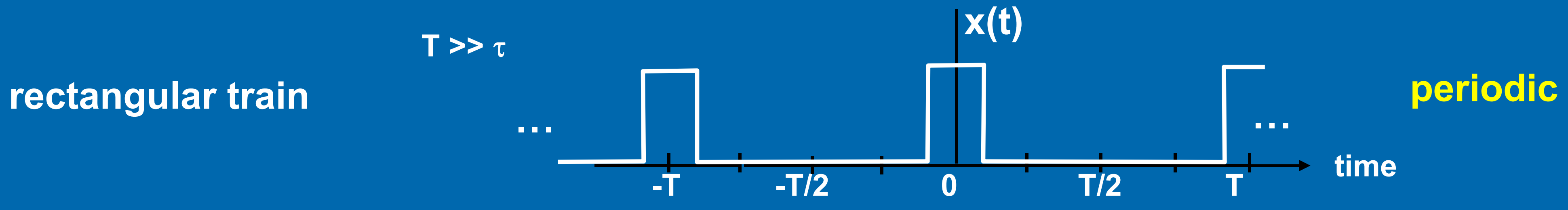
Chapter 2

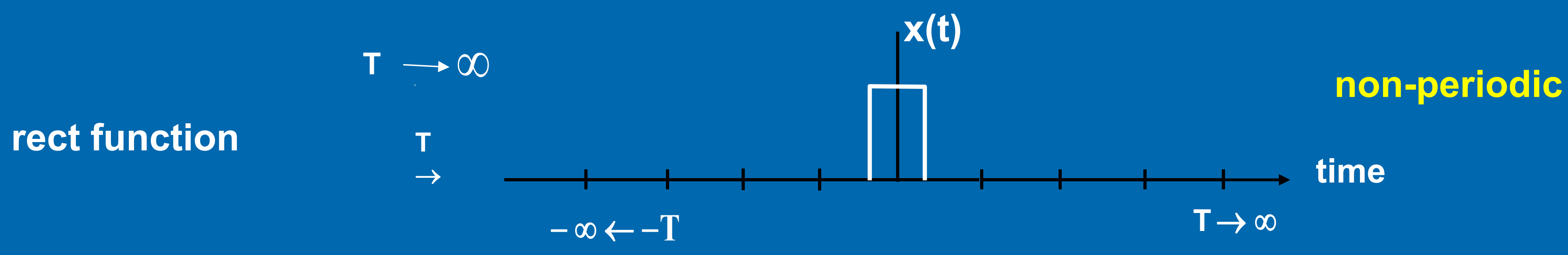
Signals and Spectra

(Part 3 of 5)

2.4 Fourier Transform

- Fourier Transform is used to obtain the frequency domain representation of the signal.
- Non-periodic signal is a limiting case of a periodic signal where T $\to \infty$





2.4 Fourier Transform

Fourier Transform of a signal x(t) is given by:

Transform Pair

$$x(t) \overset{FT}{\Longleftrightarrow} X(f)$$

continuous spectrum of signal

$|X(f)|$ **Amplitude spectrum**

$\angle X(f)$ **Phase spectrum**

Forward Transform:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

Inverse Transform:

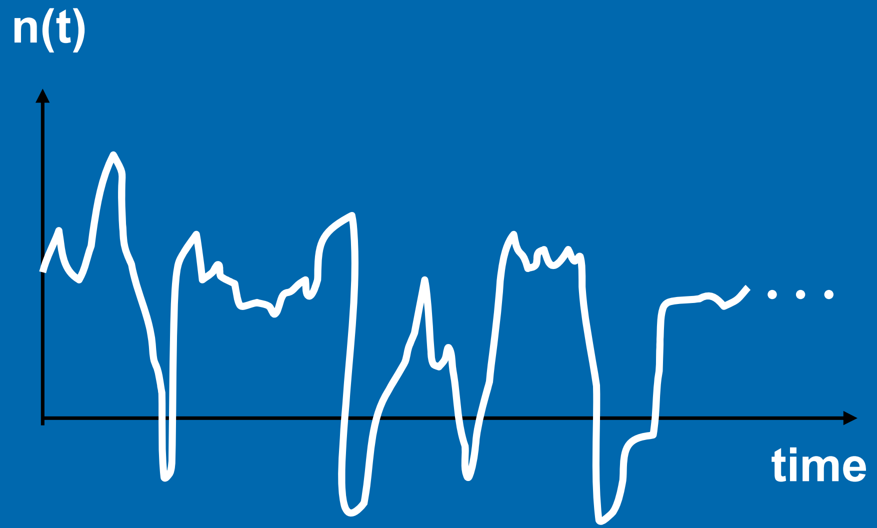
$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df$$

