4-1. Simplify the following expressions using Boolean algebra.

(a) $x = ABC + \overline{A}C$

$= C(\underbrace{AB + \overline{A}})$

$= C(\overline{A} + B)$

Recall: $x + \overline{x}y = x + y$

$x = \underline{\overline{A}C + BC}$ (SOP form)

4-1

(b) $y = (Q + R)(\overline{Q} + \overline{R})$

$= \underbrace{Q\overline{Q}}_{0} + Q\overline{R} + R\overline{Q} + \underbrace{R\overline{R}}_{0}$

$y = \underline{Q\overline{R} + \overline{Q}R}$

4-1

(c) $w = ABC + A\bar{B}C + \bar{A}$

$= AC(\underbrace{B + \bar{B}}_{1}) + \bar{A}$

$= AC + \bar{A}$

$= \underline{\bar{A} + C}$

Recall: $x + \bar{x}y = x + y$

4-1

(d) $q = \overline{RST}\,(\overline{R + S + T})$

$= (\bar{R} + \bar{S} + \bar{T})(\bar{R} \cdot \bar{S} \cdot \bar{T})$

$= (\bar{R} + \bar{S} + \bar{T})\,\bar{R}\bar{S}\bar{T}$

$= \underbrace{\bar{R}\bar{R}}_{\bar{R}}\bar{S}\bar{T} + \bar{S}\bar{R}\bar{S}\bar{T} + \bar{T}\bar{R}\bar{S}\bar{T}$

Recall: $a \cdot a = a$

$= \bar{R}\bar{S}\bar{T} + \bar{R}\bar{S}\bar{T} + \bar{R}\bar{S}\bar{T}$

Recall: $a + a + a = a$

$= \underline{\bar{R}\bar{S}\bar{T}}$

4-1 Hint: Try to get something like: $a+\bar{a}$ (which $=1$)

(e) $x = \bar{A}\bar{B}\bar{C} + \bar{A}BC + ABC + A\bar{B}\bar{C} + A\bar{B}C$

$= \overbrace{(\bar{A}+A)}^{①}\bar{B}\bar{C} + \overbrace{(\bar{A}+A)}^{②}BC + A\bar{B}C$

$= \bar{B}\bar{C} + BC + A\bar{B}C$

$= \bar{B}\bar{C} + C(B + A\bar{B})$

$= \bar{B}\bar{C} + C(B+A)$

$= \underline{\bar{B}\bar{C} + BC + AC}$

Alt. ans

$\boxed{\begin{aligned} &BC + \bar{B}(\bar{C}+AC) \\ &= BC + \bar{B}(\bar{C}+A) \\ &= \underline{BC + \bar{B}\bar{C} + A\bar{B}} \end{aligned}}$

$\boxed{\begin{aligned} &\text{Recall: } x + \bar{x}y \\ &= x + y \end{aligned}}$

4-1

(f) $z = (B+\bar{C})(\bar{B}+C) + \overline{\bar{A}+B+\bar{C}}$

$= (\underbrace{B\bar{B}}_{0} + BC + \bar{C}\bar{B} + \underbrace{\bar{C}C}_{0}) + (\bar{\bar{A}}\cdot\bar{B}\cdot\bar{\bar{C}})$

$= BC + \bar{B}\bar{C} + A\bar{B}C$

$= BC + \bar{B}(\bar{C} + AC)$

$= BC + \bar{B}(\bar{C} + A)$

$= \underline{BC + \bar{B}\bar{C} + A\bar{B}}$

Alt. ans.

