SINGAPORE POLYTECHNIC

ET0096 SEM SAMPLE ANSWERS:

Section A

A1 When N=4, k= for k = 0, 1,2 and 3, $x(n)$ = {0, 1, 1, 2}

see pages 170 to 174

$$X(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi kn}{4}}$$

$$= x(0) + x(1)e^{-j\frac{2\pi k}{4}} + x(2)e^{-j\frac{4\pi k}{4}} + x(3)e^{-j\frac{6\pi k}{4}}$$

$$X(0) = x(0) + x(1) + x(2) + x(3) = 0+1+1+2 = 4$$

$$X(1) = x(0) + x(1)e^{-j\frac{2\pi \times 1}{4}} + x(2)e^{-j\frac{4\pi \times 2}{4}} + x(3)e^{-j\frac{6\pi \times 1}{4}}$$

$$= x(0) + x(1)e^{-j\frac{\pi}{2}} + x(2)e^{-j\pi} + x(3)e^{-j\frac{3\pi}{2}}$$

$$= 0 + 1 \times (-i) + 1 \times (-1) + 2 \times (i) = -1 + i$$

A2 y(n) = {2, -4, 5, -3, 1} see page 132, ex 3.5

$Y(z) = 2 - 4z^{-1} + 5z^{-2} - 3z^{-3} + z^{-4}$

And impulse response h(n) = {1, -1, 1}

$H(z) = 1 - z^{-1} + z^{-2}$

$X(z) = Y(z)/H(z)$

$X(z) = 2 - 2z^{-1} + z^{-2}$

x(n) = {2, - 2….} The first two terms

A3 (a) $x(n)=20\sin(0.25\pi n)u(n)$ see page 126, Table 3.1

$$X(z) = \frac{20\sin(0.25\pi)z^{-1}}{1-2z^{-1}\cos(0.25\pi)+z^{-2}}$$ Or equivalent

(b) $y(n)= e^{-0.2n}\sin(0.3\pi n)u(n)$, $Y(z) = \dfrac{e^{-0.2}\sin(0.3\pi)z^{-1}}{1-2e^{-0.2}\cos(0.3\pi)z^{-1}+e^{-0.4}z^{-2}}$

A4 A linear time invariant system's response to a unit step function is given as $y(n) = e^{-n}u(n)$. See page 101, Q2-12

x(n) = u(n-1), $y(n-1) = e^{-(n-1)}u(n-1)$

Input, x(n)= u(n) – u(n-1) = δ(n),

Impulse response, y(n)=h(n) = $e^{-n}u(n) - e^{-(n-1)}u(n-1)$

For n= 0, h(0) = $e^{-0}u(0) - e^{-(0-1)}u(0-1) = 1 \times 1 - 1 \times 0 = 1$

n= 1, h(1) = $e^{-1}u(1) - e^{-(1-1)}u(1-1) = e^{-1} \times 1 - 1 \times 1$

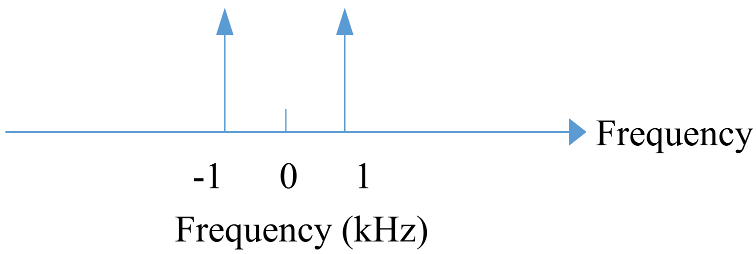
$= e^{-1} - 1 = -0.6321$

n= 2, h(2) = $e^{-2}u(2) - e^{-(2-1)}u(2-1) = e^{-2} \times 1 - e^{-1} \times 1$

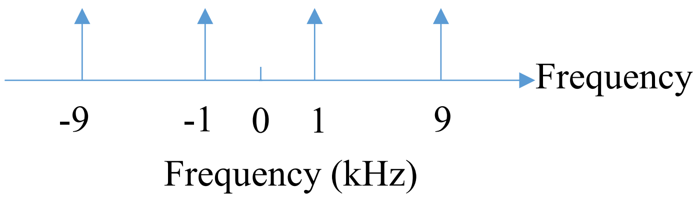
$= e^{-2} - e^{-1} = -0.2325$

A5 (a) 1 kHz

(b)



(c)



