Chapter 2

Signals and Spectra

(Part 5 of 5)

2.8 Filters

Filters

Frequency-selective circuits that can shape and limit the spectrum of signals.

Widely used to process signals

Frequency response, H(f)

- The ratio of spectrum of the output signal to the spectrum of the input signal

$$H(f) = \frac{Y(f)}{X(f)} = |H(f)| \angle H(f) \quad \text{and} \quad |H(f)| = \frac{|V_o(f)|}{|V_i(f)|}$$

$|H(f)|$ - Amplitude response, or voltage gain

$\angle H(f)$ - Phase response, or phase shift

2.8 Filters

Ideal Filters

Pass signals at certain sets of frequencies exactly and completely reject the rest.

- There are 4 types of ideal filters:
  - Low Pass
  - High Pass
  - Band Pass
  - Band Stop (not included in this module)

Transition from stopband to passband and vice versa is instantaneous.

- Each type of filter has its own unique frequency response.

- Filter parameters

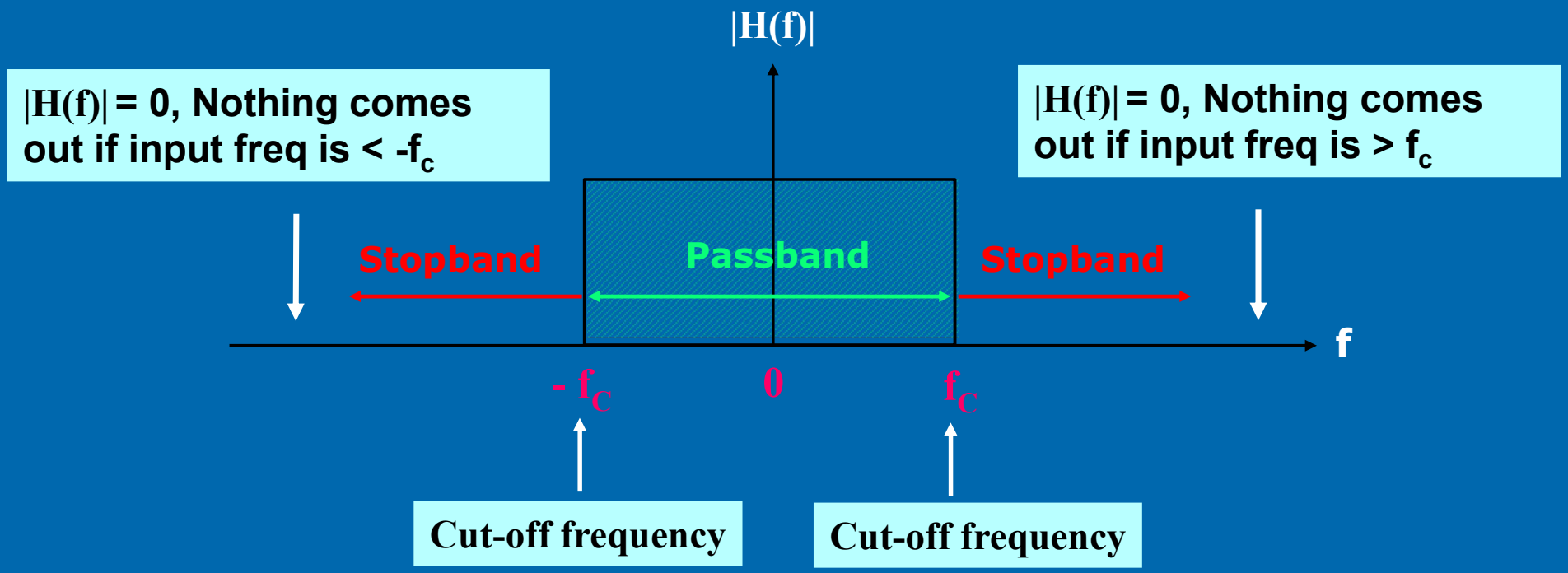
Passband - The range(s) of frequency that is passed by a filter.

Stopband - The range(s) of frequencies that is rejected by a filter.

Cut-off frequency - mark the transition from passband to stopband, or vice versa.

Amplitude Response (voltage gain) of ideal Low Pass Filter (LPF)

pass low frequency components from $-f_c$ to $+f_c$ and rejects high frequencies beyond $\pm f_c$.

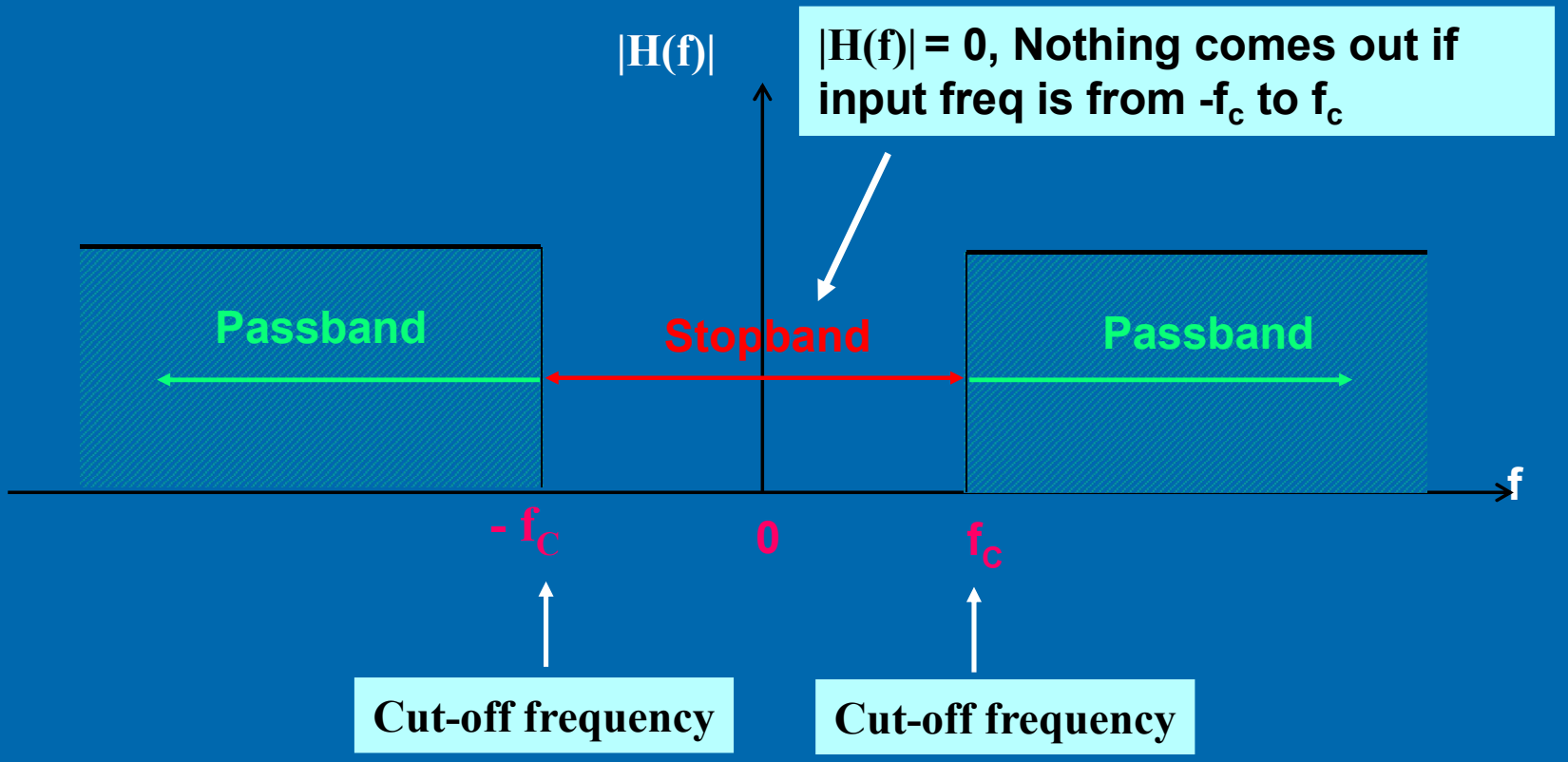


2.8 Filters

Amplitude Response (voltage gain) of High Pass Filter (HPF)

pass high frequency components beyond $\pm f_c$ but rejects frequencies from $-f_c$ to $+f_c$ (opposite of LPF).

