Revision Tutorial

Answers

I. Partial Differentiation

1(a). $5x^4 + y + \dfrac{1}{x+2y}$, $x + \dfrac{2}{x+2y}$

1(b). $2e^{2x}\sin(y)$, $e^{2x}\cos(y)$

1(c). $2x\sin^2 y$, $x^2 \sin 2y$

1(d). $3x^2 + 10xy$, $5x^2 + 6y^2$

1(e). $2xy + 2y^2 - 2$, $x^2 + 4xy$

2(a). $y + 3\sin(3x)$, $9e^{9y} + x$

2(b). $3\sqrt{x^2+5y^2} + \dfrac{3x^2}{\sqrt{x^2+5y^2}}$, $\dfrac{15xy}{\sqrt{x^2+5y^2}}$

2(c). $3y$, $3x + \dfrac{1}{y} + 4y^3$

3. $\dfrac{1}{2}$, $\cos\left(\dfrac{1}{2}\right) \approx 0.878$

4. (a) $\dfrac{3}{5}$ (b) $-\dfrac{1}{3}$

5. 11.93cm²/sec

6. 0.04 m^3/s

7. Power decreases by 375 watts

8. 3.3π cm^3

9. 4%

10. 1%

II. Integrate functions of linear functions and using trigo identities

1(a) $-\dfrac{1}{6}(1-2x)^3 + C$

1(b) $-\dfrac{2}{9}(4-3x)^{3/2} + C$

1(c) $-\dfrac{1}{8(2x-3)^4} + C$

1(d) $\dfrac{1}{8}\ln|8x+3| + C$

1(e) $-\ln|25-4x| + C$

1(f) $\dfrac{1}{3}\sin\left(3x - \dfrac{\pi}{6}\right) + C$

1(g) $-\dfrac{1}{2}\cos(2x+1) + C$

1(h) $2e^{\frac{x}{2}+5} + C$

2(a) 4

2(b) 0.2882

3(a) $-\dfrac{1}{2}\cos 2x + C$

3(b) $\dfrac{1}{2}\tan 2x + C$

3(c) $\tan 2x - 2x + C$

3(d) $\dfrac{\cos 2x}{2} - \dfrac{\cos 8x}{8} + C$

3(e) $\dfrac{3}{2}(\sin t - \dfrac{1}{4}\sin 4t) + C$

3(f) $\dfrac{\sin 3\theta}{6} - \dfrac{\sin 5\theta}{20} - \dfrac{1}{4}\sin\theta + C$

4(a) 2.41 4(b) 2

5 0.98 amp

III. Integration by substitution

1(a) $\frac{1}{10}\left(x^2-3\right)^5+C$ 1(b) $\frac{1}{2(4-x^2)}+C$

1(c) $\frac{1}{3}\sin^3\theta+C$ 1(d) $\frac{\left(x^3-10\right)^9}{9}+C$

1(e) $-\frac{1}{4}\ln\left|1-2x^2\right|+C$ 1(f) $ln\left|ln\,x\right|+C$

1(g) $-\frac{1}{4}e^{3-2t^2}+C$ 1(h) $\frac{3e^{\frac{y^2}{3}}}{2}+C$

1(i) $-5\sqrt{1-e^{2x}}+C$ 1(j) $\frac{1}{4}\cos t^4+C$

2(a) 1/24 (b) 1.07 (c) 13.33 (d) $\frac{\pi}{8}$

3(a) $2\ln\left|e^x-5\right|+C$ 3(b) $-\sqrt{1-2x^2}+C$

3(c) $-\frac{13+16t}{96\left(4t-5\right)^4}+C$ or $-\frac{1}{24\left(4t-5\right)^3}-\frac{11}{32\left(4t-5\right)^4}+C$

