

Chapter 10

Digital Modulation

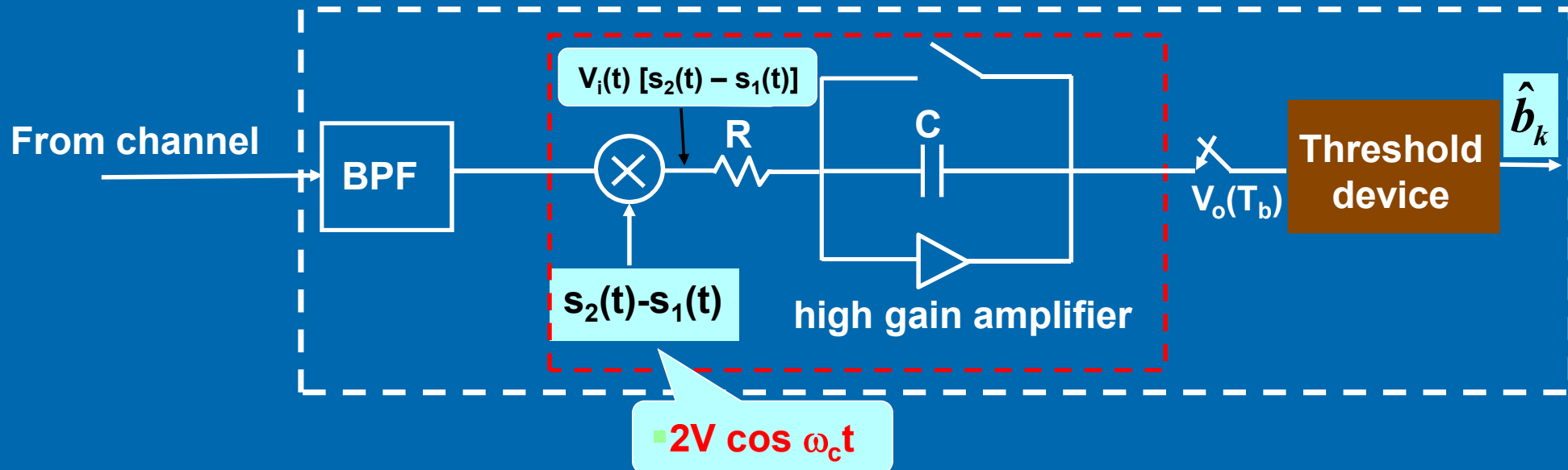
(Part 2 of 2)



10.5 Differential Phase-Shift Keying (DPSK)

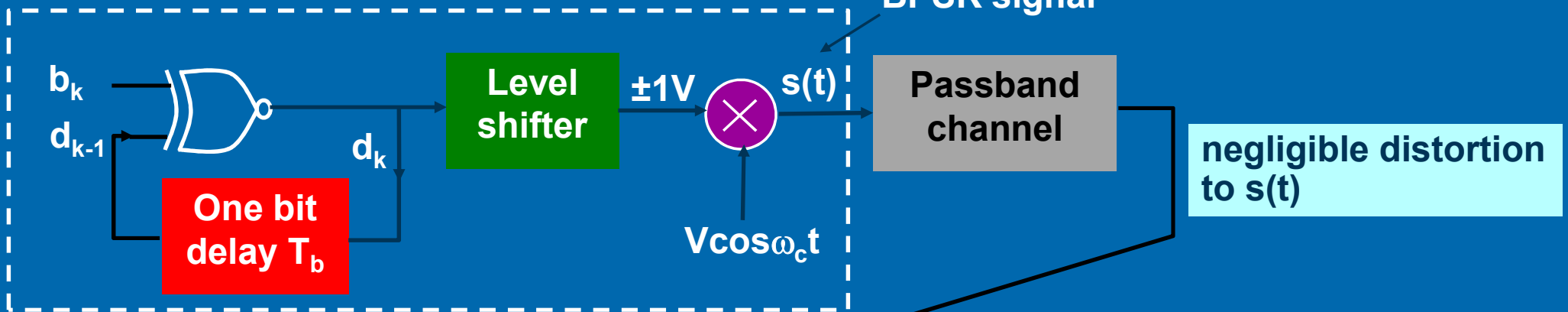
- Coherent BPSK system requires complex hardware to generate the receiver local carrier for coherent detection.
- DPSK uses non-coherent detection.

Coherent BPSK system

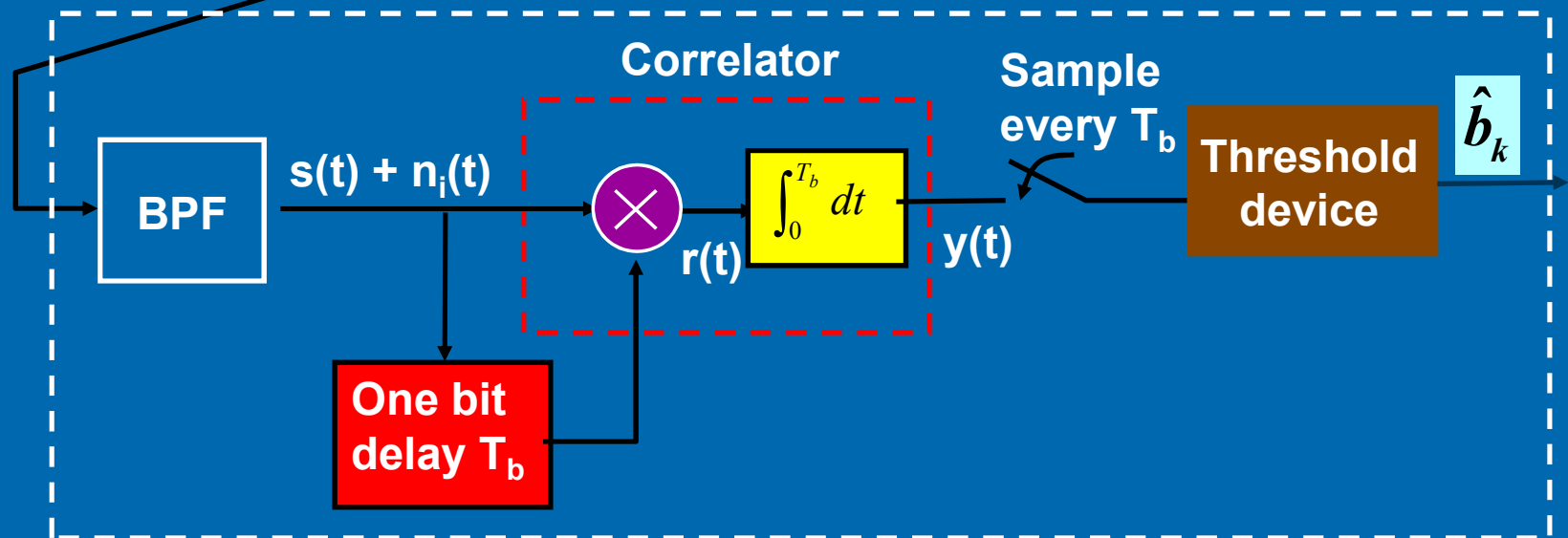


10.5 Differential Phase-Shift Keying (DPSK)

(a) Transmitter



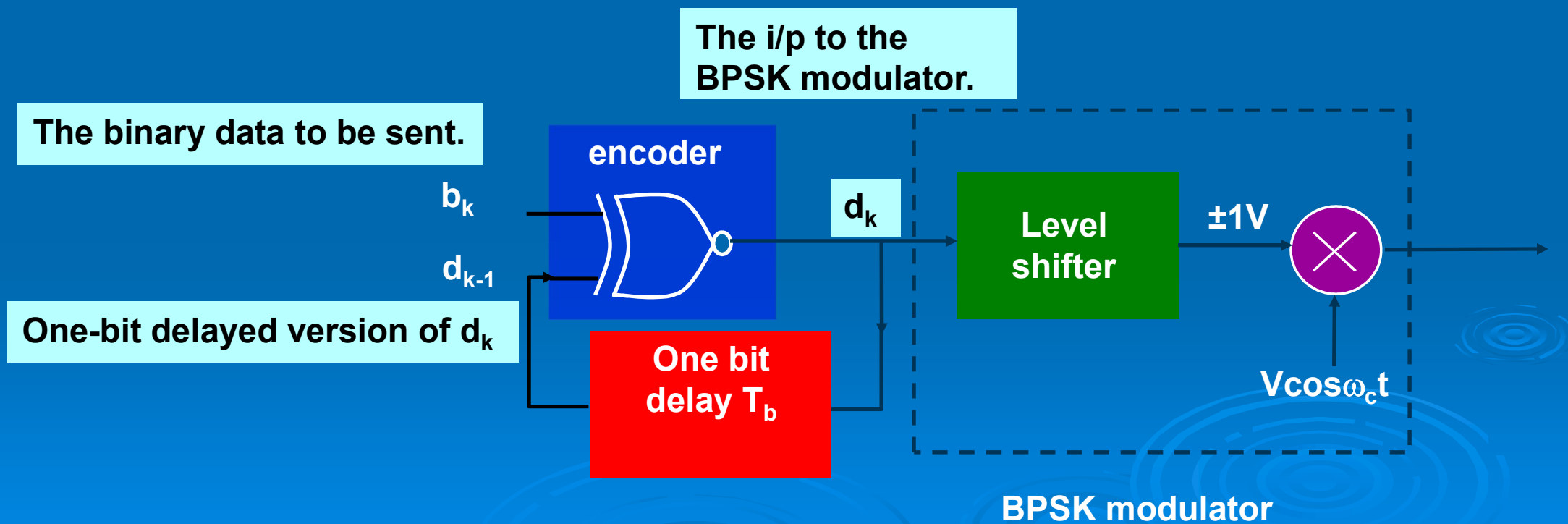
(b) Receiver



10.5 Differential Phase-Shift Keying (DPSK)

Transmitter

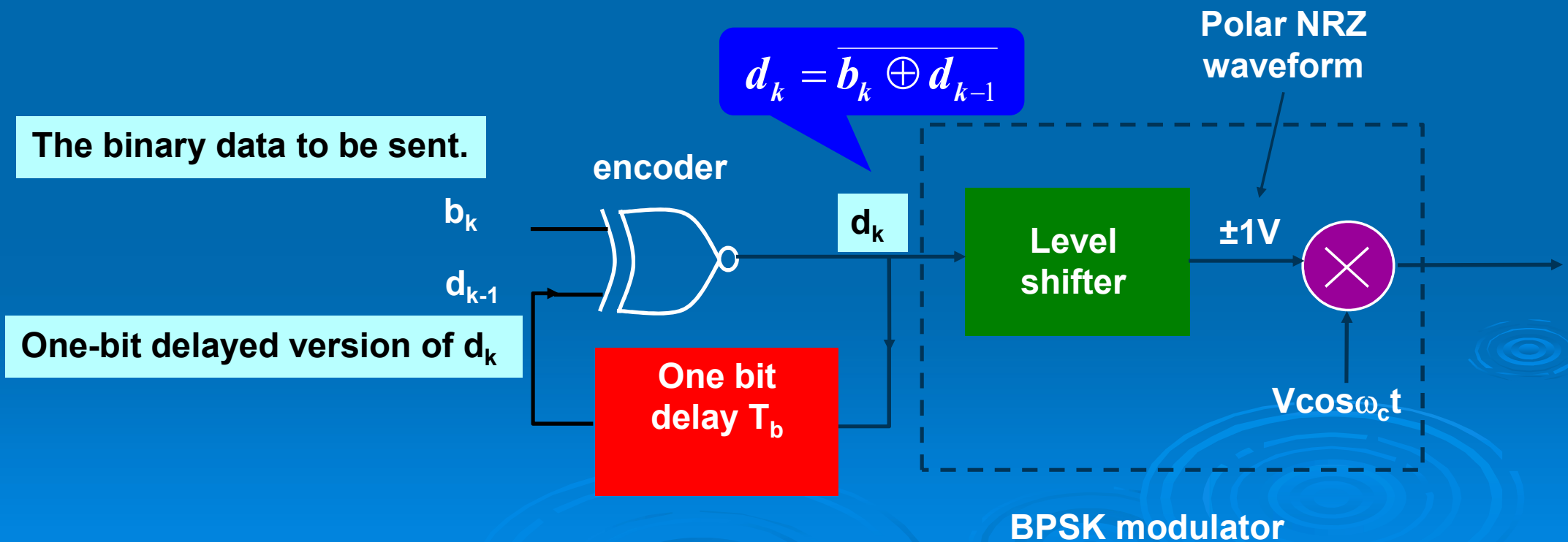
- The binary data is contained in the **difference** between the phase angles of two successive signaling elements.



10.5 Differential Phase-Shift Keying (DPSK)

Transmitter

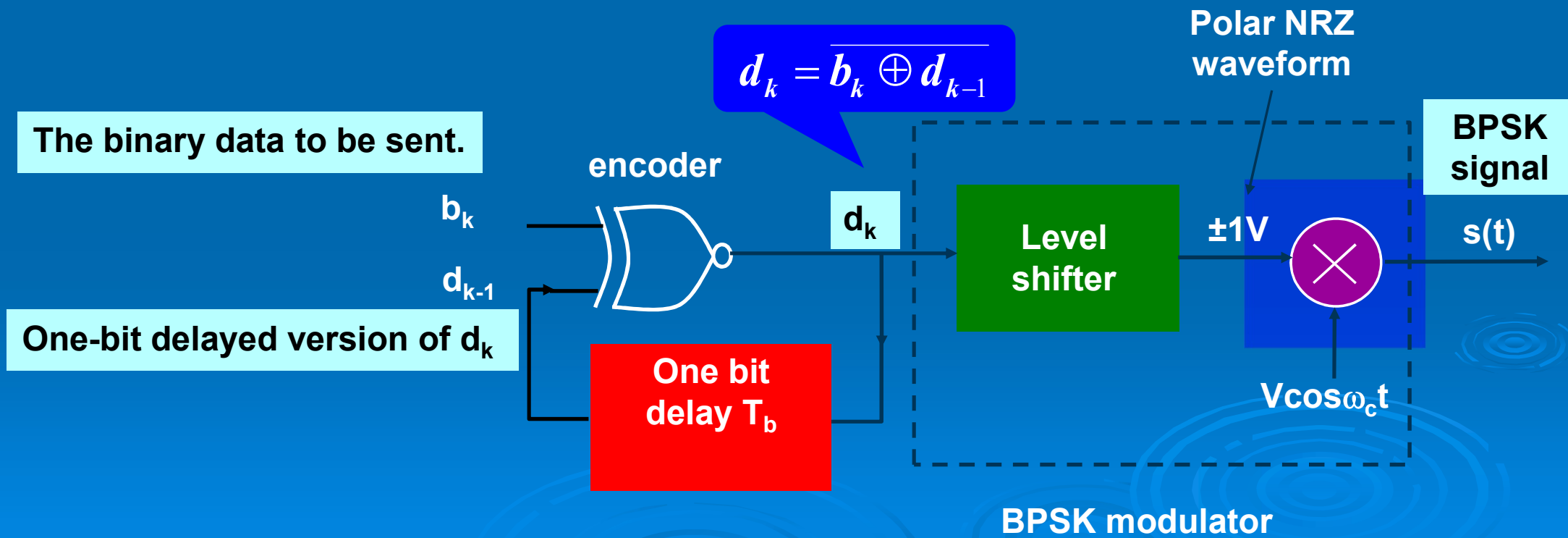
- The binary data is contained in the difference between the phase angles of two successive signaling elements.



