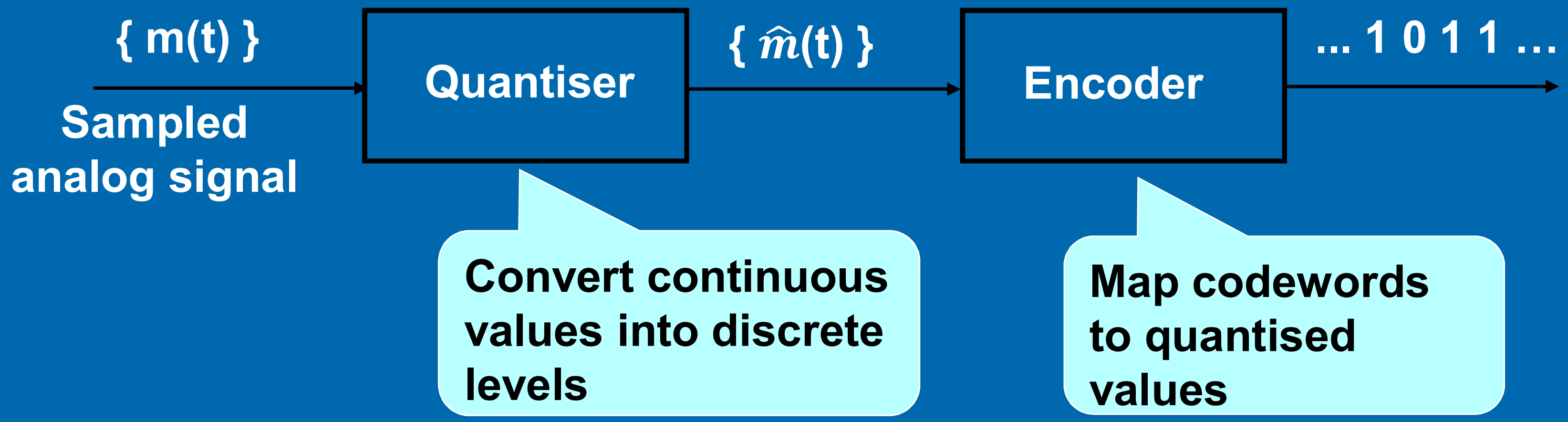
Chapter 7

Analog to Digital conversion

Part 2 of 4

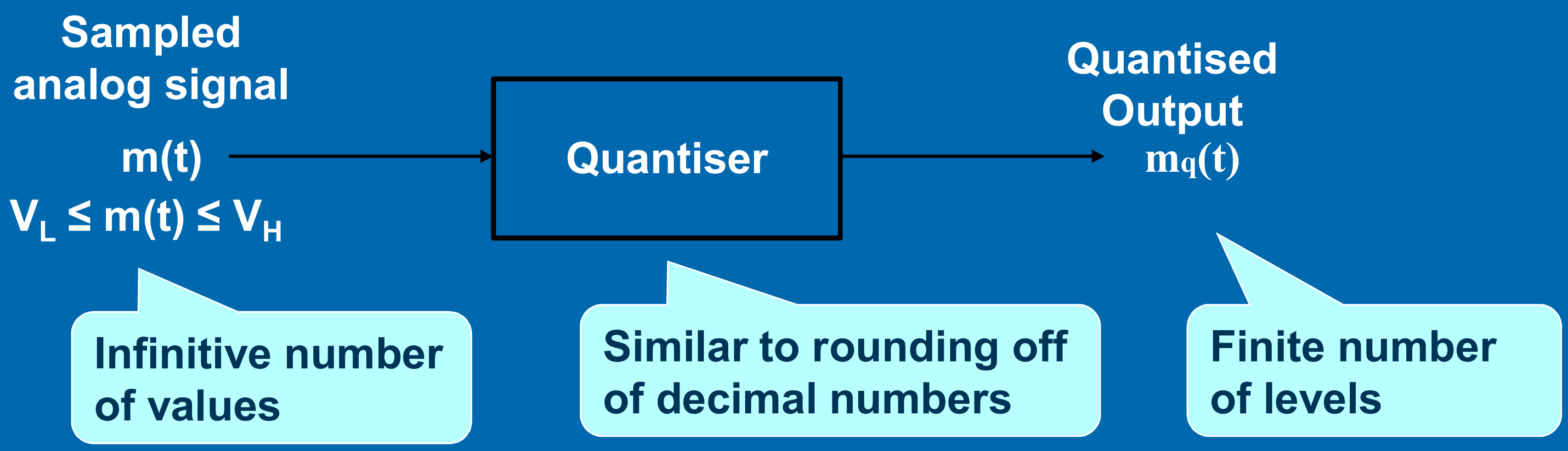
7.2 Quantization

- The sampled signal from a sampling process is still an analog signal.
- To obtain a digital representation of a sampled analog signal, quantisation and encoding are required



7.2 Quantization

- **Quantisation** - a process that converts sampled analog signal into discrete levels



7.2 Quantization

Quantisation process:

1. Divide the voltage range, V_L to V_H into M equal intervals, denoted as L_{01}, L_{02} L_{03}… L_{0M}

   The size of a intervals: $q = (V_H - V_L) / M$ **Step-size**

2. Choose the center of each interval as a quantisation level, denoted as $m_0, m_1, m_2..., m_M$.

3. Represent $m(t)$ by $m_q(t)$, where $m_q(t) \in \{ m_0, m_1, m_2..., m_M \}$

   - At any time, $m_q(t)$ has the value of a quantisation levels which is closest to $m(t)$.

