Dynamics

PRE-CLASS (1 TO 17)

IN-CLASS (19 ONWARDS)

Learning outcomes of pre-class slides

After reading the pre-class slides, students should be able to

- identify the different types of forces (contact / non-contact / normal / tension / friction / weight)
- use vector addition to determine the resultant of two or more forces acting on an object
- state Newton’s laws

Dynamics

- Dynamics describes the relationship between the motion of an object and the forces that cause the motion.
- Dynamics answers questions such as why it is harder to control a car on a wet icy road than on a dry road, or how a rocket is able to propel itself upward.
- We will study the laws of physics which describe nature in the macroscopic world.

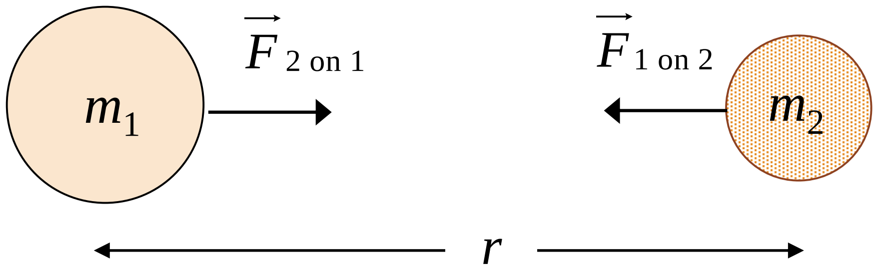
Force

- A force is an interaction between two bodies or between a body and its environment.

- E.g. the magnitude of the gravitational force between two masses, m_1 and m_2, separated by distance r is given by

$$F = G\frac{m_1 m_2}{r^2}$$

- If either mass disappears, the force between then ceases to act.



Long range force vs contact force

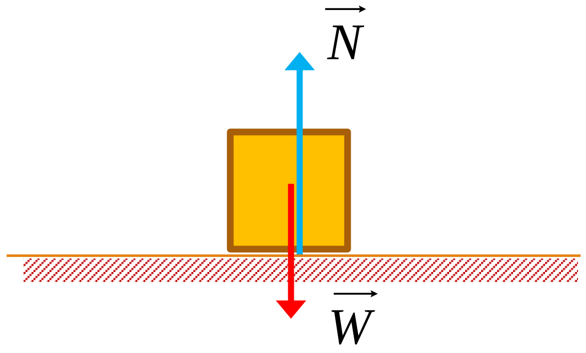
- Contact force arises when there is contact between objects. Examples are kicking a ball, pushing a cart, etc.
- The known contact forces are normal force, tension, and frictional force.
- Long-range force is felt even when the objects are not in physical contact. Examples are the gravitational pull between the Earth and the Moon, the repulsion between like electric charges, etc.
- The gravitational pull the earth exerts on you is known as your weight $\vec{W}$.

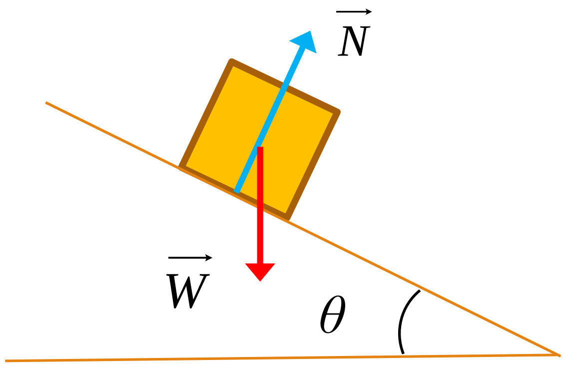
Contact force – normal force

- The perpendicular force exerted by the surface on an object is known as the normal force $\vec{N}$.

- The magnitude of the normal force (or normal reaction) is not always equal to the magnitude of the weight of the object.

- For example, if the object is on an inclined surface, its normal reaction is $N = W\cos\theta$, which is less than W.

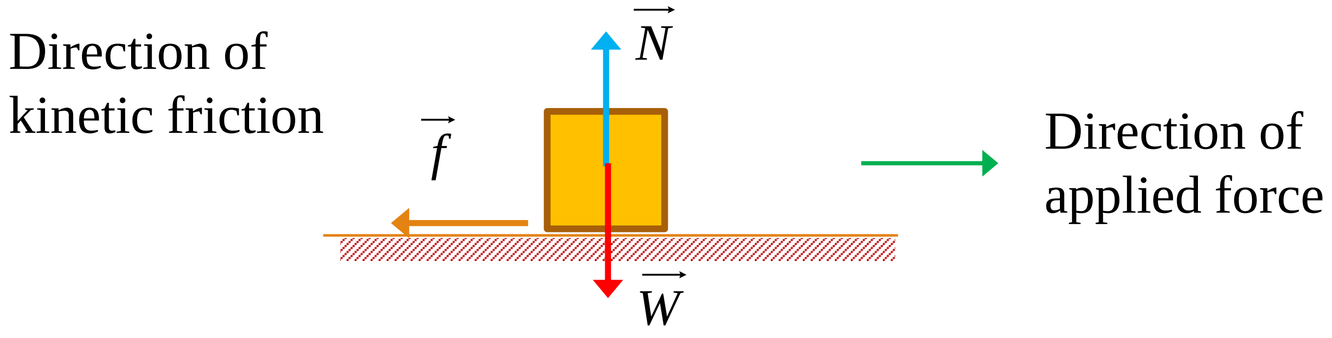




Contact force – Friction

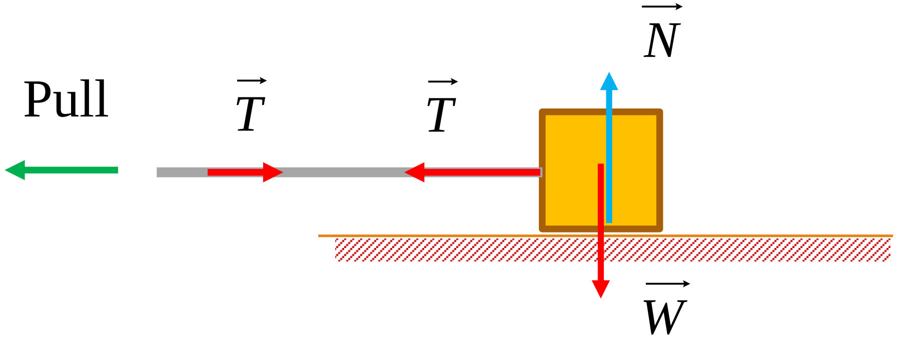
- Frictional force or just friction occurs when a surface resists motion of an object that is in contact with the surface.

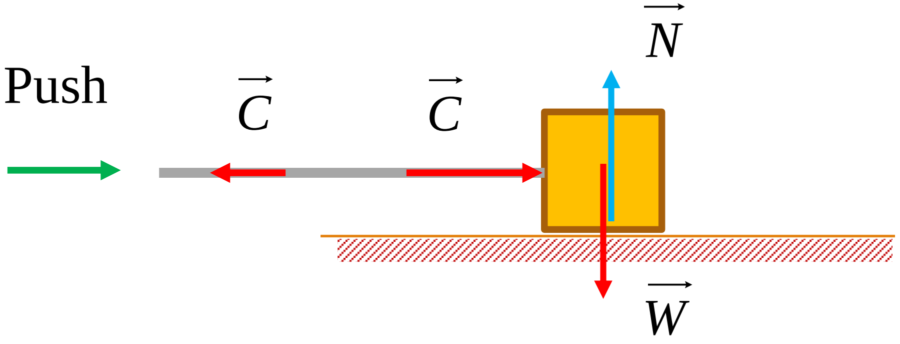
- The direction of the kinetic friction is always opposite to the direction of the force causing the motion.



Contact force – Tension and compression

- When a rod is stretched, equal and opposite forces known as tension force appears in the rod.
- When the rod is compressed, equal and opposite forces known as compression force appears in the rod.
- Within limits, compression and tension forces are governed by Hooke's law, $F = -kx$, where k is the spring constant and x is the elongation/compression.





Superposition of forces

- Force is a vector quantity. Adding forces requires vector addition. Techniques we learned from the Vectors chapter are needed.

- The principle of superposition of forces states that any number of forces acting on a body have the same effect as a single net or resultant force.

$$\vec{R} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

- If the net force is zero, the object is said to be in equilibrium.

- In equilibrium, each component of the net force is zero, i.e.

$$\sum \vec{F}_x = 0, \qquad \sum \vec{F}_y = 0, \qquad \text{and} \qquad \sum \vec{F}_z = 0$$

Superposition of forces (recap of vector addition)

- The resultant of $\vec{F}_1$ and $\vec{F}_2$ is: $\vec{R} = \vec{F}_1 + \vec{F}_2$
- The x- and y-component of the resultant is:

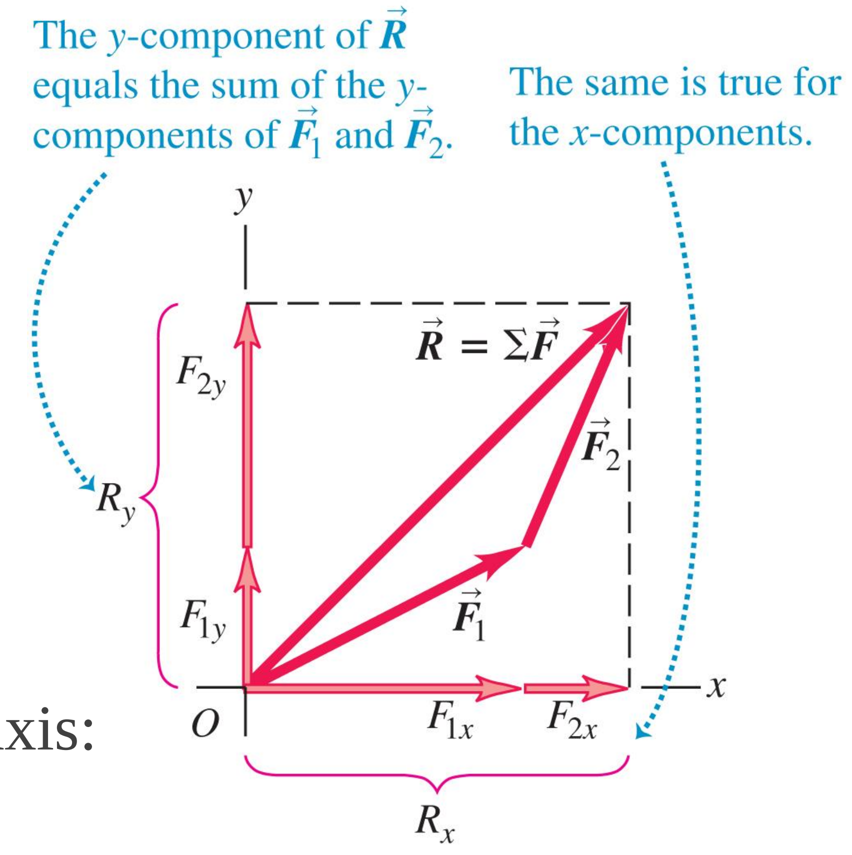
$$R_x = F_{1x} + F_{2x} \qquad R_y = F_{1y} + F_{2y}$$