10.5 Differential Phase-Shift Keying (DPSK)

Transmitter

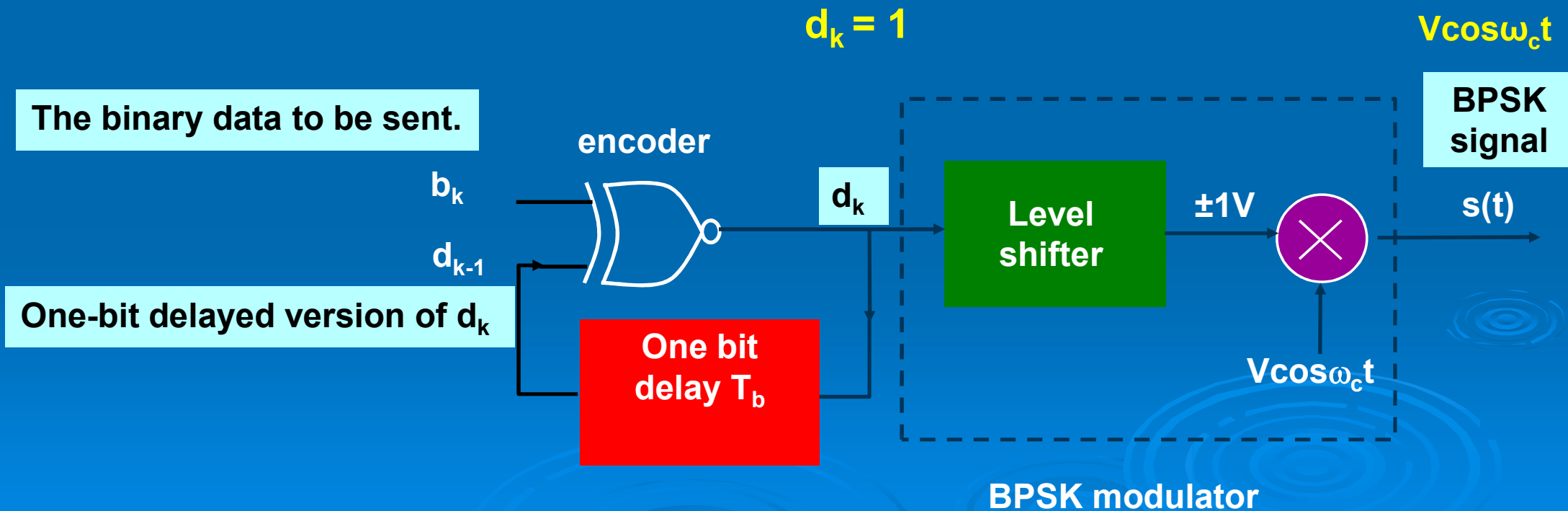
- The binary data is contained in the difference between the phase angles of two successive signaling elements.



10.5 Differential Phase-Shift Keying (DPSK)

Transmitter

$$s(t) = \begin{cases} s_2(t) = V\cos\omega_c t & 0 < t < T_b; \quad d_k = 1; \\ s_1(t) = -V\cos\omega_c t & 0 < t < T_b; \quad d_k = 0; \end{cases}$$



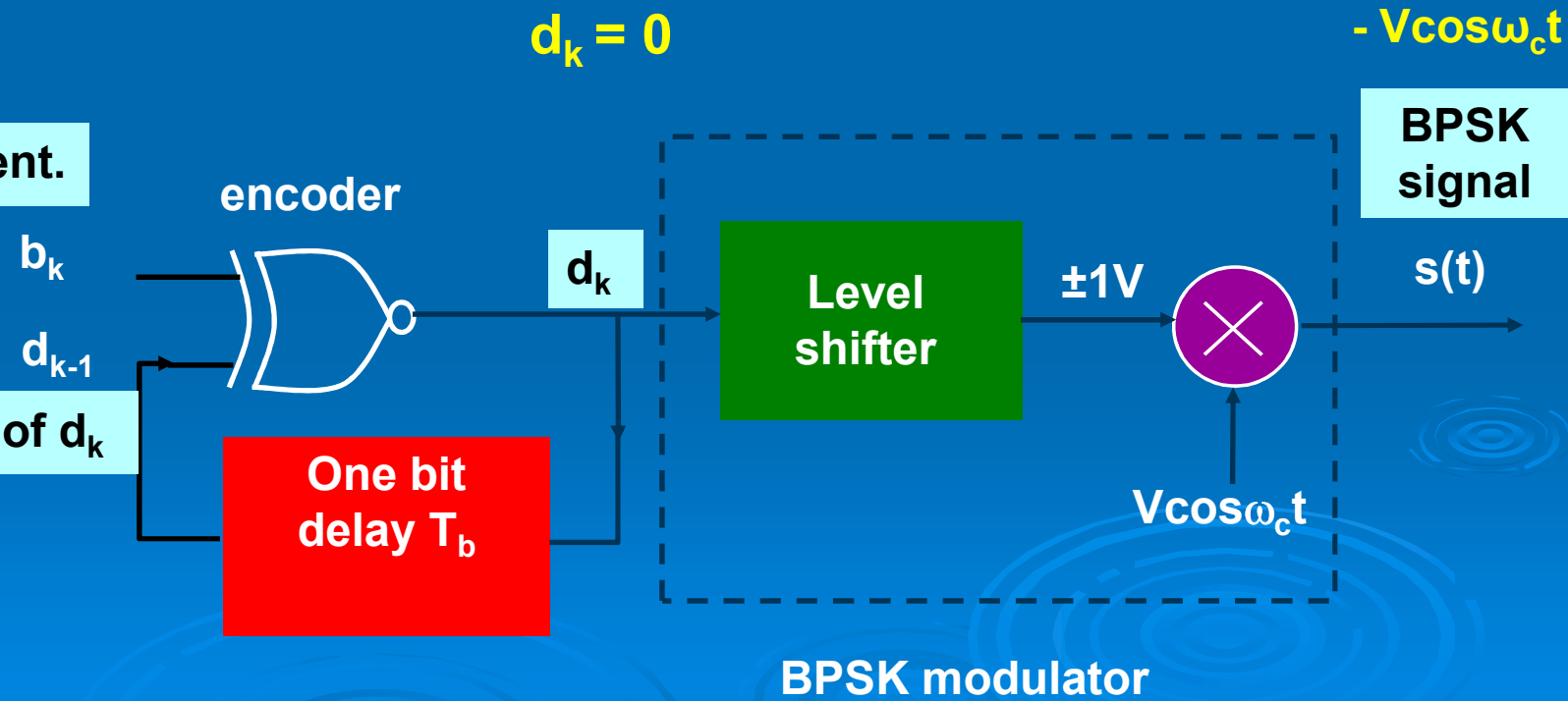
10.5 Differential Phase-Shift Keying (DPSK)

Transmitter

$$s(t) = \begin{cases} s_2(t) = V\cos\omega_c t & 0 < t < T_b; \quad d_k = 1; \\ s_1(t) = -V\cos\omega_c t & 0 < t < T_b; \quad d_k = 0; \end{cases}$$

The binary data to be sent.

One-bit delayed version of d_k



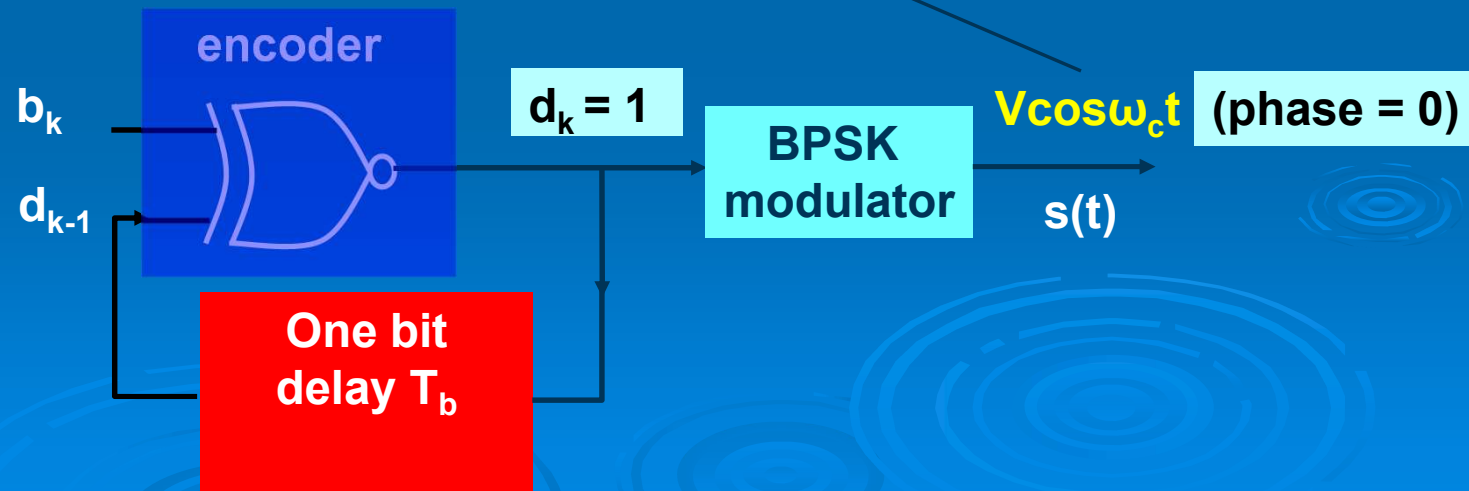
10.5 Differential Phase-Shift Keying (DPSK)

Differential encoding

Bit Time	0	1	2	3	4	5	6	7
Input sequence, b_k		1	1	0	1	0	0	0
Encoded sequence, d_k	1*	1						
Phase of transmitted $s(t)$	0	0						

*Arbitrary starting reference bit

Truth table for ex-NOR		
Inputs		output
1	1	1
0	0	1
1	0	0
0	1	0



10.5 Differential Phase-Shift Keying (DPSK)

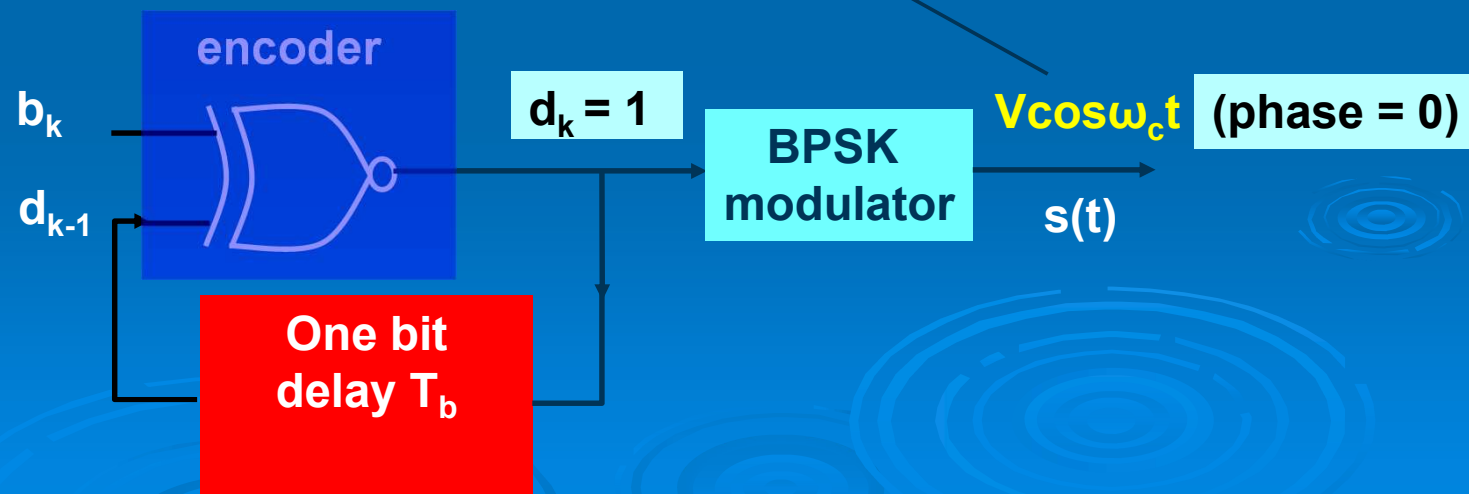
Differential encoding

Bit Time	0	1	2	3	4	5	6	7
Input sequence, b_k		1	1	0	1	0	0	0
Encoded sequence, d_k	1*	1	1					
Phase of transmitted $s(t)$	0	0	0					

*Arbitrary starting reference bit

Truth table for ex-NOR

Inputs		output
1	1	1
0	0	1
1	0	0
0	1	0



10.5 Differential Phase-Shift Keying (DPSK)

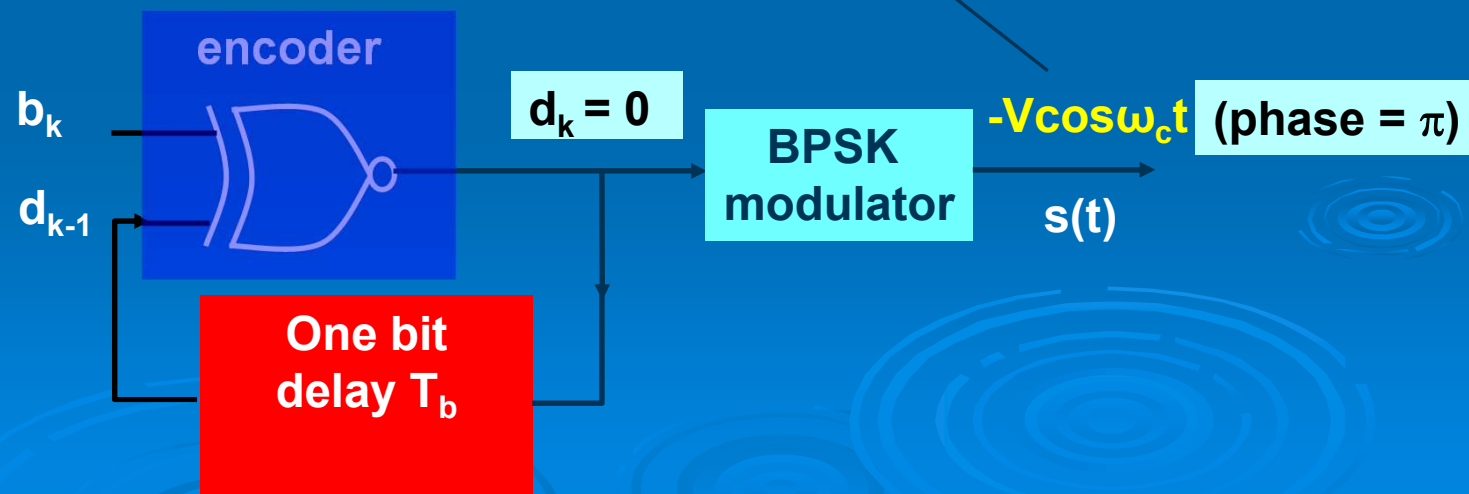
Differential encoding

Bit Time	0	1	2	3	4	5	6	7
Input sequence, b_k		1	1	0	1	0	0	0
Encoded sequence, d_k	1*	1	1	0				
Phase of transmitted $s(t)$	0	0	0	π				

*Arbitrary starting reference bit

Truth table for ex-NOR

Inputs	output
1 1	1
0 0	1
1 0	0
0 1	0



10.5 Differential Phase-Shift Keying (DPSK)

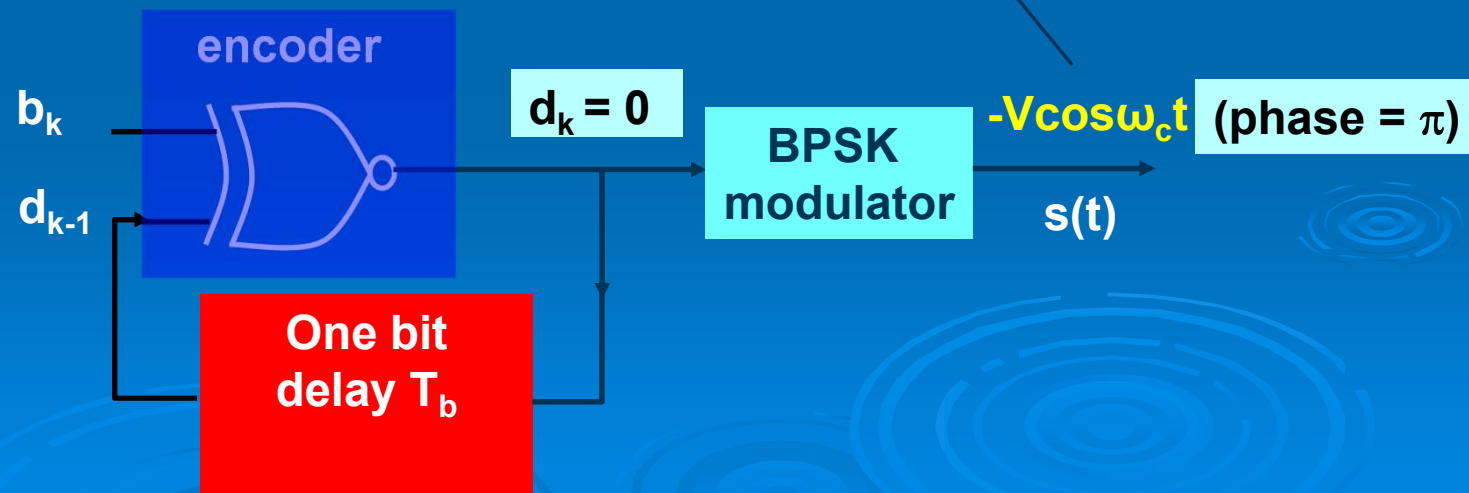
Differential encoding

Bit Time	0	1	2	3	4	5	6	7
Input sequence, b_k		1	1	0	1	0	0	0
Encoded sequence, d_k	1*	1	1	0	0			
Phase of transmitted $s(t)$	0	0	0	π	π			

