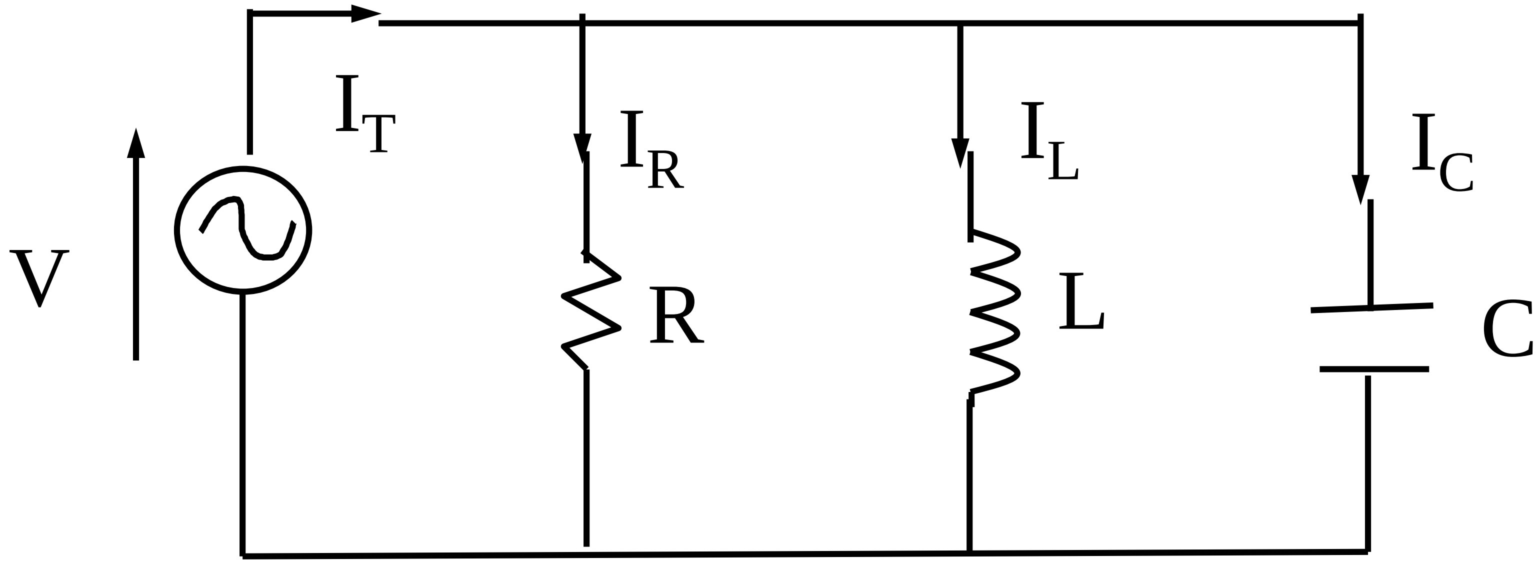
Resonant Circuits

Parallel RLC Resonant Circuit

Resonant Circuits

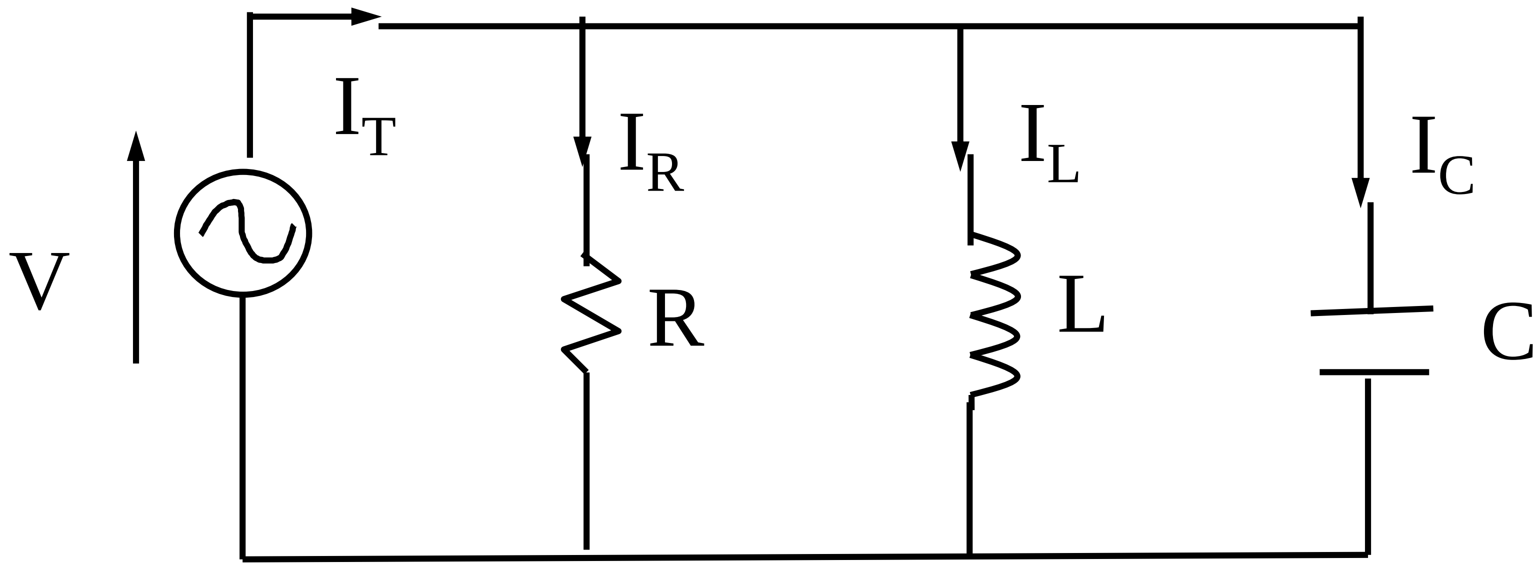
- RLC Parallel resonant circuit
  - RLC circuit in parallel
  - Phasor diagram
  - Resonance in RLC circuit
  - Graphical representation of resonance
  - Bandwidth of parallel RLC
  - Q-factor of parallel circuit

Pure RLC parallel circuit



The R, L and C are all assumed to be pure. In practice, R and C can be very close to be pure but not L.

