NODAL ANALYSIS

Objectives

- Analyse a given circuit using nodal analysis method.
- Write nodal equations by inspection and solve for the unknown loop voltages using Cramer's rule.

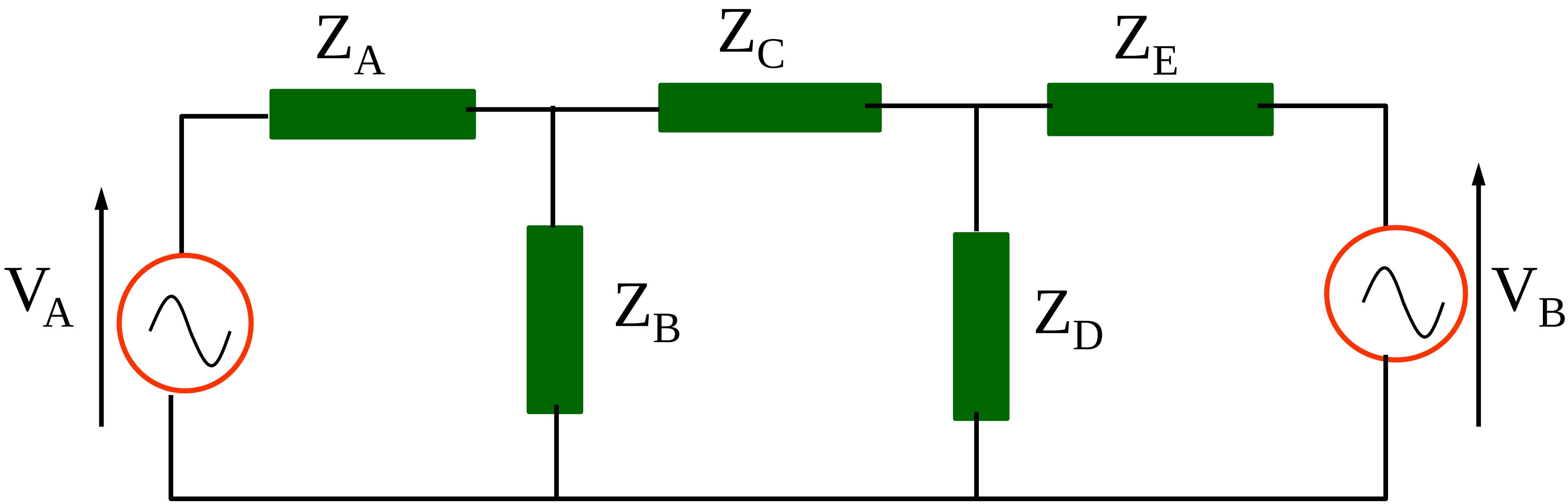
Nodal Analysis

Nodal Analysis has a lot in common with the Mesh Analysis.

Mesh Analysis - is to replace KVL

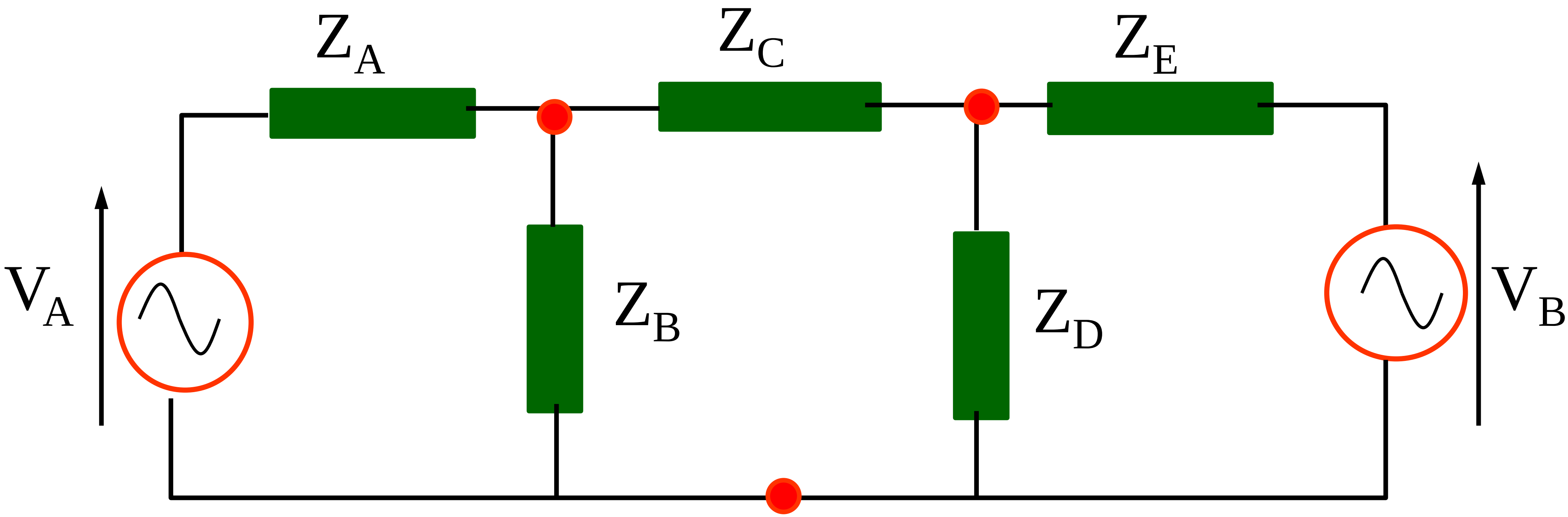
Nodal Analysis - is to replace KCL

Nodal Analysis



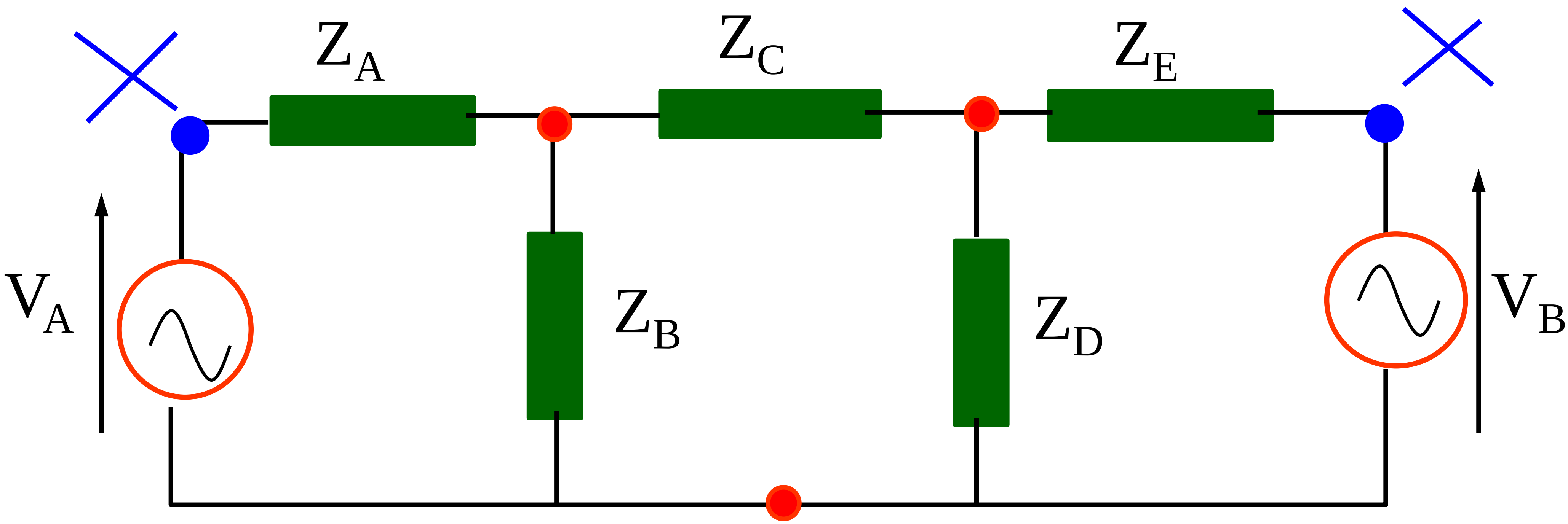
Given a circuit, first determine the number of nodes, hence determine the number of independent equations by KCL.

Nodal Analysis



Nodes are junctions of branches in a circuit.

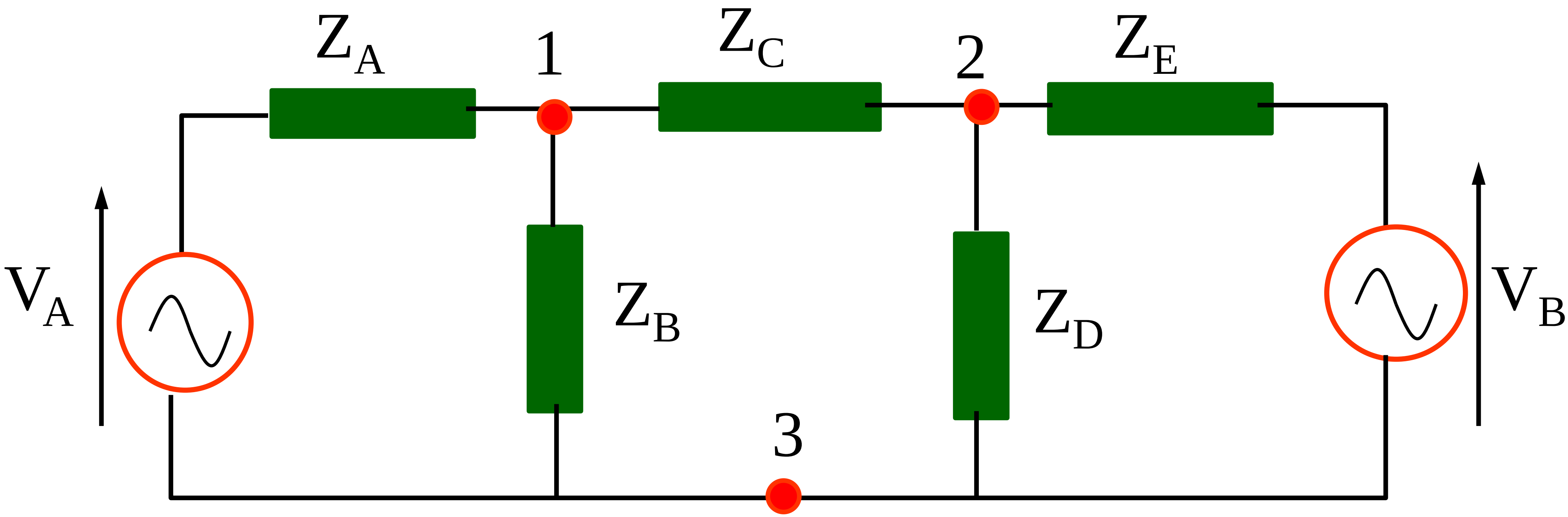
Nodal Analysis



Nodes are junctions of branches in a circuit.

Choose nodes such that no isolated voltage or current source appears between any 2 nodes.

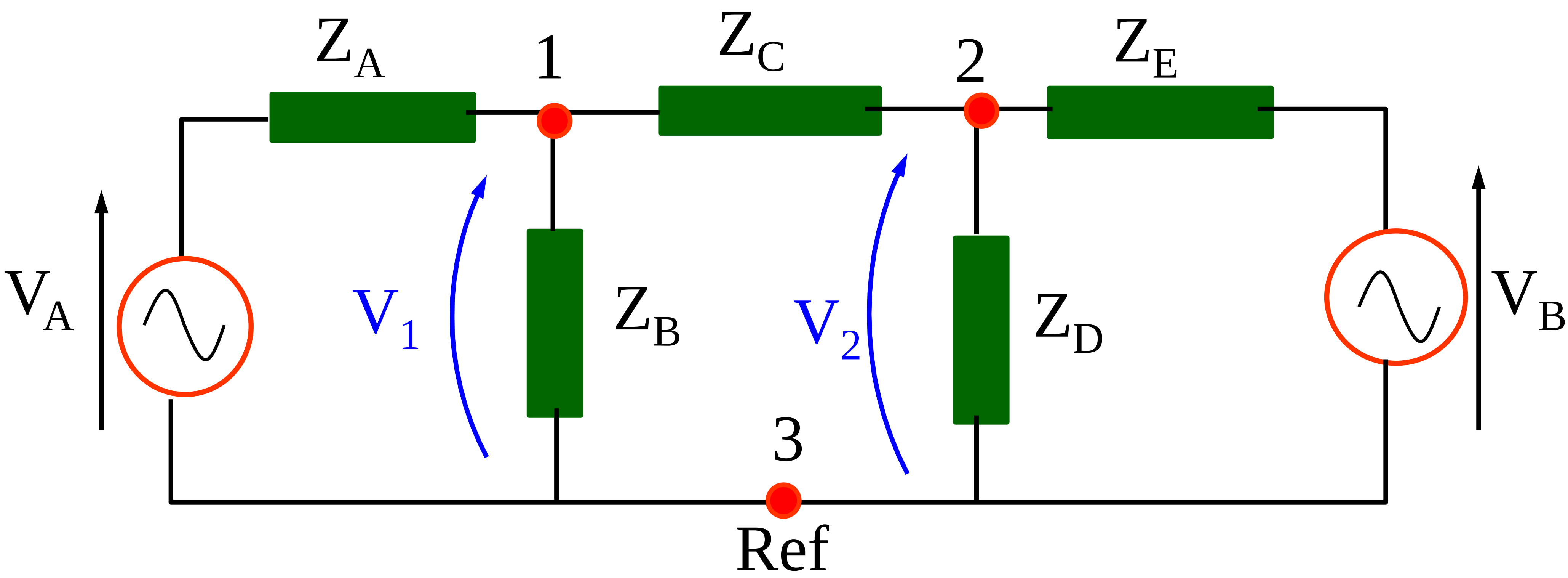
Nodal Analysis



Choose the nodes and number them.