*Arbitrary starting reference bit

Truth table for ex-NOR

Inputs	output
1 1	1
0 0	1
1 0	0
0 1	0



10.5 Differential Phase-Shift Keying (DPSK)

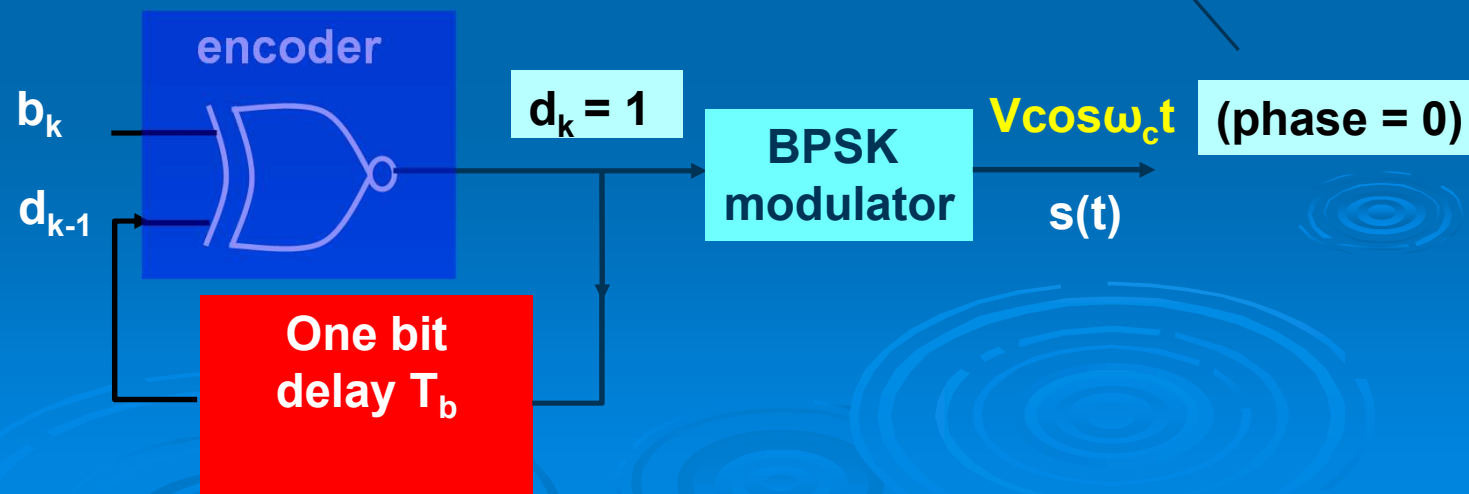
Differential encoding

Bit Time	0	1	2	3	4	5	6	7
Input sequence, b_k		1	1	0	1	0	0	0
Encoded sequence, d_k	1*	1	1	0	0	1		
Phase of transmitted $s(t)$	0	0	0	π	π	0		

*Arbitrary starting reference bit

Truth table for ex-NOR

Inputs	output
1 1	1
0 0	1
1 0	0
0 1	0



10.5 Differential Phase-Shift Keying (DPSK)

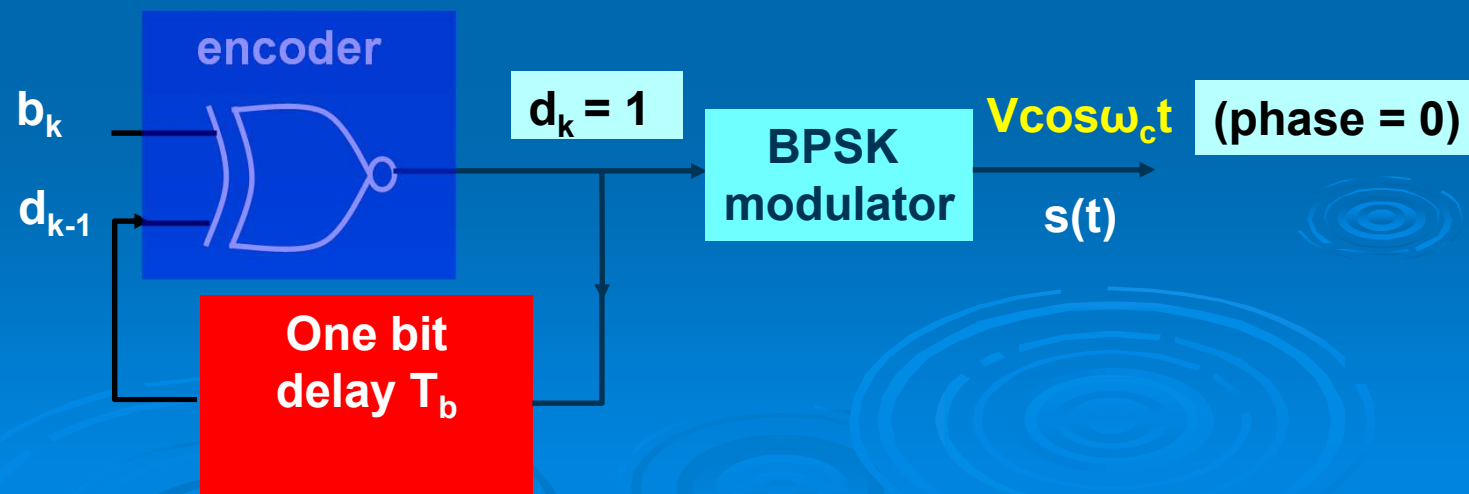
Differential encoding

Bit Time	0	1	2	3	4	5	6	7
Input sequence, b_k		1	1	0	1	0	0	0
Encoded sequence, d_k	1*	1	1	0	0	1	0	1
Phase of transmitted $s(t)$	0	0	0	π	π	0	π	0

*Arbitrary starting reference bit

Truth table for ex-NOR

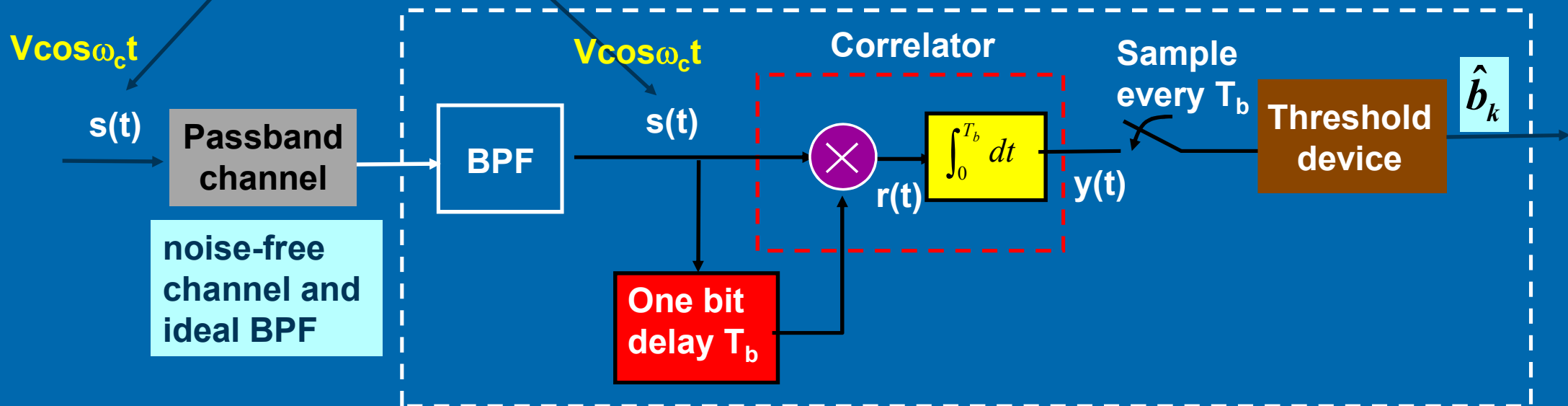
Inputs	output
1 1	1
0 0	1
1 0	0
0 1	0



10.5 Differential Phase-Shift Keying (DPSK)

Receiver

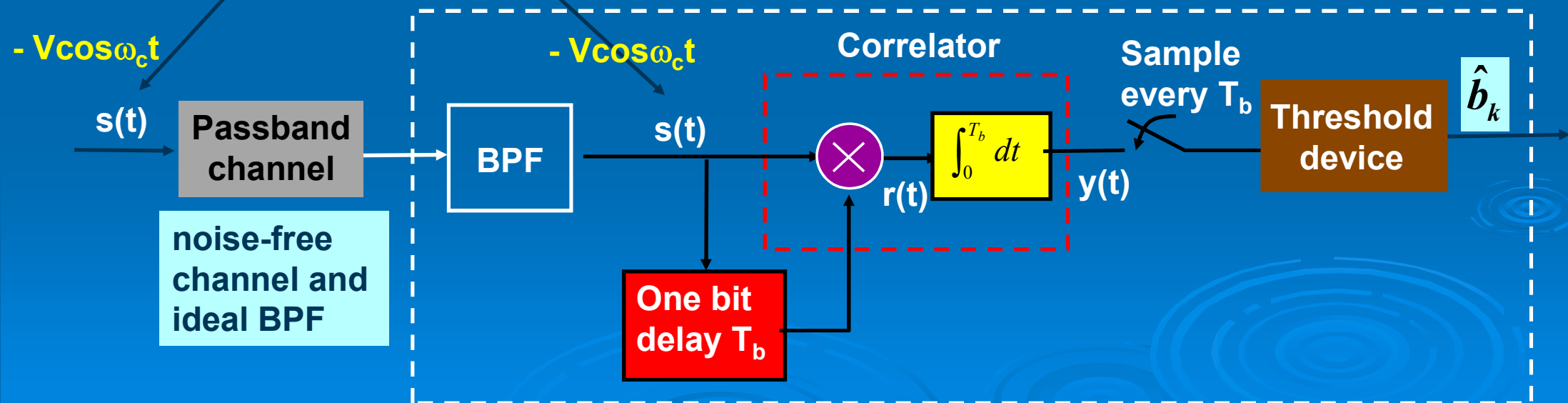
$$s(t) = \begin{cases} s_2(t) = V\cos\omega_c t & 0 < t < T_b; \quad d_k = 1; \\ s_1(t) = -V\cos\omega_c t & 0 < t < T_b; \quad d_k = 0; \end{cases}$$



10.5 Differential Phase-Shift Keying (DPSK)

Receiver

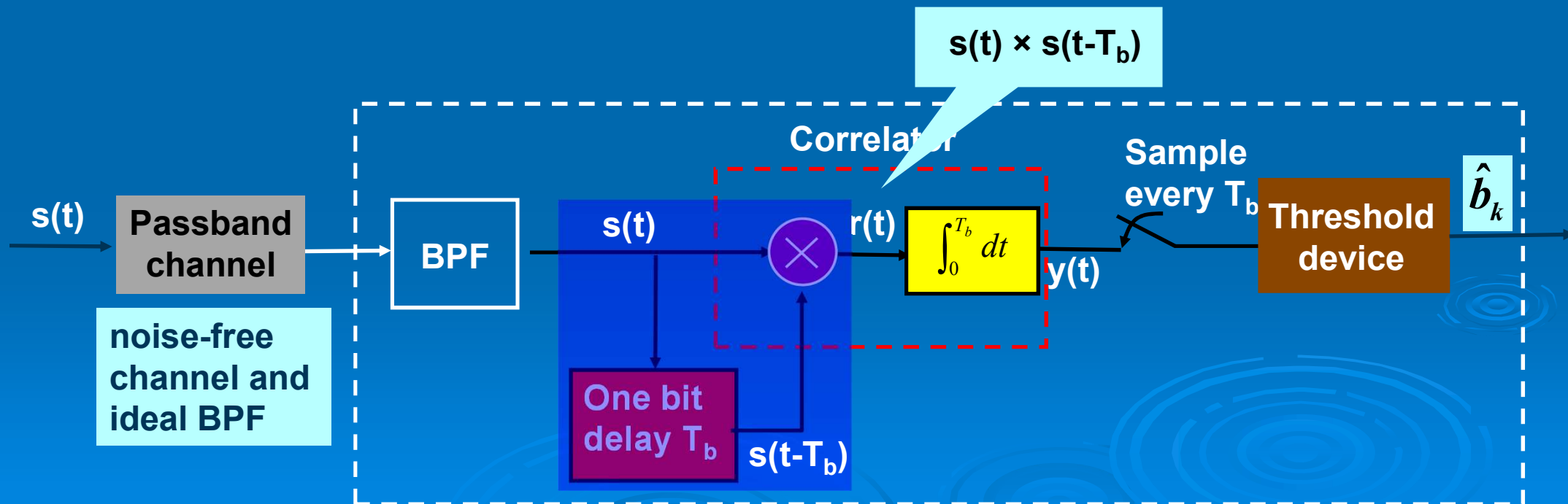
$$s(t) = \begin{cases} s_2(t) = V\cos\omega_c t & 0 < t < T_b; \quad d_k = 1; \\ s_1(t) = -V\cos\omega_c t & 0 < t < T_b; \quad d_k = 0; \end{cases}$$



10.5 Differential Phase-Shift Keying (DPSK)

Receiver

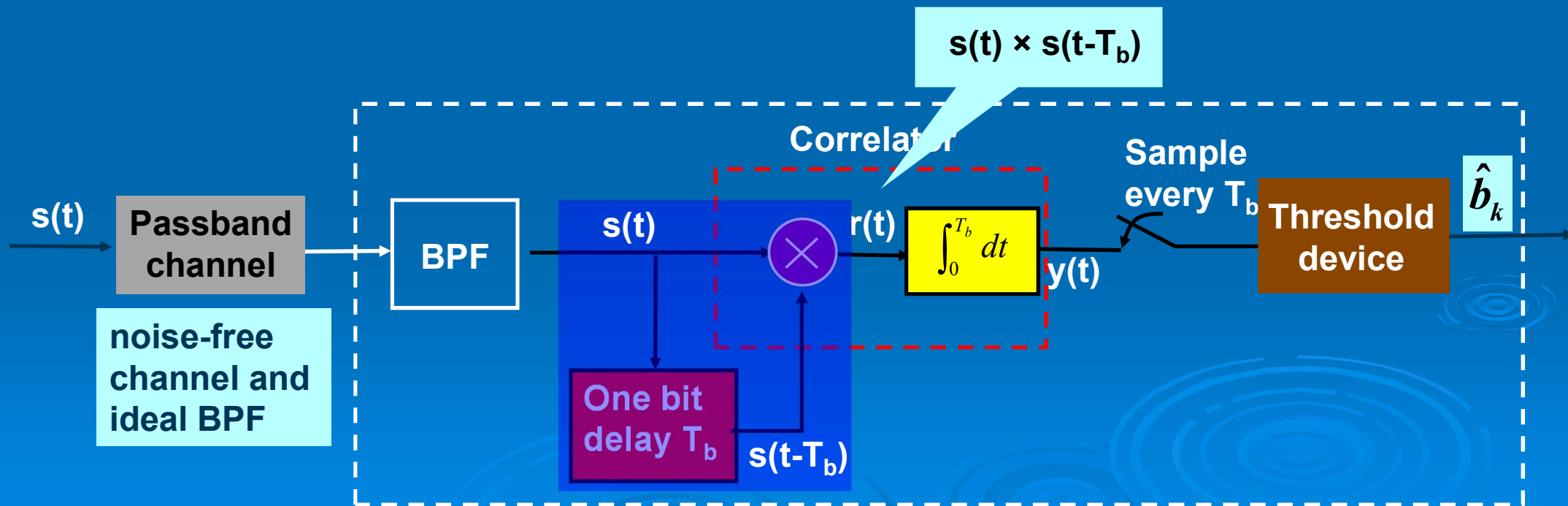
$$r(t) = s(t) \times s(t-T_b) = \begin{cases} V^2 \cos^2 \omega_c t & \text{If } s(t) = s(t-T_b) \end{cases}$$



