CHAPTER 6

Frequency Modulation

(Part 2 of 4)

6.2 Single-Tone FM

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Frequency spectrum of single-tone FM signal

$v_{FM}(t) = V_c\cos(\omega_c t + m_f \sin\omega_s t)$

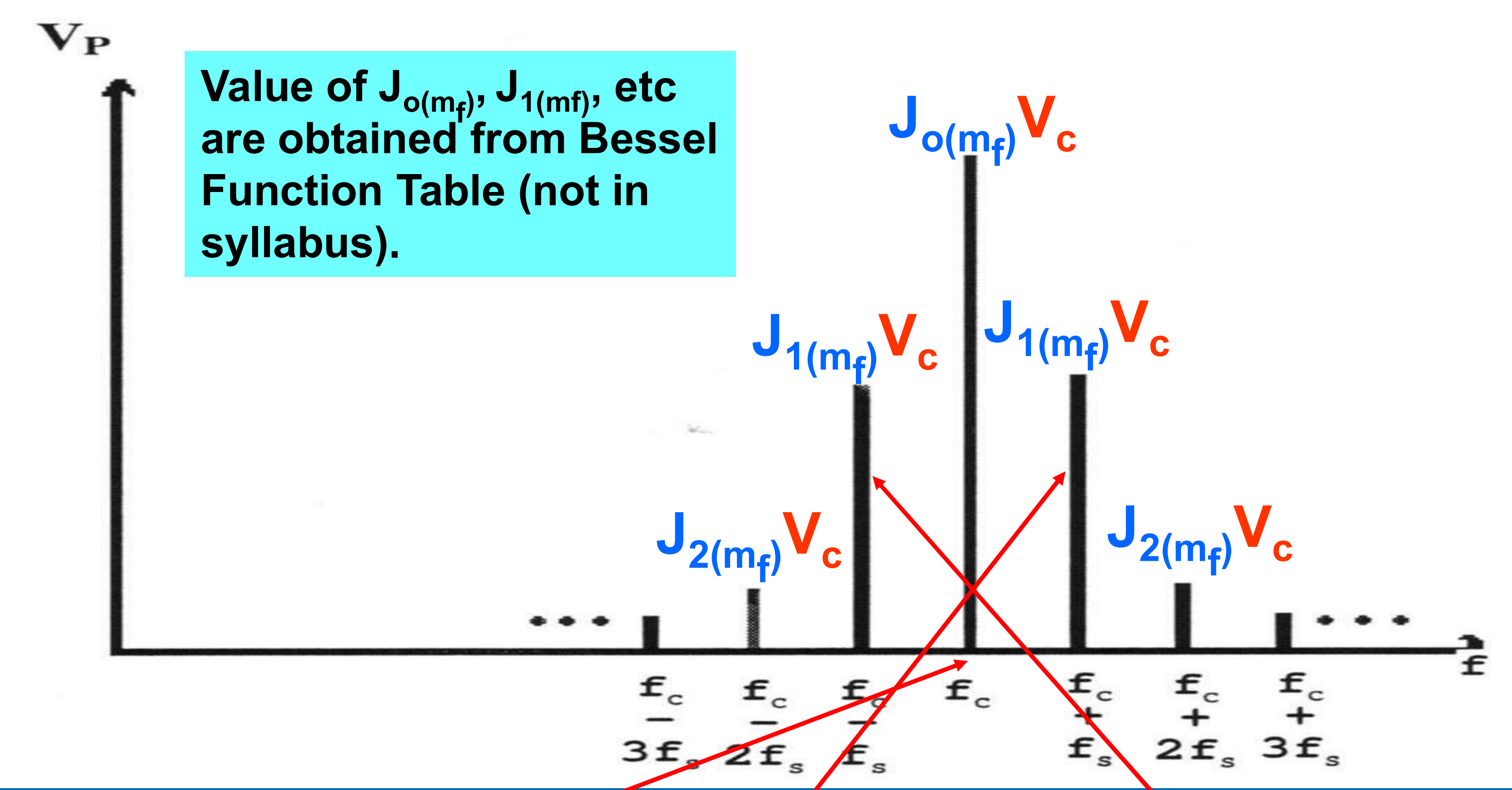
when $v_S(t)$ is sinusoid

Expand $v_{FM}(t)$ in Fourier Series:

