Chapter 2

Signals and Spectra

(Part 1 of 5)

Introduction

- Information is represented as electrical signals in electrical communication systems.

- Information may be in variety of forms

  - Wanted information, e.g. human voices, MP3 music, MPEG videos and characters/code output from a computer,

  - Unwanted signals, e.g. noise and interference

- The purpose of electrical communication system is to convey information signals from a source to the destination.

Introduction

Electrical signal

- Variation of electrical voltage/current in time and expressed as a function of time.
  - At any instant of time, there is a unique value of the function.
  - Analog, discrete (sampled analog), or digital.
  - Periodic or non-periodic.

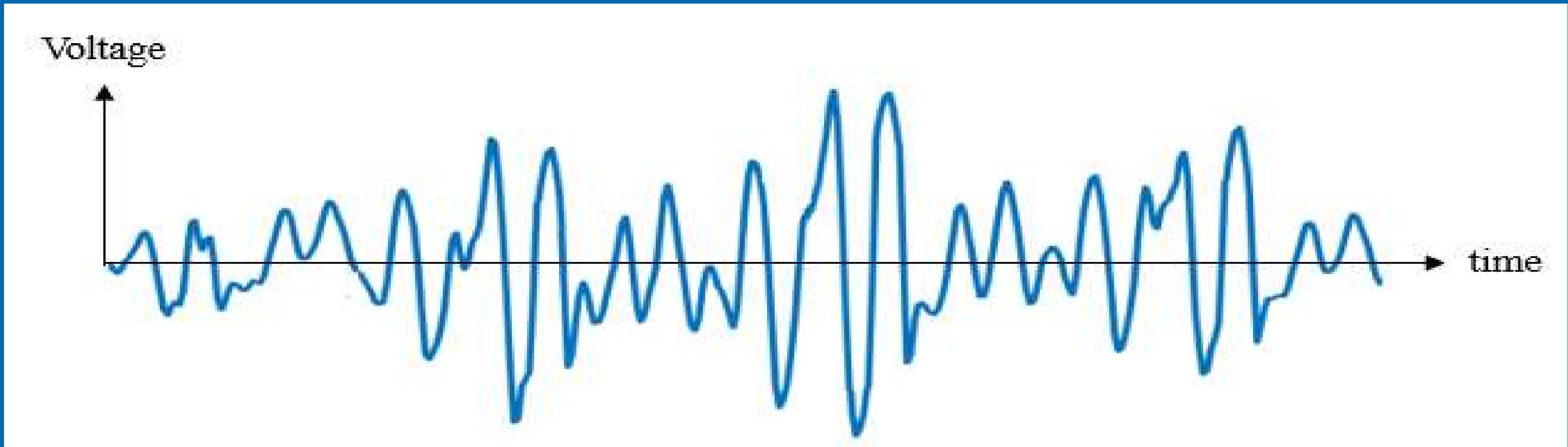
- Expressed as a function of frequency.

2.1 Signal classification

Analog and digital signals

- **Analog signals are represented as a continuous function of time and have infinite number of voltage levels.**

e.g. speech signal



2.1 Signal classification

Analog and digital signals

- Digital signals have only a finite number of possible levels.

e.g. binary signals

2.1 Signal classification

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Periodic and Non-periodic signals

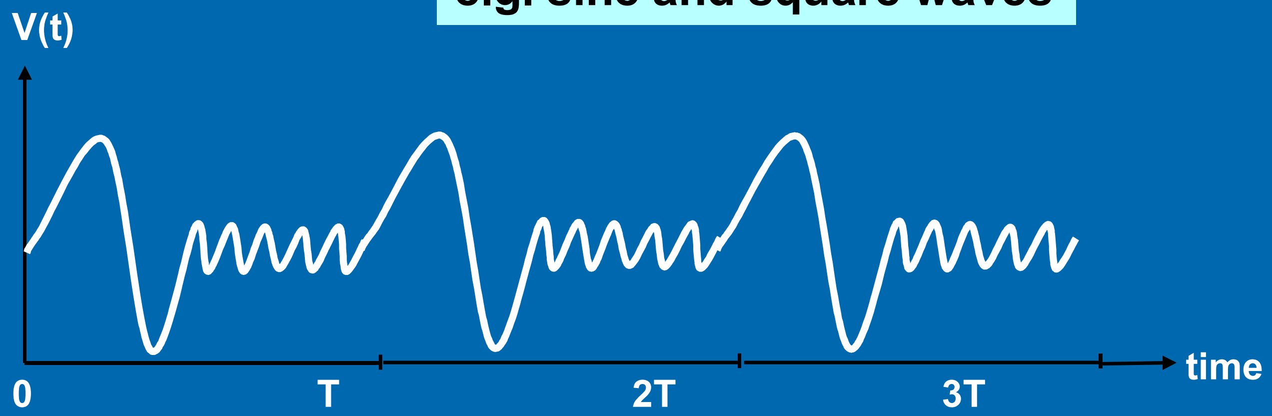
Periodic signals

- A signal v(t) is said to be periodic if it satisfies the condition:

