# SOLUTIONS/ MARKING SCHEME

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**SINGAPORE POLYTECHNIC**
**2019 / 2020 Semester 2 Exam**

**Module Name:** **Engineering Mathematics II**
**Module Code:** **MS2216/MS4216/MS6216**
**Course:** **DCHE/DASE/DCPE/DEB/DEEE/DES/DESM/DARE/DBEN/DCEP/DME/DMRO**
**Year: 2 FT** **Page 1 of 6**

<table>
<tr><th>No.</th><th>SOLUTION</th><th>TOTAL MARKS</th></tr>
<tr><td>A</td><td>b, b, c, d, a</td><td><b>10</b></td></tr>
<tr><td>B1a</td><td>

Let $u = 2x^2$ → $\frac{du}{dx} = 4x$ → $\frac{1}{4}du = xdx$

$$\int x\sin(2x^2)\,dx = \frac{1}{4}\int \sin u\,du = \frac{1}{4}(-\cos u) + C = -\frac{1}{4}\cos(2x^2) + C$$

</td><td rowspan="2"><b>10</b></td></tr>
<tr><td>B1b</td><td>

$$\int_0^{\pi} 2t\cos t\,dt$$

$$= \left[2t\sin t + 2\cos t\right]_0^{\pi}$$

$$= \left[2\pi\sin\pi + 2\cos\pi\right] - \left[(0)\sin 0 + (2)\cos 0\right]$$

$$= -2 - 2$$

$$= -4$$

$$\begin{array}{ccc} u & & dv \\ 2t & \searrow + & \cos t \\ 2 & \searrow - & \sin t \\ 0 & & -\cos t \end{array}$$

Alternate solution:

$$\int_0^{\pi} 2t\cos t\,dt = \left[2t\sin t\right]_0^{\pi} - \int_0^{\pi} 2\sin t\,dt$$

$$= \left[2\cos t\right]_0^{\pi} = -2 - 2 = -4$$

</td></tr>
<tr><td>B2</td><td>

$$h = \frac{3-0}{6} = \frac{1}{2}$$

<table>
<tr><td></td><td>$x_0$</td><td>$x_1$</td><td>$x_2$</td><td>$x_3$</td><td>$x_4$</td><td>$x_5$</td><td>$x_6$</td></tr>
<tr><td></td><td>0</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td></tr>
<tr><td>$f(x)$</td><td>1</td><td>1.118</td><td>1.414</td><td>1.803</td><td>2.236</td><td>2.693</td><td>3.162</td></tr>
</table>

Simpson's rule formula gives

$$A \approx \frac{1}{3}h(y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4))$$

$$= \frac{1}{3}\left(\frac{1}{2}\right)(1 + 3.162 + 4(1.118 + 1.803 + 2.693) + 2(1.414 + 2.236))$$

$=5.653 \approx 5.65$ (correct to 2 dp)

