SP SCHOOL OF MATHEMATICS & SCIENCE

MATHEMATICAL FORMULAE & TABLE

Algebra

Factoring Formulae

$a^2 - b^2 = (a+b)(a-b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Quadratic Formulae

If $ax^2 + bx + c = 0$, where a, b and c are real and $a \neq 0$,

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial Theorem

If n is a positive integer, then

$(a+x)^n = a^n + {}_nC_1 a^{n-1}x + {}_nC_2 a^{n-2}x^2 + {}_nC_3 a^{n-3}x^3 + \ldots + x^n$

where ${}_nC_r = \dfrac{n!}{r!(n-r)!}$

Analytic Geometry & Vectors

Analytic Geometry

Straight line passing through two points $P(x_1, y_1)$ and $Q(x_2, y_2)$:

- Equation is $y = mx + c$, where gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.
- Distance from P to Q is: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Midpoint of PQ is: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

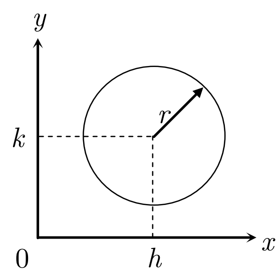
Vectors

If the following vectors are defined: $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$, $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$

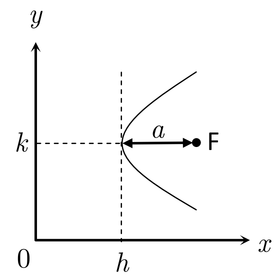
- Magnitude of $\boldsymbol{a}$ is: $|\boldsymbol{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
- Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$
- Vector Product: $|\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}|\sin\theta$, $\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Conic Sections

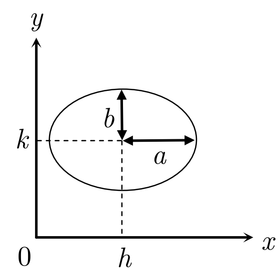
Circle: $(x-h)^2 + (y-k)^2 = r^2$



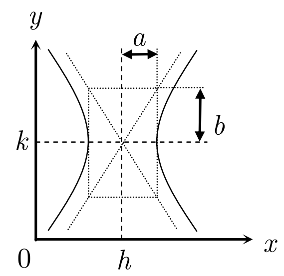
Parabola: $(y-k)^2 = 4a(x-h)$



Ellipse: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$



Hyperbola: $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$



Trigonometry

Definitions

$\sec x = \dfrac{1}{\cos x}$

$\csc x = \dfrac{1}{\sin x}$

$\tan x = \dfrac{\sin x}{\cos x}$

$\cot x = \dfrac{\cos x}{\sin x}$

Basic Identities

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\tan(-x) = -\tan x$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Compound Angle Formulae

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

Double Angle Formulae

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

Formulae for Reducing Power

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$

$\cos^2 x = \dfrac{1 + \cos 2x}{2}$

Amplitude & Phase-Angle Formulae

If a and b are positive constants,

$a\sin\theta + b\cos\theta = R\sin(\theta + \alpha)$

$a\sin\theta - b\cos\theta = R\sin(\theta - \alpha)$

$a\cos\theta + b\sin\theta = R\cos(\theta - \alpha)$

$a\cos\theta - b\sin\theta = R\cos(\theta + \alpha)$

where $R = \sqrt{a^2 + b^2}$ and $\alpha = \tan^{-1}\left(\dfrac{b}{a}\right)$

Sum to Product Identities

$\sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

$\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

$\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

$\cos x - \cos y = -2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

Product to Sum Identities

$\sin x \cos y = \dfrac{1}{2}[\sin(x+y) + \sin(x-y)]$

$\cos x \sin y = \dfrac{1}{2}[\sin(x+y) - \sin(x-y)]$

$\cos x \cos y = \dfrac{1}{2}[\cos(x+y) + \cos(x-y)]$

$\sin x \sin y = \dfrac{1}{2}[\cos(x-y) - \cos(x+y)]$

Complex Numbers

A complex number z can be expressed in one of the following forms:

- Rectangular/Cartesian form $z = a + jb$
- Trigonometric form $z = r(\cos\theta + j\sin\theta)$
- Polar form $z = r\angle\theta$
- Exponential form $z = re^{j\theta}$ (θ in radians)

where a and b are real numbers,

$j = \sqrt{-1}$ and $j^2 = -1$,

$r = |z| = \sqrt{a^2 + b^2}$, and $\theta = \arg(z)$ such that $\tan\theta = \dfrac{b}{a}$, $-\pi < \theta \le \pi$

Complex Conjugates

If $z = a + jb$, then $\overline{z} = a - jb$,

such that $z\overline{z} = a^2 + b^2$.