10.5 Differential Phase-Shift Keying (DPSK)

Receiver

$$r(t) = s(t) \times s(t-T_b) = \begin{cases} V^2 \cos^2 \omega_c t & \text{If } s(t) = s(t-T_b) \\ -V^2 \cos^2 \omega_c t & \text{If } s(t) \neq s(t-T_b) \end{cases}$$



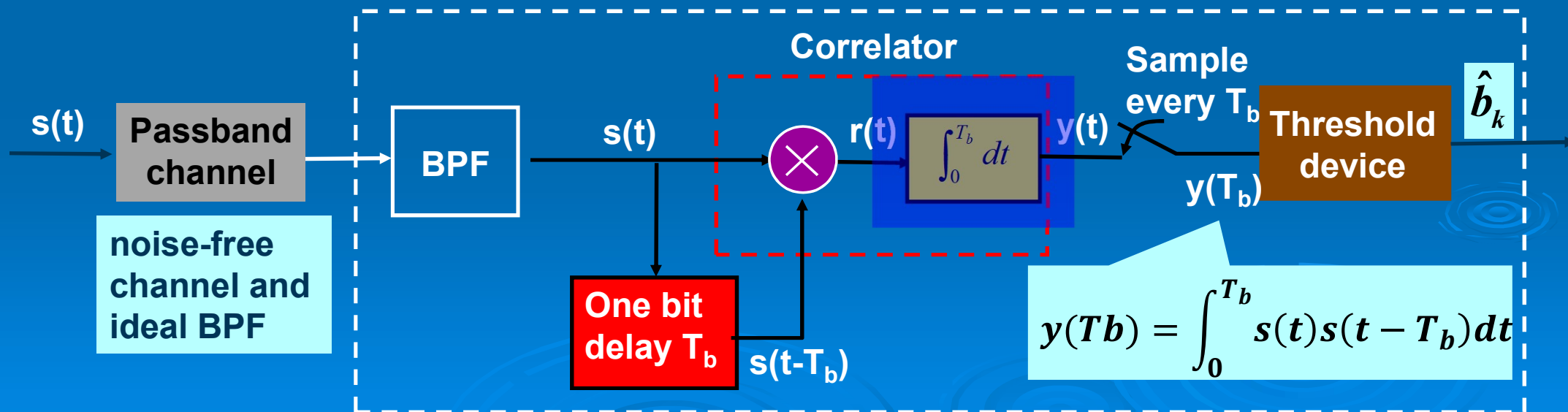
10.5 Differential Phase-Shift Keying (DPSK)

- When $r(t) = V^2 \cos^2 \omega_c t$, after integration and dump,

$$\begin{aligned}
 y(Tb) &= k \int_0^{T_b} V^2 \cos^2 \omega_c t \, dt = kV^2 \int_0^{T_b} \left(\frac{1}{2} + \frac{\cos 2 \omega_c t}{2} \right) dt \quad \text{using } \cos 2\theta = 2 \cos^2 \theta - 1 \\
 &= kV^2 \int_0^{T_b} \left(\frac{1}{2} \right) dt + kV^2 \int_0^{T_b} \left(\frac{\cos 2 \omega_c t}{2} \right) dt \\
 &= \frac{kV^2 T_b}{2}
 \end{aligned}$$

$\Rightarrow \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

Assume ideal case: whole cycles within 1 bit.



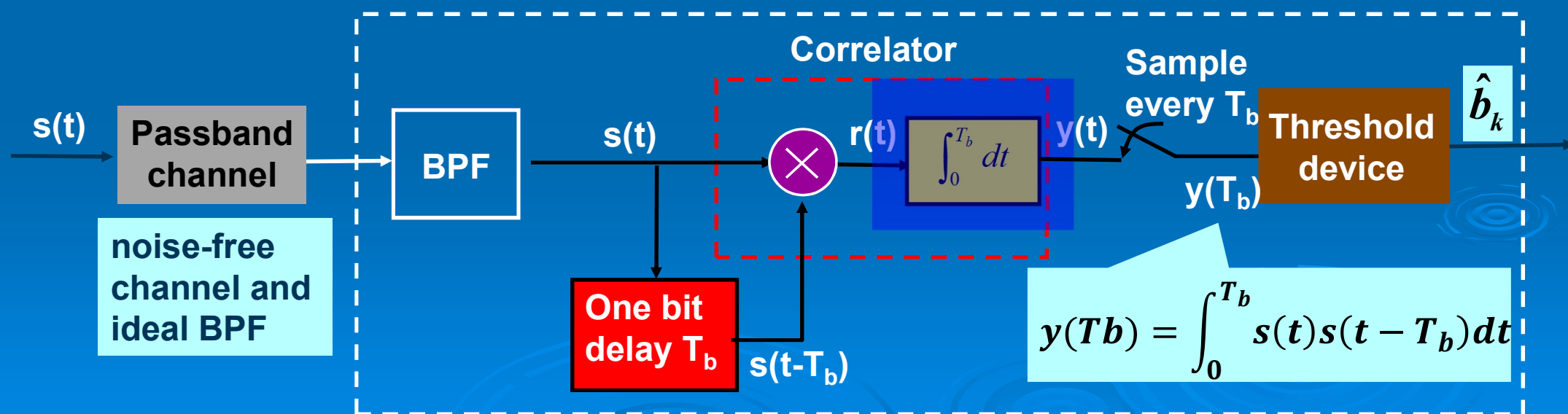
10.5 Differential Phase-Shift Keying (DPSK)

- When $r(t) = -V^2 \cos^2 \omega_c t$, after integration and dump,

$$\begin{aligned}
 y(Tb) &= k \int_0^{T_b} -V^2 \cos^2 \omega_c t \, dt = -kV^2 \int_0^{T_b} \left(\frac{1}{2} + \frac{\cos 2 \omega_c t}{2} \right) dt \\
 &= -kV^2 \int_0^{T_b} \left(\frac{1}{2} \right) dt - kV^2 \int_0^{T_b} \left(\frac{\cos 2 \omega_c t}{2} \right) dt \\
 &= -\frac{kV^2 T_b}{2}
 \end{aligned}$$

using $\cos 2\theta = 2 \cos^2 \theta - 1$
 $\Rightarrow \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

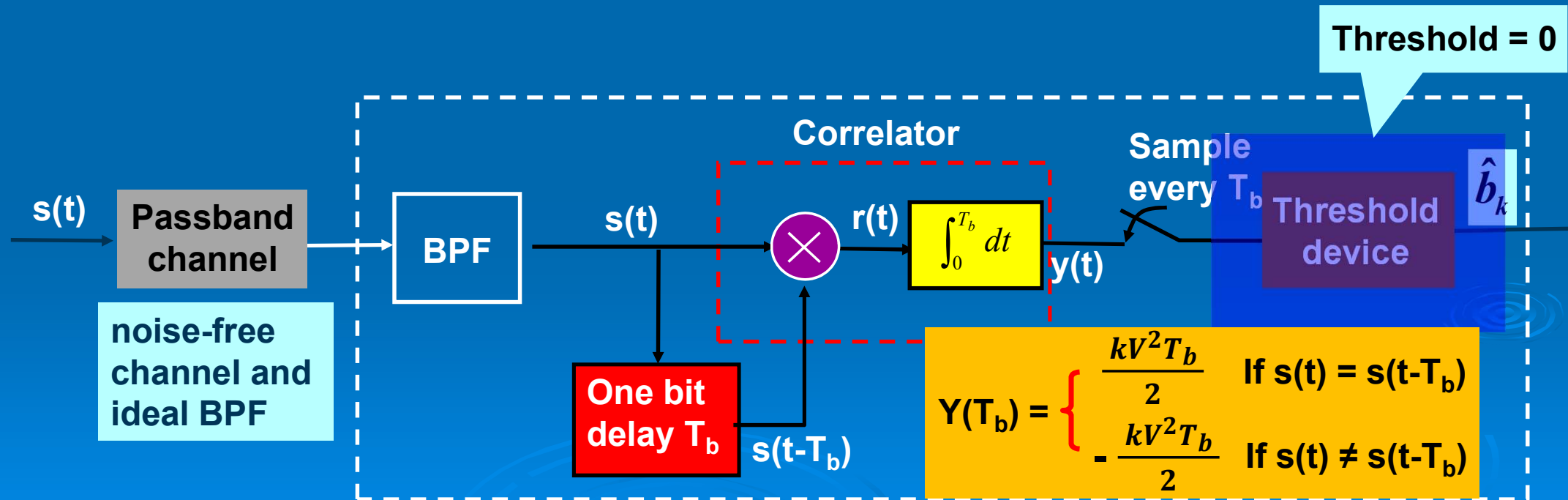
Assume ideal case: whole cycles within 1 bit.



10.5 Differential Phase-Shift Keying (DPSK)

Receiver

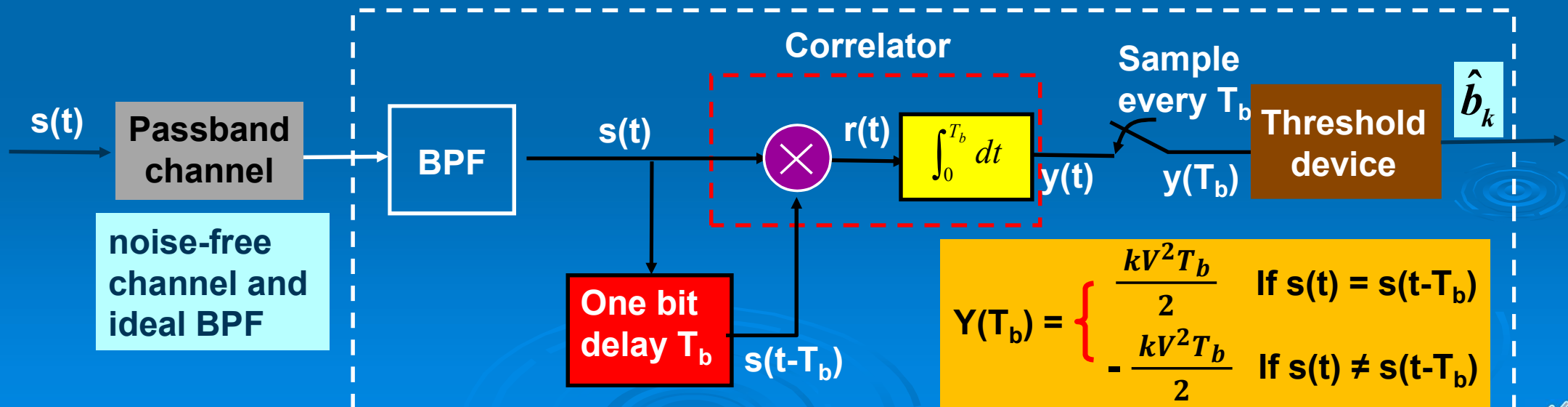
- Threshold level is therefore set at 0 V (middle value).
- Decision rule:
 - If the correlator output is positive, decode $b_k = 1$.
 - If the correlator output is negative, decode $b_k = 0$.



10.5 Differential Phase-Shift Keying (DPSK)

Differential decoding

Bit Time	0	1	2	3	4	5	6	7
Bit sequence sent		1	1	0	1	0	0	0
Phase of received $s(t)$	0	0	0	π	π	0	π	0
Correlator output (sampled value), $y(T_b)$		+	+	-	+	-	-	-
Output bit sequence recovered		1	1	0	1	0	0	0



10.5 Differential Phase-Shift Keying (DPSK)

- The probability of bit error for a DPSK system is found to be

$$P_e = \frac{1}{2} \exp\left(\frac{-V^2 T_b}{2\eta}\right) = \frac{1}{2} e^{-\left(\frac{V^2 T_b}{2\eta}\right)}$$

- Disadvantages of DPSK are:
 - For a given transmitted power, the DPSK system will give a higher error rate when compared with the coherent BPSK system.
 - Asynchronous transmission is not possible as the system need to be locked on a specific signalling speed.
 - An error will propagate to the adjacent bit. Hence higher error rate than coherent BPSK.



10.5 Differential Phase-Shift Keying (DPSK)

Example 10.1

Figure E10.1 shows the block diagram of a DPSK transmitter. The binary input is in unipolar NRZ format of amplitude 2V volt, at a bit rate of 1200 b/s. The carrier is $\sin\omega_c t$ where $\omega_c t = 4800\pi$ rad/s. Assume that the input is a long series of ...10101010... . Sketch the waveforms at points A to D as indicated in Figure E10.1 for a 1010 frame. Assume distortion-less transmission path. Also assume that the encoder output is binary 1 prior to the 1010 frame.

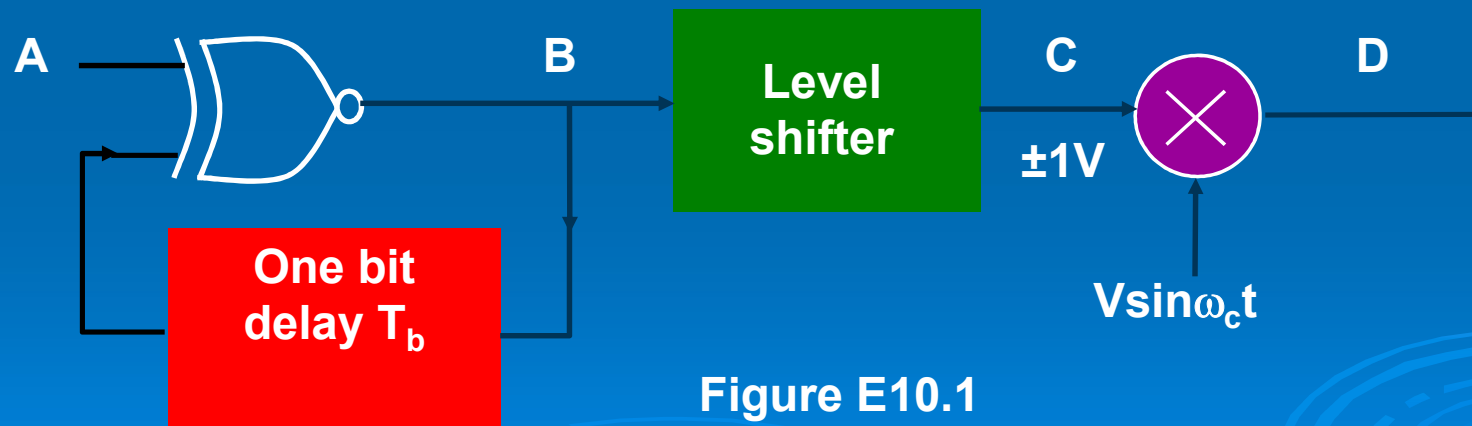


Figure E10.1



10.5 Differential Phase-Shift Keying (DPSK)

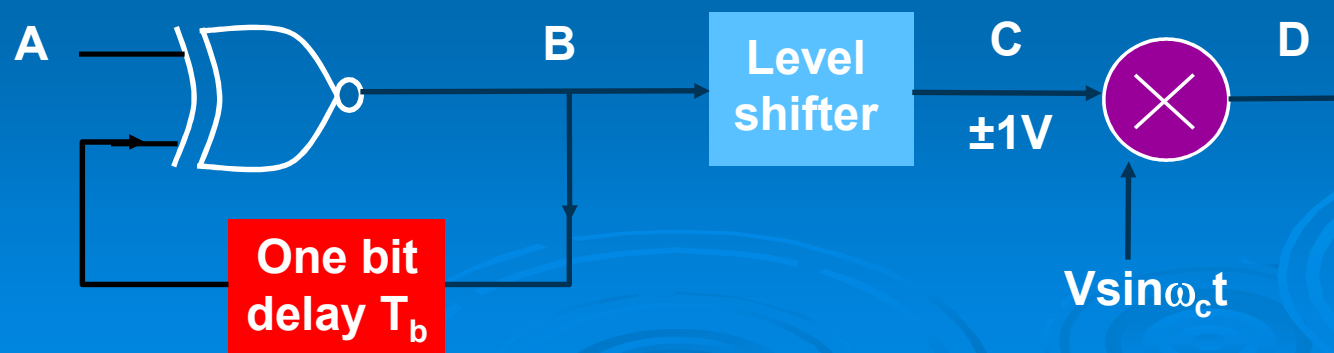
Solution:

