Sample Set 1 SEMESTER TEST

Section A

A1. Evaluate the $N = 4$-point DFT for $X(0)$ and $X(1)$ if $x(n) = \{0, 1, 1, 2\}$.

A2 Given y(n)={ 2, -4, 5, -3, 1} and impulse response h(n)={1,-1,1}, find the z-transform of y(n) and h(n), hence determine the input x(n) by using the long-division method for the 1st 2 terms.

A3 Find the z-transform of $x(n)=20\sin(0.25\pi n)u(n)$ and $y(n)= e^{-0.2n}\sin(0.3\pi n)u(n)$.

A4 A linear time invariant system's response to a unit step function is given as $y(n) = e^{-n}u(n)$. Determine the impulse response h(n) if this system and calculate the values of h(0), h(1), and h(2).

A5 A square wave having period T = 1 ms, is filtered by an ideal low pass filter having cutoff frequency f_c = 2 kHz.

(a) What are the frequency components at the output of the low pass filter?

(b) Sketch the spectrum at the output of the filter from -4 kHz to 4 kHz

(c) If it is sampled by 10 kHz, sketched the sampled signal spectrum from -10 kHz to 10 kHz.

A6 The difference equation of a particular digital network is given as:

$$y(n) = x(n) + 3\,y(n - 1)$$

(a) Find the z-transform of the transfer function, H(z).
(b) Find the impulse response of this filter, h(n).
(c) Find the values of pole and zero of this filter. Is the system stable?

SECTION B

B1 A transmitting source generates six different symbols A, B, C, D, E, and F. Each of the symbols occurs with a relative frequency of occurrence of 0.4, 0.2, 0.1, 0.1, 0.1, and 0.1 respectively and their associating set of code-words as below:

A = 11, B = 00, C = 101, D = 100, E = 011, F = 010

Compute:

(a) The information content conveyed by symbols A and B.
(b) Source entropy;
(c) The minimum number of bits required assuming fixed-length code-words and

(d) The average bit length for the code-word set.

B2 (a) Given the original symbols sequence: 11112223333311112222

(i) Express its code pairs using run-length coding (RLC);
(ii) Derive the bit stream if 3 bits are used in coding each value of the code pairs;
(iii) Calculate the total number of bits of the bit stream.

(b) The source of information A generates the symbols {a, b and c} with the corresponding probabilities {0.7, 0.2 and 0.1}. Use Arithmetic Coding technique to generate a binary representation for message “abcc”.

Appendix

The z-transform is defined as X(z) = $\sum_{n=-\infty}^{\infty} x[n]z^{-n}$

Sequence	Transform
$\delta[n]$	1
$u[n]$	$\frac{1}{1-z^{-1}}$
$\delta[n - m]$	z^{-m}
$a^n u[n]$	$\frac{1}{1-az^{-1}}$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$
$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$

Some z-transform properties:	
Sequence	Transform
$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
$x[n - m]$	$z^{-m}X(z)$

Source entropy:

$$H = \sum_{i=1}^{m} P_i \log_2\left(\frac{1}{P_i}\right)$$

Average code length:

$$\overline{n} = \sum_{i=1}^{m} n_i P(X_i)$$

Information content per symbol:

$$I(S_i) = \log_2\left(\frac{1}{P_i}\right)$$

Discrete Fourier Transform, DFT:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}$$