2015/2016 SEMESTER ONE EXAMINATION

Diploma in Electrical and Electronic Engineering
3rd Year Full Time
3rd Year Full Time Technical Elective
5th Year Evening Only

DIGITAL SIGNAL PROCESSING

Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules as set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **TWO** sections:

    Section A - 6 Short Questions, 10 marks each.
    Section B - 2 Long Questions, 20 marks each.

3. **ALL** questions are **COMPULSORY**.

4. **ALL** questions are to be answered in the answer booklet.

5. This paper consists of **6** pages, including 2 pages of mathematical formulae.

SECTION A - SHORT QUESTIONS [10 marks each]

A1 The system function of a digital system is given as:

$$H(z) = \frac{6-9z^{-1}}{1-2.5z^{-1}+z^{-2}}$$

a) Obtain its difference equation. (4 marks)

b) Determine the output y(n) of the system if the input is a unit impulse function, δ(n). (6 marks)

A2 Evaluate *N*=4 point DFT for X(0) and X(2) for the sequence x(n) ={1,1,2,2}. (10 marks)

A3 A system has an impulse response h(n) ={1,2,-1,-2} and output response,

y(n) ={1,4,6,6,1,-10,-8} when subjected to an unknown input x(n). Find the Z-transform of h(n) and y(n) and hence solve for the input sequence x(n) using long division method. (10 marks)

A4 Given the following DSP system with a sampling rate 8000 Hz

y(n) = 3x(n) + x(n-1)

where $y(n)$ is the output and $x(n)$ is the input,

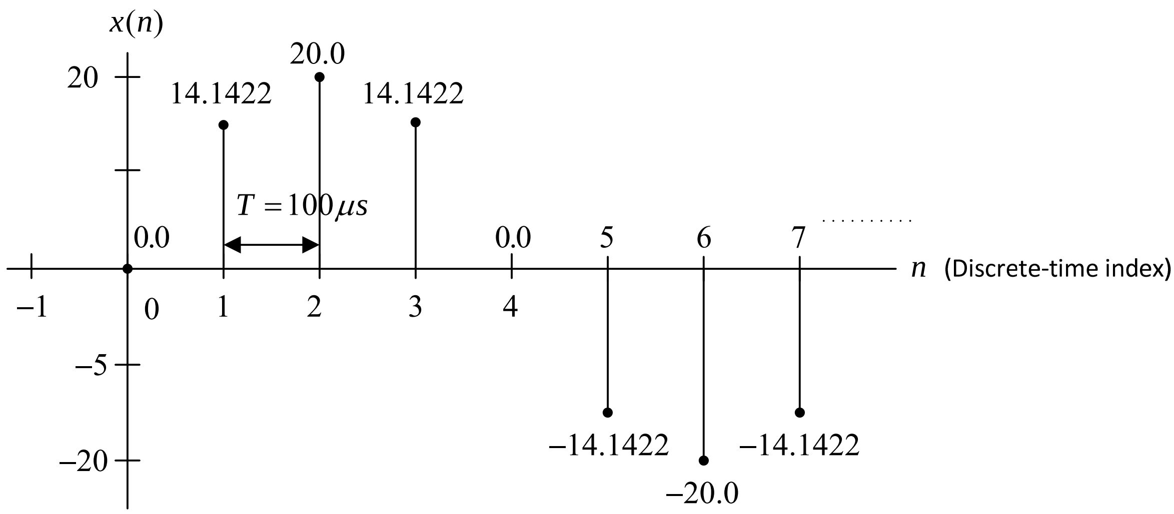
a) Determine the transfer function, H(z). (3 marks)

b) Determine the filter frequency response $|H(e^{jw})|$. (3 marks)

c) Compute the filter gain at a frequency of 2000 Hz. (4 marks)

A5 Answer the following short questions. Please note that the questions are not related to each other.

a) Given the following labelled digital signals,



(i) What is the sampling frequency and the value, x(2)? (4 marks)

(ii) If $x(n)$ is the sampled version of the analog sine wave: $x(t) = 20\sin(\omega t)$, what is the frequency of the analog sine wave? (2 marks)

b) Determine the autocorrelation of x(n) = {1,2,3,4}. (4 marks)

