Resonant Circuits

Series RLC Resonant Circuit

Resonant Circuits

- RLC Series resonant circuit
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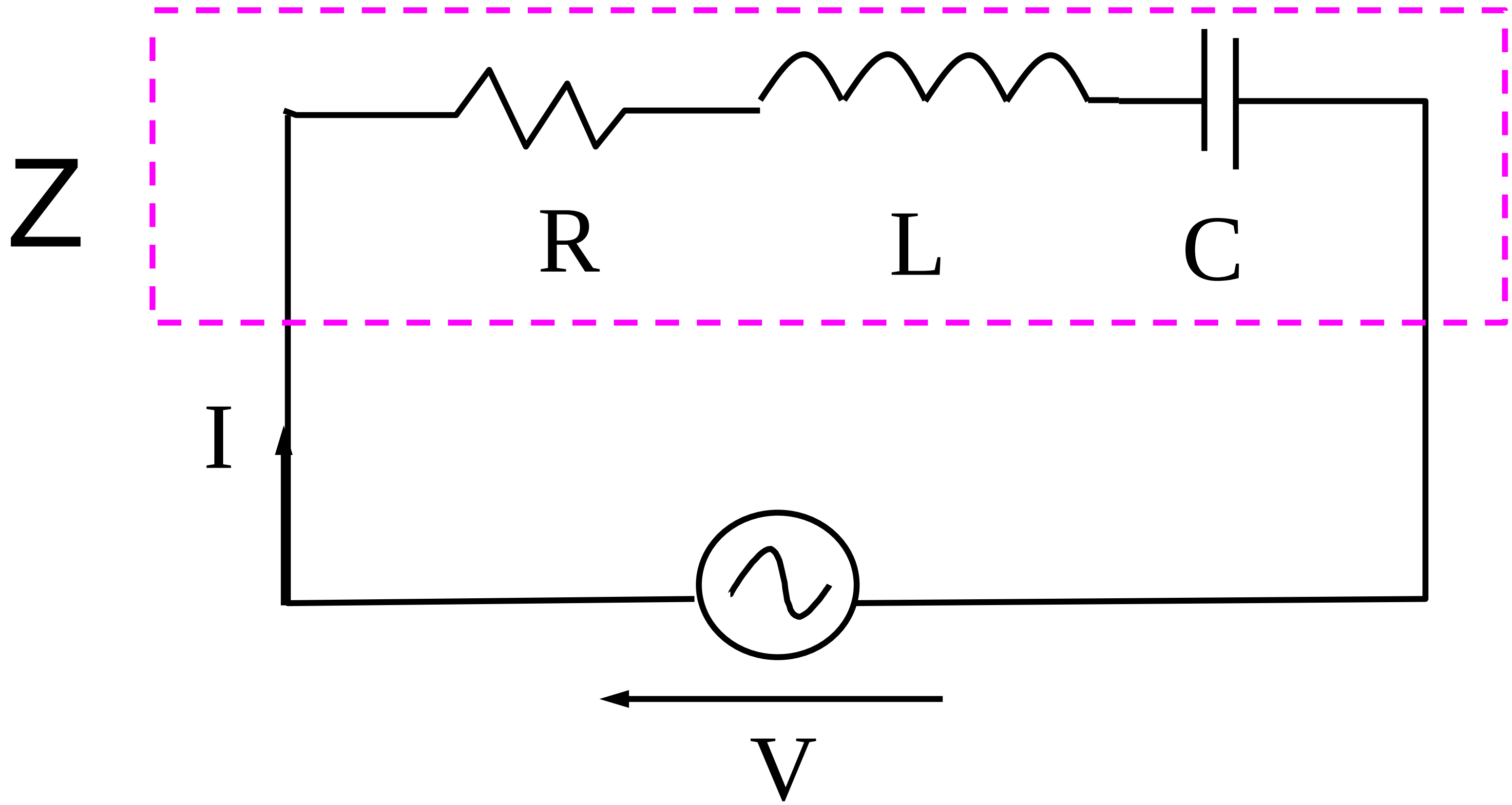
  - Phasor diagram
  - Resonance in RLC circuit
  - Graphical representation of resonance
  - Bandwidth of series RLC
  - Half power frequencies
  - Q-factor of series circuit

