Chapter 7 – Integration

Objectives:

1. *Define integration is the reverse of differentiation.*
2. *State the standard integrals.*
3. *Integrate functions involving algebraic, trigonometric, exponential and integration leading to inverse trigonometric functions.*
4. *Obtain y from* $\frac{dy}{dx} = f(x)$ or $\frac{dx}{dy} = f(y)$*, and relate it to practical problems in engineering.*

7.1 Introduction

So far, when given a function $y = f(x)$, we can differentiate it to obtain the derivative $\frac{dy}{dx}$. This procedure is called **differentiation**. In this chapter, we will look at the reverse process, that is, obtaining the function $y = f(x)$ given the derivative $\frac{dy}{dx}$. This reverse process is called **integration**. Take note that the process of integration is not as straightforward as differentiation. And in this chapter, we will only deal with some standard integrals.

7.2 Integration as the reverse of differentiation

Consider the derivative $\frac{dy}{dx} = 2x$. We will now try to find a function $y \equiv y(x)$ whose derivative is $2x$. One answer is $y = x^2$, since $\frac{dy}{dx} = \frac{d}{dx}\left(x^2\right) = 2x$.

However there are other answers too, such as $y = x^2 + 1$, $y = x^2 - 10$, and so on since

$$\frac{d}{dx}\left(x^2 + 1\right) = \frac{d}{dx}\left(x^2\right) + \frac{d}{dx}(1) = 2x + 0 = 2x,$$

$$\frac{d}{dx}(x^2 - 10) = \frac{d}{dx}(x^2) - \frac{d}{dx}(10) = 2x + 0 = 2x$$

In fact, there are infinitely many answers of the form $y = x^2 + C$ where C is a constant, as we know that $\frac{d}{dx}(C) = 0$. We say that the function $2x$ has been integrated to obtain another function $x^2 + C$ where C is a constant. And we express this as $\int 2x\,dx = x^2 + C$.
Since there is an arbitrary constant C in the expression $x^2 + C$, we say that this expression is an **indefinite integral**.

In general terms the operation of integration can be summarised as follows:

If $\dfrac{d}{dx}F(x) = f(x)$,

then $\int f(x)\,dx = F(x) + C$

where

- $\int f(x)\,dx$ is called the **indefinite integral** of $f(x)$ with respect to (w.r.t.) x,
- $f(x)$ is called the **integrand**,
- $F(x)$ is called an **anti-derivative** of the function $f(x)$, and
- C is called the **constant of integration**.

Since integration and differentiation are inverse processes of each other, we have

$$\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x) \qquad \text{and} \qquad \int\left[\frac{d}{dx}f(x)\right]dx = f(x)$$

7.3 Integration of Power Functions

We will now establish a rule for $\int x^n dx$ where n is a constant and $n \neq -1$.

Using differentiation, we have $\dfrac{d}{dx}\left(\dfrac{x^{n+1}}{n+1}\right) = \dfrac{1}{n+1}\dfrac{d}{dx}\left(x^{n+1}\right) = \dfrac{1}{n+1}(n+1)x^n = x^n$

This means that $\boxed{\int x^n dx = \dfrac{x^{n+1}}{n+1} + C}$

Important:

The above integration formula $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$ is invalid when $n = -1$. This means that the formula cannot be used to find $\int \dfrac{1}{x}\,dx$.

However we know that $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$ for $x > 0$. Hence, applying the definition of integration, we can write $\int \dfrac{1}{x}\,dx = \ln x + C$ for $x > 0$. To take care of the general case where x can be positive or negative, we have the following general result

$\boxed{\int \dfrac{1}{x}\,dx = \ln|x| + C}$ where $|x|$ is the absolute value of x.

7.4 Integration of Constants

We wish to establish a formula for $\int k\,dx$ where k is a constant.

From $k = k(1) = kx^0$, we have

$$\int k\,dx = \int kx^0\,dx = k\int x^0\,dx = k\left(\frac{x^{0+1}}{0+1}\right) + C = kx + C$$

Therefore we have $\boxed{\int k\,dx = kx + C}$

Example 1 : Find the following integrals.

