SINGAPORE POLYTECHNIC

2017/2018 SEMESTER TWO EXAMINATION

School of Architecture & the Built Environment
DCEB

School of Chemical and Life Sciences
DAPC, DCHE, DFST, DPCS

School of Digital Media & Infocomm Technology
DBIT, DDA, DISM, DIT

School of Electrical and Electronic Engineering
DASE, DESM, DCEP, DCPE, DEB, DEEE, DES, DCEP

School of Mechanical and Aeronautical Engineering
DARE, DCEP, DME, DMRO, DBEN

Singapore Maritime Academy (SMA)
DMR

ENGINEERING MATHEMATICS I Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **THREE** sections:

   Section A: 5 Multiple-Choice Questions (10 marks)
   Answer **ALL** questions.

   Section B: 7 Questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

   Section C: 3 Questions (40 marks)
   Answer **ALL** questions.

3. Unless otherwise stated, leave all answers correct to three significant figures.

4. Except for sketches, graphs and diagrams, no solution or answer is to be written in pencil.

5. This examination paper consists of **7** printed pages.

Section A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks. No mark will be deducted for incorrect answers.
Tick the choice of answer for each question in the box of the **MCQ answer sheet** provided in the answer booklet.

A1. Given $\boldsymbol{X} = \begin{pmatrix} -a & 0 \\ b & 2a \end{pmatrix}$ and $\boldsymbol{Y} = \begin{pmatrix} a & a \\ b & b \end{pmatrix}$, where a and b are non-zero constants.

Which of the following statements is **FALSE** ?

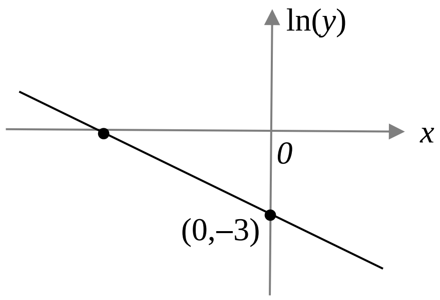
(a) $\boldsymbol{Y}$ is a singular matrix

(b) Adjoint of $\boldsymbol{Y}$ is $\begin{pmatrix} -a & b \\ a & -b \end{pmatrix}$

(c) $\boldsymbol{X}$ has an inverse

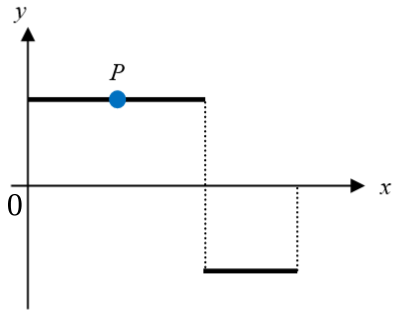
(d) Adjoint of $\boldsymbol{X}$ is $\begin{pmatrix} 2a & 0 \\ -b & -a \end{pmatrix}$

A2. The straight line graph shown below represents the equation $\ln(y) = \ln(c) - x\ln(3)$, where c is a constant. Determine the value of c .



(a) 1 (b) – 3 (c) e^{-3} (d) $\ln 3$

A3. Using the graph below and the fact that x is increasing with time, which of the following is true about the rate of change of y at point P ?



(a) $\dfrac{dy}{dt} > 0$ (b) $\dfrac{dy}{dt} < 0$ (c) $\dfrac{dy}{dt} = 0$ (d) $\dfrac{dy}{dt} = \dfrac{dx}{dt}$

A4. Given that $f(x)$ is a continuous function. Based on the information given below, which of the following is most likely a minimum point ?

Point	$f(x)$	$f'(x)$	$f''(x)$
A	0	1	0
B	1	0	–1
C	0	–1	0
D	1	0	1

(a) Point A (b) Point B (c) Point C (d) Point D

A5. If the area bounded by curve $y = f(x)$, x-axis, and the lines $x = 1$ and $x = a$ is equal to $\sqrt{a^3} - 1$ for all $a > 1$, then $y = f(x)$ is

(a) $\frac{3}{2}\sqrt{x}$ (b) $\sqrt{x^3}$ (c) $\sqrt{x}$ (d) $\frac{3}{2}\sqrt{x^3}$

Section B (50 marks)

Each question carries 10 marks.

