

Chapter 7

Analog to Digital conversion

Part 3 of 4



Pulse Code Modulation (PCM)

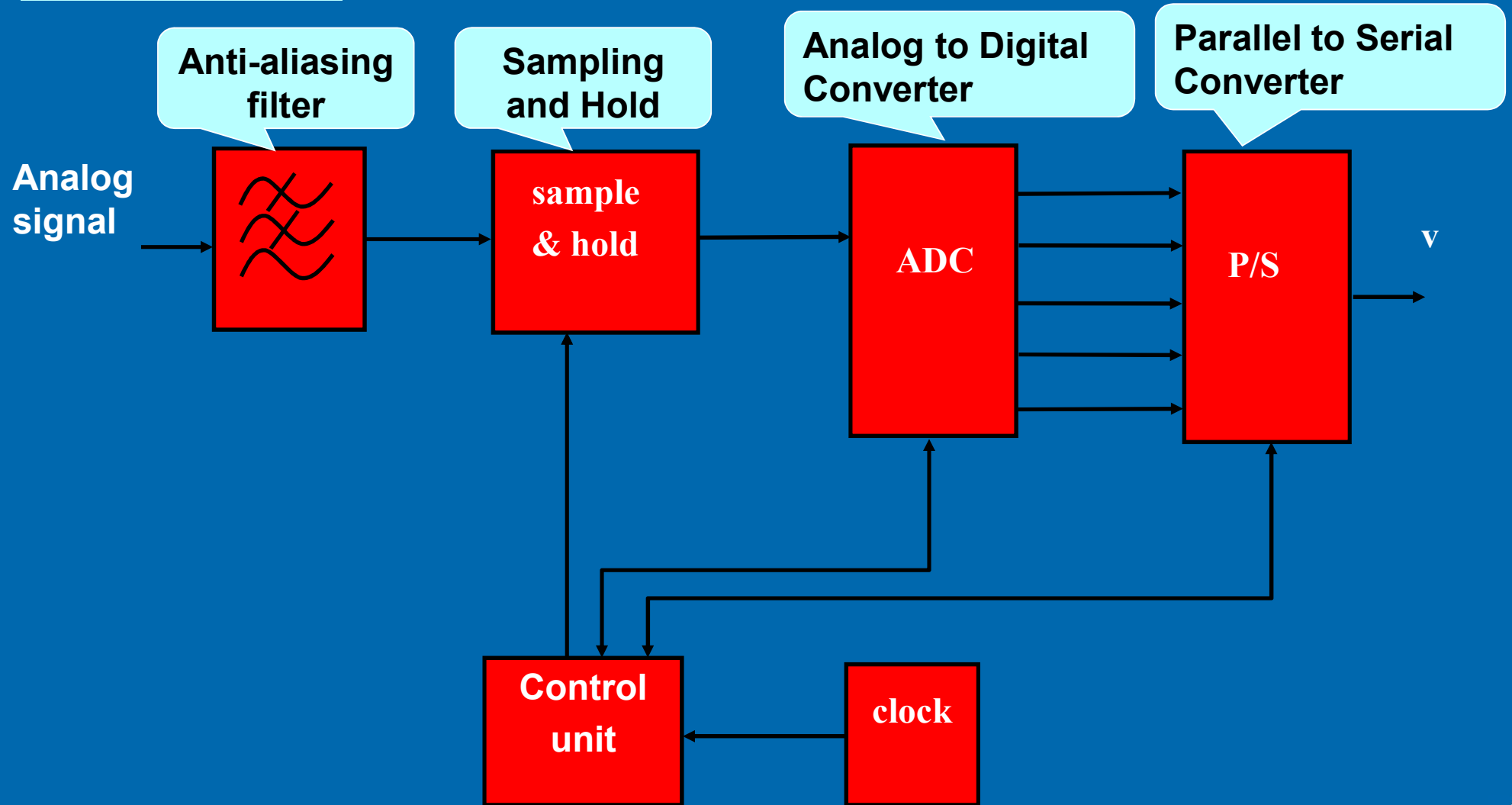
- Pulse code modulation (PCM) is the most common technique used today for **analog to digital conversion**
- PCM converts an analog signal to digital words allowing transmission of information in digital form as a serial bit stream.
- PCM applications: telephone networks, CD recording, DVD Recorder, PC audio – wav format, voice mail, etc



