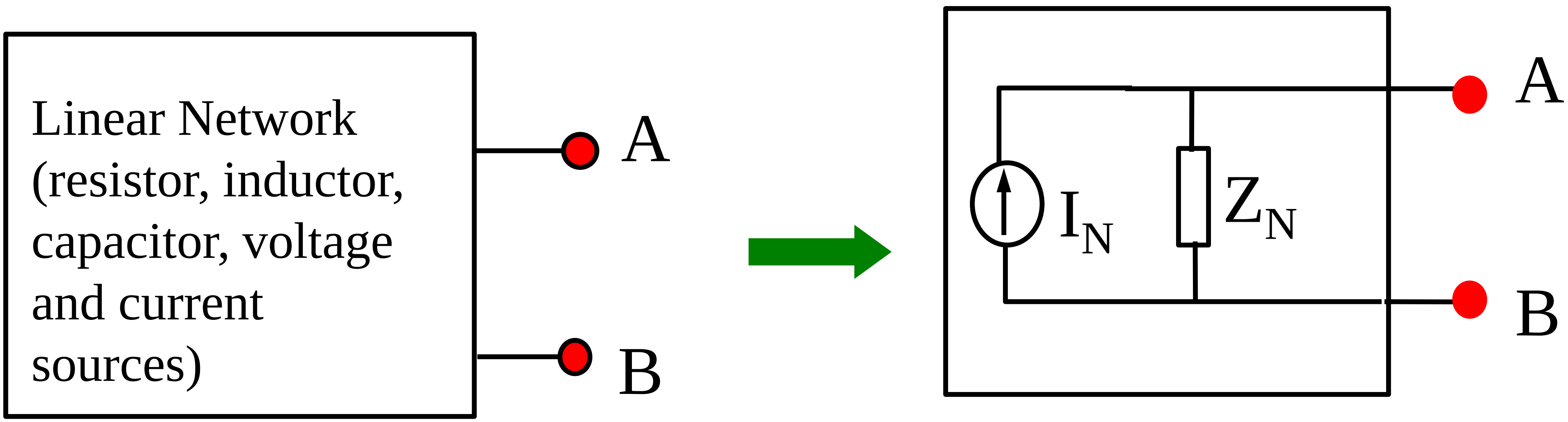
Norton's Theorem

It states that any linear circuit between two points A and B can be replaced by an equivalent circuit consisting of a current source ***in parallel*** with a single impedance.

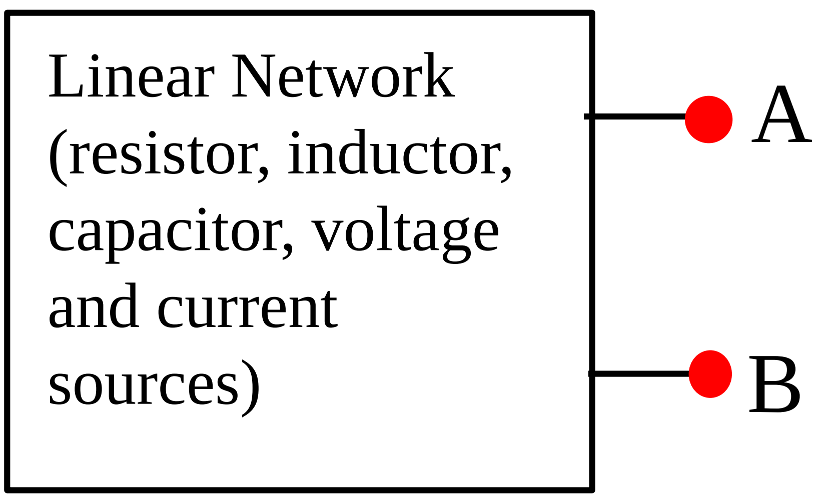
Norton's Theorem

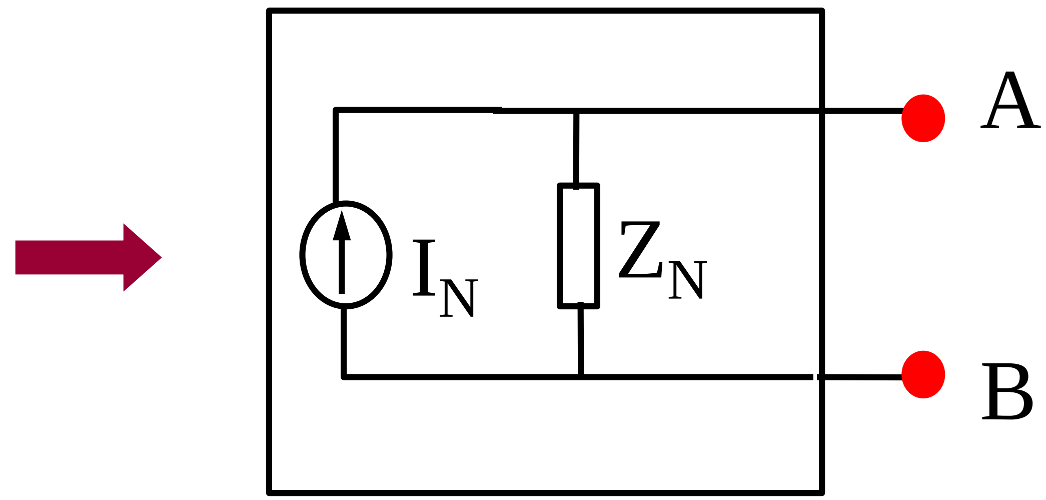


Norton's Equivalent Circuit

I_N - Norton's current, equals to the short-circuit current across terminals A & B when they are shorted in the original linear network.

Norton's Theorem

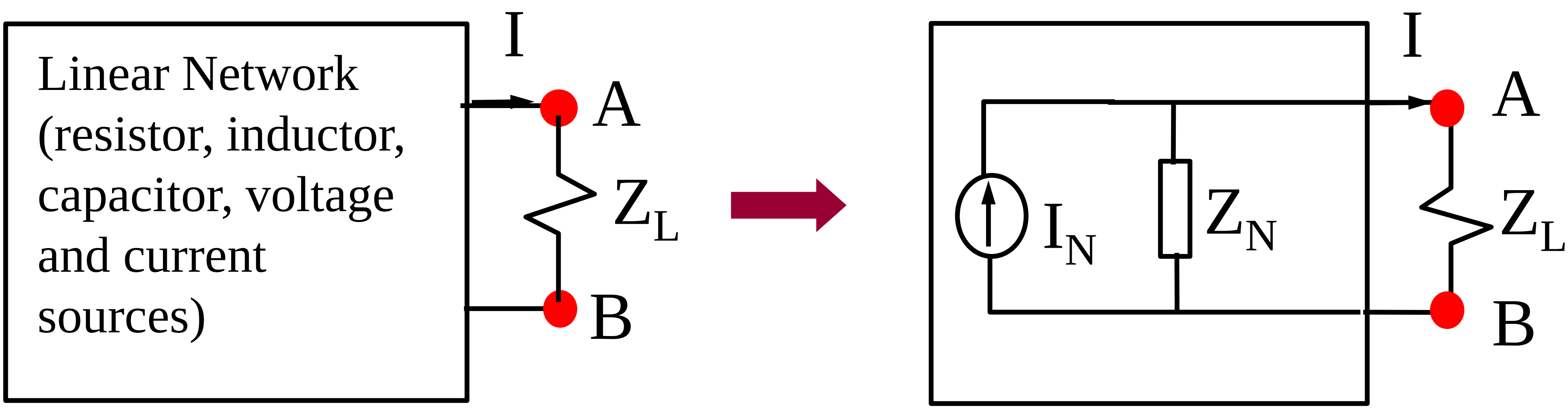




Norton's Equivalent Circuit

Z_N - Norton's impedance, equals to the impedance across terminals A & B in the original linear network, with all the voltage and current sources reduced to zero.

