Star-Delta and Delta-Star Transformation

Objectives

- Derive the relationships for star-delta and delta to star transformation.
- Use star-delta and delta to star transformation to simplify the given circuit.

Star-Delta and Delta-Star Transformation

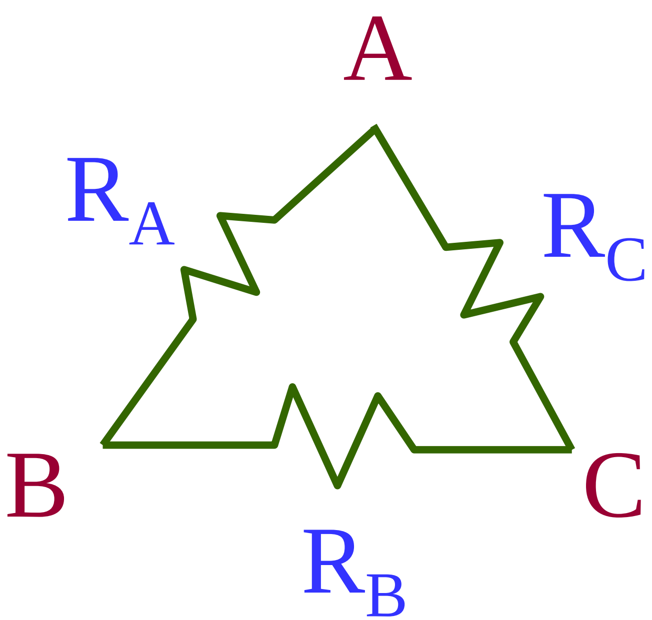
Star-Delta transformation is a mathematical tool where circuits connected in Star (Y) are converted into their Delta (Δ) equivalent, or vice versa.

$$Y \rightarrow \Delta$$

and

$$\Delta \rightarrow Y$$

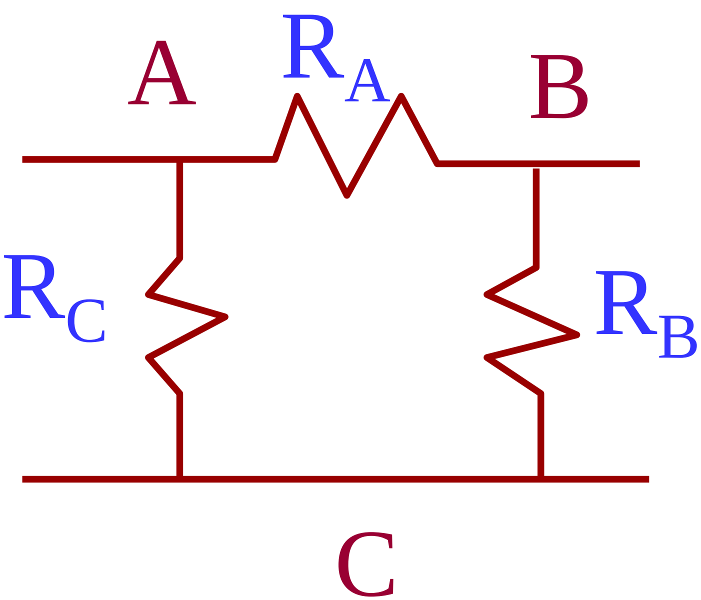
Star-Delta and Delta-Star Transformation



Delta circuit

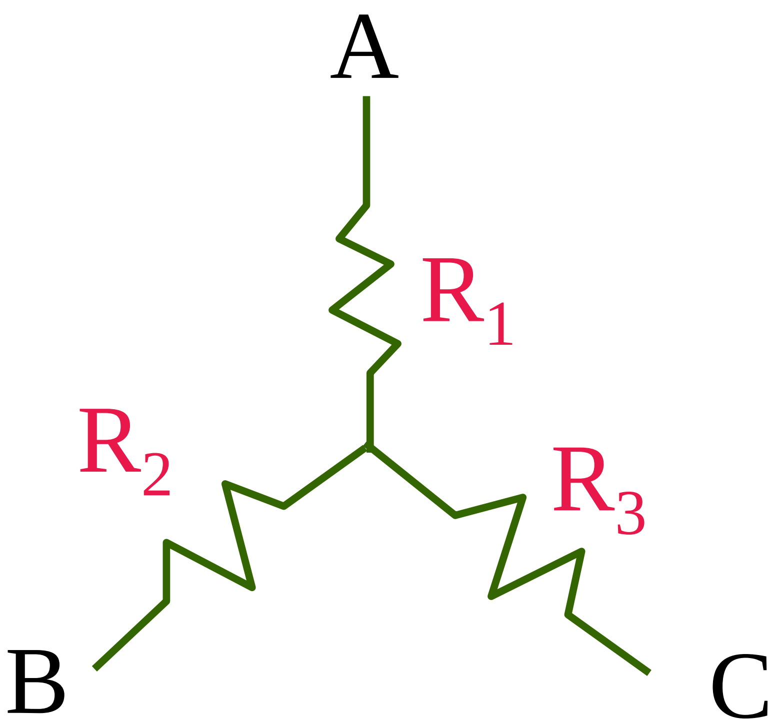
$\equiv$

also called



Π circuit

Star-Delta and Delta-Star Transformation



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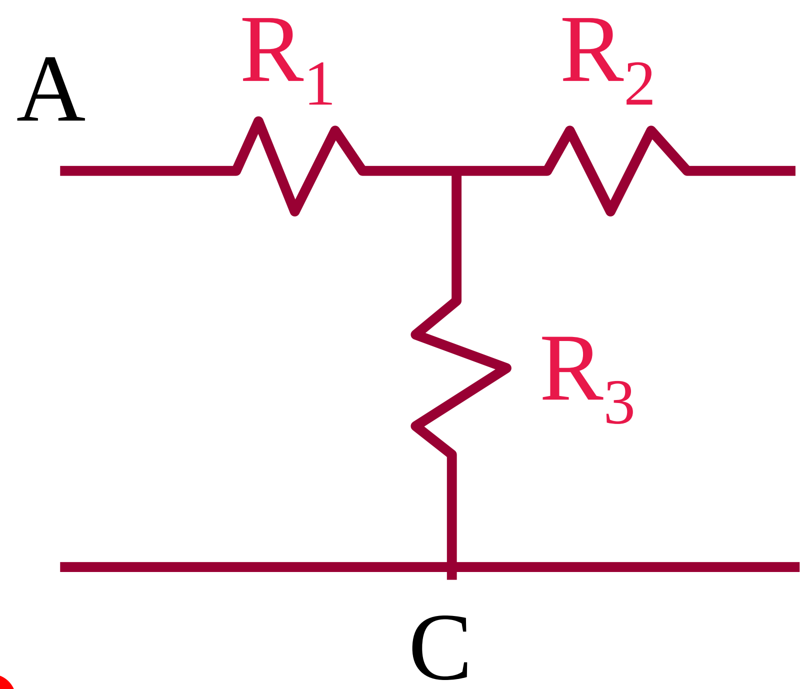
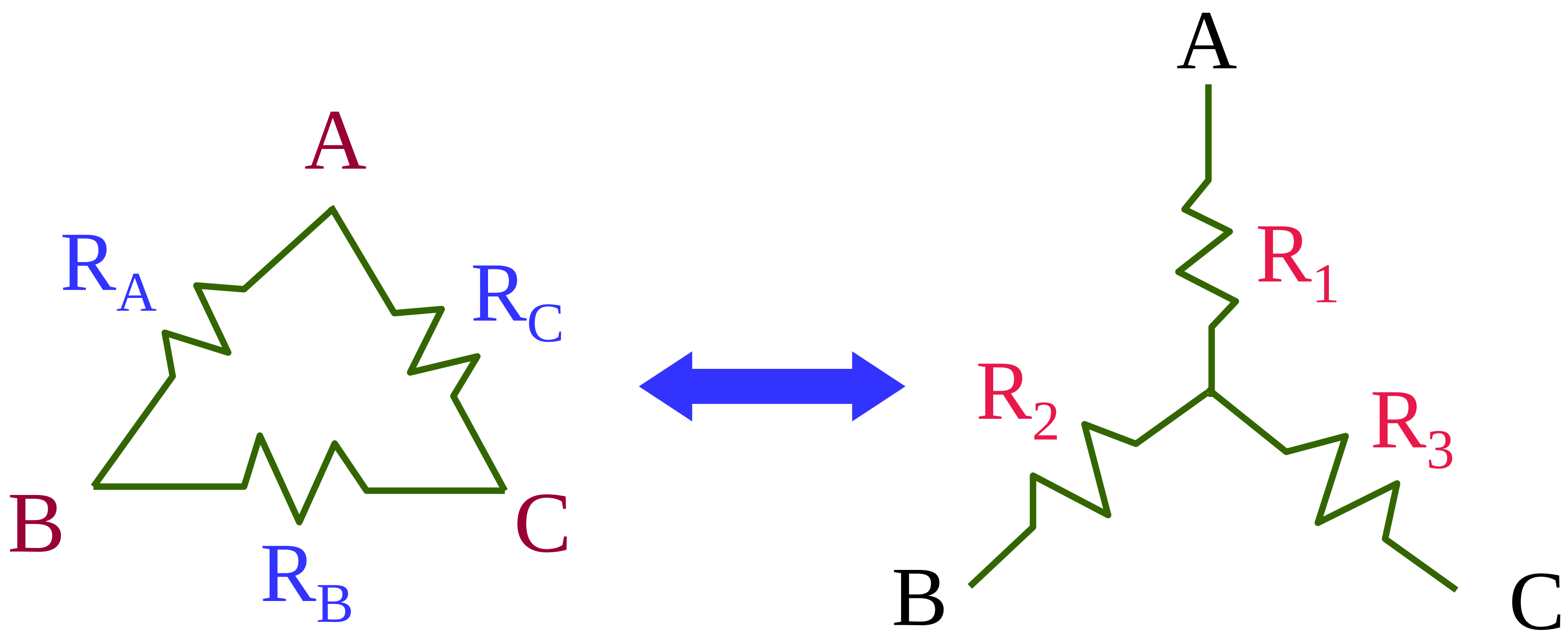


