Mid-Semester Test

Time allowed : 1 hour

Instructions

Answer all **4** questions. Each question carries **25 marks**. Take g = 9.80 m/s^2

This question paper consists of **2** pages. You can use the A4 handwritten formula sheet compiled by you.

You are reminded that cheating during test is a serious offence.

All working in support of your answer must be shown. Answers must be to appropriate significant figures.

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1. a) The acceleration a of an object is given by $ar^z = m^x v^y$ where m, v and r are mass, velocity and distance respectively. Using dimensional analysis, determine the values of x, y, and z and hence, state the formula for the acceleration.

   b) Given a magnetic field vector $\boldsymbol{B} = (3.0 \times 10^{-3}\ \mathbf{i} + 4.0 \times 10^{-3}\ \mathbf{j})$ T and an area vector $\boldsymbol{A} = (7.0\ \mathbf{i} + 24\ \mathbf{j})$ m^2. Find the dot product $\boldsymbol{B} \cdot \boldsymbol{A}$ and the angle between the two vectors.

a) $a = \dfrac{m^x v^y}{r^z}$

The dimension of a is $\dfrac{L}{T^2}$ and that of v is $\dfrac{L}{T}$

The dimension of $\dfrac{m^x v^y}{r^z}$ is $\dfrac{M^x L^y T^{-y}}{L^z}$

$$\frac{L}{T^2} = \frac{M^x L^y T^{-y}}{L^z} = \frac{M^x L^{y-z}}{T^y}$$

Equating the exponents we get

$x = 0, y = 2$

$y - z = 1,$

$z = 1$

hence $a = \dfrac{v^2}{r}$

b) $\vec{B} \cdot \vec{A} = (21 + 96) \times 10^{-3} = 117 \times 10^{-3}$ T m^2

$$\cos\theta = \frac{\vec{B} \cdot \vec{A}}{|\vec{B}||\vec{A}|} = \frac{117 \times 10^{-3}}{5 \times 10^{-3} \times 25} = 0.936$$

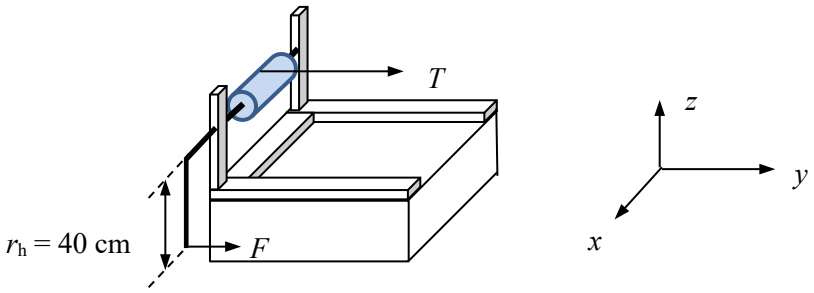
$\theta = \cos^{-1}(0.936) = 20.6^{\circ}$

2. (a) A particle has radius vector $\boldsymbol{r}(t) = (2.0t^3\ \mathbf{i} + 1.0t\ \mathbf{j} + 6.0\ \mathbf{k})$ m. At $t = 2.0$ s, find the displacement vector $\boldsymbol{r}$ and the instantaneous velocity vector $\boldsymbol{v}$.

(b) The diagram below shows a winch. The cylinder has radius $r_a = 10$ cm (not shown) and the handle has length $r_h = 40$ cm. T is 400 N and F is 100 N. When the handle is in the position shown (pointing downward) and taking $\boldsymbol{r}_a$ to be along the positive z-axis.

i) write the vectors, $\boldsymbol{r}_a$, $\boldsymbol{r}_h$, $\boldsymbol{T}$ and $\boldsymbol{F}$ in terms of **i, j** and **k** in SI units.
ii) find the cross products $\boldsymbol{r}_a \times \boldsymbol{T}$ and $\boldsymbol{r}_h \times \boldsymbol{F}$.

(c) What is the sum of the vectors $\boldsymbol{r}_a \times \boldsymbol{T}$ and $\boldsymbol{r}_h \times \boldsymbol{F}$?



(a) $\boldsymbol{r}(2.0) = 16.0\ \mathbf{i} + 2.0\ \mathbf{j} + 6.0\ \mathbf{k}$ m.

