Mid-Semester Test (AY22/23 S1)

EP0605 – Advanced Physics **Time Allowed: 1 hour 40 minutes**

Instructions to Candidates **Max Marks**: 100

1. All the Singapore Polytechnic examination rules must be strictly adhered to.
2. This paper consists of **5** questions. Take g = 9.80 m/s^2.
3. Answer all the questions in this question booklet. All working must be shown.
4. This paper consists of **6** pages (inclusive of the cover page).
5. Fill in the table below.

Name :			
Admission No :		S/No	
Class :	EL/EP0605/FT/01	Date :	

For Official Use Only	Question	Marks
	1	
	2	
	3	
	4	
	5	
	Total	

1. (a) In the below equation, the SI units of v is metre/second (m/s) and T is newton (N). Using dimensional analysis, determine the unit of μ in terms of base units.

$$v = \sqrt{\frac{T}{\mu}}$$

(b) A force $\boldsymbol{F} = (6.00\,\mathbf{i} - 2.00\,\mathbf{j})\,\mathrm{N}$ acts on a particle that undergoes a displacement $\Delta\boldsymbol{r} = (3.00\,\mathbf{i} + \mathbf{j})\,\mathrm{m}$. Find the angle between $\boldsymbol{F}$ and $\Delta\boldsymbol{r}$.

(c) Given $\boldsymbol{M} = 2\,\mathbf{i} - 3\,\mathbf{j} + \mathbf{k}$ and $\boldsymbol{N} = 4\,\mathbf{i} + 5\,\mathbf{j} - 2\,\mathbf{k}$, calculate the cross product $\boldsymbol{M} \times \boldsymbol{N}$.

(20 marks)

Solution:

(a) Dimension of v: $[L][T]^{-1}$
Dimension of T: $[M][L][T]^{-2}$
Through the given equation $v = \sqrt{T/\mu}$, we have

$$\frac{[L]}{[T]} = \sqrt{\frac{[M][L][T]^{-2}}{[\mu]}}$$

$$\frac{[L]^2}{[T]^2} = \frac{[M][L]}{[\mu][T]^2}$$

$$[\mu] = [M][L]^{-1}$$

SI unit of μ is kg/m.

(b) $\vec{F} \cdot \Delta\vec{r} = (6.00\hat{\imath} - 2.00\hat{\jmath}) \cdot (3.00\hat{\imath} + \hat{\jmath}) = 18 - 2 = 16$ J

$$\cos\theta = \frac{\vec{F} \cdot \Delta\vec{r}}{|\vec{F}||\Delta\vec{r}|} = \frac{16}{\sqrt{40}\sqrt{10}} = 0.8$$

$\theta = \cos^{-1} 0.8 = 36.9°$

(c) Cross product:

$$\vec{M} \times \vec{N} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -3 & 1 \\ 4 & 5 & -2 \end{vmatrix} = (6-5)\hat{\imath} + (4+4)\hat{\jmath} + (10+12)\hat{k} = \hat{\imath} + 8\hat{\jmath} + 22\hat{k}$$

2. The position of a particle is given by $x = 2t - 0.5t^3$, where x is in metres and t is in seconds. Let positive x means to the right of the origin.

a) Determine the average velocity of the particle between $t = 1.0$ s and $t = 4.0$ s.
b) Will the particle's velocity be zero? If yes, when?
c) When is the particle's acceleration a equal to zero?
d) Sketch the acceleration-time graph.

(20 marks)

Solution:

(a) Average velocity $v_{avg} = \frac{\Delta x}{\Delta t}$

$x(1.0) = 2(1.0) - 0.5(1.0)^3 = 1.5\ \text{m}$

$x(4.0) = 2(4.0) - 0.5(4.0)^3 = -24\ \text{m}$

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{-24 - 1.5}{4 - 1} = -8.5\ \text{m/s}$$

(b) $v = \frac{dx}{dt} = 2 - 1.5t^2$

When $v = 0$, $2 - 1.5t^2 = 0$

$t = \sqrt{\frac{2}{1.5}} = 1.15\ \text{s}$

Yes, the velocity of the particle is zero at $t = 1.15$ s.

(c) $a = \frac{dv}{dt} = -3t$

When $a = 0$, $t = 0$.

The acceleration of the particle is zero at $t = 0$.

(d) Straight line graph starting from origin with a negative gradient.

3. (a) A rock climber stands on top of a 40.0 m high cliff overhanging a pool of water. He throws two stones, A and B, vertically downwards 0.800 s apart at different speeds and observes that they cause a single splash. The initial speed of stone A was 1.00 m/s.

(i) How long after stone A leaves his hand does stone B hit the water?
(ii) What was the initial speed of stone B?

(b) A cannon ball is launched at an angle 40.0° from the horizontal with a speed of 60.0 m/s. Calculate the horizontal range and the time spent in the air of the cannon ball.

