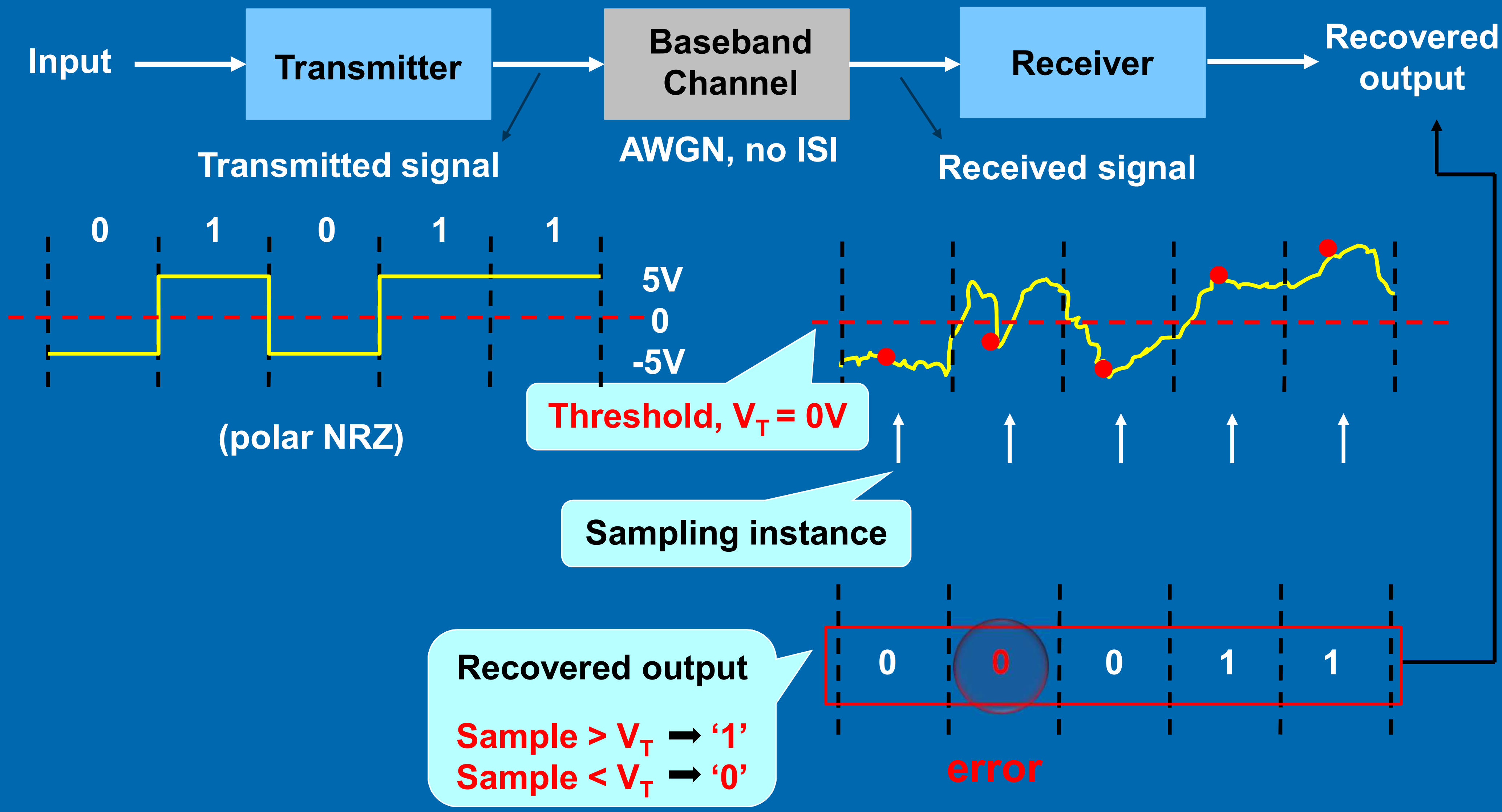
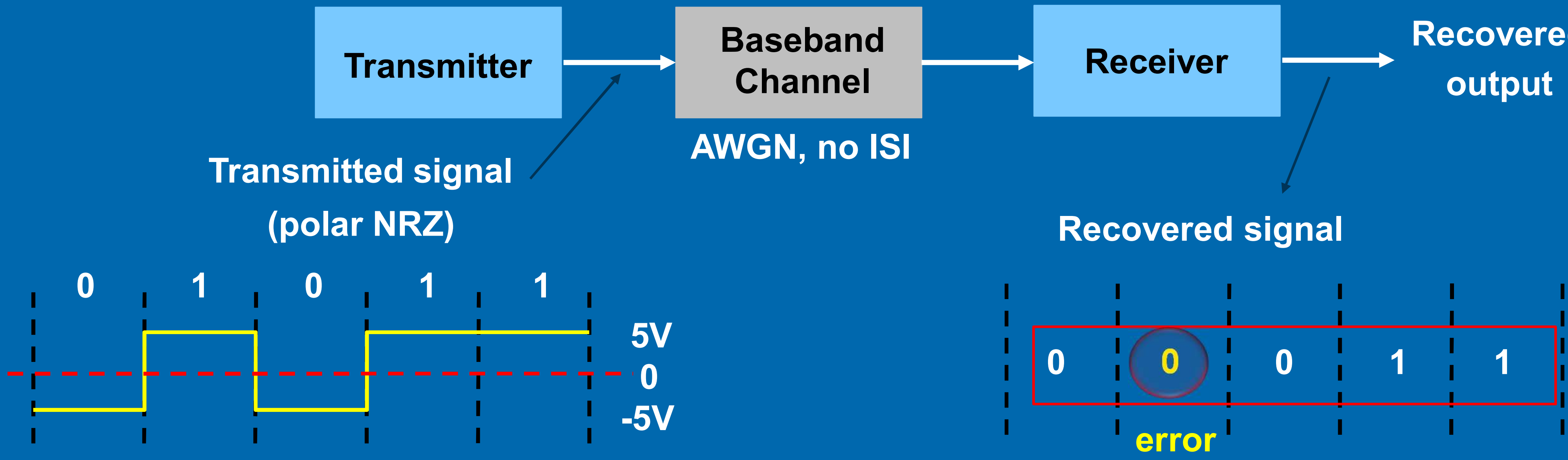
Chapter 9

Optimum Baseband Receiver

Introduction



Introduction



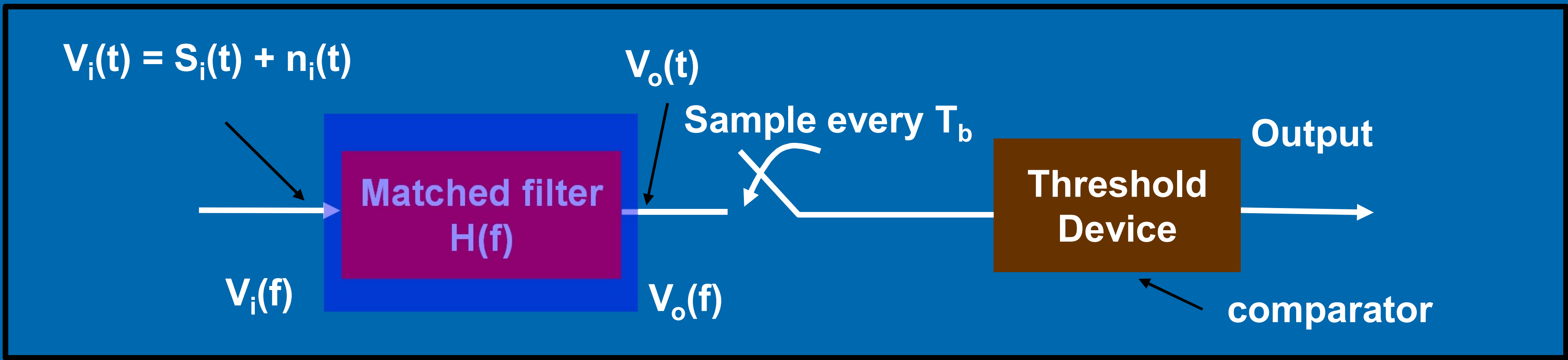
How to minimise the error bits?

Use an optimum receiver

9.1 Optimum Receiver for Binary Baseband Transmission

- Optimum receiver consists of a matched filter, a sampler and a threshold device.

Optimum Receiver Structure



9.1 Optimum Receiver for Binary Baseband Transmission

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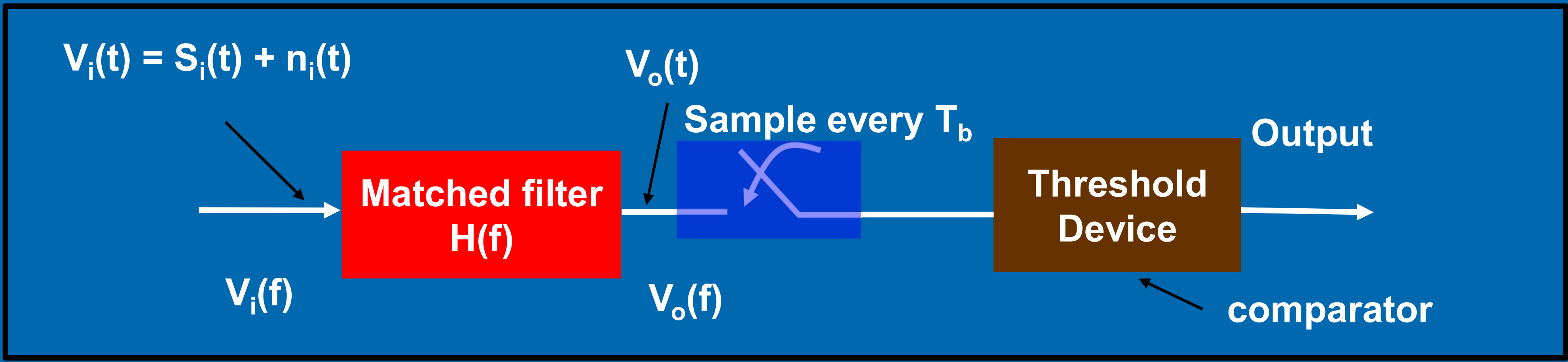
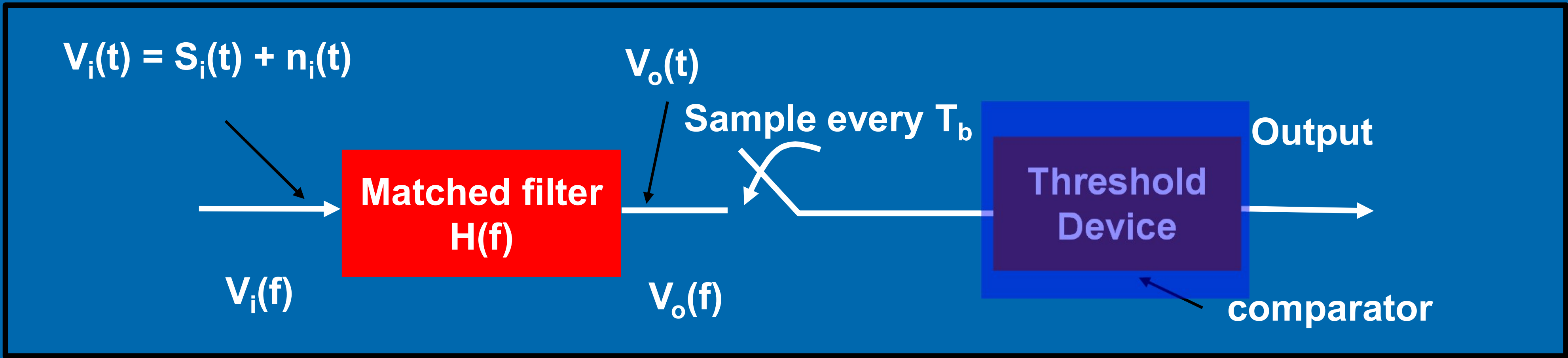
Optimum Receiver Structure



9.1 Optimum Receiver for Binary Baseband Transmission

- **Optimum receiver consists of a matched filter, a sampler and a threshold device.**

