SINGAPORE POLYTECHNIC

2019/2020 SEMESTER ONE EXAMINATION

School of Architecture & the Built Environment
DCEB

School of Chemical and Life Sciences
DAPC, DCHE, DFST, DPCS

School of Electrical and Electronic Engineering
DASE, DCEP, DCPE, DEB, DEEE

School of Mechanical and Aeronautical Engineering
DARE, DCEP, DME, DMRO, DBEN

Singapore Maritime Academy (SMA)
DMR

1st Year FT

ENGINEERING MATHEMATICS I Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **THREE** sections:

   Section A: 5 Multiple-Choice Questions (10 marks)
   Answer **ALL** questions.

   Section B: 7 Questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

   Section C: 3 Questions (40 marks)
   Answer **ALL** questions.

3. Unless otherwise stated, leave all answers correct to three significant figures.

4. Except for sketches, graphs and diagrams, no solution or answer is to be written in pencil.

5. This examination paper consists of **7** printed pages.

Section A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks. No mark will be deducted for incorrect answers.

Tick the choice of answer for each question in the box of the **MCQ answer sheet** provided in the answer booklet.

A1. Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be two non-singular square matrices. Which of the following statements is always TRUE?

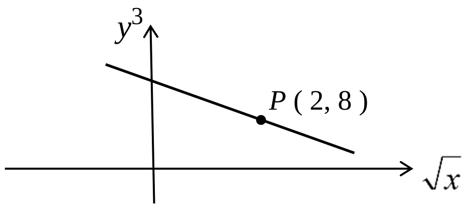
(a) $|\boldsymbol{A}|+|\boldsymbol{B}|=|\boldsymbol{A}+\boldsymbol{B}|$

(b) $2|\boldsymbol{A}|=|2\boldsymbol{A}|$

(c) $\left|\boldsymbol{A}^3\right|=|\boldsymbol{A}|^3$

(d) $|\boldsymbol{AB}|=|\boldsymbol{BA}|$

A2. The diagram below shows the graph of $y^3=a\sqrt{x}+b$, where a and b are constants.

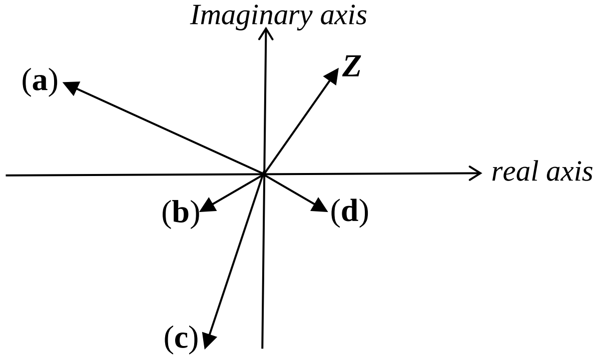


The values of x and y at point P are __________ .

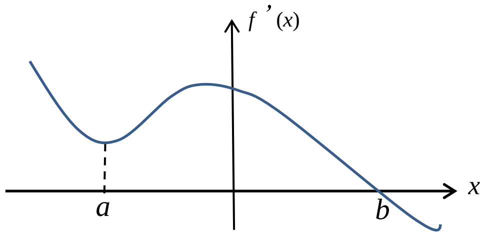
(a) $x = 4$, $y = 2$ (b) $x = 2$, $y = 8$

(c) $x = 4$, $y = 8$ (d) $x = 8$, $y = 2$

A3. The complex number $\boldsymbol{Z}$ is shown in the Argand diagram below.
Given that $|\boldsymbol{Z}| > 1$ and $45^0 < \arg(\boldsymbol{Z}) < 90^0$, which one of the four complex numbers represented by (**a**), (**b**), (**c**) and (**d**) in the diagram is possibly $\dfrac{1}{\boldsymbol{Z}^2}$?



A4. The diagram below shows the graph of the <u>derivative</u> $\frac{dy}{dx} = f\,'(x)$.



Which of the following statements best described the <u>function</u> $f(x)$?

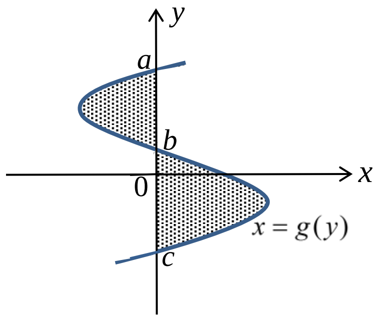
(a) $f(a)$ has positive value.

(b) $f(a)$ has a stationary point.

