Chapter 2: Understanding Kinetic Forces

At the end of the lesson, students should be able to:

- List and explain the quantities associated with uniform motion
- Explain the kinetics of motion under constant acceleration
- Explain the kinetics of uniform circular motion
- Understand and differentiate centrifugal and centripetal forces
- Understand kinetic forces in an electric car

2.1 Quantities associated with uniform motion

Kinetics is the branch of classical mechanics that is concerned with the relationship between motion and its causes, specifically, forces and torques. Forces and torques cause uniform or non-uniform motions.

Uniform motion is defined as the motion of an object in which the object travels in a straight line and its velocity remains constant along that line. That is, the object travels equal distances in equal intervals of time, irrespective of the duration of the time. For example, a car moving on a straight road without any change in its velocity.

In contrast, a body is said to be in a non-uniform motion if it travels unequal distances in equal intervals of time. For example, if we drop a ball from the roof of a tall building, we will notice that it will cover unequal distances in equal intervals of time. Like, 5 metres in the 1st second, 15 metres in the 2nd second and so on. Thus, a freely falling ball will covers smaller distances in the first '1 second' and larger distances in the later '1 second' intervals. Therefore, we can say that the motion of a freely falling body is an example of a non-uniform motion. The distance-time graph of the non-uniform motion is a curved line. Non-Uniform motion is also known as accelerated motion.

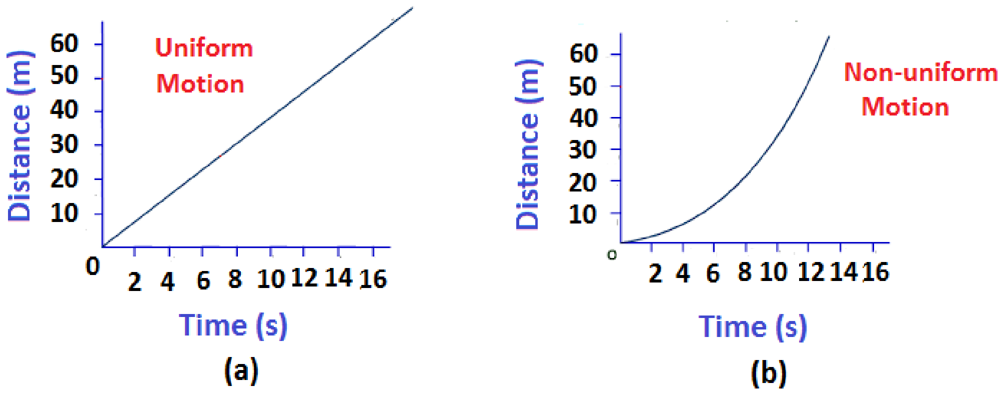


Figure 2.1: Uniform and non-uniform Motions

We shall study the common quantities associated with uniform and non-uniform motion in this section.

https://www.youtube.com/watch?v=VFfF3F-G9Uk&list=PLmdFyQYShrjcoTLhPodQGjtZKPKIWc3Vp&index=2

(a) Rectilinear motion

A motion along a straight line or along a particular direction is called rectilinear motion. Think of a car going down a straight road, or a person running on a straight track. You could also think of an object being thrown up vertically in the air and watching it fall.

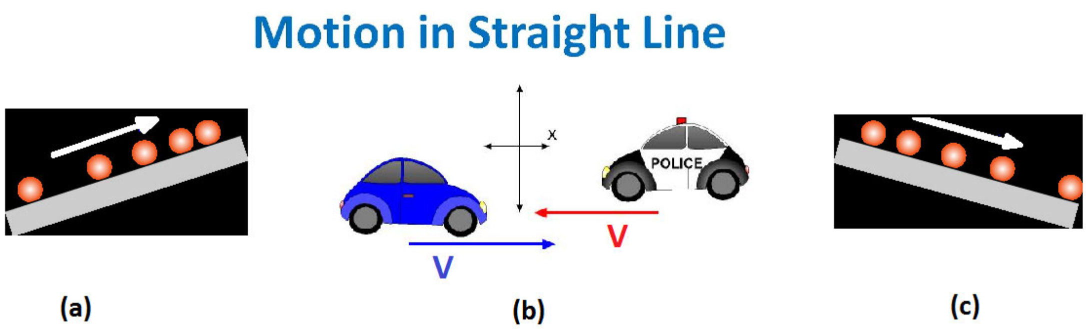


Figure 2.2: Examples of Rectilinear Motion

(b) Distance

Distance represents the magnitude of motion in terms of the "length" of the path, covered by an object during its motion. Initial and final positions of the object are start and end points of measurement and are not sufficient to determine distance.

Distance is a **scalar** quantity but with a special feature. The distance keeps increasing regardless of the direction, implies that distance is always positive. The SI unit is “meter” or m. For example, 100 m or 10 km marathon.

(c) Displacement

Displacement is the **vector** quantity extending from initial to final positions of the particle in motion during an interval.

Displacement is a measurement of change in position of the particle in motion. Its magnitude and direction are measured by the length and direction of the straight line joining initial and final positions of the particle. Obviously, the length of the straight line between the initial and final positions is the shortest distance plus direction between the points.

Actual path between two positions has no consequence in so far as the magnitude of displacement is concerned. If there is no change in the position at the end of a motion, the displacement is zero.

Once motion has begun, displacement may increase or decrease (at a slow, fast or constant rate) or may even be zero, if the object return to its initial position. Displacement is essentially a measurement of length combined with direction and its SI unit is 'meter' or m.

(d) Speed

Speed is the rate of change of distance with respect to time and is expressed as distance covered in unit time. It is a scalar quantity, as it has no direction. It tells us exactly how rapidly this change is taking place with respect to time.

$$\textbf{Speed} \quad v = \frac{ds}{dt} \quad \text{(unit is m/s)}$$

Average speed gives the overall view of the motion. It does not, however, give the details of motion.

Instantaneous speed is also defined exactly like average speed i.e. it is equal to the ratio of total distance and time interval, but with one qualification that time interval is extremely (infinitesimally) small.