Not tested

2.4 Fourier Transform

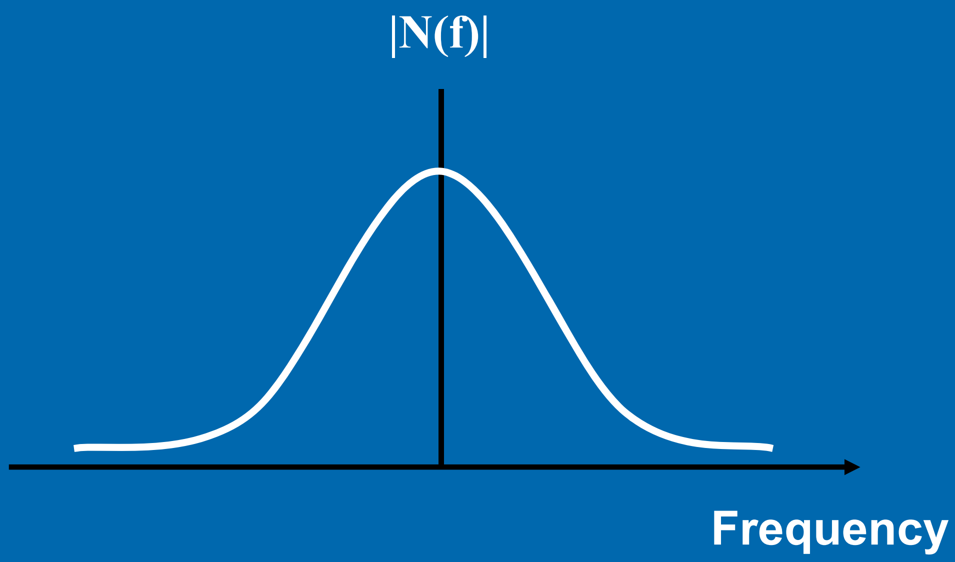
Frequency Spectrum of Non -Periodic Signals (Review)

- Example of a non-periodic signal



Time domain representation of a non-periodic signal

Continuous amplitude spectrum



Frequency domain representation of a non-periodic signal

2.4 Fourier Transform

Some Useful Properties of Fourier Transform

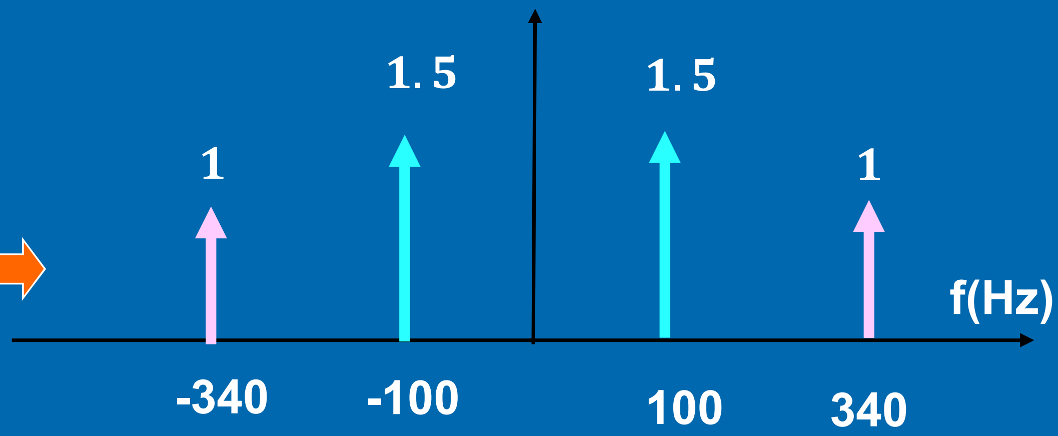
- **Addition**

If $x_1(t) \xleftrightarrow{FT} X_1(f)$ and $x_2(t) \xleftrightarrow{FT} X_2(f)$

then $\boxed{x_1(t) + x_2(t) \xleftrightarrow{FT} X_1(f) + X_2(f)}$

e.g.