- Magnitude of the resultant (net) force:

$$R = \sqrt{R_x^2 + R_y^2}$$

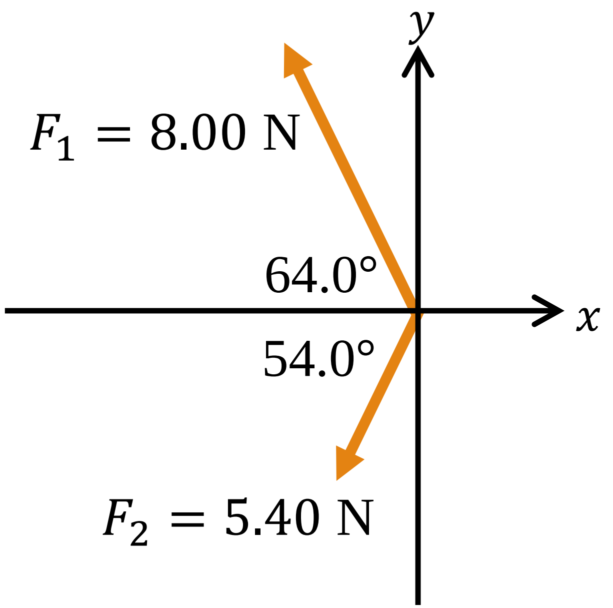
- Direction of the resultant force with the positive x-axis:

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



Example 1

Forces $\vec{F}_1$ and $\vec{F}_2$ act at a point. The magnitude of $\vec{F}_1$ is 8.00 N, and its direction is 64.0° above the *x*-axis in the 2nd quadrant. The magnitude of $\vec{F}_2$ is 5.40 N, and its direction is 54.0° below the *x*-axis in the 3rd quadrant. What are the components and magnitude of the resultant force?



Solution:

Resolving both forces into the *x* and *y* components:

$$F_{1x} = -8\cos 64.0° \qquad F_{1y} = 8\sin 64.0°$$
$$F_{2x} = -5.40\cos 54.0° \qquad F_{2y} = -5.40\sin 54.0°$$

Summing the components:

$$F_{net,x} = F_{1x} + F_{2x} = -8\cos 64.0° - 5.40\cos 54.0° = -6.68\text{ N}$$
$$F_{net,y} = F_{1y} + F_{2y} = 8\sin 64.0° - 5.40\sin 54.0° = 2.82\text{ N}$$

Magnitude of resultant force: $F_{net} = \sqrt{F_{net,x}^2 + F_{net,y}^2} = 7.25\text{ N}$

Example 2

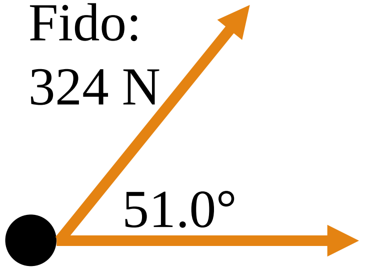
Two dogs pull horizontally on ropes attached to a post; the angle between the ropes is 51.0°. If Rover exerts a force of 288 N and Fido exerts a force of 324 N, find the magnitude of the resultant force and the angle it makes with Rover's rope.

Solution:

Since there is no mention which direction the forces are, we assume Rover exerts a force to the right.

Resolving the force by Fido into horizontal and vertical components:

$$F_{Fido,x} = 324\cos 51.0° \qquad F_{Fido,y} = 324\sin 51.0°$$



The x-component of the net force is:

$$F_{net,x} = F_{Rover} + F_{Fido,x} = 288 + 324\cos 51.0°$$

The y-component of the net force is: $F_{net,y} = 324\sin 51.0°$

Magnitude of resultant force: $F_{net} = \sqrt{F_{net,x}^2 + F_{net,y}^2} = 553\text{ N}$

Angle it makes with Rover's rope: $\theta = \tan^{-1}\left(\frac{F_{net,y}}{F_{net,x}}\right) = 27.1°$

Newton’s laws of motion

Newton's laws of motion (overview)

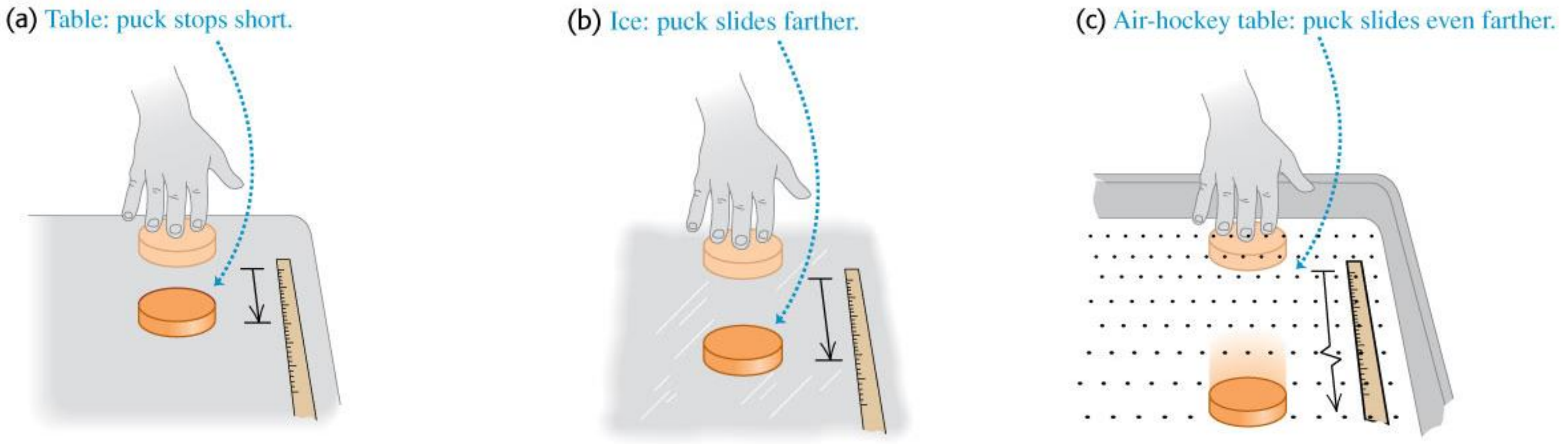
- **Newton's 1st law:** An object continues to stay at rest or move with constant velocity unless there is a net force (F_{net}) acting on it.
  - Newton's 1st law describes the motion of an object when F_{net} is 0.
- **Newton's 2nd law:** The net force acting on an object is the product of mass and acceleration. The direction of acceleration points in the direction of the F_{net}.
  - Newton's 2nd law allows us to calculate the acceleration of an object when F_{net} is not zero.
- **Newton's 3rd law:** When object A exerts a force on object B, B also exerts an equal and opposite reaction force on A.
  - Newton's 3rd law affirms that a force pair is a result of an interaction.

Newton’s first law of motion

- Every body continues in its state of rest or uniform motion (constant velocity) unless a net force acts on it to change that state.
- You do not need a force to sustain motion!
- Inertia is the property of a body which resists change in its state of motion.
- The larger the mass, the greater its inertia.

Newton's first law of motion

- In the first two scenarios below, the pucks move different distances because of difference in frictional forces.
- In the third scenario, friction is totally absent and the puck moves with constant velocity, i.e. constant speed in a straight line.



Newton’s first law as the law of inertia

- A frame of reference in which Newton’s first law is valid is known as an inertial frame of reference.
- Since we define an inertial frame of reference using Newton’s first law we refer to this law as the law of inertia.
- A non-inertial frame of reference is an accelerating frame where Newton’s first law is not valid.

End of pre-class slides

Learning outcomes

After the class, students should be able to

- solve force and motion related problems using Newton's laws
- distinguish static friction and kinetic friction
- recognize that the centripetal acceleration always point towards the centre of the circle in uniform circular motion

Newton’s second law of motion

- This law states that the magnitude of the net force acting on a body, of constant mass, is directly proportional to the magnitude of acceleration of a body.

- Newton’s second law of motion can be expressed as

$$\sum \vec{F} = \vec{F}_{net} = m\frac{d}{dt}\vec{v} = m\vec{a}$$

- In component form, $\vec{F}_{net,x} = m\vec{a}_x$, $\vec{F}_{net,y} = m\vec{a}_y$, $\vec{F}_{net,z} = m\vec{a}_z$

Definition of one newton (self-read)

- Newton's second law is used to define the magnitude of one newton.
- One newton (N) is the net force that gives a mass of one kilogram an acceleration of one metre per second square.
- Mathematically, $1\ \text{N} = 1\ \text{kg} \times 1\ \text{m/s}^2$.

Mass and weight of an object

- Note that mass and weight are not the same.
- Since an object of mass m accelerates at g m/s^2 under gravity when there is no friction, then according to Newton's second law, the magnitude of the force acting on it must be, $F = mg$.
- The force F is called the weight of the object, i.e., $F = W = mg$.
- An object in outer space is weightless but not massless.