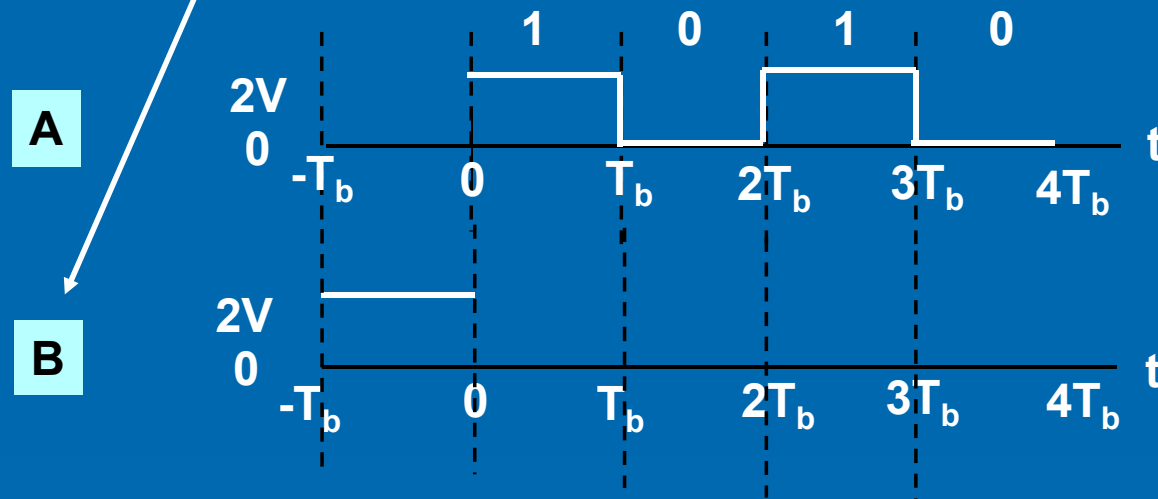
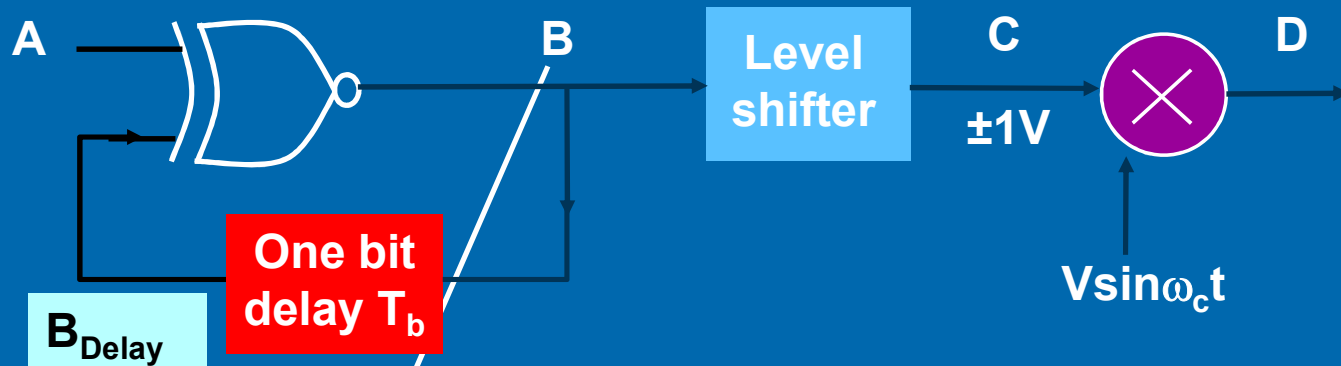
$$\omega_c = 2\pi f_c = 4800 \pi \text{ rad/s}$$

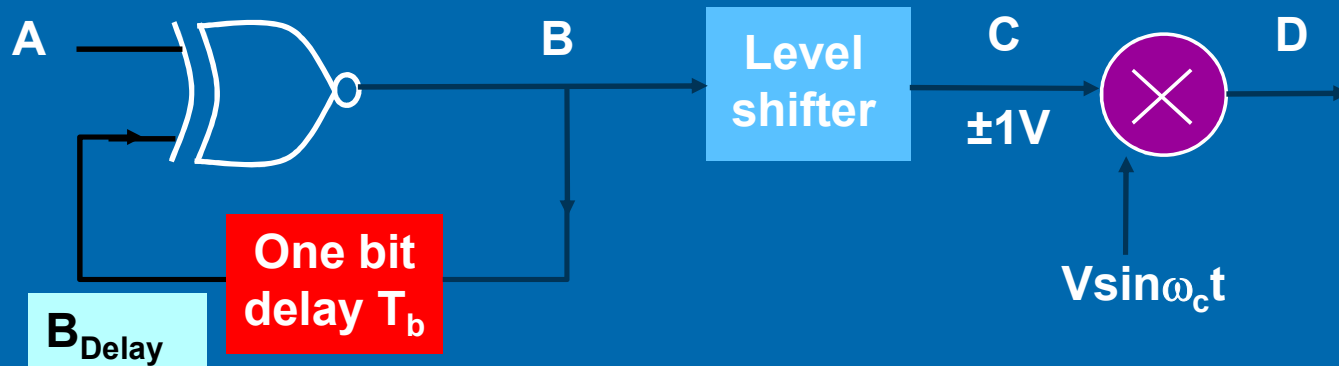
$$2f_c = 4800 \rightarrow f_c = 2400 \text{ Hz} \rightarrow \text{Carrier Period } T_c = \frac{1}{2400} \text{ sec}$$

“A” has unipolar format; bit rate $r_b = 1200 \text{ b/s}$ $T_b = \frac{1}{1200} \text{ sec}$

One bit duration $T_b = \frac{1}{r_b} = \frac{1}{1200} = 2T_c$ two carrier periods