Step size

$q = (V_H - V_L)/M$

$= (V_H - V_L)/8$

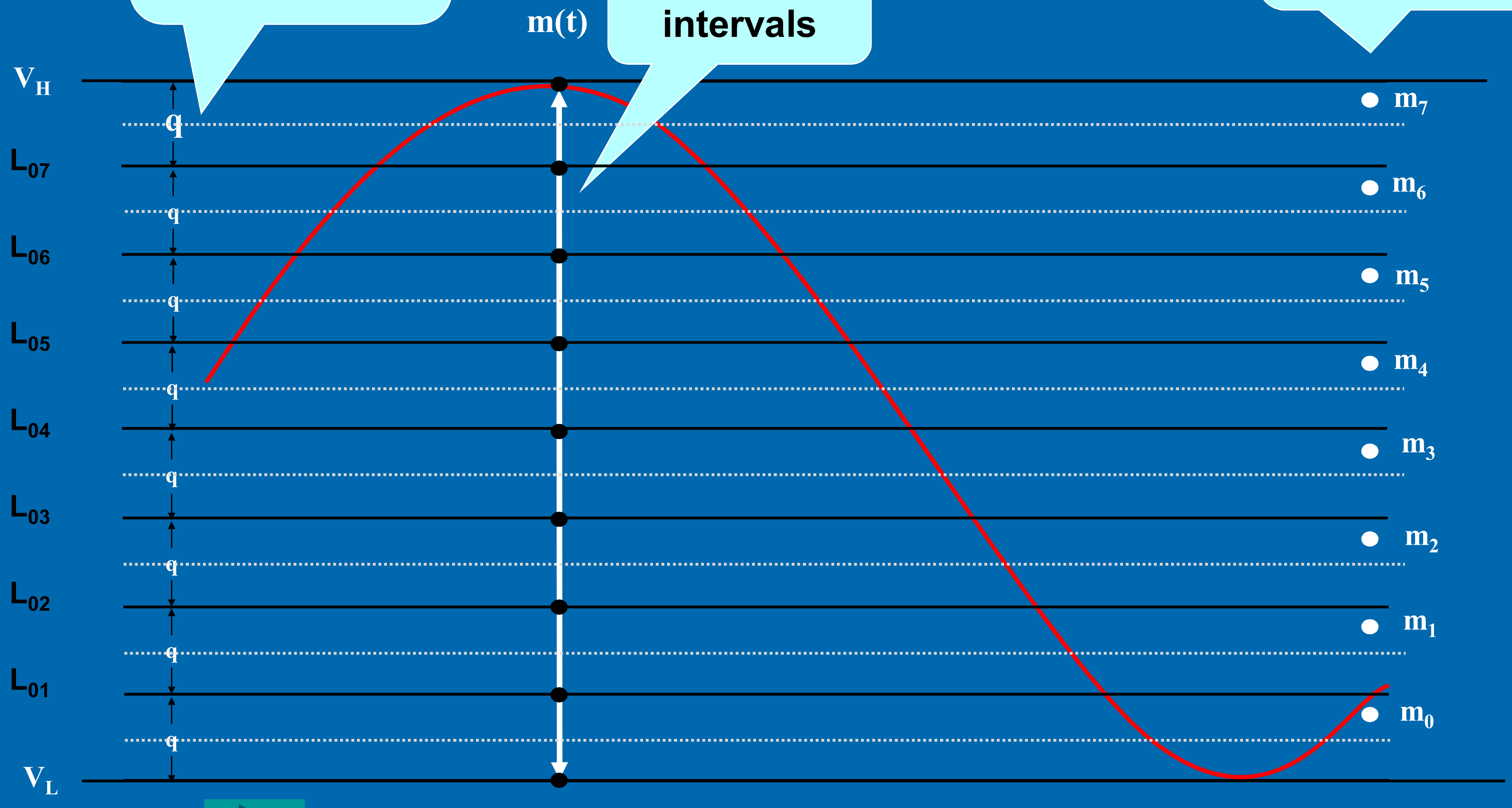
M = 8

8 equal intervals

8 quantization levels

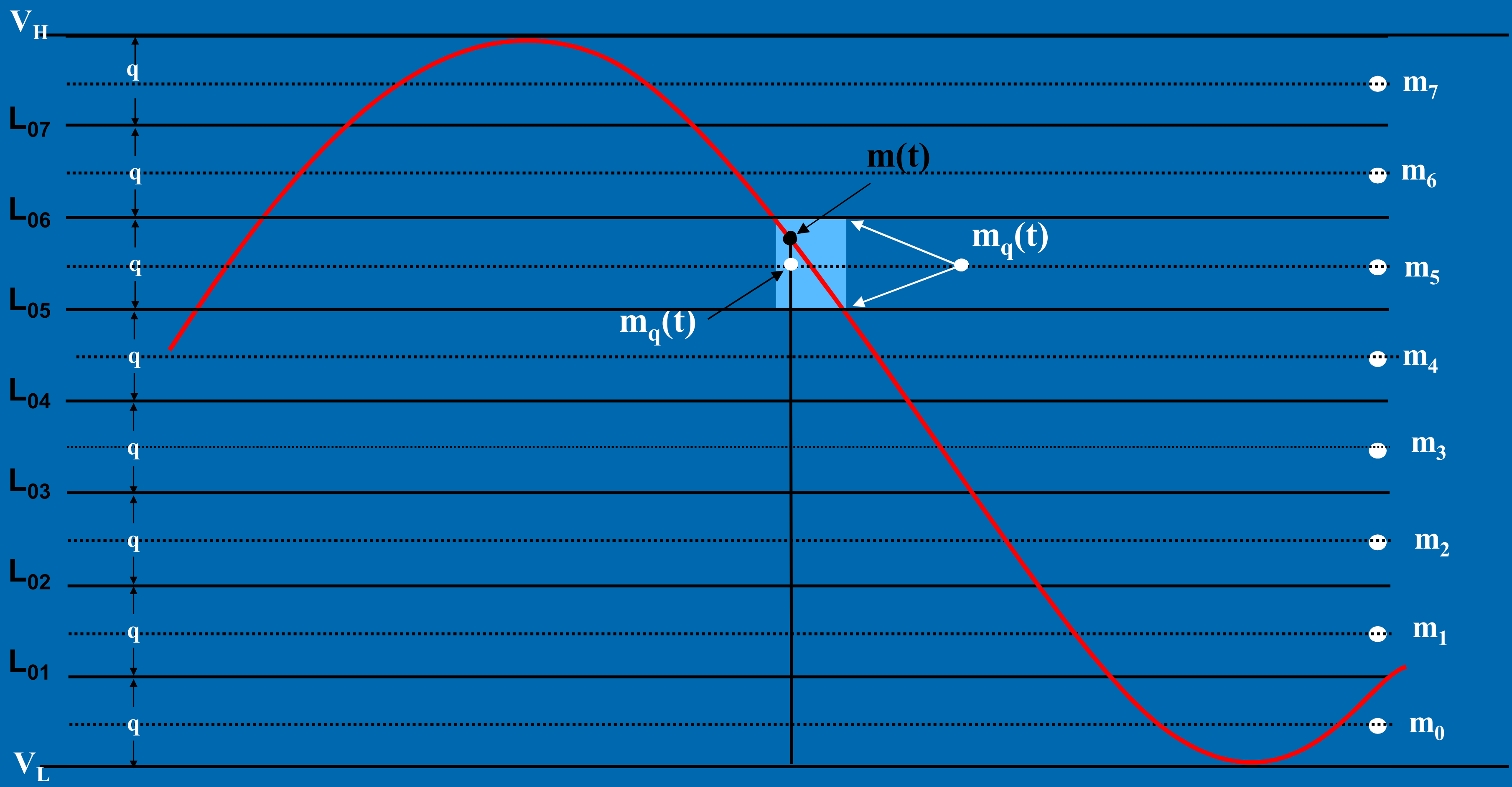
⬇

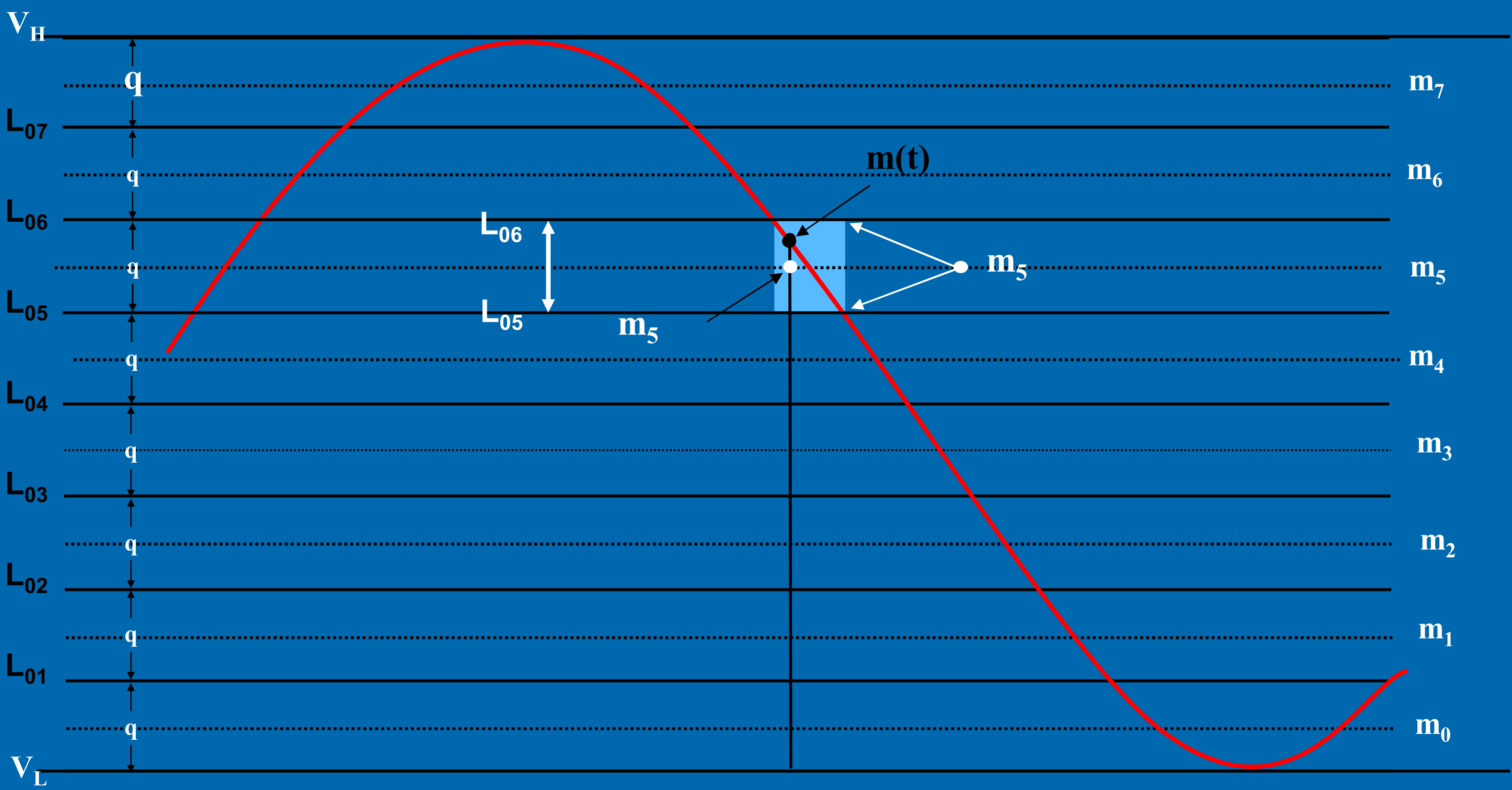
centre of each interval



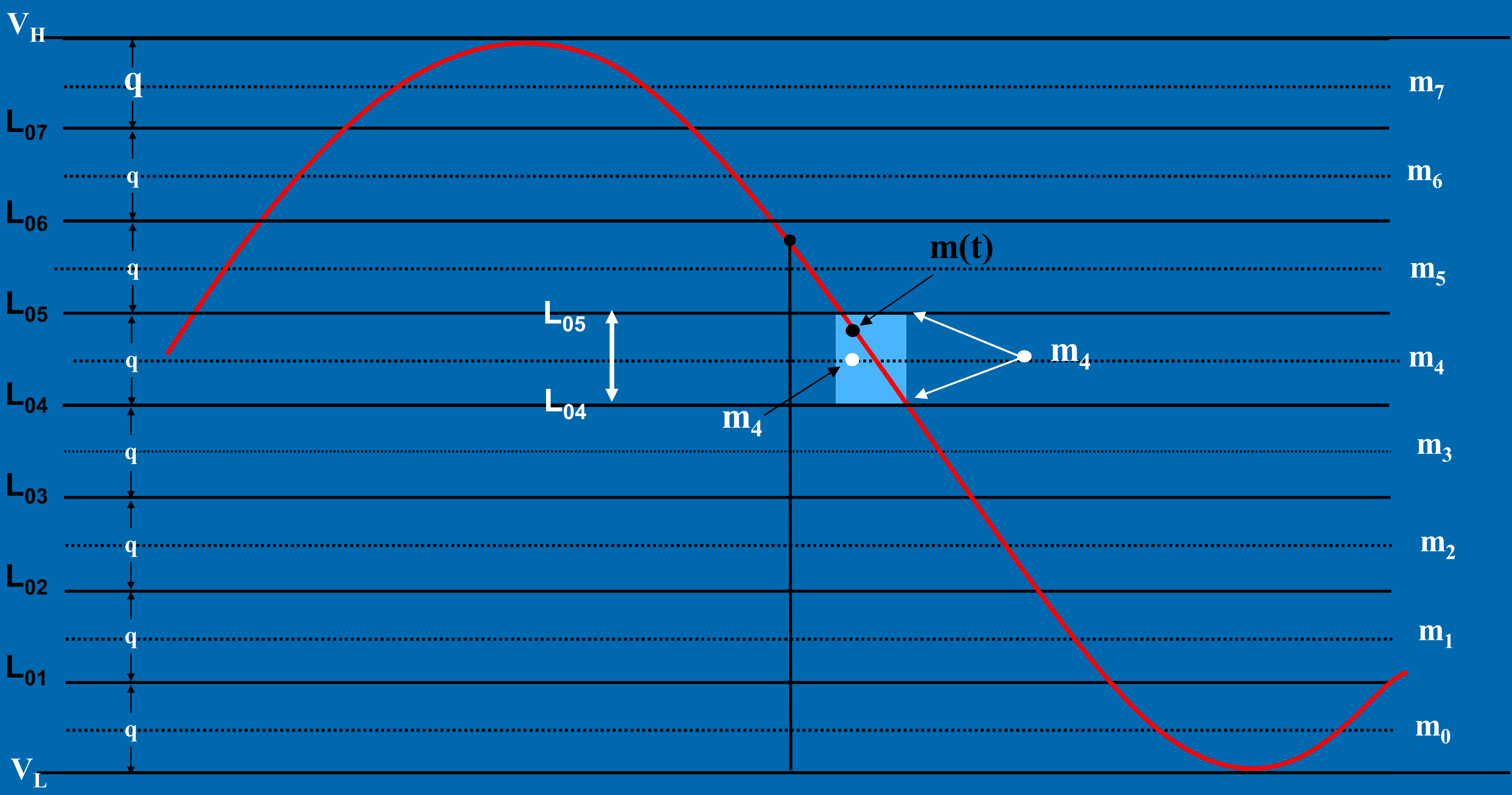
- For a sampled amplitude value m t, the quantizer rounds it up or down to make it equal to one of the 8 different quantization levels.

- The quantization level chosen is the nearest to m(t).