Newton’s laws are not valid in non-inertial frames

- Suppose person on roller skates is on a bus moving at constant velocity.
- If the bus suddenly speeds up/slows down, the person starts moving backward/forward.
- Newton’s second law does not seem to be obeyed as without any net force on the person, the velocity of the person changes.
- This is because the bus is accelerating with respect to earth and is not an inertial frame.

Newton’s third law of motion

- Newton’s third law states that if body A exerts a force on body B, then B exerts an equal but opposite reaction force on body A.
- These two equal and opposite forces are known as an action-reaction pair.
- Mathematically we write Newton's third law as $\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$.
- These two forces **DO NOT** act on the same object. They act on different objects.
- Here is one useful link: [Newton's Third Law - YouTube]()

Example 3

An apple sitting at rest on a table is in equilibrium.

a) What forces act on the apple?

b) What is the reaction force to each of the forces acting on the apple?

c) What are the action-reaction pairs?

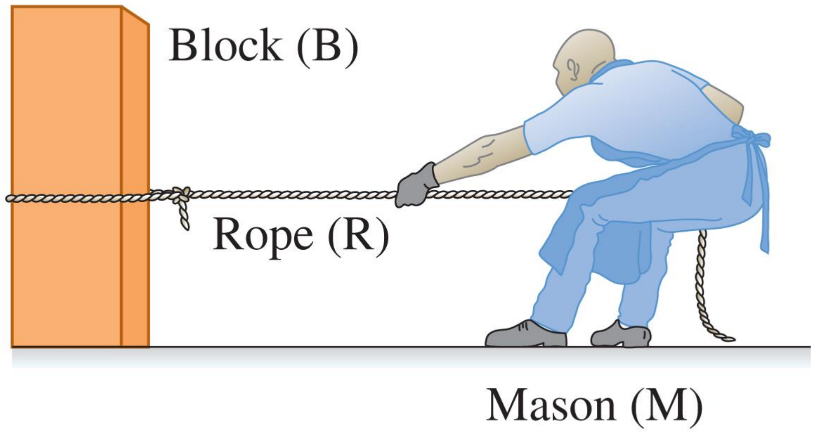
Example 4

A stone mason drags a marble block across a floor by pulling on a rope attached to the block (see figure). The block is not necessarily in equilibrium.

a) How are the various forces related?

b) What are the action-reaction pairs?

(a) The block, the rope, and the mason

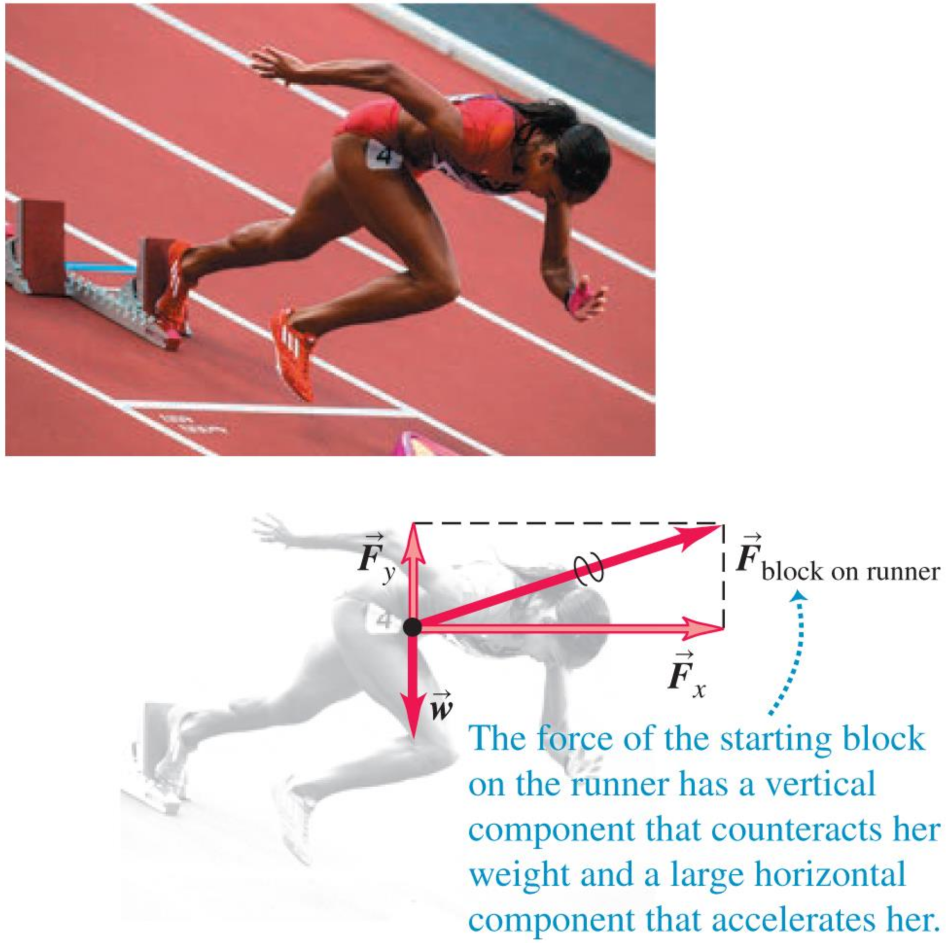


Free-body diagram

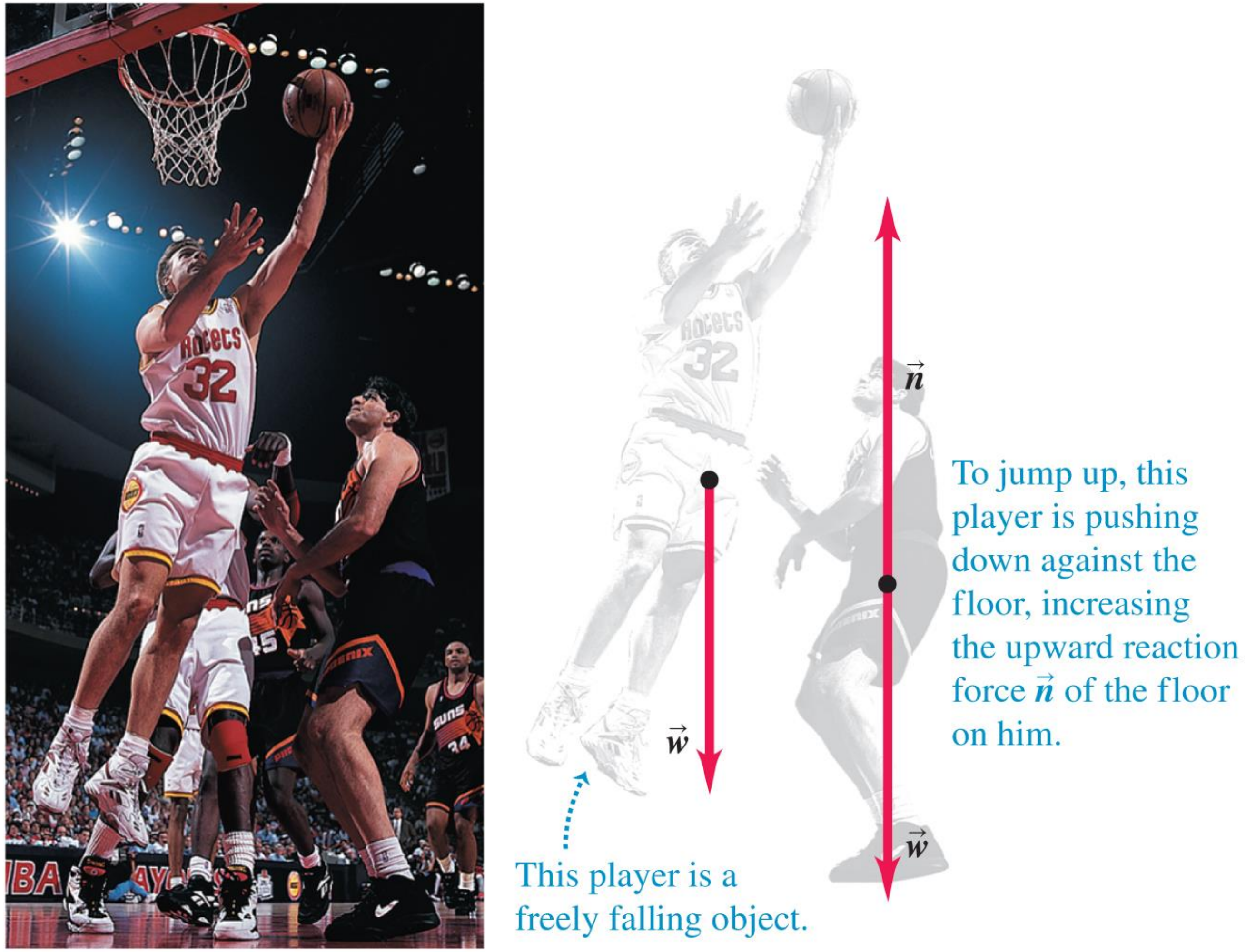
- To solve problems involving forces, we need to identify the forces acting on an object. We will draw free-body diagrams.
- Free-body diagrams are diagrams used to show the magnitude and direction of all forces acting upon an object in a given situation.
- When drawing a free-body diagram, we choose an object of interest and draw all the forces acting on it due to other bodies.
- Forces that the object of interest exerts on other bodies are **NOT** included.
- If necessary, label forces using this convention – force <u>by A on B</u> and so on.