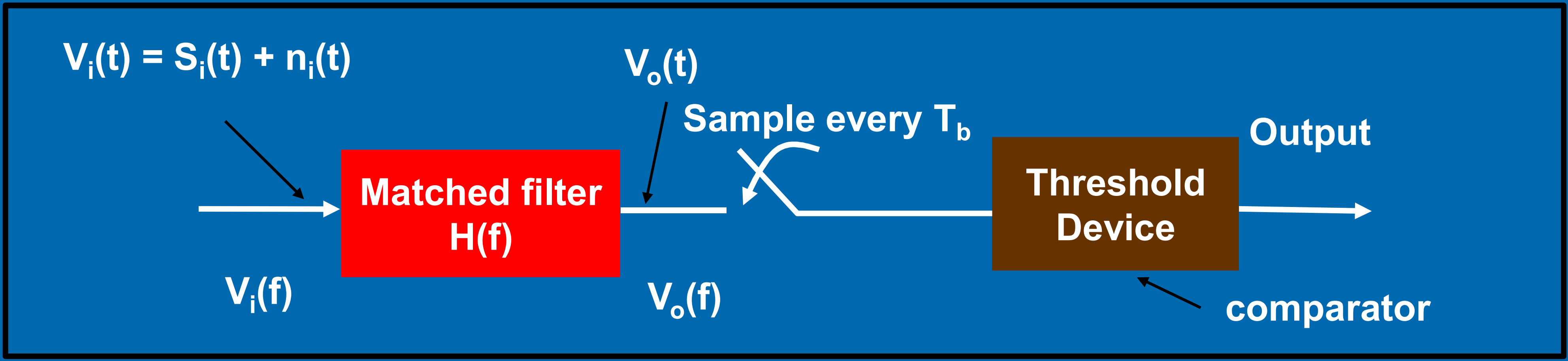
Optimum Receiver Structure



9.1 Optimum Receiver for Binary Baseband Transmission

- Optimum receiver consists of a matched filter, a sampler and a threshold device.
- Optimum receiver recovers the data from the received signal with **minimum probability of bit error, P_e.**

Optimum Receiver Structure



9.1 Optimum Receiver for Binary Baseband Transmission

- Input to the optimum receiver is a signal :

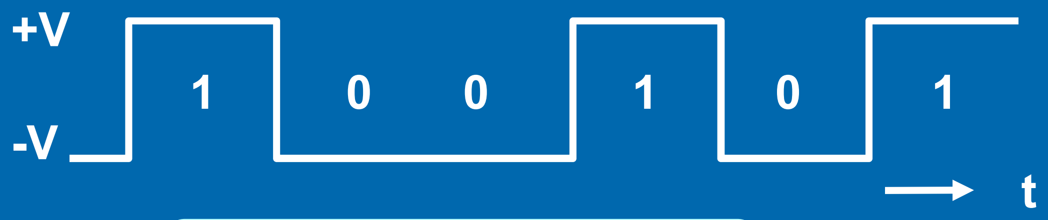
$V_i(t) = s_i(t) + n_i(t)$ → **Optimum Receiver** →

Signal component ($s_i(t)$)

Noise component ($n_i(t)$)

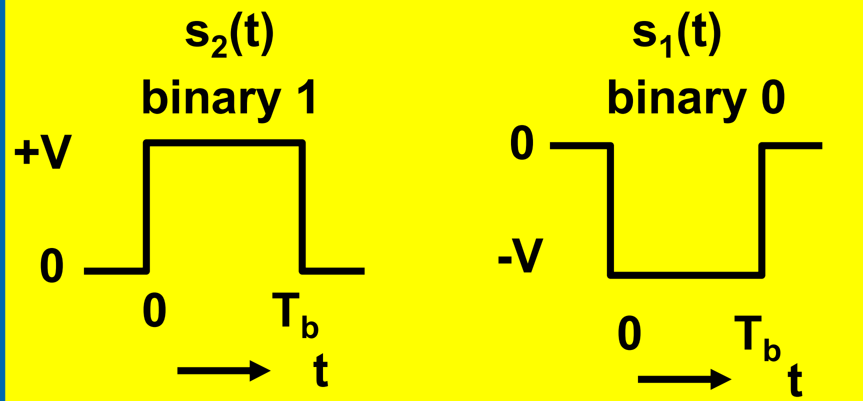
$s_1(t-2T_b)$ $s_2(t-5T_b)$

b_k: $b_0 = 1$ $b_1 = 0$ $b_2 = 0$ $b_3 = 1$ $b_4 = 0$ $b_5 = 1$



Input signal waveform to optimum receiver

Basic pulses
- polar NRZ inputs



9.1 Optimum Receiver for Binary Baseband Transmission

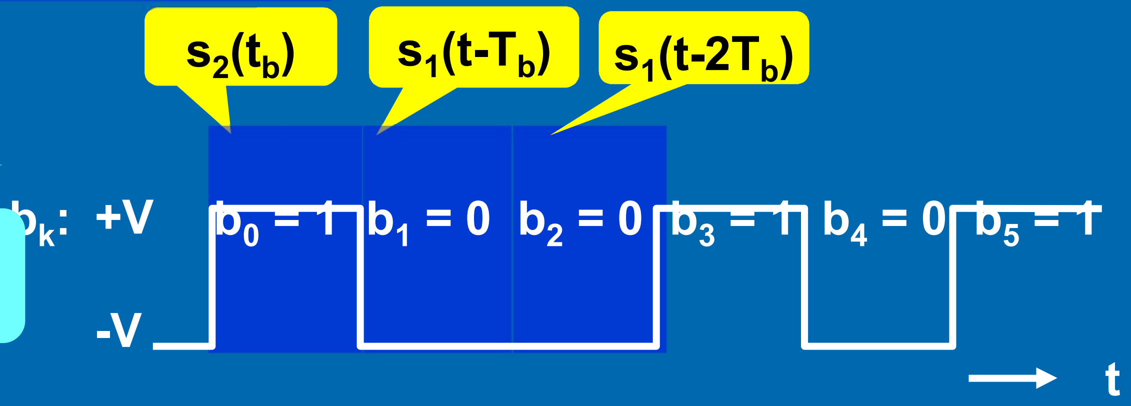
- The pulse sequence $s_i(t)$ is made up of basic pulses, $s_2(t)$ and $s_1(t)$:

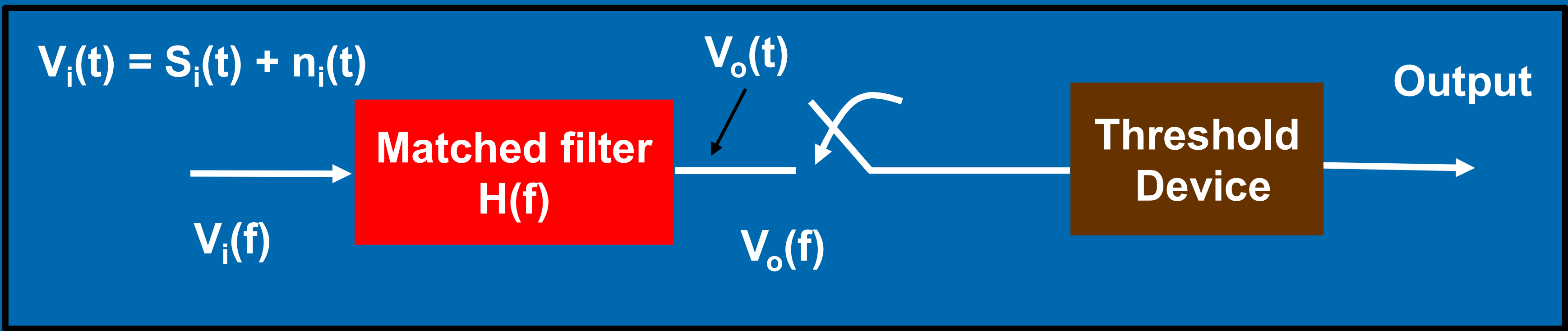
$$s_i(t) = \begin{cases} s_2(t - kT_b) = +V, & \text{if } b_k = 1 \\ s_1(t - kT_b) = -V, & \text{if } b_k = 0 \end{cases}$$

for $kT_b \leq t \leq (k+1)T_b$

k = bit number

Input signal waveform to optimum receiver





9.1 Optimum Receiver for Binary Baseband Transmission

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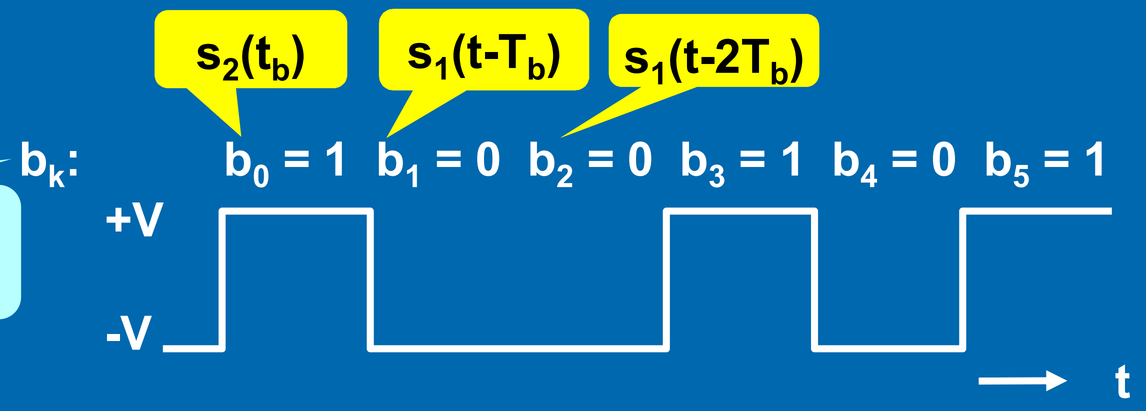
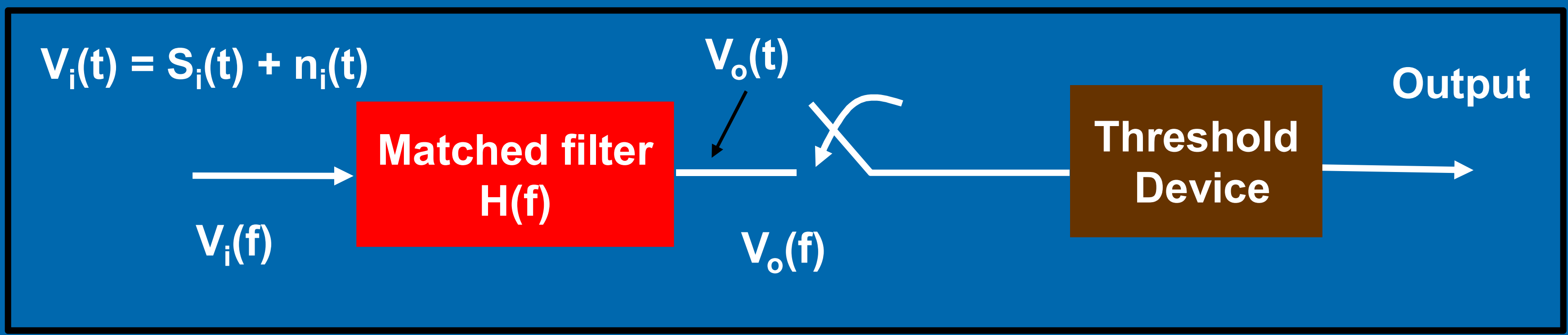


Input signal waveform to optimum receiver



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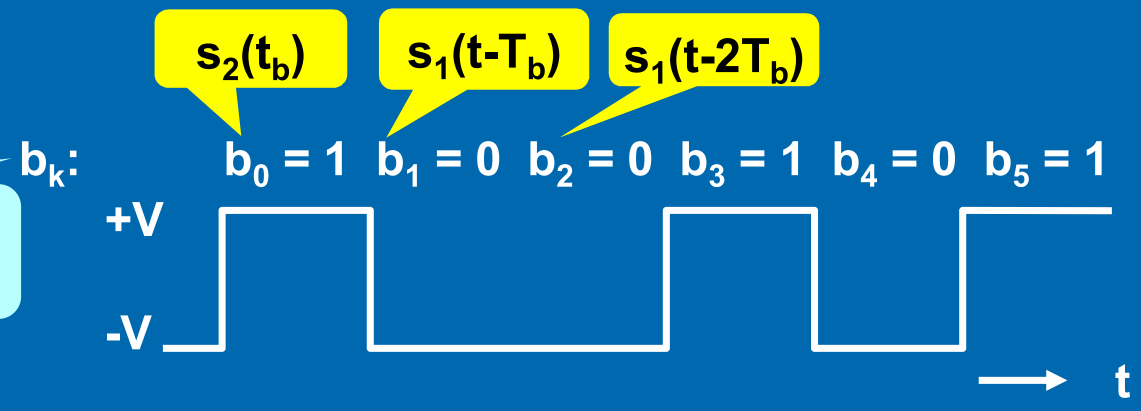
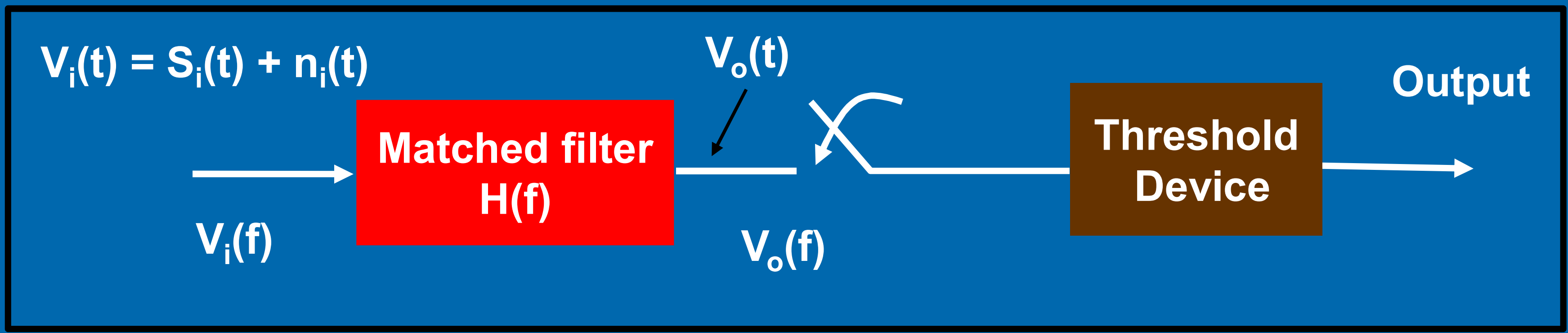
for $kT_b \le t \le (k+1)T_b$