Take one of the nodes as reference, say, node 3.

Nodal Analysis



The voltages of the other two nodes 1 & 2, namely V_1 and V_2, are then measured with respect to this reference node 3.

Nodal Analysis

Started with 3 nodes.

After taking 1 node as the reference, 2 nodes are left. This indicates that the number of equations by KCL will also be 2.

Nodal Analysis

The 2 equations are again shown here:

$$\frac{V_1 - V_A}{Z_A} + \frac{V_1}{Z_B} + \frac{V_1 - V_2}{Z_C} = 0$$

$$\frac{\mathrm{V}_2 - \mathrm{V}_1}{\mathrm{Z}_\mathrm{C}} + \frac{\mathrm{V}_2}{\mathrm{Z}_\mathrm{D}} + \frac{\mathrm{V}_2 - \mathrm{V}_\mathrm{B}}{\mathrm{Z}_\mathrm{E}} = 0$$

Rearranging to give:

$$\left(\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C}\right)V_1 - \frac{1}{Z_C}V_2 = \frac{V_A}{Z_A}$$

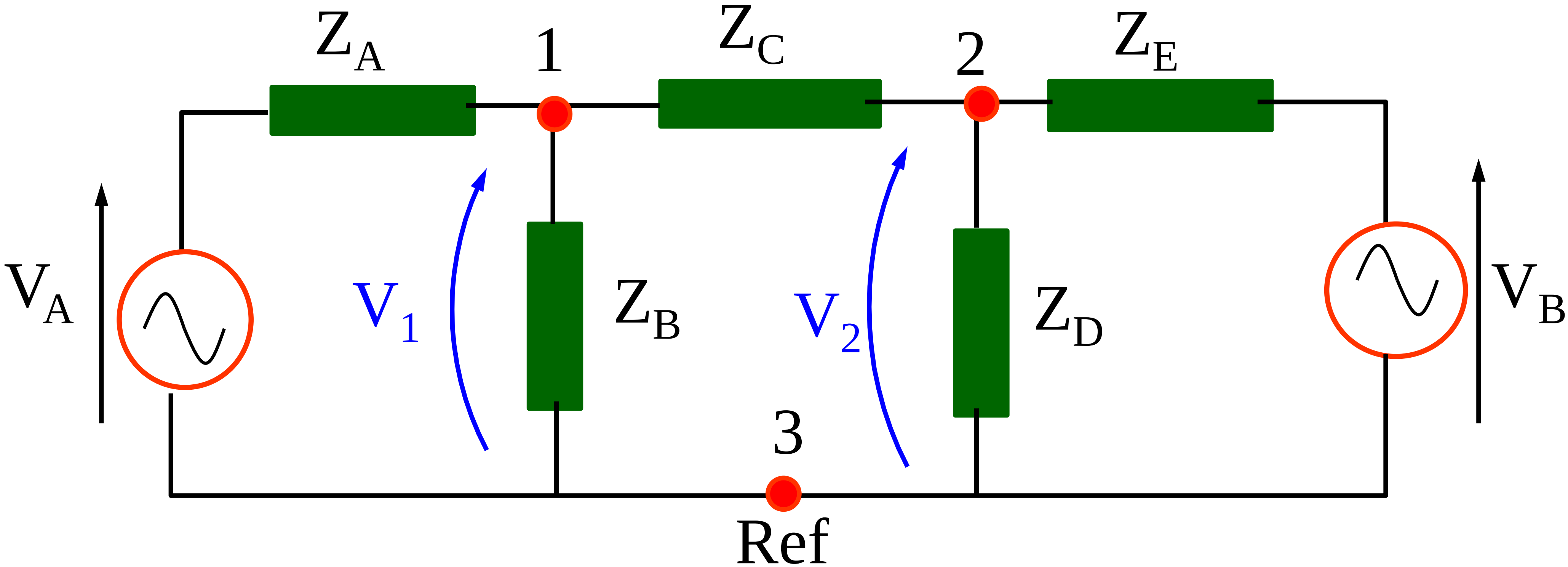
$$-\frac{1}{\mathrm{Z}_\mathrm{C}}\mathrm{V}_1 + \left(\frac{1}{\mathrm{Z}_\mathrm{C}} + \frac{1}{\mathrm{Z}_\mathrm{D}} + \frac{1}{\mathrm{Z}_\mathrm{E}}\right)\mathrm{V}_2 = \frac{\mathrm{V}_\mathrm{B}}{\mathrm{Z}_\mathrm{E}}$$

Nodal Analysis

Rewritten in matrix form to give:-

$$\begin{bmatrix} \left(\dfrac{1}{Z_A}+\dfrac{1}{Z_B}+\dfrac{1}{Z_C}\right) & -\left(\dfrac{1}{Z_C}\right) \\ -\left(\dfrac{1}{Z_C}\right) & \left(\dfrac{1}{Z_C}+\dfrac{1}{Z_D}+\dfrac{1}{Z_E}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \dfrac{V_A}{Z_A} \\ \dfrac{V_B}{Z_E} \end{bmatrix}$$

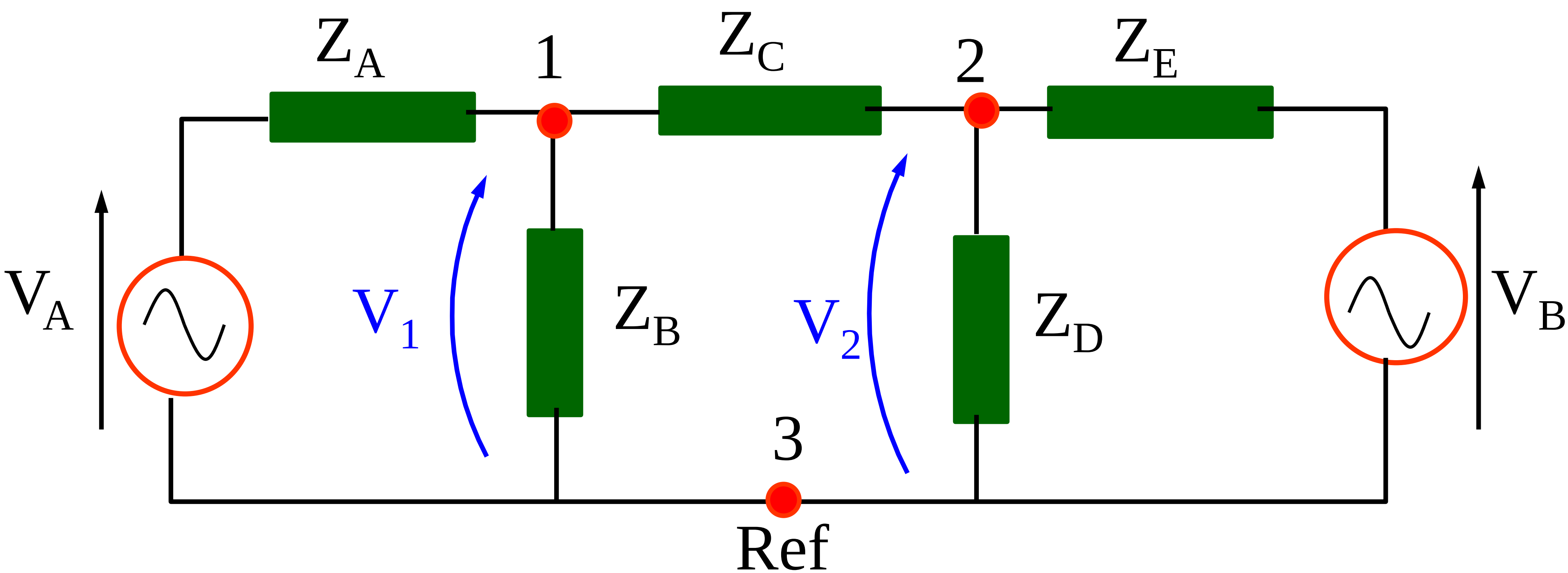
Nodal Analysis



The general nodal matrix equation is therefore:

Y x V = I where Y is a 2 x 2 admittance matrix (for a 3 nodes circuit) and V & I are 2 x 1 vectors

Nodal Analysis by Inspection



The purpose of nodal analysis is to be able to write this matrix equation Y x V = I by INSPECTION on the circuit without using KCL.

Nodal Analysis by Inspection

Let's look at the nodal matrix again:- Y x V = I

$$\begin{bmatrix} \left(\frac{1}{Z_A}+\frac{1}{Z_B}+\frac{1}{Z_C}\right) & -\left(\frac{1}{Z_C}\right) \\ -\left(\frac{1}{Z_C}\right) & \left(\frac{1}{Z_C}+\frac{1}{Z_D}+\frac{1}{Z_E}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V_A}{Z_A} \\ \frac{V_B}{Z_E} \end{bmatrix}$$

In general, the matrix Y x V = I can be expressed as:

$$\begin{bmatrix} +(Y_{11}) & -(Y_{12}) \\ -(Y_{21}) & +(Y_{22}) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Nodal Analysis by Inspection

Here comes the regulations to follow in order to write the nodal matrix equation by inspection ***without using Kirchhoff's Current Law.***

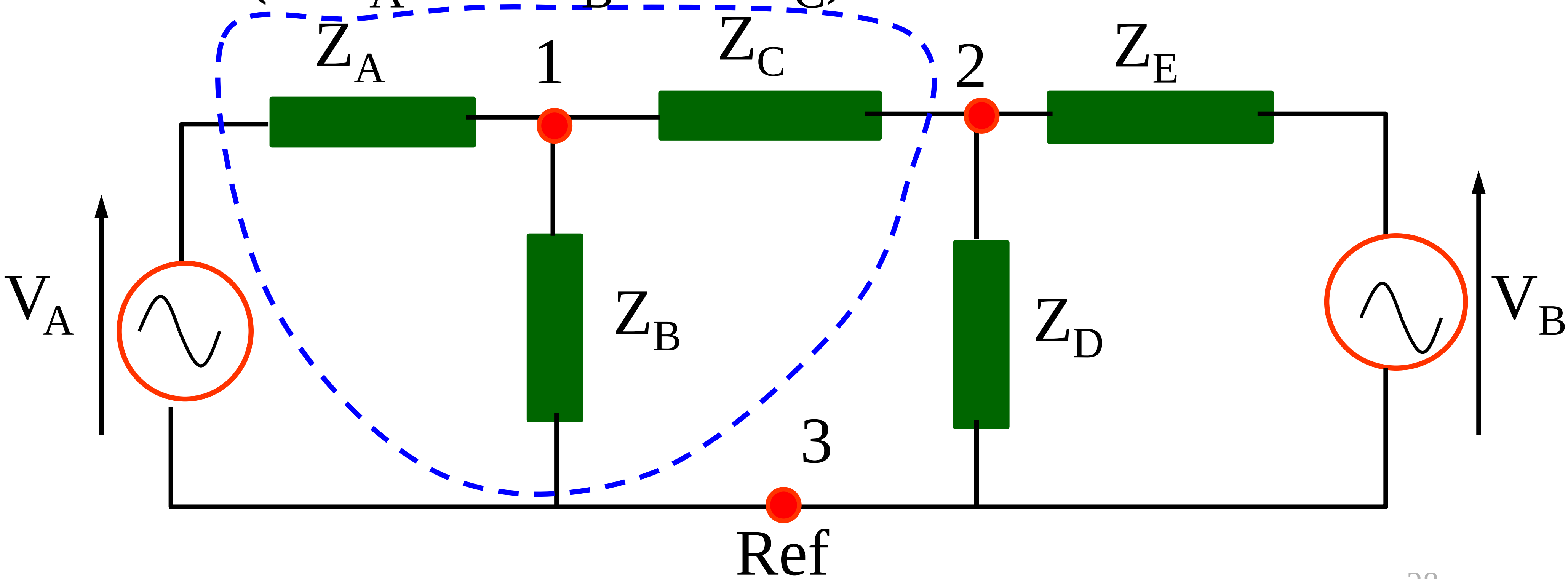
This is then followed by the use of Cramer's Rule to solve for the nodal voltages V_1, V_2 etc.