</td><td><b>10</b></td></tr>
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|---|---|---|
| B3a | $T=2\pi \to \omega_0=\frac{2\pi}{T}=1$ → $a_0=\frac{1}{T}\int_0^{T/2} f(t)\,dt=\frac{2}{2\pi}\int_0^{\pi/2}4\,dt=2$ | 10 |
| B3b | $a_n=\frac{8}{n\pi}\sin\left(\frac{n\pi}{2}\right)$ → $a_1=\frac{8}{\pi}$, $a_2=0$, $a_3=-\frac{8}{3\pi}$ | |
| B3c | $f(t)=a_0+a_1\cos\omega t+a_2\cos 2\omega t+a_3\cos 3\omega t+\cdots$<br>$=2+\frac{8}{\pi}\cos\omega t-\frac{8}{3\pi}\cos 3\omega t+\cdots$ | |
| B4a | $\frac{dy}{dx}=2xy$ → $\frac{dy}{y}=2xdx$ → $\int\frac{1}{y}dy=\int 2xdx$<br>$\ln\lvert y\rvert=x^2+C$ or $y=Ae^{x^2}$ | 10 |
| B4b | (i) $\int_4^{12}(1+2x)^2dx=\frac{1}{2}\left(\frac{(1+2x)^3}{3}\right)_4^{12}=\frac{1}{6}\left(25^3-9^3\right)=\frac{7448}{3}$<br>(ii) $y_{avg}=\frac{1}{12-4}\int_4^{12}(1+2x)^2dx=\frac{7448}{8(3)}=310.33$<br>Alternate way for (i):<br>$\int_4^{12}(1+2x)^2dx=\int_4^{12}(1+4x+4x^2)\,dx=\left(x+2x^2+4\frac{x^3}{3}\right)_4^{12}$<br>$=\left([12-4]+2[12^2-4^2]+\frac{4}{3}[12^3-4^3]\right)=\frac{7448}{3}$ | |
| B5a | $\mathcal{L}\left\{t^3-5\cos 3t\right\}=\frac{3!}{s^{3+1}}-5\cdot\frac{s}{s^2+3^2}=\frac{6}{s^4}-\frac{5s}{(s^2+9)}$ | 10 |
| B5b | $\mathcal{L}\left\{4t\cos 2t+3e^{2t}\right\}=\frac{4\left(s^2-2^2\right)}{(s^2+2^2)^2}+\frac{3}{s-2}=\frac{4\left(s^2-4\right)}{(s^2+4)^2}+\frac{3}{s-2}$ | |
| B5c | $\mathcal{L}\left\{\sin\pi t\right\}=\frac{\pi}{s^2+\pi^2}$<br>$\mathcal{L}\left\{e^{-t}\sin\pi t\right\}=\left.\frac{\pi}{s^2+\pi^2}\right\rvert_{s\to s+1}=\frac{\pi}{(s+1)^2+\pi^2}$ | |

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| B6a | $\mathcal{L}^{-1}\left\{\frac{3}{4s}+\frac{5}{s^4}-\frac{5s}{s^2+4}\right\}=\mathcal{L}^{-1}\left\{\frac{3}{4}\cdot\frac{1}{s}+\frac{5}{3!}\cdot\frac{3!}{s^{3+1}}-5\cdot\frac{s}{s^2+2^2}\right\}$<br>$=\frac{3}{4}+\frac{5}{6}t^3-5\cos 2t$ | 10 |
| B6b | $\frac{s-3}{(s-1)(s-2)}=\frac{A}{s-1}+\frac{B}{s-2}$<br>$A=\left.\frac{s-3}{s-2}\right\rvert_{s=1}=2$, $B=\left.\frac{s-3}{s-1}\right\rvert_{s=2}=-1$<br>$\mathcal{L}^{-1}\left\{\frac{s-3}{(s-1)(s-2)}\right\}=\mathcal{L}^{-1}\left\{2\cdot\frac{1}{s-1}-\frac{1}{s-2}\right\}=2e^t-e^{2t}$ | |
| B7a | $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=0$<br>Aux. equation is: $\lambda^2+4\lambda+3=0$<br>Thus: $(\lambda+1)(\lambda+3)=0\Rightarrow\lambda=-1,-3$<br>$\therefore$ the general solution is: $y=Ae^{-x}+Be^{-3x}$ | 10 |
| B7b | $y=Ae^{-x}+Be^{-3x}$ → $\frac{dy}{dx}=y'=-Ae^{-x}-3Be^{-3x}$<br>given $y(0)=2$, i.e. $2=A+B$ --- (1)<br>given $y'(0)=-4$, i.e. $-4=-A-3B$ -- (2)<br>hence $A=1$, and $B=1$<br>Thus the particular solution is: $y=e^{-x}+e^{-3x}$ | |
| C1 | Let $x$ = length, $y$ = width and $z$ = height.<br>Volume $V=xyz$<br>Then $\ln V=\ln x+\ln y+\ln z$<br>And $\frac{\partial(\ln V)}{\partial x}=\frac{1}{x}$, $\frac{\partial(\ln V)}{\partial y}=\frac{1}{y}$ *and* $\frac{\partial(\ln V)}{\partial z}=\frac{1}{z}$ | |

