Chapter 1: Understanding Static Forces

At the end of the lesson, students should be able to:

- Understand Forces and Moments
- Know the different types of mechanical forces and their applications
- Understand Stress, Strain and Elasticity and their importance
- Explain Centre of Gravity
- Understanding forces acting on an electric car

1.1 Introduction

Mechanics is a branch of physical science that deals with forces and energy and their effect on a stationary of moving body. It is the science dealing with the actions and reactions of forces on a body. What is a force?

A force is a push or a pull and it can do work.

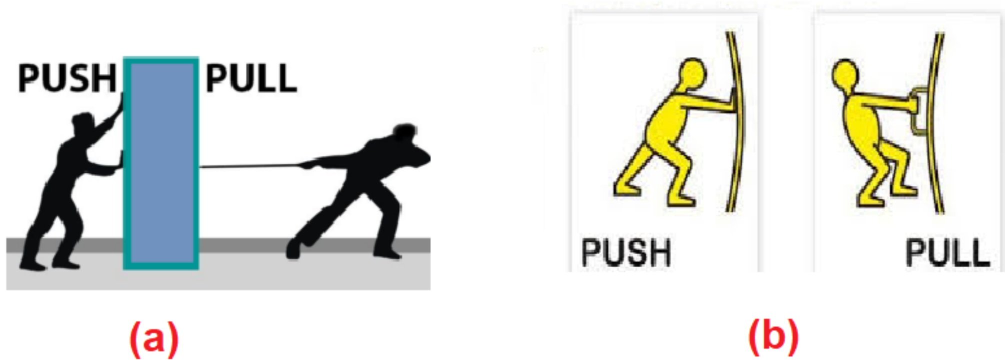

Figure 1.1: A force is a push or a pull

Mechanics is everywhere when there is an object. Examples are in the fields of vibration analysis, structures, machines, electric vehicles, spacecraft, automatic control, engine performance, fluid flow, electrical apparatus, etc.

Mechanics is conveniently divided into:

(a) Statics and
(b) Dynamics

(a) Statics

If an object is stationary, it also known to be at rest. We would say that the object is at "**static** equilibrium."

Equilibrium means the state of a body in which the forces acting upon it are so arranged that their resultant at every point is zero.

A static force refers to a constant force applied to a stationary object. A static force is too weak to move an object because it is being countered by equally strong opposite forces.

The most common example of a static force is static friction on a stationary object. If an object has some force being applied to it while it is on a surface, the force of friction will increase proportionally to the force until a certain limit. If the applied force is large enough, it can overcome the static friction and move the object. The force is then a kinetic force that is being resisted by kinetic friction.

https://www.youtube.com/watch?v=3EbUa5ZDybg

https://www.youtube.com/watch?v=9SMp-jnh8lg

(b) Dynamics

Dynamics is the study of the motion of objects (i.e. **kinematics**) and the forces responsible for that motion. It is a branch of classical mechanics, involving primarily Newton's laws of motion.

Dynamics analysis allows one to predict the motion of an object or objects, under the influence of different forces.

1.2 Newton's 3 Laws

Newton's first Law states that an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force. It may be seen as a statement about inertia, that objects will remain in their state of motion unless a force acts to change the motion.

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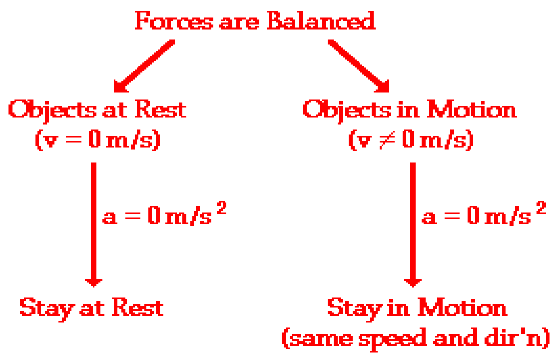


Figure 1.2: Illustration of Body at Rest and in Motion

Newton's second law of motion pertains to the behaviour of objects for which all existing forces are not balanced. The **second law** states that the net force, F, acting upon the object is the product of the mass (m) and the acceleration (a) of the object. The acceleration is in the direction of the net force.

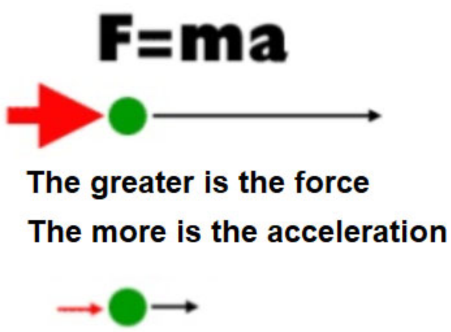


Figure 1.3: Newton's 2nd Law

https://www.youtube.com/watch?v=8YhYqN9BwB4

Newton's third law states that when one body exerts a force on a second body, the second body simultaneously exerts a force equal in magnitude and opposite in direction on the first body. Sometimes, it is simplified as for every action, there is an equal and opposite reaction.

(Action) F = F' (Reaction)

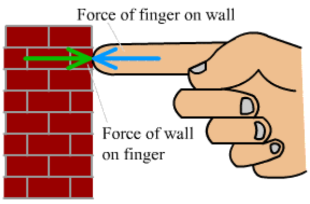


Figure 1.4: Newton's 3rd Law

https://www.youtube.com/watch?v=TVAxASr0iUY

1.3 Scalar and Vector Quantities

A **scalar quantity** has magnitude only.

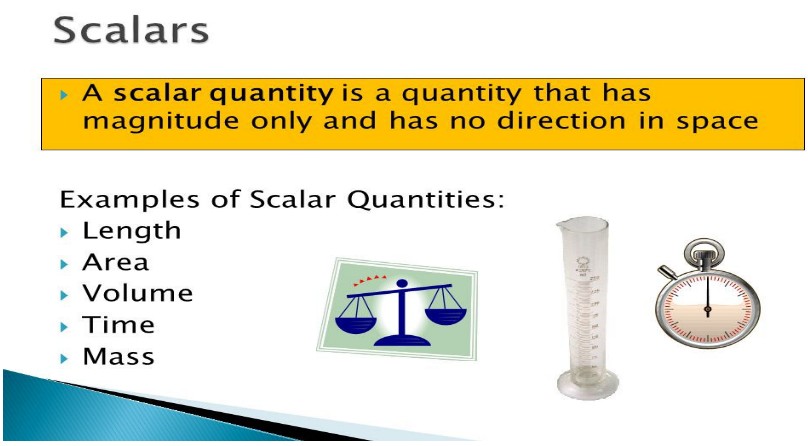


Figure 1.5: Scalar Quantities

A **vector quantity** has both magnitude and direction.