3sin 2π100t + 2sin 2π340t



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Some Useful Properties of Fourier Transform

- **Time shift**

If $x(t) \overset{FT}{\Longleftrightarrow} X(f)$ then $\boxed{x(t-\tau) \overset{FT}{\Longleftrightarrow} e^{-j\omega\tau}X(f)}$

where $e^{-j\omega\tau}$ is the phase term.

The amplitude spectrum remains unchanged when a waveform is shifted in time.
Only the phase spectrum will changed.

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Some Useful Properties of Fourier Transform

- **Frequency-shift Property**

If $x(t) \overset{FT}{\Longleftrightarrow} X(f)$ then

$$x(t)e^{j2\pi f_c} \overset{FT}{\Longleftrightarrow} X(f - f_c)$$

$$x(t)\cos 2\pi f_c t \overset{FT}{\Longleftrightarrow} \frac{1}{2}[X(f + f_c) + X(f - f_c)]$$

Multiplying x(t) with a sinusoidal signal cos $2\pi f_c t$ shifts the spectrum of x(t) to $\pm f_c$.

2.4 Fourier Transform

Some Useful Properties of Fourier Transform

- **Convolution Property**

The convolution of two signals, $x_1(t)$ and $x_2(t)$, is defined by

$$x_1 * x_2 = \int_{-\infty}^{\infty} x_1(\tau) x_2 x(t-\tau) \mathrm{d}\tau = \int_{-\infty}^{\infty} x_2(\tau) x_1(t-\tau) \mathrm{d}\tau$$

τ is another variable

where * denotes convolution

Convolution is a mathematical operation and an important analytical tool used in communications:

- Determining the response of a linear system to a input signal

- Determining the result of the interaction between two signals

2.4 Fourier Transform

<u>**Some Useful Properties of Fourier Transform**</u>

- **Convolution Property**

  - **Time Convolution Theorem**

If $x_1(t) \overset{FT}{\Longleftrightarrow} X_1(f)$ and $x_2(t) \overset{FT}{\Longleftrightarrow} X_2(f)$

then $\boxed{x_1(t)*x_2(t) \overset{FT}{\Longleftrightarrow} X_1(f).X_2(f)}$

Convolution in time domain results in multiplication in frequency domain

  - **Frequency Convolution Theorem**

If $x_1(t) \overset{FT}{\Longleftrightarrow} X_1(f)$ and $x_2(t) \overset{FT}{\Longleftrightarrow} X_2(f)$

then $\boxed{x_1(t).x_2(t) \overset{FT}{\Longleftrightarrow} X_1(f)*X_2(f)}$

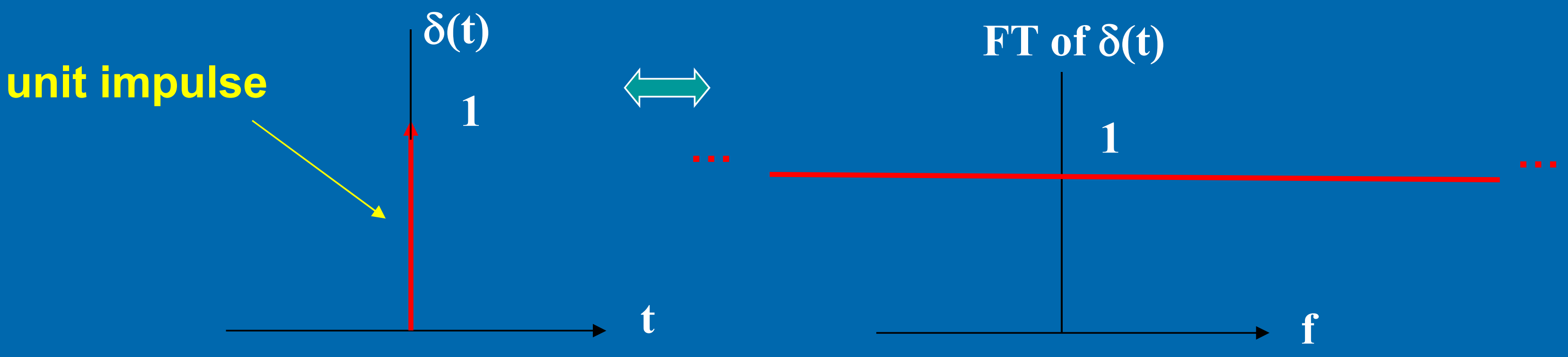
Multiplication in time domain results in convolution in frequency domain