(c) $f(x)$ is always positive over the interval $a < x < b$.

(d) $f(x)$ increases with x over the interval $a < x < b$.

A5.



Referring to the diagram above, which of the following represents the area of the shaded region bounded by the curve $x = g(y)$?

(a) Total area $= \int_b^a g(y)dy - \int_c^b g(y)dy$

(b) Total area $= \int_c^b g(y)dy - \int_b^a g(y)dy$

(c) Total area $= \int_b^c g(y)dy - \int_a^b g(y)dy$

(d) Total area $= \int_b^a g(y)dy + \int_c^b g(y)dy$

Section B (50 marks)

Each question carries 10 marks.

The total mark of the questions in this section is 70 marks.

You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. The currents I_1 , I_2 and I_3 in an electrical circuit fulfil the following equations :

$$\begin{aligned} I_1 + I_2 + I_3 &= 0 \\ k\,I_1 \qquad - 5\,I_3 &= 8 \\ 10\,I_2 - 5\,I_3 &= 8 \end{aligned}$$

Use Cramer's rule to find the value of I_2 in terms of the constant k.
(Detailed workings of evaluating a determinant must be clearly shown.)

B2. Given $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ -4 & 3 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 5 & 8 \\ 6 & -7 \end{pmatrix}$.

(a) Evaluate $5\boldsymbol{A} - \boldsymbol{B}$.

(b) Find $\boldsymbol{A}^{-1}$.

(c) Find matrix $\boldsymbol{C}$, given that $\boldsymbol{AC} = \boldsymbol{B}^{T}$.

B3. (a) Given $z_1 = 2\angle 60^o$ and $z_2 = -1 - j$.

(i) Express z_1 in rectangular form and z_2 in polar form.

(ii) Determine the magnitude and argument of $\left(z_1 + z_2\right)$.

(iii) Evaluate $\overline{z_1 z_2}$, leaving the answer in polar form.

(b) Solve $x + 1 + jy = 3x + j4$ for real numbers x and y.

B4. Experimental values of x and y were measured and shown in the table below. The values are related by the formula $y = a\,x^n$ where a and n are constants.

x	1.0	1.5	2.5	4.0	5.5
y	50	112	310	800	1510

(a) By using logarithm to the base 10, rearrange the formula such that a best fit straight line can be drawn.

(b) State the variables that should be plotted on the vertical and horizontal axes of a graph.

(c) Hence compute in a table, the values of the variables to be plotted on the horizontal and vertical axes, correct to 3 significant figures. ***Do not plot the values***.

(d) Suppose the best fit straight line passes through the points $(0.1, 1.9)$ and $(0.8, 3.3)$. Use these two points to estimate the gradient and the vertical intercept. Hence, determine the values of a and n.

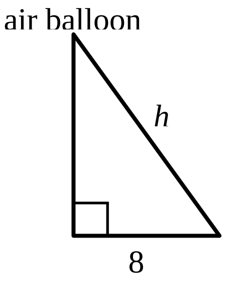
B5. Find $\dfrac{dy}{dx}$ for each of the following :

(a) $y = 2\tan^{-1}(3x) - \ln(2+3x)$

(b) $e^{2x} + 3y^2 = x - 4y$

B6. (a) Suppose that the demand function for a product is given by $D(t) = \dfrac{80000}{1.6t+9}$ where t is in days.
Find the rate of change of the quantity demanded when $t = 100$ days.

(b) An air balloon is rising vertically at a constant speed of 2 m/s from a location at ground level which is 8 metres away from the person handling the string of the air balloon.
Let h be the distance from the air balloon to the person.
Find the rate of change of h with respect to time t when $h = 10$ metres.



B7. (a) Find the following integrals:

(i) $\displaystyle\int \left(\frac{1}{x^3} - \frac{5}{x} + \sec(3x)\tan(3x) \right) dx$

(ii) $\displaystyle\int \frac{3}{\sqrt{64 - x^2}}\, dx$

(b) The reaction rate to a new drug is given by

$$\frac{dR}{dt} = e^{-0.05t} + 0.01$$

where R is the total reaction to the drug and t is the time (in hours) after the drug is administered. If no drug is administered at time $t = 0$, find the total reaction when $t = 6$ hours.

Section C (40 marks)
Answer ALL **THREE** questions.

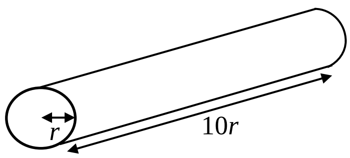
C1. (a) It is given that the second derivative of a curve is $\dfrac{d^2y}{dx^2} = -6x$ and it has a relative maximum value of 34 when $x = 3$. Find the equation of the curve.

