Chapter 2

Signals and Spectra

(Part 4 of 5)

2.4 Fourier Transform

- The following 2 Fourier transform pairs are commonly used in the study of communication systems :

	Time domain		Frequency domain
1.	$A\,\mathrm{rect}\,\dfrac{t}{\tau}$	$\overset{FT}{\Longleftrightarrow}$	$A\,\tau\,\mathrm{sinc}\,f\tau$
2.	$A\,\mathrm{sinc}\,\dfrac{t}{\tau}$	$\overset{FT}{\Longleftrightarrow}$	$A\tau\,\mathrm{rect}\,f\tau$

2.4 Fourier Transform

Time domain

Frequency domain

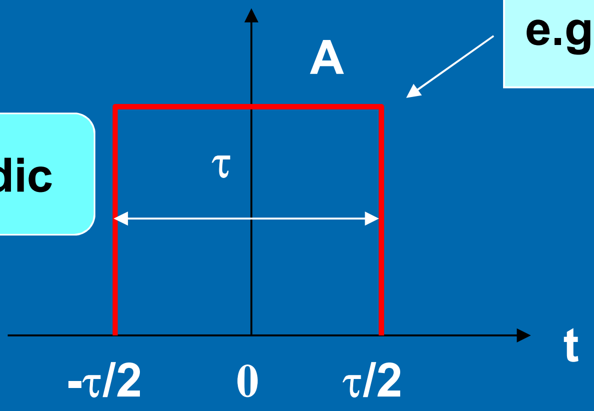
Rectangular pulse

Spectrum of rectangular pulse

$$A\,\mathrm{rect}\,\frac{t}{\tau} \;\overset{FT}{\Longleftrightarrow}\; A\,\tau\,\mathrm{sinc}\,f\tau$$

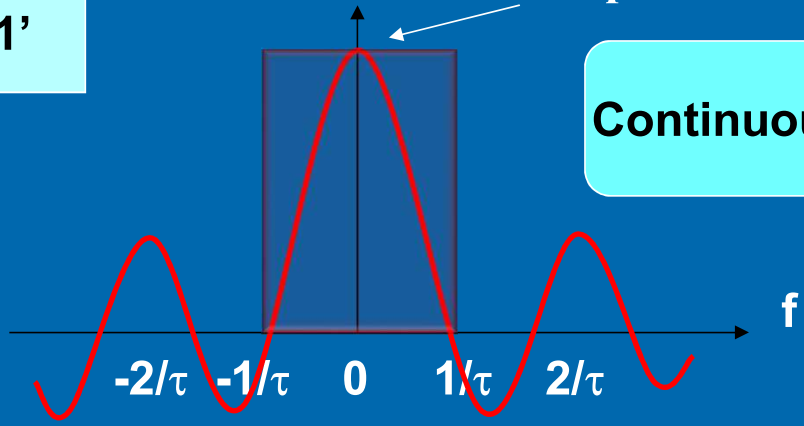
e.g. binary ‘1’

