MCQ

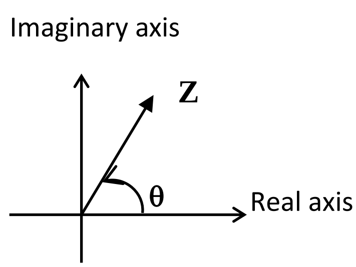
1) Given that **A**, **B** and **X** are 2x2 matrices and that $\mathbf{A}^{-1}$ exists. If **XA** = **AB**, then

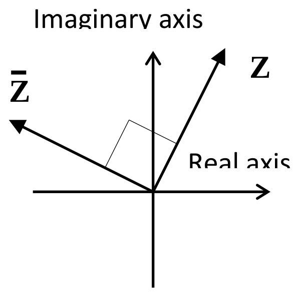
(a) $\mathbf{X} = \mathbf{ABA}^{-1}$ (b) $\mathbf{X} = \mathbf{A}^{-1}\mathbf{BA}$
(c) $\mathbf{X} = \mathbf{ABA}$ (d) $\mathbf{X} = \mathbf{B}$

2) Given $\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, which of the following is true?

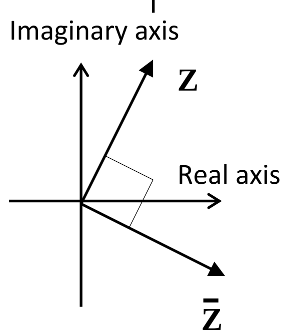
(a) **A** is a zero matrix (b) **A** is a diagonal matrix
(c) **A** is a symmetric matrix (d) **A** is an identity matrix

3) Which of the following illustrates the conjugate of **Z**, as shown below?





(a)



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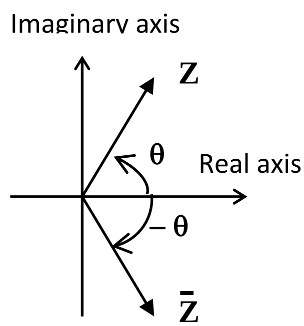
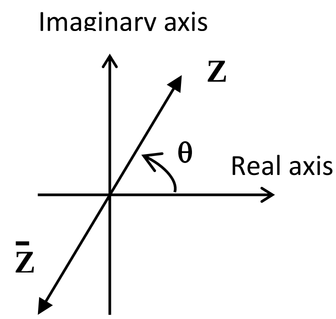


(c)



(d)

4) If $\overline{\mathbf{Z}}$ is the conjugate of the complex number **Z** , then

(a) $\arg(\mathbf{Z}) = \arg(\overline{\mathbf{Z}})$ (b) $\dfrac{\mathbf{Z}}{\overline{\mathbf{Z}}}$ is real

(c) $\mathbf{Z}\,\overline{\mathbf{Z}}$ is real (d) $\mathbf{Z} - \overline{\mathbf{Z}} = 0$

5) Given that $\mathbf{Z} = r\angle\theta$, where θ is a positive acute angle. Then

(a) $j\mathbf{Z} = r\angle\theta + 90^\circ$ (b) $j\mathbf{Z} = r\angle 90^\circ$

(c) $j\mathbf{Z} = r\angle\theta - 90^\circ$ (d) $j\mathbf{Z} = r\angle -\theta$

6) The curve $y = f(x)$ has a maximum point at $x = a$. Which of the following statements can be true at $x = a$?

(a) $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} = 0$ (b) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$

(c) $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$ (d) $\frac{dy}{dx} < 0$ and $\frac{d^2y}{dx^2} = 0$

7) Given that P (3 , 7) is a stationary point on the curve $y = f(x)$. If $f''(3) > 0$, then P is

(a) a point of inflexion (b) a minimum point

(c) a maximum point (d) none of these

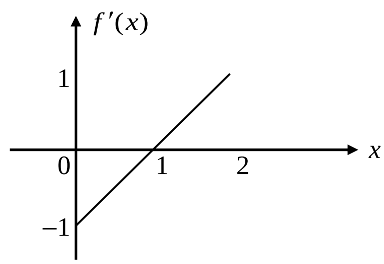
8) If $y = f(x)$ and $\frac{dy}{dt}$ is the rate of change of *y* w.r.t. time *t*, then the rate of change of *x* w.r.t *t* can be expressed as

