

SINGAPORE POLYTECHNIC

2019/2020 SEMESTER TWO MID SEMESTER TEST

ENGINEERING MATHEMATICS II

Time allowed: 1.5 hrs

1st / 2nd Year Full-Time

School of Chemical and Life Sciences
DCHE

School of Electrical and Electronic Engineering
DASE, DCEP, DCPE, DEB, DEEE, DES, DESM

School of Mechanical and Aeronautical Engineering
DARE, DBEN, DCEP, DME, DMRO

Instructions:

1. The Singapore Polytechnic Examination rules are to be complied with.
 2. This paper consists of 3 **printed** pages.
 3. Answer all the 7 questions.
 4. Unless otherwise stated, correct all your decimal answers to **two** decimal places.
 5. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to do so will result in loss of marks.
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1. (a) Given that $z = \ln(x+y) - 2x \cos y$, find the partial derivative $z_y(x,y)$ and hence compute the value $z_y(2,1)$.
- (b) At a quarry, sand is poured down from a certain height at a rate of $0.5 \text{ m}^3/\text{s}$. It formed a heap shaped like a right circular cone with a base radius of r metre and a height h metre on the ground.
- If the base radius r is increasing at a rate of 0.03 m/s , find the rate at which the height h is changing when the radius $r = 3 \text{ m}$ and the height $h = 2 \text{ m}$.
- (Hint: volume of circular cone $V = \frac{1}{3} \pi r^2 h$) (17 marks)
2. (a) Find the following integrals:
- (i) $\int \frac{1}{4x-3} dx$
- (ii) $\int 6(3x+1)^4 dx$
- (iii) $\int 2 \sin 6x \sin 2x dx$ [Hint: use product-to-sum formula]
- (b) Find the root mean square (RMS) value of the function $y = 3t + 2$ from $t = 0$ to $t = 1$. (20 marks)
3. Find:
- (a) $\int x^2 e^{x^3} dx$ by using the substitution $u = x^3$;
- (b) $\int_1^e \frac{\ln x}{x(1+\ln x)^2} dx$ by using a suitable substitution. (11 marks)
4. (a) If $\frac{2x}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2}$, find the values of constants A and B .
- Hence, find $\int \frac{2x}{(x+5)(x-2)} dx$.
- (b) By completing the square for $x^2 + 2x + 5$, find $\int \frac{3}{x^2 + 2x + 5} dx$. (13 marks)

5. Use integration by parts to find the following integrals.

(a) $\int 2x \sin x \, dx$

(b) $\int \frac{\ln x}{x^4} \, dx$ (13 marks)

6. By using Simpson's rule with 6 equal intervals, find the approximate value of

$$\int_0^{\pi} \sqrt{\cos(x)+2} \, dx, \text{ accurate to 2 decimal places. (Show your workings clearly.)}$$

(10 marks)

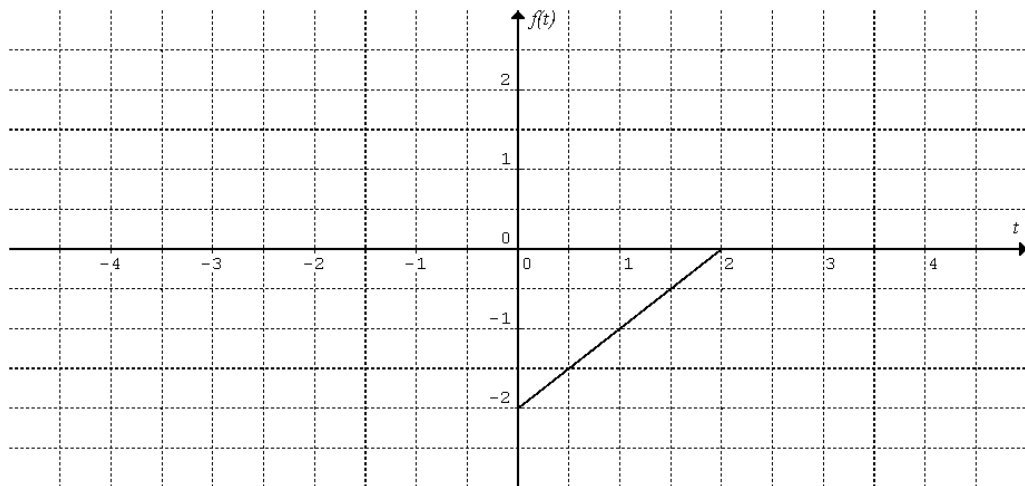
7. A periodic function $f(t)$ of period 4 is defined over half a period as:

$$f(t) = t - 2, \quad 0 < t < 2, \quad f(t+4) = f(t).$$

- (a) The graph of $f(t)$ over $0 < t < 2$ is shown below. Sketch the graph for (i) and (ii) in your answer booklet.

- (i) If $f(t)$ is an odd function, sketch the graph of $f(t)$ over $-2 < t < 0$.

- (ii) Hence complete sketching the graph over $f(t)$ for $-4 < t < 4$.



- (b) Find the Fourier series of $f(t)$ as far as the third harmonic.
 (c) Hence determine the Fourier series of $g(t)$, which is an odd and periodic function, defined over half period as $g(t) = \frac{3}{2}t - 6$ $0 < t < 4$.

(16 marks)

- End of Paper -