2018/2019 SEMESTER TWO EXAMINATION

Diploma in Electrical and Electronic Engineering
3rd Year Full Time

DIGITAL SIGNAL PROCESSING

Time Allowed: 2 Hours

Instructions to Candidates

1. The examination rules as set out on the last page of the answer booklet are to be complied with.

2. This paper consists of **TWO** sections:

Section A	-	6 Short Questions, 10 marks each.
Section B	-	2 Long Questions, 20 marks each.

3. **ALL** questions are **COMPULSORY**.

4. **ALL** questions are to be answered in the answer booklet.

5. This paper consists of **5** pages, including 2 pages of mathematical formulae.

SECTION A - SHORT QUESTIONS [10 marks each]

A1. Use partial fraction to find the impulse response of the system described by the following equation,

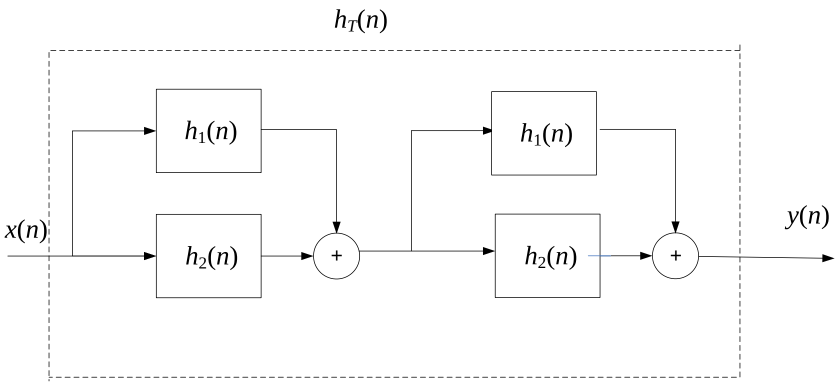
$$H(z) = \frac{2z}{3z^2 - 4z + 1}$$ (10 marks)

A2 Using z-transform and long-division method, find the input x(n) given y(n)={1, 4, 7, 6} and h(n)={1,2} (10 marks)

A3 Find the z-transform of $x_1(n) = \delta(n-2) + (0.5)^n u(n)$ and $x_2(n) = (0.5)^n \cos(\pi n/3) u(n)$. (10 marks)

A4 Determine the 4–point DFT of the following sequence, x(n)={1,0,2,0} (10 marks)

A5 The block diagram of a digital system is given as:



(a) Find the overall impulse response of the system, $h_T(n)$ in terms of $h_1(n)$ and $h_2(n)$. Find the z-transform of $h_T(n)$, $H_T(z)$. (6 marks)

(b) If $h_1(n)$ ={2,1} and $h_2(n)$ = {1,2} respectively, find $h_T(n)$. (4 marks)

A6 The difference equation of a particular digital network is given as:

$$y(n) = x(n) - x(n-1) – 0.5\, y(n-2)$$

(a) Sketch the digital network. (5 marks)

(b) Find the z-transform of the transfer function, H(z). (3 marks)

(c) Given the input x(n) is a unit step function, find y(n) (2 marks)

SECTION B - LONG QUESTIONS [20 marks each]

B1. For the digital filter described by the digital network shown in Fig. B1.

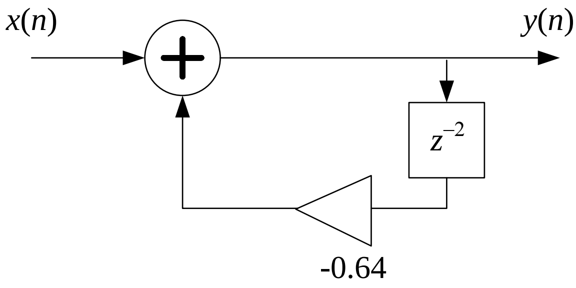


Figure B1

Determine the following:

(a) The difference equation. (4 marks)

(b) The system function. (4 marks)

(c) The gain of the filter at 0 kHz, 2 kHz and 4 kHz if the sampling frequency used is 8 kHz? (9 marks)

(d) What filtering function does this digital filter perform? (3 marks)

B2. A FIR low pass filter must be designed using the windowing technique with the following specification given below:

Sampling frequency: 10 kHz
Pass band: 0 to 1.5 kHz
Stop band: 2.25 kHz to 5 kHz
Peak approximation error: 0.02

(a) Determine the type of Window functions to be used. (4 marks)

(b) Compute the centre of the transition band. (2 marks)

(c) Compute the first and the centre tap coefficients. (6 marks)

(d) Draw the digital network of the FIR filter. (4 marks)

(e) Comment on the shape and stability of the FIR filter. (4 marks)

-End of Paper-

Appendix

The z-transform is defined as $X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$

Sequence	Transform
$\delta[n]$	1
$u[n]$	$\frac{1}{1-z^{-1}}$
$\Delta[n-m]$	z^{-m}
$a^n u[n]$	$\frac{1}{1-az^{-1}}$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$
$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$
$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$
$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$

Some z-transform properties:	
Sequence	Transform
$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
$x[n-m]$	$z^{-m}X(z)$

Some trigonometric identities:

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

Series:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

Discrete Fourier Transform:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn}$$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{N}kn}$$

Complex number theory:

$$z = a + jb = r\angle\theta = re^{j\theta}$$
$$r = \sqrt{a^2+b^2}$$
$$\theta = \tan^{-1}\frac{b}{a}$$

Quadratic equation solution:

If $ax^2 + bx + c = 0$

then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

The characteristics of the different windowing functions:

Window Type	Peak approximation Error 20 $\log_{10} \delta$dB	Transition Band $\Delta\omega$
Rectangular: $w(n) = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	-21	$\frac{4\pi}{M+1}$
Bartlett: $w[n] = \begin{cases} \frac{2n}{M} & 0 \le n \le \frac{M}{2} \\ 2 - \frac{2n}{M} & \frac{M}{2} \le n \le M \\ 0 & \text{otherwise} \end{cases}$	-25	$\frac{8\pi}{M}$
Hanning: $w(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	-44	$\frac{8\pi}{M}$
Hamming: $w(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	-53	$\frac{8\pi}{M}$
Blackman: $w(n) = \begin{cases} 0.42 - 0.5\cos\left(\frac{2\pi n}{M}\right) + 0.08\cos\left(\frac{4\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	-74	$\frac{12\pi}{M}$

The impulse response of an ideal low pass filter is: $h_d(n) = \dfrac{\sin\left(\omega_c\left[n - \frac{M}{2}\right]\right)}{\pi\left(n - \frac{M}{2}\right)}$