(a) $\frac{dx}{dt} = \frac{dy}{dt} \cdot \frac{dt}{dx}$ (b) $\frac{dx}{dt} = \frac{dy}{dx} \div \frac{dy}{dt}$

(c) $\frac{dx}{dt} = \frac{dy}{dt} \cdot \frac{dx}{dy}$ (d) $\frac{dx}{dt} = \frac{dy}{dt} \div \frac{dx}{dy}$

9) The diagram of $f'(x)$ over $0 \le x \le 2$ is shown.
At *x* = 1, $f(x)$ has

(a) a maximum value
(b) a minimum value
(c) a point of inflexion
(d) none of these



10) If $f(x) > 0$ for all *x*, $\int_{-1}^{1} f(x)\, dx = 3$ and $\int_{-1}^{6} f(x)\, dx = 7$, then $\int_{1}^{6} 2f(x)\, dx =$

(a) 4 (b) 6

(c) 8 (d) 14

11) If *c* is a value such that $a < c < b$, then $\int_a^b f(x)\, dx =$

(a) $\int_a^b f(x)\, dx - \int_c^b f(x)\, dx$ (b) $\int_c^b f(x)\, dx - \int_a^b f(x)\, dx$

(c) $\int_a^c f(x)\, dx + \int_c^b f(x)\, dx$ (d) $-\int_a^b f(x)\, dx - \int_c^b f(x)\, dx$

Basic Question

1. (a) Find x if $\begin{vmatrix} x & x \\ 1 & x-1 \end{vmatrix} = 15$. (b) Find x if $\begin{vmatrix} 2 & x & 3 \\ 1 & 3 & -1 \\ 2 & -2 & 5 \end{vmatrix} = 9$.

2. Use Cramer's Rule to solve for x **only**, given that:

$$\begin{aligned} x - y + z &= 0 \\ 4x + 6y \quad &= 8 \\ 6y + 2z &= 4 \end{aligned}$$

3. The currents in amperes in a certain electrical circuit satisfy the following equations. Use Cramer's Rule to solve for I_2 **only**.

$$\begin{aligned} 2I_1 + 4I_2 + I_3 &= 5 \\ I_1 - 6I_2 + 2I_3 &= 0 \\ 6I_2 + 3I_3 &= 6 \end{aligned}$$

4. Use inverse matrix matrix to solve the system of equations.

$$\begin{aligned} 2x + y &= 23 \\ 4x - 3y &= 11 \end{aligned}$$

5. Calculate the product $\begin{pmatrix} 3 & 1 & 5 \\ 6 & -3 & 4 \end{pmatrix}\begin{pmatrix} 0 & -1 & 5 \\ 1 & 0 & 2 \\ 4 & 9 & 2 \end{pmatrix}$.

6. Find the matrix A such that $A - 3\begin{pmatrix} 1 & -2 \\ 5 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 8 & 7 \end{pmatrix}$.

7. Given a singular matrix $\mathbf{A} = \begin{pmatrix} x-2 & -2 \\ -5 & x+1 \end{pmatrix}$, find the value(s) of x.

8. If $A = \begin{pmatrix} 2 & 6 \\ -1 & p \end{pmatrix}$

   (i) For what value(s) of p is A^2 is a diagonal matrix ?

   (ii) With the value of p found in part (i) , does matrix A has an inverse ? Justify your answer.

9. Find the value of $\boldsymbol{k}$ such that the matrix $\begin{bmatrix} 1 & 0 & k+1 \\ 0 & k & 2 \\ 2k-1 & 2 & 3 \end{bmatrix}$ is a symmetric matrix.

10. If $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -1 & -3 \\ 5 & 6 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -1 & 0 & 1 \\ 2 & 3 & -5 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ -6 & 3 \end{pmatrix}$, $\mathbf{D} = (-1 \quad 2)$ and $\mathbf{X} = (x \quad y)$, find

    (a) $\mathbf{2A + B^T}$ (b) $\mathbf{AC}$ (c) the values of x and y if $\mathbf{XC = D}$.