7.3 Pulse Code Modulation

PCM system

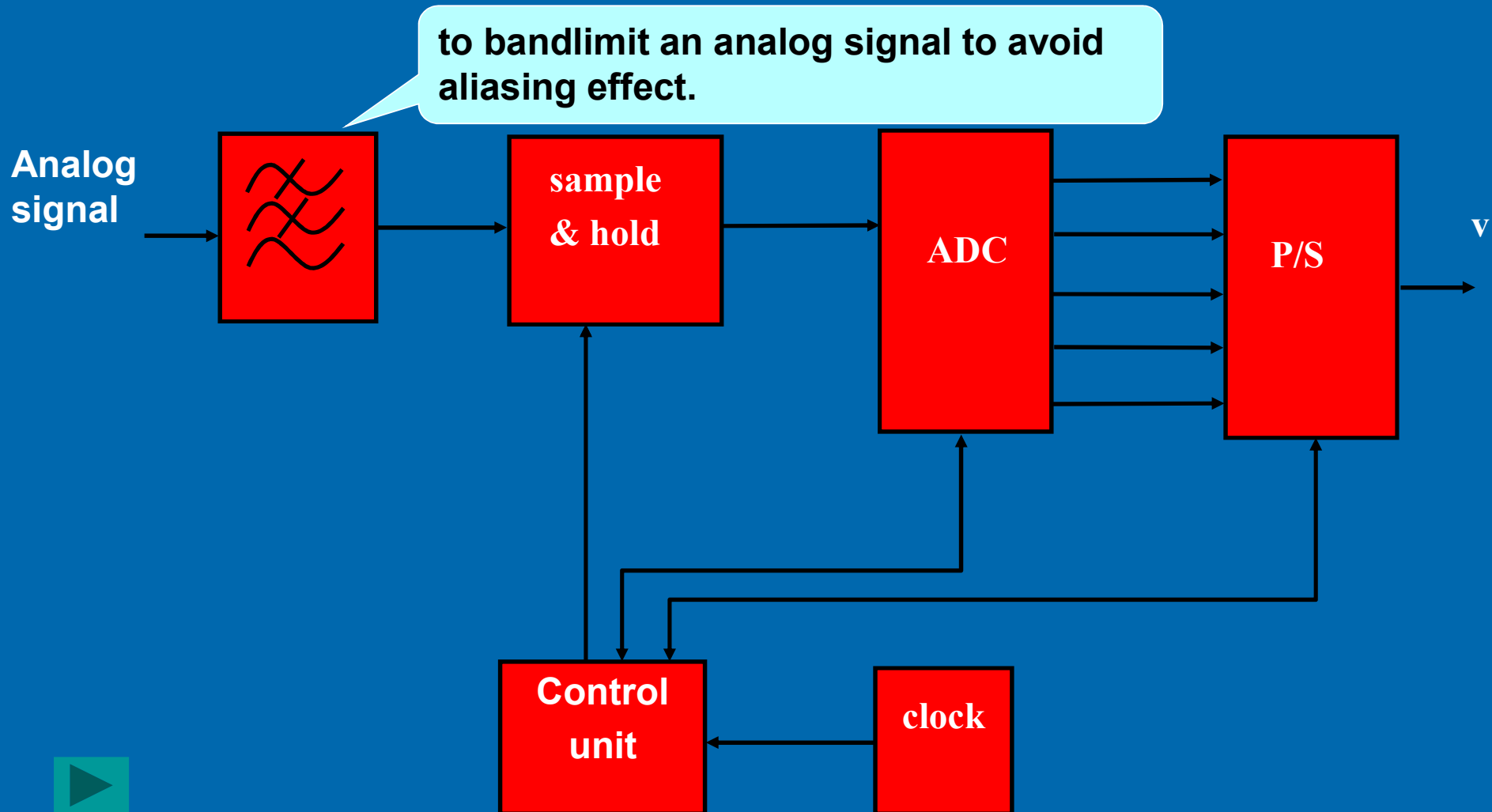
PCM Transmitter



7.3 Pulse Code Modulation

PCM system

PCM Transmitter

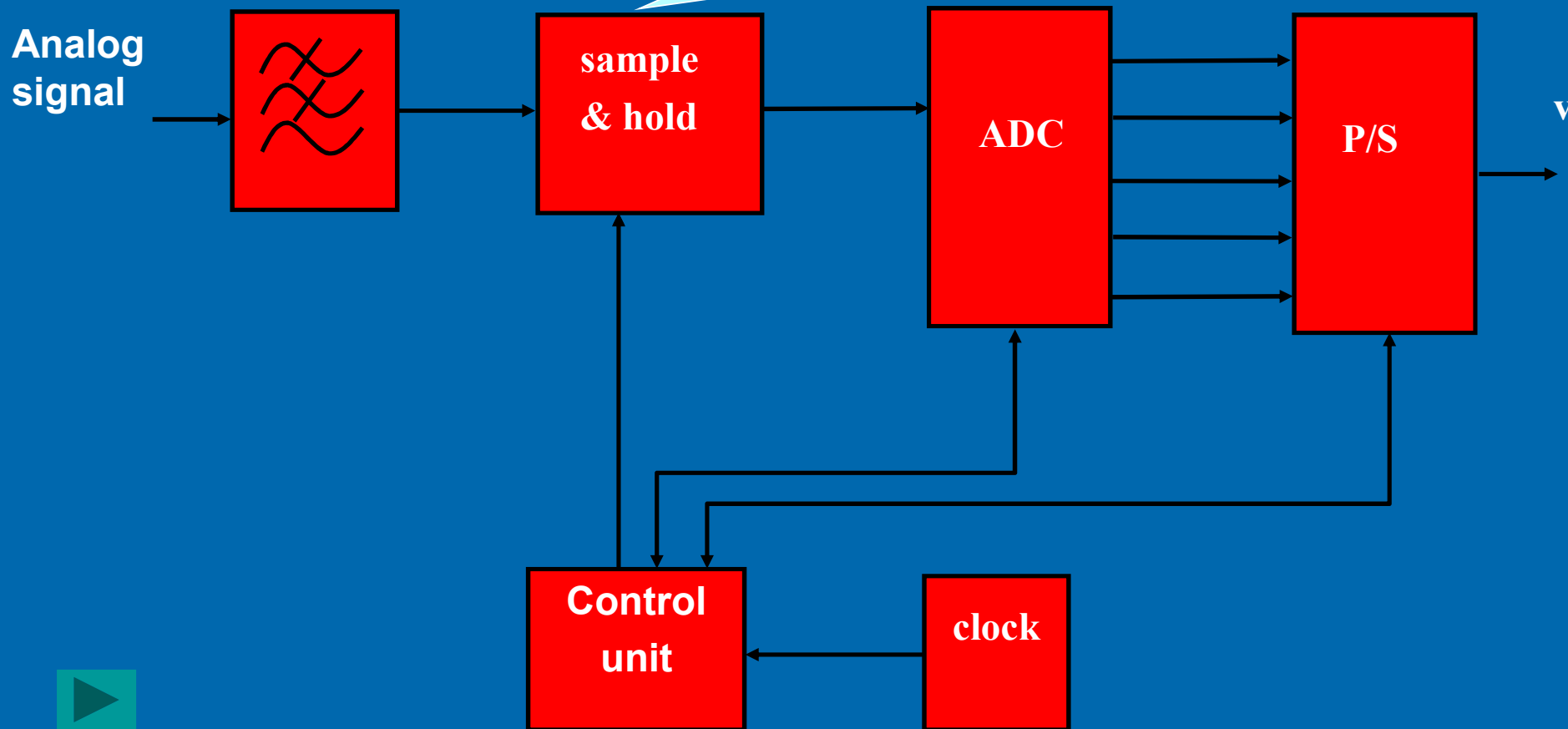


7.3 Pulse Code Modulation

PCM system

PCM Transmitter

The bandlimited analog signal is then sampled

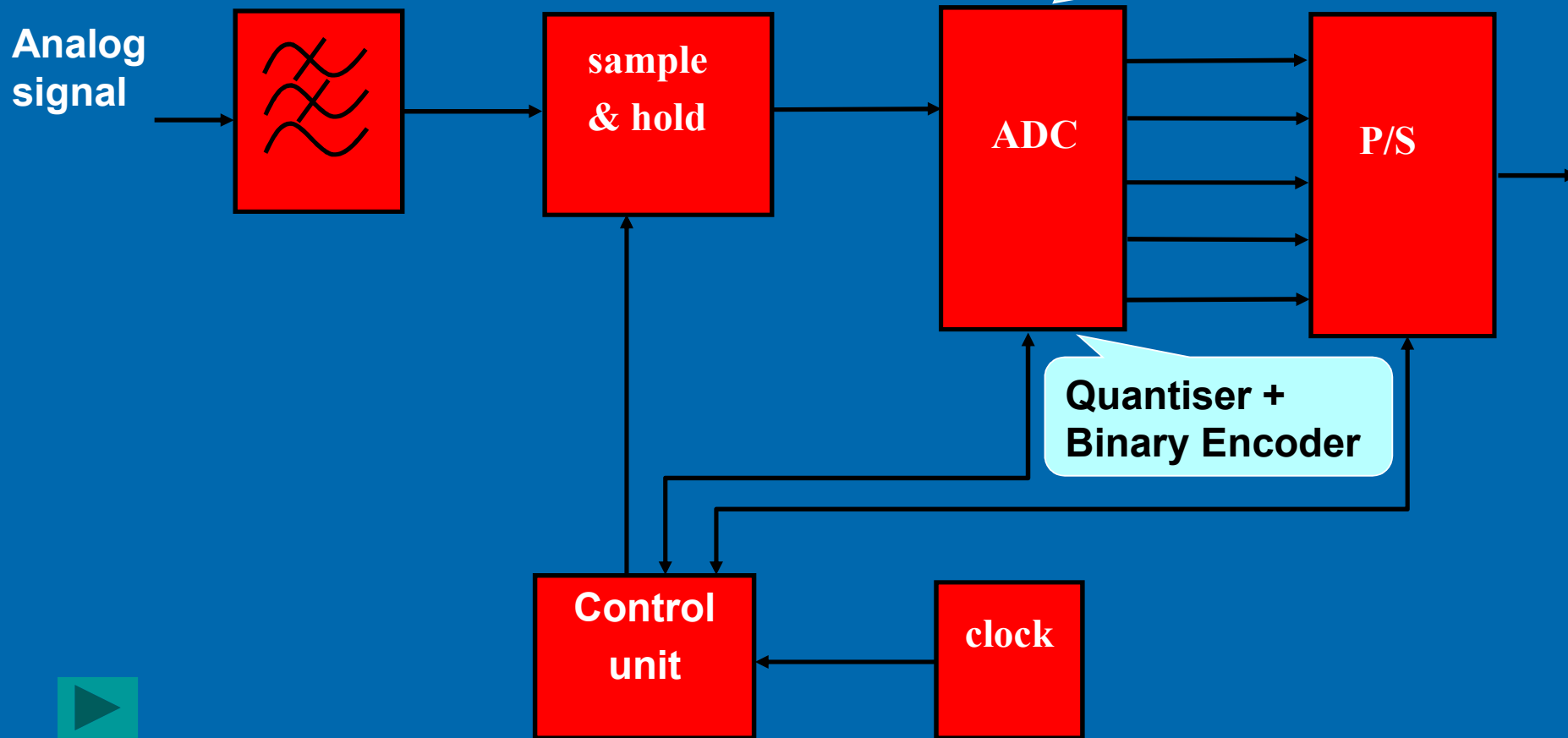


7.3 Pulse Code Modulation

PCM system

PCM Transmitter

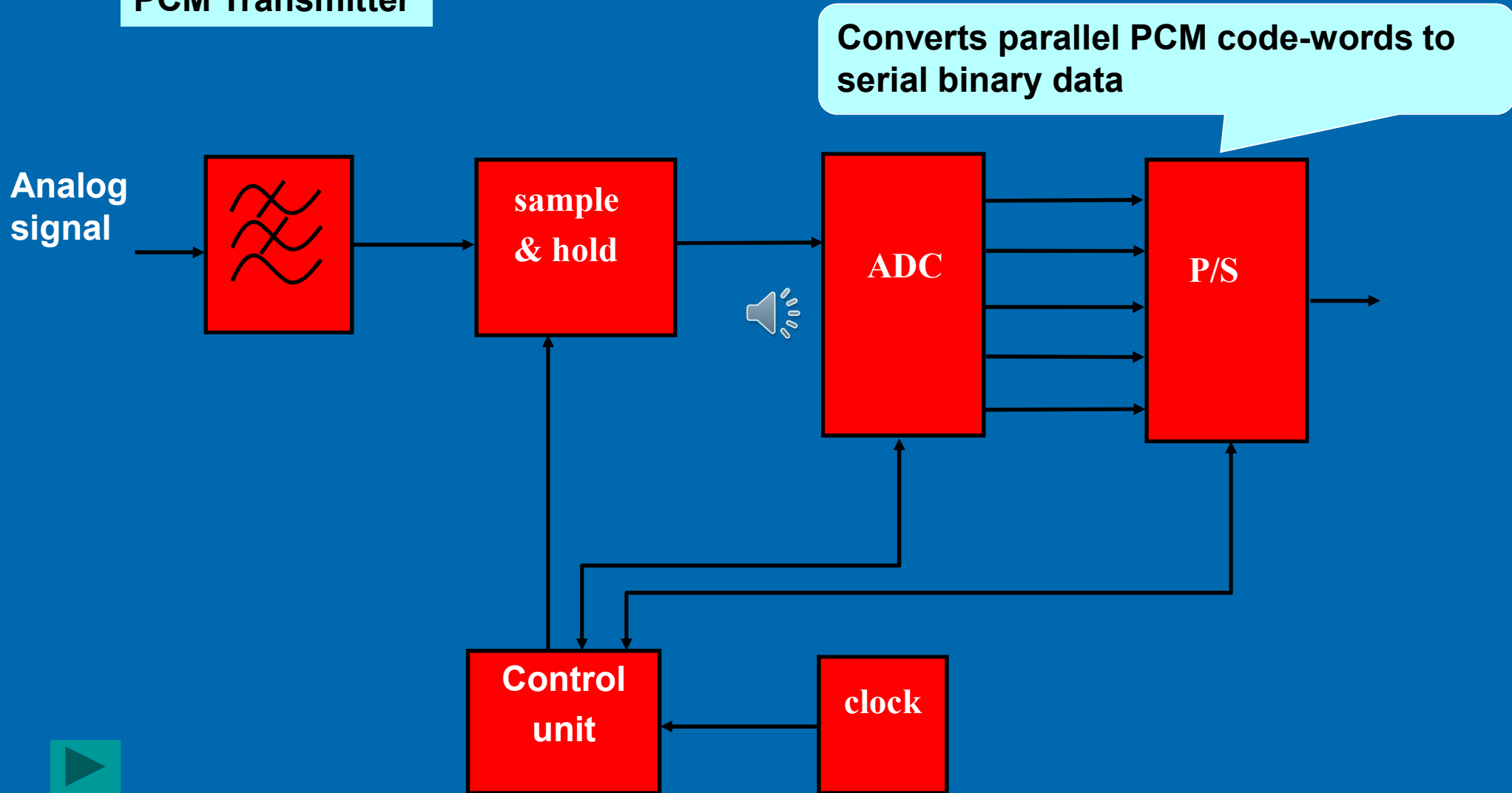
converts the analogue samples to PCM codewords