$$v(t) = v(t + T)$$

Repetitive waveforms

e.g. sine and square waves



T is the duration of one cycle or Period

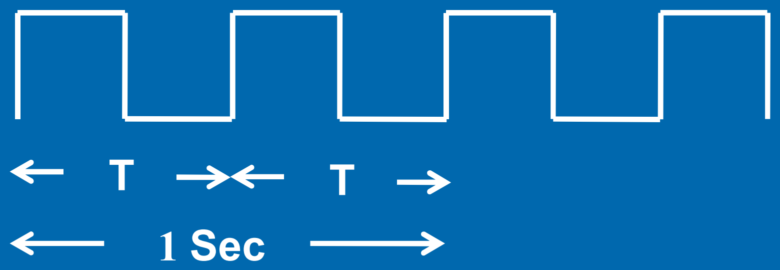
2.1 Signal classification

Periodic and Non-periodic signals

Fundamental frequency of periodic signals

- The number of cycles per second measured in cycles per second or Hz.
- Rate at which a periodic signal repeats its waveform.

f = 1/T



The period T is 0.5 sec.
The fundamental frequency f is 2 Hz (or cycles/sec).

2.1 Signal classification

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<u>**Periodic and Non-periodic signals**</u>

Harmonics

Multiples of fundamental frequency

Angular frequency, ω

Frequency expressed in radians/sec, ω :

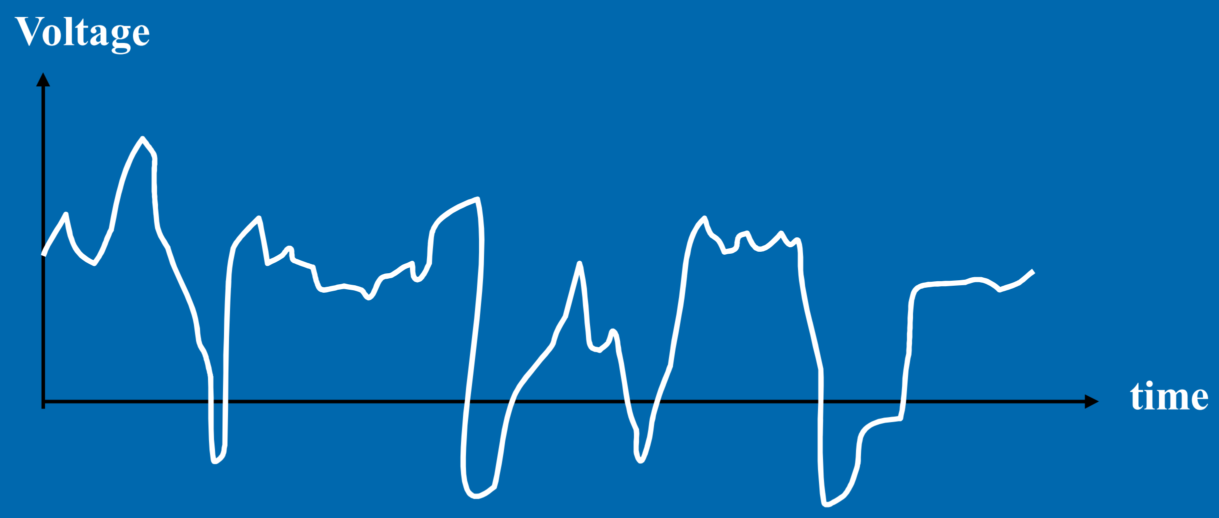
$\omega = 2\pi \times$ number of cycles/sec $= 2\pi f$ radians/sec.

2.1 Signal classification

Periodic and Non-periodic signals

Non-periodic signals

- Signals that are not repetitive. **e.g. speech and music signals**



2.1 Signal classification

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Continuous-time and discrete-time signals

Continuous-time signals

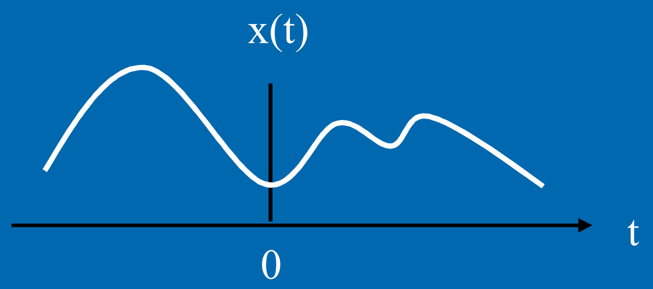
- Defined at all instants of time. **e.g. sine, speech and music signals**
- Naturally produced by a transducer when converting a physical signal into an electrical signal.
  - e.g. produced when a microphone converts a sound wave into electrical signal.
- May have zero values at some instants of time or even for certain interval of time.