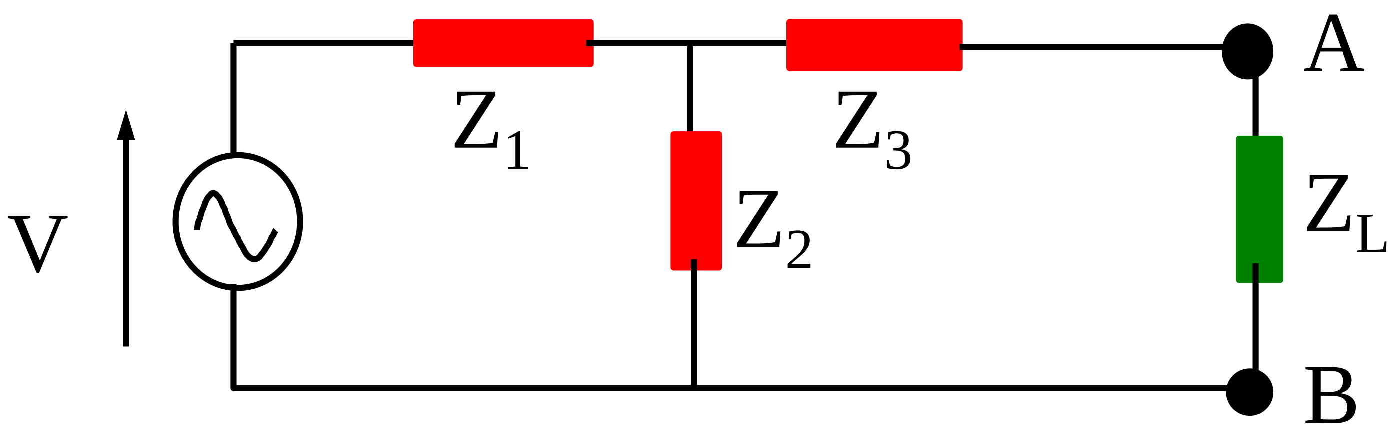
Norton's Theorem



Norton's Equivalent Circuit

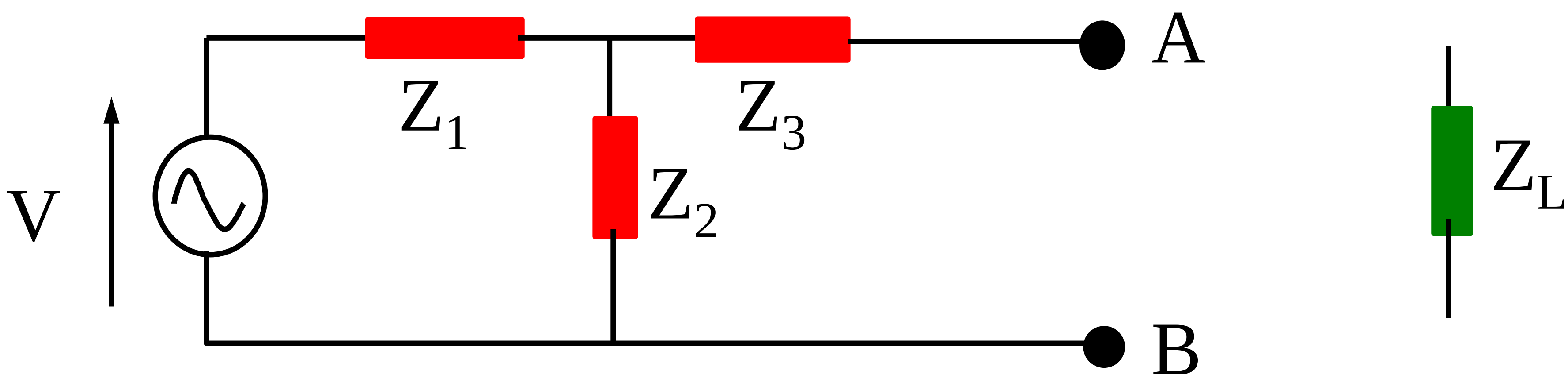
A load impedance Z_L will experience the same current by connecting across AB in the original circuit and in the Norton's equivalent circuit.

Norton's Theorem



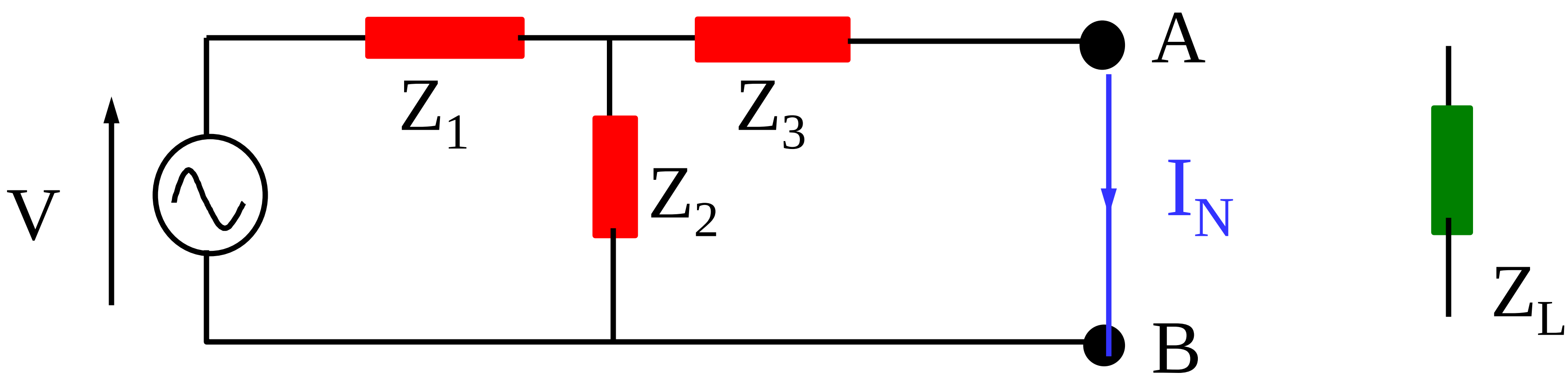
Given this original circuit, find the Norton's equivalent circuit across terminals A & B.

Procedure in applying Norton's Theorem



Step 1 - Disconnect the load Z_L from the circuit such that terminals A & B are now open-circuit.

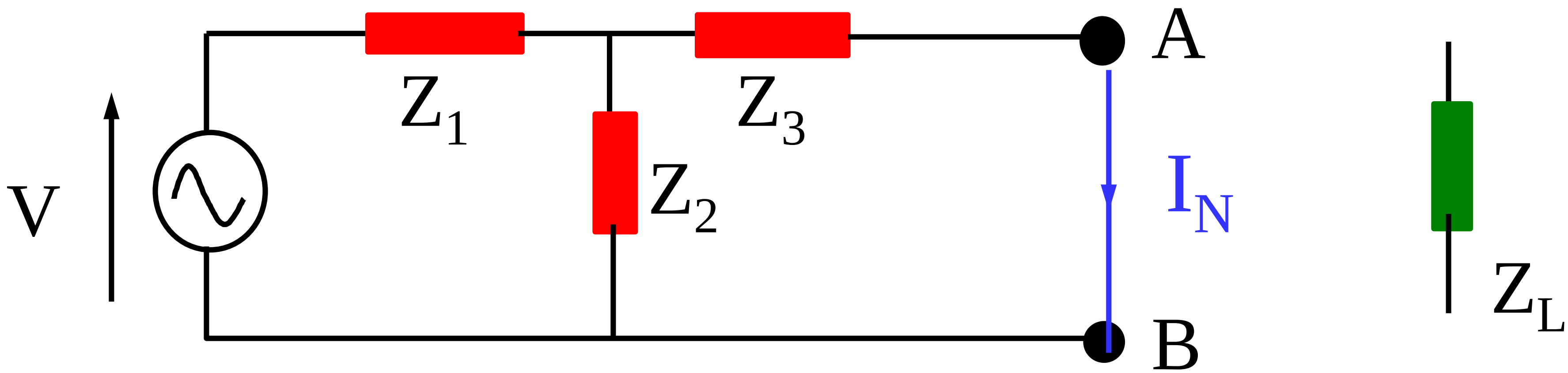
Procedure in applying Norton's Theorem



Step 2 - put a short-circuit across terminals A & B and then calculate the short-circuit current. This is the Norton's current I_N.

For computing you can only use either the application of Ohm's Law or Mesh/Loop Analysis method.