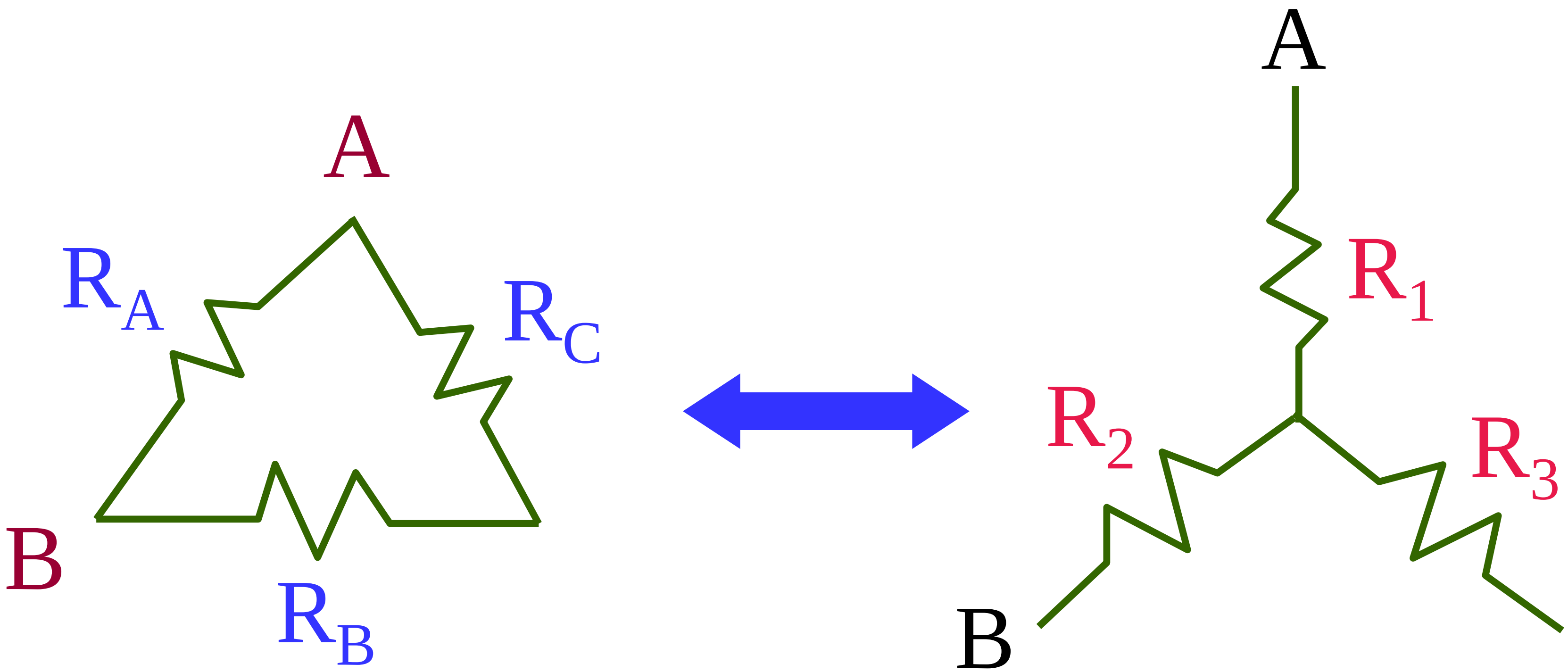
Star circuit ***also called*** T circuit

Star-Delta and Delta-Star Transformation



To make the Δ and the Y equivalent, the impedance across any two terminals in the Δ must be equal to that across the corresponding terminals in the Y.

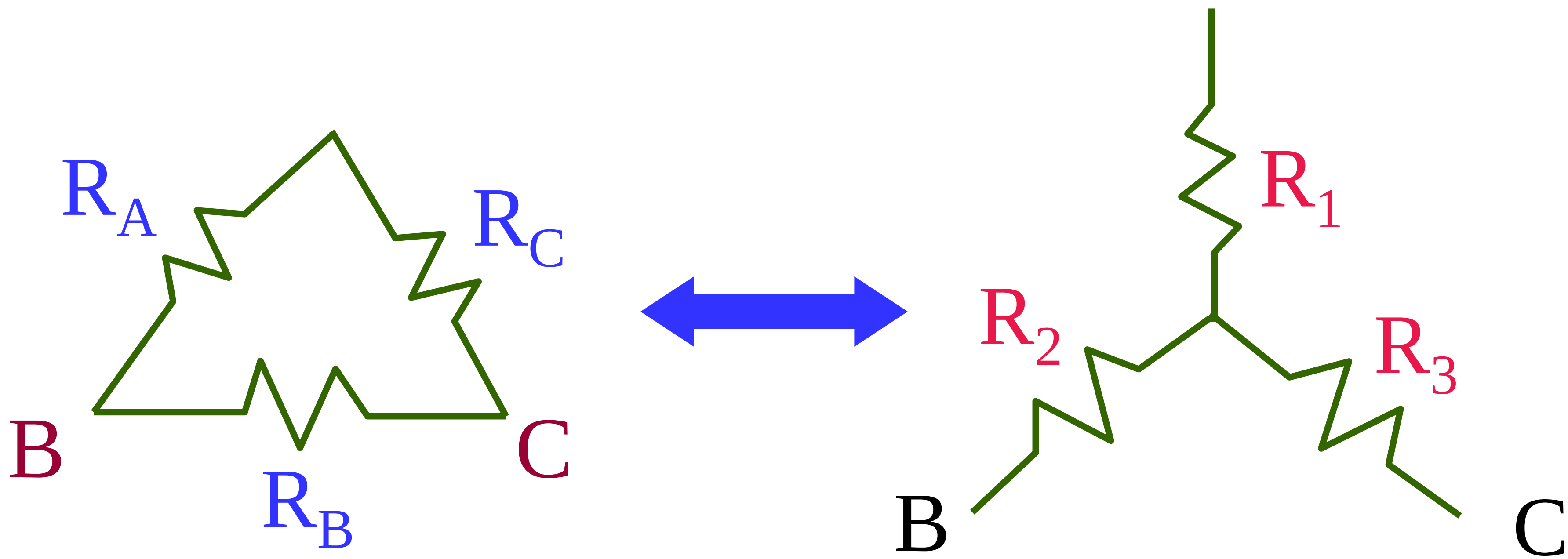
Star-Delta and Delta-Star Transformation



