Revision Tutorial

I. Partial Differentiation

MCQ

1. Which of the following is **TRUE**?

(a) The partial derivative $\frac{\partial z}{\partial x}$ represents the rate of change of $z = f(x,y)$ with respect to z.

(b) Suppose that $z = f(x,y,t)$ where $x = g(t)$ and $y = h(t)$, then $\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial z}{\partial t}$.

(c) The partial derivative of $f(x,y)$ with respect to y, written as $\frac{\partial f}{\partial y}$, is the derivative of $f(x,y)$, where y is treated as the constant and $f(x,y)$ is treated as a function of x alone.

(d) If A is a function of b and c and $\frac{\partial A}{\partial b} > 0$ implies that a decrease in b will cause in increase in A, when c is kept constant.

2. Given that $z = f(x,y)$. Which one of the following statements is **FALSE**?

(a) $\Delta z \approx \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y$

(b) $f_y(a,b) = \left.\frac{\partial z}{\partial x}\right|_{\substack{x=a \\ y=b}}$

(c) $\frac{\Delta z}{z} \times 100\% \approx \frac{1}{z}\left(\frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y\right) \times 100\%$

(d) $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$, where $x = g(t)$ and $y = h(t)$.

Structured questions

Basic Questions

1. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the functions below.

(a) $f(x,y) = x^5 + x^3 y^2 + 3xy^4$

(b) $f(x,y) = x^3 + 5x^2 y + 2y^3 + 6$

(c) $f(x,y) = x^3 y^2 + \frac{y}{x}$

2. (a) If $f(x,y) = \ln(xy)$, evaluate $f_x(1,2)$.

(b) If $h(x,y) = \left(1 + x^2 y\right)e^{3y}$, evaluate $h_y(1,0)$.

Intermediate and/or Challenging Questions

3. Find the first partial derivatives of the function.

(a) $f(r,s) = r \cdot \ln\left(r^2 + s^2\right)$ (b) $h(u,v) = \ln\sqrt{u^2 - v^2}$

(c) $z = x^2 \sin(xy) - 3y^3$

4. The diameter and height of a right circular cylinder are found by measurement to be 8 cm and 12.5 cm respectively, with possible error of +0.05 cm in each measurement. Use partial differentiation to find the possible approximate error in the computed volume.

5. The inductance L (microhenrys) of a certain wire in free space is

$$L = 0.00021\left(\ln\frac{2h}{r} - 0.75\right)$$

where h is the length (mm) of the wire and r (mm) is the radius of a circular cross section. Use partial differentiation to approximate L when $r = 2 \pm \frac{1}{16}$ mm and $h = 100 \pm \frac{1}{100}$ mm.

6. The radius r and height h of a right circular cylinder are measured with possible errors of 4% and 2% respectively. Use partial differentiation to approximate the possible percentage error in measuring the volume.

7. Electrical power P is given by $P = \frac{E^2}{R}$, where E is voltage and R is resistance. Approximate the percent error in calculating power if the percentage errors in measuring E and R are 2% and 3%, respectively.

II. Integration Techniques

MCQ

1. To find the integral $\int \frac{x-2}{\sqrt{x^2 - 4x + 1}}\,dx$ by substitution method, we should let

(a) $u = x - 2$ (b) $u = x^2 - 4x + 1$

(c) $u = 2x - 4$ (d) $u = x$

2. Which of the following integrals **cannot** be found using the substitution method?

(a) $\int \frac{1}{1+x^2}\,dx$ (b) $\int \frac{x}{1+x^2}\,dx$

(c) $\int x^2 e^{x^3}\,dx$ (d) $\int 4\cos^2 x \sin x\,dx$

3. To find $\int x\sqrt{x^2+1}\,dx$,

(a) let $u = x$ (b) let $u = \sqrt{x}$

(c) let $u = x+1$ (d) let $u = x^2+1$

4. The maximum number of partial fractions that $\dfrac{x^4-16}{(2x+1)^3(x^2-1)}$ can be expressed to is ____________________