(9 marks)

(b) There is a function $f(x)$ such that $\int_0^1 (x-a)^2 f(x)\, dx = 0$, where a is a non-zero real number. If $\int_0^1 x^2 f(x)\ dx = a^2$ and $\int_0^1 f(x)\ dx = 1$, find $\int_0^1 x\, f(x)\ dx$ in terms of a. (6 marks)

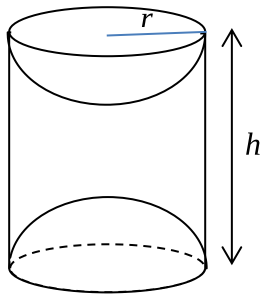
C2. (a) The figure below shows a circular cylindrical metal rod. The volume of the rod increases at a constant rate of $0.025 \text{ cm}^3/\text{s}$ when it is heated.

Find $\dfrac{dr}{dt}$ when the radius of the rod is 3 cm.



(5 marks)

(b) The figure on the right shows a container which is made up of a cylinder with a hemisphere carved out at each end of the cylinder. The total surface area of the container is fixed at 640 cm^2 and its radius is allowed to vary.



(i) Show that the volume of the container is

$$V = 320r - \frac{10}{3}\pi r^3 \quad \text{cm}^3$$

(ii) Find the radius of the container for its volume to be at its maximum.

(12 marks)

[Note : Volume of cylinder = $\pi r^2 h$

Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

where r is the radius and h is the height of the cylinder .]

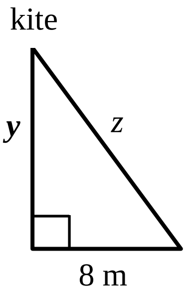
C3. Suppose z and w are complex numbers such that $\bar{z} = j\,\bar{w}$ and $\arg(z\,w) = -\pi$.

