Kinematics 1 – Motion in 1D

PRE-CLASS (1 TO 16)

IN-CLASS (18 ONWARDS)

Learning outcomes for pre-class slides

At the end of the pre-class slides, students are to be able to

- define position, displacement, distance, average velocity, and average speed in 1D
- distinguish displacement and distance, average velocity and average speed
- interpret position-time graphs and solve relevant problems using the definitions

Kinematics

- Kinematics is a part of mechanics that enables us to describe motion.
- We need to define some physical quantities which can help describe motion in one, two and three dimensions.

Motion in one dimension (1D)

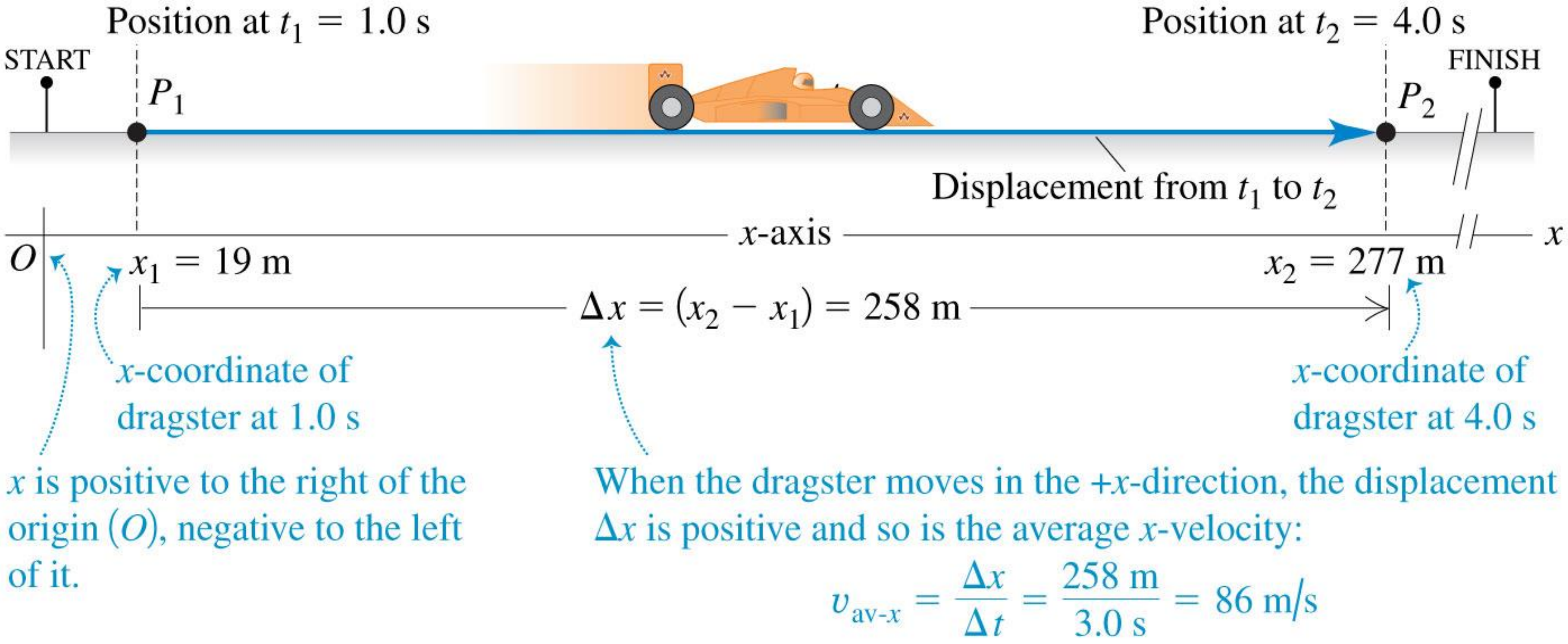
- The simplest kind of motion is motion in a straight line (one dimension).
- Examples of this kind of motion include an object moving on a straight road and a stone thrown vertically upward.
- We will assume that the objects are represented by point particles instead of considering them as extended objects.

Motion of a particle in one-dimension (1D)

- To describe the motion of a particle, we need a coordinate system.
- We choose the x-axis with an origin O to lie along the road.
- The change in the particle's x-coordinate over a time interval will describe the motion of the particle over that time interval.
- The x-coordinate of the particle is called the position of the particle.

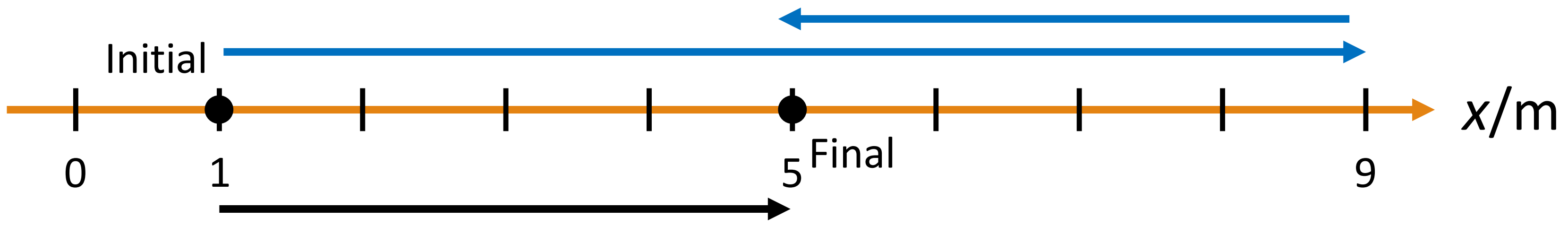
Displacement

- The displacement of the particle is a vector pointing from P_1 (initial position) to P_2 (final position).
- The x-component of the displacement is given by the change in the particle's position i.e. $\Delta x = x_2 - x_1$.



Distance vs displacement

- Distance is the length of the path travelled by an object.
- Displacement is the change in position. It is a vector pointing from the initial position to the final position.
- Distance is a scalar and displacement is a vector.
- For example, the blue arrow refers to the path travelled by the ball and the distance travelled by the ball is 12 m.
- However, the displacement is 4 m to the right.



Example 1

A dog (represented by the dot) runs from points $A \rightarrow B \rightarrow C \rightarrow D$. Each tick mark is 1 m.

a) What is the total distance travelled by the dog? (Ans: 14 m)

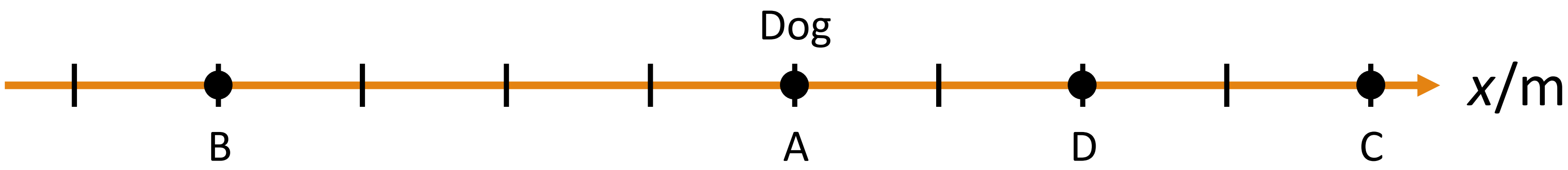
b) What is the displacement of the dog when it travels from

   i. A to B? (Ans: 4 m to the left)

   ii. B to C? (Ans: 8 m to the right)