(a) 2 (b) 3
(c) 4 (d) 5

5. The expression $\dfrac{x}{(x-2)(x+1)}$ (in partial fractions) is equivalent to ____________________

(a) $\dfrac{1}{3}\left[\dfrac{2}{x-2}-\dfrac{1}{x+1}\right]$ (b) $\dfrac{1}{3}\left[\dfrac{2}{x-2}+\dfrac{1}{x+1}\right]$

(c) $\dfrac{1}{3}\left[\dfrac{1}{x+1}-\dfrac{2}{x-2}\right]$ (d) $-\dfrac{1}{3}\left[\dfrac{2}{x-2}+\dfrac{1}{x+1}\right]$

6. $\dfrac{x+3}{(2x-1)(x^2+9)}$ can be expressed in the form ____________________

(a) $\dfrac{A}{2x-1}+\dfrac{B}{x+3}$ (b) $\dfrac{A}{2x-1}+\dfrac{B}{x+3}+\dfrac{C}{(x+3)^2}$

(c) $\dfrac{A}{2x-1}+\dfrac{Bx}{x^2+9}$ (d) $\dfrac{A}{2x-1}+\dfrac{Bx+C}{x^2+9}$

7. $\dfrac{x(3x-1)}{(x+1)(x^2+4)}$ can be expressed in the partial fractions as ____________________

(a) $\dfrac{A}{x+1}+\dfrac{B}{x^2+4}$ (b) $\dfrac{A}{x+1}+\dfrac{Bx}{x^2+4}$

(c) $\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+4}$ (d) $\dfrac{A}{x+1}+\dfrac{B}{x+2}+\dfrac{C}{(x+2)^2}$

8. To find $\int x\sec^2(5x)\,dx$ using 'integration by parts', we choose

(a) $u = x$ and $dv = \sec^2(5x)dx$ (b) $u = \sec^2(5x)$ and $dv = x\,dx$

(c) $u = xdx$ and $dv = \sec^2(5x)$ (d) $u = \sec^2(5x)dx$ and $dv = x$

Structured questions

Basic Questions

1. Find the following using appropriate methods:

(a) integrate functions of linear function:

(i) $\displaystyle\int \frac{1}{(2x-3)^5}\,dx$ (ii) $\displaystyle\int \sqrt{4-3x}\;\;dx$ (iii) $\displaystyle\int \frac{1}{8x+3}\,dx$

(b) integrate by using suitable substitutions:

(i) $\int x\left(x^2+1\right)^4 dx$, by letting $u = x^2+1$

(ii) $\int \sin^2 x \cos x \, dx$, by letting $u = \sin x$

(iii) $\int \frac{dx}{x \ln x}$, by letting $u = \ln x$

(iv) $\int \frac{5e^{2x}}{\sqrt{1-e^{2x}}} dx$, by letting $u = 1-e^{2x}$

(c) integrate by using partial fractions:

(i) $\int \frac{-x+7}{(x+3)(3x-1)} dx$ (ii) $\int \frac{x^2-6x+2}{(x+1)(2x-1)^2} dx$ (iii) $\int \frac{3s^2-s+8}{s(s^2+4)} ds$

(d) integrate by completing squares:

(i) $\int \frac{2}{x^2-2x+2} dx$ (ii) $\int \frac{1}{x^2-10x+50} dx$

(e) integrate by using trigonometric identities:

(i) $\int \sin 3x \cos 5x \, dx$ (ii) $\int \sin^2 2x \, dx$ (iii) $\int \cos^2 3x \, dx$

(f) integrate by parts:

(i) $\int (x^2+x) e^{2x} dx$ (ii) $\int x^2 \sin 3x \, dx$ (iii) $\int e^{5x} \cos 2x \, dx$

2. Evaluate the definite integrals with the appropriate integration techniques:

(a) functions of linear function:

(i) $\int_{4.5}^{10.5} \frac{2}{\sqrt{2x-5}} dx$ (ii) $\int_{-2/3}^{0} \frac{1}{e^{3x+2}} dx$

(b) substitution method:

(i) $\int_0^{1/2} y\sqrt{\frac{1}{4}-y^2}\, dy$, let $u = \frac{1}{4}-y^2$

(ii) $\int_1^2 \frac{e^{1/t}}{t^2} dt$, let $u = \frac{1}{t}$

(c) integration by parts:

(i) $\int_0^1 x e^{-5x} dx$ (ii) $\int_1^e x^2 \ln x \, dx$

3. Find the RMS (root-mean-square) value of the following functions:

(a) $y = 2x+1$ over the interval $1 \le x \le 4$

(b) $f(t) = 1+3e^{-t}$ over the interval $0 \le t \le 2$

(c) $y = 2(\sin t + \cos t)$ over the interval $0 \le t \le \pi$

[hint: $(\sin t + \cos t)^2 = \sin^2 t + 2\sin t \cos t + \cos^2 t = 1 + \sin 2t$]

Intermediate and/or Challenging Questions

4. Find the integrals

(a) $\int \frac{1}{\sqrt{x}+x}\,dx$ (b) $\int \sin^2\theta\cos 3\theta\,d\theta$

5. Evaluate the definite integrals

(a) $\int_0^4 \frac{4x}{\sqrt{2x+1}}\,dx$ (b) $\int_0^{\pi/2} \sin^4 x\,dx$

III. Simpson's Rule & Fourier Series

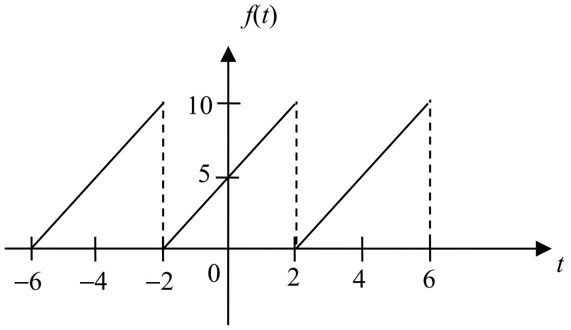
MCQ

1. The number of panels or strips to be considered in Simpson's rule must be ________.
   (a) Odd (b) Even

2. The exact solution of a definite integral can be obtained using the Simpson's rule.
   (a) True (b) False

3. A definite integral $\int_0^3 \sqrt{1-x^2}\ dx$ is evaluated using the Simpson's rule with 8 strips. Which of the following could be used to increase the accuracy of the final answer?
   (a) Evaluate the definite integral by integrating the function $\sqrt{1-x^2}$ and substituting the limits of integration.
   (b) Use the trapezoid method instead of Simpson's rule using the same number of strips.
   (c) Reduce the number of strips from 8 to 4.
   (d) Increase the number of strips from 8 to 16.

4.



In the figure above, $f(t)$ is a periodic function. The period of $f(t)$ is
   (a) 2 (b) 4
   (c) 6 (d) 10

5. The d.c. component a_0 of the trigonometric Fourier series of $f(t)$ (as shown in the figure in MCQ 4) is

(a) 0 (b) 2
(c) 5 (d) 10

6. For the given periodic function $f(t)=\begin{cases}2 & 0<t<2\\ 1 & 2<t<4\end{cases}$, $f(t+4)=f(t)$, which has a period $T=4$, the amplitude of the 2nd cosine component (a_2) of the Fourier series associated with $f(t)$ is
(a) 0 (b) 1
(c) -1 (d) $\dfrac{\pi}{2}$

7 The trigonometric Fourier series representation of the periodic function $f(t)$ of period 2π is given by $f(t)=\dfrac{4}{\pi^2}\left(\cos t+\dfrac{1}{9}\cos 3t+\dfrac{1}{25}\cos 5t+\cdots\right)+\dfrac{1}{\pi}\left(\sin t-2\sin 3t+3\sin 5t+\cdots\right)+\cdots$.
Then $f(t)$ is
(a) an even function (b) an odd function
(c) an odd function plus constant (d) a function with no symmetry