Opposite of LPF



2.8 Filters

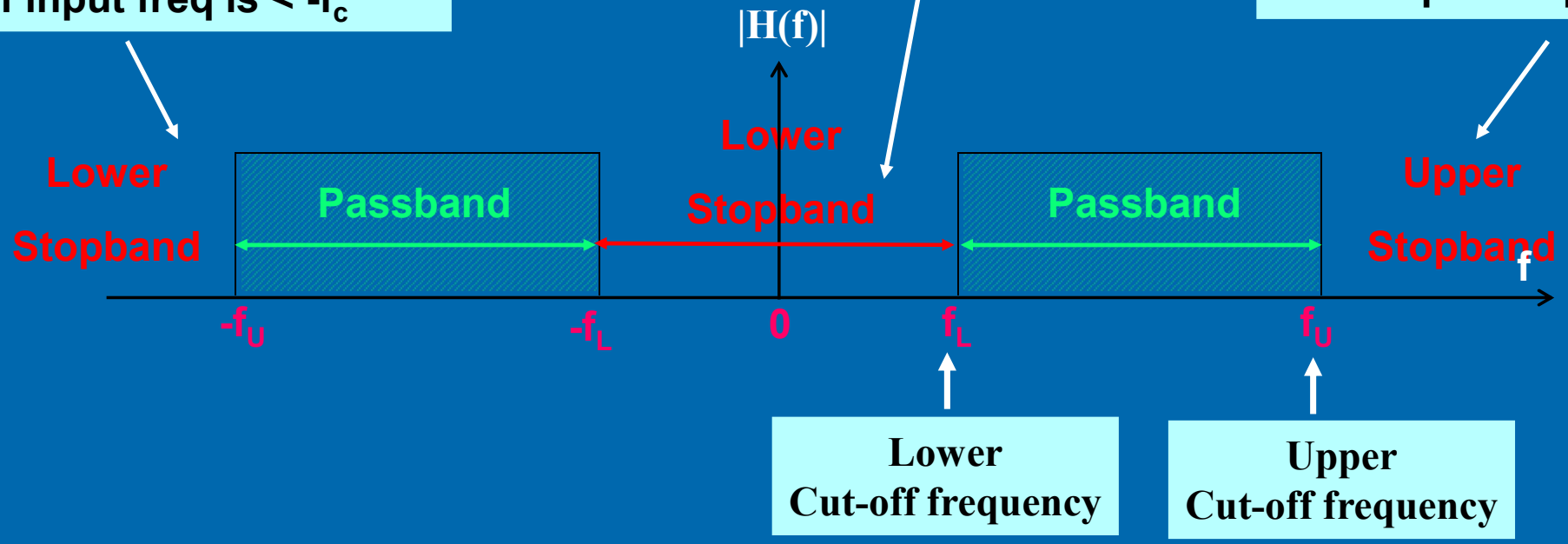
Amplitude Response (voltage gain)of Band Pass Filter (BPF)

Passes frequencies in the range of $-f_L$ to $-f_U$, f_L to f_U and rejects all frequencies outside this range.

$|H(f)| = 0$, Nothing comes out if input freq is in the range of $-f_c$ to f_c

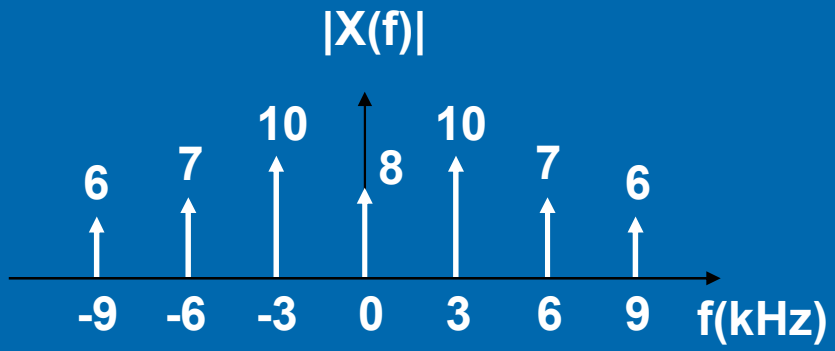
$|H(f)| = 0$, Nothing comes out if input freq is < $-f_c$

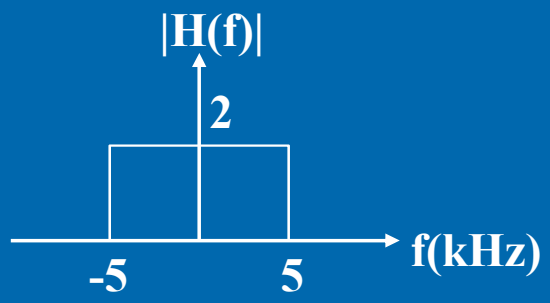
$|H(f)| = 0$, Nothing comes out if input freq is > f_c



Example 2.16

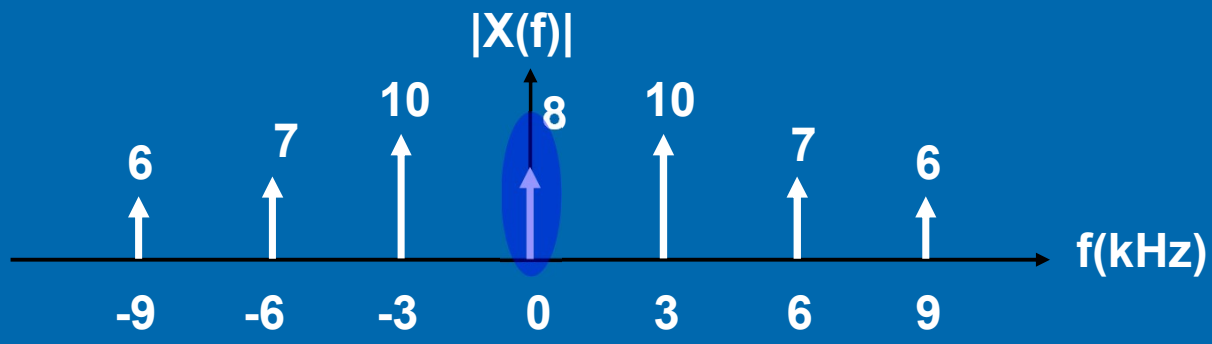
The amplitude spectrum of the input signal of an ideal LPF is shown below. The LPF has a cut-off frequency of 5 kHz. Determine the amplitude spectrum of the filter output signal.



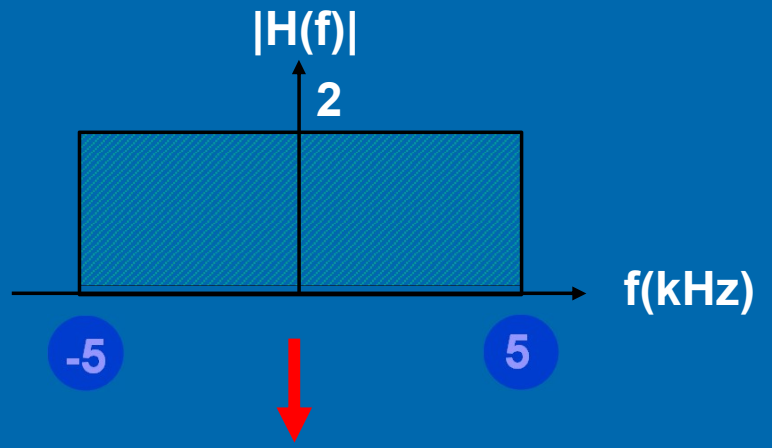


Solution

Input spectrum

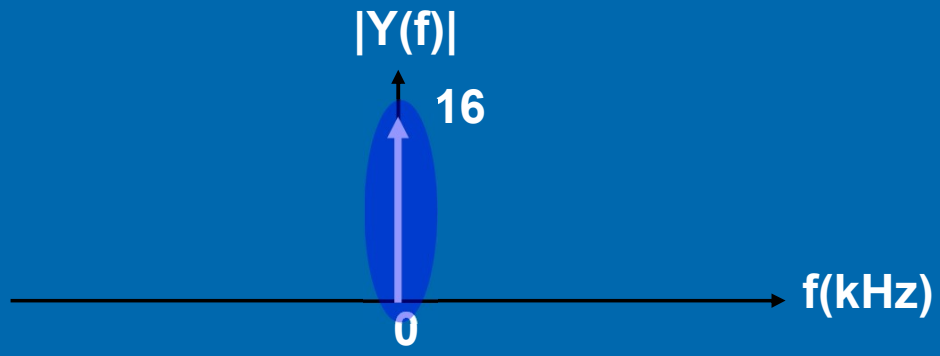


At f= 0 kHz, |H(f)| = 2

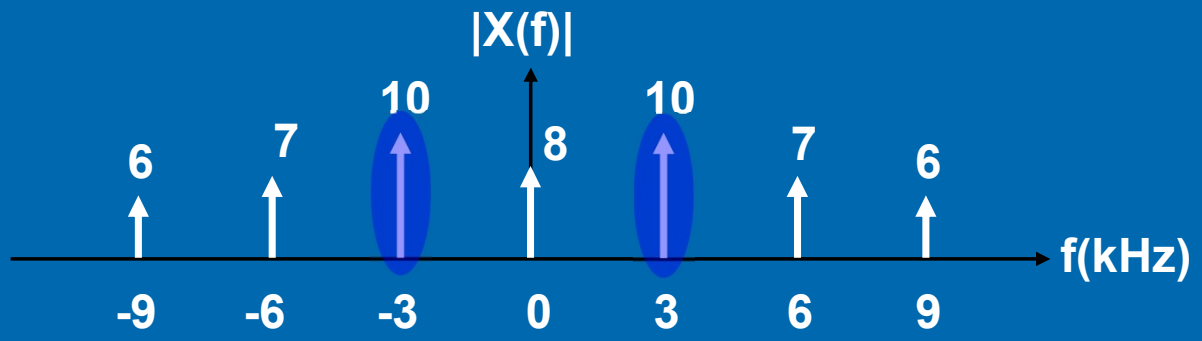


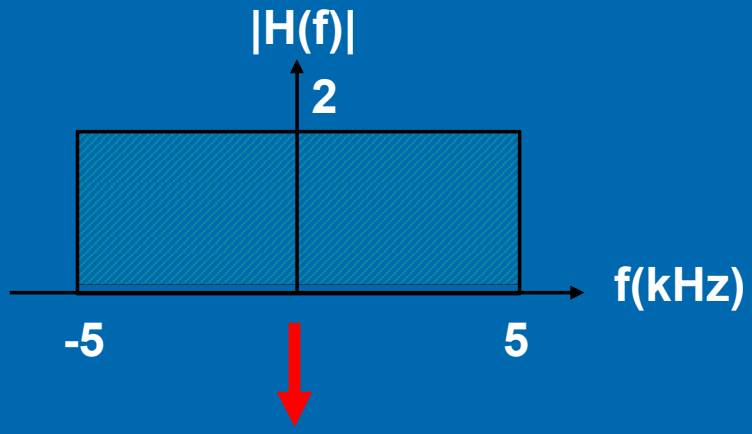
$|Y(f)| = |X(f)| \times |H(f)| = 8 \times 2 = 16$

