SOLUTIONS

SINGAPORE POLYTECHNIC

2018 / 2019 Semester 2 Exam

Module Name: **Engineering Mathematics II**

Module Code: **MS2216/MS4216/MS6216**

No.	SOLUTION	TOTAL MARKS
A	c, c, d, d, b	10
B1	$f(x,y)=5y^2-e^{xy}+\ln x$ $\frac{\partial f}{\partial x}=0+\left(-e^{xy}\right)y+\frac{1}{x}=-ye^{xy}+\frac{1}{x}$ At (1, 0) $\frac{\partial f}{\partial x}=-0\left(e^0\right)+\frac{1}{1}=1$ $\frac{\partial f}{\partial y}=5(2y)-\left(e^{xy}\right)x+0=10y-xe^{xy}$ At (1, 0) $\frac{\partial f}{\partial y}=10\left(0\right)-1\left(e^0\right)=-1$	10
B2a	$\int\left(4\cos 5t\sin 2t-2\cos^2 3t\right)dt$ $=\int\left(2\left[\sin 7t-\sin 3t\right]-\left[1+\cos 6t\right]\right)dt$ $=2\left(-\frac{1}{7}\cos 7t+\frac{1}{3}\cos 3t\right)-\left(t+\frac{1}{6}\sin 6t\right)+C$ $=-\frac{2}{7}\cos 7t+\frac{2}{3}\cos 3t-t-\frac{1}{6}\sin 6t+C$ (optional)	10
B2b	$\int_1^2\frac{3}{(5x-2)^2}dx=\int_1^2 3(5x-2)^{-2}dx=\left[-\frac{3}{5}(5x-2)^{-1}\right]_1^2$ $=-\frac{3}{5}\left[(5\left(2\right)-2)^{-1}-(5\left(1\right)-2)^{-1}\right]=\frac{1}{8}$ or 0.125 (0.13 to 2 dp)	
B3	$h=\frac{1.5-0}{6}=0.25$ $\begin{array}{c c c c c c c c} \hline x & 0 & 0.25 & 0.5 & 0.75 & 1 & 1.25 & 1.5 \\ \hline y=e^{x^2+1} & 2.7183 & 2.8936 & 3.4903 & 4.7707 & 7.3891 & 12.9682 & 25.7903 \\ \hline \end{array}$ $\int_0^{1.5}e^{x^2+1}dx\approx\frac{1}{3}\left(0.25\right)[2.7183+25.7903+4(2.8936+4.7707+12.9682)+2(3.4903+7.3891)]$ ≈ 11.07	10

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B4	$\frac{dy}{dx}=x+2y \to \frac{dy}{dx}-2y=x \to P(x)=-2, \quad Q(x)=x$ $\mu(x)=e^{\int -2\,dx}=e^{-2x}$ $\int \mu(x)Q(x)dx=\int e^{-2x}x\,dx$ $=-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}+C$ $\begin{array}{ccc} u & & dv \\ x & \overset{+}{\searrow} & e^{-2x} \\ 1 & \overset{-}{\searrow} & -\frac{1}{2}e^{-2x} \\ 0 & & \frac{1}{4}e^{-2x} \end{array}$ $\therefore ye^{-2x}=-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}+C$ thus $y=-\frac{1}{2}x-\frac{1}{4}+Ce^{2x}$	10
B5a	$\mathcal{L}\left\{\pi-5t^2+3t\sin 2t\right\}=\frac{\pi}{s}-5\cdot\frac{2!}{s^{2+1}}+3\cdot\frac{2(2)s}{(s^2+2^2)^2}$ $=\frac{\pi}{s}-\frac{10}{s^3}+\frac{12s}{(s^2+4)^2}$	10
B5b	$\mathcal{L}\left\{4e^{-3t}+9e^{2t}\cos\pi t\right\}=\frac{4}{s-(-3)}+9\cdot\left.\frac{s}{s^2+\pi^2}\right\|_{s\to s-2}$ $=\frac{4}{s+3}+\frac{9(s-2)}{(s-2)^2+\pi^2}$	
B6a	$\mathcal{L}^{-1}\left\{\frac{1}{3s}+\frac{2}{s^4}-\frac{2}{5(s+1)}\right\}=\mathcal{L}^{-1}\left\{\frac{1}{3}\cdot\frac{1}{s}+\frac{1}{3!}\cdot\frac{2(3!)}{s^{3+1}}-\frac{2}{5}\cdot\frac{1}{s-(-1)}\right\}$ $=\frac{1}{3}+\frac{1}{3}t^3-\frac{2}{5}e^{-t}$	10
B6b	$\mathcal{L}^{-1}\left\{\frac{s^2-3}{\left(s^2+3\right)^2}-\frac{s-6}{(s-1)^2+25}\right\}=\mathcal{L}^{-1}\left\{\frac{s^2-(\sqrt{3})^2}{\left(s^2+(\sqrt{3})^2\right)^2}-\frac{(s-1)-5}{(s-1)^2+25}\right\}$ $=t\cos\sqrt{3}t-e^t\mathcal{L}^{-1}\left\{\frac{s-5}{s^2+5^2}\right\} \quad =t\cos\sqrt{3}t-e^t(\cos 5t-\sin 5t)$	