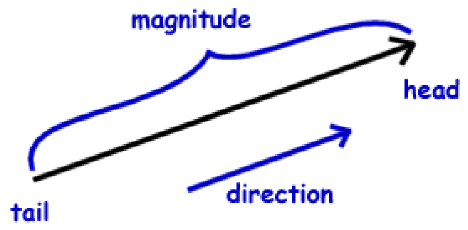


Figure 1.6: A Vector Quantity

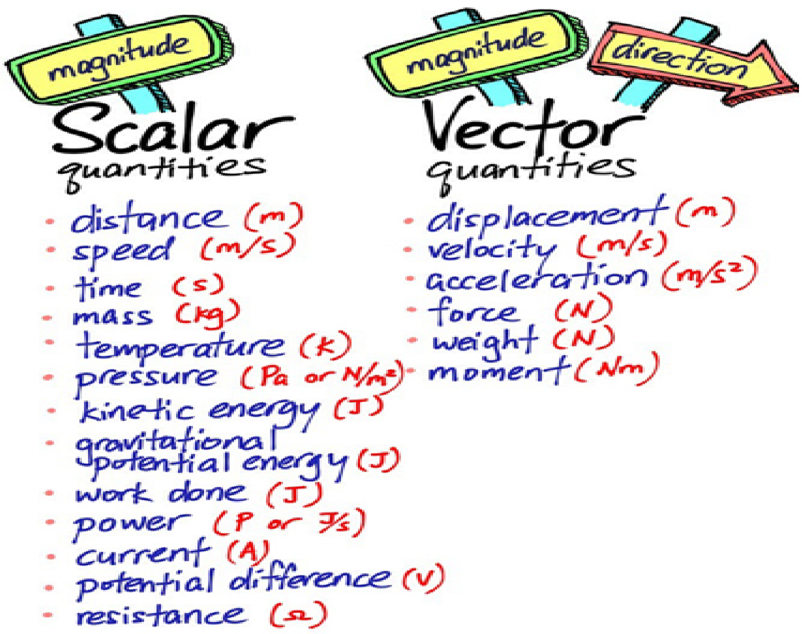


Figure 1.7: Scalar and Vector Quantities

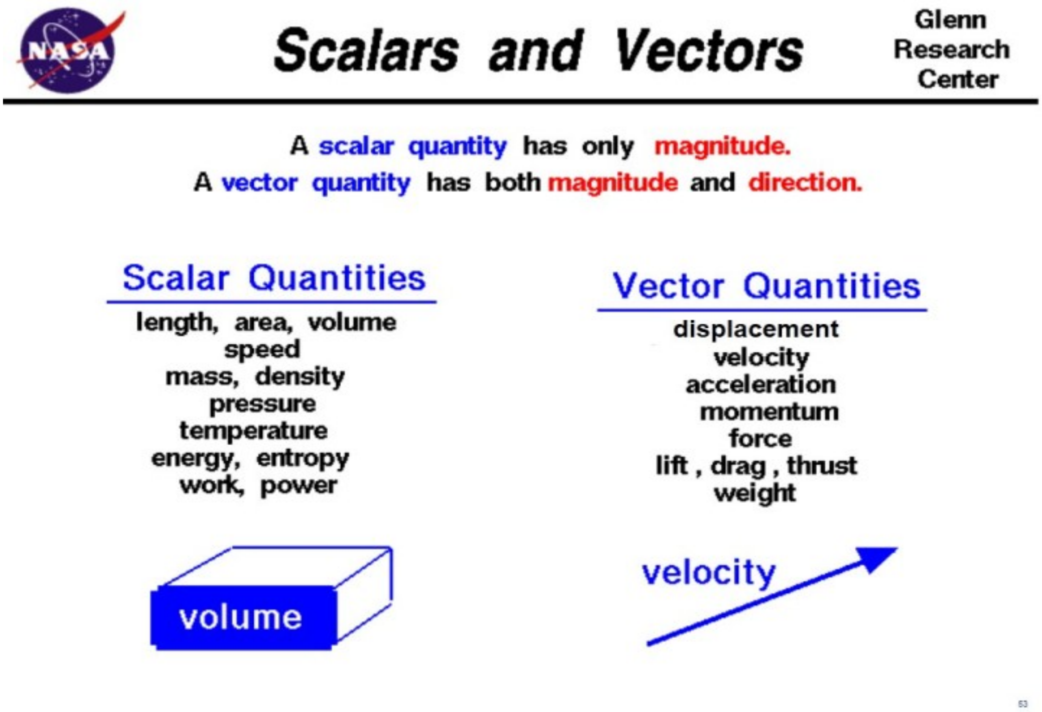


Figure 1.8: Scalar and Vector Quantities

https://www.youtube.com/watch?v=Pj8Zh0A-uLU&list=PLmdFyQYShrjcoTLhPodQGjtZKPKIWc3Vp&index=3

1.4 Resultant Force on a body

When more than one force are being applied to an object, there will be a **resultant force.** Since force is a vector, the resultant force can always be found by drawing or by using trigonometry. The direction of the arrow of each force is very important to determine the resultant force. The unit for force is Newton.

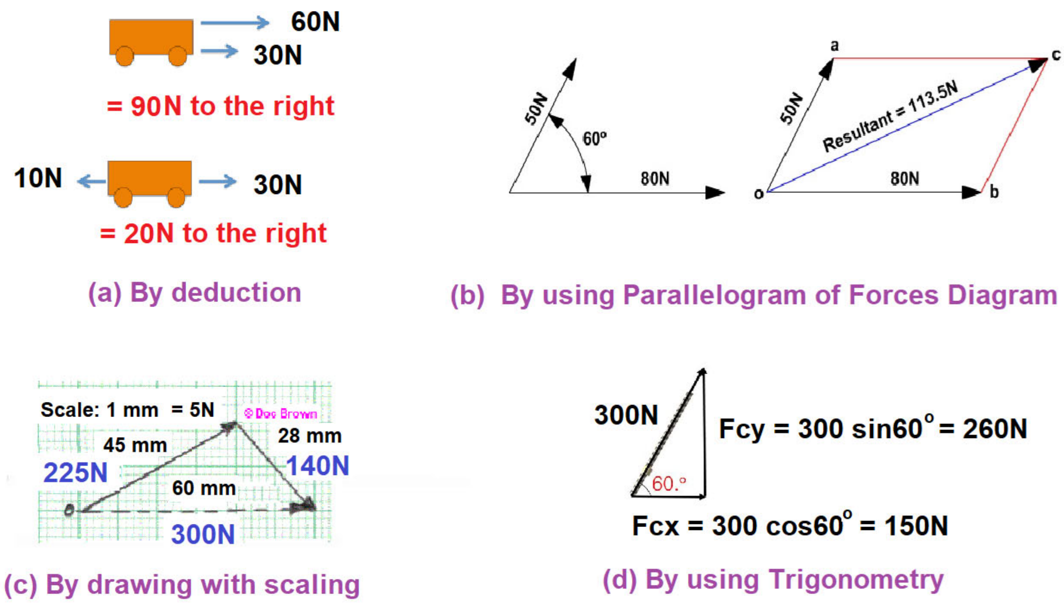


Figure 1.9: Finding Resultant Force

https://www.youtube.com/watch?v=U8z8WFhOQ_Y

1.5 Resolution of forces on a body

Graphical solutions to find the resultant force problems are sufficiently accurate for most engineering problems. However, if more accuracy is needed, especially if decimals are involved, mathematical method known as the resolution of forces has to be used.

Consider a resultant force F acting on a body A as in Figure 1.10. The force F may be replaced by two forces P and Q, acting at right angles to each other, **which together have the same effect on the bolt.** It can be shown that

$$\mathbf{P = F\sin\theta} \quad \text{and} \quad \mathbf{Q = F\cos\theta}$$

F cos θ is known as the horizontal component of F and Fsin θ is known as the vertical component of F.

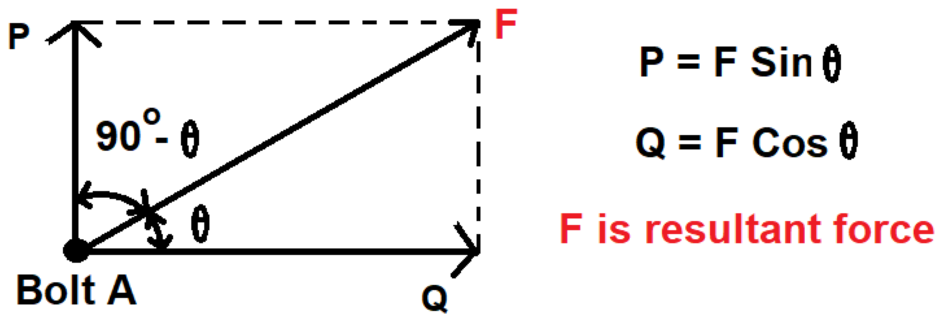


Figure 1.10: Resultant Force and its Components

Example 1.1

Referring to Figure 1.11, what is the resultant of a 5N force directed horizontally to the right and a 12N force directed vertically downward?