The total mark of the questions in this section is 70 marks.

You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. The following equations show the volumes of three types of petrol that are blended to produce 20 litres of 97 octane petrol with an ethanol content of 4.5%.

$$\begin{aligned} A + B + C &= 20 \\ 100A + 95B + 90C &= 1935 \\ 0.05A + 0.10B &= 0.90 \end{aligned}$$

By using Cramer's Rule, find the value of A only.

(Detailed workings of evaluating a determinant should be clearly shown.)

B2. Given $\boldsymbol{A}=\begin{pmatrix}6 & -4\\ 0 & 6\end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix}-4 & 2\\ -1 & 2\end{pmatrix}$, $\boldsymbol{C}=\begin{pmatrix}0 & x\\ y & 1\end{pmatrix}$ and $\boldsymbol{D}=\begin{pmatrix}-1 & 0 & 7\\ 5 & 3 & 1\end{pmatrix}$.

(i) Evaluate $\frac{1}{2}\boldsymbol{A}-\boldsymbol{B}$.

(ii) Find $\boldsymbol{D}^T$ and hence evaluate $\boldsymbol{D}^T\boldsymbol{B}$.

(iii) Find $\boldsymbol{B}^{-1}$.

(iv) Find the values of x and y such that $\boldsymbol{AC}$ is a diagonal matrix.

B3. (a) Given $Z_1 = 1 + j4$, $Z_2 = 2\angle 45°$ and $Z_3 = 3\angle 40°$.
Evaluate the following and leave your answers in polar form.

(i) $Z_1 + Z_2$

(ii) $\dfrac{Z_1 + Z_2}{Z_3}$

(b) Find the real numbers x and y if $x - jy = (2 - j5)(3 + j2)$.

B4. A given mass of air expanding adiabatically is believed to follow a law of the form $P = kV^n$, where k and n are constants, P (cm of mercury) is the pressure and V (cm^3) is the volume. The following table gives corresponding values of P and V :

P	155	60	33	22
V	50	100	150	200

(i) By taking logarithm on both sides of the law $P = kV^n$, reduce the law to a linear form.

(ii) State the terms that should be plotted on the vertical and horizontal axes of a graph so that a best fit line can be drawn to show that P and V obey the law.

(iii) Hence compute and show the new values to be plotted on the horizontal and vertical axes in a table, correct to 4 significant figures. _Do not plot the values_.

(iv) Suppose the best fit line passes through the first and the third points of the new values, use these two points to find the value of n.

B5. Find $\frac{dy}{dx}$ for each of the following.

(a) $y = \ln(2x+1) + 3\sin^{-1}\left(4x^2\right)$

(b) $3x^2 - 4y^2 + 5 = 2x + e^{3y}$

B6. The population $P(t)$ of a certain bacteria is given as

$$P(t) = 1500\left(1 + \frac{5t}{t^2+30}\right)$$

where t is the time in hours .

(i) Find the rate of the population , $\frac{dP}{dt}$, at $t = 3$ hours.

(ii) Is the population increasing or decreasing at $t = 3$ hours ? Justify your answer.

(iii) Suppose the rate of growth becomes zero at $t = a$ hours. Determine a.

B7. (a) Find the following integrals:

(i) $\int\left(2x + \frac{7}{x} - 3\sec^2(2x)\right)\,dx$ (ii) $\int \frac{1}{x^2+8}\,dx$

(b) A certain type of herbal plant grows in such a way that its height *h(t)* in centimetres after t months is changing at the rate of

$$h'(t) = \frac{5}{2\sqrt{t}} - 0.6e^{-0.3t} \qquad \text{centimetres/month}$$

where $0 \le t \le 30$ months.

The plant was 5 centimetres tall when it was first planted. How tall will the plant be in 24 months?

Section C (40 marks)

Answer ALL **THREE** questions.

C1. (a) The combined impedance for a series circuit is given by

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

where ω, C, L and R are real numbers, $\omega > 0$.

