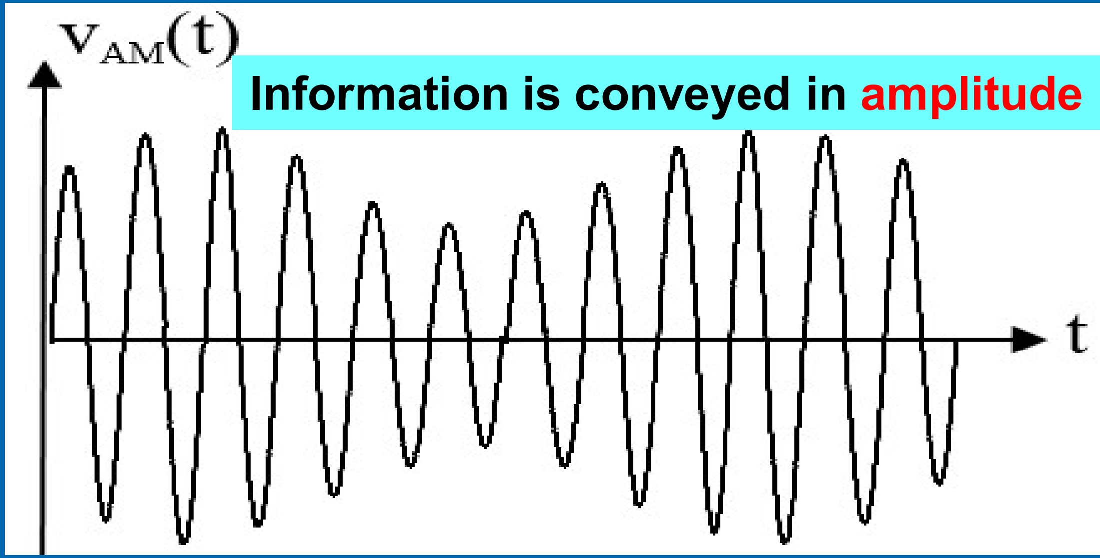
CHAPTER 6

Frequency Modulation

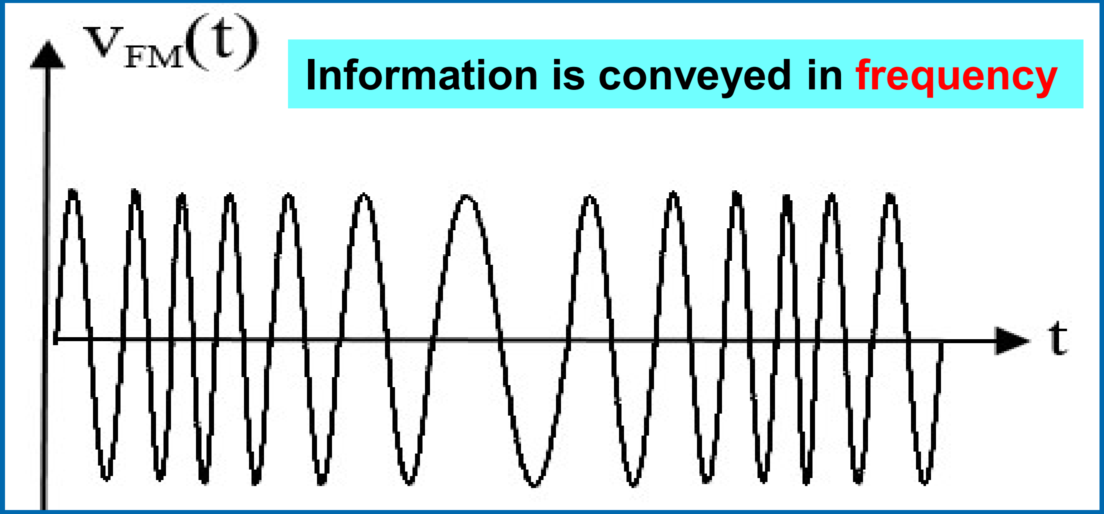
(Part 1 of 4)

Introduction



AM signal

Frequency constant
Amplitude varies



FM signal

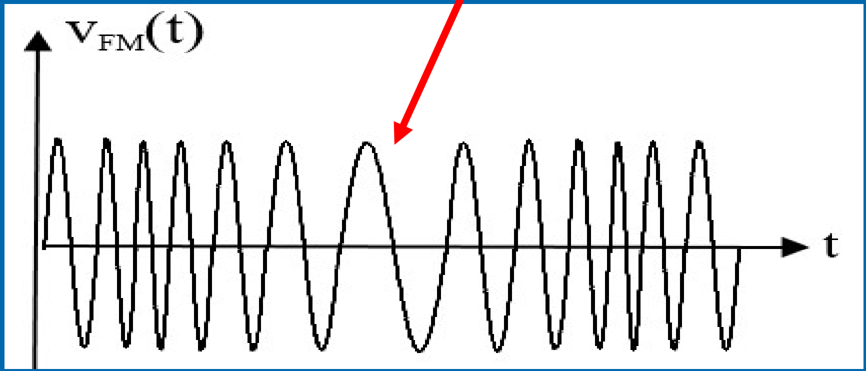
Amplitude constant
Frequency varies

6.1 Basic concepts of FM

FM signal

Frequency of FM signal is constantly changing

$$v_{FM}(t) = V_c \cos \theta(t) = V_c \cos 2\pi \left(f_c t + k_f \int_0^t v_s(\tau)\, d\tau \right)$$



When $v_s(t)=0$, FM waveform becomes a pure sinusoidal carrier: $V_c \cos(2\pi f_c t)$

The frequency at any instant in time is known as the **instantaneous frequency**, $f_i(t)$.