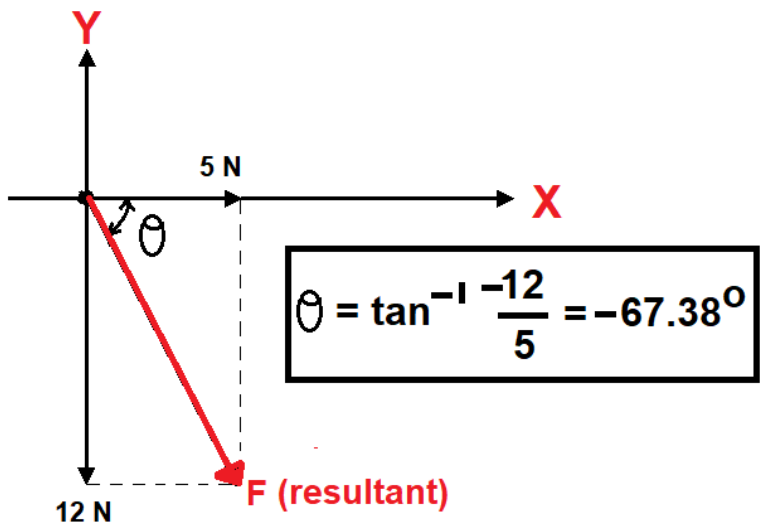


Figure 1.11

F_x = 5N

F_y = 12N

$F_R^2 = 5^2 + 12^2 = 169N$ (by Pythagoras theorem)

Therefore, resultant force F_R = 13N at - 67.38° (4th quadrant)

Example 1.2

Find the resultant force acting on the object at the origin shown in Figure 1.12.

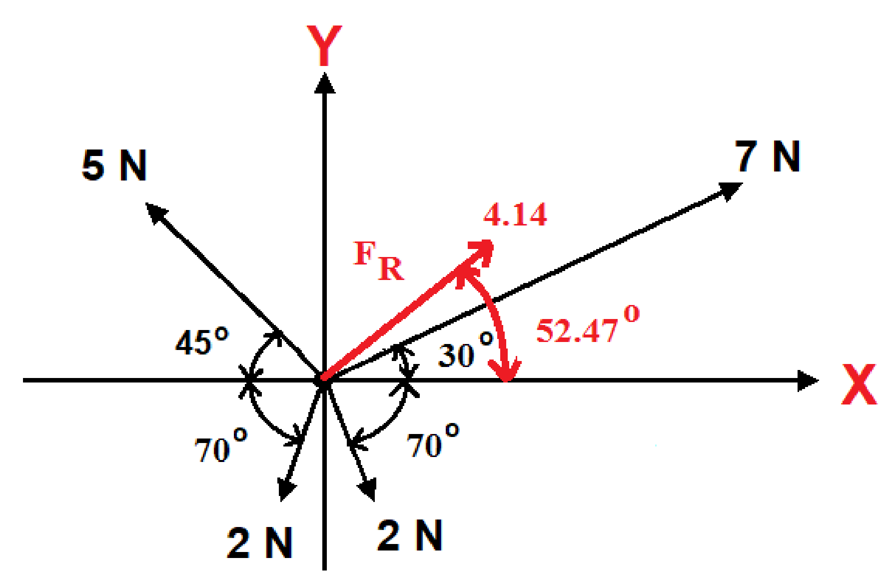


Figure 1.12

F(N)	Fx (N) – X Axis	Fy(N) – Y Axis
7	7 Cos 30 = 6.06	7 Sin 30 = 3.50
5	-5 Cos 45 = -3.54	5 Sin 45 = 3.54
2 (3rd quadrant)	-2 Cos 70 = - 0.68	-2 Sin 70 = -1.88
2 (4th quadrant)	2 cos 70 = 0.68	- 2 sin 70 = -1.88
Total	**(6.06 -3.54 -0.68 +0.68) = 2.52**	**(3.50 +3.54-1.88-1.88) = 3.28**

Therefore, $F_R^2 = 2.52^2 + 3.28^2 = 6.35 + 10.76 = 17.11$N.

Therefore, $\mathbf{F_R = 4.14}$**N**; $\theta = \tan^{-1} (| F_y | / | F_x |) = \tan^{-1} = 3.28/2.52 =$ **52.47°** in the first quadrant.

Example 1.3

Find the resultant of given force in Figure 1.13 by finding the components of the forces in the x and y directions first. Show your calculations clearly.

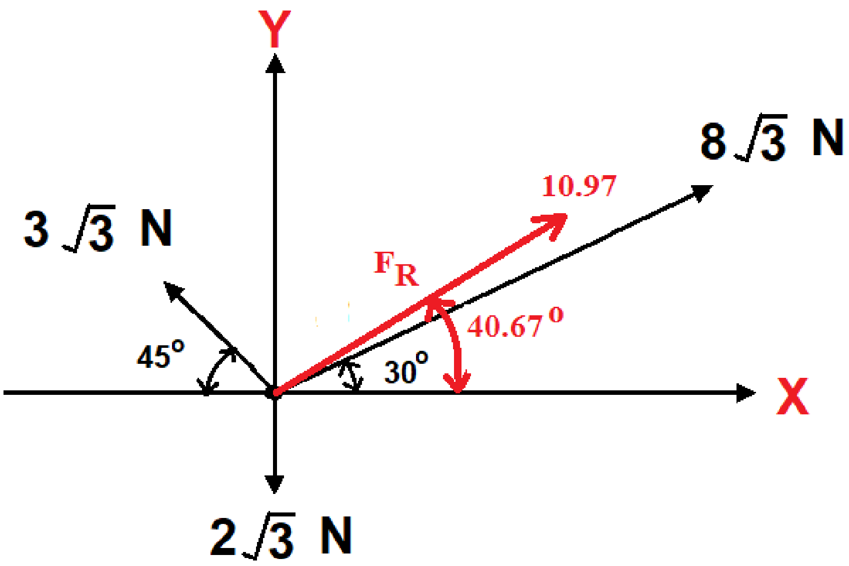


Figure 1.13

F(N)	Fx (N) – X Axis	Fy(N) – Y Axis
8 √3 =13.86	12	6.93
3 √3 = 5.20	-3.68	3.68
2√3 =3.46	0	-3.46
Total	**12-3.68 = 8.32**	**6.93 + 3.68 – 3.46 = 7.15**

Therefore, **F_R = 10.97N;** **θ = 40.67°** in the first quadrant.

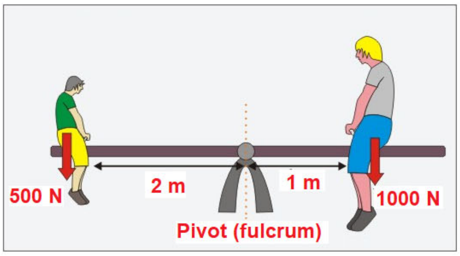
1.6 Moments and Torque

We have already understood that a force is a push or a pull on a body. If there are more than one force acting on a body, the resultant force should be found. If the resultant force is greater, the acceleration will be higher for the same body.

In an extended body, force may also cause rotation or deformation of the body. Rotational effects and deformation are determined respectively by the torques and stresses that the forces create.

(a) Moments

The turning effect of a **force** is known as the **moment**. It is the product of the **force** multiplied by the perpendicular distance from the line of action of the **force** to the pivot or point where the object will turn. The unit of moment is N-m or Nm.