k = bit number

Input signal waveform to optimum receiver

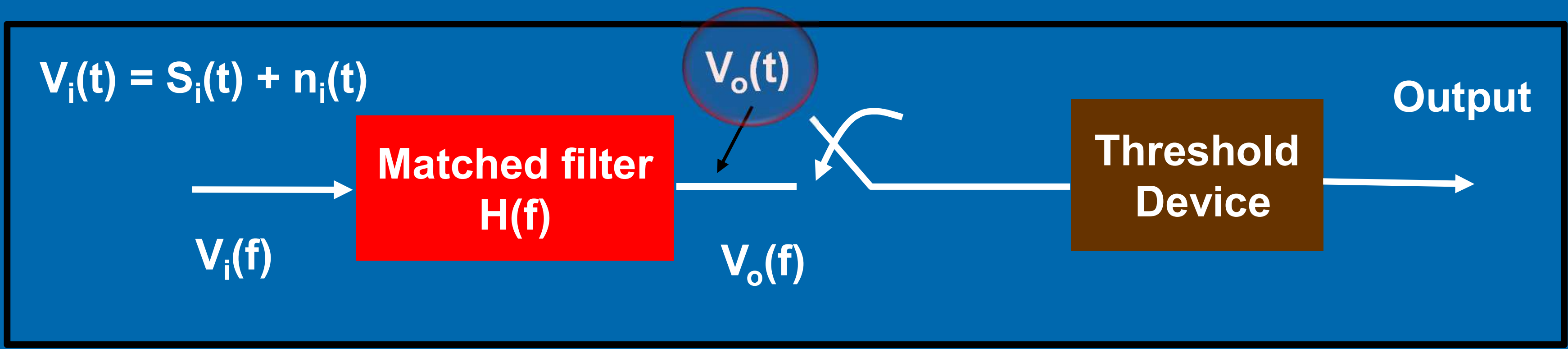




9.1 Optimum Receiver for Binary Baseband Transmission

- **The output of the matched filter and its spectrum**

$$V_o(t) = V_i(t) * h(t)$$



9.1 Optimum Receiver for Binary Baseband Transmission

- The output of the matched filter and its spectrum

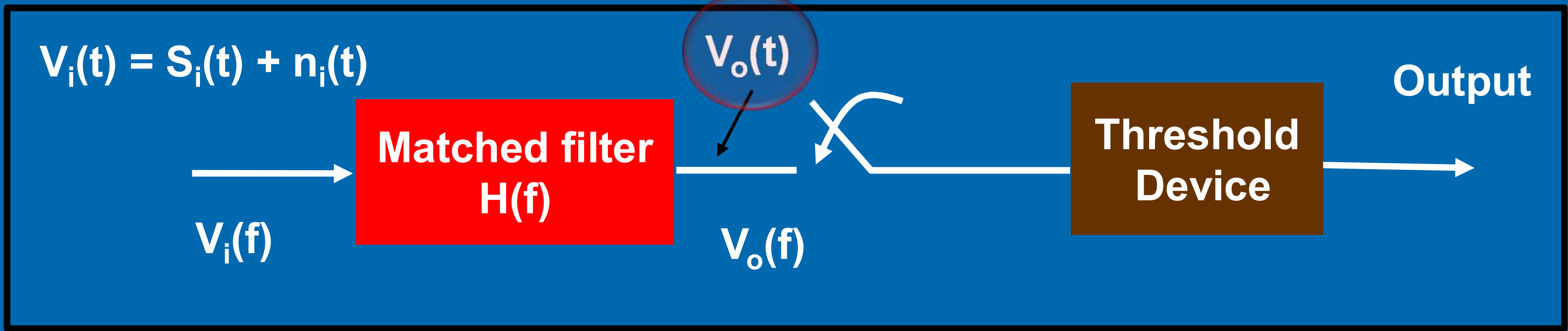
$$V_o(t) = V_i(t) * h(t)$$

$$V_o(f) = H(f) \times V_i(f)$$

- From definition of convolution in Chapter 2 : $f(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau) d\tau$

$$V_o(t) = \int_{-\infty}^{\infty} V_i(t) h(t-\tau) d\tau$$

where $V_i(t) = s_i(t) + n_i(t)$



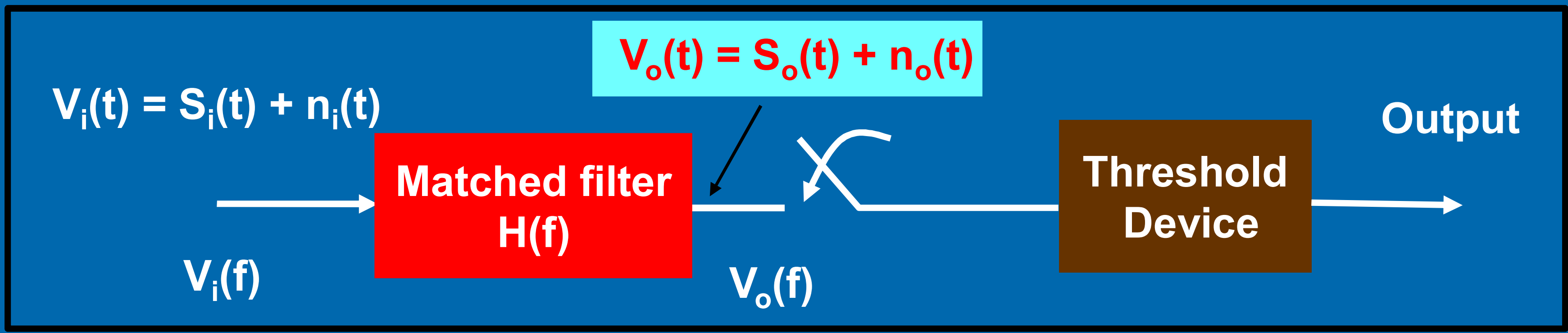
9.1 Optimum Receiver for Binary Baseband Transmission

- $V_o(t)$ also has a signal and noise components due to $s_i(t)$ and $n_i(t)$, respectively:

$$V_o(t) = s_o(t) + n_o(t)$$

Signal component

Noise component



9.1 Optimum Receiver for Binary Baseband Transmission

- $V_o(t)$ also has a signal and noise components due to $s_i(t)$ and $n_i(t)$, respectively:

$$V_o(t) = s_o(t) + n_o(t)$$

Signal component

Noise component

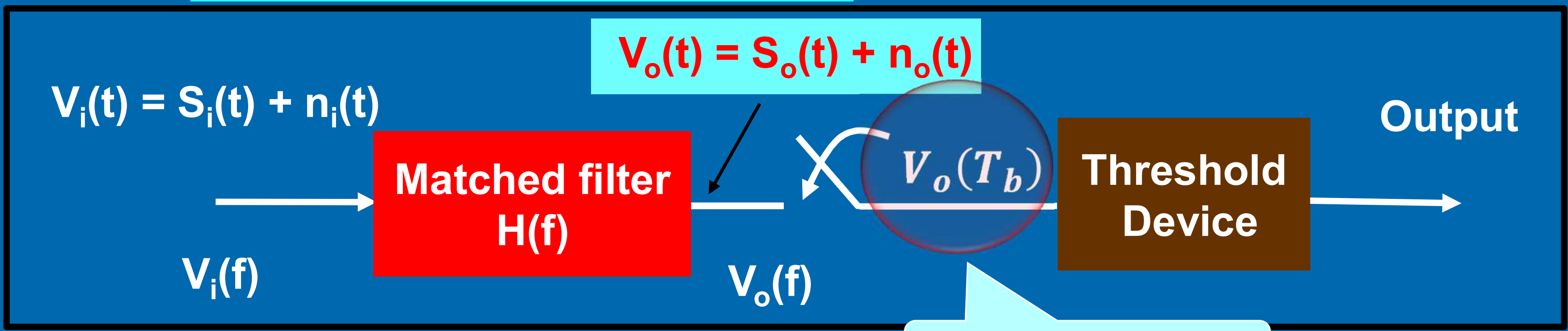
- As the output $V_o(t)$ is sampled at the end of every T_b, it can be written as:

Sampled Output

$$V_o(T_b) = \int_0^{T_b} V_i(\tau)h(T_b - \tau)d\tau$$

$$= s_o(T_b) + n_o(T_b)$$

$$V_o(t) = \int_{-\infty}^{\infty} V_i(t)h(t - \tau)d\tau$$



Sampled Output

9.1 Optimum Receiver for Binary Baseband Transmission

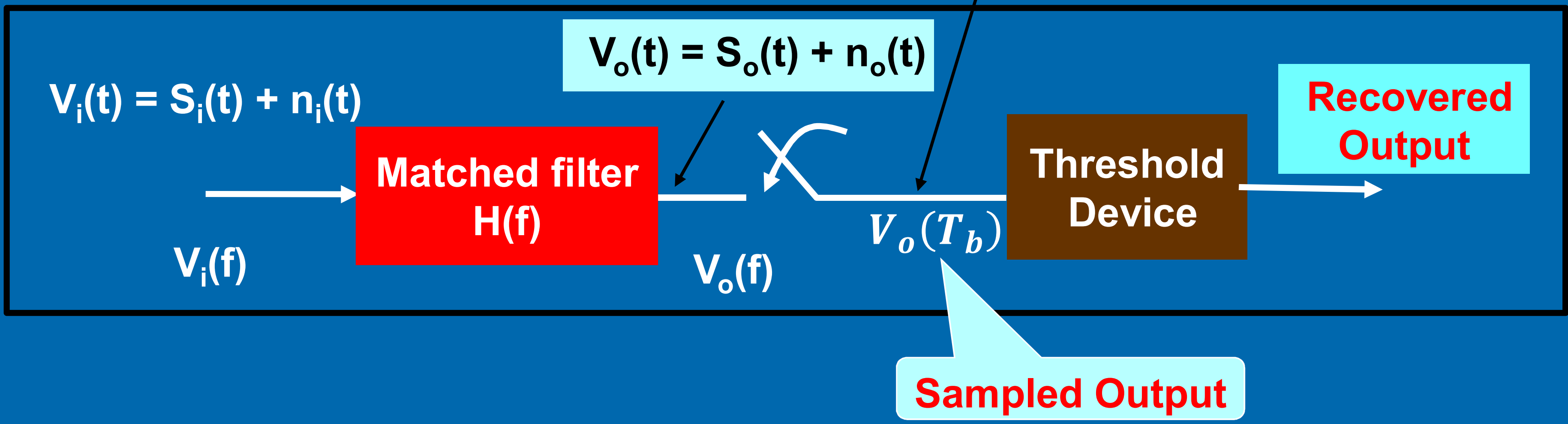
- The output is determined by comparing the sampled value $V_o(T_b)$ against the threshold voltage, V_T:

Recovered output

Sampled Output $> V_T$ ➡ '1'

Sampled Output $< V_T$ ➡ '0'

$$V_o(T_b) = \int_0^{T_b} V_i(\tau)h(T_b - \tau)d\tau$$



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9.1 Optimum Receiver for Binary Baseband Transmission

Matched Filter

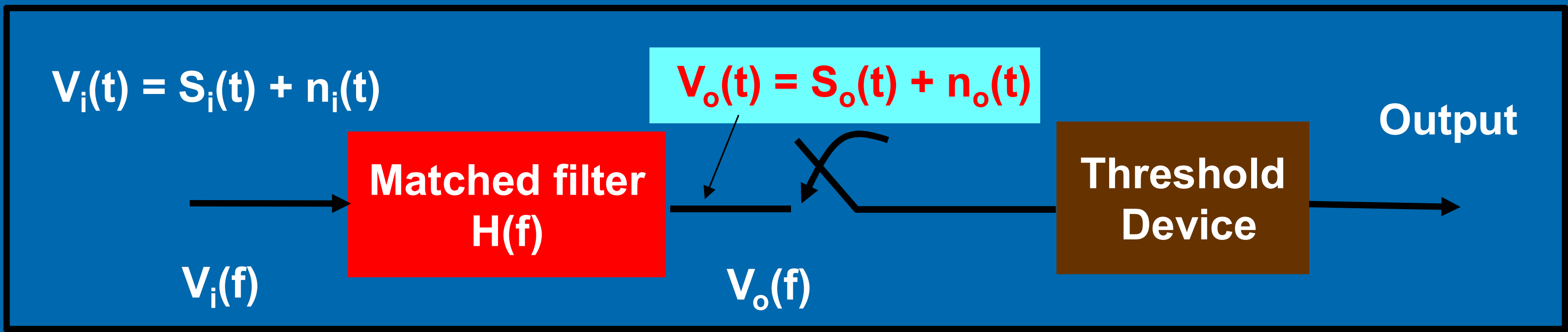
Matched filter impulse response

- To minimise the probability of bit error, the matched filter should have an impulse response, h(t), related to $s_1(t)$ and $s_2(t)$ by

$$h(t) = s_2(T_b - t) - s_1(T_b - t)$$

$s_2(t)$: binary 1

$s_1(t)$: binary 0



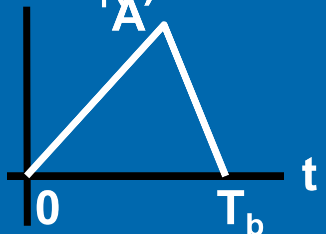
9.1 Optimum Receiver for Binary Baseband Transmission

Matched Filter

The process of obtaining h(t) from $s_2(t)$ and $s_1(t)$

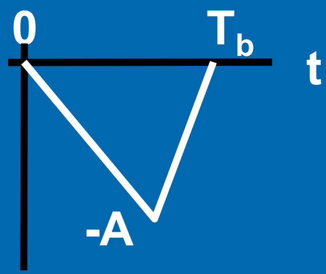
- If $s_2(t)$ and $s_1(t)$ have the forms as shown in (a) and (b), then h(t) is as shown in (e).