Example 2.1

Convert (a) 450 knots into km/hr and (b) 120 m/s into km/hr.

(a) 1 knot = 1.852 km/hr

450 x 1.852 = 833.4 km/hr

(b) 120 m/s = (120 x 60 x 60) / 1000 = 432 km/hr.

(e) Distance versus time

Motion of an object over a period may vary. These variations are conveniently represented on a distance - time plot as shown below.

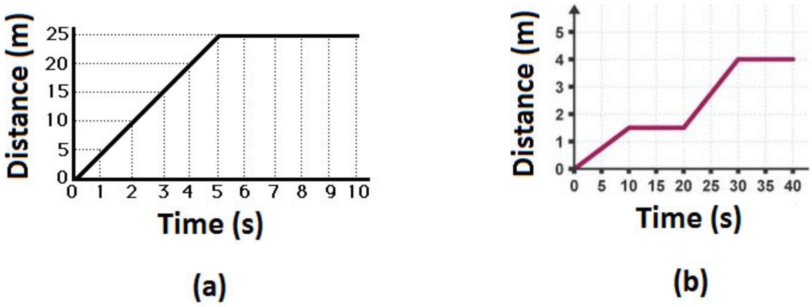


Figure 2.3: Distance versus Time

The plot is always drawn in the first quadrant, as distance cannot be negative. Distance versus time plot is ever increasing during the motion. It means that the plot cannot decrease from any level at a given instant.

When the object is at rest, the distance becomes constant and plot is a horizontal line parallel to time axis. Note that this portion of plot does not add to the distance and the vertical segment representing distance remains constant during the motion.

(f) Speed versus time

The distance, s, covered in the small period, dt, is given by:

$$\text{Speed, } v = \frac{ds}{dt}$$

$$ds = v\,dt$$

$$\text{distance, } s = \int_{t_1}^{t_2} v\,dt$$

The right hand side of the integral represents an area on a plot drawn as shown in Figure 2.4. The area between the speed versus time is the distance travelled.

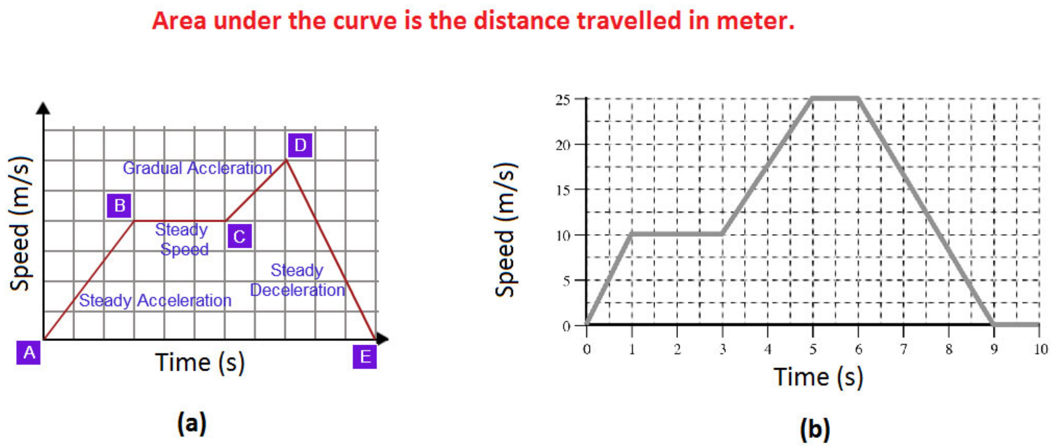


Figure 2.4 Speed versus time plot

(g) Velocity

Velocity is the rate of change of displacement with respect to time and is expressed as the ratio of **displacement** and time. It has direction.

$$\mathbf{v} = \frac{\textbf{Displacement}}{\Delta \mathbf{t}}$$

If the ratio of displacement and time is evaluated for finite time interval, we call the ratio "average" velocity, whereas if the ratio is evaluated for infinitesimally small time interval ($\Delta t \to 0$), then we call the ratio "instantaneous" velocity. The SI unit is meter/second (m/s).

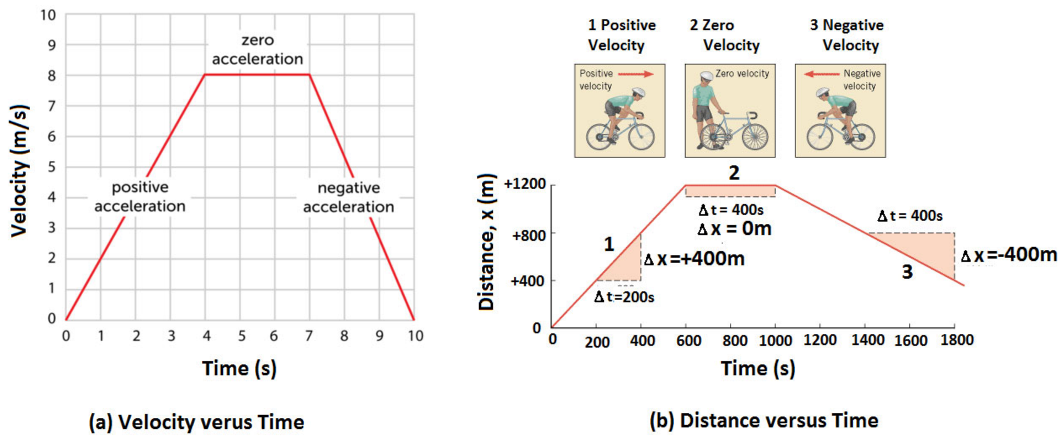


Figure 2.5: Velocity versus time and Distance Versus Time Plots

Example 2.2

A particle completes a motion in two parts. It covers a straight distance of 10 m in 1 s in the first part along the positive x-direction and 20 m in 5 s in the second part along negative x-direction (See Figure 2.6). Find average speed and velocity.