Series RLC Resonant Circuit



Impedance $Z = R + jX_L - jX_C$

Conditions for Series Resonance

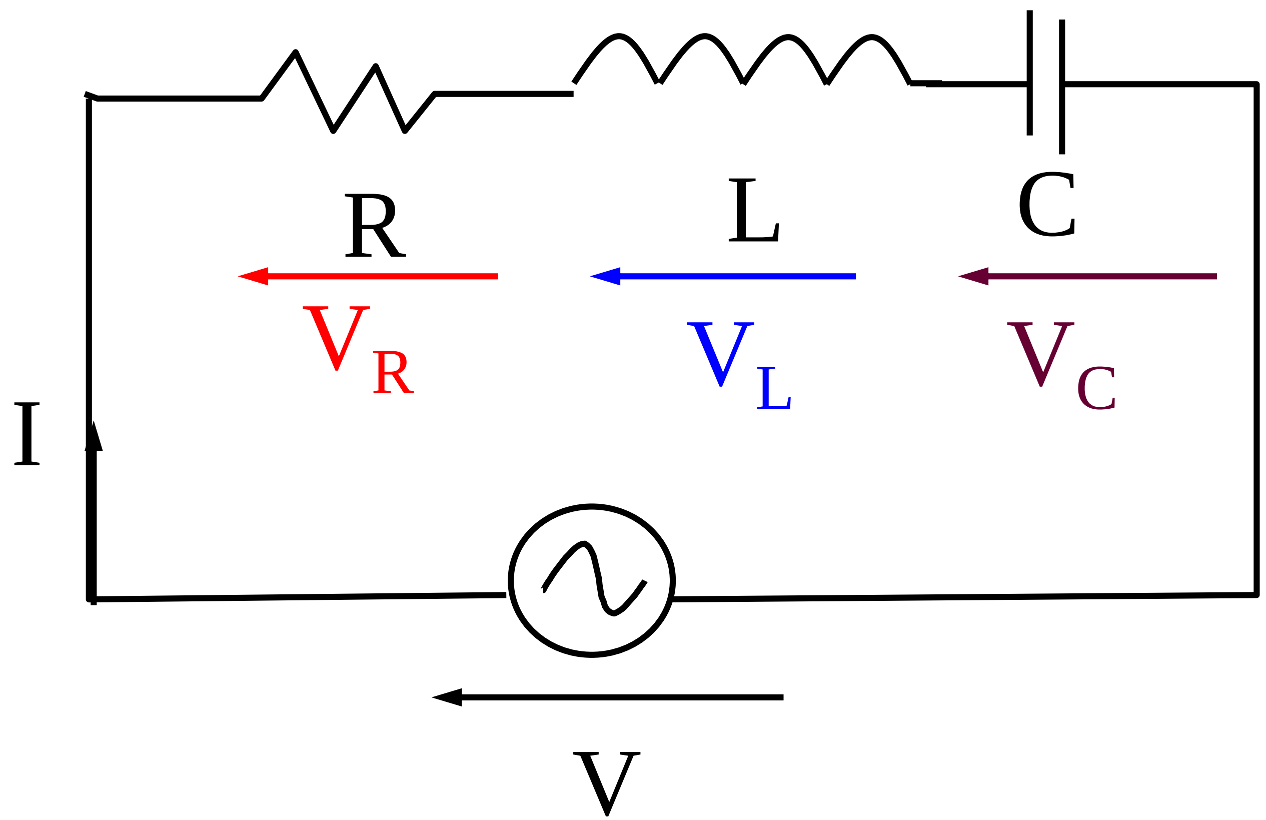
Impedance $Z = R + jX_L - jX_C$

- When Z becomes real i.e. when Z consists of only the resistive part i.e. $Z = R$
- $X_L - X_C = 0$
- Magnitude of Z, i.e. $|Z|$, is a minimum

This resonance is known as

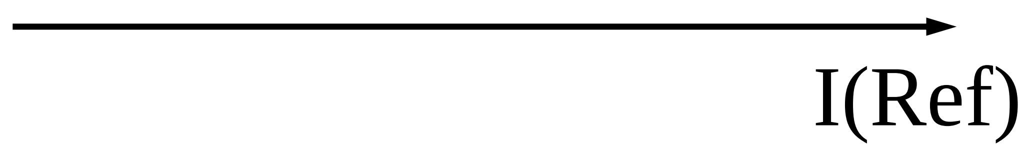
Low Impedance Resonance

Phasor Diagram of a RLC series circuit at resonance



$V = V_R + V_L + V_C$ (vector addition by KVL)