$R_{AB} = R_A \,//\, (R_B + R_C)$ ***Equals*** $R_{AB} = R_1 + R_2$

$$= \frac{R_A(R_B + R_C)}{R_A + R_B + R_C}$$

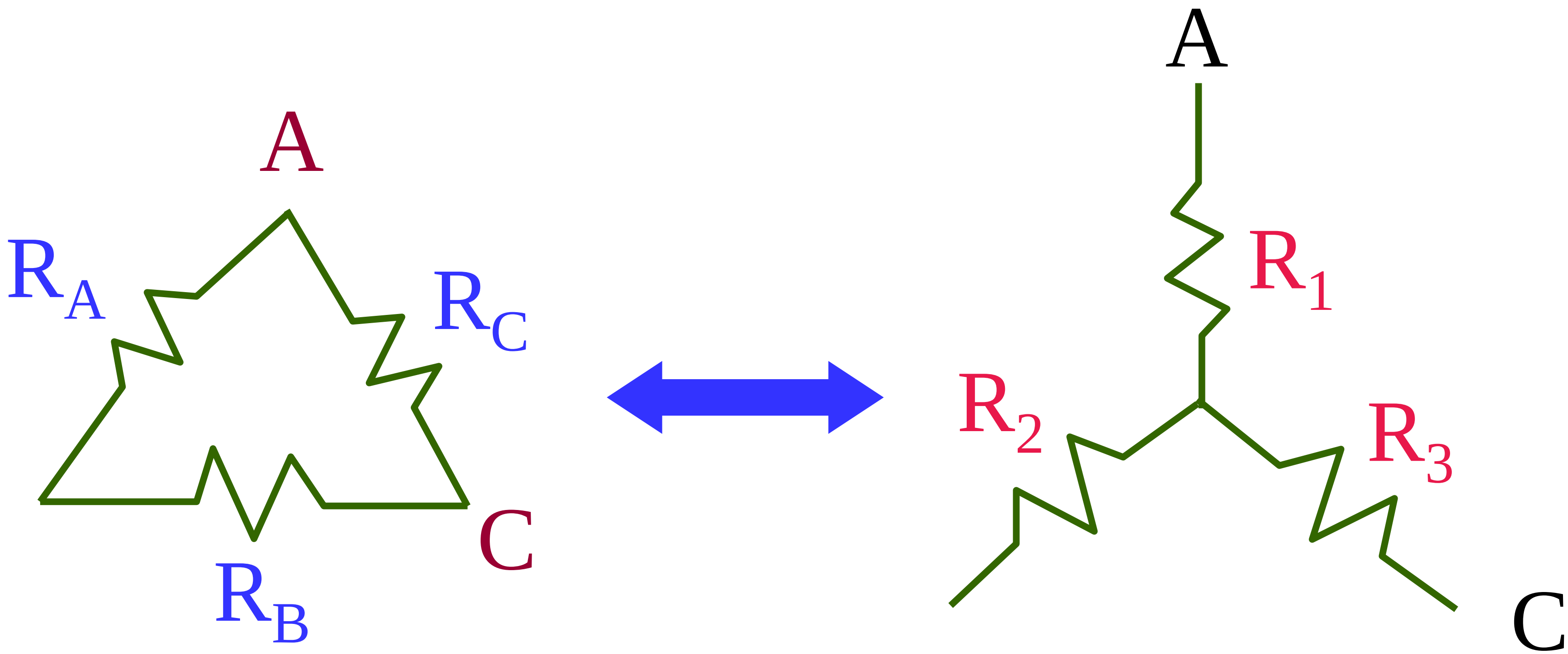
Star-Delta and Delta-Star Transformation



$$\mathbf{R_{BC} = R_B \,//\, (R_A + R_C)} \quad \textbf{\textit{Equals}} \quad \mathbf{R_{BC} = R_2 + R_3}$$

$$= \frac{R_B(R_A + R_C)}{R_A + R_B + R_C}$$

Star-Delta and Delta-Star Transformation



$R_{CA} = R_C \,//(R_A + R_B)$ ***Equals*** $R_{CA} = R_1 + R_3$

$$= \frac{R_C(R_A + R_B)}{R_A + R_B + R_C}$$

Star-Delta and Delta-Star Transformation

For the Δ and the Y to be equivalent, the following three equations must therefore be satisfied ***at the same time.***

For terminals AB $\dfrac{R_A R_B + R_A R_C}{R_A + R_B + R_C} = R_1 + R_2$....(12)

For terminals BC $\dfrac{R_A R_B + R_B R_C}{R_A + R_B + R_C} = R_2 + R_3$....(13)

For terminals CA $\dfrac{R_A R_C + R_B R_C}{R_A + R_B + R_C} = R_1 + R_3$....(14)

Delta-Star Transformation

Equations (12) - (13) + (14) results in

$$2R_1 = \frac{2R_A R_C}{R_A + R_B + R_C} \quad \text{or} \quad R_1 = \frac{R_A R_C}{R_A + R_B + R_C} \quad ...(15)$$

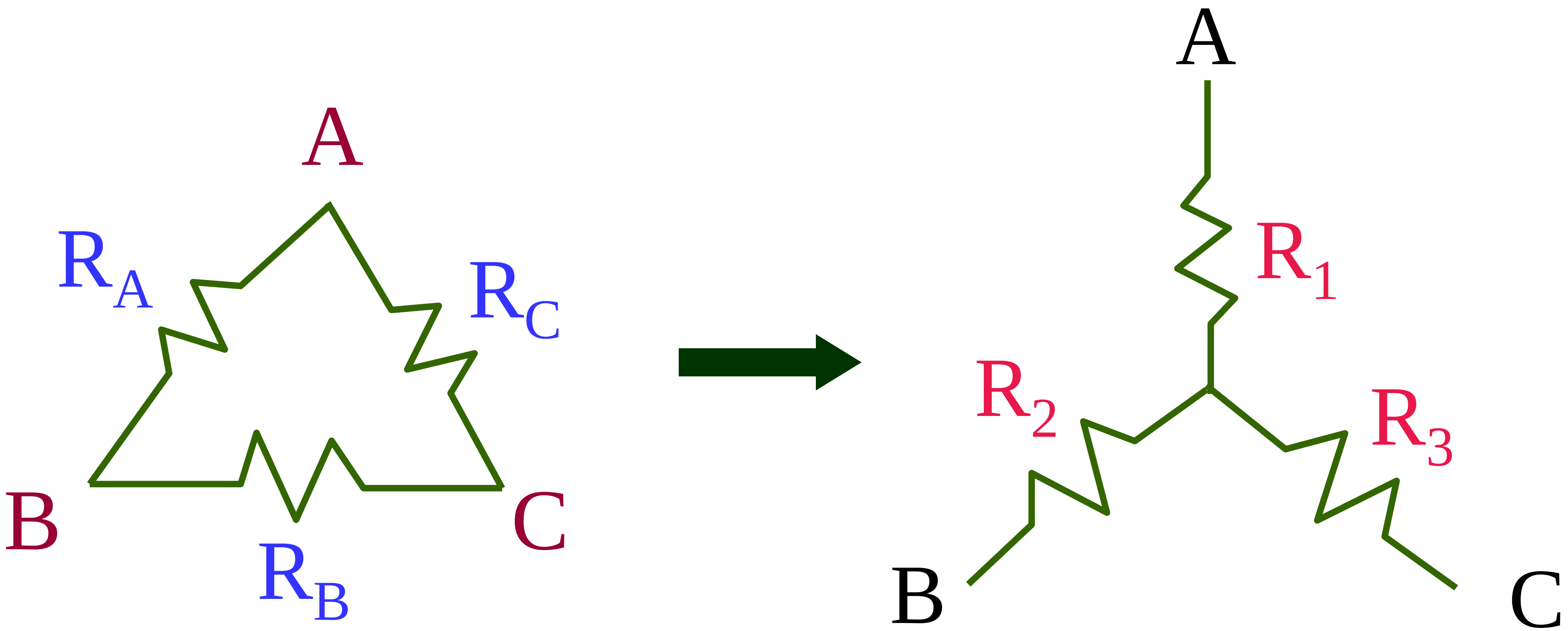
Equations (13) - (14) + (12) results in

$$2R_2 = \frac{2R_A R_B}{R_A + R_B + R_C} \quad \text{or} \quad R_2 = \frac{R_A R_B}{R_A + R_B + R_C} \quad ...(16)$$

Equations (14) - (12) + (13) results in

$$2R_3 = \frac{2R_B R_C}{R_A + R_B + R_C} \quad \text{or} \quad R_3 = \frac{R_B R_C}{R_A + R_B + R_C} \quad ...(17)$$

Equations for Delta to *Star Transformation*



$$R_1 = (R_A R_C) / (R_A + R_B + R_C) \ldots\ldots(15)$$

$$R_2 = (R_A R_B) / (R_A + R_B + R_C) \ldots\ldots(16)$$

$$R_3 = (R_B R_C) / (R_A + R_B + R_C) \ldots\ldots(17)$$

Star-Delta Transformation

Equations (17) divided by (15) will result in

$$R_B = \frac{R_A R_3}{R_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

Equations (17) divided by (16) will result in

$$R_C = \frac{R_A R_3}{R_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

Substituting (18) and (19) into (17) will result in

$$R_3 = \frac{\left(\frac{R_A R_3}{R_1}\right)\left(\frac{R_A R_3}{R_2}\right)}{R_A + \frac{R_A R_3}{R_1} + \frac{R_A R_3}{R_2}} = \frac{R_A{}^2 R_3{}^2}{R_A R_1 R_2 + R_A R_2 R_3 + R_A R_1 R_3}$$

