1.8 Problems

Q1-1

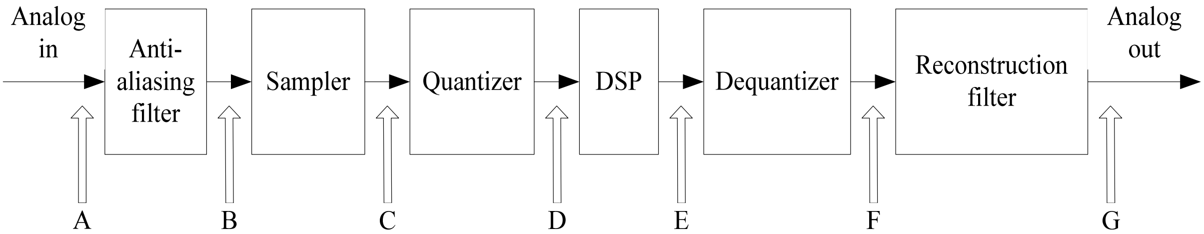

Figure 1.35 A typical DSP system

In Figure 1.35, where can we find

(a) continuous-time signals
(b) continuous-amplitude signals
(c) discrete-time signals
(d) discrete-amplitude signals?

Q1-2 A continuous-time analog signal is represented by $x(t) = \cos(2000\pi t)$. This signal is sampled at different sampling frequencies with the first sample taken at $t = 0$.

(a) Sketch the analog signal $x(t)$ for $0 \leq t \leq 4$ ms.
(b) Sketch the sampled signal for $0 \leq t \leq 4$ ms if the sampling frequency is 8 kHz.
(c) Sketch the sampled signal for $0 \leq t \leq 4$ ms if the sampling frequency is 2 kHz.
(d) Sketch the sampled signal for $0 \leq t \leq 4$ ms if the sampling frequency is 1 kHz.
(e) From the results of (b), (c) and (d), which sampling frequency would yield samples which resemble closely to (a)? Which sampling frequency would produce the worst result?

Q1-3 The following signals are sampled at the rate of 2000 samples per second with the first sample taken at 0.25 ms after the positive going zero-crossing.

(i) A 500 Hz sine wave with an amplitude of 1.414 volt.
(ii) A 500 Hz square wave with an amplitude of 1 volt.

(a) For each of the signals in (i) and (ii), sketch two cycles of the sampled signals.
(b) Are the two sampled signals similar? Why?

Q1-4 (a) State Nyquist sampling theorem.
(b) Explain what is aliasing.
(c) How can aliasing be prevented and what are the trade-offs?

Q1-5 Assume that the cut-off frequency of the anti-aliasing filter in Figure 1.35 is 10 kHz. The output magnitude spectrum of the signal at the output of the anti-aliasing filter is shown in Figure 1.36.
(a) What is the Nyquist frequency of the DSP system in Figure 1.35?
(b) Determine the Nyquist rate.
(c) Determine the required minimum sampling frequency for the DSP system.
(d) If the signal at point B in Figure 1.35 is sampled with the minimum sampling frequency, sketch the frequency spectrum after sampling. Indicate on the spectrum whether aliasing has occurred.

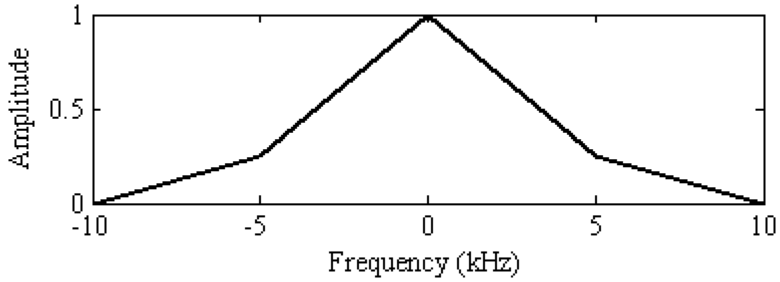

Figure 1.36 Magnitude spectrum of the signal at the output of the anti-aliasing filter

Q1-6 Assume that the quantizer in Figure 1.35 cannot operate with sampling frequency f_s > 15 kHz. Figure 1.36 shows the magnitude spectrum of the signal at the output of the anti-aliasing filter.
(a) Sketch the magnitude spectrum of the sampled signal if f_s = 15 kHz for –25 kHz ≤ f ≤ 25 kHz. Indicate on the spectrum whether aliasing has occurred.
(b) Propose two methods to solve the aliasing problem. Comment on which is the better method for this particular problem.
(c) Having applied the better solution in (b), sketch the magnitude spectrum of the sampled signal.

Q1-7 A sinusoidal signal with unknown frequency is sampled at 16000 samples per second. The resultant magnitude spectrum is shown in Figure 1.37.
(a) If there is no aliasing distortion, what is the frequency of the sinusoidal signal?
(b) If aliasing has taken place, what is one possible frequency of the sinusoidal signal?