Non-periodic



Aτ = pulse area

Continuous spectrum



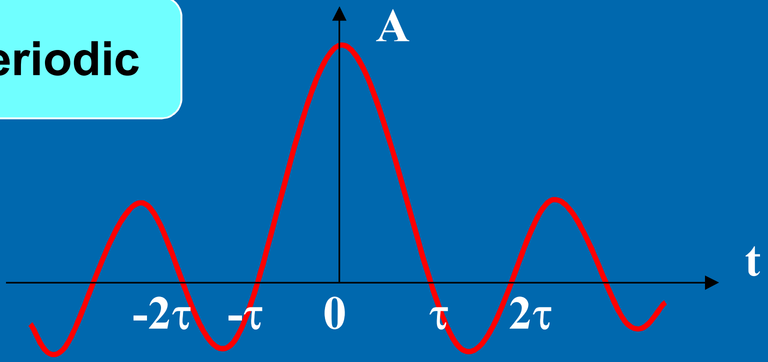
2.4 Fourier Transform

Time domain

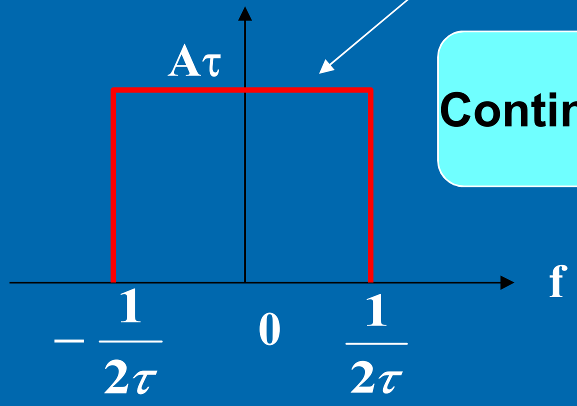
Frequency domain

$$A\,\text{sinc}\,\frac{t}{\tau} \;\overset{FT}{\Longleftrightarrow}\; A\tau\,\text{rect}\,f\tau$$

Non-periodic



e.g. LPF



Continuous spectrum

Example 2.11

Sketch the amplitude spectrum of $x(t) = 3\text{rect}\,20t$

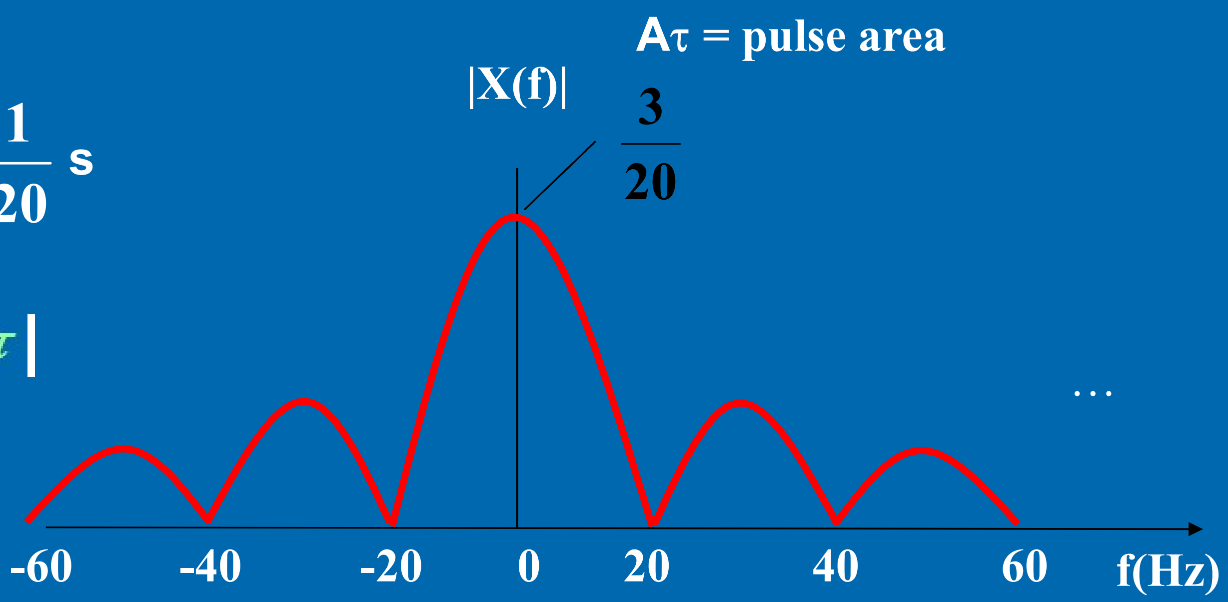
Solution

Comparing with the standard expression of rectangular pulse,

$$x(t) = 3\text{rect}\,20t \equiv A\text{rect}\,\frac{t}{\tau}$$

Hence, A = 3, and $\frac{1}{\tau} = 20$ or $\tau = \frac{1}{20}$ s

The amplitude spectrum is $|A\tau\,\text{sinc}\,f\tau|$



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Fourier Transform of an unit impulse train

