SINGAPORE POLYTECHNIC

2018/2019 SEMESTER TWO EXAMINATION

School of Architecture & the Built Environment
DCEB

School of Chemical and Life Sciences
DAPC, DCHE, DFST, DPCS

School of Computing
DBIT, DDA, DISM, DIT

School of Electrical and Electronic Engineering
DASE, DCEP, DCPE, DEB, DEEE, DES, DESM

School of Mechanical and Aeronautical Engineering
DARE, DCEP, DME, DMRO, DBEN

Singapore Maritime Academy (SMA)
DMR

1st Year FT

ENGINEERING MATHEMATICS I Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules set out on the last page of the answer booklet are to be complied with.
2. This paper consists of **THREE** sections:

   Section A: 5 Multiple-Choice Questions (10 marks)
   Answer **ALL** questions.

   Section B: 7 Questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

   Section C: 3 Questions (40 marks)
   Answer **ALL** questions.

3. Unless otherwise stated, leave all answers correct to three significant figures.
4. Except for sketches, graphs and diagrams, no solution or answer is to be written in pencil.
5. This examination paper consists of **6** printed pages.

Section A (10 marks)

Answer ALL **FIVE** questions. Each question carries 2 marks. No mark will be deducted for incorrect answers.

Tick the choice of answer for each question in the box of the **MCQ answer sheet** provided in the answer booklet.

A1. Let $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Which of the following statements is true?

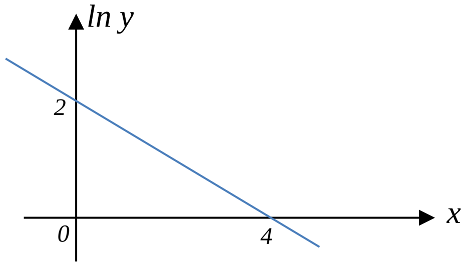
(a) $\mathbf{A}$ is a diagonal matrix

(b) $\mathbf{A}$ is a symmetric matrix

(c) $\mathbf{A}$ is an identity matrix

(d) $\mathbf{A}$ is a singular matrix

A2. Variables x and y are related by the equation $y = ae^{bx}$, where a and b are constants. The equation is reduced to linear form and a best fit line is drawn as shown in the diagram below.



Determine the value of a.

(a) $a = 2$ (b) $a = 4$

(c) $a = e^2$ (d) $a = e^4$

A3. If z and w are complex numbers and $\bar{z}$ and $\bar{w}$ are the conjugates of z and w respectively, which of the following statements is TRUE ?

(a) $\overline{zw} = -zw$ (b) $\overline{zw} = zw$

(c) $\overline{zw} = -\bar{z}\,\bar{w}$ (d) $\overline{zw} = \bar{z}\,\bar{w}$

A4. Suppose that $\dfrac{dy}{dx} = 2 + \sin x$. Which of the following statements is TRUE ?

(a) The curve $y = f(x)$ is always increasing.

(b) The curve $y = f(x)$ is sometimes increasing and sometimes decreasing.

(c) The curve $y = f(x)$ has at least one minimum point.

(d) The curve $y = f(x)$ has only one inflection point.

A5. Given $\int_0^a f(x)\ dx = 3$ and $\int_0^b f(x)\ dx = 8$, where $a < b$.

Then $5\int_a^b f(x)\ dx =$ ____________ .

(a) 5 (b) 15

(c) 25 (d) 40

Section B (50 marks)

Each question carries 10 marks.

The total mark of the questions in this section is 70 marks.

You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you may obtain from this section is 50 marks.

B1. The number of units M_1 , M_2 and M_3 produced by three machines in an hour for a certain part of a computer is given by the following system of equations.

$$\begin{array}{rcrcrcr} M_1 & + & M_2 & + & M_3 & = & 650 \\ k\,M_1 & - & M_2 & + & 2M_3 & = & 10 \\ 3M_1 & + & 2M_2 & + & 2M_3 & = & 1550 \end{array}$$

Use Cramer's rule to find the value of M_3 in terms of the constant k .
(Detailed workings of evaluating a determinant must be clearly shown.)

B2. Given $\boldsymbol{A} = \begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$.

(a) Evaluate $2\boldsymbol{A} + \boldsymbol{B}$.

(b) Find $\boldsymbol{A}^{-1}$.

