SINGAPORE POLYTECHNIC

<u>ET0096 SEM SAMPLE 2 ANSWERS:</u>

Section A

A1

$$y(n) = x(n) - 0.866x(n-1) + 1.732y(n-1) - y(n-2)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 0.866z^{-1}}{1 - 1.732z^{-1} + z^{-2}}$$

$$= \frac{1 - \cos\left(\frac{\pi}{6}\right)z^{-1}}{1 - 2\cos\left(\frac{\pi}{6}\right)z^{-1} + z^{-2}}$$

$$h(n) = \cos\left(\frac{n\pi}{6}\right)u(n)$$

Since the impulse response is a constant amplitude cosine function, the system is marginally stable (or unstable).

A2 y(n)={ 2, 3, 1, 6}

$Y(z) = 2 + 3z^{-1} + z^{-2} + 6z^{-3}$

and impulse response h(n)={1,2}

$H(z) = 1+2z^{-1}$

X(z) = Y(z)/H(z)

$$\begin{array}{r|l} & 2 - z^{-1} + 3z^{-2} \\ \hline 1+2z^{-1} & 2 + 3z^{-1} + z^{-2} + 6z^{-3} \\ & \underline{2 + 4z^{-1}} \\ & \text{- - - - - - - - - - - - -} \end{array}$$

$X(z) = 2 - z^{-1} + 3z^{-2}$

x(n) = { 2, -1, 3}

A3 When N=4, k= for k = 0, 1,2 and 3

$$X(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi kn}{4}}$$

$$= x(0) + x(1)e^{-j\frac{2\pi k}{4}} + x(2)e^{-j\frac{4\pi k}{4}} + x(3)e^{-j\frac{6\pi k}{4}}$$

$X(0)= 0$

$X(2) = 0$

A4

$$x_1(n) = e^{-2n}\sin(3n)u(n)$$

$$X_1(z) = \frac{e^{-2}\sin(3)\, z^{-1}}{1 - 2e^{-2}cos(3) + e^{-4}z^{-2}}$$

Or equivalent

and $x_2(n)=n\ 5^{n-1}u(n)$

$$X_2(z) = \frac{z^{-1}}{(1-5z^{-1})^2}$$

Or equivalent

A5

(a) $h_T(n) =\{ h_1(n)+ h_2(n)\} * \{h_3(n)+ h_4(n)\}$

$H_T(z) = \{ H_1(z)+ H_2(z)\}\ X\ \{H_3(z)+ H_4(z)$

(b) $h_T(n) =\{ 2, 2\}*(2,2) =\{4,8,4\}$

A6 Applying the partial fraction expansion leads to

$$X(z) = \frac{z}{3z^2 - 4z + 1}$$

$$\frac{X(z)}{z} = \frac{1}{(3z-1)(z-1)} = \frac{A}{3z-1} + \frac{B}{z-1}$$

$$A = \left. \frac{X(z)}{z}(3\mathrm{z} - 1) \right| z = 1/3$$

$$A = -1.5$$

$$B = \left. \frac{X(z)}{z}(\mathrm{z} - 1) \right| z = 1$$

B=0.5

$$X(z) = \frac{0.5}{1-z^{-1}} - \frac{0.5}{1-\frac{1}{3}z^{-1}}$$

x(n) = 0.5u(n) – 0.5 $(1/3)^n$ u(n) or equivalent

Section B

B1 **(a)**

$$H(X) = \sum_{1}^{5} P_i \log_2 \frac{1}{P_i}$$

$= 0.51\log_2(1/0.51) + 0.26\log_2(1/0.26) + 0.12\log_2(1/0.12) + 0.11\log_2(1/0.11)$

$= 1.7181$ bits/symbol

(b)

A	0.51	0.51	0.51
B	0.26	0.26	0.49
C	0.12	0.23	
D	0.11		

A = 1, B = 01, C = 001, D = 000

Average bit length = $\bar{n} = \sum_{1}^{4} n_i P(x_i)$ = 1 x 0.51 + 2 x 0.26 + 3 x 0.12 + 3 x 0.11

= <u>1.72 bits/symbol</u>

B2 Original symbols sequence: 255,255,…..255

(i) (255,40),(100,4),(0,1),(100,10), (0,1),(100,4),(255,40)

(ii) Total number of bits of the bit stream = 14 x 8 = 112 bits.

(iii) Compression ratio = 100/14 = 3.25