Find $\arg(z)$ and $\arg(w)$. (8 marks)

~~~ END OF PAPER ~~~

<table>
<tr><th>No.</th><th>SOLUTION</th></tr>
<tr><td>A</td><td>A1) c A2) a A3) b A4) d A5) b</td></tr>
<tr><td></td><td></td></tr>
<tr><td>B1</td><td>$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ k & 0 & -5 \\ 0 & 10 & -5 \end{vmatrix} = 0+0+10k-0-(-50)-(-5k)$$
$$= 50+15k$$
$$\Delta_{I_2} = \begin{vmatrix} 1 & 0 & 1 \\ k & 8 & -5 \\ 0 & 8 & -5 \end{vmatrix} = -40+0+8k-0-(-40)-(0)$$
$$= 8k$$
$$I_2 = \frac{\Delta_{I_2}}{\Delta} = \frac{8k}{50+15k}$$</td></tr>
<tr><td>B2
(a)</td><td>$$\boldsymbol{5A} - \boldsymbol{B} = \begin{pmatrix} 5 & 10 \\ -20 & 15 \end{pmatrix} - \begin{pmatrix} 5 & 8 \\ 6 & -7 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ -26 & 22 \end{pmatrix}$$</td></tr>
<tr><td>(b)</td><td>$$|\boldsymbol{A}| = 11$$
$$adj(\boldsymbol{A}) = \begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix}$$
$$\boldsymbol{A}^{-1} = \frac{1}{11}\begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix}$$</td></tr>
<tr><td>(c)</td><td>$\boldsymbol{AC = B^T}$
$\boldsymbol{C = A^{-1} B^T}$
$$\boldsymbol{B}^T = \begin{pmatrix} 5 & 6 \\ 8 & -7 \end{pmatrix}$$
$$\boldsymbol{C} = \frac{1}{11}\begin{pmatrix} 3 & -2 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 8 & -7 \end{pmatrix} = \frac{1}{11}\begin{pmatrix} -1 & 32 \\ 28 & 17 \end{pmatrix}$$</td></tr>
</table>

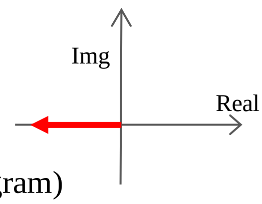
No.	SOLUTION
B3 (a)i	$z_1 = 2\angle 60^o = 1 + j\sqrt{3}$ $z_2 = -1 - j = \sqrt{2}\angle -135^o$
(ii)	$z_1 + z_2 = \left(1 + j\sqrt{3}\right) + \left(-1 - j\right) \quad = 0 + j0.732$ $\lvert z_1 + z_2 \rvert = 0.732$ $\arg\left(z_1 + z_2\right) = 90^o$
(iii)	$\overline{z_1 z_2} = \overline{\left(2\angle 60^o\right)\left(\sqrt{2}\angle -135^o\right)} = \overline{2\sqrt{2}\angle -75^o} \quad = 2\sqrt{2}\angle 75^o$
(b)	$x + 1 + jy = 3x + j4$ Equate real parts : $x + 1 = 3x$ $x = ½$ Equate imaginary part : $y = 4$
B4 (a)	$\log y = \log ax^n$ $\log y = \log a + n\log x$ $\log y = n\log x + \log a \rightarrow Y = mX + c$
(b)	$\log y$ should be plotted on vertical axis. $\log x$ should be plotted on horizontal axis.
(c)	<table><tr><td>log x</td><td>0</td><td>0.176</td><td>0.398</td><td>0.602</td><td>0.740</td></tr><tr><td>log y</td><td>1.70</td><td>2.05</td><td>2.49</td><td>2.90</td><td>3.18</td></tr></table>
(d)	Gradient $n = \dfrac{3.3 - 1.9}{0.8 - 0.1} = 2$ $\log y = 2\log x + \log a$ At (0.1, 1.9) $1.9 = 2(0.1) + \log a$ $\log a = 1.7 \quad \therefore a = 50.1$ Hence $y = 50.1\,x^2$
B5 (a)	$\dfrac{dy}{dx} = 2\left(\dfrac{3}{1+9x^2}\right) - \dfrac{3}{2+3x}$
(b)	$2e^{2x} + 3\left(2y\dfrac{dy}{dx}\right) = 1 - 4\dfrac{dy}{dx}$ $6y\dfrac{dy}{dx} + 4\dfrac{dy}{dx} = 1 - 2e^{2x}$ $\dfrac{dy}{dx} = \dfrac{1 - 2e^{2x}}{6y + 4}$

No.	SOLUTION
B6a	$D(t)=\dfrac{80000}{1.6t+9}$ $D=80000(1.6t+9)^{-1}$ $\dfrac{dD}{dt}=-80000(1.6t+9)^{-2}(1.6)=-\dfrac{128000}{(1.6t+9)^2}$ $\left.\dfrac{dD}{dt}\right\|_{t=100}=-\dfrac{128000}{(160+9)^2}=-4.48$ units/day
B6b	Let t be time, then $y=2t$  By Pythagoras theorem, $z^2=8^2+(2t)^2$ $z=\sqrt{64+4t^2}=(64+4t^2)^{\frac{1}{2}}$ ------(1) $\dfrac{dz}{dt}=\dfrac{1}{2}(64+4t^2)^{-\frac{1}{2}}8t=\dfrac{4t}{\sqrt{64+4t^2}}$ From equation (1), when $z=10$, $t=3$. $\left.\dfrac{dz}{dt}\right\|_{t=3}=1.2$ m/s
B7 (a)i	$\int\left(\dfrac{1}{x^3}-\dfrac{5}{x}+\sec(3x)\tan(3x)\right)dx=\dfrac{-1}{2x^2}-5\ln\|x\|+\dfrac{\sec(3x)}{3}+C$