(c) Find matrix $\boldsymbol{C}$, given that $\boldsymbol{AC} = \begin{pmatrix} 5 & 2 \\ 10 & 1 \end{pmatrix}$.

B3. Given $z = 3\angle 50°$ and $w = 4 - j$.

Evaluate the following and leave your answers in polar form.

(i) $z + w$ (ii) $z\,w$ (iii) $\dfrac{z}{\overline{w}}$ (iv) w^3

(Detailed workings must be clearly shown.)

B4. In the study of heat flow for a certain metal plate, the pressure y on the plate is related to the volume x of the plate by the formula

$$y^2 = ax^2 + b$$

where a and b are constants.
The readings collected are shown in the table below.

x	0	0.25	0.5	0.75	1
y	2	1.97	1.87	1.70	1.41

(a) Re-arrange the formula such that a best fit straight line can be drawn.
(b) State the variables that should be plotted on the vertical and horizontal axes of a graph .
(c) Hence compute in a table, the values of the variables to be plotted on the horizontal and vertical axes, correct to 4 significant figures. *Do not plot the values.*
(d) Suppose the best fit straight line passes through the second and the third points of the new values, use these two points to estimate the gradient and the vertical intercept. Hence, determine the values of a and b.

B5. Find $\dfrac{dy}{dx}$ for each of the following.

(a) $y = 2\ln(1+3x) + \sin^{-1}(2x)$

(b) $2y^3 + 3x^2 = \sin(4x) + 5e^{3y}$

B6. (a) The displacement y (cm) of a weight oscillating on a spring at any time t (s) is described by the equation

$$y = 0.25 + 0.25\cos(6t) + 0.4\sin(3t)$$

Find the rate of change of the displacement of the oscillation $\frac{dy}{dt}$ at $t = 1$ s.

(b) The current i flowing through an inductor L henries at time t seconds is given by the formula

$$i = t^2 e^{-t} \quad \text{amperes}$$

Determine the voltage as a function of time t if $L = 0.10$ henry.

[Note : voltage, $v = L\frac{di}{dt}$]

B7. (a) Find the following integrals:

(i) $\int \left(\frac{3}{x^2} + \frac{4}{x} - \sec^2(2x) \right) dx$

(ii) $\int \frac{1}{x^2 + 16}\, dx$

(b) An object starts moving from rest when it is 3 metres from a fixed point. Its velocity is given by

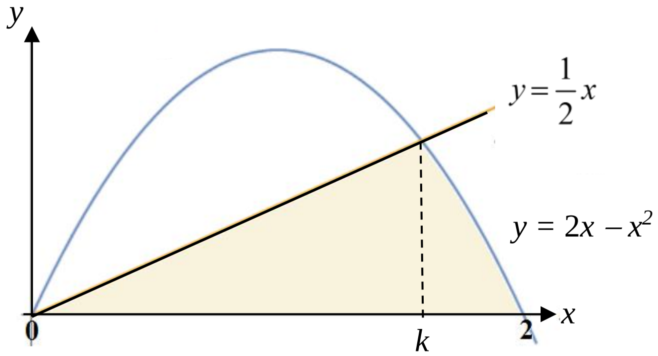
$$v = 5e^{0.05t} + \sqrt{t} - 5 \quad \text{metres/second}$$

Find the displacement of the object from the fixed point when $t = 5$ seconds.

(Note: $s = \int v\, dt$)

Section C (40 marks)
Answer ALL **THREE** questions.

C1. The diagram below shows the graphs of $y=2x-x^2$ and the line $y=\frac{1}{2}x$.



(a) Find the value of the constant k. (3 marks)

(b) Hence find the shaded area. (10 marks)

C2. (a) A metal cube dissolves in acid such that an edge of the cube decreases at the rate of 0.50 mm/min. Find the rate of change of the volume of the cube when the edge is 9 mm . (5 marks)

(b) The power output P (in kW) of a certain turbine varies with the flow rate r (in m^3/s) of the water to the turbine according to the equation

$$P = 0.03r^3 - 2.6r^2 + 71r - 200$$

Determine the value of r such that the rate of the power output is at its smallest. (10 marks)

C3. The complex number w is given as $w = \cos\theta + j\sin\theta$ where $0 < \theta < \frac{\pi}{2}$.