$\boxed{\begin{aligned} &C(B + A\bar{B}) + \bar{B}\bar{C} \\ &= C(B+A) + \bar{B}\bar{C} \\ &= \underline{BC + AC + \bar{B}\bar{C}} \end{aligned}}$

4-1

(g) $y = \overline{(C+D)} + \bar{A}C\bar{D} + A\bar{B}\bar{C} + \bar{A}\bar{B}CD + AC\bar{D}$

$$= \bar{C}\cdot\bar{D} + \bar{A}C\bar{D} + A\bar{B}\bar{C} + \bar{A}\bar{B}CD + AC\bar{D}$$

$$= \bar{C}\bar{D} + C\bar{D}\underbrace{(\bar{A}+A)}_{1} + A\bar{B}\bar{C} + \bar{A}\bar{B}CD$$

$$= \underbrace{(\bar{C}+C)}_{1}\bar{D} + A\bar{B}\bar{C} + \bar{A}\bar{B}CD$$

$$= \bar{D} + \bar{A}\bar{B}CD + A\bar{B}\bar{C}$$

$$= \underline{\bar{D} + \bar{A}\bar{B}C + A\bar{B}\bar{C}}$$

$\boxed{\text{Recall: } x + \bar{x}y = x + y}$

4-1

(h) $x = AB(\overline{\bar{C}D}) + \bar{A}BD + \bar{B}\bar{C}\bar{D}$

$$= AB(\bar{\bar{C}} + \bar{D}) + \bar{A}BD + \bar{B}\bar{C}\bar{D}$$

$$= AB(C + \bar{D}) + \bar{A}BD + \bar{B}\bar{C}\bar{D}$$

$$= \underline{ABC + AB\bar{D} + \bar{A}BD + \bar{B}\bar{C}\bar{D}}$$

✓4-4. Design the logic circuit corresponding to the truth table shown in Table 4-9.

Step 1 : Truth-Table (already given in this problem)

A	B	C	x	
0	0	0	1	— $\bar{A}\cdot\bar{B}\cdot\bar{C}$
0	0	1	0	
0	1	0	1	— $\bar{A}\,B\,\bar{C}$
0	1	1	1	— $\bar{A}\,B\,C$
1	0	0	1	— $A\,\bar{B}\,\bar{C}$
1	0	1	0	
1	1	0	0	
1	1	1	1	— $A\,B\,C$

Step 2

Write down the product Terms which output = 1.

Step 3 : Add the above Terms to form the SOP expression:

$$x = \underset{000}{\bar{A}\bar{B}\bar{C}} + \underset{010}{\bar{A}B\bar{C}} + \underset{011}{\bar{A}BC} + \underset{100}{A\bar{B}\bar{C}} + \underset{111}{ABC}$$

Step 4: Simplify the above SOP expression: (either using algebra or K-map)

$$x = \underbrace{\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C}}_{1} + \underbrace{\bar{A}BC}_{2} + A\bar{B}\bar{C} + \underbrace{ABC}_{2}$$

$$= \overbrace{\bar{A}\bar{C}\,(\underbrace{\bar{B}+B}_{1})}^{1} + \overbrace{BC\,(\underbrace{\bar{A}+A}_{1})}^{2} + A\bar{B}\bar{C}$$

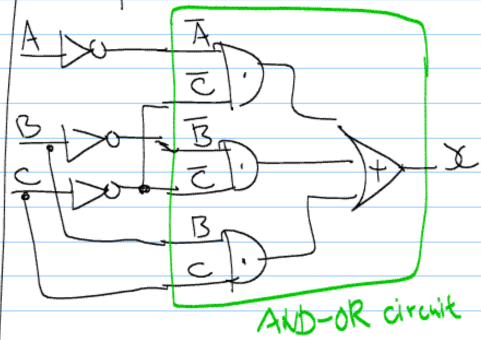
$$= \bar{A}\bar{C} + BC + A\bar{B}\bar{C}$$

$$= \bar{C}\,(\bar{A} + A\bar{B}) + BC$$

$$= \bar{C}\,(\bar{A} + \bar{B}) + BC$$

$$\boxed{x = \bar{A}\bar{C} + \bar{B}\bar{C} + BC}$$

Step 5 : Circuit



Alt: Simplify with K-map

$$x = \overset{000}{\bar{A}\bar{B}\bar{C}} + \overset{010}{\bar{A}B\bar{C}} + \overset{011}{\bar{A}BC} + \overset{100}{A\bar{B}\bar{C}} + \overset{111}{ABC}$$