Star-Delta Transformation

$$1 = \frac{R_A R_3}{R_1R_2 + R_2R_3 + R_1R_3}$$

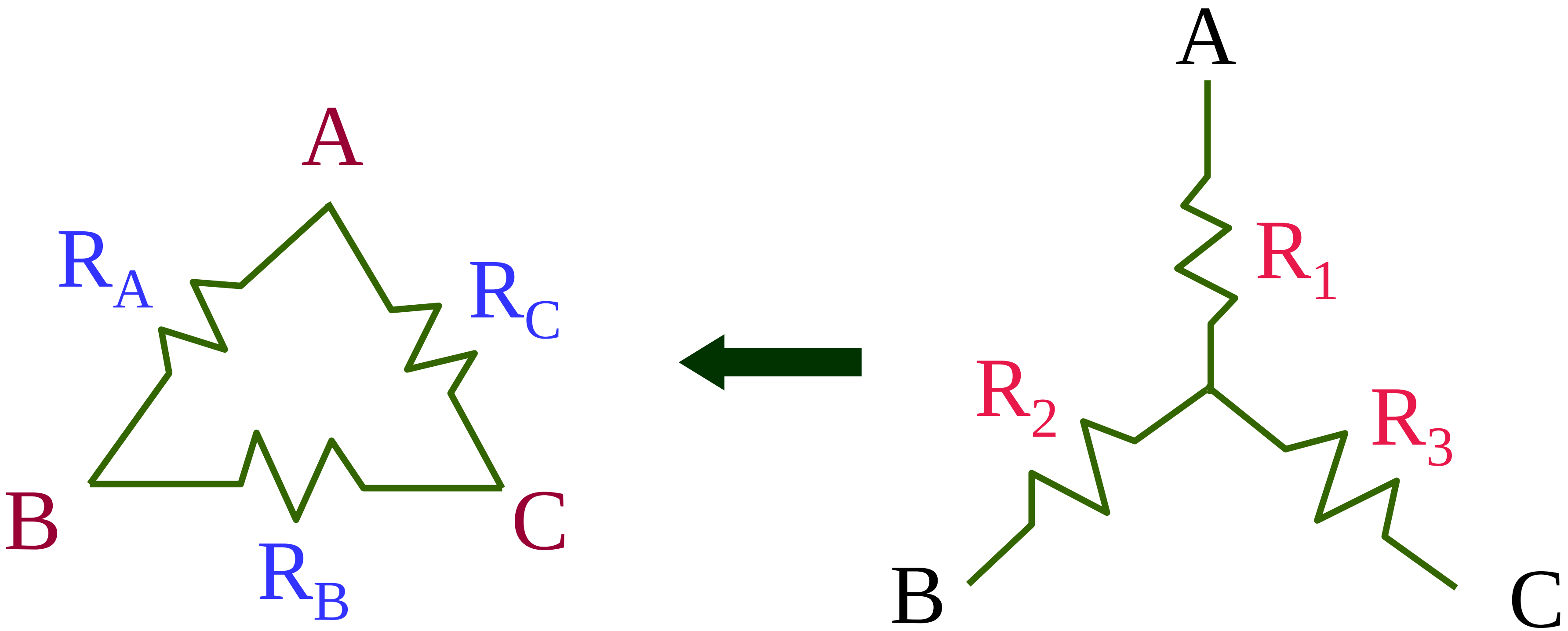
$$R_A = R_1 + R_2 + \frac{R_1R_2}{R_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

Similarly with the same approach, we can solve for R_B and $R_{C.}$

$$R_B = R_2 + R_3 + \frac{R_2R_3}{R_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

$$R_C = R_1 + R_3 + \frac{R_1R_3}{R_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

Equations for *Star to Delta Transformation*



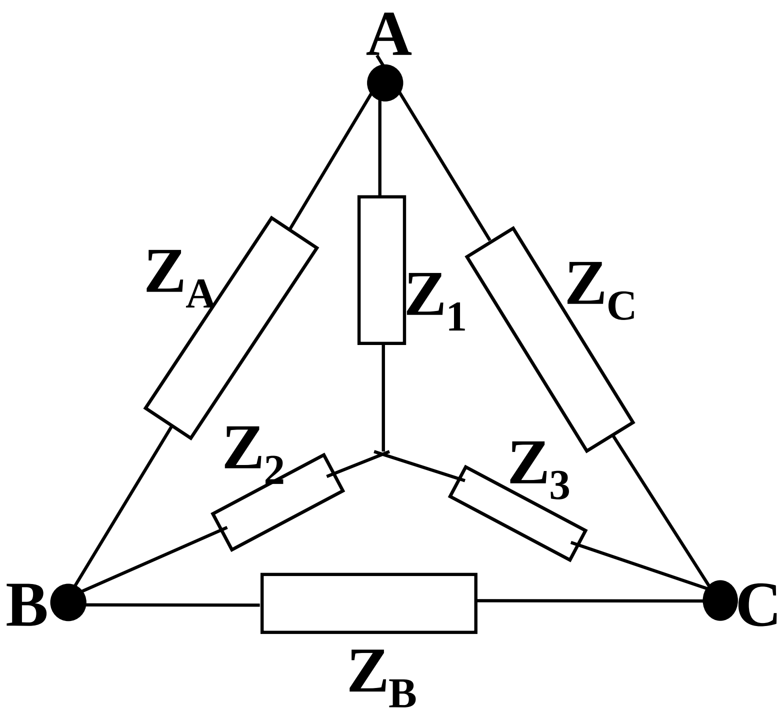
$$R_A = R_1 + R_2 + (R_1R_2) / R_3 \ldots\ldots\ldots\ldots (20)$$

$$R_B = R_2 + R_3 + (R_2R_3) / R_1 \ldots\ldots\ldots\ldots (21)$$

$$R_C = R_1 + R_3 + (R_1R_3) / R_2 \ldots\ldots\ldots\ldots (22)$$

Summary of transformation rules:

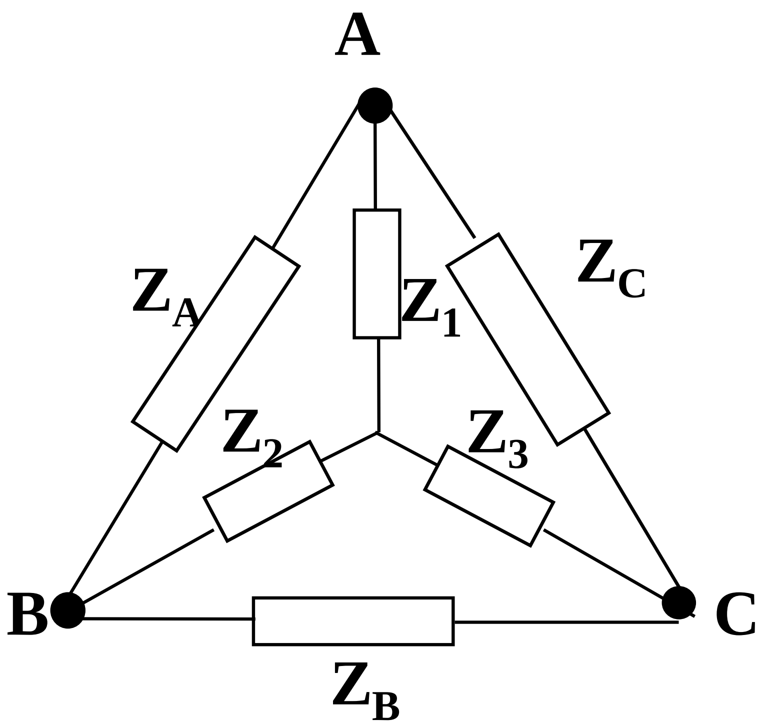
Delta to star



$$Z_1 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C} \qquad Z_2 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C} \qquad Z_3 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C}$$

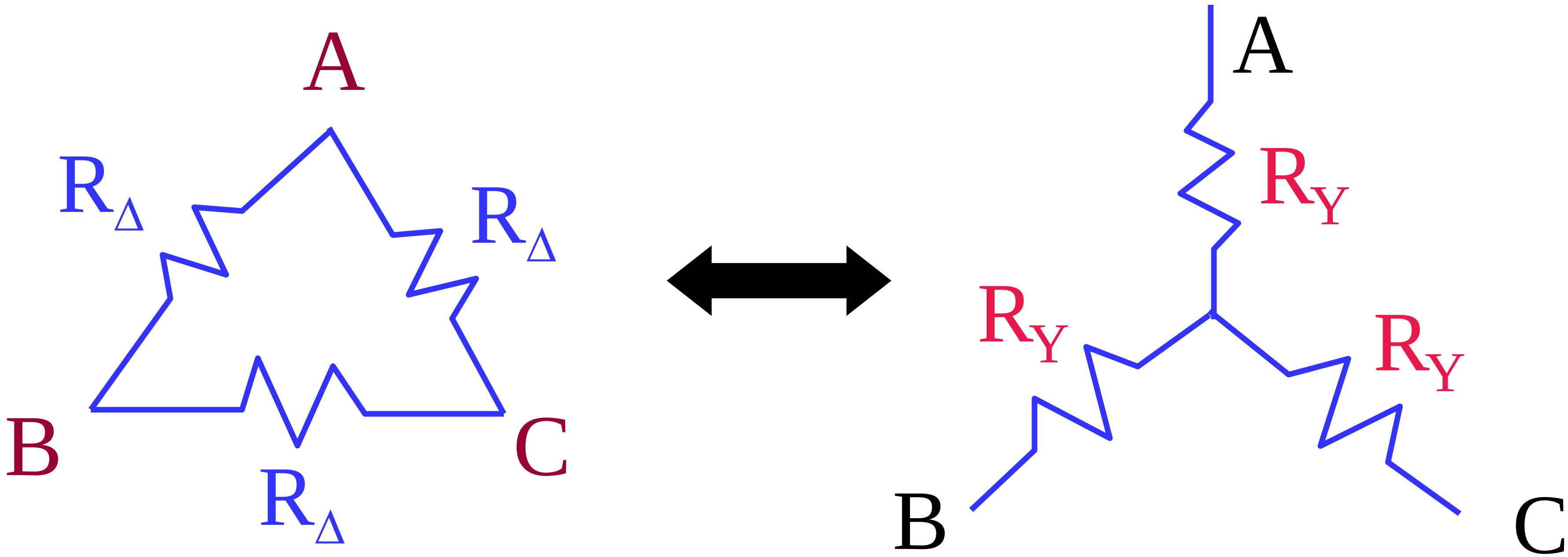
Summary of transformation rules:

Star to delta



$$Z_A = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3} \qquad Z_B = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1} \qquad Z_C = Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2}$$

