

No.	SOLUTION
1a(i)	$a = 200$ $d = 225 - 200 = 25$
(ii)	$T_{17} = 200 + 16(25)$ $= \$600$ $S_{17} = \frac{17}{2} [2(200) + 16(25)]$ $= \$6,800$ $S_n = \frac{n}{2} [2(200) + (n-1)25] = 20,000$ $n[2(200) + (n-1)25] = 40000$ $375n^2 + 25n^2 - 40000 = 0$ $n^2 + 15n - 1600 = 0$ $n = 33.19 \text{ or } -33.195 \text{ (N.A.)}$ <p>On his 34th birthday, the amount will first exceed 20000.</p>
1(b)	$f(x) = \begin{cases} \frac{1}{2}x, & -4 \leq x < -2 \\ \sqrt{4-x^2}, & -2 \leq x \leq 2 \\ -1, & 2 < x \leq 4 \end{cases}$

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2a	$\frac{dx}{dt} = -\frac{2}{t^2}$ $\frac{dy}{dt} = 2t$ $\frac{dy}{dt} = 2t \times -\frac{t^2}{2}$ $= -t^3$
(b)	$\frac{dy}{dt} = -t^3$ $x = 2, \frac{dy}{dt} = -8$ $x = 2, t = 2$ $y = (2)^2 + 1 = 5$ <p>Gradient of normal = $\frac{1}{8}$</p> <p>Equ of normal: $y - 5 = \frac{1}{8}(x - 2)$</p> $8y - 40 = x - 2$ $8y = x + 38$ <p>Cut y-axis: $x=0, y = \frac{19}{4}$</p> <p>Coordinate the normal at $x=2$ cuts the y-axis at $(0, 19/4)$</p>
(c)	$2y - x(y - 1) = 2$ $2(t^2 + 1) - \left(\frac{2}{t} + 1\right)(t^2 + 1 - 1) = 2$ $2t^2 + 2 - 2t - t^2 = 2$ $t^2 - 2t = 0$ $t = 0 \text{ or } t = 2$
(d)	$x = \frac{2}{t} + 1 \Rightarrow t = \frac{2}{x-1}$ $y = \left(\frac{2}{x-1}\right)^2 + 1$ $D = (-\infty, 1) \cup (1, \infty)$ $R = (1, \infty)$

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<p>3a</p> <p>(i)</p> <p>(ii)</p> <p>(iii)</p>	$\frac{w}{h} = \frac{16}{9}$ $h = \frac{9}{16}w \dots (1)$ $d^2 = w^2 + h^2$ $d = \sqrt{w^2 + h^2} \text{ or } -\sqrt{w^2 + h^2} \text{ (N. A.)}$ <p>From (1)</p> $d = \sqrt{w^2 + \left(\frac{9}{16}w\right)^2}$ $= \sqrt{w^2 \left(1 + \frac{81}{256}\right)}$ $= \frac{\sqrt{337}}{16}w$ <p>when $w = 16$,</p> $d = \frac{\sqrt{337}}{16}(16)$ $= \sqrt{337}$ <p>when $w = 24$,</p> $d = \frac{\sqrt{337}}{16}(24)$ $= \frac{3\sqrt{337}}{2}$ $R_d = \left[\sqrt{337}, \frac{3\sqrt{337}}{2} \right]$ <p>$\max w = 24$</p> $\max h = \frac{9}{16}(24)$ $= 13.5$ <p>Min. wall space needed is $24 \times 13.5 = 324 \text{ in}^2$</p>

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3(b)	$f(-x) = (-x)^4 - 2(-x)^2 + 1$
(i)	$= x^4 - 2x^2 + 1$
	$= f(x)$
	Hence, f(x) is an even function
(ii)	$f(-x) = \sin(-2x)$
	$= -\sin(2x)$
	$= -f(x)$
	Hence, f(x) is an odd function
(iii)	$f(-x) = \frac{e^{-x} - 1}{e^{-x} + 1}$
	$= \frac{\frac{1}{e^x} - 1}{\frac{1}{e^x} + 1}$
	$= \frac{1 - e^x}{1 + e^x}$
	$= -\left(\frac{e^x - 1}{e^x + 1}\right)$
	$= -f(x)$
	Hence, f(x) is an odd function
(c)	$\text{let } y = \frac{e^x - 1}{e^x + 1}$
	$ye^x + y = e^x - 1$
	$e^x(y - 1) = -1 - y$
	$e^x = \frac{-1 - y}{y - 1}$
	$e^x = \frac{y + 1}{1 - y}$
	$x = \ln\left(\frac{y + 1}{1 - y}\right) \Rightarrow f^{-1}(x) = \ln\left(\frac{x + 1}{1 - x}\right)$

