2017/2018 SEMESTER ONE EXAMINATION

Diploma in Electrical and Electronic Engineering
3rd Year Full Time

DIGITAL SIGNAL PROCESSING

Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules as set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **TWO** sections:

   Section A - 6 Short Questions, 10 marks each.
   Section B - 2 Long Questions, 20 marks each.

3. **ALL** questions are **COMPULSORY**.

4. **ALL** questions are to be answered in the answer booklet.

5. This paper consists of **5** pages, including 2 pages of mathematical formulae.

SECTION A - SHORT QUESTIONS [10 marks each]

A1 A DSP system with no anti-aliasing filter samples a 1.5 kHz sine wave signal with a sampling frequency of 2.5 kHz.

a) Sketch the magnitude spectrum of the sampled signal for $-5\text{ kHz} \le f \le 5\text{ kHz}$. (7 marks)

b) If a reconstruction filter with cutoff frequency at half sampling frequency is used to recover the original analog signal from the samples, what signal will be recovered? (3 marks)

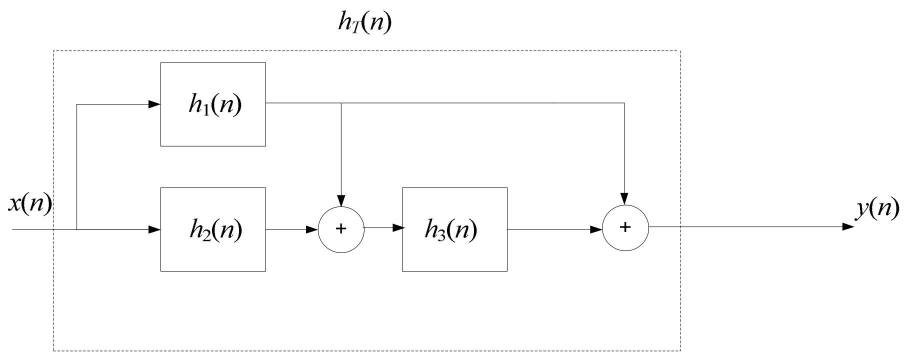
A2 When a unit step function is applied to the input of a linear time-invariant digital system, it produces an output of $y(n)$ = { 2, 4, 8, 3, 2, 1, 1, 1, 1, 1, 1, …}. Determine the impulse response of this system. Hence, or otherwise, explain briefly whether the system is stable. (10 marks)

A3 Given a digital system with an impulse response function, $h(n) = \left(0.8^n \cos\left[\frac{n\pi}{3}\right]\right)u(n)$,

calculate the output $y(n)$, if the input to the system is $x(n) = (0.4)^n u(n-1)$. (10 marks)

A4 Evaluate the N = 4-point DFT for $X(0)$ and $X(2)$ if $x(n)$ = {0, 2, 1, 2}. (10 marks)

A5 The block diagram of a digital system is given as:



a) Find the overall impulse response of the system, $h_T(n)$ in terms of $h_1(n)$, $h_2(n)$ and $h_3(n)$. (4 marks)

b) If $h_1(n) = (0.5)^n u(n)$, $h_2(n) = (0.2)^n u(n)$, $h_3(n) = \delta(n)$ respectively, find the z-transform of $h_T(n)$, $H_T(z)$. (6 marks)

A6 The system function of a digital system is given as:

$$X(z) = \frac{z^2 + 2z}{z^2 - 3z - 4}$$

Using partial fraction, find $x(n)$. (10 marks)

SECTION B - LONG QUESTIONS [20 marks each]

B1 You are required to design a digital FIR low-pass filter to reject the high frequency noise found in a telemetry signal. The specifications of the filter are as follow:

Passband: 0 to 3 kHz
Stopband: 4 to 20 kHz
Sampling frequency: 8 kHz
Peak approximation error: 0.002

To design this filter, determine

(a) the width of transition band in π radians, (4 marks)
(b) the windowing function that you would choose, (3 marks)
(c) the number of tap coefficients that you would need, (5 marks)
(d) the values of the first 2 and the last 2 tap coefficients. (8 marks)