Binary 1

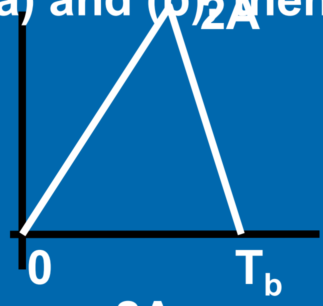


(a) $s_2(t)$

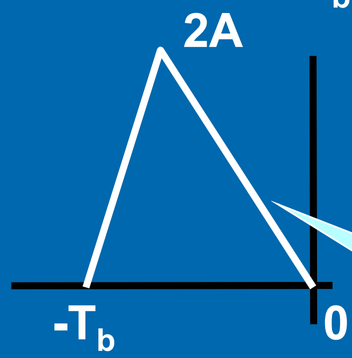
Binary 0



(b) $s_1(t)$

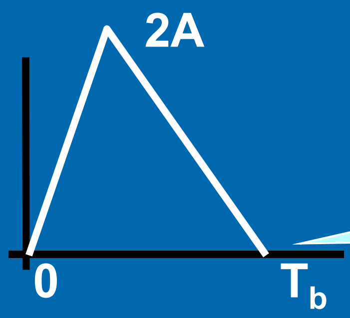


(c) $s_2(t) - s_1(t)$



(d) $s_2(-t) - s_1(-t)$

Fold about the vertical axis



(e) $h(t) = s_2(T_b-t) - s_1(T_b-t)$

Shift to the right by T_b

9.1 Optimum Receiver for Binary Baseband Transmission

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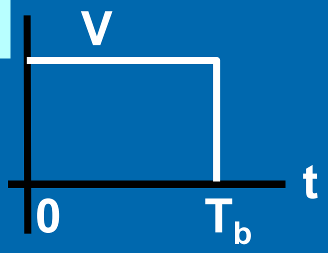
Matched Filter

The process of obtaining h(t) from $s_2(t)$ and $s_1(t)$

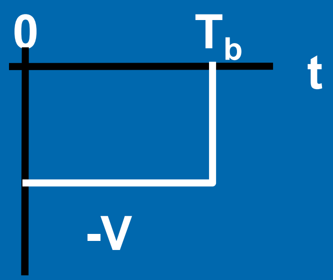
Polar NRZ

Binary 1

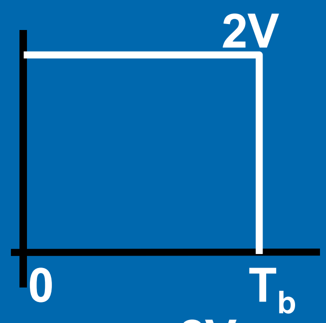


(a) $s_2(t)$

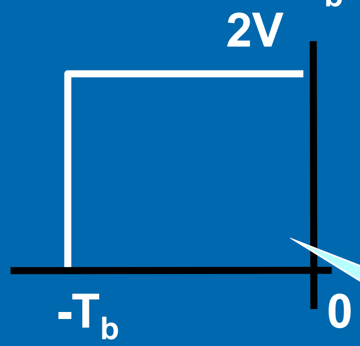
Binary 0



(b) $s_1(t)$



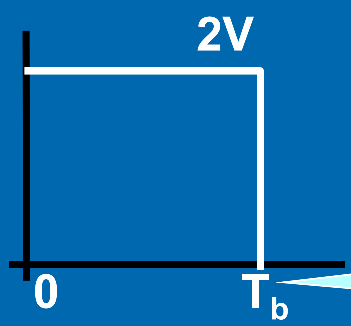
(c) $s_2(t) - s_1(t)$



(d) $s_2(-t) - s_1(-t)$

Fold about the vertical axis

h(t) for signals in polar NRZ



(e) $h(t) = s_2(T_b - t) - s_1(T_b - t)$

Shift to the right by T_b

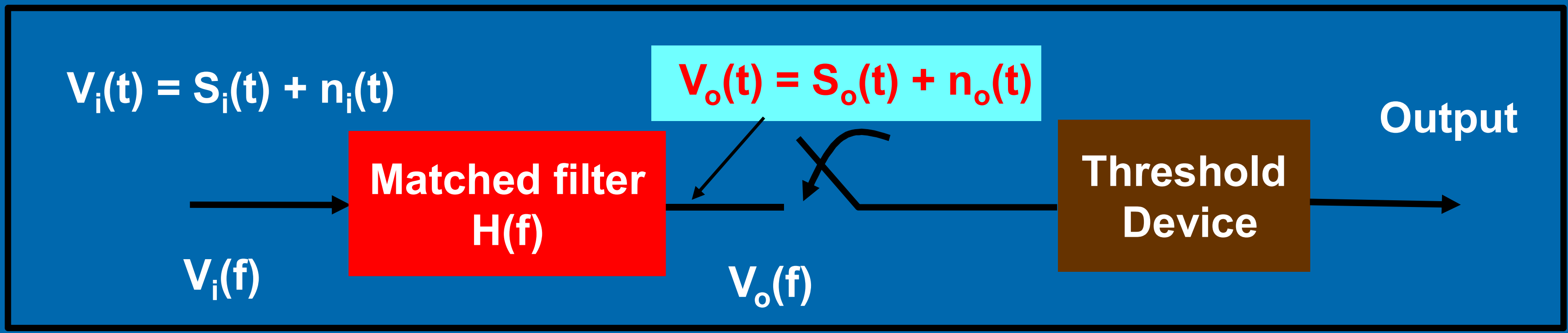
9.1 Optimum Receiver for Binary Baseband Transmission

Matched Filter

- For polar NRZ inputs, where $s_2(t) = +V$ and $s_1(t) = -V$,

Matched filter impulse response

$$h(t) = \begin{cases} 2\text{V for } 0 \le t \le T_b \\ 0\text{V for other } t \end{cases}$$



9.1 Optimum Receiver for Binary Baseband Transmission

Probability of bit error for optimum receiver

- The probability of bit error, P_e for a matched filter receiver is given by

$$P_e = \frac{1}{2} erfc\left(\frac{\gamma}{2\sqrt{2}}\right)$$

where $$\gamma^2 = \frac{2}{\eta}\int_0^{T_b} [s_2(t) - s_1(t)]^2 \, dt$$

and $\frac{\eta}{2}$ is the double-sided power spectral density of the white channel noise.

η is the single-sided power spectral density of the white channel noise.

9.1 Optimum Receiver for Binary Baseband Transmission

Probability of bit error for optimum receiver

For polar NRZ inputs:

$s_2(t) = +V$ ‘ 1 ‘

$s_1(t) = -V$ ‘ 0 ’

$$\gamma^2 = \frac{2}{\eta}\int_0^{T_b} [s_2(t) - s_1(t)]^2\,dt$$

$$\gamma^2 = \frac{2}{\eta}\int_0^{T_b} (V - (-V))^2\,dt = \frac{2}{\eta}\int_0^{T_b} 4V^2\,dt = \frac{2.4V^2}{\eta}\int_0^{T_b} dt$$

$$= \frac{8}{\eta}V^2\,[t]_0^{T_b} = \frac{8}{\eta}V^2 T_b$$

Hence, $\gamma = V\sqrt{\frac{8T_b}{\eta}}$

9.1 Optimum Receiver for Binary Baseband Transmission

Probability of bit error for optimum receiver

Polar NRZ input

Therefore

$$P_e = \frac{1}{2} erfc\left(\frac{\gamma}{2\sqrt{2}}\right) \qquad \gamma = V\sqrt{\frac{8T_b}{\eta}}$$

$$= \frac{1}{2} erfc\left[\frac{V\sqrt{\frac{\cancel{8}T_b}{\eta}}}{\cancel{2\sqrt{2}}}\right]$$

$\sqrt{8} = 2\sqrt{2}$

$$P_e = \frac{1}{2} erfc\left\{V\sqrt{\frac{T_b}{\eta}}\right\} = \frac{1}{2} erfc\left\{\sqrt{\frac{V^2 T_b}{\eta}}\right\}$$

9.1 Optimum Receiver for Binary Baseband Transmission

Probability of bit error for optimum receiver

Example 9.1

A polar NRZ binary signal, s(t), is a +1 V or –1 V pulse during the interval (0, T_b). The transmission rate of the signal is 100 bps. AWGN noise having two-sided power spectral density of 10^{-3} W/Hz is added to the signal. If the received signal is detected with a matched filter, calculate the bit error probability.

Solution

Given: r_b = 100 bps, $\eta/2 = 10^{-3}$

therefore $T_b = 1/r_b = 1/100$

and $\eta = 2 \times 10^{-3}$ W/Hz

9.1 Optimum Receiver for Binary Baseband Transmission

Probability of bit error for optimum receiver

- **For matched filter with polar NRZ inputs**

$$P_e = \frac{1}{2} erfc\left\{\sqrt{\frac{V^2 T_b}{\eta}}\right\}$$

$$= \frac{1}{2} erfc\left\{\sqrt{\frac{1^2 x0.01}{2x10^{-3}}}\right\}$$

$$= \frac{1}{2} erfc\left(2.236\right)$$

9.1 Optimum Receiver for Binary Baseband Transmission

Probability of bit error for optimum receiver

- **For matched filter with polar NRZ inputs**

$$P_e = \frac{1}{2} erfc\left\{\sqrt{\frac{V^2 T_b}{\eta}}\right\}$$

$$= \frac{1}{2} erfc\left\{\sqrt{\frac{1^2 x0.01}{2x10^{-3}}}\right\}$$

$$= \frac{1}{2} erfc(2.236)$$

Round to 2.23 worst case P_e

$= 0.5 \times 0.1612 \times 10^{-2}$

$= 8.1 \times 10^{-4}$

Z	erfc(Z)
2.21	0.177556D-02
2.22	0.169205D-02
2.23	0.161217D-02
2.24	0.153577D-02
2.25	0.146272D-02
2.26	0.139288D-02

9.1 Optimum Receiver for Binary Baseband Transmission

Implementation of optimum receiver

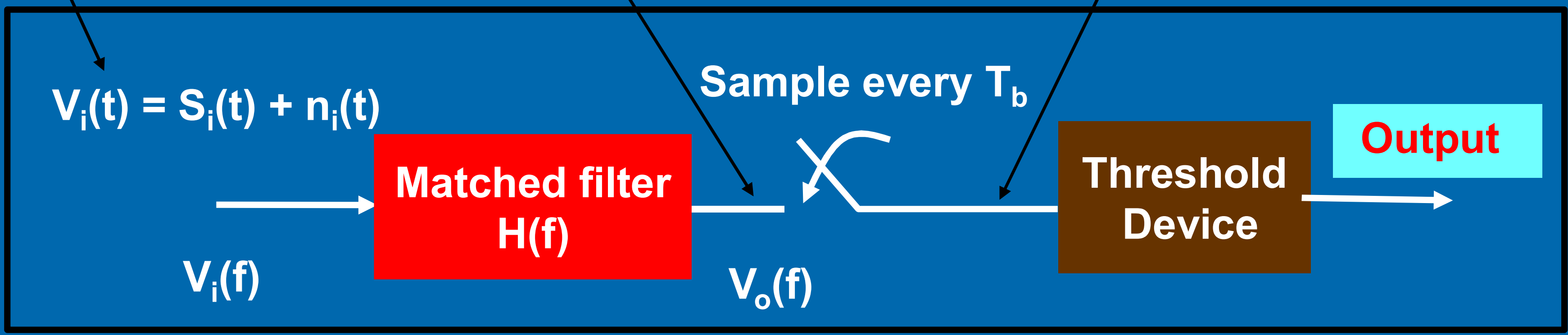
What is the practical circuit for optimum receiver?