$$f_i(t) = f_c + k_f v_s(t)$$

changing range

$f_c + k_f [\min v_s(t)]$ **Minimum**

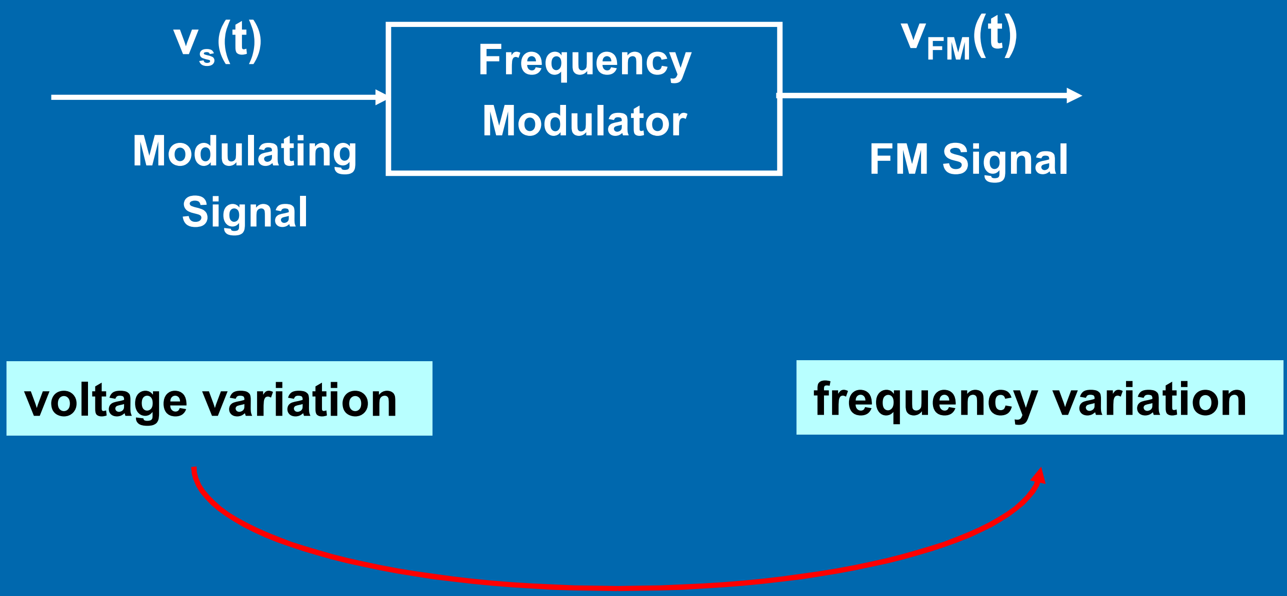
to

$f_c + k_f [\max v_s(t)]$ **Maximum**

Conversion gain (Hz/V)

Relates frequency changes to instantaneous values of $v_s(t)$

6.1 Basic concepts of FM



6.2 Single-Tone FM

<u>Single-tone FM signal</u>

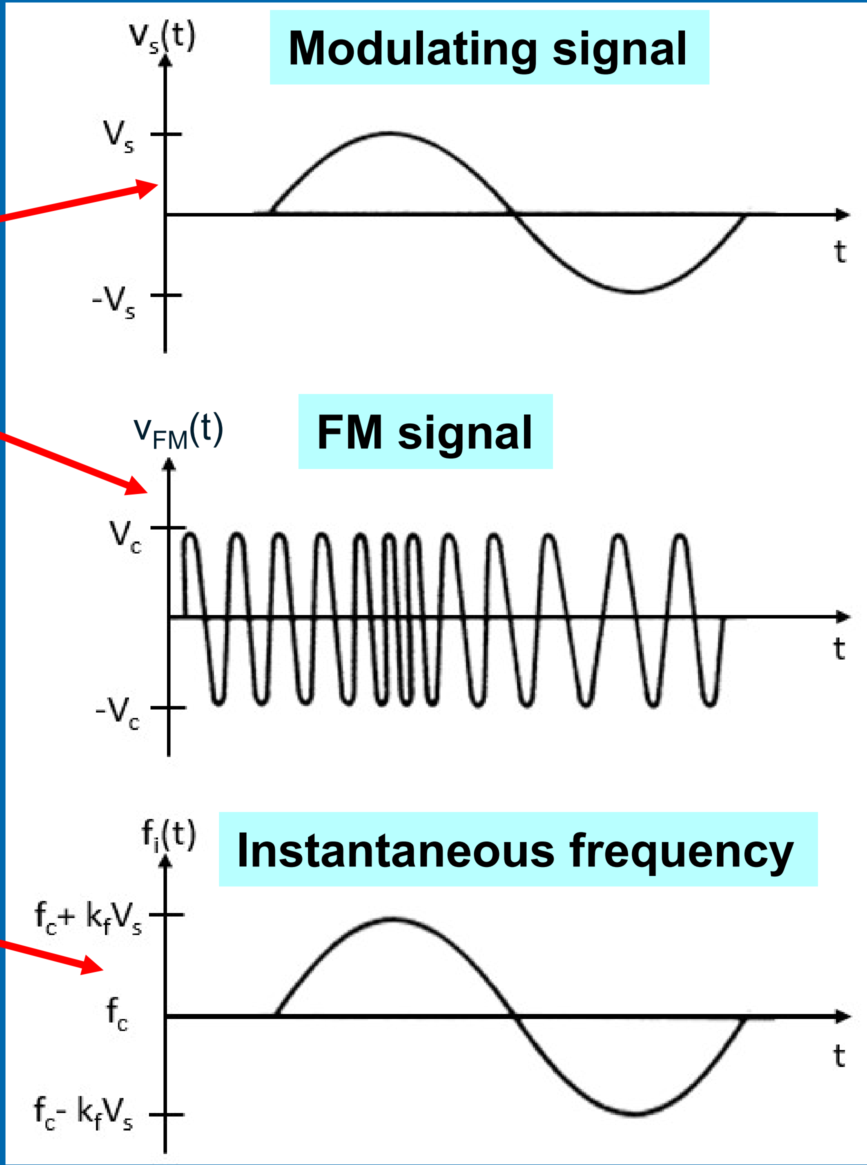
Modulating signal: $v_s(t) = V_s \cos 2\pi f t = V_s \cos\omega_s t$

FM signal: $v_{FM}(t) = V_c \cos(\omega_c t + m_f \sin\omega_s t)$

Modulation index

The instantaneous frequency:

$$f_i(t) = f_c + k_f v_s(t) = f_c + k_f V_s \cos\omega_s t$$



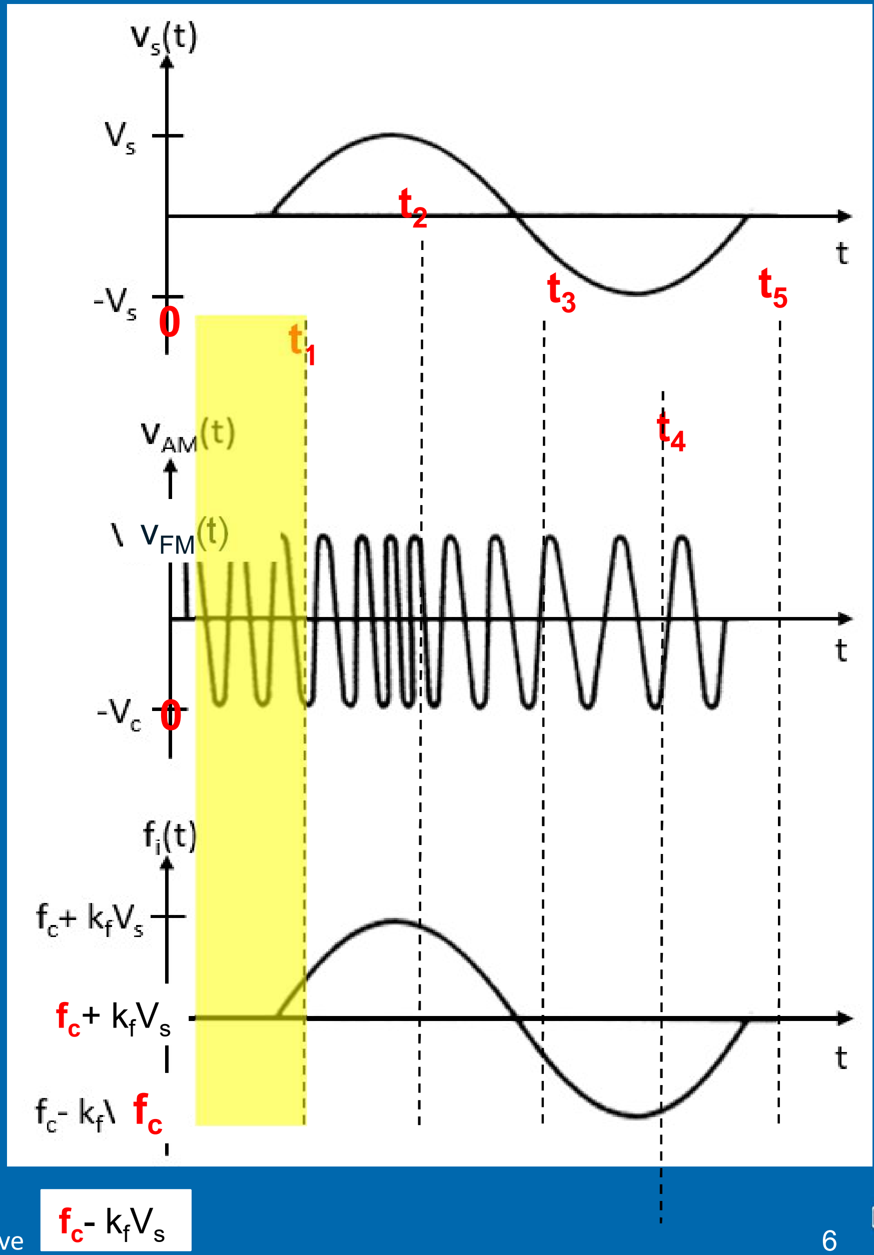
6.2 Single-Tone FM

$f_i(t) = f_c + k_f v_s(t)$

$= f_c + k_f V_s \cos\omega_s t$

follows the changes in $v_s(t)$

From 0 to t_1 ($v_s(t) = 0V$): $f_i(t) = f_c$



6.2 Single-Tone FM

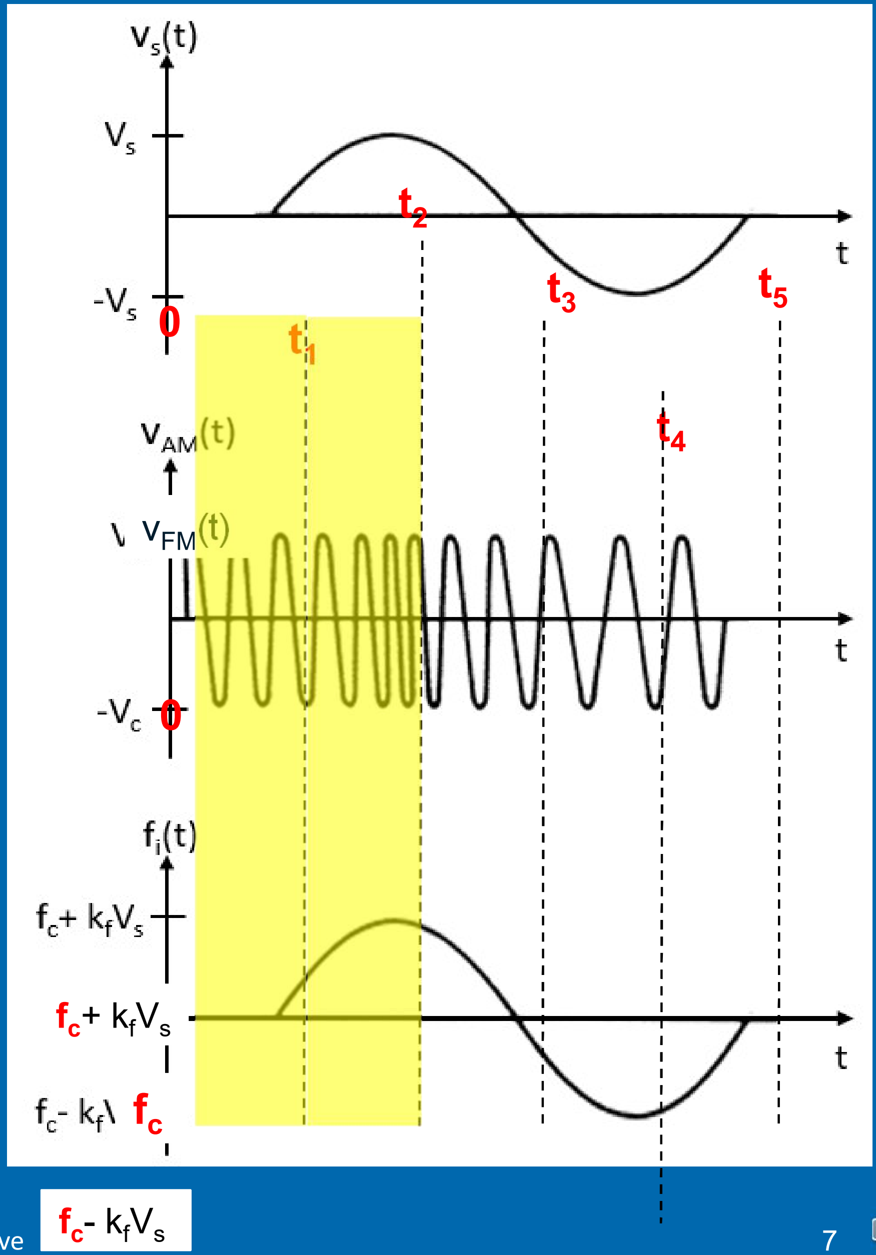