2.4 Fourier Transform

Fourier transform of an Impulse

- Fourier Transform of $\delta(t)$ is given by

$$\delta(t) \xleftrightarrow{FT} 1$$



- Fourier Transform of A $\delta(t)$ is

$$A\delta(t) \xleftrightarrow{FT} A$$

- Fourier transform of a DC signal V_0 is

$$x(t)=V_o \xleftrightarrow{FT} X(f) = V_o\,\delta(f)$$

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Fourier transform of an Impulse

Convolution of any function *f*(*t*) with a unit impulse function δ(*t*) gives the function *f*(*t*) itself.

$$f(t) \;*\; \delta(t) \;=\; f(t)$$

Proof :

Not tested

Since $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$ and $\int_{-\infty}^{\infty} f(t)\delta(t)\,dt = f(0)$

it follows, $\int_{-\infty}^{\infty} f(t)\delta(t-t_0)\,dt = f(t_0)$

Then, $f(t) * \delta(t) = \int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)\,d\tau = f(t)$

Similar result follows in the frequency domain:

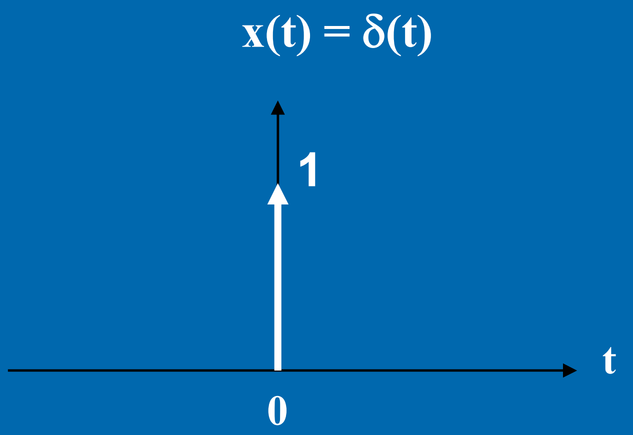
Convolution of any function *X*(*f*) with a unit impulse function δ(*f*) gives the function X(*f*) itself.

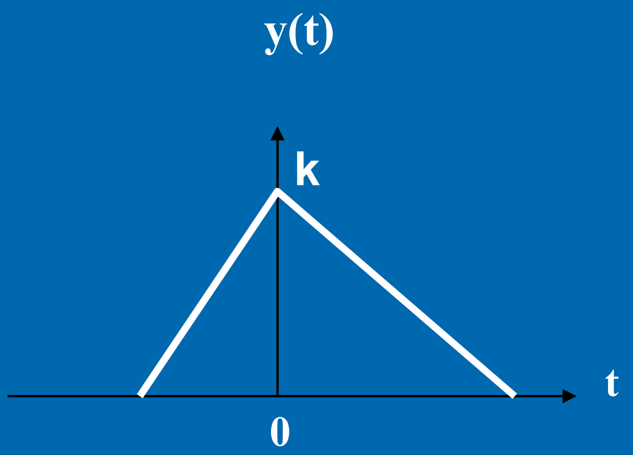
$$X(f) * \delta(f) = X(f)$$

Example 2.8

Two signals *x*(*t*) and *y*(*t*) are shown below.

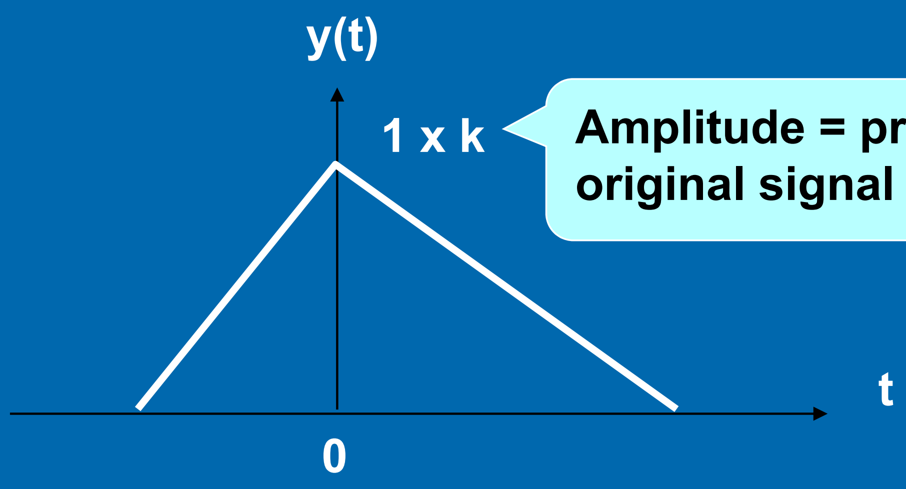
Obtain the graphical convolution of *x*(*t*) and *y*(*t*).