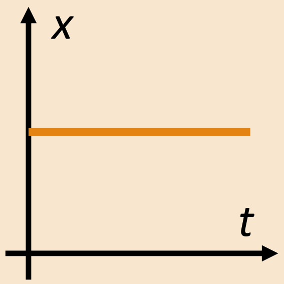
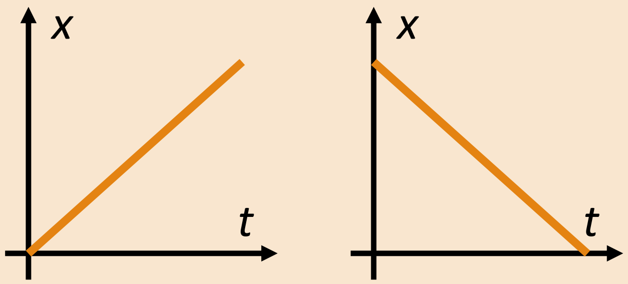
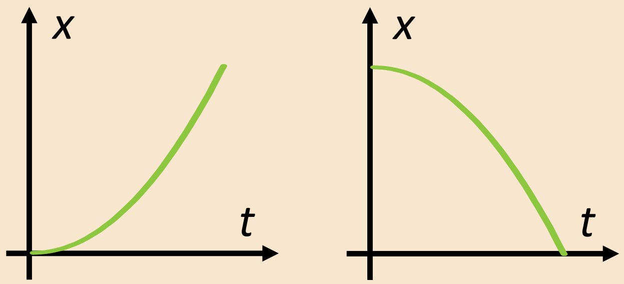
   iii. C to D? (Ans: 2 m to the left)

   iv. A to D? (Ans: 2 m to the right)



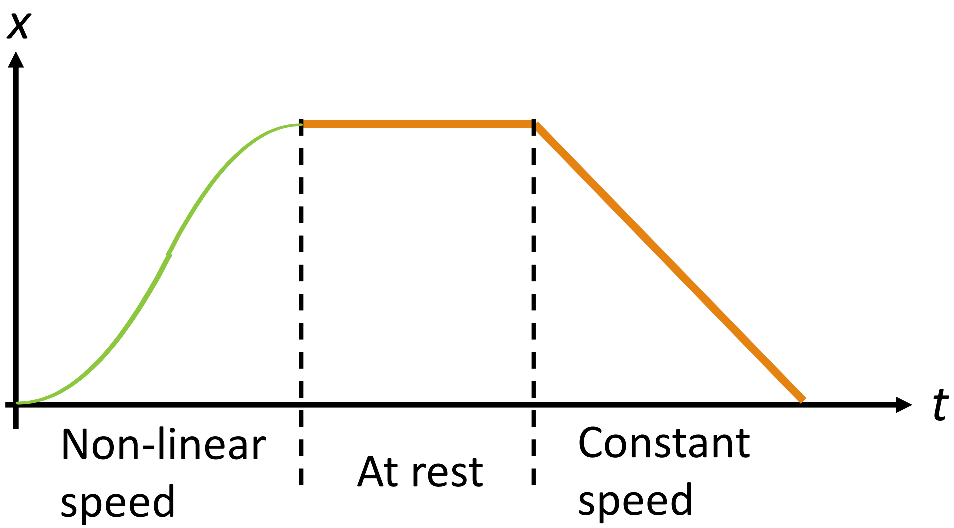
Position vs. time (x-t) graph

- The motion of the object can be represented on the position vs. time graph.
- Here are some examples of position-time graphs:

Object at rest	**Object moving at constant speed**	**Object moving with non-uniform speed**
		
The position of the object does not change with time.	The position of the object changes linearly with time.	The position of the object changes non-linearly with time.

Position vs. time graph

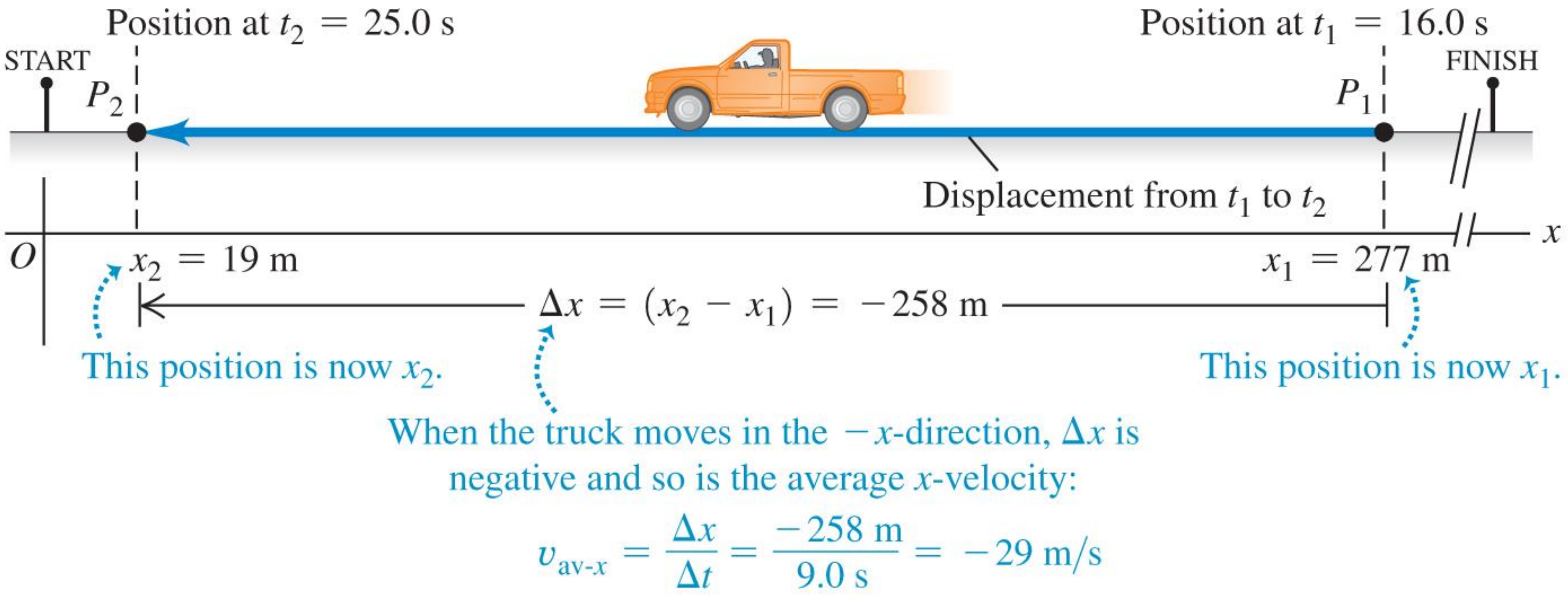
- However, the motion of the object can be multi-staged.
- For example,