$f_i(t) = f_c + k_f v_s(t)$

$= f_c + k_f V_s \cos\omega_s t$

follows the changes in $v_s(t)$

From 0 to t_1 ($v_s(t) = 0V$): $f_i(t) = f_c$

From t_1 to t_2: $f_i(t)$ increases from f_c to $f_c+k_fV_s$



6.2 Single-Tone FM

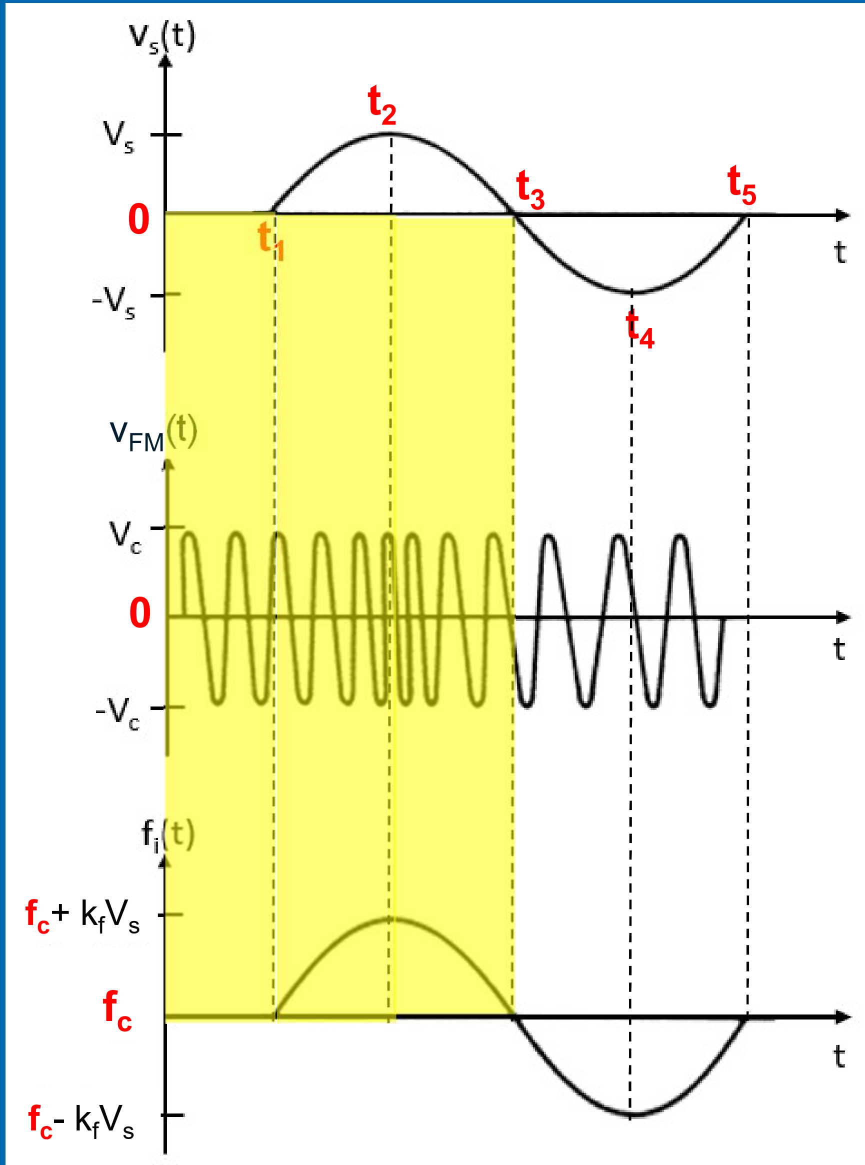
$f_i(t) = f_c + k_f v_s(t)$
$= f_c + k_f V_s \cos\omega_s t$