2.1 Signal classification

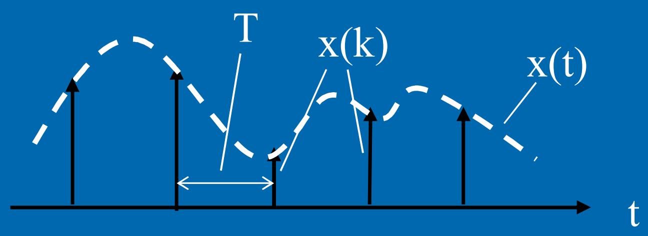
Continuous-time and discrete-time signals

Discrete-time signal, x(k)

- Defined only at discrete instants of time.
- The time variable, k takes discrete values only. i.e. it takes values in a set of integers.
- Usually derived from a continuous time signal x(t) by a sampling process.



continuous time signal x(t)



Discrete-time signal x(k)

2.2 Basic Signals

Sinusoidal signals

$$x(t) = V_p \cos(2\pi f t + \varphi) = V_p \sin(2\pi f t + \varphi')$$ Continuous-time

$$x(k) = V_p \cos(2\pi f k + \varphi) = V_p \sin(2\pi f k + \varphi')$$ Discrete-time

or

$$x(t) = V_p \cos(2\pi f t + \varphi) = V_p \cos(\omega t + \varphi)$$

$$x(k) = V_p \cos(2\pi f k + \varphi) = V_p \cos(\omega k + \varphi)$$

$\omega = 2\pi f$

- **Fully described by three characteristics:**

Peak amplitude (V_P): the maximum voltage

Frequency (f): the rate at which a sinusoidal wave repeats its waveform.

Initial phase (φ): the position of the waveform at $t = 0$.

2.2 Basic Signals

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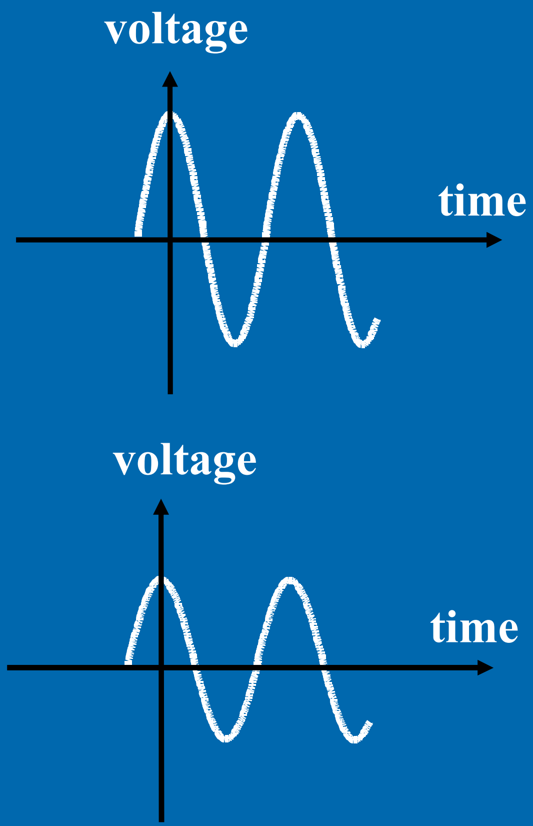
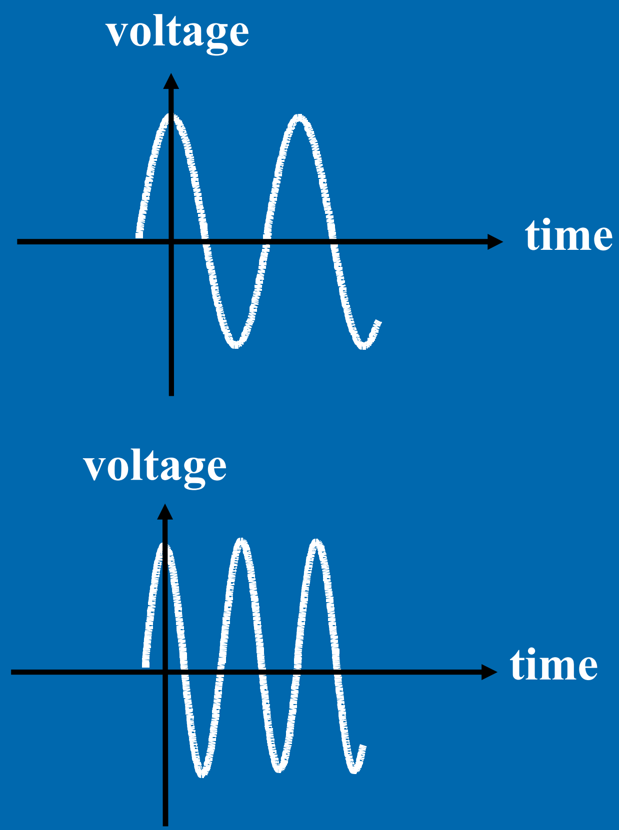
Sinusoidal signals

