SINGAPORE POLYTECHNIC

2018/2019 SEMESTER TWO EXAM

ENGINEERING MATHEMATICS II Time allowed: 2 hrs

2nd Year Full-Time

School of Chemical and Life Sciences
DCHE

School of Electrical and Electronic Engineering
DASE, DCPE, DEB, DEEE, DES, DESM

School of Mechanical and Aeronautical Engineering
DARE, DCEP, DME, DMRO

Instructions to Candidates:

1. The examination rules set out on the last page of the answer booklet are to be complied with.
2. **The questions are printed on Pages 2 – 4**.
3. This paper consists of THREE (3) sections.

   Section A: 5 multiple-choice questions (MCQ), 2 marks each.
   Answer ALL five (5) questions in this section.

   Section B: 7 short questions, 10 marks each.
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all the questions you answered will be added, but the maximum mark you can obtain from this section is 50 marks.

   Section C: 3 questions, total of 40 marks.
   Answer ALL three (3) questions in this section.

3. All answers are to be written in the answer booklet provided.
4. Unless otherwise stated, leave your decimal answers correct to **two** decimal places.
5. A 'mathematical formulae and tables' card is provided for your reference.
   Please do not write anything on the card, and return it at the end of the examination.
6. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply will result in loss of marks.

Section A (10 marks):

Answer ALL multiple choice questions on the MCQ answer sheet of the answer booklet.

A1. Given that $f(x,y)=0$, then $\dfrac{dy}{dx}$ equals to ___________

(a) $\dfrac{f_x}{f_y}$ (b) $\dfrac{f_y}{f_x}$

(c) $-\dfrac{f_x}{f_y}$ (d) $-\dfrac{f_y}{f_x}$

A2. Which of the following methods can be used to solve the integral $\int e^{2x}\sin 2x\,dx$?

I. Integration by parts, let $u=e^{2x}$ and $dv=\sin 2x\,dx$

II. Integration by parts, let $u=\sin 2x$ and $dv=e^{2x}\,dx$

III. Integration by substitution and let $u=2x$

(a) I only (b) II only

(c) I & II (d) I, II & III

A3. If T is the period of a periodic function, then the function

(a) has $T=2\pi$. (b) has $T=\pi$.

(c) satisfies $f(t+T)=-f(t)$. (d) satisfies $f(t+T)=f(t)$.

A4. If $\mathcal{L}\{f(t)\}=F(s)$, then $F(s-7)=$ _______

(a) $e^{7s}\mathcal{L}\{f(t)\}$ (b) $\mathcal{L}\{e^{t}f(t)\}$

(c) $e^{s-7}\mathcal{L}\{f(t)\}$ (d) $\mathcal{L}\{e^{7t}f(t)\}$

A5. For a spring-damped motion system governed by the differential equation $ay''-3y'+4y=0$ to be underdamped, the value(s) of a should be _______.

(a) $a=\dfrac{9}{16}$ (b) $a>\dfrac{9}{16}$

(c) $a<\dfrac{9}{16}$ (d) $a\neq\dfrac{9}{16}$

<u>Section B</u> (50 marks):

Each question carries 10 mark. The total marks of the questions in this section is 70. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you can obtain from this section is 50 marks.

B1. Let $f(x,y)=5y^2-e^{xy}+\ln x$. Evaluate $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at (1, 0).

B2. (a) Find the integral $\int\left(4\cos 5t\sin 2t-2\cos^2 3t\right)dt$.

(b) Evaluate the definite integral $\displaystyle\int_1^2 \frac{3}{(5x-2)^2}\,dx$.

B3. Use Simpson's rule to evaluate $\int_0^{1.5} e^{x^2+1}dx$ using 6 strips, correct to 2 decimal places.

B4. Given the differential equation $\dfrac{dy}{dx}=x+2y$.

(a) Find the integrating factor.

(b) Hence, find the general solution for y in terms of x.

B5. (a) Find $\mathcal{L}\left\{\pi-5t^2+3t\sin 2t\right\}$.

(b) Find $\mathcal{L}\left\{4e^{-3t}+9e^{2t}\cos\pi t\right\}$.

B6. (a) Find $\mathcal{L}^{-1}\left\{\dfrac{1}{3s}+\dfrac{2}{s^4}-\dfrac{2}{5(s+1)}\right\}$.

(b) Find $\mathcal{L}^{-1}\left\{\dfrac{s^2-3}{\left(s^2+3\right)^2}-\dfrac{s-6}{(s-1)^2+25}\right\}$.

B7. Given the differential equation $y''-4y'-5y=0$, find

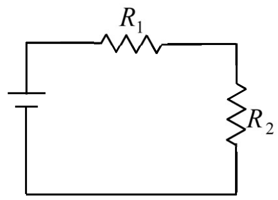
(a) the general solution;

(b) a particular solution, if $y(0)=1$ and $y'(0)=2$.

Section C (40 marks):

Answer all THREE questions below.

C1. The power delivered to the load resistance R_1 for the circuit shown on the right is given by $P = \dfrac{10R_1}{(R_1+R_2)^2}$. If R_1 increases by 2% while R_2 decreases by 3%, find the percentage change in P when $R_1 = 1000\,\Omega$ and $R_2 = 2000\,\Omega$. (12 marks)



C2. A mercury thermometer is removed from a room where the temperature is $20\,^\circ\mathrm{C}$ and is taken outside, where the air temperature is maintained at $35\,^\circ\mathrm{C}$. After two minutes, the thermometer reads $27\,^\circ\mathrm{C}$. According to Newton's law of cooling/heating, the rate at which the temperature of a body changes is proportional to the difference between the temperature of the body and the temperature of the surrounding medium.

(a) Set up the differential equation that model the temperature change in the thermometer.

(b) Find the thermometer reading at $t = 1$ min.

(c) How long will it take for the thermometer to reach $32\,^\circ\mathrm{C}$? (12 marks)

C3. (a) Given that $\dfrac{34s+68}{(s^2+4s+20)(s^2+16)}$ can be resolved into partial fractions of the form $\dfrac{s+A}{2(s^2+4s+20)} + \dfrac{Bs+9}{s^2+16}$, find the values of the constants A and B.

(b) A mass of 2 kg is attached to a spring with a spring constant of 32 N/m. The mass is initially at rest in its equilibrium position. An external force $F(t) = 68e^{-2t}\cos 4t$ (N) is applied to the mass.

(i) Let $x(t)$ be the position of the mass of the system at any time t. Set up the differential equation that model the motion of the mass of the system in terms of $x(t)$. State also its initial conditions.

(ii) By using the result obtained in part (a), solve the differential equation for $x(t)$. (16 marks)

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