Procedure in applying Norton's Theorem

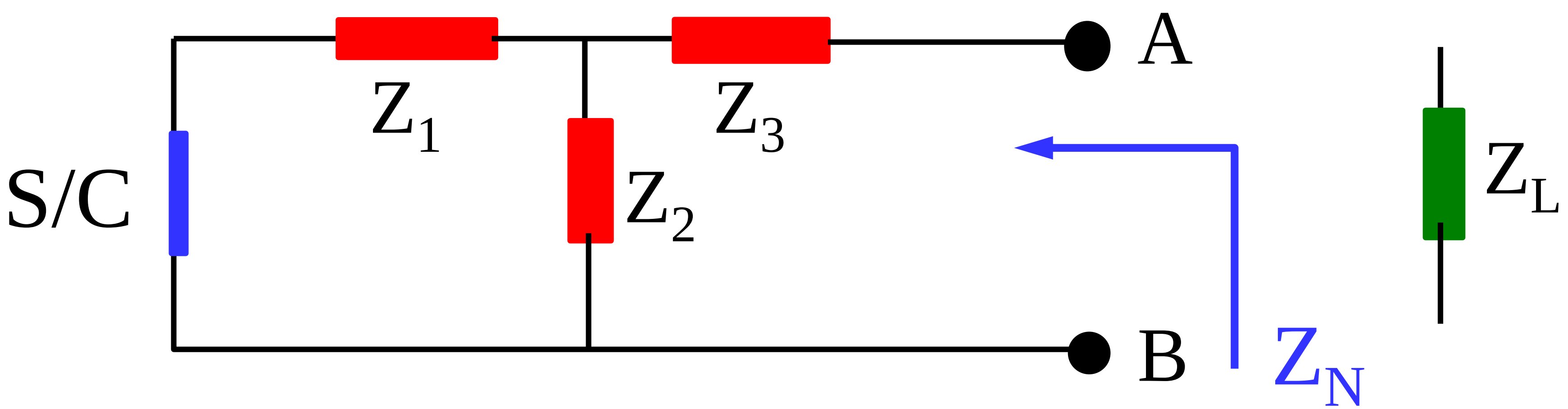


In this example, $Z_T = Z_1 + (Z_2 // Z_3)$

$I_T = V/Z_T$

$I_N = I_T \times Z_2 / (Z_2 + Z_3)$

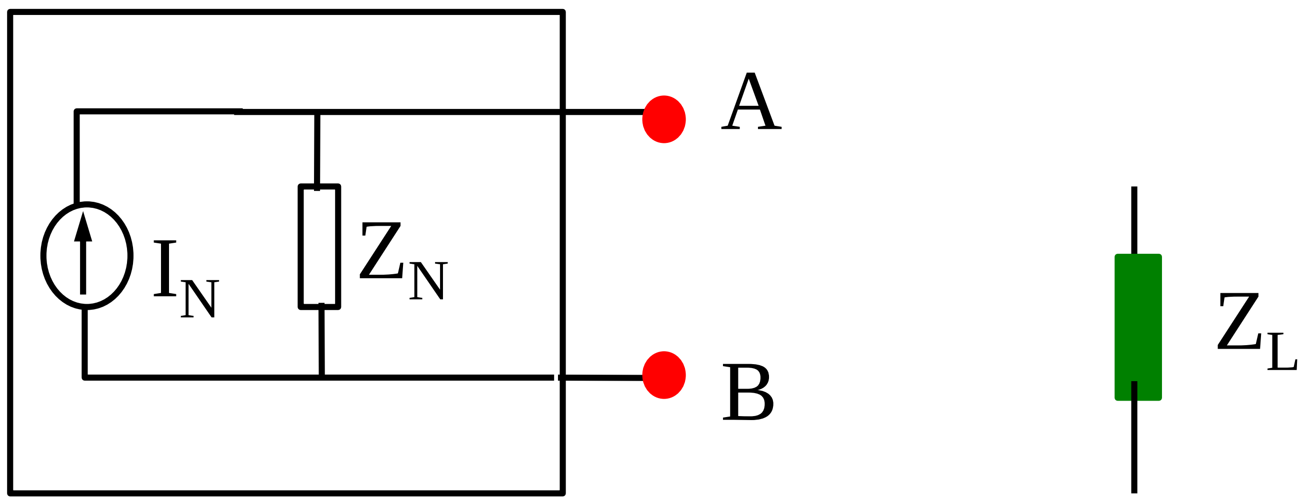
Procedure in applying Norton's Theorem



Step 3 - Replace the voltage source by a short circuit (current source by open circuit) and calculate the impedance across A & B by looking into the source free circuit. **This is the Z_N.**

For this example, $Z_N = (Z_1 \,//\, Z_2) + Z_3$

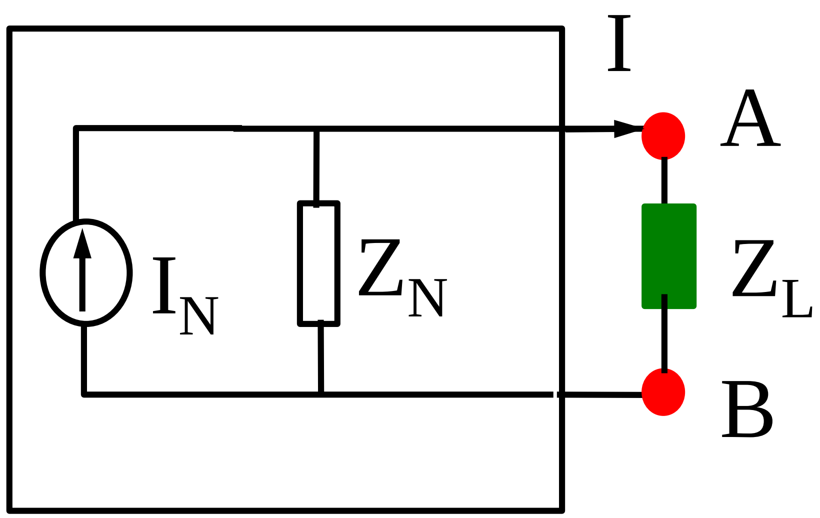
Procedure in applying Norton's Theorem



Norton's Equivalent Circuit

Step 4 - Knowing I_N and Z_N, the Norton's equivalent circuit can now be formed.

Procedure in applying Norton's Theorem



$I = I_N \times Z_N / (Z_N + Z_L)$

This value of I will have the same value when Z_L is connected in the original linear circuit.

If it is the current through Z_L you want to find, then Z_L should be reconnected across A & B.

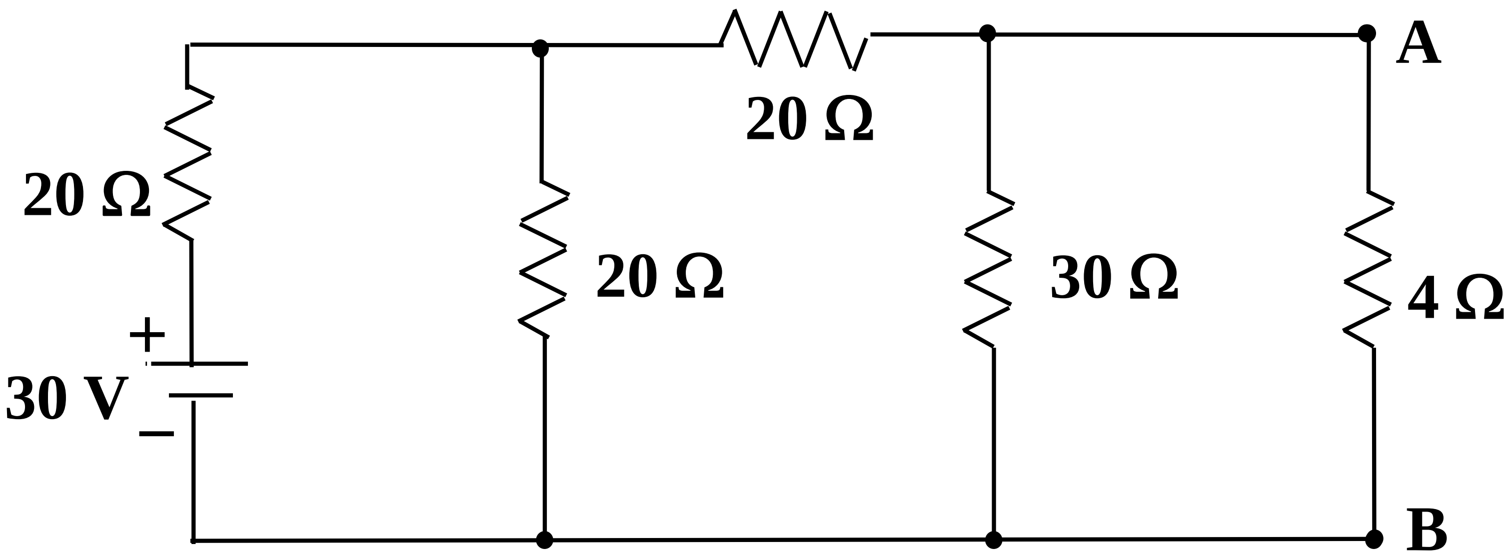
Procedure in applying Norton's Theorem

NOTE:

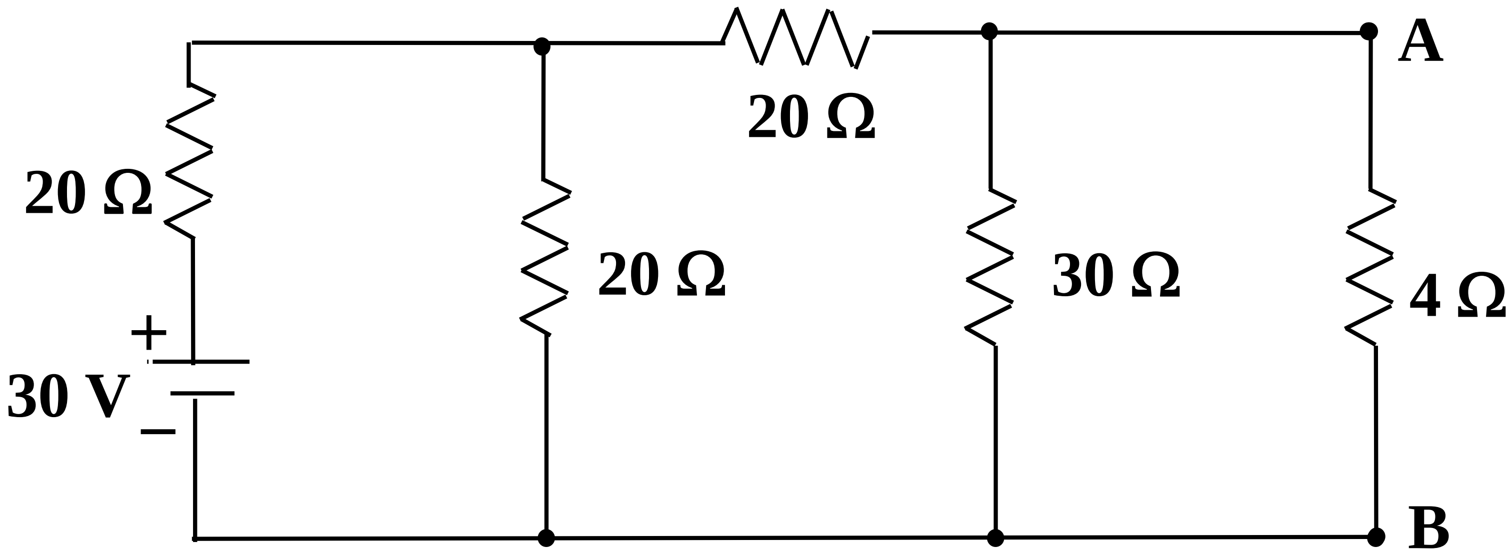
Source Conversion is not allowed.