Structured Questions

Basic Questions

1. Estimate the following integrals by Simpson's rule, using the number of intervals indicated:

(a) $\int_0^1 \sqrt{1+x^3}\,dx$ (n = 8) (b) $\int_0^{\pi/2} \sqrt{\sin\theta}\,d\theta$ (n = 6)

(c) $\int_{\pi/2}^{3\pi/2} \dfrac{\sin x}{x}\,dx$ (n = 6) (d) $\int_0^2 e^{x^2}\,dx$ (n=4)

2. The table below gives the values of a current i (mA) flowing through a 33 μF capacitor at different instants of time t (s).

t (s)	0	0.2	0.4	0.6	0.8	1.0	1.2
i (mA)	0	0.198	0.380	0.496	0.476	0.310	0.117

By using Simpson's Rule, calculate the amount of charge (mC) stored in the capacitor from $t=0$ to $t=1.2$. (Hint: $q=\int i\,dt$)

3. A periodic function $f(t)$ of period 4 is defined as

$$f(t)=\begin{cases}t-1 & , \quad -1<t<1\\ 2 & , \quad 1<t<3\end{cases}$$

Find:
(a) the d.c. component (i.e. a_0)
(b) the second sine harmonic (i.e. $b_2\sin(2\omega t)$), and
(c) the third cosine harmonic (i.e. $a_3\cos(3\omega t)$) of the Fourier series of $f(t)$.

4. A periodic function $f(t)$ of period 2π is defined as

$$f(t)=\begin{cases}0, & -\pi \le t < -\dfrac{\pi}{2} \\ 4, & -\dfrac{\pi}{2} \le t < \dfrac{\pi}{2} \\ 0, & \dfrac{\pi}{2} \le t < \pi\end{cases}, \qquad f(t+2\pi)=f(t).$$

(a) Find the d.c. component (i.e. a_0).

(b) If $a_n=\dfrac{8}{n\pi}\sin\left(\dfrac{n\pi}{2}\right)$, find a_1, a_2 and a_3.

(c) Given that $f(t)$ is an even function, use the results from parts (a) and (b), write down the Fourier series of $f(t)$ as far as the 3rd harmonic.

Intermediate and/or Challenging Questions

5. Sketch one cycle of the function

$$f(t)=\begin{cases}-0.5 & , \quad -2<t<-1 \\ 0.5 & , \quad -1<t<1 \\ -0.5 & , \quad 1<t<2\end{cases} \qquad \text{and} \quad f(t+4)=f(t)$$

Is $f(t)$ an even function?

6. A periodic function $f(t)$ of period 4 is defined over one period as

$$f(t)=\begin{cases}t+2 & -2<t<-1 \\ 0 & -1<t<1 \\ t-2 & 1<t<2\end{cases}$$

(a) Sketch the graph of $f(t)$ for the interval $-2<t<2$, hence determine whether it is even, odd or neither.

(b) Find the Fourier series of $f(t)$ up to and including the third harmonic.

IV. 1st ODE & Applications

MCQ

1. Which of the following differential equations cannot be solved by separating the variables?

(a) $\dfrac{dy}{dx}=\dfrac{y}{x}$ (b) $\dfrac{dy}{dx}=\dfrac{x}{y}$

(c) $\dfrac{dy}{dx}=xy$ (d) $\dfrac{dy}{dx}=x+y$

2. Which of the following differential equations can be solved by separating the variables?

(a) $\frac{dy}{dx} = \frac{xe^x \sin y}{\cos y}$ (b) $\frac{dy}{dx} = \frac{x^2 + x - 1}{xe^y - \sin y}$

(c) $\frac{dy}{dx} = \frac{e^{x^2}}{\tan y}$ (d) $\frac{dy}{dx} = \frac{x+y}{x-y}$