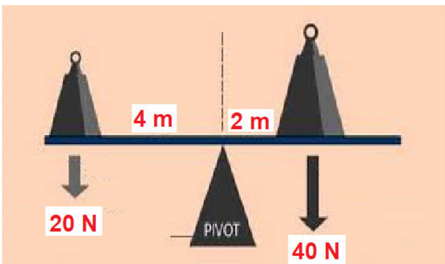


Clockwise moment = 1000 N x 1 m = 1000 N-m

Anti clockwise moment = 500 N x 2 m = 1000 N-m

Clockwise moment = Anti Clockwise moment

(a) Moments in Equilibrium



Clockwise moment = 40 N x 2 m = 80 N-m

Anticlockwise moment = 20 N x 4 m = 80 N-m

Clockwise moment = Anti Clockwise moment

(b) Moments in Equilibrium

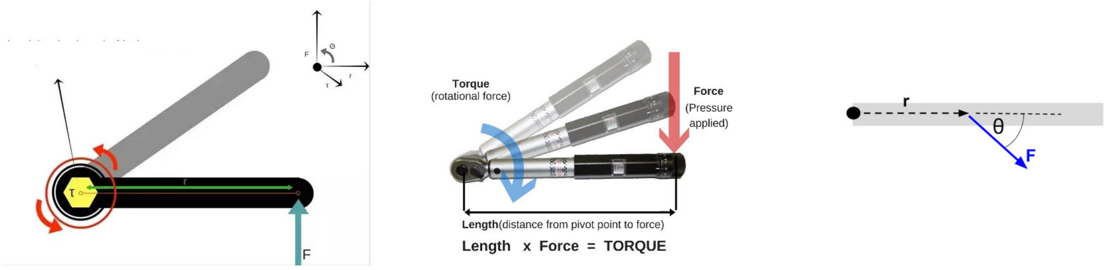
FIGURE 1.14: FINDING MOMENTS IN EQUILIBRIUM

https://www.youtube.com/watch?v=p7QS4cz-Avs

HTTPS://WWW.YOUTUBE.COM/WATCH?V=KK6T0M9WMNU

(b) Torque

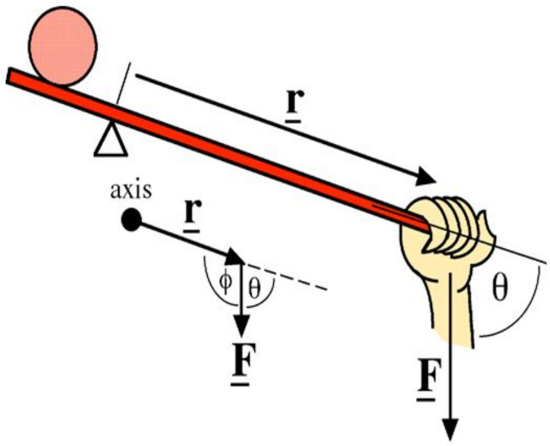
Torque is the tendency of a force to turn or twist. The force applied to a lever, multiplied by the distance from the lever's fulcrum, multiplied again by the sine of the angle created, is described as **torque**. This is also known "force times fulcrum distance time’s sine theta."



(a) (b) (c)

Figure 1.15: Forces producing Torque

Example: Calculate torque on lever exerted by hand:



If you're given r and θ, use formula for torque (magnitude)

torque $\tau = r F \sin\theta$

Figure 1.16: Torque Calculation

https://www.youtube.com/watch?v=SyoksbkTuWQ

Example 1.4

Figure1.18 below shows a vehicle control system crank lever ABC pivoted at B. AB is 20 cm and BC is 30 cm. Calculate the magnitude of the vertical rod force, F at C, required to balance the horizontal control rod force of magnitude 10 KN applied at A.

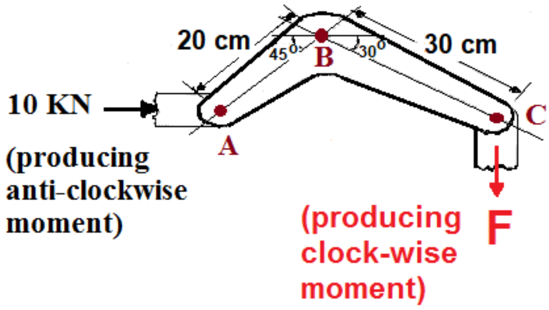


Figure 1.17: Torque Calculation

Torque due to 10 KN = 10 x 0.2 sin 45^o = 1.414 KN

Torque due to F = F x 0.3 sin 60^o = 0.26 F

Equating the torques, we get 1.414 KN = 0.26 F.

Therefore **F = 5.44 KN**.

Example 1.5

A 20 kg child and a 30 kg child sit at opposite ends of a 4 m seesaw that is pivoted at its center. Where should another 20 kg child sit in order to balance the seesaw?

Clockwise moment = Anticlockwise moment

(20 x 2) + (20 x d) = (30 x 2)

Where d = distance of 2nd 20 kg child from the pivot.

Therefore, **d = 1 m.**

1.7 Other types of mechanical forces that may be experienced by a body

A **force** is a pushing or a pulling action which moves, or tries to move, an object. Engineers design **structures**, such as buildings, dams, planes and bicycle frames to hold up weight and withstand forces that are placed on them. An engineer's job is to first determine the **loads** or external forces that are acting on a structure. Whenever external forces are applied to a structure, **internal stresses** (internal forces) develop inside the materials that resist the outside forces and fight to hold the structure together. Once an engineer knows what loads will be acting on a structure, they have to calculate the resulting internal stresses, and design each **structural member** (piece of the structure) so it is strong enough to carry the loads without breaking (or even coming close to breaking).

The 5 types of loads/ forces that can act on a structure are:

(a) Tension
(b) Compression
(c) Shear
(d) Bending
(e) Torsion

(a) Tension

Tension occurs when 2 opposing forces are trying to pull an object apart. For example, pulling on a rope, a car towing another car with a chain – the rope and the chain are in tension or are "being subjected to a tensile forces or loads.

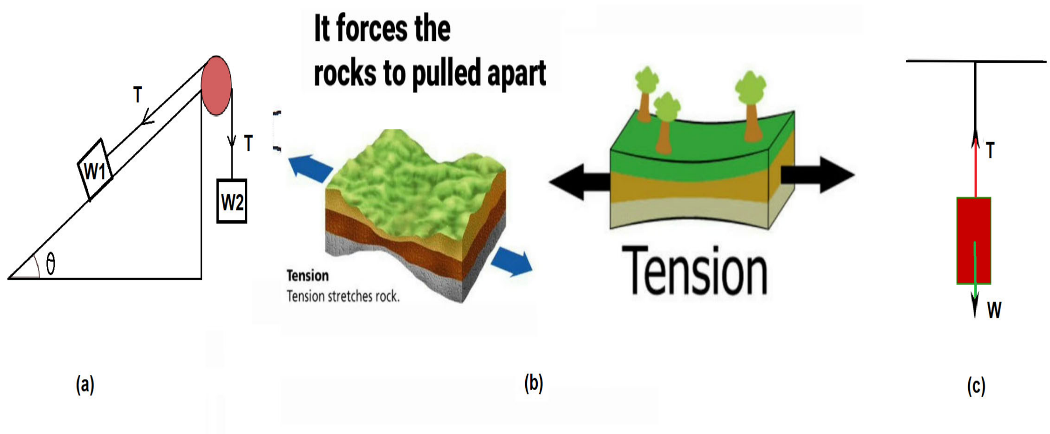


Figure 1.18: Tension Forces

https://www.youtube.com/watch?v=t2RjGcvNSas

(b) Compression

Compression is the application of balanced inward forces to different points on a material or structure. Compression tends to reduce the size of the object in one or more directions.

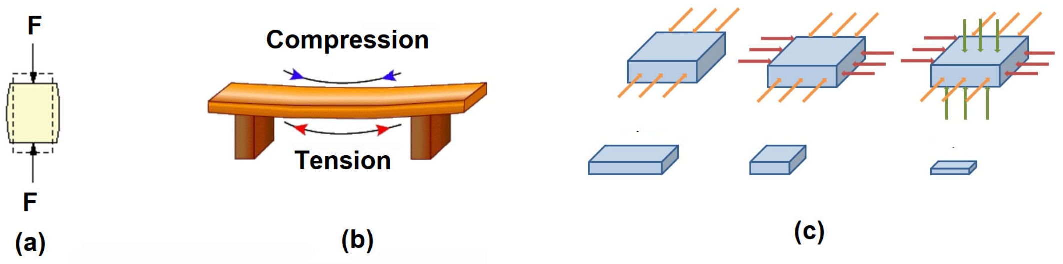


Figure 1.19: Compression Forces

https://www.youtube.com/watch?v=c6ndD5kTkP4

(c) Shear

A **shear force** is applied perpendicularly to a surface, in opposition to an offset **force** acting in the opposite direction. When a structural member experiences failure by **shear**, two parts of it are pushed in different directions to separate them. Papers are cut by scissors by shearing forces.