7.3 Pulse Code Modulation

PCM system

PCM Transmitter



7.3 Pulse Code Modulation

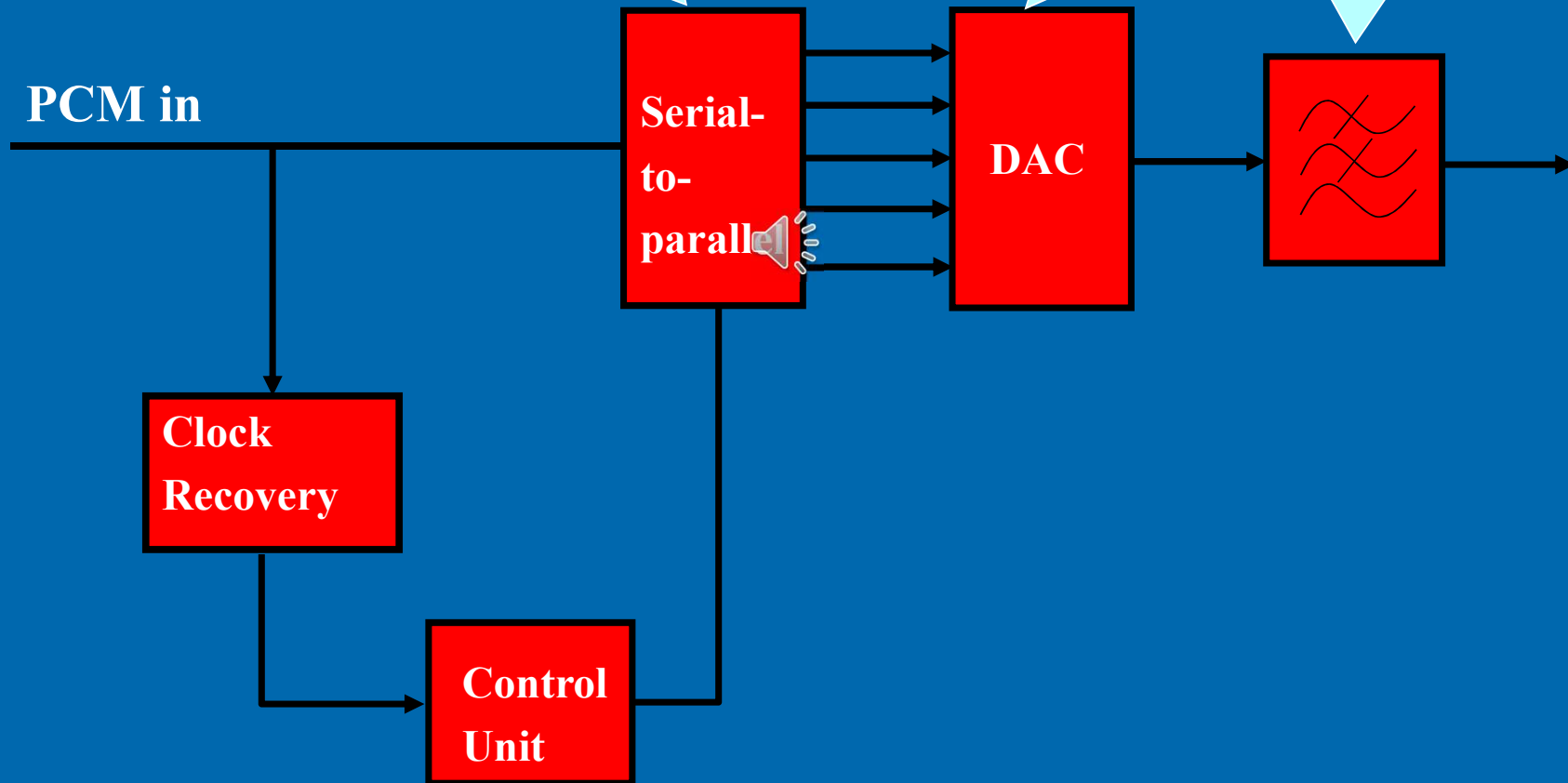
PCM system

PCM Receiver

Serial to Parallel Converter

Digital to analog converter

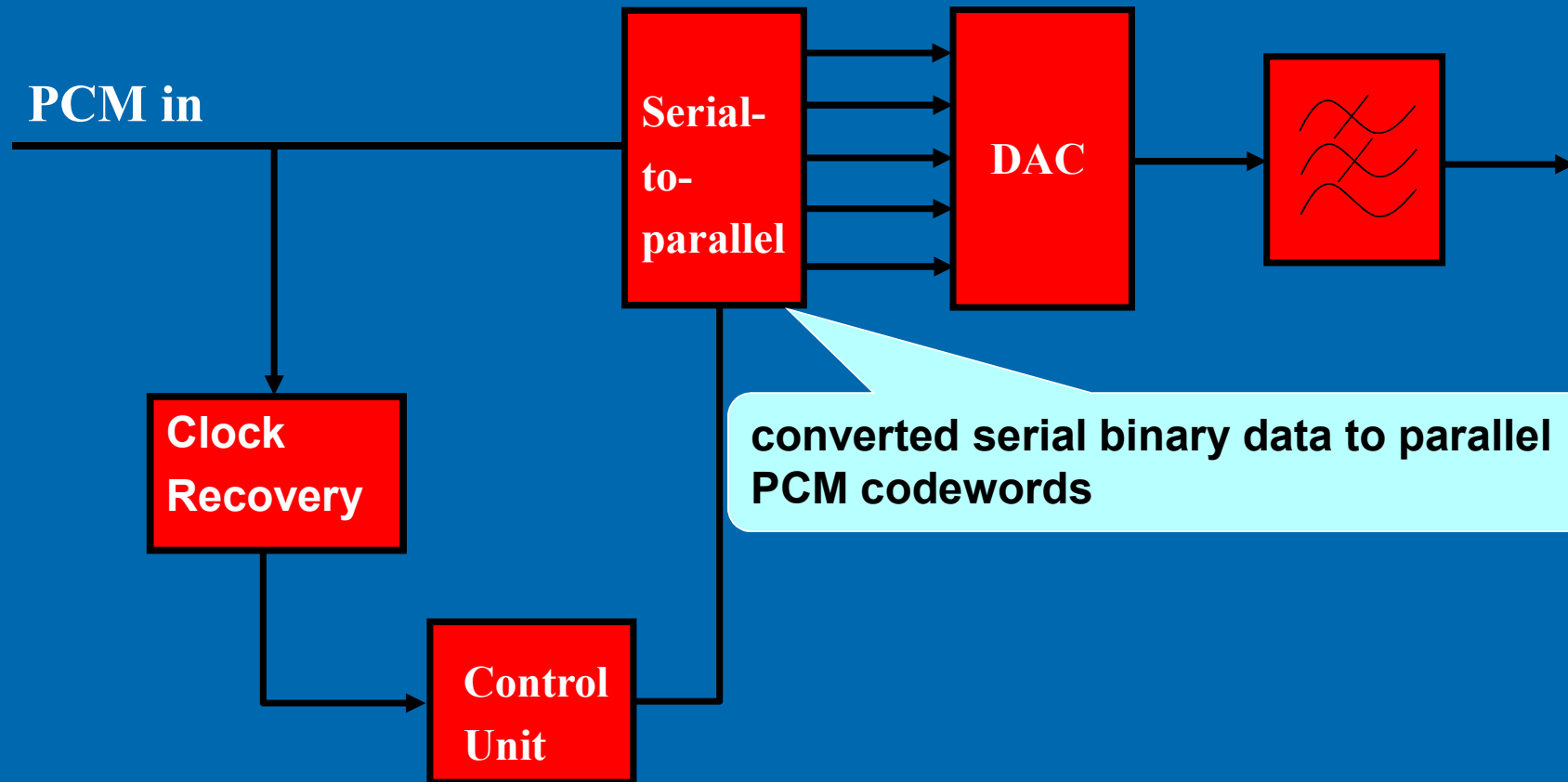
Low pass filter



7.3 Pulse Code Modulation

PCM system

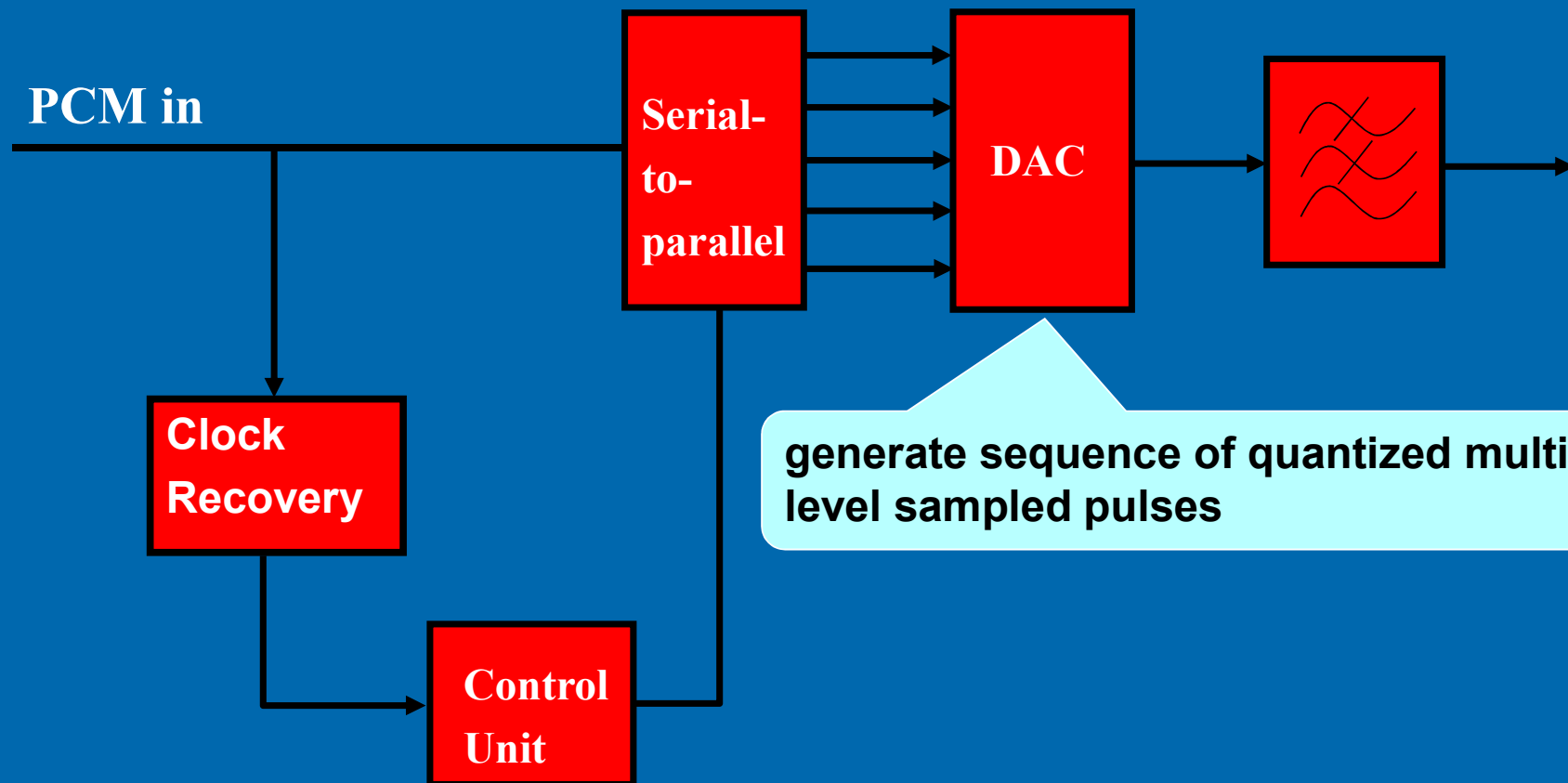
PCM Receiver



7.3 Pulse Code Modulation

PCM system

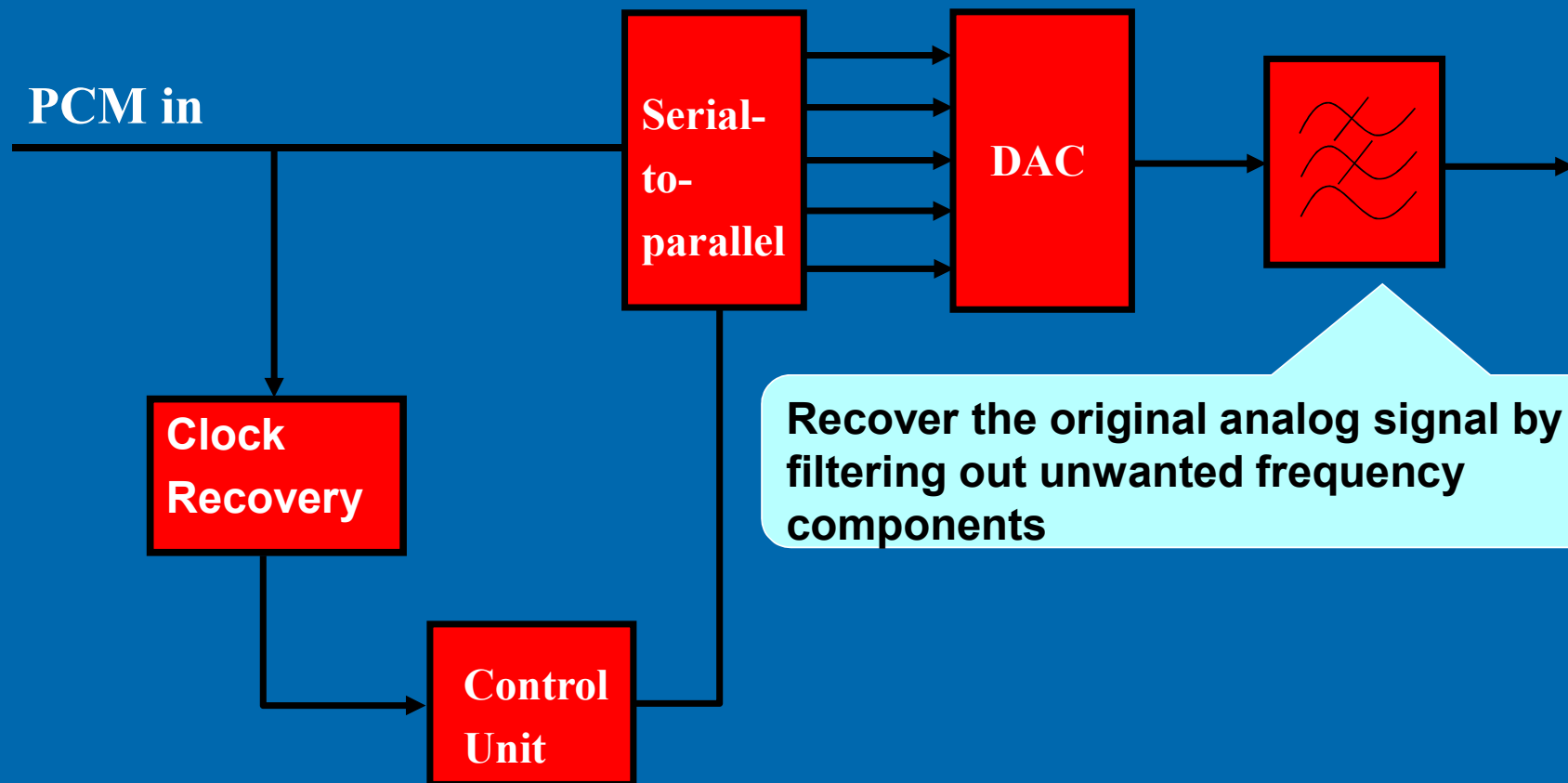
PCM Receiver



7.3 Pulse Code Modulation

PCM system

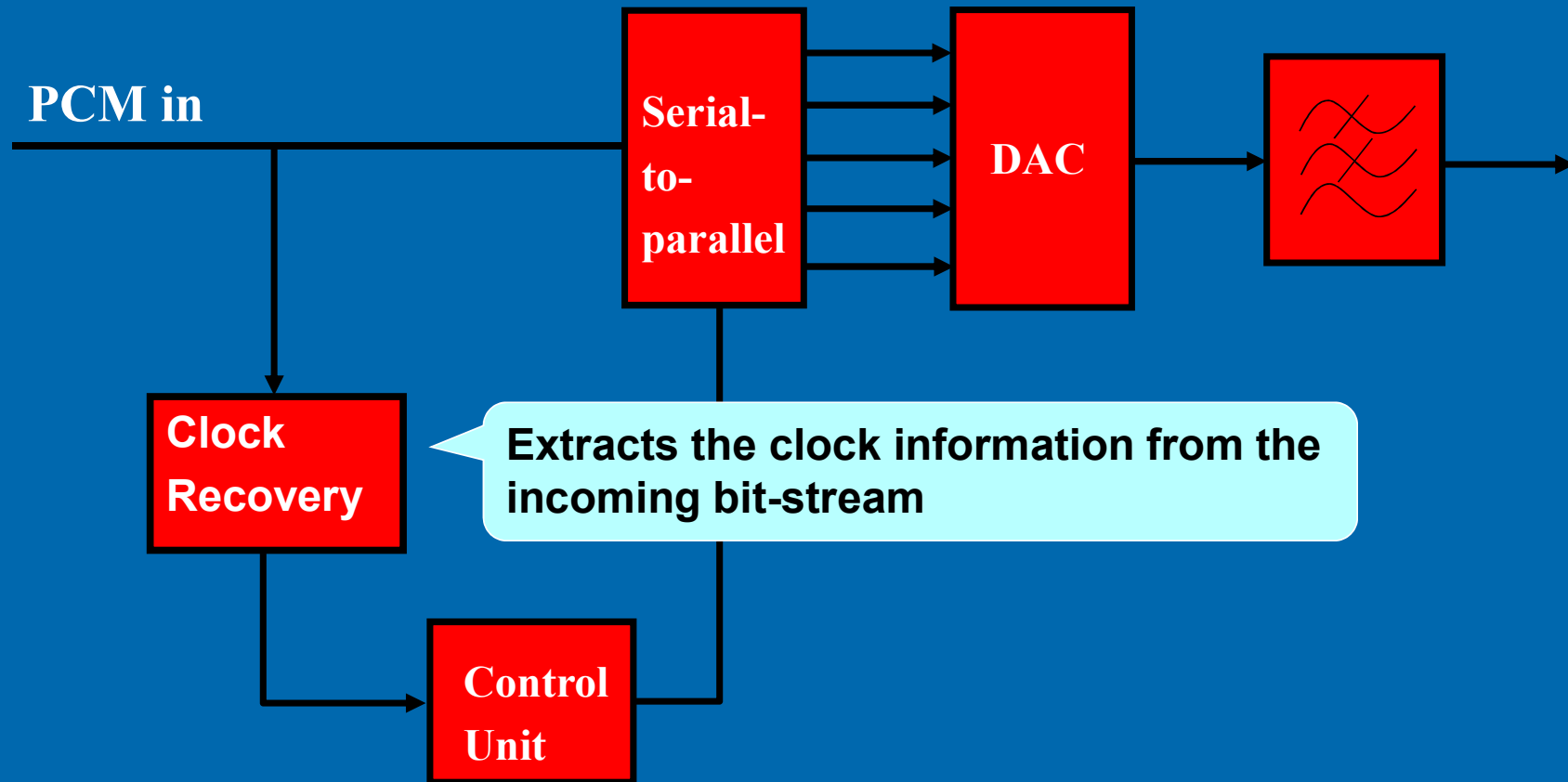
PCM Receiver



7.3 Pulse Code Modulation

PCM system

PCM Receiver



7.3 Pulse Code Modulation

Minimum PCM transmission bandwidth

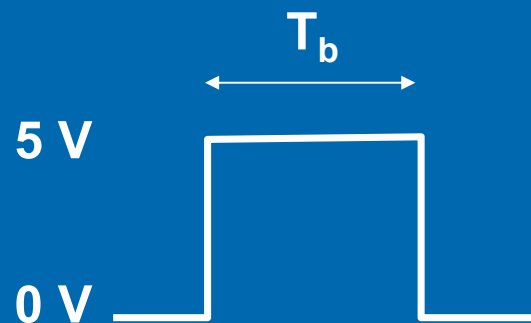
- The minimum BW occupied by a PCM bit-stream is

$$BW = R/2$$

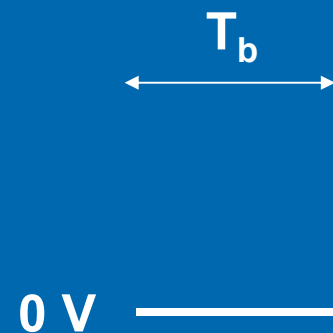
where R is the bit rate and assuming NRZ (non-return-to-zero) signalling format.

NRZ signalling format

T_b = bit duration



Binary 1



Binary 0



7.3 Pulse Code Modulation