Average velocity

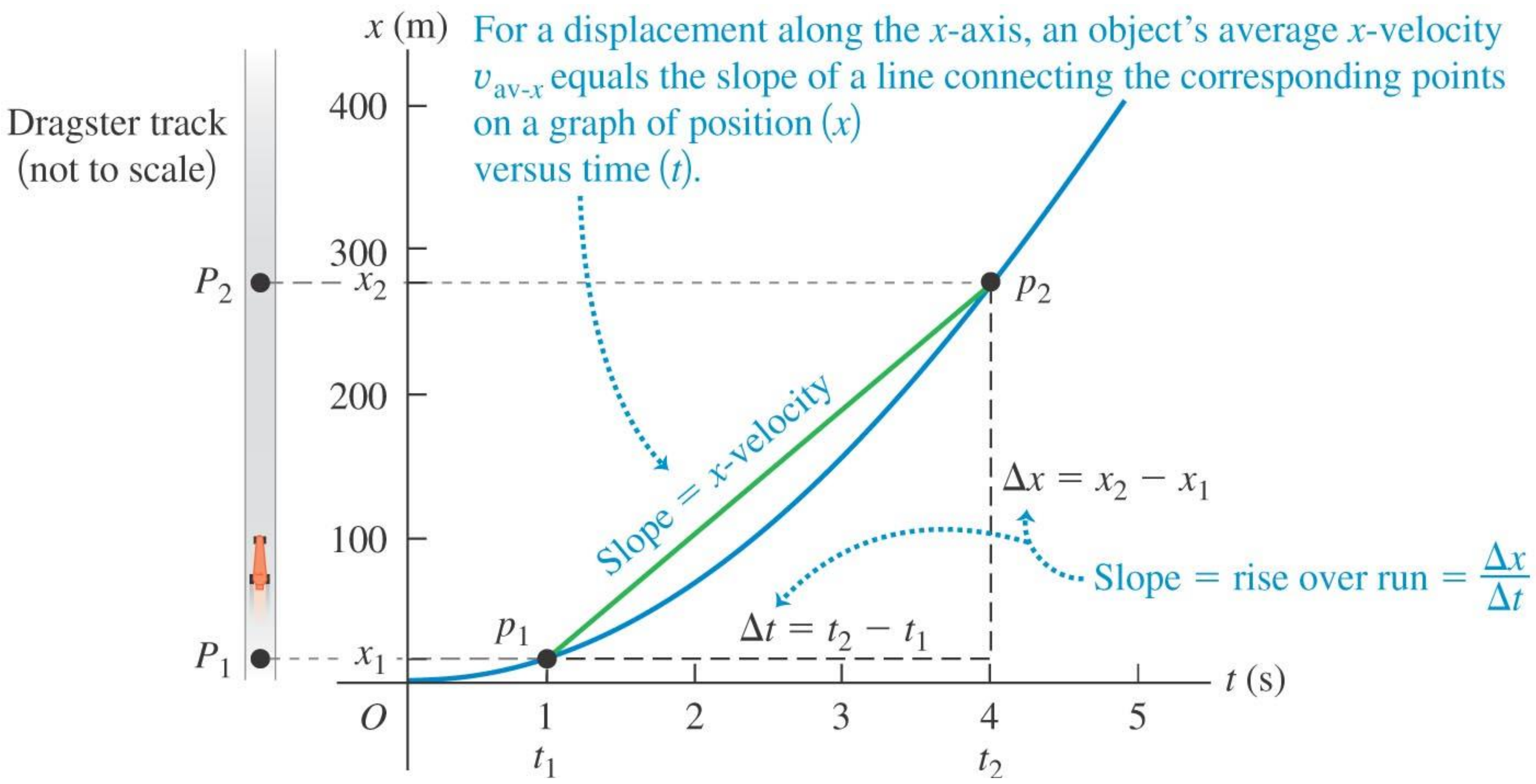
- The average velocity is a vector whose *x*-component is defined as

$$v_{av,x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$



Average velocity

- The average x-velocity $v_{av,x}$ is the slope of an x-t graph.



Average speed vs. average velocity

- Definition of average speed:

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Time taken}}$$

- Definition of average velocity:

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}$$

- Average speed is a scalar quantity; average velocity is a vector quantity.

- The average speed is greater than or equal to the magnitude of the average velocity.

Example 2

A dog (represented by the dot) runs from points A to B in 2 s, points B to C in 6 s, points C to D in 2 s. Each tick mark is 1 m.

a) What is the average speed of the dog when it travels from A to D? (Ans: 1.4 m/s)

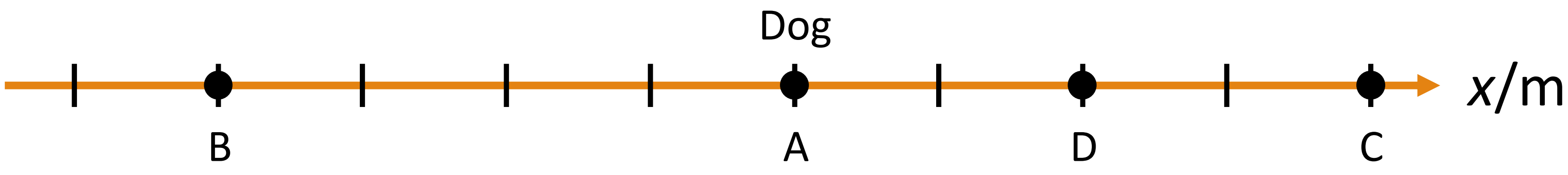
b) What is the average velocity of the dog when it travels from

   i. A to B? (Ans: 2 m/s to the left)

   ii. B to C? (Ans: 1.33 m/s to the right)

   iii. C to D? (Ans: 1 m/s to the left)

   iv. A to D? (Ans: 0.2 m/s to the right)



Example 3

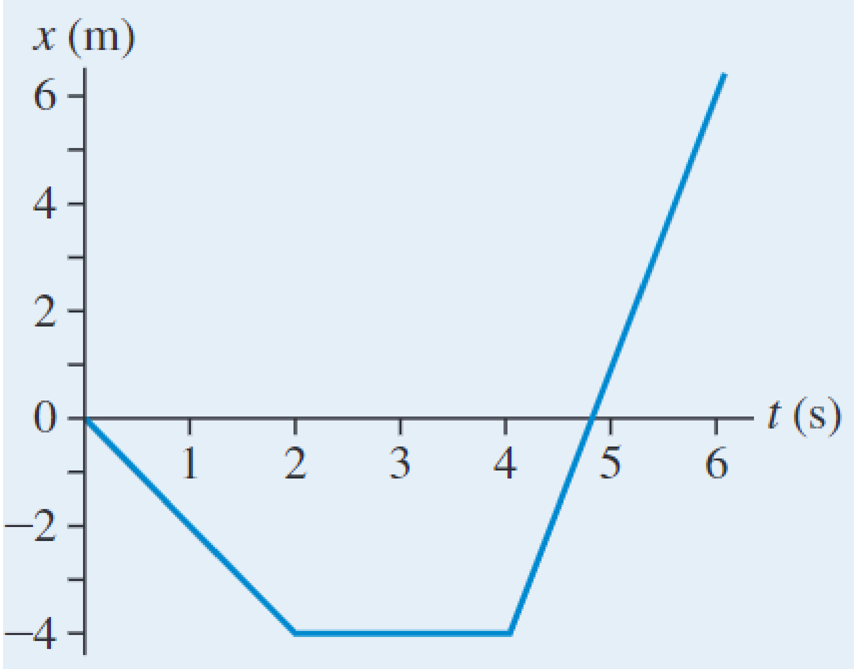
The graph at right shows the position vs. time graph of a car. Calculate the average velocity of the car

a) from $t = 0$ s to $t = 2$ s (Ans: -2 m/s)

b) from $t = 2$ s to $t = 4$ s (Ans: 0 m/s)

c) from $t = 4$ s to $t = 6$ s (Ans: 5 m/s)

d) from $t = 0$ s to $t = 6$ s (Ans: 1 m/s)



