m/s}^3)} = 13.3\text{ s}$

EVALUATE: $v_x(t)$ for this motion says the car starts from rest, speeds up, and then slows down again.

9. **IDENTIFY:** Assume that the acceleration is constant and apply the constant acceleration kinematic equations. Set $|a_x|$ equal to its maximum allowed value.

SET UP: Let $+x$ be the direction of the initial velocity of the car. $a_x = 2\,250\text{ m/s}^2$. 105 km/h = 29.17 m/s.

EXECUTE: $v_{0x} = 1\,29.17\text{ m/s}$. $v_x = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = \dfrac{0 - (29.17\text{ m/s})^2}{2(-250\text{ m/s}^2)} = 1.70\text{ m}.$

EVALUATE: The car frame stops over a shorter distance and has a larger magnitude of acceleration. Part of your 1.70 m stopping distance is the stopping distance of the car and part is how far you move relative to the car while stopping.

10. **IDENTIFY:** In (a) the hip pad must reduce the person's speed from 2.0 m/s to 1.3 m/s over a distance of 2.0 cm, and we want the acceleration over this distance, assuming constant acceleration. In (b) we want to find out how the acceleration in (a) lasts.

SET UP: Let $+y$ be downward. $v_{0y} = 2.0\text{ m/s}$, $v_y = 1.3\text{ m/s}$, and $y - y_0 = 0.020\text{ m}$. The equations $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ apply for constant acceleration.

EXECUTE: (a) Solving $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ for a_y gives

$a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \dfrac{(1.3\text{ m/s})^2 - (2.0\text{ m/s})^2}{2(0.020\text{ m})} = -58\text{ m/s}^2 = -5.9g.$

(b) $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $t = \dfrac{2(y - y_0)}{v_{0y} + v_y} = \dfrac{2(0.020\text{ m})}{2.0\text{ m/s} + 1.3\text{ m/s}} = 12\text{ ms}.$

EVALUATE: The acceleration is very large, but it only lasts for 12 ms so it produces a small velocity change.

11. **IDENTIFY:** Two stones are thrown up with different speeds. (a) Knowing how soon the faster one returns to the ground, how long it will take the slow one to return? (b) Knowing how high the slower stone went, how high did the faster stone go?

SET UP: Use subscripts f and s to refer to the faster and slower stones, respectively. Take $+y$ to be upward and $y_0 = 0$ for both stones. $v_{0f} = 3v_{0s}$. When a stone reaches the ground, $y = 0$. The constant-acceleration formulas $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ and $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ both apply.

EXECUTE: (a) $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ gives $a_y = -\dfrac{2v_{0y}}{t}$. Since both stones have the same a_y, $\dfrac{v_{0f}}{t_f} = \dfrac{v_{0s}}{t_s}$ and $t_s = t_f\left(\dfrac{v_{0s}}{v_{0f}}\right) = \left(\frac{1}{3}\right)(10\text{ s}) = 3.3\text{ s}$.

(b) Since $v_y = 0$ at the maximum height, then $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y = -\dfrac{v_{0y}^2}{2y}$. Since both have the same a_y, $\dfrac{v_{0f}^2}{y_f} = \dfrac{v_{0s}^2}{y_s}$ and $y_f = y_s\left(\dfrac{v_{0f}}{v_{0s}}\right)^2 = 9H$.

Evaluate: The faster stone reaches a greater height so it travels a greater distance than the slower stone and takes more time to return to the ground.

12. (a) **IDENTIFY** and **SET UP:** From $\vec{r}$ we can calculate x and y for any t.

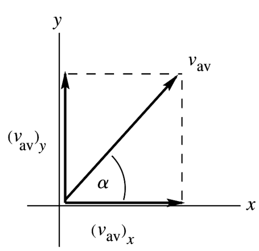
EXECUTE: $\vec{r} = [4.0\text{ cm} + (2.5\text{ cm/s}^2)t^2]\hat{i} + (5.0\text{ cm/s})t\hat{j}$

At $t = 0$, $\vec{r} = (4.0\text{ cm})\,\hat{i}$.

At $t = 2.0$ s, $\vec{r} = (14.0\text{ cm})\,\hat{i} + (10.0\text{ cm})\,\hat{j}$.

$$(v_{av})_x = \frac{\Delta x}{\Delta t} = \frac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$$

$$(v_{av})_y = \frac{\Delta y}{\Delta t} = \frac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$$



$$v_{av} = \sqrt{(v_{av})_x^2 + (v_{av})_y^2} = 7.1\text{ cm/s}$$

$$\tan\alpha = \frac{(v_{av})_y}{(v_{av})_x} = 1.00$$

$$\theta = 45°.$$

EVALUATE: Both x and y increase, so $\vec{v}_{av}$ is in the 1st quadrant.

(b) **IDENTIFY** and **SET UP:** Calculate $\vec{r}$ by taking the time derivative of $\vec{r}(t)$.