$\boldsymbol{v}(t) = 6.0t^2\ \mathbf{i} + 1.0\ \mathbf{j}$ m/s
$\boldsymbol{v}(2.0) = 24.0\ \mathbf{i} + 1.0\ \mathbf{j}$ m/s

(b) i) $\vec{r}_a = 0.10\ \hat{k}$ m

$\vec{r}_h = -0.40\ \hat{k}$ m

$\vec{T} = 400\ \hat{j}$ N

$\vec{F} = 100\ \hat{j}$ N

ii) $\vec{\tau}_a = \vec{r}_a \times \vec{T} = 0.10\ \hat{k} \times 400\ \hat{j}$

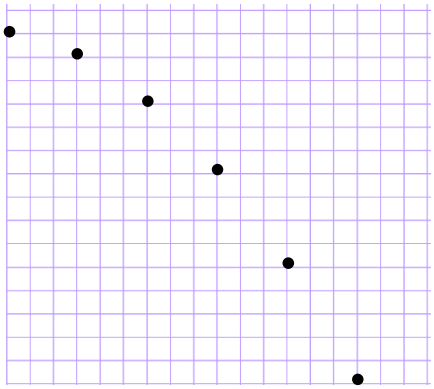
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0.1 \\ 0 & 400 & 0 \end{vmatrix} = -40\ \hat{i}\ \text{N m}$$

$\vec{\tau}_h = \vec{r}_h \times \vec{F} = -0.40\ \hat{k} \times 100\ \hat{j}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -0.4 \\ 0 & 100 & 0 \end{vmatrix} = 40\ \hat{i}\ \text{N m}$$

iii) The sum of $\vec{\tau}_a$ and $\vec{\tau}_h$ is zero.

3. a) To find the acceleration due to gravity g of a planet, a piece of stone was projected horizontally from a height of 50 m on the planet. The diagram shows a snapshot of the horizontal and vertical distance travelled by the stone every 1.0 s starting from $t = 0$. Each square is 1.0 m.



i) How do you tell that air resistance is negligible?
ii) What is the average vertical velocity of the object between $t = 2.0$ and $t = 3.0$ s and between $t = 3.0$ and $t = 4.0$ s?
iii) Taking the average velocities in (ii) as the instantaneous velocity at $t = 2.5$ s and $t = 3.5$ s, what is the vertical average acceleration between $t = 2.5$ s and $t = 3.5$ s?

b) We can also find g by using the formula for the period T of a simple pendulum, i.e. $T = 2\pi\sqrt{\dfrac{l}{g}}$, where l is the length of the string. Will the two results be exactly same? Explain.

c) Back on earth where $g = 9.80$ m/s^2, a stone is projected at 30^o with initial speed 20 m/s. Assuming no air resistance, find

i) the maximum height reached by the stone.
ii) the horizontal range of the stone.

a) i) The horizontal velocity is constant at 3.0 m/s.

ii) Between $t = 2.0$ and $t = 3.0$ s, the average speed is 3.0 m/s.
Between $t = 3.0$ and $t = 4.0$ s, the average speed is 4.0 m/s.

iii) Average acceleration = 1.0 m/s^2.

b) They will differ due to different errors in measurement.

c) i) $v_y^{\,2} = v_{0y}^{\,2} - 2g(y - y_0)$

At max ht, $v_y = 0$ and $y_0 = 0$.

$0 = (20\sin 30^o)^2 - 2\times 9.8y$

$s = 5.1$ m

ii) $y = x\tan\theta_0 - \dfrac{gx^2}{2v_0^2\cos^2\theta_0}$

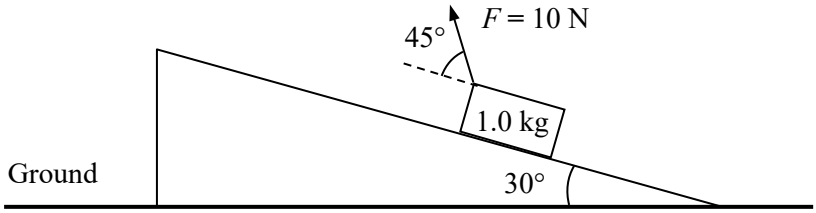
$0 = x\tan 30^o - \dfrac{9.8x^2}{2\times 20^2\cos^2 30^o}$

$x = 0$ m (reject)

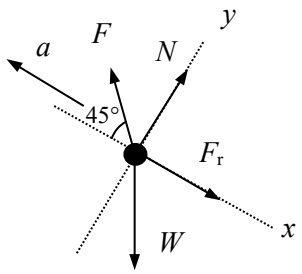
$x = 35$ m

4. A 1.0 kg block is pulled up a fixed incline by a 10 N force as shown in the diagram below. The coefficient of kinetic friction between the block and the incline is 0.20.

a) Draw the free body diagram of the block.
b) Find the magnitude of the normal force due to the incline on the block.
c) Find the magnitude of the frictional force on the block.
d) Find the acceleration of the block.



(a)



(b) $N = W\cos30^\circ - F\sin45^\circ = 9.8\cos30^\circ - 10\sin45^\circ = 1.41$ N

(c) $F_r = \mu N = 0.2 \times 1.41 = 0.28$ N

(d) Net force $= F\cos45^\circ - F_r - W\sin30^\circ = ma$
$= 10\cos45^\circ - 0.28 - 9.8\sin30^\circ$
$= 1.9$
$a = 1.9/1.0 = 1.9$ m/s^2

********* End ********