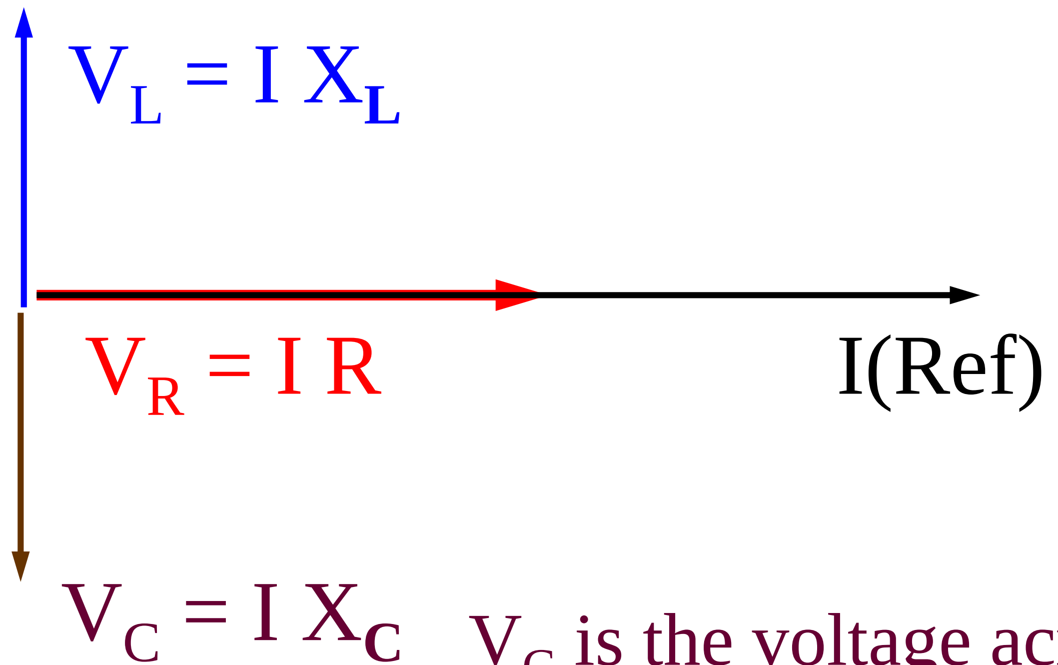
Phasor Diagram of a RLC series circuit at resonance



There are 4 voltages but only 1 current in the circuit. The current is therefore usually chosen as the Reference.

Phasor Diagram of a RLC series circuit at resonance

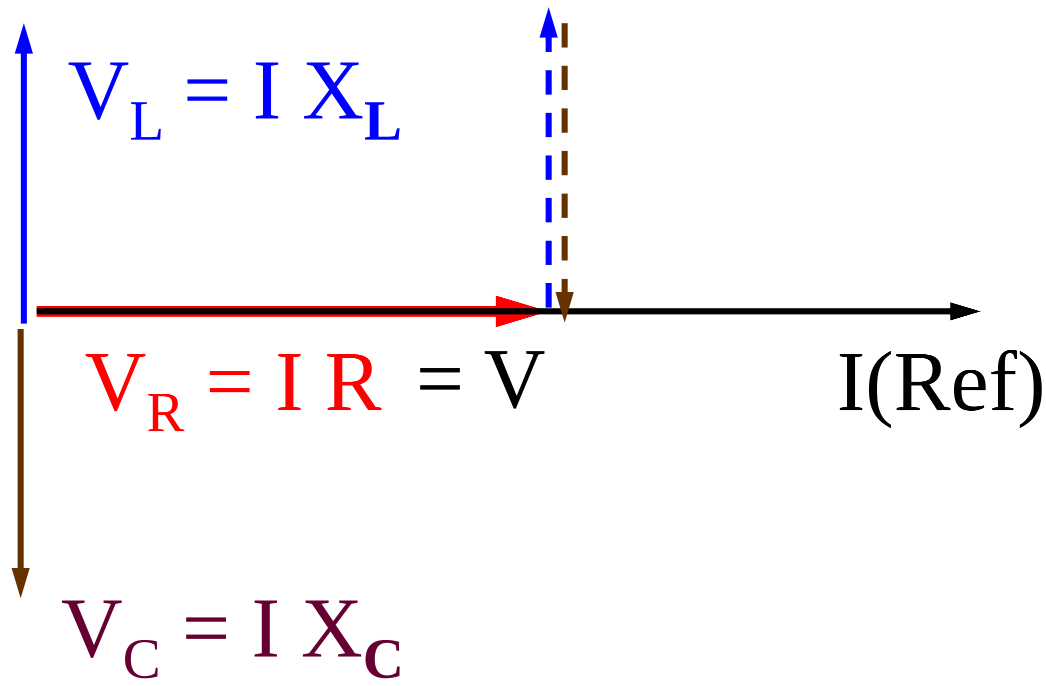
V_R is the voltage across a pure resistor and is therefore in phase with I.



V_L is the voltage across a pure inductor and is therefore leading the current I by 90°.

V_C is the voltage across a pure capacitor and is therefore lagging behind the current I by 90°.