Equations for Balanced Star to Delta or Balanced Delta to Star Transformation



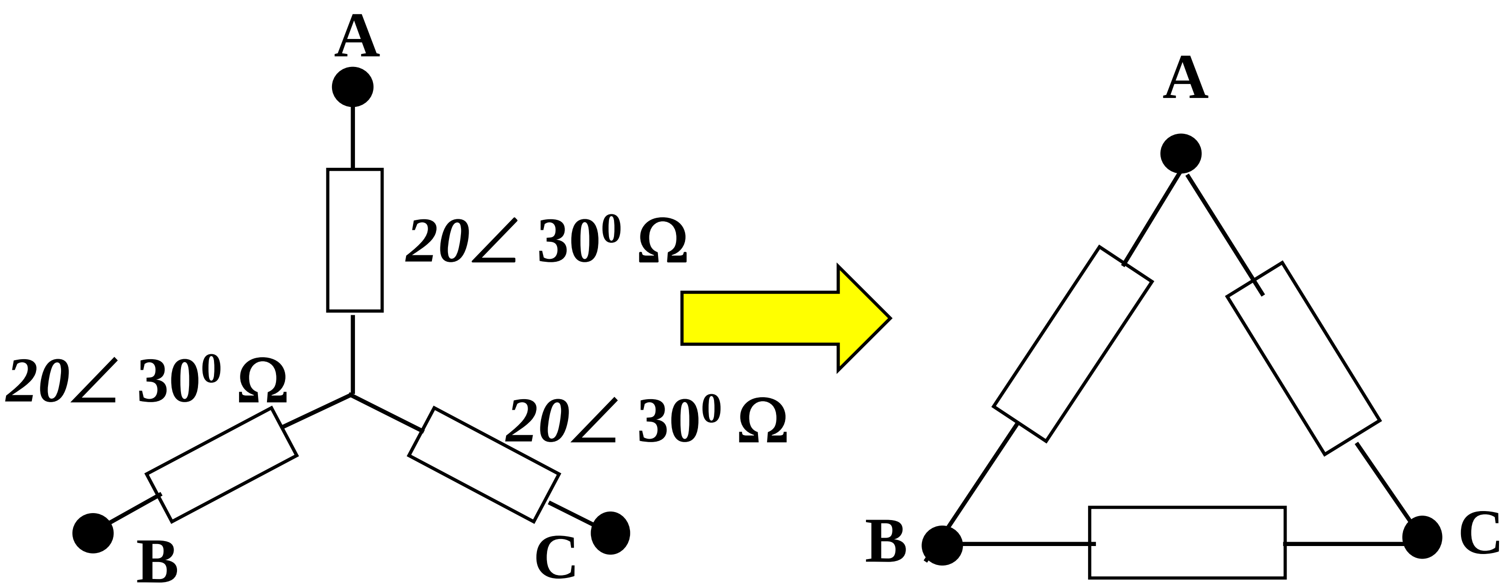
When the Δ or the Y circuit is balanced

$$R_\Delta = R_Y + R_Y + (R_Y R_Y) / R_Y$$

giving $R_\Delta = 3 R_Y$ or $Z_\Delta = 3 Z_Y$

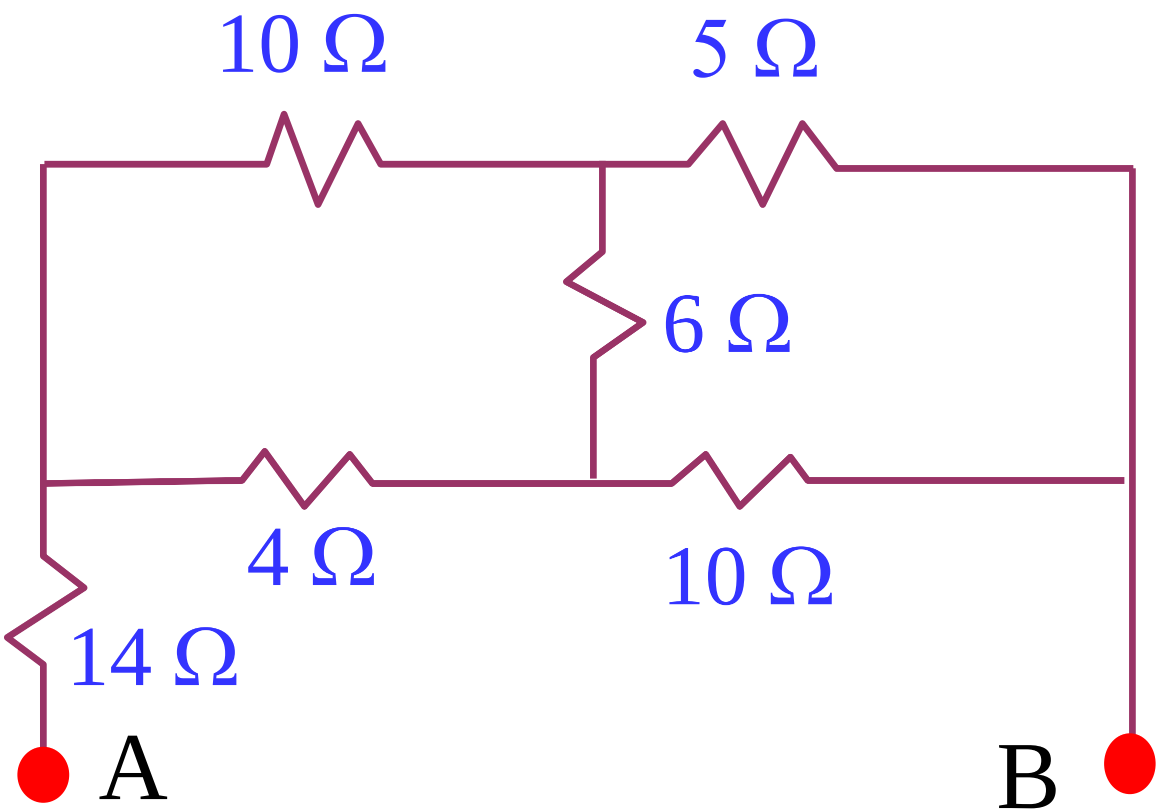
Example 1.7

- Find the delta equivalent of the balanced star network.



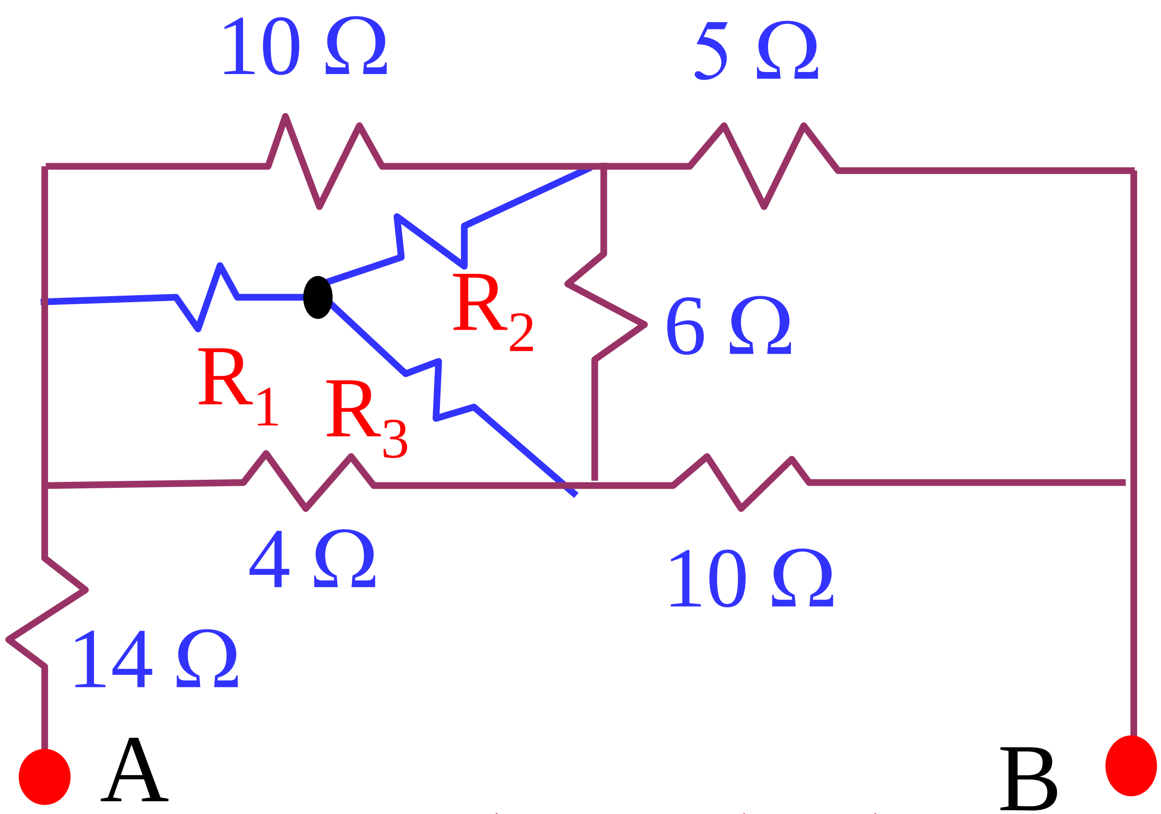
☑Solution: $Z_D = 3xZ_S = 3 \text{ x } 20\angle 30^o\ \Omega = 60\angle 30^o\ \Omega$