End of pre-class slides

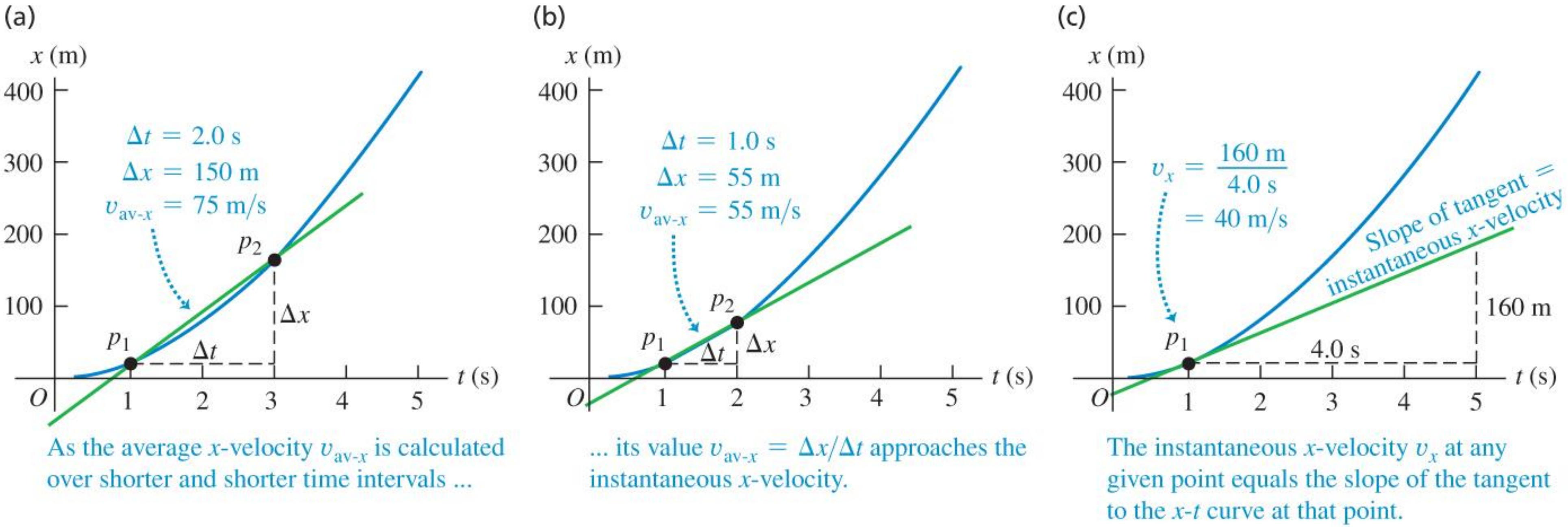
Learning outcomes

At the end of the lesson, students are to be able to

- define instantaneous velocity, average acceleration and instantaneous acceleration in 1D
- distinguish the directions of velocity and acceleration of an object in different situations (constant velocity, speeding up, slowing down)
- interpret motion graphs (position-time, velocity-time, acceleration-time graphs) and solve relevant problems
- apply the kinematics equations to solve problems related to constant acceleration motion including free-fall situations

Instantaneous velocity for 1D motion

- The average velocity of a particle during a time interval can’t tell us how fast or in what direction the particle was moving at any given time during the interval.
- Therefore we introduce the concept of **instantaneous velocity** which is the velocity at a specific instant of time or specific point along the path.



Instantaneous velocity for 1D motion

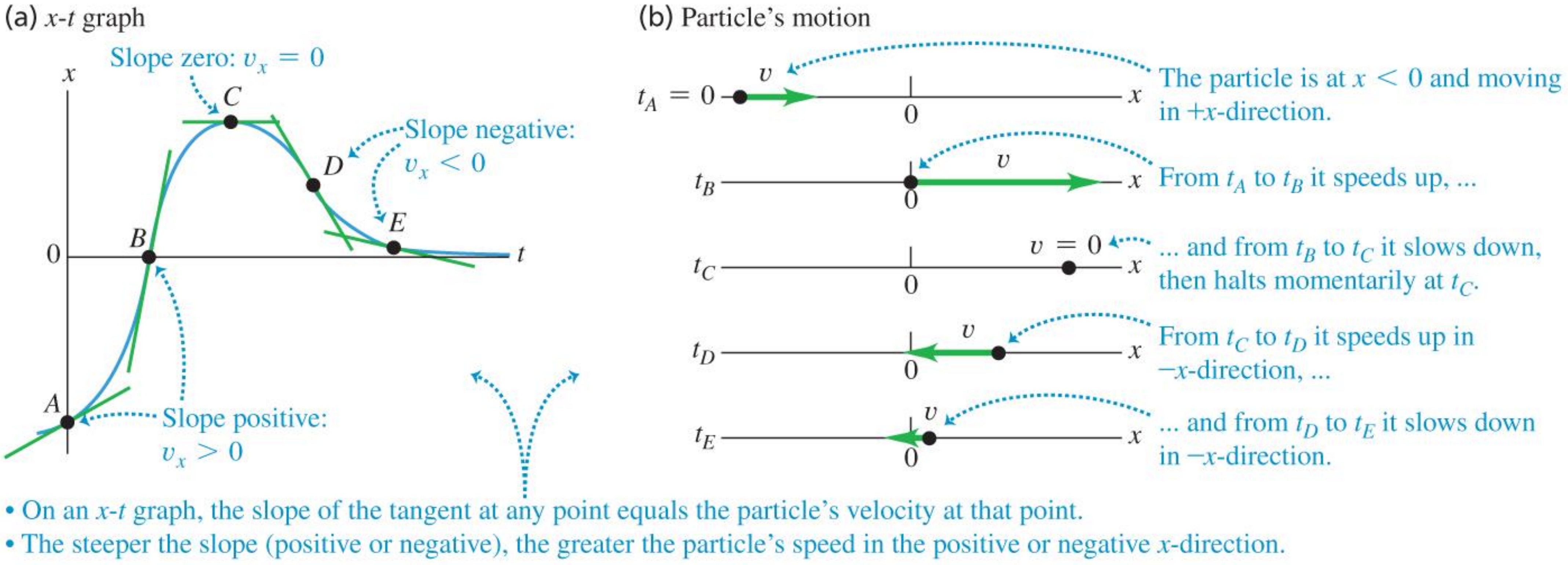
- The instantaneous velocity is the limit of the average velocity as the time interval approaches zero.

