2016/2017 SEMESTER ONE EXAMINATION

Diploma in Electrical and Electronic Engineering
3rd Year Full Time
3rd Year Full Time Technical Elective
5th Year Evening Only

DIGITAL SIGNAL PROCESSING

Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules as set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **TWO** sections:

   Section A - 6 Short Questions, 10 marks each.
   Section B - 2 Long Questions, 20 marks each.

3. **ALL** questions are **COMPULSORY**.

4. **ALL** questions are to be answered in the answer booklet.

5. This paper consists of **7** pages, including 2 pages of mathematical formulae.

SECTION A - SHORT QUESTIONS [10 marks each]

A1 An analog signal is represented by $x(t) = \cos(1000\pi t) + 2\sin(3000\pi t + \pi/4)$ and Fs is the sampling frequency used.

(a) What are the fundamental frequency components that x(t) contains? (2 marks)

(b) What is the minimum sampling frequency Fs used to avoid the aliasing problem? (2 marks)

(c) Sketch the magnitude spectrum of the sampled signal for $0< f <4$ kHz if Fs = 4kHz (6 marks)

A2 Evaluate N=4 point DFT for X(0) and X(2) for x(n) ={2,1,1,2} (10marks)

A3 Find the input signal x(n) to a system if its impulse response and output signal are given as:

$h(n) = \delta(n) - \delta(n-1)$ and $y(n) = 0.5\delta(n) - \delta(n-1) + 0.75\delta(n-2) - 0.25\delta(n-3)$. (10 marks)

A4 Given the following DSP system with a sampling rate of 8000 Hz

$$y(n) = 0.2x(n) - 0.2x(n-1)$$

where $y(n)$ is the output and $x(n)$ is the input.

(a) Determine the transfer function, H(z). (3 marks)

(b) Determine the magnitude of the filter frequency response $|H(e^{jw})|$. (3 marks)

(c) Compute the filter gain at the frequency of 0 Hz and 4000 Hz respectively. (4 marks)

A5 Compute the inverse Z-transform for the followings:

(a) $X(z) = \frac{5z}{z-1} + 4$ (2 marks)

(b) $X(z) = \frac{5z}{(z-1)^2} + z^{-5}$ (3 marks)

(c) $X(z) = \frac{10(z-0.7071)}{z^2-1.4142z+1}$ (3 marks)

(d) $X(z) = \frac{7z}{z-0.5}$ (2 marks)

A6 Given that a difference equation of a system is $y(n) = x(n) - 0.2x(n-2)$,

(a) determine its impulse response; (2 marks)

(b) compute $y(n)$ when $x(n) = u(n) - u(n-3)$; (4 marks)

(c) draw the digital network representing a discrete-time system whose difference equation is given by $y(n)$. (4 marks)

SECTION B - LONG QUESTIONS [20 marks each]

B1 A linear-phase digital FIR low pass filter is to be designed using the windowing technique. The specifications of the filter is given below:

Sampling frequency = 20 kHz
Pass band = 0 – 4.6 kHz
Stop band = 5.0 – 10 kHz
Peak approximation error < 0.0003

(a) Determine the type of windowing function to be used. (3 marks)

(b) Determine the number of tap coefficients, N, needed for this filter. (4 marks)

(c) Compute the impulse response of the filter for $\left(\frac{N}{2}-2\right) < n < \left(\frac{N}{2}+2\right)$. (10 marks)

(d) If the number of tap coefficients cannot exceed 100, explain how you can sacrifice one of the filter specifications to meet this additional requirement. (3 marks)

Given:

$$h_d(n) = \frac{\sin \omega_c (n - \frac{M}{2})}{\pi(n - \frac{M}{2})}$$

Window Type	Peak Approximation error, 20 log δ dB	Transition Band, Δω
Rectangular	-21	$\frac{4\pi}{M+1}$
Bartlett	-25	$\frac{8\pi}{M}$
Hanning	-44	$\frac{8\pi}{M}$
Hamming	-53	$\frac{8\pi}{M}$
Blackman	-74	$\frac{12\pi}{M}$