Example 1.8



Find R_{AB}

Example 1.8



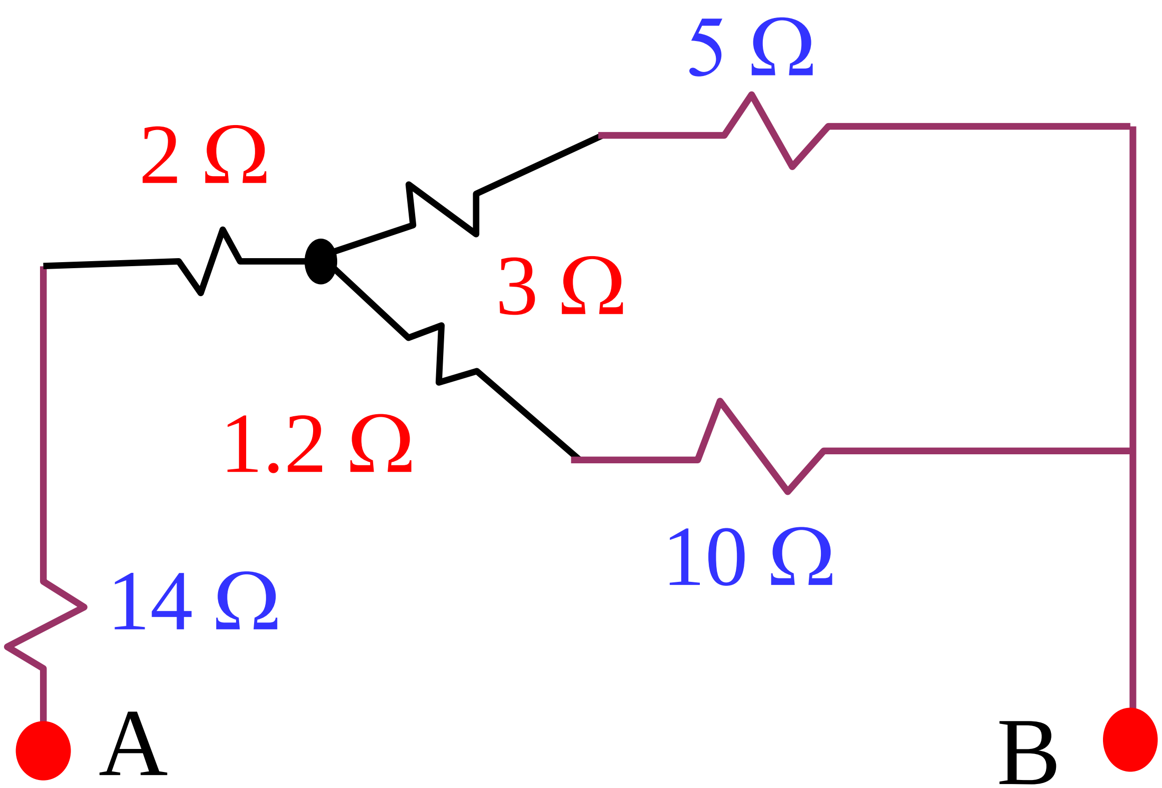
The Δ formed by the 10 Ω, 6 Ω and 4 Ω resistors is transformed to Y formed by R_1, R_2 and R_3

$R_1 = (10 \times 4) / (10 + 6 + 4) = 2\ \Omega$

$R_2 = (10 \times 6) / (10 + 6 + 4) = 3\ \Omega$

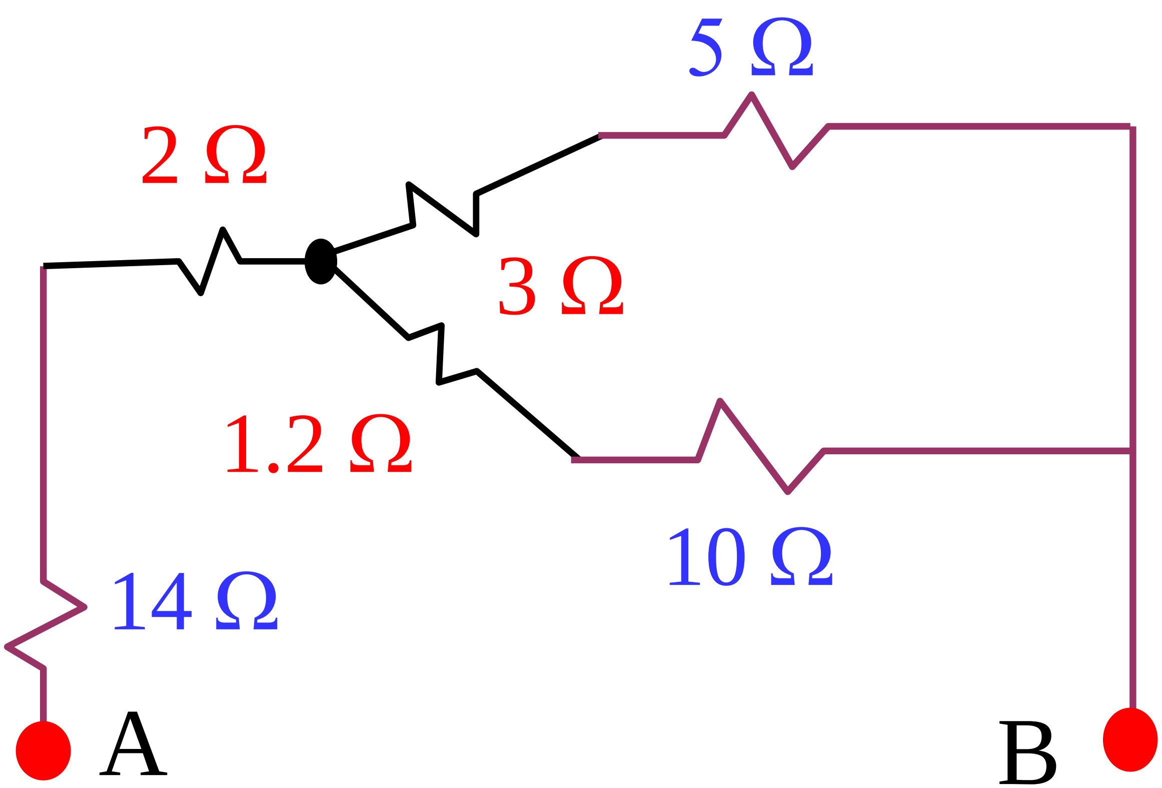
$R_3 = (4 \times 6) / (10 + 6 + 4) = 1.2\ \Omega$

Example 1.8



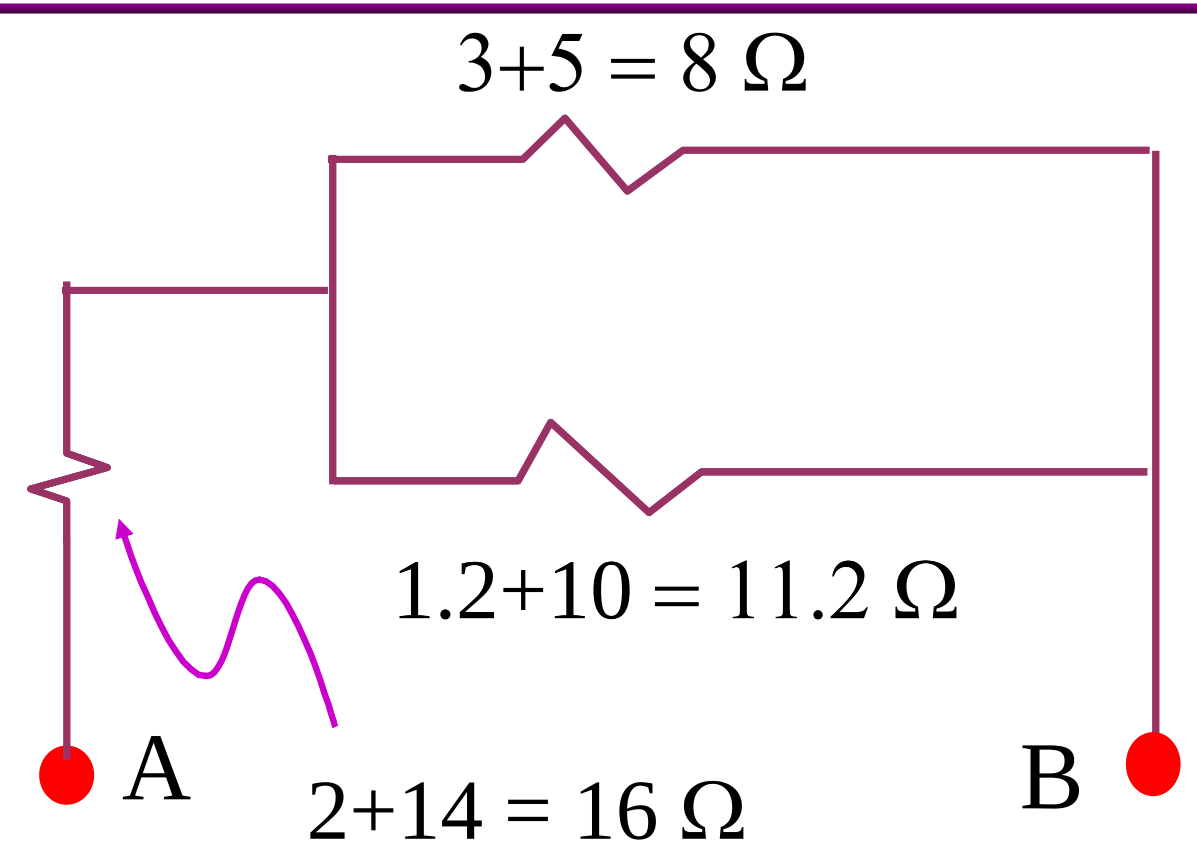
The 10 Ω, 6 Ω and 4 Ω resistors in Delta are now replaced by the R_1, R_2 and R_3 in Star

Example 1.8



Now add the series resistors together, the circuit then becomes.......