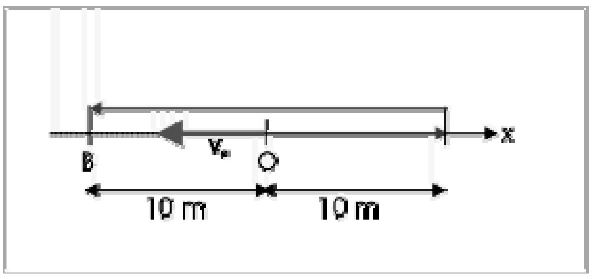


Figure 2.6

In order to find the average speed, we need to find the displacement and time. Here total time is 1 + 5 = 6 s and total distance covered is 10 + 20 = 30 m. Hence,

Average speed v_a = 30 / 6 = 5 m/s (no direction indicated)

The displacement is equal to the linear distance between initial and final positions. The linear and final position are at a linear distance = -10 m. The value is taken as negative as final position falls on the opposite side of the origin. Hence,

Average velocity, v_A = -10 / 6 = -1.66 m/s (has direction indicated)

The negative value indicates that the average velocity is directed in the opposite direction to that of the velocity in the first part of the motion. Also note that average velocity vector is not rooted at any position and can be shown anywhere between initial and final position. Conventionally, however, we chose to shown it with respect to the initial position.

Example 2.3

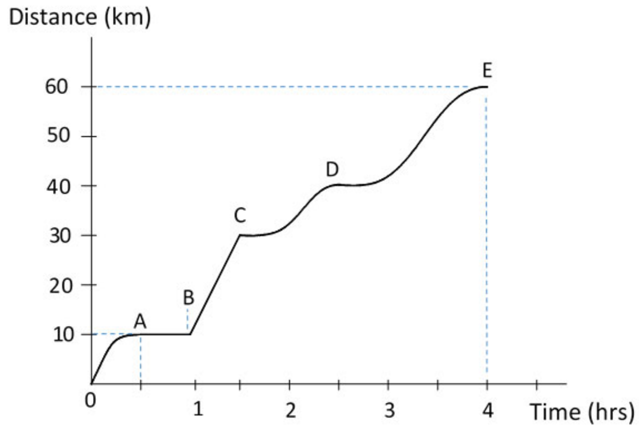


Figure 2.7

Figure 2.7 is the Distance-Time graph of a cyclist on a long distance ride from home and returned. Based on the riding profile, determine:

a. the total distance covered. (60 km)
b. the total displacement covered. (0 m)
c. his average speed. (15 km/hr)
d. his average velocity. (0 km/hr)
e. the distance covered at point C. (30 km)
f. the displacement at point C. (cannot be determined)
g. his speed from point A to point B. (0 km/hr)
h. his speed from 1 hour mark to 1.5 hour mark. (40 km/hr)
i. his velocity from point B to point C. (cannot be determined)
j. his velocity from point B to point C given that he was covering a straight and flat path.

(cannot be determined as no direction is given)

k. the magnitude of his velocity from point B to point C given that he was covering a straight and flat path. (40 km/hr)
l. at the point where he decided to turn back for home. (cannot be determined)
m. at which hour mark he reached home. (4 hour mark)
n. if it is possible that the path was straight and flat from B to D. (yes, it is possible)

Given that he decided to turn back at point D, answer the following questions:

a. Was he riding faster, on the average, before he decided to return home compare to his trip home? (Yes, 16 km/hr vs 13.3 km/hr)

b. When he decided to return home, was it possible that he took the same route home? Why?
(Not possible, the reason being he needed to cover the same distance if the same route was used.)

(h) Components of velocity

The component of velocity is a powerful concept that makes it possible to treat a two or three-dimensional motion as composition of component straight-line motions. To illustrate the point, consider the case of two-dimensional parabolic motions as in Figure 2.8. Here, the velocity of the body is resolved in two mutually perpendicular directions; treating motion in each direction independently and then combining the component directional attributes, using rules of vector addition.

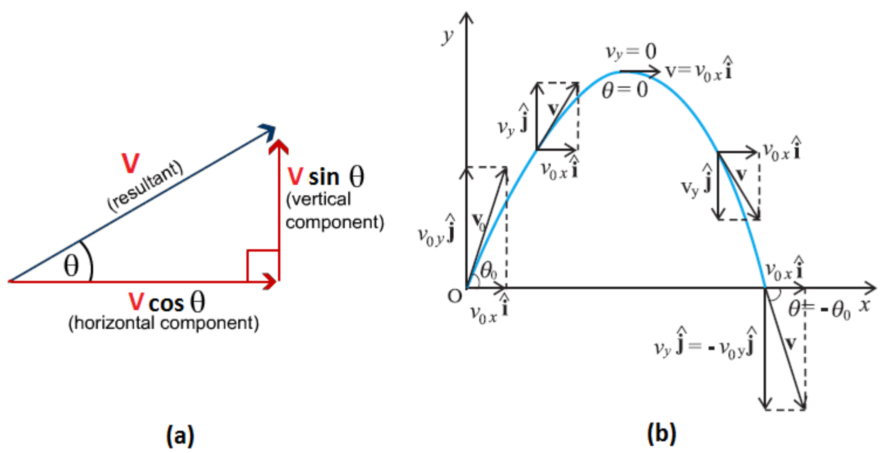


Figure 2.8: Motion is treated separately in two perpendicular directions

2.2 Kinetics of motion under constant acceleration

Acceleration of an object can be derived or defined in 2 ways. From Newton's 2nd law, force on an object F = ma where m is the mass and a is the acceleration. Here, acceleration is force divided by mass. Acceleration can also be obtained from the rate of change of velocity.