Multiplication & Division

$z_1 z_2 = (r_1\angle\theta_1)(r_2\angle\theta_2) = r_1 r_2 \angle(\theta_1 + \theta_2)$

$\dfrac{z_1}{z_2} = \dfrac{(r_1\angle\theta_1)}{(r_2\angle\theta_2)} = \dfrac{r_1}{r_2}\angle(\theta_1 - \theta_2)$

De Moivre's Theorem

$(r\angle\theta)^n = r^n\angle n\theta = r^n(\cos n\theta + j\sin n\theta)$

4. Euler's Formula

$e^{j\theta} = \cos\theta + j\sin\theta$

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Differentiation

Standard Derivatives

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$

$\frac{d}{dx}(\ln x)=\frac{1}{x}$

$\frac{d}{dx}\left(a^x\right)=a^x\ln a$

$\frac{d}{dx}\left(e^x\right)=e^x$

$\frac{d}{dx}(\sin x)=\cos x$

$\frac{d}{dx}(\cos x)=-\sin x$

$\frac{d}{dx}(\tan x)=\sec^2 x$

$\frac{d}{dx}(\cot x)=-\csc^2 x$

$\frac{d}{dx}(\sec x)=\sec x\tan x$

$\frac{d}{dx}(\csc x)=-\csc x\cot x$

$\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\tan^{-1}x\right)=\frac{1}{1+x^2}$

Rules of Differentiation

Let $u\equiv u(x)$, $v\equiv v(x)$ and $y\equiv y(u)$

- Constant Multiple Rule $\frac{d}{dx}(ku)=k\frac{du}{dx}$
- Sum Rule $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$
- Product Rule $\frac{d}{dx}(u\cdot v)=u\frac{dv}{dx}+v\frac{du}{dx}$
- Quotient Rule $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
- Chain Rule $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$

Approximation Formula

If $y=f(x)$, then $\Delta y\approx\frac{dy}{dx}\Delta x$

If $u=f(x_1,x_2,\ldots,x_n)$,

then $\Delta u\approx\frac{\partial u}{\partial x_1}\Delta x_1+\frac{\partial u}{\partial x_2}\Delta x_2+\ldots+\frac{\partial u}{\partial x_n}\Delta x_n$

Newton's Method

Newton's method of Approximation to a root of the equation $f(x)=0$ is

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

where $f'(x_n)=\left.\frac{df}{dx}\right|_{x=x_n}$

Integration

Standard Integrals

$\int x^n\,dx=\frac{x^{n+1}}{n+1}+C$

where C is a constant

$\int\frac{1}{x}\,dx=\ln|x|+C$

$\int a^x\,dx=\frac{a^x}{\ln a}+C$

where a is a positive constant

$\int e^x\,dx=e^x+C$

$\int\sin x\,dx=-\cos x+C$

$\int\cos x\,dx=\sin x+C$

$\int\tan x\,dx=-\ln|\cos x|+C$

$\int\cot x\,dx=\ln|\sin x|+C$

$\int\sec x\,dx=\ln|\sec x+\tan x|+C$

$\int\csc x\,dx=-\ln|\csc x+\cot x|+C$

$\int\sec^2 x\,dx=\tan x+C$

$\int\csc^2 x\,dx=-\cot x+C$

$\int\sec x\tan x\,dx=\sec x+C$

$\int\csc x\cot x\,dx=-\csc x+C$

$\int\frac{1}{x^2+a^2}\,dx=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$

$\int\frac{1}{x^2-a^2}\,dx=\frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C$

$\int\frac{1}{\sqrt{a^2-x^2}}\,dx=\sin^{-1}\frac{x}{a}+C$

$\int\frac{1}{\sqrt{x^2+a^2}}\,dx=\ln\left|x+\sqrt{x^2+a^2}\right|+C$

$\int\frac{1}{\sqrt{x^2-a^2}}\,dx=\ln\left|x+\sqrt{x^2-a^2}\right|+C$

Integration by Parts

$\int u\,dv=uv-\int v\,du$

Mean Value

$\bar{y}=\frac{1}{b-a}\int_a^b y\,dx$

Root Mean Square Value

$y_{rms}=\sqrt{\frac{1}{b-a}\int_a^b y^2\,dx}$

Area & Volume Formula

- Area enclosed by the curve $y=f(x)$, the x-axis, and the lines $x=a$ and $x=b$, where $f(x)>0$ for $a\le x\le b$, is $A=\int_a^b y\,dx$.
- Volume of solid of revolution of $y=f(x)$ about the x-axis between $x=a$ and $x=b$ is $V=\pi\int_a^b y^2\,dx$.