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- The instantaneous velocity is the rate of change of position.
- Unless specified average velocity, you can assume that the term **velocity** implies instantaneous velocity.
- The SI unit of velocity is m/s.
- Instantaneous speed = magnitude of instantaneous velocity

Instantaneous velocity from *x*-*t* graph for 1D motion

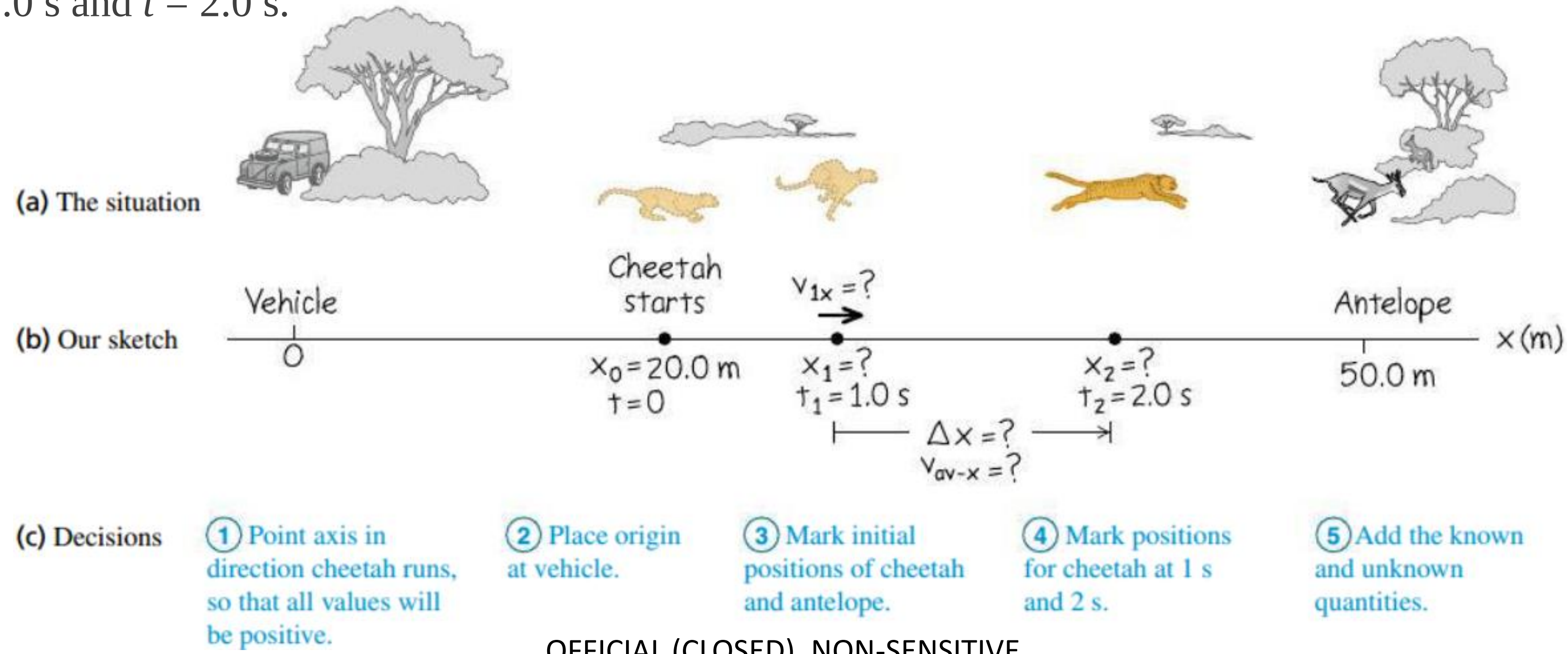
- The velocity at any point on *x*-*t* graph is equal to the slope of the tangent at that point.



Example 4

A cheetah is crouched 20 m to the east of an observer sitting in a vehicle (see below figure). At $t = 0$ the cheetah begins to run due eastward toward an antelope that is 50 m to the east of the observer. During the first 2.0 s of the attack, the cheetah's coordinate x varies with time according to $x = 20\text{ m} + (5.0\text{ m/s}^2)t^2$.

a) Find the cheetah's displacement between $t_1 = 1.0$ s and $t_2 = 2.0$ s.
b) Find the average velocity during that interval.
c) Derive an expression for the cheetah's instantaneous velocity as a function of time and use it to find v_x at $t = 1.0$ s and $t = 2.0$ s.

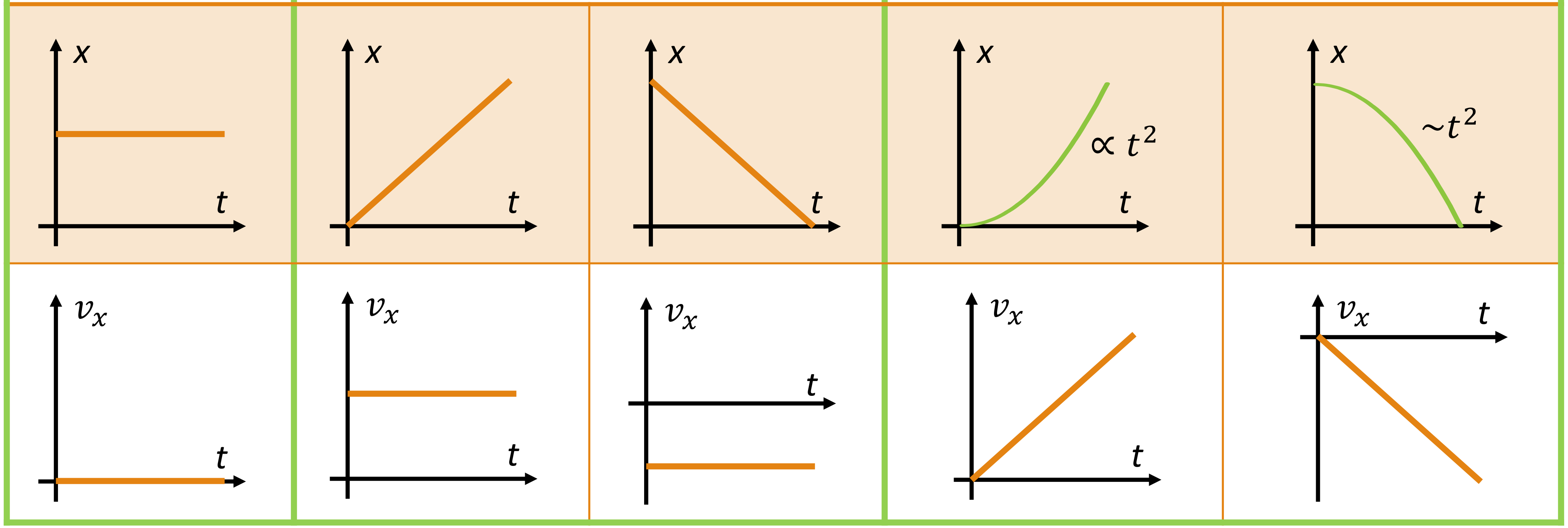


From x-t graphs to v-t graphs

- Velocity at any point on x-t graph = the slope of the tangent at that point.

$$v_x = \frac{dx}{dt}$$

Object at rest	Object moving at constant velocity	Object moving with non-uniform velocity



Acceleration

Average acceleration for 1D motion

- The average acceleration, $a_{av,x}$ of the particle is a vector whose *x*-component is defined as the change in *x*-component of the velocity divided by the time interval. That is,

$$a_{av,x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}$$

- The SI unit of average acceleration is m/s^2.

