SINGAPORE POLYTECHNIC

2018/2019 SEMESTER TWO MID SEMESTER TEST

ENGINEERING MATHEMATICS II Time allowed: 1.5 hrs

2nd Year Full-Time

School of Chemical and Life Sciences
DCHE

School of Electrical and Electronic Engineering
DASE, DCPE, DEB, DEEE, DES, DESM

School of Mechanical and Aeronautical Engineering
DARE, DCEP, DME, DMRO

Instructions:

1. The Singapore Polytechnic Examination rules are to be complied with.
2. This paper consists of 3 **printed** pages.
3. Answer all the 7 questions.
4. Unless otherwise stated, correct all your decimal answers to **two** decimal places.
5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to do so will result in loss of marks.

1. (a) Find $f_x(x,y)$ and $f_y(x,y)$ for each of the following.

(i) $f(x,y)=\ln(x^2-y)$

(ii) $f(x,y)=x\cos y+ye^x$

(b) If $z=x^2+3xy^2$ where $x=\cos t$ and $y=e^{2t}$, find $\dfrac{dz}{dt}$ when $t=0$.

(c) The height of a timber tree increases at a rate of 0.5 meter per year and the circumference increases at 0.2 meter per year. At what rate is the volume of timber increasing when the height is 6 meter and the circumference is 1.5 meter? (Assume the tree trunk to be a circular cylinder). (22 marks)

2. (a) Evaluate the following integrals:

(i) $\int 4(5x-2)^3\,dx$ (ii) $\int \dfrac{3}{6u+1}\,du$

(iii) $\int 5\cos 3t\cos 2t\,dt$

(b) Find the root mean square value of $y=7e^{2t+3}$ over the interval $t=0$ to $t=2$. (11 marks)

3. (a) Given that $\dfrac{2}{(x+3)^2(x+2)}=\dfrac{A}{x+3}+\dfrac{B}{(x+3)^2}+\dfrac{C}{x+2}$, find the value of constants A, B and C.

Hence, determine $\int \dfrac{2}{(x+3)^2(x+2)}\,dx$.

(b) By completing the square for $x^2+2x+17$, find $\int \dfrac{1}{x^2+2x+17}\,dx$.

Hence, determine $\int \dfrac{2x+1}{x^2+2x+17}\,dx$. (17 marks)

4. By using the method of substitution, find the following integrals.

(a) $\int x^2\cos x^3\,dx$

(b) $\int_0^{\frac{1}{\sqrt{2}}} \sqrt{1-2x^2}\,dx$ using the substitution $x=\dfrac{1}{\sqrt{2}}\sin\theta$.

(15 marks)

5. Using integration by parts, find the following integrals.

(a) $\int (2x-1)e^x \, dx$

(b) $\int x^3 \ln x \, dx$ (10 marks)

6. By using Simpson's rule with 6 equal intervals, find the approximate value of

$\int_0^{\pi/3} \ln(\sec x) \, dx$, accurate to 3 decimal places. (Show your workings clearly.)

(10 marks)

7. A periodic function $f(t)$ of period 2π seconds over the period $-\pi < t < \pi$ is defined as follows:

$$f(t) = \begin{cases} -1 & \text{for } -\pi \le t < 0 \\ 1 & \text{for } 0 \le t < \pi \end{cases}$$

(a) Sketch the graph of $f(t)$ for $-2\pi \le t < 2\pi$.

(b) Find the Fourier series of $f(t)$ as far as the third harmonic.

(15 marks)

- End of Paper -