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B7	(a) $y''-4y'-5y=0$ Aux. equation is: $\lambda^2-4\lambda-5=0$ $(\lambda-5)(\lambda+1)=0\rightarrow\lambda=5,-1$ $\therefore$ the general solution is: $y=Ae^{5t}+Be^{-t}$ (b) $y=Ae^{5t}+Be^{-t}$ → $y'=5Ae^{5t}-Be^{-t}$ given $y(0)=1$, i.e. $1=A+B$ --- (1) given $y'(0)=2$, i.e. $2=5A-B$ -- (2) hence $A=\frac{1}{2}$, and $B=\frac{1}{2}$ Thus the particular solution is: $y=\frac{1}{2}\left[e^{5t}+e^{-t}\right]$	10
C1	$P=\frac{10R_1}{(R_1+R_2)^2}$, $\frac{\Delta R_1}{R_1}\times 100\%=2\%$, $\frac{\Delta R_2}{R_2}\times 100\%=-3\%$ $\frac{\partial P}{\partial R_1}=\frac{10(R_1+R_2)^2-10R_1\cdot 2(R_1+R_2)}{(R_1+R_2)^4}=\frac{10(R_2-R_1)}{(R_1+R_2)^3}$ $\frac{\partial P}{\partial R_2}=10R_1\cdot(-2)(R_1+R_2)^{-3}=\frac{-20R_1}{(R_1+R_2)^3}$ $\Delta P\approx\frac{\partial P}{\partial R_1}\cdot\Delta R_1+\frac{\partial P}{\partial R_2}\cdot\Delta R_2$ $\frac{\Delta P}{P}\times 100\%\approx\frac{\partial P}{\partial R_1}\cdot\frac{\Delta R_1}{P}\times 100\%+\frac{\partial P}{\partial R_2}\cdot\frac{\Delta R_2}{P}\times 100\%$ $\approx\frac{R_2-R_1}{R_1+R_2}\cdot\frac{\Delta R_1}{R_1}\times 100\%-\frac{2R_2}{R_1+R_2}\cdot\frac{\Delta R_2}{R_2}\times 100\%$ $\approx\frac{1000}{3000}\cdot 2\%-\frac{4000}{3000}\cdot(-3\%)=\frac{14}{3}\%\approx 4.67\%$, i.e. P will increase by approximately 4.67%.	12