Instantaneous acceleration for straight line motion

- The instantaneous acceleration is the rate of change of velocity with time and is given by :

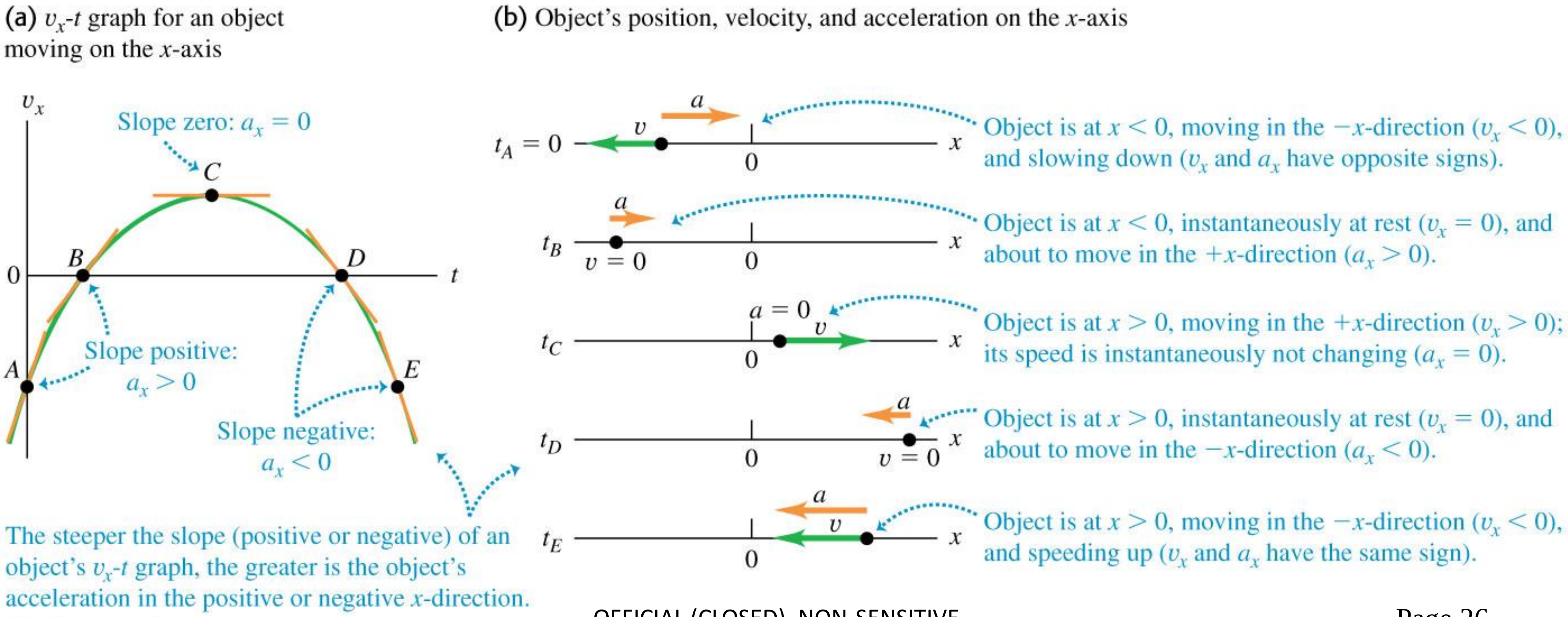
$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

- Unless specified average acceleration, you can assume that the term acceleration implies instantaneous acceleration.

- The SI unit of acceleration is (m/s/s) or simply m/s^2.

Instantaneous acceleration from v_x-t graph for 1D motion

- The acceleration at any point on the *v*-*t* graph is equal to the **slope** of the **tangent** to the curve at that point.

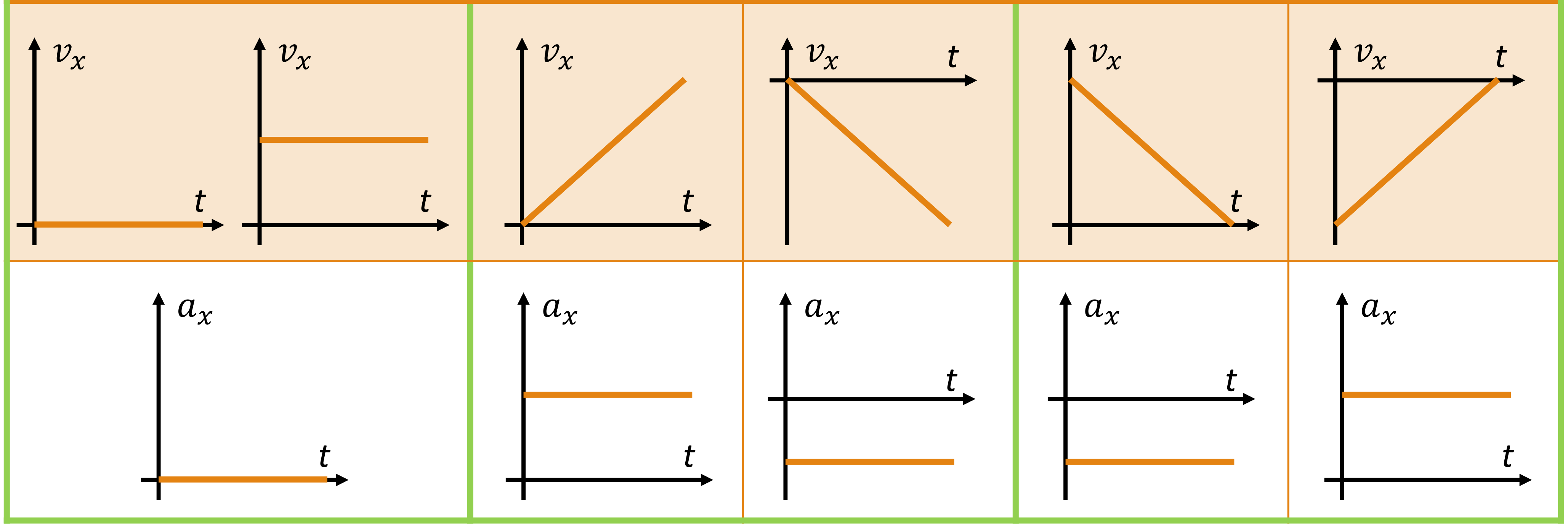


From v-t graphs to a-t graphs

- Acceleration at any point on v-t graph = the slope of the tangent at that point.