Minimum PCM transmission bandwidth

Example 7.6 – Determine the bandwidth required for transmitting a 4 kHz audio signal using 8-bit PCM.

Solution

Sampling frequency, $f_s = 2 \times 4 \text{ kHz} = 8 \text{ kHz}$.

Bit rate = $f_s \times \text{no. of bits/sample} = 8 \text{ kHz} \times 8 \text{ bits} = 64 \text{ kbits/s}$

Assuming NRZ format,

$\text{BW} = \text{bit rate} / 2 = 32 \text{ kHz}$

exceeds the bandwidth of a telephone voice channel (3.3 kHz).



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

- When a large number of PCM signals are to be transmitted over a common channel, time division multiplexing (TDM) of these PCM signals is required.

Pulse Amplitude Modulation (PAM)-TDM system

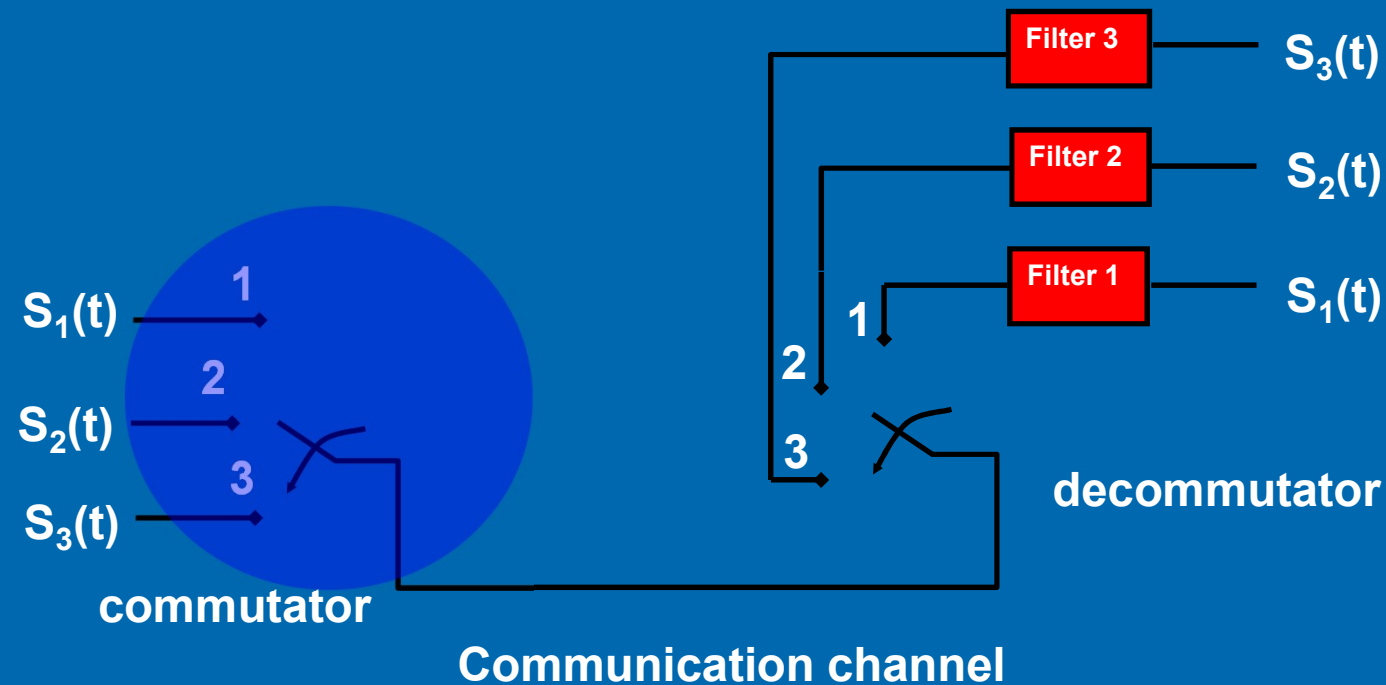
- TDM is the time interleaving of samples from several sources so that the information from these sources can be transmitted serially over a single communication channel.
- A technique used to share a communication channel among multiple signals