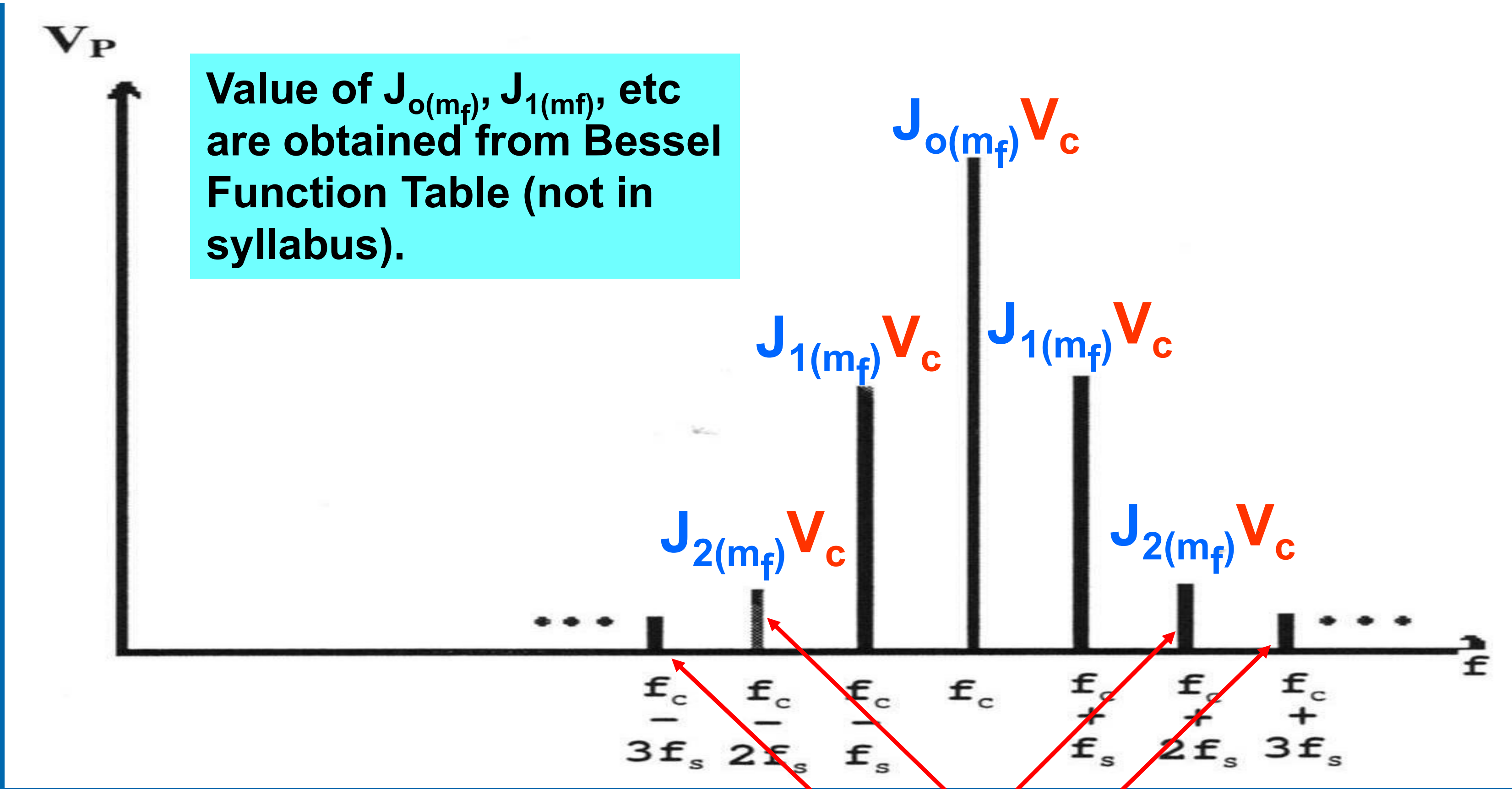
Obtain single-sided spectrum

$$v_{FM}(t) = J_{o(m_f)}V_c\cos 2\pi f_c t + J_{1(m_f)}V_c\cos 2\pi(f_c + f_s)t + J_{1(m_f)}V_c\cos 2\pi(f_c - f_s)t$$
$$+ J_{2(m_f)}V_c\cos 2\pi(f_c + 2f_s)t + J_{2(m_f)}V_c\cos 2\pi(f_c - 2f_s)t$$
$$+ J_{3(m_f)}V_c\cos 2\pi(f_c + 3f_s)t + J_{3(m_f)}V_c\cos 2\pi(f_c - 3f_s)t$$
$$+ \ldots\ldots\ldots$$

Value of $J_{o(m_f)}$, $J_{1(mf)}$, etc are obtained from Bessel Function Table (not in syllabus)



$$V_{FM}(t) = J_{o(m_f)}V_c\cos 2\pi f_c t + J_{1(m_f)}V_c\cos 2\pi(f_c + f_s)t \quad + J_{1(m_f)}V_c\cos 2\pi(f_c - f_s)t$$



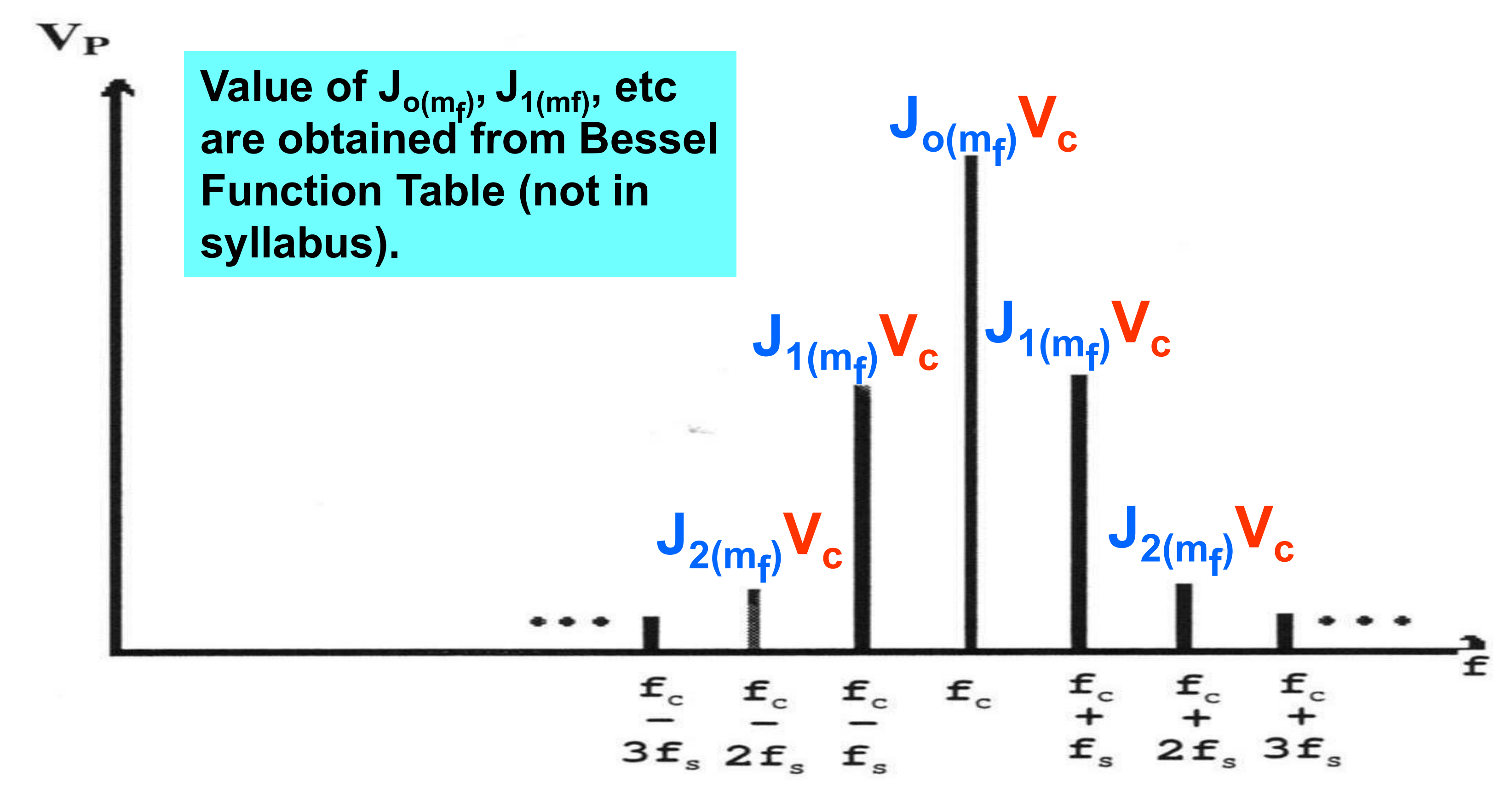
Value of $J_{o(m_f)}$, $J_{1(mf)}$, etc are obtained from Bessel Function Table (not in syllabus).