(20 marks)

Solution:

(a) (i) Using $\Delta y = v_{0y}t - \frac{1}{2}gt^2$,

$$-40 = -t - \frac{1}{2}(9.8)t^2$$
$$4.9t^2 + t - 40 = 0$$
$$t = \frac{-1 \pm \sqrt{1^2 + 4(4.9)(40)}}{9.8} = 2.76\text{ s or} - 2.96\text{ s (NA)}$$

Time taken for stone A to hit the water = 2.76 s

(ii) Time taken for stone B to hit the water = 2.76 s – 0.80 s = 1.96 s

Using $\Delta y = v_{0y}t - \frac{1}{2}gt^2$,

$$v_{0y} = \frac{1}{t}\left[\Delta y + \frac{1}{2}gt^2\right] = \frac{1}{1.96}\left[-40 + \frac{1}{2}(9.8)(1.96)^2\right] = -10.9\text{ m/s}$$

Initial speed of stone B = 10.9 m/s

(b) Using the range formula,

$$R = \frac{v^2 \sin 2\theta}{g} = \frac{60.0^2 \sin 2(40.0°)}{9.80} = 362\text{ m}$$

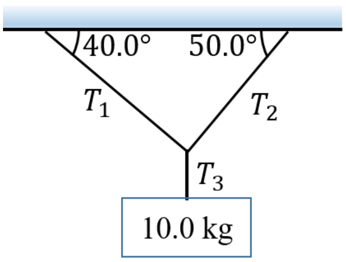
Using the time-of-flight formula,

$$t = \frac{2v\sin\theta}{g} = \frac{2(60.0)\sin(40.0°)}{9.80} = 7.87\text{ s}$$

4. (a) Three forces acting on a 4.00-kg object are given by $\boldsymbol{F}_1 = (-2.00\ \mathbf{i} + 2.00\ \mathbf{j})$ N, and $\boldsymbol{F}_2 = (5.00\ \mathbf{i} - 3.00\ \mathbf{j})$ N, and $\boldsymbol{F}_3 = (-4.00\ \mathbf{i})$ N. The object is at rest initially. Determine

(i) the acceleration in unit vector notation and its magnitude,
(ii) the velocity in unit vector notation at $t = 5.00$ s.

(b) In the figure at right, a 10.0 kg load is hung from the ceiling of an elevator that is moving at constant velocity. Find the tension in each of the three cords supporting the load.



(20 marks)

Solution:

(a) (i) Net force, $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$$\vec{F}_{net} = (-2\hat{\imath} + 2\hat{\jmath}) + (5\hat{\imath} - 3\hat{\jmath}) - 4\hat{\imath} = (-\hat{\imath} - \hat{\jmath})\ \mathrm{N}$$

Newton's 2nd law: $\vec{F}_{net} = m\vec{a}$

$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{1}{4}(-\hat{\imath} - \hat{\jmath})\ \mathrm{m/s^2}$$

Magnitude of acceleration, $a = \sqrt{0.25^2 + 0.25^2} = 0.354\ \mathrm{m/s^2}$

(ii) Using $\vec{v} = \vec{v}_0 + \vec{a}t$,

$$\vec{v} = \frac{1}{4}(-\hat{\imath} - \hat{\jmath}) \times 5 = \frac{5}{4}(-\hat{\imath} - \hat{\jmath})\ \mathrm{m/s}$$

(b) Constant velocity means acceleration = 0

$$T_3 = Mg = 10(9.80) = 98.0\ \mathrm{N}$$

$\Sigma F_x = 0$: $T_1 \cos 40.0° = T_2 \cos 50.0°$

$$T_1 = T_2 \frac{\cos 50.0°}{\cos 40.0°}$$

$\Sigma F_y = 0$: $T_1 \sin 40.0° + T_2 \sin 50.0° = 98$

$$T_2(\cos 50°)(\tan 40°) + T_2 \sin 50° = 98$$

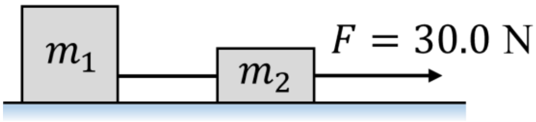
$$T_2 = \frac{98}{\cos 50° \tan 40° + \sin 50°} = 75.0\ \mathrm{N}$$

$$T_1 = 75.0 \frac{\cos 50.0°}{\cos 40.0°} = 63.0\ \mathrm{N}$$

5. The diagram below shows two blocks that are connected by a string of negligible mass, placed on a rough surface. Block 2 is pulled by a 30.0 N force to the right.

Given that $m_1 = 4.00$ kg and $m_2 = 2.00$ kg, and the coefficient of kinetic friction between each block and the horizontal surface is 0.20.

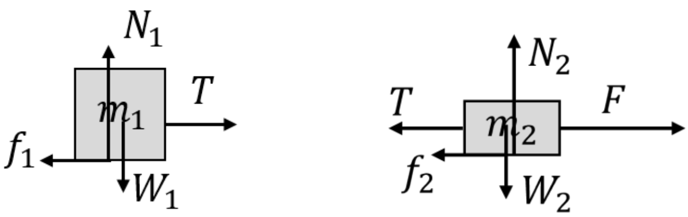
(a) Draw free-body diagrams for blocks 1 and 2.
(b) Calculate the acceleration of the blocks.
(c) What is the tension in the string?



(20 marks)

Solution:

(a) Free-body diagrams of m_1 and m_2:



(b) The acceleration of each block is to the right, writing equations using Newton's 2nd law using blocks 1 and 2 as 1 system,

$\Sigma F_x = (m_1 + m_2)a$: $\quad F - f_1 - f_2 = (m_1 + m_2)a$

Frictions: $f_1 = \mu_k m_1 g$, $f_2 = \mu_k m_2 g$

$$a = \frac{F - f_1 - f_2}{m_1 + m_2} = \frac{30.0 - 0.2(2.00 + 4.00)(9.80)}{2.00 + 4.00} = 3.04 \text{ m/s}^2$$

(c) Using the free-body diagram for block 1, Newton's 2nd law gives

$\Sigma F_x = m_1 a$: $\qquad T - f_1 = m_1 a$
$T = m_1 a + f_1 = m_1 a + \mu_k m_1 g = 4.00(3.04) + 0.20(4.00)(9.80) = 20.0$ N