$$\text{acceleration, } a = \frac{F}{m} \ m/s^2$$

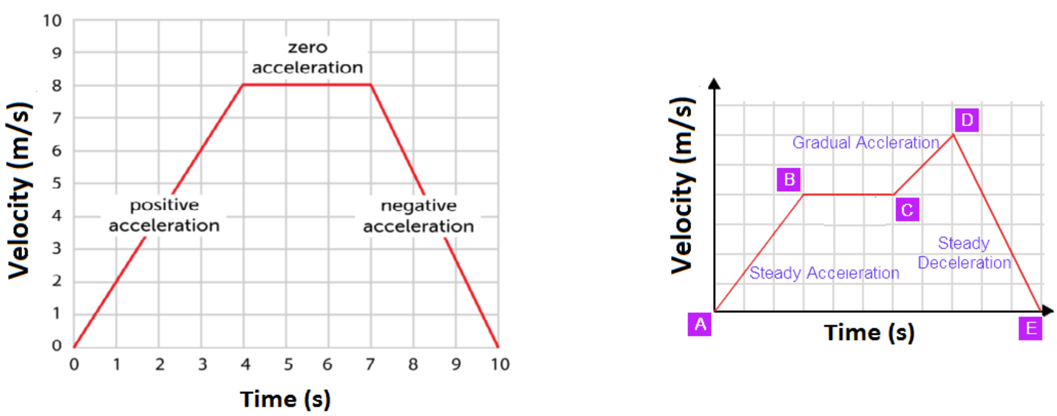
OR

$$\text{acceleration, } a = \frac{dv}{dt} \ m/s^2$$

Acceleration, like velocity, is a vector quantity, so any change in the velocity or / and any change in the direction of a moving body is also an acceleration.
An increase in the magnitude of the velocity of a moving body (or an increase in speed in a given direction) is called a positive acceleration; a decrease in speed in the negative gradient direction is called a negative acceleration or deceleration.

Average acceleration gives the overall acceleration over a finite interval of time. The magnitude of the average acceleration tells us the rapidity with which the velocity of the object changes in a given time interval.

https://www.youtube.com/watch?v=bqf8m7xNvLg



(a) (b)

Figure 2.9: Velocity versus Time Plots

(a) Constant acceleration

The constant acceleration is a special case of accelerated motion. Here, the rate of change of velocity of an objection is constant. For example, constant acceleration occurs whenever an object is dropped under constant acceleration 9.81 m/s^2, under the influence of gravity.

Two of the most important forces controlling motions in our daily life are force due to gravity and frictional force. Gravity and frictional force can produce constant acceleration to an object.

Consider the motion of a ball dropped from the top of a tower. The ball moves under constant acceleration of gravity during its flight to the ground. In the same manner, motion on a rough plane is acted upon by the force of friction in the direction opposite to the motion. The force of friction is a constant force for the moving body with a constant mass, m, and characteristic of the surfaces in contact. As a result, the object slows down at a constant rate.

The important point of a constant acceleration is that its magnitude has a constant value and its direction is fixed. A change in either of the two attributes, constituting acceleration, shall render acceleration variable. **Let us look at some graphs to understand acceleration**