Solution

$$y(t)*x(t) = y(t)*\delta(t) = y(t)$$



2.4 Fourier Transform

Fourier transform of a sinusoid signal

- A sinusoidal signal of a frequency of f_0 and an amplitude of V_p can be expressed as a complex exponential function:

$$x(t) = V_p cos(2\pi f_0 t)$$

$$= V_P \left(\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right)$$

- Applying the frequency-shift property, the Fourier transform of a sinusoidal x(t) is

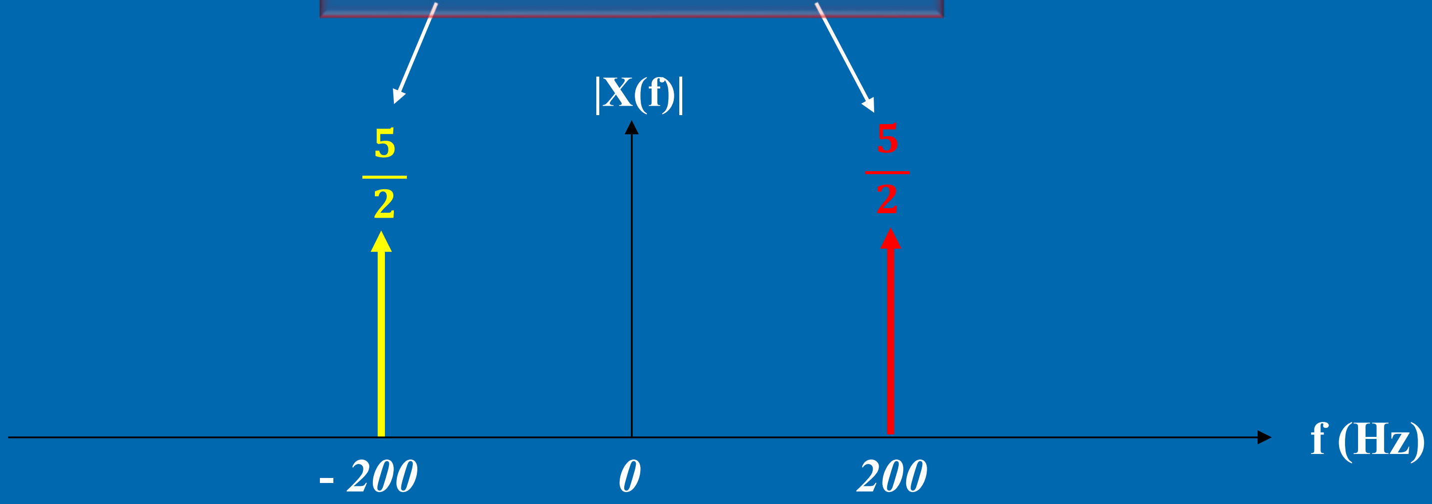
$$x(t) = V_p cos(2\pi f_0 t) \stackrel{FT}{\Longleftrightarrow} X(f) = \boxed{\frac{V_p}{2}} [\delta(f + f_0) + \delta(f - f_0)]$$

Example 2.9

Plot the amplitude spectrum of signal *x*(*t*) = 5cos400πt

Solution

$$x(t) = 5\cos(400\pi t) \xLeftrightarrow{FT} X(f) = \frac{5}{2}\left[\delta(f+200) + \delta(f-200)\right]$$



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Fourier transform of an amplitude modulated signal

- One type of amplitude modulation is achieved by multiplying x(t) with a high frequency sinusoidal signal:

$$y(t) = x(t)\cos 2\pi f_c t$$

- Applying the frequency-shift property, the Fourier transform of y(t) is

If $$x(t) \overset{FT}{\Longleftrightarrow} X(f)$$

The amplitude is halved

then $$y(t) = x(t)\cos 2\pi f_c t \overset{FT}{\Longleftrightarrow} Y(f) = \frac{1}{2}\boxed{[X(f+f_c) + X(f-f_c)]}$$