Nodal Analysis by Inspection

Self-admittances (those in the diagonal)

Y_{11} = sum of all admittances connected to node 1

$= (1/Z_A + 1/Z_B + 1/Z_C)$

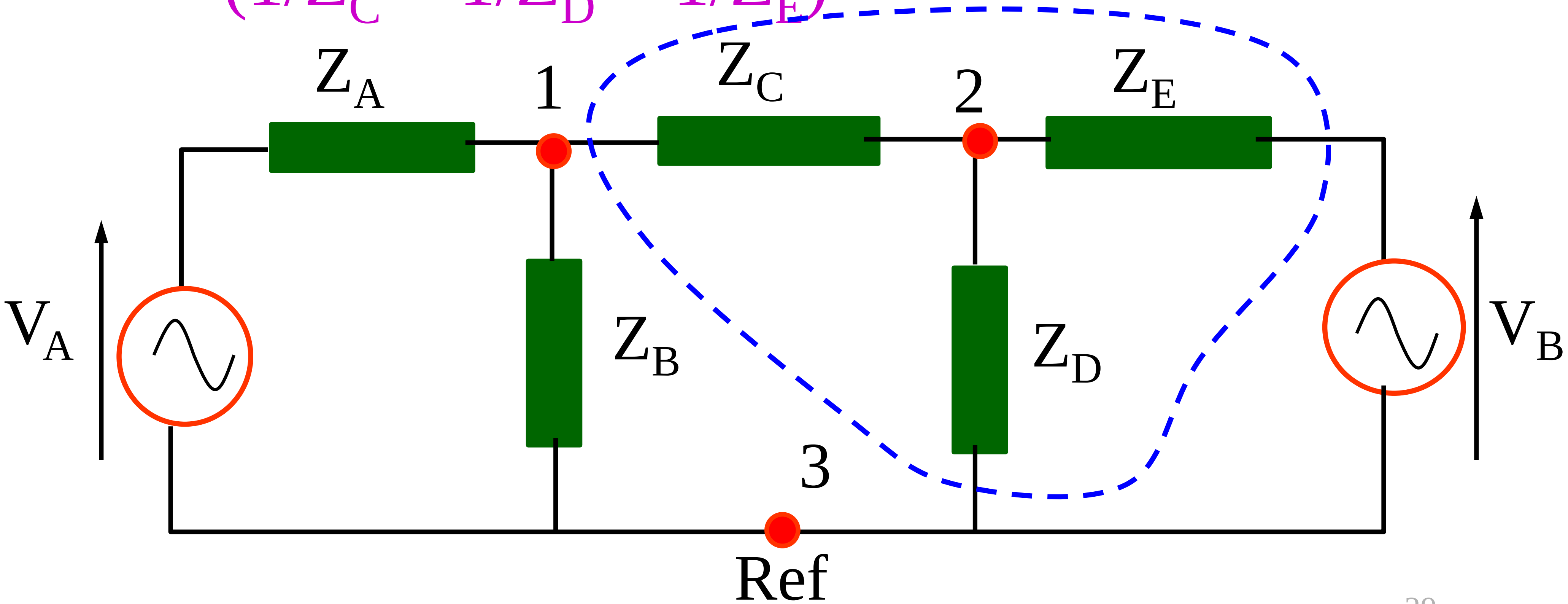


Nodal Analysis by Inspection

Self-admittances (those in the diagonal)

Y_{22} = sum of all admittances connected to node 2

$= (1/Z_C + 1/Z_D + 1/Z_E)$



Nodal Analysis by Inspection

Self-admittances (those in the diagonal)

$Y_{11} = (1/Z_A + 1/Z_B + 1/Z_C)$ = sum of all admittances connected to node 1

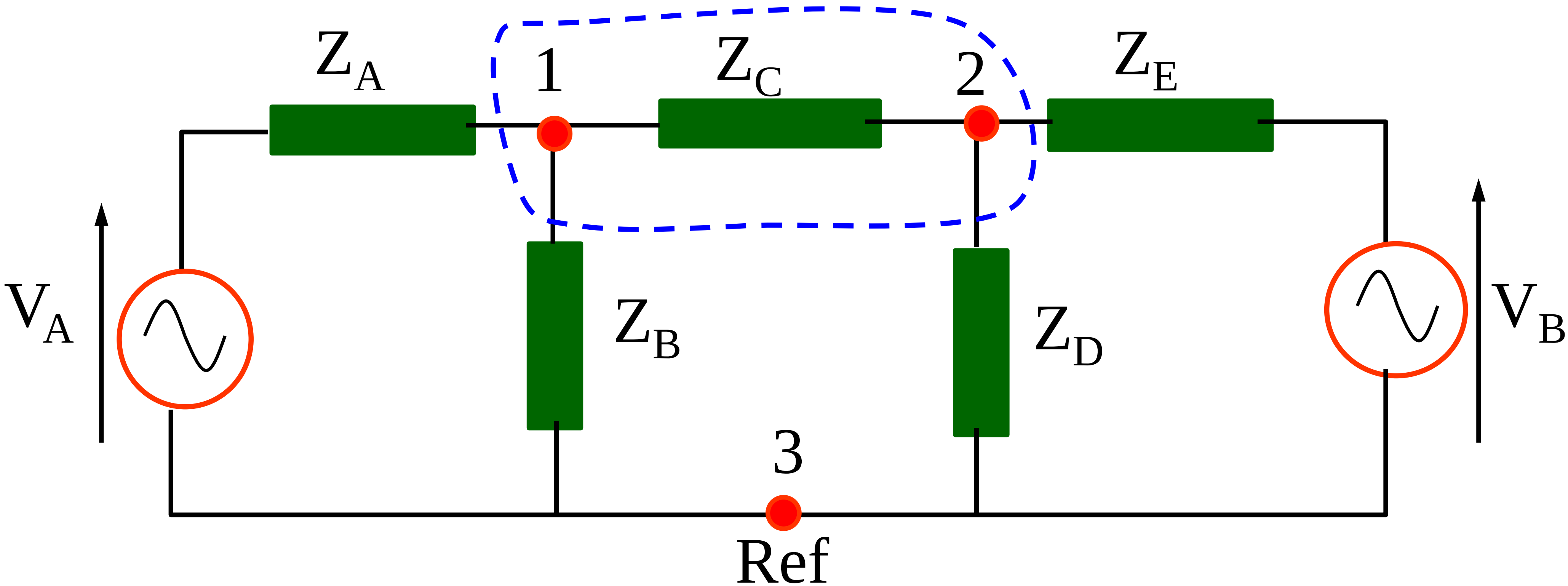
$Y_{22} = (1/Z_C + 1/Z_D + 1/Z_E)$ = sum of all admittances connected to node 2

If the admittance matrix were a 3 x 3 matrix (for a four nodes circuit, taking node 4, say, as the reference), then

Y_{33} = sum of all admittances connected to node 3

Nodal Analysis by Inspection

Coupling-admittances (those off the diagonal) $Y_{12} = Y_{21}$ = ***minus*** the sum of all admittances connected between node 1 & 2 $= -1/Z_C$



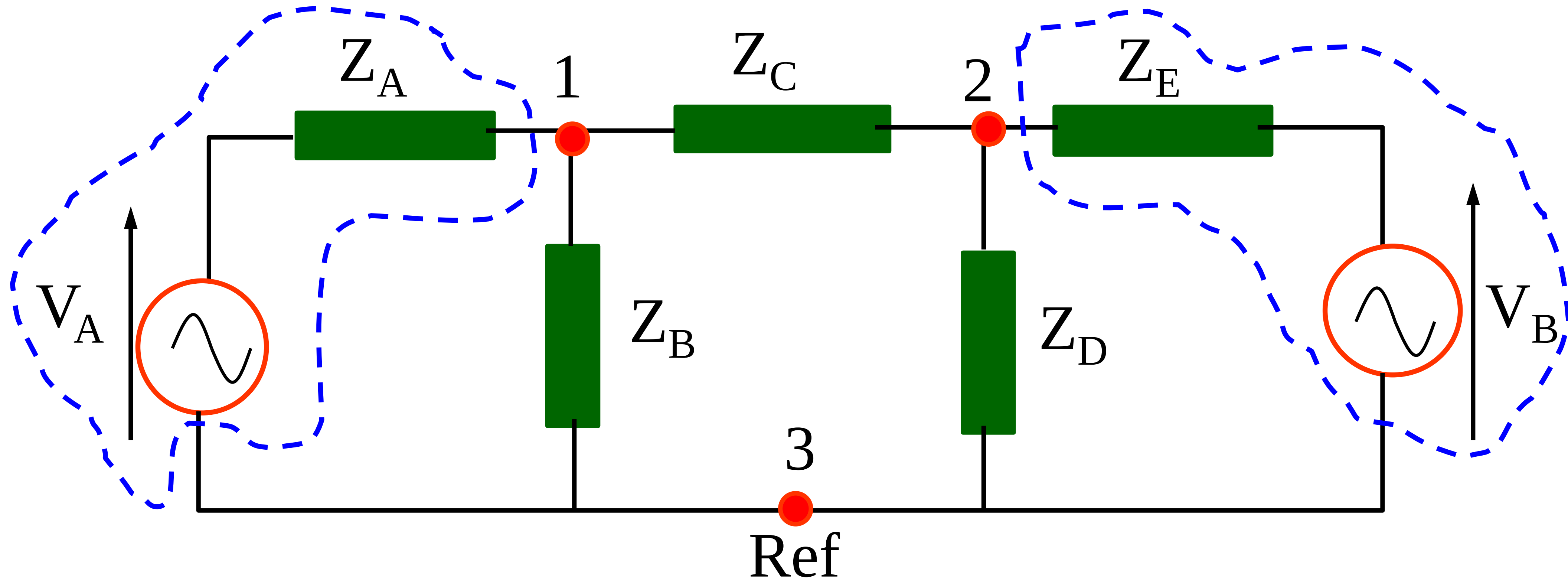
Nodal Analysis by Inspection

If the admittance matrix were a 3 x 3 matrix (for a four nodes circuit, taking node 4 as the reference), then

$Y_{13} = Y_{31} =$ ***minus*** the sum of all admittances connected between nodes 1 & 3

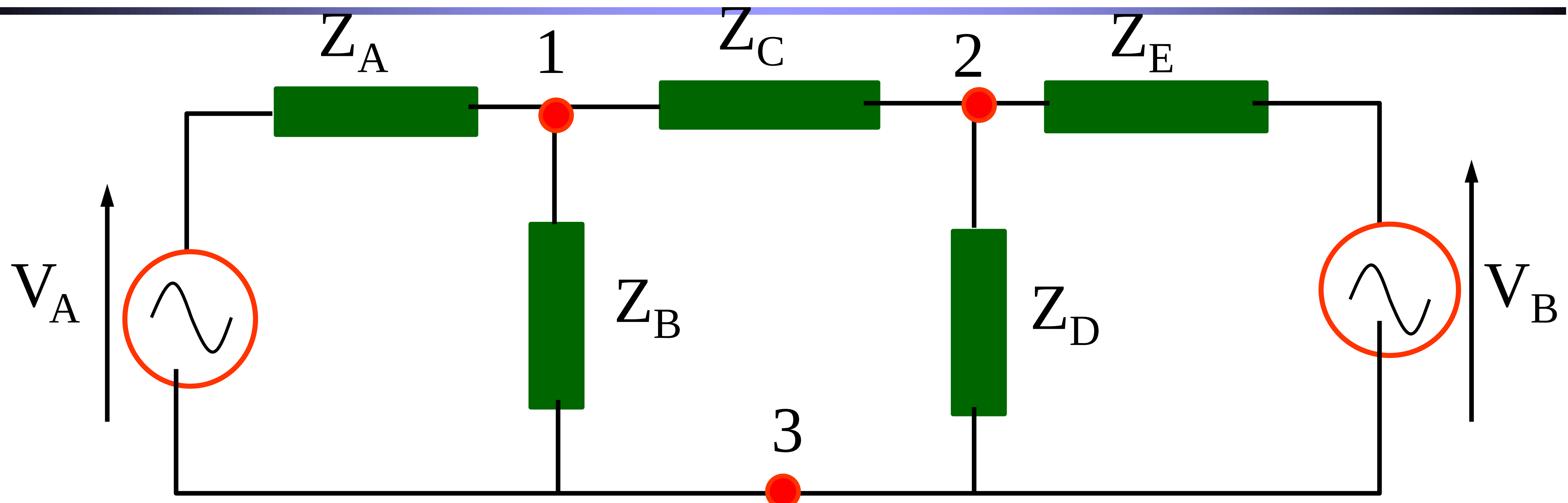
$Y_{23} = Y_{32} =$ ***minus*** the sum of all admittances connected between nodes 2 & 3

