SOLUTIONS/ MARKING SCHEME

SINGAPORE POLYTECHNIC
2019 / 2020 Semester 2 MST

Module Name: **Engineering Mathematics II**

Module Code: MS2216/MS4216/MS6216

Course: DCHE/DASE/DCPE/DEB/DEEE/DES/DESM/DARE/DCEP/DME/DMRO **Year: 2 FT**

No.	SOLUTION
1a	$z=\ln(x+y)-2x\cos y$ $z_y(x,y)=\dfrac{1}{x+y}\cdot 1-2x(-\sin y)=\dfrac{1}{x+y}+2x\sin y$ $z_y(2,1)=\dfrac{1}{2+1}+2(2)\sin(1)\approx 3.70$
1b	$\dfrac{dV}{dt}=0.5\,\text{m}^3/\text{s},\quad \dfrac{dr}{dt}=0.03\,\text{m/s},\quad V=\dfrac{1}{3}\pi r^2 h$ $\dfrac{\partial V}{\partial r}=\dfrac{1}{3}\pi(2r)h=\dfrac{2}{3}\pi rh,\quad \dfrac{\partial V}{\partial h}=\dfrac{1}{3}\pi r^2$ $\dfrac{dV}{dt}=\dfrac{\partial V}{\partial r}\dfrac{dr}{dt}+\dfrac{\partial V}{\partial h}\dfrac{dh}{dt}$ $0.5=\dfrac{2}{3}\pi rh(0.3)+\dfrac{1}{3}\pi r^2\dfrac{dh}{dt}$ Given $r=3$ and $h=2$ $0.5=\dfrac{2}{3}\pi(3)(2)(0.03)+\dfrac{1}{3}\pi(3^2)\dfrac{dh}{dt} \rightarrow 0.5=0.12\pi+3\pi\dfrac{dh}{dt}$ $\dfrac{dh}{dt}=\dfrac{0.5-0.12\pi}{3\pi}=0.013\,\text{m/s}$
2a (i)	$\displaystyle\int\frac{1}{4x-3}\,dx=\frac{1}{4}\ln\lvert 4x-3\rvert+C$
2a (ii)	$\displaystyle\int 6(3x+1)^4dx=6\int(3x+1)^4\,dx=\frac{6}{3}\left[\frac{(3x+1)^5}{5}\right]+C=\frac{2}{5}(3x+1)^5+C$
2a (iii)	$\displaystyle\int 2\sin 6x\sin 2x\,dx=\int[\cos 4x-\cos 8x]\,dx=\frac{\sin 4x}{4}-\frac{\sin 8x}{8}+C$
2b	$y^2=(3t+2)^2=9t^2+12t+4$ $y_{ms}=\int_0^1(9t^2+12t+4)\,dt=\left[3t^3+6t^2+4t\right]_0^1=13$ $y_{rms}=\sqrt{13}\approx 3.61$

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3a	Let $u = x^3$ $\frac{du}{dx} = 3x^2 \to du = 3x^2\ dx \to \frac{1}{3}du = x^2 dx$ $\int x^2 e^{x^3} dx = \frac{1}{3}\int e^u du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3} + C$
3b	Let $u = 1 + \ln x \rightarrow \ln x = u - 1$ $\frac{du}{dx} = \frac{1}{x} \to du = \frac{1}{x}dx$ When $x = 1,\quad u = 1\ ;\ x = e,\quad u = 2$ $\int_1^e \frac{\ln x}{x(1+\ln x)^2}dx = \int_1^e \frac{\ln x}{(1+\ln x)^2}\frac{1}{x}dx = \int_1^2 \frac{u-1}{u^2}du = \int_1^2 \left(\frac{1}{u} - \frac{1}{u^2}\right)du$ $= \left(\ln\lvert u\rvert + \frac{1}{u}\right)_1^2 = \ln 2 - \ln 1 + \frac{1}{2} - 1 = 0.19$
4a	Use ‘cover-up” method: $A = \left.\frac{2x}{x-2}\right\rvert_{x=-5} = \frac{10}{7} \qquad B = \left.\frac{2x}{x+5}\right\rvert_{x=2} = \frac{4}{7}$ $\therefore \frac{2x}{(x+5)(x-2)} = \frac{10/7}{x+5} + \frac{4/7}{x-2}$ $\int \frac{2x}{(x+5)(x-2)}dx = \int\left(\frac{10/7}{x+5} + \frac{4/7}{x-2}\right)dx$ $= \frac{10}{7}\ln\lvert x+5\rvert + \frac{4}{7}\ln\lvert x-2\rvert + C$
4b	$x^2 + 2x + 5 = (x+1)^2 - (1)^2 + 5 = (x+1)^2 + 4$ $\int \frac{3}{x^2+2x+5}dx = \int \frac{3}{(x+1)^2+4}dx = 3\int \frac{1}{(x+1)^2+(2)^2}dx = \frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C$