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|  | Thus<br>$\frac{\Delta V}{V} \approx \frac{\partial(\ln V)}{\partial x}\Delta x + \frac{\partial(\ln V)}{\partial y}\Delta y + \frac{\partial(\ln V)}{\partial z}\Delta z = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$<br>The estimated percentage error for the volume is given by<br>$\frac{\Delta V}{V}\cdot 100\% \approx \frac{\Delta x}{x}\cdot 100\% + \frac{\Delta y}{y}\cdot 100\% + \frac{\Delta z}{z}\cdot 100\%$<br>$= 1\% + 1\% + 1\% = 3\%$<br>The estimated percentage error for the volume is approximately 3% too large.<br>**Alternate solution:**<br>Let $x$ = length, $y$ = width and $z$ = height.<br>Volume $V = xyz$<br>And $\frac{\partial V}{\partial x} = yz,\ \frac{\partial V}{\partial y} = xz\ and\ \frac{\partial V}{\partial z} = xy$<br>Thus<br>$\frac{\Delta V}{V} \approx \frac{\partial V}{\partial x}\frac{\Delta x}{V} + \frac{\partial V}{\partial y}\frac{\Delta y}{V} + \frac{\partial V}{\partial z}\frac{\Delta z}{V}$<br>$= yz\frac{\Delta x}{xyz} + xz\frac{\Delta y}{xyz} + xy\frac{\Delta z}{xyz} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$<br>The estimated percentage error for the volume is given by<br>$\frac{\Delta V}{V}\times 100\% \approx \frac{\Delta x}{x}\times 100\% + \frac{\Delta y}{y}\times 100\% + \frac{\Delta z}{z}\times 100\%$<br>$= 1\% + 1\% + 1\%$<br>$= 3\%$<br>The estimated percentage error for the volume is approximately 3% too large | **11**<br>**(11)** |

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| C2a | Let $T$ be the temperature of the body<br>According to Newton's law of cooling,<br>$\frac{dT}{dt} = -k\left(T - T_s\right) = -k(T-20)$<br>$\int \frac{1}{T-20}\,dT = \int -k\,dt$<br>$\ln\left\|T-20\right\| = -kt + C$<br>$T(t) = 20 + e^{-kt+C} = 20 + Ae^{-kt}$<br>We know that $T(10) = 75, T(20) = 50$, hence<br>$75 = 20 + Ae^{-10k} \quad \rightarrow \quad 55 = Ae^{-10k} \quad (1)$<br>$50 = 20 + Ae^{-20k} \quad \rightarrow \quad 30 = Ae^{-20k} \quad (2)$<br>Solving for $k$ and $A$ by dividing (1) by (2):<br>$\frac{55}{30} = e^{10k} \quad \rightarrow \quad k = \frac{1}{10}\ln\left(\frac{55}{30}\right) = 0.06$<br>Sub into (1): $55 = Ae^{-10k} \quad \rightarrow \quad A = 55e^{10k} = 100.83$<br>$\therefore T(t) = 20 + 100.83e^{-0.06t}$ | |
| C2b | $T(0) = 20 + 100.83e^{-0.06(0)} = 120.83°\text{C}$ | |
| C2c | Now the rate of change of temperature has an additional term (−5 °C/min)<br>$\frac{dT}{dt} = -k\left(T - T_s\right) - 5$ | **14** |
| C3a | $\frac{d^2x}{dt^2} + 4x = 3\cos\left(2t\right)$<br>$x(0) = 0.5$ and $x'(0) = 0$ | |

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| C3b | Let $X = \mathcal{L}\{x(t)\}$<br>$s^2X - sx(0) - x'(0) + 4X = \frac{3s}{s^2+4}$<br>since $x(0) = \frac{1}{2}$ and $x'(0) = 0$,<br>$s^2X - \frac{1}{2}s + 4X = \frac{3s}{s^2+4}$<br>$\left(s^2+4\right)X = \frac{3s}{s^2+4} + \frac{1}{2}s$<br>$X = \frac{3s}{(s^2+4)^2} + \frac{s}{2\left(s^2+4\right)}$<br>$x(t) = \mathcal{L}^{-1}\left\{\frac{3}{4}\frac{(2)(2)s}{(s^2+2^2)^2} + \frac{1}{2}\frac{s}{s^2+2^2}\right\}$<br>$= \frac{3}{4}t\sin(2t) + \frac{1}{2}\cos(2t)$ | |
| C3c | When $t = 1$ sec,<br>$x = \frac{3}{4}\sin(2) + \frac{1}{2}\cos(2) = 0.47\text{m}$<br>The mass will be 0.47 m below the equilibrium position. | 15 |