(a) $\int 3x^2 dx$ (b) $\int \frac{1}{x^2}\,dx$ (c) $\int \frac{1}{u}\,du$

7.5 Integration of Trigonometric Functions

7.5.1 Standard Integration Formula for Trigonometric Functions

From $\frac{d}{dx}(\sin x) = \cos x$, we have $\boxed{\int \cos x\,dx = \sin x + C}$

Similarly, the following differentiation formulas give rise to the following corresponding integration formulas (where x is in radians) :

$$\frac{d}{dx}(\cos x) = -\sin x \quad \rightarrow \quad \boxed{\int \sin x\,dx = -\cos x + C}$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \quad \rightarrow \quad \boxed{\int \sec^2 x\,dx = \tan x + C}$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x \quad \rightarrow \quad \boxed{\int \csc^2 x\,dx = -\cot x + C}$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \quad \rightarrow \quad \boxed{\int \sec x \tan x\,dx = \sec x + C}$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x \quad \rightarrow \quad \boxed{\int \csc x \cot x\,dx = -\csc x + C}$$

7.5.2 Integration for Trigonometric Functions of Multiple Angles

What about the indefinite integral $\int \cos 2x\,dx$? What is the rule for integration when there is a coefficient (in this case 2) attach to the variable x?

Our knowledge of differentiation tells us that

$$\frac{d}{dx}(\sin 2x) = 2\cos 2x \text{ or } \frac{d}{dx}\left(\frac{\sin 2x}{2}\right) = \cos 2x .$$

This latter result shows us that.

$$\int \cos 2x\,dx = \frac{1}{2}\sin 2x + C$$

In general, if m is a nonzero constant, then $\frac{d}{dx}\left(\frac{\sin mx}{m}\right) = \cos mx$ and therefore

$$\boxed{\int \cos mx\,dx = \frac{\sin mx}{m} + C}$$

Similarly, we can obtain the following integration formulae (m is a nonzero constant.):

$$\boxed{\int \sin mx\,dx = -\frac{\cos mx}{m} + C}$$

$$\boxed{\int \sec^2 mx\,dx = \frac{\tan mx}{m} + C}$$

$$\boxed{\int \csc^2 mx\,dx = -\frac{\cot mx}{m} + C}$$

$$\boxed{\int \sec mx \tan mx\,dx = \frac{\sec mx}{m} + C}$$

$$\boxed{\int \csc mx \cot mx\,dx = -\frac{\csc mx}{m} + C}$$

Example 2 : Find:

(a) $\int \cos(\pi x)\,dx$

(b) $\int \sin\left(\frac{x}{2}\right)dx$

(c) $\int \sec(\pi x)\tan(\pi x)\,dx$

7.6 Integration of Exponential Functions

Let a be a positive constant and $a \neq 1$. From the differentiation formula:

$$\frac{d}{dx}a^x = a^x \ln a \qquad \text{or} \qquad \frac{d}{dx}\left(\frac{a^x}{\ln a}\right) = a^x$$

we obtain the integration formula $\boxed{\int a^x dx = \frac{a^x}{\ln a} + C}$

For most applications, the positive constant a is equal to the base of the natural logarithms e . Using the above result, we have

$$\boxed{\int e^x dx = \frac{e^x}{\ln e} + C = e^x + C} \qquad \text{where } \ln e = 1$$

If we substitute x by mx where m is a nonzero constant, the integration formulas become:

$$\boxed{\int a^{mx} dx = \int \left(a^m\right)^x dx = \frac{\left(a^m\right)^x}{\ln\left(a^m\right)} + C = \frac{a^{mx}}{m \ln a} + C}$$

and

$$\boxed{\int e^{mx} dx = \frac{e^{mx}}{m} + C}$$

Example 3 : Find: (a) $\int 2^{3x} dx$ (b) $\int e^{2x} dx$

7.7 Integration Leading to Inverse Trigonometric Functions

Recall that $\dfrac{d}{dx}\left[\sin^{-1} x\right] = \dfrac{1}{\sqrt{1-x^2}}$

Hence, $\dfrac{d}{dx}\left[\sin^{-1}\dfrac{x}{a}\right] = \dfrac{1}{\sqrt{1-\left[\dfrac{x}{a}\right]^2}}\left[\dfrac{1}{a}\right]$, where a is a constant.

$$= \sqrt{\frac{a^2}{a^2-x^2}}\left[\frac{1}{a}\right] \qquad = \frac{1}{\sqrt{a^2-x^2}}$$

Since integration is the reverse process of differentiation, we can use the above result to get

$$\boxed{\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left[\frac{x}{a}\right] + C}$$ where a is any constant.