AB		C = 0 ($\bar{C}$)	C = 1 (C)
00	$\bar{A}\bar{B}$	1	0
01	$\bar{A}B$	1	1
11	AB	0	1
10	$A\bar{B}$	1	0

$$x = \bar{B}\cdot\bar{C} + B\cdot C + \bar{A}B$$

or

$$x = \bar{B}\cdot\bar{C} + B\cdot C + \bar{A}\bar{C}$$

Both are correct

→ 4-8. Figure 4-63 shows a diagram for an automobile alarm circuit used to detect certain undesirable conditions. The three switches are used to indicate the status of the door by the driver's seat, the ignition, and the headlights, respectively. Design the logic circuit with these three switches as inputs so that the alarm will be activated whenever either of the following conditions exists:

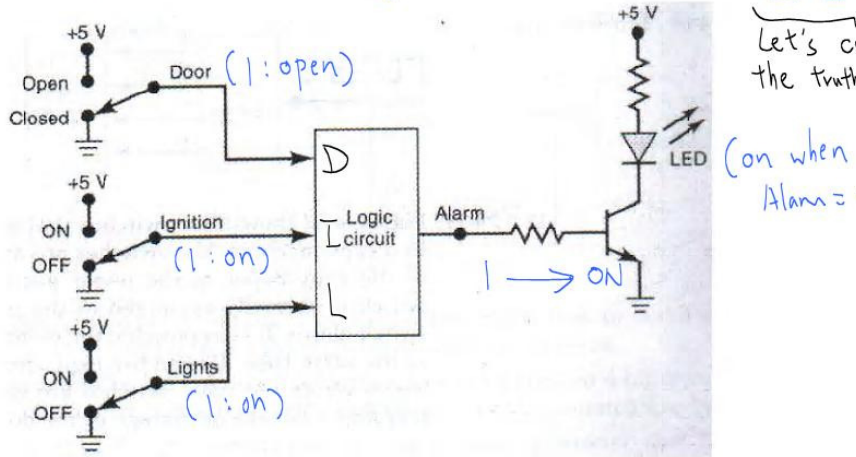
OR {
- ① The headlights are on while the ignition is off.
- ② The door is open while the ignition is on.

ie AND

From logic reasoning:

$$\text{Alarm} = \overset{①}{L\cdot\bar{I}} + \overset{②}{D\cdot I}$$

Let's compare with the truth-table method.



Step 1: Truth-table

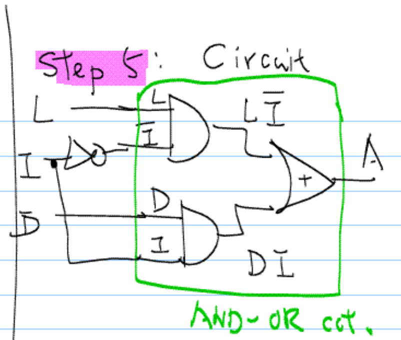
D	I	L	Alarm	
0	0	0	0	
0	0	1	1	— $\bar{D}\bar{I}L$
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	1	— $D\bar{I}L$
1	1	0	1	— $DI\bar{L}$
1	1	1	1	— DIL

Step 2

Light on, Ignition off

Door open, Ignition On

Step 5: Circuit



Step 3 (SoP): $A = \bar{D}\bar{I}L + D\bar{I}L + DI\bar{L} + DIL$

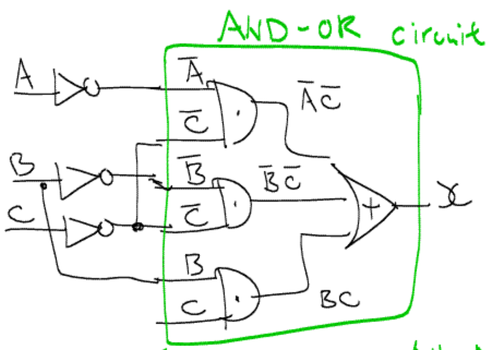
Step 4 (Simplify): $= (\bar{D}+D)\bar{I}L + DI(\bar{L}+L)$

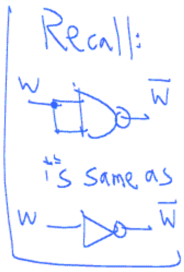
$$A = \bar{I}L + DI$$

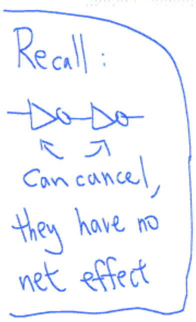
→ 4-9. Implement the circuit of Problem 4-4 using all NAND gates.

