SINGAPORE POLYTECHNIC

2020/2021 SEMESTER ONE EXAM

ENGINEERING MATHEMATICS II Time allowed: 2 hrs

1st/2nd Year Full-Time

School of Chemical and Life Sciences
DCHE

School of Electrical and Electronic Engineering
DASE, DCPE, DEB, DEEE

School of Mechanical and Aeronautical Engineering
DARE, DBEN, DCEP, DME, DMRO

Instructions to Candidates:

1. The examination rules set out on the last page of the answer booklet are to be complied with.
2. **The questions are printed on Pages 2 – 5**.
3. This paper consists of THREE (3) sections.

   Section A: 5 Multiple Choice Questions (10 marks)
   Answer **ALL** questions.

   Section B: 7 Structured Questions (50 marks)
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all the questions you answered will be added, but the maximum mark you can obtain from this section is 50 marks.

   Section C: 3 Structured Questions (40 marks)
   Answer **ALL** questions.

4. Unless otherwise stated, leave your decimal answers correct to **two** decimal places.
5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply will result in loss of marks.

Section A (10 marks):

Answer ALL multiple choice questions on the MCQ answer sheet of the answer booklet.

A1. Which one of the following integrals **cannot** be integrated using only the method of substitution?

(a) $\int x^{-3}e^{x^{-2}}\,dx$ (b) $\int x\sin 2x\,dx$

(c) $\int \frac{4x}{\sqrt{2x+1}}\,dx$ (d) $\int \tan^3 x \sec x\,dx$

A2. The values of $f(t)$ are listed in the table below.

t	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$f(t)$	25.0	26.0	27.0	28.1	36.0	49.0	64.0	81.0

Which one of the following integrals can be approximated using Simpson's rule with the data provided in the table?

(a) $\int_0^{0.6} f(t)\,dt$ (b) $\int_0^{0.7} f(t)\,dt$

(c) $\int_{0.1}^{0.6} f(t)\,dt$ (d) $\int_{0.2}^{0.7} f(t)\,dt$

A3. Which one of the following statements is TRUE for an even periodic function $f(t)$?

(a) The d.c. component in the Fourier series of $f(t)$ is equal to zero.

(b) The coefficients of the cosine terms in the Fourier series of $f(t)$ are equal to zero.

(c) The coefficients of the sine terms in the Fourier series of $f(t)$ are equal to zero.

(d) The graph of $f(t)$ is symmetrical about the origin.

A4. What is the Laplace transform of the piecewise function $f(t)=\begin{cases}1 & , \quad 1\le t\le 2\\ 0 & , \quad \text{otherwise}\end{cases}$?

(a) $\int_0^{\infty} e^{-st}\,dt$ (b) $\int_0^{\infty} te^{-st}\,dt$

(c) $\int_1^{2} e^{-st}\,dt$ (d) $\int_1^{2} te^{-st}\,dt$

A5. What is the value of the constant k in the 2nd order differential equation

$$9\frac{d^2y}{dx^2}+k\frac{dy}{dx}+y=0, \text{ if } y=e^{\frac{1}{3}x}(Ax+B) \text{ is its general solution?}$$

(a) 6 (b) −3
(c) 3 (d) −6

<u>Section B</u> (50 marks):

Each question carries 10 mark. The total marks of the questions in this section is 70. You may answer as many questions as you wish. The marks from all the questions you answered will be added, but the maximum mark you can obtain from this section is 50 marks.

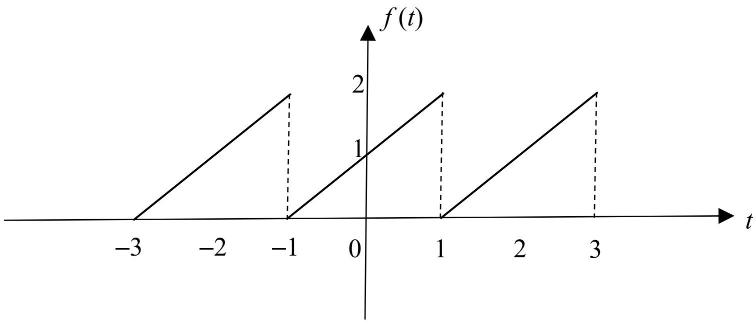
B1. (a) Find f_x and f_y when $f(x,y)=\pi x\sin\left(y^2\right)$.

(b) If $g(x,y)=(x^2-2)e^{4y}$, evaluate $g_x(1,0)$.

B2. (a) Find $\int 10x^4e^{x^5}\,dx$, by using the substitution $u=x^5$.

(b) Evaluate $\int_1^2 t\ln t\,dt$, by using integration by parts.

B3. A sawtooth function of period 2 is defined as shown in the figure below:



It is known that $f(t)=t+1$, for $-1<t<1$ and $f(t+2)=f(t)$.