Pure RLC parallel circuit



$$\text{Admittance } Y = 1/R + 1/jX_L + 1/(-jX_C)$$

$$= 1/R + 1/j\omega L + 1/(1/j\omega C)$$

$$= 1/R + j(\omega C - 1/\omega L)$$

Pure RLC parallel circuit - at resonance

Admittance $Y = 1 / R + 1 / j\omega L + 1 / (1 / j\omega C)$

$= 1 / R + j(\omega C - 1 / \omega L)$

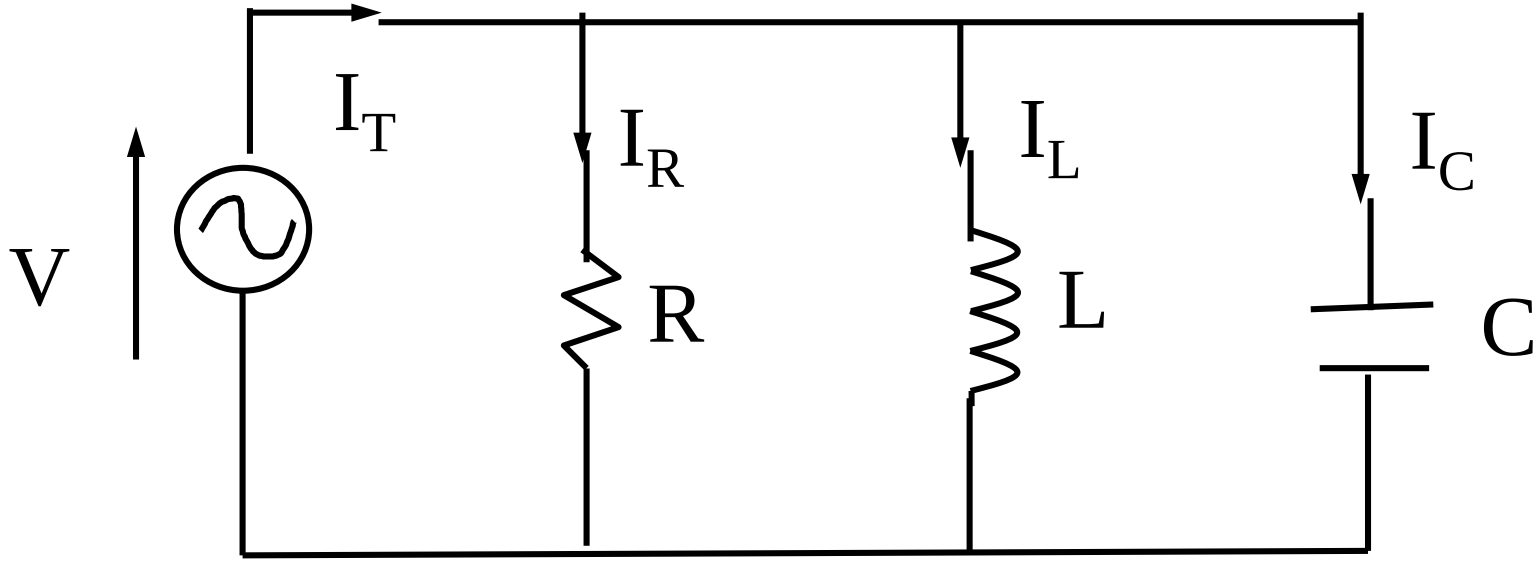
$= G + jB$ (unit - siemen, S)

where G is the conductance and B the susceptance.

Resonance occurs when B = 0, i.e. when Y = G only and is a minimum (Z = 1 / Y is at maximum)

Known as High Impedance Resonance

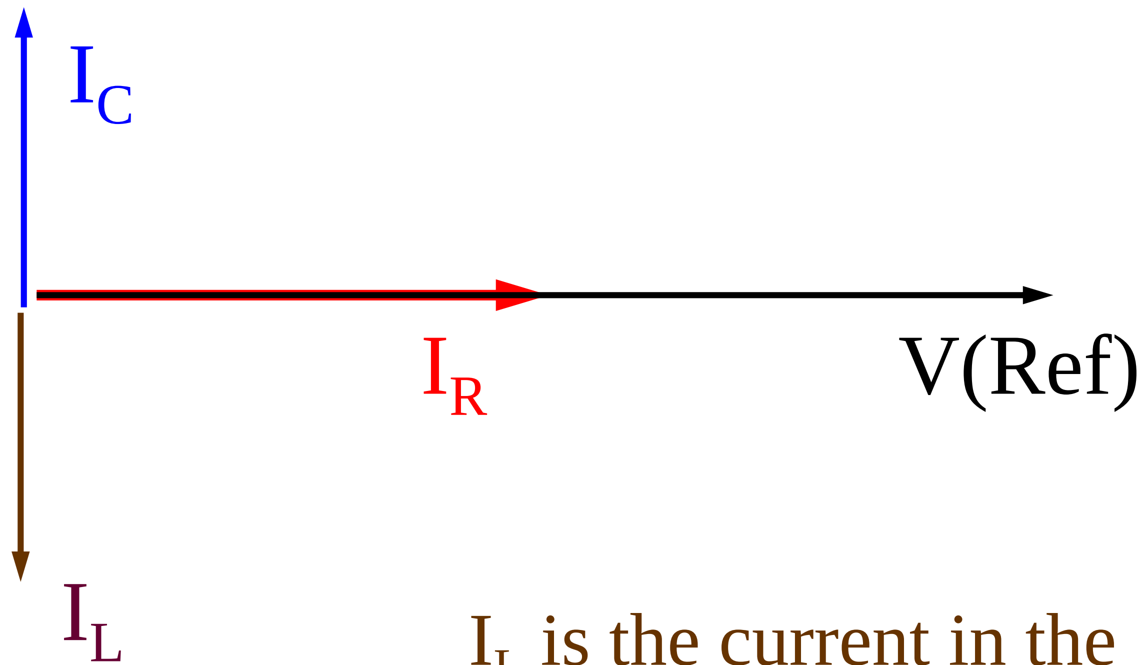
Pure RLC parallel circuit - at resonance