3(d) $-\cos x+\frac{\cos^3 x}{3}+C$

3(e) $\frac{2}{5}(4-x)^{5/2}-\frac{8}{3}(4-x)^{3/2}+C$

3(f) $\frac{1}{40}(1+4e^x)^{5/2}-\frac{1}{24}(1+4e^x)^{3/2}+C$

IV. Integration by partial fraction

1(a). $-\ln\left|x+3\right|+\frac{2}{3}\ln\left|3x-1\right|+C$

1(b). $-\ln\left|x+1\right|+\ln\left|x-2\right|+C$

1(c). $-\ln\left|x\right|+\frac{3}{2}\ln\left|x-3\right|-\frac{1}{2}\ln\left|x+1\right|+C$

2(a). $\ln\left|x+1\right|-\frac{3}{4}\ln\left|2x-1\right|+\frac{1}{4(2x-1)}+C$

2(b). $2\ln\left|x\right|+\frac{1}{2}\ln\left|x^2+4\right|-\frac{1}{2}\tan^{-1}\frac{x}{2}+C$

3(a). $\frac{4}{3\left(1-x\right)}-\frac{17x+20}{3\left(x^2+2\right)}$

3(b). $\frac{1}{6}\left[-20\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-8\ln\left|1-x\right|-17\ln\left|2+x^2\right|\right]+C$

4(a). $\frac{1}{x-1}+\frac{2}{\left(x+2\right)^2}-\frac{1}{x+2}$

4(b). $\ln|x-1| - \dfrac{2}{x+2} - \ln|x+2| + C$

5(a). $2\ln|x| - \dfrac{1}{2}\ln|x^2+3| - \sqrt{3}\tan^{-1}\dfrac{x}{\sqrt{3}} + C$

5(b). 0.349

V. Integration by completing the square

1(a). $\sqrt{3}\tan^{-1}\left(\dfrac{x+3}{\sqrt{3}}\right) + C$

1(b). $\dfrac{1}{2}\ln\left([x-5]^2 + 25\right) + C$

2(a). $a = 3, b = 4$

2(b). 0.1865

3. $\dfrac{1}{\sqrt{3}}\tan^{-1}\dfrac{(2x-1)}{\sqrt{3}} - \dfrac{1}{2}\ln|x^2 - x + 1| + C$

4. $\dfrac{1}{8}\tan^{-1}\left(\dfrac{x-2}{8}\right) + C$, $\dfrac{1}{16}\tan^{-1}\left(\dfrac{x-2}{8}\right) + C$

5(a). $\dfrac{1}{\sqrt{3}} tan^{-1}\left(\dfrac{x-3}{\sqrt{3}}\right) + C$

5(b). $x + \dfrac{1}{\sqrt{3}} tan^{-1}\left(\dfrac{x-3}{\sqrt{3}}\right) + C$

VI. Integration by parts

1(a) $-\dfrac{x^2}{3}\cos 3x + \dfrac{2}{9}x\sin 3x + \dfrac{2}{27}\cos 3x + C$ 1(b) 0.0384 1(c) 4.575

1(d) $\dfrac{1}{29}e^{5x}(2\sin 2x + 5\cos 2x) + C$ 1(e) $x\,\ell n(1-4x) - x - \dfrac{1}{4}\ell n(1-4x) + C$

2(a) $-\dfrac{\ln|x|}{4(2x+1)^2} + \dfrac{1}{4}\left[\ln|x| - \ln|2x+1| + \dfrac{1}{2x+1}\right] + C_1$

2(b) $\dfrac{1}{4}\left[-\sin^{-1}(2x)\sqrt{1-4x^2} + 2x\right] + C$

3. $\dfrac{1}{3}\left(2\sqrt{3}\,\pi - \dfrac{\pi}{2} - 2 + \ln 2\right)$ or 2.668

VII. Simpson's Rules

1(a). 1.3001

1(b). Simpson's Rule cannot be calculated using 7 strips since n = 7 is odd.