$$V_o(T_b) = \int_0^{T_b} V_i(\tau)h(T_b - \tau)d\tau$$

Received signal

$V_o(t) = S_o(t) + n_o(t)$

Output samples



9.1 Optimum Receiver for Binary Baseband Transmission

Implementation of optimum receiver

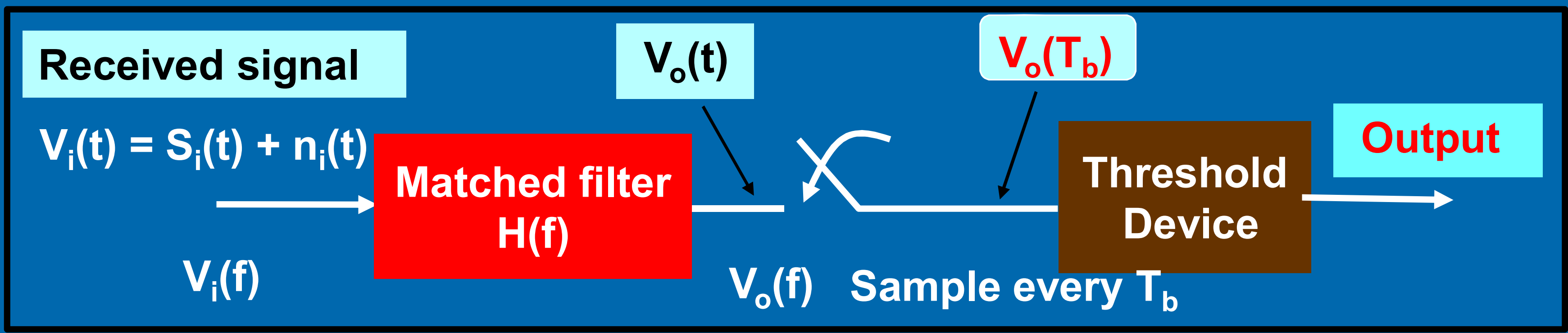
Matched filter

- The output of the matched filter at the end of each bit frame is $V_o(T_b) = \int_0^{T_b} V_i(\tau)h(T_b-\tau)d\tau$

For matched filter $h(t) = s_2(T_b - t) - s_1(T_b - t)$

Let $t = T_b - \tau$, we have $h(T_b - \tau) = s_2(T_b - (T_b - \tau)) - s_1(T_b - (T_b - \tau))$

$$h(T_b - \tau) = s_2(\tau) - s_1(\tau)$$



9.1 Optimum Receiver for Binary Baseband Transmission

Implementation of optimum receiver

- **The output of the matched filter at the end of each bit frame is**

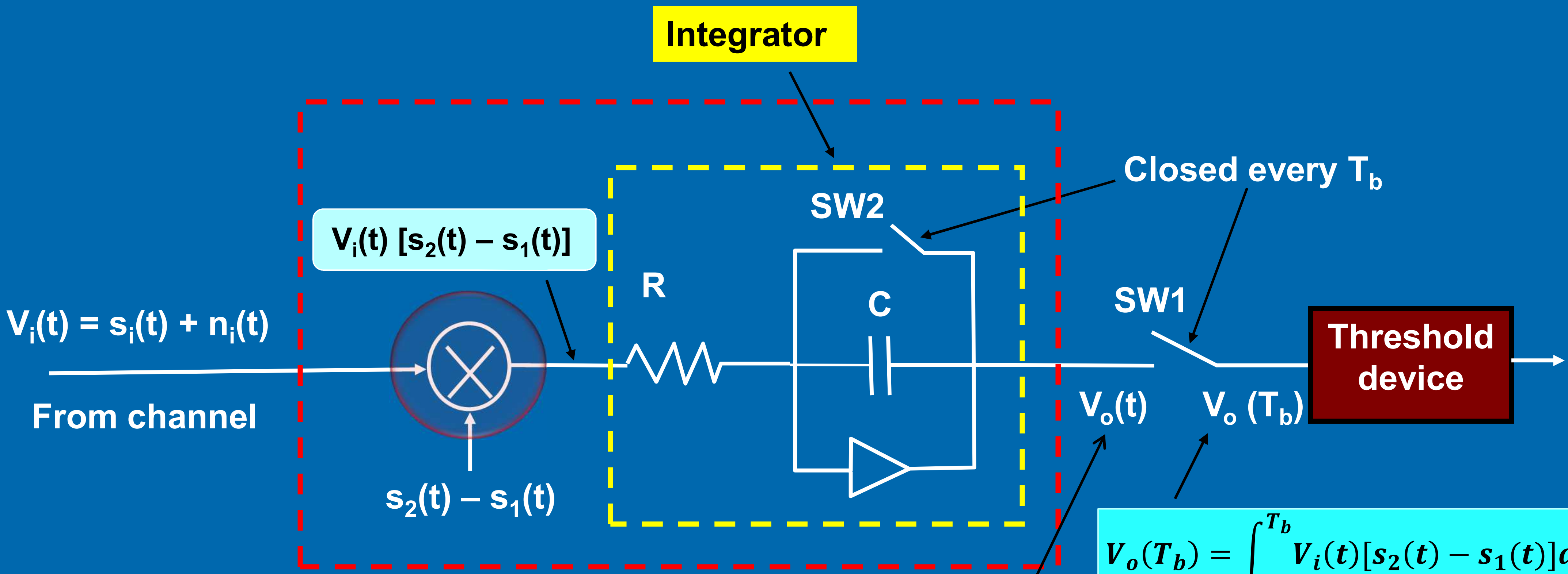
$$V_o(T_b) = \int_0^{T_b} V_i(t)[s_2(t) - s_1(t)]dt$$

Can be implemented by the Integrate-and-Dump Correlation receiver.

9.1 Optimum Receiver for Binary Baseband Transmission

Implementation of optimum receiver

Integrate-and-Dump Correlation Receiver



$$V_o(T_b) = \int_0^{T_b} V_i(t)[s_2(t) - s_1(t)]dt$$

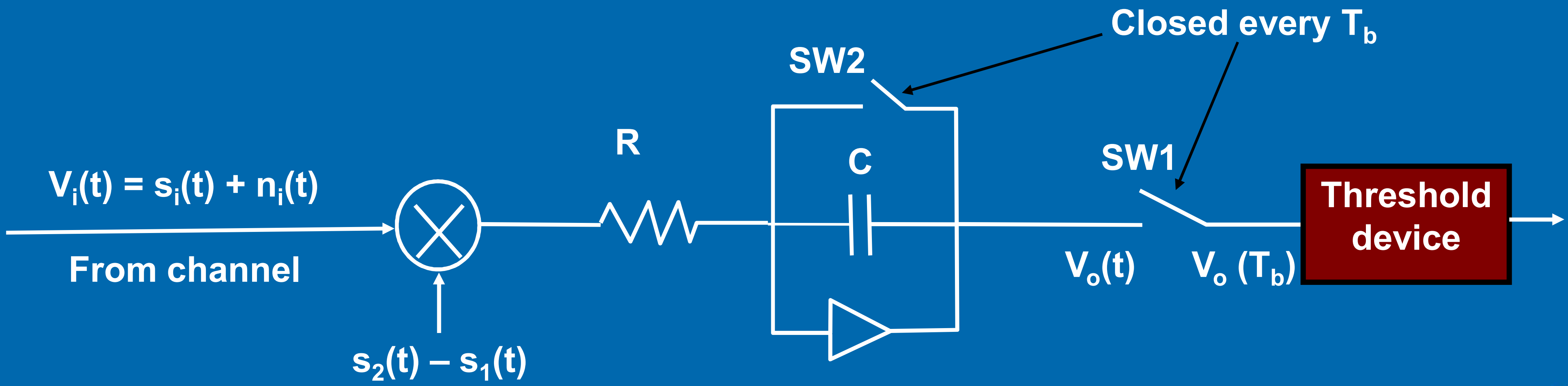
$$V_o(t) = \int V_i(t)[s_2(t) - s_1(t)]dt$$

9.1 Optimum Receiver for Binary Baseband Transmission

Implementation of optimum receiver

Integrate-and-Dump Correlation Receiver

- SW1 and SW2 are closed (and opened) at the end of each bit interval, T_b.
- SW1 is used to sample $V_o(T_b)$
- SW2 is closed to reset (dump) the integrator to zero initial condition before the occurrence of the next bit.



9.1 Optimum Receiver for Binary Baseband Transmission

Example

Received signal in polar NRZ format with no channel noise

$$s_i(t) = \begin{cases} s_2(t - kT_b) = +V, & \text{if } b_k = 1 \\ s_1(t - kT_b) = -V, & \text{if } b_k = 0 \end{cases}$$

for $kT_b \leq t \leq (k+1)T_b$.

$$V_o(T_b) = \begin{cases} \boxed{k\int_0^{T_b} 2V^2 dt = 2kV^2T_b \text{ for + V input}} & \text{Binary '1'} \\ k\int_0^{T_b} -2V^2 dt = -2kV^2T_b \text{ for - V input} & \text{Binary '0'} \end{cases}$$

$V_i(t) = V$

$$V_o(T_b) = K\int_0^{T_b} V_i(t)\underbrace{[s_2(t) - s_1(t)]}dt$$

K = circuit constant

binary 1 is transmitted

$$= K\int_0^{T_b} V\,[\,V - (-V)]\,dt$$

for noise-free channel

$$= K\int_0^{T_b} 2V^2 dt = 2V^2\,\mathrm{K}\int_0^{T_b} dt = 2V^2K[t]_0^{T_b}$$

varies linearly with t over $0 \le t \le T_b$

$$= 2kV^2T_b$$

$$V_o(T_b) = \begin{cases} k\int_0^{T_b} 2V^2 dt = 2kV^2 T_b \text{ for } + \text{V input} & \textbf{Binary '1'} \\ \boxed{k\int_0^{T_b} -2V^2 dt = -2kV^2 T_b \text{ for - V input} \quad \textbf{Binary '0'}} \end{cases}$$

$V_i(t)$ = -V

$$V_o(T_b) = K\int_0^{T_b} V_i(t)[\underbrace{s_2(t) - s_1(t)}]dt$$

***K* = circuit constant**

binary 0 is transmitted

$$= K\int_0^{T_b} (-V)[\,V - (-V)]\,dt$$

for noise-free channel

$$= K\int_0^{T_b} \left(-2V^2\right)dt = -2V^2\,\mathrm{K}\int_0^{T_b} dt = -2V^2 K[t]_0^{T_b}$$

varies linearly with t over $0 \le t \le T_b$

$$= -2kV^2 T_b$$

9.1 Optimum Receiver for Binary Baseband Transmission

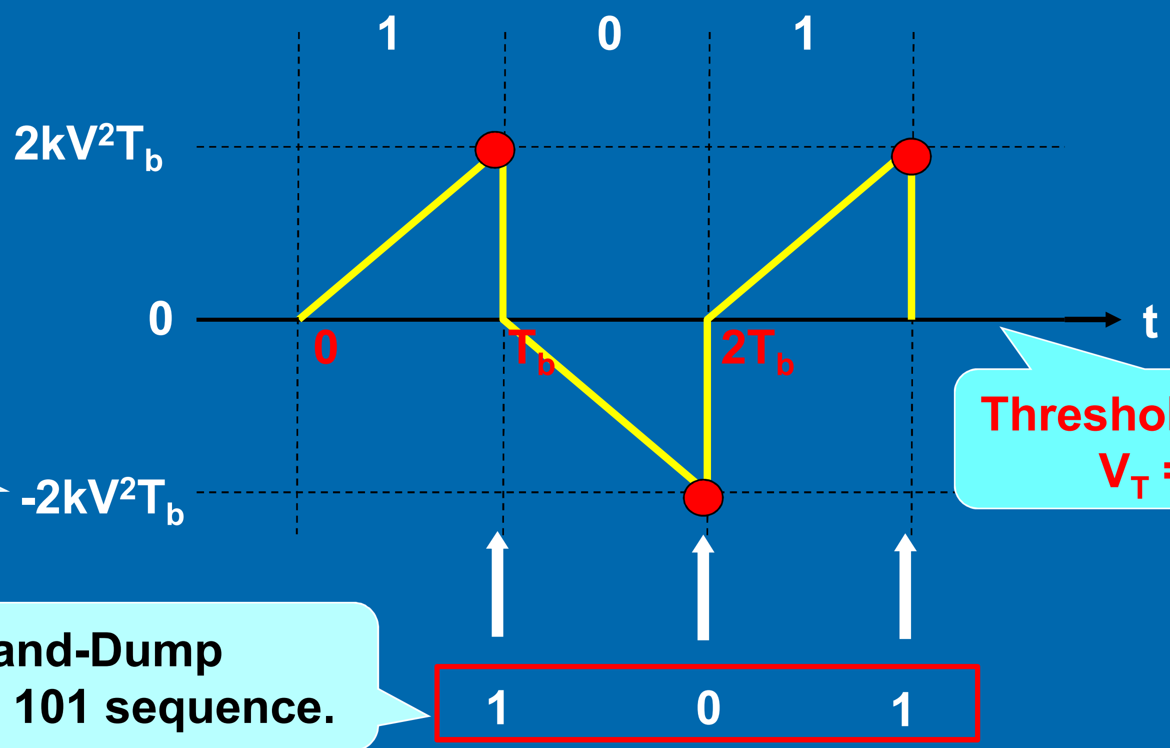
Example

Received signal in polar NRZ format with no channel noise

- The output $V_o(t)$ for a received signal, 1 0 1.