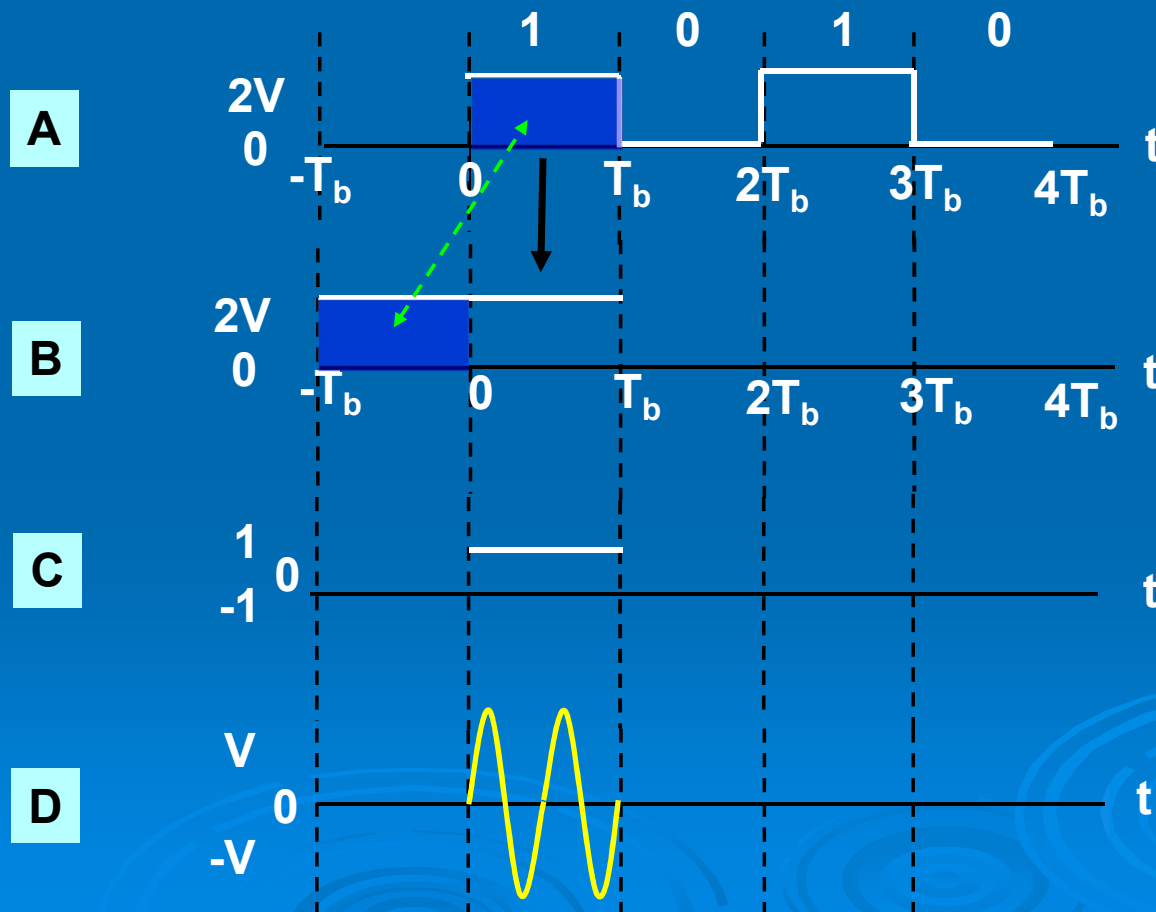


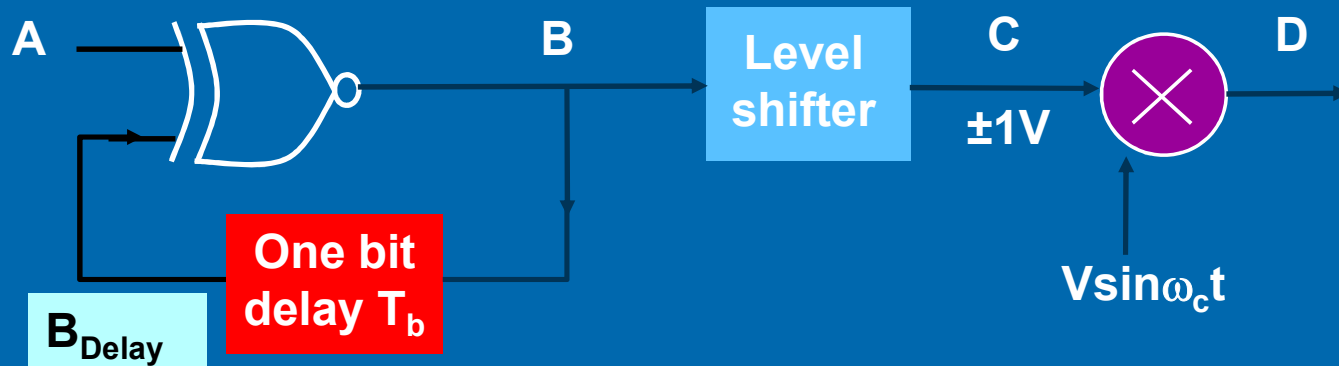




X-NOR

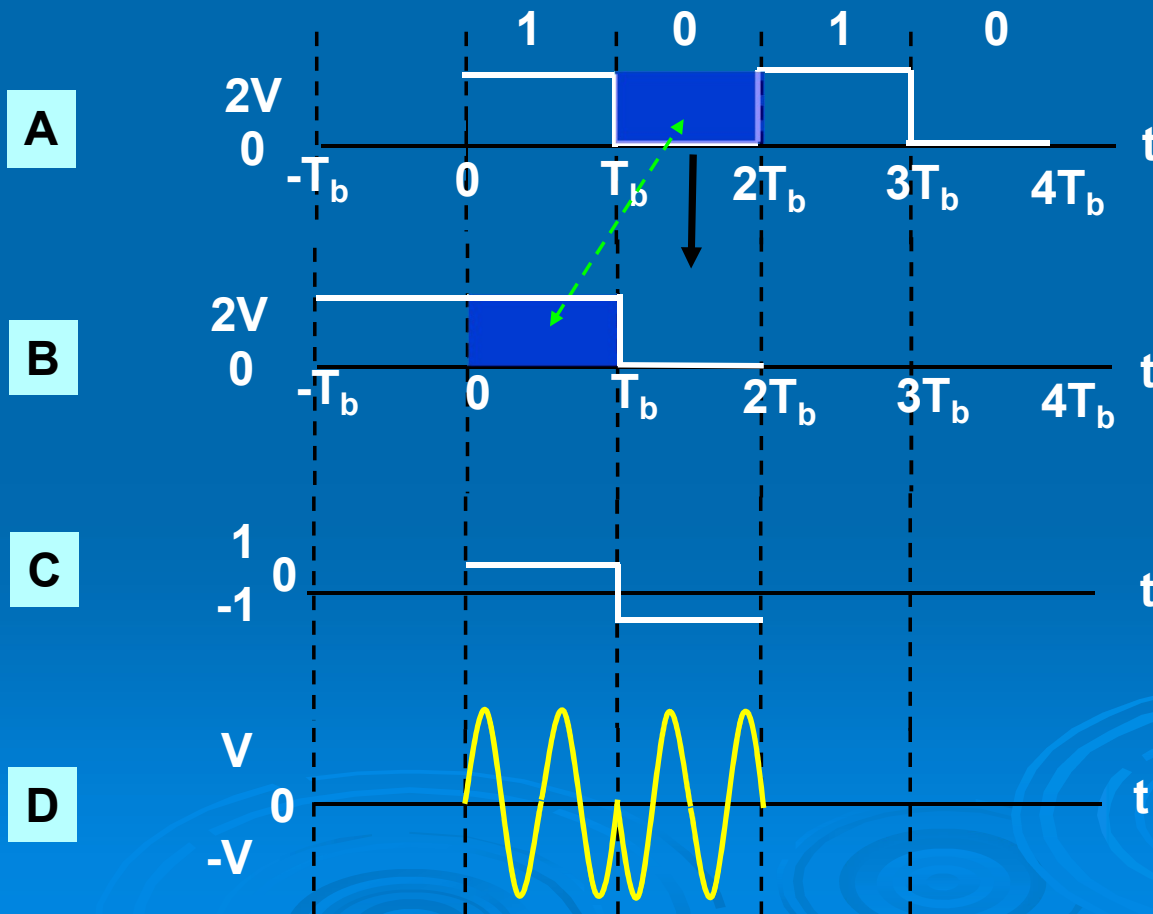
X	Y	Z(o/p)
0	0	1
0	1	0
1	0	0
1	1	1

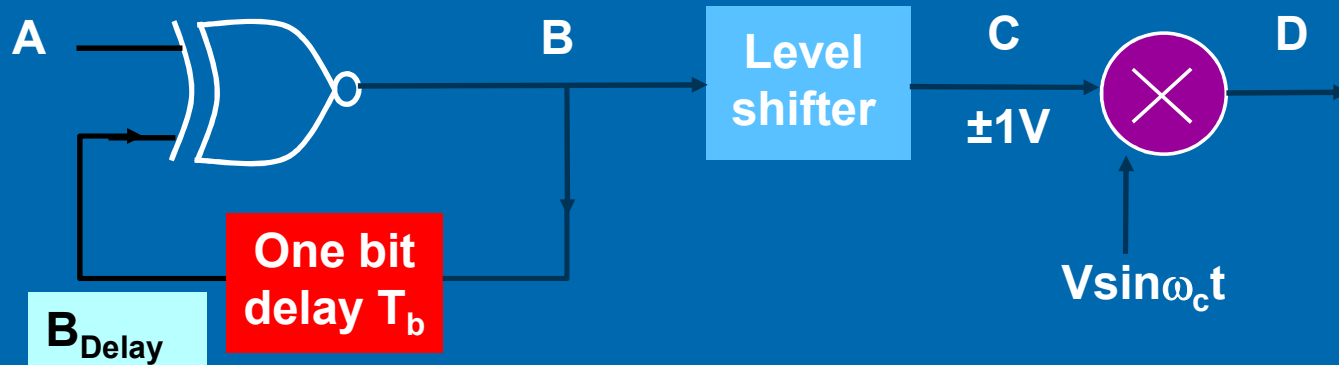




X-NOR

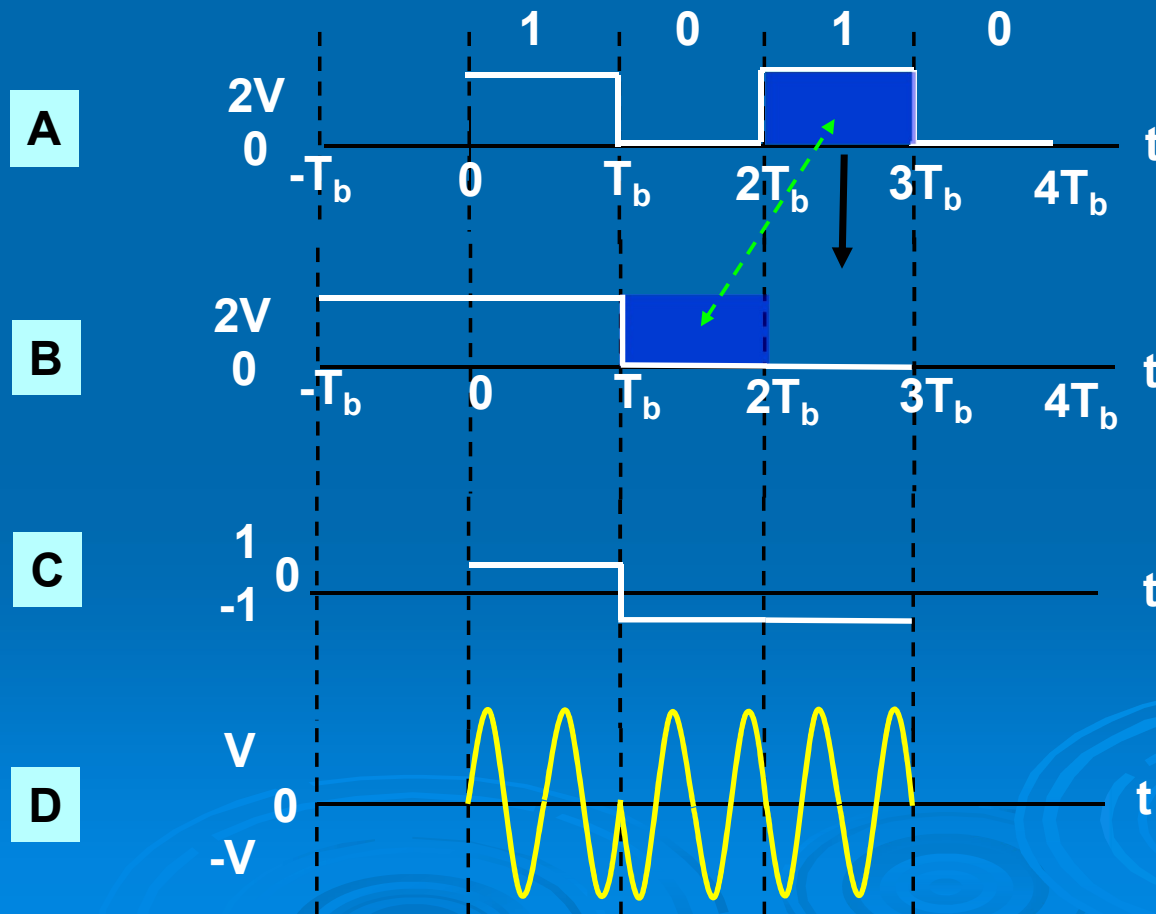
X	Y	Z(o/p)
0	0	1
0	1	0
1	0	0
1	1	1

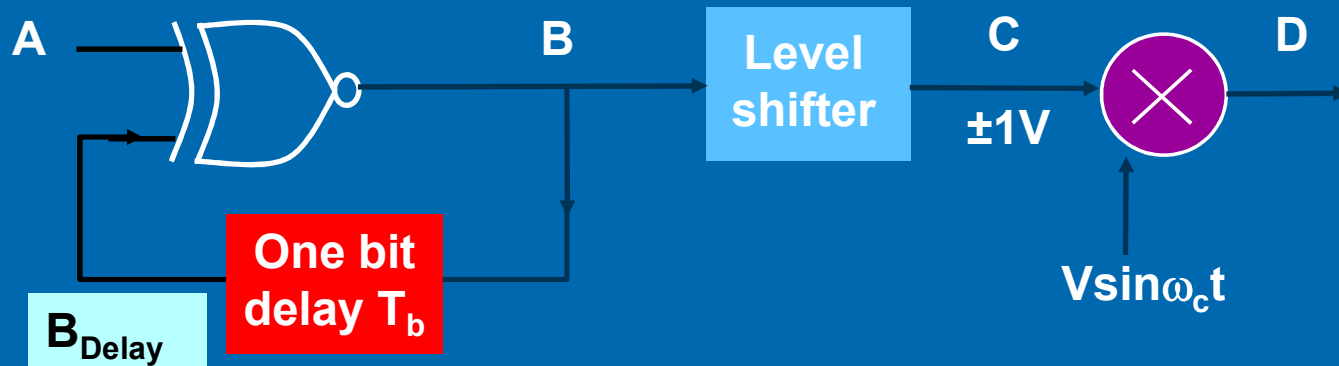




X-NOR

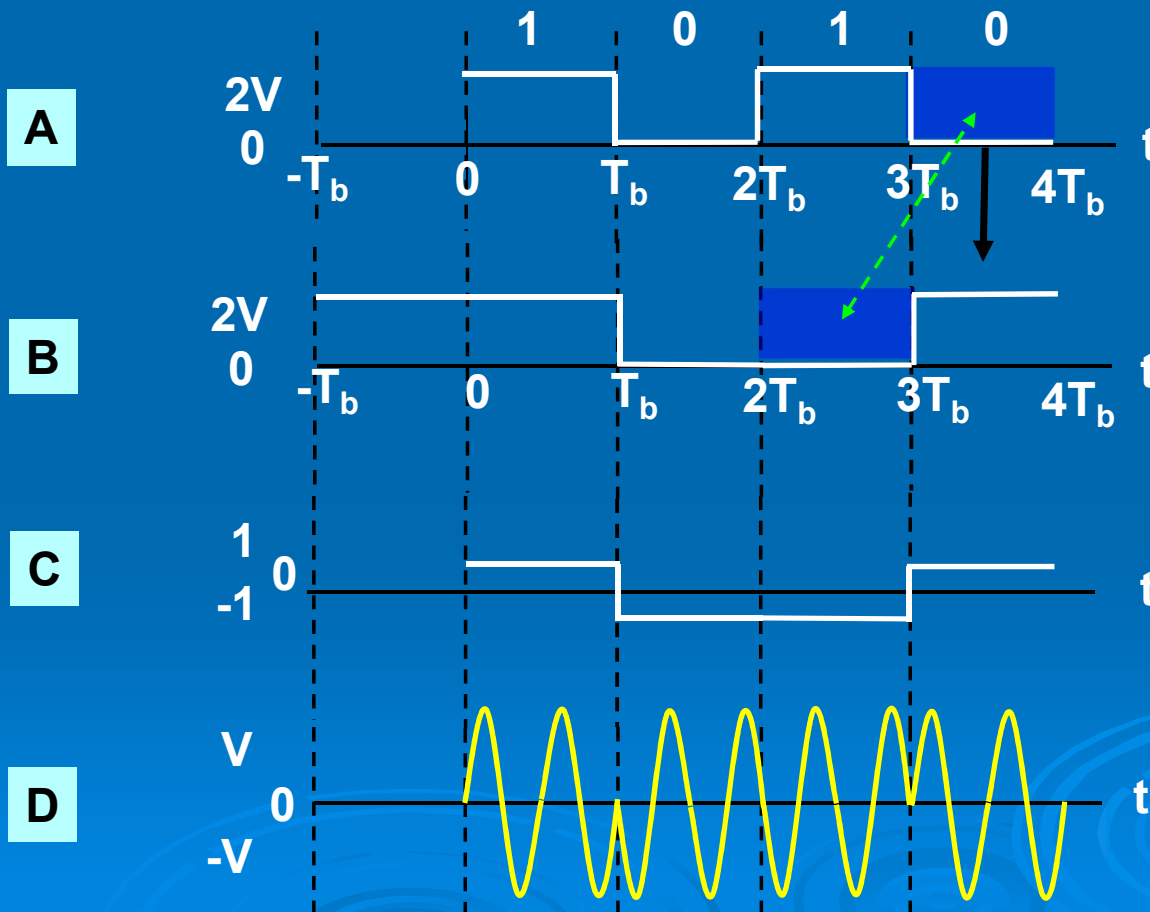
X	Y	Z(o/p)
0	0	1
0	1	0
1	0	0
1	1	1





X-NOR

X	Y	Z(o/p)
0	0	1
0	1	0
1	0	0
1	1	1



10.6 Quadrature Phase Shift Keying (QPSK)

- Limitation of binary modulation techniques - require high transmission bandwidth.

Each bit is transmitted individually

- One way of improving bandwidth utilisation - use quadrature multiplexing e.g. *quadrature phase shift keying* (QPSK).
- In QPSK, two bits are lumped together to form a symbol.
 - There are **four** distinct symbols corresponding to the two-bit sequences 00, 01, 10 and 11.
 - A symbol is transmitted as a carrier with an initial phase angle of four possible values.



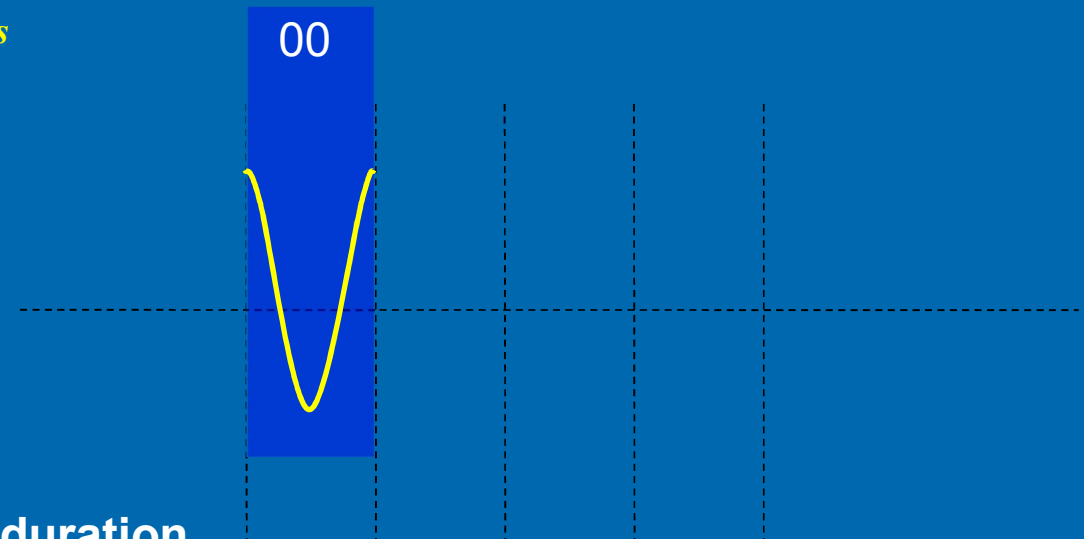
10.6 Quadrature Phase Shift Keying (QPSK)

- The four possible phase angles are the four equally spaced angles.

e.g.

$$s_i(t) = \begin{cases} A \cos(2\pi f_c t + \phi_i) & 0 \leq t \leq T_s \\ \phi_i = \begin{cases} 0 & \text{for "00"} \\ 90^\circ & \text{for "01"} \\ 180^\circ & \text{for "10"} \\ 270^\circ & \text{for "11"} \end{cases} \end{cases}$$

where T_s ($T_s = 2T_b$) is the symbol duration.

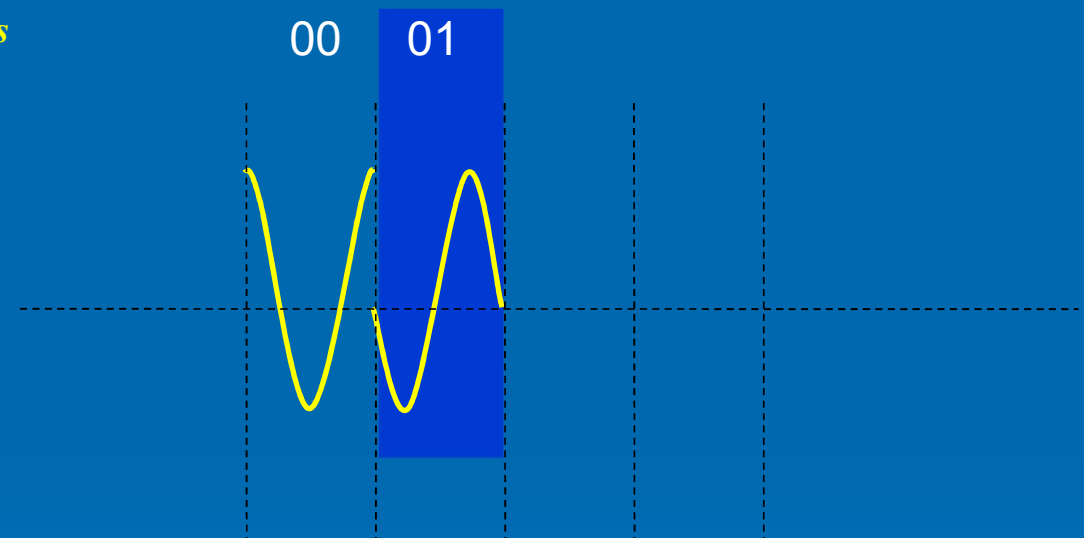


10.6 Quadrature Phase Shift Keying (QPSK)

- Each symbol is a carrier that takes on one of the four equally spaced initial phase angles. e.g.

$$s_i(t) = \begin{cases} A \cos(2\pi f_c t + \phi_i) & 0 \leq t \leq T_s \\ \phi_i = \begin{cases} 0 & \text{for "00"} \\ 90^\circ & \text{for "01"} \\ 180^\circ & \text{for "10"} \\ 270^\circ & \text{for "11"} \end{cases} \end{cases}$$

where T_s is the symbol duration.

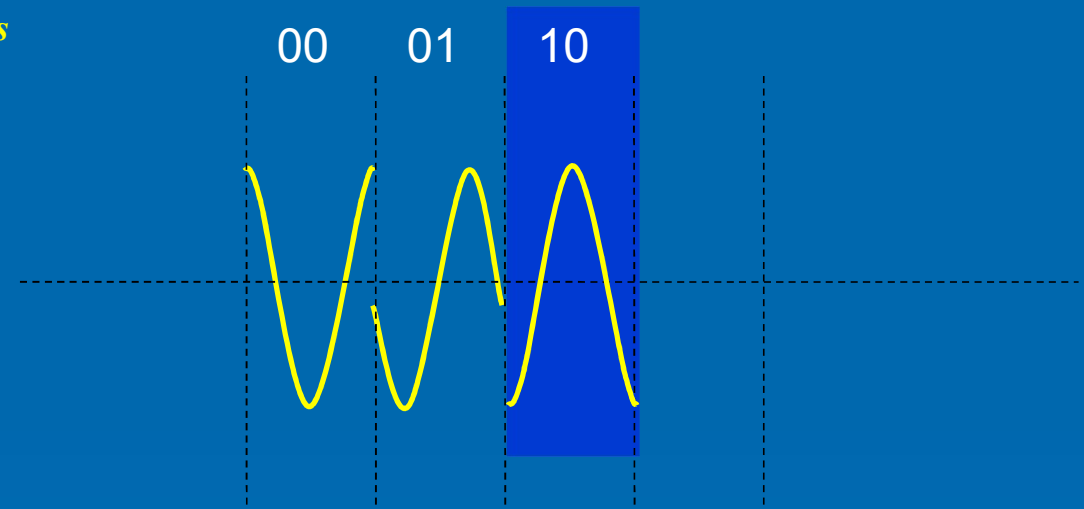


10.6 Quadrature Phase Shift Keying (QPSK)

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where T_s is the symbol duration.