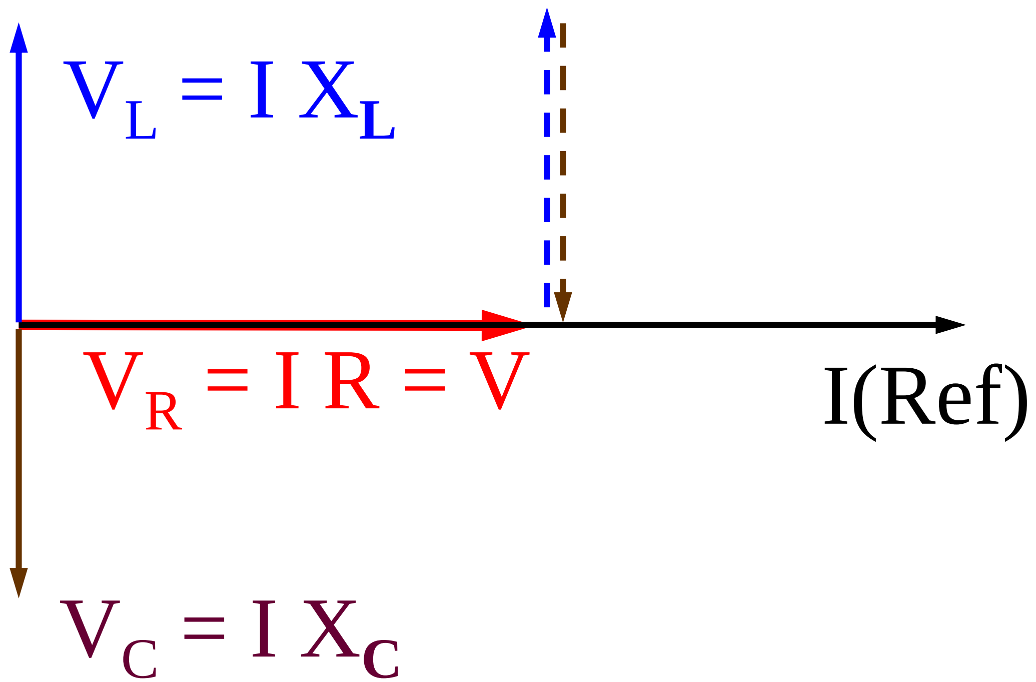
Phasor Diagram of a RLC series circuit at resonance



$V = V_R + V_L + V_C$ (By KVL)

At resonance, $|V_L| = |V_C|$, making $V_L + V_C = 0$, leaving only $V = V_R$

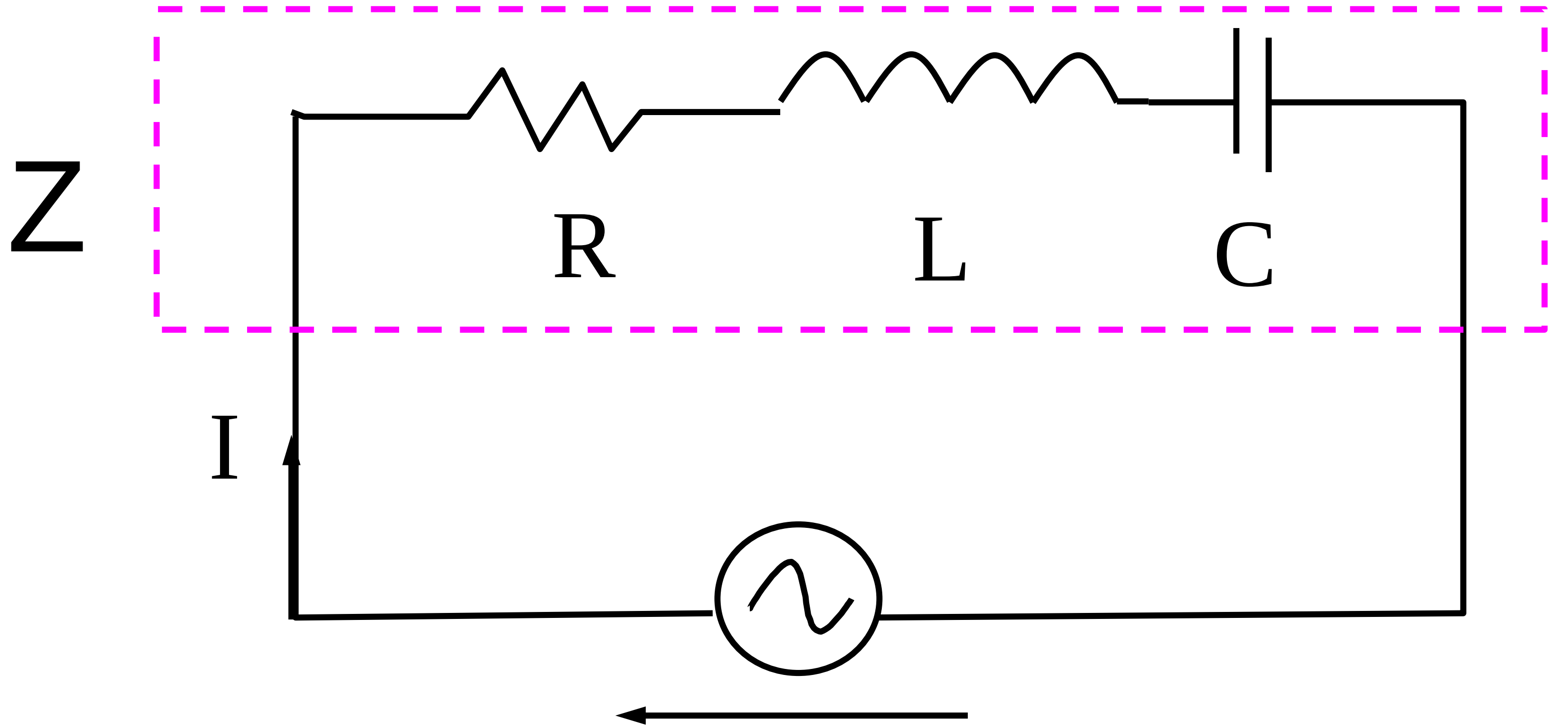
Phasor Diagram of a RLC series circuit at resonance



At resonance, the applied voltage V and current I are in phase.