Arc Length

$S=\int_a^b\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

Centroid of Area

$\bar{x}=\frac{\int_a^b xy\,dx}{\int_a^b y\,dx}$, $\bar{y}=\frac{\frac{1}{2}\int_a^b y^2\,dx}{\int_a^b y\,dx}$

Numerical Integration

Let $y=f(x)$ and $y_0,y_1,\ldots,y_{n-1},y_n$ be the values of $f(x)$ at $x_0=a$, $x_1=a+h,\ \ldots,\ x_{n-1}=a+(n-1)h,\ x_n=a+nh=b$ where $h=\frac{b-a}{n}$.

- Trapezoidal Rule:

$$\int_a^b f(x)\,dx\approx\frac{1}{2}h\left[y_0+y_n+2\left(y_1+y_2+\ldots+y_{n-1}\right)\right]$$

- Simpson's Rule:

$$\int_a^b f(x)\,dx\approx\frac{1}{3}h\left[y_0+y_n+4\left(y_1+y_3+\ldots+y_{n-1}\right)+2\left(y_2+y_4+\ldots+y_{n-2}\right)\right]$$

where n is an even positive integer.

Series

Arithmetic Series

$a+(a+d)+(a+2d)+(a+3d)+\ldots$

The n^{th} term is: $u_n=a+(n-1)d$

The sum of the first n terms is: $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

Geometric Series

$a+ar+ar^2+ar^3+\ldots$

The n^{th} term is: $u_n=ar^{n-1}$

The sum of the first n terms is: $S_n=\frac{a\left(1-r^n\right)}{1-r}$

If $-1<r<1$, then the sum to infinity is: $S_\infty=\frac{a}{1-r}$

Taylor's Series about x = a

$$f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\ldots$$

Maclaurin's Series

$$f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\ldots$$

Fourier Series

If $f(t)$ is a periodic function of period T, then its trigonometric Fourier series is given by:

$$f(t)=a_0+\sum_{n=1}^{\infty}\left(a_n\cos n\omega t+b_n\sin n\omega t\right)$$

where $\omega=\frac{2\pi}{T}$, $a_0=\frac{1}{T}\int_k^{k+T}f(t)\,dt$, $a_n=\frac{2}{T}\int_k^{k+T}f(t)\cos n\omega t\,dt$, $b_n=\frac{2}{T}\int_k^{k+T}f(t)\sin n\omega t\,dt$

Fourier Transform

The Fourier transform $F(\omega)$ of $f(t)$ is: $F(\omega)=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}\,dt$

Standard Power Series

- Binomial Series: $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\ldots$

  where $-1<x<1$ and n is not a positive integer
- Logarithm Series: $\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots$, $-1<x\le 1$
- Exponential Series: $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$
- Sine & Cosine Series: $\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots$, $\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots$

Differential Equations

First Order Linear ODE

$$\frac{dy}{dx}+P(x)y=Q(x)$$

Integrating factor:

$$\mu(x)=e^{\int P(x)\,dx}$$

General solution:

$$y\cdot\mu(x)=\int\mu(x)Q(x)\,dx$$

Second Order Homogeneous ODE with Constant Coefficients

$$a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$$

Auxiliary equation: $a\lambda^2+b\lambda+c=0$, where $\lambda_{1,2}=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

General solution:

Case 1: $b^2-4ac>0$	Case 2: $b^2-4ac=0$	Case 3: $b^2-4ac<0$
2 real roots: λ_1 and λ_2	2 equal roots: $\lambda_1=\lambda_2=\lambda$	2 complex roots: $\lambda=\alpha\pm j\beta$
$y=Ae^{\lambda_1 x}+Be^{\lambda_2 x}$	$y=e^{\lambda x}\left(Ax+B\right)$	$y=e^{\alpha x}\left(A\cos\beta x+B\sin\beta x\right)$

where A and B are arbitrary constants.