Similarly, $\dfrac{d}{dx}\left[\tan^{-1}\dfrac{x}{a}\right] = \dfrac{1}{1+\left[\dfrac{x}{a}\right]^2}\left[\dfrac{1}{a}\right]$

$$= \frac{a^2}{a^2+x^2}\left[\frac{1}{a}\right] \qquad = \frac{a}{a^2+x^2}$$

$$\frac{d}{dx}\left[\frac{1}{a}\tan^{-1}\frac{x}{a}\right] = \frac{1}{a^2+x^2}$$

Thus,

$$\boxed{\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left[\frac{x}{a}\right] + C}$$ where a is any constant.

Example 4 : Find : (a) $\displaystyle\int \frac{dx}{\sqrt{4-x^2}}$ (b) $\displaystyle\int \frac{dt}{7+t^2}$

7.8 Rules of Integration

Indefinite integrals have the following properties:

(i) $\int k\, f(x)\,dx = k\int f(x)\,dx + C$, where k is a constant.

(ii) $\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

(iii) $\int [f(x)-g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx$

In other words, the process of integration satisfies the linearity property

$$\int [a\,f(x)+b\,g(x)]\,dx = a\int f(x)\,dx + b\int g(x)\,dx$$

where a and b are constants.

Example 5 : Find :

(a) $\int \left(\sqrt{x}+\frac{3}{x}\right)dx$

(b) $\int \left(2+3x^4-\frac{3}{\sqrt{x}}\right)dx$

(c) $\int (\pi+\sin 2x)\,dx$

(d) $\int \left(5\sec^2 3\theta+\frac{2}{\theta^2}\right)d\theta$

(e) $\int \left(2e^{x/2}-e^{-x}\right)dx$

(f) $\int \left(3r+10^{5r}-\frac{1}{e^{2r}}\right)dr$

(g) $\int x(3x-2)\,dx$

(h) $\int (2x-3)^2\,dx$

7.9 Some Applications of Indefinite Integrals

7.9.1 Charge in a Circuit

The current i in amperes (A) is the time rate of change of the charge q in coulombs (C) flowing through a circuit. That is

$$i = \frac{dq}{dt}$$

Applying the definition of integration, the charge can be expressed as

$$q = \int i\,dt$$

Example 6 : **(Electrical)** The current i in a charging circuit is given by

$i = 10e^{-100t}$ amperes.

(i) Find the general solution for the charge q as a function of time.
(*Note : General solution is an equation whereby the value of the arbitrary constant C is not found yet*)

(ii) If there is initially no charge in the circuit, determine the particular solution for the charge.
(*Note : Particular solution is an equation whereby the value of the arbitrary constant C is found*)

(iii) Calculate the charge after 10 milliseconds,

Note : $i = \frac{dq}{dt}$ where q is the charge in the circuit.

7.9.2 Voltage Across a Capacitor

If v is the voltage in volts (V) across the plates of a capacitor with capacitance C in farads (F) and i is the current in amperes across the capacitor at time t in seconds, then

$$i = C\frac{dv}{dt}$$

Hence the voltage across the capacitor is given by

$$v = \frac{1}{C}\int i\,dt$$

Example 7 : **(Electrical)** The current i charging a capacitor of capacitance $C = 0.001$ farad is given by $i = 10e^{-100t}$ amperes.

(i) Find the general solution for voltage across the capacitor as a function of time.

(ii) Determine the voltage at t=10 milliseconds if the initial voltage is zero.

7.9.3 Current Across an Inductor

A voltage v in volts across the inductor with inductance L in henrys induces a time varying current i in amperes to flow across it where

$$v = L\frac{di}{dt}$$

Hence the current across the inductor is given by

$$i = \frac{1}{L}\int v\,dt$$

Example 8 : **(Electrical)** Find the current flowing through an inductor of inductance $L = 0.0001$ Henry, 10 milliseconds after a sinusoidal voltage $v = 50\sin 100\pi t$ volts is connected across it. Assume that there is no initial current across the inductor.

The voltage v across the inductor is given by $v = L\dfrac{di}{dt}$.