Nodal Analysis by Inspection



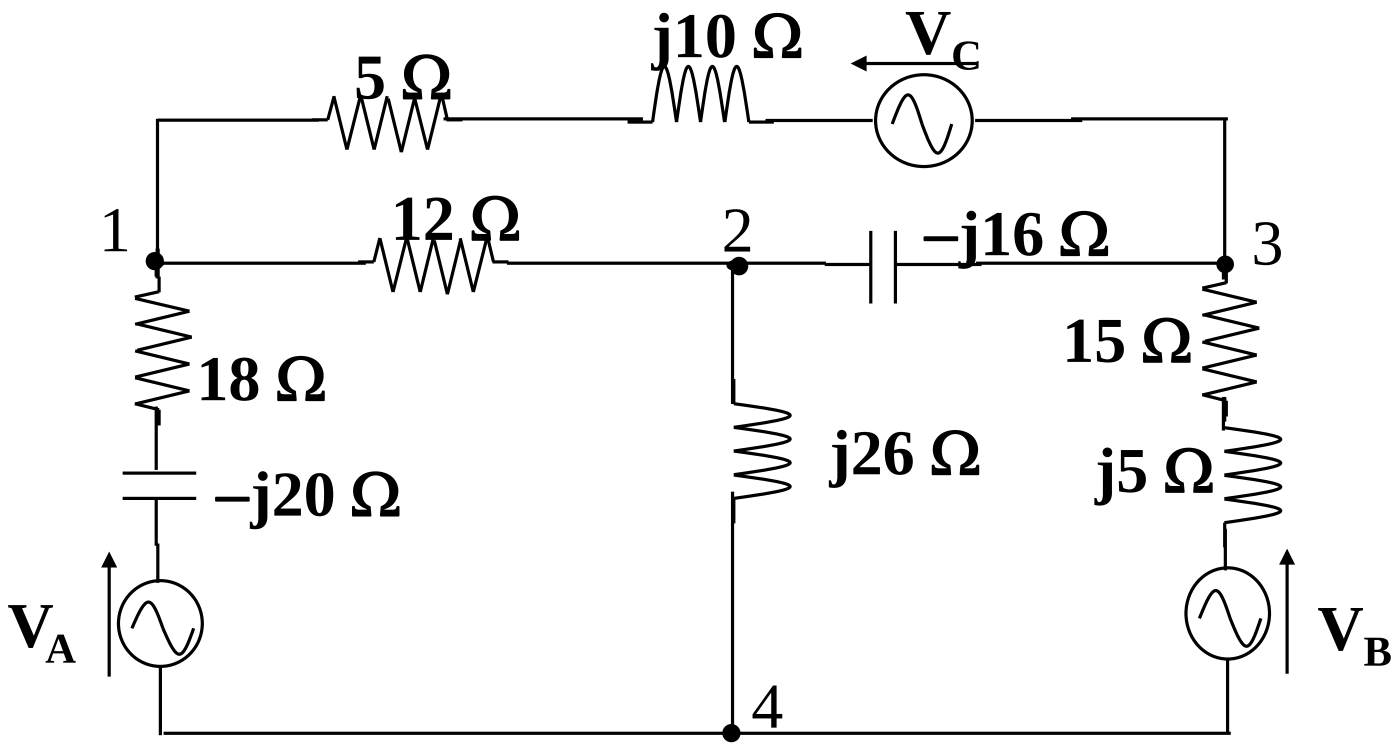
A **positive** sign should be applied on a current source if the current is flowing toward the node. **Negative** when flowing away from the node.

Nodal Analysis by Inspection



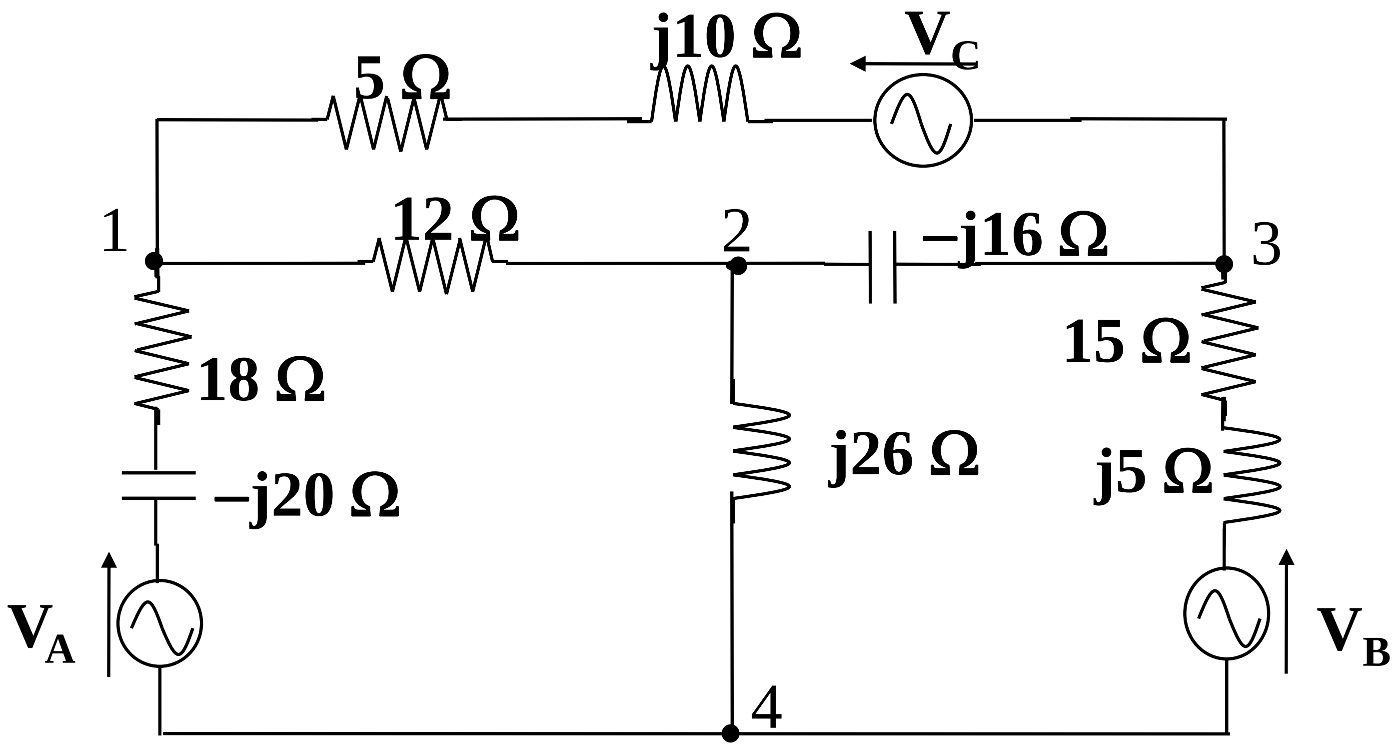
$$\begin{bmatrix} \left(\frac{1}{Z_A}+\frac{1}{Z_B}+\frac{1}{Z_C}\right) & -\left(\frac{1}{Z_C}\right) \\ -\left(\frac{1}{Z_C}\right) & \left(\frac{1}{Z_C}+\frac{1}{Z_D}+\frac{1}{Z_E}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{V_A}{Z_A} \\ \frac{V_B}{Z_E} \end{bmatrix}$$

Example 1.5



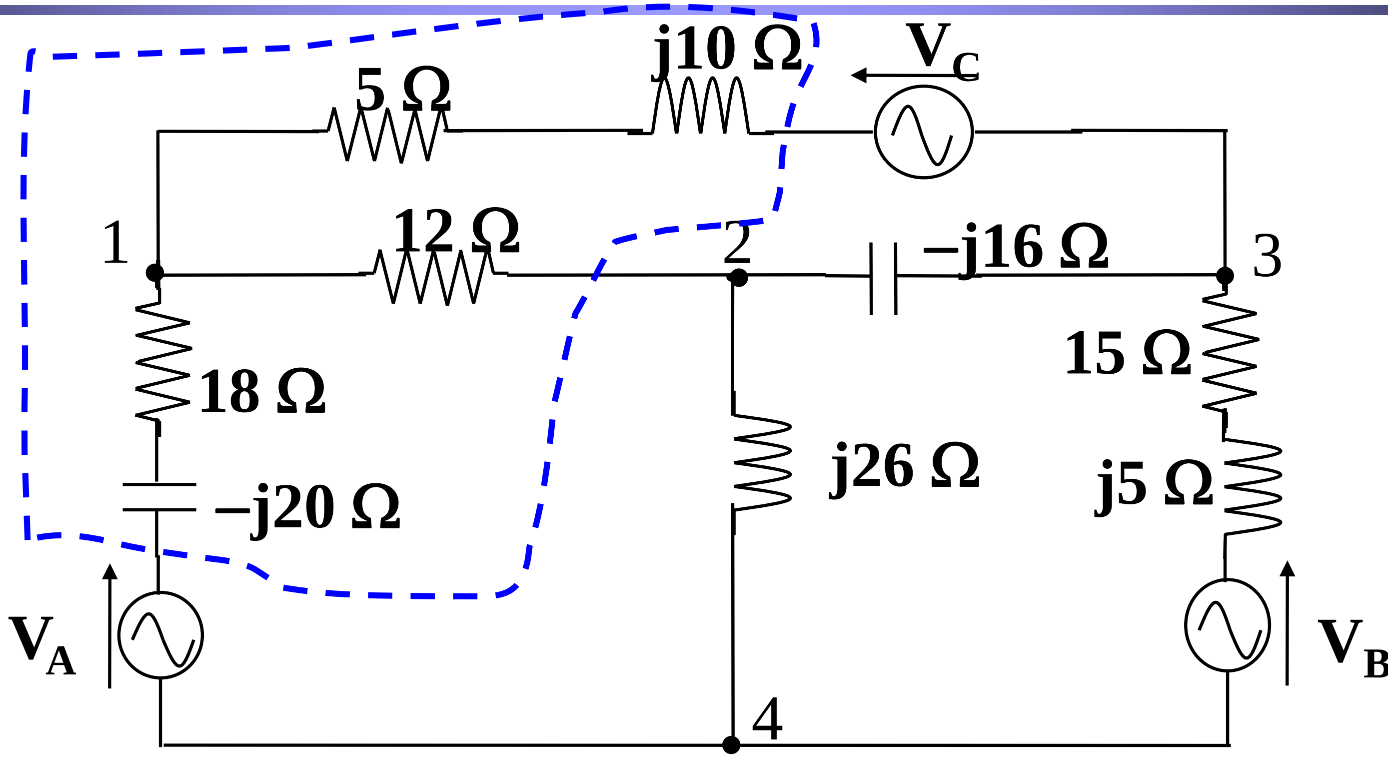
Write the nodal voltage equation by inspection for the circuit shown.