e.g. $2kV^2T_b = 25$

e.g. $-2kV^2T_b = -25$



Threshold voltage $V_T = 0$ v

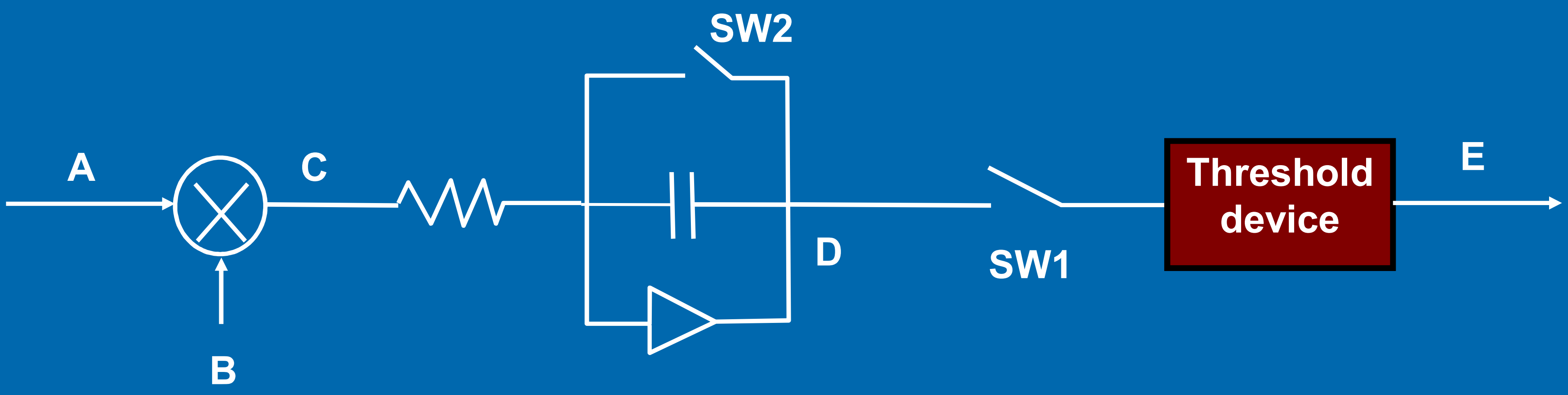
Output of Integrate-and-Dump receiver for a binary 101 sequence.

9.1 Optimum Receiver for Binary Baseband Transmission

<u>**Example 9.2**</u>

An integrate and dump correlation receiver is shown.

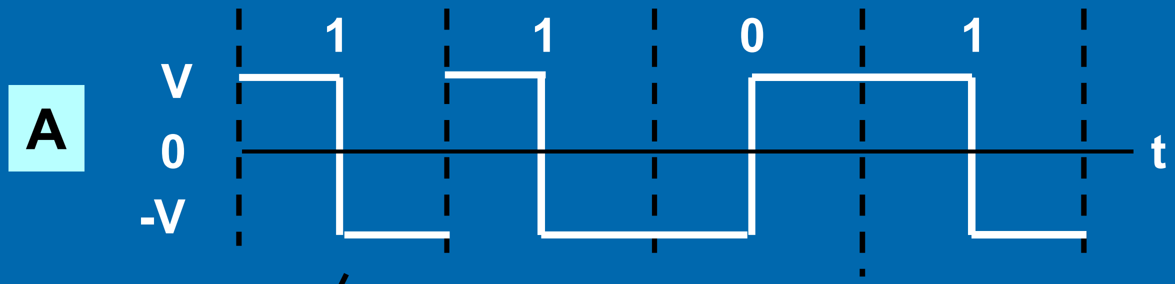
If its input is a Manchester code waveform of amplitude V volt, sketch the waveforms at A to E for a 1101 sequence. Explain the operations of SW1 and SW2.

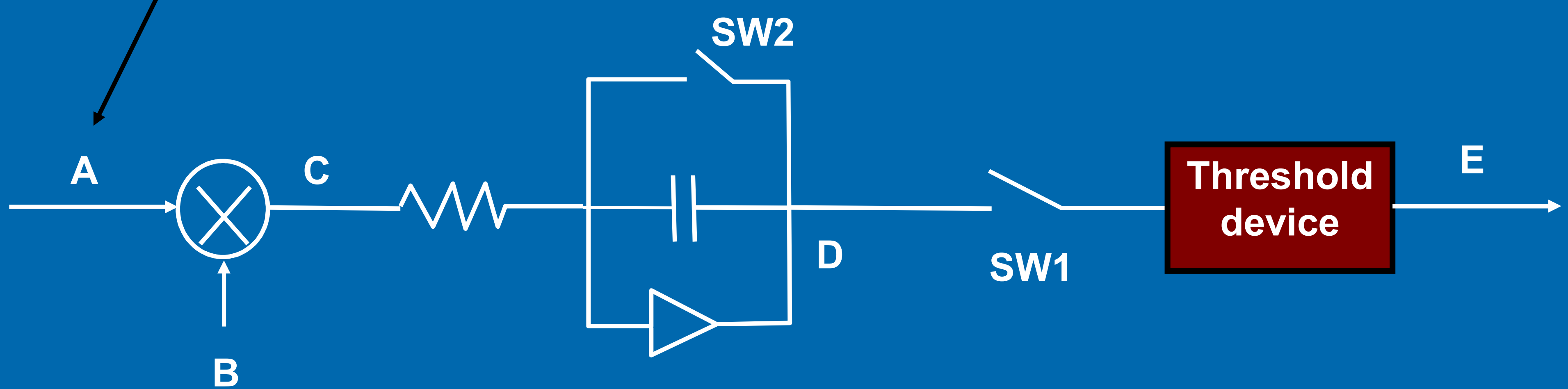


9.1 Optimum Receiver for Binary Baseband Transmission

Solution

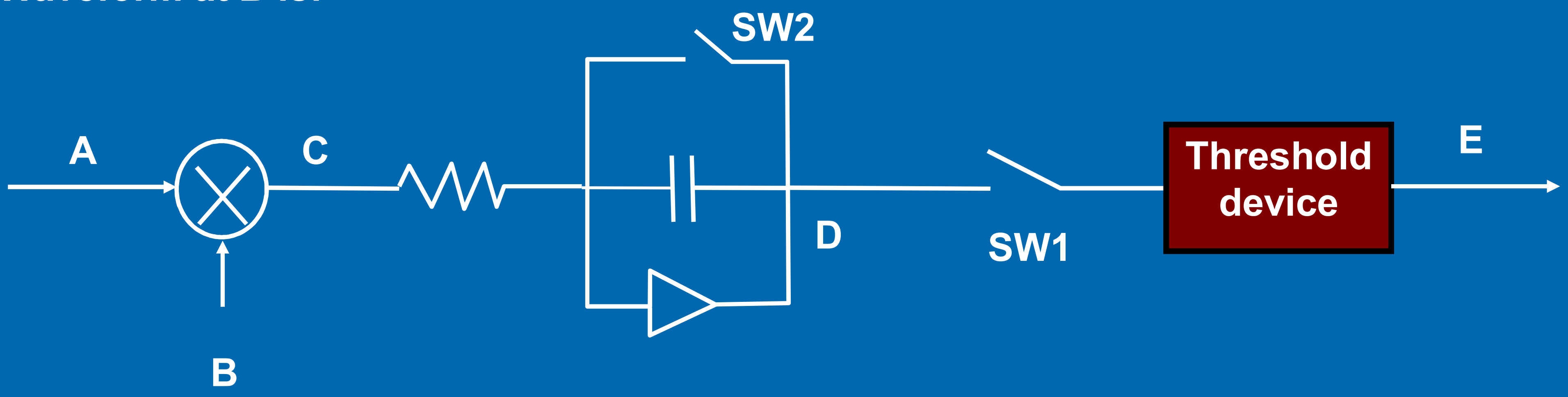
For binary sequence {1 1 0 1}, Manchester code waveform at A is:

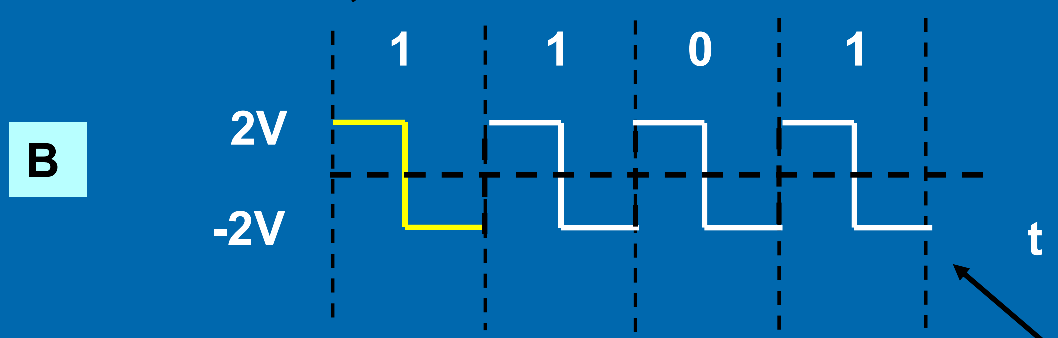




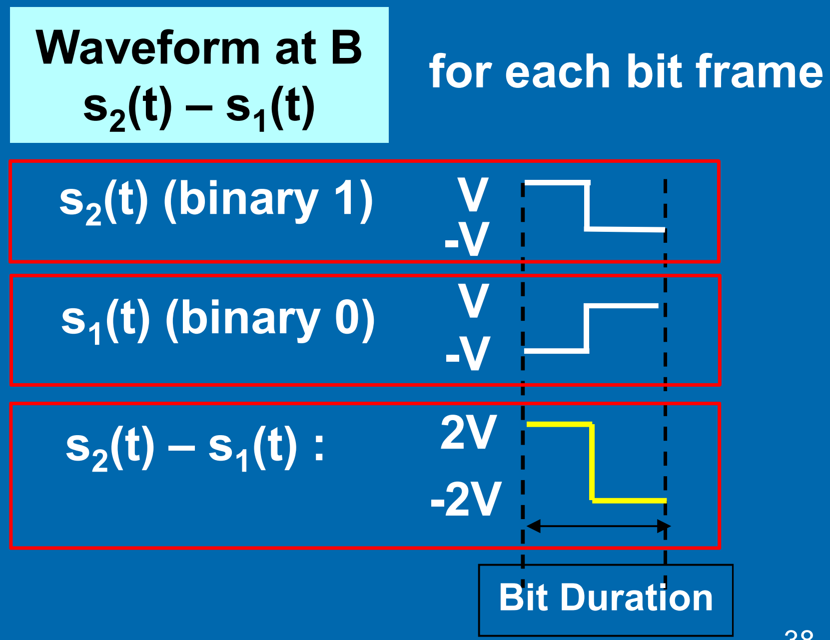
Solution

Waveform at B is:



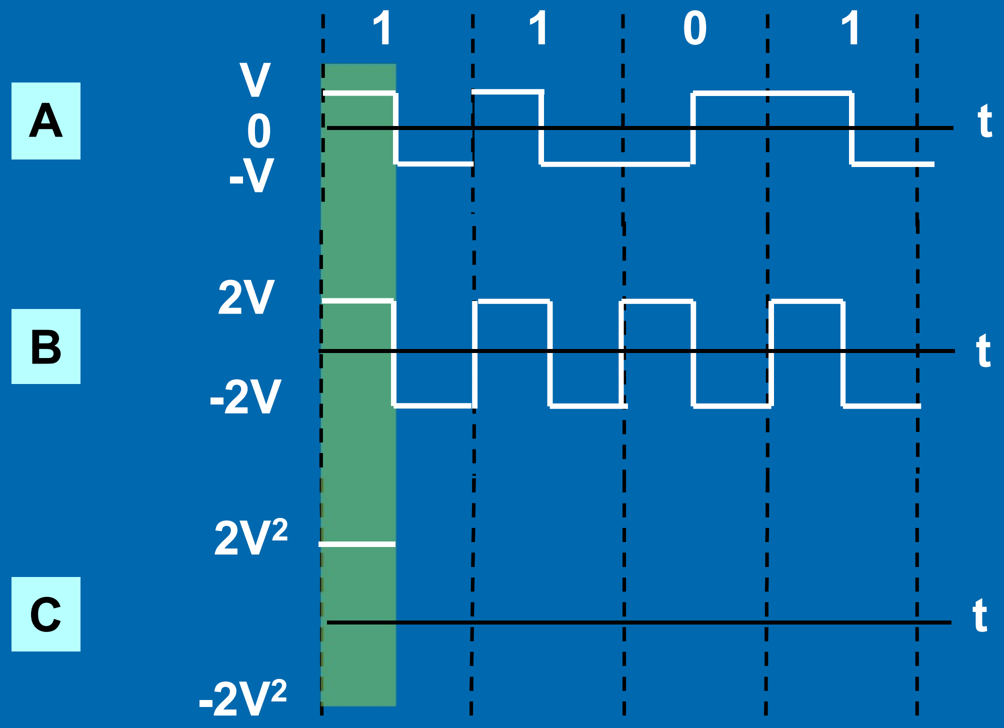


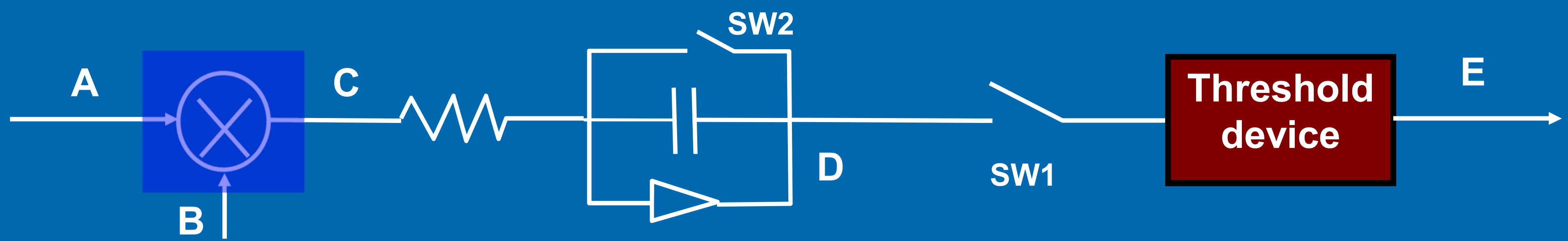
Waveform at B, repeat $s_2(t) - s_1(t)$ pattern for every bit frame.