Output spectrum



Input spectrum

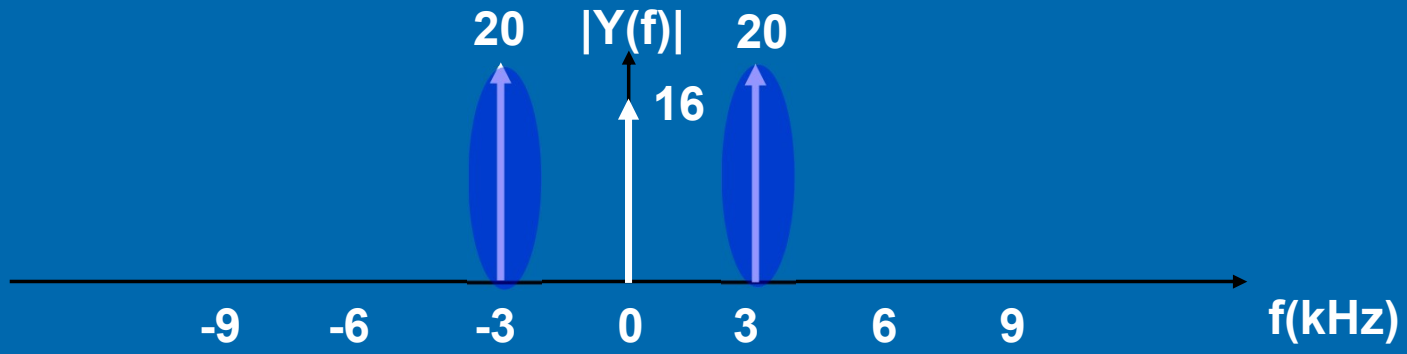




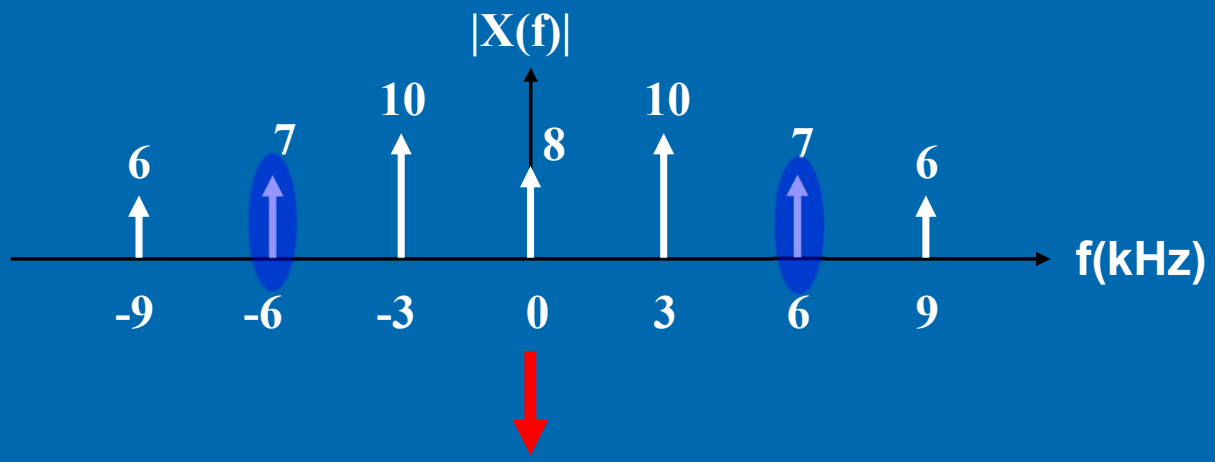
At f= ±3 kHz, |H(f)| = 2

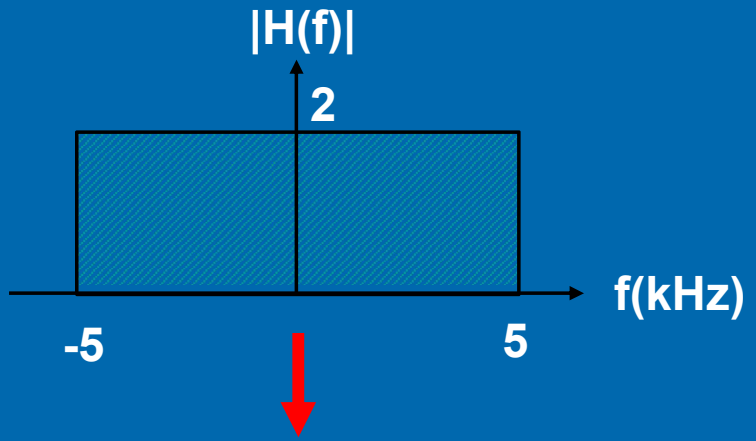
|Y(f)| = |X(f)| × |H(f)| = 10 × 2 =20

Output spectrum



Input spectrum

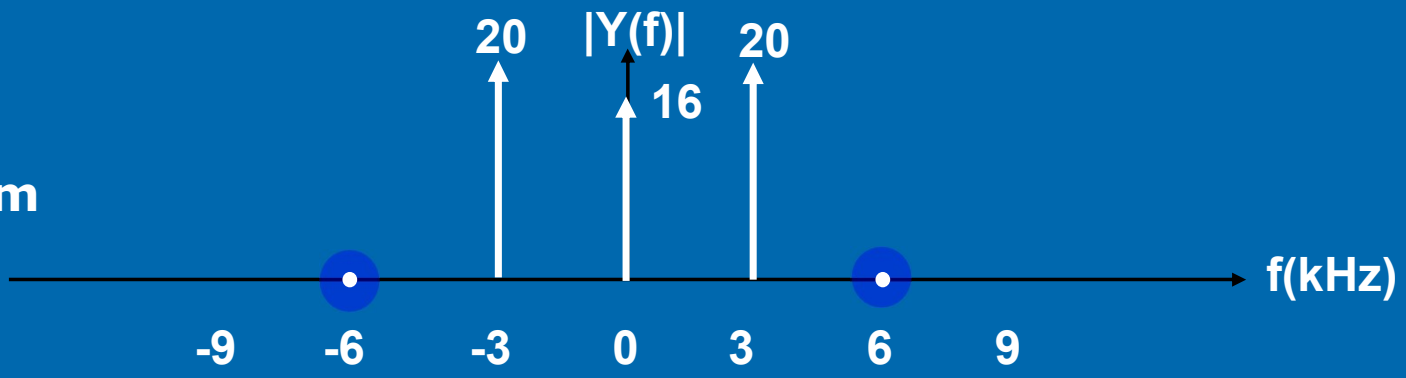




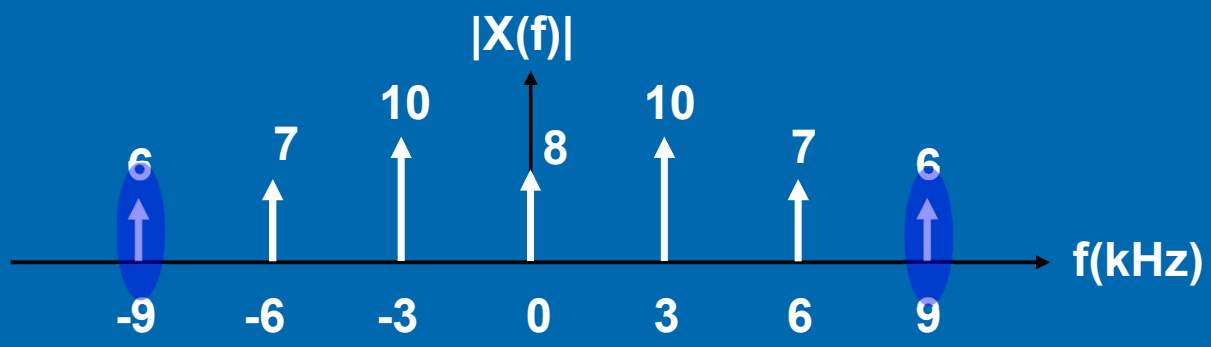
At f= ±6 kHz, $|H(f)| =$ 0

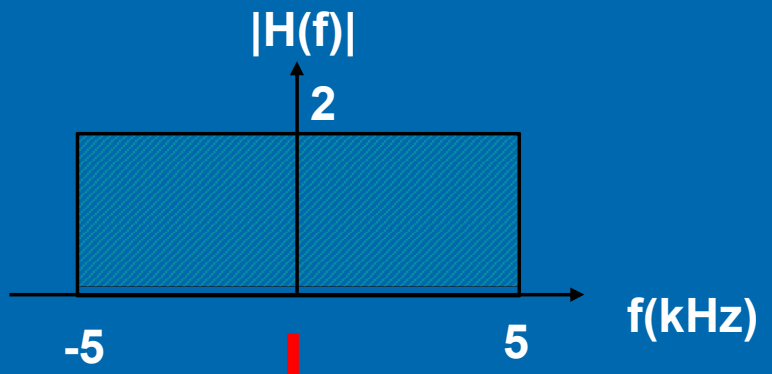
$|Y(f)| = |X(f)| \times |H(f)| = 7 \times 0 = 0$

