Magnetism in-class assignment

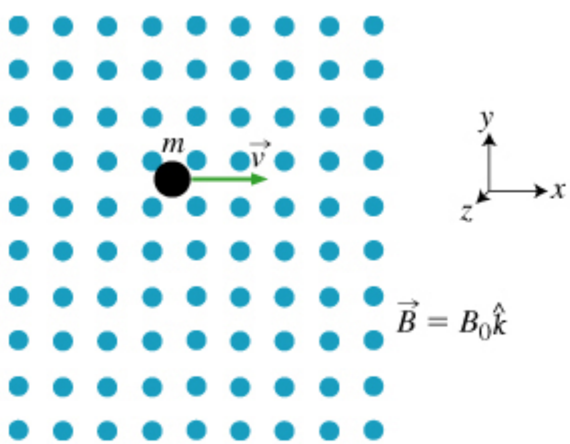
Due: 11:59pm on Sunday, August 14, 2022

You will receive no credit for items you complete after the assignment is due. Grading Policy

Charge Moving in a Cyclotron Orbit

Learning Goal:
To understand why charged particles move in circles perpendicular to a magnetic field and why the frequency is an invariant.

A particle of charge and mass moves in a region of space where there is a uniform magnetic field (i.e., a magnetic field of magnitude in the +*z* direction).



In this problem, neglect any forces on the particle other than the magnetic force.

Part A

At a given moment the particle is moving in the +*x* direction (and the magnetic field is always in the +*z* direction). If is positive, what is the direction of the force on the particle due to the magnetic field?

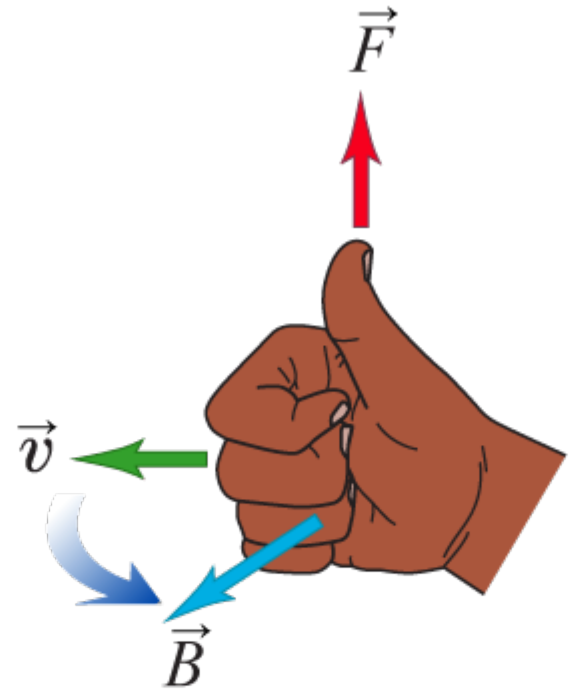
Hint 1. The right-hand rule for magnetic force

A charged particle moving through a region of magnetic field experiences a magnetic force. This force is directed perpendicular to both the velocity vector and the magnetic field vector at the point of interaction. The requirement that the force be perpendicular to both of the other vectors specifies the direction of the force to within an algebraic sign. This algebraic sign is determined by the right-hand rule. To employ the right-hand rule:

1. Straighten your fingers so that they point in the direction of the velocity.

2. Bend your fingers and orient your hand, so that your fingers curl in the direction of the magnetic field.
3. Your thumb should be maintained perpendicular to your fingers.

If the charge is positive, your thumb is now pointing in the direction of the force, , as shown in the figure.



If the charge is negative, the force is in the direction opposite your thumb.

ANSWER:

- *x* direction
- *x* direction
- *y* direction
- *y* direction
- *z* direction
- *z* direction

Correct

Part B

This force will cause the path of the particle to curve. Therefore, at a later time, the direction of the force will ____________.

ANSWER:

- have a component along the direction of motion
- remain perpendicular to the direction of motion
- have a component against the direction of motion
- first have a component along the direction of motion; then against it; then along it; etc.

Correct

Part C

The fact that the magnetic field generates a force perpendicular to the instantaneous velocity of the particle has implications for the work that the field does on the particle. As a consequence, if only the magnetic field acts on the particle, its kinetic energy will ____________.