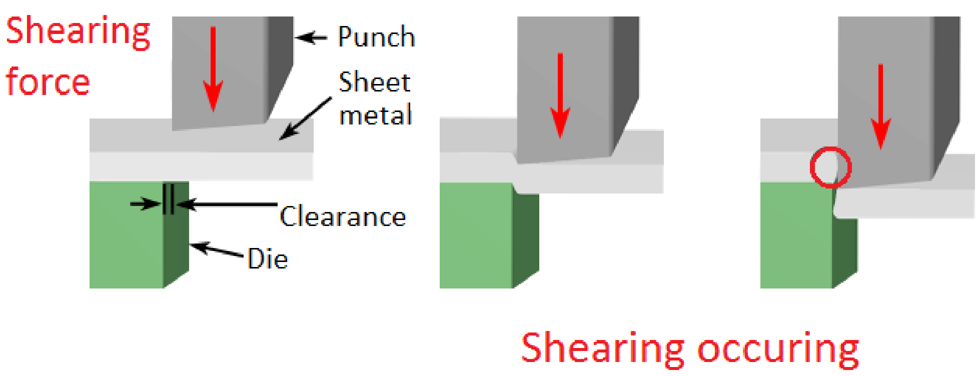


Figure 1.20: Shear Forces

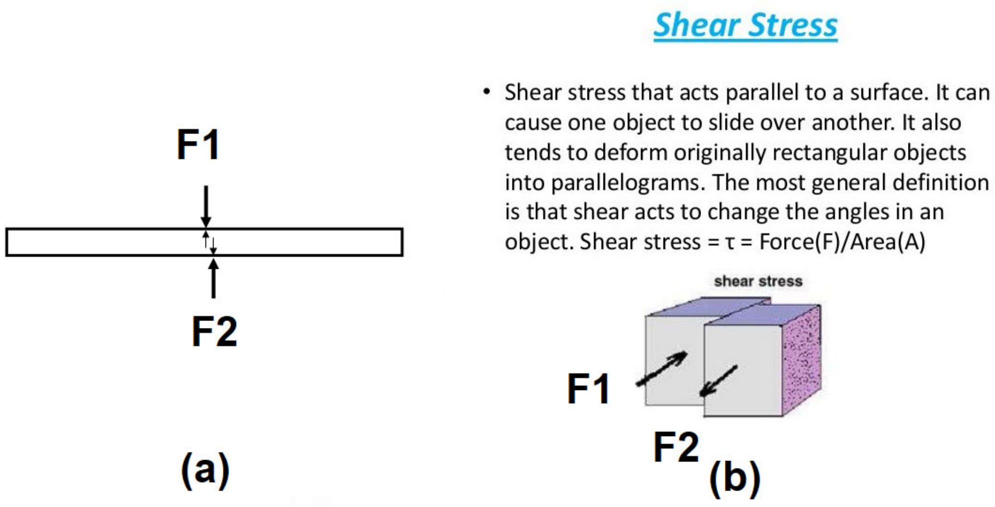


Figure 1.21: Shearing Forces creating Shearing Stress

(d) Bending Forces

Bending forces are shown in Figure 1.22 to solid materials. Bending forces can caused compression on one side and tension on the opposite side. An excessive bending force can cause material failure, especially for materials that have corroded. If one continues to bend and object in both direction continuously, the material may break due to fatigue failure.

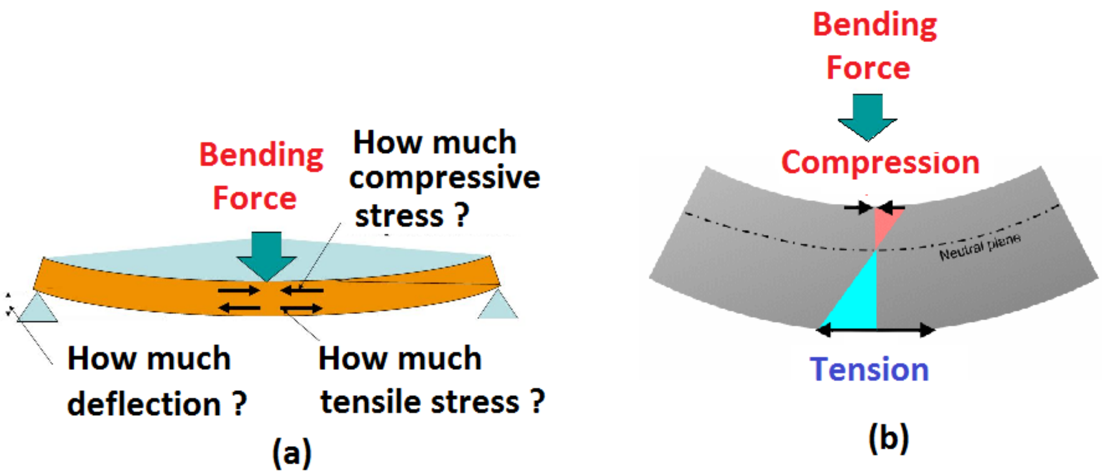


Figure 1.22: Bending Force causing Compression and Tension to occur

(e) Torsion Forces

Torsion occurs when an object, such as a bar with a cylindrical or square cross section is twisted. The twisting **force** acting on the object is known as torque, and the resulting stress is known as Shear stress.

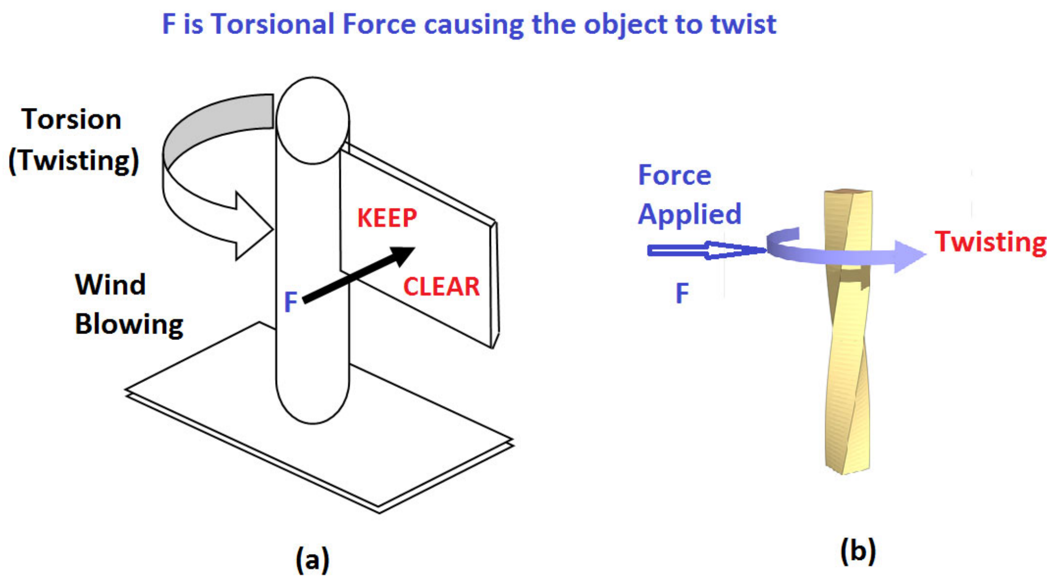
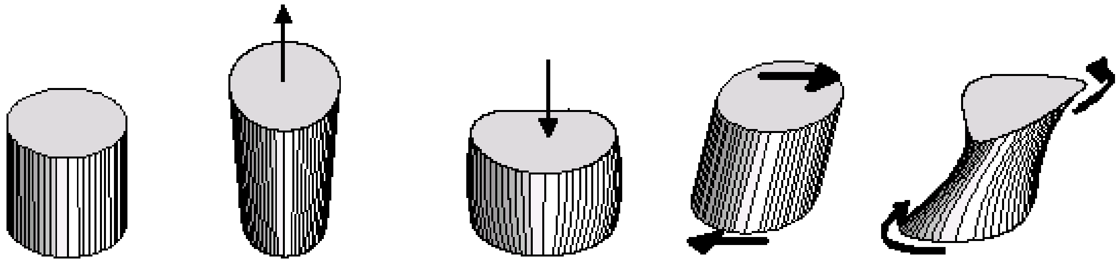