Z = R and
the power factor of the circuit is unity (equals to 1).

Series RLC Circuit at Resonant



At resonance, V & I are in phase and Z = R. The circuit is still an RLC circuit but is now ***behaving*** like a pure resistor given by R

Equations for Series Resonant Frequency

Resonant Frequency (f_o) is the frequency at which the resonance occurs.

At resonance, Z is real (= R)

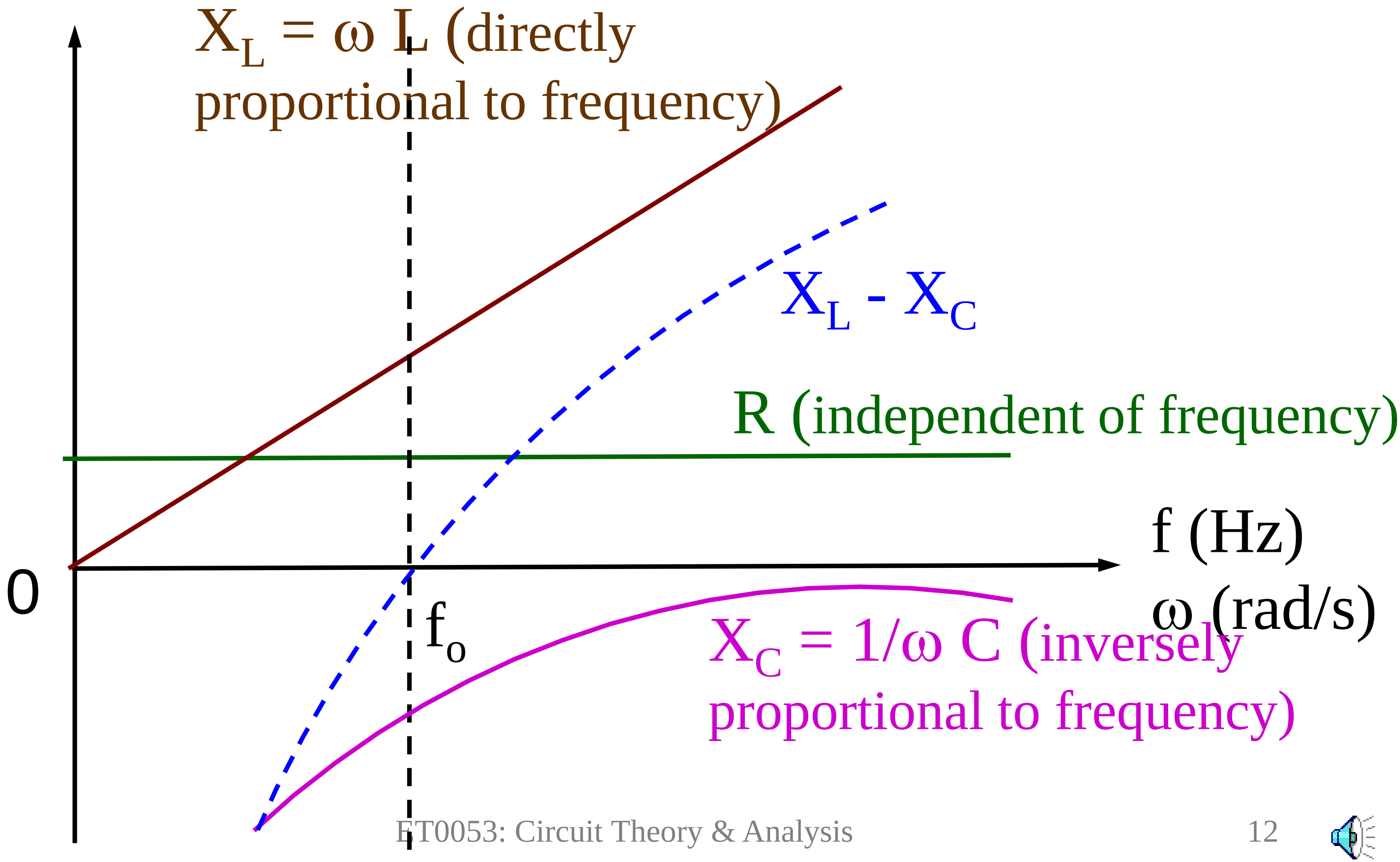
Giving $X_L - X_C = 0$

Therefore $\omega L = 1 / \omega C$,

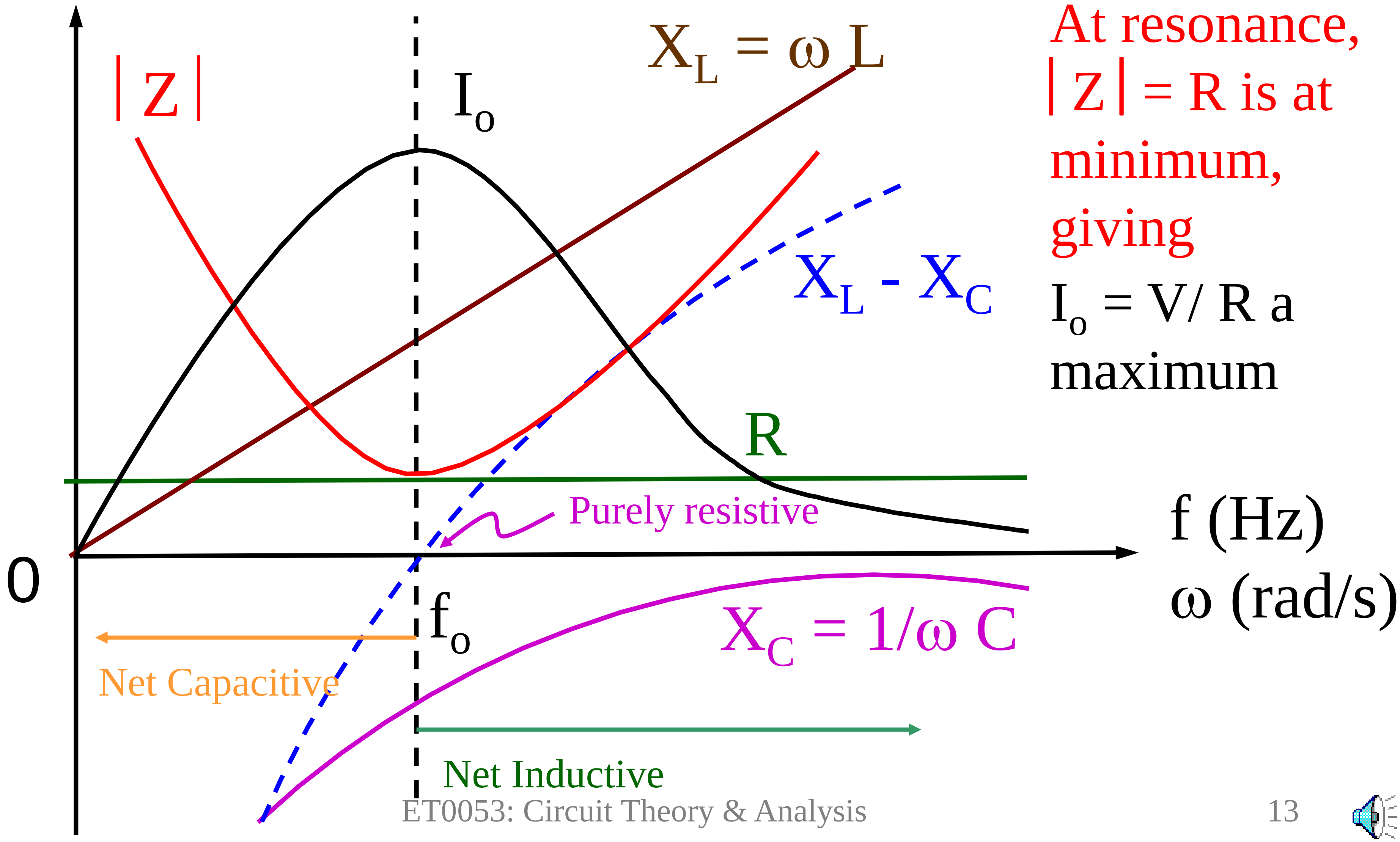
giving $\omega = 1 / \sqrt{LC}$ rad/s = ω_o (= $2 \pi f_o$)