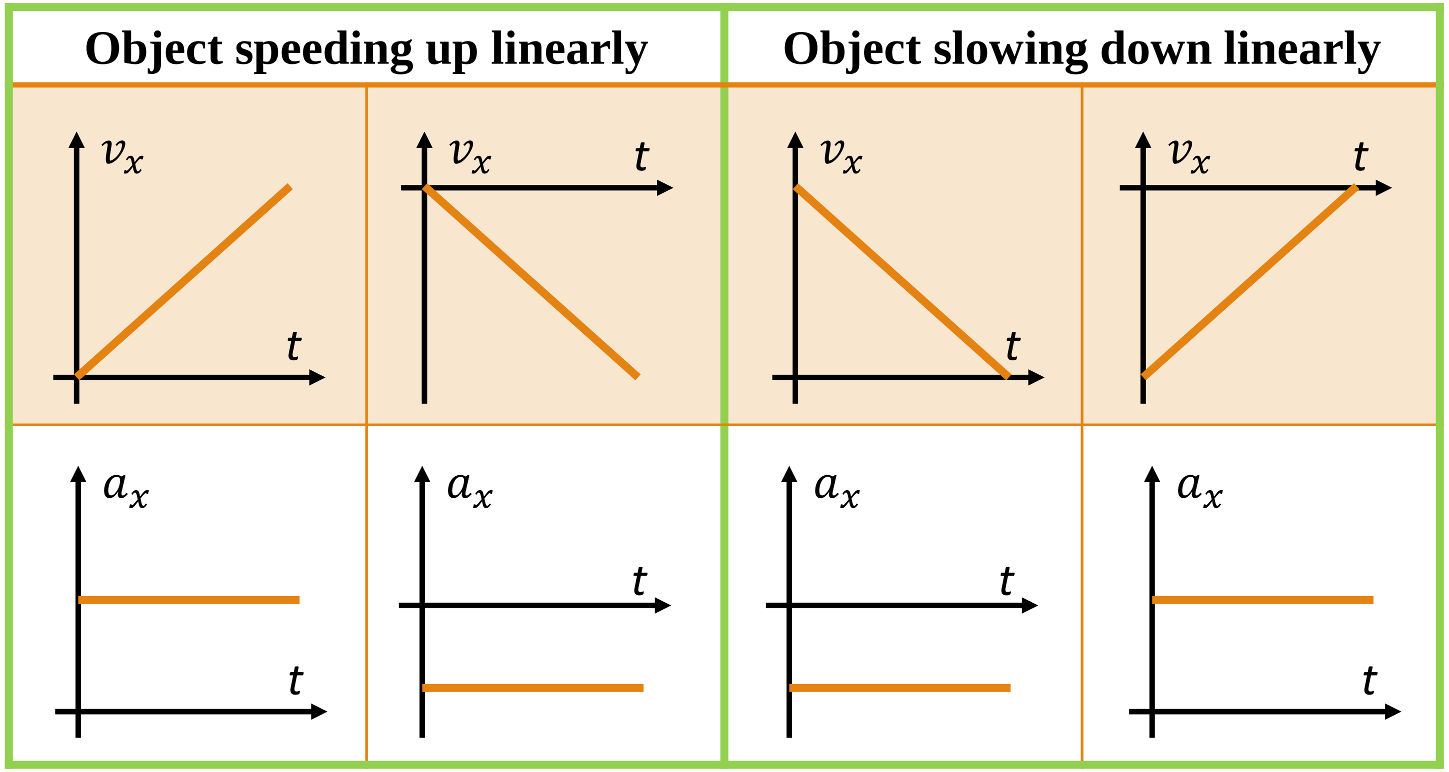
$$a_x = \frac{dv}{dt}$$

$v = 0$ or $v =$ constant	Object speeding up linearly	Object slowing down linearly



Rules for the sign of x-acceleration

- Deceleration means a decrease in speed. Negative acceleration does not always mean a decrease in speed.



If x-velocity is:	. . . x-acceleration is:
Positive & increasing (getting more positive)	Positive: Particle is moving in $+x$-direction & speeding up
Positive & decreasing (getting less positive)	Negative: Particle is moving in $+x$-direction & slowing down
Negative & increasing (getting less negative)	Positive: Particle is moving in $-x$-direction & slowing down
Negative & decreasing (getting more negative)	Negative: Particle is moving in $-x$-direction & speeding up

Note: These rules apply to both the average x-acceleration $a_{\text{av-}x}$ and the instantaneous x-acceleration a_x.

Example 5

The x-velocity v_x, of a car at time t is given by the equation $v_x = 60\ \mathrm{m/s} + \left(0.50\ \mathrm{m/s}^3\right) t^2$.

a) Find the change in x-velocity of the car in the time interval $t_1 = 1.0$ s and $t_2 = 3.0$ s.

b) Find the average x-acceleration during this time interval.

c) Derive an expression for the instantaneous acceleration as a function of time and use it to find a_x at $t = 1.0$ s and $t = 3.0$ s.

Problem solving strategy involving $x(t)$ or $v(t)$ equations

- Given $x(t)$ function:

Quantity to be determined	**Instructions**
Displacement between t_1 and t_2	$x(t_2) - x(t_1)$
Average velocity between t_1 and t_2	$v_{av} = \dfrac{x(t_2) - x(t_1)}{t_2 - t_1}$
Instantaneous velocity at t_1	Find $v(t) = \frac{dx}{dt}$, then substitute t_1 into it
Change in velocity between t_1 and t_2	1. Find $v(t) = \frac{dx}{dt}$ 2. Substitute t_1 and t_2 into $v(t)$ 3. Find the difference $v(t_2) - v(t_1)$
Acceleration at t_1	1. Find $v(t) = \frac{dx}{dt}$ 2. Find $a(t) = \frac{dv}{dt}$ 3. Substitute t_1 into $a(t)$

Problem solving strategy involving $x(t)$ or $v(t)$ equations

- Given $v(t)$ function:

Quantity to be determined	**Instructions**
Instantaneous velocity at t_1	Substitute t_1 into it
Change in velocity between t_1 and t_2	$v(t_2) - v(t_1)$
Acceleration at t_1	1. Find $a(t) = \frac{dv}{dt}$ 2. Substitute t_1 into $a(t)$

1D motion with constant acceleration

- Let a particle have a velocity v_{0x} at time $t = 0$ which we call the initial velocity and v_x at a later time t which we call the final velocity.

- From the definition of average acceleration we get

$$a_{av-x} = a_x = \frac{v_x - v_{0x}}{t - 0}$$

- Rearranging the above to get the relation gives

$$v_x = v_{0x} + a_x t$$

1D motion with constant acceleration

- If the position of the particle at $t = 0$ is x_0 and moves to position x at time t, then the average velocity is

$$v_{av-x} = \frac{x - x_0}{t - 0}$$

- We also know that the average velocity can be written as

$$v_{av-x} = \frac{1}{2}(v_{0x} + v_x)$$

- The above two equations can be combined to yield

$$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$$

1D motion with constant acceleration

- Simplifying we get a useful equation that relates the final position with the initial position, initial velocity, acceleration and time.

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

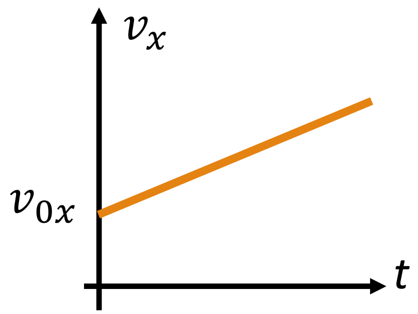
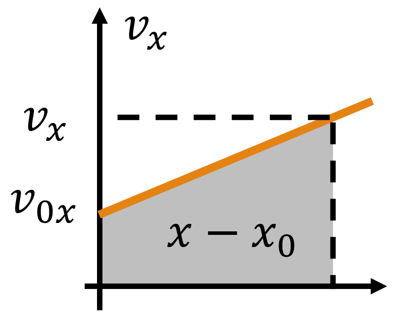
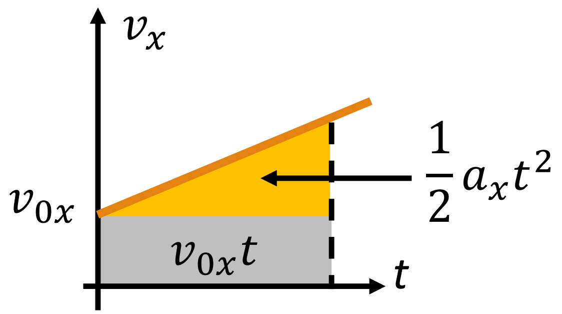
- We can eliminate time using $t = \frac{v_x - v_{0x}}{a_x}$ and on substituting in the above equation we get

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

- Note that the equations we have derived are applicable only when the acceleration is constant.