$$v_{FM}(t) = J_{o(m_f)}V_c\cos 2\pi f_c t + J_{1(m_f)}V_c\cos 2\pi(f_c + f_s)t + J_{1(m_f)}V_c\cos 2\pi(f_c - f_s)t$$

$$+ J_{2(m_f)}V_c\cos 2\pi(f_c + 2f_s)t + J_{2(m_f)}V_c\cos 2\pi(f_c - 2f_s)t$$

$$+ J_{3(m_f)}V_c\cos 2\pi(f_c + 3f_s)t + J_{3(m_f)}V_c\cos 2\pi(f_c - 3f_s)t$$

$$+ \dots\dots\dots$$



$$V_{FM}(t) = J_{o(m_f)}V_c\cos 2\pi f_c t + J_{1(m_f)}V_c\cos 2\pi(f_c + f_s)t + J_{1(m_f)}V_c\cos 2\pi(f_c - f_s)t$$
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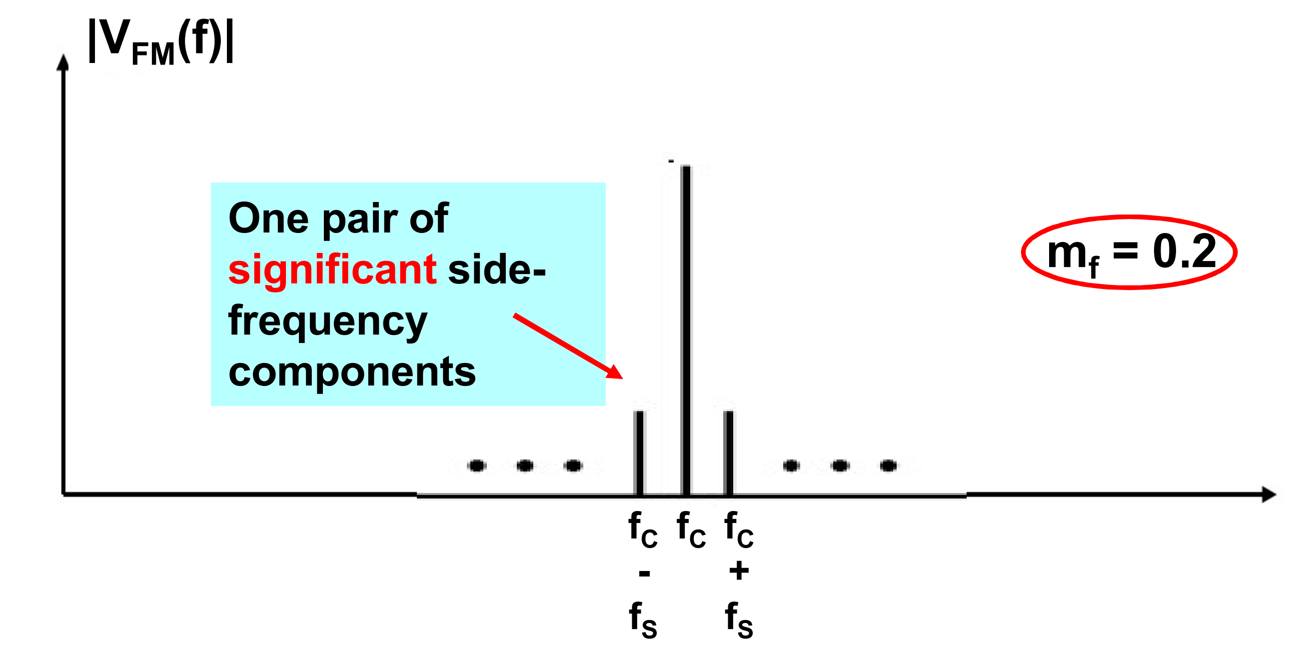
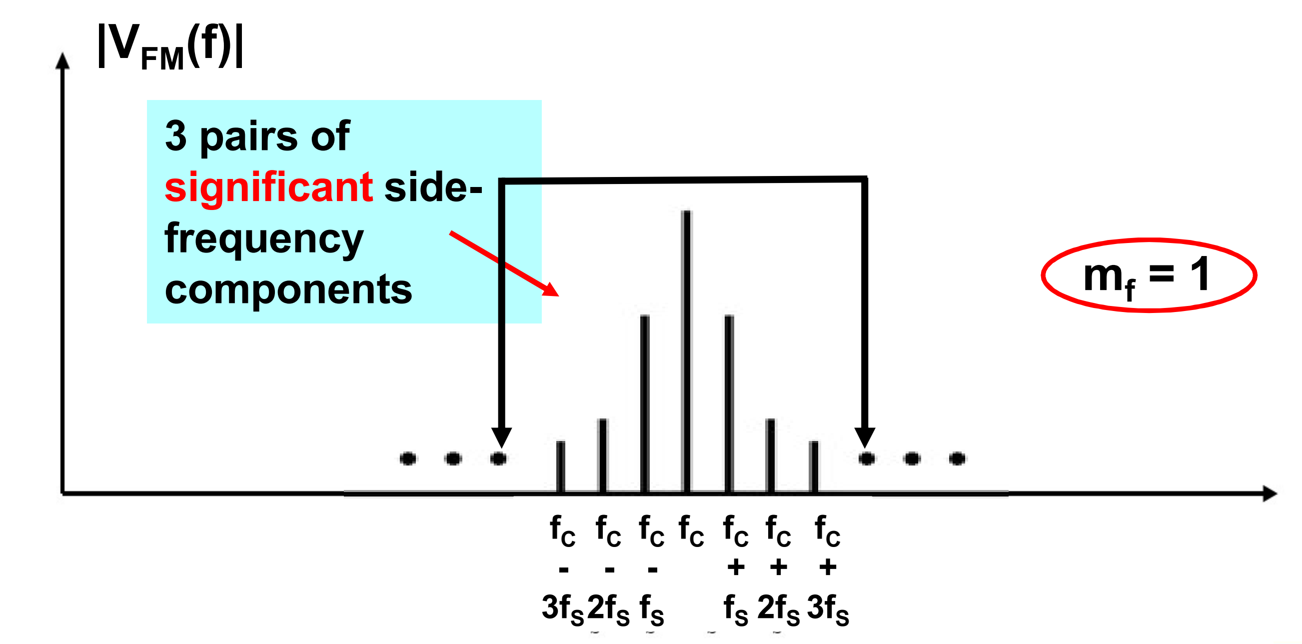
$$+ J_{3(m_f)}V_c\cos 2\pi(f_c + 3f_s)t + J_{3(m_f)}V_c\cos 2\pi(f_c - 3f_s)t$$
$$+ \ldots\ldots\ldots$$

- Single-tone FM consists of infinite number of pairs of side-frequency components.