or $f_o = 1 / 2\pi \sqrt{LC}$ Hz

Variations of Resistance, Impedance and Current with Frequency - Series Circuit



Variations of Resistance, Impedance and Current with Frequency - Series Circuit



At resonance, |Z| = R is at minimum, giving

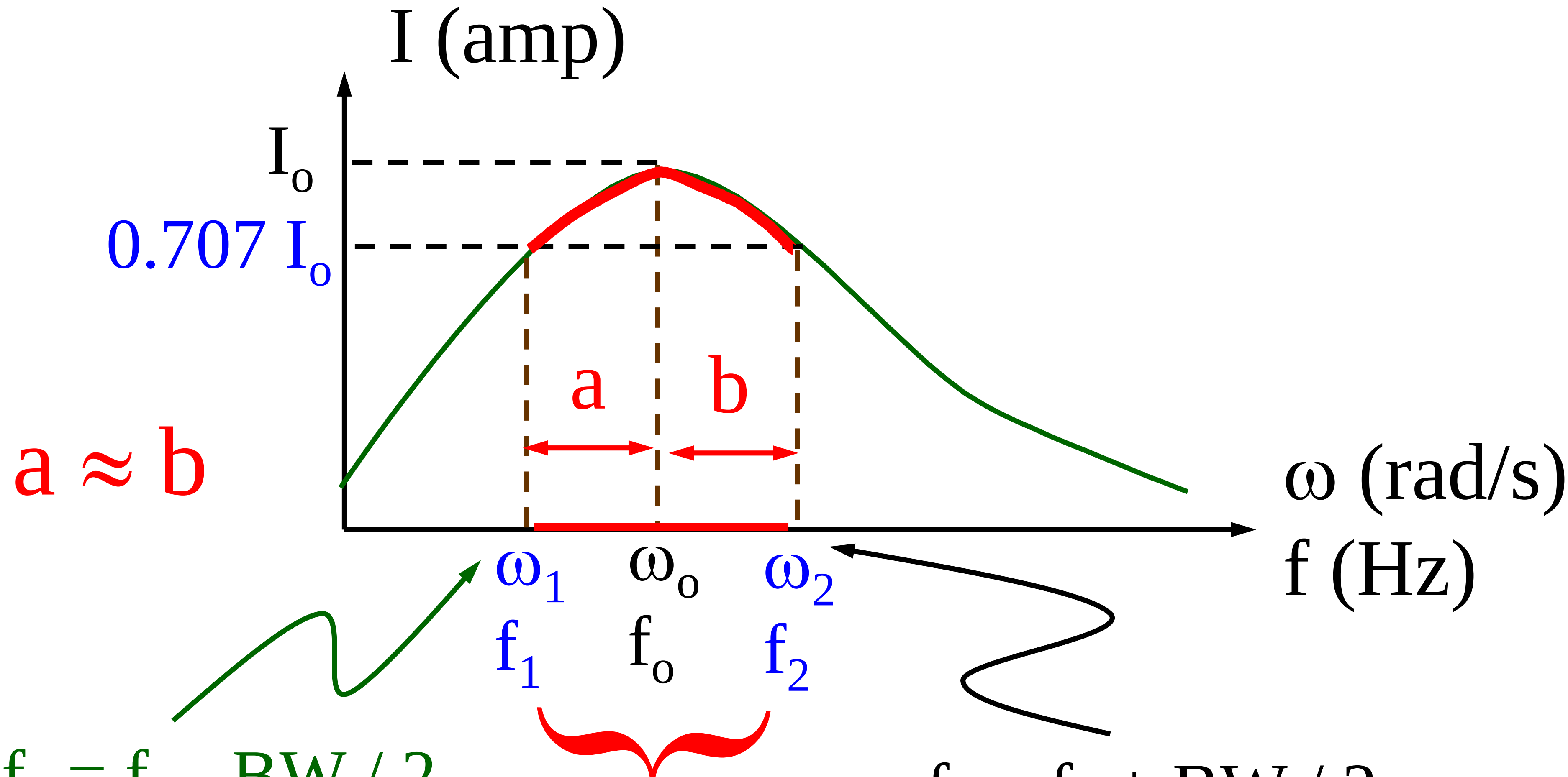
I_o = V/ R a maximum

Half Power Frequencies and Bandwidth of a series resonant circuit

f_2 and f_1 are frequencies at which the power dissipation of the circuit is half the power dissipated at resonance. f_2 and f_1 are known as upper and lower half power frequencies (or cut-off frequencies) respectively.

BW is the range of frequency within which the current I is equal to or greater than 70.7 % of its value at resonance (Io)

Bandwidth and Half Power Frequencies of a series resonant circuit



$f_1 = f_o - BW / 2$

$f_2 = f_o + BW / 2$

$$\text{Bandwidth (BW)} = f_2 - f_1 \text{ (Hz)}$$

Bandwidth of a series resonant circuit

$BW = f_2 - f_1$ (in Hz) or $= \omega_2 - \omega_1$ (in rad/s)

BW serves as a measure of the sharpness of the peak.

Quality Factor or Q Factor of Series Resonance Circuit

The ratio of the capacitor or inductor voltage at resonance to the supply voltage is a measure of the quality of a resonance circuit.

This ratio is termed the Q factor of the circuit, and is also known as the voltage magnification factor.