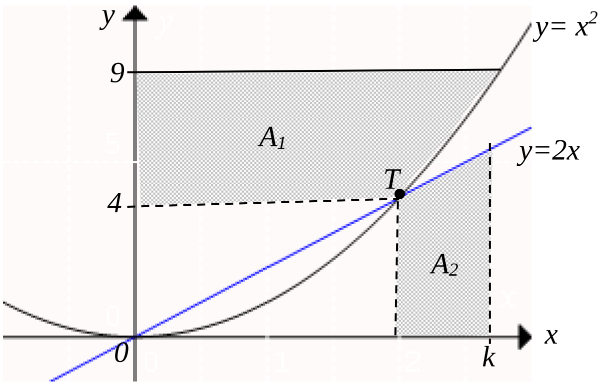
Find ω in terms of C and L for Z to be purely real. (5 marks)

(b) Let w and z be two complex numbers such that

$$|z|^2 = 1 \text{ and } w = \frac{1}{1-z}$$

By letting $z = x + jy$ and knowing that $z \neq 1$, determine the real part of w. (7 marks)

C2.



The figure above shows the graphs of $y = x^2$ and $y = 2x$.

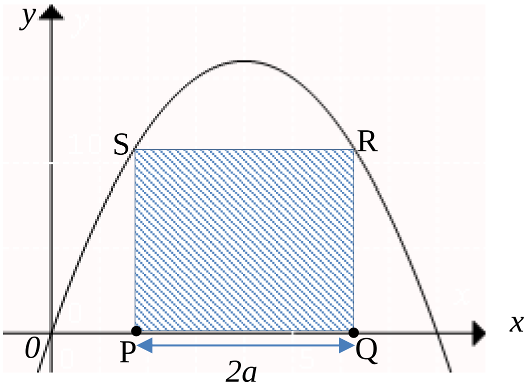
(i) Find the value of x at the intersection point T. (2 marks)

(ii) Given the area of the shaded region is

$$A_1 + A_2 = \frac{53}{3} \quad \text{square units}$$

Find the value of k. (13marks)

C3. The diagram below shows a rectangle PQRS enclosed within the curve $y = 8x - x^2$ and the x-axis.



(i) If length PQ = $2a$, show that the area A of the rectangle PQRS is

$$A = 32a - 2a^3$$

(6 marks)