Example 1.5



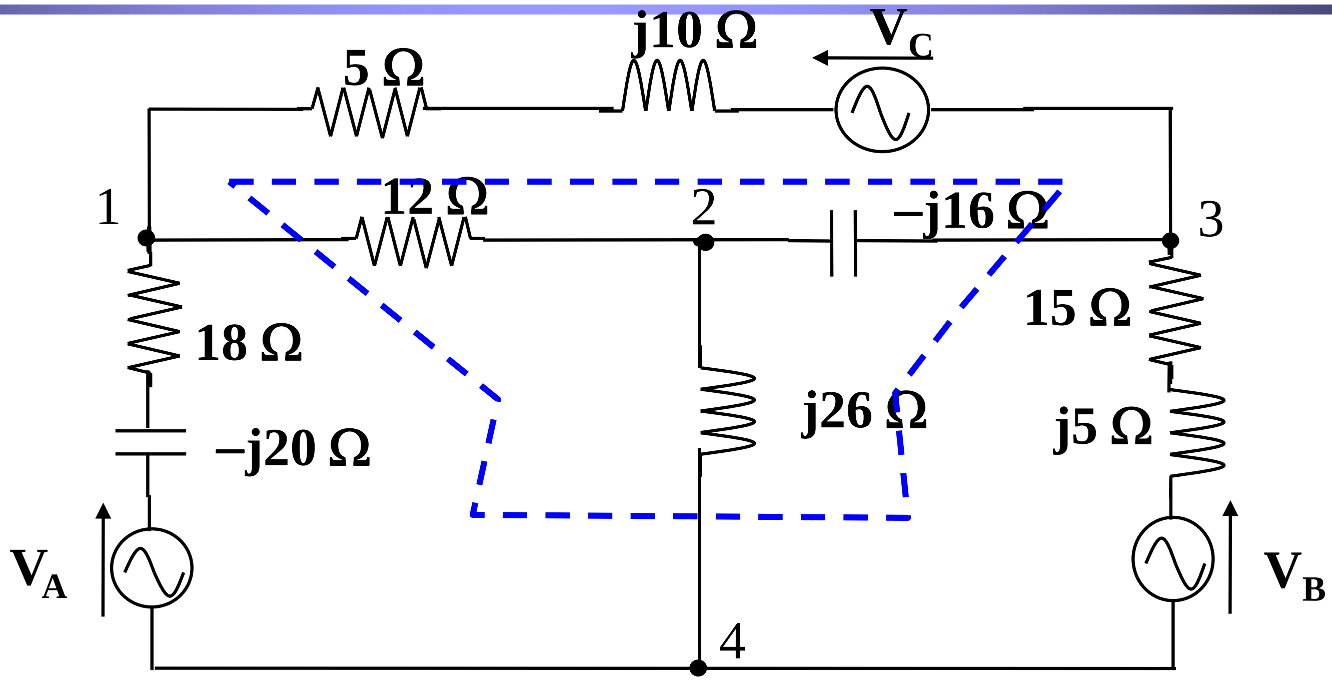
For a four nodes circuit, taking node 4 as the reference, the admittance matrix is a 3 x 3 matrix

Example 1.5



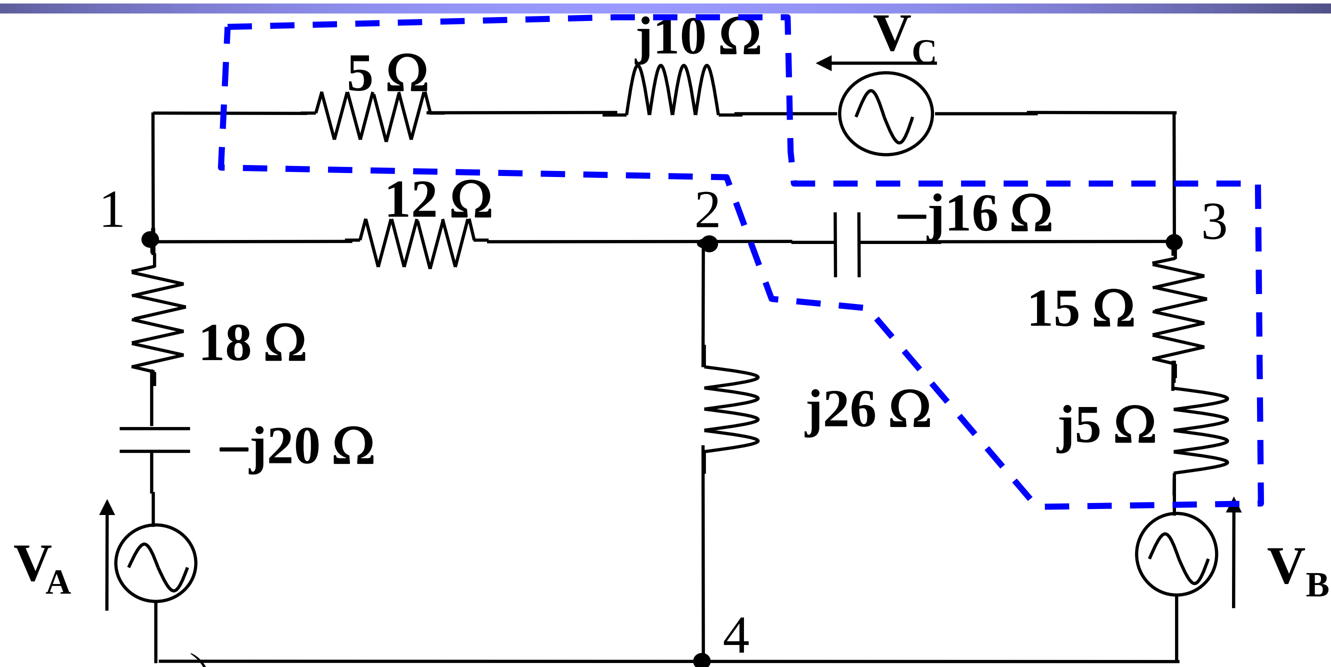
$$\left[\begin{matrix} \left(\dfrac{1}{18 - j20} + \dfrac{1}{12} + \dfrac{1}{5 + j10}\right) \\ \\ \\ \end{matrix}\right] \left[\begin{matrix} \\ \\ \end{matrix}\right] = \left[\begin{matrix} \\ \\ \end{matrix}\right]$$

Example 1.5



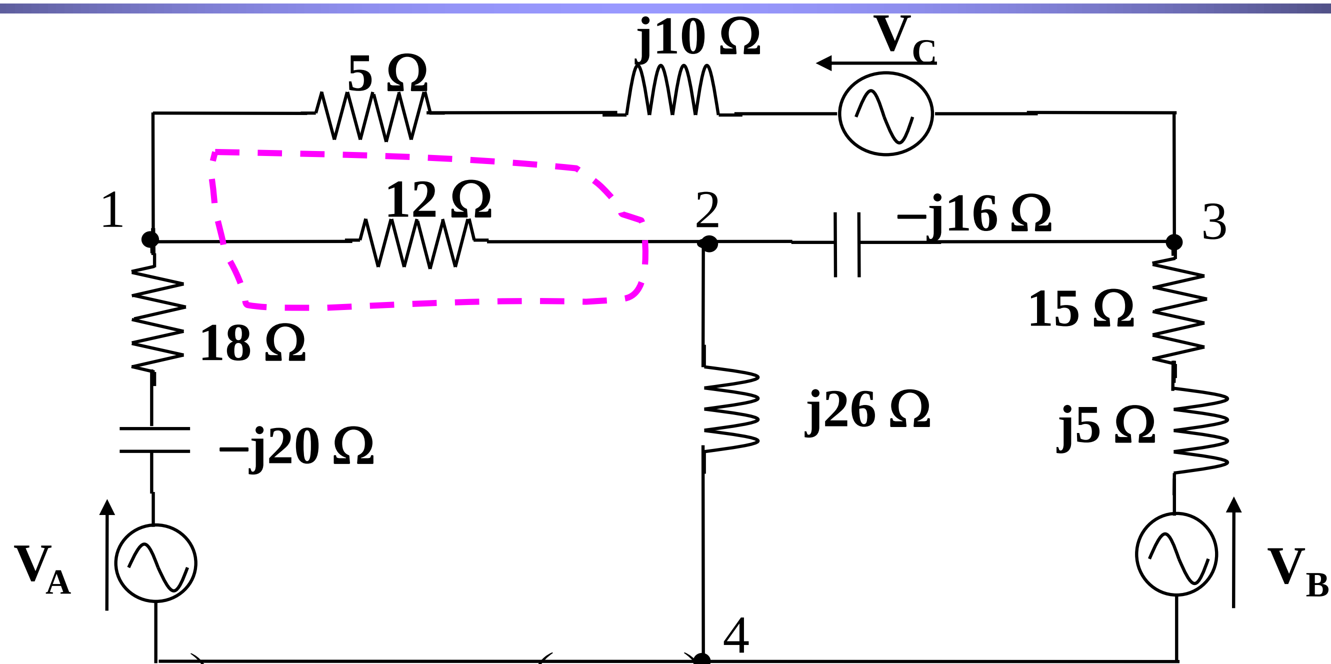
$$\begin{bmatrix} \left(\frac{1}{18 - j20} + \frac{1}{12} + \frac{1}{5 + j10}\right) & & \\ & \left(\frac{1}{12} + \frac{1}{-j16} + \frac{1}{j26}\right) & \\ & & \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

Example 1.5



$$\begin{bmatrix} \left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & & \\ & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & \\ & & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right)\end{bmatrix}\begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Example 1.5



$$\begin{bmatrix} \left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & -\left(\frac{1}{12}\right) & \\ & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & \\ & & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right) \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

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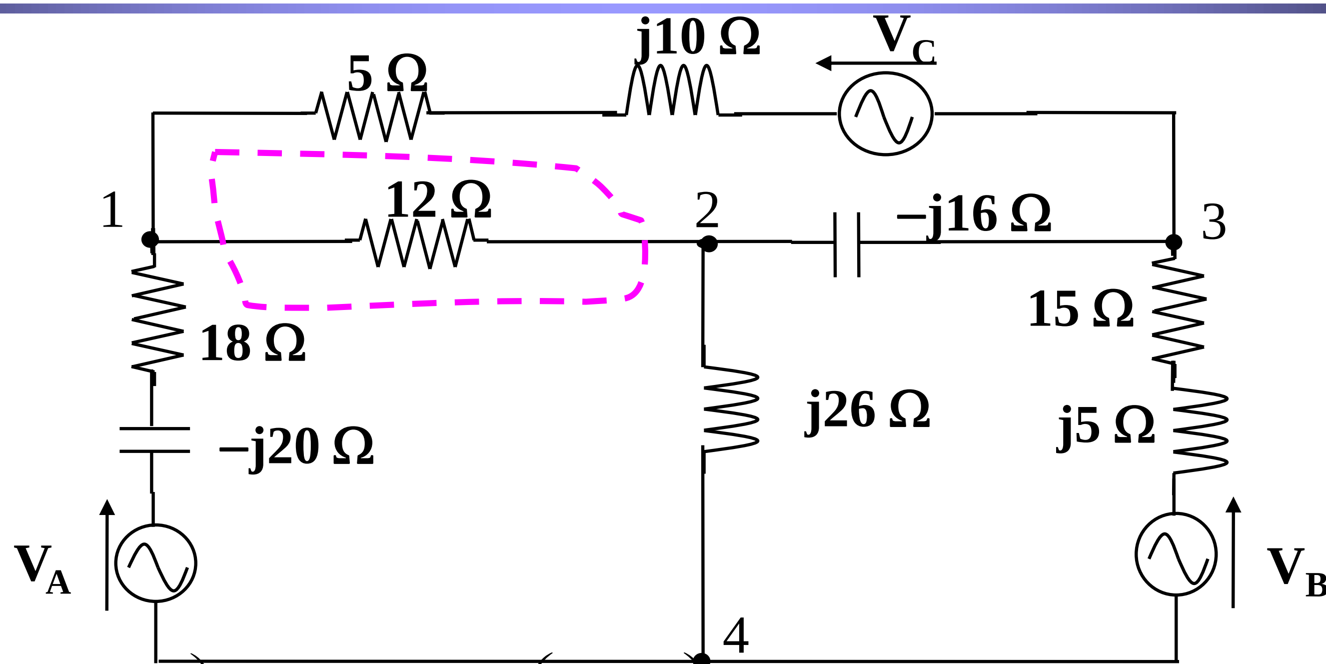

$$\begin{bmatrix} \left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & -\left(\frac{1}{12}\right) & \\ -\left(\frac{1}{12}\right) & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & \\ & & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right) \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