(a)ii	$\int\dfrac{3}{\sqrt{64-x^2}}\,dx=3\sin^{-1}\left(\dfrac{x}{8}\right)+C$
B7 (b)	$R=\int\left(e^{-0.05t}+0.01\right)dt$ $=\dfrac{e^{-0.05t}}{-0.05}+0.01t+C$ When $t=0$, $R(0)=0$, $\therefore C=20$ Hence when $t=6$ $R=\dfrac{e^{-0.05(6)}}{-0.05}+0.01(6)+20$ $=5.24$

No.	SOLUTION
C1 (a)	$\int \frac{d^2y}{dx^2}\,dx = \int -6x\,dx$ $\frac{dy}{dx} = -6\left(\frac{x^2}{2}\right) + C_1$ The curve has a maximum value at $x=3$, => $\left.\frac{dy}{dx}\right\|_{x=3} = 0$ $-3\left(3^2\right) + C_1 = 0$ $C_1 = 27$ $\frac{dy}{dx} = -3x^2 + 27$ $\int \frac{dy}{dx}\,dx = \int \left(-3x^2 + 27\right)dx$ $y = -x^3 + 27x + C_2$ At $x = 3$, $y = 34$: $34 = -3^3 + 27(3) + C_2$ $C_2 = -20$ There for equation of the curve is $y = -x^3 + 27x - 20$
C1 (b)	$\int_0^1 (x-a)^2\; f(x)dx = 0$ $\int_0^1 (x^2 - 2ax + a^2)\; f(x)dx = 0$ $\int_0^1 x^2\; f(x)dx - 2a\int_0^1 x\; f(x)dx + a^2\int_0^1\; f(x)dx = 0$ $a^2 - 2a\int_0^1 x\; f(x)dx + a^2 = 0$ $2a\int_0^1 x\; f(x)dx = 2a^2$ $\int_0^1 x\; f(x)dx = a$

No.	SOLUTION
C2 (a)	Given $\frac{dV}{dt} = 0.025$ cm³/second, Volume of the rod , $V = \pi r^2 \cdot 10r = 10\pi r^3$ $\frac{dV}{dt} = 30\pi r^2 \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{\left(\frac{dV}{dt}\right)}{30\pi r^2}$ $\left.\frac{dr}{dt}\right\|_{r=3} = \frac{0.025}{30\pi(3)^2} = 2.95\times10^{-5}\text{cm}/s$ **OR** $\frac{dV}{dr} = 30\pi r^2$ $\frac{dr}{dt} = \frac{dr}{dV}\frac{dV}{dt} \quad = \left(\frac{1}{30\pi r^2}\right)(0.025)$ $\left.\frac{dr}{dt}\right\|_{r=3} = \left(\frac{1}{30\pi(3)^2}\right)(0.025) \quad = \quad 2.95\times10^{-5}\text{cm}/s$
C2 (b) i ii	Volume, V = Volume of cylinder – volume of sphere $V = \pi r^2 h - \frac{4}{3}\pi r^3$ Surface area, A = surface are of sphere + surface area of cylinder $A = 4\pi r^2 + 2\pi rh = 640$ $h = \frac{640 - 4\pi r^2}{2\pi r}$ Therefore , $V = \pi r^2\left(\frac{640-4\pi r^2}{2\pi r}\right) - \frac{4}{3}\pi r^3$ $= 320r - 2\pi r^3 - \frac{4}{3}\pi r^3 \quad = 320r - \frac{10}{3}\pi r^3$ $\frac{dv}{dr} = 320 - 10\pi r^2$ Let $320 - 10\pi r^2 = 0$ $r = \sqrt{\frac{32}{\pi}}$ *radius >0* $\frac{d^2v}{dr^2} = -20\pi r < 0$ Therefore V is maximum when $r = \sqrt{\frac{32}{\pi}}$ cm

No.	**SOLUTION**
C3	Let arg(z) = α and arg(ω) = θ Since $\bar{z} = j\bar{w}$ $\arg(\bar{z}) = \arg(j\bar{w})$ $-\alpha = \frac{\pi}{2} + (-\theta)$ Since $\arg(zw) = -\pi$ $\alpha + \theta = -\pi$ $-\left(\frac{\pi}{2} - \theta\right) + \theta = -\pi$ $2\theta = \frac{-\pi}{2}$ $\theta = \frac{-\pi}{4}$ $-\alpha = \frac{\pi}{2} + \left(\frac{\pi}{4}\right)$ $\alpha = -\frac{3\pi}{4}$

No.	**Alternate SOLUTION to C3**
C3	Let $w = a + jb$ and $\bar{w} = a - jb$ $\bar{z} = j\bar{w} = j(a - jb) = ja - j^2 b$ $\bar{z} = b + ja$ $\therefore z = b - ja$
	$zw = (b - ja)(a + jb)$ $= ab + jb^2 - ja^2 - j^2 ab = ab + ab + j\left(b^2 - a^2\right) = 2ab + j\left(b^2 - a^2\right)$
	Since arg $-\pi, j = 0$ $b^2 - a^2 = 0$ $(b - a)(b + a) = 0$ $\therefore b = a$ (Reject) OR $b = -a$ (Reject, as either a or b is $-ve$ based on Argand diagram) 
	$w = a + jb = a - ja$ => *w is in the* 4^{th} *quadrant since* $\arg(zw) = -\pi$ => *IM(w) must be negative* Arg $w = \tan^{-1}\left(\frac{-a}{a}\right) = \tan^{-1}(-1) = -45°$ OR $\frac{-\pi}{4}$
	Arg $z = -180° - $ Arg w $= -180° - (-45°) = -135°$ OR $-\frac{3\pi}{4}$ OR Arg $z = $ Arg $w - 90°$ $= -45° - 90° = -135°$ OR $-\frac{3\pi}{4}$