ANSWER:

- increase over time
- decrease over time
- remain constant (selected)
- oscillate

Correct

Part D

If the resulting trajectory of the charged particle is a circle, what is , the angular frequency of the circular motion?

Express in terms of , , and .

Hint 1. How to approach the problem

This is a circular dynamics problem. Set to solve the problem. Note that angular speed and angular frequency are the same physical quantity.

Hint 2. Determine the magnetic force

If the particle is moving with velocity of magnitude , what is , the magnitude of the magnetic force on the particle?

Express in terms of and other given variables.

ANSWER:

=

Hint 3. Determine the acceleration of the particle

If the particle moves in a circular orbit of radius with uniform speed , what is the radial component of the particle's acceleration ?

Express in terms of and other given variables

ANSWER:

=

Hint 4. Express the angular speed in terms of the linear speed

What is the angular speed in terms of , the tangential speed?

Express in terms of and , the radius of the circle.

Hint 1. Relationship between and

The linear speed of a particle located a distance from the center of rotation is given by , where is the angular speed of the particle.

ANSWER:

=

ANSWER:

=

Correct

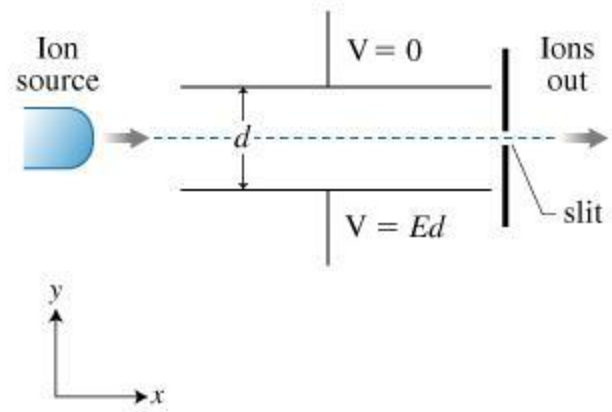
Note that this result for the frequency does not depend on the radius of the circle. Although it appeared in the equations of force and motion, it canceled out. This implies that the frequency (but not the linear speed) of the particle is invariant with orbit size.

The first particle accelerator built, the *cyclotron*, was based on the fact that the frequency of a charged particle orbiting in a uniform field is independent of the radius. In the cyclotron, radio frequency voltage is applied across a gap between the two sides of the conducting vacuum chamber in which the protons circulate owing to an external magnetic field. Particles in phase with this voltage are accelerated each time they cross the gap (because the field reverses while they make half a circle) and reach energies of millions of electron volts after several thousand round trips.

Electromagnetic Velocity Filter

When a particle with charge moves across a magnetic field of magnitude , it experiences a force to the side. If the proper electric field is simultaneously applied, the electric force on the charge will be in such a direction as to cancel the magnetic force with the result that the particle will travel in a straight line. The balancing condition provides a relationship involving the velocity of the particle. In this problem you will figure out how to arrange the fields to create this balance and then determine this relationship.

Part A



Consider the arrangement of ion source and electric field plates shown in the figure. The ion source sends particles with velocity along the positive *x* axis. They encounter electric field plates spaced a distance apart that generate a uniform electric field of magnitude in the +*y* direction. To cancel the resulting electric force with a magnetic force, a magnetic field (not shown) must be added in which direction? Using the right-hand rule, you can see that the positive *z* axis is directed out of the screen.

Choose the direction of .

Hint 1. Method for determining direction

Assume a sign for the charge. Since both the electric force and magnetic force depend on , in particular, they also depend on its sign. So the sign doesn't matter here. Apply the right-hand rule to the equation for the magnetic force, .

Hint 2. Right-hand rule

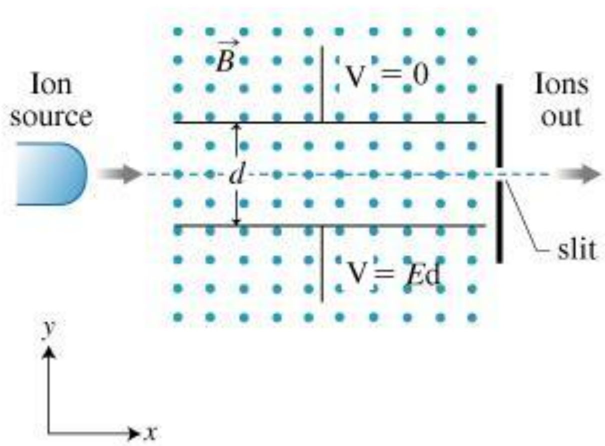
Curl the fingers of your right hand from the first vector to the second in the product. Your outstretched thumb then points in the direction of the cross-product vector.