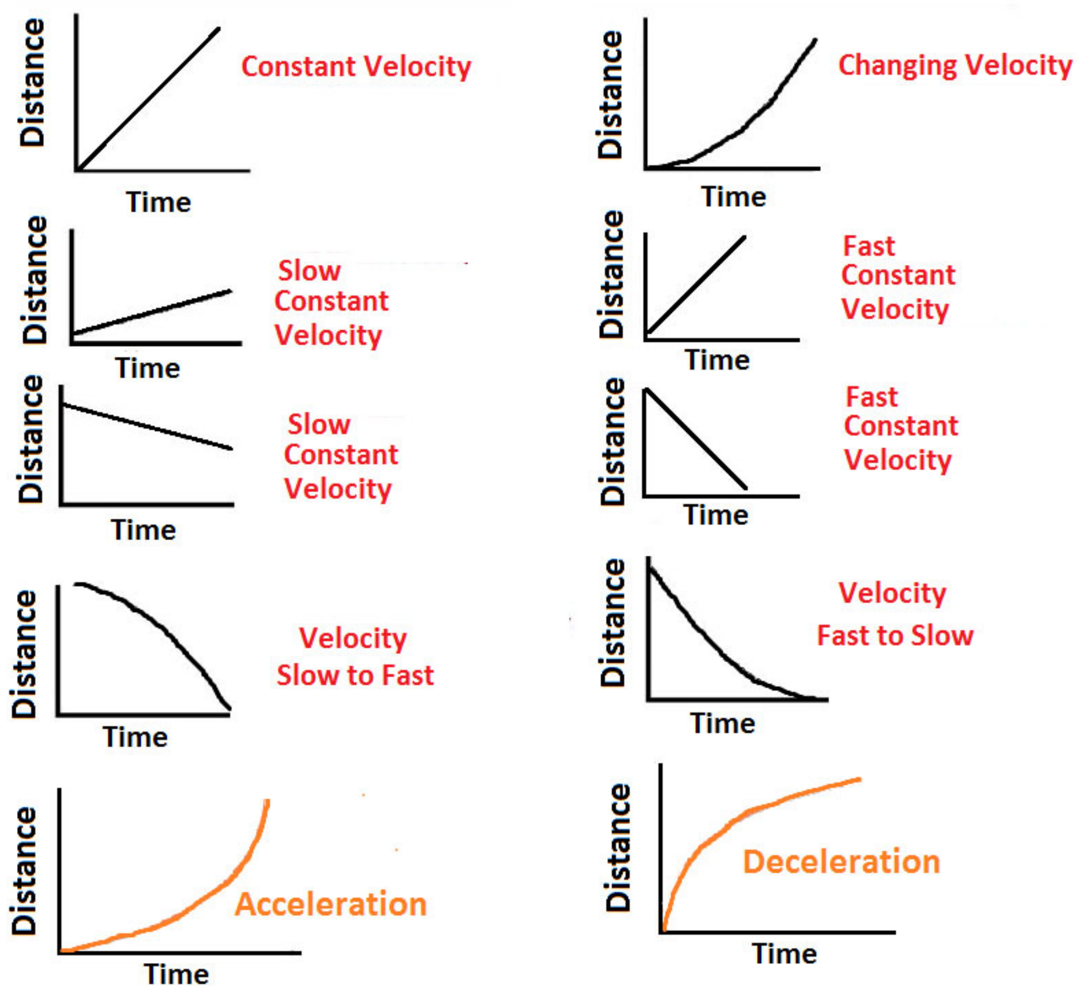


Figure 2.10: Distance versus Time Plots

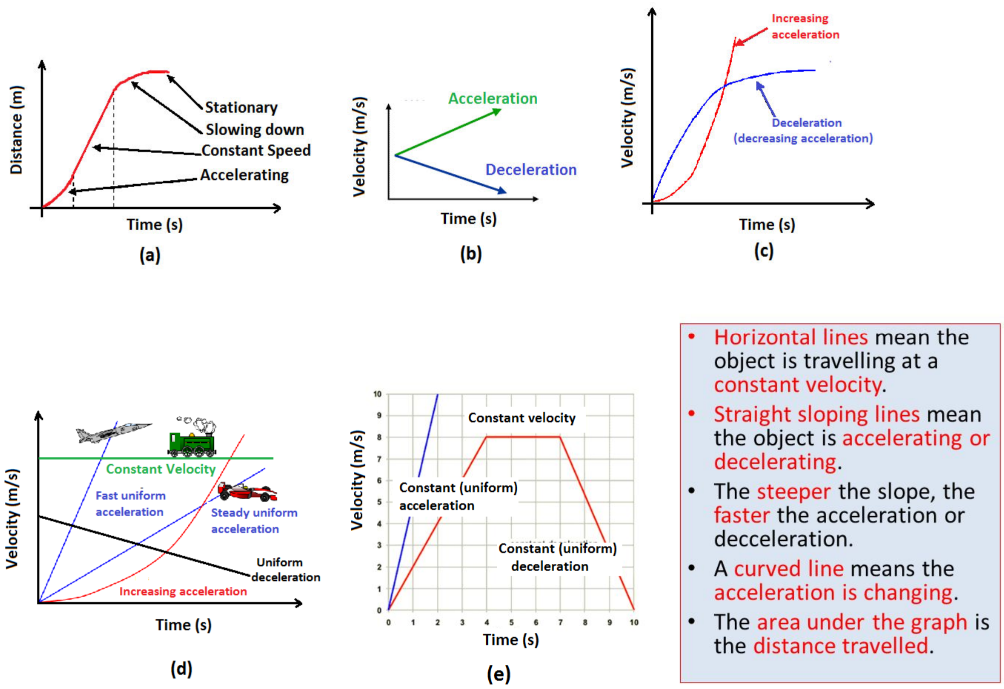


Figure 2.11: Understanding Velocity, Acceleration and Deceleration

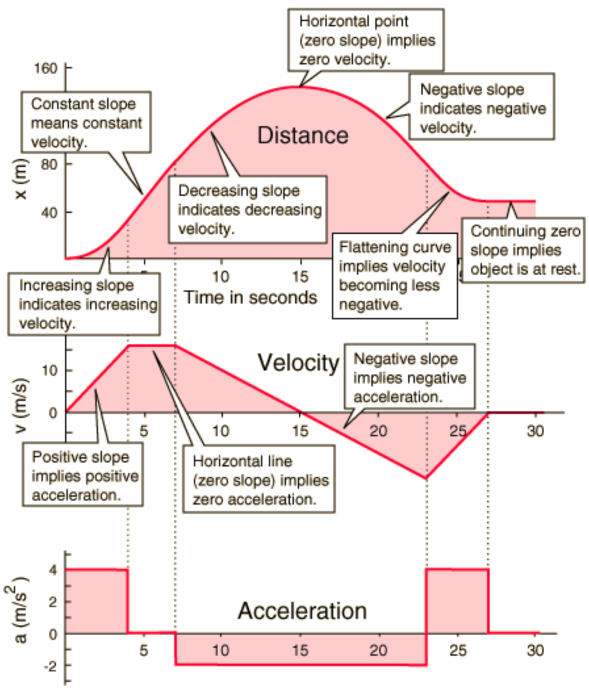


Figure 2.12: Understanding Position, Velocity and Acceleration

(b) Equation of Motion under constant acceleration

The motion under **constant acceleration** allows us to describe accelerated motion, using simple mathematical equations. The equations are:

$$\mathbf{v = u + at} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$
$$\mathbf{s = ut + \tfrac{1}{2}\, at^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$
$$\mathbf{v^2 = u^2 + 2as} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

In these equations,

u is the initial velocity
v is the final velocity
a is the constant acceleration
t is the time interval of motion under consideration
s is the distance covered.

Example 2.4

A light aircraft of mass 1965 kg accelerates from 160 km/hr to 240 km/hr in 3.5 s. If the air resistance is 2000N/tonne, find the :

(i) average acceleration
(ii) force required to produce the acceleration, a
(iii) force on the aircraft
(iv) propulsive force on the aircraft.

(a) u = 160 km/hr = 160 x 1000 /3600 = 44.4 m/s

v = 240 km/hr = 240 x1000/3600 = 66.6 m/s

a = (v-u) /t = (66.6 – 44.4) / 3.5 = 6.34 m / s^2.

(b) F = ma = 1965 x 6.34 = 12.46 KN

(c) Applying Newton's 3rd Law, Force on the aircraft = 12.46 KN

(d) The propulsive force must be sufficient to overcome the inertia force and that of the force due to the air resistance.

Force due to air resistance = (1965/1000) x 2000 = 3930N

Propulsive force = Inertia force + Force due to air resistance

= 12.46 + 3.93 = 16.39 kN.

Example 2.5

An autonomous electric car is travelling at a speed of 270 km/h. Then it decelerates at 4.5 m/s^2 . What is the distance travelled before it stops completely?

u = 270 km/h = (270 x1000) / 3600 =75 m/s, v = 0, a = - 4.5 m/s^2. s = ?