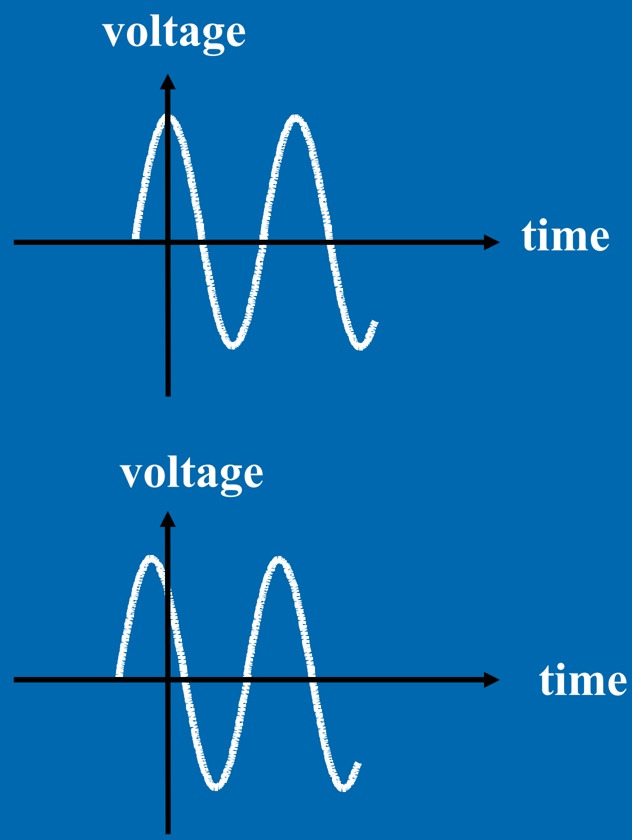
Sinusoidal signals with different amplitude, frequency and initial phase



(a)Sinusoidal signals with different amplitude



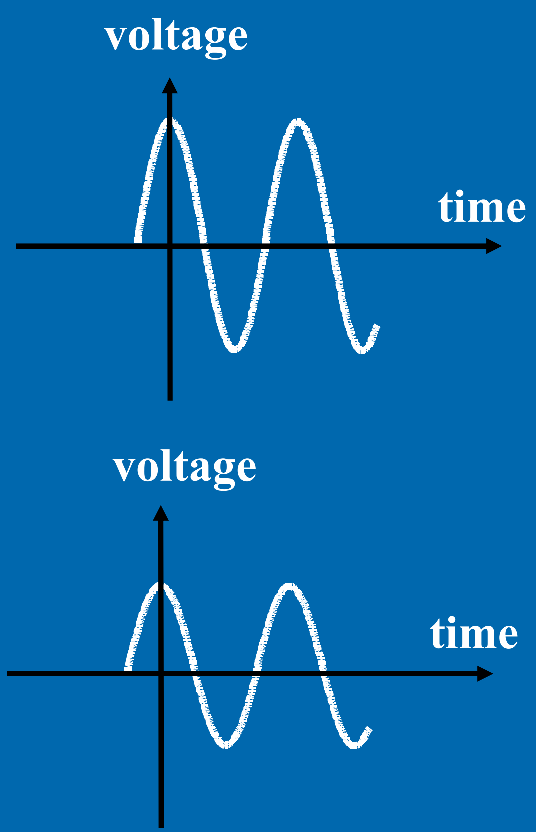
(b) Sinusoidal signals with different frequency



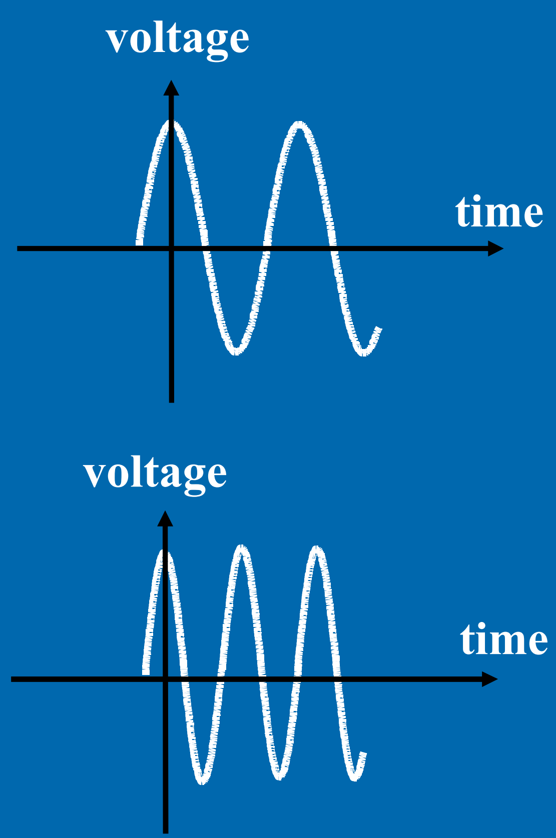
(c) Sinusoidal signals with different initial phase