- The amplitude of the components decrease as these are away from f_c.

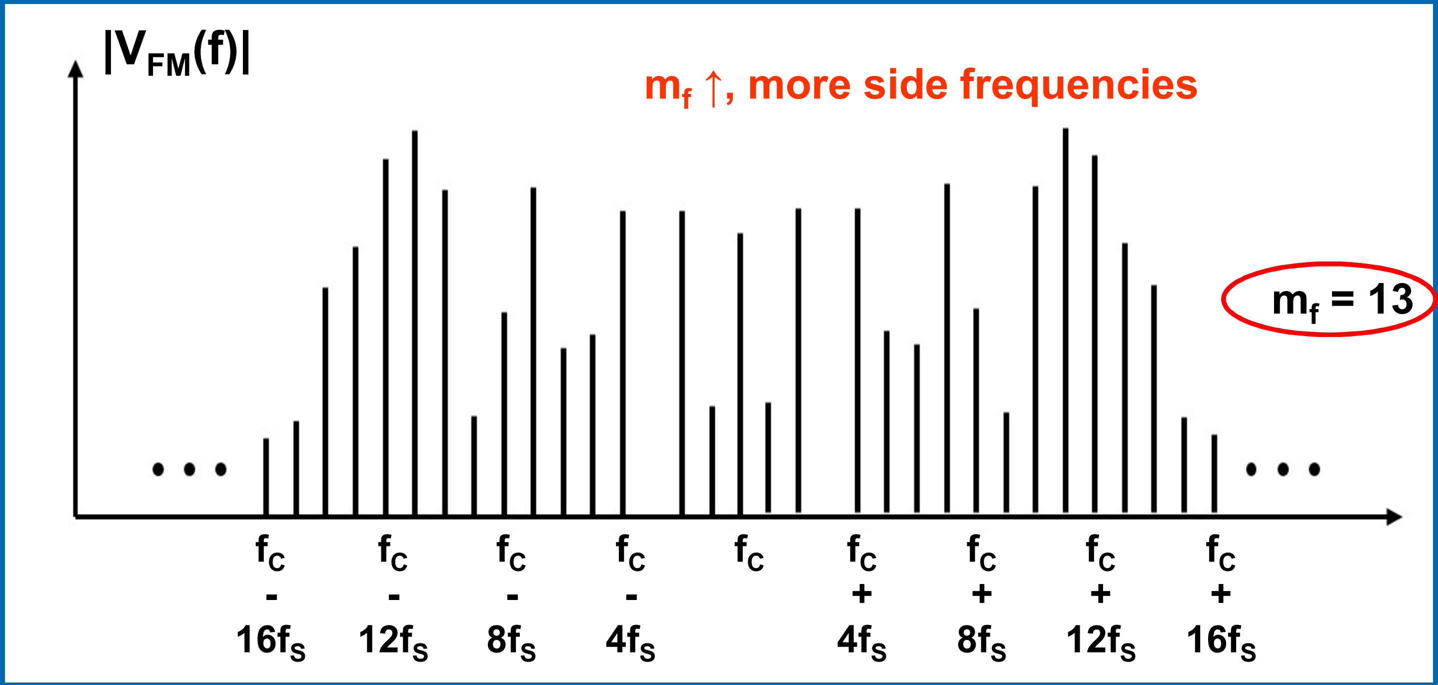
The frequency components far away from f_c have very low amplitudes.





- The number of pairs increase as m_f, but not proportionally.

when m f increases 13 times, the number of side frequencies does not increase by 13 times.



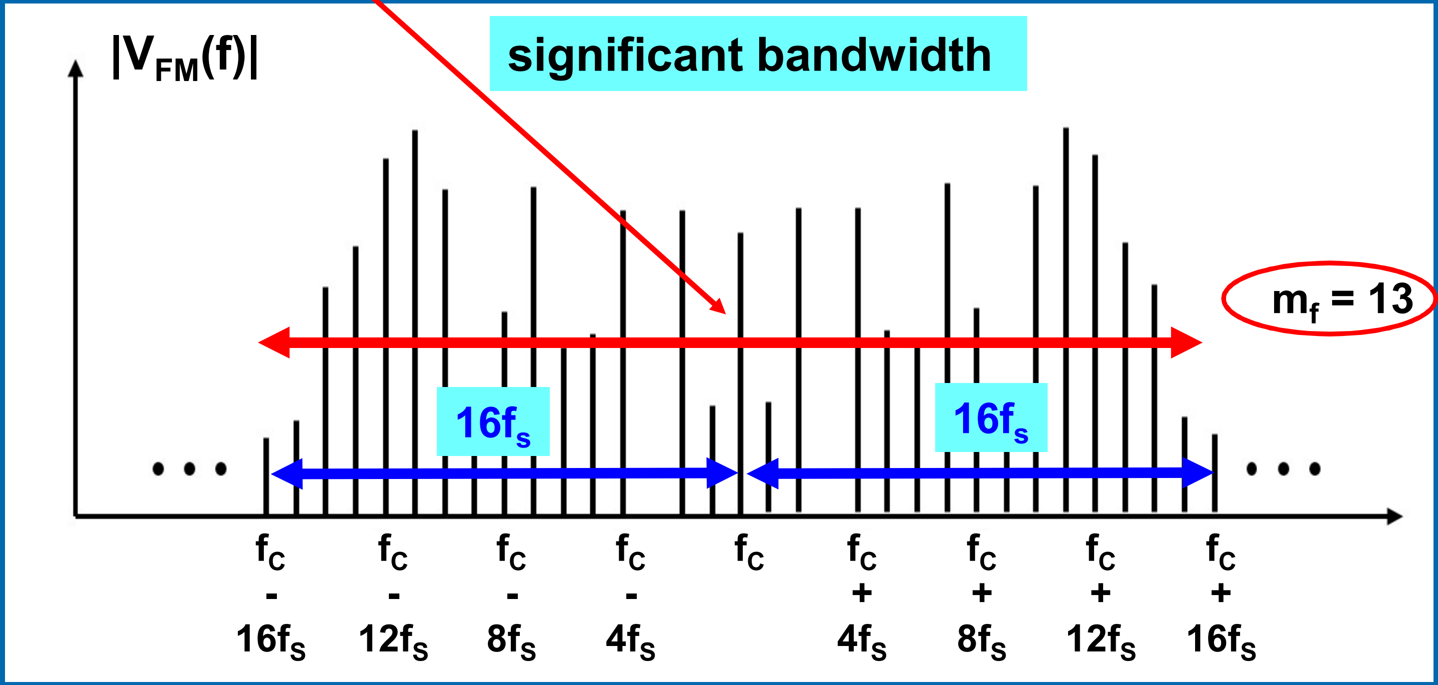
- **The significant bandwidth of FM signal is determined by ignoring the higher pairs of side frequencies that have low amplitudes.**

- **Bandwidth of the FM signal is an EVEN multiple of f_S.**

$B_{FM} = 32f_s$

Need to use Bessel Function Table to determine which higher pairs of side frequencies to ignore.