3. Which of the following is not a solution to the differential equation $\frac{dy}{dx} = ky$, where k is a constant?

(a) $\ln y = kx + c$ (b) $y = ce^x + k$

(c) $\ln(cy) = kx$ (d) $y = ce^{kx}$

4. The expression $e^{\frac{1}{2}\ln|1+x|}$ can be simplified as

(a) $\frac{1}{2}(1+x)$ (b) $\sqrt{1+x}$

(c) $e^{\frac{1}{2}}(1+x)$ (d) $\frac{1}{\sqrt{1+x}}$

5. Reduce $x\frac{dy}{dx} - \frac{y}{x^2} = \ln x$ to linear form and identify $P(x)$ and $Q(x)$.

(a) $P(x) = -\frac{1}{x^2}$ and $Q(x) = \ln x$ (b) $P(x) = -\frac{1}{x^3}$ and $Q(x) = \ln x$

(c) $P(x) = -\frac{1}{x^2}$ and $Q(x) = \frac{\ln x}{x}$ (d) $P(x) = -\frac{1}{x^3}$ and $Q(x) = \frac{\ln x}{x}$

6. Given $f'(x) = x^3 f(x)$, $f(0) = 1$. Then $f(1) =$ ____________________.

(a) e (b) $\sqrt[4]{e}$

(c) $\sqrt{e}$ (d) $\frac{e}{4}$

Structured Questions

Basic Questions

1. Solve the following differential equations by separating the variables:

(a) $\frac{dy}{dx} = \frac{y^2}{4x^2+1}$ (b) $\left(y^2 + 3y\right)\frac{dy}{dx} = y\sin 3x \cos x$

(c) $\left(x^2 + 9\right)\frac{dy}{dx} = \sin(2y)$ (d) $xy\frac{dy}{dx} + 1 - y^2 = 0$

(e) $\left(1 + x^2\right)\frac{dy}{dx} = xy$ (f) $\frac{dy}{dx} - x^2 + 1 = 0$

(g) $2x^2 y\dfrac{dy}{dx} = -(y+1)$, $y(1) = 0$

2. Solve the following differential equations by using the integrating factor

(a) $\dfrac{dy}{dx} + 2y = e^x$ (b) $x\dfrac{dy}{dx} + y = 2x$, $y(1) = 2$

(c) $(x+1)\dfrac{dy}{dx} + y = \dfrac{x+1}{x+3}$ (d) $\dfrac{dy}{dx} + 2y = e^{4x-1}$, $y\left(\dfrac{1}{6}\right) = 0$

(e) $\dfrac{dy}{dx} + \dfrac{3y}{x} = 6x^2$ (f) $\dfrac{dy}{dx} + 3y = 3x^2 e^{-3x}$

Intermediate and/or Challenging Questions

3. Solve the following differential equations

(a) $\cos y + (1+e^{-x})\sin y\dfrac{dy}{dx} = 0$, $y(0) = \dfrac{\pi}{4}$

(b) $y' + \dfrac{y}{x} - \sin^2 x = 0$

(c) $\dfrac{dy}{dx} + 5x = x - xy$, $y(0) = 1$

4. A cup of boiling coffee is allowed to cool in a room where the temperature is maintained constant at $25°C$. The cooling process follows Newton's law of cooling. If after 2 minutes, the coffee temperature is dropped to $80°C$.
(a) Set up the differential equation that depicts the cooling process of the coffee;
(b) Find the particular solution of the differential equation in part (a);
(c) Find the coffee temperature after 8 minutes.

5. If a body cools from $100°C$ to $80°C$ in 10 minutes in air, which is maintained at $20°C$. The cooling process follows Newton's law of cooling.
(a) Set up the differential equation that depicts the cooling process of the body.
(b) Solve the equation in part (a) using given conditions.
(c) How long will it takes the body to cool down from $80°C$ to $60°C$?

6. A voltage source is connected in series with a resistor and a capacitor. The charge q on the capacitor satisfies the differential equation

$$R\frac{dq}{dt} + \frac{q}{C} = E$$

where $R = 1\text{K}\Omega$, $C = 1\mu\text{F}$ and $E = 10\text{V}$.
If the initial charge on the capacitor is zero, find
(i) the charge and current at any time t.
(ii) the voltage across the resistor when $t = 5\,\text{ms}$.