Sinusoidal signals

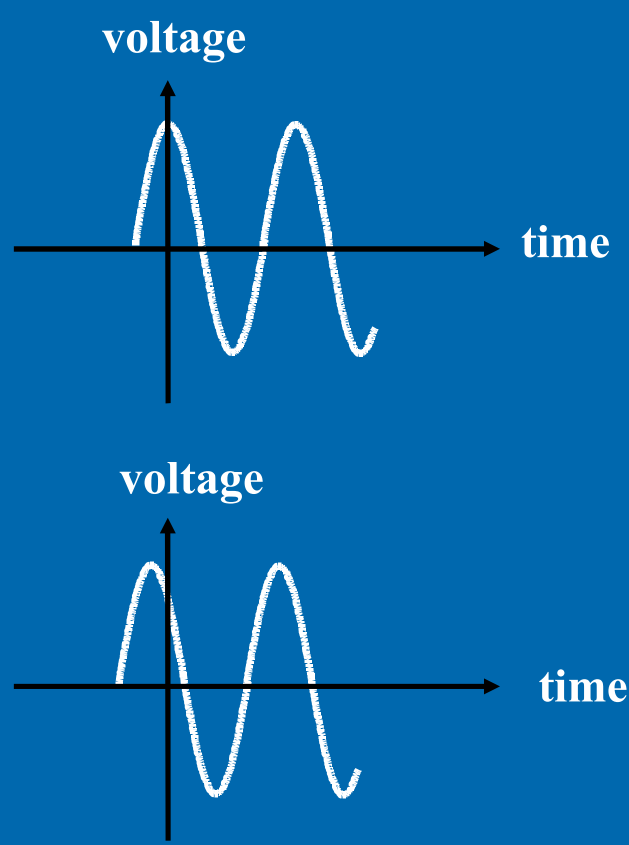
Sinusoidal signals with different amplitude, frequency and initial phase



(a)Sinusoidal signals with different amplitude



(b) Sinusoidal signals with different frequency



(c) Sinusoidal signals with different initial phase

2.2 Basic Signals

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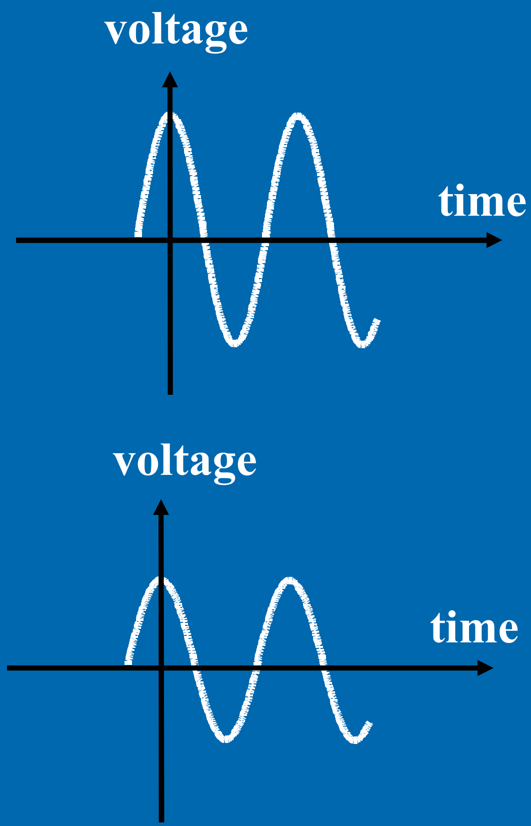
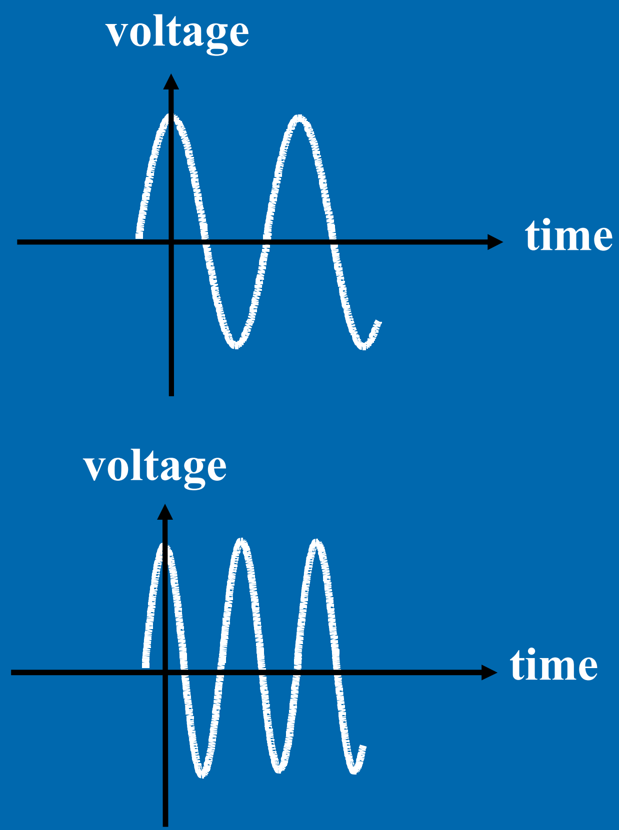
Sinusoidal signals

