No.	SOLUTION
1	Let P_n be the statement $1+2\times 2^1+3\times 2^2+\cdots+n\times 2^{n-1}=(n-1)2^n+1$ Step 1: Prove that P_1 is true. When n = 1, LHS = 1 $\text{RHS}=(1-1)2^1+1=1=\text{LHS}$ Hence, P_1 is true. Step 2: Assume that P_n is true for an arbitrary $n\in \mathbb{Z}^+$. $P_n:\quad 1+2\times 2^1+3\times 2^2+\cdots+n\times 2^{n-1}=(n-1)2^n+1$ Step 3: Prove that P_{n+1} is true $P_{n+1}:\quad 1+2\times 2^1+3\times 2^2+\cdots+(n+1)\times 2^n=n2^{n+1}+1$ LHS: $1+2\times 2^1+3\times 2^2+\cdots+(n+1)\times 2^n$ $=(n-1)2^n+1+(n+1)\times 2^n$ $=2n\times 2^n+1$ $=n2^{n+1}+1$ $=\text{RHS}$ Hence P_n is true implies P_{n+1} is true. Since P_1 is true, it follows by the principle of mathematical induction that P_n is true for all $n\in \mathbb{Z}^+$.

No.	SOLUTION
2(a)	Let u = x^2 + x $\frac{du}{dx}=2x+1$ $\int(2x+1)e^{x^2+x}\,dx$ $=\int e^u du$ $=e^u+C$ $=e^{x^2+x}+C$
2(b)	$y=\int xe^{2x}dx$ $=x\frac{e^{2x}}{2}-\int\frac{e^{2x}}{2}dx$ $=x\frac{e^{2x}}{2}-\frac{e^{2x}}{4}+C$ $(0,-1/4):-\frac{1}{4}=0-\frac{1}{4}+C$ $C=0$ $y=x\frac{e^{2x}}{2}-\frac{e^{2x}}{4}$

No.	SOLUTION
3	$\text{Area} = \int_0^{\pi} \sin^2 x \, dx$ $= \int_0^{\pi} \frac{1-\cos 2x}{2} \, dx$ $= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$ $= \frac{1}{2}\left[\left(\pi - \frac{\sin 2(\pi)}{2}\right) - (0-0)\right]$ $= \frac{\pi}{2}$

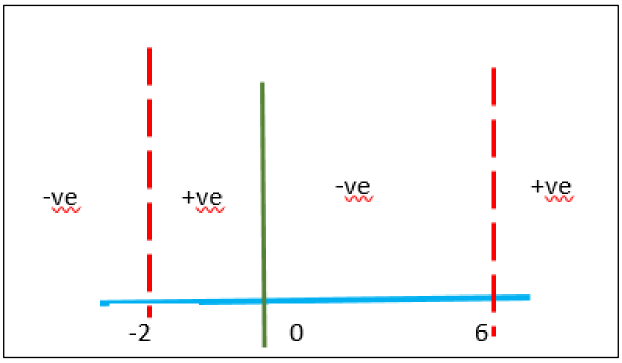
No.	SOLUTION
4	$e^x = \frac{3}{e^x} + 2$ $(e^x)^2 = 3 + 2e^x$ $(e^x)^2 - 2e^x - 3 = 0$ $(e^x + 1)(e^x - 3) = 0$ $e^x = -1$ (No solution) or $e^x = 3$ $x = \ln 3$ (shown) (b) Volume of solid of revolution of R about the x-axis $= \pi \int_0^{\ln 3} (3e^{-x} + 2)^2 - (e^x)^2 dx$ $= \pi \int_0^{\ln 3} (9e^{-2x} + 12e^{-x} + 4 - e^{2x}) dx$ $= \pi \left[\frac{9e^{-2x}}{-2} + \frac{12e^{-x}}{(-1)} + 4x - \frac{e^{2x}}{2}\right]_0^{\ln 3}$ $= \pi \left[-\frac{9}{2e^{2x}} - \frac{12}{e^x} + 4x - \frac{e^{2x}}{2}\right]_0^{\ln 3}$ $= \pi \left[\left(-\frac{9}{2e^{2\ln 3}} - \frac{12}{e^{\ln 3}} + 4\ln 3 - \frac{e^{2\ln 3}}{2}\right) - \left(-\frac{9}{2} - 12 - \frac{1}{2}\right)\right]$ $= \pi \left[-\frac{9}{2(9)} - \frac{12}{3} + 4\ln 3 - \frac{9}{2} + \frac{9}{2} + 12 + \frac{1}{2}\right]$ $= \pi(8 + 4\ln 3)$ $= 4\pi(2 + \ln 3)$ unit^3