11. If $z_1 = 6e^{j0.4}$ and $z_2 = 1.27 - j2.72$, evaluate the following and express your answers in exponential form.

(a) $z_1 z_2$ (b) $\dfrac{z_1}{z_2}$ (c) $z_1 - z_2$ (d) $\left(\overline{z_2}\right)^4$

12. If $Z = 2\angle 35°$ and $W = 3\angle -50°$, evaluate the following and express your answers in polar form.

(a) $\dfrac{\overline{Z}}{W}$ (b) $W^3 Z$ (c) $2Z + W$

13. The total impedance of a parallel connection is given by $Z = \dfrac{Z_1 Z_2}{Z_1 + Z_2}$, where $Z_1 = 1 + j$ and $Z_2 = \sqrt{2}\angle -45°$.

(a) Show that Z is real. (b) Plot Z_1 and Z_2 in the same Argand diagram.

14. Given that $Z_1 = -3 + j4$ and $Z_2 = 5\angle 90°$, express $\dfrac{\overline{Z_1}}{Z_2}$, Z_1Z_2 and $Z_2^{\,5}$ in exponential form.

15. Simplify $j6 + j^2 12 + j^3 2$.

16. The total impedance Z_T of a circuit is given as $Z_T = \dfrac{z_1^{\,2} + z_2}{2z_1 + z_3}$ Ω.

If $z_1 = 3\angle 0.5\ \Omega$, $z_2 = 1.76 + j0.96\ \Omega$, and $z_3 = 0.88 - j0.48\ \Omega$, find

(i) $z_1^{\,2}$ and $2z_1$, expressing your answers in rectangular form.

(ii) $z_1^{\,2} + z_2$ and express your answer in polar form.

(iii) $2z_1 + z_3$ and express your answer in polar form.

(iv) Hence find Z_T in polar form.

17. Solve for z in the following equations where $z = x + jy$:

(a) $z^2 + j2z + 3 = 0$ $\boxed{\text{(b)}}$ $z^2 + j2z - j = z$ (c) $z + 3\overline{z} = 8 - j6$

18. Solve the following equations for real values of x and y.

(a) $2x - jy = \dfrac{1}{4 - j}$ (b) $x + jy = (2 + j3)(3 - j4)$

(c) $x - jy = \dfrac{3 + j2}{j}$ (d) $3x + 2 = j(y + 2)$

19. Find $\dfrac{dy}{dx}$ and simplify your answers where possible.

(a) $y = 6x^2 + 8\sqrt{x} - \dfrac{1}{x^3} + 3$ (b) $y = 6x^5 + \sqrt{x} + \dfrac{1}{x}$ (c) $y = 4\sqrt{x} + 2\tan\left(\dfrac{x}{2}\right)$

(d) $y = 4\cos x + \dfrac{6}{x} - \ln x$ (e) $y = x^2 e^{3x}$ (f) $y = 2x^3 \cos 5x$

(g) $y = \dfrac{\ln x}{\sin x}$ (h) $y = (7+5x)^5$ (i) $y = 5\cos(2x)$

(j) $y = \dfrac{e^x}{1+x}$ (k) $3x^2 + y^2 = 9$ (l) $y^2 - x^2 - 3y = 0$

(m) $x^2 + 2y^2 + 3x - 7y = 9$ (n) $y^2 + 6x = (x+3)^2$ (o) $x^2 - 3xy^2 = 4$

20. Find

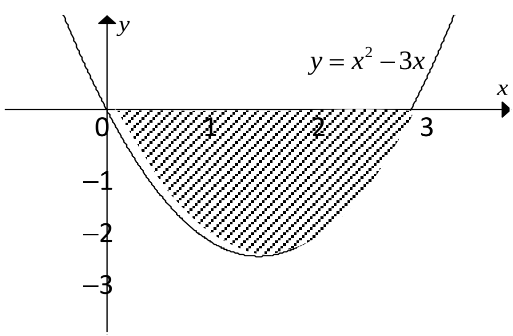
(a) $\int (4 + \sin 2x)\,dx$ (b) $\int \left(2x^3 - \dfrac{1}{x} + \sec^2 x\right) dx$ (c) $\int \left(5\tan x \sec x + \dfrac{2}{x^2}\right) dx$

(d) $\int (3e^{2x} - \dfrac{1}{2}\cos 2x)\,dx$ (e) $\int \left(\sqrt{x} - \dfrac{3}{x} + \csc^2 4x\right) dx$