Example 1.11

Apply Norton's theorem to find the current in the 4 Ω resistor.

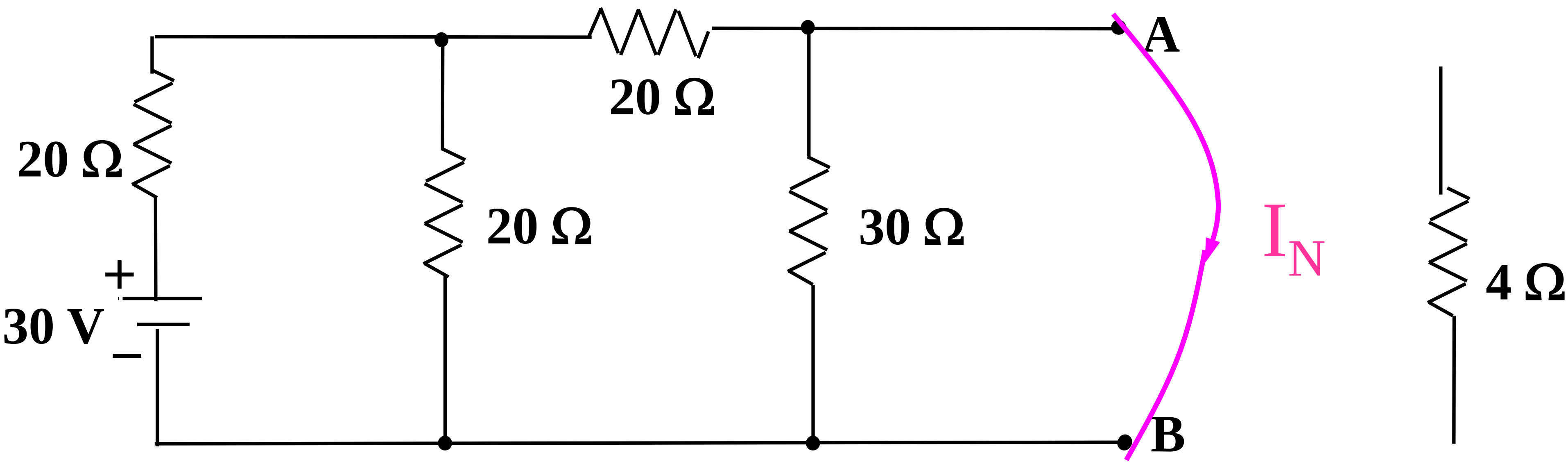


Example 1.11



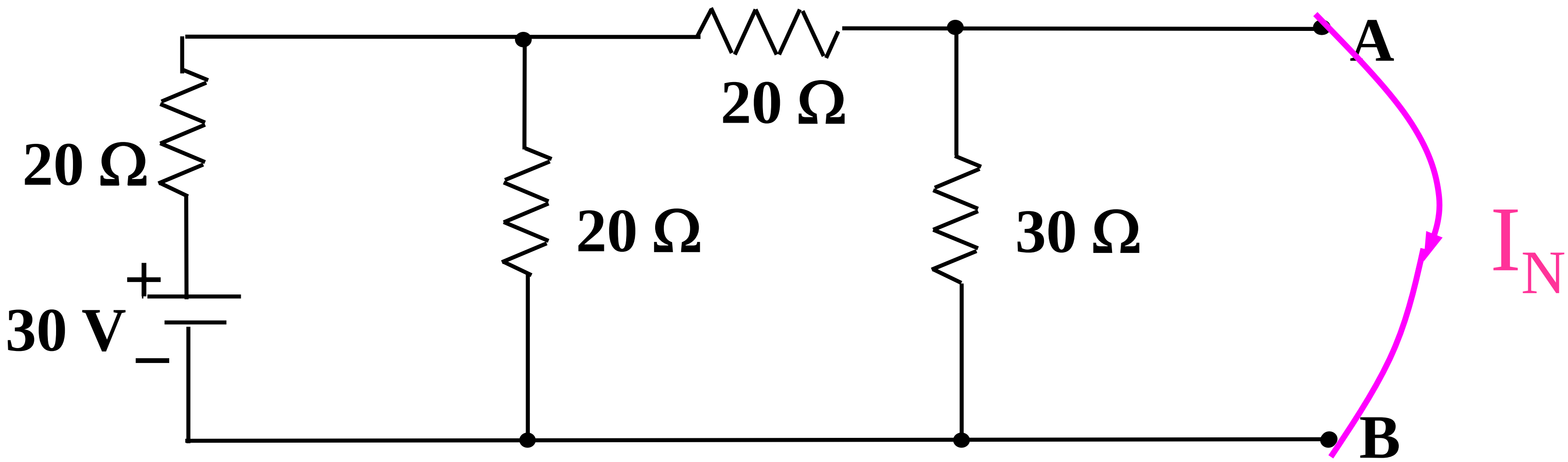
To find the Norton's equivalent circuit across AB, the first thing to do is to remove R_L such that terminals A & B are now open-circuit.

Example 1.11



To find the Norton's current sources I_N, put a short circuit across terminals AB. The current I_N flows through the short circuit is the Norton's current source.

