SOLUTIONS/ MARKING SCHEME

SINGAPORE POLYTECHNIC
2019 / 2020 Semester 1 Exam

Module Name: **Engineering Mathematics II**

Module Code: MS2216/MS4216/MS6216

Course: DCHE/DASE/DCPE/DEB/DEEE/DES/DESM/DARE/DBEN/DCEP/DME/DMRO

Year: 2 FT

No.	SOLUTION
A	c, b, d, a, d

B1a

$$f(x,y) = xy^2 + 3xy - x + 2$$

$$\frac{\partial f}{\partial x} = y^2 + 3y - 1, \qquad \frac{\partial f}{\partial y} = 2xy + 3x$$

B1b

$$g(x,y) = (y^2 + x)e^{-xy}$$

$$g_y(x,y) = -x\left(y^2 + x\right)e^{-xy} + 2ye^{-xy} = e^{-xy}\left(2y - xy^2 - x^2\right)$$

$$g_y(0,2) = 4$$

B2a

$$\int x^2 \cos x \, dx$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + C$$

u		dv
x^2	$+$	$\cos x$
$2x$	$-$	$\sin x$
2	$+$	$-\cos x$
0		$-\sin x$

B2b

$$\int_0^1 \frac{x^2}{(x^3+1)^2}\,dx \underset{u=x^3+1}{=} \frac{1}{3}\int_1^2 \frac{1}{u^2}\,du = \frac{1}{3}\left[-\frac{1}{u}\right]_1^2 = -\frac{1}{3}\left[\frac{1}{2} - 1\right] = \frac{1}{6}$$

B3

$$h = \frac{2-0}{4} = 0.5$$

x	0	0.5	1.0	1.5	2.0
$\sin\left(x^2\right)$	0	0.2474	0.8415	0.7781	−0.7568

$$\int_0^2 \sin\left(x^2\right)dx \approx \frac{1}{6}[0 - 0.7568 + 4(0.2474 + 0.7781) + 2(0.8415)] \simeq 0.84$$

B4a

$$e^{-(2x-1)}\frac{dy}{dx} = e^{-y} \quad \rightarrow \quad e^y dy = e^{(2x-1)}dx$$

$$\rightarrow \quad \int e^y \, dy = \int e^{(2x-1)}dx$$

$$\rightarrow \quad e^y = \frac{e^{(2x-1)}}{2} + C$$

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B4b	$\int_0^{2\pi} v^2(t)\,dt = \int_0^{2\pi} 36\sin^2 t\,dt = \frac{36}{2}\int_0^{2\pi} 1-\cos 2t\,dt = \frac{36}{2}\left[t-\frac{\sin 2t}{2}\right]_0^{2\pi} = 36\pi$ $v_{rms} = \sqrt{\frac{36\pi}{2\pi}} = \sqrt{18} = 3\sqrt{2} \simeq 4.24\text{ V}$
B5a	$\mathcal{L}\left\{3t^3 - 2e^{-5t} + 7\right\} = 3\frac{3!}{s^{3+1}} - 2\frac{1}{s-(-5)} + \frac{7}{s} = \frac{18}{s^4} - \frac{2}{s+5} + \frac{7}{s}$
B5b	$\mathcal{L}\left\{f(t)\right\} = \mathcal{L}\left\{2t\sin 4t\right\} = \frac{16s}{\left(s^2+16\right)^2}$ $\mathcal{L}\left\{e^{\pi t} f(t)\right\} = \left.\frac{16s}{\left(s^2+16\right)^2}\right\|_{s\to s-\pi} = \frac{16(s-\pi)}{\left[(s-\pi)^2+16\right]^2}$
B5c	$\mathcal{L}\left\{\cos(3t+\pi)\right\} = \mathcal{L}\left\{\cos 3t\cos\pi - \sin 3t\sin\pi\right\} = -\mathcal{L}\left\{\cos 3t\right\} = -\frac{s}{s^2+9}$
B6a	$\mathcal{L}^{-1}\left\{\frac{3}{s^2} + \frac{2s}{s^2+9} + \frac{5}{s^2+4}\right\} = \mathcal{L}^{-1}\left\{\frac{3\times 1!}{s^{1+1}} + \frac{2s}{s^2+3^2} + \frac{5}{2}\left(\frac{2}{s^2+2^2}\right)\right\} = 3t + 2\cos 3t + \frac{5}{2}\sin 2t$
B6b	$\mathcal{L}^{-1}\left\{\frac{3}{\left(s-1\right)^2+9}\right\} = \mathcal{L}^{-1}\left\{\left.\frac{3}{s^2+3^2}\right\|_{s\to s-1}\right\} = e^t\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\} = e^t\sin 3t$
B6c	$\mathcal{L}^{-1}\left\{\frac{5}{(s-2)(s+2)}\right\} = \mathcal{L}^{-1}\left\{-\frac{5}{4}\left(\frac{1}{s+2}\right) + \frac{5}{4}\left(\frac{1}{s-2}\right)\right\} = -\frac{5}{4}e^{-2t} + \frac{5}{4}e^{2t}$
B7 (i)	$y'' + 4y' + 4y = 0$ The auxiliary equation is $\lambda^2 + 4\lambda + 4 = 0$ $\rightarrow\ (\lambda+2)^2 = 0\ \rightarrow$ one repeated root $\lambda = -2$. $\therefore$ The general solution is $y(t) = e^{-2t}(At+B)$.