Determinants & Matrices

Determinants

Order 2: $\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix}=a_{11}a_{22}-a_{12}a_{21}$

Order 3: $\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}=a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}$

where A_{11}, A_{12} and A_{13} are cofactors of elements a_{11}, a_{12} and a_{13} respectively, and given by

$$A_{11}=+\begin{vmatrix}a_{22} & a_{23}\\ a_{32} & a_{33}\end{vmatrix}=+\left(a_{22}a_{33}-a_{23}a_{32}\right),$$

$$A_{12}=-\begin{vmatrix}a_{21} & a_{23}\\ a_{31} & a_{33}\end{vmatrix}=-\left(a_{21}a_{33}-a_{23}a_{31}\right),$$

$$A_{13}=+\begin{vmatrix}a_{21} & a_{22}\\ a_{31} & a_{32}\end{vmatrix}=+\left(a_{21}a_{32}-a_{22}a_{31}\right)$$

Inverse Matrix

If $|A|\neq 0$, then inverse of 3×3 matrix A is: $A^{-1}=\dfrac{1}{|A|}\operatorname{adj}A$, where $\operatorname{adj}A=\begin{bmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{bmatrix}^T$

Cramer's Rule

For a system of 3 linear equations:

$$\begin{aligned}a_{11}x_1+a_{12}x_2+a_{13}x_3&=k_1\\ a_{21}x_1+a_{22}x_2+a_{23}x_3&=k_2\\ a_{31}x_1+a_{32}x_2+a_{33}x_3&=k_3\end{aligned}$$

The solutions are:

$$x_1=\frac{1}{|A|}\begin{vmatrix}k_1 & a_{12} & a_{13}\\ k_2 & a_{22} & a_{23}\\ k_3 & a_{32} & a_{33}\end{vmatrix},\quad x_2=\frac{1}{|A|}\begin{vmatrix}a_{11} & k_1 & a_{13}\\ a_{21} & k_2 & a_{23}\\ a_{31} & k_3 & a_{33}\end{vmatrix},\quad x_3=\frac{1}{|A|}\begin{vmatrix}a_{11} & a_{12} & k_1\\ a_{21} & a_{22} & k_2\\ a_{31} & a_{32} & k_3\end{vmatrix},\text{where } |A|=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$$

Probability & Statistics

Statistical Measure (For Grouped Data)

Mean: $\mu=\dfrac{\sum f_i x_i}{N}$

Standard deviation: $\sigma=\sqrt{\dfrac{\sum f_i\left(x_i-\mu\right)^2}{N}}$

Median: $\tilde{x}=L_m+\dfrac{\frac{N}{2}-F_c}{f_m}C$

where x_i = class mark of the i^{th} class,
f_i = frequency of the i^{th} class,
L_m = lower class boundary of the median class,
$N=\sum f_i$ = total frequency,
F_c = sum of frequencies of all classes below the median class,
f_m = frequency of the median class,
C = class width.

Probability Rules

Addition Rule: $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

Subtraction Rule: $P(A)=1-P(\bar{A})$

Multiplication Rule: $P(A\cap B)=P(A)P(B)$
if A and B are independent events.

Conditional Probability: $P(A\,|\,B)=\dfrac{P(A\cap B)}{P(B)}$

Bayes' Theorem:

$$P(A)=P(A\,|\,B_1)P(B_1)+P(A\,|\,B_2)P(B_2)$$

$$P(B_1\,|\,A)=\frac{P(A\,|\,B_1)P(B_1)}{P(A\,|\,B_1)P(B_1)+P(A\,|\,B_2)P(B_2)}$$

where $B_1\cap B_2=\varnothing$ and $B_1\cup B_2=S$ the sample space.

Sample Statistics

Mean: $\bar{x}=\dfrac{\sum f_i x_i}{n}$

Standard deviation: $s=\sqrt{\dfrac{\sum f_i\left(x_i-\bar{x}\right)^2}{n-1}}$

where f = frequency and n = sample size.

Sampling Distribution

Mean: $\mu_{\bar{x}}=\mu$

Standard error:

- for finite population: $\sigma_{\bar{x}}=\dfrac{\sigma}{\sqrt{n}}\sqrt{\dfrac{N-n}{N-1}}$
- for infinite population: $\sigma_{\bar{x}}=\dfrac{\sigma}{\sqrt{n}}$

Test Statistics

Test for Population Mean	Test for Difference of Means	Test for Proportions
$z=\dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$	$z=\dfrac{\left(\bar{x}-\bar{y}\right)-\left(\mu_x-\mu_y\right)}{\sqrt{\dfrac{\sigma_x^2}{n_x}+\dfrac{\sigma_y^2}{n_y}}}$	$z=\dfrac{\hat{p}-p_0}{\sqrt{p_0\left(1-p_0\right)/n}}$