No.	SOLUTION
5(a) (i)	$\vec{F} = 10\frac{\langle 4,3,0\rangle}{\sqrt{4^2+3^2}}=\langle 8,6,0\rangle$
(ii)	Displacement vector $\vec{S}=\langle 2,0,4\rangle - \langle 1,-1,6\rangle=\langle 1,1,-2\rangle$ Work done = $\vec{F}\cdot\vec{S}$ =14 N
(b)	When in same direction $\theta = 0$. Work done =$\vec{F}\cdot\vec{S} = \lvert\vec{F}\rvert\lvert\vec{S}\rvert\cos\theta.\ cos\,\theta = 1$ when $\theta = 0$. Hence work done is maximized.

No.	SOLUTION		
6(a)	Vector parallel to $L_1=\langle -1,1,0\rangle$ Hence parametric equation of $L_1$is $x = 3 - t, y = -1 + t, z = 6$		
(b)	$\begin{pmatrix} 3-t \\ -1+t \\ 6 \end{pmatrix} = \begin{pmatrix} 3+\mu \\ 2+\mu \\ 1-\frac{10\mu}{3} \end{pmatrix}$ Solving we get $t=\frac{3}{2}, \mu=-\frac{3}{2}$ Hence point of intersection is $\left(\frac{3}{2}, \frac{1}{2}, 6\right)$		

No.	SOLUTION
7(a)	Vector parallel to line is $\langle 2, 1, -1\rangle$ Hence parametric equation of line is $x = 0 + \lambda 2$ $y = -6 + \lambda 1$ $z = 0 - \lambda$ Subst equ of line to equ of plane to find intersection point:

	$2(2\lambda)+(-6+\lambda)-(-\lambda)=0$ $\lambda=1$ Hence point of intersection is $(2,\ -5,-1)$
(b)	Vector parallel to the line is $\langle 2,\ 2,\ 4\rangle$ Vector perpendicular to the plane (normal) is $\langle -1,\ 5,\ -2\rangle$ $\begin{pmatrix}2\\2\\4\end{pmatrix}\bullet\begin{pmatrix}-1\\5\\-2\end{pmatrix}=-2+10-8$ $=0$ The line is perpendicular to the normal of the plane. Hence, the line and the plane is **parallel**.

No.	SOLUTION
8(a)	$x^3\ \geq x\,(4x+12)$ $x^3\ -x\,(4x+12)\geq 0$ $x[x^2\ -(4x+12)]\geq 0$ $x[x^2\ -4x-12]\geq 0$ x(x-6)(x+2) ≥ 0 $x\geq 6$ Or $\quad -2\leq x\leq 0$ 
(b)	$\dfrac{\lvert 3x-2\rvert}{\lvert x-3\rvert}\geq 1$ Hence (x≠ 3) $\lvert 3x-2\rvert\geq\lvert x-3\rvert$ $\lvert 3x-2\rvert^2\geq\lvert x-3\rvert^2$ $9x^2+4-12x\geq x^2+9-6x$ $8x^2-6x-5\geq 0$ $(4x-5)(2x+1)\geq 0$ $x\neq 3$ Ans (from number line): $\frac{5}{4}\leq x<3$ or x >3 or $x\leq -1/2$

No.	SOLUTION