Output spectrum



Input spectrum

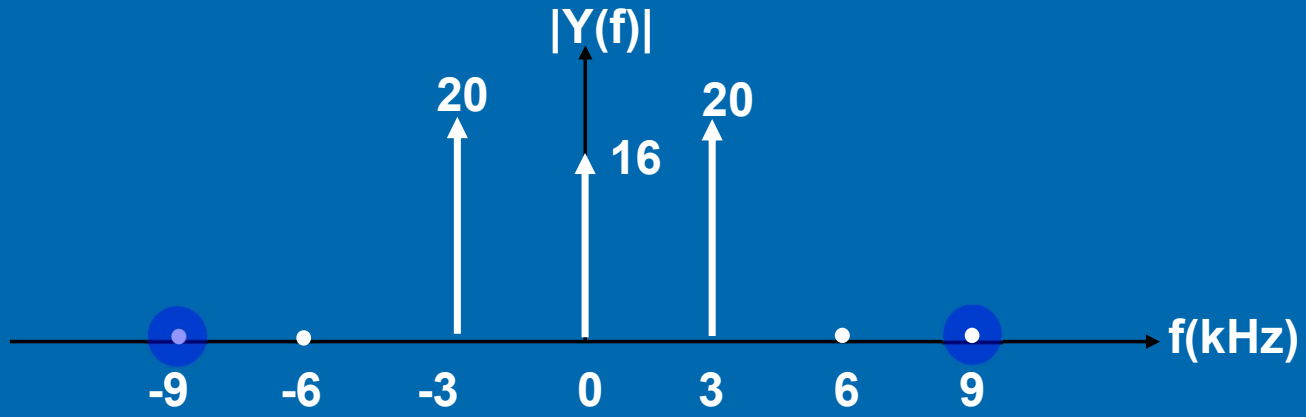




At f= ±9 kHz, |H(f)| = 0

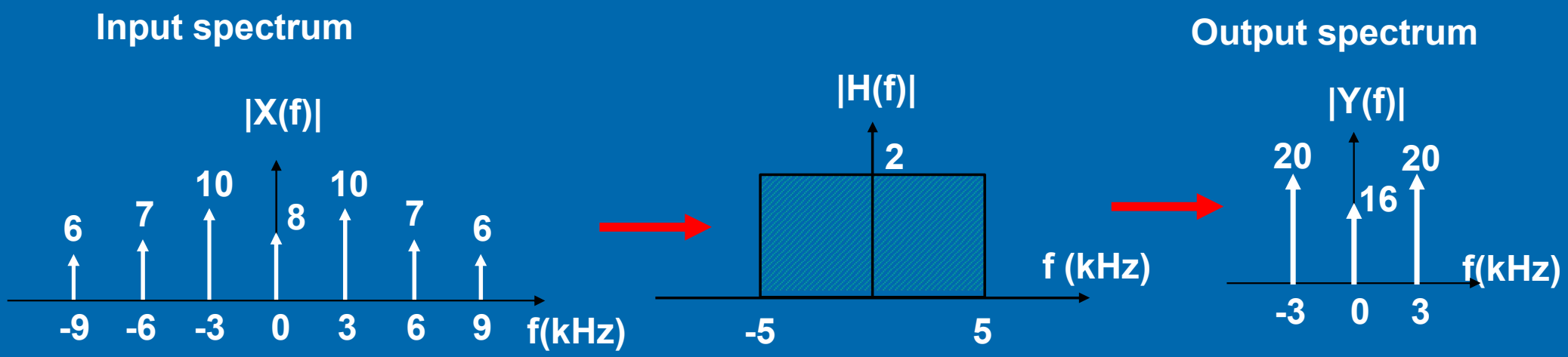
|Y(f)| = |X(f)| × |H(f)| = 6 × 0 = 0

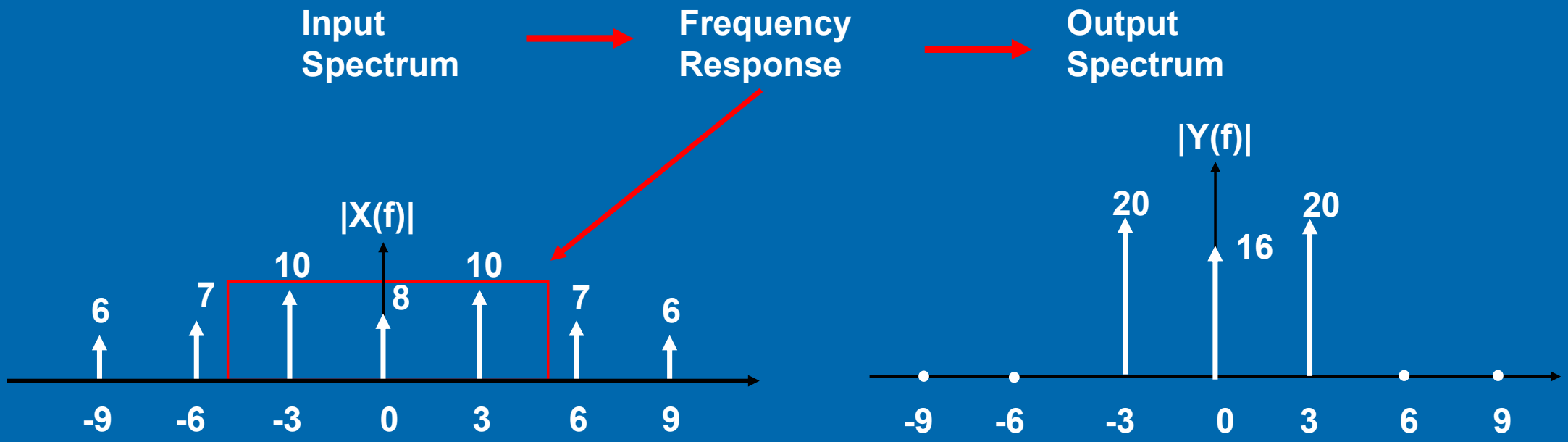
Output spectrum



Solution

Ideal LPF, cut-off frequency, = f_c 5 kHz.

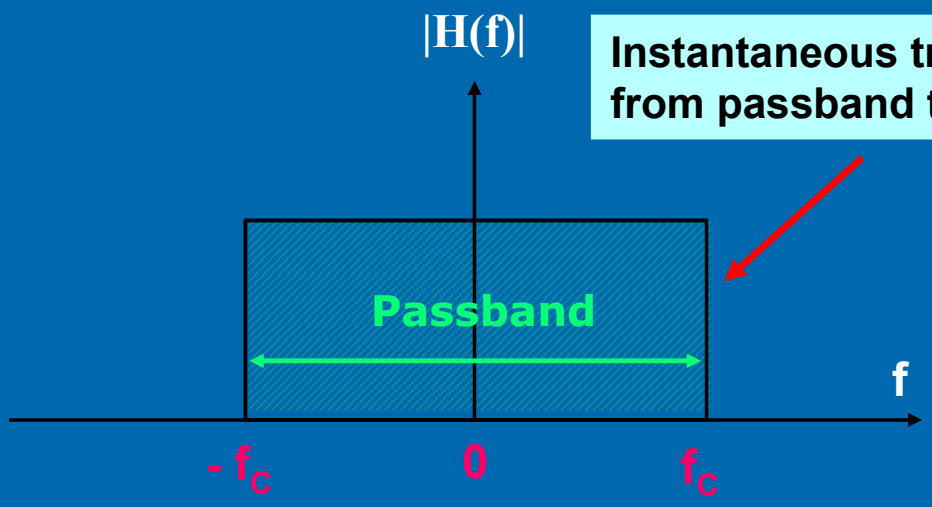



Input Spectrum
Frequency Response
Output Spectrum
|X(f)|
6
7
10
8
10
7
6
-9
-6
-3
0
3
6
9
|Y(f)|
20
16
20
-9
-6
-3
0
3
6
9