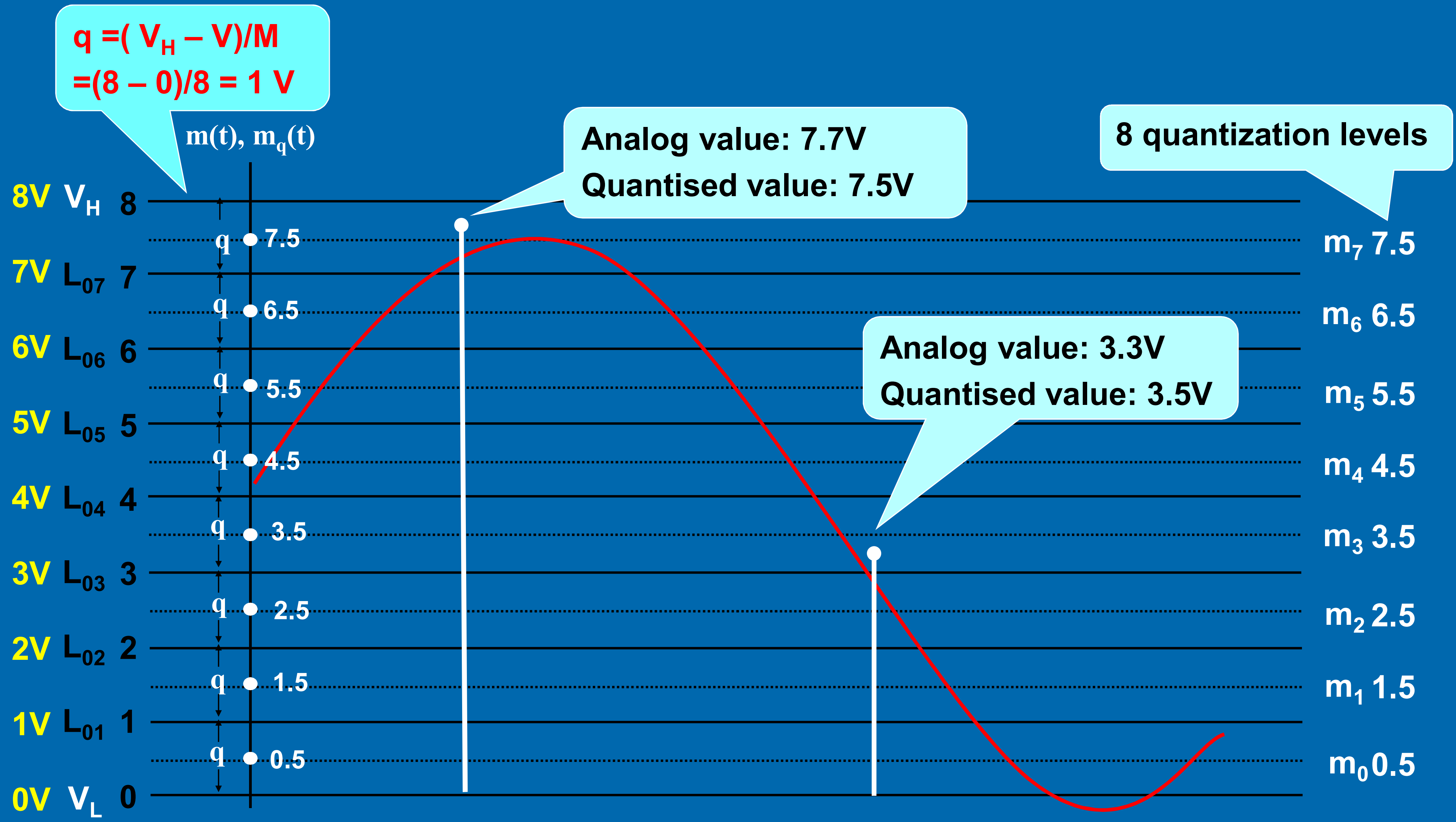


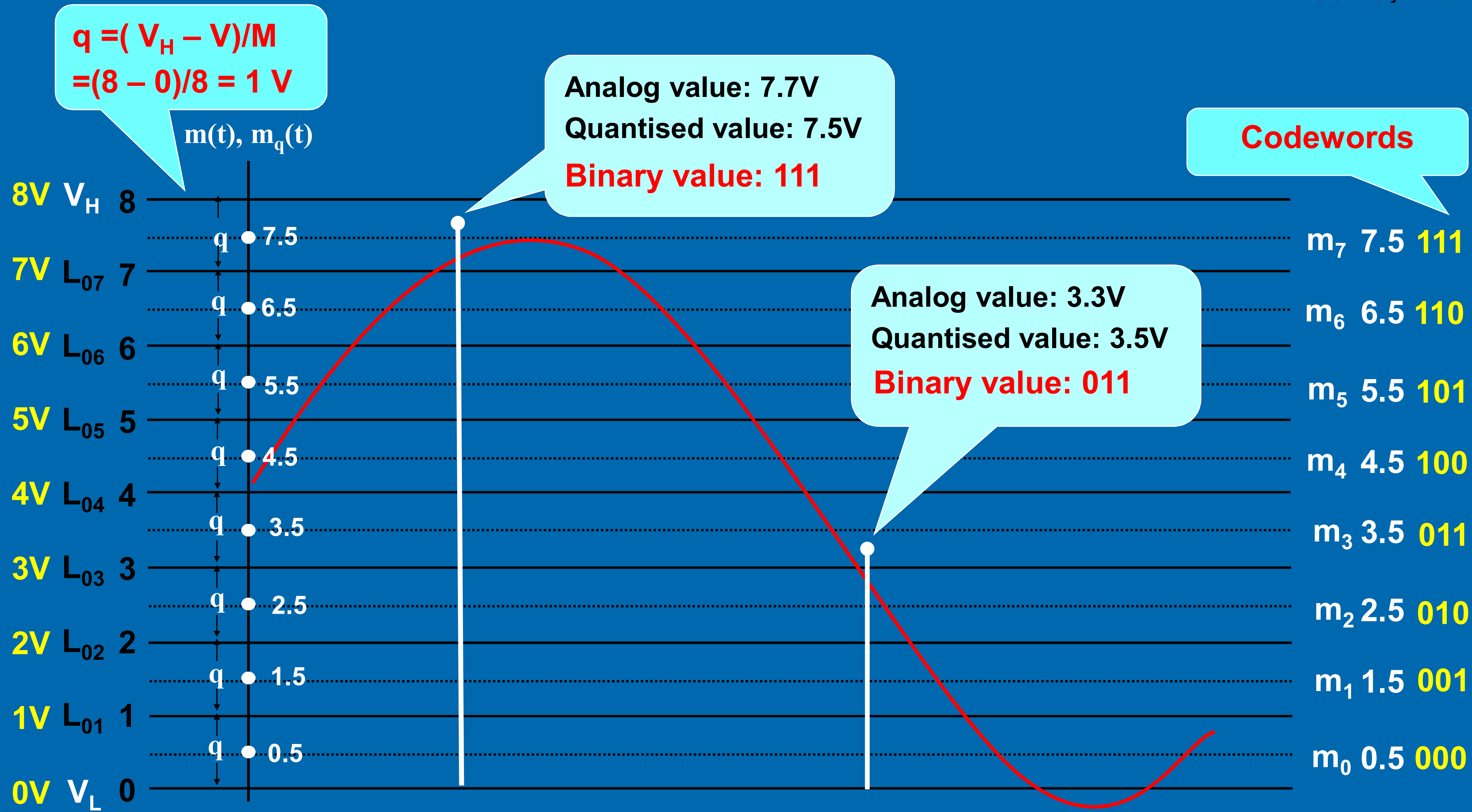


V_H
q
L_{07}
q
L_{06}
q
L_{05}
q
L_{04}
q
L_{03}
q
L_{02}
q
L_{01}
q
V_L
m_7
m_6
m_5
m_4
m_3
m_2
m_1
m_0
m(t)
L_{06}
L_{05}
m_5
m_5



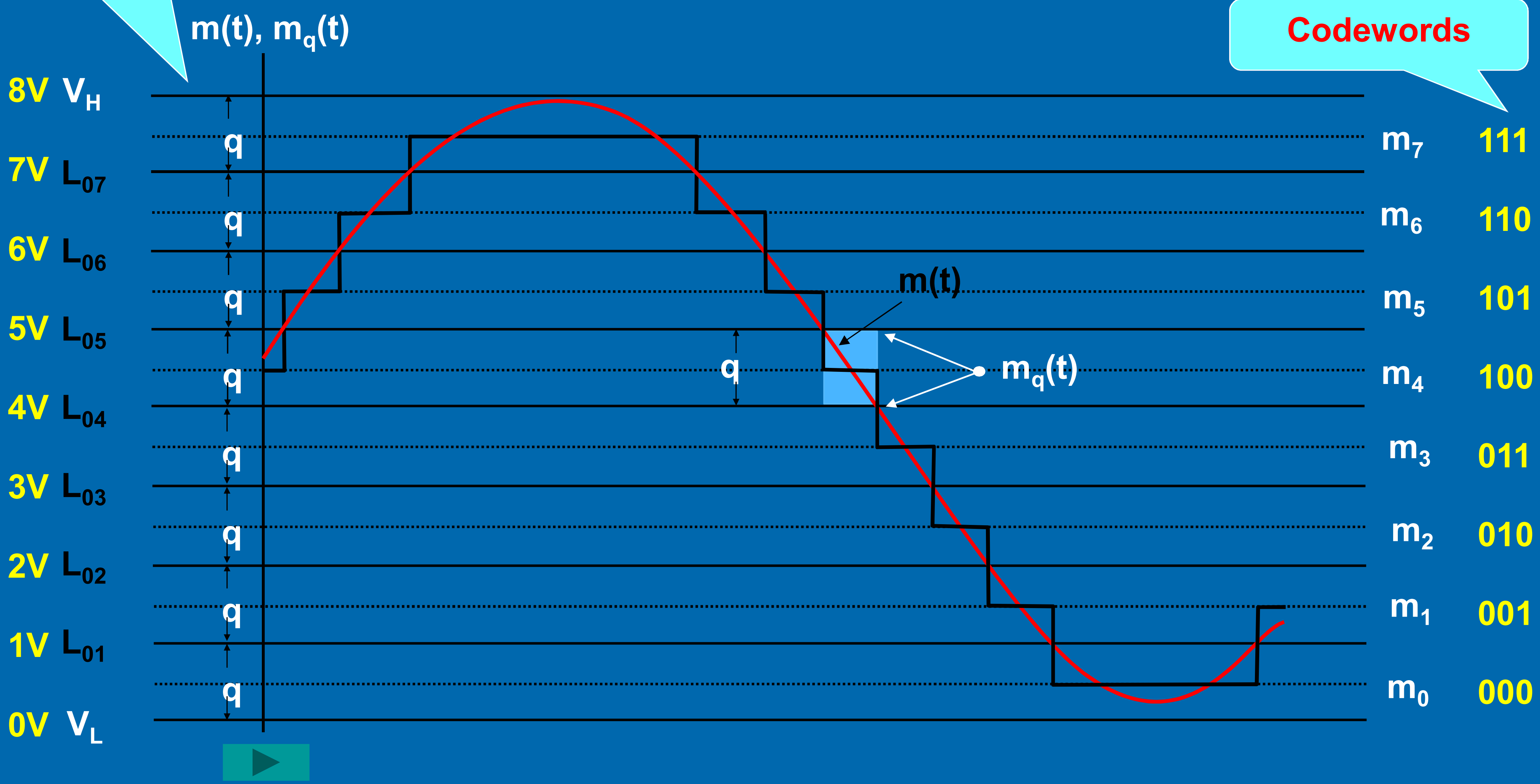
VH
q
L07
q
L06
q
L05
q
L04
q
L03
q
L02
q
L01
q
VL
m7
m6
m5
m4
m3
m2
m1
m0
m(t)
L05
L04
m4
m4


q =(V_H – V)/M
=(8 – 0)/8 = 1 V
m(t), m_q(t)
Analog value: 7.7V
Quantised value: 7.5V
8 quantization levels
Analog value: 3.3V
Quantised value: 3.5V
8V V_H 8
7V L_07 7
6V L_06 6
5V L_05 5
4V L_04 4
3V L_03 3
2V L_02 2
1V L_01 1
0V V_L 0
q
7.5
6.5
5.5
4.5
3.5
2.5
1.5
0.5
m_7 7.5
m_6 6.5
m_5 5.5
m_4 4.5
m_3 3.5
m_2 2.5
m_1 1.5
m_0 0.5



q =(V_H – V)/M
=(8 – 0)/8 = 1 V
m(t), m_q(t)
Analog value: 7.7V
Quantised value: 7.5V
Binary value: 111
Analog value: 3.3V
Quantised value: 3.5V
Binary value: 011
Codewords
8V V_H 8
7V L_07 7
6V L_06 6
5V L_05 5
4V L_04 4
3V L_03 3
2V L_02 2
1V L_01 1
0V V_L 0
q
7.5
6.5
5.5
4.5
3.5
2.5
1.5
0.5
m_7 7.5 111
m_6 6.5 110
m_5 5.5 101
m_4 4.5 100
m_3 3.5 011
m_2 2.5 010
m_1 1.5 001
m_0 0.5 000

$q = (V_H - V)/M$
$= (8 - 0)/8 = 1\ V$



m(t), mq(t)
8V VH
7V L07
6V L06
5V L05
4V L04
3V L03
2V L02
1V L01
0V VL
q
m(t)
mq(t)
Codewords
m7 111
m6 110
m5 101
m4 100
m3 011
m2 010
m1 001
m0 000

7.2 Quantization

- The quantisation process always introduces error as it **approximates** the sampled analog signal using quantization levels.

- The quantisation error is defined as:

$$\text{Quantisation error} = m_q(t) - m(t)$$

- At any time instant, the quantisation error magnitude, $|m_q(t) - m(t)|$ is equal or less than q/2.

$$|m_q(t) - m(t)| \le q/2 \quad \text{or} \quad -q/2 \le m_q(t) - m(t) \le q/2$$

- The quantisation error is regarded as noise and is also called **quantisation noise**.