Shift X(f) left by f_c

Shift X(f) right by f_c

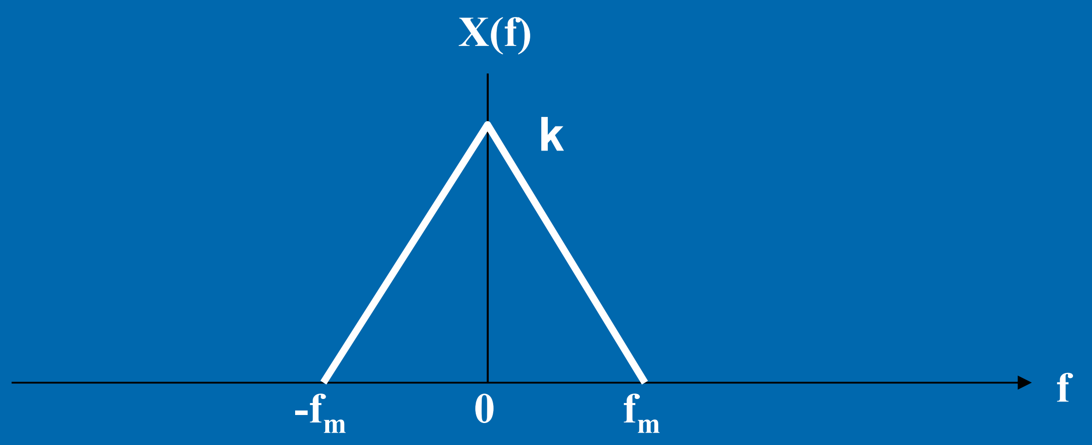
The spectrum of an amplitude modulated signal, x(t)cos $2\pi f_c t$, consists of two frequency shifted version of X(f).

Example 2.10

A signal $x(t)$ is multiplied by a carrier cos $2\pi f_c t$, *giving the resultant signal* y(t) = $x(t)\cos 2\pi f_c t$.

Obtain the frequency spectrum of the resultant signal y(t).

The frequency spectrum of $x(t)$ is as shown.

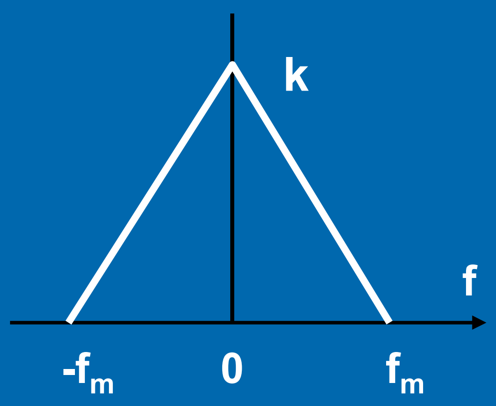


Solution

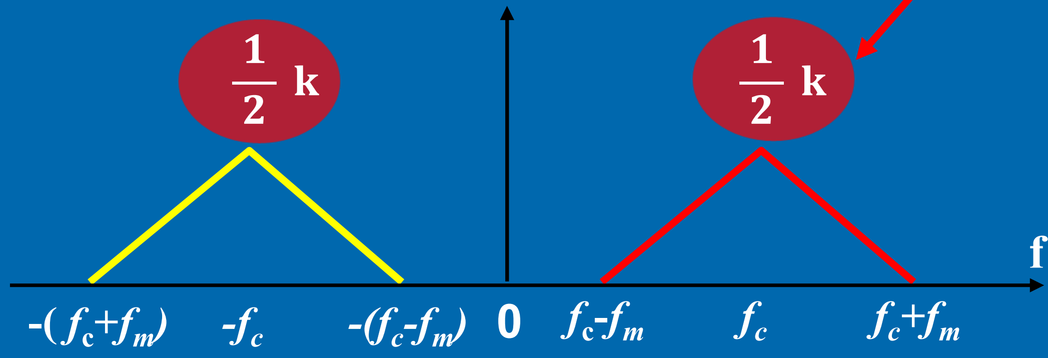
$$x(t) \overset{FT}{\Longleftrightarrow} X(f)$$

$$y(t) = x(t)\cos 2\pi f_c t \overset{FT}{\Longleftrightarrow} Y(f) = \frac{1}{2}\boxed{[X(f+f_c) + X(f-f_c)]}$$

Multiplying x(t) with cos $2\pi f_c t$ shifts X(f) to $\pm f_c$.



The amplitude is halved



$\frac{1}{2}X(f+f_c)$

$\frac{1}{2}X(f-f_c)$

Shift X(f) left by f_c

Shift X(f) right by f_c

End

CHAPTER 2

(Part 3 of 5)