Rectangular $w(n) = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$

Bartlett $$w(n) = \begin{cases} \dfrac{2n}{M} & 0 \le n \le \dfrac{M}{2} \\ 2 - \dfrac{2n}{M} & \dfrac{M}{2} \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

Hanning $$w(n) = \begin{cases} 0.5 - 0.5\cos\left(\dfrac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

Hamming $$w(n) = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

Blackman $$w(n) = \begin{cases} 0.42 - 0.5\cos\left(\dfrac{2\pi n}{M}\right) + 0.08\cos\left(\dfrac{4\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

B2 An integrator can be digitally simulated by the discrete-time linear system described by the following difference equation:

$$y(n) = y(n-1) + x(n)$$

where $x(n)$ and $y(n)$ are the input and output to the integrator respectively and $y(n) = 0$ when $n < 0$.

(a) Based on the difference equation given above, construct a digital network in the form of a block diagram that realizes the integrator. (4 marks)

(b) Compute and sketch the waveform of the sampled impulse response of the integrator for $0 \le n \le 10$ on your answer booklet (make sure that all the variables and scales involved are clearly labelled). (6 marks)

(c) If $x(n)$ is a zero mean, unity amplitude square pulse train sequence as shown in Figure B2, sketch the waveform of $y(n)$ for 2 consecutive cycles, i.e. for $0 \le n \le 20$, on your answer booklet (make sure that all the variables and scales involved are clearly labelled). (10 marks)

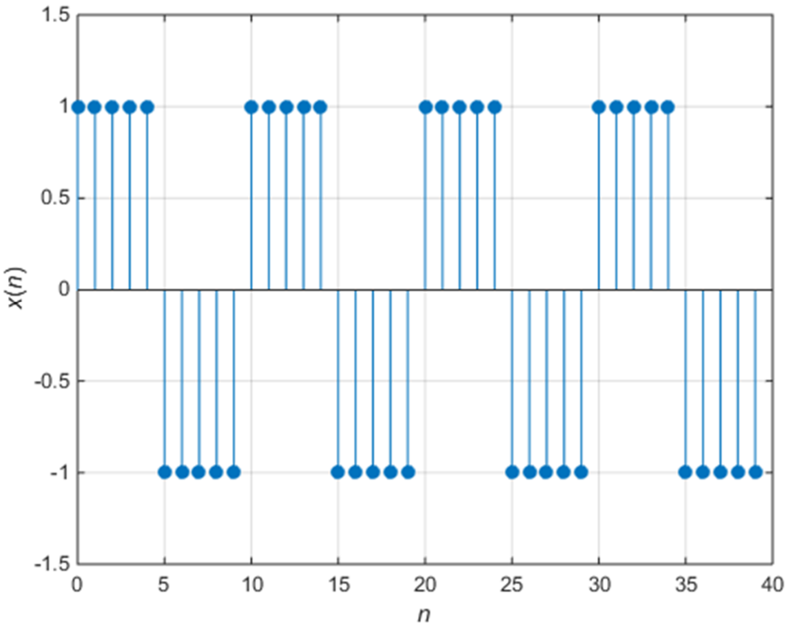

Figure B2

-End of Paper-

Appendix

The z-transform is defined as X(z) = $\sum_{n=-\infty}^{\infty} x[n]z^{-n}$

Sequence	Transform
$\delta[n]$	1
u[n]	$\frac{1}{1-z^{-1}}$
$\delta[n - m]$	z^{-m}
$a^n u[n]$	$\frac{1}{1-az^{-1}}$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$
$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$

Some z-transform properties:	
Sequence	Transform
$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
$x[n - m]$	$z^{-m}X(z)$

Some trigonometric identities:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

Discrete Fourier Transform:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{N}kn}$$

Complex number theory:

$z = a + jb = r\angle\theta = re^{j\theta}$

$r = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}\frac{b}{a}$

Quadratic equation solution:

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$