EXECUTE: $\vec{v} = \dfrac{d\vec{r}}{dt} = ([5.0\text{ cm/s}^2]t)\hat{i} + (5.0\text{ cm/s})\hat{j}$

$t = 0$: $v_x = 0$, $v_y = 5.0$ cm/s; $v = 5.0$ cm/s and $\theta = 90°$

$t = 1.0$ s: $v_x = 5.0$ cm/s, $v_y = 5.0$ cm/s; $v = 7.1$ cm/s and $\theta = 45°$

$t = 2.0$ s: $v_x = 10.0$ cm/s, $v_y = 5.0$ cm/s; $v = 11$ cm/s and $\theta = 27°$

13. **IDENTIFY:** Consider the horizontal and vertical components of the projectile motion. The water travels 45.0 m horizontally in 3.00 s.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. $v_{0x} = v_0 \cos\theta_0$, $v_{0y} = v_0 \sin\theta_0$.

EXECUTE: (a) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = v_0(\cos\theta_0)t$ and $\cos\theta_0 = \dfrac{45.0\text{ m}}{(25.0\text{ m/s})(3.00\text{ s})} = 0.600$; $\theta_0 = 53.1°$

(b) At the highest point $v_x = v_{0x} = (25.0\text{ m/s})\cos 53.1° = 15.0\text{ m/s}$, $v_y = 0$ and $v = \sqrt{v_x^2 + v_y^2} = 15.0\text{ m/s}$. At all points in the motion, $a = 9.80\text{ m/s}^2$ downward.

(c) Find $y - y_0$ when $t = 3.00\text{s}$:

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (25.0\text{ m/s})(\sin 53.1°)(3.00\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(3.00\text{ s})^2 = 15.9\text{ m}$

$v_x = v_{0x} = 15.0\text{ m/s}$, $v_y = v_{0y} + a_y t = (25.0\text{ m/s})(\sin 53.1°) - (9.80\text{m/s}^2)(3.00\text{ s}) = -9.41\text{ m/s}$, and

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.0\text{ m/s})^2 + (-9.41\text{ m/s})^2} = 17.7\text{ m/s}$

EVALUATE: The acceleration is the same at all points of the motion. It takes the water $t = -\dfrac{v_{0y}}{a_y} = -\dfrac{20.0\text{ m/s}}{-9.80\text{ m/s}^2} = 2.04\text{ s}$ to reach its maximum height. When the water reaches the building it has passed its maximum height and its vertical component of velocity is downward.

14. **IDENTIFY** and **SET UP:** The stone moves in projectile motion. Its initial velocity is the same as that of the balloon. Use constant acceleration equations for the *x* and *y* components of its motion. Take $+y$ to be downward.

EXECUTE: (a) Use the vertical motion of the rock to find the initial height.

$t = 6.00\text{ s}$, $v_{0y} = +20.0\text{ m/s}$, $a_y = +9.80\text{ m/s}^2$, $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = 296\text{ m}$

(b) In 6.00 s the balloon travels downward a distance $y - y_0 = (20.0\text{ m/s})(6.00\text{ s}) = 120\text{ m}$. So, its height above ground when the rock hits is $296\text{ m} - 120\text{ m} = 176\text{ m}$.

(c) The horizontal distance the rock travels in 6.00 s is 90.0 m. The vertical component of the distance between the rock and the basket is 176 m, so the rock is $\sqrt{(176\text{ m})^2 + (90\text{ m})^2} = 198\text{ m}$ from the basket when it hits the ground.

(d) (i) The basket has no horizontal velocity, so the rock has horizontal velocity 15.0 m/s relative to the basket. Just before the rock hits the ground, its vertical component of velocity is $v_y = v_{0y} + a_y t = 20.0\text{ m/s} + (9.80\text{ m/s}^2)(6.00\text{ s}) = 78.8\text{ m/s}$, downward, relative to the ground. The basket is moving downward at 20.0 m/s, so relative to the basket the rock has a downward component of velocity 58.8 m/s.

(ii) horizontal: 15.0 m/s; vertical: 78.8 m/s

Evaluate: The rock has a constant horizontal velocity and accelerates downward.

15. **IDENTIFY:** Apply the relative velocity relation.

SET UP: The relative velocities are $\vec{v}_{C/E}$, the canoe relative to the earth, $\vec{v}_{R/E}$, the velocity of the river relative to the earth and $\vec{v}_{C/R}$, the velocity of the canoe relative to the river.

EXECUTE: $\vec{v}_{C/E} = \vec{v}_{C/R} + \vec{v}_{R/E}$ and therefore $\vec{v}_{C/R} = \vec{v}_{C/E} - \vec{v}_{R/E}$. The velocity components of $\vec{v}_{C/R}$ are $-0.50\text{ m/s} + (0.40\text{ m/s})/\sqrt{2}$, east and $(0.40\text{ m/s})/\sqrt{2}$, south, for a velocity relative to the river of 0.36 m/s, at $52.5°$ south of west.

Evaluate: The velocity of the canoe relative to the river has a smaller magnitude than the velocity of the canoe relative to the earth.