7.2 Quantization

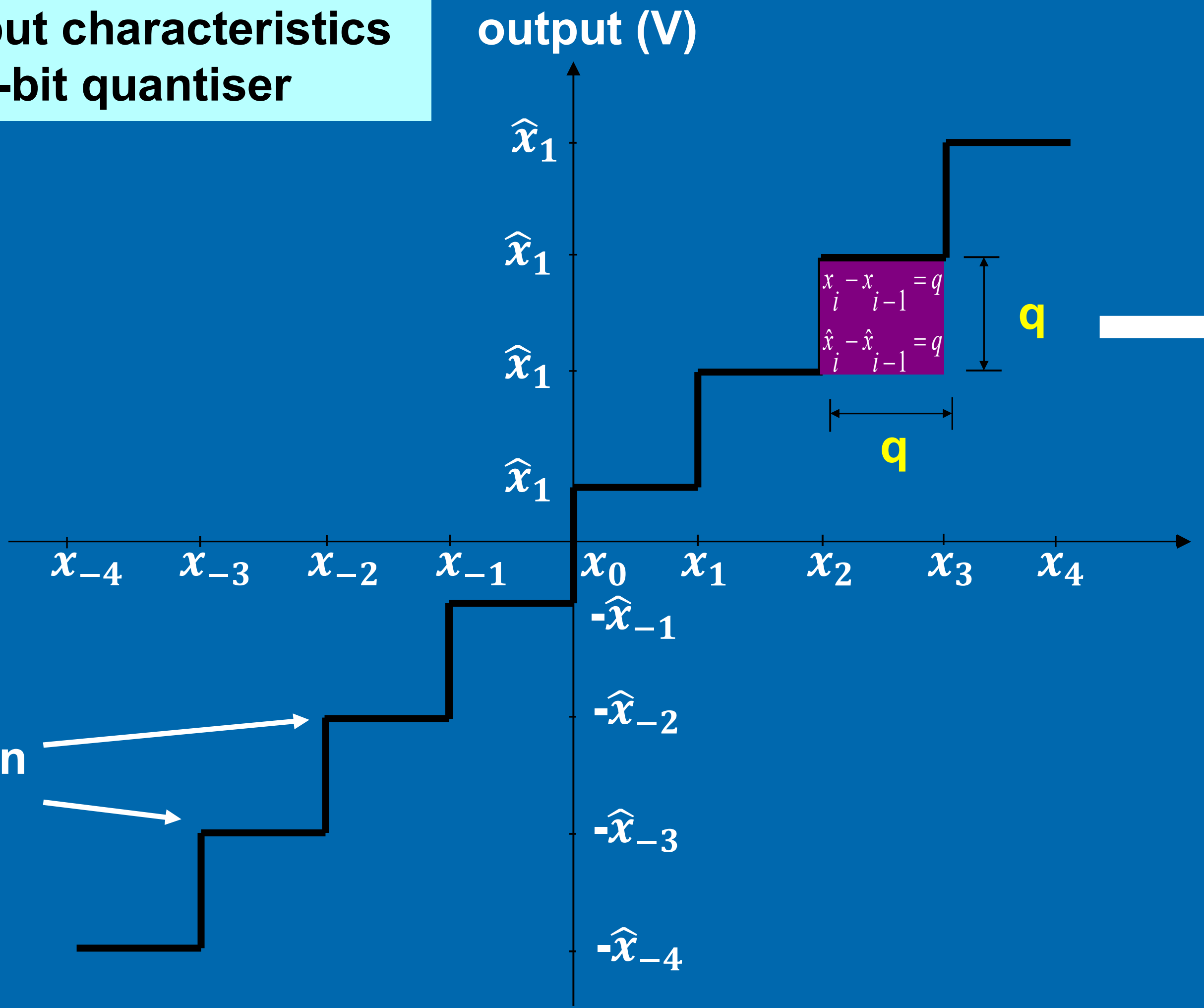
- The quality of the approximation of the quantisation process is improved by
  - reducing the step size
  - increasing the number of allowable levels.
- Depending on intended applications, different quantisation steps and levels may be chosen.
- E.g. voice telephony: 8 bits/sample → 2^8 = 256 levels
  Audio CD: 16 bits/sample → 2^{16} = 65,536 levels
- HowStuffWorks "How DVDs Work"

7.2 Quantization

Uniform quantisation

- A quantiser can be defined by its input-output characteristics.

Input-output characteristics of a 3-bit quantiser



input (V)

uniform quantiser

$$x_i - x_{i-1} = q$$

$$\hat{x}_i - \hat{x}_{i-1} = q$$

Quantisation levels

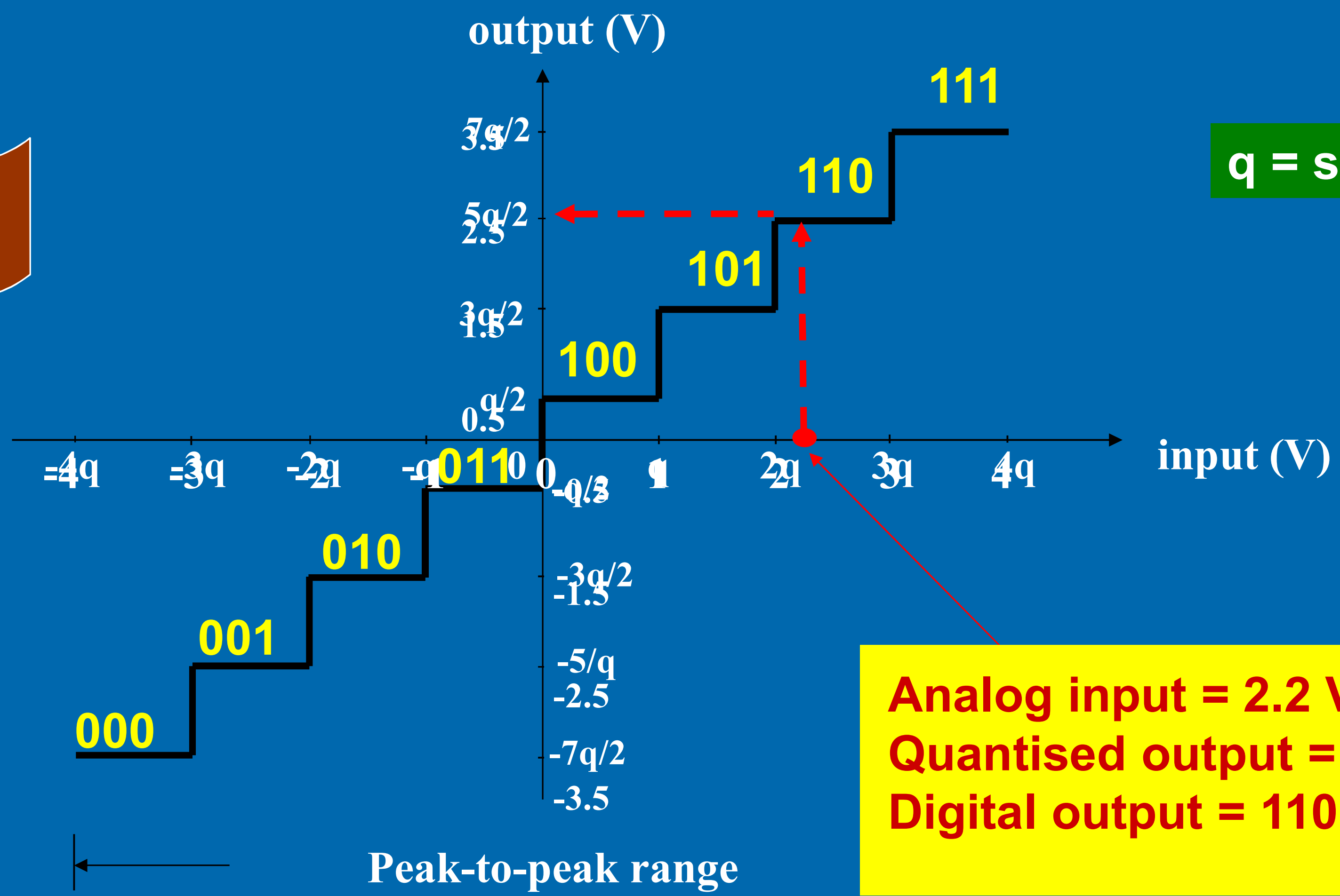
$\hat{x}_1$ x_{-3}

7.2 Quantization

Uniform quantisation

- A common type of uniform quantiser characteristic: **mid-riser type**

E.g. q = 1V



q = step size

Analog input = 2.2 V
Quantised output = 2.5 V
Digital output = 110

7.2 Quantization

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Uniform quantisation

- A uniform quantiser is defined by two parameters:

  - number of levels

  - step size

- The number of levels, M is generally chosen to be = 2^B to make the most efficient use of B-bit binary codewords. i.e. $M = 2^B$ E.g. $B = 4$, $M = 2^4 = 16$ levels

- q and B must be chosen so as to cover the entire range of input samples which means we should set:

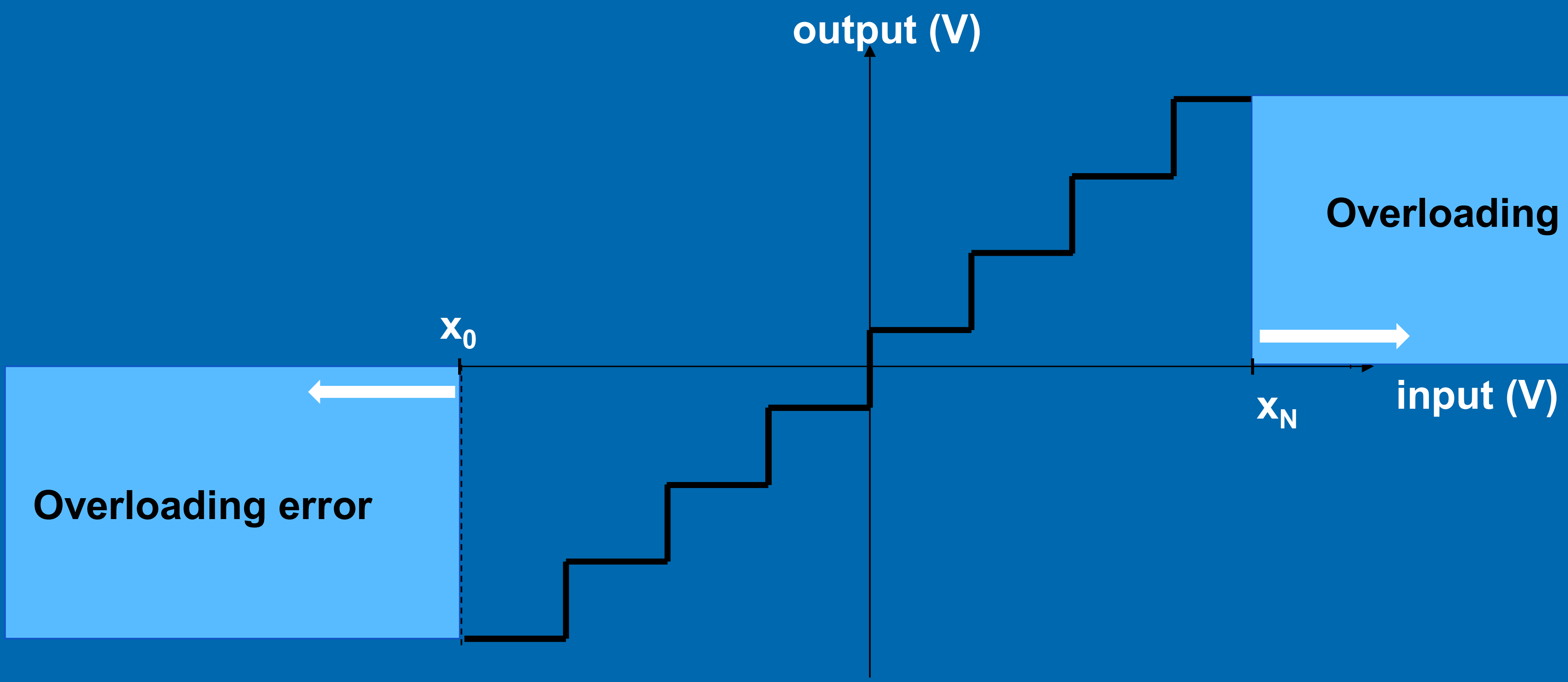
peak-to-peak signal amplitude = input range of quantizer, i.e. $2\,X_{max} = q2^B$

