

ET0096 SEM SAMPLE 2 ANSWERS:

Section A

A1

$$y(n) = x(n) - 0.866x(n-1) + 1.732y(n-1) - y(n-2)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 0.866z^{-1}}{1 - 1.732z^{-1} + z^{-2}}$$

$$= \frac{1 - \cos\left(\frac{\pi}{6}\right)z^{-1}}{1 - 2\cos\left(\frac{\pi}{6}\right)z^{-1} + z^{-2}}$$

$$h(n) = \cos\left(\frac{n\pi}{6}\right)u(n)$$

Since the impulse response is a constant amplitude cosine function, the system is marginally stable (or unstable).

A2 $y(n) = \{2, 3, 1, 6\}$

$$Y(z) = 2 + 3z^{-1} + z^{-2} + 6z^{-3}$$

and impulse response $h(n) = \{1, 2\}$

$$H(z) = 1 + 2z^{-1}$$

$$X(z) = Y(z)/H(z)$$

$$1+2z^{-1} \overline{\begin{array}{r} 2 - z^{-1} + 3z^{-2} \\ 2 + 3z^{-1} + z^{-2} + 6z^{-3} \\ 2 + 4z^{-1} \\ \hline \end{array}}$$

$$X(z) = 2 - z^{-1} + 3z^{-2}$$

$$x(n) = \{2, -1, 3\}$$

A3 When $N=4$, $k=$ for $k = 0, 1, 2$ and 3

$$X(k) = \sum_{n=0}^3 x(n) e^{-j \frac{2\pi kn}{4}}$$

$$= x(0) + x(1) e^{-j \frac{2\pi k}{4}} + x(2) e^{-j \frac{4\pi k}{4}} + x(3) e^{-j \frac{6\pi k}{4}}$$

$$X(0) = 0$$

$$X(2) = 0$$

A4

$$x_1(n) = e^{-2n} \sin(3n) u(n)$$

$$X_1(z) = \frac{e^{-2} \sin(3) z^{-1}}{1 - 2e^{-2} \cos(3) + e^{-4} z^{-2}}$$

Or equivalent

and $x_2(n) = n 5^{n-1} u(n)$

$$X_2(z) = \frac{z^{-1}}{(1 - 5z^{-1})^2}$$

Or equivalent

A5

(a) $h_T(n) = \{ h_1(n) + h_2(n) \} * \{ h_3(n) + h_4(n) \}$

$$H_T(z) = \{ H_1(z) + H_2(z) \} X \{ H_3(z) + H_4(z) \}$$

(b) $h_T(n) = \{ 2, 2 \} * (2, 2) = \{ 4, 8, 4 \}$

A6 Applying the partial fraction expansion leads to

$$X(z) = \frac{z}{3z^2 - 4z + 1}$$

$$\frac{X(z)}{z} = \frac{1}{(3z-1)(z-1)} = \frac{A}{3z-1} + \frac{B}{z-1}$$

$$A = \left. \frac{X(z)}{z} (3z-1) \right|_{z=1/3}$$

$$A = -1.5$$

$$B = \left. \frac{X(z)}{z} (z-1) \right|_{z=1}$$

$$B=0.5$$

$$X(z) = \frac{0.5}{1-z^{-1}} - \frac{0.5}{1-\frac{1}{3}z^{-1}}$$

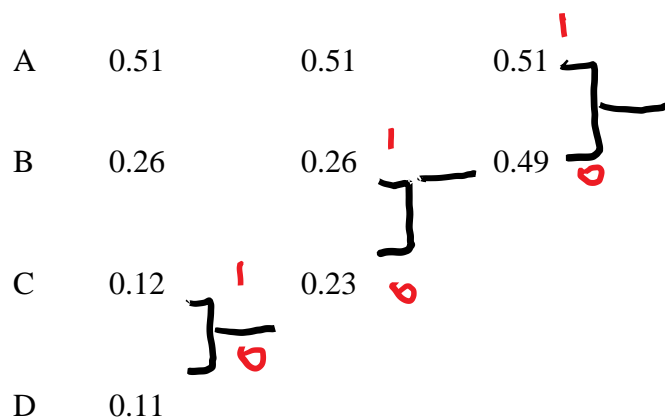
$$x(n) = 0.5u(n) - 0.5 (1/3)^n u(n) \text{ or equivalent}$$

Section B

B1 (a)

$$\begin{aligned}
 H(X) &= \sum_1^5 P_i \log_2 \frac{1}{P_i} \\
 &= 0.51 \log_2(1/0.51) + 0.26 \log_2(1/0.26) + 0.12 \log_2(1/0.12) + 0.11 \log_2(1/0.11) \\
 &= 1.7181 \text{ bits/symbol}
 \end{aligned}$$

(b)



A = 1, B = 01, C = 001, D = 000

$$\begin{aligned}
 \text{Average bit length} &= \bar{n} = \sum_1^4 n_i P(x_i) = 1 \times 0.51 + 2 \times 0.26 + 3 \times 0.12 + 3 \times 0.11 \\
 &= \underline{1.72 \text{ bits/symbol}}
 \end{aligned}$$

B2 Original symbols sequence: 255,255,.....255

- (i) (255,40),(100,4),(0,1),(100,10), (0,1),(100,4),(255,40)
- (ii) Total number of bits of the bit stream = $14 \times 8 = 112$ bits.
- (iii) Compression ratio = $100/14 = 3.25$