Example 1.11



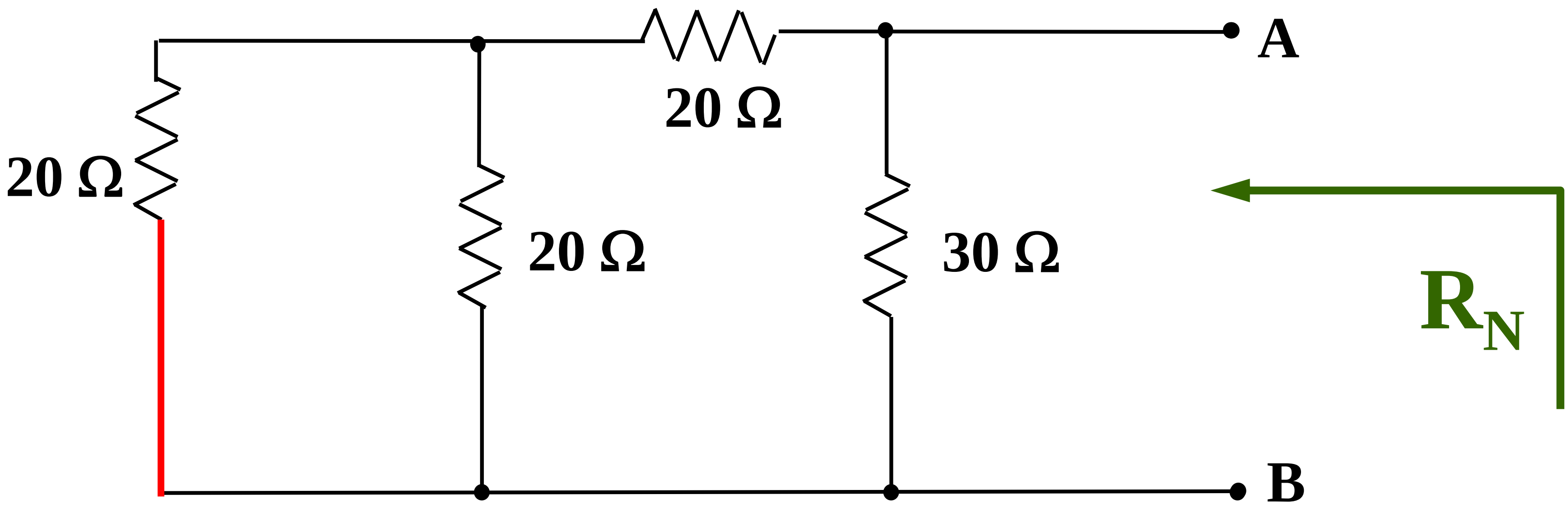
$$R_{//} = 10\,\Omega$$

$$R_T = 10 + 20 = 30\,\Omega$$

$$I_T = \frac{30}{30} = 1\,A$$

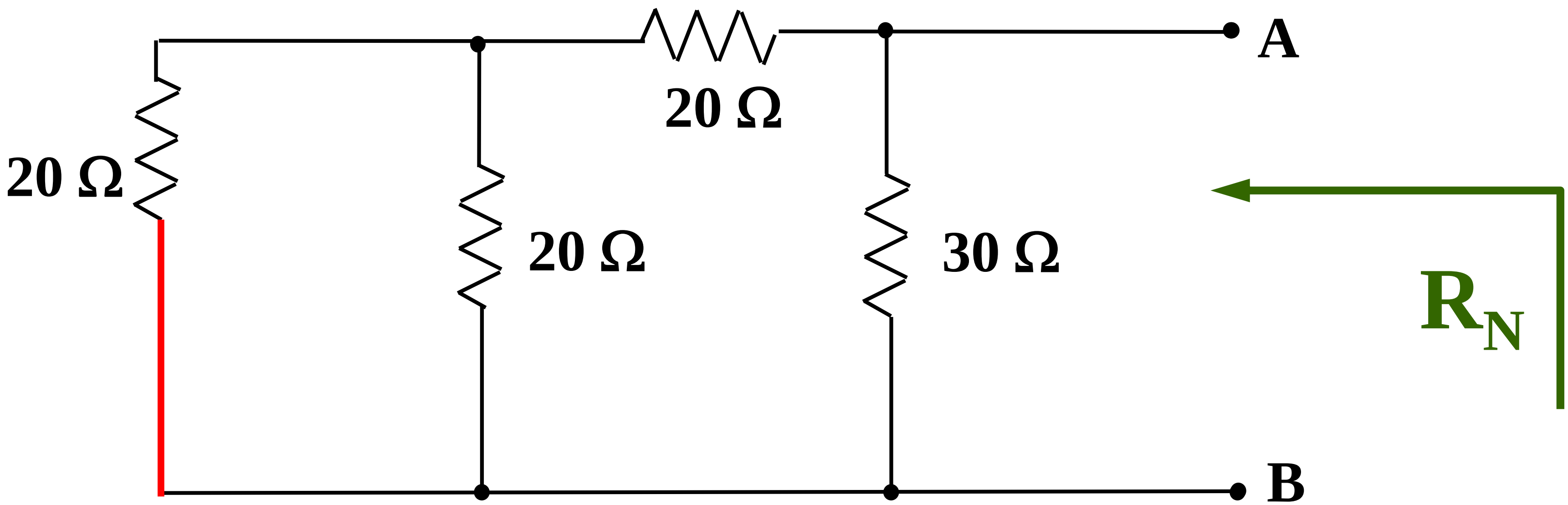
$$I_N = \frac{1}{2} = 0.5\,A$$

Example 1.11



Norton's equivalent resistance R_N is the resistance measured between A & B with all voltage sources replaced by short circuits, and current sources by open circuits.

Example 1.11



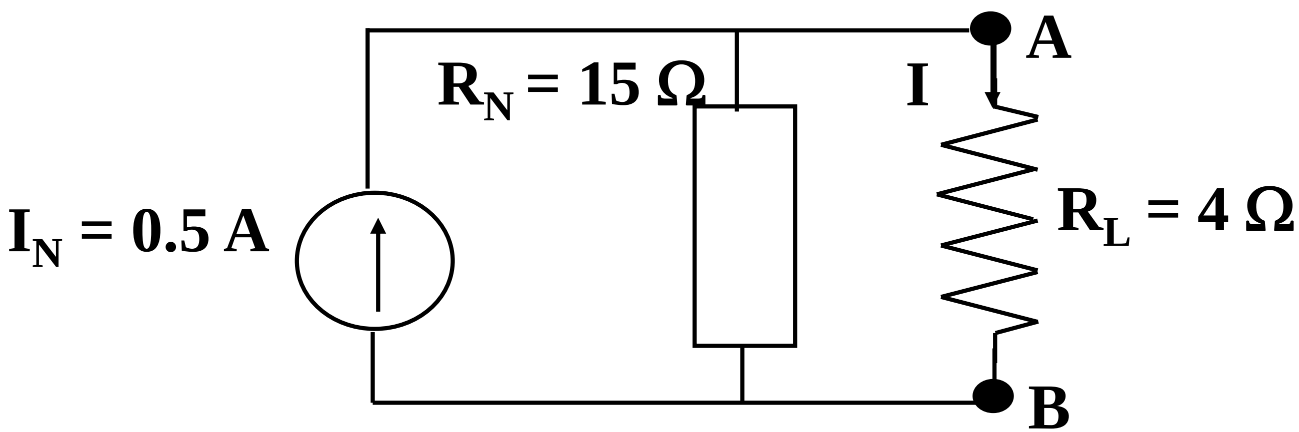
$$R_{//} = 10\,\Omega$$

$$R_N = R_{AB} = (10+20)//30 = 15\,\Omega$$

So I_N = 0.5 A and R_N = 15 Ω, the Norton's equivalent circuit can now be drawn.

Example 1.11

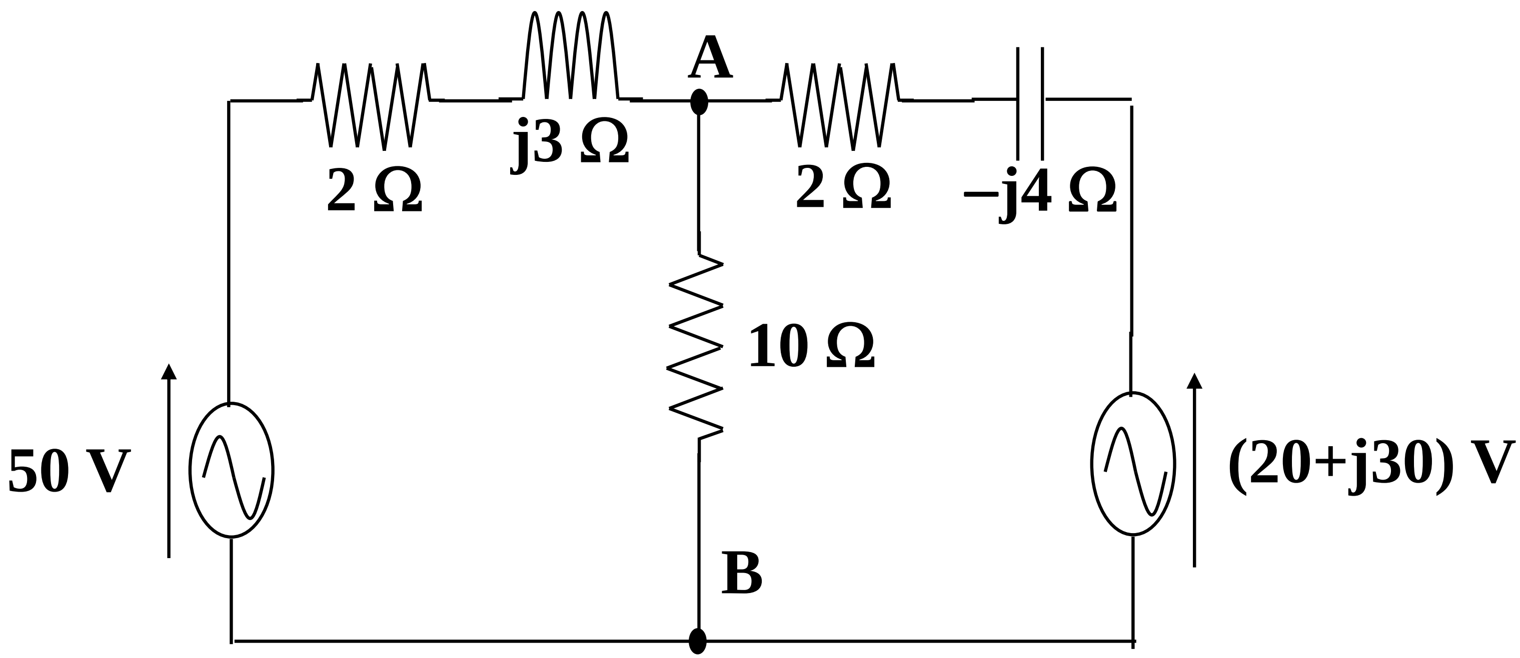
Norton's equivalent circuit with load (R_L)