i.e. $q = 2\,X_{max} / 2^B$ where X_{max} is the peak signal amplitude

7.2 Quantization

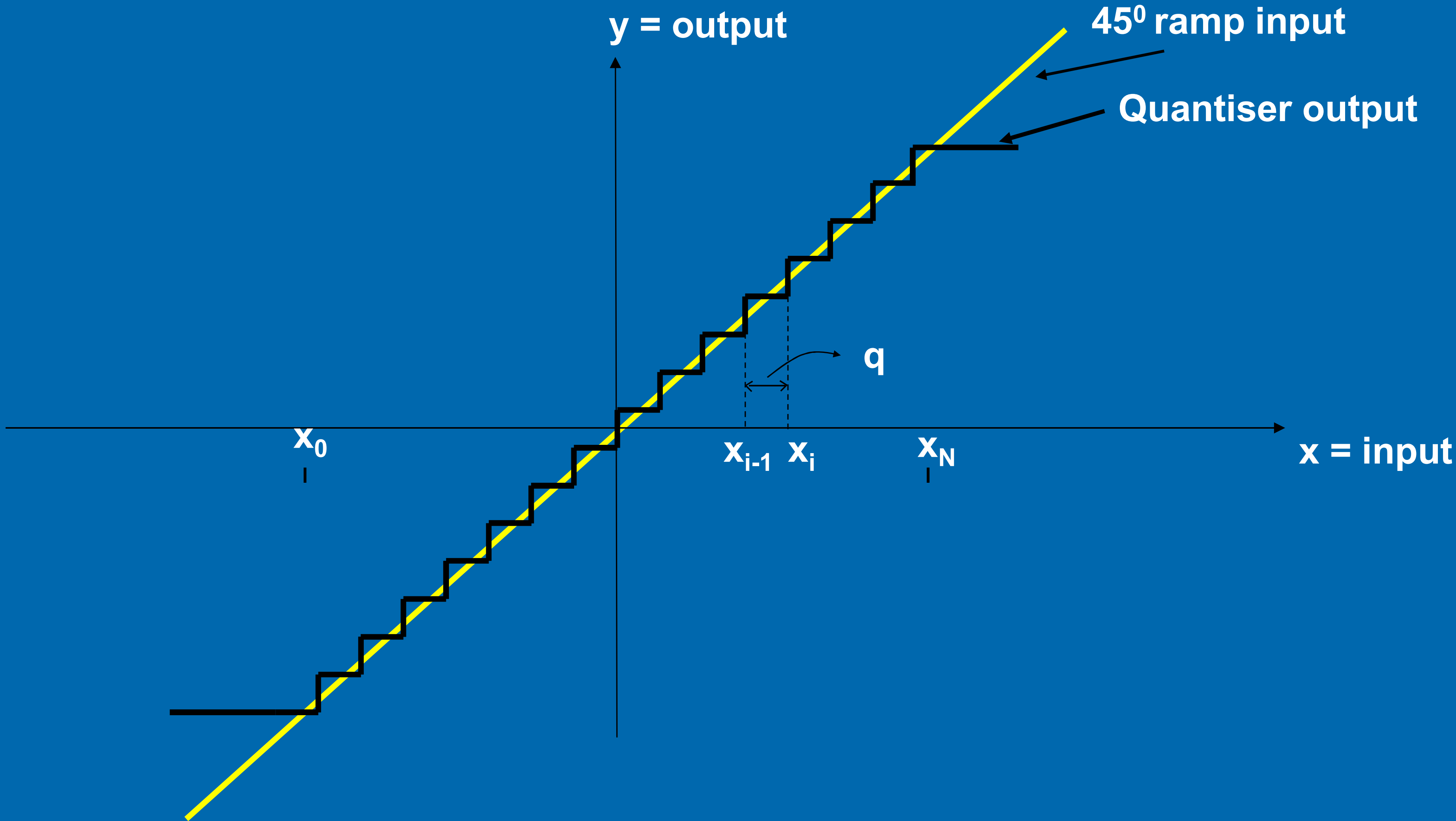
Overload Error

- Besides quantisation error or quantising noise the quantisation process also causes **overload error or clipping.**



7.2 Quantization

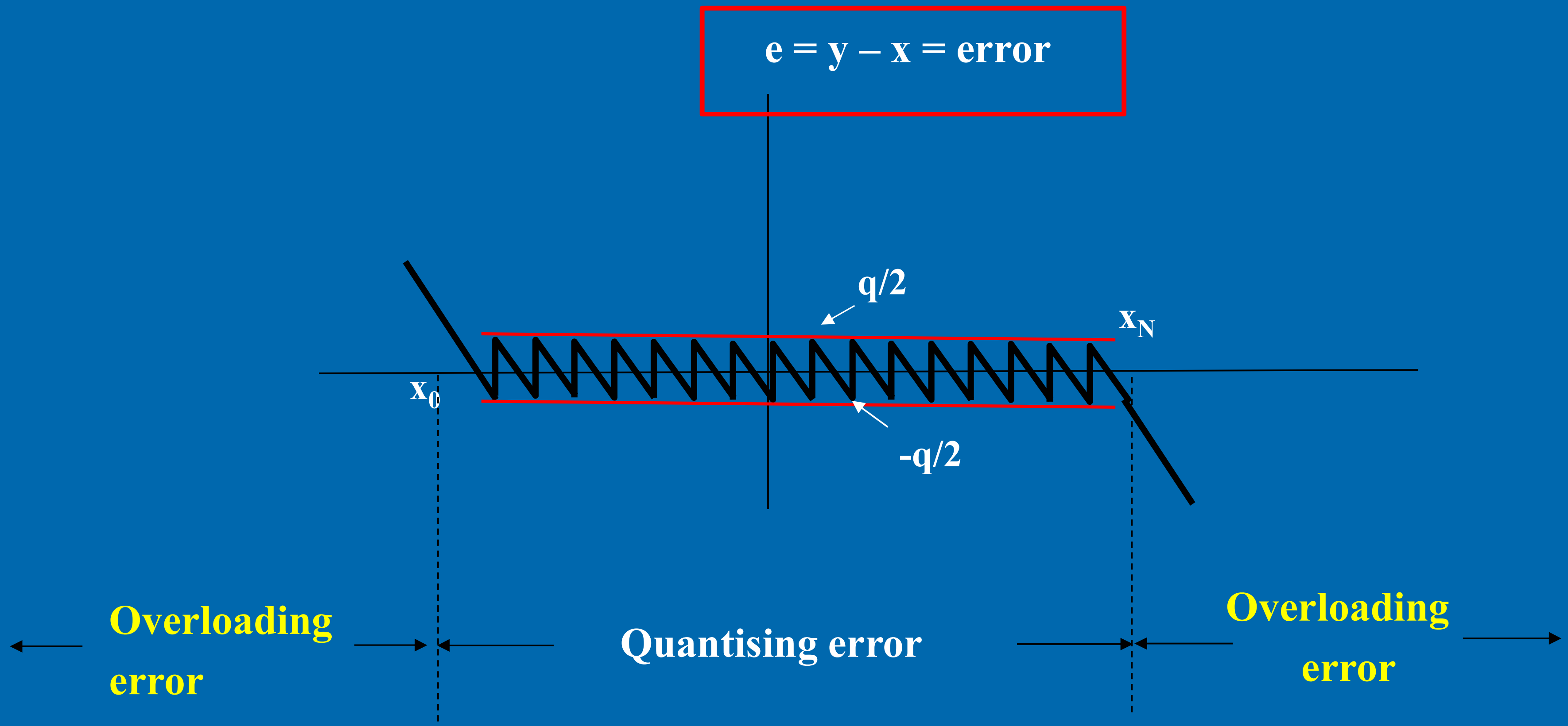
<u>Quantisation noise power</u>



7.2 Quantization

Quantisation noise power

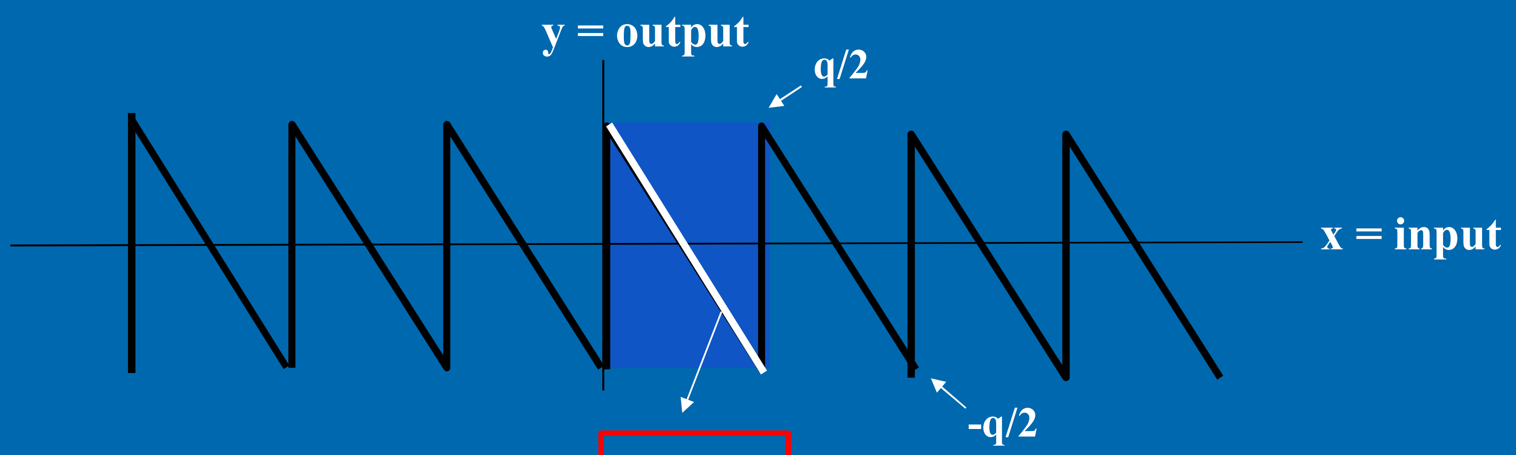
- The difference between the quantised output and input waveform gives the quantisation noise (waveform), denoted as e.



7.2 Quantization

Quantisation noise power

Quantisation Noise Waveform



Quantising noise: $e = \boxed{-x + q/2}$

rms value of the quantisation noise signal:

$$e_{rms} = \sqrt{\frac{1}{q}\int_0^q \left(-x + \frac{q}{2}\right)^2 dx} = \sqrt{\frac{q^2}{12}}$$

7.2 Quantization

Quantisation noise power

- The quantisation noise power (over a 1Ω load) is

$$N_q = \frac{e_{rms}^2}{R} = e_{rms}^2 \quad (\text{as} \quad \mathrm{R} = 1)$$

$$N_q = \frac{q^2}{12} \quad \text{watts} \quad (R = 1)$$

- The result is applicable to any input to an uniform quantiser.

- The same result can be obtained when the quantisation noise is a random signal with an uniform distribution in the interval -q/2 to +q/2, i.e.

$$\mathrm{p_e(e)} = \frac{1}{\mathrm{q}}, \qquad -\frac{\mathrm{q}}{2} \leq \mathrm{e} \leq +\frac{\mathrm{q}}{2}$$

$$= 0, \qquad \text{otherwise}$$

7.2 Quantization

Signal to quantising noise (S/N_q)

- The performance of a quantiser is measured by **signal-to-noise ratio** that takes both quantising error and overload error into account.

$$SNR = \frac{S}{\underbrace{N_o}_{0} + N_q} \approx \frac{S}{N_q}$$

Overload error is neglected for simplicity.

- Quite often SNR calculation for a quantiser is based on sinusoidal inputs because SNR results for speech and sinusoidal inputs are quite close.

- Use of sinusoidal make the measurements and calculation of S/N_q easier.

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Signal to quantising noise (S/N_q)

Derive formula for S/N_q

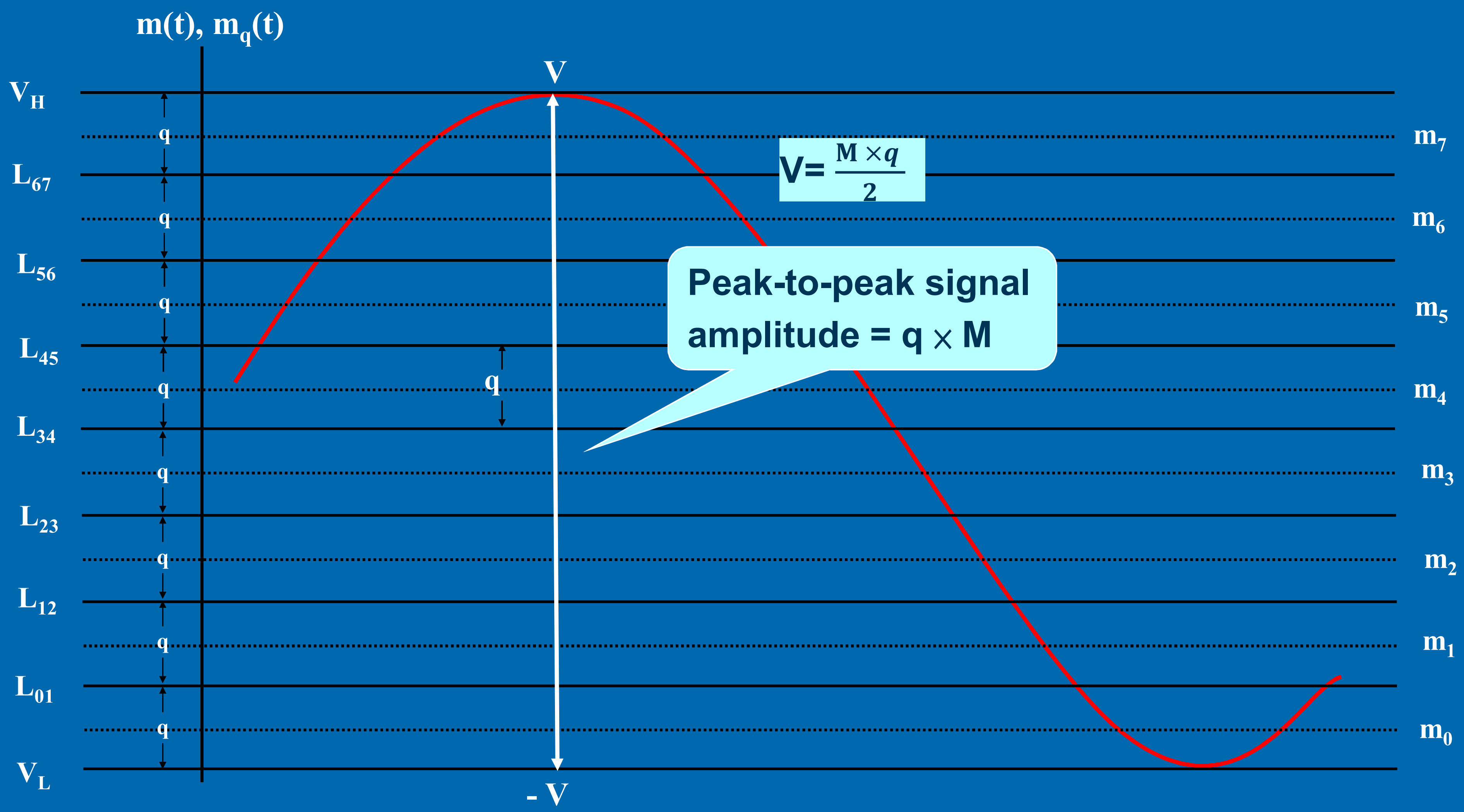
- For a full range (-V to +V) sinusoidal input that has zero overload error, the average signal power is