Figure 1.23: Torsional Forces

1.8 Stress, Strength, Strain and Elasticity

When a sufficient load is applied to a **metal** or other structural material, it will cause the material to change shape. This change in shape is called **deformation**. Deformations can be produced by forces that cause a body to be stretched, compressed, bend, shear or twisted.

Deformation is caused by stress. Temporary deformation is also called **elastic deformation** while the permanent deformation is called **plastic deformation.**



Unloaded	Stretched	Squeezed	Cut	Twisted
	(Tension)	**(Compression)**	**(Simple shear)**	**(Torsion)**

Figure 1.24: Various Form of Distortion causes by forces

https://www.youtube.com/watch?v=AnwrItEvqh4

(a) Stress

Stress is the force exerted on a body per unit cross sectional area. By stretching a body using a force (in the above case the force is weight), the *tensile* stress (in the direction of elongation), becomes

$$Stress = \frac{Force}{Cross - sectional\ area}$$

Therefore, if the force applied is 100N (Newton), and the cross sectional area measures 0.0004 m^2 (square metres), the stress becomes

$$\frac{100}{0.0004} = 250{,}000 N/m^2$$

Or 250 kN/m^2, or 0.25 MN/m^2 .If the force doubles (200N), stress will increase accordingly to 500 kN/m^2. We could also double the level of stress by reducing the cross sectional area to half of its original value, i.e. to 0.0002 m^2.

If the same weights were placed on the rectangular specimens to cause a contraction in the longitudinal direction, the resulting stress would be called *compressive* stress.

Apart from tensile and compressive stress, the other common type of stress encountered in mechanics is *shear* stress. This relates to the force which distorts rather than extends a body.

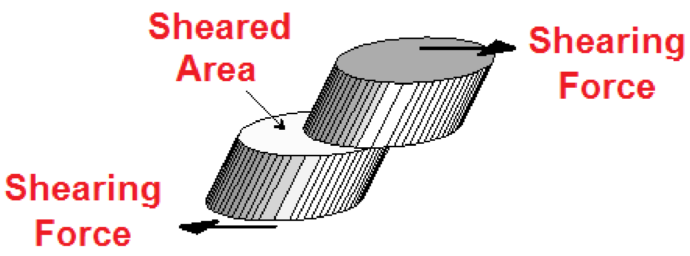


Figure 1.25: Cylindrical specimen subjected to simple shear

In the above example where a solid section is sheared,

$$\text{Shearing Stress} = \frac{\text{Shearing Force}}{\text{Cross Sectional Area}}$$

Shear forces can also result in failure.

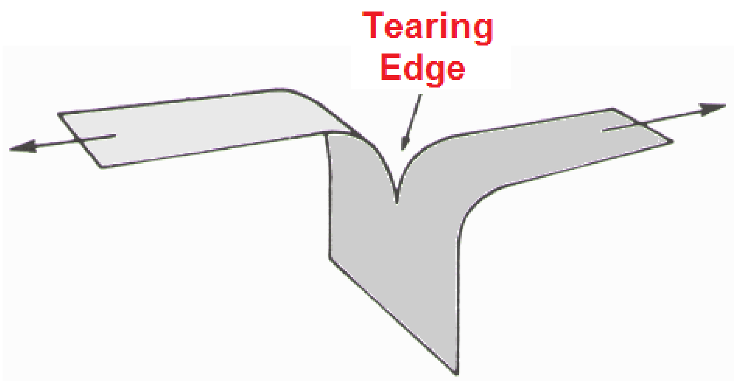


Figure 1.26: Simple example of shear failure

(b) Strength

Strength is defined as the highest stress that a material can withstand before it completely fails to perform structurally. If the applied force is tensile (stretch), the ultimate stress is known as ***tensile strength*** (i.e., maximum tensile stress that the material can tolerate). Others types of strength are related to the mode of the applied force, i.e. compressive, shear and torsional.

- A *strong* material is one that can withstand a very high force per unit area before it fails.

- A *weak* material is one that markedly deteriorates or fails at relatively low levels of applied forces.

Whether a material is strong or weak is relative, according to whether comparisons are attempted within a narrow group or across group or class boundaries. For example, in polymers, the tensile strength of nylon is higher than that of polyethylene, but it is far lower than that of mild steel, which itself is lower than that of a high-strength steel.

(c) Strain

Strain is the change in one dimension produced because of an applied force and it is expressed as the ratio of the amount of deformation to the sample's original dimension. In the case of tension,

$$\textbf{Strain} = \frac{\textbf{Extension}}{\textbf{Original Length}} \times 100\%$$

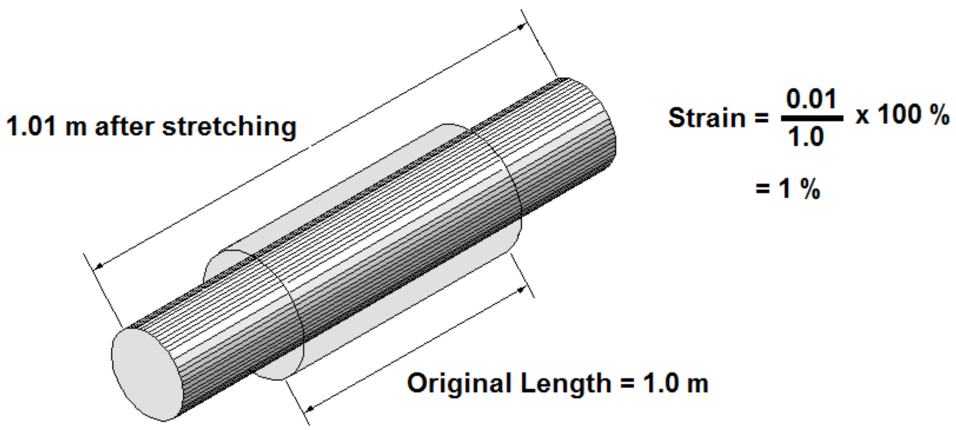


Figure 1.27: Straining by stretching a circular bar

If the change in dimension occurs in the opposite direction, the resulting strain is compressive.

If the deformation manifests itself as a distortion of geometry of angle θ, then strain is defined as shear strain. **Shear strain** is the ratio of deformation to original dimensions or is defined as the tangent of the angle of deformation.