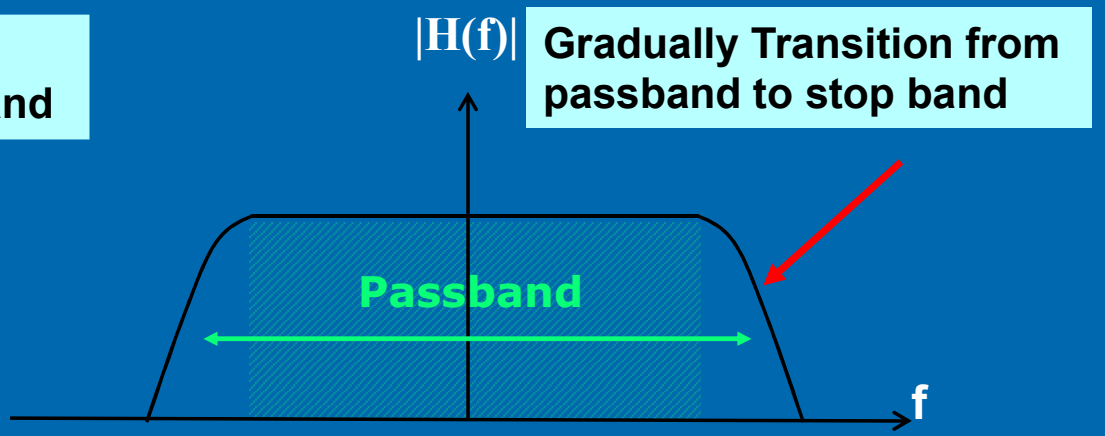
2.8 Filters

Practical Filters

Ideal Filters



Practical Filters

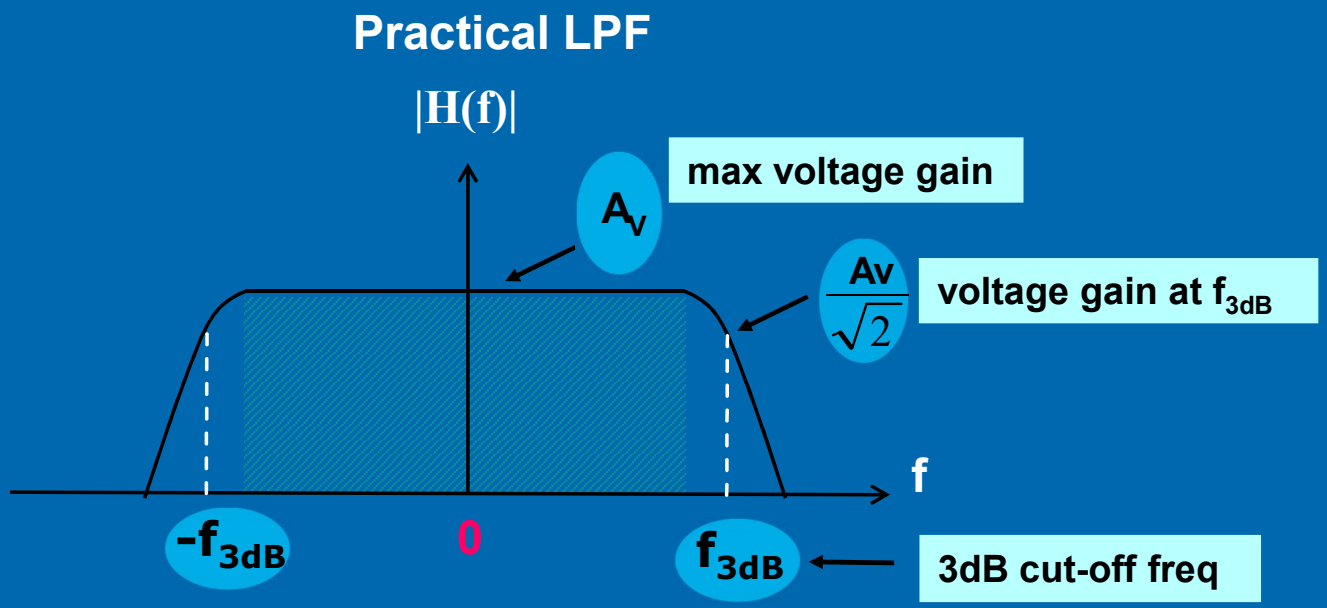


2.8 Filters

Filter parameters of practical filters

Cut-off frequency f_{3dB}

- Defined as the frequency at which the filter voltage gain has drop to 1/ $\sqrt{2}$, or 0.707 of its maximum voltage gain, A_v.
- Known as the 3 dB cut-off frequency.

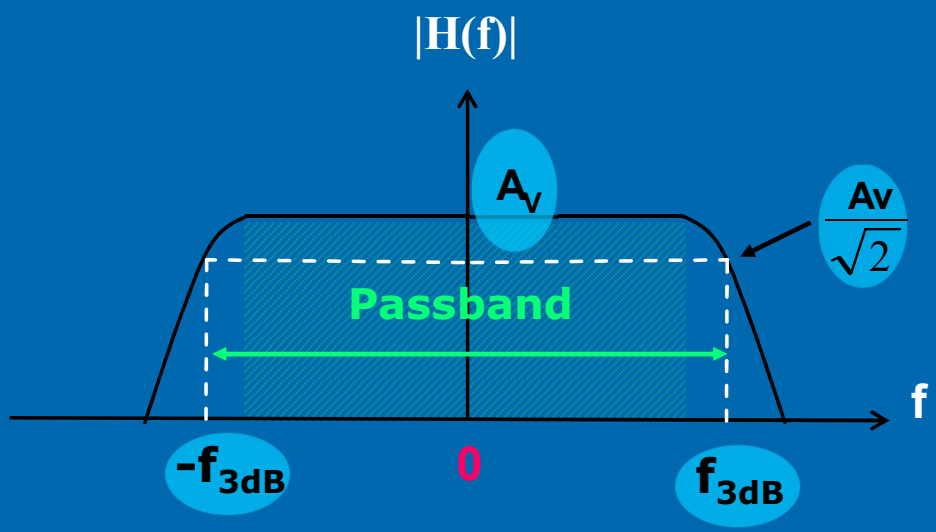


Filter parameters of practical filters

Passband

- The range of frequency that the filter will pass with a specified gain, traditionally, $|H(f)| = A_v/\sqrt{2}$ or $0.707A_v$.

Practical LPF

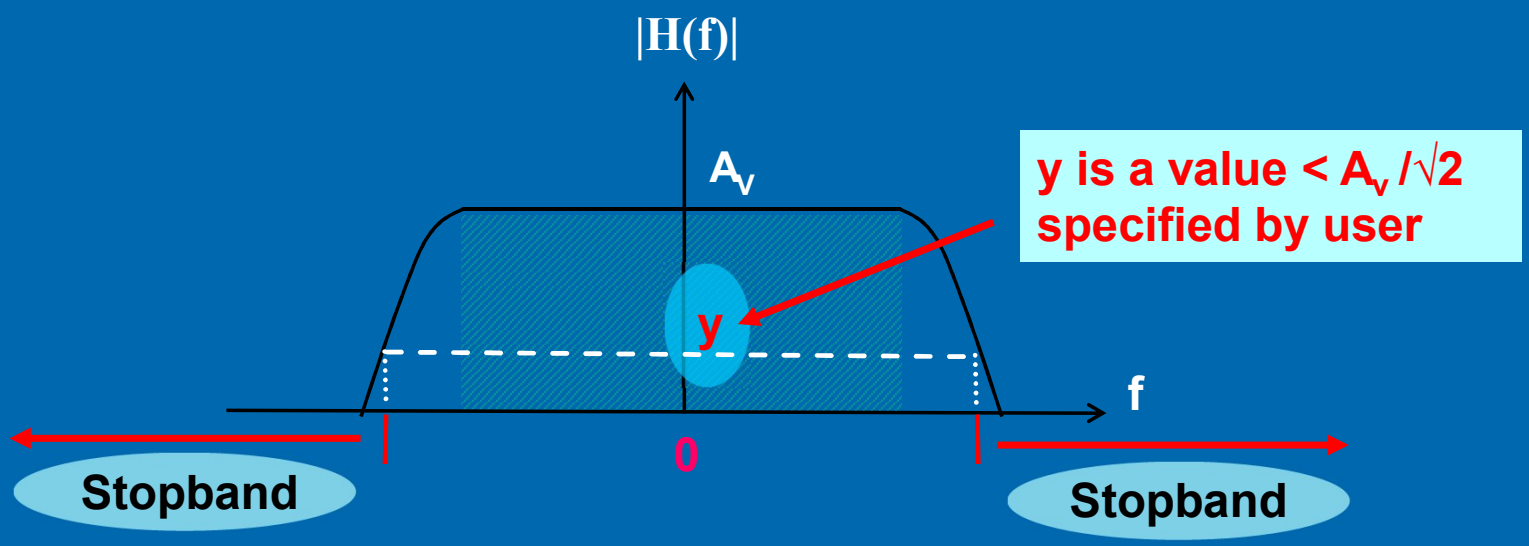


Filter parameters of practical filters

Stopband

- The band of frequencies in which the voltage gain |H(f)| is below a value y specified by users.
- Frequency components within the stop band will be amplified much less than those frequencies in the passband.