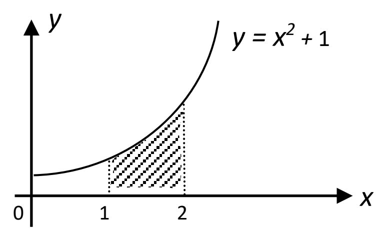
(f) $\int \left(5\cos\dfrac{x}{2} + \dfrac{2}{e^{2x}}\right) dx$ (g) $\int (3x^5 - 2\sin(\pi x) + e)\,dx$

21. Find the shaded area:

(a)



(b)



22. The charge q (coulombs) on a conductor varies with time t (seconds) is given as $q = 10\,(1 - e^{-\frac{t}{10}})$
Find the current i at t = 10s. (note: $i = \dfrac{dq}{dt}$)

23. The current flowing to a $L = 15$ mH-inductor at time t (second) is given by
$i = 0.005\sin\left(1000t + \dfrac{5}{8}\right)$ ampere. The voltage (volt) across the inductor is $V_L = L\dfrac{di}{dt}$. Find V_L.

24. The electric current i (A) in a circuit at time t (s) is given by $i = \dfrac{1}{4}\left[2\sqrt{t} - t\right]$

Assume that the initial charge is zero, (i) find the charge q as a function of t. $\{ i = \dfrac{dq}{dt} \}$

(ii) hence find q at $t = 2$ s.

25. The current i (A) flowing to a $0.01\ F$ capacitor at time t (s) is given by $i = \cos t + 6\sin 3t$

If the initial voltage is zero, (i) find the voltage as a function of t. $\left\{\text{Hint: } i = C\dfrac{dv}{dt}\right\}$

(ii) calculate the voltage at $t = 0.1$s.

26. The voltage v_L across a 0.1 H inductor at time t seconds is given by $v_L = 1 + \sin 5t - \cos 10t$ (volts). Find the current i flowing in the circuit after 0.1 second, if initially there is no current.

(Note: $v_L = L\dfrac{di}{dt}$)

27. At time t seconds, the current i flowing in a certain circuit is given by $i = \dfrac{7}{4}\ 1 - e^{-10t}$ amperes. Given that initially the charge q on the capacitor is zero, find the charge at time t seconds. (Note: $i = \dfrac{dq}{dt}$).

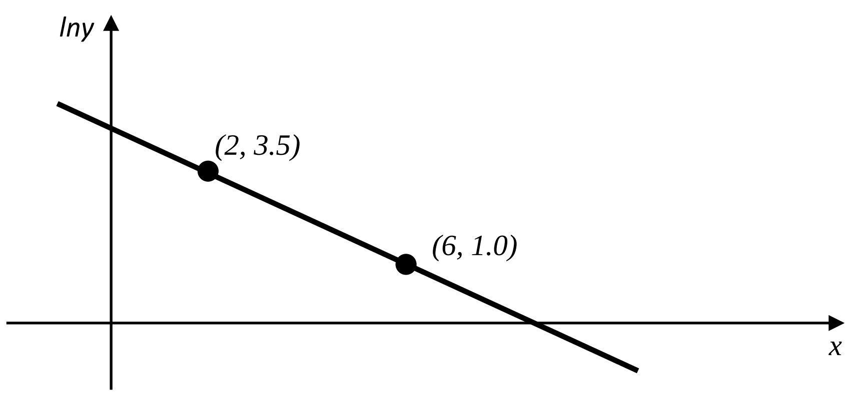
28. Given the function $y = e^{\sin x + \cos x}$, find

(i) the slope of the tangent line at $x = \dfrac{\pi}{2}$. (ii) the equation of the tangent line at $x = \dfrac{\pi}{2}$.

29. Find the equation of the tangent line to the curve $xy + y^2 + 2 = 0$ at $y = 1$.

30. During the testing of an electrical component, the current I amperes and voltage V volts were found to be related by the equation $2i + 1 = kV^n$, whre k and n are constants.
Rewrite the equation into a form suitable for a straight line graph. State clearly what should be plotted on each axis. Explain how the constants k and n can be obtained from the graph.