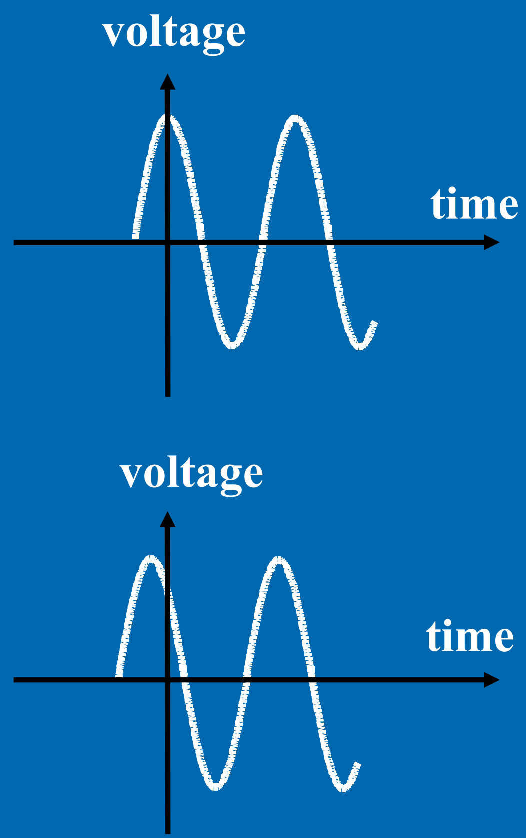
Sinusoidal signals with different amplitude, frequency and initial phase



(a)Sinusoidal signals with different amplitude



(b) Sinusoidal signals with different frequency



(c) Sinusoidal signals with different initial phase

Example 2.1

Extract the parameters from the following sine waves.

v(t)	Peak Voltage	freq(Hz) = $\frac{\omega}{2\pi}$	Phase φ
$6\sin(4\pi f_0 t + \frac{\pi}{4})$			
$4\sin(6\pi f_0 t + \frac{\pi}{2})$			
$8\sin 5000\pi t$			
$\sin 3000t$			

Solution

Standard expression: $V_p \sin(2\pi ft + \varphi)$

$V_p = 6$

$2\pi ft = 4\pi f_0 t$

$f = 2f_0$

$$6\sin\left(4\pi f_0 t + \frac{\pi}{4}\right)$$

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Solution

Standard expression: $V_p \sin(2\pi f t + \varphi)$

$V_p = 6$ $f = 2f_0$ $\varphi = \frac{\pi}{4}$

$$6 \sin\left(4\pi f_0 t + \frac{\pi}{4}\right)$$

Solution

Standard expression: $V_p \sin(2\pi ft + \varphi)$

v(t)	Peak Voltage	freq(Hz) $= \frac{\omega}{2\pi}$	Phase φ
$6\sin(4\pi f_0 t + \frac{\pi}{4})$	6	$2f_0$	$\frac{\pi}{4}$
$4\sin(6\pi f_0 t + \frac{\pi}{2})$	4	$3f_0$	$\frac{\pi}{2}$
$8\sin 5000\pi t$	8	2500	0
$\sin 3000t$	1	$1500/\pi$	0

2.2 Basic Signals

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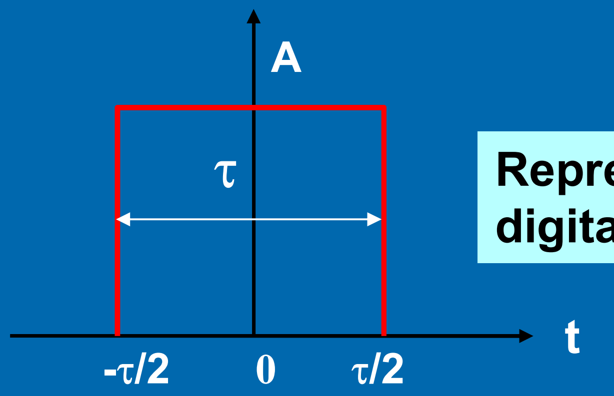
Rectangular pulse (rect function)

$$x(t) = A\,rect\left(\frac{t}{\tau}\right) = \begin{cases} A, & |t| \le \frac{\tau}{2} \\ 0, & |t| > \frac{\tau}{2} \end{cases}$$



Represent binary 1 in digital communications.

- **A rectangular-shaped pulse centred at t = 0 with a width of τ and height of A.**

Example 2.2

Sketch x(t) = 4 rect (0.5t)

Solution

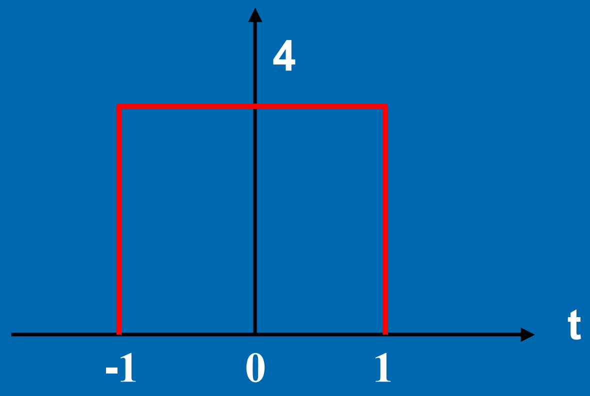
Standard expression: $x(t) = A rect(\frac{t}{\tau})$