(a) Show that $w^2 - 1 = (j2\sin\theta)w$. (6 marks)

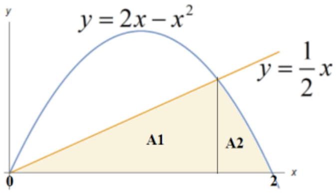
(b) Find the <u>magnitude</u> and <u>argument</u> of $w^2 - 1$.
Leave the answers in terms of θ. (6 marks)

~~~ END OF PAPER ~~~

No.	**SOLUTION**
A	A1) **b** A2) **c** A3) **d** A4) **a** A5) **c**
B1	$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ k & -1 & 2 \\ 3 & 2 & 2 \end{vmatrix} = -2+6+2k-(-3)-4-(2k) = 3$ $\Delta_{M_3} = \begin{vmatrix} 1 & 1 & 650 \\ k & -1 & 10 \\ 3 & 2 & 1550 \end{vmatrix} = -1550+30+(1300k)-(-1950)-20-(1550k) = 410-250k$ $M_3 = \dfrac{410-250k}{3}$
B2 (a)	$2\boldsymbol{A}+\boldsymbol{B} = \begin{pmatrix} 2 & -4 \\ 2 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 3 & 9 \end{pmatrix}$
(b)	$\lvert\boldsymbol{A}\rvert = \left\lvert\begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix}\right\rvert = 6$ $adj(\boldsymbol{A}) = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$ $\boldsymbol{A}^{-1} = \dfrac{1}{6}\begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$
(c)	$\boldsymbol{C} = \boldsymbol{A}^{-1}\begin{pmatrix} 5 & 2 \\ 10 & 1 \end{pmatrix}$ $= \dfrac{1}{6}\begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 5 & 2 \\ 10 & 1 \end{pmatrix}$ $= \dfrac{1}{6}\begin{pmatrix} 40 & 10 \\ 5 & -1 \end{pmatrix}$
B3(a)(i)	$z+w = (1.928+j2.298) + (4-j) = 5.928+j1.298 = 6.07\angle 12.4^o$
(ii)	$zw = (3\angle 50°)(4.123\angle -14.04°) = 12.4\angle 35.96°$ or $12.4\angle 36.0°$
(iii)	$\dfrac{z}{\overline{w}} = \dfrac{3\angle 50°}{4.123\angle 14.04°} = 0.728\angle 35.96°$
iv	$w^3 = (4.123\angle -14.04°)^3 = 70.1\angle -42.12°$

<table>
<tr><th>No.</th><th>SOLUTION</th></tr>
<tr><td>B4(a)
(b)

(c)</td><td>$y^2 = ax^2 + b$
Vertical axis is y^2
Horizontal axis is x^2
<table><tr><td>x^2</td><td>0.00</td><td>0.06</td><td>0.25</td><td>0.56</td><td>1.00</td></tr><tr><td>y^2</td><td>4.00</td><td>3.88</td><td>3.50</td><td>2.88</td><td>2.00</td></tr></table>$a = gradient$
$= \dfrac{3.50-3.88}{0.25-0.06} \quad = -2.00$
Take point (0.06, 3.88) and substitute into $y^2 = -2x^2 + b$
$3.88 = -2(0.06) + b$
$b = 4.00$</td></tr>
<tr><td>B5
(a)</td><td>$\dfrac{dy}{dx} = 2\left(\dfrac{1}{1+3x}\right)(3) + \left(\dfrac{1}{\sqrt{1-(2x)^2}}\right)(2)$</td></tr>
<tr><td>B5
(b)</td><td>$2(3y^2)\dfrac{dy}{dx} + 3(2x) = 4\cos(4x) + 5(e^{3y})(3)\left(\dfrac{dy}{dx}\right)$
$6y^2\dfrac{dy}{dx} - 15(e^{3y})\left(\dfrac{dy}{dx}\right) = 4\cos(4x) - 6x$
$\dfrac{dy}{dx}(6y^2 - 15e^{3y}) = 4\cos(4x) - 6x$
$\dfrac{dy}{dx} = \dfrac{4\cos(4x) - 6x}{6y^2 - 15e^{3y}}$</td></tr>
<tr><td>B6a</td><td>$\dfrac{dy}{dt} = 0 + 0.25(-\sin(6t)(6)) + 0.4(\cos(3t))(3)$
$\left.\dfrac{dy}{dt}\right|_{t=1} = -1.5\sin(6) + 1.2\cos(3) \quad = -0.769$ cm/s</td></tr>