ANSWER:

Correct

Part B

Now find the magnitude of the magnetic field that will cause the charge to travel in a straight line under the combined action of electric and magnetic fields.



Express the magnetic field that will just balance the applied electric field in terms of some or all of the variables , , and .

Hint 1. Find the magnetic force

What is , the magnitude of the force due to a magnetic field (with a magnitude of) interacting with a charge moving at velocity (a speed of)?

Express in terms of some or all of the variables , , and .

ANSWER:

=

Hint 2. Find the force due to the electric field

What is , the magnitude of a force on a charge due to an applied electric field ?

Express in terms of one or both of the variables and .

ANSWER:

=

ANSWER:

=

Correct

Part C

It may seem strange that the selected velocity does not depend on either the mass or the charge of the particle. (For example, would the velocity of a neutral particle be selected by passage through this device?) The explanation of this is that the mass and the charge control the resolution of the device--particles with the wrong velocity will be accelerated away from the straight line and will not pass through the exit slit. If the acceleration depends strongly on the velocity, then particles with just slightly wrong velocities will feel a substantial transverse acceleration and will not exit the selector. Because the acceleration depends on the mass and charge, these influence the sharpness (resolution) of the transmitted particles.

Assume that you want a velocity selector that will allow particles of velocity to pass straight through without deflection while also providing the best possible velocity resolution. You set the electric and magnetic fields to select the velocity . To obtain the best possible velocity resolution (the narrowest distribution of velocities of the transmitted particles) you would want to use particles with ___________.

Assume that the selector is short enough so that particles that move away from the axis do not have time to come back to it.

Hint 1. Use Newton's law

If the velocity is "wrong" the forces won't balance and the resulting transverse force will cause a transverse acceleration. Use to determine how this acceleration will depend on and . You want particles with the incorrect velocity to have the maximum possible deviation in the *y* direction so that they will not go through a slit placed at the right end. This means that the acceleration should be maximum.

ANSWER:

- both and large
- large and small
- small and large
- both and small

Correct

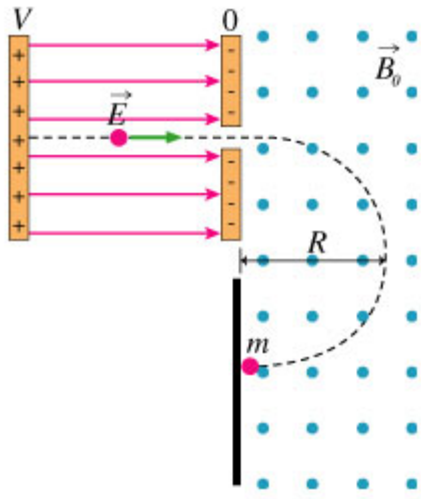
You want particles with the incorrect velocity to have the maximum possible deviation in the *y* direction so that they will not go through a slit placed at the right end. The deviation will be maximum when the acceleration is maximum. The acceleration is directly proportional to and inversely proportional to :

.

So for maximum deviation, should be large and small.

Mass Spectrometer

J. J. Thomson is best known for his discoveries about the nature of cathode rays. Another important contribution of his was the invention, together with one of his students, of the mass spectrometer. The ratio of mass to (positive) charge of an ion may be accurately determined in a mass spectrometer. In essence, the spectrometer consists of two regions: one that accelerates the ion through a potential difference and a second that measures its radius of curvature in a perpendicular



magnetic field as shown in .

The ion begins at potential and is accelerated toward zero potential. When the particle exits the region with the electric field it will have obtained a speed .

Part A

With what speed does the ion exit the acceleration region?

Find the speed in terms of , , , and any constants.

Hint 1. Suggested general method

Perhaps the easiest method to use for solving this problem is conservation of energy.

Hint 2. Initial energy

Find the initial total mechanical energy (which includes electric potential energy) of the particle.

Express in terms of , , , and any constants.