31. The linear equatin of the graph below is of the form $lny = kx + lna$, where a and k are constants. From the graph (which is not scale), determine the values of a and k.



Hence express the above equation into the original laws not involving logarithms. (Leave the equation in terms of a and k)

Intermediate/ Challenging Questions

1) The impedance of an electrical circuit is given by

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

where R , ω , L and C are real.
Find ω (ω > 0) in terms of L and C if Z is real.

2) Given that $\frac{1}{R} + j\omega C = \frac{j\omega C_1}{1 + j\omega R_1 C_1}$, find R in terms of ω , R_1 and C_1 .

3) The resultant impedance Z in a certain circuit is given by the formula

$$\frac{1}{Z} = \frac{1}{(1-j2)^2} + \frac{1}{b(1+j)} ,$$

where b is a constant.
Given that Z is a real number, find the value of b.

4) The condition of balance of an AC bridge is

$$Z_1 Z_3 = Z_2 Z_4 ,$$

where $Z_1 = R + jX$, $Z_2 = 2 - j$, $Z_3 = 1 + j$ and $Z_4 = j8$.
Find the values of the resistance R and the reactance X.

5) Given that $|z - 3| = 4$ and the argument of z is $\frac{3\pi}{4}$, find z in rectangular form.

6) The voltage *v* (volts) produced by a current *i* (amperes) in a wire is given by $v = \frac{3i}{4}$. Find the rate at which current is increasing if $\frac{dv}{dt} = 90\ mV/s$.

7) The total voltage *v* across a series RC circuit is given by $v = \sqrt{{v_R}^2 + {v_c}^2}$, where v_R and v_c are voltages across the resistor R and the capacitor C respectively. If v_R is constant at 4 volts and v_c is decreasing at a rate of 0.5 volts/second. Find

(i) $\frac{dv}{dv_c}$ in terms of v_c .

(ii) the rate at which *v* is changing when v_c is 3 volts.

8). The electric field E at a point is given by

$$E = x + \frac{4}{x-2}$$

where x is the distance from the centre of the charge ($x > 0$).
Find the value of x for E to be a minimum.

9) The power P is given by $P = \dfrac{12R}{25 + R^2}$ where R is the variable resistance.
Find the value of R at the point where maximum or minimum power occur.

10) The power P (watts) delivered to a load is $P = 120I - 5I^2$, where I (amperes) is the current to the load. If the current is changing at a rate of 1.5 A/s, find the rate of change of the power when I = 10 amperes.

11) The energy E stored in a constant inductance L is related to the current i flowing through it as
$E = \dfrac{1}{2}Li^2$ joules

(i) Find $\dfrac{dE}{di}$ in terms of L and i.

(ii) It is given that at a certain instant, L = 15 H, i = 2 A and $\dfrac{di}{dt} = 0.5$ A/s. Find the rate of change of E at that instant.

Answers :

(MCQ)

1 (a)	2. (c)	3. (c)	4. (c)	5. (a)	
6. (b)	7. (b)	8. (c)	9. (b)	10. (c)	11. (c)

(Basic Question)

1) (a) $x = 5,\ x = -3$ (b) $x = -1$ (2) $x = 10/11$

(3) $I_2 = 0.5A$ (4) $x = 8$, $y = 7$ (5) $\begin{pmatrix} 21 & 42 & 27 \\ 13 & 30 & 32 \end{pmatrix}$

(6) $\begin{pmatrix} 2 & -3 \\ 23 & 7 \end{pmatrix}$ (7) $x = 4$, -3

(8) (i) $p = -2$ 8(ii) Yes, because $|A| \neq 0$ (9) $k = 2$

10)(a) $\begin{pmatrix} 1 & 6 \\ -2 & -3 \\ 11 & 7 \end{pmatrix}$ (b) $\begin{pmatrix} -11 & 8 \\ 17 & -11 \\ -31 & 28 \end{pmatrix}$ (c) $x = 3/5$, $y = 4/15$

(11) (a) $18e^{-j0.734}$ (b) $2e^{j1.534}$ (c) $6.61e^{j0.87}$ (d) $81e^{-j1.75}$

(12) (a) $\frac{2}{3}\angle 15°$ (b) $54\angle -115°$ (c) $5.2\angle -0.04°$

13) $z = 1$ *or* $1\angle 0°$ (14) $e^{j2.5}$, $25e^{-j2.5}$, $3125e^{j\frac{\pi}{2}}$ (15) $-12 + j4$