At resonance, V and I_T are in phase and the RLC circuit ***behaves*** like a pure resistor. With Y now equals to 1 / R, therefore $I_T = VY = V / R$

Phasor Diagram of a pure RLC parallel circuit at resonance

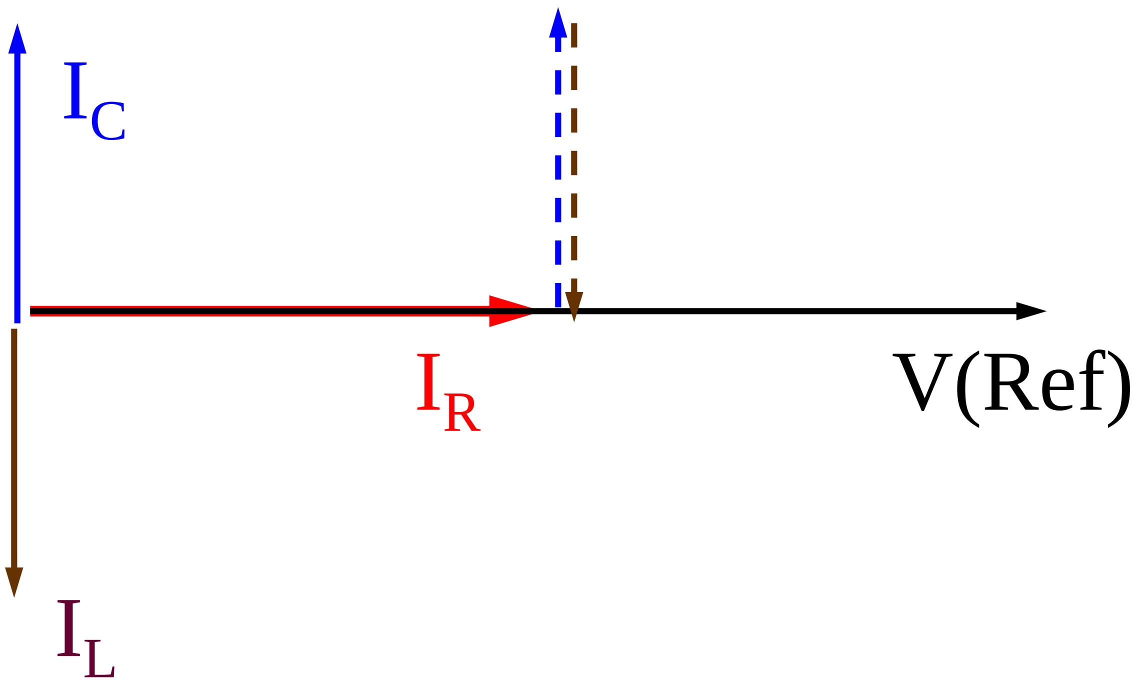
I_R is the current in the pure resistor and is therefore in phase with the voltage V.