$$x(t) = 4\,\mathrm{rect}(0.5\,t)$$

$$= 4\,\mathrm{rect}\left(\frac{t}{(1/0.5)}\right)$$

$$= 4\,\mathrm{rect}\left(\frac{t}{2}\right)$$

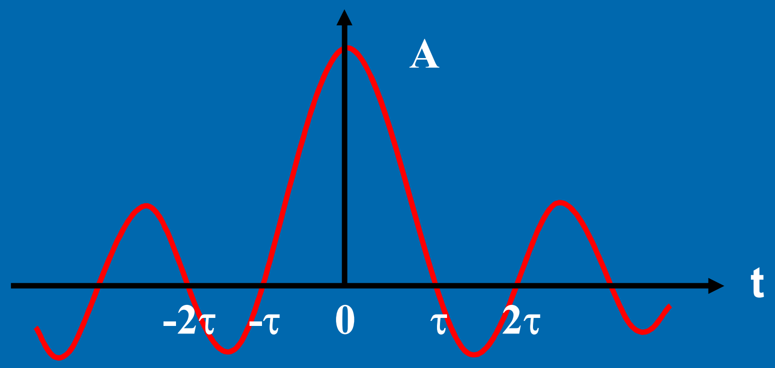
A = 4 and $\tau = 2$



2.2 Basic Signals

Sinc Function

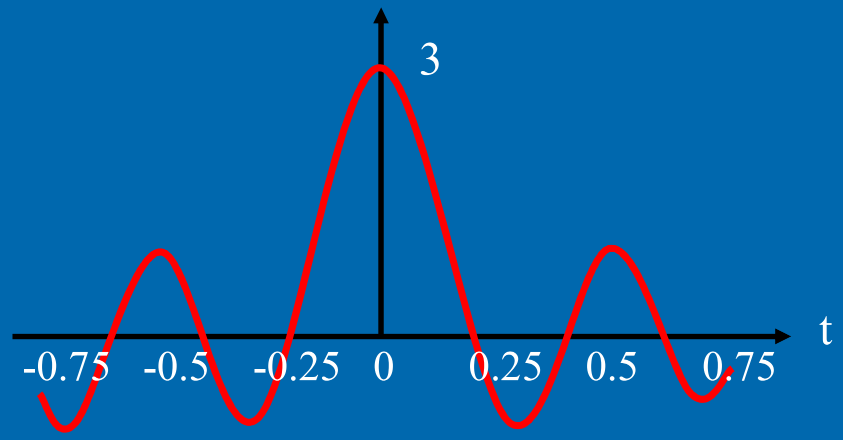
$$x(t) = A\text{sinc}\left(\frac{t}{\tau}\right) = \frac{A\sin\left(\pi\frac{t}{\tau}\right)}{\pi\frac{t}{\tau}}$$



- An even function passes through zero at all positive and negative multiples of τ i.e. $t = \pm\tau, \pm2\tau, \ldots$
- Reaches maximum of A at t = 0.