V. Laplace Transform & Inverse Laplace Transform

MCQ

1. $\mathcal{L}\left\{e^{-3t-5}\right\}$ is equal to

(a) $\dfrac{1}{e^3(s+5)}$ (b) $\dfrac{e^3}{s+5}$

(c) $\dfrac{1}{e^5(s+3)}$ (d) $\dfrac{e^5}{s+3}$

2. $\mathcal{L}\left\{\left(1-e^{-t}\right)\cos 2t\right\}$ is equal to

(a) $\left(\dfrac{1}{s}-\dfrac{1}{s+1}\right)\left(\dfrac{s}{s^2+4}\right)$ (b) $\dfrac{s}{s^2+4}-\dfrac{s}{(s+1)\left(s^2+4\right)}$

(c) $\dfrac{s}{s^2+4}-\dfrac{s}{(s+1)^2+4}$ (d) $\dfrac{s}{s^2+4}-\dfrac{s+1}{(s+1)^2+4}$

3. $\mathcal{L}\left\{\dfrac{d}{dt}\left(e^t\cos 2t\right)\right\}$ is equal to

(a) $\dfrac{s}{(s-1)^2+4}$ (b) $\dfrac{s-1}{(s-1)^2+4}$

(c) $\dfrac{s-1}{(s-1)^2+4}-1$ (d) $\dfrac{s(s-1)}{(s-1)^2+4}-1$

4. If $f(t)=te^{3t}$ and $\mathcal{L}\{f(t)\}=F(s)$, then $\mathcal{L}^{-1}\{F(s+5)\}=$____________.

(a) te^{-t} (b) te^{t}

(c) te^{-2t} (d) te^{2t}

5. The function $f(t)=\begin{cases}1 & 1\le t<2\\ 0 & \text{otherwise}\end{cases}$ has the following Laplace transform:

(a) $\int_0^\infty e^{-st}\,dt$ (b) $\int_0^\infty te^{-st}\,dt$

(c) $\int_1^2 e^{-st}\,dt$ (d) $\int_1^2 te^{-st}\,dt$

6. If $\mathcal{L}\{f(t)\}=\dfrac{s}{(s-1)^2+1}+\dfrac{1}{(s-1)^2+1}$, then $f(t)=$____________ .

(a) $e^t(\cos t+\sin t)$ (b) $e^{-t}(\cos t+\sin t)$

(c) $e^t(\cos t+2\sin t)$ (d) $e^{-t}(\cos t+2\sin t)$

Structured Questions

Basic Questions

1. Find the following Laplace transforms:

(a) $\mathcal{L}\left\{4-9e^{-4t}\right\}$ (b) $\mathcal{L}\left\{5t^3+3\sin 2t\right\}$ (c) $\mathcal{L}\left\{t\,e^{2t}\cos 5t\right\}$

(d) Expand $(t+1)(t+2)$, hence find $\mathcal{L}\{(t+1)(t+2)\}$

(e) Express e^{2t+3} as a product using laws of indices, hence find $\mathcal{L}\left\{e^{2t+3}\right\}$

(f) Use compound angle formula to expand $\sin\left(t+\dfrac{\pi}{6}\right)$, hence find $\mathcal{L}\left\{\sin\left(t+\dfrac{\pi}{6}\right)\right\}$.

(g) Use reducing power formula to simplify $\cos^2 3t$, hence find $\mathcal{L}\left\{t\cos^2 3t\right\}$

(h) Use product to sum formula to simplify $\sin 2t\sin 5t$, hence find $\mathcal{L}\{t\sin 2t\sin 5t\}$

2. Find the following inverse Laplace transforms:

(a) $\mathcal{L}^{-1}\left\{\dfrac{2}{s}-\dfrac{8}{s^3}+\dfrac{16}{s^5}\right\}$ (b) $\mathcal{L}^{-1}\left\{\dfrac{1}{s+6}-\dfrac{3s}{s^2+25}+\dfrac{1}{s^2+49}\right\}$