$v^2 = u^2 + 2as$

$0 = 75^2 + (2)(-4.5)(s).$

$s = 625$ m

Example 2.6

A cyclist travelling on the road accelerates uniformly at 1.5 m/s^2 . If his initial velocity is 3 m/s, find:

(a) how far he travels in 8s.
(b) how far he travels before reaching a velocity of 7 m/s.

(a) a = 1.5 m/s^2 , u = 3 m/s, t= 8 s. s = ?

$\mathbf{s = ut + \frac{1}{2} at^2}$ = (3x 8) + (1/2) (1.5) (8x8) = 72 m

(b) v = 7 m/s; s =?

$\mathbf{v^2 = u^2 + 2as;}$ $7^2 = 3^2 + 2 \times 1.5 \times s$; s = 13.33m

Example 2.7

A ball is thrown vertically upwards with a velocity of 14.7 m/s from a platform 19.6m above the ground level. Find:

(a) the time taken for the ball to reach the ground.
(b) the velocity of the ball when it hits the ground.

(a) $u = 14.7$ m/s, $a = -9.8$ m/s^2, $s = -19.6$ m;

$$\mathbf{s = ut + \tfrac{1}{2}\, at^2}$$

$$-19.6 = (14.7\,t) + (1/2)(-9.8)\,t^2$$

$$t^2 - 3t - 4 = 0$$
$$(t-4)(t+1) = 0$$

Therefore, $t = 4$ s and $t = -1$s (invalid).

Therefore $t = 4$ s

(b) $v = u + at = (14.7) + (-9.8) \times 4 = -24.5$ m/s

Example 2.8

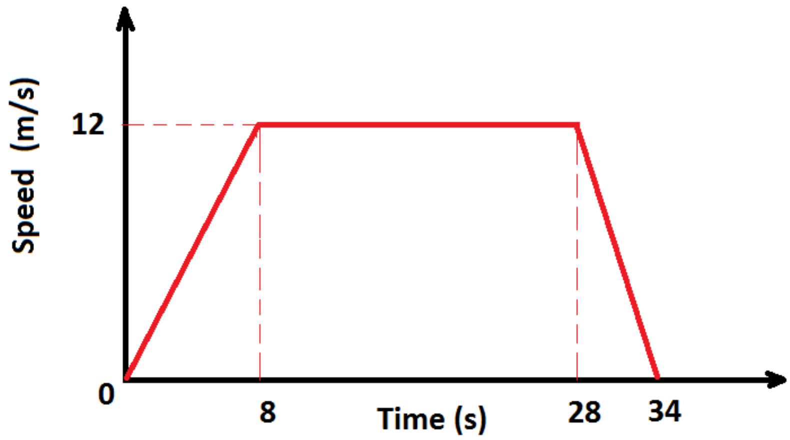
A cyclist rides along a straight road from point A to a point B. He starts from rest at A and accelerates uniformly to reach a speed of 12 m/s in 8 seconds. He maintains this speed for a further 20 seconds and then uniformly decelerates to rest at B. If the whole journey takes 34 seconds,

a) draw a velocity-time graph for the motion.

From the graph find

b) his acceleration for the first part of the motion
c) his deceleration for the last part of the motion
d) the total distance travelled.

a)



b) a = 12/8 = 1.5 m / s^2

c) Deceleration = 12 /6 = 2 m/s^2

d) Total distance = area under curve

= {(1/2) (12) (8) } + { 12 x(28-8)} + {(1/2)(12)(6)} = 324 m

(c) Motion under gravity

We have observed that when a feather and an iron ball are released from a height, they reach earth surface with different velocity and at different times. These objects are under the action of different forces like gravity, friction, wind and buoyancy force. In case forces other than gravity are absent like in vacuum, the objects are only acted by the gravitational pull towards earth. In such situation, acceleration due to gravity, denoted by g, is the only acceleration.

The acceleration due to gravity near the earth surface is nearly constant and equal to **9.8 m / s ^{2}**.

When only acceleration due to gravity is considered, neglecting other forces, each of the objects (feather and iron ball) starting from rest is accelerated at the same rate. Velocity of each bodies increases by 9.8 m/s at the end of every second. As such, the feather and the iron ball reach the surface at the same time and at the same velocity.

(d) Equation of Motion under constant acceleration (information only)

We consider a ball thrown upwards from ground with an initial speed of 30 m/s as in Figure 2.13.

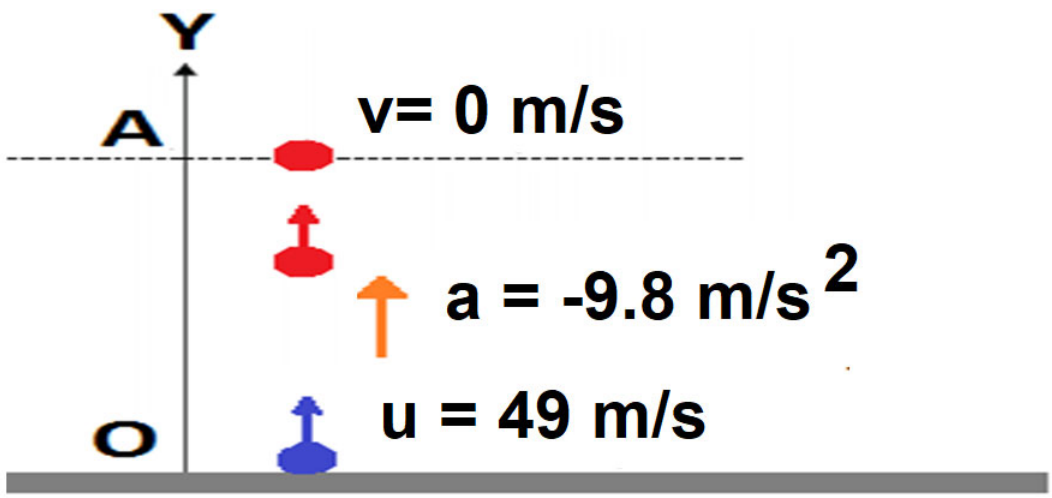


Figure 2.13: Velocity of ball is zero at maximum height

Figure 2.13: Velocity of ball is zero at maximum height

Find the

(i) time taken,t, to reach maximum height,s
(ii) maximum height ,s, reached
(iii) time taken, t_2, for the ball to drop back from the maximum height to the ground

$$v = u + at \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$
$$s = ut + \tfrac{1}{2}\, at^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$
$$v^2 = u^2 + 2as \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