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4a.	<p>(i) $y = \frac{2x+3}{4x^2+7}$</p> $\frac{dy}{dx} = \frac{(4x^2+7)(2) - (2x+3)(8x)}{(4x^2+7)^2}$ $= \frac{8x^2+14-16x^2-24x}{(4x^2+7)^2}$ $= \frac{14-24x-8x^2}{(4x^2+7)^2}$ <p>(ii) $\frac{dy}{dx} = \frac{14-24x-8x^2}{(4x^2+7)^2}$</p> $= \frac{2(7-12x-4x^2)}{(4x^2+7)^2}$ $= -\frac{2(4x^2+12x-7)}{(4x^2+7)^2}$ <p>Since $4x^2+12x-7 < 0$, then $\frac{dy}{dx}$ is always > 0 due to the negative sign and the perfect square at the denominator.</p> <p>(iii) $x=0, y = \frac{3}{7}$</p> $\frac{dy}{dx} = \frac{2}{7}$ <p>Equ of tangent: $y - \frac{3}{7} = \frac{2}{7}(x-0)$</p> $y = \frac{2}{7}x + \frac{3}{7} \dots (1)$ <p>$y = x + k \dots (2)$. Solve (1) and (2):</p> $\frac{14}{49}x + \frac{3}{7} = x + k$ <p>$x = 1$:</p> $\frac{14}{49}(1) + \frac{3}{7} = 1 + k \Rightarrow k = -0.29$

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4(b))	$f(x) = 2x^2 - x + 5$ $f'(x) = \lim_{\Delta x \rightarrow 0} \left(\frac{f(x + \Delta x) - f(x)}{\Delta x} \right)$ $f'(x) = \lim_{\Delta x \rightarrow 0} \left(\frac{2(x + \Delta x)^2 - (x + \Delta x) + 5 - (2x^2 - x + 5)}{\Delta x} \right)$ $= \lim_{\Delta x \rightarrow 0} \left(\frac{2x^2 + 4x\Delta x + 2\Delta x^2 - x - \Delta x + 5 - 2x^2 + x - 5}{\Delta x} \right)$ $= \lim_{\Delta x \rightarrow 0} \left(\frac{4x\Delta x + 2\Delta x^2 - \Delta x}{\Delta x} \right)$ $= \lim_{\Delta x \rightarrow 0} (4x + 2\Delta x - 1)$ $= 4x - 1$

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(c)	<div data-bbox="399 246 1053 627" data-label="Figure"> </div> <p data-bbox="279 638 534 672">Expression for area:</p> <p data-bbox="311 672 470 705">Area = $4xy$</p> <p data-bbox="295 716 454 761">$x^2 + y^2 = 5^2$</p> <p data-bbox="367 772 821 828">$y = \sqrt{25 - x^2}$ or $-\sqrt{25 - x^2}$ (N.A.)</p> <p data-bbox="279 884 566 940">Area, $A = 4x\sqrt{25 - x^2}$</p> <p data-bbox="343 952 774 1041">$\frac{dA}{dx} = 4x \left(\frac{-x}{\sqrt{25 - x^2}} \right) + 4\sqrt{25 - x^2}$</p> <p data-bbox="391 1052 646 1153">$= \frac{-4x^2 + 100 - 4x^2}{\sqrt{25 - x^2}}$</p> <p data-bbox="391 1164 566 1254">$= \frac{-8x^2 + 100}{\sqrt{25 - x^2}}$</p> <p data-bbox="343 1265 646 1355">$\frac{dA}{dx} = 0: \frac{-8x^2 + 100}{\sqrt{25 - x^2}} = 0$</p> <p data-bbox="343 1366 470 1411">$8x^2 = 100$</p> <p data-bbox="367 1422 726 1478">$x = \sqrt{12.5}$ or $-\sqrt{12.5}$ (N.A)</p> <p data-bbox="279 1512 598 1568">Max Area = $4x\sqrt{25 - x^2}$</p> <p data-bbox="422 1568 678 1624">$= 4\sqrt{12.5}\sqrt{25 - 12.5}$</p> <p data-bbox="422 1624 550 1668">$= 50 \text{ cm}^2$</p>