Example 1.8



Finally, $R_{AB} = 16 + (8 \,//\, 11.2) = 20.67\ \Omega$

Tutorial 3, Question 1

A network is arranged as shown in Figure 1. Calculate the equivalent resistance between A & C using star-delta transformation.

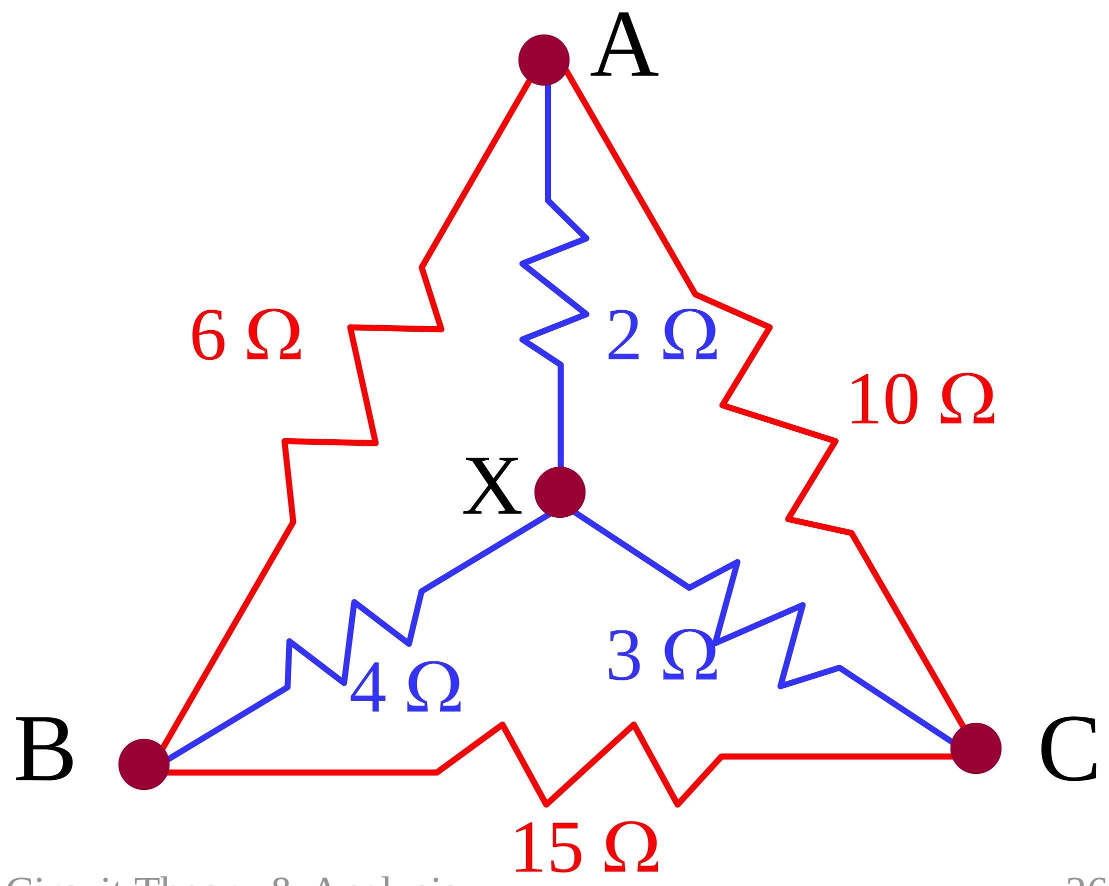
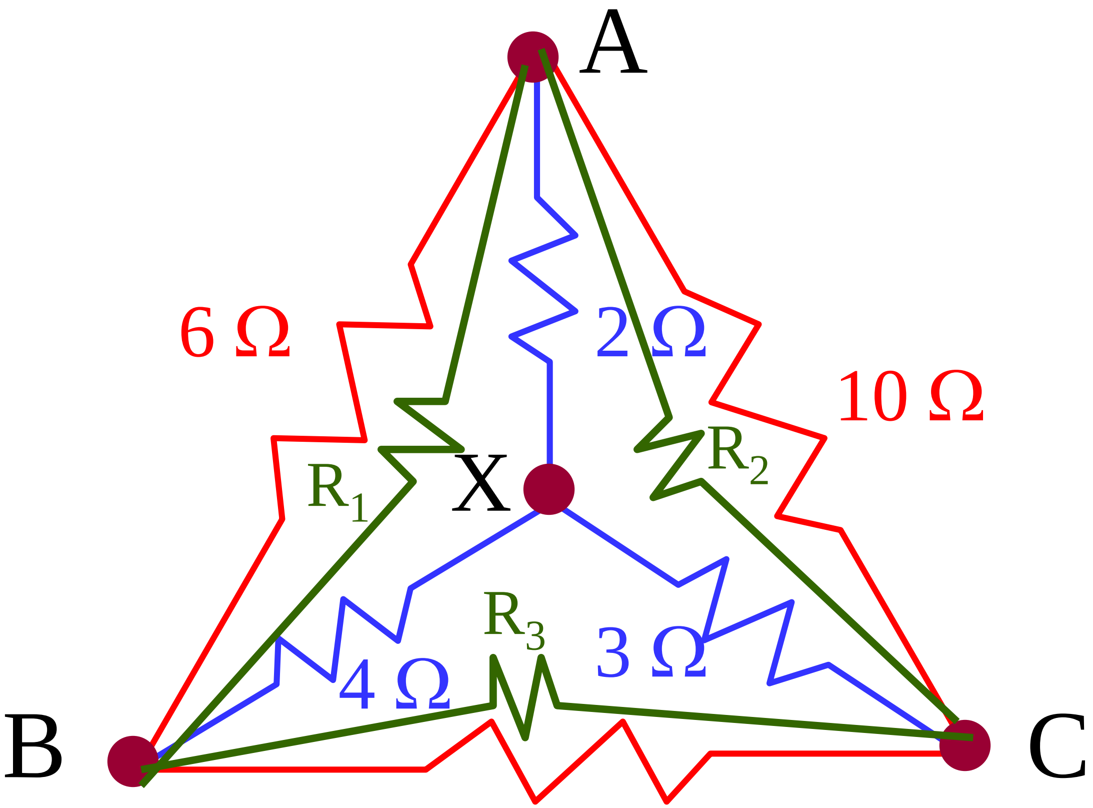


Figure 1

Tutorial 3, Question 1



Transform the blue star into the **green** delta.

