SOLUTIONS

SINGAPORE POLYTECHNIC
2020 / 2021 Semester 1 Exam

Module Name: **Engineering Mathematics II**

Module Code: **MS2216/MS4216/MS6216**

Course: **DCHE/DASE/DCPE/DEB/DEEE/DARE/DBEN/DCEP/DME/DMRO**

Year: 2 FT

No.	SOLUTION	Total
A	b, a, c, c, d	10
B1a	$f(x,y)=\pi x\sin\left(y^2\right)$ $f_x=\pi\sin\left(y^2\right)$ $\quad$ $f_y=2\pi xy\cos\left(y^2\right)$	10
B1b	$g(x,y)=(x^2-2)e^{4y}$ $g_x=2xe^{4y}$ At (1, 0), $g_x(1,0)=2(1)e^{4(0)}=2$	
B2a	$u=x^5 \rightarrow \dfrac{du}{dx}=5x^4 \rightarrow 2du=10x^4dx$ $\int 10x^4e^{x^5}\,dx=2\int e^u du=2e^u+C=2e^{x^5}+C$	10
B2b	$\int_1^2 t\ln t\,dt$ $\quad$ Let $u=\ln t\rightarrow du=\dfrac{1}{t}dt$ $=\left(\dfrac{t^2}{2}\ln t\right)_1^2-\int_1^2\dfrac{1}{t}\cdot\dfrac{t^2}{2}dt$ $\quad$ $dv=t\,dt\rightarrow v=\int t\,dt=\dfrac{t^2}{2}$ $=2\ln 2-0-\dfrac{1}{4}\left[t^2\right]_1^2$ $=2\ln 2-\dfrac{1}{4}(4-1)=2\ln 2-\dfrac{3}{4}\quad or\quad 0.64$	
B3a	$f(t)$ is neither even nor odd	10
B3b	$T=2\rightarrow\omega_0=\dfrac{2\pi}{T}=\pi$ $a_0=\dfrac{1}{T}\int_{-1}^{1}f(t)\,dt=\dfrac{1}{2}\int_{-1}^{1}(t+1)\,dt=\dfrac{1}{2}\left(\dfrac{(t+1)^2}{2}\right)_{-1}^{1}=1$	
B3c	$b_n=\dfrac{-2}{n\pi}\cos(n\pi)$ $b_1=\dfrac{-2}{\pi}\cos\pi=\dfrac{2}{\pi},\quad b_2=-\dfrac{1}{\pi},\quad b_3=\dfrac{2}{3\pi}$	
B3d	$f(t)=a_0+b_1\sin\pi t+b_2\sin 2\pi t+b_3\sin 3\pi t+\cdots$ $=1+\dfrac{2}{\pi}\sin\pi t-\dfrac{1}{\pi}\sin 2\pi t+\dfrac{2}{3\pi}\sin 3\pi t+\cdots$	