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

- By sampling and TDM techniques, three signals can be transmitted over a single communication channel.



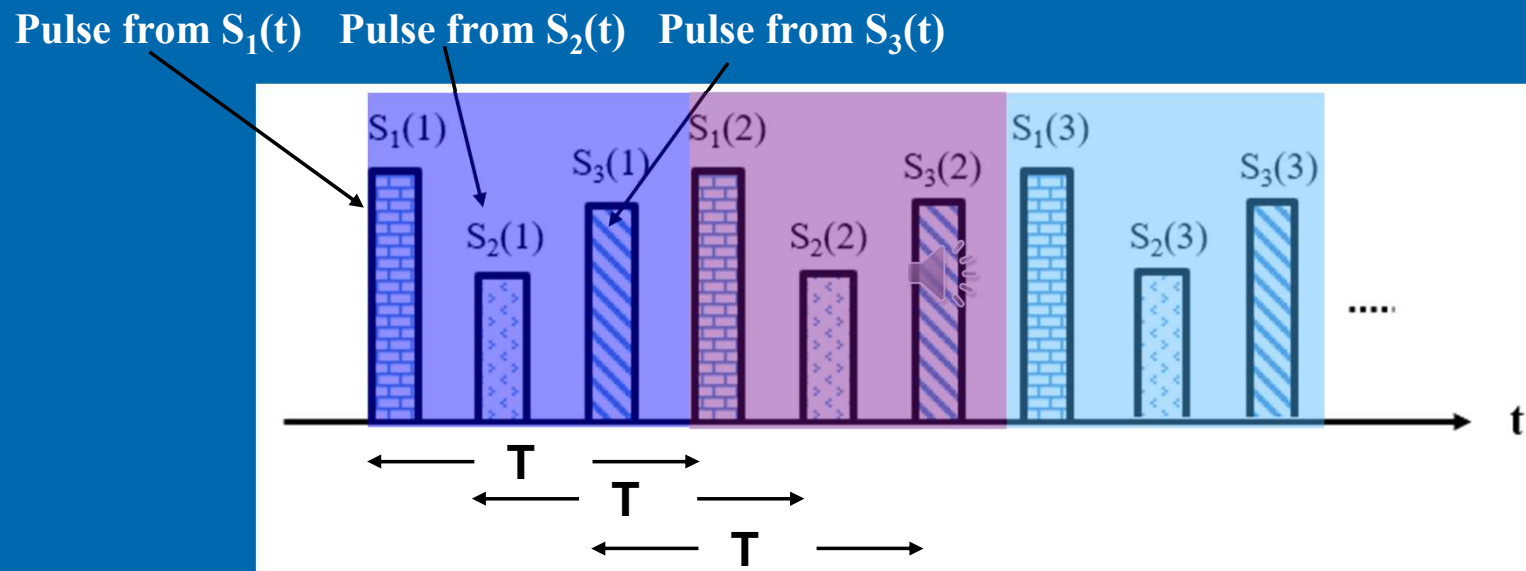
Time-division multiplexing of three signals



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

Time interlacing of three baseband signals



T = sampling interval = time taken to go through one cycle

$1/T$ = sampling frequency = commutator speed



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

- The commutator performs both the sampling and multiplexing.
- The commutator must operate at rate that satisfies the sampling theorem for all signals.

The signal with highest frequency determines the commutator speed.

- Suppose the maximum frequencies for the three input signals are $f_1 = 4$ kHz, $f_2 = 12$ kHz, and $f_3 = 4$ kHz. **Hence $f_{s1} = 8$ kHz, $f_{s2} = 24$ kHz, and $f_{s3} = 8$ kHz.**
- The commutator speed must be 24 kHz.