<tr><td>B6b</td><td>$\dfrac{di}{dt} = t^2 e^{-t}(-1) + 2te^{-t}$
$v = L\dfrac{di}{dt} \quad = 0.1(2te^{-t} - t^2 e^{-t})$</td></tr>
</table>

No.	SOLUTION
B7 (a)i	$\int \left(\frac{3}{x^2} + \frac{4}{x} - \sec^2(2x) \right) dx = 3\left(\frac{x^{-1}}{-1} \right) + 4\ln\lvert x\rvert - \frac{1}{2}\tan(2x) + C$
B7 (a)ii	$\int \frac{1}{x^2+16}\, dx = \frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right) + C$
B7 (b)	$s = \int (5e^{0.05t} + \sqrt{t} - 5)dt \;=\; \frac{5e^{0.05t}}{0.05} + \frac{2}{3}t^{\frac{3}{2}} - 5t + C$ Since $s = 3$ when $t = 0$, $C = -97$ Therefore when $t = 5$ $s = 100e^{0.05(5)} + \frac{2}{3}(5)^{1.5} - 5(5) - 97 = 13.9$ m
C1 (a)	 To find points of intersection, let $2x - x^2 = \frac{1}{2}x$ $2x - x^2 - \frac{1}{2}x = 0$ $x(\frac{3}{2} - x) = 0$ $\Rightarrow x = 0$ or $x = \frac{3}{2}$ $\therefore\; k = 1.5$
(b)	Consider A1. When $x = \frac{3}{2}, y = \frac{1}{2}\left(\frac{3}{2}\right) = \frac{3}{4}$ $\therefore \text{A1} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times \frac{3}{2} \times \frac{3}{4} = \frac{9}{16}$ sq unit Consider A2. $\text{A2} = \int_{3/2}^{2} (2x - x^2)dx$ $= \left[x^2 - \frac{x^3}{3} \right]_{\frac{3}{2}}^{2}$ $= \left(4 - \frac{8}{3}\right) - \left(\frac{9}{4} - \frac{1}{3}\cdot\frac{27}{8}\right) = \frac{4}{3} - \frac{9}{8} = \frac{5}{24}$ sq unit total area = A1+A2 $= \frac{9}{16} + \frac{5}{24} \approx 0.771$ sq unit

No.	SOLUTION
C2 a	Let the edge of the cube be x . Volume of the cube , $V = x^3$ $\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$ $\left.\frac{dV}{dt}\right\|_{x=9} = 3(9)^2(-0.5) \quad = -121.5 \text{ mm}^3/\text{min}$ **OR** $\frac{dV}{dx} = 3x^2$ $\frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} \quad = (3x^2)(-0.5)$ $\left.\frac{dV}{dt}\right\|_{x=9} = 3(9)^2(-0.5) \quad = -121.5 \text{ mm}^3/\text{min}$
C2 b	$\frac{dP}{dr} = 0.09r^2 - 5.2r + 71$ Let Rate of P be $R = \frac{dP}{dr} = 0.09r^2 - 5.2r + 71$ $\frac{dP^2}{dr^2} = \frac{dR}{dr} = 0.18r - 5.2$ At max/min, $\frac{dR}{dr} = 0$ $0.18r - 5.2 = 0$ $r = 28.89 \quad \text{m}^3/s$ $\frac{d^2R}{dr^2} = 0.18 > 0$ Therefore Rate of Power is slowest when r = 28.89 m^3/s
C3 (a)	$w^2 - 1 = (\cos\theta + j\sin\theta)^2 - 1$ $= \cos^2\theta + j2\cos\theta\sin\theta + j^2\sin^2\theta - 1$ $= \cos^2\theta + j2\cos\theta\sin\theta - \sin^2\theta - 1$ $= (1 - \sin^2\theta) - \sin^2\theta - 1 + j2\cos\theta\sin\theta$ $= -2\sin^2\theta + j2\cos\theta\sin\theta$ $= j2\sin\theta(\boldsymbol{\cos\theta + j\sin\theta})$ $= j2\boldsymbol{w}\sin\theta$ (shown)
	$w^2 - 1 = \left(2\sin\theta\angle\frac{\pi}{2}\right)(1\angle\theta) \quad = 2\sin\theta\angle\frac{\pi}{2} + \theta$ $\|w^2 - 1\| = 2\sin\theta$ $\arg(w^2 - 1) = \frac{\pi}{2} + \theta$