7.9.4 Distance Travelled by an Object

The speed v is the rate of change of the distance s travelled.

$$v = \frac{ds}{dt}$$

Hence the distance travelled is given by

$$s = \int v\,dt$$

The acceleration a is the rate of change of the speed v.

$$a = \frac{dv}{dt}$$

Hence the speed is given in terms of the acceleration by

$$v = \int a\,dt$$

Example 9 : **(Mechanical)** An object falls to the ground due to gravity. The acceleration g due to gravity is approximately $9.8\ \text{m/s}^2$. An object initially at rest is released 1 km from the ground. Find the time taken for the object to hit the ground.

The speed at time t is given by $v = \int g\,dt$.

7.9.5 Amount of excretion of biochemical compound

The rate of excretion of a biochemical compound is $f'(t)$.

Hence, the amount excreted by time can be expressed as

$$f(t) = \int f'(t)\,dt$$

Example 10 : **(Chemical)** If the rate of excretion of a biochemical compound is given by $f'(t) = 0.01e^{-0.01t}$, the total amount excreted by time t in minutes is $f(t)$. Find an expression for $f(t)$. If no units are excreted at time t = 0, how many units are excreted in 10 minutes?

<u>Tutorial</u> : MCQ

1. Given the function $y = f(x)$ and its derivative $\frac{dy}{dx} = 1$, which of the following best describes the function $y = f(x)$?

   (a) $y = f(x)$ is a trigonometric function (b) $y = f(x)$ is a linear function
   (c) $y = f(x)$ is a logarithmic function (d) $y = f(x)$ is an exponential function

2. If $\frac{d}{dx}F(x) = f(x)$, then ________.

   (a) $\int F'(x)\,dx = f(x) + C$ (b) $\int f'(x)\,dx = F(x) + C$
   (c) $\int F(x)\,dx = f(x) + C$ (d) $\int f(x)\,dx = F(x) + C$

Tutorial 7

1. Find the following indefinite integrals:

   (a) $\int 10\,dx$ (b) $\int t^5 dt$ (c) $\int \frac{1}{2x}\,dx$

   (d) $\int \frac{x+1}{x}\,dx$ (e) $\int \left(x + \frac{1}{x}\right)^2 dx$ (f) $\int \frac{x^4 - 3x + 2}{x^2}\,dx$

   (g) $\int \cos 3x\,dx$ (h) $\int 2\sin\left(\frac{3u}{2}\right) du$ (i) $\int (\sec^2 2x + \csc^2 3x)\,dx$

   (j) $\int \frac{4}{9 + x^2}\,dx$ (k) $\int \frac{2}{\sqrt{3 - x^2}}\,dx$

2. Find the following indefinite integrals:

   (a) $\int 3^x\,dx$ (b) $\int e^{3y}\,dy$ (c) $\int e^x\left(e^x + \frac{2}{e^{2x}}\right) dx$

   (d) $\int \frac{1 - 2e^{-x} + e^x}{e^x}\,dx$ (e) $\int (e^x - e^{-x})^2\,dx$ (f) $\int (x^e + e^x + e)\,dx$

   *(g) $\int e^x\left(10^x + e\right) dx$ (h) $\int \frac{1}{\sqrt{(4-w)(4+w)}}\,dw$

   (i) $\int \left(\frac{1}{\sqrt{25 - 3t^2}} - \frac{1}{3t^2 + 25}\right) dt$

3. **(Electrical)** The rate of change of an electric current i in a circuit is given by $\frac{di}{dt} = 4t - 0.6t^2$.
   Find : (i) the general solution for the current i as a function of the time t.
   (ii) the particular solution for the current if $i = 2$ amperes when $t = 0$ second.

4. **(Electrical)** The rate of change of a power supply P is given as
$\frac{dP}{dt} = 100e^{-0.005t}$ watts per second.
(i) Express the power supply P as a function of the time t
(ii) If the initial power supply is 100 watts, determine the particular solution for the power P.

5. **(Electrical)** The rate of change of the charge q is given by
$$\frac{dq}{dt} = t^2 - 1 \text{ coulombs/ second.}$$
If $q = \frac{4}{3}$ coulombs when $t = 1$ second, express q in term of t.