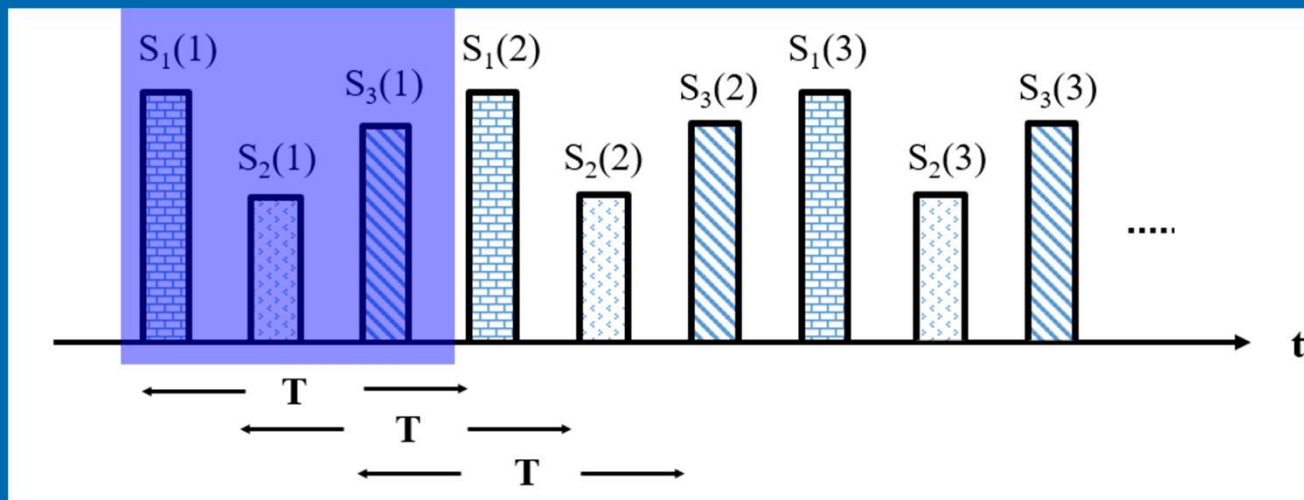
7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

Hence, $T = 1/(24 \text{ kHz}) = 41.67 \mu\text{s}$

The gross channel pulse rate = 3 pulses/per cycle x 24 k cycles/s = 72 k pulses/s

3 pulses per cycle



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

- In general, the channel transmission pulse rate is given by:

Channel transmission pulse rate = number of signal inputs x commutator speed.



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

- Multiplexing of many signals will require relatively high pulse rate/bitrate transmission systems.
- To minimize the transmission bandwidth required, an optimum commutator structure is necessary.

Generates the lowest pulse rate/bitrate

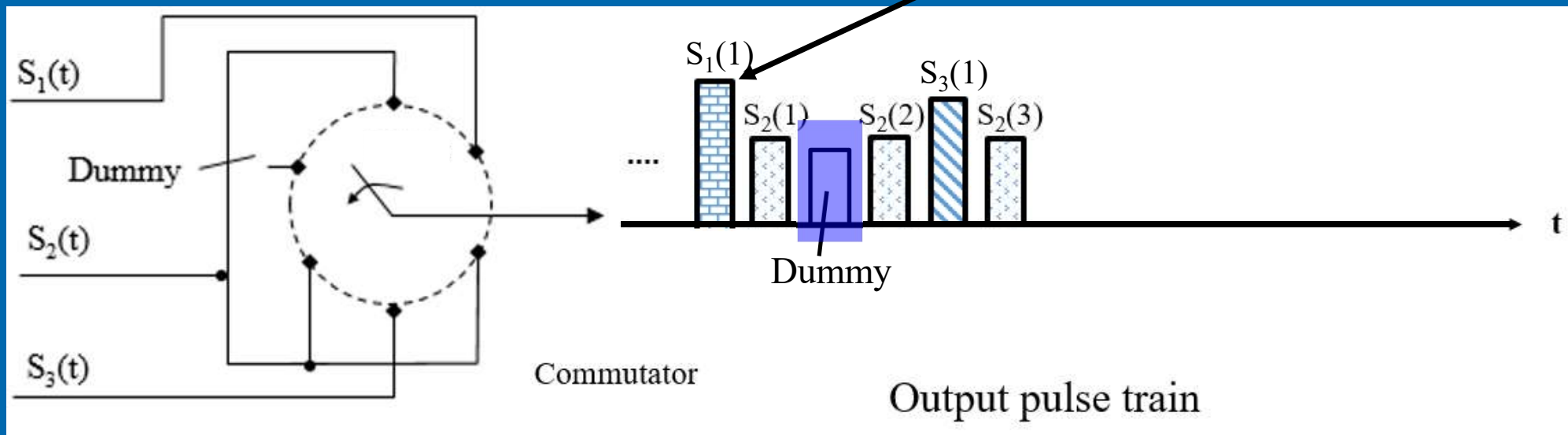


7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

An optimum Commutator structure for minimum channel pulse rate

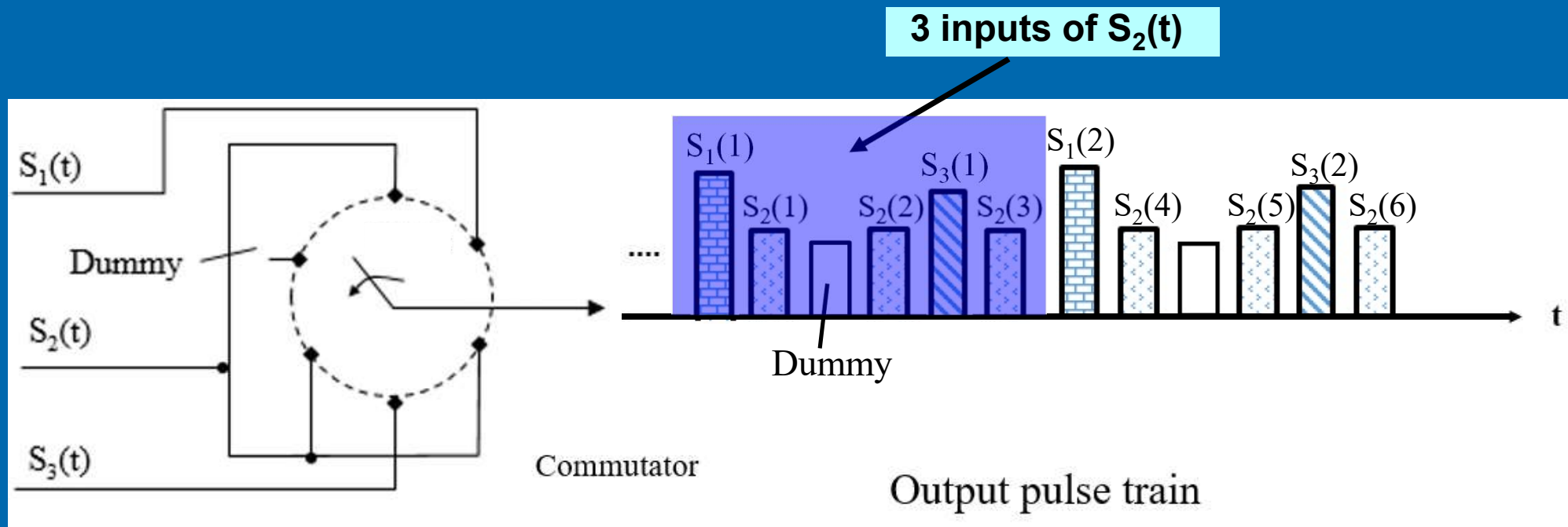
Insertion of signals $S_1(t)$, $S_3(t)$, and a dummy between inputs of $S_2(t)$.



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

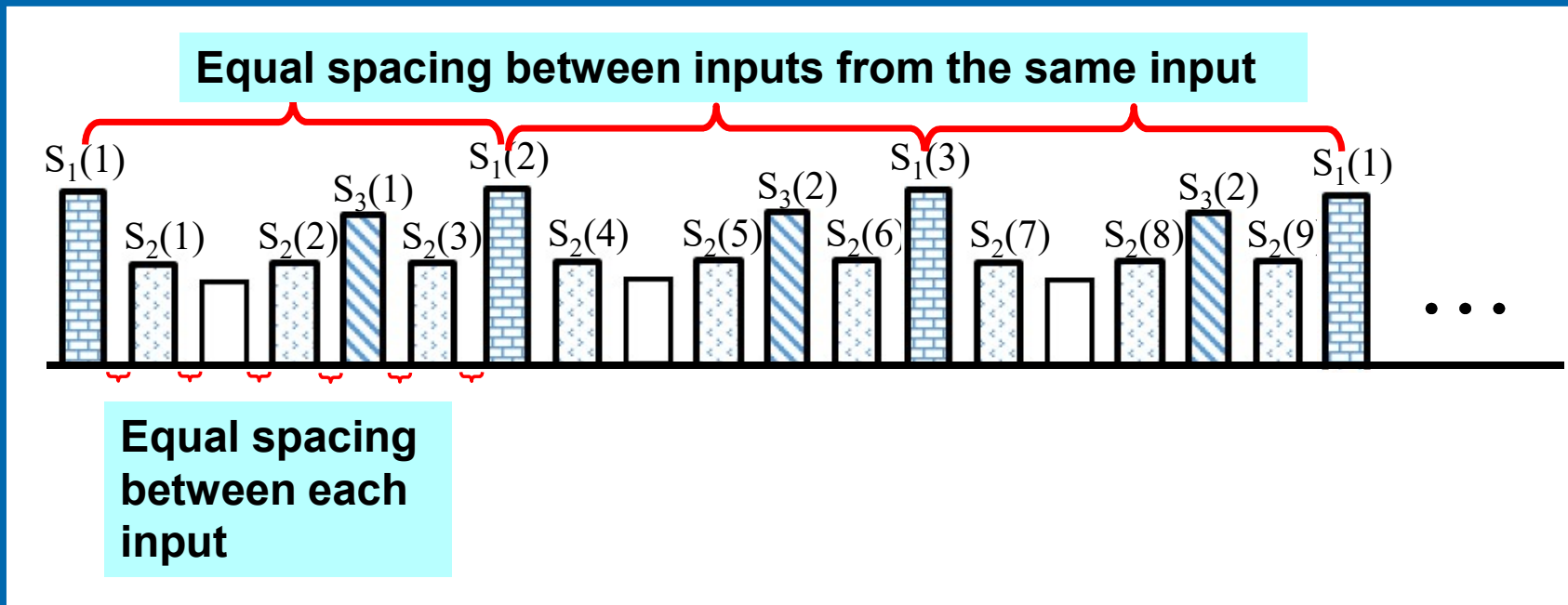
An optimum Commutator structure for minimum channel pulse rate