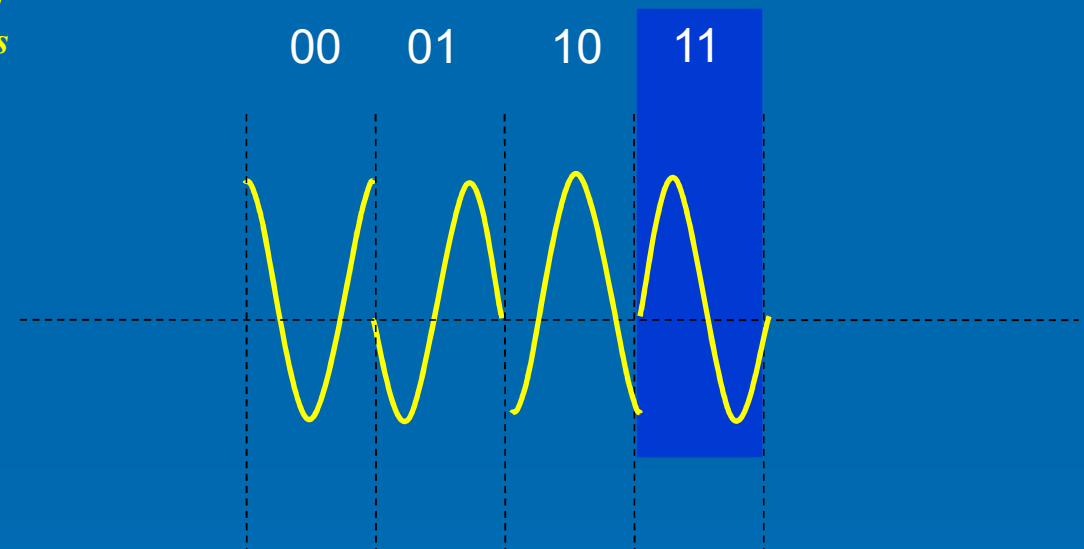


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$$s_i(t) = \begin{cases} A \cos(2\pi f_c t + \phi_i) & 0 \leq t \leq T_s \\ \phi_i = \begin{cases} 0 & \text{for "00"} \\ 90^\circ & \text{for "01"} \\ 180^\circ & \text{for "10"} \\ 270^\circ & \text{for "11"} \end{cases} \end{cases}$$

where T_s is the symbol duration.



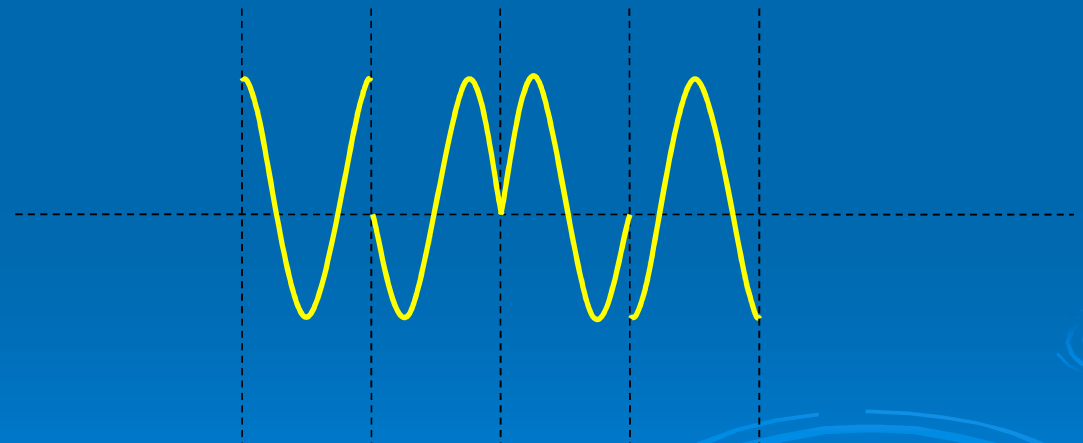
10.6 Quadrature Phase Shift Keying (QPSK)

- The QPSK waveform of a bit stream of 01111000 generated by a QPSK modulator (LSB at the right most). Note: LSB is transmitted first.

01 11 10 00
LSB is transmitted first

$$s_i(t) = \begin{cases} A \cos(2\pi f_c t + \phi_i) & 0 \leq t \leq T_s \\ \phi_i = \begin{cases} 0 & \text{for "00"} \\ 90^\circ & \text{for "01"} \\ 180^\circ & \text{for "10"} \\ 270^\circ & \text{for "11"} \end{cases} \end{cases}$$

00 01 11 10



10.6 Quadrature Phase Shift Keying(QPSK)

- The probability of bit error for a QPSK system is given by

$$P_e = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{V^2 T_b}{2\eta}} \right)$$

Same P_e as in BPSK

- QPSK has the same probability of bit error as that of BPSK.
But the bandwidth required by QPSK is half of that is required by BPSK.
- QPSK is the preferred system if smaller channel bandwidth is desired.



10.7 M-ary PSK (MPSK)

- In MPSK, the initial phase angle of the carrier takes on one of M possible values:

$$S_i(t) = A \cos (2\pi f_c t + \phi_i) \quad \phi_i = 360i/M, \text{ where } i = 0, 1, \dots, M-1.$$

- During each symbol interval T_s , one of M possible symbols is sent.

$$s_i(t) = A \cos \left(2\pi f_c t + \frac{360i}{M} \right) \quad i = 0, 1, \dots, M-1$$

- QPSK is an example of MPSK with $M = 4$.



10.7 M-ary PSK (MPSK)

- In MPSK, as M is increased, bandwidth efficiency is improved.
 - Trade-off is increase in transmitted power or an increase in P_e .
- Among MPSK, QPSK offers the best trade-off between power and bandwidth requirements. Thus, QPSK is the first to be widely used in practice.
- For $M > 8$,
 - Higher transmitted power is required.
 - More complex equipment is required.



10.8 Quadrature Amplitude Modulation (QAM)

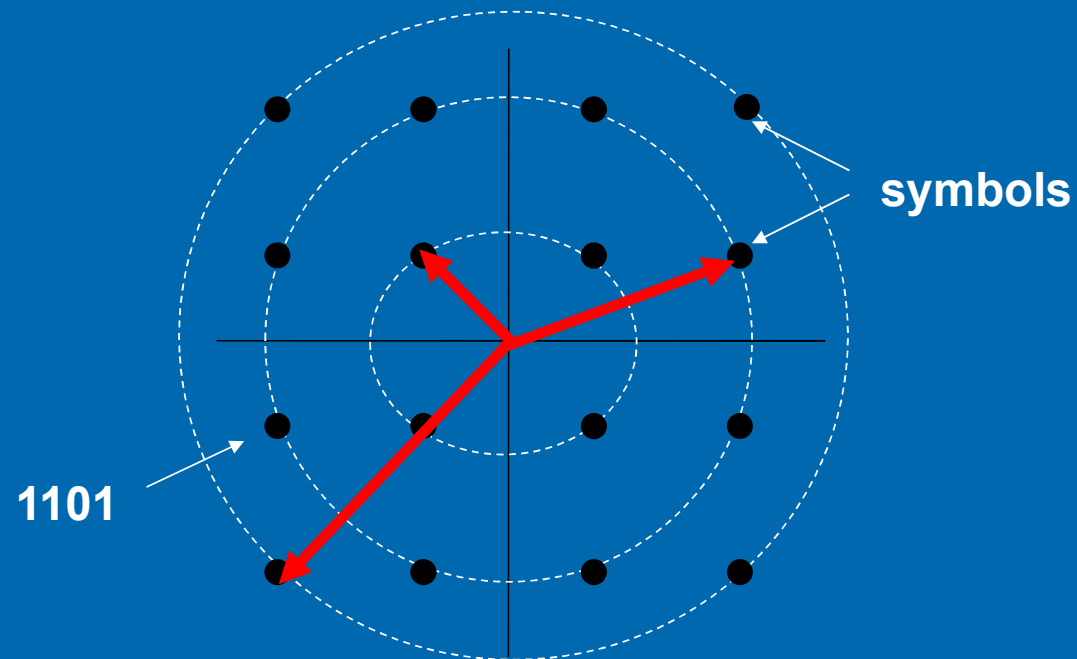
- Channel bandwidth utilisation can be improved further by the hybrid use of amplitude and phase modulation.
- A common form of this hybrid scheme is QAM. The principle of QAM is illustrated using 16-QAM.
- In 16-QAM there are 16 possible symbols, each representing a 4-bit data.



10.8 Quadrature Amplitude Modulation (QAM)

- A possible way to represent these 16 symbols is by a constellation diagram shown.
- The constellation uses three different amplitudes, two with four phases and one with eight phases.

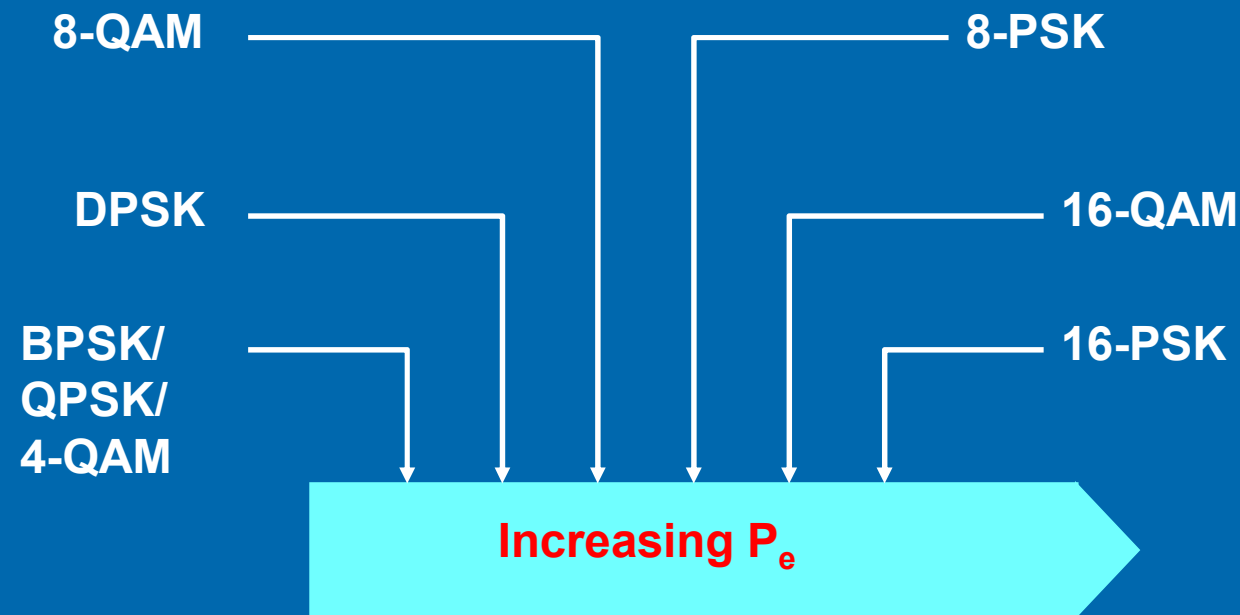
Used in 9600 bits/s modems



10.9 Comparison of digital modulation systems

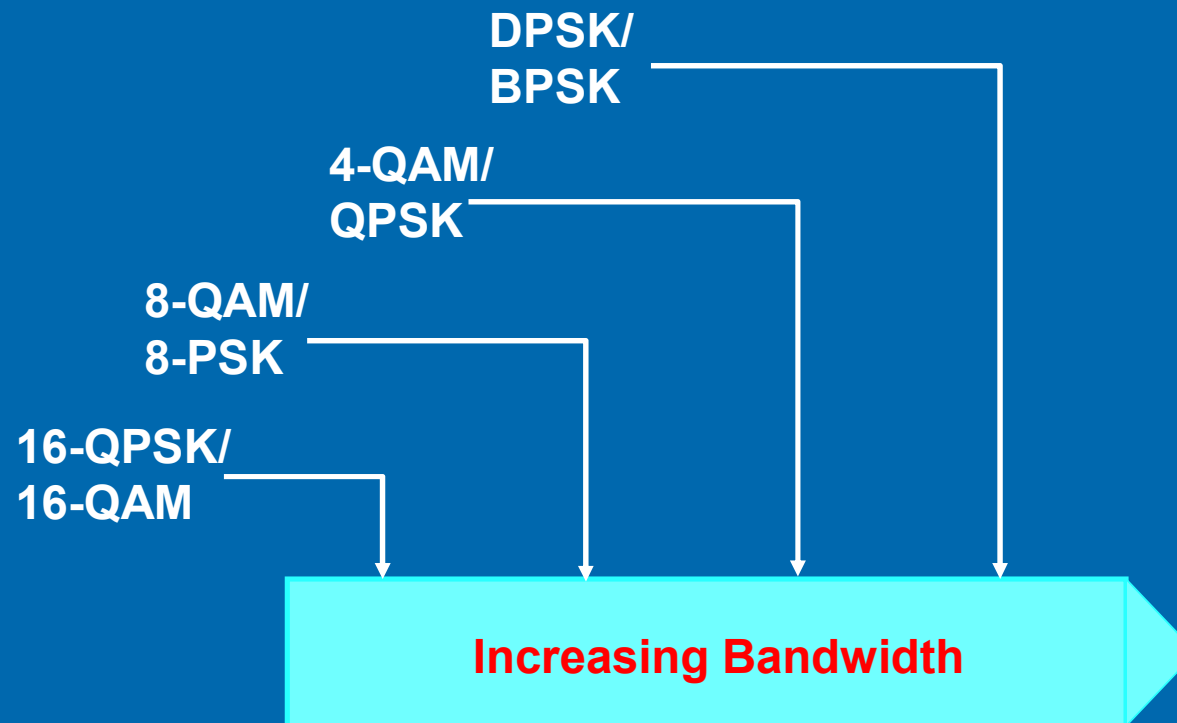
- Choice of digital modulation methods depends on
 - error performance,
 - bandwidth efficiency (in bps/Hz)
 - equipment complexity.