$$S = \frac{V_{rms}^2}{R} \qquad V_{rms} = \frac{V}{\sqrt{2}}$$

V = peak signal amplitude

$$S = \frac{V^2}{2} \qquad (R = 1)$$

Peak-to-peak signal amplitude	=	input range of quantiser



$V = \frac{M \times q}{2}$

Peak-to-peak signal amplitude = q × M

7.2 Quantization

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<u>Signal to quantising noise (S/N_q)</u>

Derive formula for S/N_q

- For a full range (-V to +V) sinusoidal input that has zero overload error, the average signal power is

$$S = \frac{V_{rms}^2}{R} = V_{rms}^2 \qquad (R = 1)$$

$$S = \frac{V^2}{2} \qquad \text{Since} \quad V_{rms} = \frac{V}{\sqrt{2}}$$

- The peak value V of the sinusoid can be expressed in terms of step size q and number of levels in the quantiser M, as follows:

$$2\mathrm{V} = \mathrm{qM}$$

$$\mathrm{V} = \frac{\mathrm{qM}}{2}$$

$$\therefore rms\ value = V_{rms} = \frac{V}{\sqrt{2}} = \frac{qM}{2\sqrt{2}}$$

7.2 Quantization

Signal to quantising noise (S/N_q) **Derive formula for S/N_q**

- Hence, the average signal power is

$$S = V_{rms}^2 = \left(\frac{qM}{2\sqrt{2}}\right)^2 = \frac{q^2M^2}{8}$$

Combining eqs for S and N_q

$$N_q = \frac{q^2}{12}$$

$$\frac{S}{N_q} = \frac{q^2M^2}{8} \cdot \frac{12}{q^2} = 1.5M^2$$

$$\left[\frac{S}{N_q}\right]_{\text{dB}} = 10\log_{10}\left(1.5M^2\right) = \underbrace{10\log_{10}(1.5)}_{1.76} + 10\log_{10}\left(M^2\right)$$

$$\left[\frac{S}{N_q}\right]_{dB} = 1.76 + 20\log_{10}M$$

7.2 Quantization

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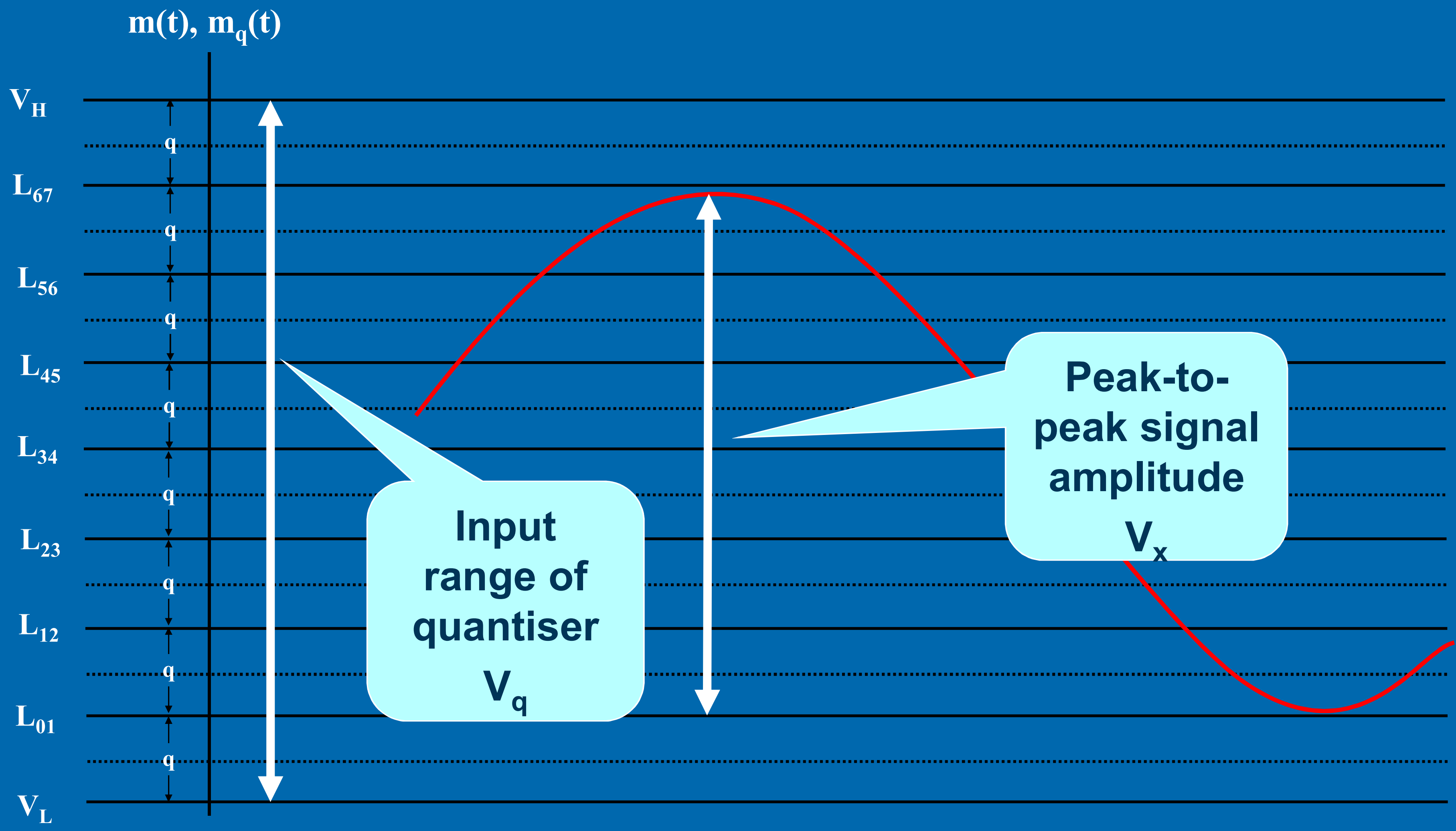
<u>Signal to quantising noise (S/N_q)</u> **Derive formula for S/N_q**

Since $M = 2^B$,

$$\left[\frac{S}{N_q}\right]_{dB} = 1.76 + 20log_{10}\left(2^B\right) = 1.76 + B\times 20log_{10}(2)$$

$$\left[\frac{S}{N_q}\right]_{dB} = 1.76 + 6B \quad dB$$

for a sinusoid whose amplitude range *coincides* with the range of the quantiser.


m(t), $m_q(t)$
V_H
L_{67}
L_{56}
L_{45}
L_{34}
L_{23}
L_{12}
L_{01}
V_L
q
Input range of quantiser V_q
Peak-to-peak signal amplitude V_x

7.2 Quantization

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Signal to quantising noise (S/N_q)

- For sinusoidal inputs whose amplitude, V_x, is less than the full input range of the quantiser, V, then

$$\left[\frac{S}{N_q}\right]_{dB} = 1.76 + 6B + 20\log_{10}\frac{V_x}{V_q} \quad dB$$

Peak-to-peak signal amplitude

Input range of quantiser

7.2 Quantization

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Non-uniform quantization

- For signals with large variance in strength over time, it is preferable to use a non-uniform quantiser i.e. a quantiser that has **variable step size.**

E.g. Speech signal strength can vary largely from one speech segment to another.

- If the quantiser is designed to accommodate strong signals (like vowels), the quantisation step size will be large.
- Weaker signals (like consonants) will be subjected to a larger quantisation error.

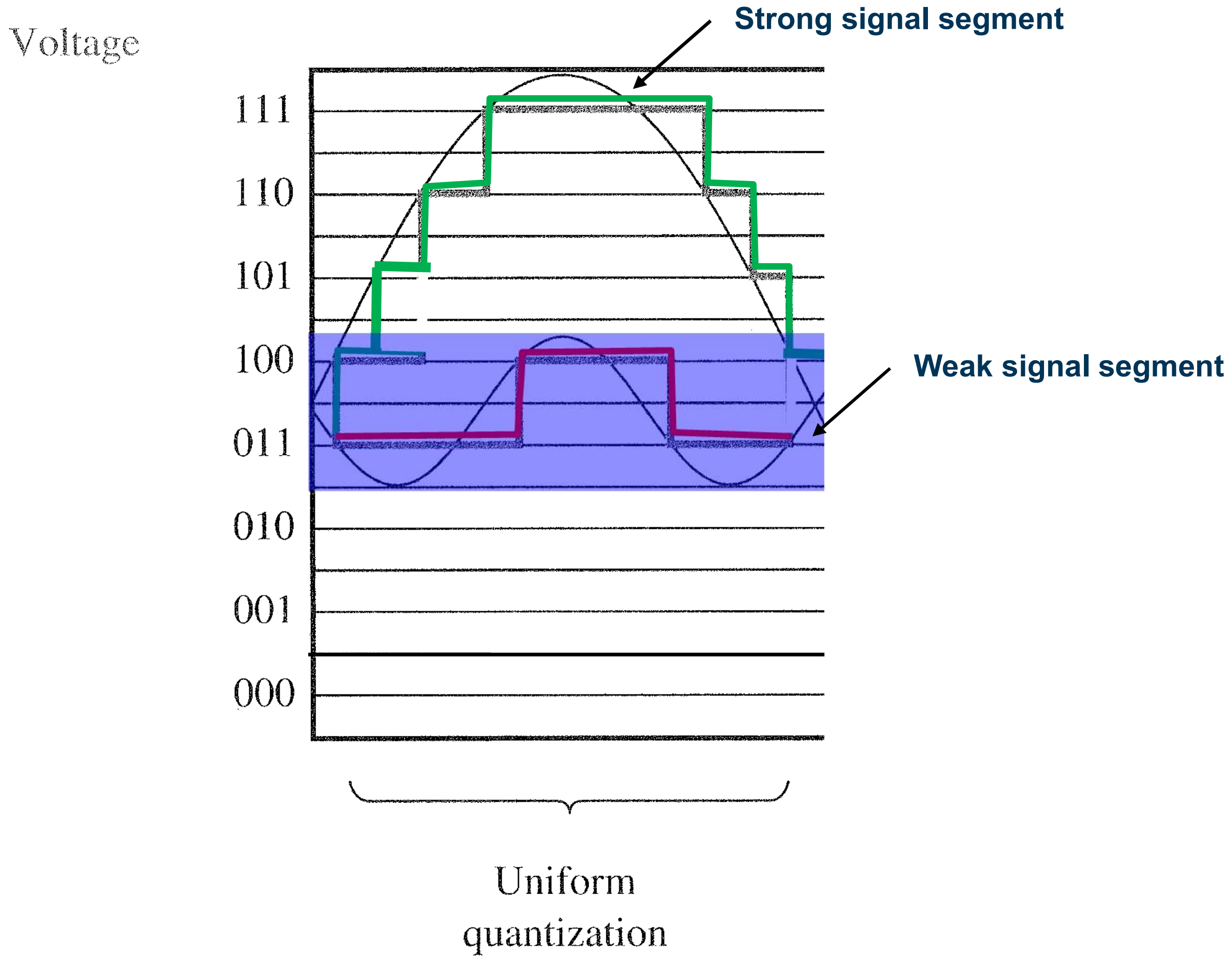
Strong signal → S/N_q = SNR high SNR

Weak signal → S/N_q = SNR low SNR

- The quality of the speech will be affected.