$\mathbf{R_1} = 2 + 4 + (2 \times 4) / 3$

$= 8.667\ \Omega$

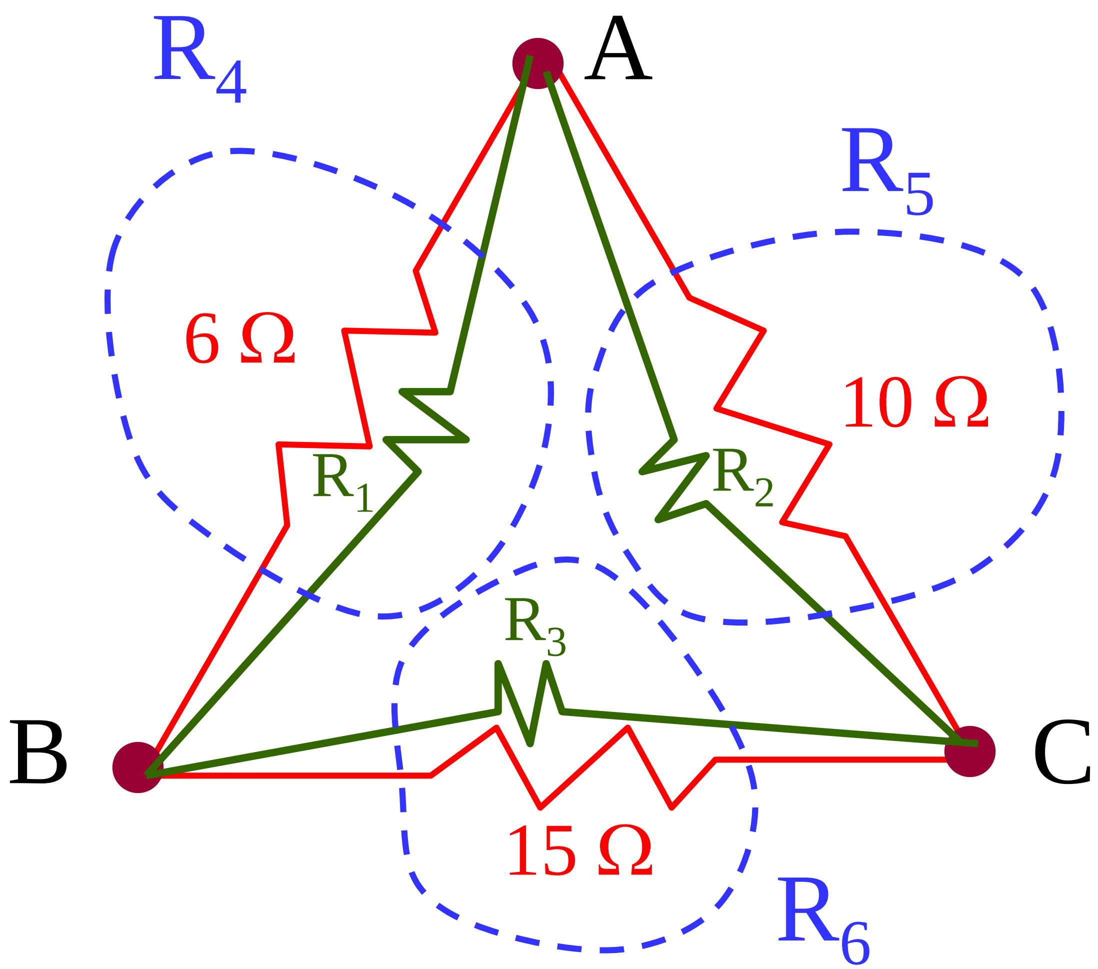
$\mathbf{R_2} = 2 + 3 + (2 \times 3) / 4$

$= 6.5\ \Omega$

$\mathbf{R_3} = 3 + 4 + (3 \times 4) / 2$

$= 13\ \Omega$

Tutorial 3, Question 1



Now the 6 Ω resistor is parallel to R_1. Similarly for the 10 Ω & R_2 and 15 Ω & R_3

R_4 = (6x8.667)/(6+8.667)

= 3.55 Ω

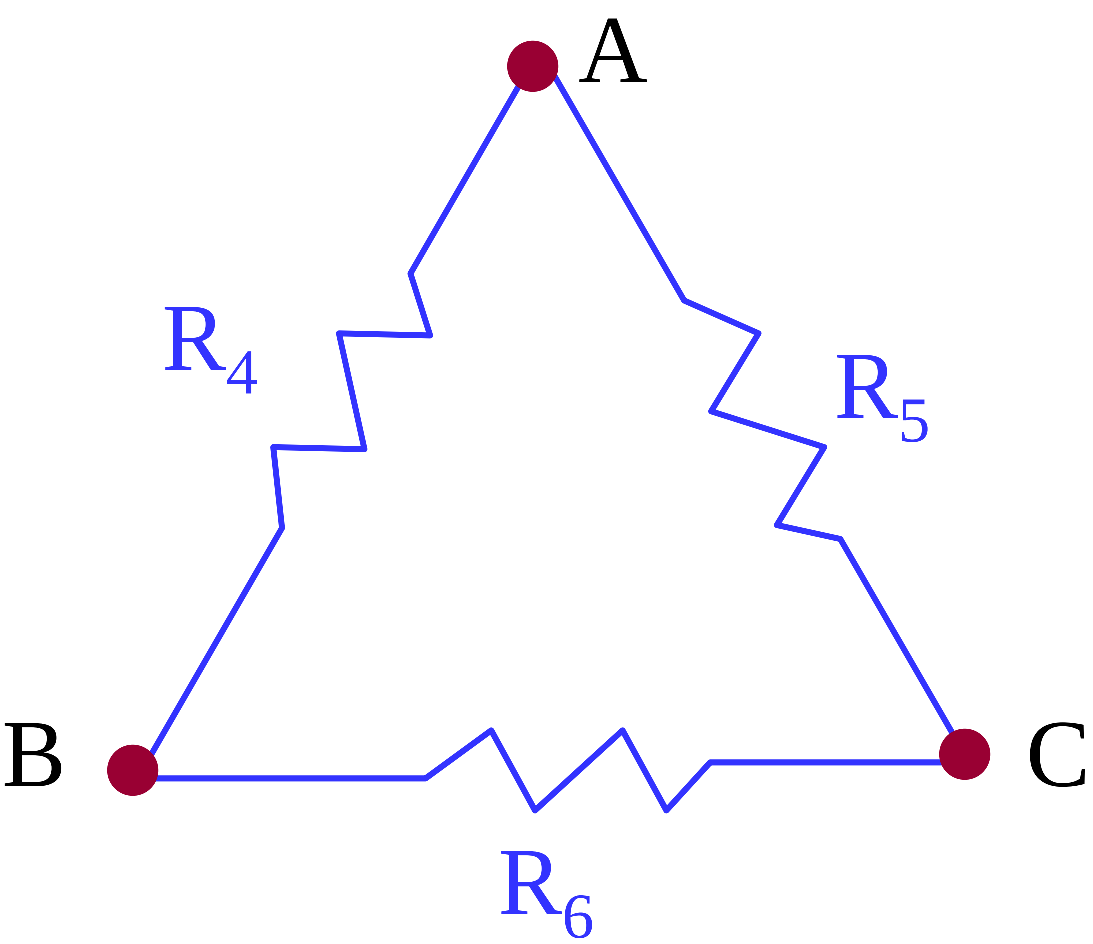
R_5 = (10x6.5)/(10+6.5)

= 3.94 Ω

R_6 =(15x13)/(15+13)

= 6.96 Ω

Tutorial 3, Question 1



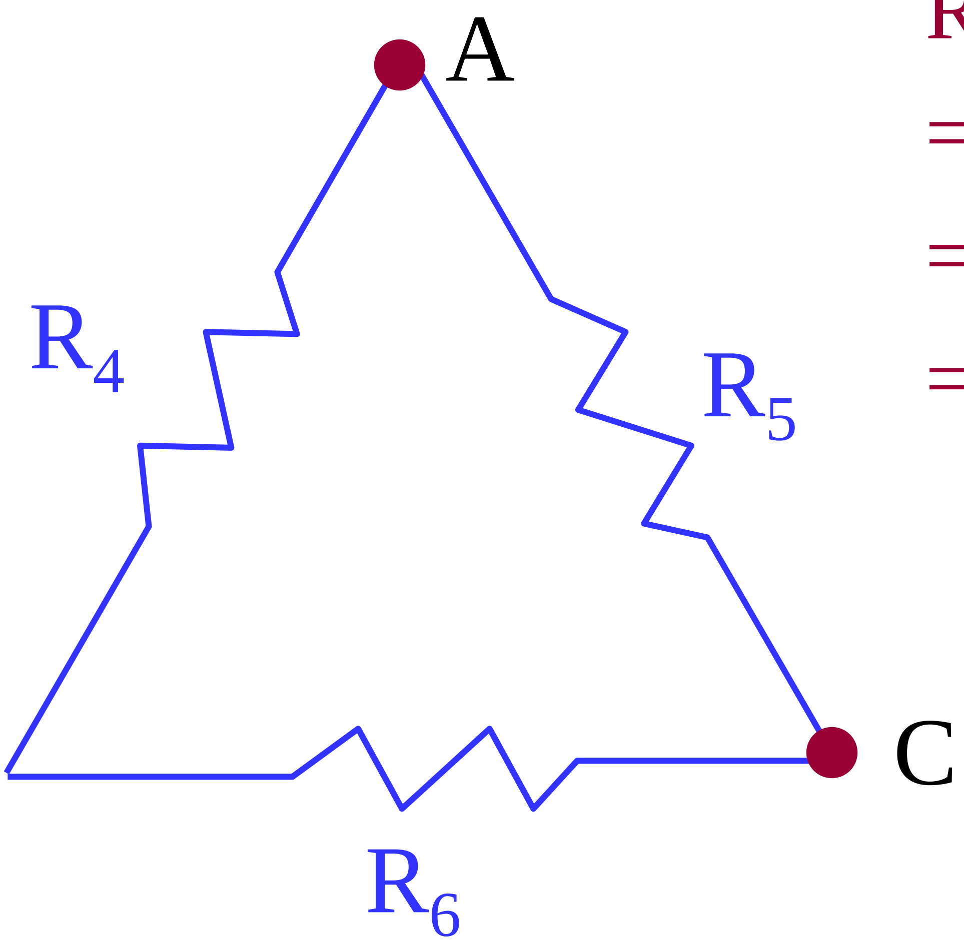
And the circuit becomes

$R_4 = 3.55\ \Omega$

$R_5 = 3.94\ \Omega$

$R_6 = 6.96\ \Omega$

Tutorial 3, Question 1



Resistance between A & C

$= (R_4 + R_6) // R_5$

$= 10.51 \times 3.94/(10.51+3.94)$

$= 2.86\ \Omega$

…next topic

Thevenin's Theorem

Nurturing Curious Minds, Producing Passionate Engineers

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