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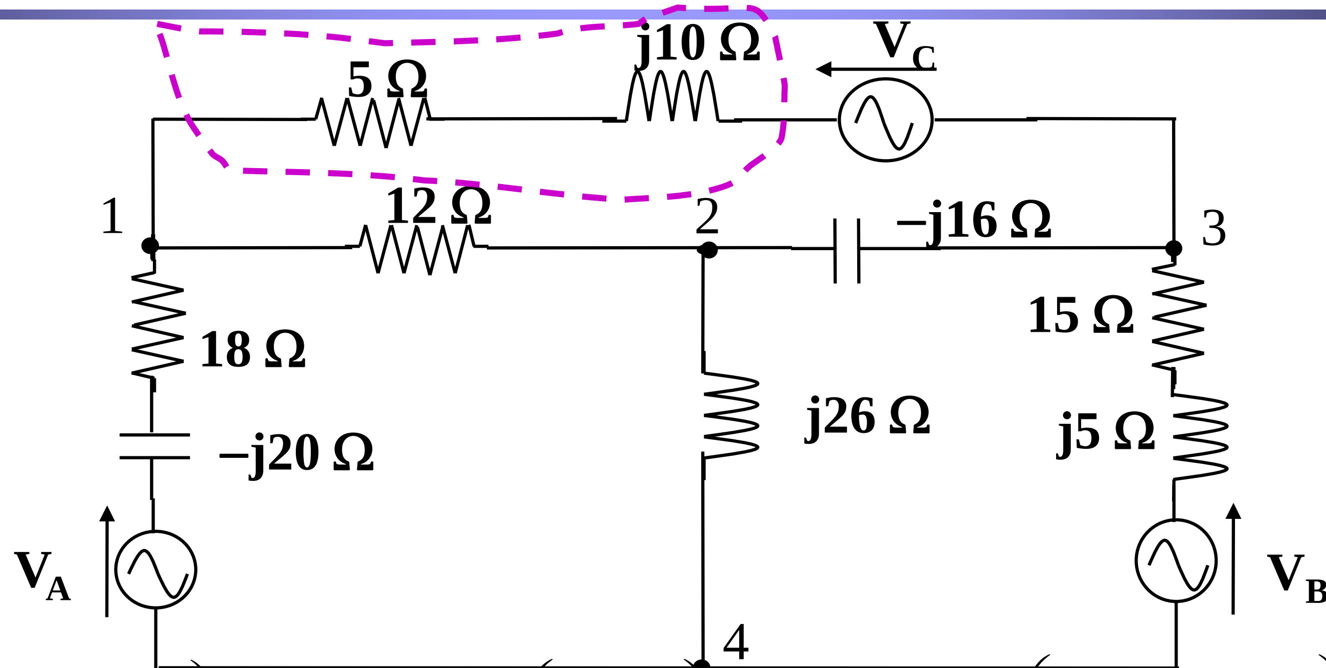
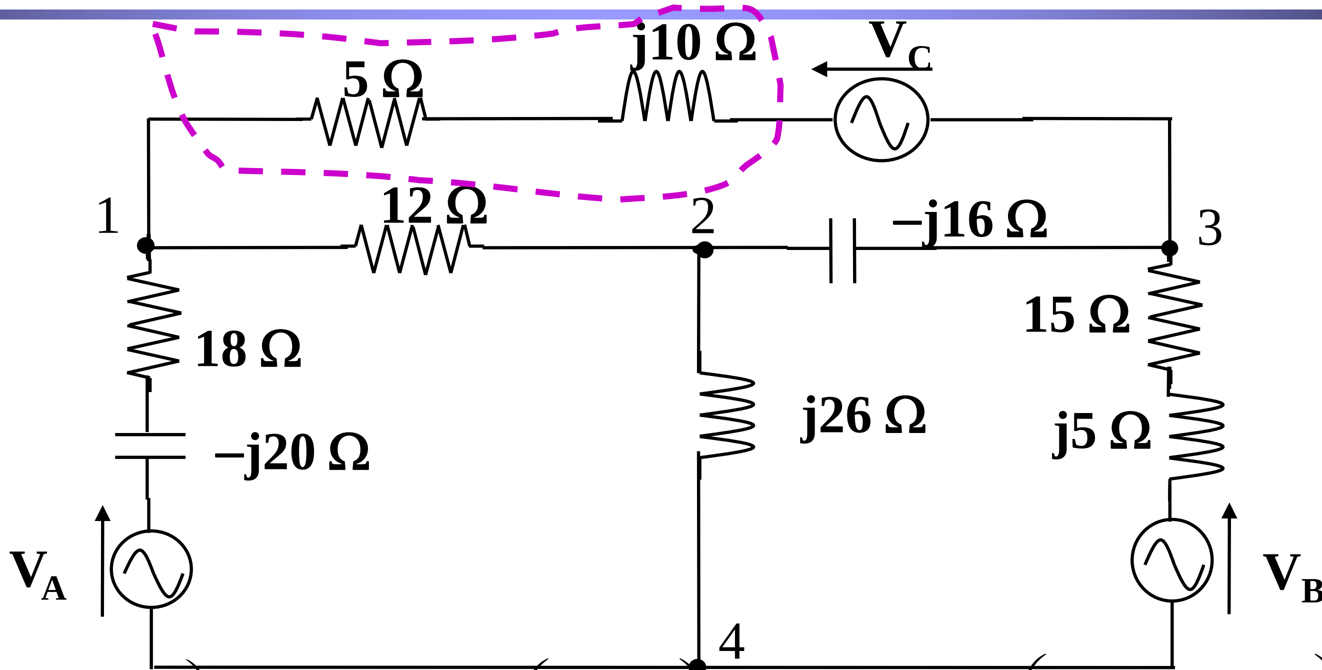


$$
\begin{bmatrix}
\left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & -\left(\frac{1}{12}\right) & -\left(\frac{1}{5+j10}\right) \\
-\left(\frac{1}{12}\right) & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & \\
 & & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right)
\end{bmatrix}
\begin{bmatrix} \\ \\ \end{bmatrix}
=
\begin{bmatrix} \\ \\ \end{bmatrix}
$$

Example 1.5



$$\begin{bmatrix} \left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & -\left(\frac{1}{12}\right) & -\left(\frac{1}{5+j10}\right) \\ -\left(\frac{1}{12}\right) & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & \\ -\left(\frac{1}{5+j10}\right) & & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right) \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

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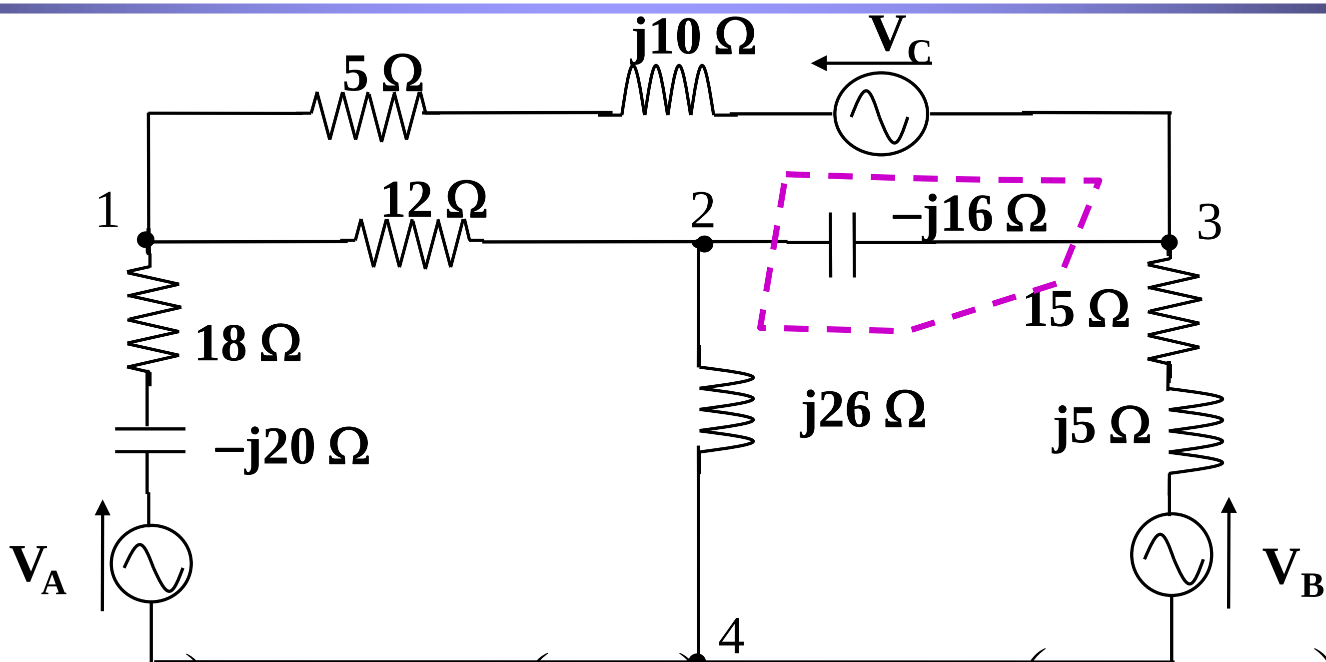

$$\begin{bmatrix} \left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & -\left(\frac{1}{12}\right) & -\left(\frac{1}{5+j10}\right) \\ -\left(\frac{1}{12}\right) & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & -\left(\frac{1}{-j16}\right) \\ -\left(\frac{1}{5+j10}\right) & & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right) \end{bmatrix} \begin{bmatrix} \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

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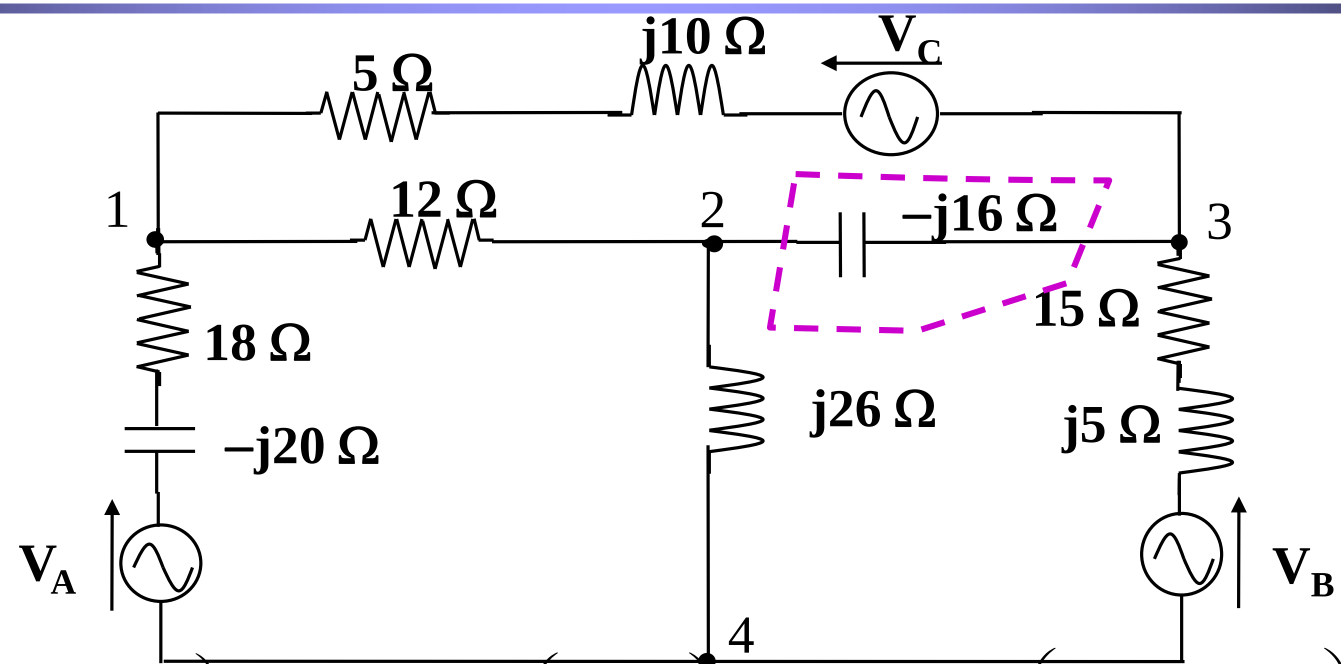
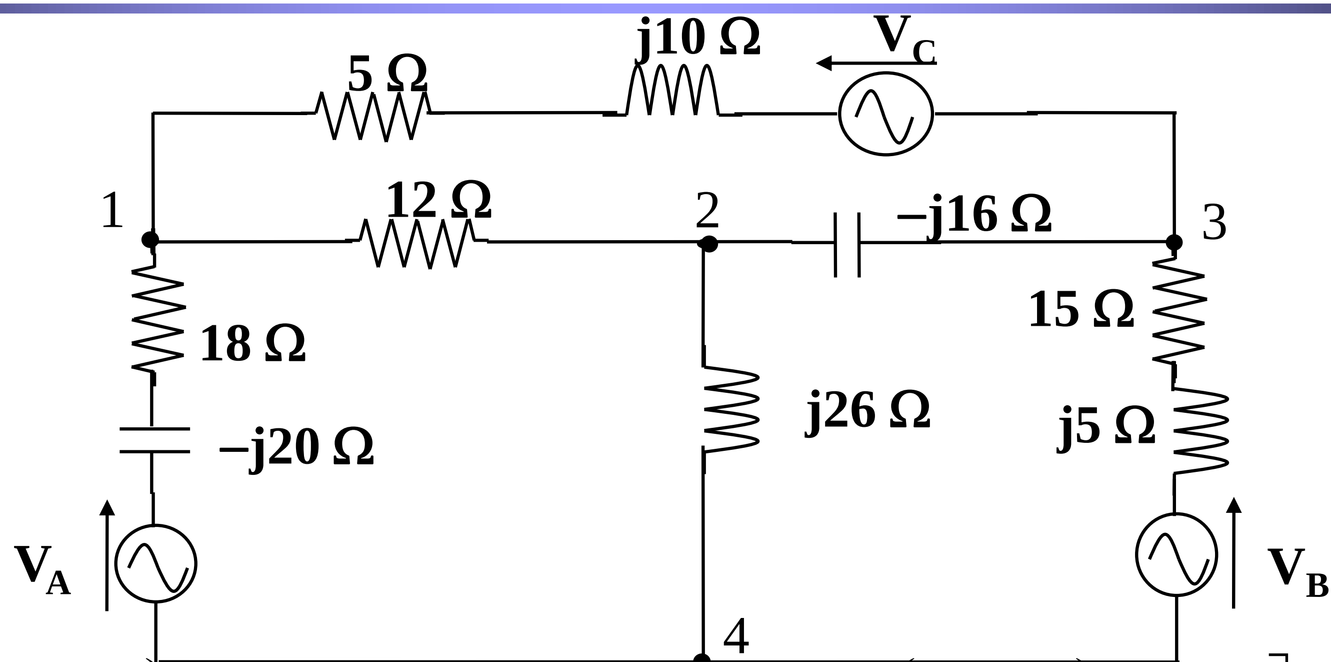