(i) Using equation (1)

$$0 = 49 + (-9.8)\, t$$

Therefore time taken to reach maximum height, t = (-49)/(-9.8) = **5 sec**

(ii) Using equation (2)

Distance, $s = (49)(5) + \tfrac{1}{2}\,(-9.8)\,(5)^2$ = **122.5 m**

(iii) Using equation (2) again,

Distance, $\mathbf{s = ut + \tfrac{1}{2}\, at^2}$ = **122.5 m**

Distance, $s = 0 + \tfrac{1}{2}\,(9.8)\,(t)^2$ = **122.5 m**

$t = [(2)(122.5)/9.8]^{1/2}$

= **5 sec** (from maximum height to ground)

2.3 Kinetic of uniform circular motion

Uniform circular motion denotes motion of a particle along a circular arc or a circle with **constant speed.**

(a) Requirement of uniform circular motion

The uniform circular motion represents the basic form of rotational motion. Uniform circular motion involves continuous change in the direction of velocity without any change in its magnitude (v). A change in the direction of motion is a change in velocity and acceleration. It also means that a uniform circular motion is associated with an acceleration and hence force. Thus, uniform circular motion indicates "presence" of force. This is indicated in Figure 2.14.

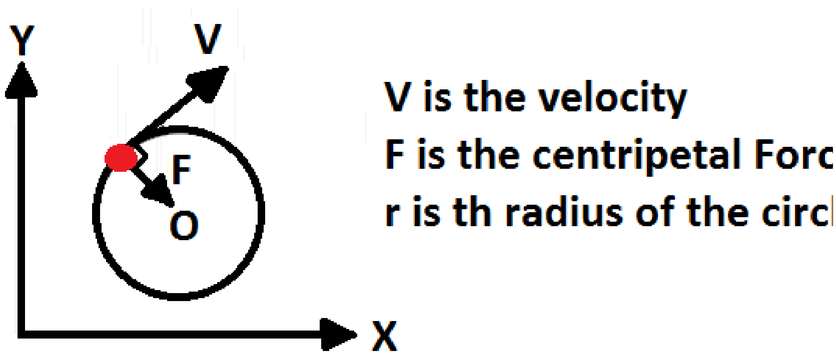


Figure 2.14: Force is perpendicular to the direction of the velocity

Uniform circular motion (UCM) needs a **force, which is always perpendicular to the direction of velocity.** Since the direction of velocity is continuously changing, the direction of force, being perpendicular to velocity, also changes continuously.

The direction of velocity along the circular trajectory is tangential. The perpendicular direction to the circular trajectory is, therefore, radial direction. It implies that **force (and hence acceleration) in uniform direction motion is radial.** For this reason, acceleration in UCM is recognized to seek center i.e. centripetal (seeking center).

The important features of the uniform circular motion are:

- The trajectory of uniform circular motion is circular arc or a circle and hence planar or two-dimensional.
- The speed (v) is a constant.
- The velocity (v) is variable and is tangential or circumferential in direction.
- Centripetal force (F) is required to maintain uniform circular motion against the natural tendency of the bodies to move linearly.
- Centripetal force (F) is variable and is radial in direction.

https://www.youtube.com/watch?v=SZj6DuB0vvo

(b) Equations of uniform circular motion

The magnitude of acceleration and the centripetal force, F, is derived from Newton's 2nd Law of motion. Referring to Figure 2.17,

$$\text{acceleration, } a = \frac{V^2}{r}$$

$$\text{Hence, centripetal force, } F = ma = m\frac{V^2}{r}$$

A particle under uniform circular motion covers a constant distance in completing a circular trajectory in one revolution, which is equal to the perimeters, of the circle.

Further, the particle covers the perimeter with a constant speed. It means that the particle travels the circular trajectory in a constant time given by its time, T, as:

$$\text{Circumference, } s = 2\pi r$$

$$\text{Period, } T = \frac{s}{V} = \frac{2\pi r}{V}$$

Example 2.9

A cyclist negotiated a curvature of 20 m with a speed of 20 m/s. What is the magnitude of his acceleration?

The acceleration of cyclist is the centripetal acceleration required to move the cyclist along a circular path.

Here, v = 20 m/s and r = 20 m

$$a = 20^2 / 20 \quad = 20 \text{ m} / \text{s}^2.$$

This example points to an interesting aspect of circular motion. The centripetal acceleration of the cyclist is almost two (2) times that of acceleration due to gravity (g = 9.8 m / s ^{2}).

