2016/2017 SEMESTER TWO EXAMINATION

Diploma in Electrical and Electronic Engineering
3rd Year Full Time
3rd Year Full Time Technical Elective

DIGITAL SIGNAL PROCESSING

Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules as set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **TWO** sections:

   Section A - 6 Short Questions, 10 marks each.
   Section B - 2 Long Questions, 20 marks each.

3. **ALL** questions are **COMPULSORY**.

4. **ALL** questions are to be answered in the answer booklet.

5. This paper consists of **6** pages, including 2 pages of mathematical formulae.

SECTION A - SHORT QUESTIONS [10 marks each]

A1 The system function of a digital system is given as:

$$H(z) = \frac{z^2-1}{z^2+0.5z}$$

a) Obtain its difference equation. (4 marks)

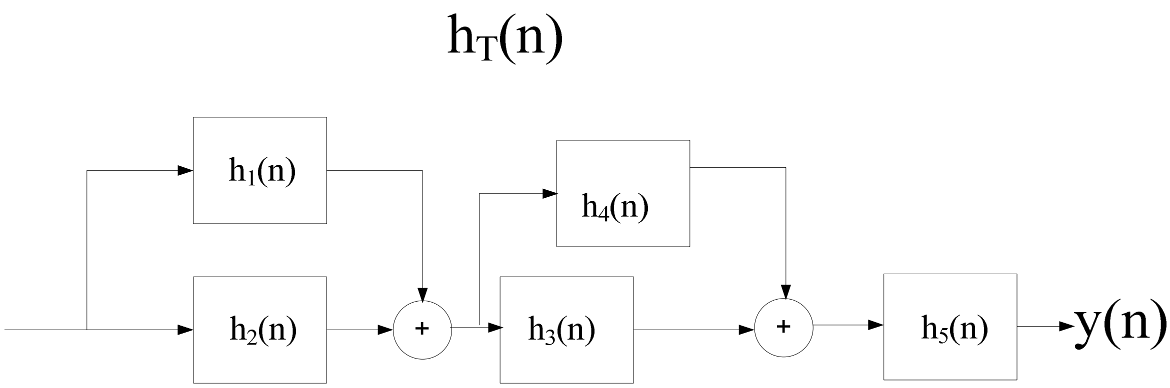
b) Determine the output y(n) of the system if the input is a unit step. (4 marks)

c) If the output y(n) from (b) is cascaded in series with another system, $G(z)=z^{-1}+z^{-2}$, find the new output, $k_T(n)$. (2 marks)

A2 Evaluate N=4 point DFT for X(0), X(1) and X(2) for x(n) ={0,1,0,1}. (10 marks)

A3 A system has an output response, y(n) ={-2,5,-3,1,2} when subjected to an unknown input x(n) and an impulse response h(n)={1, -1, 1} Find the Z-transform of h(n) and y(n) and hence solve for the input sequence x(n) up to the 1st 3 terms using long division method. (10 marks)

A4 The block diagram of a digital system is given as:



a) Find the overall system function, $h_T(n)$ in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$, $h_4(n)$ and $h_5(n)$. (4 marks)

b) If z-transform of $h_1(n)= H_1(z)$, $h_2(n)= H_2(z)$, $h_3(n)= H_3(z)$, $h_4(n)= H_4(z)$ and $h_5(n)= H_5(z)$ respectively and $h_1(n)= h_2(n) =h_3(n)= h_4(n)= h_5(n)= \{2,-1\}$, find the system function, $h_T(n)$.

(6 marks)

A5 The system function of a digital system is given as:

$$X(z) = \frac{4-0.6z^{-1}+0.2z^{-2}}{(1-0.5z^{-1})(1+0.4z^{-1})}$$

Using partial fraction, find x(n). (10 marks)

Hint:

$$X(z) = A + \frac{B}{1-0.5z^{-1}} + \frac{C}{1+0.4z^{-1}}$$

A6 Find the inverse z transform of the following signals.

a) $X(z) = \frac{2}{1+z^{-1}} - \frac{1}{1+2z^{-1}}$ (5 marks)

b) $Y(z) = 1 + \frac{2z^{-1}}{(1-z^{-1})^2} + \frac{1+z^{-1}}{1+2z^{-1}+z^{-2}}$ (5 marks)

SECTION B - LONG QUESTIONS [20 marks each]

B1 You are required to design a digital FIR low-pass filter to reject the high frequency noise found in a telemetry signal. The specifications of the filter are as follow:

Passband: 0 to 10 kHz
Stopband: 12 to 20 kHz

To minimise the effect of the noise, they need to be suppressed by at least **42** dB. The sampling frequency for the digital filter is chosen to be 2 times the Nyquist frequency.

To design this filter, determine

(a) the windowing function that you would choose, (3 marks)
(b) the number of tap coefficient that you would need, (5 marks)
(c) the value of the first 4 tap coefficients, (8 marks)
(d) 2 disadvantage for using IIR filter and 2 disadvantages for using FIR filter. (4 marks)

B2 Given the difference equation y(n) =x(n) – 0.7071x(n-1) + 1.4142y(n-1) – y(n-2).

(a) Determine the network diagram of the filter. (4 marks)
(b) From the filter system function, determine its impulse response using inverse z-transform. (8 marks)
(c) Find the gain of this filter at 2 kHz if the sampling frequency used for the filter is 8 kHz (*Hint*: $e^{-j\frac{\pi}{2}} = -j$). (8 marks)

-End of Paper-

Appendix

The z-transform is defined as $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$

Sequence	Transform
$\delta[n]$	1
$u[n]$	$\frac{1}{1-z^{-1}}$
$\delta[n - m]$	z^{-m}
$a^n u[n]$	$\frac{1}{1-az^{-1}}$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$
$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$

Some z-transform properties:	
Sequence	Transform
$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
$x[n - m]$	$z^{-m}X(z)$

Some trigonometric identities:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

Discrete Fourier Transform:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{N}kn}$$

Complex number theory:

$$z = a + jb = r\angle\theta = re^{j\theta}$$
$$r = \sqrt{a^2 + b^2}$$
$$\theta = \tan^{-1}\frac{b}{a}$$

Quadratic equation solution:

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The characteristics of the different windowing functions:

Window Type	Peak approximation Error $20\log_{10}\delta$ dB	Transition Band $\Delta\omega$
Rectangular $w[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–21	$\frac{4\pi}{M+1}$
Bartlett $w[n] = \begin{cases} \frac{2n}{M} & 0 \le n \le \frac{M}{2} \\ 2 - \frac{2n}{M} & \frac{M}{2} \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–25	$\frac{8\pi}{M}$
Hanning $w[n] = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–44	$\frac{8\pi}{M}$
Hamming $w[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–53	$\frac{8\pi}{M}$
Blackman $w[n] = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{M}\right) + 0.08\cos\left(\frac{4\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–74	$\frac{12\pi}{M}$

The impulse response of an ideal low pass filter is: $h_d(n) = \dfrac{\sin(\omega_c(n - \frac{M}{2}))}{\pi(n - \frac{M}{2})}$