A6 Find the inverse z transform of the following casual signals

a) $X_1(z) = 1 + \dfrac{2z}{z-1} + \dfrac{5z}{(z-1)^2} + z^{-10}$ (5 marks)

b) $X_2(z) = \dfrac{10(z-0.7071)}{z^2 - 1.4142z + 1}$ (5 marks)

SECTION B - LONG QUESTIONS [20 marks each]

B1. A linear phase low-pass filter is to be designed using the FIR technique. The filter has the following specification where peak approximation error (δ) = 0.015 and $\omega_{pass} = 0.514\pi$, $\omega_{stop} = 0.714\pi$. The sampling frequency of the system is given as 14kHz.

To strictly meet the specification, determine

a) the windowing function that you would choose. (2 marks)
b) the values for ω_c and $\Delta\omega$. (4 marks)
c) the number of tap coefficients that you would need. (4 marks)
d) the value of tap coefficient for h(16) and h(17). (6 marks)
e) the value of tap coefficient for h(20). (2 marks)
f) the type of windowing function you will choose if peak approximation error (δ) = 0.05. (2 marks)

B2. The difference equation of system A in Figure B2a is given as:

$$y(n) - 0.5y(n-1) + y(n-2) = x(n) - 0.25x(n-1)$$

a) Draw the digital network diagram for the system. (5 marks)
b) Is this system a FIR or IIR system? (2 marks)
c) State One advantage for FIR and One advantage for IIR filter. (2 marks)
d) Using Z transform, determine the system function H(z) for system A. (4 marks)
e) Using inverse Z transform, determine the impulse response. (4 marks)

f) The output of system A is now fed as input to an additional system named system B as shown in Figure B2.

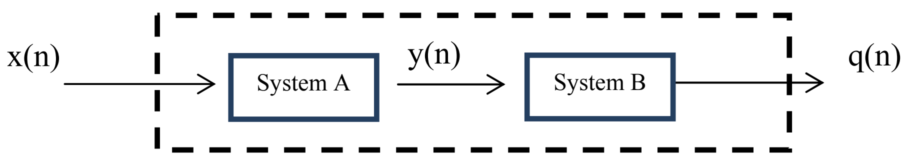


Figure B2

Given the difference equation of system B to be q(n) = 0.5y(n) + 0.25y(n-1), determine the new system function for this cascaded system. (3 marks)

-End of Paper-

Appendix

The z-transform is defined as X(z) = $\sum_{n=-\infty}^{\infty} x[n]z^{-n}$

Sequence	Transform
$\delta[n]$	1
$u[n]$	$\frac{1}{1-z^{-1}}$
$\delta[n-m]$	z^{-m}
$a^n u[n]$	$\frac{1}{1-az^{-1}}$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$
$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$

Some z-transform properties:	
Sequence	Transform
$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
$x[n-m]$	$z^{-m}X(z)$

Some trigonometric identities:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

Discrete Fourier Transform:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{N}kn}$$

Complex number theory:

$z = a + jb = r\angle\theta = re^{j\theta}$

$r = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}\dfrac{b}{a}$

Quadratic equation solution:

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The characteristics of the different windowing functions:

Window Type	Peak approximation Error $20\log_{10}\delta$ dB	Transition Band $\Delta\omega$
Rectangular $w[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–21	$\dfrac{4\pi}{M+1}$
Bartlett $w[n] = \begin{cases} \dfrac{2n}{M} & 0 \le n \le \dfrac{M}{2} \\ 2 - \dfrac{2n}{M} & \dfrac{M}{2} \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–25	$\dfrac{8\pi}{M}$
Hanning $w[n] = \begin{cases} 0.5 - 0.5\cos\left(\dfrac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–44	$\dfrac{8\pi}{M}$
Hamming $w[n] = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–53	$\dfrac{8\pi}{M}$
Blackman $w[n] = \begin{cases} 0.42 - 0.5\cos\left(\dfrac{2\pi n}{M}\right) + 0.08\cos\left(\dfrac{4\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	–74	$\dfrac{12\pi}{M}$

The impulse response of an ideal low pass filter is: $h_d(n) = \dfrac{\sin(\omega_c(n - \frac{M}{2}))}{\pi(n - \frac{M}{2})}$