ANSWER:

=

Hint 3. Final energy

Find , the total mechanical energy of the ion as it exits the region with the electric field (i.e., when it reaches the region of zero electric potential and enters the magnet).

Express in terms of , , and any needed constants.

ANSWER:

=

ANSWER:

=

Correct

Part B

After being accelerated, the particle enters a uniform magnetic field of strength and travels in a circle of radius (determined by observing where it hits on a screen--as shown in the figure). The results of this experiment allow one to find in terms of the experimentally measured quantities such as the particle radius, the magnetic field, and the applied voltage.
What is ?

Express in terms of , , , and any necessary constants.

Hint 1. Cyclotron frequency

Find the cyclotron frequency , which is the (angular) frequency of the orbital motion of the ion in the magnetic field. There is a skill builder problem on this if you need help.

Express the cyclotron frequency in terms of , , and .

Hint 1. General method to find

The motion of a charged particle with velocity perpendicular to a uniform magnetic field yields uniform circular motion in which the magnetic force is equal to the force from the centripetal acceleration. Balancing these forces will allow one to find .

ANSWER:

=

Hint 2. Relationship of and

What is the relationship between the cyclotron frequency and the kinematic variables and ? (This is the definition of angular speed.)

Express in terms of and .

ANSWER:

=

Hint 3. Putting it all together

Eliminate from the two equations from the previous hints, and then eliminate using the result of Part A. The result should give in terms of the experimentally measured parameters.

ANSWER:

=

Correct

By sending atoms of various elements through a mass spectrometer, Thomson's student, Francis Aston, discovered that some elements actually contained atoms with several different masses. Atoms of the same element with different masses can only be explained by the existence of a third subatomic particle in addition to protons and electrons: the neutron.

Exercise 27.3

In a 1.35 magnetic field directed vertically upward, a particle having a charge of magnitude 8.40 and initially moving northward at 4.76 is deflected toward the east.

Part A

What is the sign of the charge of this particle?

ANSWER:

- ○ The charge is negative.
- ◉ The charge is positive.

Correct

Part B

Find the magnetic force on the particle.

ANSWER:

= 5.40×10^{-2}

Correct

Cyclotron Motion

A charged particle enters a uniform magnetic field with a velocity at right angles to the field. It moves in a circle with period .

Part A

If a second particle, with the same electric charge but ten times as massive, enters the field with the same velocity , what is its period?

Hint 1. How to approach the problem

When a charged particle enters a region of uniform magnetic field with an initial velocity perpendicular to the field, the particle is acted on by the magnetic force and follows a circular path. The radius of the circular path can be found by applying Newton's laws, while the particle's period, which is related to its angular speed , can be found from the relation , where is the particle's speed.

To solve this problem you must use proportional reasoning to find a relation among the variables.

- Find the simplest equation that contains the variables and other known quantities from the problem.
- Write this equation twice, once to describe the motion of the lighter particle, and again to describe the motion of the heavier particle.
- You need to write each equation so that all the constants are on one side and the variables are on the other. Since the unknown variable is the period in this problem, you want to write your equations in the form .
- To finish, you need to compare the two cases presented in the problem. For this question you should compare the periods of the two particles.

Hint 2. Find an expression for the period of a particle in a magnetic field

Which of the following expressions correctly gives the period of a particle moving in a magnetic field with an initial velocity perpendicular to the field? Let , , and denote the mass, the magnitude of the charge, and the speed of the particle, while is the magnitude of the magnetic field.

Hint 1. Period and angular speed

The period of a particle that moves along a circular path with angular speed is given by

.

Hint 2. Angular and linear speed

The relation between the angular speed and the linear speed of a particle moving along a circular path of radius is .

Hint 3. Radius of a circular orbit in a magnetic field

When a charged particle of mass and charge of magnitude enters a region of uniform magnetic field of magnitude with an initial velocity perpendicular to the field, the particle is acted on by the magnetic force and follows a circular path. The radius of the circular path is given by

,

where is the speed of the particle.

ANSWER:

- ( )
- (•)
- ( )
- ( )

ANSWER:

- ( )
- ( )
- ( )
- (•)

Correct

The period of revolution of a particle moving in a magnetic field is proportional to its mass. In other words, for a given initial velocity, more-massive particles take longer to complete one revolution. This is because the radius of their orbit is larger than those of less-massive particles, resulting in a longer trajectory.