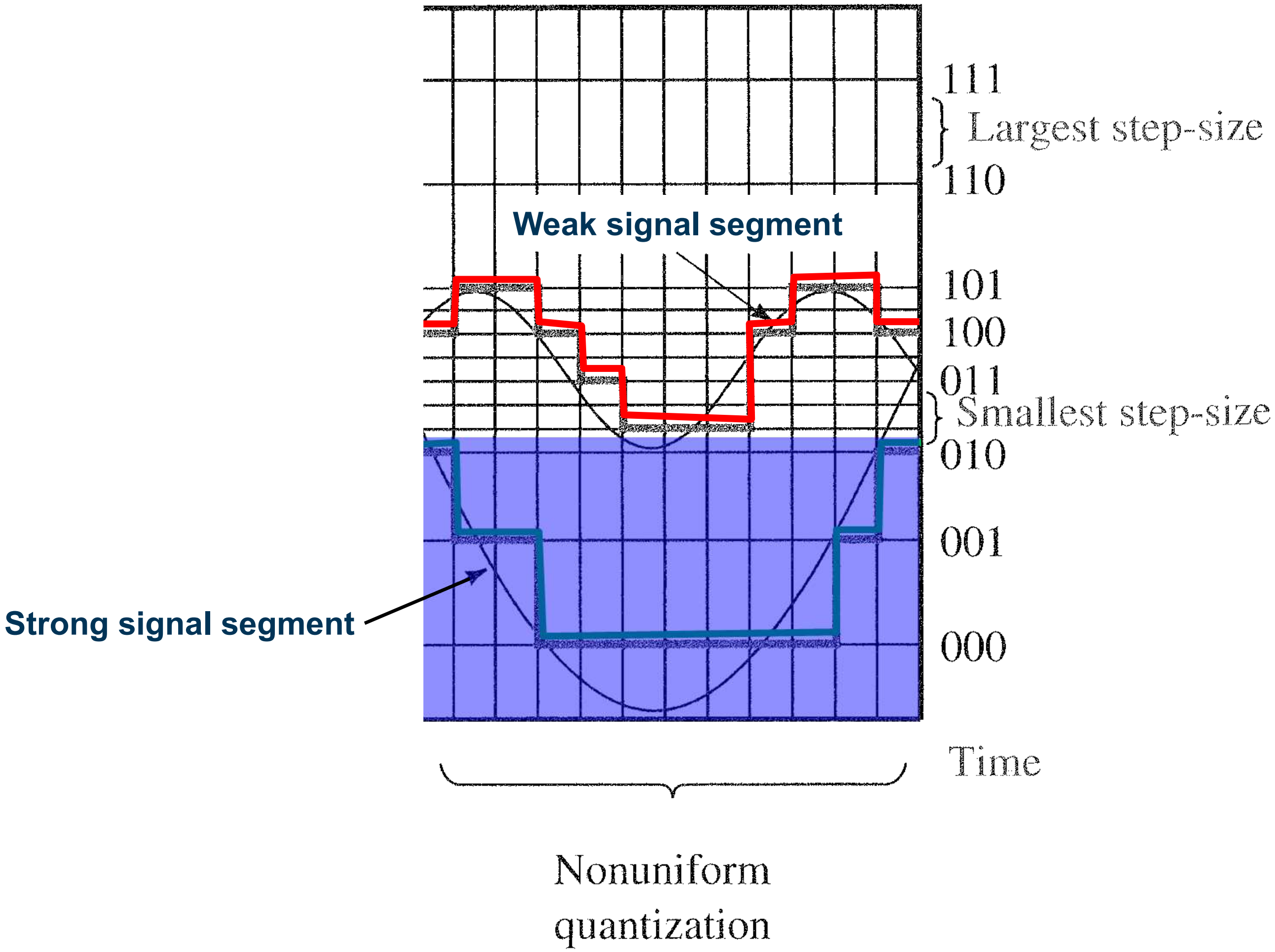
7.2 Quantization

Non-uniform quantization



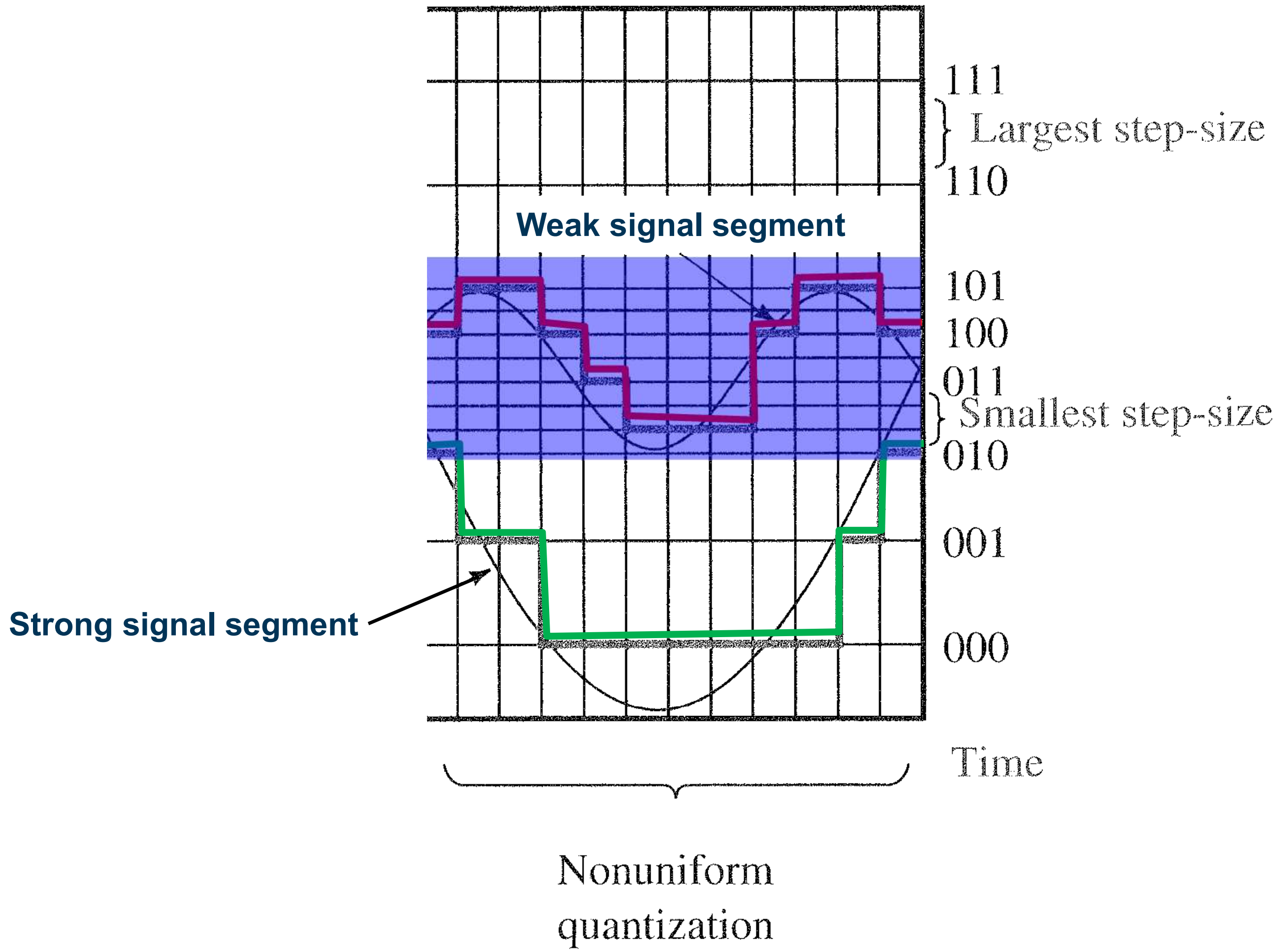
7.2 Quantization

Non-uniform quantization



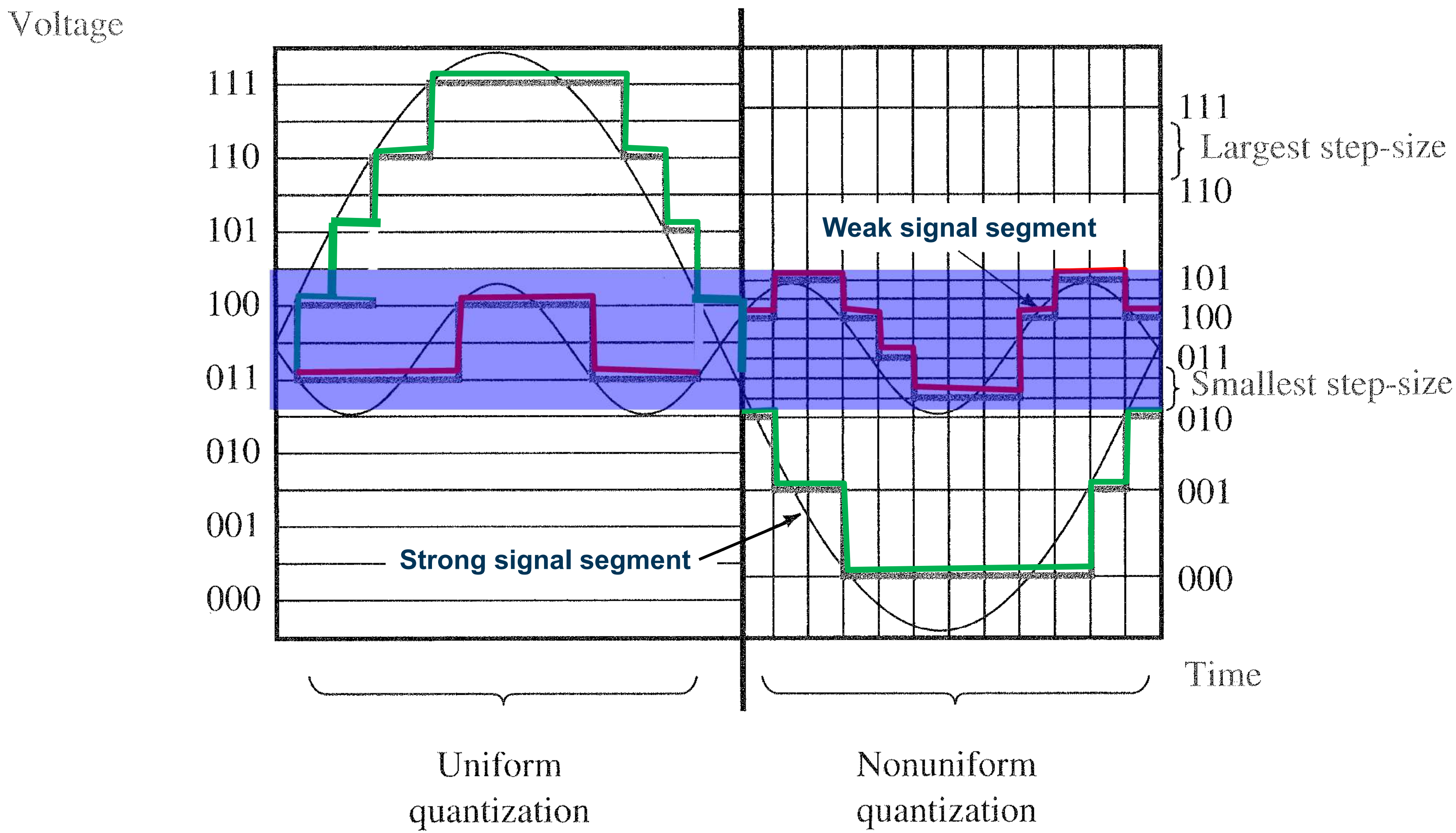
7.2 Quantization

Non-uniform quantization



7.2 Quantization

Non-uniform quantization

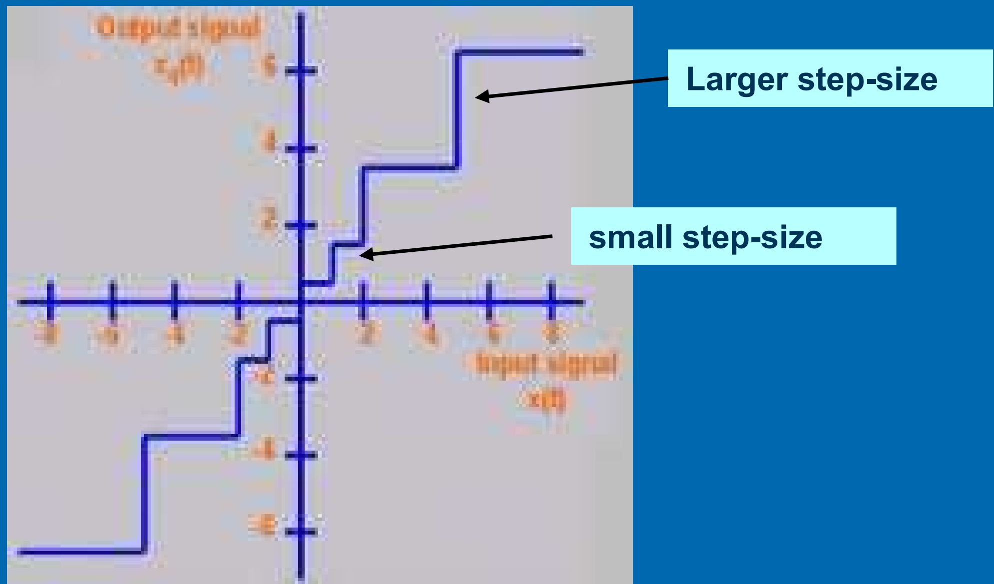


7.2 Quantization

Non-uniform quantization

- **For signals with large varying amplitude, a suitable non-uniform quantiser would be a quantiser whose step size increases with the signal amplitude.**

I/O characteristic of a non-uniform quantiser



7.2 Quantization

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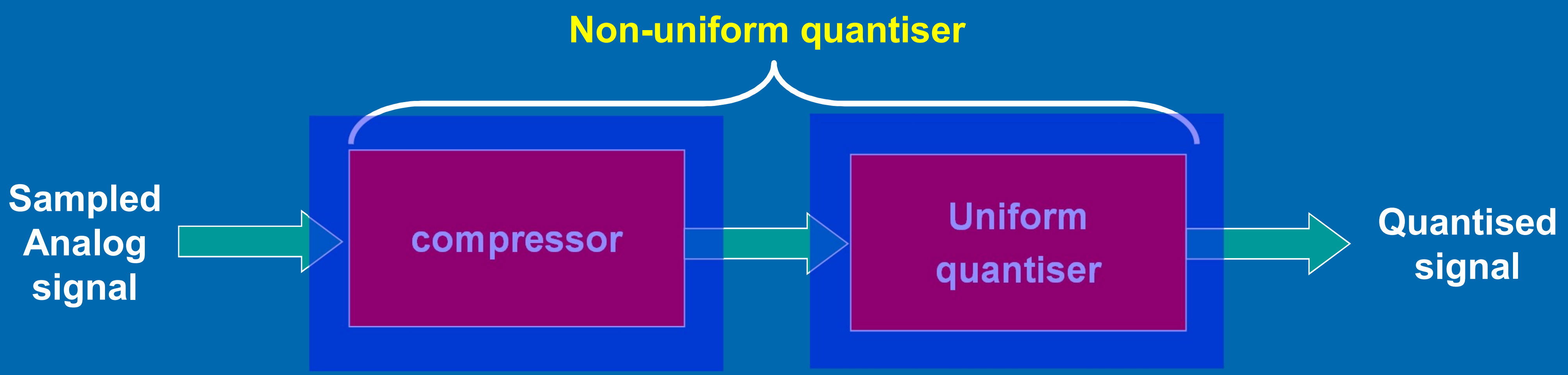
Non-uniform quantization

- In practice, a non-uniform quantizer is implemented by combining a uniform quantizer with a compressor.