follows the changes in $v_s(t)$

From 0 to t_1 ($v_s(t) = 0V$): $f_i(t) = f_c$

From t_1 to t_2: $f_i(t)$ increases from f_c to $f_c + k_f V_s$

From t_2 to t_3: $f_i(t)$ decreases from $f_c + k_f V_s$ to f_c



6.2 Single-Tone FM

$f_i(t) = f_c + k_f v_s(t)$

$= f_c + k_f V_s \cos\omega_s t$

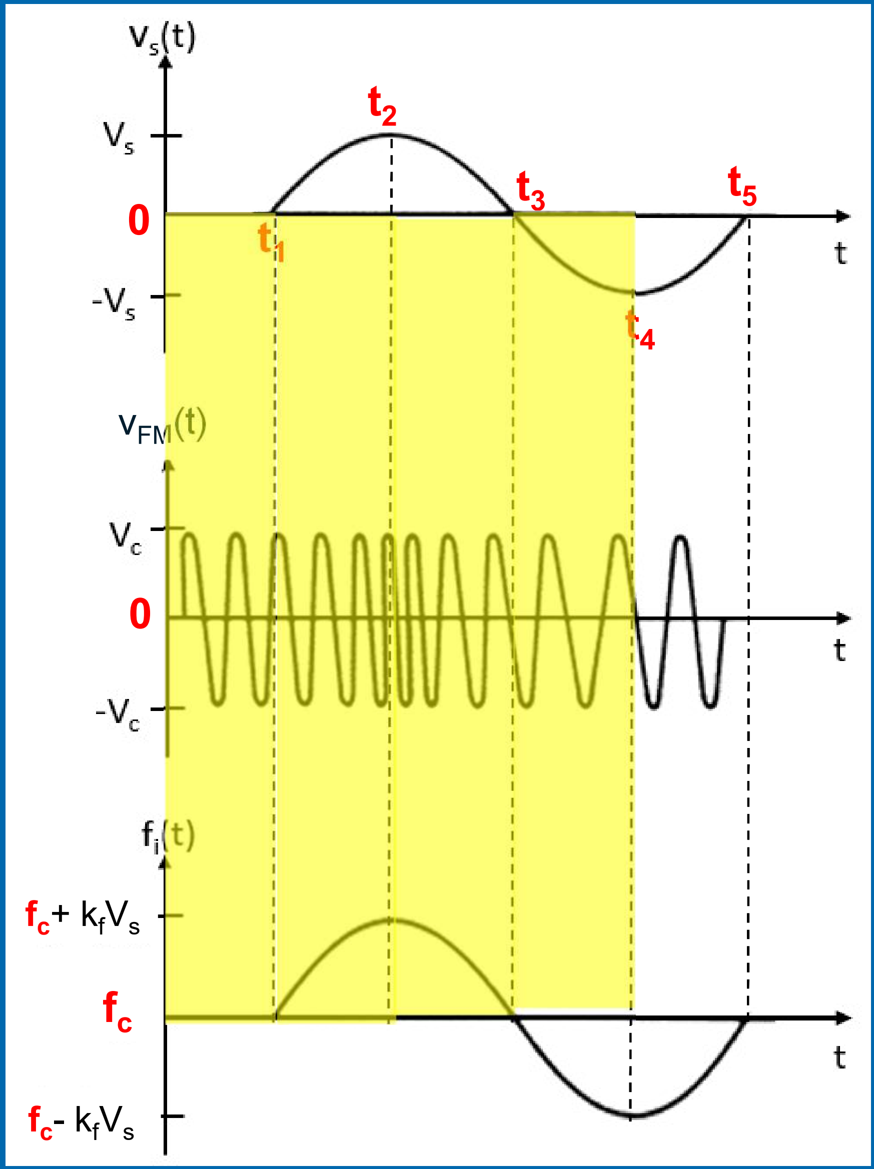
follows the changes in $v_s(t)$

From 0 to t_1 ($v_s(t) = 0V$): $f_i(t) = f_c$

From t_1 to t_2: $f_i(t)$ increases from f_c to $f_c+k_fV_s$

From t_2 to t_3: $f_i(t)$ decreases from $f_c+k_fV_s$ to f_c

From t_3 to t_4: $f_i(t)$ decreases from f_c to $f_c-k_fV_s$



6.2 Single-Tone FM

$$f_i(t) = f_c + k_f v_s(t)$$
$$= f_c + k_f V_s \cos\omega_s t$$

follows the changes in $v_s(t)$

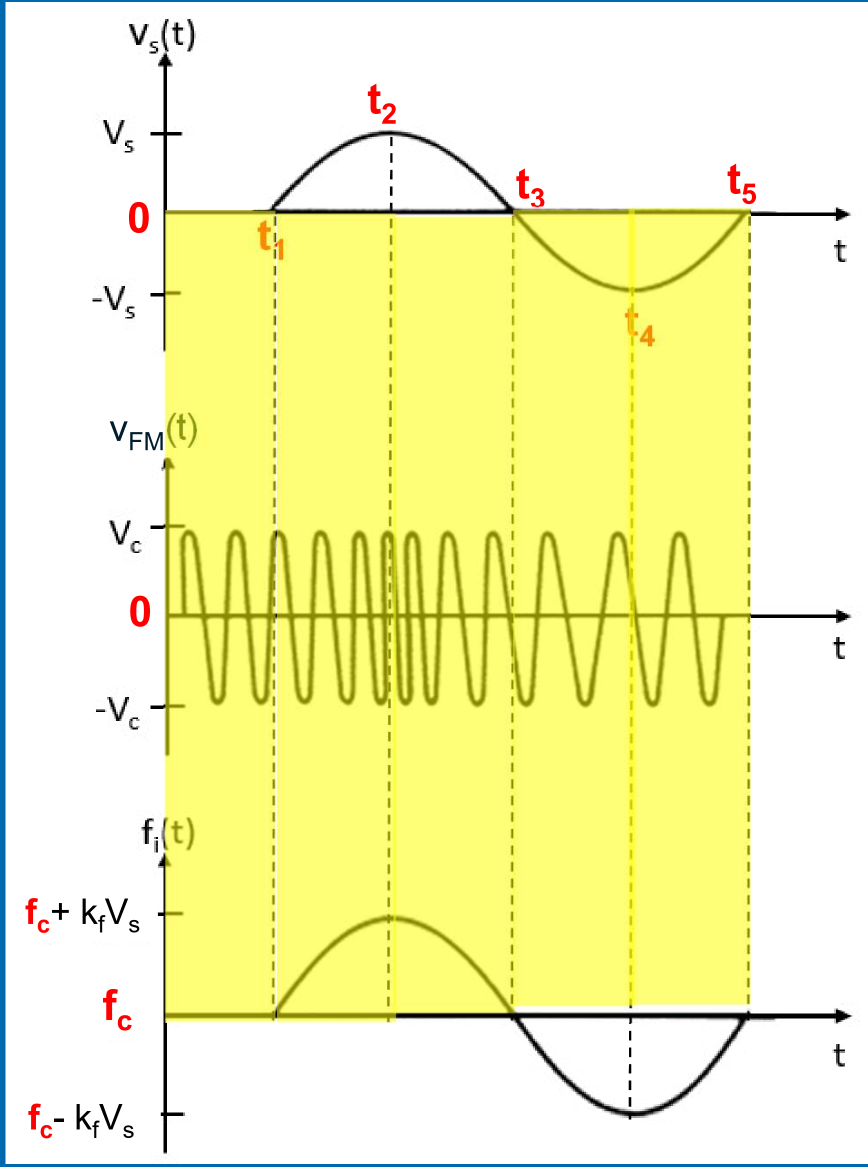
From 0 to t_1 ($v_s(t) = 0V$): $f_i(t) = f_c$

From t_1 to t_2: $f_i(t)$ increases from f_c to $f_c+k_fV_s$

From t_2 to t_3: $f_i(t)$ decreases from $f_c+k_fV_s$ to f_c

From t_3 to t_4: $f_i(t)$ decreases from f_c to $f_c-k_fV_s$

From t_4 to t_5: $f_i(t)$ increases from $f_c-k_fV_s$ to f_c.



$f_i(t)$ has the same shape as $v_S(t)$

6.2 Single-Tone FM

Frequency deviation

The amount of frequency change away from f_c at any instant in time

Frequency Deviation = $k_f \times v_s(t)$

Proportional to the modulating voltage at that instant in time

Peak frequency deviation, Δ_f

The maximum frequency change on either side of carrier frequency f_c

$$\Delta_f = k_f \text{ x peak modulating voltage } = k_f V_S$$

$$f_{i(max)}(t) = f_C + \Delta_f$$

$$f_{i(min)}(t) = f_C - \Delta_f$$

6.2 Single-Tone FM

Frequency modulation index

Modulation index of signal-tone FM signal, m_f

$$m_f = \frac{\Delta_f}{f_S} \qquad \text{for } v_S(t) = V_S\cos\omega_S t,$$

AM	FM
$m = \frac{V_s}{V_c}$ • expresses the size of the envelope • $m \leq 1$	$m_f = \frac{\Delta_f}{f_S}$ • expresses the amount (Δ_f) and speed (fS) of frequency change. • Δ_f can be set independent of f_S and therefore m_f can be larger than 1. • Δ_f must not be larger than f_C.