This is TEDIOUS!



6.2 Single-Tone FM

Carson's Rule

Carson's Rule allows us to estimate the bandwidth of the FM signal without using Bessel Function Table.

$$B_{FM} \approx 2(m_f + 1)f_s$$

$$\approx 2(\Delta_f + f_s)$$

B_{FM} is an even multiple of f_s

Estimation (not exact)

Carson's Rule can only be used if m_f is an integer.

Example:

FM signal with $m_f = 1$

B_{FM} calculated from Carson's Rule = 4f_s

B_{FM} calculated from the spectrum = 6f_s

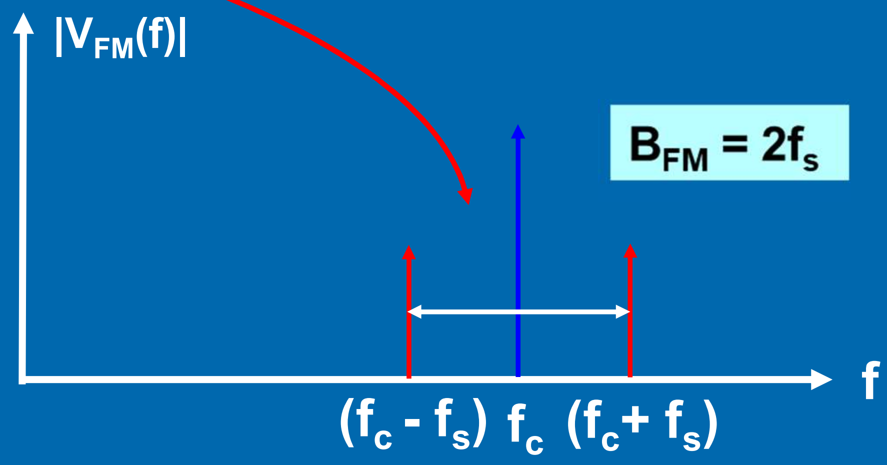
6.3 Wideband and Narrowband FM

- B_{FM} changes with m_f.

- When m_f reduces to < 0.5, number of frequency components reduces to ONE pair only.

- FM systems with $m_f < 0.5$ are known as

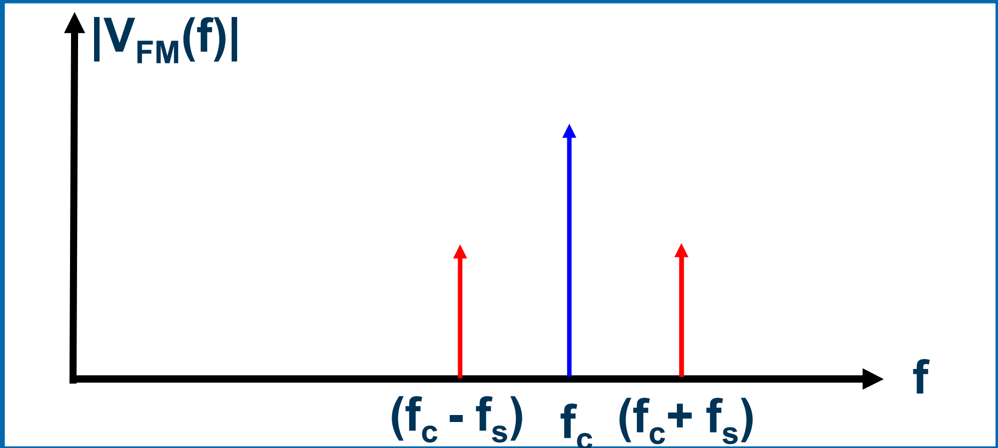
  Narrowband FM (NBFM) systems



- Conversely, FM systems with $m_f \geq 0.5$ are known as

  wideband FM (WBFM) systems → larger bandwidth

6.3 Wideband and Narrowband FM

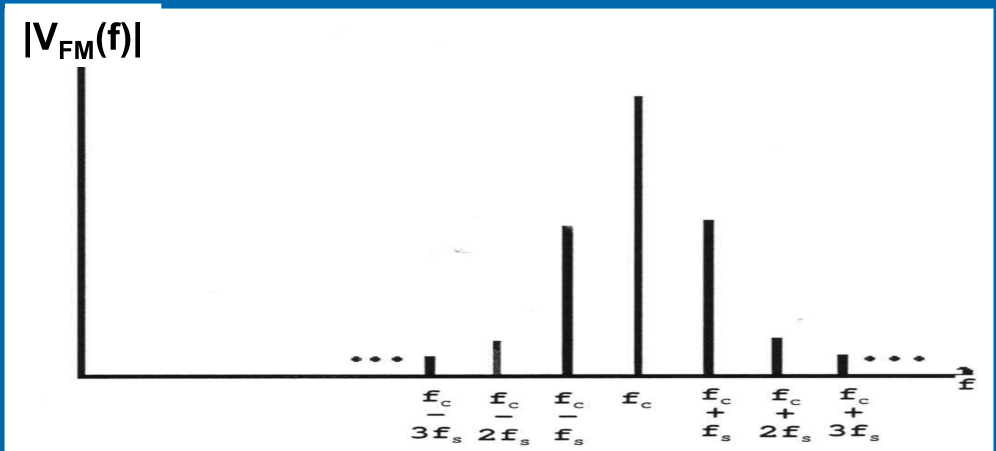


NBFM:

$m_f < 0.5$, $B_{FM} = 2f_s$

$B_{FM} = B_{AM}$

Do NOT use Carson's rule to determine B_{FM}



WBFM:

$m_f \geq 0.5$, $B_{FM} > 2f_s$

$B_{FM} > B_{AM}$

Use Carson's rule to determine B_{FM}

6.4 FM Demodulation

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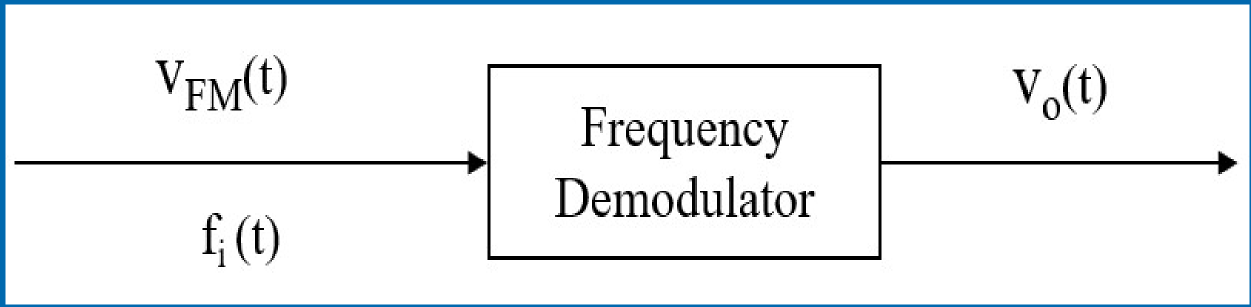
Convert frequency change to voltage change

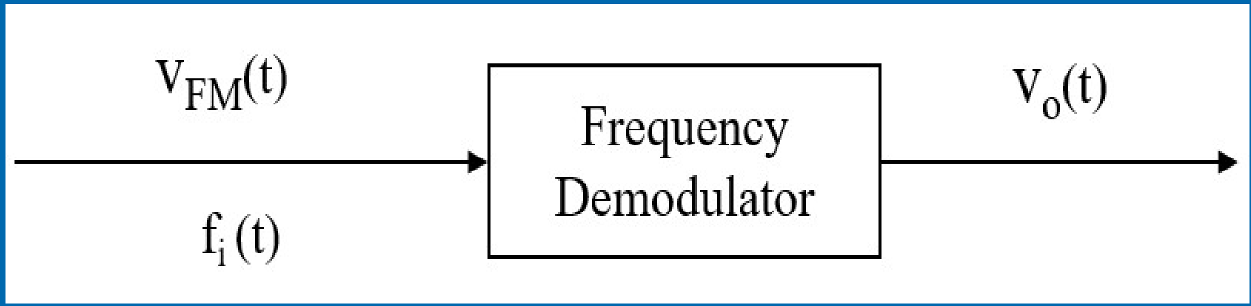


Conversion gain, k_d = V/Hz

$f_i = f_c$	$v_o(t) = 0V$	
$f_i \uparrow$ above f_c	$v_o(t) \uparrow$ above 0V	$V_{o(max)} = k_d\Delta_f$
$f_i \downarrow$ below f_c	$v_o(t) \downarrow$ below 0V	$V_{o(min)} = -k_d\Delta_f$

6.4 FM Demodulator

Convert frequency change to voltage change



$v_o(t) = k_d(f_i - f_c)$

recall $f_i(t) = f_C + k_f v_S(t)$

Therefore, $v_O(t) = k_d k_f v_S(t) \propto v_S(t)$

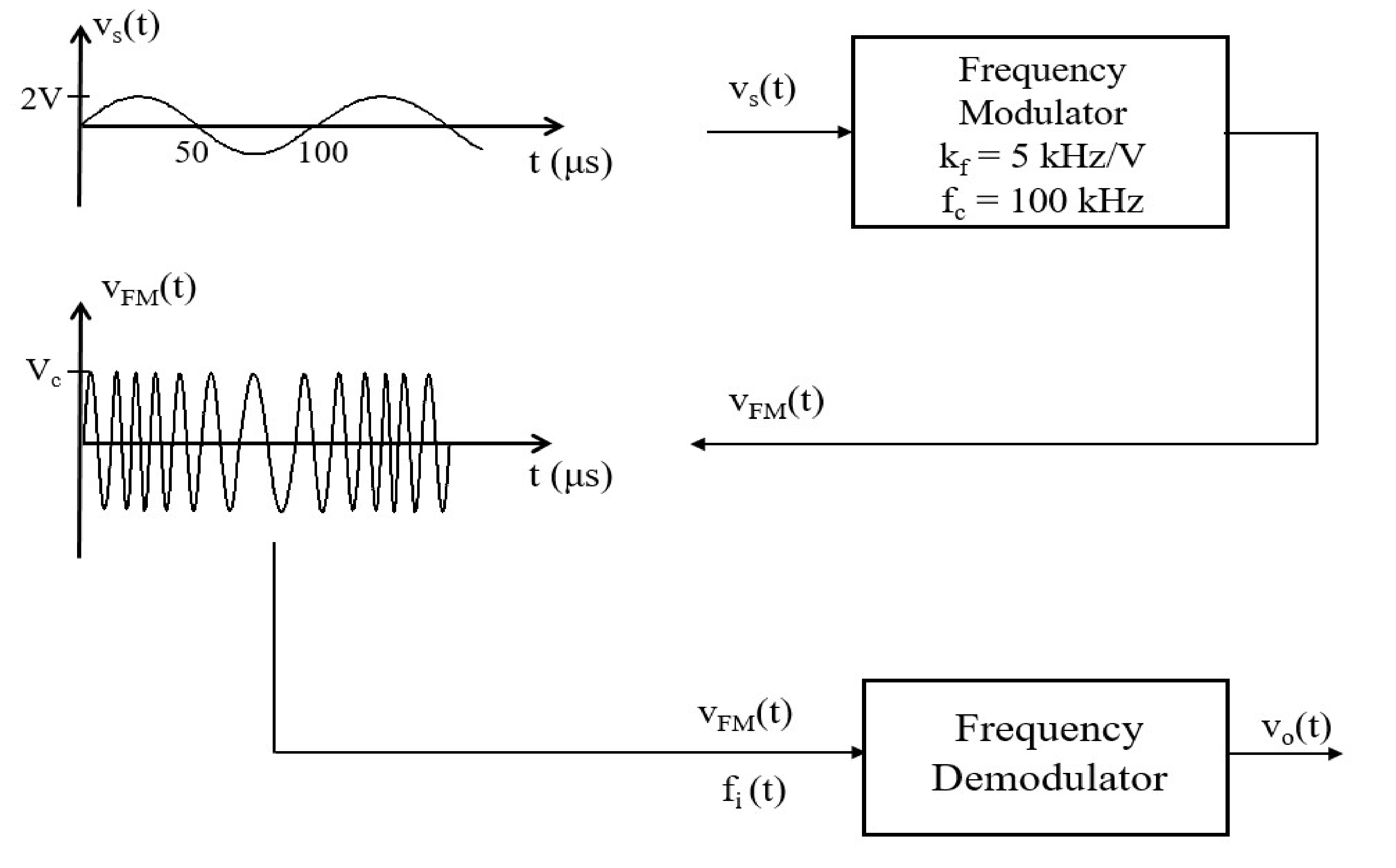
$v_{o(t)} \propto v_{s(t)}$

$v_o(t)$ has the same shape as $v_s(t)$ but need not be the same size as it is adjustable