I_C is the current in the pure capacitor and is therefore leading the voltage by 90°

I_L is the current in the pure inductor and is therefore lagging behind the voltage by 90°

Phasor Diagram of a pure RLC parallel circuit at resonance



$I_T = I_R + I_L + I_C$ (By KCL)

At resonance, $|\mathbf{I}_L| = |\mathbf{I}_C|$, making $I_L + I_C = 0$, leaving only $I_T = I_R$

Phasor Diagram of a pure RLC parallel circuit at resonance

At resonance, $| I_C | = | I_L |$

Therefore $V / X_C = V / X_L$, giving

$X_C = X_L$

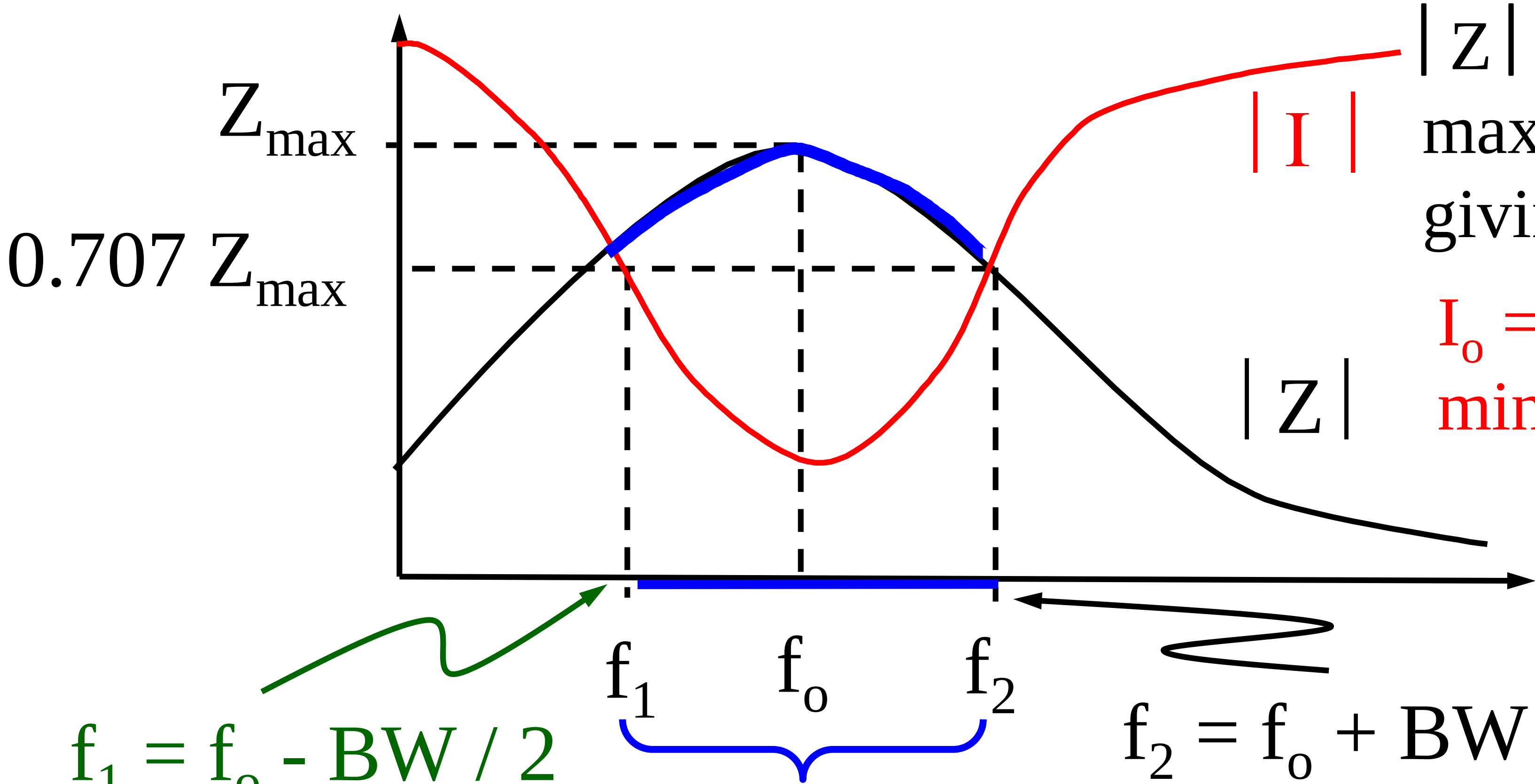
$$2 \pi f_o L = 1 / 2 \pi f_o C$$

$$4 \pi f_o^2 = 1 / LC$$

$$f_o = 1 / 2\pi \sqrt{LC} \quad \text{Hz}$$

Bandwidth of a parallel resonant circuit

At resonance, $|Z| = R$ is at maximum, giving $I_o = V/R$ a minimum



$f_1 = f_o - BW / 2$

$f_2 = f_o + BW / 2$

$$\text{Bandwidth (BW)} = f_2 - f_1 \ \text{(Hz)}$$

Q factor for a pure RLC 3-branch resonant circuit

Q factor $= I_L / I_T = (V / X_L) / (V / R) = R / X_L$

or

Q factor $= I_C / I_T = (V / X_C) / (V / R) = R / X_C$

With $X_L = X_C$ at resonance, these two Q factors actually have the same numerical value.

Q here is also called the ***current magnification factor.***

Relationship between Bandwidth, resonant frequency and Q factor

$$BW = f_o / Q \quad (Hz)$$

or

$$BW = 2\pi f_o / Q \quad (rad/s)$$

Example

A three branch parallel resonant circuit consists of an inductor of 4 mH, a resistor of 10 kΩ and a capacitor of 0.001 μF. The supply for the tuned circuit is 20 V AC. Determine:

(a) the resonant frequency, maximum impedance Q factor and bandwidth and

(b) supply current, capacitor current and inductor current at resonance.

Example

Using $f_O = \dfrac{1}{2\pi\sqrt{LC}}$

$$\mathrm{f_O} = \frac{1}{2\pi\sqrt{4\times10^{-3}\,\mathrm{x}0.001\times10^{-6}}} = 79.577\ kHz$$

$$\mathrm{Z_{max}} = R = 10\,k\Omega$$

$$Q = \frac{R}{X_L} = \frac{10000}{2\pi f_O L} = \frac{10000}{2\times\pi\times79.577\times10^3\times4\times10^{-3}} = 5$$

$$Bandwidth\ \mathrm{BW} = \frac{\mathrm{f_O}}{\mathrm{Q}} = \frac{79.577\times1000}{5} = 15.91\,kHz$$

Example

$$Supply\ current\ \mathrm{I_T} = \frac{V}{R} = \frac{20}{10 \times 1000} = 2\,mA$$

$$Capacitor current\,\mathrm{I_C} = Q \times I_T = 5 \times 2m = 10\,mA$$

$$Inductor\ current\ \mathrm{I_L} = Q \times I_T = 5 \times 2m = 10\,\mathrm{m}A$$

SUMMARY

Comparison of Series and Parallel Resonant Circuits

Parameter	Series RLC Circuit	Parallel RLC Circuit
1. Impedance	Z = R (minimum)	Z = R (maximum)
2. Current	I = V/R (maximum)	I = V/R (minimum)
3. Power Factor	Unity	
4. Resonant Frequency	$f_o = \dfrac{1}{2\pi\sqrt{LC}}$	
5. Type of Magnification, Q Factor	Voltage	Current
6. Q Factor	$Q = X_L / R = X_C / R$ $= V_L / V = V_C / V$	$Q = R / X_L = R / X_C$ $= I_L / I = I_C / I$
7. Bandwidth	$BW = \dfrac{f_o}{Q_o} = f_2 - f_1$	
8. Half power frequencies	$f_1 = f_o - \dfrac{BW}{2}$ $f_2 = f_o + \dfrac{BW}{2}$	