Impulse train

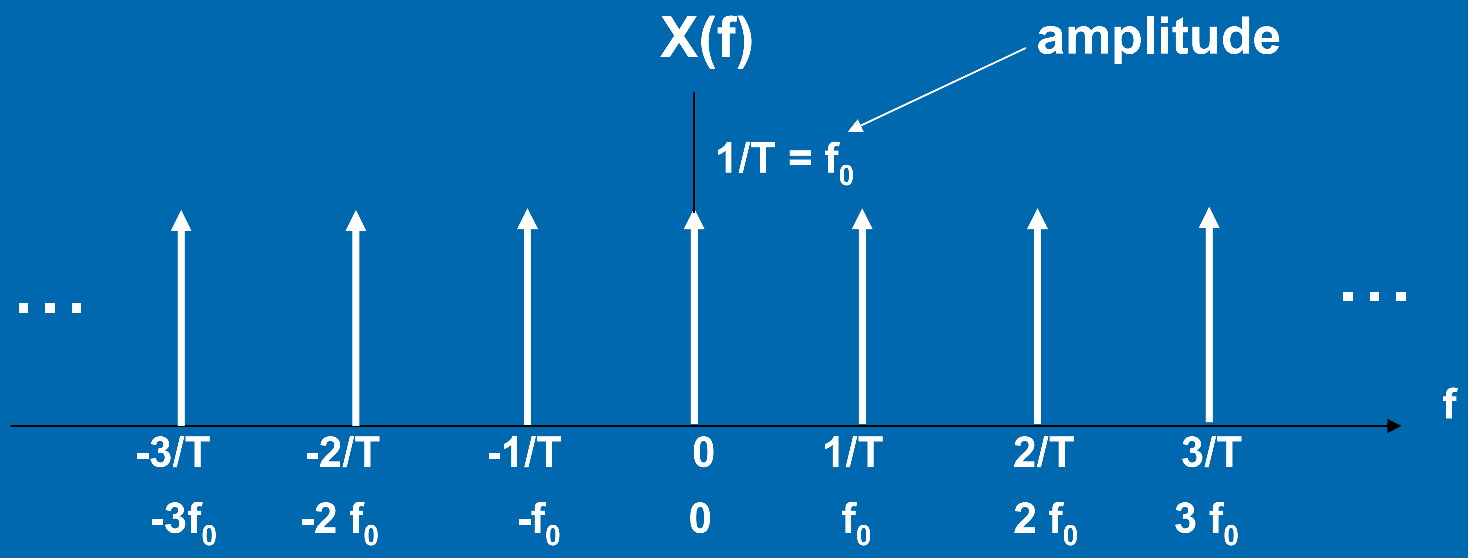
$$\mathbf{x(t)} = \sum_{\mathbf{n}=-\infty}^{\mathbf{n}=\infty} \boldsymbol{\delta}(\mathbf{t}-\mathbf{nT})$$

$$= \ldots + \underbrace{\delta(t+2T)}_{n\,=\,-2} + \underbrace{\delta(t+T)}_{n\,=\,-1} + \underbrace{\delta(t)}_{n\,=\,0} + \underbrace{\delta(t-T)}_{n\,=\,1} + \underbrace{\delta(t-2T)}_{n\,=\,2} \ldots$$

Fourier transform of a unit impulse train

In appendix part D, it is shown its FT is given by

$$\therefore \quad x(t) \overset{FT}{\Leftrightarrow} X(f) = \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(f - nf_0\right) \quad where\ f_o = \frac{1}{T}$$