Error Performance Comparison



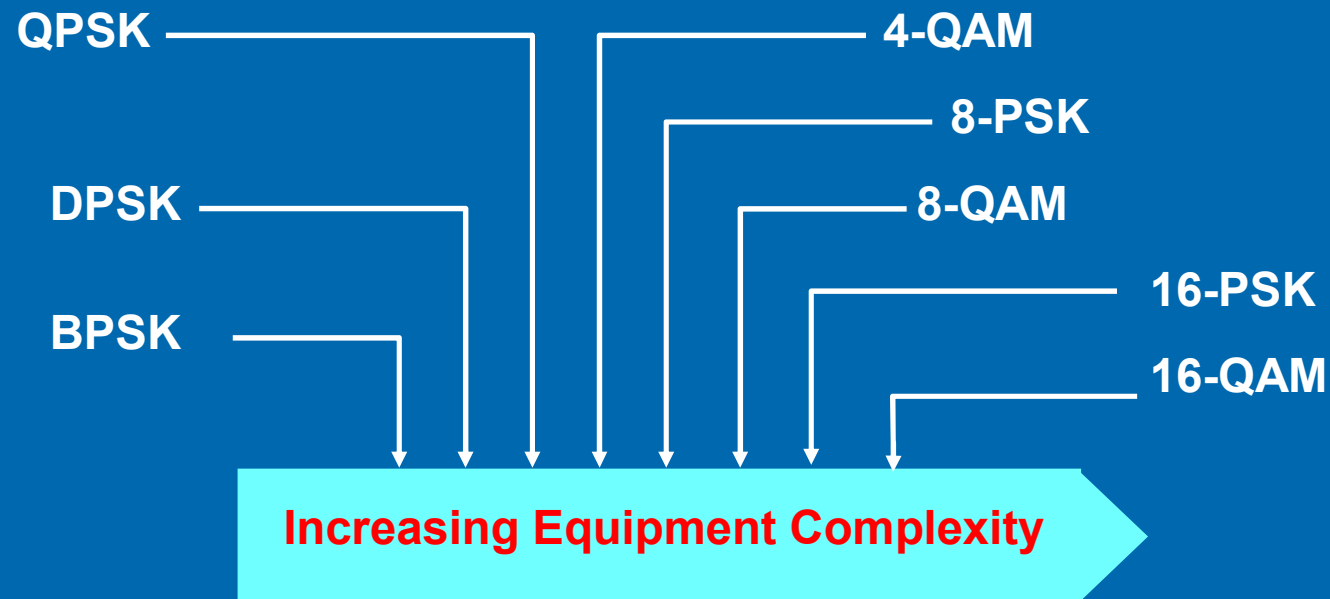
10.9 Comparison of digital modulation systems

Bandwidth Comparison



10.9 Comparison of digital modulation systems

Equipment Complexity Comparison

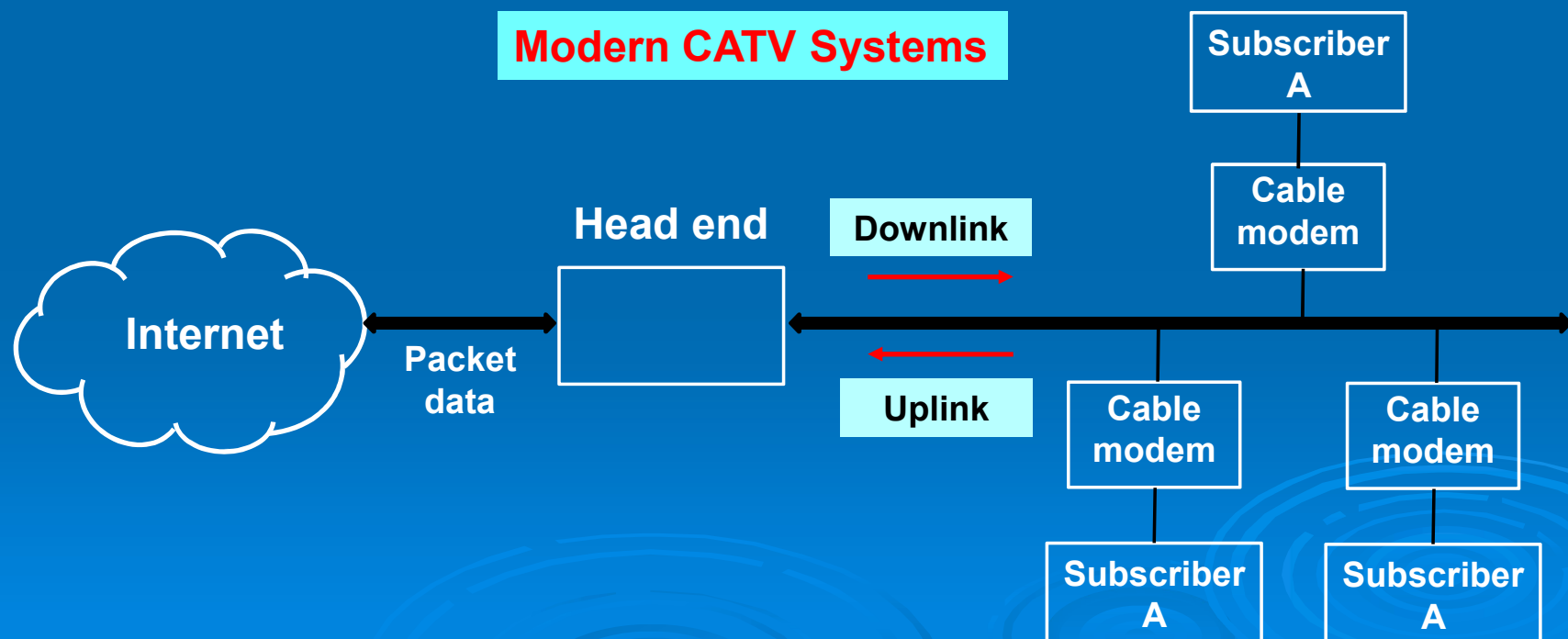


10.10 Applications

Cable Data Modem

- Modern CATV systems provides a high-speed internet connection via a cable modem distributing the TV and data signals from the CATV head end to the neighbourhood of customer.
- The signals are converted from light to RF signal and transmitted to individual home via coaxial cable.

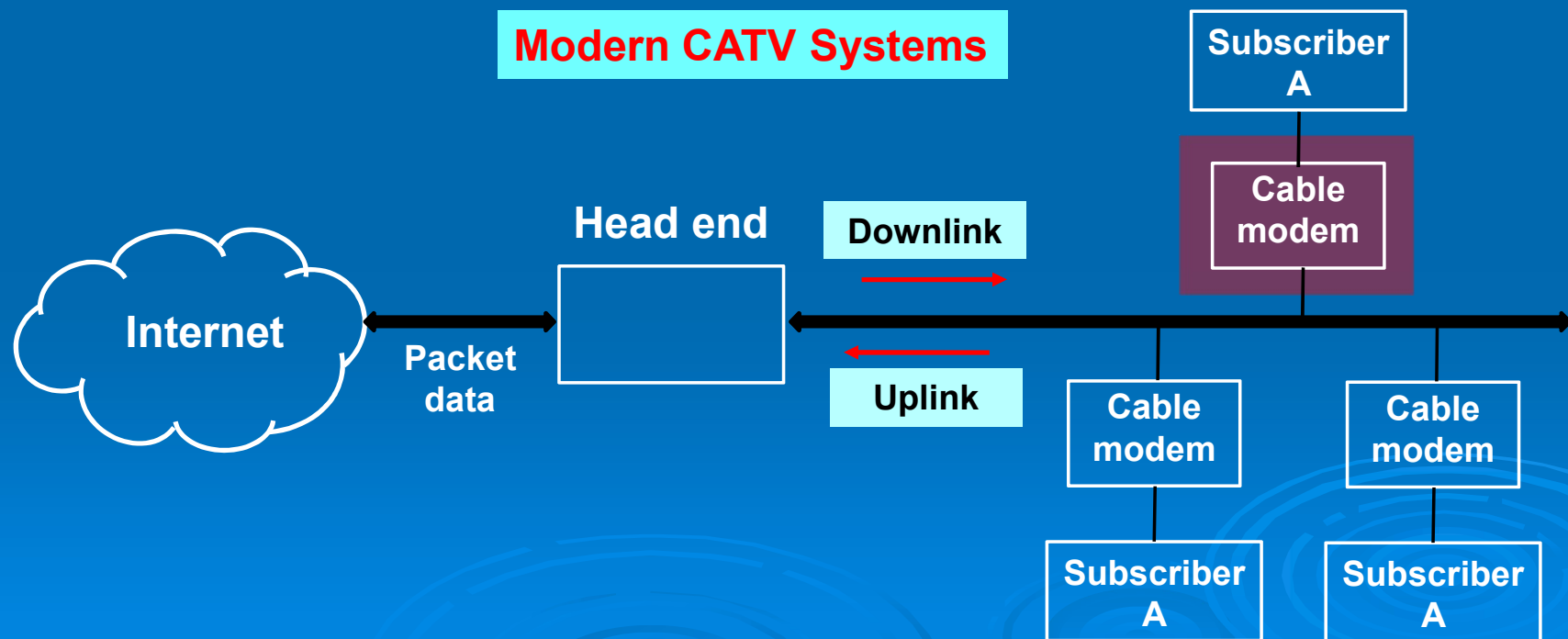
Operate up to 800 MHz



10.10 Applications

Cable Data Modem

- The cable modem, usually connected to a PC or in-house data network via an Ethernet line, demodulates the downlink data and modulates the uplink data:
 - The downlink is usually around 3 Mega bits/s
 - The uplink speed is around 500 kbits/s.



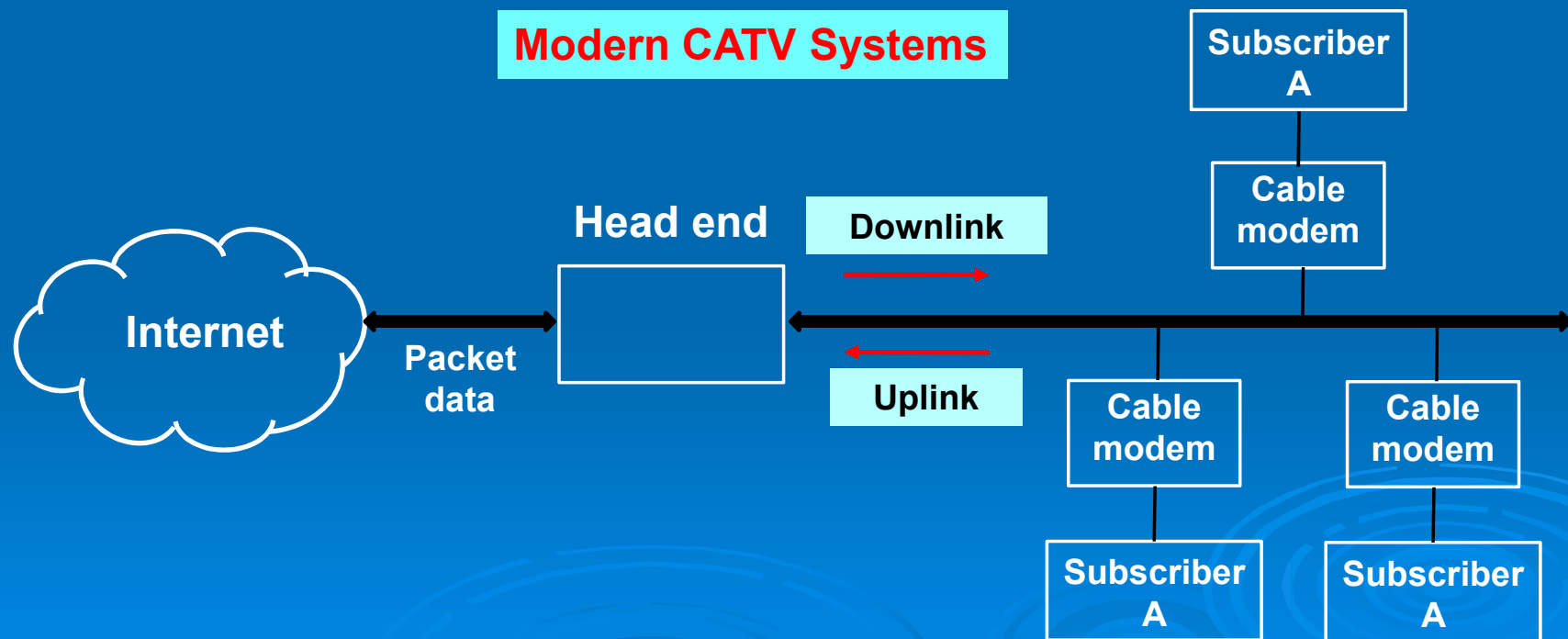
10.10 Applications

Cable Data Modem

- A single downlink 6 Mega hertz-wide channel can support a combined downstream data rate:

27 Mbits/s if 64 QAM is used.

36 Mbits/s if 256 QAM is used.



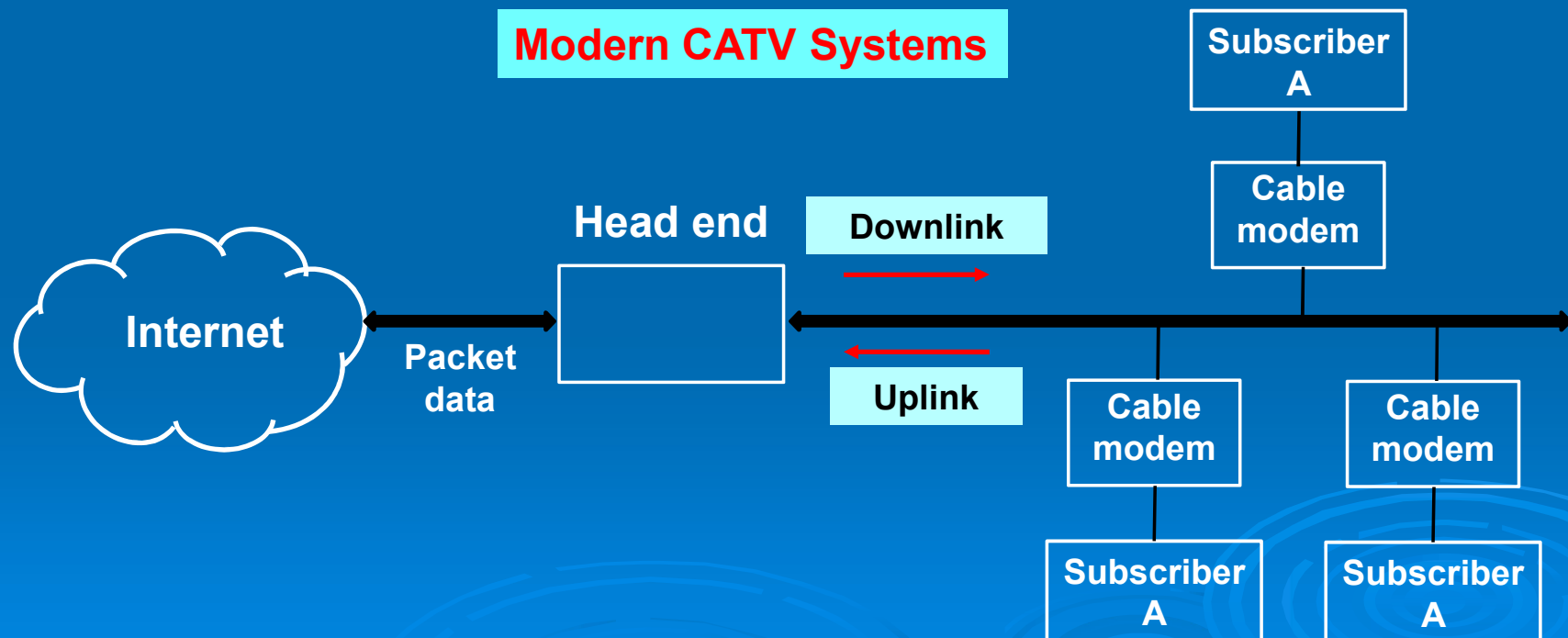
10.10 Applications

Cable Data Modem

- One uplink channel supports a combined data rate:

10 Mbits/s if QPSK is used

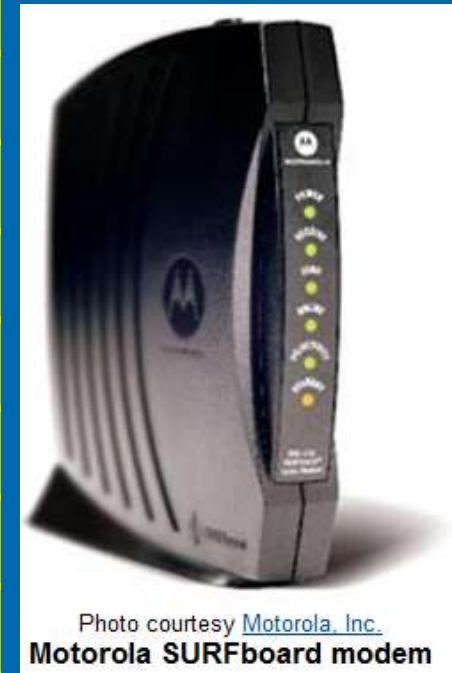
30 Mbits/s if 16QAM is used



10.10 Applications

■ Cable Modem Standards

Component	Downstream	Upstream
Carrier frequency range	50 - 750 MHz	5 – 42 MHz
Channel bandwidth	6 MHz	6 MHz or 2 MHz
Modulation	64-QAM or 256-QAM	QPSK or 16-QAM
Composite data rate	27 Mb/s or 36 Mb/s	10 – 30 Mb/s
Subscriber data rate	1.5 – 6 Mb/s	256 kb/s – 1.5 Mb/s
Coding	Block code (Reed Solomon)	Block code (RS)
Encryption	DES	DES



10.10 Applications

- Digital Radio
Microwave radio links using multilevel QAM.
- Digital Communications by satellite
QPSK is often used.



End

CHAPTER 10

(Part 2 of 2)