Example 2.3

Sketch $x(t) = 3\,\text{sinc}(4t)$



Solution

Standard expression: $x(t) = A\,\text{sinc}\left(\frac{t}{\tau}\right)$

$$x(t) = 3\,\text{sinc}(4t)$$

$$= 3\,\text{sinc}\left(\frac{t}{(1/4)}\right)$$

$$= 3\,\text{sinc}\left(\frac{t}{0.25}\right)$$

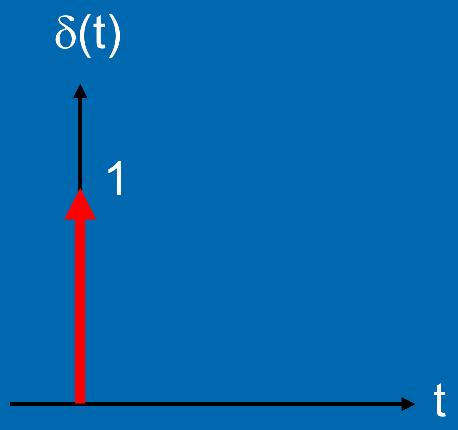
$A = 3$ and $\tau = 0.25$

2.2 Basic Signals

Unit Impulse

$$x(t) = \delta(t) = 0 \quad t \neq 0$$

and $$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

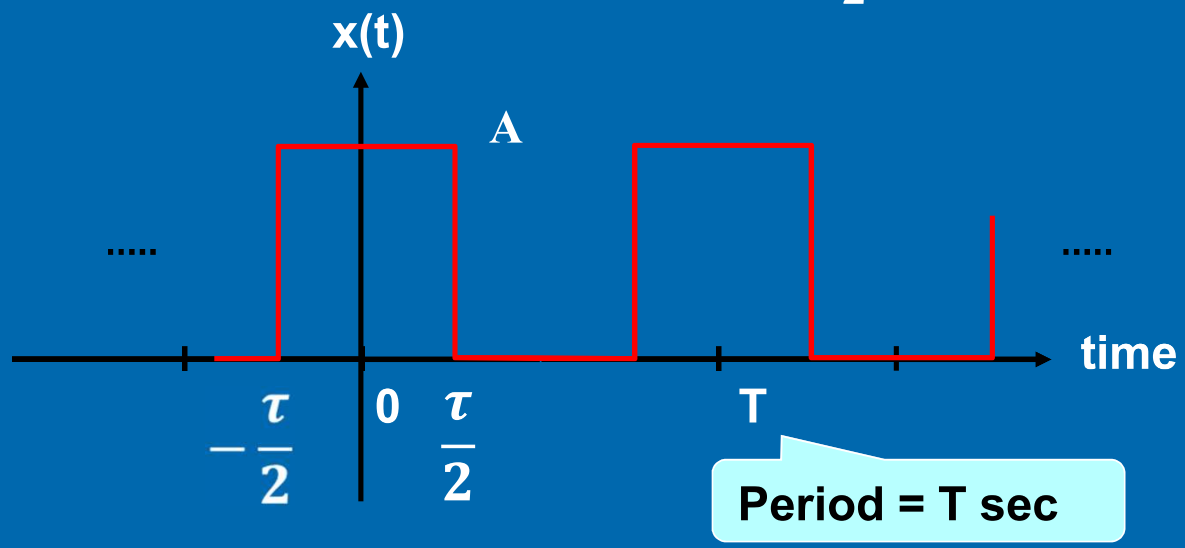


- **A pulse with zero width and the area under the pulse is unity**
- **An idealization of a signal that occurs in an extremely short period of time with an extremely large amplitude.**
- **Though not existed in practice, it is very useful for simplifying complicated analysis of communication systems.**

2.2 Basic Signals

Periodic pulse train

$$y(t) = \sum_{n=-\infty}^{\infty} x(t - nT), \text{ where } x(t) = \begin{cases} A, & |t| \le \frac{\tau}{2} \\ 0, & |t| > \frac{\tau}{2} \end{cases}$$

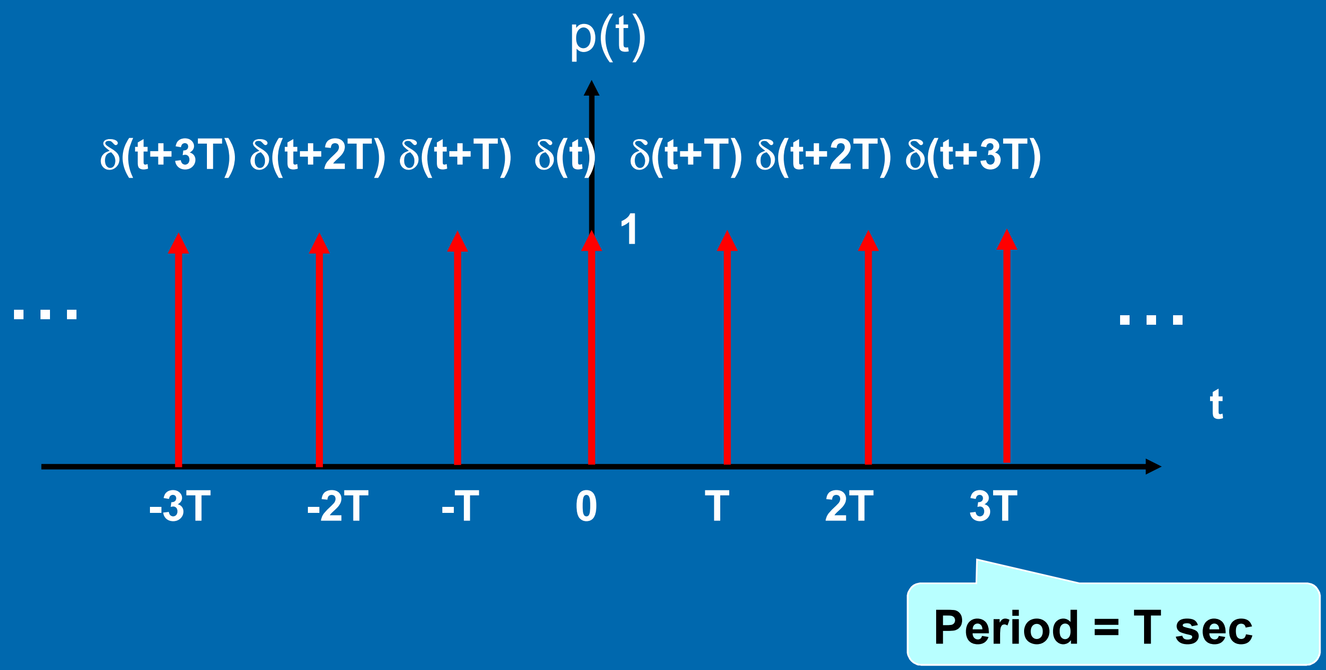


- Rectangular pulses with duration τ uniformly spaced apart
- The duty cycle of a periodic pulse train: $\frac{\tau}{T}$

2.2 Basic Signals

Impulse train

$$p(t) = \sum_{n=-\infty}^{n=\infty} \delta(t - nT)$$



- **Impulses uniformly spaced T seconds apart**

End

CHAPTER 2

(Part 1 of 5)