2. 0.993

3. 0.984

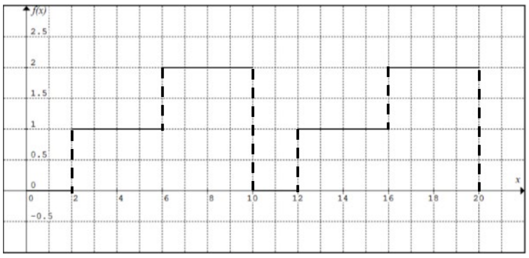
4. 0.3697 or 0.370 (3 dp)

5. 159.62

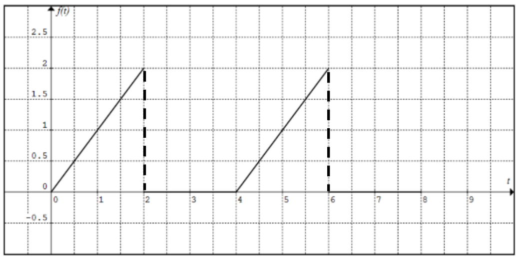
6. 14.77 mA

VIII. Fourier Series

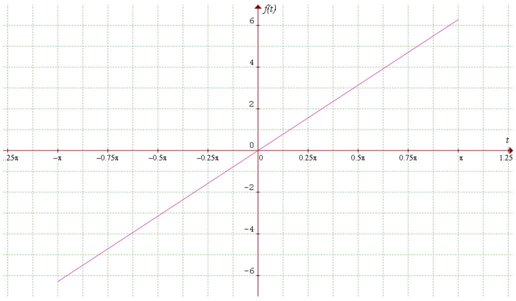
1. (a) neither



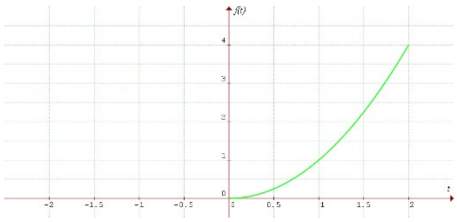
(b) neither



(c) odd



(d) neither



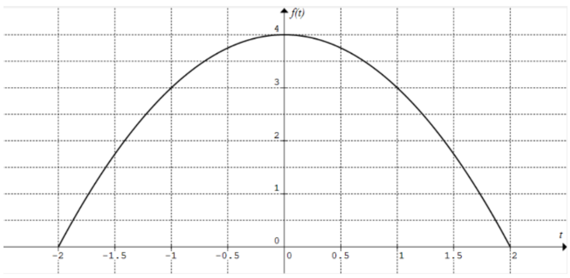
2. (a) $\dfrac{1}{2}$ (b) $\dfrac{1}{\pi}\sin(\pi t)$ (c) $\dfrac{2}{\pi}\cos\left(\dfrac{3\pi}{2}t\right)$

3. $a_0 = 1$, $a_n = 0$, $b_n = \dfrac{4}{n\pi}\left[1-\cos(n\pi)\right]$, $f(t) = 1 + \dfrac{8}{\pi}\sin t + \dfrac{8}{3\pi}\sin 3t + \cdots$

4(a). $f(t) = \dfrac{1}{2} - \dfrac{4\cos(\pi t)}{\pi^2} - \dfrac{4\cos(3\pi t)}{9\pi^2} + \ldots$

4(b). $f(t) = \dfrac{12\sin(\pi t)}{\pi^3} - \dfrac{3\sin(2\pi t)}{2\pi^3} + \dfrac{4\sin(3\pi t)}{9\pi^3} + \ldots$

5(i).



6. Assume that the function is continuous with period $T = 2$.

$$i(t) = \begin{cases} -20t + 10, & 0 < t < 1 \\ 20t - 30, & 1 < t < 2 \end{cases} \qquad i(t+2) = i(t)$$

$i(t) = \dfrac{80\cos(\pi t)}{\pi^2} + \ldots$, and $V(t) = -\dfrac{8\sin(\pi t)}{\pi} + \ldots$