Practical LPF

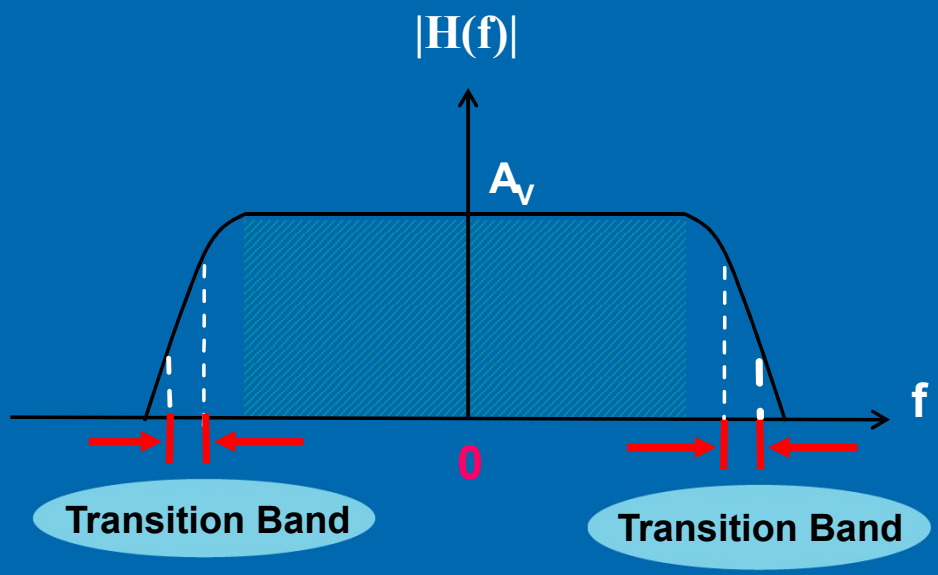


Filter parameters of practical filters

Transition band

- The band of frequencies between the passband and the stop band and vice-versa

Practical LPF

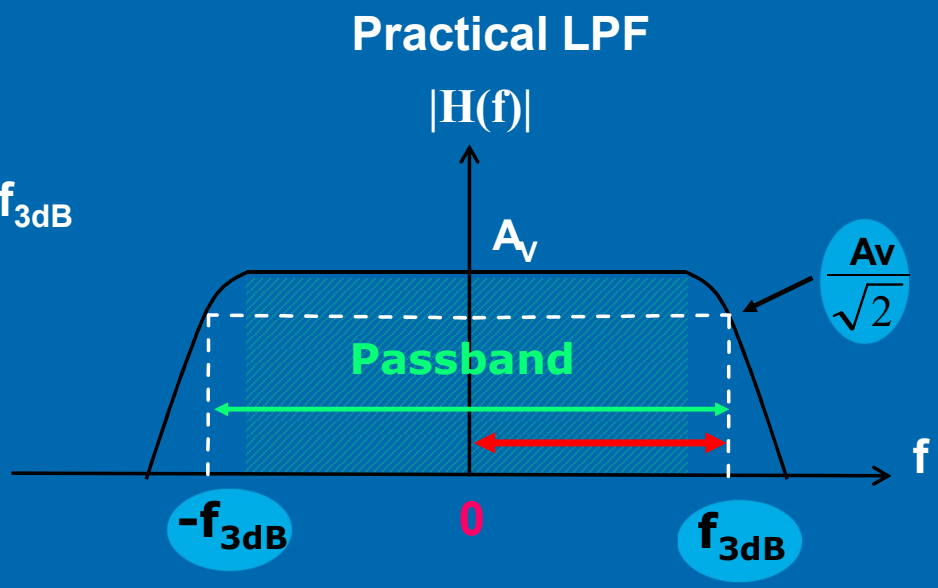


Filter parameters of practical filters

Filter bandwidth band

- The width of the passband of **positive frequencies** for LPF, HPF and BPF

Bandwidth = $f_{3dB} - 0 = f_{3dB}$

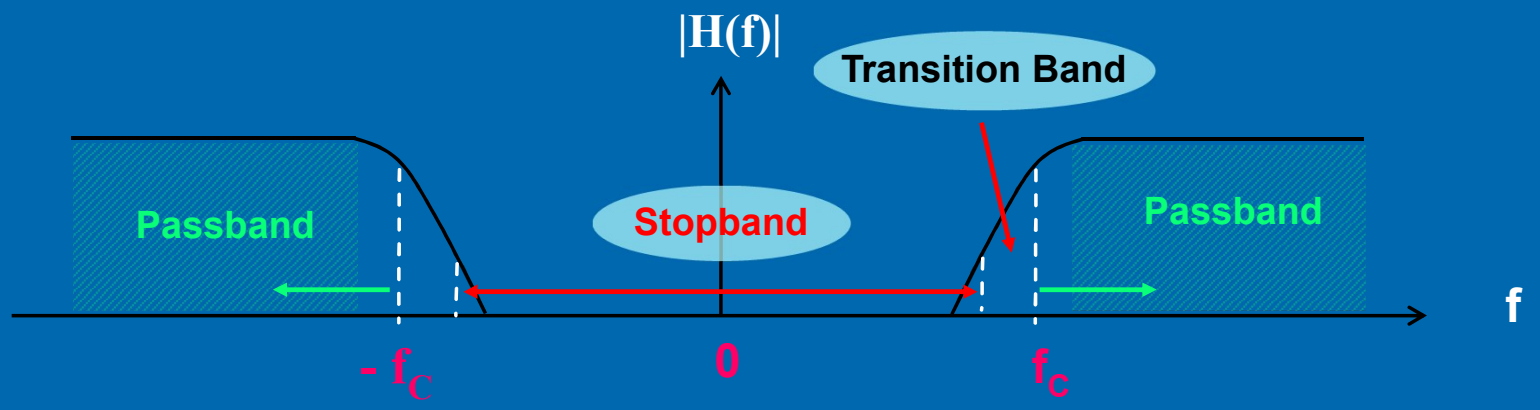


2.8 Filters

Filter parameters of practical filters

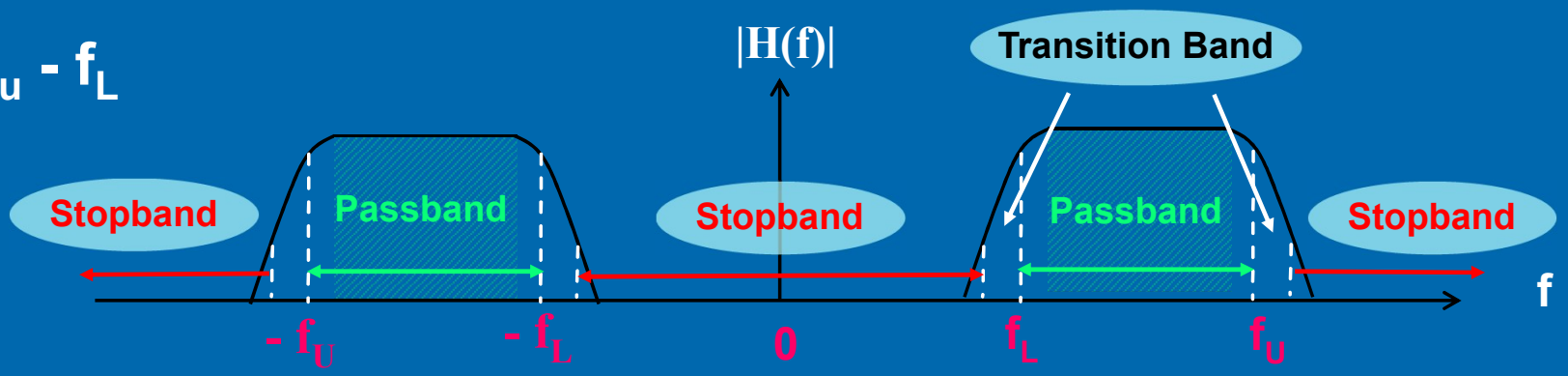
Practical HPF

Bandwidth = ∞



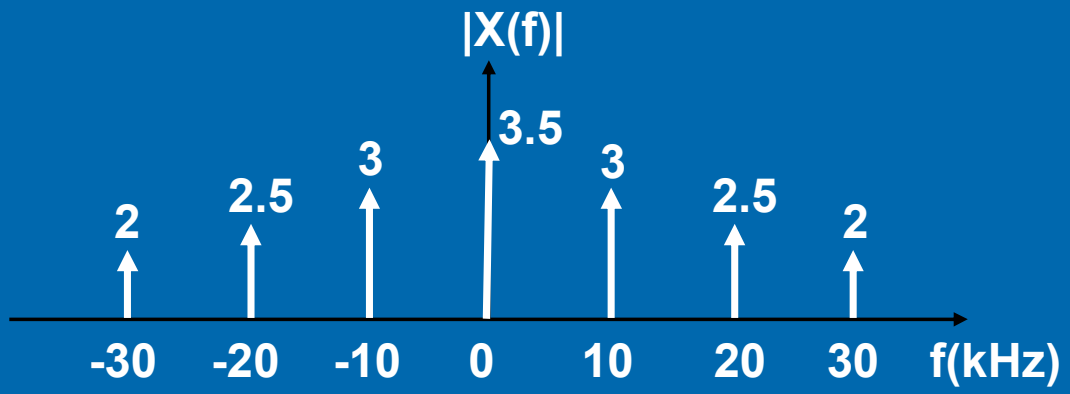
Practical BPF

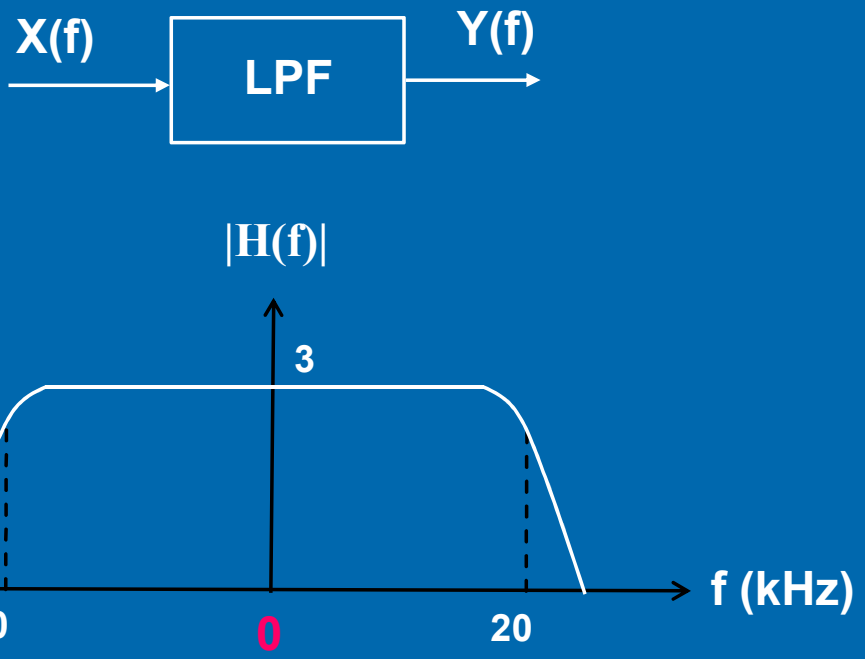
Bandwidth = f_u - f_L



Example 2.17

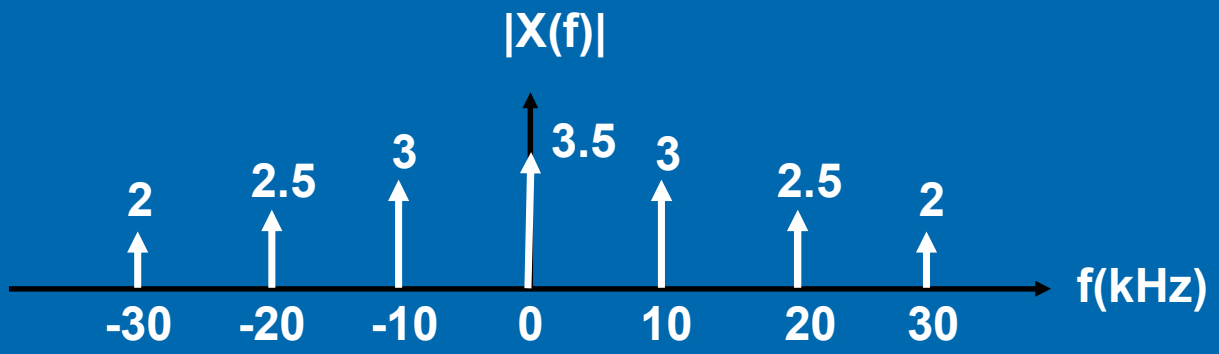
The amplitude spectrum of the input signal of a LPF is shown below. The LPF has a 3dB cut-off frequency of 20 kHz. Sketch the amplitude spectrum of the filter output signal.