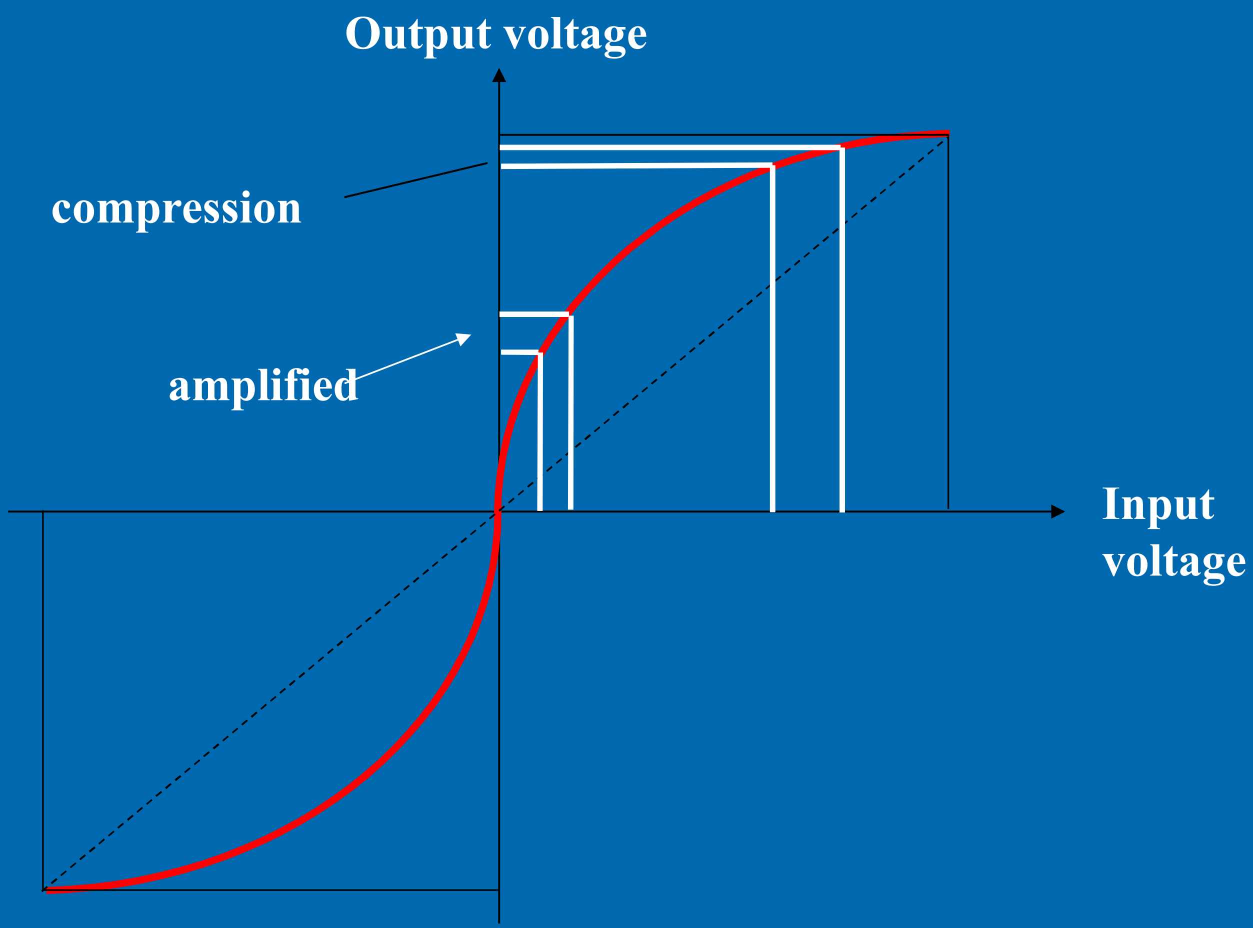
- A sampled analog signal is first input to a compressor and then to a uniform quantiser.
- The compressor can be viewed as a variable-gain amplifier that amplifiers the signal at low amplitude and attenuate the signal at high amplitude.

- The compressor and uniform quantiser work jointly to form a non-uniform quantiser.



7.2 Quantization

Non-uniform quantization



Input-output characteristic of a compressor

7.2 Quantization

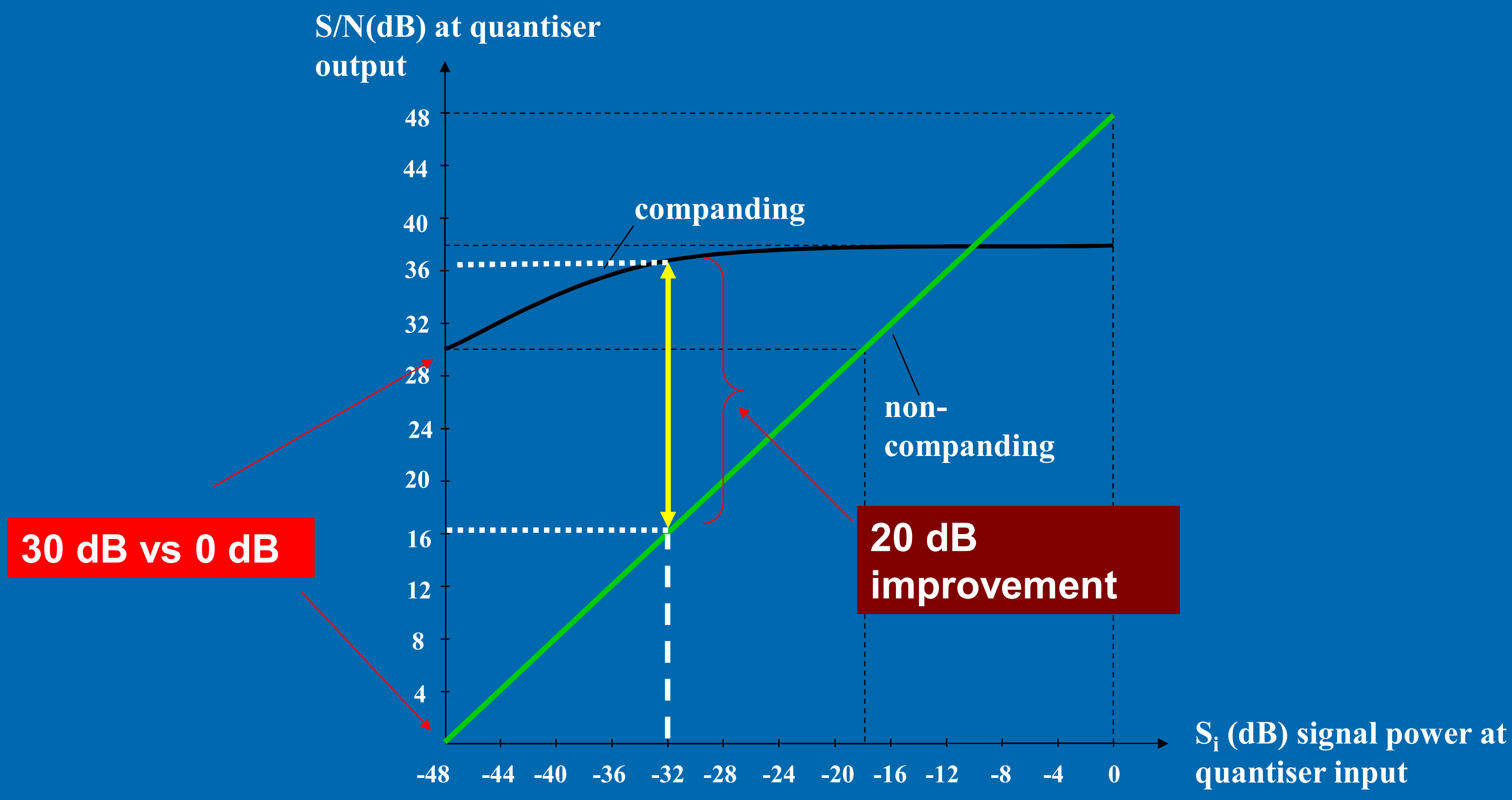
Non-uniform quantization

- At the receiver the signal is reconstructed by the reverse process. i.e. by expanding it.

- This process of compression-expansion is called COMPANDING. (COMpressing-exPANDING)

7.2 Quantization

Non-uniform quantization



END

CHAPTER 7

(Part 2 of 4)