6. **(Electrical)** A capacitor with an initial charge of 0.05 coulomb is charged by a current
$$i = 10\left(1 - e^{-100t}\right) \text{ amperes.}$$
Find : (i) the general solution for the charge as a function of time.
(ii) the particular solution for the charge .
(iii) the charge due to the current after 100 milliseconds.

[Note : $i = \frac{dq}{dt}$ **]**

7. **(Electrical)** The current i charging a capacitor of capacitance $C = 0.01$ farad is given by
$$i = 10\left(1 - e^{-100t}\right) \text{ amperes.}$$
Find : (i) the general solution for voltage v as a function of time.
(ii) the particular solution for voltage in terms of time if, initially, the capacitor has no voltage.
(iii) the voltage after 100 milliseconds.

[Note : $i = C\frac{dv}{dt}$ **]**

8. **(Electrical)** Find the current i flowing through an inductor one millisecond after a sinusoidal voltage $v = 50\cos 100\pi t$ volts is connected across it. Assume that there is no initial current across the inductor and the inductance $L = 10$ henrys. **[** Note : $v = L\frac{di}{dt}$ **]**

9. **(Electrical)** A voltage $v = 10\sin 100\pi t$ volts is applied to a coil of inductance $L = 0.2$ henry. If the current $i = 0$ ampere at time $t = 0$ second, express i as a function of t.

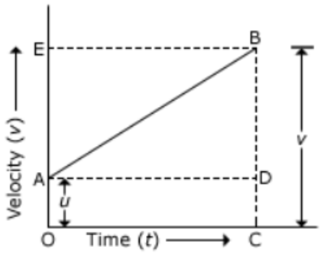
[Note : $v = L\frac{di}{dt}$ **]**

10. **(Electrical)** The charging current $i = 0.2e^{-3t} + 12\sin 2t$ amperes in a circuit deposits charge on a capacitor of capacitance C=1000 μF . Initially, the capacitor has no voltage. Find the voltage across the capacitor at time $t = 0.1$ second. **[** Note : $i = C\frac{dv}{dt}$ **]**

11. **(Electrical)** The voltage pulse $v = 50\sin 100\pi t$ is applied to a 0.1 millihenry inductor. If there is no initial current in the inductor, find the time t when the induced current is 5 amperes.

[Note : $v = L\frac{di}{dt}$ **]**

12. (a) Consider the velocity-time graph shown below.



Let u be the initial velocity at point A and v be the final velocity at point B, assuming the velocity changes at a uniform rate from A to B during time t.

(i) Show that $v=u+at$, where a is the accleration (average rate of change of velocity).

(ii) Use integration to show that the displacement is $s = ut + \frac{1}{2}at^2$.

(b) **(Mechanical)** An object 2 km from the ground falls with an initial speed of 10 m/s .

(i) Find the time taken for the object to hit the ground. The acceleration g due to gravity is approximately $9.8\ m/s^2$.

(ii) Find the velocity at the instant the object hit the ground.

13. **(Mechanical)** An object has an initial speed of $5m/s$. It then moves with acceleration $a = 18 - 9t^2\ m/s^2$.

Use integration to find : (a) the speed of the object after 2 seconds.

(b) the distance the object travelled after 2 seconds.

Will you get the same answers if the formulae in 12 (a) is used ? Why ?

14. **(Mechanical)** The velocity of a robotic welding device at time t (seconds) is given by

$$v = 2t - \frac{12}{2+t^2}$$

Find the expression for the :

(i) acceleration a (m/s^2) as a function.

(ii) displacement s (metres) as a function of t if $s=0$ when $t = 0$.

*15. **(Mechanical)** An object is hurled vertically upwards from the ground with speed 25 m/s . Find the time taken for the object to hit the ground again. The acceleration g due to gravity is approximately $9.8\ m/s^2$.

*16. The rate of change of the gradient with respect to x is given as $4e^{2x} - 9\sin(3x)$. Find the equation of the function y if the gradient at $(0, 3)$ is equal to 5 .

*17. The current flowing to a capacitor at t seconds is given by $i = 0.02t - 0.01t^2$ amperes.

(a) Find the general solution for the charge q as a function of t .

(b) Find the time at which the charge q on the capacitor is maximum . [Note : $i = \frac{dq}{dt}$]

Problem-solving Assignment
The goal of this series of problem-solving assignments is to develop problem-solving skills, not just to test your ability to get the answer. It's ok to try hard and not succeed at first (only your effort is evaluated), but <u>you must try</u>.