(ii) Find the value of a for which A is a maximum. (7 marks)

~~~ END OF PAPER ~~~

No.	SOLUTION
A	A1) **b** A2) **c** A3) **c** A4) **d** A5) **a**
B1	$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 100 & 95 & 90 \\ 0.05 & 0.10 & 0 \end{vmatrix} = 0.05\begin{vmatrix} 1 & 1 \\ 95 & 90 \end{vmatrix} - 0.10\begin{vmatrix} 1 & 1 \\ 100 & 90 \end{vmatrix} = -0.25 + 1 = 0.75$ $\Delta_A = \begin{vmatrix} 20 & 1 & 1 \\ 1935 & 95 & 90 \\ 0.90 & 0.10 & 0 \end{vmatrix} = 0.90\begin{vmatrix} 1 & 1 \\ 95 & 90 \end{vmatrix} - 0.10\begin{vmatrix} 20 & 1 \\ 1935 & 90 \end{vmatrix} = -4.5 + 13. = 9$ $A = \frac{\Delta_A}{\Delta} = \frac{9}{0.75} = 12$
B2 (i)	$\frac{1}{2}\begin{pmatrix} 6 & -4 \\ 0 & 6 \end{pmatrix} - \begin{pmatrix} -4 & 2 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 0 & 3 \end{pmatrix} - \begin{pmatrix} -4 & 2 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & -4 \\ 1 & 1 \end{pmatrix}$
(ii)	$\boldsymbol{D}^T = \begin{pmatrix} -1 & 5 \\ 0 & 3 \\ 7 & 1 \end{pmatrix}$ $\therefore \boldsymbol{D}^T\boldsymbol{B} = \begin{pmatrix} -1 & 5 \\ 0 & 3 \\ 7 & 1 \end{pmatrix}\begin{pmatrix} -4 & 2 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} -1 & -2+10 \\ 0-3 & 0+6 \\ -28-1 & 14+2 \end{pmatrix} = \begin{pmatrix} -1 & 8 \\ -3 & 6 \\ -29 & 16 \end{pmatrix}$
(iii)	$\boldsymbol{B}^{-1} = \frac{1}{-8+2}\begin{pmatrix} 2 & -2 \\ 1 & -4 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ -\frac{1}{6} & \frac{2}{3} \end{pmatrix}$
(iv)	$\boldsymbol{AC} = \begin{pmatrix} 6 & -4 \\ 0 & 6 \end{pmatrix}\begin{pmatrix} 0 & x \\ y & 1 \end{pmatrix} = \begin{pmatrix} -4y & 6x-4 \\ 6y & 6 \end{pmatrix}$ Since $\boldsymbol{AC}$ is a diagonal matrix, $6x-4=0$ and $6y=0$ $\Rightarrow x = \frac{2}{3}$, $y=0$

<table>
<tr><th>No.</th><th>SOLUTION</th></tr>
<tr><td>B3(a)
(i)</td><td>$Z_1 + Z_2 = 1 + j4 + (1.414 + j1.414)$
$= 2.414 + j5.414$
$= 5.928\angle 65.97^o$</td></tr>
<tr><td>(ii)</td><td>$\dfrac{Z_1 + Z_2}{Z_3} = \dfrac{5.928\angle 65.97^o}{3\angle 40^\circ} = 1.98\angle 25.97^o$</td></tr>
<tr><td>B3
(b)</td><td>$x - jy = (2 - j5)(3 + j2)$
$= 6 + j4 - j15 - j^2 10$
$x - jy = 16 - j11$
$\therefore x = 16, \quad y = 11$</td></tr>
<tr><td>B4</td><td>$(i) \quad P = kV^n$
$\log P = \log\left(kV^n\right)$
$\log P = \log(k) + n\log(V)$

(ii) Vertical axis is $\log P$
Horizontal axis is $\log V$

(iii)
<table><tr><td>logP</td><td>2.191</td><td>1.778</td><td>1.519</td><td>1.342</td></tr><tr><td>logV</td><td>1.699</td><td>2.000</td><td>2.176</td><td>2.301</td></tr></table>
$(iv) \quad n = \dfrac{1.519 - 2.191}{2.176 - 1.699}$
$= \dfrac{-0.672}{0.477} = -1.409$</td></tr>
<tr><td>B5
(a)</td><td>$y = \ln(2x+1) + 3\sin^{-1}\left(4x^2\right)$
$\dfrac{dy}{dx} = \dfrac{1}{2x+1} \times 2 + \dfrac{3}{\sqrt{1-16x^4}} \times 8x = \dfrac{2}{2x+1} + \dfrac{24x}{\sqrt{1-16x^4}}$</td></tr>
<tr><td>B5
(b)</td><td>$3x^2 - 4y^2 + 5 = 2x + e^{3y}$