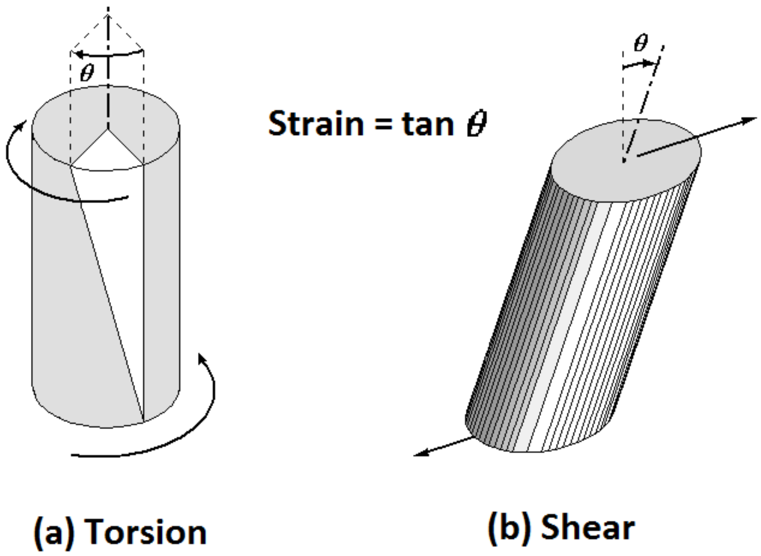


Figure 1.28: Straining by Torsion and Shearing

(d) Stress-Strain Relationship (below Failure Conditions)

Materials deform *elastically* or *inelastically*. During elastic deformation, the stress in a body is directly related to the strain, and vice-versa. Therefore, when the force is removed (i.e. when stress becomes zero) then strain returns to zero. The plot of stress against strain produces a straight line as shown in Figure 1.29. The stress can be increased or decreased, and stresses are always proportional to each other.

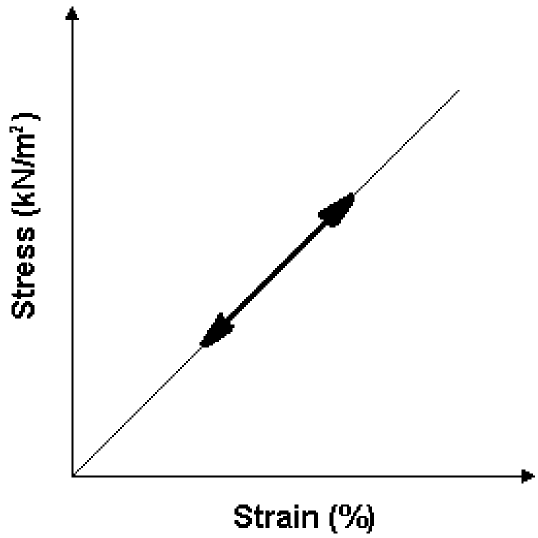


Figure 1.29: Linear elastic stress-strain relationship

For ductile materials (materials that can be stretched), increasing the stress above a certain limit will give rise to inelastic deformations, known as *yielding*. In other words, when the stress is removed, the strain does not return to zero (and the original shape is not fully restored) since some deformation has permanently set in. The stress level at which this occurs is referred to as the *yield stress* or *yield point.*

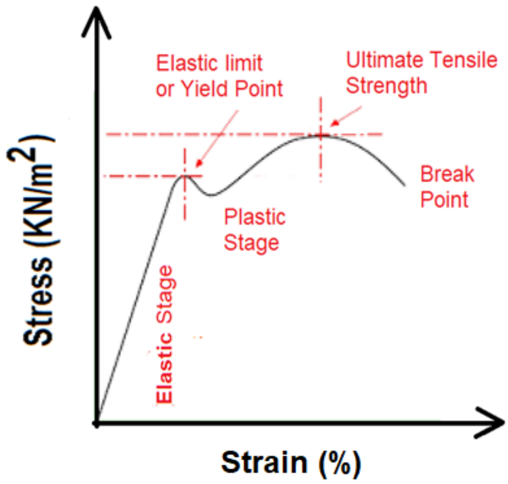


Figure 1.30: The Stress-Strain Curve showing Yield Point

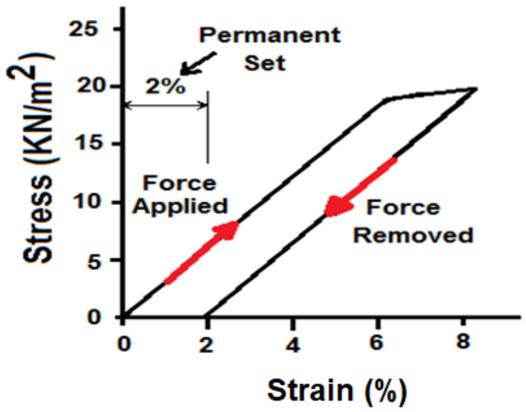


Figure 1.31: The amount of permanent deformation is evident after the force applied is removed

The relationship between stress and strain is expressed in terms of a property called the *Modulus* (or ***Young Modulus,*** named after the originator).

The linear portion of the stress-strain curve can be used to determine the modulus which correspond to the slope of the curve before the yield point, up to which all deformation is elastic and, therefore, recoverable. In other words,

$$\text{Modulus} = \frac{\text{Stress}}{\text{Strain}}$$

The slope (modulus) at any point in the linear portion of the line gives the same result. The modulus, in effect, denotes stiffness or rigidity for any kind of applied load, i.e. tension, compression or shear.

- **Stiff** materials have a high modulus. This means it is not easy to cause deformation or strain.

- **Flexible** materials have a low modulus. They undergo large deformations with relatively low applied forces.

Example 1.6

A telephone wire 120 m long with 0.22 cm radius is stretched by a force of 380N. What is the longitudinal stress? If the length after stretching is 120.10 m, what is the longitudinal strain? Determine Young's modulus, Y, for the wire?

Stress = Force / area; Force = 380N;

Area = $\pi r^2 = \pi (0.22 \times 10^{-2})^2 = 0.1521 \times 10^{-4}\ m^2$.

Stress = $(380 / 1.52 \times 10^{-4}) = 2498 \times 10^4\ Nm^2$.

Strain = (Change in length / Original length) x100%

= $(0.10 / 120) \times 100 = 8.33 \times 10^{-2}$ %

Y = Stress / Strain = $2498 \times 10^4 / 8.33 \times 10^{-4}$

Y = $299.9 \times 10^8\ N/m^2$

1.9 Centre of Gravity

The **center of mass** of a system of particles is a specific point at which, the system's mass behaves as if it were concentrated. In the case of a rigid body, the position of its center of mass is fixed in relation to the object. In the context of an entirely uniform gravitational field, the center of mass is often called the **center of gravity**- the point where gravity can be said to act.

Examples

- The center of mass of a two-particle system lies on the line connecting the particles (or, more precisely, their individual centers of mass). The center of mass is closer to the more massive object;
- The center of mass of a ring is at the center of the ring (in the air).
- The center of mass of a solid triangle lies on all three medians and therefore at the centroid, which is also the average of the three vertices.
- The center of mass of a rectangle is at the intersection of the two diagonals.
- More generally, for any symmetry of a body, its center of mass will be a fixed point of that symmetry.

https://www.youtube.com/watch?v=R8wKV0UQtlo

Locating the center of mass of an arbitrary 2D physical shape

This method is useful when one wishes to find the center of gravity of a complex planar object with unknown dimensions.

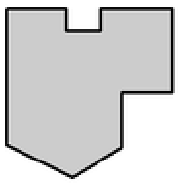
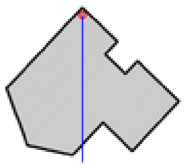
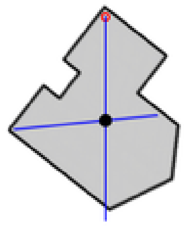
		
Step 1: An arbitrary 2D shape.	**Step 2:** Suspend the shape from a location near an edge. Drop a plumb line and mark on the object.	**Step 3:** Suspend the shape from another location not too close to the first. Drop a plumb line again and mark. The intersection of the two lines is the center of gravity.