Solution

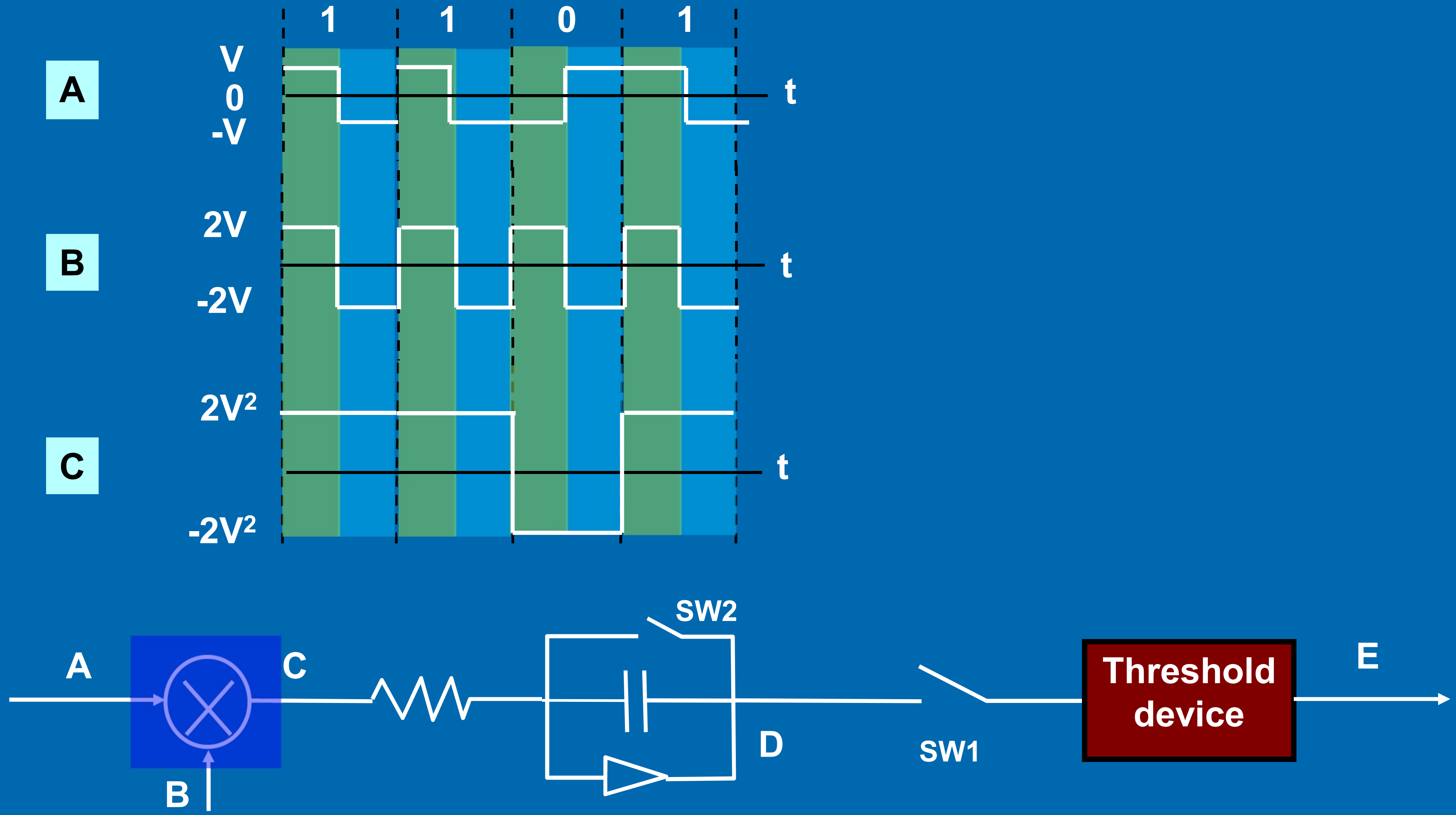
Waveform C is the multiplication of Waveform A and Waveform B:





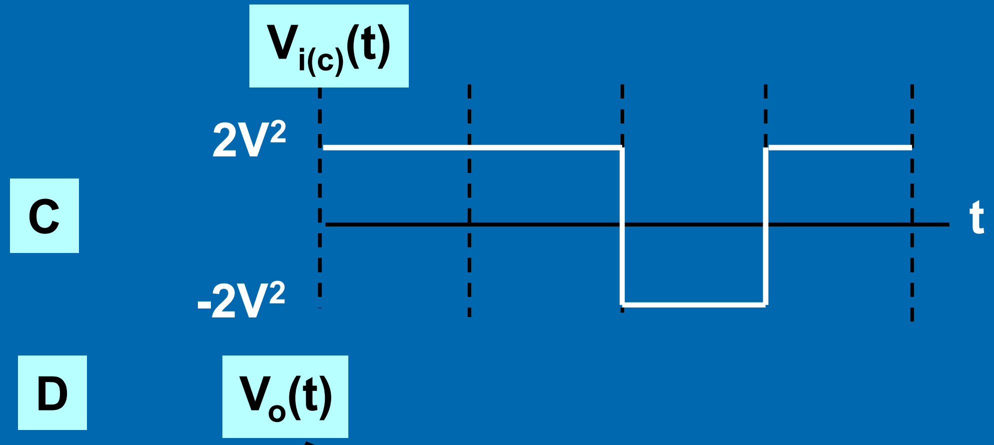
Solution

Waveform C is the multiplication of Waveform A and Waveform B:

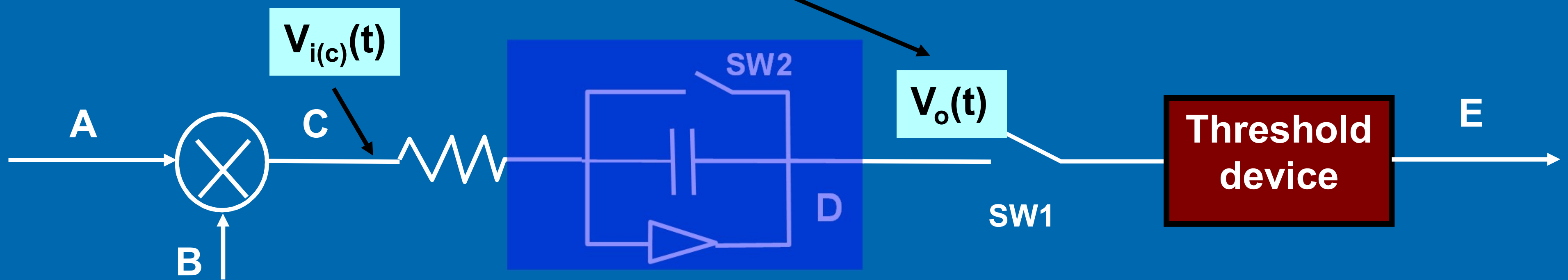


Solution

Waveform D is the integration of Waveform C.



$$V_{i(c)}(t) = \begin{cases} 2V^2 \\ or \\ -2V^2 \end{cases}$$ for each bit frame

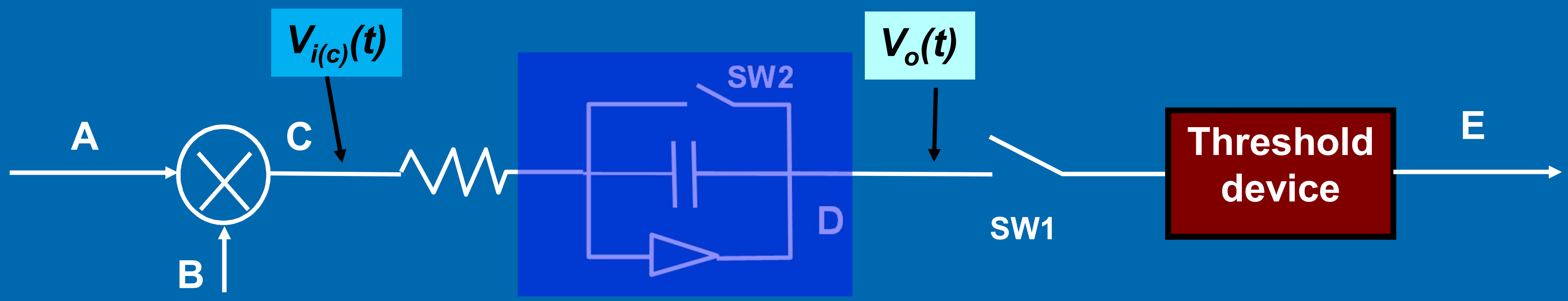


$$V_{i(c)}(t) = \begin{cases} 2V^2 \\ or \\ -2V^2 \end{cases}$$ for each bit frame

$$V_o(t) = K\int_0^{T_b} V_{i(c)}(t)\, dt$$

$$V_o(t) = \begin{cases} K\int_0^{T_b} 2V^2 dt = 2V^2K[t]_0^{T_b} = 2KV^2T_b & \text{for } V_{i(c)}(t) = 2V^2 \\ or \\ K\int_0^{T_b} (-2V^2) dt = -2V^2K[t]_0^{T_b} = -2KV^2T_b & \text{for } V_{i(c)}(t) = -2V^2 \end{cases}$$

varies linearly with t over $0 \le t \le T_b$

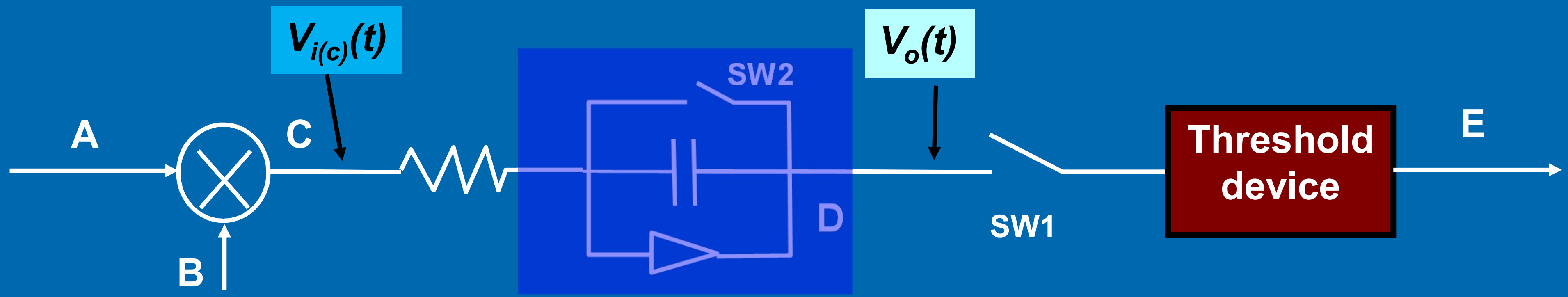


$$V_{i(c)}(t) = \begin{cases} 2V^2 \\ or \\ -2V^2 \end{cases} \quad \text{for each bit frame}$$

$$V_o(t) = K\int_0^{T_b} V_{i(c)}(t)\,dt$$

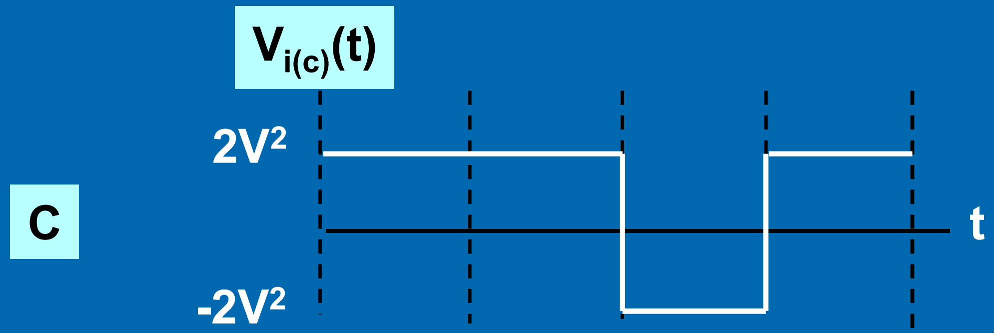
$$V_o(t) = \begin{cases} K\int_0^{T_b} 2V^2 dt = 2V^2K[t]_0^{T_b} = 2KV^2T_b & \text{for } V_{i(c)}(t) = 2V^2 \\ or & \\ K\int_0^{T_b} (-2V^2) dt = -2V^2K[t]_0^{T_b} = -2KV^2T_b & \text{for } V_{i(c)}(t) = -2V^2 \end{cases}$$

varies linearly with t over $0 \le t \le T_b$

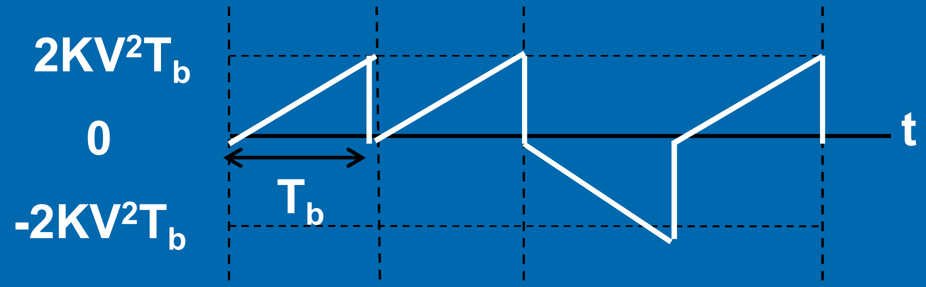


Solution

Waveform D is the integration of Waveform C.