Example 6.1

For the FM waveform shown below

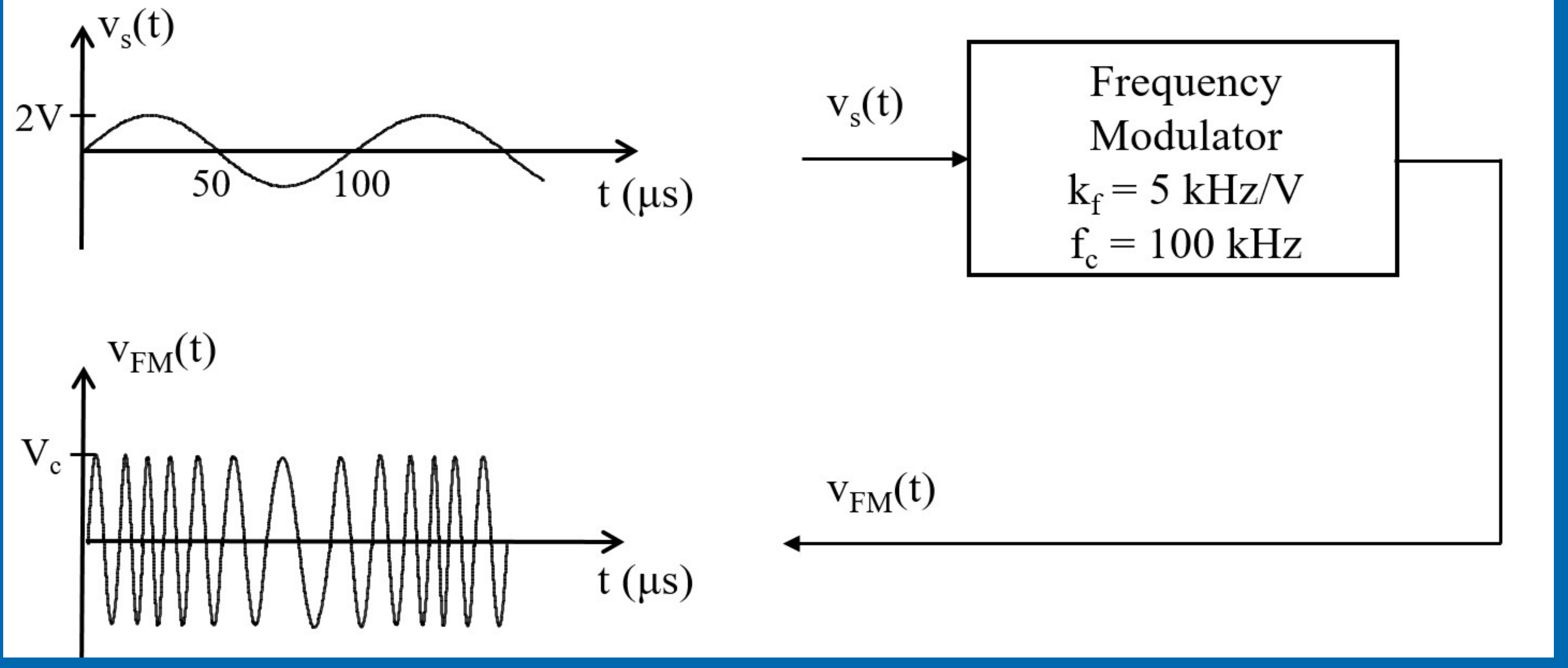
(i) calculate the instantaneous frequency at t = 50 μs.

(ii) determine Δ_f.

(iii) determine $f_{i(max)}$ and specify when it occurs.

(iv) determine $f_{i(min)}$ and specify when it occurs.

(v) show how $f_i(t)$ changes with time.

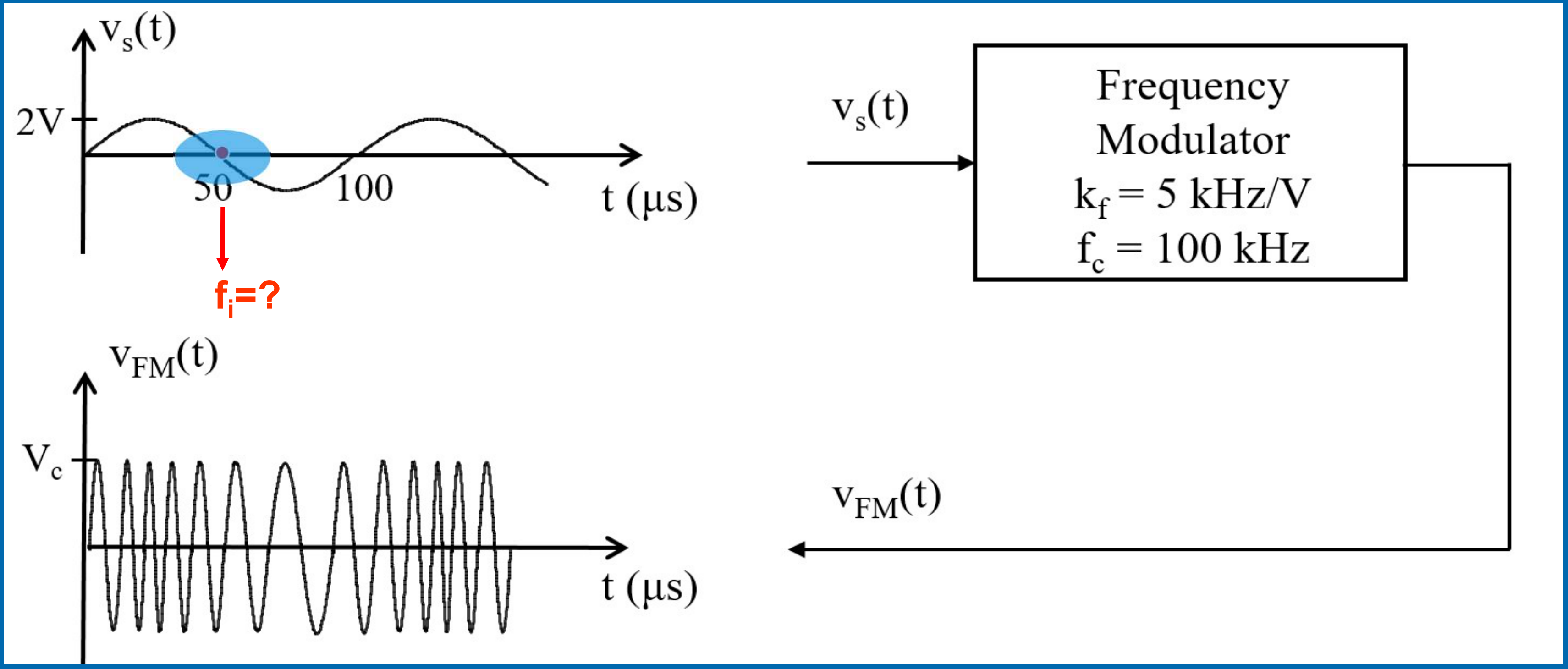


Solution

(i) Calculate the instantaneous frequency at t = 50 μs.