Examples of free-body diagrams

(a)



(b)



Applying Newton’s laws

Problem solving strategy involving Newton's 2^{nd} law

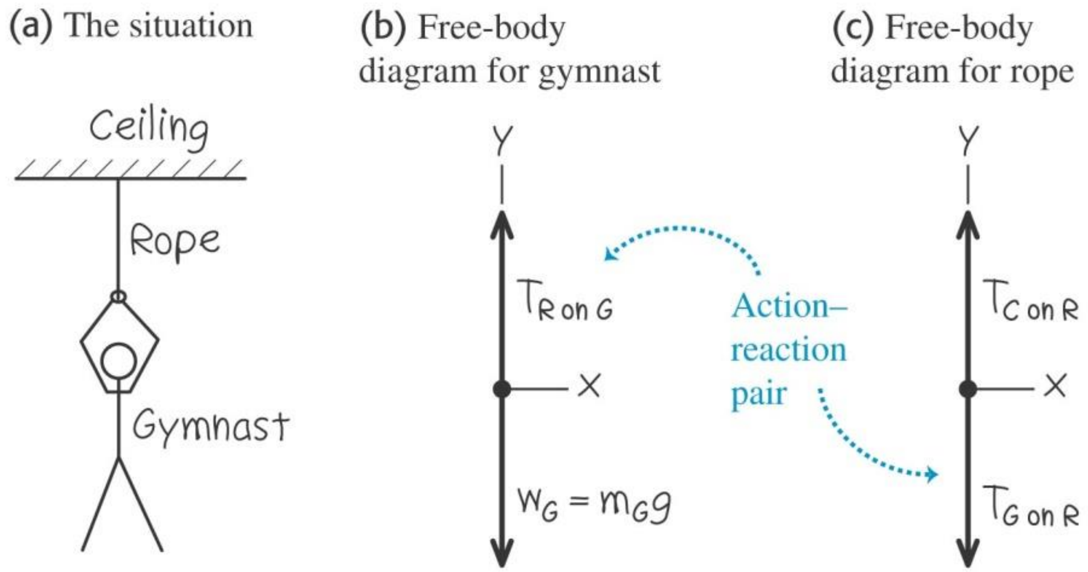
1. Draw the free-body diagrams of the objects of interest.
2. Identify the direction of acceleration (a).
3. Draw a coordinate system.
   - If $a = 0$, choose a coordinate system such that you resolve less forces.
   - If $a \neq 0$, choose a coordinate system such that one of the axes points in the direction of a.
4. Write down equations using Newton's 2^{nd} law in the x and y directions.
5. Solve for the unknowns.

Example 5

a) A gymnast with mass m_G = 50.0 kg suspends herself from the lower end of a hanging rope of negligible mass. The upper end of the rope is attached to the ceiling.

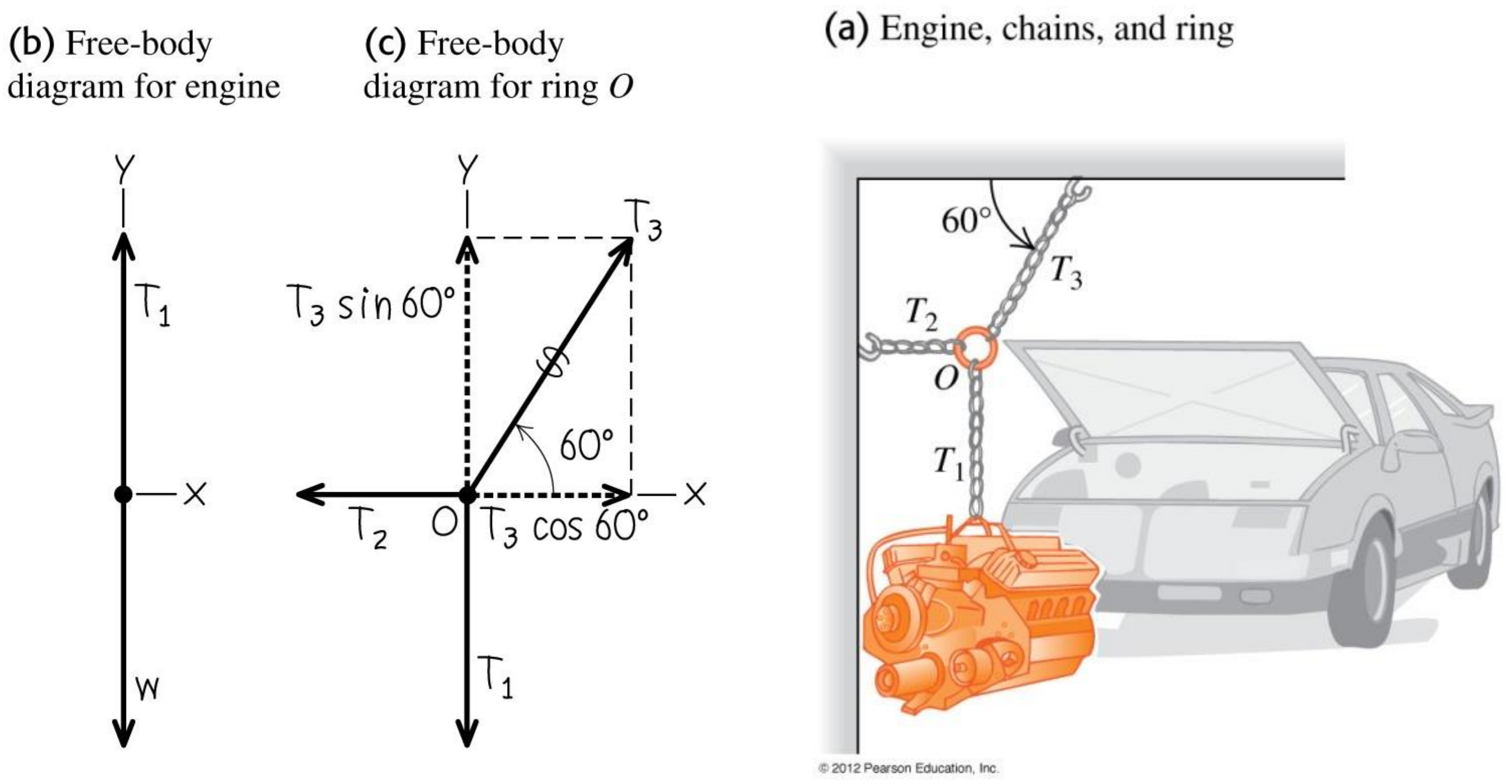
- i) What is the gymnast's weight?
- ii) What force (magnitude and direction) does the rope exert on her?
- iii) What is the tension at the top of the rope?

b) Find the tension at each end of the rope if the weight of the rope is 120 N.



Example 6

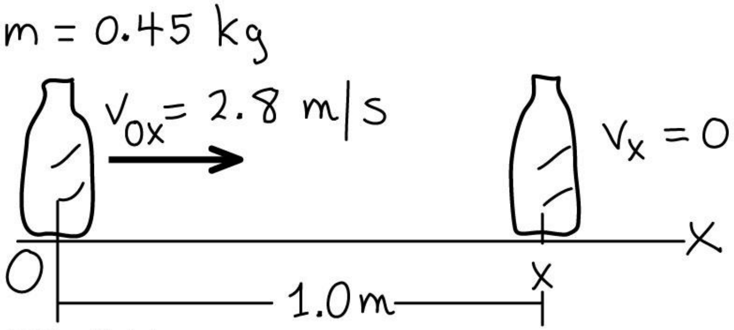
A car engine with weight w hangs from a chain that is linked at ring O to two other chains that are attached to the ceiling and wall respectively (see figure). Find expressions for the tension in each of the three chains in terms of w. The weights of the ring and chains are negligible compared with the weight of the engine.

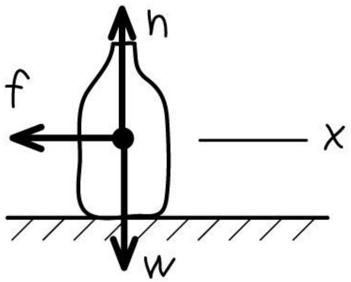


Example 7

An object with mass 0.45 kg with initial velocity of 2.8 m/s moves on a rough table. The friction force slows down the object which comes to rest after travelling 1.0 m. What are the magnitude and direction of the friction force acting on the object? You can assume a constant friction force.

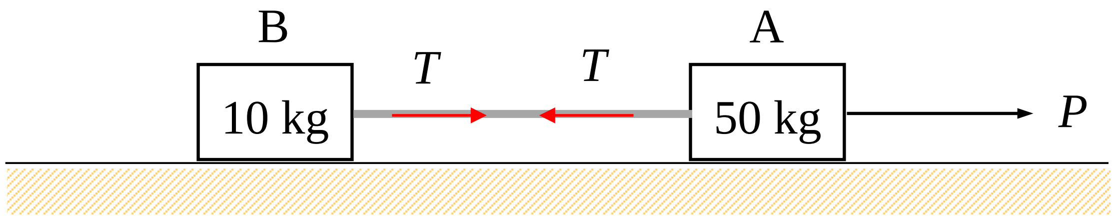
We draw one diagram for the bottle's motion and one showing the forces on the bottle.





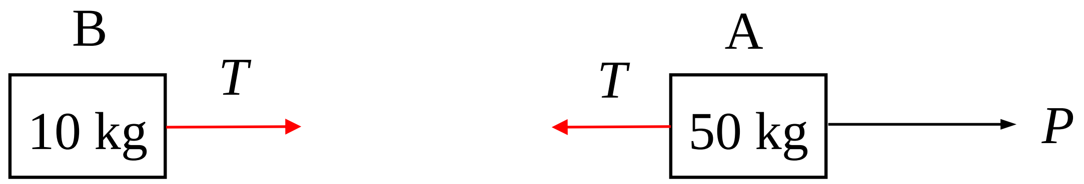
Example 8

Two blocks A and B are connected as in figure below on a smooth horizontal floor and pulled to the right with an acceleration of 2.0 m/s^2 by force P. If M_A = 50 kg and M_B = 10 kg, what are the values of T and P?



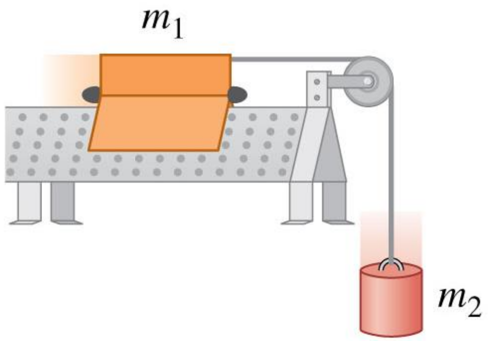
Mass B, it is acted on by only the tension T. Mass A is acted on by the tension T and the force P.