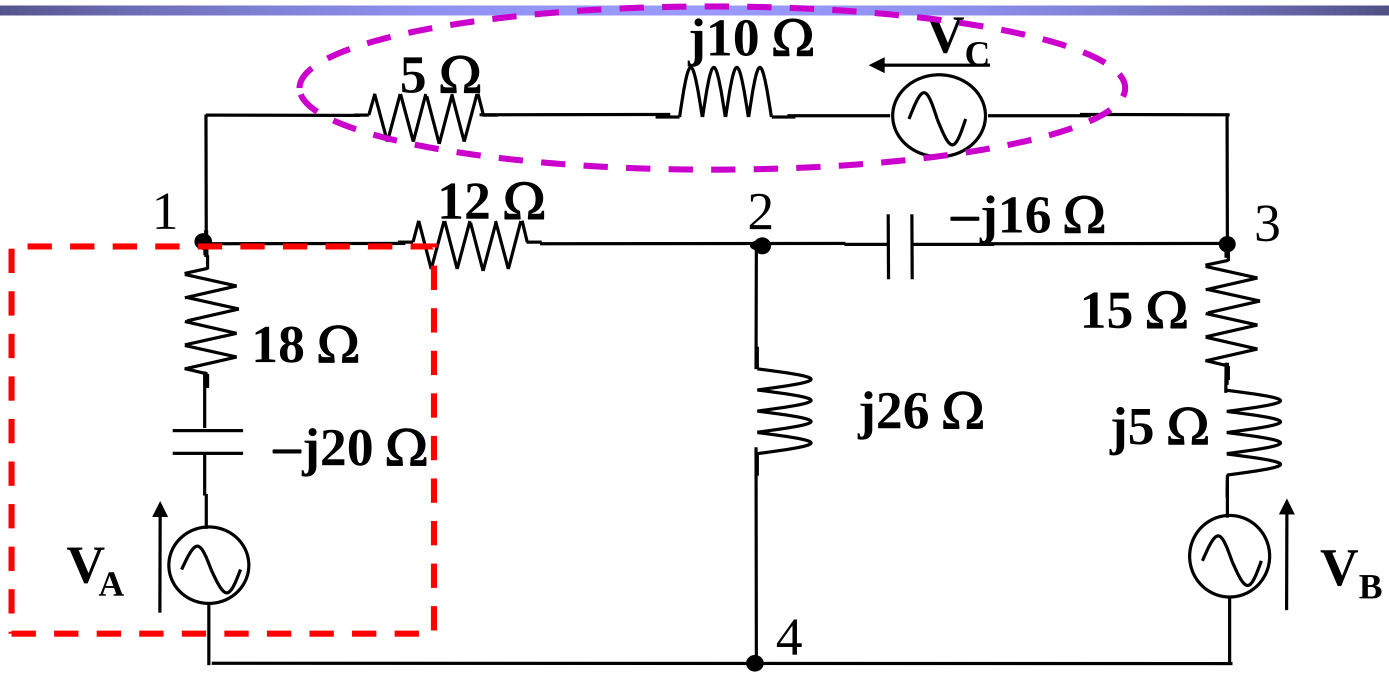
$$\begin{bmatrix} \left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & -\left(\frac{1}{12}\right) & -\left(\frac{1}{5+j10}\right) \\ -\left(\frac{1}{12}\right) & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & -\left(\frac{1}{-j16}\right) \\ -\left(\frac{1}{5+j10}\right) & -\left(\frac{1}{-j16}\right) & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right) \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

Example 1.5



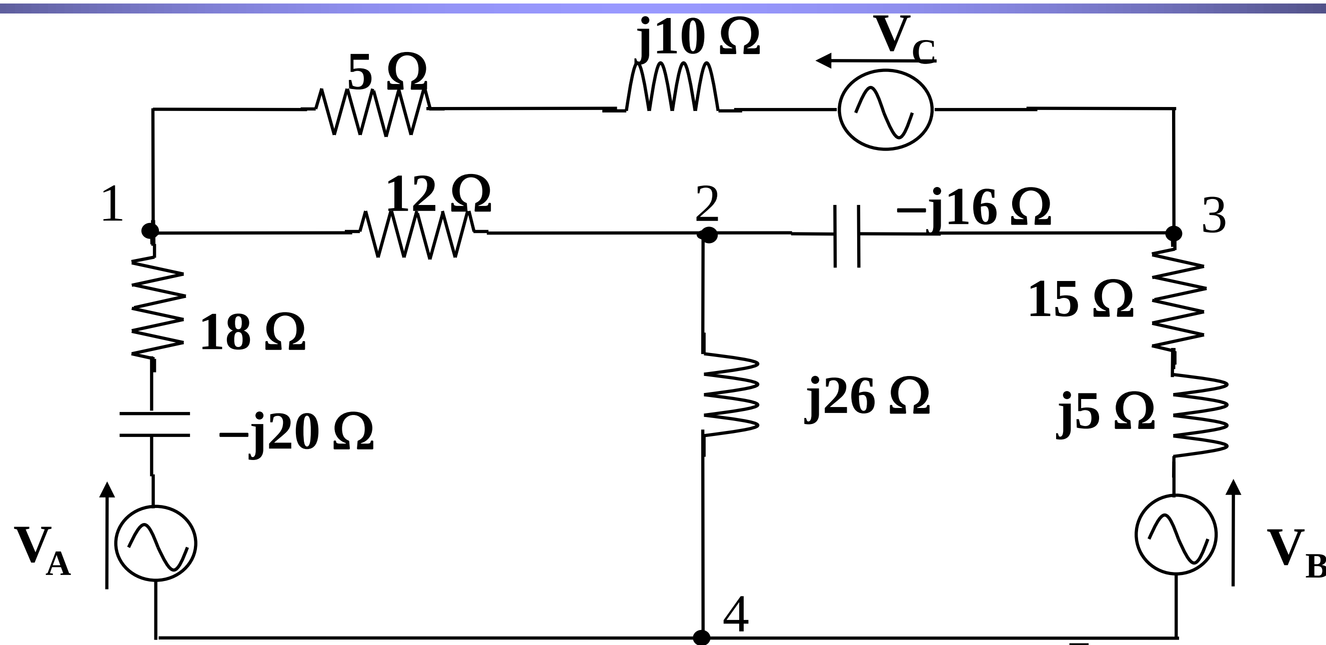
$$\begin{bmatrix} \left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & -\left(\frac{1}{12}\right) & -\left(\frac{1}{5+j10}\right) \\ -\left(\frac{1}{12}\right) & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & -\left(\frac{1}{-j16}\right) \\ -\left(\frac{1}{5+j10}\right) & -\left(\frac{1}{-j16}\right) & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Example 1.5



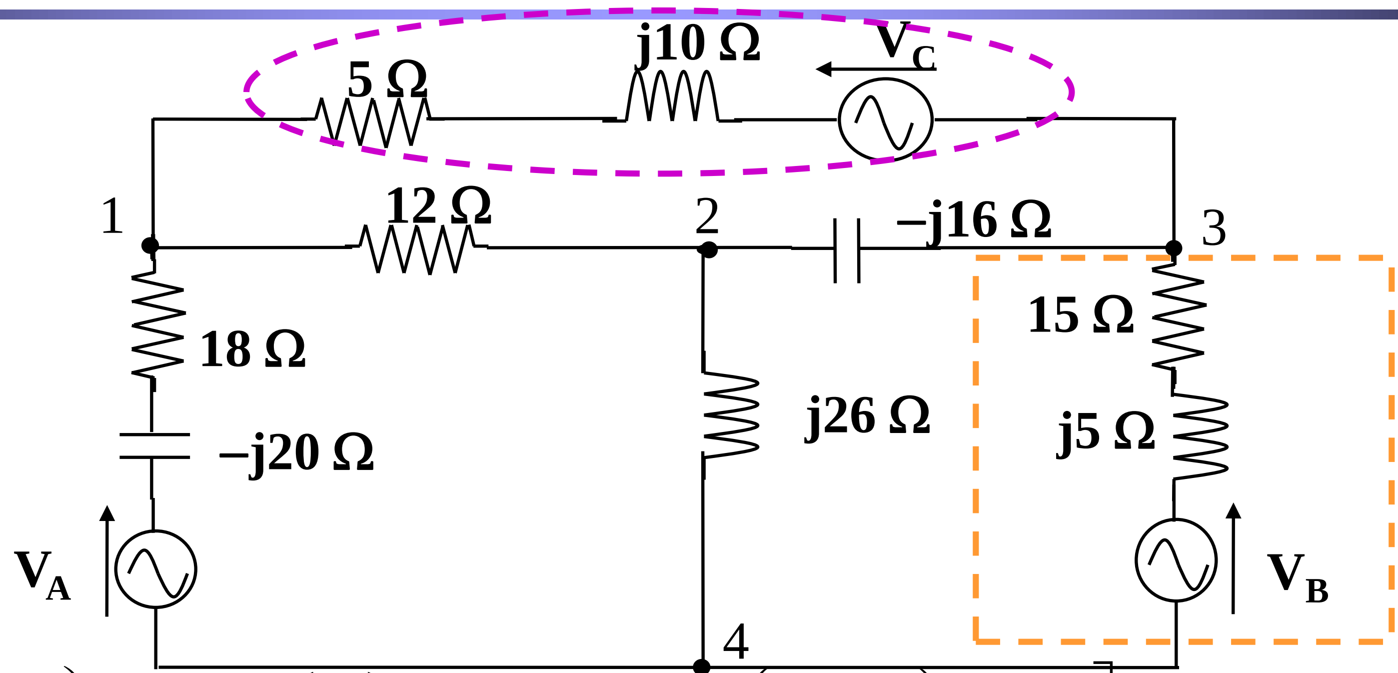
$$\begin{bmatrix} \left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & -\left(\frac{1}{12}\right) & -\left(\frac{1}{5+j10}\right) \\ -\left(\frac{1}{12}\right) & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & -\left(\frac{1}{-j16}\right) \\ -\left(\frac{1}{5+j10}\right) & -\left(\frac{1}{-j16}\right) & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{V_A}{18-j20}+\frac{V_C}{5+j10} \\ \\ \\ \end{bmatrix}$$