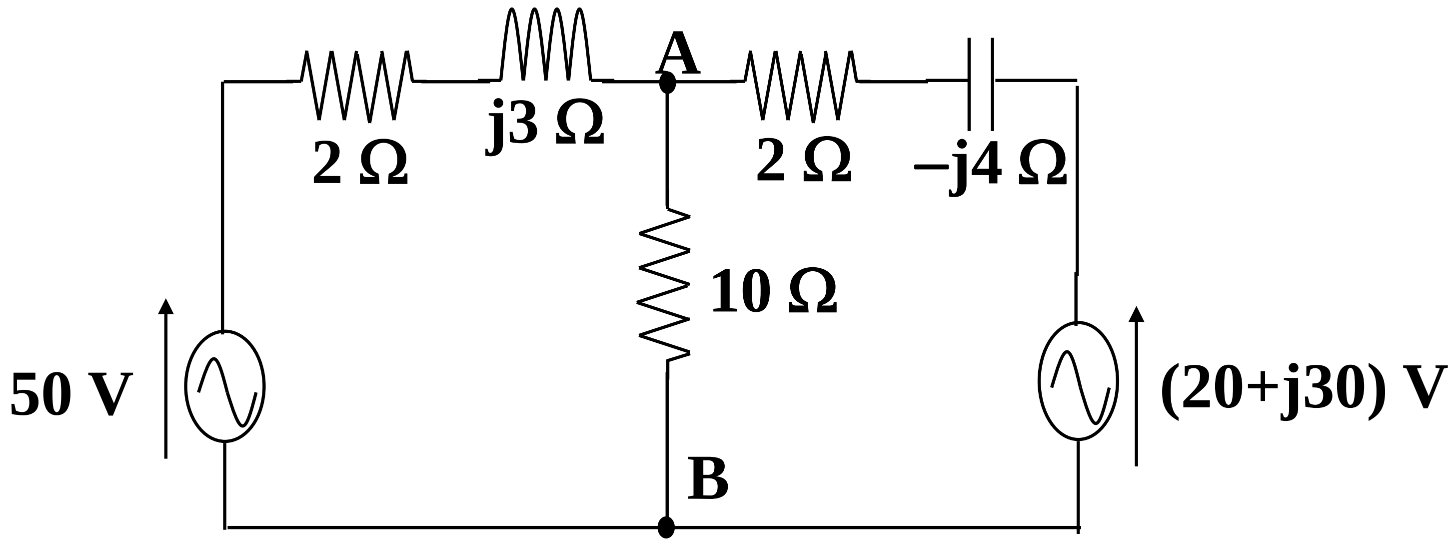
$$\text{Load current } I = \frac{I_N R_N}{R_N + R_L} = \frac{0.5 \times 15}{19} = 0.395 \text{ A}$$

Example 1.12

Apply Norton's theorem and calculate the current in the 10 ohm resistor.

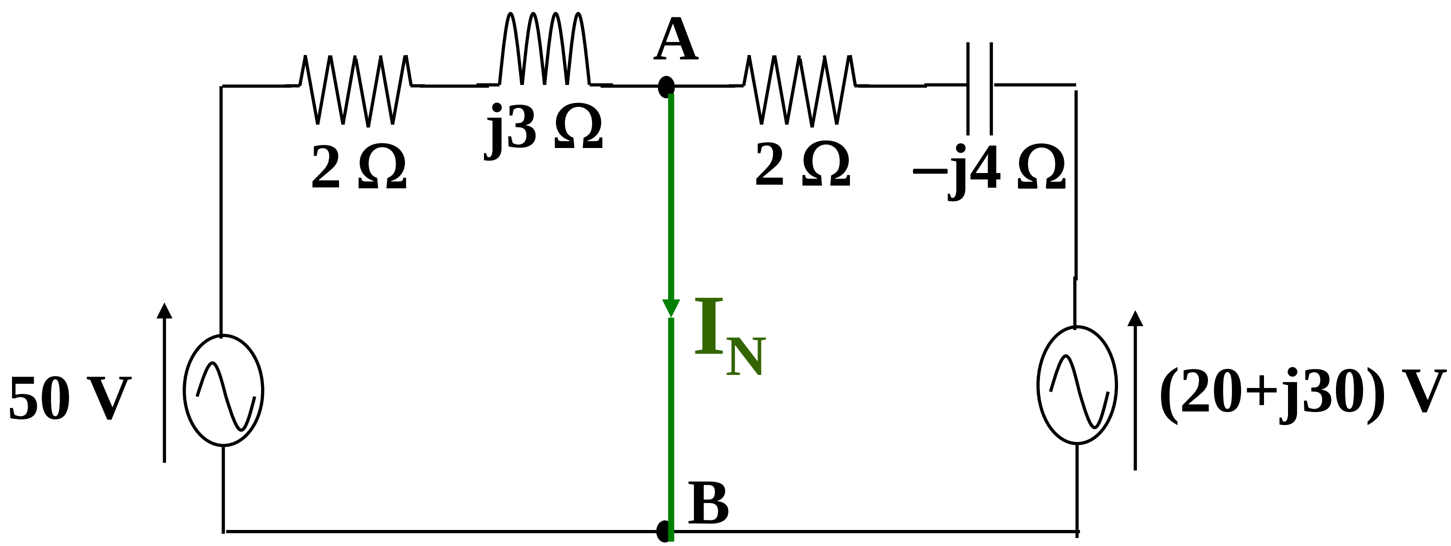


Example 1.12



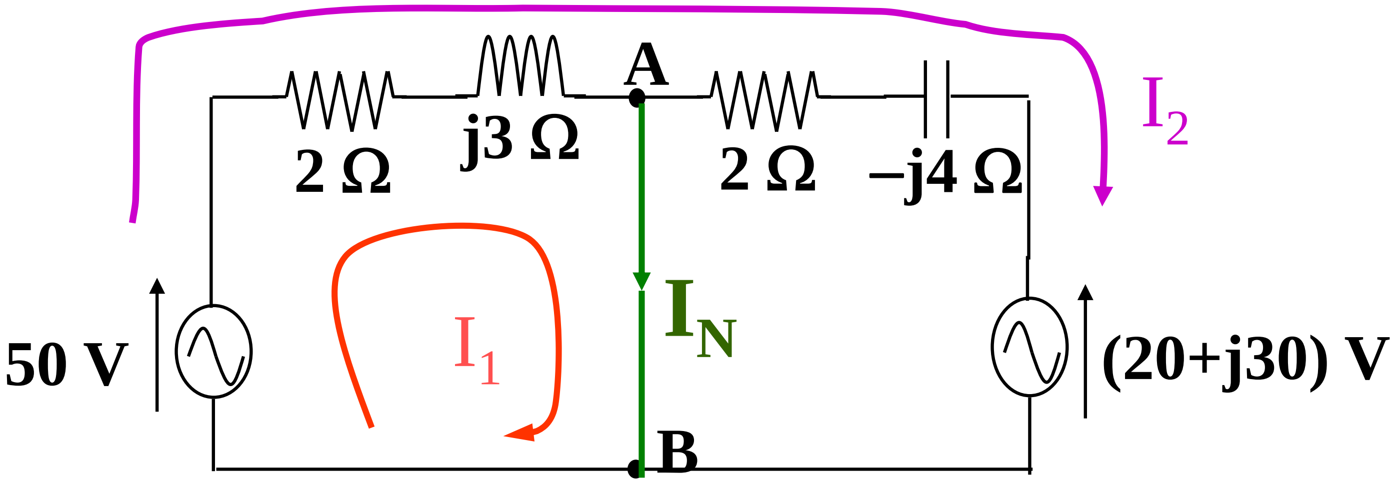
To find the Norton's equivalent circuit across AB, remove R_L such that terminals A & B are now open-circuit.

Example 1.12



Put a short circuit across terminals AB. The current I_N flows through the short circuit is the Norton's current source.

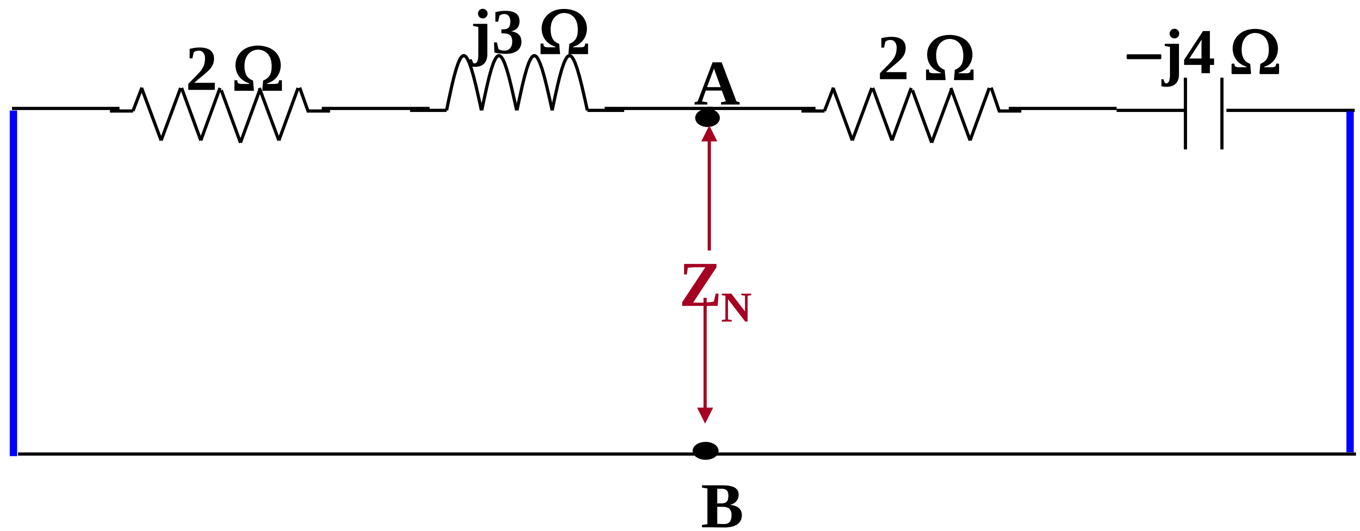
Example 1.12



$$\begin{bmatrix} 2+j3 & 2+j3 \\ 2+j3 & 4-j1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50+j0 \\ 50+j0-(20+j30) \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} 50 & 2+j3 \\ 30-j30 & 4-j1 \end{vmatrix}}{\begin{vmatrix} 2+j3 & 2+j3 \\ 2+j3 & 4-j1 \end{vmatrix}} = 5.84\angle -50.81^0 \text{ A}$$