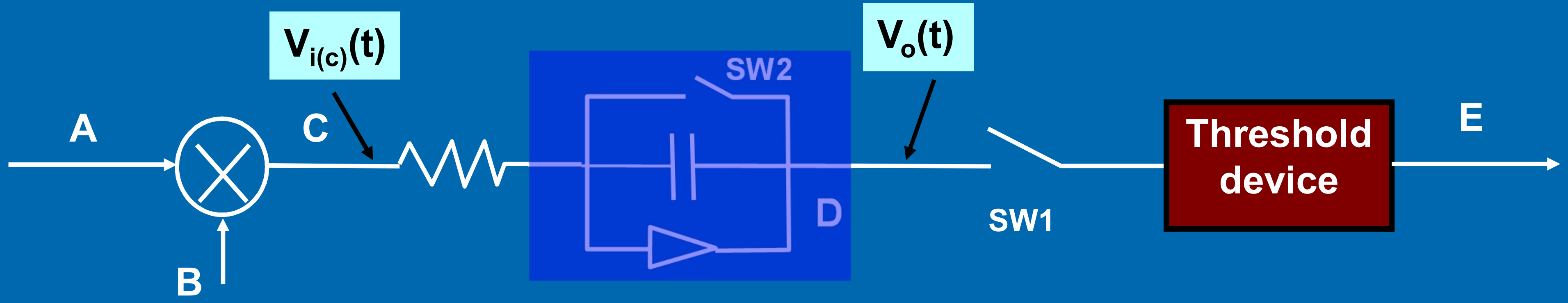
$$V_{i(c)}(t) = \begin{cases} 2V^2 \\ or \\ -2V^2 \end{cases} \text{ for each bit frame}$$



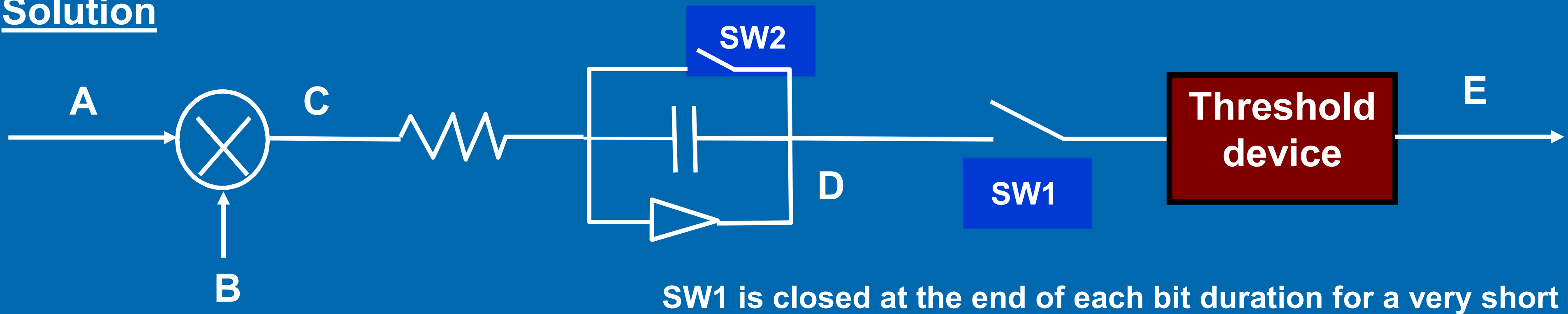
Value of D at $t = T_b$

$$V_o(t) = \begin{cases} 2KV^2T_b & \text{for } V_{i(c)}(t) = 2V^2 \\ -2KV^2T_b & \text{for } V_{i(c)}(t) = -2V^2 \end{cases}$$

varies linearly with t over $0 \le t \le T_b$

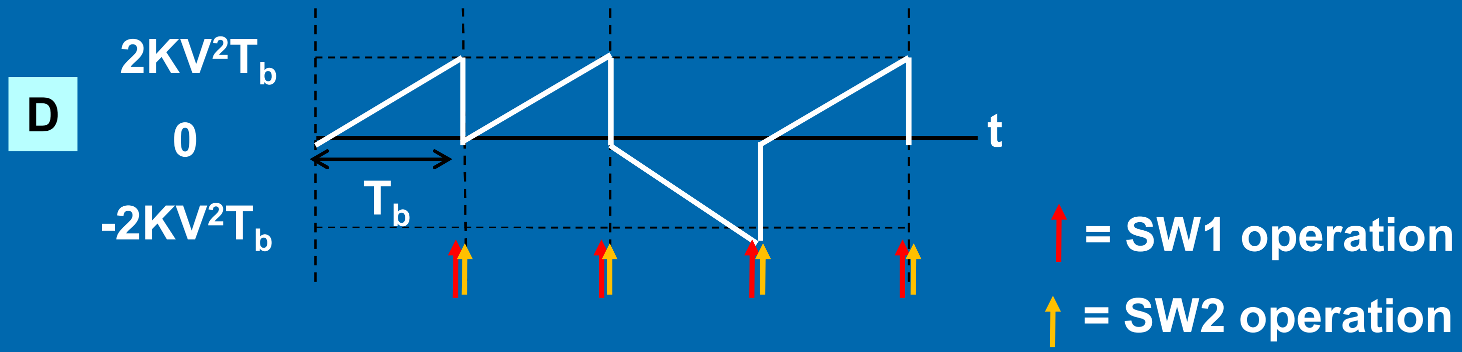


Solution

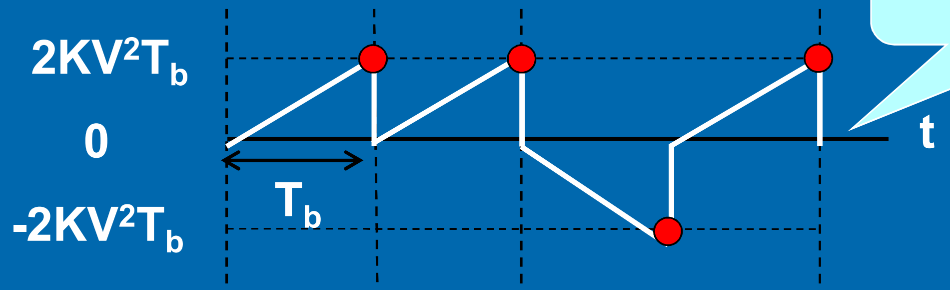


SW1 is closed at the end of each bit duration for a very short duration to sample the D waveform.

After sampling waveform at D, SW1 is opened again followed by the short closure of SW2 to discharge the capacitor so that D waveform drops to zero to initialize the start of the next bit-frame waveform of D.



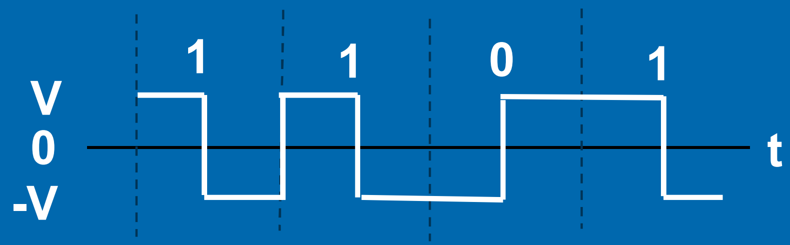
D



Threshold voltage
V_T = 0 v

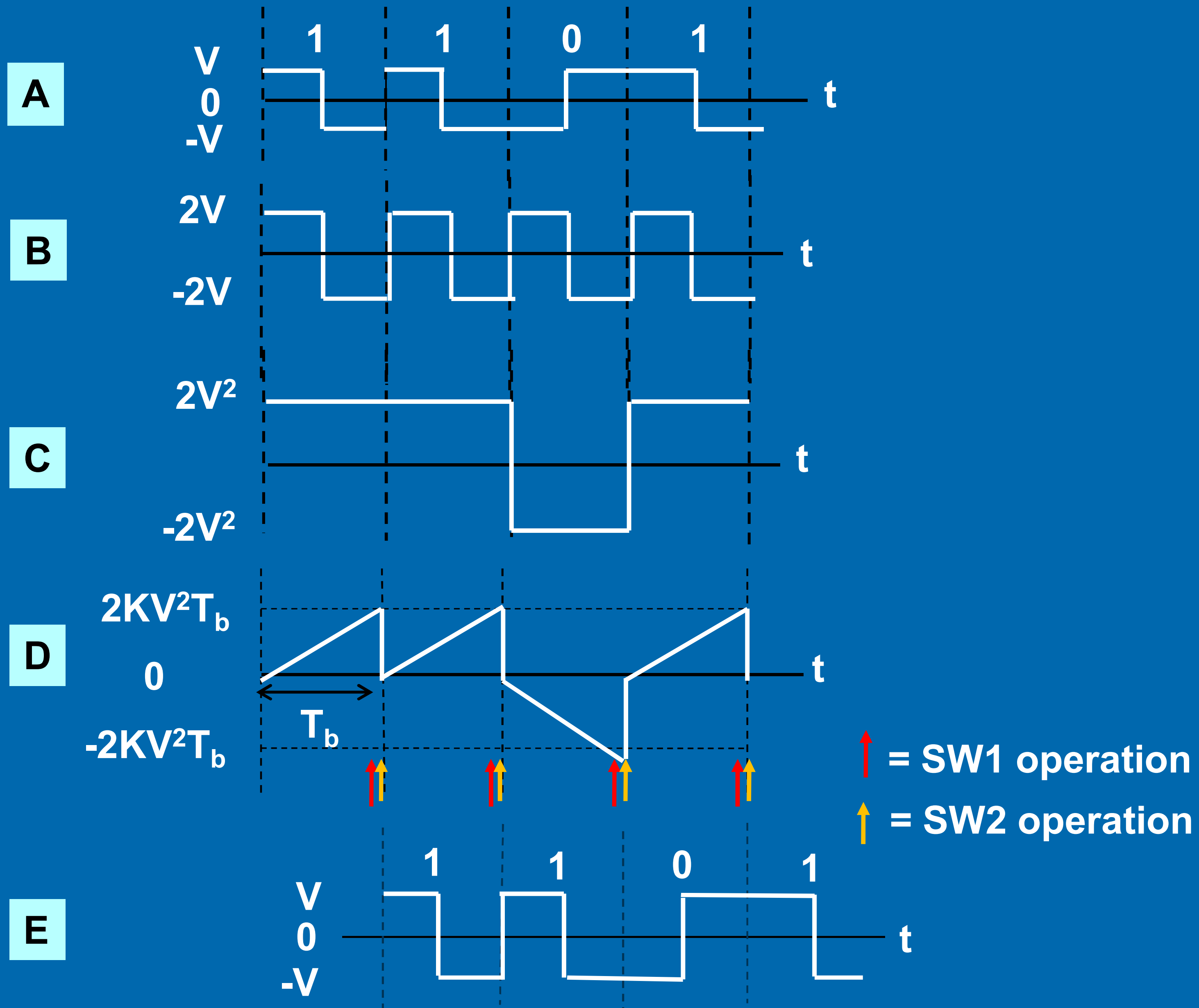
Sampled output > V_T ➡ '1'
Sampled output < V_T ➡ '0'

E



Recovered output

Waveform A to E:



End

Chapter 9