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No.	SOLUTION

5a

$$\begin{array}{ccc} u & & dv \\ 2x & \searrow^{+} & \sin x \\ 2 & \searrow^{-} & -\cos x \\ 0 & & -\sin x \end{array}$$

$$\therefore \int 2x \sin x \, dx = -2x\cos x + 2\sin x + C$$

5b

$$\begin{array}{ccc} u & & dv \\ \ln x & \searrow^{+} & x^{-4} \\ \dfrac{1}{x} & \xrightarrow{-\int} & -\dfrac{x^{-3}}{3} \end{array}$$

$$\therefore \int \frac{\ln x}{x^4} dx = -\frac{1}{3}x^{-3}\ln x + \int \frac{1}{x}\cdot\frac{1}{3}x^{-3}\,dx = -\frac{1}{3x^3}\ln x - \frac{1}{9x^3} + C$$

6

$$h = \frac{\pi - 0}{6} = \frac{\pi}{6}$$

Let $y = \sqrt{\cos(x) + 2}$

x	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	$\frac{6\pi}{6} = \pi$
y	1.732	1.6929	1.5811	1.4142	1.2247	1.0649	1

$$\int_0^{\pi} \sqrt{\cos x + 2}\, dx \approx \frac{1}{3}\left(\frac{\pi}{6}\right) \times [1.7321 + 1 + 4(1.6929 + 1.4142 + 1.0649) + 2(1.5811 + 1.2247)]$$

$$\approx 4.3767 \approx 4.38$$

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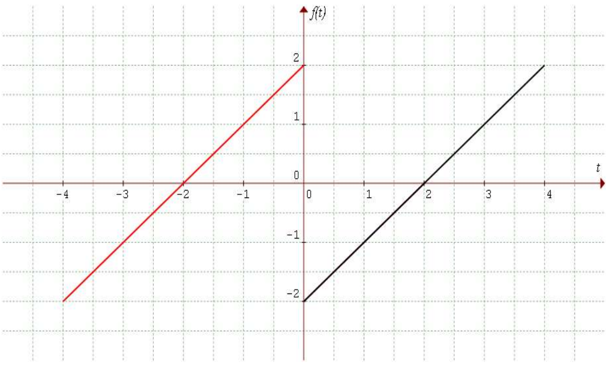
7(a)	
7(b)	Since $f(t)$ is an odd function, $a_0 = 0, \quad a_n = 0$. $T = 4 \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$ $b_n = \frac{4}{4}\int_0^2 (t-2)\,sin\frac{n\pi t}{2}\,dt = \left[-\frac{2(t-2)}{n\pi}cos\frac{n\pi t}{2} + \frac{4}{n^2\pi^2}sin\frac{n\pi t}{2}\right]_0^2$ $= \underbrace{(0+0)}_{\text{when } t=2} - \underbrace{\left(\frac{4}{n\pi}+0\right)}_{\text{when } t=0} = -\frac{4}{n\pi}$ $\therefore \quad b_1 = -\frac{4}{\pi}, \quad b_2 = -\frac{2}{\pi}, \quad b_3 = -\frac{4}{3\pi}$ $\therefore \quad f(t) = -\frac{4}{\pi}sin\frac{\pi t}{2} - \frac{2}{\pi}sin\,\pi t - \frac{4}{3\pi}sin\frac{3\pi t}{2} + \ldots$
7(c)	$g(t) = \frac{3t}{2} - 6 = 3\left(\frac{t}{2} - 2\right) = 3f\left(\frac{t}{2}\right)$ $= -\frac{12}{\pi}sin\frac{\pi t}{4} - \frac{6}{\pi}sin\frac{\pi t}{2} - \frac{4}{\pi}sin\frac{3\pi t}{4} + \ldots$