2015/2016 SEMESTER TWO EXAMINATION

Diploma in Electrical and Electronic Engineering
3rd Year Full Time
3rd Year Full Time Technical Elective
5th Year Evening Only

DIGITAL SIGNAL PROCESSING

Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules as set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **TWO** sections:

Section A	-	6 Short Questions, 10 marks each.
Section B	-	2 Long Questions, 20 marks each.

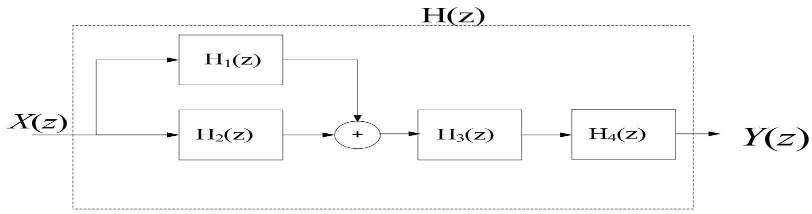
3. **ALL** questions are **COMPULSORY**.

4. **ALL** questions are to be answered in the answer booklet.

5. This paper consists of **6** pages, including 2 pages of mathematical formulae.

SECTION A - SHORT QUESTIONS [10 marks each]

A1 The block diagram of a digital system is given as:



a) Find the overall system function, H(z) in terms of $H_1(z)$, $H_2(z)$, $H_3(z)$ and $H_4(z)$.

(4 marks)

b) If the inverse z-transform of $H_1(z)= h_1(n)$, $H_2(z)= h_2(n)$, $H_3(z) =h_3(n)$ and $H_4(z) =h_4(n)$ and $h_1(n)= h_2(n) =h_3(n)= h_4(n)=\{1,1\}$, find the impulse response, h(n).

(6 marks)

A2 A first-order high-pass IIR digital Butterworth filter has the following difference equation.

$$y(n) = 0.7071\ x(n-1) + 1.414y(n-1) -y(n-2)$$

a) Draw the digital network diagram for the system. (4 marks)
b) Using Z transform, determine the system function H(z). (3 marks)
c) Using inverse Z transform, determine the impulse response. (3 marks)

A3 Find the difference equation and impulse response of the following system. (Hint: Use partial fraction method).

$$H(z) = \frac{z}{(z^2+0.2z-0.08)}$$

(10 marks)

A4 Given the sampling frequency is 10 kHz. The output y(n) of a particular filter system to the input x(n) is

$$y(n) = x(n) - 2x(n-1) + x(n-3)$$

a) Determine the transfer function, H(z). (3 marks)

b) Determine the filter frequency response $|H(e^{jw})|$. (3 marks)

c) Compute the filter gain at dc and at a frequency of 5 kHz. (4 marks)

A5 The continuous-time signal $x(t)=3\cos(600\pi t) + 4\sin(1200\pi t) + 5\cos(4600\pi t)$ is sampled at a 4-kHz rate generating the sequence x[n].

(i) Determine the expression x[n] (3 marks)

(ii) Find x[0] and x[2]. (2 marks)

(iii) If the sampled signal passed through an ideal low pass filter with a cut-off frequency of a 2kHz, generating a continuous signal y(t), what will be the frequency components of y(t)?

(5 marks)

A6 Find the inverse z transform of the following causal signals

a) $X_1(z) = \dfrac{2z^{-4}}{z-1} + \dfrac{2z^{-1}}{(z-1)^2} + z^{-7} + \dfrac{z^{-3}}{z-0.2}$ (5 marks)

b) $X_2(z) = \dfrac{10z(z-0.4854)}{z^2 - 0.9708z + 0.36}$ (5 marks)

SECTION B - LONG QUESTIONS [20 marks each]

B1. Design a low pass filter with the following specification

. Sampling frequency = 12500 Hz
Passband = 2500Hz
Peak approximation error = 0.006
Filter length = 31

Determine

a) To strictly meet the specifications, what should be the Window function used in this design? (3 marks)
b) Based on (a), what is the transition bandwidth? (2 marks)
c) What is the centre frequency and stopband? (4 marks)
d) What is the maximum ripple for this filter (3 marks)
e) Calculate the value of tap coefficient for h(10) (5 marks)

f) Explain how will the transition band of the filter be change if you are able to compute infinite number of tap coefficient to represent the filter? (3 marks)

B2. A discrete time signal is given by

$$x(n) = 3 + \frac{\sqrt{2}}{2}\delta(n-1) - \frac{\sqrt{2}}{2}\delta(n-3)$$

a) Verify that the N = 4 point DFT of x(n) for k = 0,1,2,3 is

X(k) = {3, 3 – 1.414j, 3, 3 + 1.414j} (8 marks)

b) Compute corresponding amplitude and phase of X(k). (8 marks)

c) Comment how the magnitude spectrum from (b) can be derived from the magnitude spectrum of $\left|X(e^{j\omega})\right|$, for $0 \le w \le 2\pi$ where $X(e^{j\omega})$ is the fourier transform of x(n). (4 marks)

-End of Paper-

Appendix

The z-transform is defined as X(z) = $\sum_{n=-\infty}^{\infty} x[n]z^{-n}$

Sequence	Transform
$\delta[n]$	1
$u[n]$	$\frac{1}{1-z^{-1}}$
$\delta[n - m]$	z^{-m}
$a^n u[n]$	$\frac{1}{1-az^{-1}}$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[r^n \cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2 z^{-2}}$
$[r^n \sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2 z^{-2}}$

Some z-transform properties:	
Sequence	Transform
$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
$x[n - m]$	$z^{-m}X(z)$

Some trigonometric identities:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

Discrete Fourier Transform:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{N}kn}$$

Complex number theory:

$$z = a + jb = r\angle\theta = re^{j\theta}$$

$$r = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

Quadratic equation solution:

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The characteristics of the different windowing functions:

Window Type	Peak approximation Error $20\log_{10}\delta$ dB	Transition Band $\Delta\omega$
Rectangular $w[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–21	$\frac{4\pi}{M+1}$
Bartlett $w[n] = \begin{cases} \frac{2n}{M} & 0 \le n \le \frac{M}{2} \\ 2 - \frac{2n}{M} & \frac{M}{2} \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–25	$\frac{8\pi}{M}$
Hanning $w[n] = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–44	$\frac{8\pi}{M}$
Hamming $w[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–53	$\frac{8\pi}{M}$
Blackman $w[n] = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{M}\right) + 0.08\cos\left(\frac{4\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–74	$\frac{12\pi}{M}$

The impulse response of an ideal low pass filter is: $h_d(n) = \dfrac{\sin(\omega_c(n - \frac{M}{2}))}{\pi(n - \frac{M}{2})}$