Example 1.5



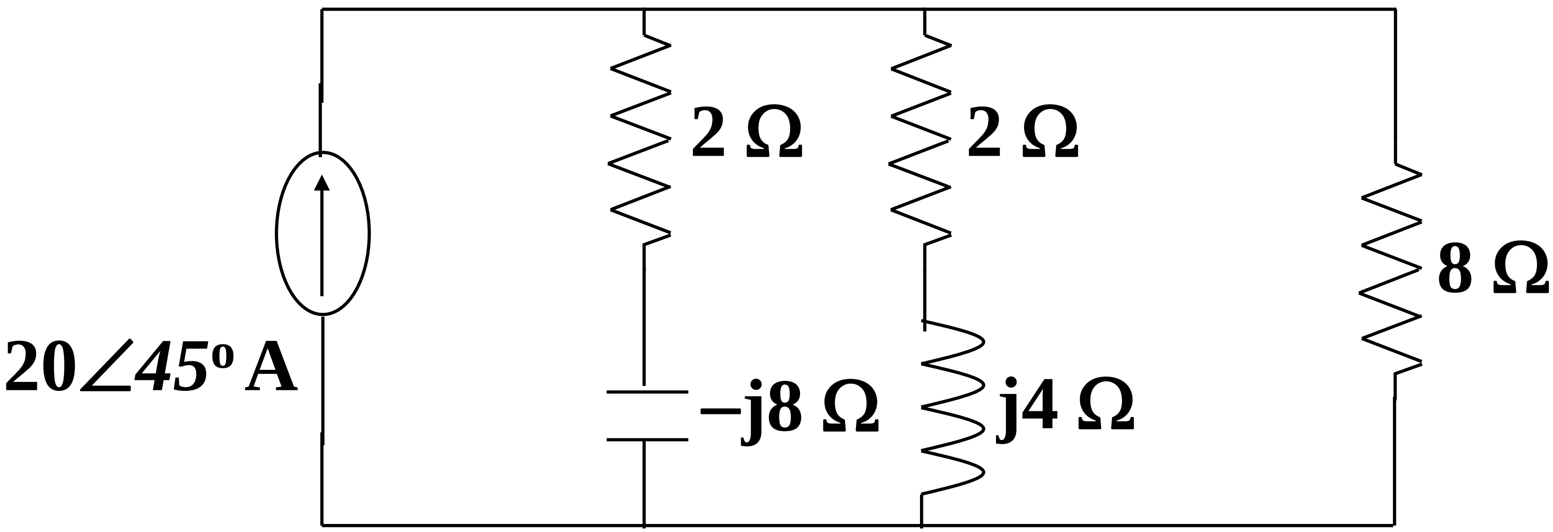
$$\begin{bmatrix} \left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & -\left(\frac{1}{12}\right) & -\left(\frac{1}{5+j10}\right) \\ -\left(\frac{1}{12}\right) & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & -\left(\frac{1}{-j16}\right) \\ -\left(\frac{1}{5+j10}\right) & -\left(\frac{1}{-j16}\right) & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{V_A}{18-j20}+\frac{V_C}{5+j10} \\ 0 \\ \\ \end{bmatrix}$$

Example 1.5



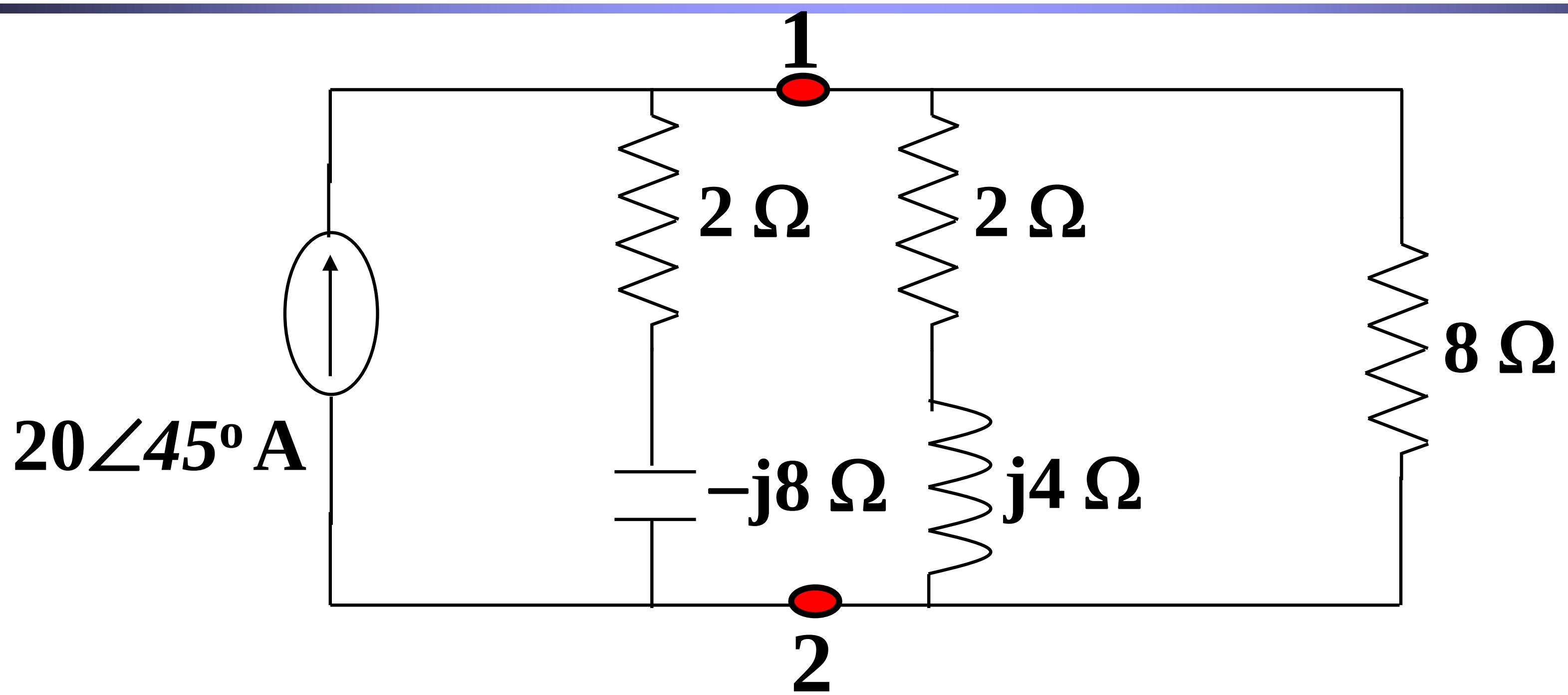
$$\begin{bmatrix} \left(\frac{1}{18-j20}+\frac{1}{12}+\frac{1}{5+j10}\right) & -\left(\frac{1}{12}\right) & -\left(\frac{1}{5+j10}\right) \\ -\left(\frac{1}{12}\right) & \left(\frac{1}{12}+\frac{1}{-j16}+\frac{1}{j26}\right) & -\left(\frac{1}{-j16}\right) \\ -\left(\frac{1}{5+j10}\right) & -\left(\frac{1}{-j16}\right) & \left(\frac{1}{5+j10}+\frac{1}{-j16}+\frac{1}{15+j5}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{V_A}{18-j20}+\frac{V_C}{5+j10} \\ 0 \\ \frac{V_B}{15+j5}-\frac{V_C}{5+j10} \end{bmatrix}$$

Example 1.6



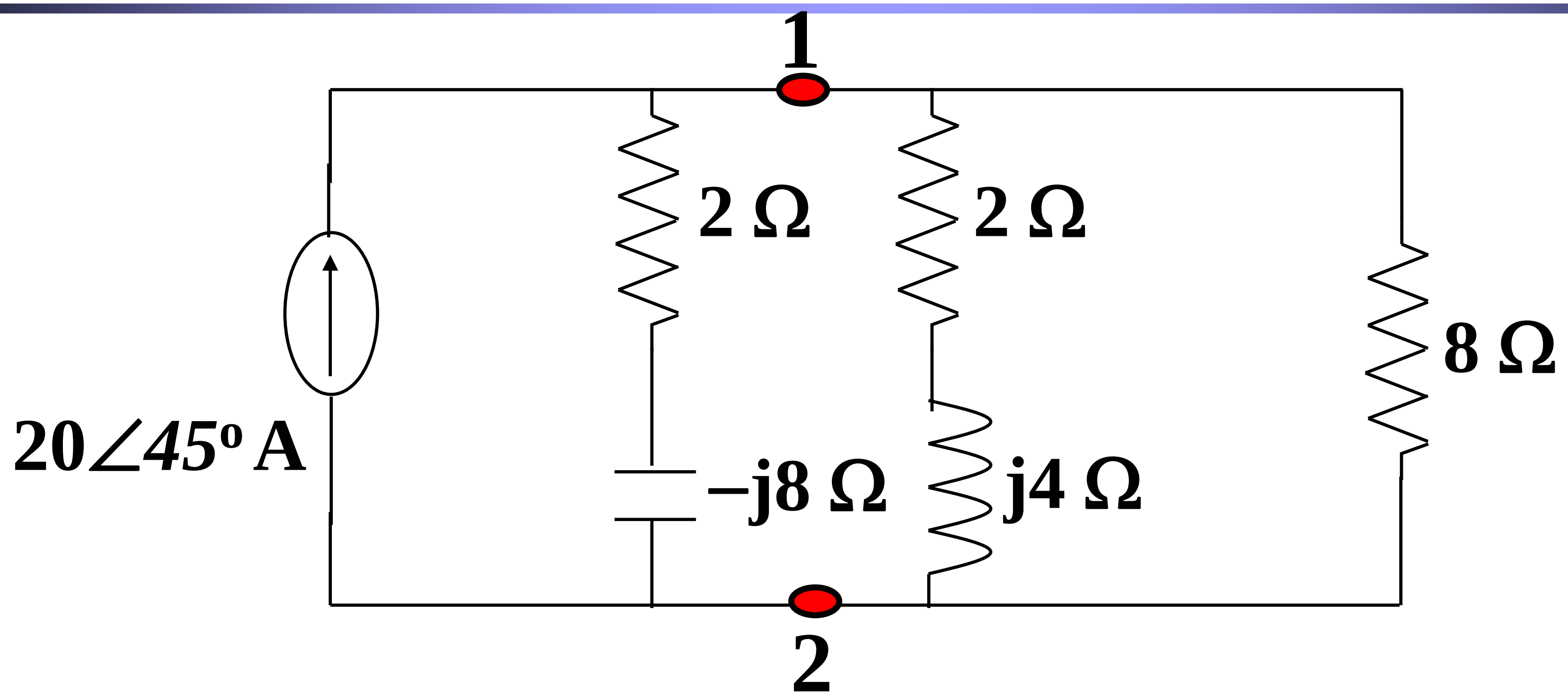
Find the current in the 8 Ω resistor using node voltage analysis method.

Example 1.6



Two nodes and taking node 2 as the reference, the admittance matrix is a 1 x 1 matrix.

Example 1.6



$$\left[\frac{1}{2-j8}+\frac{1}{2+j4}+\frac{1}{8}\right]\left[V_1\right]=\left[20\angle 45^\circ\right]$$

Example 1.6

$$\left[\frac{1}{2-j8}+\frac{1}{2+j4}+\frac{1}{8}\right]V_1 = 20\angle 45^\circ$$

$$\left[\frac{1}{8.24\angle -75.96^0}+\frac{1}{4.47\angle 63.43^0}+0.125\right]V_1 = 20\angle 45^\circ$$

$$\left[0.121\angle 75.96^0 + 0.224\angle -63.43^0 + 0.125\right]V_1 = 20\angle 45^\circ$$

$$\left[0.029 + j0.117 + 0.1 - j0.2 + 0.125\right]V_1 = 20\angle 45^\circ$$

Example 1.6

$$(0.254 - j0.083)V_1 = 20\angle 45^\circ$$

$$\therefore\ V_1 = \frac{20\angle 45^\circ}{0.254 - j0.083} = \frac{20\angle 45^\circ}{0.267\angle -18.09} = 74.9\angle 63.09^\circ$$

$$\text{Hence } I_{8\Omega} = \frac{V_1}{8} = 9.36\angle 63.09^\circ\,\text{A}$$

…next topic

Star-Delta & Delta-Star Transformation

Nurturing Curious Minds, Producing Passionate Engineers

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