At t = 50μs, $v_s(t)$ = 0V

Therefore, $f_i = f_c$ =100kHz



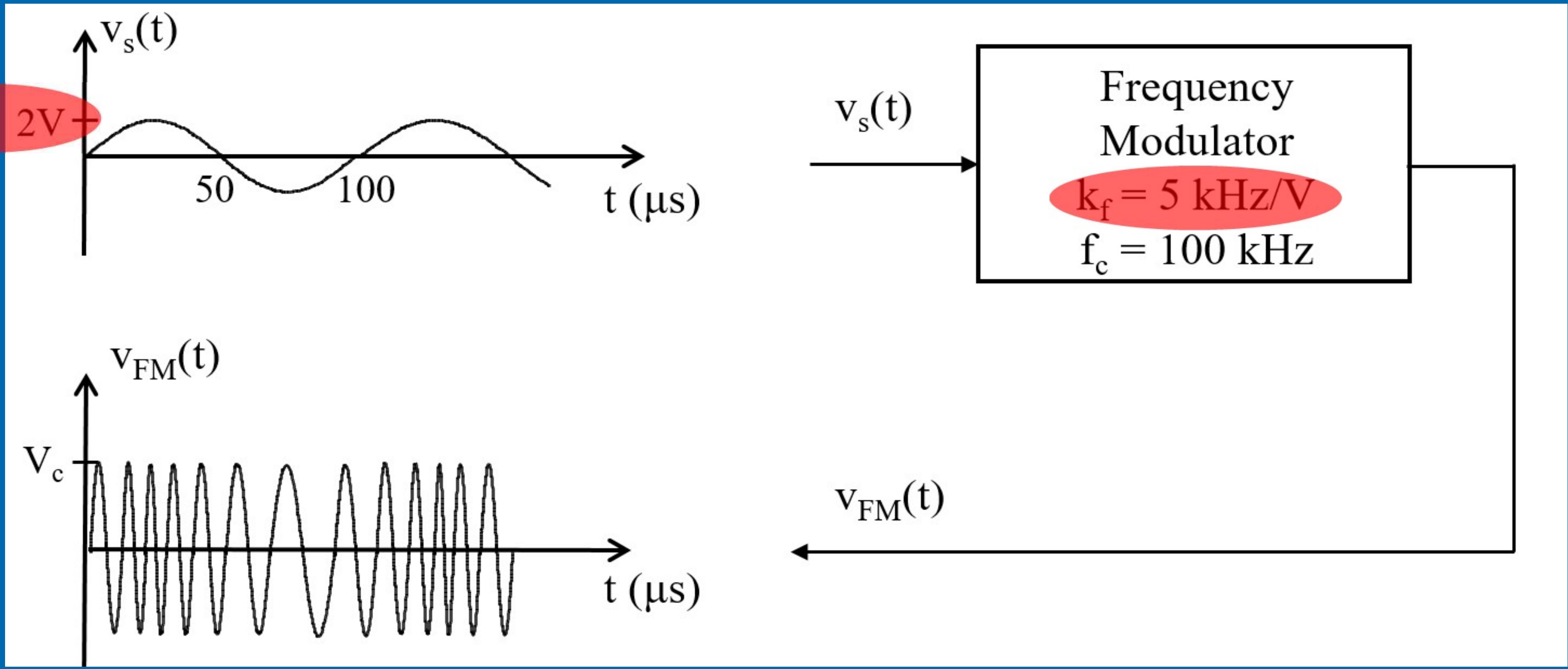
Solution

(ii) **Determine Δ_f.**

$\Delta_f = k_f V_s$

$= 5kHz/V \times 2V = 10kHz$

$V_s =$

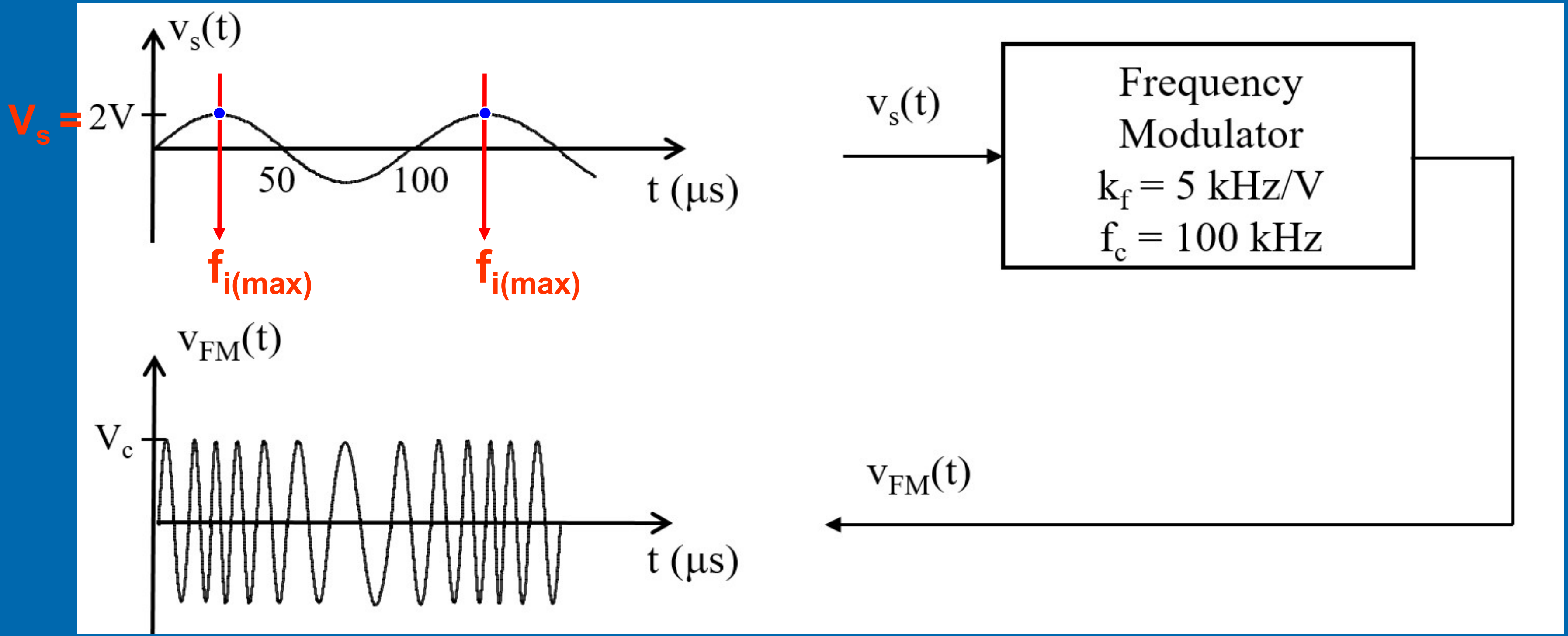


Solution

(iii) determine $f_{i(max)}$ and specify when it occurs

$f_{i(max)} = f_C + \Delta_f = 100 + 10 = \underline{110\ \text{kHz}}$, when $v_s(t) = V_s$