$6x - 8y\dfrac{dy}{dx} + 0 = 2 + e^{3y}\left(3\dfrac{dy}{dx}\right)$
$\left(8y + 3e^{3y}\right)\dfrac{dy}{dx} = 6x - 2$
$\dfrac{dy}{dx} = \dfrac{6x - 2}{8y + 3e^{3y}}$</td></tr>
</table>

No.	SOLUTION
B6 (i)	$P(t)=1500+\frac{7500t}{t^2+30}$ $P'(t)=0+\frac{(t^2+30).7500-7500t.2t}{(t^2+30)^2}$ $=\frac{7500\left[(t^2+30)-2t^2\right]}{(t^2+30)^2}=\frac{7500(30-t^2)}{(t^2+30)^2}$ $P'(3)=\frac{7500(30-3^2)}{(3^2+30)^2}\approx 104$ bacteria/hr
(ii)	Increasing since $P'(3)>0$
(iii)	When $P'(t)=0$, $\frac{7500(30-t^2)}{(t^2+30)^2}=0$ $\Rightarrow 30-t^2=0$ $\Rightarrow t=\sqrt{30}\approx 5.48$ hrs $(t>0)$ i.e. $a\approx 5.48$ hrs
B7 (a)i	$\int\left(2x+\frac{7}{x}-3\sec^2 2x\right)\,dx=2\left(\frac{x^2}{2}\right)+7\ln\lvert x\rvert-\frac{3}{2}\tan(2x)+C$
B7 (a)ii	$\int\frac{1}{x^2+8}\,dx=\frac{1}{\sqrt{8}}\tan^{-1}\left(\frac{x}{\sqrt{8}}\right)+C$
B7 (b)	$h'(t)=\frac{5}{2\sqrt{t}}-0.6e^{-0.3t}$
	$h(t)=\int\left(\frac{5}{2\sqrt{t}}-0.6e^{-0.3t}\right)dt$
	$h(t)=5t^{\frac{1}{2}}+2e^{-0.3t}+C$
	when t = 0 , $C=3$
	$h(24)=5(24)^{\frac{1}{2}}+2e^{-(0.3)(24)}+3=27.50$ cm.

No.	**SOLUTION**
C1 (a)	$Z = R + j\omega L + \frac{1}{j\omega C}\left(\frac{j}{j}\right) \quad = R + j\omega L + \frac{j}{j^2\omega C}$ $= R + j\left(\omega L - \frac{1}{\omega C}\right)$ For Z to be purely real, Im(Z) = 0 $\omega L - \frac{1}{\omega C} = 0$ $\omega L = \frac{1}{\omega C}$ $\omega^2 = \frac{1}{LC}$ $\omega = \frac{1}{\sqrt{LC}}$ (rej negative value)
C1 (b)	$w = \frac{1}{1-(x+jy)}$ $= \frac{1}{(1-x)-jy}\left(\frac{(1-x)+jy}{(1-x)+jy}\right)$ $= \frac{(1-x)+jy}{(1-x)^2+y^2}$ $= \frac{(1-x)+jy}{1-2x+x^2+y^2}$ $= \frac{(1-x)+jy}{1-2x+1}$ $= \frac{(1-x)+jy}{2(1-x)}$ $= \frac{(1-x)}{2(1-x)} + \frac{jy}{2(1-x)}$ $= \frac{1}{2} + \frac{jy}{2(1-x)}$ Re(w)= 1/2

No.	SOLUTION
C2 (i)	At point $T, y = 4$ $2x = 4$ $x = 2$ OR $x^2 = 2x$ $x^2 - 2x = 0$ $x(x-2) = 0$ $x = 2 \quad or \quad 0$ (reject)
C2 (ii)	$A_1 = \int_4^9 x\, dy = \int_4^9 \sqrt{y}\, dy = \left[\frac{2}{3} y^{\frac{3}{2}}\right]_4^9$ $= \frac{2}{3}\left(\left[(9)^{\frac{3}{2}}\right] - \left[(4)^{\frac{3}{2}}\right]\right) = \frac{38}{3}$
	$A_2 = \int_2^k y\, dx = \int_2^k 2x\, dx = \left[\frac{2}{2} x^2\right]_2^k$ $= \left[(k)^2\right] - \left[(2)^2\right]$ $= k^2 - 4$
	$A_1 + A_2 = \frac{53}{3}$ $\frac{38}{3} + k^2 - 4 = \frac{53}{3}$ $k^2 = \frac{53}{3} - \frac{26}{3}$ $k = 3$ or -3 (rej)
C3 (i)	The curve cuts the x-axis at : $8x - x^2 = 0$ $x(8-x) = 0$ $x = 0\ ,\ 8$ Mid-point of curve is at $x=4$, Point P is at $x=4-a$ At $x=4-a$, $y = 8(4-a) - (4-a)^2$ $=16 - a^2$ $\therefore$ length of rectangle = $2a$ Breadth of rectangle = $16 - a^2$ Area , A = $(2a)(16 - a^2) = 32a - 2a^3$
C3 (ii)	$\frac{dA}{da} = 32 - 6a^2$ Let $32 - 6a^2 = 0$ $a = \frac{4}{\sqrt{3}}$ or $-\frac{4}{\sqrt{3}}$ (rej) $\frac{d^2A}{da^2} = -12a < 0$ Therefore, area of rectangle is maximum at $a = \frac{4}{\sqrt{3}}$