(c) $\mathcal{L}^{-1}\left\{\dfrac{s^2-100}{\left(s^2+100\right)^2}-\dfrac{4s}{\left(s^2+81\right)^2}\right\}$ (d) $\mathcal{L}^{-1}\left\{\dfrac{1}{2s-3}\right\}$

(e) Rewrite $\dfrac{3(1+s)}{s^5}$ as sum of two fractions, hence find $\mathcal{L}^{-1}\left\{\dfrac{3(1+s)}{s^5}\right\}$

(f) Rewrite $\dfrac{3s+2}{s^2+36}$ as sum of two fractions, hence find $\mathcal{L}^{-1}\left\{\dfrac{3s+2}{s^2+36}\right\}$

(g) Find $\mathcal{L}^{-1}\left\{\dfrac{6}{s^3}\right\}$, hence use first shift theorem to find $\mathcal{L}^{-1}\left\{\dfrac{6}{(s-1)^3}\right\}$

(h) Find $\mathcal{L}^{-1}\left\{\dfrac{3}{s^2+9}\right\}$, hence use first shift theorem to find $\mathcal{L}^{-1}\left\{\dfrac{3}{(s-2)^2+9}\right\}$

(i) Find $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+25}\right\}$, hence use first shift theorem to find $\mathcal{L}^{-1}\left\{\dfrac{s+2}{(s+2)^2+25}\right\}$

(j) By partial fraction, find $\mathcal{L}^{-1}\left\{\dfrac{s^2-2s+3}{s(s-1)(s-2)}\right\}$

(k) By partial fraction, find $\mathcal{L}^{-1}\left\{\dfrac{s^2+1}{(s-1)(s^2+2)}\right\}$

Intermediate Questions

3. Find the following:

(a) $\mathcal{L}\left\{\dfrac{dv}{dt}+3v-13\sin 2t\right\},\quad v(0)=6$

(b) $\mathcal{L}\left\{\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}+5y-e^{-2t}\cos 3t\right\},\quad y(0)=1,\quad y'(0)=-2$

(c) $\mathcal{L}^{-1}\left\{\dfrac{s-1}{s^2-4s+20}\right\}$

(d) $\mathcal{L}^{-1}\left\{\dfrac{s+2}{s^2+6s+9}\right\}$

VI. 2nd ODE & Applications

MCQ

1. If the differential equation $4\dfrac{d^2y}{dx^2}+8\dfrac{dy}{dx}+ky=0$ has a general solution of the form $y=e^{\alpha x}\left[A\cos(\beta x)+B\sin(\beta x)\right]$, where α, β, A and B are constants, then the value of the constant k is ________.

(a) <4
(b) ≤ 4
(c) >4
(d) ≥ 4

2. $\mathcal{L}\left\{\dfrac{d^3y}{dt^3}\right\}$ is given by

(a) $s^3\mathcal{L}\{y\}-s^2y''(0)-sy'(0)-y(0)$
(b) $s^3\mathcal{L}\{y\}-s^2y'(0)-sy''(0)-y'''(0)$
(c) $s^3\mathcal{L}\{y\}-s^2y(0)-sy'(0)-y''(0)$
(d) $s^3\mathcal{L}\{y\}-s^2y(0)-sy'''(0)-y''(0)$

3. If the motion of an engineering system is described by $y=\dfrac{1}{2}[e^{-2t}\cos(t)+3e^{-2t}\sin(t)-e^{-t}]$, the motion is considered ___________.

(a) un-damped
(b) under-damped
(c) critically-damped
(d) over-damped

Structured Questions

Basic Questions

1. Find the general solution to each differential equation, using auxiliary equation method:

(a) $\dfrac{d^2y}{dx^2}-\dfrac{dy}{dx}-6y=0$
(b) $\dfrac{d^2y}{dx^2}-4\dfrac{dy}{dx}+4y=0$
(c) $\dfrac{d^2y}{dx^2}+4\dfrac{dy}{dx}+13y=0$

2. By auxiliary equation method, find the particular solution for each differential equation below, using the given boundary conditions:

(a) $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=0$, $y(0)=5$ and $y'(0)=-9$
(b) $\dfrac{d^2y}{dx^2}-4y=0$, $y(0)=1$ and $y'(0)=-1$

(c) $2\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=0$, $y(0)=1$ and $y'(0)=1$

(d) $\frac{d^2y}{dt^2}-2\frac{dy}{dt}=4$, $y(0)=-1$, $y'(0)=2$

Intermediate and/or Challenging Questions

3. Solve the following differential equation using Laplace transform method:
$q'' + 9q = 0$, where $q(0) = 0$ and $q'(0) = 2$

4. (a) Resolve $\frac{8}{(s+2)^2(s^2+4)}$ into partial fractions of the form $\frac{A}{s+2}+\frac{B}{(s+2)^2}+\frac{Cs+D}{s^2+4}$.

(b) Hence, use the result from part (a) to solve the differential equation for v(t):
$v'' + 4v' + 4v = 4\sin 2t$, where $v(0) = 1$ and $v'(0) = 0$

5. A mass of 0.6 kg is attached to the lower end of a vertical spring of stiffness 200 N/m. The mass is raised 3 cm above the equilibrium position, i.e. $x(0)=-3$ cm, and released from rest, i.e. $v(0)=x'(0)=0$ cm/s. Assuming no air resistance,
(a) describe the motion of the mass;
(b) set up the differential equation to model the displacement $x(t)$, and indicate clearly the initial conditions;
(c) find the position of the mass 5 seconds after it is released; and
(d) determine the frequency of the motion. ($g = 10$ m/s^2)

6. A mass of 10 kg is suspended from a spring of spring constant 300 N/m. The mass is pushed up 15 cm above its equilibrium position and released from rest. Assuming there is no damping force,
(a) set up the differential equation to model the displacement $x(t)$, and indicate clearly the initial conditions ;
(b) find the position of the mass after 1 second ;
(c) the amplitude, period and frequency of the vibration.

7. A 1 kg mass is attached to the lower end of a vertical spring of stiffness 25 N/m. The mass is set into motion from rest at the equilibrium position by an external force $F(t) = \sin(5t)$ (N). If the resistance to the motion is numerically equal to $8v$ (N) where v (m/s) is the velocity of the mass at time t (s),
(a) set up the differential equation to model the displacement $x(t)$, and indicate clearly the initial conditions ;
(b) find the displacement x (m) of the mass at any time t (s) ,
(c) indicate the amplitude of the steady-state vibration of the mass ;
(d) what is the ratio of the displacement in the steady-state motion to that in the transient-state motion when $t = 0.5$s?

8. A spring has a spring constant of 125 Nm^{-1}. A mass of 5 kg is suspended from the spring and, after it has come to equilibrium, is pulled down 20 cm and released from rest. Assuming that there is a damping force numerically equal to $30v$, where v (m/s) is the instantaneous velocity at time t (s),

(a) set up the differential equation to model the displacement $x(t)$, and indicate clearly the initial conditions ;

(b) find the position and the velocity of the mass at any time.

9. Find the charge on the capacitor in the *RLC*-series circuit when $L = 0.25$ H, $R = 20\ \Omega$, $C = \frac{1}{300}$ F, $E(t) = 0$ V, $q(0) = 4$ C and $q'(0) = 0$.

10. In a RLC circuit, it is known that R = 10 ohms, L = 5/3 henry, C = 1/30 farad, and the electromotive force $E(t) = 300$ volts. If initially, there is no current flowing thru the circuit, and the rate of change of the current is 180 amp/sec,

(a) set up the differential equation to model the current in the circuit, and indicate clearly the initial conditions;

(b) hence, find the current $i(t)$.

11. In a RLC circuit, it is known that R = 10 ohms, L = 0.5 henry, C = 0.01 farad, and the electromotive force $E(t) = 150$ volts. If initially, the charge on the capacitor is 1 coulomb, and there is no current,

(a) set up the differential equation to model the charge on the capacitor, and indicate clearly the initial conditions;

(b) hence, find the charge $q(t)$.