Sample Set 2 SEMESTER TEST

Section A

A1. Using z-transform, determine the impulse response of the digital system shown in Figure A1. Comment on the stability of the system.

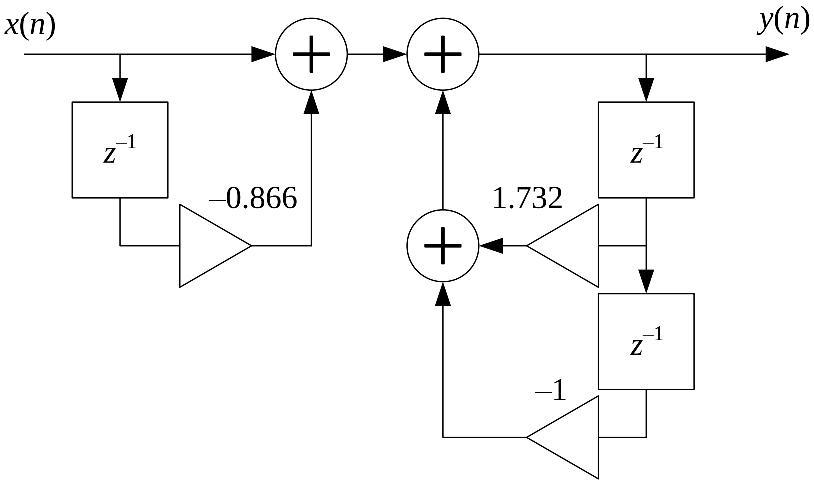


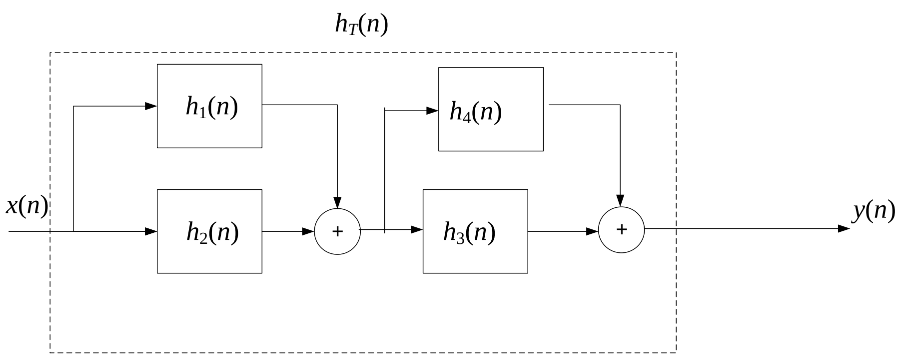
Figure A1

A2 Using z-transform and long-division method, find the input x(n) given y(n)={2, 3,1,6} and h(n)={1,2}.

A3 Evaluate the N = 4-point DFT for $X(0)$ and $X(2)$ if $x(n) = \{0, 2, 0, -2\}$.

A4 Find the z-transform of $x_1(n) = e^{-2n}\sin(3n)u(n)$ and $x_2(n)=n\ 5^{n-1}u(n)$.

A5 The block diagram of a digital system is given as:



a) Find the overall impulse response of the system, $h_T(n)$ in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$ and $h_4(n)$. Find the z-transform of $h_T(n)$, $H_T(z)$.

b) If $h_1(n) = h_2(n) = h_3(n) = h_4(n)=\{1,1\}$ respectively, find $h_T(n)$.

A6 The system function of a digital system is given as:

$$X(z) = \frac{z}{3z^2 - 4z + 1}$$

Using partial fraction, find $x(n)$.

SECTION B

B1 A source is producing sequences of independent symbols A, B, C and D with the following probabilities: A=0.51, B=0.26, C=0.12 and D =0.11.

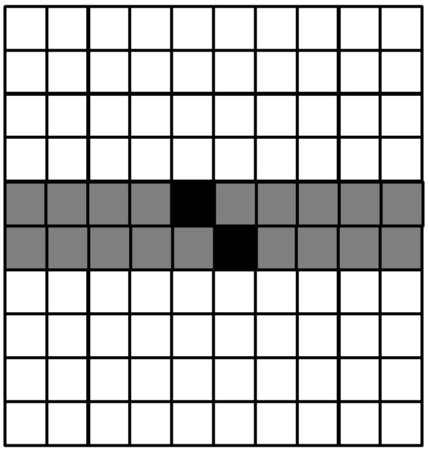
Compute:

(a) Source entropy;

(b) Design a binary Huffman code such that binary one is sent as often as possible;

(c) The average bit length for the code-word set.

B2 The following **Figure B2** shows a 10×10 image with 3 different grey levels (black, grey, white). The image is scanned row by row and the values are put together to form a 1 dimensional data stream. The intensity value of the white, the black and the gray colour is 255, 0, 100 respectively. Each value is encoded by 8 bits before or after coding.



(i) Express its code pairs using run-length coding (RLC);
(ii) Calculate the total number of bits of the bit stream.
(iii) Compute the compression ratio.

Appendix

The z-transform is defined as X(z) = $\sum_{n=-\infty}^{\infty} x[n]z^{-n}$

Sequence	Transform
$\delta[n]$	1
u[n]	$\frac{1}{1-z^{-1}}$
$\delta[n - m]$	z^{-m}
$a^n u[n]$	$\frac{1}{1-az^{-1}}$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$
$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$

Some z-transform properties:	
Sequence	Transform
x[n] $x_1[n]$ $x_2[n]$	X(z) $X_1(z)$ $X_2(z)$
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
x[n - m]	$z^{-m}X(z)$

Source entropy:

$$H = \sum_{i=1}^{m} P_i \log_2\left(\frac{1}{P_i}\right)$$

Average code length:

$$\bar{n} = \sum_{i=1}^{m} n_i P(X_i)$$

Information content per symbol:

$$I(S_i) = \log_2\left(\frac{1}{P_i}\right)$$

Discrete Fourier Transform, DFT:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{N}kn}$$