Discrete Probability Distributions

Mean: $\mu=E(X)=\sum_{\text{all }x}x\,P(X=x)$

Variance: $\sigma^2=\operatorname{Var}(X)=E\left(X^2\right)-\mu^2$

where $E\left(X^2\right)=\sum_{\text{all }x}x^2P(X=x)$

Standard deviation: $\sigma=\sqrt{\operatorname{Var}(X)}$

- Binomial Distribution: $X\sim B(n,p)$

  $P(X=x)={}_nC_x p^x q^{n-x}$

  Mean: $\mu=np$, standard deviation: $\sigma=\sqrt{npq}$

- Poisson Distribution: $X\sim P(\lambda)$

  $P(X=x)=\dfrac{\lambda^x}{x!}e^{-\lambda}$

  Mean: $\mu=\lambda$, standard deviation: $\sigma=\sqrt{\lambda}$

Continuous Probability Distributions

$P(a<X<b)=\int_a^b f(x)\,dx$

Mean: $\mu=E(X)=\int_{-\infty}^{\infty}x\,f(x)\,dx$

Variance: $\sigma^2=\operatorname{Var}(X)=E\left(X^2\right)-\mu^2$

where $E\left(X^2\right)=\int_{-\infty}^{\infty}x^2 f(x)\,dx$

Standard deviation: $\sigma=\sqrt{\operatorname{Var}(X)}$

- Normal Distribution: $X\sim N\left(\mu,\sigma^2\right)$

  $f(x)=\dfrac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

  Mean $=\mu$, standard deviation $=\sigma$

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Simple Linear Regression

Least Squares Line ($y = mx + c$)

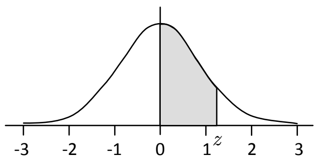
$$m = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{n\sum x^2 - \left(\sum x\right)^2} \quad , \quad c = \frac{\sum y - m\sum x}{n}$$

Correlation coefficient: $r = \dfrac{\sum (x-\bar{x})(y-\bar{y})}{\sqrt{\sum (x-\bar{x})^2 \cdot \sum (y-\bar{y})^2}}$

Standard Normal Table

Area under the Standard Normal Curve from 0 to z

$$z = \frac{x-\mu}{\sigma}$$



z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000

Hyperbolic Functions

Definitions

$$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$$

$$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

Basic Identities

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{csch}^2 x$$

Laplace Transforms

Definition

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = F(s)$$

Table of Laplace Transforms

Function $f(t)$	Laplace Transform $F(s)$
1	$\dfrac{1}{s}$
t^n n is a positive integer	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
$\sin at$	$\dfrac{a}{s^2+a^2}$
$\cos at$	$\dfrac{s}{s^2+a^2}$
$t\sin at$	$\dfrac{2as}{\left(s^2+a^2\right)^2}$
$t\cos at$	$\dfrac{s^2-a^2}{\left(s^2+a^2\right)^2}$
First Shift Theorem $e^{at} f(t)$	$F(s-a)$
$\dfrac{dy}{dt}$	$s\mathcal{L}\{y\} - y(0)$
$\dfrac{d^2y}{dt^2}$	$s^2\mathcal{L}\{y\} - sy(0) - y'(0)$
$\int_0^t f(t)\,dt$	$\dfrac{1}{s}\mathcal{L}\{f(t)\}$
Unit Step Function $u(t-c)$	$\dfrac{e^{-cs}}{s}$
Second Shift Theorem $f(t-c)u(t-c)$	$e^{-cs}\mathcal{L}\{f(t)\}$
$f(t)u(t-c)$	$e^{-cs}\mathcal{L}\{f(t+c)\}$
Unit Impulse Function $\delta(t-c)$	e^{-cs}
$f(t)\delta(t-c)$	$f(c)e^{-cs}$

Boolean Algebra

Commutative Laws	$x\cdot y = y\cdot x$ $x+y=y+x$
Associative Laws	$x\cdot(y\cdot z)=(x\cdot y)\cdot z = x\cdot y\cdot z$ $x+(y+z)=(x+y)+z=x+y+z$
Distributive Laws	$x+(y\cdot z)=(x+y)\cdot(x+z)$ $x\cdot(y+z)=(x\cdot y)+(x\cdot z)$
Identity Laws	$x\cdot 1 = x$ $x+0=x$
Complement Laws	$x\cdot\overline{x}=0$ $x+\overline{x}=1$
Involution Law	$\overline{\overline{x}}=x$
Idempotent Laws	$x\cdot x = x$ $x+x=x$
Bound Laws	$x\cdot 0=0$ $x+1=1$
De Morgan's Theorem	$\overline{(x\cdot y)}=\overline{x}+\overline{y}$ $\overline{(x+y)}=\overline{x}\cdot\overline{y}$
Absorption Laws	$x\cdot(x+y)=x$ $x\cdot(\overline{x}+y)=x\cdot y$ $x+(x\cdot y)=x$ $x+(\overline{x}\cdot y)=x+y$