Example 1.12

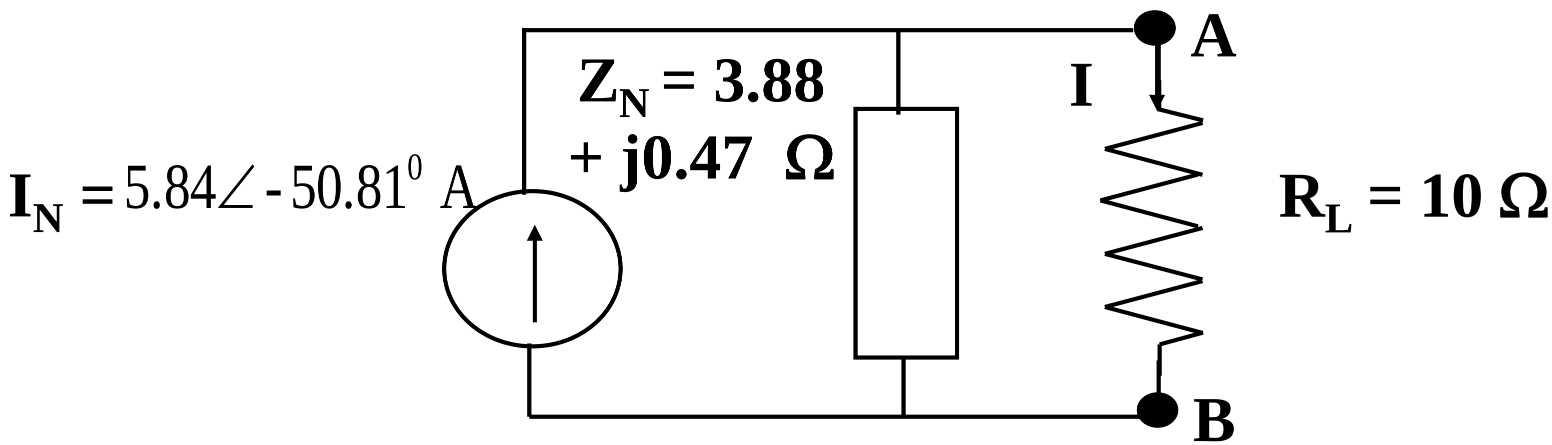


Norton's equivalent impedance Z_N is the impedance measured between A & B with all voltage sources replaced by short circuits, and current sources by open circuits.

$$Z_{AB} = Z_N = \frac{(2+j3)(2-j4)}{(2+j3)+(2-j4)} = (3.88+j0.47)\Omega$$

Example 1.12

Norton's equivalent circuit with load (R_L)