Part B

If the frequency of revolution (the number of revolutions per unit time) of the lighter particle is , what is the frequency of revolution of the more massive particle?

Hint 1. How to approach the problem

Again, to solve this problem use proportional reasoning to find a relation between frequency and period .

- Find the simplest equation that contains these variables and other known quantities from the problem.
- Write this equation twice, once to describe the frequency and period of the lighter particle and again to relate the frequency and period of the

heavier particle.

- You need to write each equation so that all the constants are on one side and your variables are on the other. Since the variable is in this problem, you should write your equations in the form .
- To finish, you need compare the two cases presented in the problem. For this question you should compare the frequencies of revolution of the two particles.

Hint 2. Relationship between frequency and period

Recall that the frequency is the multiplicative inverse of the period :

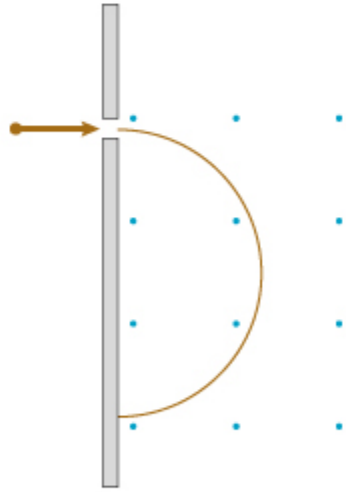
.

ANSWER:

Correct

Since frequency is always the inverse of period, the frequency of revolution of a particle moving in a magnetic field is inversely proportional to its mass. In other words, for a given initial velocity, more-massive particles perform a smaller number of revolutions per unit time than less-massive particles.

Mass Spectrometer Conceptual Questions

A beam of singly ionized uranium atoms (and) is injected into the mass spectrometer shown in the figure. The ions all have the same velocities and charges. The ions follow the trajectory illustrated  .

Part A

Will the ions strike the collecting plate above, below, or at the same location as the ions?

Hint 1. Description of a mass spectrometer

In a mass spectrometer, charged particles are injected into a region of uniform magnetic field (all with the same speed), where they travel along circular trajectories and, in this example, are collected after completing one-half of a complete circular orbit. If different mass isotopes are injected, they will trace different paths and be collected at different locations.

Hint 2. Dependence of radius on other parameters

Once inside the spectrometer, the particles are acted on by a magnetic force and they trace circular paths. Applying Newton's second law to their motion results in the equation

.

In our scenario, the velocity and field vectors are perpendicular, so . Also, since the particle moves along a circular path, the acceleration must equal the expression for centripetal acceleration:

.

This can be solved for the radius to yield

.

The diameter of their path, which is the distance from the opening of the spectrometer to their "landing spot" on the collecting plate is therefore

.

Thus, the position on the collecting plate is directly proportional to the mass of the particle.

ANSWER:

- (selected) above
- below
- same location

Correct

Part B

If the magnetic field strength in the spectrometer is increased, will the spacing between where the and ions strike increase, decrease, or stay the same?

Hint 1. Spacing

The relationship
describes the diameter of any one particle's path in terms of the other parameters in the system. If two different isotopes are injected, the difference in their diameters is the spacing that appears on the collecting plate:

,

,

.

Thus, the spacing is proportional to the mass difference and velocity of the isotopes and inversely proportional to the field strength in the spectrometer.

ANSWER:

- increase
- decrease
- stay the same

Correct

Part C

If the initial velocity of the ions as they enter the spectrometer is increased, will the spacing between where the and ions strike increase, decrease, or stay the same?

ANSWER:

- increase
- decrease
- stay the same

Correct

Part D

If a beam of singly ionized carbon atoms (and) is injected into the same mass spectrometer as the uranium ions (with the same speed), will the spacing between the carbon ions be greater than, less than, or equal to the spacing between the uranium ions?

Hint 1. Relating spacing to mass

The relationship
describes the diameter of any one particle's path in terms of the other parameters in the system. If two different isotopes are injected, the difference in their diameters is the spacing that appears on the collecting plate:

,

,

.

Notice that the spacing is proportional to . The absolute mass of the elements has no bearing on the spacing.

ANSWER:

The spacing of atoms will be