2.4 Understand and differentiate centrifugal and centripetal forces

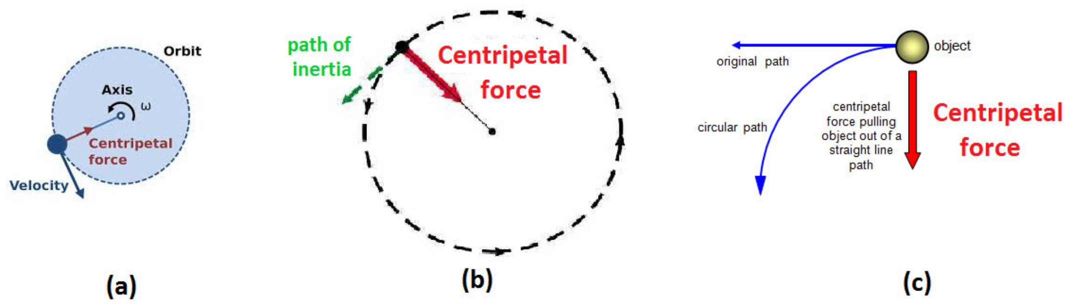


Figure 2.15: Example of Centripetal Force

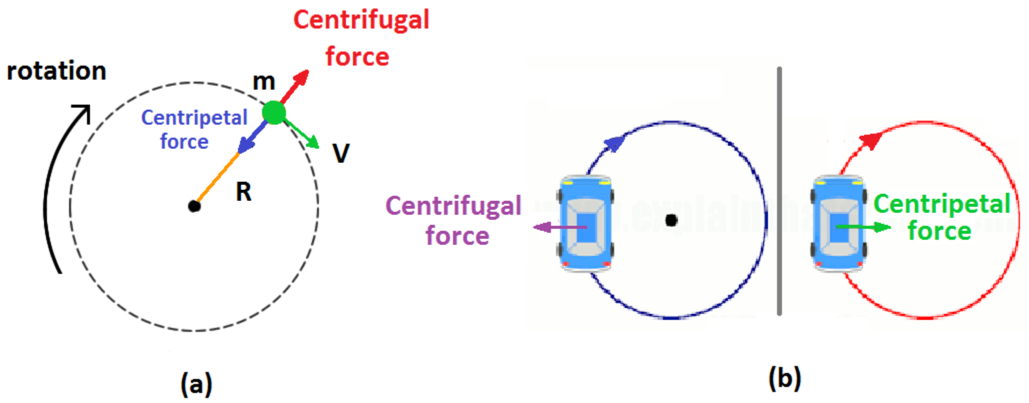


Figure 2.16: Centripetal and Centrifugal Forces

https://www.youtube.com/watch?v=9s1IRJbL2Co

Example 2.10

An aircraft with a mass of 80000 kg is in a steady turn of radius 300 m, flying at 800km/hr. Determine the centripetal force required to hold the aircraft in the turn.

m = 80000 kg, r = 300 m, v = 800 km/hr = (800) (1000)/3600 = 222.2 m/s, a= ?, F=?

$a = v^2 / r = (222.2)^2 / 300 = 164.58 \text{ m/s}^2$.

F = 80000 x 164.58 = 13166 KN

Example 2.11

How much centripetal force is needed to keep a 0.5 kg stone moving in a horizontal circle of radius 1 m at a velocity of 4 m/s?

$F = m v^2 / r = (0.5)(4^2) / 1 = 8N$.

2.5 Understand kinetic forces in an electric car in a circular motion

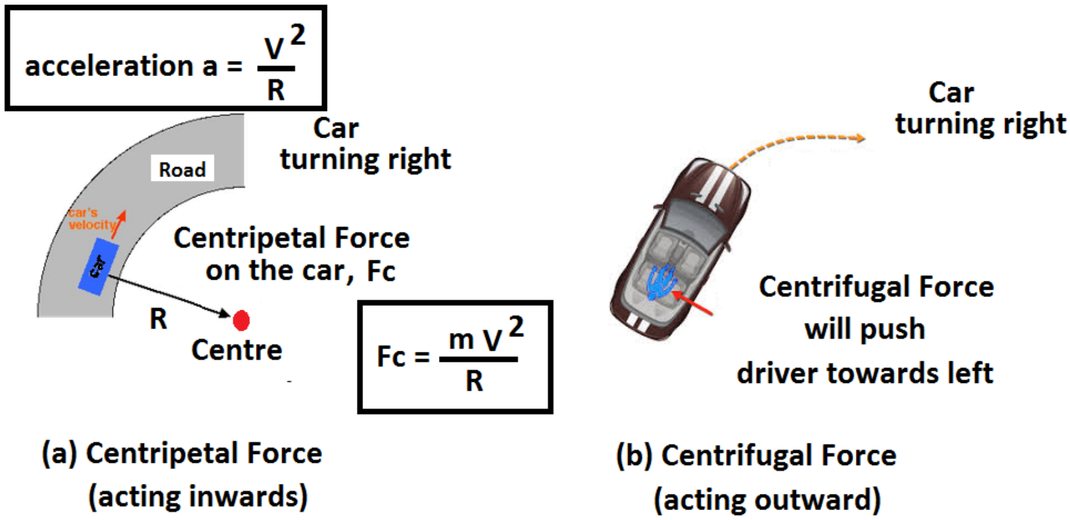


Figure 2.17: Centripetal and Centrifugal forces on a car executing a circular motion

Centrifugal force = Centripetal Force

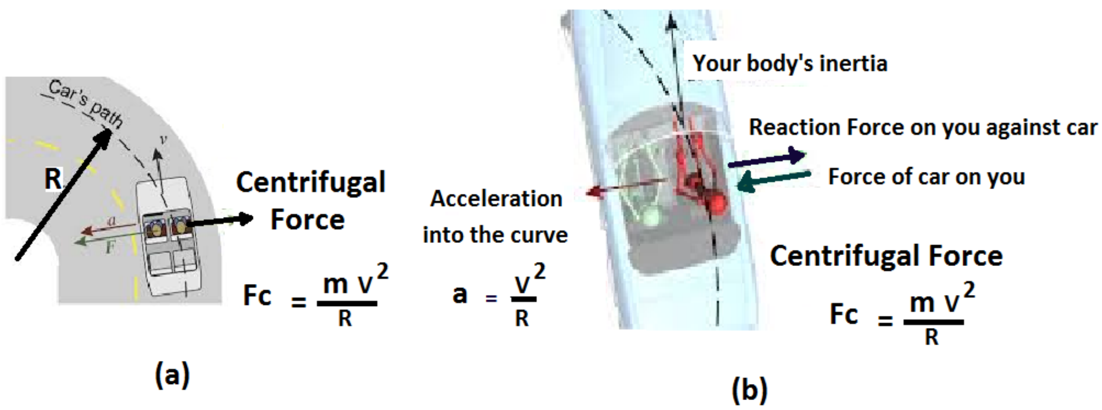


Figure 2.18: Centripetal and Centrifugal forces experienced by a car in a circular motion

Acknowledgement:

1. Images, pictures and photo from Google Images
2. Information from Google search and Wikipedia