No.	SOLUTION	TOTAL MARKS
	Alternate solution: $\ln P = \ln 10 + \ln R_1 - 2\ln(R_1 + R_2)$, $\dfrac{\Delta R_1}{R_1} = \dfrac{2}{100}$, $\dfrac{\Delta R_2}{R_2} = -\dfrac{3}{100}$ $\dfrac{\partial}{\partial R_1}(\ln P) = \dfrac{1}{R_1} - \dfrac{2}{R_1 + R_2}$, $\dfrac{\partial}{\partial R_2}(\ln P) = -\dfrac{2}{R_1 + R_2}$ $\Delta(\ln P) = \dfrac{\Delta P}{P} \approx \dfrac{\partial}{\partial R_1}(\ln P)\Delta R_1 + \dfrac{\partial}{\partial R_2}(\ln P)\Delta R_2$ $= \left(\dfrac{1}{R_1} - \dfrac{2}{R_1 + R_2}\right)\Delta R_1 - \dfrac{2}{R_1 + R_2}\Delta R_2$ $= \dfrac{\Delta R_1}{R_1} - \dfrac{2R_1}{R_1 + R_2}\dfrac{\Delta R_1}{R_1} - \dfrac{2R_2}{R_1 + R_2}\dfrac{\Delta R_2}{R_2}$ $= \left(1 - \dfrac{2}{3}\right)\dfrac{2}{100} - \dfrac{4}{3}\left(-\dfrac{3}{100}\right) = \dfrac{2}{300} + \dfrac{12}{300} = \dfrac{14}{300}$ $\dfrac{\Delta P}{P} \times 100\% \approx \dfrac{14}{300} \times 100\% = \dfrac{14}{3}\% \approx 4.67\%$	**(12)**
C2	(a) $\dfrac{dT}{dt} = k(T - 35)$ We know that: $T(0) = 20$, $T(2) = 27$ $\displaystyle\int \frac{dT}{T - 35} = \int k\,dt$ $\rightarrow$ $\ln\lvert T - 35\rvert = kt + C \rightarrow T(t) = 35 + e^{kt+C} = 35 + Ae^{kt}$ $T(0) = 20 = 35 + A \rightarrow A = -15 \rightarrow T(t) = 35 - 15e^{kt}$ $T(2) = 27 = 35 - 15e^{2k} \rightarrow e^{2k} = 8 \rightarrow k = \dfrac{1}{2}\ln\left(\dfrac{8}{15}\right) = -0.314$ $\therefore T(t) = 35 - 15e^{-0.314t}$ $T(1) = 35 - 15e^{-0.314} = 24.04°\text{C}$	

No.	SOLUTION	TOTAL MARKS
	(b) Let $T(t_1) = 32\,^\circ\text{C}$: $32 = 35 - 15e^{-0.314t_1} \to e^{-0.314t_1} = 0.2 \to t_1 = 5.13 \text{ min}$ i.e. it takes about 5.13 min for the thermometer to reach $32\,^\circ\text{C}$	12
C3a	$\frac{34s+68}{(s^2+4s+20)(s^2+16)} = \frac{s+A}{2(s^2+4s+20)} + \frac{Bs+9}{s^2+16}$ $34s+68 = \frac{1}{2}(s+A)(s^2+16) + (Bs+9)(s^2+4s+20)$ s^3 term: $0 = \frac{1}{2} + B \rightarrow B = -\frac{1}{2}$ const: $68 = 8A + 180 \rightarrow A = -14$ $\therefore \frac{34s+68}{(s^2+4s+20)(s^2+16)} = \frac{s-14}{2(s^2+4s+20)} + \frac{-\frac{1}{2}s+9}{s^2+16}$	16
C3b	(i) $m = 2$ kg, $c = 0$, $k = 32$ N/m, and $F(t) = 68e^{-2t}\cos 4t$ $2x''(t) + 32x(t) = 68e^{-2t}\cos 4t$ $x(0) = x'(0) = 0$	
	(ii) Take Laplace transform on both sides of the equation Let $X = \mathcal{L}\{x(t)\}$ $\left[s^2X - sx(0) - x'(0)\right] + 16X = \frac{34(s+2)}{(s+2)^2+4^2}$ $(s^2+16)X = \frac{34s+68}{s^2+4s+20} \rightarrow X = \frac{34s+68}{(s^2+4s+20)(s^2+16)}$ From (a): $X = \frac{34s+68}{(s^2+4s+20)(s^2+16)} = \frac{s-14}{2(s^2+4s+20)} + \frac{-\frac{1}{2}s+9}{s^2+16}$ $= \frac{(s+2)-16}{2\left[(s+2)^2+16\right]} - \frac{1}{2}\left(\frac{s-18}{s^2+16}\right)$ $\therefore x(t) = \mathcal{L}^{-1}\{X\}$ $= \frac{1}{2}e^{-2t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4^2} - 4\cdot\frac{4}{s^2+4^2}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+4^2} - \frac{9}{2}\frac{4}{s^2+4^2}\right\}$ $= \frac{1}{2}e^{-2t}\left(\cos 4t - 4\sin 4t\right) - \frac{1}{2}\cos 4t + \frac{9}{4}\sin 4t$	