Note: Important result used in analysing sampling process

2.4 Fourier Transform

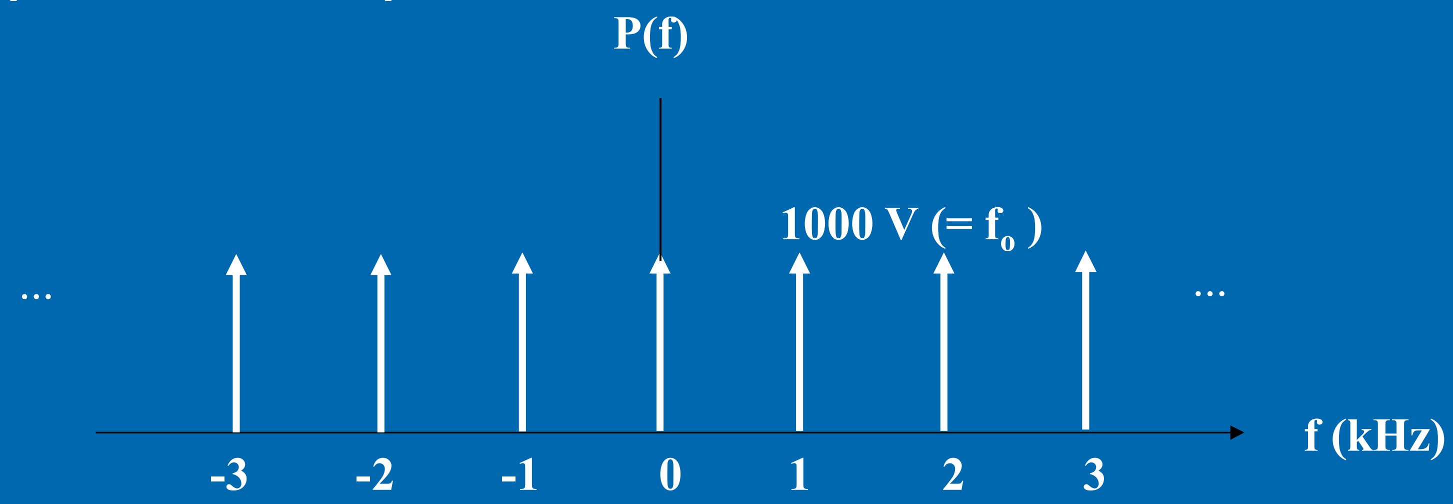
Example 2.12

Sketch the amplitude spectrum of a unit impulse train p(t) of frequency 1 kHz.

Solution

Given $f_0 = \frac{1}{T} = 1$ kHz.

The amplitude spectrum is an impulse train:



2.5 Signal Bandwidth

<u>Signal bandwidth (B)</u>

The width of positive frequencies contained in a signal.

- **Bandwidth of a signal with positive frequencies ranging from f_L to f_H:**

  $B = f_H - f_L$ (Hz)
  where to $0 \leq f_L \leq f_H$

- **For non-bandlimited signls, bandwidth is the width of significant spectrum.**

e.g.
Natural speech has a significant spectrum of 100 Hz – 10 kHz.
Thus, the bandwidth of natural speech is approximately 10 kHz.

Example 2.13

The significant frequency range of a telephone signal is between 300 Hz and 3.4 kHz. What is the bandwidth of the signal?

Solution

Bandwidth (B):

$$B = f_H – f_L = 3.4 – 0.3 = 3.1 \text{ (kHz)}$$

2.6 Signal Power Measurement

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Signal power in decibels (dB):

A relative measure of two different power level in base 10 logarithmic measure.

A dimensionless unit convenient for representing very large or small numbers.

- **Expressions for measurement of**
  - **Gain/loss of a system**
  - **Attenuation of signal power**
  - **Signal to noise ratios**

2.6 Signal Power Measurement

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Power gain in dB: G (dB)

- A number indicating the relative value of output power with respect to the input power:

$$G(dB) = 10log\frac{P_o}{P_i}$$

where P_o and P_i are measured in same units (watts or milliwatts).

- If the dB value is known, the power ratio can be obtained by

$$\frac{P_o}{P_i} = 10^{\frac{G(dB)}{10}}$$

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Example 2.14

Express the power gain/loss in dB in the table below.

P_1 (Watts)	P_2 (Watts)	$\frac{P_2}{P_1}$	dB $(10\log\frac{P_2}{P_1})$	Remarks
2	4			
2	20			
2	2			

Solution

P_1 (Watts)	P_2 (Watts)	$\frac{P_2}{P_1}$	dB $(10\log\frac{P_2}{P_1})$	Remarks
2	4	2		
2	20	10		
2	2	1		

Solution

P_1 (Watts)	P_2 (Watts)	$\frac{P_2}{P_1}$	dB ($10\log\frac{P_2}{P_1}$)	Remarks
2	4	2	3	
2	20	10	10	
2	2	1	0	

Solution

P_1 (Watts)	P_2 (Watts)	$\frac{P_2}{P_1}$	dB $(10\log\frac{P_2}{P_1})$	Remarks
2	4	2	3	Power gain is 3 dB
2	20	10	10	Power gain is 10 dB
2	2	1	0	Power gain is 0 dB