B2

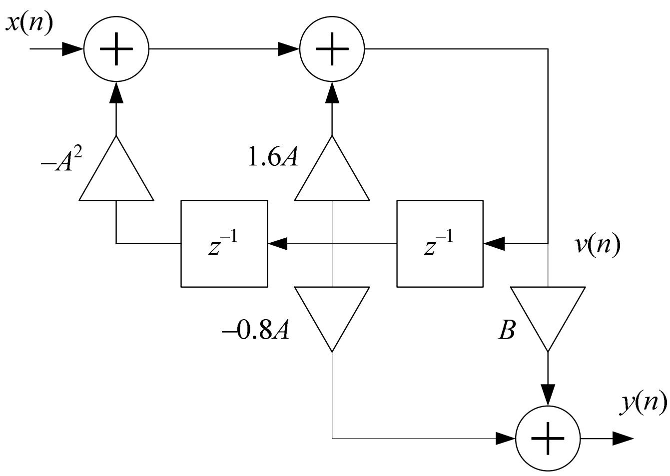


Figure B2

The digital network of a certain digital system is shown in figure B2.

(a) Show that the system function, $H(z) = \dfrac{B - 0.8Az^{-1}}{1 - 1.6Az^{-1} + A^2 z^{-2}}$. (6 marks)
(b) If $A = B = 1$, determine the impulse response of the system. (5 marks)
(c) Assuming that $A = B = 1$, determine the frequency response of the system in terms of $e^{j\omega}$. (4 marks)
(d) Based on the results in (c), or otherwise, find the gain of the system at 1024 Hz if the sampling frequency used is 10 kHz. (5 marks)
[*Hint*: $e^{-j0.2048\pi} = 0.8 - j0.6$]

-End of Paper-

Appendix

The z-transform is defined as $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

Sequence	Transform
$\delta[n]$	1
$u[n]$	$\dfrac{1}{1-z^{-1}}$
$\delta[n-m]$	z^{-m}
$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$
$na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$
$[\cos \omega_0 n]u[n]$	$\dfrac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[\sin \omega_0 n]u[n]$	$\dfrac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[r^n \cos \omega_0 n]u[n]$	$\dfrac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$
$[r^n \sin \omega_0 n]u[n]$	$\dfrac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$

Some z-transform properties:	
Sequence	Transform
$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
$x[n-m]$	$z^{-m}X(z)$

Some trigonometric identities:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Complex number theory:

$$z = a + jb = r\angle\theta = re^{j\theta}$$

$$r = \sqrt{a^2+b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

Series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

Discrete Fourier Transform:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}$$

Quadratic equation solution:

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

The characteristics of the different windowing functions:

Window Type	Peak approximation Error $20\log_{10}\delta$ dB	Transition Band $\Delta\omega$
Rectangular: $w(n)=\begin{cases}1 & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$	−21	$\frac{4\pi}{M+1}$
Bartlett: $w[n]=\begin{cases}\frac{2n}{M} & 0\le n\le\frac{M}{2}\\ 2-\frac{2n}{M} & \frac{M}{2}\le n\le M\\ 0 & \text{otherwise}\end{cases}$	−25	$\frac{8\pi}{M}$
Hanning: $w(n)=\begin{cases}0.5-0.5\cos\left(\frac{2\pi n}{M}\right) & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$	−44	$\frac{8\pi}{M}$
Hamming: $w(n)=\begin{cases}0.54-0.46\cos\left(\frac{2\pi n}{M}\right) & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$	−53	$\frac{8\pi}{M}$
Blackman: $w(n)=\begin{cases}0.42-0.5\cos\left(\frac{2\pi n}{M}\right)+0.08\cos\left(\frac{4\pi n}{M}\right) & 0\le n\le M\\ 0 & \text{otherwise}\end{cases}$	−74	$\frac{12\pi}{M}$

The impulse response of an ideal low pass filter is: $h_d(n)=\dfrac{\sin\left(\omega_c\left[n-\frac{M}{2}\right]\right)}{\pi\left(n-\frac{M}{2}\right)}$