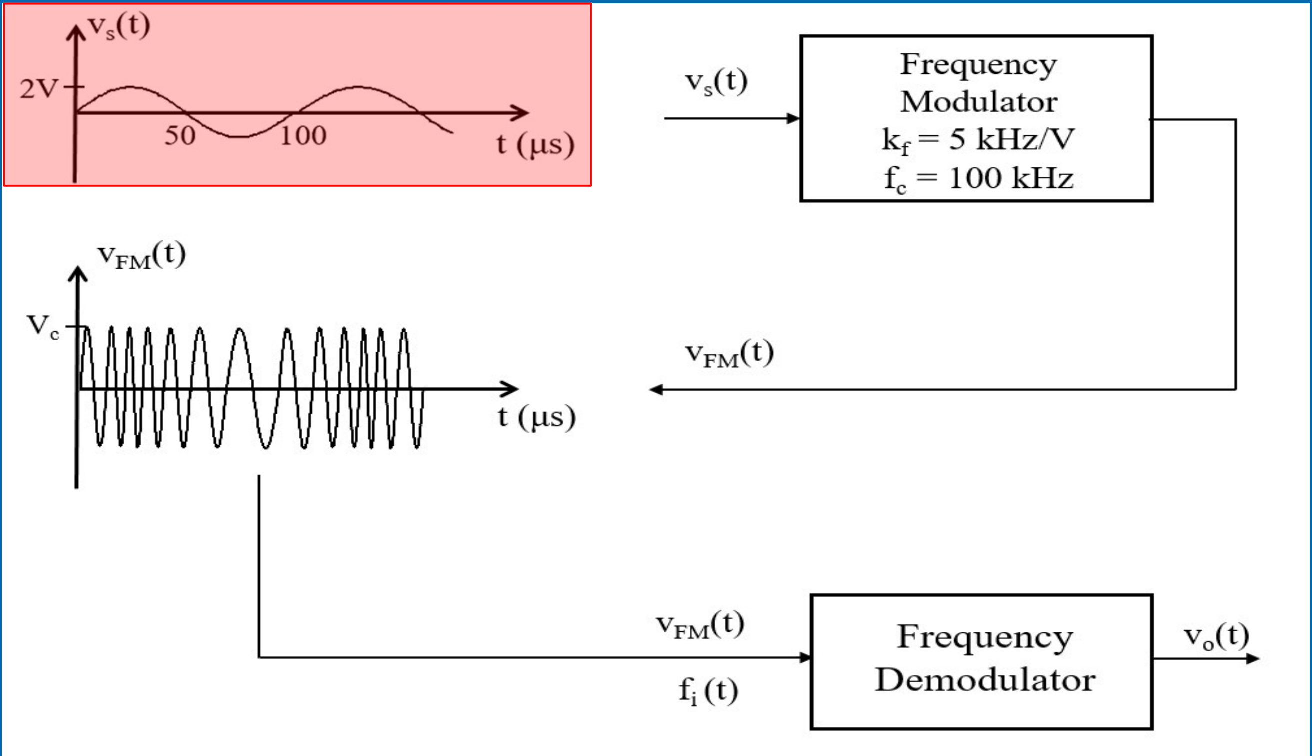
The modulating signal successfully recovered.

Example 6.2

Sketch the demodulated output waveform if k_d = 0.1 V/kHz for the FM system shown below.