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B4a	$\frac{dy}{dx}+\frac{2}{x}y=\frac{\sin(3x)}{x^2}$ Integrating Factor $\mu(x)=e^{\int \frac{2}{x}dx}=e^{2\ln x}=x^2$ $x^2\frac{dy}{dx}+2xy=\sin(3x) \quad \rightarrow \quad \frac{d}{dx}\left(x^2 y\right)=\sin(3x)$ $x^2 y=\int \sin(3x)\,dx=-\frac{1}{3}\cos(3x)+C \quad or \quad y=\frac{1}{x^2}\left[-\frac{1}{3}\cos(3x)+C\right]$	10
B4b	$x^2 y=-\frac{1}{3}\cos(3x)+C$ given $y(\pi)=\frac{1}{\pi^2}$: $\pi^2\cdot\frac{1}{\pi^2}=-\frac{1}{3}\cos(3\pi)+C \quad \rightarrow \quad C=\frac{2}{3}$ Thus, the particular solution is: $y=\frac{1}{3x^2}\left[2-\cos(3x)\right]$	
B5a	$\mathcal{L}\left\{\frac{1}{2}-3t^2+e^{-t}\right\}=\frac{1}{2}\cdot\frac{1}{s}-3\cdot\frac{2!}{s^{2+1}}+\frac{1}{s-(-1)}=\frac{1}{2s}-\frac{6}{s^3}+\frac{1}{s+1}$	10
B5b	$\sin(2t+\pi)=\sin 2t\cos\pi+\cos 2t\sin\pi=-\sin 2t$ $\mathcal{L}\left\{\sin(2t+\pi)\right\}=-\frac{2}{s^2+2^2}=-\frac{2}{s^2+4}$	
B5c	$\mathcal{L}\left\{t\cos 2t\right\}=\frac{s^2-2^2}{\left(s^2+2^2\right)^2}=\frac{s^2-4}{\left(s^2+4\right)^2}$ $\mathcal{L}\left\{e^{3t}\,t\cos 2t\right\}=\left.\frac{s^2-4}{\left(s^2+4\right)^2}\right\rvert_{s\to s-3}=\frac{(s-3)^2-4}{\left((s-3)^2+4\right)^2}$	
B6a	$\mathcal{L}^{-1}\left\{\frac{2}{s-3}+\frac{2}{s^3}+\frac{1}{s^2+4}\right\}=\mathcal{L}^{-1}\left\{2\cdot\frac{1}{s-3}+\frac{2!}{s^{2+1}}+\frac{1}{2}\cdot\frac{2}{s^2+2^2}\right\}=2e^{3t}+t^2+\frac{1}{2}\sin 2t$	10
B6b	$\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\}=\mathcal{L}^{-1}\left\{\frac{s}{s^2+3^2}\right\}=\cos 3t$ $\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+9}\right\}=\mathcal{L}^{-1}\left\{\left.\frac{s}{s^2+9}\right\rvert_{s\to s+1}\right\}=e^{-t}\mathcal{L}^{-1}\left\{\frac{s}{s^2+3^2}\right\}=e^{-t}\cos 3t$	

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<table>
<tr><th>No.</th><th>SOLUTION</th><th>Total</th></tr>
<tr><td>B7</td><td>

$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=0$

a Aux. equation is: $\lambda^2+2\lambda-3=0$

Thus: $(\lambda-1)(\lambda+3)=0 \Rightarrow \lambda=1,-3$

b $\therefore$ the general solution is: $y=Ae^{x}+Be^{-3x}$

c $y(x)=Ae^{x}+Be^{-3x} \rightarrow y'(x)=Ae^{x}-3Be^{-3x}$

given $y(0)=3$, i.e. $3=A+B$ --- (1)

given $y'(0)=1$, i.e. $1=A-3B$ -- (2)

hence $A=\frac{5}{2}$, and $B=\frac{1}{2}$

Thus the particular solution is: $y(x)=\frac{5}{2}e^{x}+\frac{1}{2}e^{-3x}$

</td><td>10</td></tr>
<tr><td>C1a</td><td>

t (s)	0	0.3	0.6	0.9	1.2	1.5	1.8
v (volts)	1	0.9828	0.9306	0.8409	0.7071	0.5087	0
v^2	1	0.9659	0.8660	0.7071	0.5000	0.2588	0

$h=\frac{1.8-0}{6}=0.3$

Simpson's rule formula gives

$\int_0^{1.8} v^2 dt \approx \frac{1}{3}h(y_0+y_6+4(y_1+y_3+y_5)+2(y_2+y_4))$

$=\frac{1}{3}(0.3)(1+0+4(0.9659+0.7071+0.2588)+2(0.8660+0.5000))$

$=1.1459$

</td><td>11</td></tr>
<tr><td>C1b</td><td>

$v_{rms}=\sqrt{\frac{1}{1.8-0}\int_0^{1.8} v^2 dt}=\sqrt{\frac{1.1459}{1.8}}=0.79789\approx 0.80$ volts