(16) (i) $4.8627 + j7.5732$, $5.2655 + j2.8766$ (ii) $10.8017\angle 0.9108$
(iii) $6.5963\angle 0.3718$ (iv) $1.64\angle 0.54$

(17) (a) j, $-j3$ (b) $Z = 0.5 - \mathrm{j}\ 0.134$ or $0.5 - \mathrm{j}\ 1.866$ (c) $2 + j3$

(18) (a) $x = 2/17$, $y = -1/17$ (b) x = 18, y = 1 (c) 2 ,3 (d) -2/3 , -2

(19) (a) $y' = 12x + \frac{4}{\sqrt{x}} + \frac{3}{x^4}$ (b) $y' = 30x^4 + \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$ (c) $y' = \frac{2}{\sqrt{x}} + \sec^2\left(\frac{x}{2}\right)$

(d) $y' = -4\sin x - \frac{6}{x^2} - \frac{1}{x}$ (e) $y' = xe^{3x}(3x+2)$ (f) $y' = 2x^2(3\cos 5x - 5x\sin 5x)$

(g) $y' = \frac{\sin x - x\cos x \ln x}{x\sin^2 x}$ (h) $y' = 25(7+5x)^4$ (i) $y' = -10\sin 2x$

(j) $y' = \frac{xe^x}{(1+x)^2}$ (k) $\frac{dy}{dx} = -\frac{3x}{y}$ (l) $\frac{dy}{dx} = \frac{2x}{2y-3}$

(m) $\frac{dy}{dx} = \frac{-2x-3}{4y-7}$ or $\frac{2x+3}{7-4y}$ (n) $\frac{dy}{dx} = \frac{x}{y}$ (o) $\frac{dy}{dx} = \frac{2x-3y^2}{6xy}$

(20) (a) $4x - \frac{1}{2}\cos 2x + C$ (b) $\frac{x^4}{2} - \ln|x| + \tan x + C$ (c) $5\sec x - \frac{2}{x} + C$

(d) $\frac{3}{2}e^{2x} - \frac{1}{4}\sin 2x + C$ (e) $\frac{2}{3}x^{\frac{3}{2}} - 3\ln|x| - \frac{1}{4}\cot 4x + C$ (f) $10\sin\frac{x}{2} - e^{-2x} + C$

(g) $\frac{1}{2}x^6 + \frac{2}{\pi}\cos\pi x + ex + C$

(21) (a) 4.5 (b) 3.33 (22) 0.368

(23) $V_L = 0.075\cos\left(1000t + \frac{5}{8}\right)$ (24) (i) $q = \frac{1}{3}t^{3/2} - \frac{1}{8}t^2$ (ii) 0.44

(25) (i) $v_c = 100(\sin t - 2\cos 3t + 2)$ (ii) 18.92

(26) 0.4 (27) $q = \frac{7}{4}\left(t + \frac{1}{10}e^{-10t}\right) - \frac{7}{40}$

(28) (i) Slope at $x = \frac{\pi}{2}$, $\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}} = -e$ (ii) Equation is $y = e + \frac{\pi}{2}e - ex$ or $y = 6.99 - ex$

(29) $y = x + 4$

(30) $\ln(2i + 1) = \ln k + n \ln V$; $\ln k$ = intercept on Vertical-axis ; n = gradient of the straight line

(31) $k = -0.625$, $a = 115.58$; $y = ae^{kx}$

<u>(Intermediate/Challenging Questions)</u>

(1) $\omega = \sqrt{\frac{1}{LC}}$ (2) $R = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2}$ (3) $b = \frac{25}{8}$ (4) $R = 12$, $X = 4$

(5) $Z = -0.898 + j\,0.898$ (6) $\frac{di}{dt} = 120$ mA/s

(7) (i) $\frac{dv}{dv_c} = \frac{v_c}{\sqrt{16 + v_c^2}}$ (ii) $\frac{dv}{dt} = -0.3$ V/s (8) $x = 4$ (9) *R= 5 ohms*

(10) 30 W/s (11) (i) $\frac{dE}{di} = Li$ (ii) 15