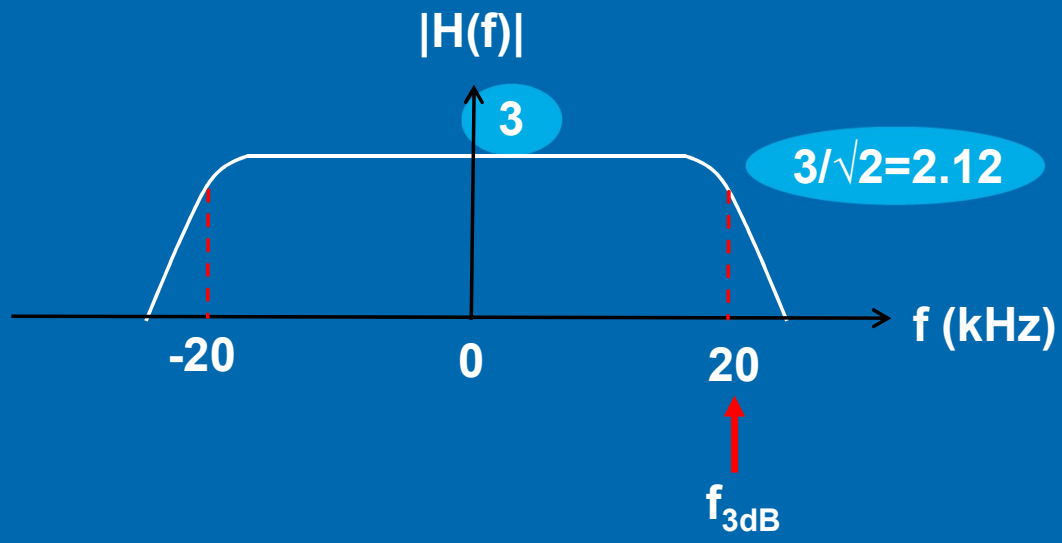


Solution

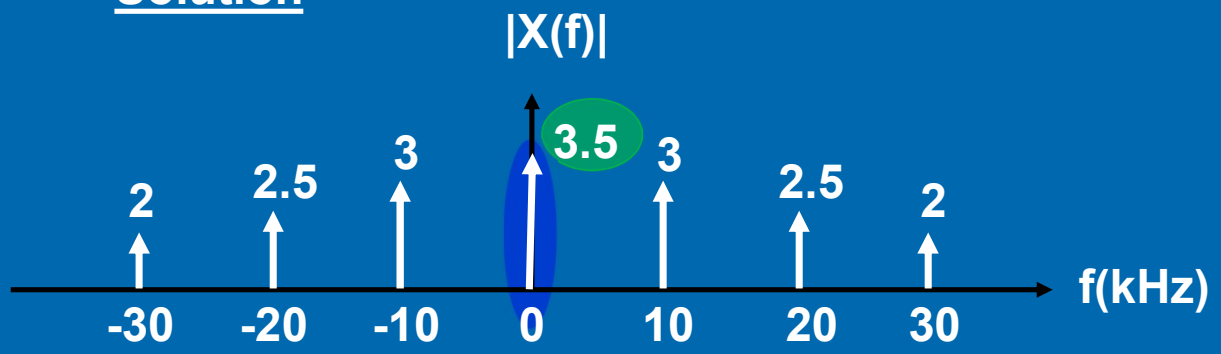
Input spectrum

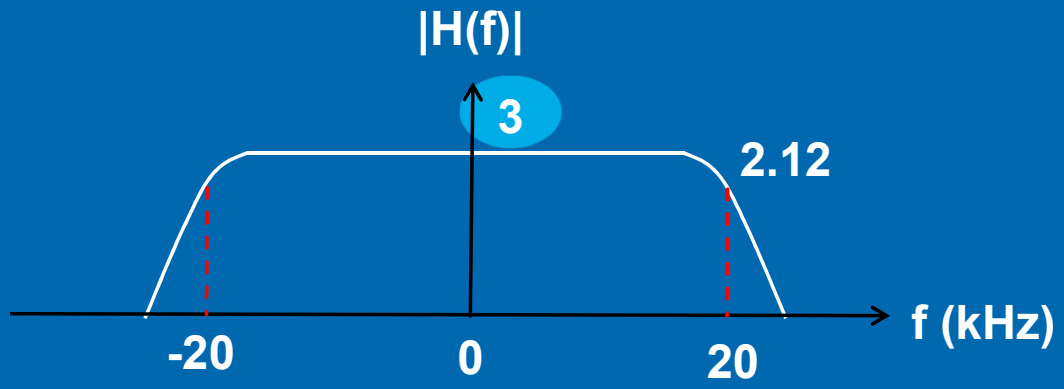


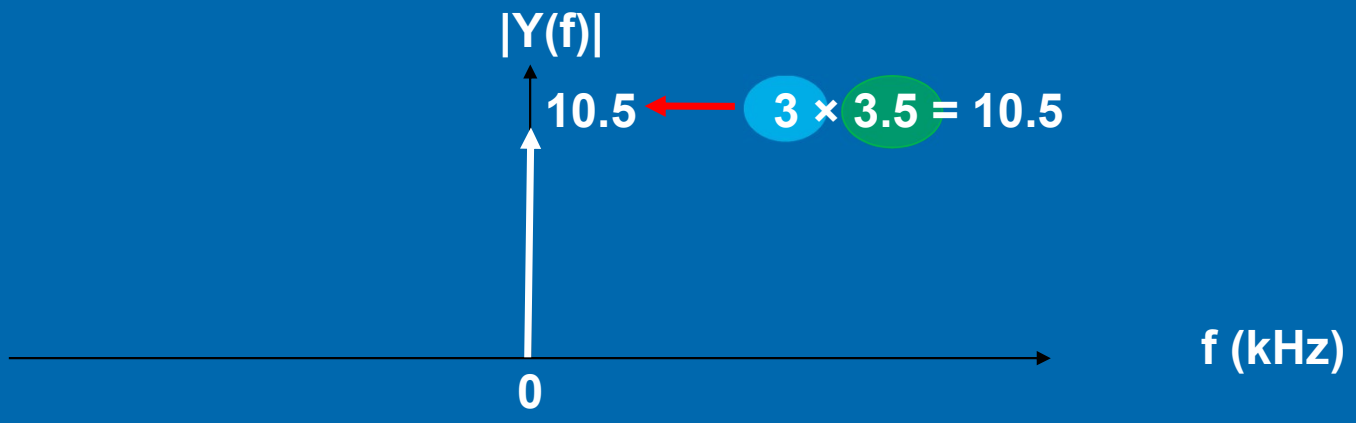
LPF Freq response



Solution







Input spectrum

|X(f)|

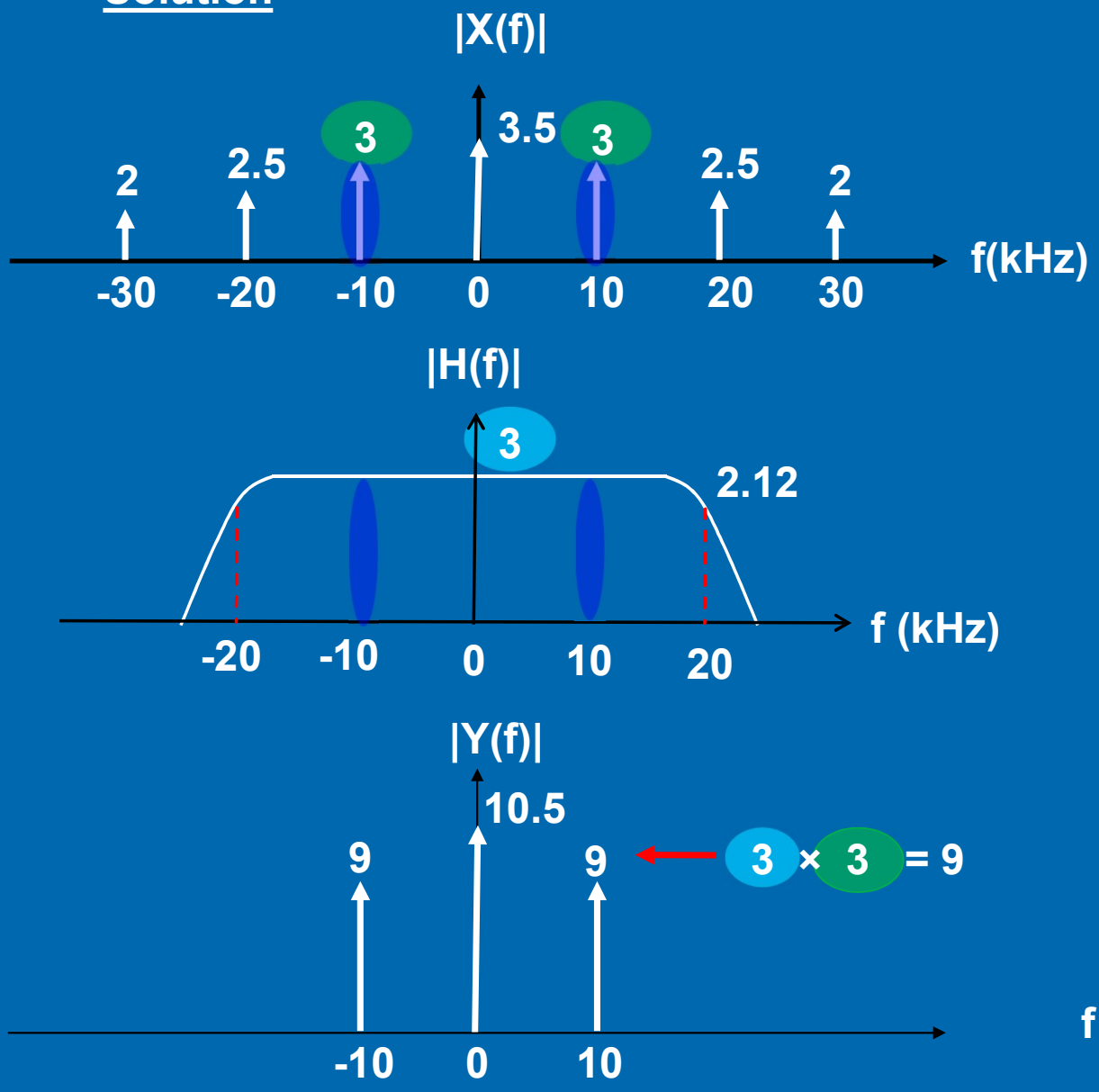
Freq response

|H(f)|

output spectrum

$|Y(f)| = |X(f)| \times |H(f)|$

Solution



Input spectrum

|X(f)|

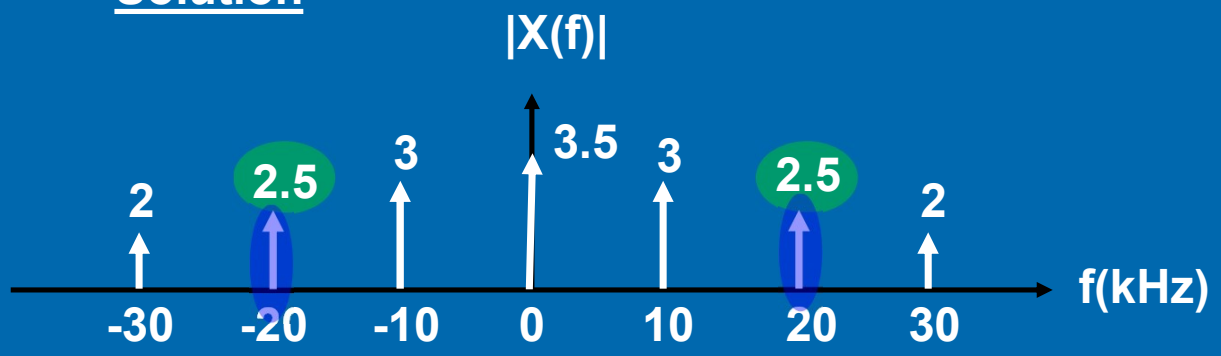
Freq response

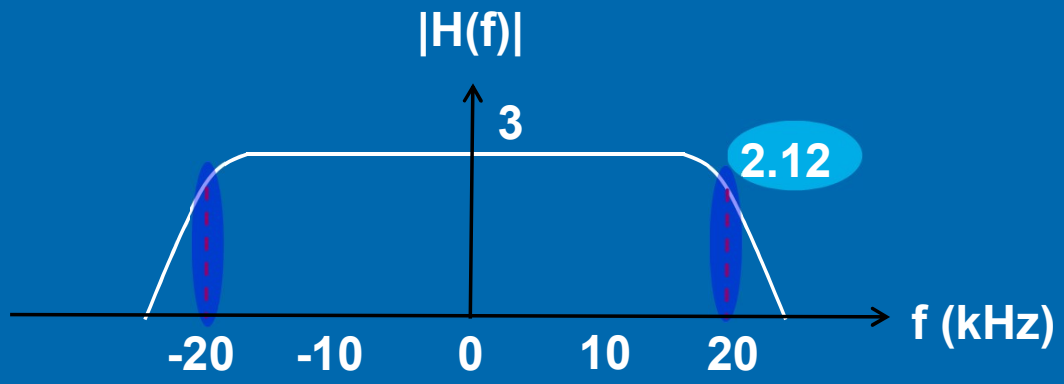
