EP0604/MS837M/MS8179 MST Solutions (AY19/20 S2)

No.	SOLUTION
1	$a = 500, r = 1.05$ $T_{15} = 500(1.05)^{14} = \$990$ Solve for n: $S_n = \dfrac{500(1-1.05^n)}{1-1.05} = 104,674$ $1.05^n = 11.4674$ $n = 50$ He has donated for 50 years.
2(a) (i)	$x = 2e^t,\ y = \ln(t+1)$ $\dfrac{dx}{dt} = 2e^t$ $\dfrac{dy}{dt} = \dfrac{1}{t+1}$ $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$ $= \dfrac{1}{t+1} \div 2e^t$ $= \dfrac{1}{2e^t(t+1)}$
(ii)	$t = 0, \dfrac{dy}{dx} = \dfrac{1}{2}$ $x = 2, y = 0$ $\dfrac{y-0}{x-2} = \dfrac{1}{2} \Rightarrow y = \dfrac{1}{2}x - 1$
(iii)	$x = 2e^t$ $e^t = \dfrac{x}{2}$ $t = \ln\left(\dfrac{x}{2}\right)$ $y = \ln\left(\ln\left(\dfrac{x}{2}\right) + 1\right)$

<table>
<tr><th>No.</th><th>SOLUTION</th></tr>
<tr><td>2(b)</td><td>

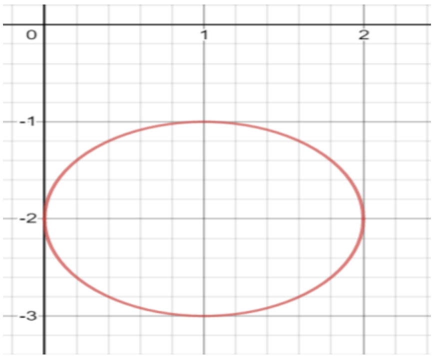
$x = \sin t + 1, \qquad y = \cos t - 2$

$\sin t = x - 1$

$\cos t = y + 2$

$(x-1)^2 + (y+2)^2 = 1$

Curve C is a circle with centre(1,-2) and radius of 1 unit.



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<tr><th>No.</th><th>SOLUTION</th></tr>
<tr><td>3a
(i)</td><td>

$f(x) = \dfrac{3}{x^2 + 5x + 6}$

$g(x) = 3 + \sqrt{x-2}$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x = -3, -2$

$D_f = \{x : x \neq -3 \text{ or } x \neq -2\}$

$= (-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$

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<tr><td>(ii)</td><td>

$D_g = \{x : x \geq 2\}$

$= [2, \infty)$

$R_g = \{g(x) : g(x) \geq 3\}$

$= [3, \infty)$

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<tr><td>(iii)</td><td>

$(f \circ g)(x) = f\left(3 + \sqrt{x-2}\right)$

$= \dfrac{3}{\left(3 + \sqrt{x-2}\right)^2 + 5\left(3 + \sqrt{x-2}\right) + 6}$

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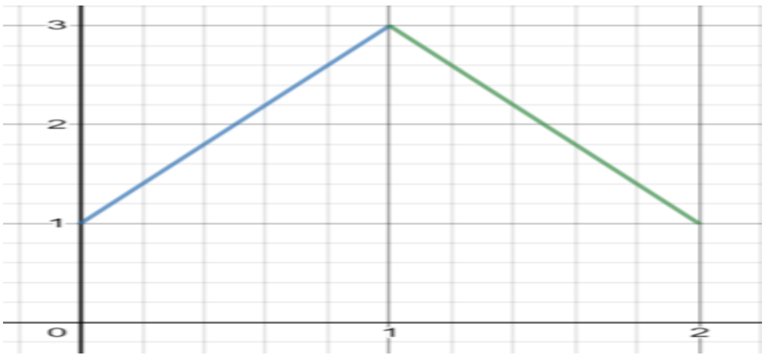
No.	SOLUTION
3(a) (iv)	$let\ y = 3+\sqrt{x-2}$ $x = (y-3)^2 + 2$ $g^{-1}(x) = (x-3)^2 + 2$ $D_{g^{-1}} = R_g$ $= [3,\infty)$ $R_{g^{-1}} = D_g$ $= [2,\infty)$
3b (i)	
(ii)	$R_f = [1,3]$
(iii)	f passes the vertical line test.

No.	SOLUTION
4a. (i)	$y = 3\sin(2x) + \frac{7}{x^3}$ $\frac{dy}{dx} = 6\cos(2x) - \frac{21}{x^4}$
(ii)	$y = \log_5(3x+1)$ $= \frac{\ln(3x+1)}{\ln 5}$ $\frac{dy}{dx} = \frac{3}{(3x+1)\ln 5}$
(b) (i)	$y = \frac{2}{(2x+1)^3}$ $\frac{dy}{dx} = -\frac{12}{(2x+1)^4}$ $x = 0, \frac{dy}{dx} = -12$ gradient of normal $= \frac{1}{12}$ $\frac{y-2}{x} = \frac{1}{12}$ $12y - 24 = x$ $12y = x + 24$
(c) (i)	Perimeter $= 2x + 2\pi r = 400$ $x = \frac{400 - 2\pi r}{2}$ $A = x(2r) + \pi r^2$ $= 2r\left(\frac{400 - 2\pi r}{2}\right) + \pi r^2$ $= 400r - 2\pi r^2 + \pi r^2$ $= 400r - \pi r^2$

No.	SOLUTION
(c) (ii)	$A = 400r - \pi r^2$ $\dfrac{dA}{dr} = 400 - 2\pi r$ $\dfrac{dA}{dr} = 0: \quad 400 - 2\pi r = 0$ $r = \dfrac{400}{2\pi}$ $\dfrac{d^2A}{dr^2} = -2\pi < 0$ (max) $x = \dfrac{400 - 2\pi r}{2}$ $= 200 - \pi\left(\dfrac{400}{2\pi}\right)$ $= 0$ Since x =0 m when A is max, there are no straight sections.