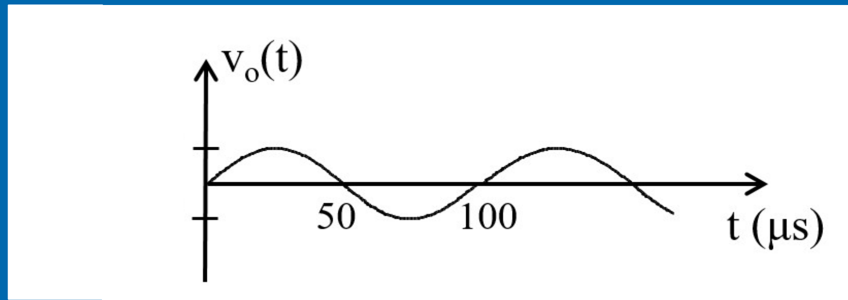


Solution

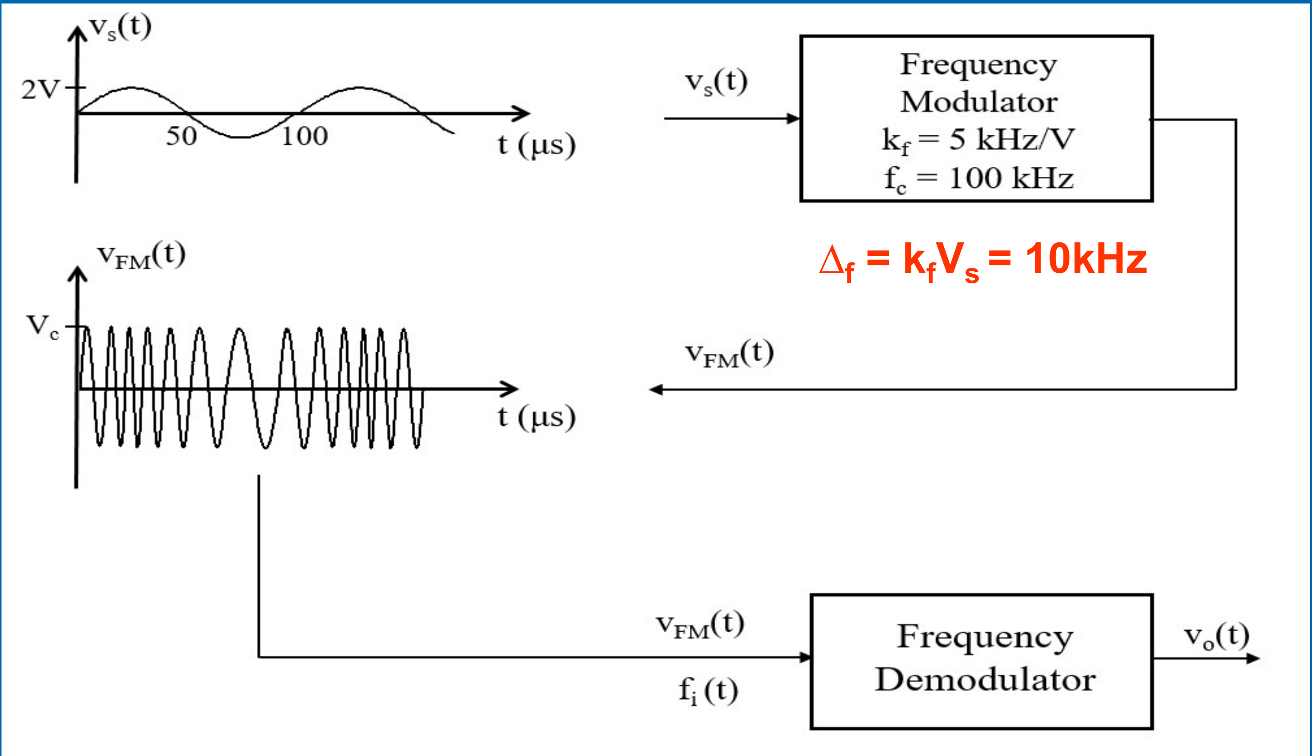


$v_{o(t)}$ **has the same shape as** $v_s(t)$

$v_{o(t)} \propto v_s(t)$



Solution

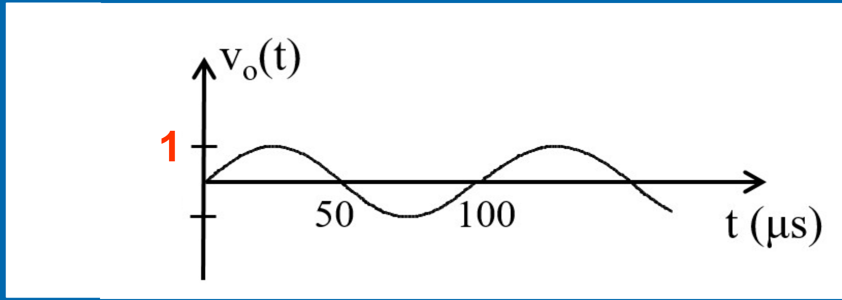


$\Delta_f = k_f V_s = 10\text{kHz}$

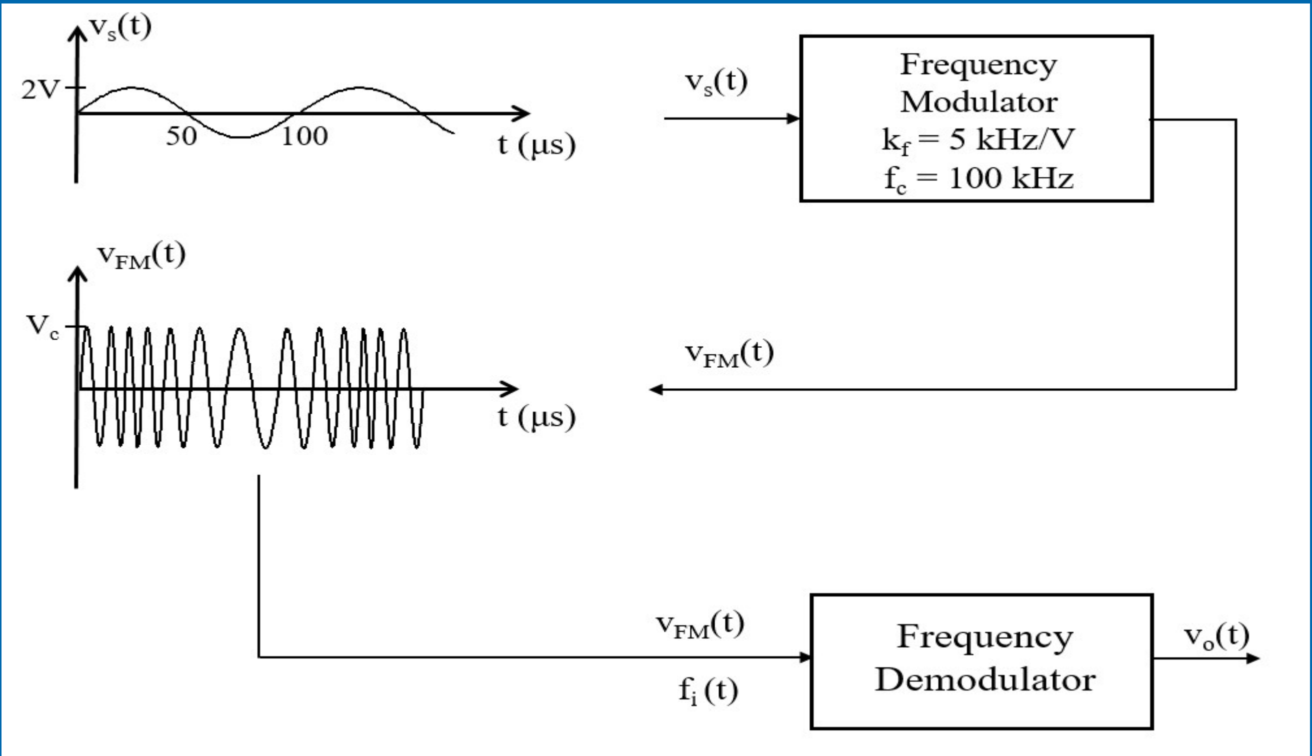
$V_{o(max)} = k_d \Delta_f$

$= 0.1$ V/kHz x 10kHz

$= 1$ V



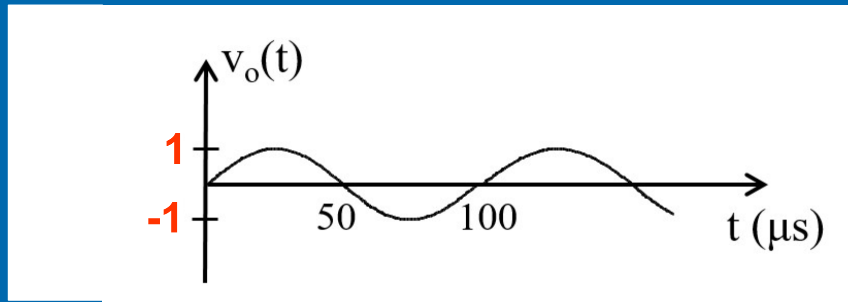
Solution



$$V_{o(min)} = -k_d\Delta_f$$

$$= -0.1 \text{ V/kHz} \times 10\text{kHz}$$

$$= -1 \text{ V}$$



End

CHAPTER 6

(Part 2 of 4)