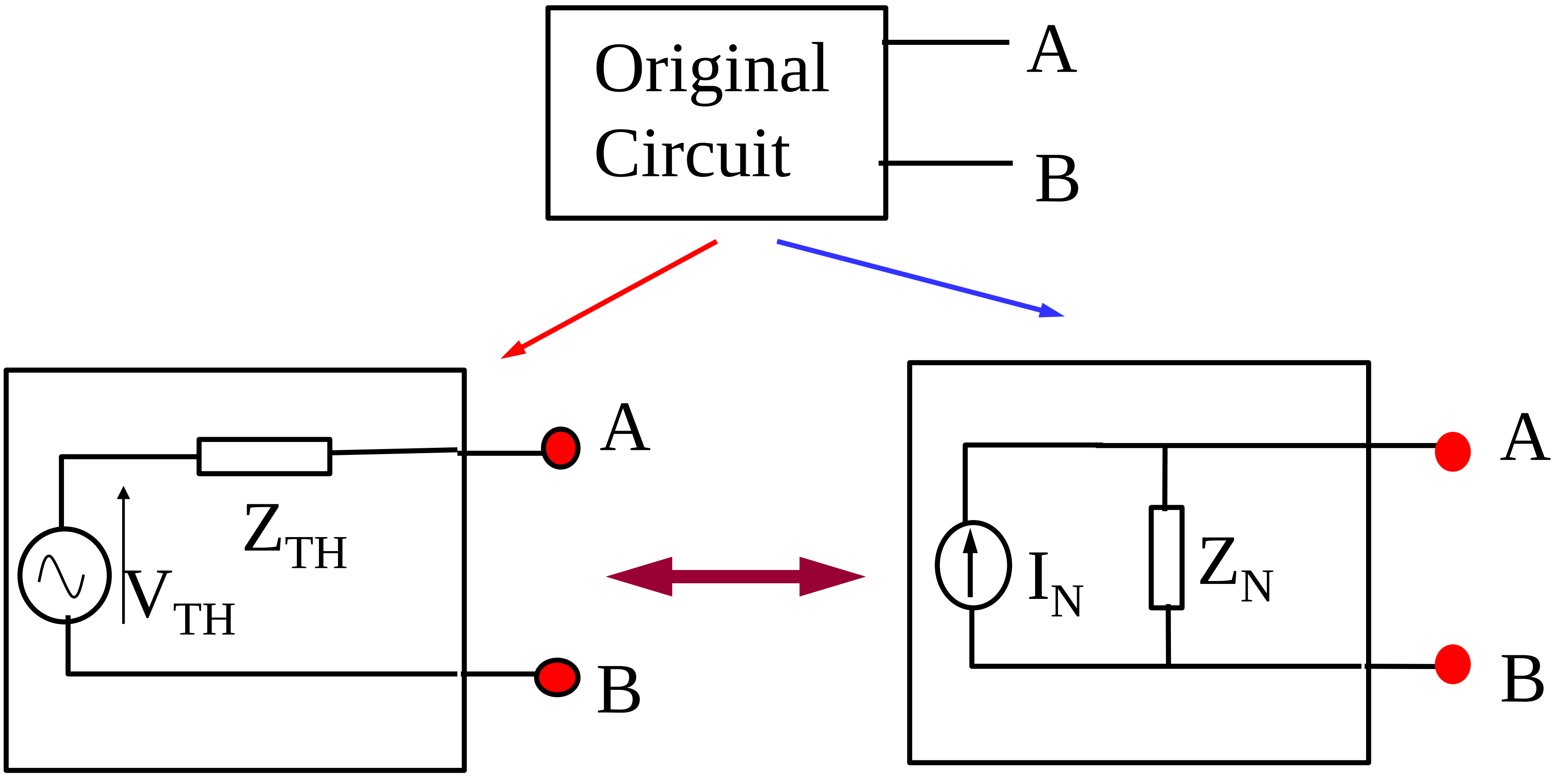


$$\text{Load current } I = \frac{I_N Z_N}{Z_N + R_L}$$

$$= \frac{5.84\angle-50.81^0 \times (3.88 + j0.47)}{13.88 + j0.47}$$

$$= 1.64\angle-45.84^0 \text{ A}$$

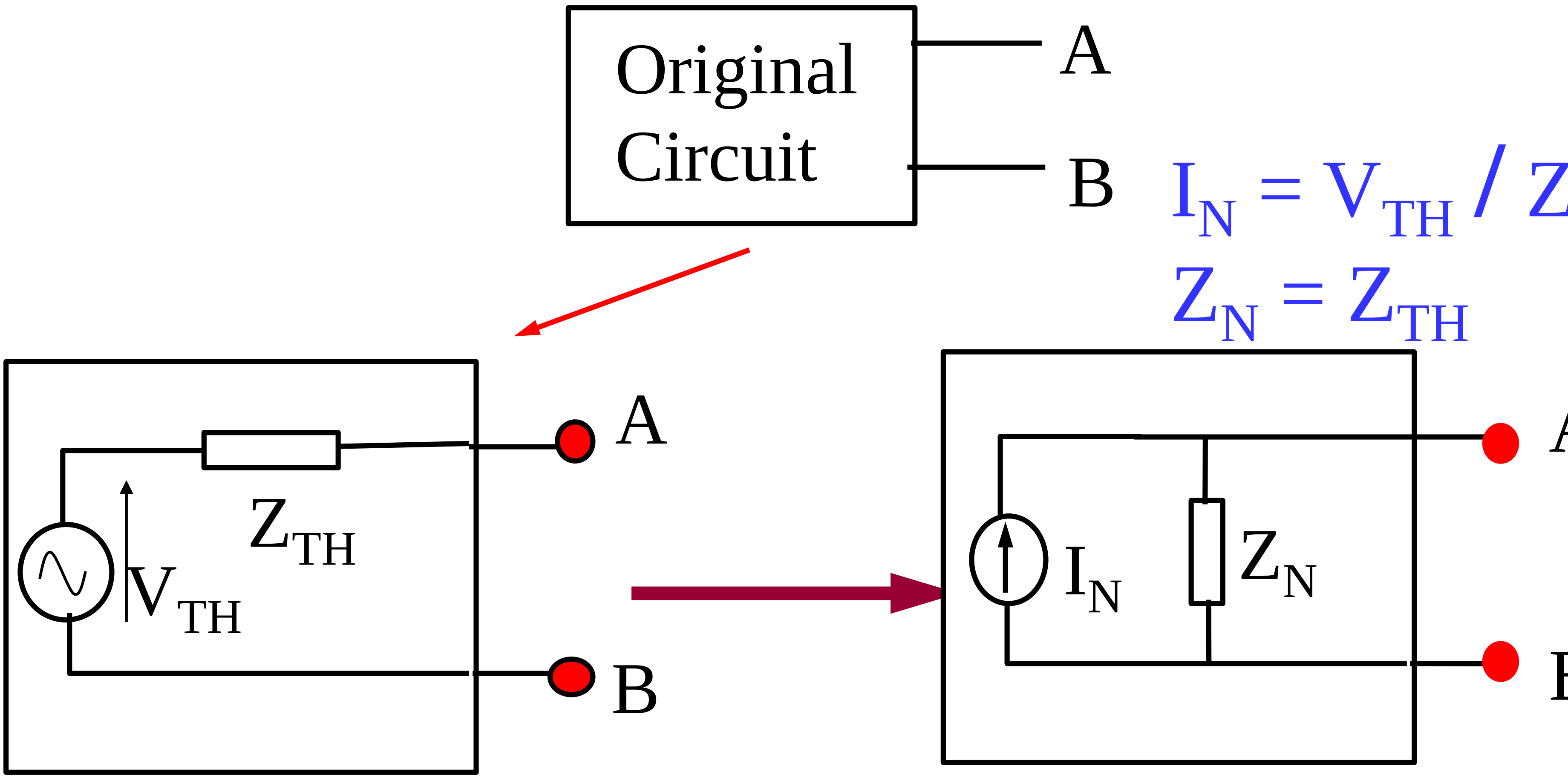
Relationship between Thevenin and Norton Equivalent Circuits



Thevenin's Equivalent Circuit

Norton's Equivalent Circuit

Relationship between Thevenin and Norton Equivalent Circuits



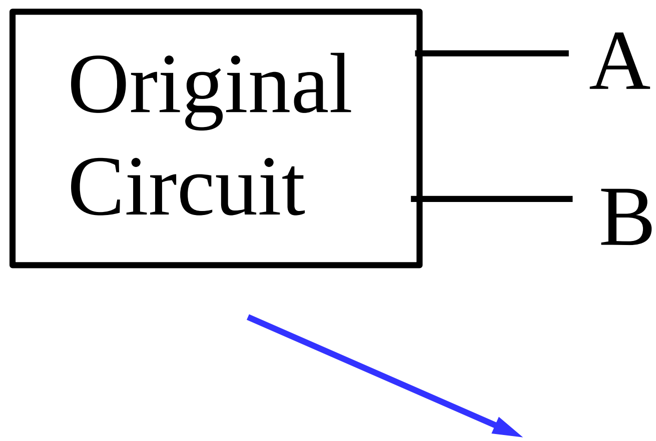
$I_N = V_{TH} / Z_{TH}$

$Z_N = Z_{TH}$

Thevenin's Equivalent Circuit

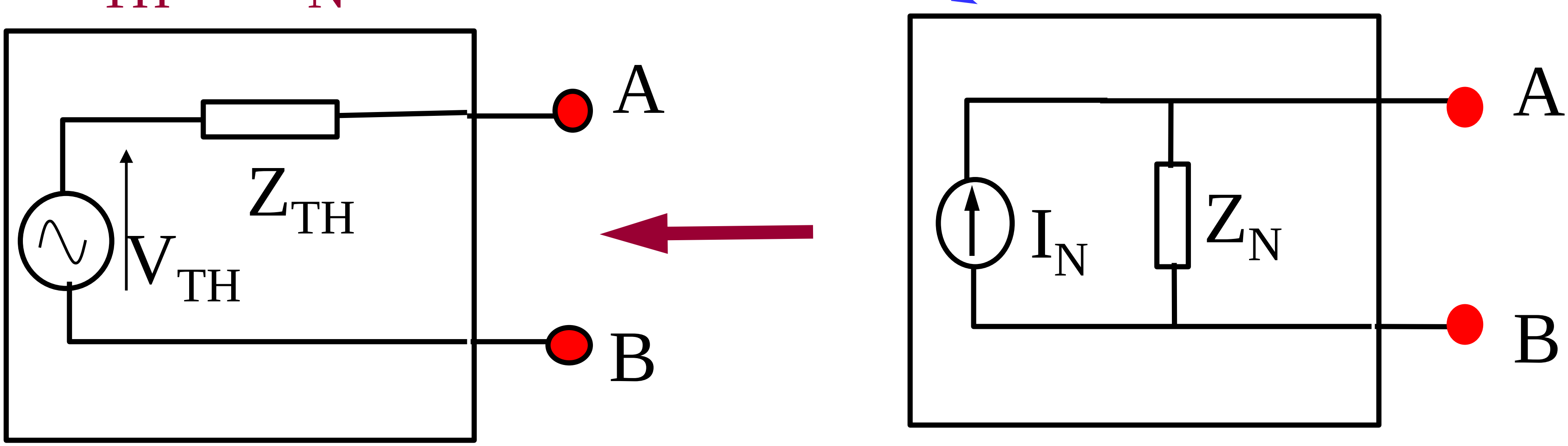
Norton's Equivalent Circuit

Relationship between Thevenin and Norton Equivalent Circuits



$V_{TH} = I_N \times Z_N$

$Z_{TH} = Z_N$

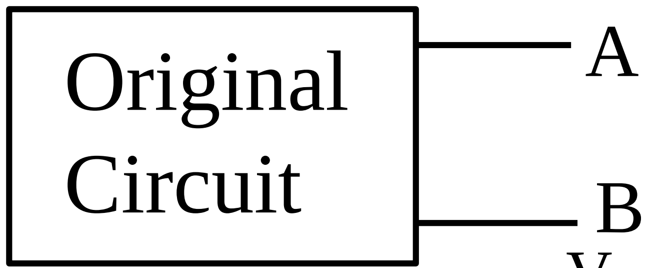


Thevenin's Equivalent Circuit

Norton's Equivalent Circuit

Relationship between Thevenin and Norton Equivalent Circuits

Example 1.10 & 1.12 (same circuit)



From Example 1.10,

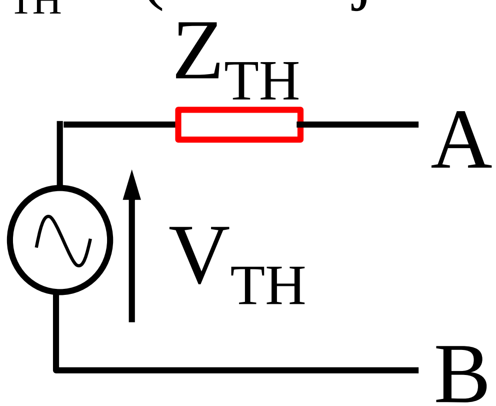
$$V_{TH} = 22.88\angle -43.96^0 \text{ V}$$

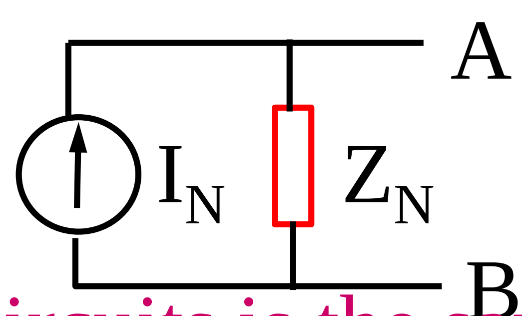
$$Z_{TH} = (3.88 + j0.47)\,\Omega$$

$$I_N = \frac{V_{TH}}{Z_{TH}} = \frac{22.88\angle -43.96^0}{3.88 + j0.47}$$

$$= \frac{22.88\angle -43.96^0}{3.91\angle 6.91^0} = 5.85\angle -50.87^0 \text{ A}$$

$$Z_N = Z_{TH} = (3.88 + j0.47)\Omega$$





Same answer as Example 1.12

The load current for both circuits is the same.

…next topic

Three Phase Supply Generation

Nurturing Curious Minds, Producing Passionate Engineers

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