|H(f)|

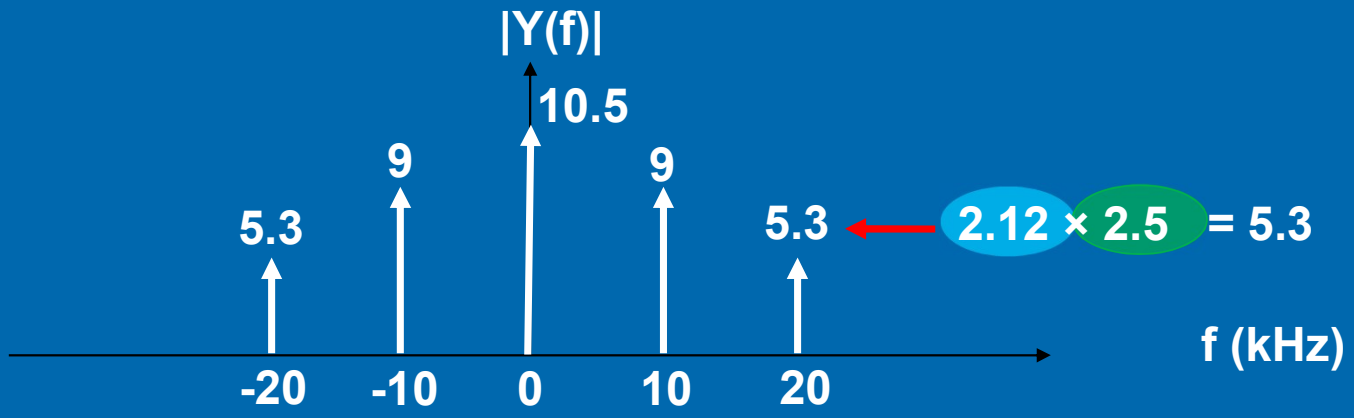
output spectrum

$|Y(f)| = |X(f)| \times |H(f)|$

Solution







$2.12 \times 2.5 = 5.3$

Input spectrum

$|X(f)|$

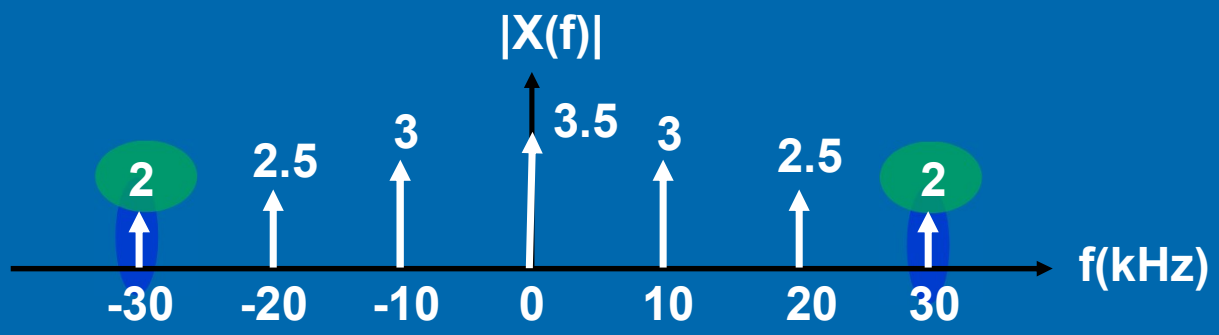
Freq response

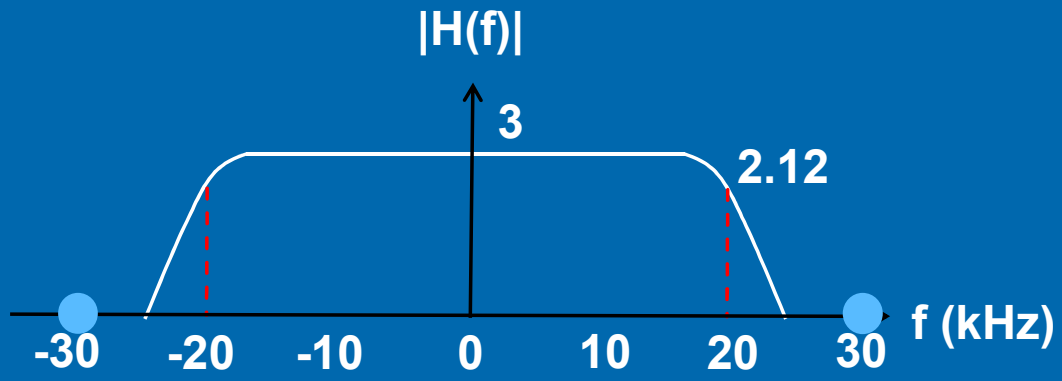
$|H(f)|$

output spectrum

$|Y(f)| = |X(f)| \times |H(f)|$

Solution





Input spectrum

$|X(f)|$

Freq response

$|H(f)|$

output spectrum

$|Y(f)| = |X(f)| \times |H(f)|$

End

CHAPTER 2

(Part 5 of 5)