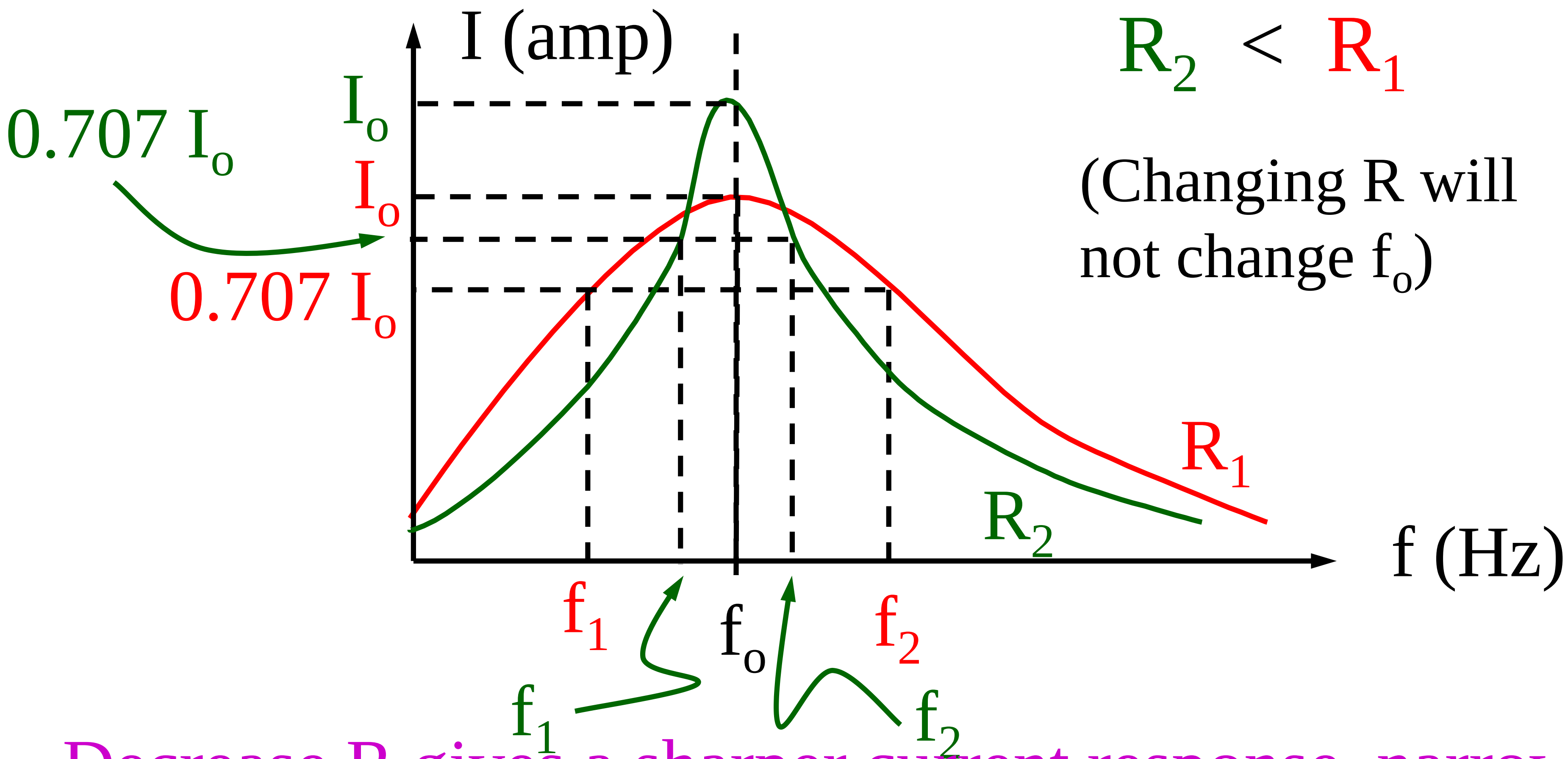
Effects of varying R, L and C on the Q factor of series RLC circuit

$$Q \text{ factor} = V_L / V = \omega_o L / R$$

$$\text{or} \quad = V_c / V = 1 / \omega_o RC$$

Q factor of a circuit at resonance can be improved (increased) by reducing the effective resistance R of the circuit.

Effect of changing R on bandwidth and Q factor



Decrease R gives a sharper current response, narrower bandwidth and higher Q factor

Relationship between Bandwidth, resonant frequency and Q factor

$$BW = f_o \,/\, Q \quad (Hz)$$

or

$$BW = 2\pi f_o \,/\, Q \quad (rad/s)$$

Example 3.1

A series resonant circuit has the following parameters: frequency at resonance = $5000/2\pi$ Hz; impedance at resonance = 56 Ω; Q factor = 25.

(a) Assuming that the capacitor is pure, calculate

(i) the capacitance value

(ii) the inductance value

(b) Determine the upper and lower half power frequencies.

Example 3.1

Frequency at resonance = $5000/2\pi$ Hz = f_o

giving $\omega_o = 2\pi f_o = 5000$ rad/s

impedance at resonance = 56 Ω = R

Q factor = 25

(i) $Q = 25 = \omega_o L / R = 5000 \times L / 56$

giving $L = 25 \times 56 / 5000 = 0.28$ H

(ii) $Q = 25 = 1/\omega_o RC = 1/5000(56)C$

giving $C = 1/(25 \times 56 \times 5000) = 0.143\ \mu F$

Example 3.1

Frequency at resonance = $5000/2\pi$ Hz = $f_o = 795.8$ Hz

Q factor = 25

(b) Bandwidth BW = $f_2 - f_1$

$= f_o / Q = (5000/2\pi) / 25$

$= 31.83$ Hz

$f_1 = f_o - BW / 2 = 795.8 - 31.83/2 = 779.9$ Hz

$f_2 = f_o + BW / 2 = 795.8 + 31.83/2 = 811.7$ Hz