The weights and normal reactions of A and B are not shown because they are perpendicular to their directions of motion.

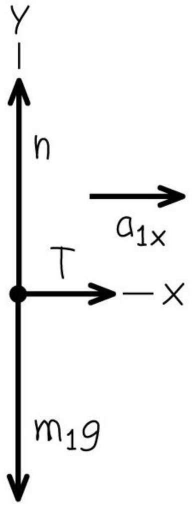


Example 9

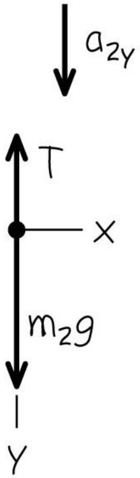
An air-track glider with mass m_1 moves on a level, frictionless surface. The glider is connected to another object of mass m_2 through a light, flexible, non-stretching string that passes over a stationary frictionless pulley (see figure). Find the acceleration of the glider and the object as well as the tension in the string.



(b) Free-body diagram for glider



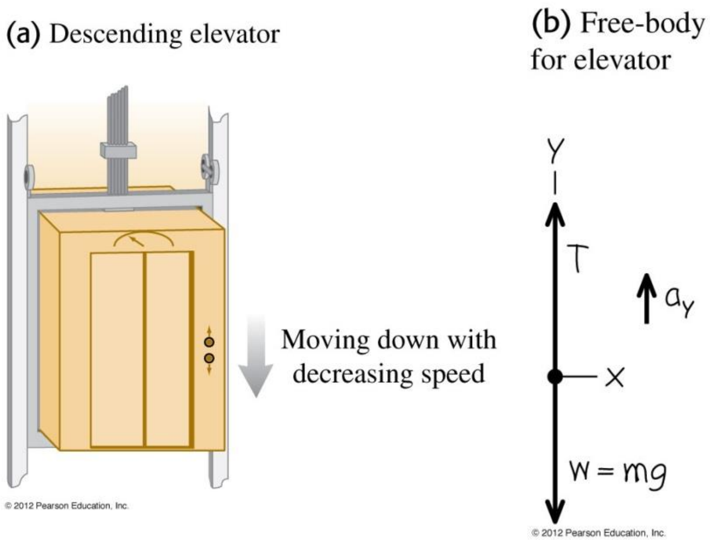
(c) Free-body diagram for weight

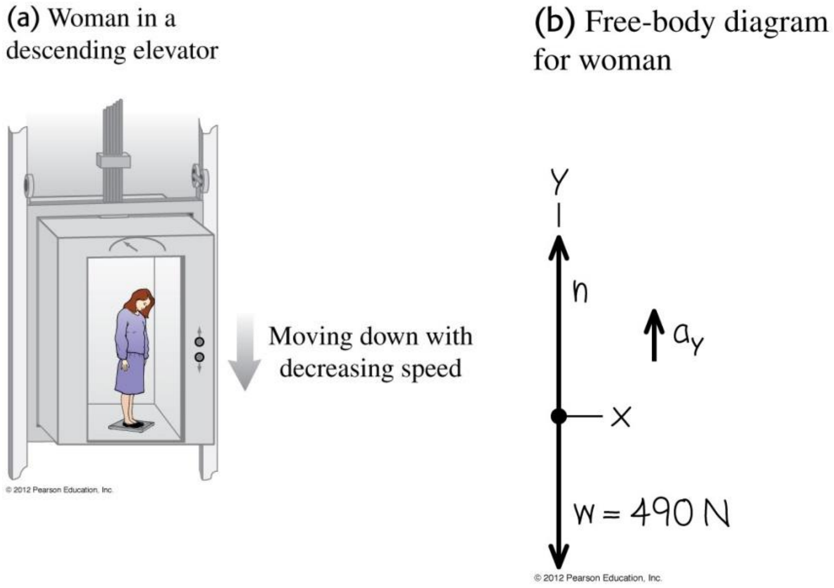


Example 10

a) An elevator and its load have a combined mass of 800 kg. The elevator is initially moving downward at 10.0 m/s; it slows to a stop with constant acceleration in a distance of 25.0 m. What is the tension T in the supporting cable while the elevator is being brought to rest?

b) If a 50.0 kg woman stands on a bathroom scale while riding in the elevator. What is the reading on the scale?





Static and kinetic friction

Kinetic friction

- When an object moves on a surface, the friction force that acts is known as kinetic friction force (f_k).
- In some scenarios the kinetic friction force is approximately proportional to normal force (N) and can be represented as

$$f_k = \mu_k N$$

- Note that the above equation is a scalar equation.
- μ_k is known as the coefficient of kinetic friction.

Static friction

- Friction forces may act even there is no relative motion. This friction is called a static friction force f_s.

- The static friction force on a stationary object increases until a maximum value before the objects starts moving.

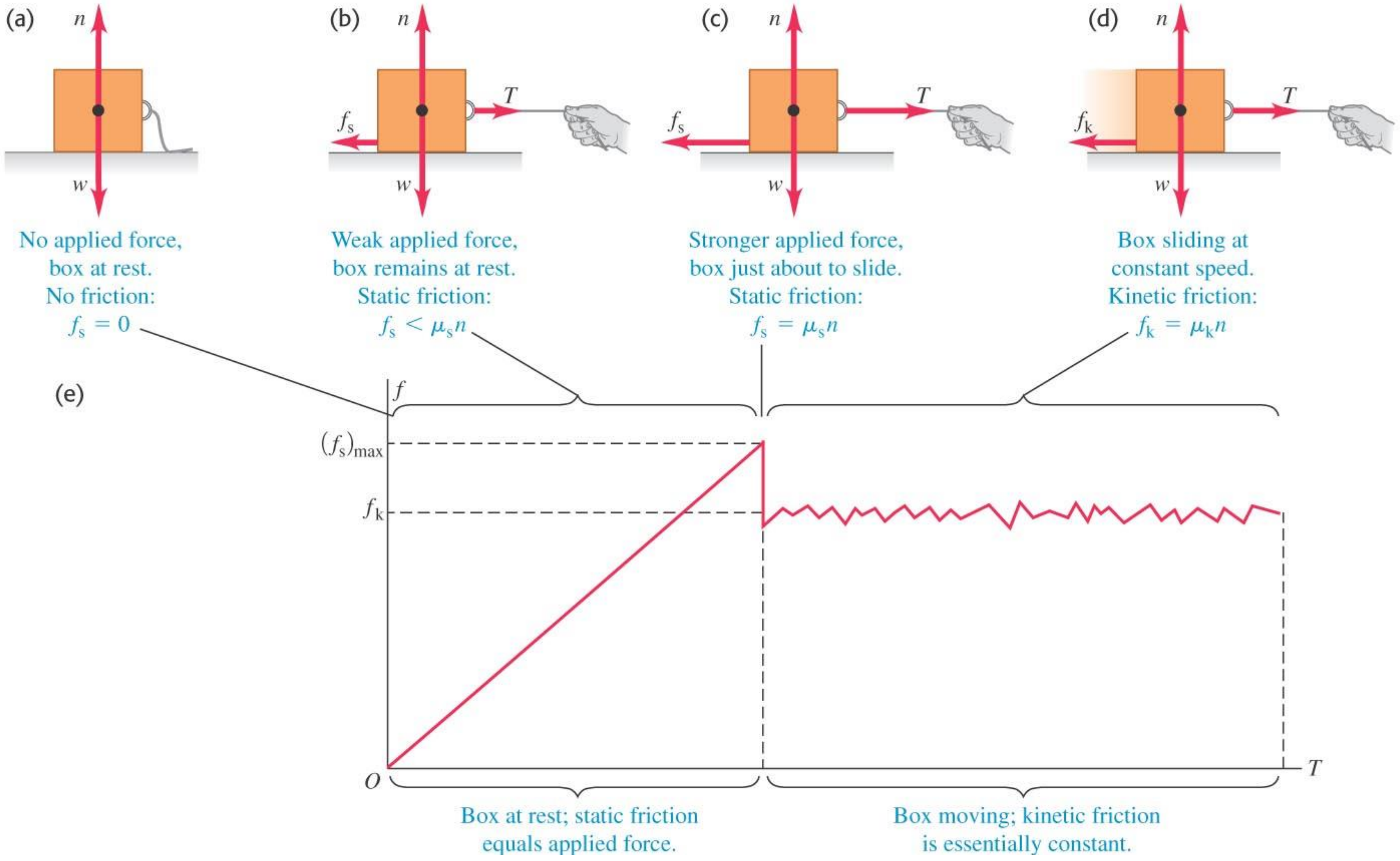
- The maximum static friction force is given by

$$f_{s,max} = \mu_s N$$

where μ_s is known as the coefficient of static friction. Note that $\mu_s > \mu_k$.

From static to kinetic friction

- The figures at right show the various scenarios and the friction force involved.
- We can see $f_k < f_{s,max}$, which means $\mu_k < \mu_s$.