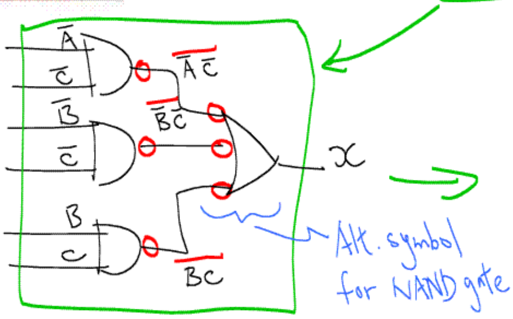
A	B	C	x
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

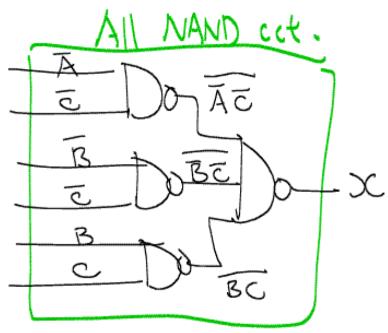
From 4-4 :











3→4-11. Determine the minimum expression for each K map in Figure 4-64. Pay particular attention to step 5 for the map in (a).

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	1	1	0	0
AB	0	0	0	1
$A\bar{B}$	0	0	1	1

(a)

Not needed, as all '1's in this group are already in other groups.

$$\underbrace{\bar{A}\cdot\bar{C} + \bar{B}\cdot C + A\cdot C\bar{D}}_{\text{Ans.}} + \bar{A}\bar{B}$$

→4-11. Determine the minimum expression for

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	1	1
$\bar{A}B$	1	0	0	1
AB	0	0	0	0
$A\bar{B}$	1	0	1	1

(b)

$$\bar{B}\cdot\bar{D} + \bar{B}\cdot C + \bar{A}\cdot\bar{D}$$

4-11. Determine the minimum expression for

	$\bar{C}$	C
$\bar{A}\bar{B}$	0	1
$\bar{A}B$	0	0
AB	1	0
$A\bar{B}$	1	X

(c)

Don't care

4-12. Simplify the expression in Problem 4-1(e) using a K map.

$$\overset{000}{} \quad \overset{011}{} \quad \overset{111}{} \quad \overset{100}{} \quad \overset{101}{}$$

(e) $x = \bar{A}\bar{B}\bar{C} + \bar{A}BC + ABC + A\bar{B}\bar{C} + A\bar{B}C$

C

AB	$\bar{C}$	C
00 $\bar{A}\bar{B}$	1	0
01 $\bar{A}B$	0	1
11 AB	0	1
10 $A\bar{B}$	1	1

$x = \bar{B}\bar{C} + A\bar{B} + BC$

4-13. Simplify the expression in Problem 4-1(g) using a K map.

(Have to change to SOP form first)

(g) $y = \overline{(C + D)} + \overline{A}C\overline{D} + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}CD + AC\overline{D}$

Step 1

$= \bar{C}\bar{D} + \bar{A}C\bar{D} + A\bar{B}\bar{C} + \bar{A}\bar{B}CD + AC\bar{D}$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1		1	1
$\bar{A}B$	1			1
AB	1			1
$A\bar{B}$	1	1		1

$\bar{A}$ for these 2 rows

A for these 2 rows

$\bar{C}$ for these 2 columns

Step 2

Re-group the 1's :

AB \ CD	00	01	11	10
00	1		1	1
01	1			1
11	1			1
10	1	1		1

$$y = \underbrace{\bar{D} + A\bar{B}\bar{C} + \bar{A}\bar{B}C}_{\text{Simplified SOP}}$$