Question
A vehicle starts from rest and continues with speed $v = 6t \ m/s$.
How far will the vehicle travel in 5 seconds?

1. Understand the problem • State the given conditions and quantities. • Identify the unknown that you are asked to find. • If applicable, draw a diagram to describe the scenario.	
2. Devise a plan • Identify which are the relevant concepts that can be applied.	
3. Implement the plan • Carry out the plan, showing each step clearly. • Any graph or diagram should be clearly labelled.	
4. Look back • Ask yourself - "Does it answer the question that was asked?" - "Does the answer make sense?" • Determine if there is any other easier way of finding the solution.	

ANSWERS

Eg 6 : 0.0632 coulomb   *Eg 7* : 63.2 volts   *Eg 8* : $\frac{10000}{\pi}$ amperes

Eg 9 : 14.3 s   *Eg10* : 0.095 units

MCQ

1. (b) 2. (d)

Tutorial 7

1. (a) $10x+C$ (b) $\frac{t^6}{6}+C$ (c) $\frac{1}{2}\ln|x|+c$

(d) $x+\ln|x|+C$ (e) $\frac{x^3}{3}+2x-\frac{1}{x}+C$ (f) $\frac{x^3}{3}-3\ln|x|-\frac{2}{x}+C$

(g) $\frac{\sin 3x}{3}+C$ (h) $-\frac{4}{3}\cos\left(\frac{3u}{2}\right)+C$ (i) $\frac{\tan 2x}{2}-\frac{\cot 3x}{3}+C$

(j) $\frac{4}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$ (k) $2\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)+C$

2. (a) $\frac{3^x}{\ln 3}+C$ (b) $\frac{e^{3y}}{3}+C$ (c) $\frac{e^{2x}}{2}-\frac{2}{e^x}+C$

(d) $-e^{-x}+e^{-2x}+x+C$ (e) $\frac{e^{2x}}{2}-2x-\frac{e^{-2x}}{2}+C$ (f) $\frac{x^{e+1}}{e+1}+e^x+ex+C$

(g) $\frac{e^x 10^x}{1+\ln 10}+e^{x+1}+C$ (h) $\sin^{-1}\left(\frac{w}{4}\right)+C$

(i) $\frac{1}{\sqrt{3}}\sin^{-1}\left(\frac{\sqrt{3}\,t}{5}\right)-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}\,t}{5}\right)+C$

3. (i) $\boldsymbol{i}=2\boldsymbol{t}^2-0.2\boldsymbol{t}^3+\boldsymbol{C}$ (ii) $i=2t^2-0.2t^3+2$ amperes

4. (i) $\boldsymbol{P}=-20000\boldsymbol{e}^{-0.005\boldsymbol{t}}+\boldsymbol{C}$ watts (ii) $P=-20000e^{-0.005t}+20100$

5. $q=\frac{t^3}{3}-t+2$ coulombs

6. (i) $\boldsymbol{q}=10\boldsymbol{t}+0.1\boldsymbol{e}^{-100\boldsymbol{t}}+\boldsymbol{C}$ (ii) $\boldsymbol{q}=10\boldsymbol{t}+0.1\boldsymbol{e}^{-100\boldsymbol{t}}-0.05$ (iii) 0.95 coulomb

7. (i) $\boldsymbol{v}=1000\boldsymbol{t}+10\boldsymbol{e}^{-100\boldsymbol{t}}+\boldsymbol{C}$ (ii) $\boldsymbol{v}=1000\boldsymbol{t}+10\boldsymbol{e}^{-100\boldsymbol{t}}-10$ (iii) 90 volts

8. 0.005 ampere 9. $i=\frac{1}{2\pi}(1-\cos 100\pi t)$ amperes 10. 136.9 volts

11. 0.00025 second 12. (b) (i) 19.2 seconds (ii) 198.16 m/s

13. 17 m/s , 34 m

14. (i) $a=2+\frac{24t}{\left(2+t^2\right)^2}$ (ii) $s=t^2-6\sqrt{2}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)$ 15. 5.1 seconds

16. $e^{2x}+\sin(3x)+2$ 17. (a) $\frac{1}{100}t^2-\frac{1}{300}t^3+C$ (b) *t* = *2* s

Problem-solving Assignment : 75 metres