Mid-Semester Test

Time allowed : 1 hour

Instructions

Answer all **4** questions. Each question carries **25 marks**.

This question paper consists of **2** pages. You can use the A4 handwritten formula sheet compiled by you.

You are reminded that cheating during test is a serious offence.

All working in support of your answer must be shown. Answers must be to appropriate significant figures. Take g = 9.80 m/s^2.

---

1. a) In dimensional analysis, what is meant by a homogenous equation?

   b) In the equation below, the SI units of x and x_0 are metres, t and t_0 are seconds, v_0 is m/s and a is m/s^2. Show whether this equation is homogenous or not.

   $$x = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

   c) The force on a current carrying conductor is given by $\boldsymbol{F} = c\ (\boldsymbol{L} \times \boldsymbol{B})$, where $c = 2.0$ amperes. Determine the force $\boldsymbol{F}$ on a conductor whose length vector is $\boldsymbol{L} = 1.0\ \mathbf{i} + 2.0\ \mathbf{j}$ and the conductor is in a magnetic field $\boldsymbol{B} = 0.10\ \mathbf{k}$. The SI units of $\boldsymbol{L}$ and $\boldsymbol{B}$ are metre and tesla respectively.

a) A homogenous equation is one which is dimensionally consistent.

b) LHS $= x$

The dimension of LHS is $[L]$

$$\text{RHS} = x_0 + v_0(t - t_0) + \frac{1}{2}a(t - t_0)^2$$

The dimension of RHS is $[L] + [L][T]^{-1}[T] + [L][T]^{-2}[T]^2$

$= [L] + [L] + [L] = [L] = \text{LHS}$

Since the equation is dimensionally consistent, it is homogenous..

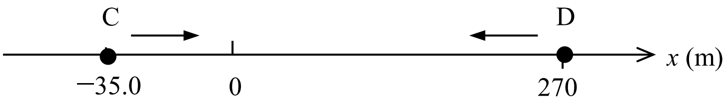
c) $$\boldsymbol{F} = c\ (\boldsymbol{L} \times \boldsymbol{B}) = (2.0)\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.0 & 2.0 & 0 \\ 0 & 0 & 0.10 \end{vmatrix}$$

$= 0.40\ \mathbf{i} - 0.20\ \mathbf{j}$ N

2. In the figure below, particles C and D move towards each other along the x-axis. At time $t = 0$, C is at $x = -35.0$ m and accelerates uniformly from rest at 2.00 m/s^2 while D is at $x = 270$ m and moving at constant speed 20.0 m/s.

a) When do the particles meet?
b) Where do the particles meet?
c) What is the speed of C when it meets D?
d) Sketch the position-time graphs of C and D using the same set of x-t axes.

(25 marks)



a) Let t be the time when the particles meet at point x.
The equations for the particles would be

$$x_C = -35.0 + \frac{1}{2}(2.0)t^2$$

$$x_D = 270 - 20.0 \times t$$

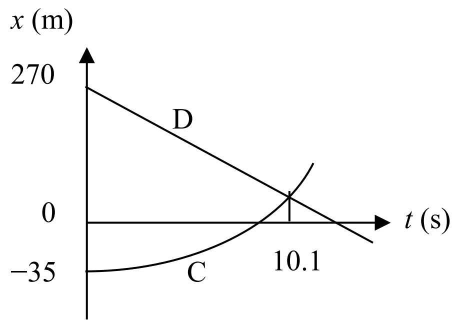
Solving the above equations we get $t = 10.1\text{s}$ when the particles meet.

b) Substituting t in one of the above two equations we get $x = 68$ m where the particles meet.

c) The velocity of C when it meets D can be calculated as

$$v = v_0 + at$$
$$= 0 + 2.0 \times 10.1$$
$$= 20.2 \text{ m/s}$$

d)



3. The position vector of a particle of mass 4.0 kg moving on the *x*-*y* plane is $\boldsymbol{r}(t) = 2t\,\mathbf{i} + t^2\,\mathbf{j}$, with $\boldsymbol{r}$ in metres and t in seconds. Calculate in component form the particle's

   a) average velocity from $t = 0$ to $t = 1.0$ s.
   b) instantaneous velocity at $t = 1.0$ s.
   c) instantaneous acceleration at $t = 1.0$ s.
   d) net force acting on the particle at $t = 1.0$ s.