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B7 (ii)	$y(0)=1 \ \rightarrow \ y(0)=e^{-2(0)}(A(0)+B)=1 \ \rightarrow \ B=1$ $y'(t)=e^{-2t}A-2e^{-2t}(At+B)$ $y'(0)=-2 \ \rightarrow \ y'(0)=e^{-2(0)}A-2e^{-2(0)}(A(0)+B)=-2$ $\rightarrow \ A-2B=-2 \ \rightarrow \ A=-2+2B \ \rightarrow \ A=0$ Thus the particular solution is $y(t)=e^{-2t}$
C1	$I=\dfrac{V}{R}$; $\dfrac{\Delta V}{V}=0.01$; $\dfrac{\Delta R}{R}=0.04$ $\Delta I \simeq \dfrac{\partial I}{\partial V}\Delta V+\dfrac{\partial I}{\partial R}\Delta R$ $\dfrac{\Delta I}{I} \simeq \dfrac{1}{I}\dfrac{\partial I}{\partial V}\Delta V+\dfrac{1}{I}\dfrac{\partial I}{\partial R}\Delta R=\dfrac{\partial(\ln I)}{\partial V}\Delta V+\dfrac{\partial(\ln I)}{\partial R}\Delta R$ $\ln I=\ln V-\ln R$, $\dfrac{\partial(\ln I)}{\partial V}=\dfrac{1}{V}$, $\dfrac{\partial(\ln I)}{\partial R}=-\dfrac{1}{R}$. $\therefore \ \dfrac{\Delta I}{I} \simeq \dfrac{\Delta V}{V}-\dfrac{\Delta R}{R}=0.01-0.04=-0.03$ or -3%
C2a	$\dfrac{dT}{dt}=-k\left(T-R\right)$
C2b	Since $k=0.1$ and $R=30°$ C, $\dfrac{dT}{dt}=-0.1\left(T-30\right)$, $\displaystyle\int\frac{dT}{T-30}=\int -0.1\,dt$ $\ln\lvert T-30\rvert=-0.1t+C$ $T-30=Ae^{-0.1t}$ or $T=Ae^{-0.1t}+30$ At $t=0$, $T=120°$ C: $120-30=Ae^{-0.1(0)} \rightarrow A=90$ Thus $T=90e^{-0.1t}+30$
C2c	$T(2)=90e^{-0.1(2)}+30=103.69°C$

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C3a	$\dfrac{5000}{(s^2+100)(s^2+10s+100)}=\dfrac{As}{s^2+100}+\dfrac{5s+D}{s^2+10s+100}$ $5000=As(s^2+10s+100)+(5s+D)(s^2+100)$ s^3 term: $0=A+5 \rightarrow A=-5$ const: $5000=100D \rightarrow D=50$ $\therefore \dfrac{5000}{(s^2+100)(s^2+10s+100)}=\dfrac{-5s}{s^2+100}+\dfrac{5s+50}{s^2+10s+100}$
C3b	(i) $E(t)=500\sin 10t$ $Lq''(t)+Rq'(t)+\dfrac{q(t)}{C}=500\sin 10t$ $q(0)=q'(0)=0$ (as $q'(t)=i(t)$)
	(ii) $L=1$ henry, $R=10\ \Omega$, $C=0.01$ farad, $q''(t)+10q'(t)+100q(t)=500\sin 10t$ Take Laplace transform on both sides of the equation Let $Q=\mathcal{L}\{q\}$ $\left[s^2Q-sq(0)-q'(0)\right]+10\left[sQ-q(0)\right]+100Q=\dfrac{5000}{s^2+10^2}$ $(s^2+10s+100)Q=\dfrac{5000}{s^2+100} \rightarrow Q=\dfrac{5000}{(s^2+100)(s^2+10s+100)}$ (iii) From (a): $Q=\dfrac{-5s}{s^2+100}+\dfrac{5s+50}{s^2+10s+100}$ $=-5\left(\dfrac{s}{s^2+10^2}\right)+\dfrac{5(s+5)+25}{(s+5)^2+75}$ $\therefore q(t)=\mathcal{L}^{-1}\{Q\}$ $=-5\cos 10t+e^{-5t}\mathcal{L}^{-1}\left\{\dfrac{5s}{s^2+\left[\sqrt{25(3)}\right]^2}+\dfrac{25}{s^2+\left[\sqrt{25(3)}\right]^2}\right\}$ $=-5\cos 10t+e^{-5t}\left(5\cos 5\sqrt{3}t+\dfrac{5\sqrt{3}}{3}\sin 5\sqrt{3}t\right)$