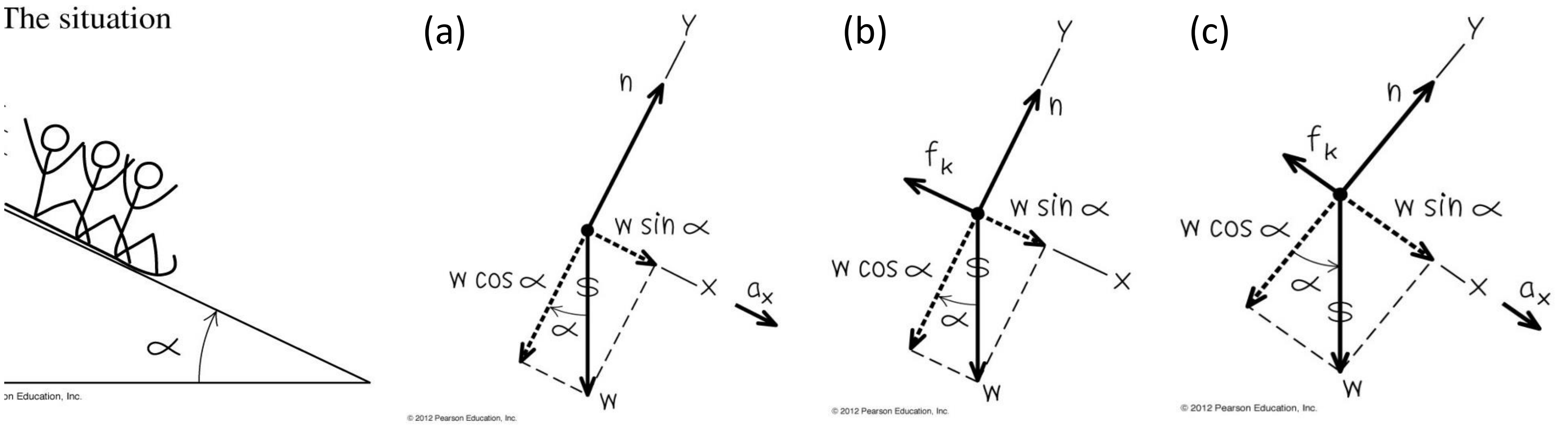
Example 11

A toboggan loaded with students (total weight w) slides down a snow-covered slope.

a) Assuming no friction what is the acceleration of toboggan if the hill slopes at a constant angle α?

b) Assuming friction with coefficient of kinetic friction μ_k, what should be the angle so that the toboggan slides at constant velocity?

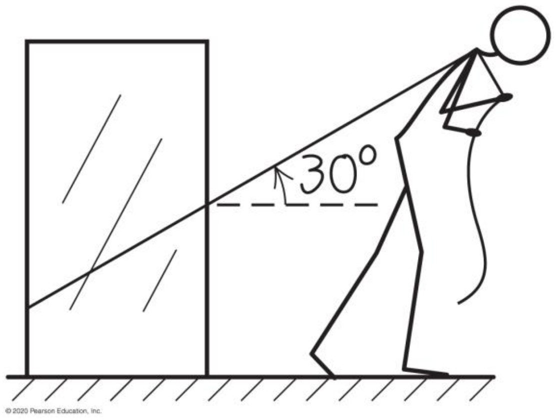
c) Assuming friction with coefficient of kinetic friction μ_k, what should be the acceleration of the toboggan in terms of α, μ_k, g, and w?



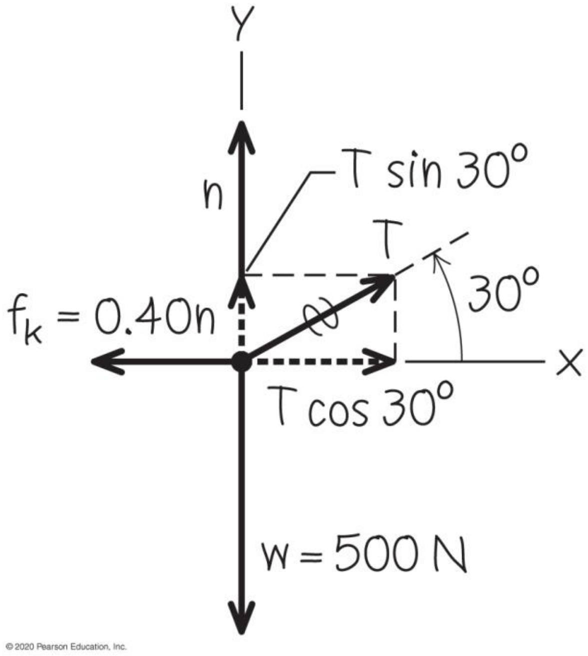
Example 12

Suppose you move a 500 N crate across a level floor by pulling upward on the rope at an angle of 30° above the horizontal. How hard must you pull to keep it moving with constant velocity? Assume $\mu_k = 0.40$.

(a) Pulling a crate at an angle



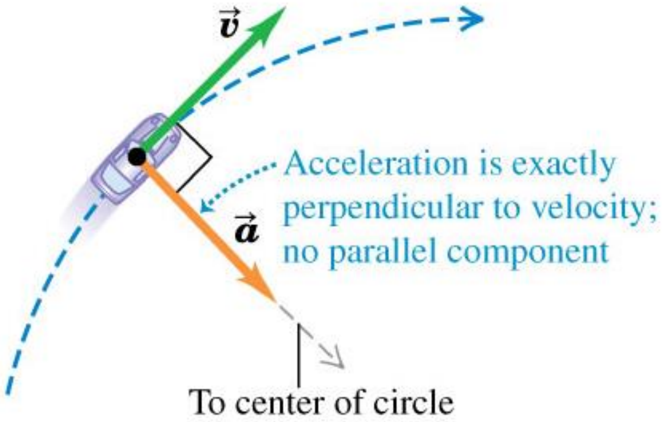
(b) Free-body diagram for moving crate



Circular motion

Uniform circular motion

- When a particle moves in a circle with constant speed, the motion is called uniform circular motion.
- There is only the perpendicular component of acceleration directed inward, which is a result of the change in direction of the particle.



Centripetal acceleration

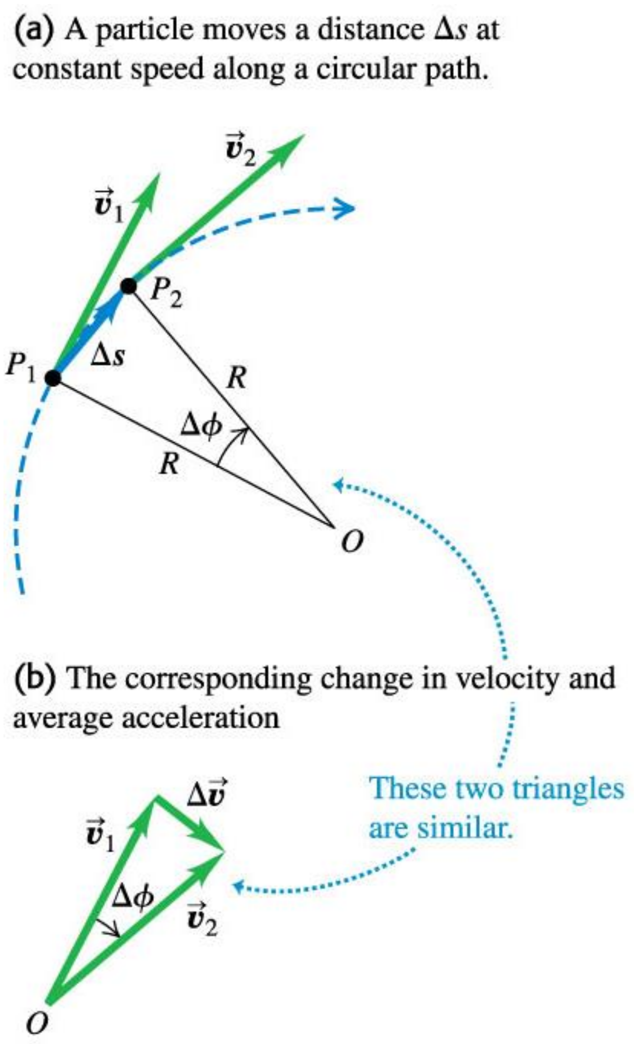
- The angle $\Delta\phi$ is the same for both triangles because $\vec{v}_1$ is perpendicular to line OP_1 while $\vec{v}_2$ is perpendicular to line OP_2.

- Since the two triangles are similar,

$$|\vec{v}_1| = |\vec{v}_2| = v$$

$$\frac{|\Delta\vec{v}|}{v} = \frac{\Delta s}{R}$$

$$|\Delta\vec{v}| = v\frac{\Delta s}{R}$$



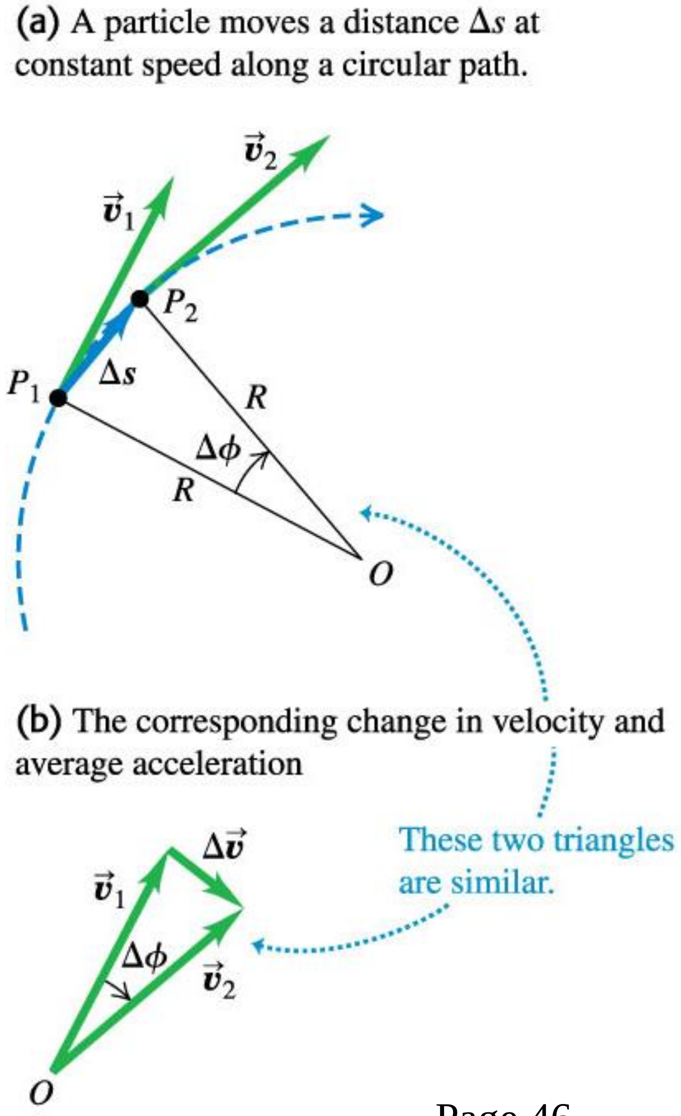
Centripetal acceleration

- The magnitude of the average acceleration is

$$a_{av} = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{v\Delta s}{R\Delta t}$$