(25 marks)

a) $\vec{r}(t) = 2t\,\mathbf{i} + t^2\,\mathbf{j}$

$\vec{r}(0) = 0\,\mathbf{i} + 0\,\mathbf{j}$

$\vec{r}(1.0) = 2\,\mathbf{i} + \mathbf{j}$

$$\vec{v}_{\text{ave}}(t) = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}(1.0) - \vec{r}(0.0)}{1.0 - 0.0}$$

$= (2\,\mathbf{i} + \mathbf{j})$ m/s

b) $\vec{v}(t) = 2\mathbf{i} + 2t\,\mathbf{j}$

$\vec{v}(1.0) = 2\,\mathbf{i} + 2\,\mathbf{j}$

c) $\vec{a}(t) = 2\,\mathbf{j}$

$\vec{a}(1.0) = 2\,\mathbf{j}$ m/s^2

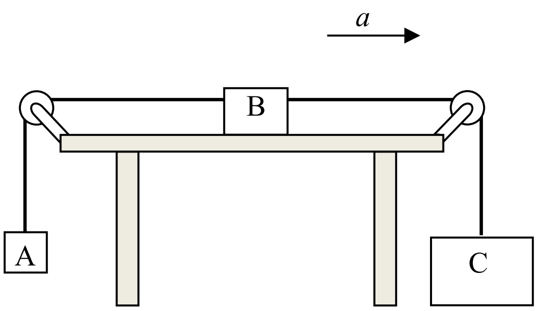
d) $\vec{F} = m\vec{a} = 8.0\,\mathbf{j}$ N

4. a) Three forces act on a particle of mass 3.0 kg such that it is at rest. Two of the forces are $\boldsymbol{F}_1 = 2.0\ \mathbf{i} - 7.0\ \mathbf{j} + 4.0\ \mathbf{k}$ N and $\boldsymbol{F}_2 = 4.0\ \mathbf{i} + 1.0\ \mathbf{k}$ N while the third force $\boldsymbol{F}_3$ is unknown.

i) Write the relationship between $\boldsymbol{F}_1$, $\boldsymbol{F}_2$ and $\boldsymbol{F}_3$.
ii) Find $\boldsymbol{F}_3$.

b) The diagram below shows three blocks A, B and C attached by chords that loop over frictionless pulleys. Block B lies on a frictionless table. The masses of A, B and C are m_1, m_2 and m_3 respectively and that $m_3 > m_2 > m_1$. The tension in the chord connecting A and B is T_1 while the tension in the chord connecting B and C is T_2. When the blocks are released, they accelerate with a as shown. Find a and T_2 in terms of m_1, m_2 and m_3 and g, the acceleration due to gravity.

(25 marks)



a) i) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

ii) $2.0\ \mathbf{i} - 7.0\ \mathbf{j} + 4.0\ \mathbf{k} + 4.0\ \mathbf{i} + 1.0\ \mathbf{k} + F_x\mathbf{i} + F_y\ \mathbf{j} + F_z\ \mathbf{k} = 0$

$2.0 + 4.0 + F_x = 0$

$-7.0 + F_y = 0$

$4.0 + 1.0 + F_z = 0$

Hence $\vec{F}_3 = (-6.0\,\mathbf{i} + 7.0\,\mathbf{j} - 5.0\,\mathbf{k})\,\mathrm{N}$

b) For C, $m_3 g - T_2 = m_3 a$ (1)

For B, $T_2 - T_1 = m_2 a$ (2)

For A, $T_1 - m_1 g = m_1 a$ (3)

Add eqns (2) and (3),

$T_2 - m_1 g = (m_1 + m_2)a$

$T_2 = m_1 g + (m_1 + m_2)a$ (4)

Substituting for T_2 in eqn (1)

$m_3 g - (m_1 g + (m_1 + m_2)a) = m_3 a$

$$a = \frac{(m_3 - m_1)}{(m_1 + m_2 + m_3)} g$$

Therefore T_2 from eqn (4) is

$$T_2 = m_1 g + \frac{(m_3 - m_1)(m_1 + m_2)}{(m_1 + m_2 + m_3)} g$$