(a) State whether $f(t)$ is even, odd or neither.

(b) Find the d.c. component (i.e. a_0).

(c) If $b_n=\frac{-2}{n\pi}\cos(n\pi)$, find b_1, b_2 and b_3.

(d) Given that $a_n=0$ (where $n\neq 0$), use the results from parts (b) and (c), write down the Fourier series of $f(t)$ as far as the third harmonic.

B4. Given $\dfrac{dy}{dx}+\dfrac{2}{x}y=\dfrac{\sin(3x)}{x^2}$,

(a) solve the differential equation, by using an integrating factor;

(b) hence, find the particular solution if $y(\pi)=\dfrac{1}{\pi^2}$.

B5. (a) Find $\mathcal{L}\left\{\dfrac{1}{2}-3t^2+e^{-t}\right\}$.

(b) By using the compound angle formula for $\sin(2t+\pi)$, find $\mathcal{L}\{\sin(2t+\pi)\}$.

(c) Find $\mathcal{L}\{t\cos 2t\}$. Hence, use First Shift Theorem to find $\mathcal{L}\{e^{3t}\,t\cos 2t\}$.

B6. (a) Find $\mathcal{L}^{-1}\left\{\dfrac{2}{s-3}+\dfrac{2}{s^3}+\dfrac{1}{s^2+4}\right\}$.

(b) Find $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+9}\right\}$. Hence, use the First Shift Theorem to find $\mathcal{L}^{-1}\left\{\dfrac{s+1}{(s+1)^2+9}\right\}$.

B7. Given the differential equation $y''+2y'-3y=0$, find

(a) the auxiliary equation;

(b) the general solution;

(c) the particular solution if $y(0)=3$ and $y'(0)=1$.

<u>Section C</u> (40 marks):

Answer all THREE questions below.

C1. The table below gives the values of voltage v (volts) measured at regular interval of time t (seconds):

t (s)	0	0.3	0.6	0.9	1.2	1.5	1.8
v (volts)	1	0.9828	0.9306	0.8409	0.7071	0.5087	0

(a) By using Simpson's rule with 6 strips, evaluate the integral $\int_0^{1.8} v^2 dt$. Give your answer correct to 2 decimal places.

(b) Hence, find the r.m.s. value of v over the complete interval of 1.8 s. (11 marks)

C2. Jim cooked a big pot of soup late at night, the soup had just boiled at $100\,°\mathrm{C}$. Jim discovered that by cooling the pot in a sink of cold water, which was maintained at a constant temperature of $5\,°\mathrm{C}$, the temperature of the soup would drop down to $60\,°\mathrm{C}$ in ten minutes.

Let $T(t)$ be the temperature ($°\mathrm{C}$) of the soup at time t (min) after the cooling process has started in the sink.

(a) Apply Newton's law of cooling and set up the differential equation that models the cooling process of the soup. Indicate clearly the surrounding temperature T_s.

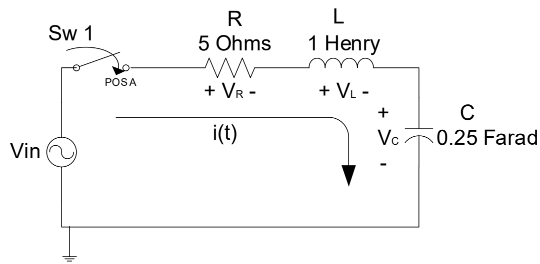
(Newton's law of cooling states that the rate of change of temperature is proportional to the temperature difference between the object and the surrounding environment.)

(b) Solve the equation in part (a) using the given conditions.

(c) How much longer did Jim have to wait for the soup's temperature to drop further to $20\,°\mathrm{C}$, so that it could be stored in the fridge? (14 marks)

C3. (a) Given $\dfrac{40}{(s+1)(s+4)(s^2+1)} = \dfrac{A}{s+1} + \dfrac{B}{s+4} + \dfrac{Cs+D}{s^2+1}$.

If $A = \dfrac{20}{3}$ and $C = -\dfrac{100}{17}$, find the constants B and D.



(b) In the *RLC* circuit shown on the right, the capacitor is fully discharged with no initial energy stored. At $t = 0$, switch Sw 1 is moved to position A. The total voltage drop across the circuit can be derived as

$$V_{in}'(t) = R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C} i .$$

(i) If $V_{in}(t) = -40\cos t$, show that the differential equation for the current $i(t)$ is

$$\frac{d^2 i}{dt^2} + 5\frac{di}{dt} + 4i = 40\sin t .$$

(ii) If $i'(0) = i(0) = 0$, find $\mathcal{L}\{i(t)\}$.

(iii) Hence, by using the result obtained in part (a), solve the differential equation for the current $i(t)$.

(c) Determine the steady-state current and the transient state current of $i(t)$.

(15 marks)

- End of Paper -