1D motion with constant acceleration

- Another way to look at the kinematics equations:

Equation and its interpretation	$v_x = v_{0x} + a_x t$ The equation of the line on the v-t graph, a_x is the gradient, v_{0x} is the vertical intercept.	$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$ Area of the trapezium under the v-t graph is $x - x_0$.	$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ Area of the trapezium under the v-t graph is split into a rectangle ($v_{0x}t$) and triangle ($\frac{1}{2}a_x t^2$).
Graph	 	 	

1D motion with constant acceleration

- Summary of equations for constant acceleration motion along horizontal:

$v_x = v_{0x} + a_x t$	$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

- If the acceleration along the vertical motion is constant, then we have:

$v_y = v_{0y} + a_y t$	$y - y_0 = \frac{1}{2}(v_{0y} + v_y)t$
$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$	$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

Problem solving strategy involving constant a motion

1. Sketch a diagram to describe the motion of the object of interest. Sometimes, a graph can be sketched to describe the motion.

2. Indicate the given quantities and sometimes their directions (position, velocity, acceleration, times) on the diagram.

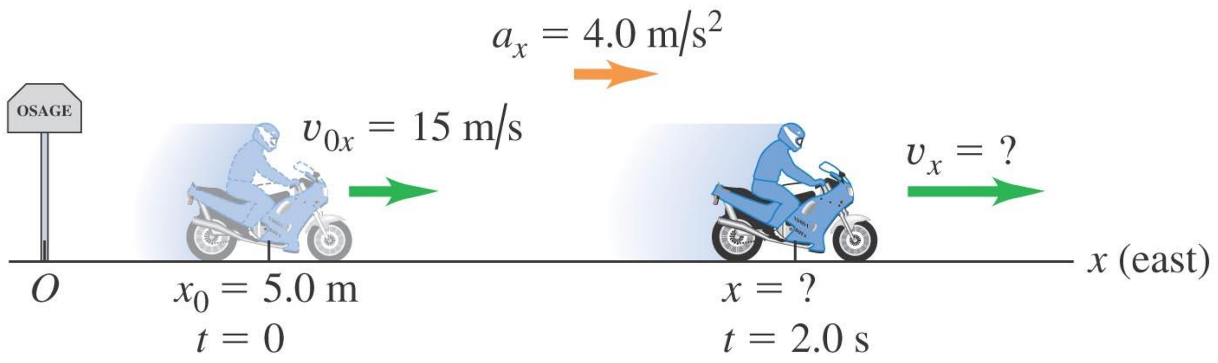
3. Pay attention to phrases such as “at rest”, “constant speed”, “speeding up”, “slowing down”. They convey information about the velocities and accelerations.

4. Pay attention to the unknowns and use the appropriate kinematics equations to solve for them.

Example 6

A motorcyclist heading east through a small town accelerates at a constant 4.0 m/s^2 after he leaves the city limits (see below figure). At time $t = 0$ he is 5.0 m east of the city-limits signpost, moving east at 15 m/s.

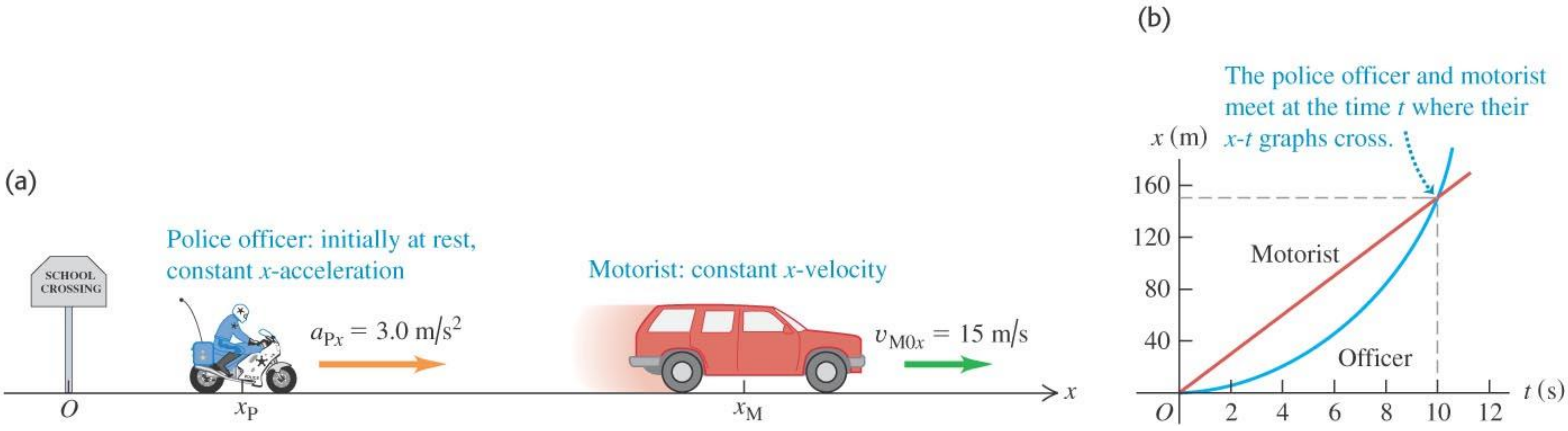
a) Find his position and velocity at $t = 2.0$ s.
b) Where is he when his velocity is 25 m/s?



Example 7

A motorist travelling with a constant speed of 15 m/s passes a school-crossing corner where the speed limit is 10 m/s. Just as the motorist passes the school-crossing sign, a police officer on a motorcycle, initially at rest, starts in pursuit with a constant acceleration of 3.0 m/s^2 (see below figure).

a) How much time elapses before the officer passes the motorist?
b) At that time, what distance has each vehicle travelled?



Free fall – 1D motion with constant acceleration

- Free fall motion is when an object falls under earth's gravity assuming that
  - there is no air resistance,
  - the distance of the fall is small compared to the radius of the earth,
  - the rotation of the earth is not important.
- The acceleration due to gravity is taken to be a constant irrespective of the size and mass of the object.
- The magnitude of the acceleration due to gravity on Earth's surface is 9.80 m/s^2. The direction is **downward**.
- Video: https://youtu.be/E43-CfukEgs

Free fall – 1D motion with constant acceleration

- By convention, we take vectors pointing upwards to be positive, so $a_y = -g$.
- The kinematics equations become:

$v_y = v_{0y} - gt$	$y - y_0 = \frac{1}{2}(v_{0y} + v_y)t$
$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$	$v_y^2 = v_{0y}^2 - 2g(y - y_0)$

Example 8

You throw a ball vertically upward from the roof of a tall building. The ball leaves your hand at a point even with the roof railing with an upward speed of 15.0 m/s. On its way back down, it just misses the railing.

a) Find the ball’s position and velocity 1.00 s and 4.00 s after leaving your hand.
b) Find the ball’s velocity when it is 5.00 m above the railing.
c) Find the maximum height reached.
d) Find the ball’s acceleration when it is at its maximum height.
e) At what time after being released has the ball fallen 5.00 m below the roof railing?

Example 8 - cont