</td><td></td></tr>
<tr><td>C2a</td><td>

Let T be the temperature of the body

According to Newton's law of cooling,

$\frac{dT}{dt}=-k\left(T-T_s\right)=-k(T-5)$

</td><td>14</td></tr>
</table>

No.	SOLUTION	Total
C2b	$\int \frac{1}{T-5}\,dT = \int -k\,dt$ $\ln\lvert T-5\rvert = -kt + C$ $T(t) = 5 + e^{-kt+C} = 5 + Ae^{-kt}$ We know that $T(0)=100, T(10)=60$, hence $100 = 5 + Ae^{0} \quad \to \quad 95 = A$ $60 = 5 + Ae^{-10k} \quad \to \quad 55 = 95e^{-10k}$ $\frac{55}{95} = e^{-10k} \quad \to \quad k = -\frac{1}{10}\ln\left(\frac{55}{95}\right) = 0.0547$ $\therefore T(t) = 5 + 95e^{-0.0547t}$	
C2c	$T(t) = 5 + 95e^{-0.0547t}$ $T(\tau) = 20^\circ\text{C}$ $20 = 5 + 95e^{-0.0547\tau} \quad \to \quad 15 = 95e^{-0.0547\tau}$ $\frac{15}{95} = e^{-0.0547\tau} \quad \to \quad \tau = \frac{1}{-0.0547}\ln\left(\frac{15}{95}\right) = 33.745\ \text{min}$ i.e. Jim need to wait for another 23.745 min.	
C3a	$\frac{40}{(s+1)(s+4)(s^2+1)} = \frac{A}{s+1} + \frac{B}{s+4} + \frac{Cs+D}{s^2+1}$, $A = \frac{20}{3}$ and $C = -\frac{100}{17}$ $40 = A(s+4)(s^2+1) + B(s+1)(s^2+1) + (Cs+D)(s+1)(s+4)$ $s=-4: \quad 40 = B(-3)(17) \quad \to B = -\frac{40}{51}$ Constant term: $40 = 4A + B + 4D$ $D = \frac{1}{4}\left(40 - 4\left(\frac{20}{3}\right) + \frac{40}{51}\right) = \frac{1}{4}\left(\frac{2040-1360+40}{51}\right) = \frac{60}{17}$ $\therefore \frac{40}{(s+1)(s+4)(s^2+1)} = \frac{20}{3}\cdot\frac{1}{s+1} - \frac{40}{51}\cdot\frac{1}{s+4} - \frac{100}{17}\cdot\frac{s}{s^2+1} + \frac{60}{17}\cdot\frac{1}{s^2+1}$	15

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C3b	(i) $V_{in}'(t) = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i$, $V_{in}(t) = -40\cos t$ $V_{in}'(t) = -40(-\sin t) = 40\sin t$ $R = 5, \quad L = 1, \quad C = 0.25$ $\therefore \frac{d^2i}{dt^2} + 5\frac{di}{dt} + 4i = 40\sin t$ (ii) Let $I(s) = \mathcal{L}\{i(t)\}$ $s^2I - si(0) - i'(0) + 5\left(sI - i(0)\right) + 4I = \frac{40}{s^2+1}$ since $i(0) = i'(0) = 0$, $s^2I + 5sI + 4I = \frac{40}{s^2+1} \rightarrow (s+1)(s+4)I = \frac{40}{s^2+1}$ $I = \frac{40}{(s+1)(s+4)(s^2+1)}$ (iii) $I = \frac{40}{(s+1)(s+4)(s^2+1)}$ $= \frac{20}{3}\cdot\frac{1}{s+1} - \frac{40}{51}\cdot\frac{1}{s+4} - \frac{100}{17}\cdot\frac{s}{s^2+1} + \frac{60}{17}\cdot\frac{1}{s^2+1}$ $i(t) = \mathcal{L}^{-1}\left\{\frac{20}{3}\cdot\frac{1}{s+1} - \frac{40}{51}\cdot\frac{1}{s+4} - \frac{100}{17}\cdot\frac{s}{s^2+1} + \frac{60}{17}\cdot\frac{1}{s^2+1}\right\}$ $= \frac{20}{3}e^{-t} - \frac{40}{51}e^{-4t} - \frac{100}{17}\cos t + \frac{60}{17}\sin t$	
C3c	$i_{\text{steady-state}} = \frac{60}{17}\sin t - \frac{100}{17}\cos t$ $i_{\text{transient-state}} = \frac{20}{3}e^{-t} - \frac{40}{51}e^{-4t}$	