- The magnitude of the instantaneous acceleration is

$$a_{rad} = \lim_{\Delta t \to 0} \frac{|\Delta \vec{v}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{v\Delta s}{R\Delta t}$$
$$= \frac{v}{R} \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{v^2}{R}$$



Period of revolution

- If T is the time taken by the particle to complete one revolution, then the particle travels a distance equal to the circumference, i.e.

$$v = \frac{2\pi R}{T}$$

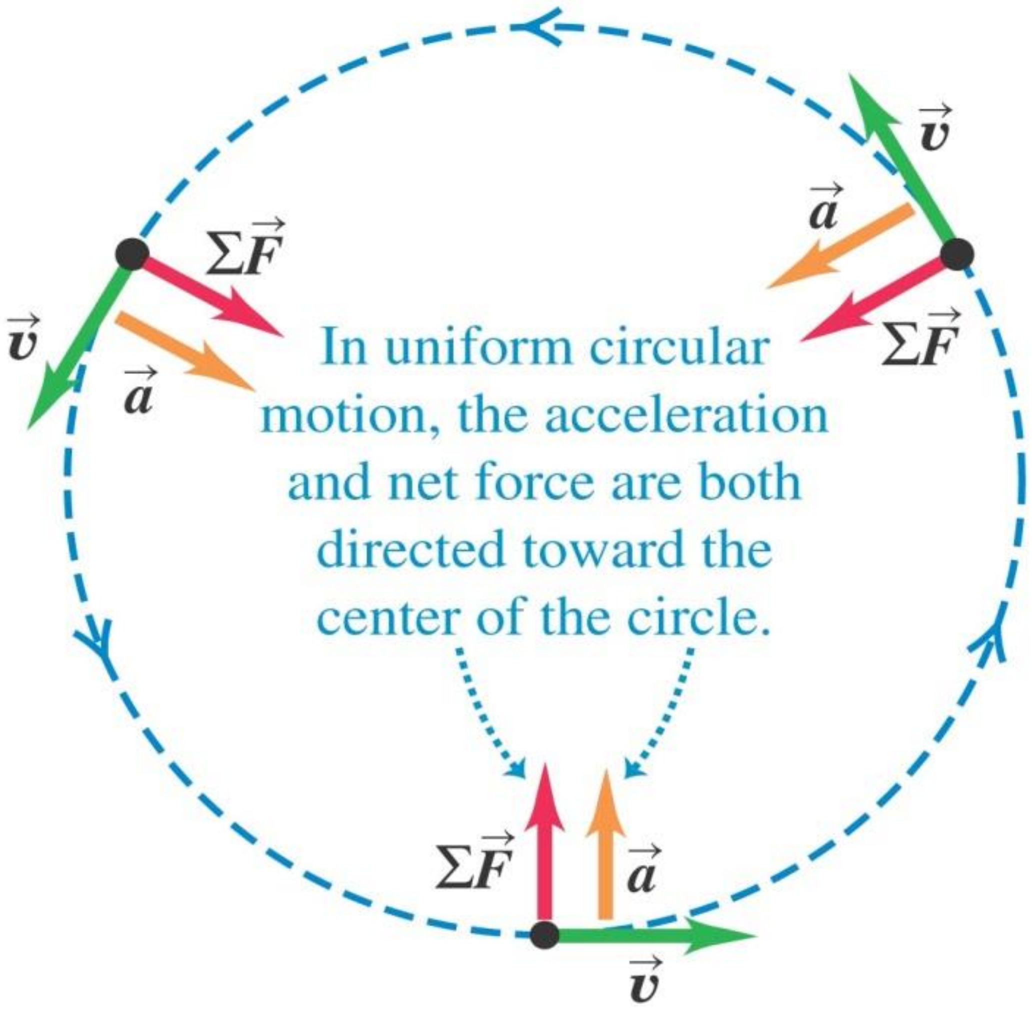
- The centripetal acceleration can be written in terms of T as

$$a_{rad} = \frac{v^2}{R} = \left(\frac{2\pi R}{T}\right)^2 \frac{1}{R} = \frac{4\pi^2 R}{T^2}$$

Centripetal force

- If there is a centripetal acceleration, then by Newton's second law, a net force must be present.
- We call this net force the centripetal force and it acts towards the centre of the circle (see figure).
- Its magnitude is given by

$$F_{net} = ma = \frac{mv^2}{R}$$



Some useful things to remember

- The centripetal acceleration is directed towards the centre of the circular path.
- The centripetal force
  - points towards the centre of the circular path.
  - prevents the particle from travelling in a tangential direction.
  - is perpendicular to the displacement of the particle and does no work on the particle.

Example 13

a) An object moves along a circle or a circular arc with a constant speed is accelerating.
   i) Why is it accelerating?
   ii) What is its acceleration?

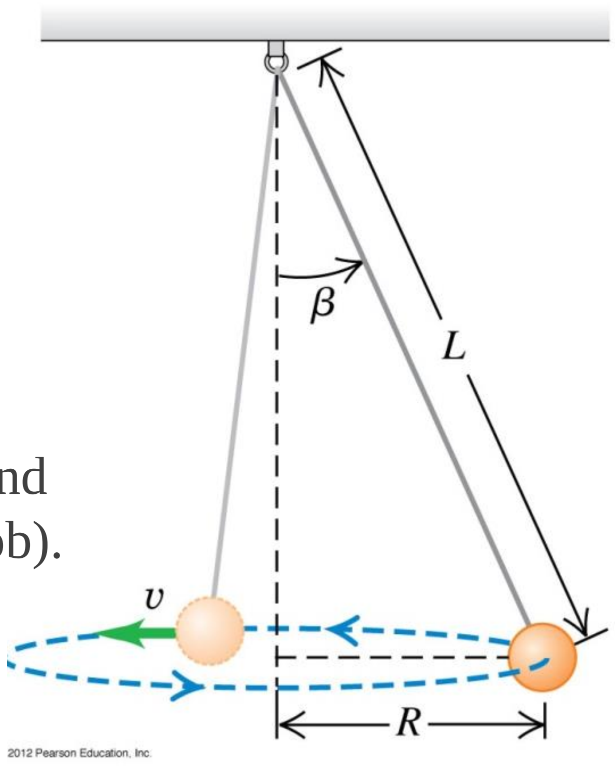
b) For a body to move in a circular path, a force must act on it from outside the body.
   i) What is this required force?
   ii) What is providing this required force?

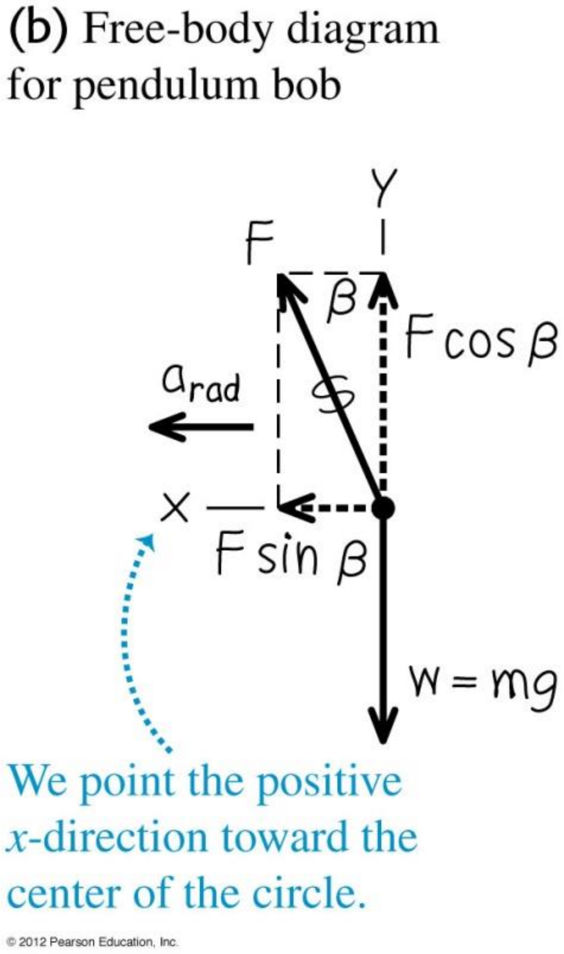
Example 14

When an aircraft is moving in a horizontal plane at a constant speed of 650 m/s, its turning circle has a radius of 80 km. What is the ratio of the centripetal force to the weight of the aircraft? Take g = 9.80 m/s^2.