2.6 Signal Power Measurement

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<u>Power in dBm</u>

Power measurement relative to 1 mW

$$10log\frac{P(mW)}{1\ mW} = X\ dBm$$

- **0 dBm equivalent to one milliwatt, and 1 dBm is equivalent to 1.259 mW.**
- **When the dBm value is known, the power in mW can be obtained by**

$$P = 10^{\frac{X}{10}} \quad (mW)$$

2.6 Signal Power Measurement

Power in dBW

Power measurement relative to 1 W

$$10 log \frac{P\ (Watt)}{1\ Watt} = X\ dBW$$

- 0 dBW is equivalent to one watt, and 1 dBW is equivalent to 1.259 W.
- When the dBW value is known, the power in Watt can be obtained by

$$P = 10^{\frac{X}{10}} \quad (W)$$

2.6 Signal Power Measurement

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<u>Example 2.15</u>

Express the power in table in terms of dBW, dBm.

Power	dBW	dBm
30 W		
3 μW		

2.6 Signal Power Measurement

Solution

P = 30 W

P (dBW) = $10log\frac{P\ (Watt)}{1\ Watt}$ =10log(30) = 14.77 dBW

P (dBm) = $10\log\frac{\mathrm{P(mW)}}{1\ \mathrm{mW}}$ = 10log(30×10^{3}) = 14.77 + 30 = 34.77 dBm

P = 3 μW

P (dBW) = $10log\frac{P\ (Watt)}{1\ Watt}$ =10log(3×10^{-6}) = -55.23 dBW

P (dBm) = $10\log\frac{\mathrm{P(mW)}}{1\ \mathrm{mW}}$ = 10log(3×10^{-3}) = -25.23 dBm

Power	dBW	dBm
30 W	14.77 dBW	34.77 dBm
3 μW	-55.23 dBW	-25.23 dBm

2.7 Signal Transmission Through Linear Time Invariant Systems

Linear Time Invariant (LTI) Systems

- The output due to a sum of different inputs is the sum of the corresponding individual outputs.

$$ax_1(t) \longleftrightarrow ay_1(t)$$

$$bx_2(t) \longleftrightarrow by_2(t)$$

$$ax_1(t) + bx_2(t) \longleftrightarrow ay_1(t) + by_2(t)$$

- The input-output relationship of a system does not change with time.

2.7 Signal Transmission Through Linear Time Invariant Systems

Linear Time Invariant (LTI) Systems

- A LTI system is described by its impulse response, h(t)

Input signal

x(t) → **LTI System** $h(t) \overset{FT}{\Longleftrightarrow} H(f)$ → y(t)

Output signal

impulse signal | **impulse response**

$x(t) = \delta(t) \longleftrightarrow y(t) = h(t)$

2.7 Signal Transmission Through Linear Time Invariant Systems

Linear Time Invariant (LTI) Systems

Input signal

x(t) → LTI System

$h(t) \overset{FT}{\Longleftrightarrow} H(f)$

→ Output signal

y(t)

$x(t) = \delta(t)$ ⟷ $y(t) = h(t)$

impulse signal **impulse response**

- The output of an A LTI system can be obtain by

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\mathrm{d}\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\mathrm{d}\tau = \boxed{x(t) * h(t)}$$

convolution integral

2.7 Signal Transmission Through Linear Time Invariant Systems

Linear Time Invariant (LTI) Systems

Input signal	LTI System	Output signal
$x(t)$ →	$h(t) \overset{FT}{\Longleftrightarrow} H(f)$	→ $y(t)$

impulse signal

$x(t) = \delta(t)$ ⟷ $y(t) = h(t)$

impulse response

Spectrum of x(t)

$X(f)$

Spectrum of y(t)

$Y(f)$

- The output signal spectrum is obtained by

$$Y(f) = X(f) \cdot H(f)$$

H(f) - frequency response of the system

2.7 Signal Transmission Through Linear Time Invariant Systems

Linear Time Invariant (LTI) Systems

- The impulse response and frequency response are a Fourier transform pair:

$$h(t) \overset{FT}{\Longleftrightarrow} H(f)$$

- The frequency response, H(f), consists of amplitude and phase response

$$H(f) = \frac{Y(f)}{X(f)} = |H(f)| \angle H(f)$$

$$|H(f)| = \frac{|V_o(f)|}{|V_i(f)|}$$ Amplitude response

$\angle H(f)$ Phase response

End

CHAPTER 2

(Part 4 of 5)