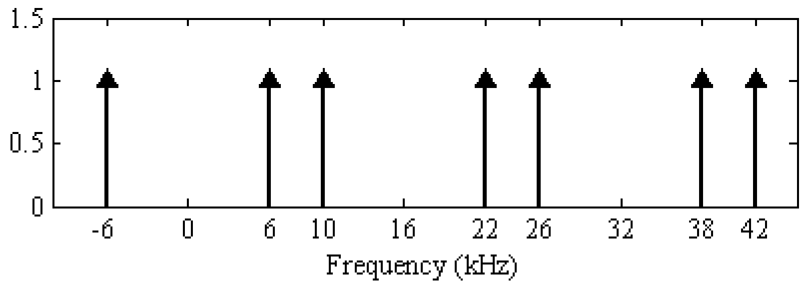

Figure 1.37 Magnitude spectrum of an unknown sinusoidal signal

Q1-8

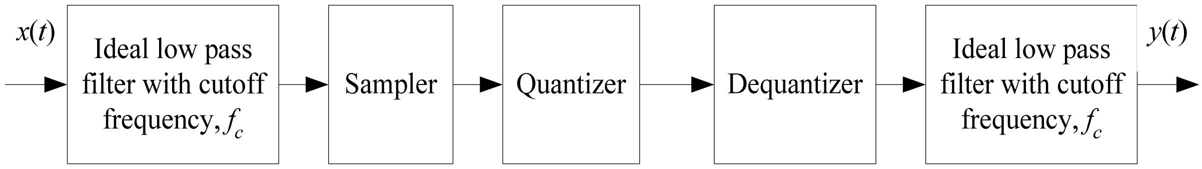

Figure 1.38 A certain DSP system

Figure 1.38 shows the block diagram of a certain DSP system. Given that f_s is the sampling frequency used and:

$$x(t) = \cos(2000\pi t) + 2\cos(4000\pi t) + 3\cos(10000\pi t)$$

(a) In Figure 1.38, if $f_c = f_s = 8$ kHz, sketch the spectrum of the signal $y(t)$ for $0 \le f \le 8$ kHz. Briefly point out the possible problems or limitations for processing $x(t)$, if any.

(b) If $f_c = 4$ kHz and $f_s = 8$ kHz, sketch the spectrum of the signal $y(t)$ for $0 \le f \le 8$ kHz. Briefly point out the possible problems or limitations for processing $x(t)$, if any.

(c) If $f_c = 6$ kHz and $f_s = 12$ kHz, sketch the spectrum of the signal $y(t)$ for $0 \le f \le 8$ kHz. Briefly point out the possible problems or limitations for processing $x(t)$, if any.

(d) If $f_c = \infty$ (no filters) and $f_s = 12$ kHz, briefly point out the possible problems or limitations for processing $x(t)$, if any.

Q1-9* An analog signal $x(t)$ is given as $x(t) = 2\cos(2000\pi t)$ and it is sampled with 8 kHz sampling frequency. If the first sample is taken at $t = 0$, what is the magnitude of the 1000th sample?

Q1-10* Signal $x(n)$ is composed of three sine waves:

$$x(n) = \cos(0.5\pi n) + 3\sin(0.25\pi n) - \sin(0.1\pi n)$$

(a) Find the magnitude of the first two samples, i.e., $x(0)$ and $x(1)$.

(b) If it is known that $x(n)$ is obtained with a sampling frequency of 8 kHz, derive the equation for the continuous time signal $x(t)$.

(c) If signal $x(n)$ is applied to a low pass filter shown in Figure 1.39, sketch the frequency spectrum of the signal at the output.

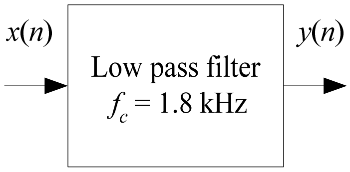

Figure 1.39 Low pass filter

Q1-11* A continuous signal $x(t) = 1.4\cos\left(2000\pi t + \frac{\pi}{4}\right)$ is sampled with a sampling frequency of 1.8 kHz.

(a) Sketch the sampled signal for $0 \le t \le 2\,\text{ms}$. Assume that the first sample is taken at 0.1 ms.

(b) Sketch the magnitude spectrum of the sampled signal for $-4\,\text{kHz} \le f \le 4\,\text{kHz}$.

(c) Do you think the sampling frequency used is appropriate?

Q1-12* Magnitude of a sample is given as 2.345V. If this sample is quantized with a quantization step size of Δ = 0.15V, what will be the quantization error if one of the quantized values is 0V?

Q1-13* A continuous time signal $x(t)$ is converted to discrete-time signal $x(nT_s)$ with a sampling interval T_s = 0.5 ms. If it is known that sampling frequency used is at Nyquist rate, what is the highest possible frequency component of $x(t)$?

Q1-14* The maximum dynamic range of a 4-bit quantizer is from –2.4V to +2.1V. If a sample of the input signal has a magnitude of 1.56V, what will be the corresponding code after quantization?