7.3 Pulse Code Modulation

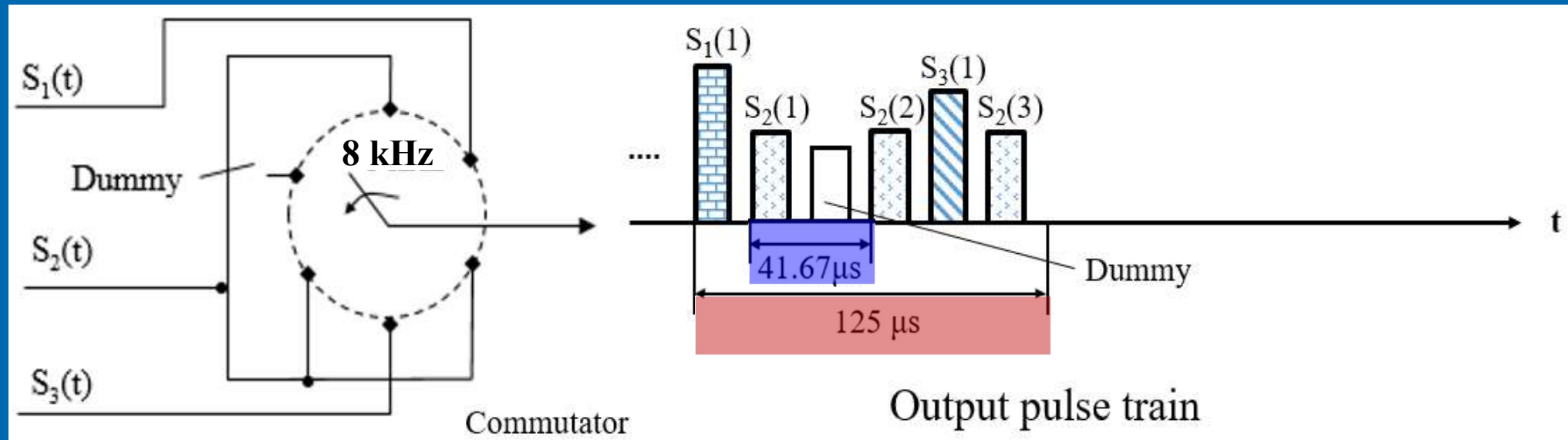
Time Division Multiplexing of PCM signals

An optimum Commutator structure for minimum channel pulse rate



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals



The duration between two samples of $s_2 = 1 / 24 \text{ kHz} = 41.67 \mu s$

One pulse duration = $41.67 / 2 \text{ pulses } \mu s = 20.835 \mu s$

One cycle duration = $(20.835 \mu s) \times (5+1 \text{ pulses/cycle}) \mu s = 125 \mu s$

Commutator speed = $1 / \text{One cycle duration} = 8 \text{ kHz}$



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

For signal $S_2(t)$

$cs = 8 \text{ kHz} \longrightarrow 1 \text{ second} \longrightarrow 8000 \text{ cycles}$

$1 \text{ cycle} \longrightarrow 3 \text{ samples}$

$1 \text{ second} \longrightarrow 8000 \times 3 = 24000 \text{ samples}$

Hence, sampling freq $f_s = 24000 \text{ samples/sec} = 24 \text{ kHz}$

Sampling frequency for a signal = commutator speed x no. of inputs for that signal.

$$f_{s2} = 8 \text{ kHz} \times 3 = 24 \text{ kHz}$$

and, $f_{s1} = f_{s3} = 8 \text{ kHz} \times 1 = 8 \text{ kHz}$



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

- Channel pulse rate = $8 \text{ kHz} \times (5+1) \text{ pulses/cycles} = 48 \text{ k pulses/s}$.
- **Optimum vs basic configuration**
 - Output : 48 kpulses/s vs 72 kpulses/s >>> smaller transmission bandwidth
 - Commutator speed : 8 kHz vs 24 kHz >>> power savings



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

- Optimize a commutator structure with minimise channel pulse rate:
 - Choose a commutator speed (CS)/rotation rate that equals to the sampling rate of the signal with minimum bandwidth.
 - For signals with larger bandwidth, allocate multiple-equally-spaced inputs in one cycle of commutator rotation determined by sampling theorem.

The sampling rates of all the signals will be multiples of CS.

Note:

Decreasing the commutator speed will result in a more complex commutator structure.

Sometimes, it is necessary to trade-off between the channel pulse rate and the complexity of commutator structure.



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

Example 7.7

A PAM-TDM system is used to multiplex four signals:

$$m_1(t) = \cos w_0 t,$$

$$m_2(t) = 0.5 \cos w_0 t,$$

$$m_3(t) = 2 \cos 2w_0 t,$$

$$m_4(t) = \cos 4w_0 t, \text{ where } w_0 = 2000\pi \text{ radians/s.}$$

- a) If each signal is sampled at Nyquist rate, sketch the optimum commutator structure assuming uniform sampling.
- b) What is the commutator speed?



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

Solution

$$\omega_0 = 2000\pi = 2\pi f_0 \gg f_0 = 1000 \text{ Hz}$$

$$f_{m1} = 1000 \text{ Hz}$$

$$f_{m2} = 1000 \text{ Hz}$$

$$f_{m3} = 2000 \text{ Hz}$$

$$f_{m4} = 4000 \text{ Hz}$$



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

Solution

a)

<u>Signal</u>	<u>f_m (kHz)</u>	<u>f_s (kHz)</u>
m_1	1	2
m_2	1	2
m_3	2	4
m_4	4	8



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

Solution

a)

$$\text{No of inputs} = f_s / cs$$

<u>Signal</u>	<u>f_m (kHz)</u>	<u>f_s (kHz)</u>	<u>No. of inputs</u>
m_1	1	2	$2/2 = 1$
m_2	1	2	$2/2 = 1$
m_3	2	4	$4/2 = 2$
m_4	4	8	$8/2 = 4$

Total no of inputs = 8

Choose the lowest f_s in this column to be the cs
i.e. **cs = 2 kHz**



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

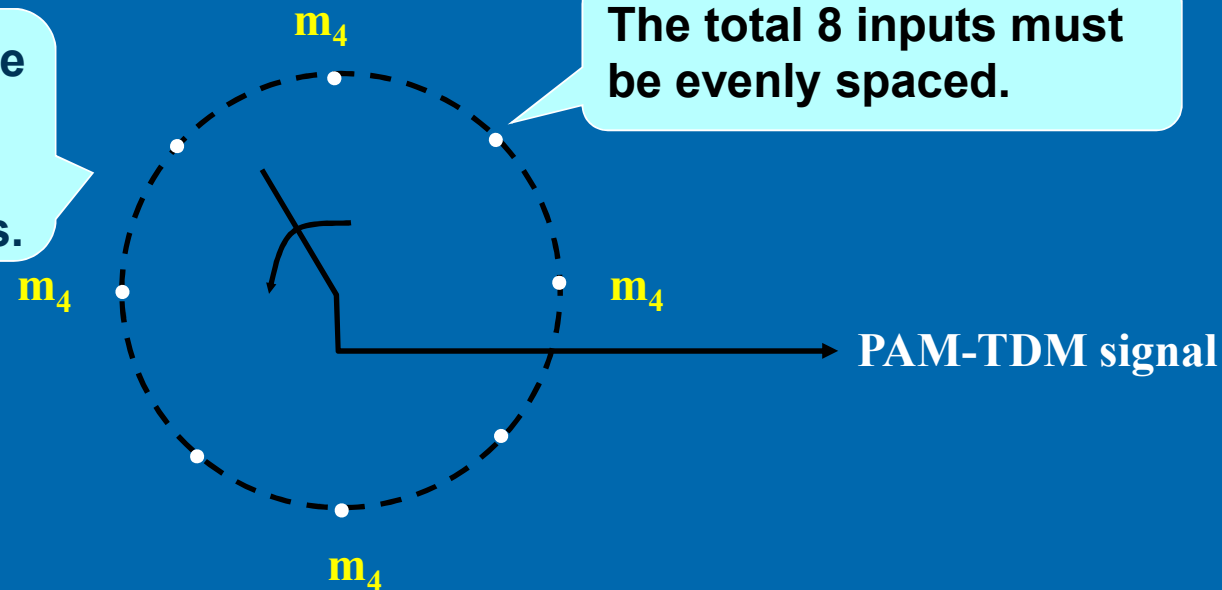
Solution

a)

Total no of inputs = 8

All 4 m_4 inputs should be evenly spaced which divides the commutator structure into 4 sections.

The total 8 inputs must be evenly spaced.