Example 15

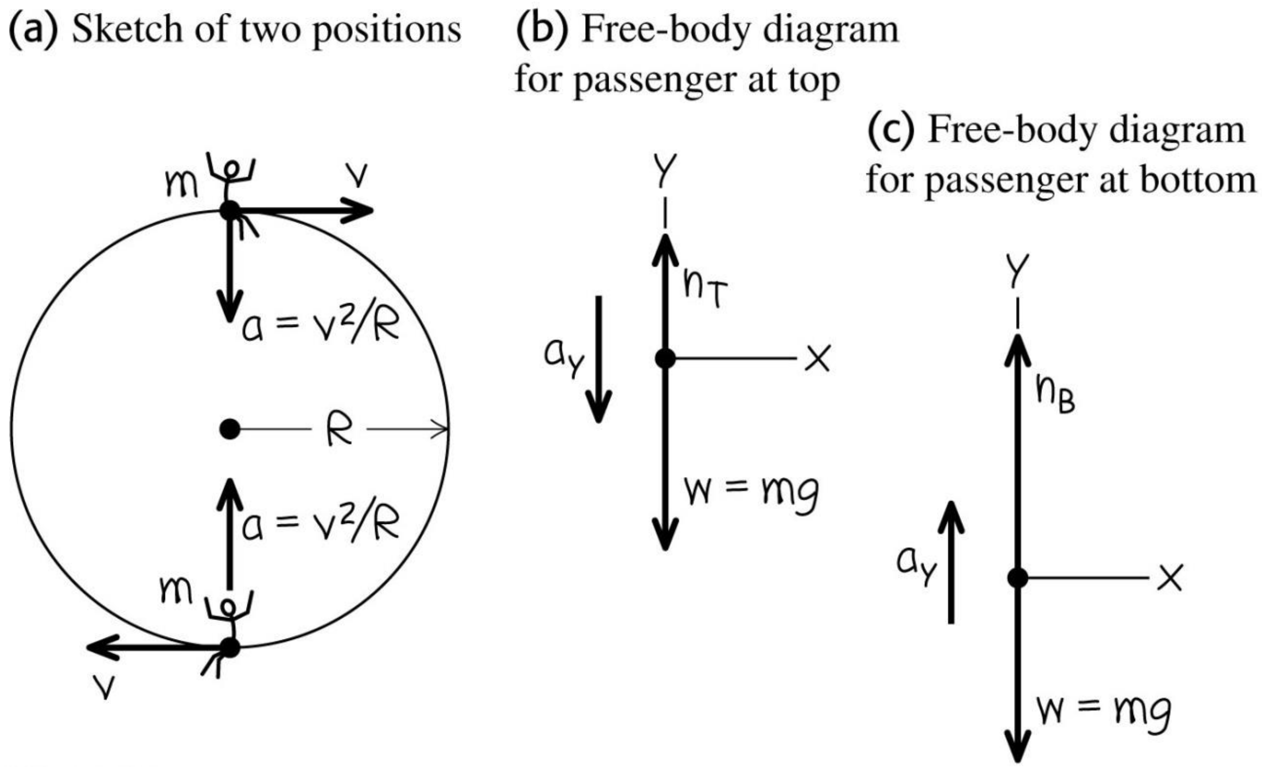
A inventor designs a pendulum clock using a bob with mass m at the end of a thin wire of length L. Instead of swinging back and forth, the bob is to move in a horizontal circle with constant speed v with the wire making a fixed angle β with the vertical direction (see figure). This is called a *conical pendulum* because the suspending wire traces out a cone. Find the tension F in the wire and the period T (the time for one revolution of the bob).





Example 16

A passenger on a carnival Ferris wheel moves in a vertical circle of radius *R* with constant speed *v*. The seat remains upright during the motion. Find expressions for the force the seat exerts on the passenger at the top of the circle and at the bottom.



Optional slides (not tested)

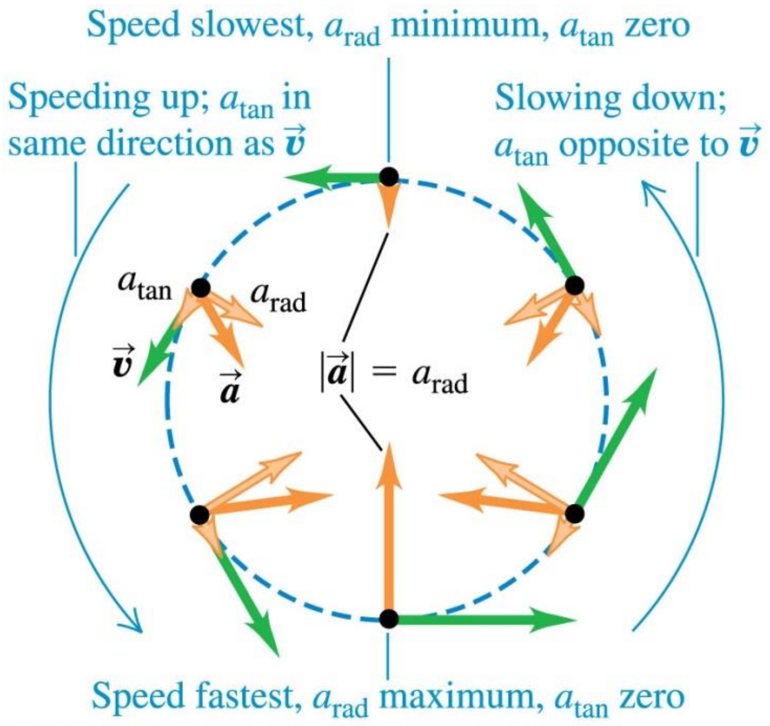
Non-uniform circular motion

- If the speed of the particle on a circle varies, it is known as non-uniform circular motion.

- In this motion, there will also be a tangential acceleration component, $a_{\tan}$ that is parallel to the instantaneous velocity given by

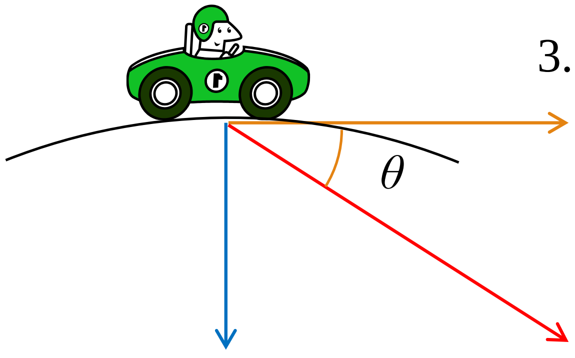
$$a_{\tan} = \frac{dv}{dt}$$

- The total acceleration is therefore $\vec{a} = \vec{a}_{\tan} + \vec{a}_{rad}$.



Example 17

A car exhibits a constant acceleration of 3.00 m/s^2 parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius 50.0 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6.00 m/s. What are the magnitude and direction of the total acceleration vector for the car at this instant?



Solution:

$$a_{\tan} = 3.00\ \mathrm{m/s^2}$$

$$a_{rad} = \frac{v^2}{R} = \frac{6.00^2}{50.0} = 0.72\ \mathrm{m/s^2}$$

$$a^2 = a_{\tan}^2 + a_{rad}^2 = 3.00^2 + 0.72^2$$

$$a = \sqrt{3.00^2 + 0.72^2} = 3.09\ \mathrm{m/s^2}$$

$$\theta = \tan^{-1}\frac{a_{rad}}{a_{\tan}} = \tan^{-1}\frac{0.72}{3.00} = 13.5°$$

Fluid resistance force

- If an object moves in a fluid, it experiences resistance to its motion.

- For small objects with very low speeds the magnitude of the fluid resistance force f depends directly on the velocity as

$$f = kv$$

  where k is a constant that depends on the shape and size of the body and properties of the fluid.

- Examples of small objects are dust particles falling in air or a ball bearing falling in oil.

Terminal velocity

- Consider a metal ball falling in oil such that the resistance force is $f = kv$.
- According to Newton's second law, $\sum F_y = mg + (-kv_y) = ma_y$
- As the ball falls, the speed increases, which will also increase the resistance force.
- At a certain point of time, the weight of the object will equal the resistance force.

$$\sum F_y = mg + (-kv_y) = 0 \quad \text{or} \quad v_y = v_t = \frac{mg}{k}$$

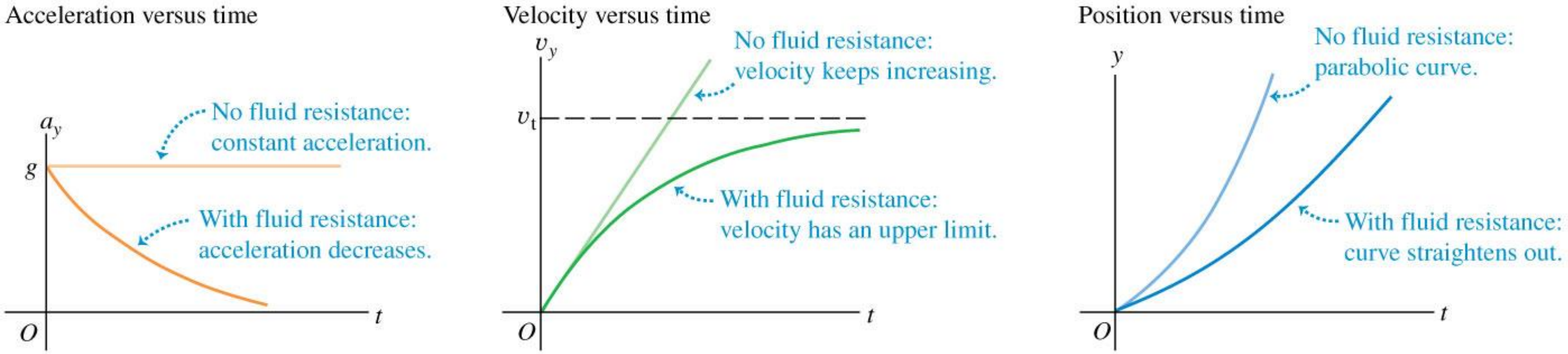
Terminal velocity – detailed calculation

- Before reaching terminal velocity, $\sum F_y = mg + (-kv_y) = ma_y$

$$m\frac{dv_y}{dt} = mg - kv_y$$

- Solving the above equation with $v_y = 0$, when $t = 0$, we get

$$v_y = v_t\left[1 - e^{-(k/m)t}\right], a_y = \frac{dv_y}{dt} = ge^{-(k/m)t}, y = v_t\left[t - \frac{m}{k}\left(1 - e^{-(k/m)t}\right)\right]$$



Air drag

- For larger objects moving through air at high speeds, the resisting force is approximately proportional to v^2.

- This kind of resisting force is called air drag or simply drag. Airplanes, falling rain drops and bicyclists all experience air drag.

- Air drag can be expressed as $f = Dv^2$

  where '*D*' is a constant that depends on the shape and size of the body and the density of air.

- The terminal velocity of objects under air drag is given by $v_t = \sqrt{\frac{mg}{D}}$

End of chapter