Hence, $f_{i(max)}$ occurs at t = 25μs and 125 μs

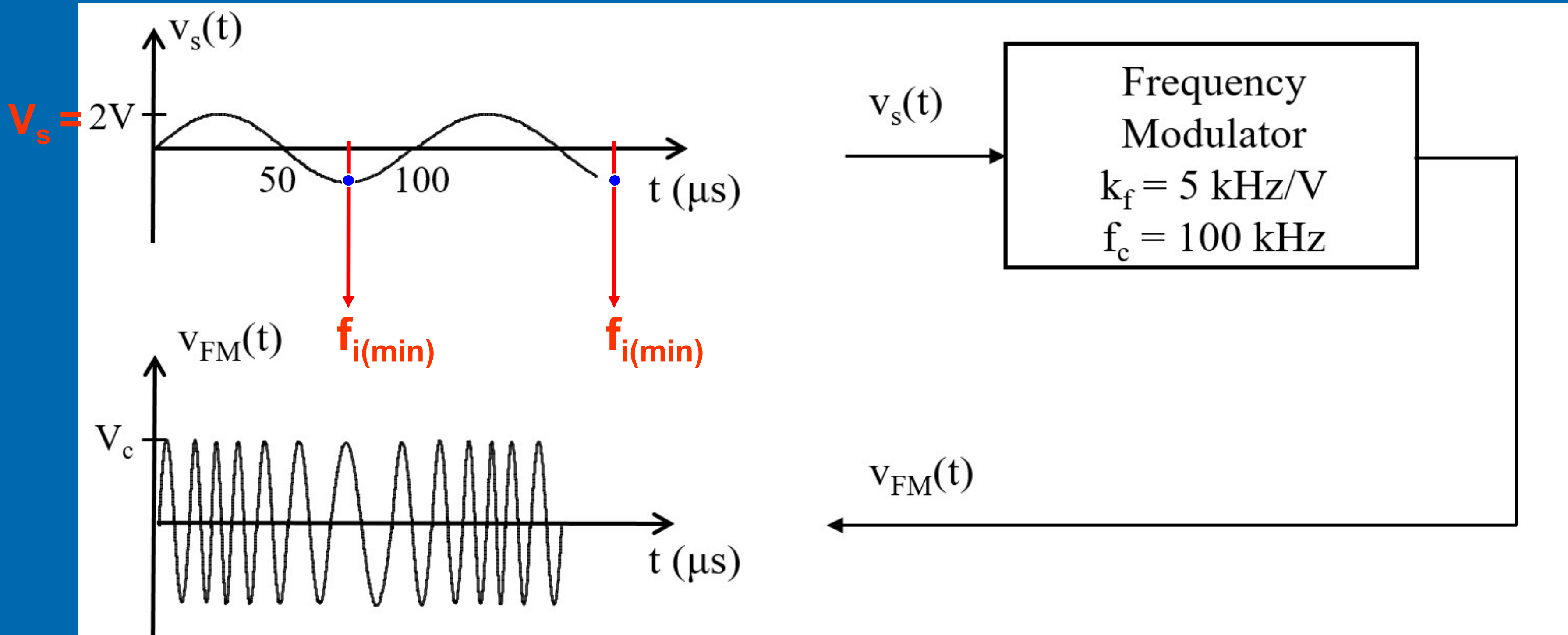


Solution

(iv) determine $f_{i(min)}$ and specify when it occurs

$f_{i(min)} = f_C - \Delta_f = 100 - 10 = \underline{\underline{90\ kHz}}$, when $v_s(t) = -V_s$

Hence, $f_{i(min)}$ occurs at t = 75μs and 175 μs

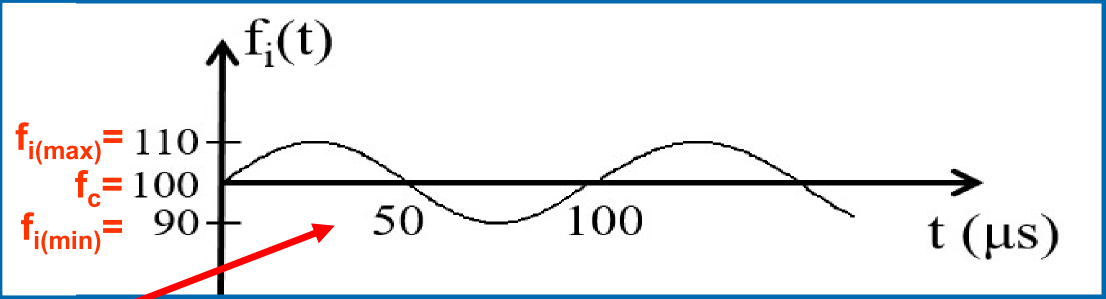


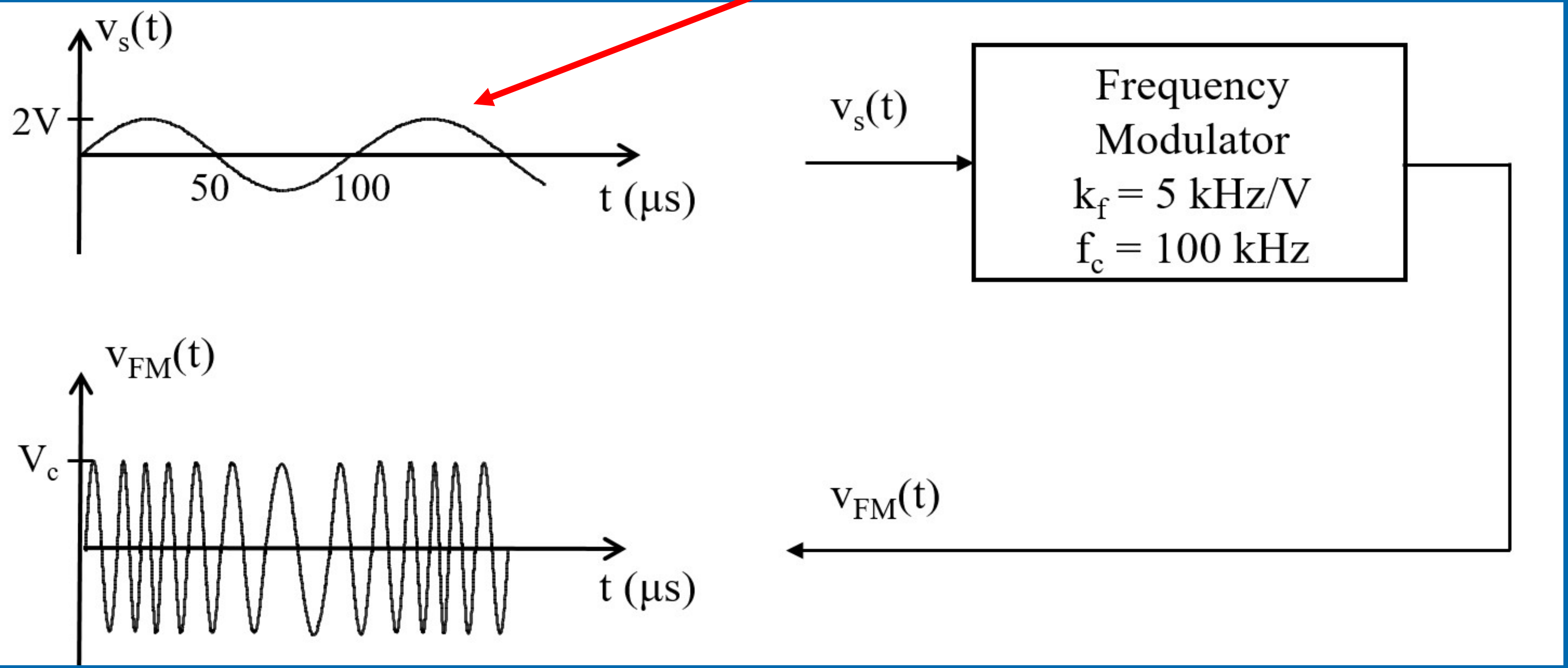
Solution

(v) show how $f_i(t)$ changes with time.

$f_i(t) = f_c + k_f v_s(t)$

$f_i(t)$ changes in the same way as $v_s(t)$





End

CHAPTER 6

(Part 1 of 4)