Figure 1.32: Locating the center of gravity

1.10 Forces acting on a car in a straight-line motion

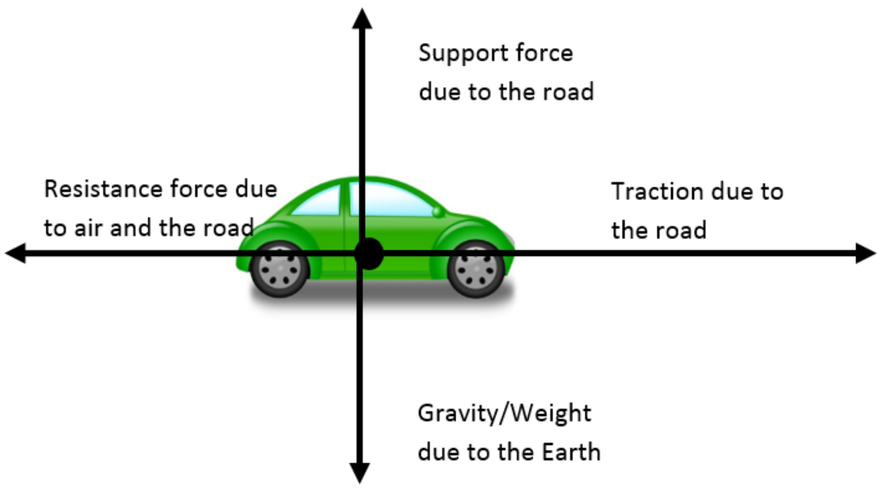
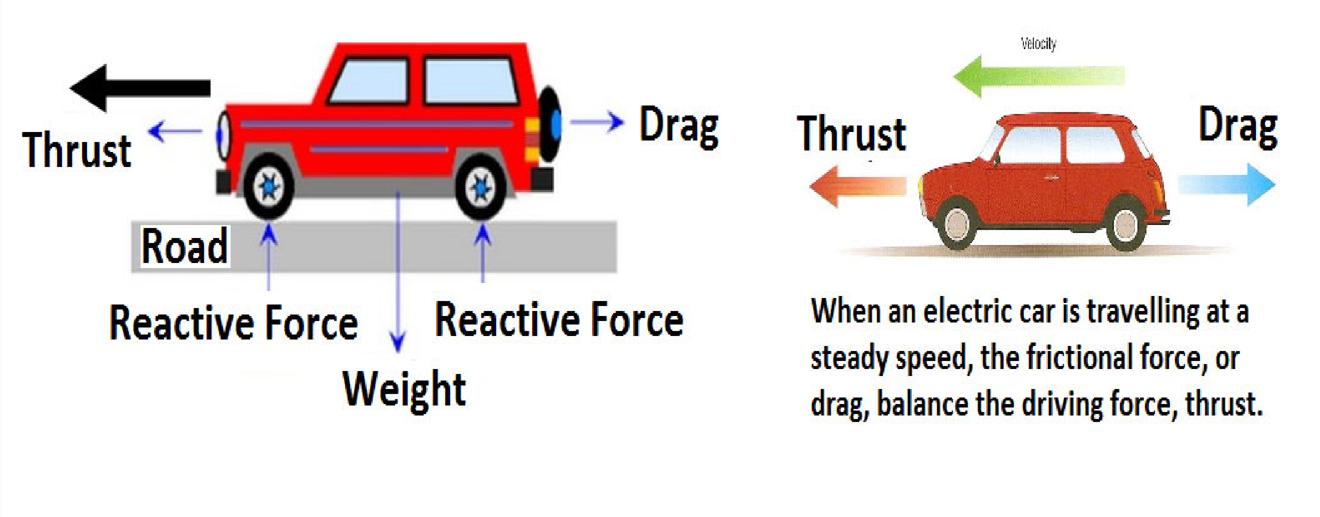


Figure 1.33: Forces acting on an electric car on a flat surface road



(a) Forces experience by an electric vehicle on a road

(b) An electric car travelling at steady speed

Figure 1.34: Forces acting on an electric car on a flat surface road

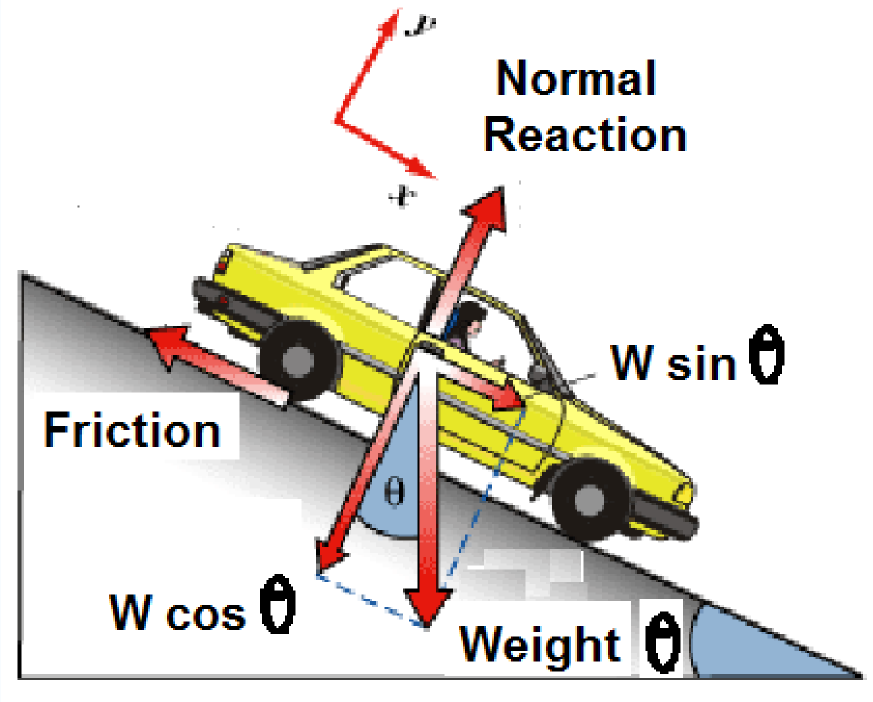


Figure 1.35 Forces on a car going down a slope

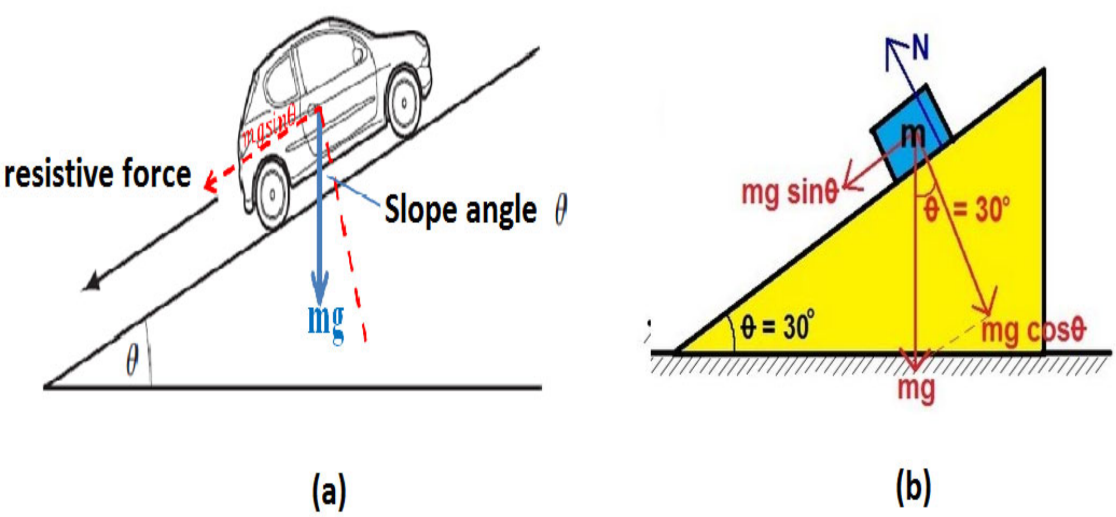


Figure 1.36: Forces on a car going up a slope

https://www.youtube.com/watch?v=4Bwwq1munB0&list=PLmdFyQYShrjcoTLhPodQGjtZKPKIWc3Vp&index=10

Acknowledgement:

1. Images, pictures and photo from Google Images
2. Links to videos provided from Youtube
3. Information from Google search and Wikipedia