- Note that to get uniform sampling:
 - all the signal inputs should be evenly spaced.
 - for the same signal all inputs must be evenly.
 - total number of inputs is a multiple of 4.

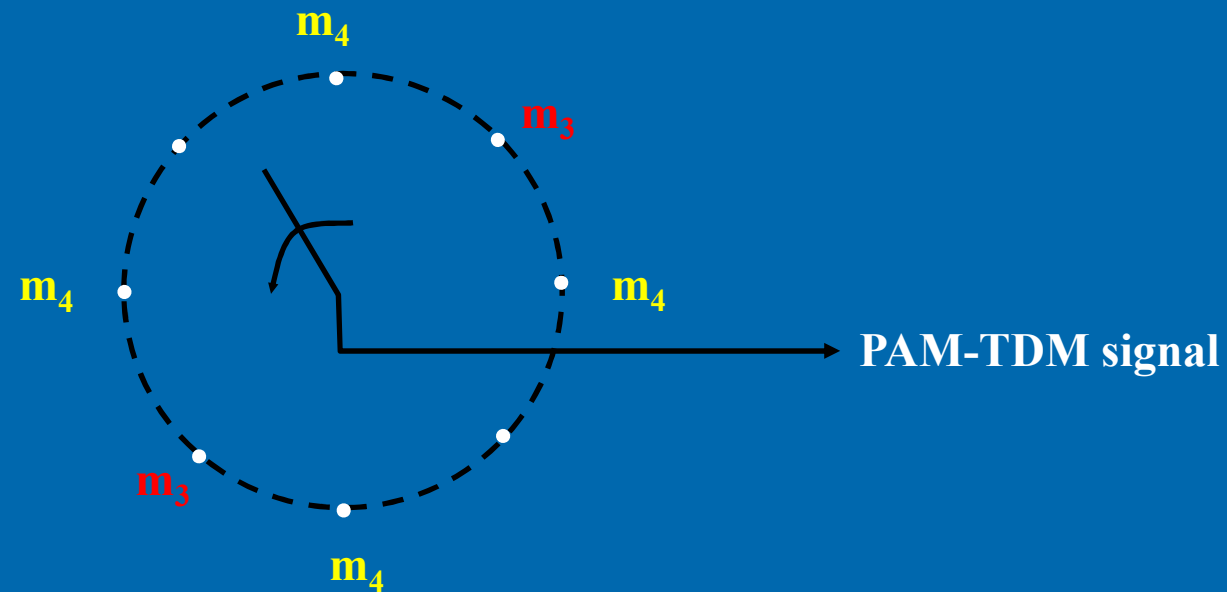


7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

Solution

a)

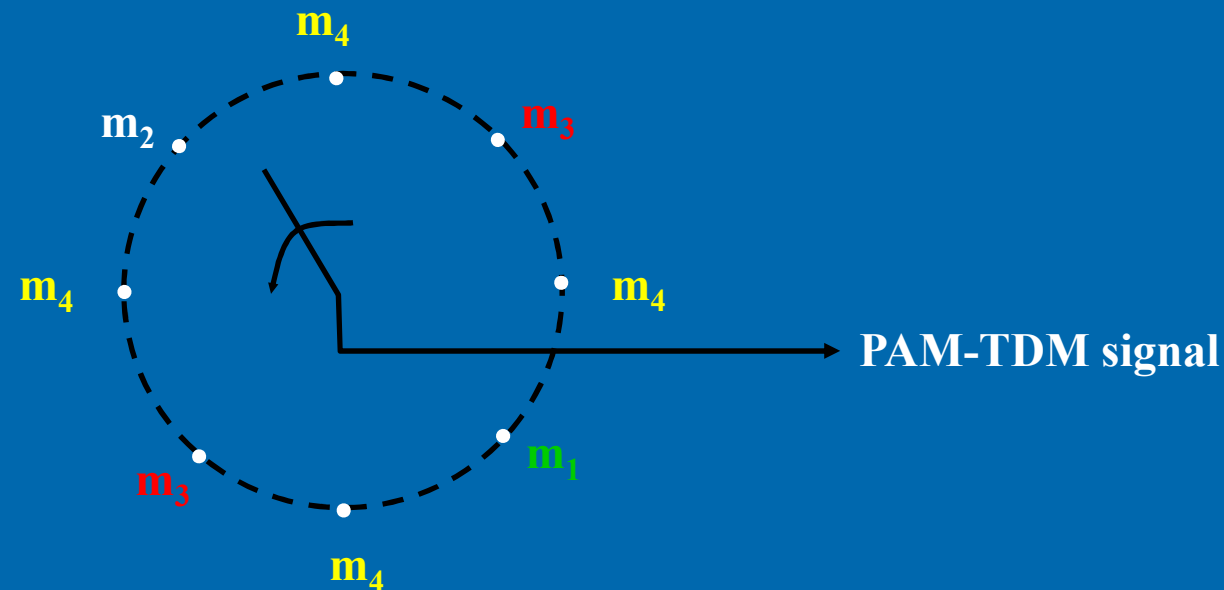


7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

Solution

a)



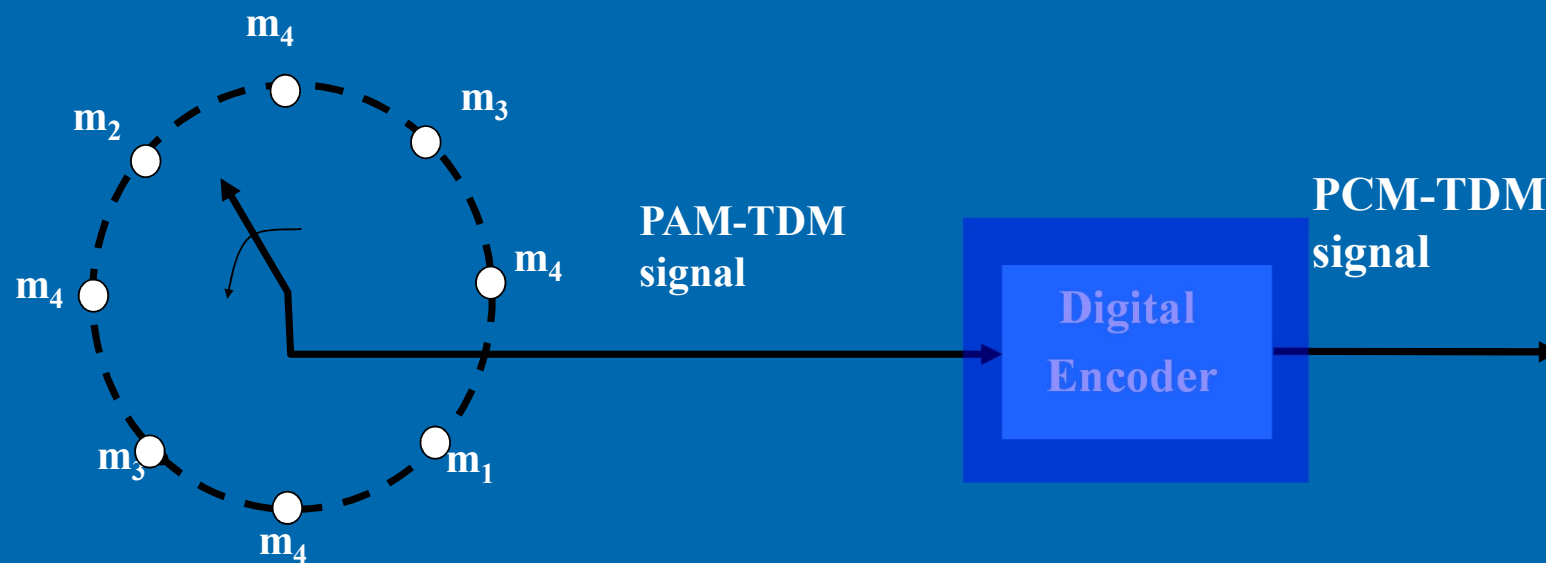
b) commutator speed = 2 k cycles/s



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

- TDM of PCM signals follows the same concepts as just an additional digital encoder is needed



Commutator structure for PCM-TDM signal



7.3 Pulse Code Modulation

Time Division Multiplexing of PCM signals

- The gross channel output bit rate is:

Gross channel output bit rate, R = commutator speed x no. of inputs x no. of bits per sample



7.3 Pulse Code Modulation

Example 7.8

A 4 kHz signal is transmitted through a PCM system. The system employ a 8-bit quantiser which has a step size of 5 mV.

- a) Calculate the quantisation noise power and the maximum output signal-to-quantisation noise ratio (in dB) of the system.
- b) If five similar systems are time-multiplexed, what is the minimum transmission bandwidth required by the multiplexed system?



7.3 Pulse Code Modulation

Solution

$$\begin{aligned}\text{a) Quantisation noise power} = N_q &= \frac{q^2}{12} = \frac{(5 \times 10^{-3})^2}{12} \\ &= 2.08 \times 10^{-6} \text{ W}\end{aligned}$$

$$\text{Signal-to-quantisation noise} = 1.76 + 6B = 1.76 + 6 \times 8 \text{ dB} = 49.76 \text{ dB}$$

$$\text{b) } f_m = 4 \text{ kHz, } f_s = 2f_m = 8 \text{ kHz}$$

For similar signals, $cs = f_s$

$$\begin{aligned}\text{Gross output bit rate, } R &= \text{commutator speed} \times \text{no. of inputs} \times \text{no. of bits per sample} \\ &= 8000 \times 5 \times 8 = 320 \text{ kbps}\end{aligned}$$

$$\begin{aligned}\text{Hence, minimum transmission bandwidth} &= R/2 \\ &= 320/2 = 160 \text{ kHz}\end{aligned}$$



END

CHAPTER 7

(Part 3 of 4)