A6 (a) $Y(z) = X(z) + 3\ z^{-1}\ Y(z)$ see pages 148 and 151

$$H(z) = 1/(1 - 3\ z^{-1}) = \frac{1}{(1 - 3z^{-1})}$$ see page 126, Table 3.1

(b) $h(n) = 3^n\, u(n)$

(c) $Z_{pole} = 3$ and $Z_{zero} = 0$ UNSTABLE, outside unity circle

Extra info

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>> b=[1];
>> a=[1 -3];
>> zplane(b,a);
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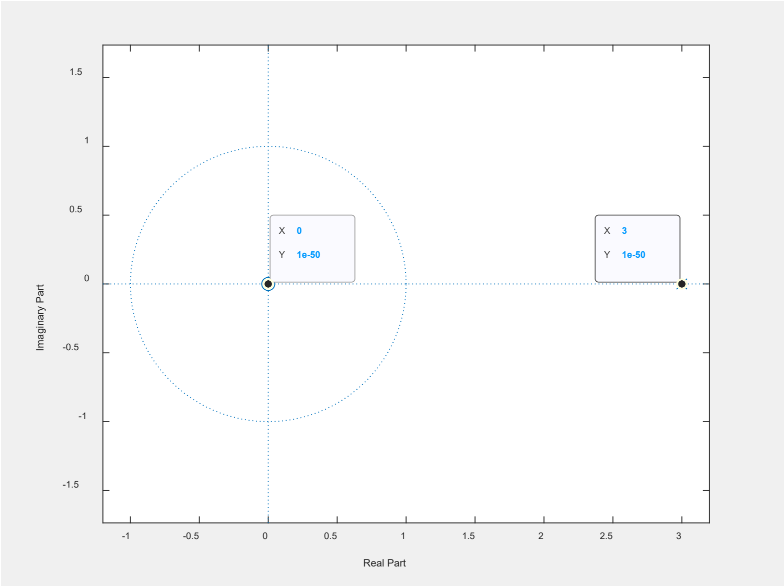


Section B

B1 (a) Information conveyed by each symbol:

$I(A) = \log_2(1/0.4) = 1.3219$ bits.

$I(B) = \log_2(1/0.20) = 2.3219$ bits.

(b) Entropy of source:

$H = 0.4(1.3219) + 0.2(2.3219) + 0.1(3.3219) \times 4 = 2.3219$ bits/symbol

(c) Since there are 6 different symbols, using fixed-length code- words would need a minimum of 3 bits. ($2^3 - 8$ symbols)

(d) Average bit length = $0.4(2) + 0.2(2) + 0.1(3)\times 4 = 2.4$ bits/symbol

B2 (a) Original symbols sequence: 11112223333331111 2222

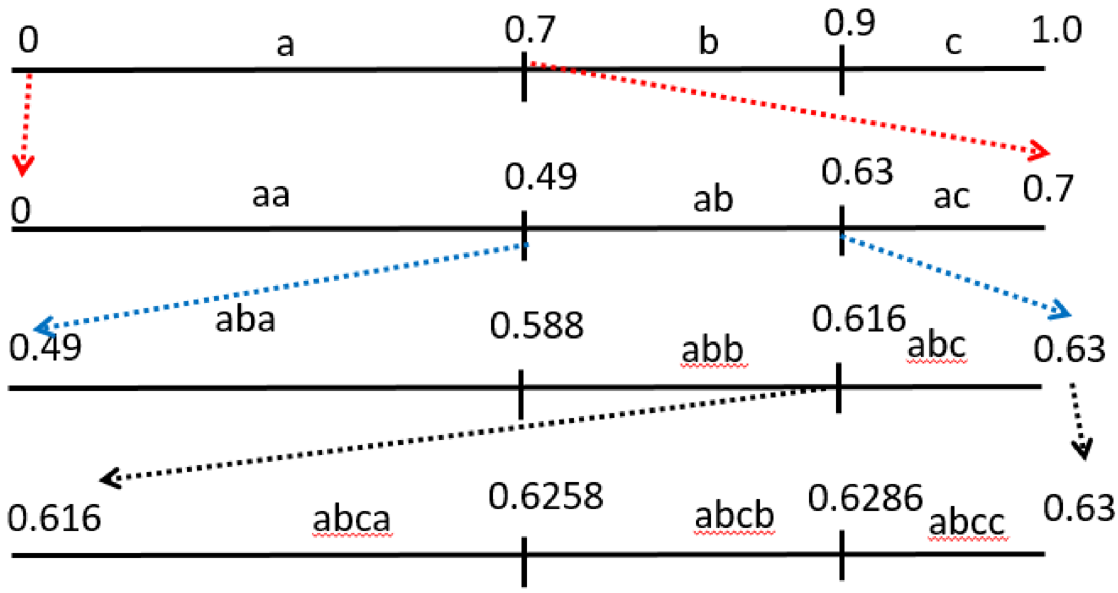
(i) (1,4),(2,3),(3,6),(1,4),(2,4)

(ii) Bit stream generated:

001 100 010 011 011 110 001 100 010 100

(iii) Total number of bits of the bit stream = 5 x 2 x 3 = 30 bits.

(b) The source of information A generates the symbols {a, b and c} with the corresponding probabilities {0.7, 0.2 and 0.1}. Use Arithmetic Coding technique to generate a binary representation for message "abcc".



Any number, such as 0.6289 within [0.6286, 0.63) will be acceptable for encoding "abcc". The binary representation of this number is 0.10100001