SINGAPORE POLYTECHNIC
2020/2021 SEMESTER TWO END OF SEMESTER TEST

**EP0604/MS837M FURTHER MATHEMATICS**
Time Allowed: 1 hour 30 min + 10 min reading time

Instructions to Candidates

1. The Singapore Polytechnic examination rules are to be complied with.
2. This examination paper consists of FOUR printed pages.
3. Answer **ALL** the questions.
4. Give all non-exact answers to 3 significant figures.
5. A mathematical formulae and tables card is provided for reference.

**Additional Formulae**

**Absolute value Inequalities**: (i) $|x-a|<k$ is equivalent to $-k<x-a<k$

(ii) $|x-a|>k$ is equivalent to $x-a>k$ or $x-a<-k$

**VECTOR EQUATION OF A LINE**

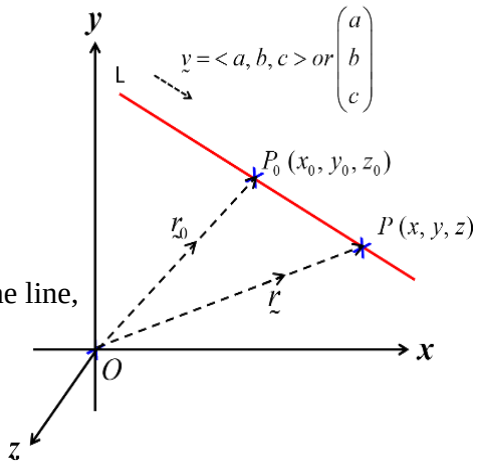
$$\underset{\sim}{r}=\underset{\sim}{r}_0+\lambda\underset{\sim}{v} \quad , \quad \lambda\in\mathbb{R}$$

where

$\underset{\sim}{r} = \langle x,y,z\rangle$ is the position vector of any point on the line,

$\underset{\sim}{r}_0 = \langle x_0,y_0,z_0\rangle$ is the position vector of a known point on the line,

$\underset{\sim}{v} = \langle a,b,c\rangle$ is a non-zero vector parallel to the line.




**PARAMETRIC EQUATIONS OF A LINE**

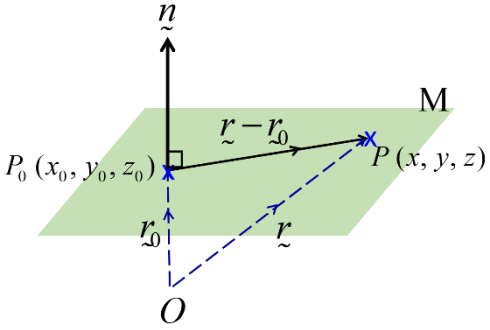
$$x=x_0+\lambda a,\quad y=y_0+\lambda b,\quad z=z_0+\lambda c \quad \text{where } \lambda\in\mathbb{R}$$

**EQUATION OF A PLANE**

The plane in $\mathbb{R}^3$ that passes through the point $P_0(x_0,y_0,z_0)$ and is normal to the non-zero vector $\underset{\sim}{n}=\langle a,b,c\rangle=a\underset{\sim}{i}+b\underset{\sim}{j}+c\underset{\sim}{k}$ has equations:

In vector form: $\underset{\sim}{n}\bullet\overrightarrow{P_0P}=0$ or $\underset{\sim}{r}\bullet\underset{\sim}{n}=\underset{\sim}{r}_0\bullet\underset{\sim}{n}$

In point-normal form: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$



1. Prove using mathematical induction that

$$1+2\times 2^1+3\times 2^2+\ldots+n\times 2^{n-1}=(n-1)2^n+1 \quad \text{for } n\in\mathbb{Z}^+$$ (15 marks)

2. (a) Find $\int(2x+1)e^{x^2+x}dx$ by using an appropriate substitution. (5 marks)

(b) Find the equation of the curve passing through the point $\left(0,\ -\frac{1}{4}\right)$, given that the slope to the curve at any point is $xe^{2x}$. (7 marks)

3. The graph of $f(x)=\sin^2 x$ in the interval $0\le x\le\pi$ is shown in figure 1. Find the area enclosed by the curve $f(x)$ and the $x$-axis. (10 marks)

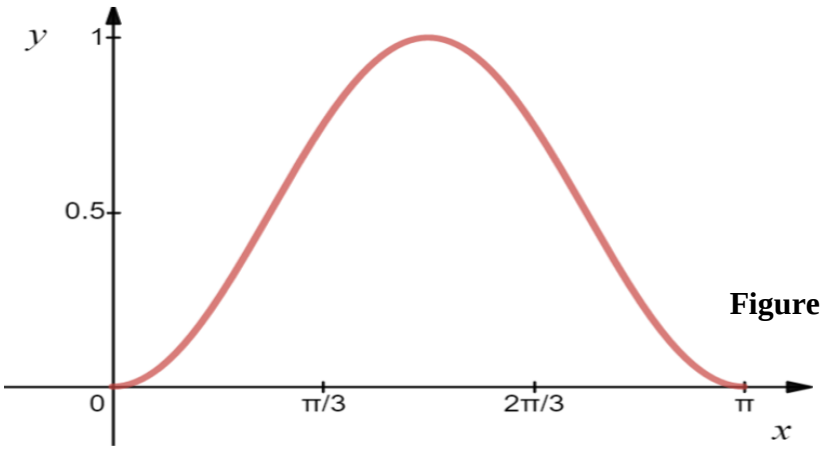



**Figure 1**

4. The figure below shows the shaded region $R$ bounded by the curves $y = e^x$ , $y = 3e^{-x} + 2$ and the $y-$axis. $A$ is the point of intersection between the two curves.

(a) Show that the $x-$coordinate of $A$ is $\ln 3$.

(b) Hence, find the exact volume of the solid generated when $R$ is rotated through one revolution about the $x-$axis.

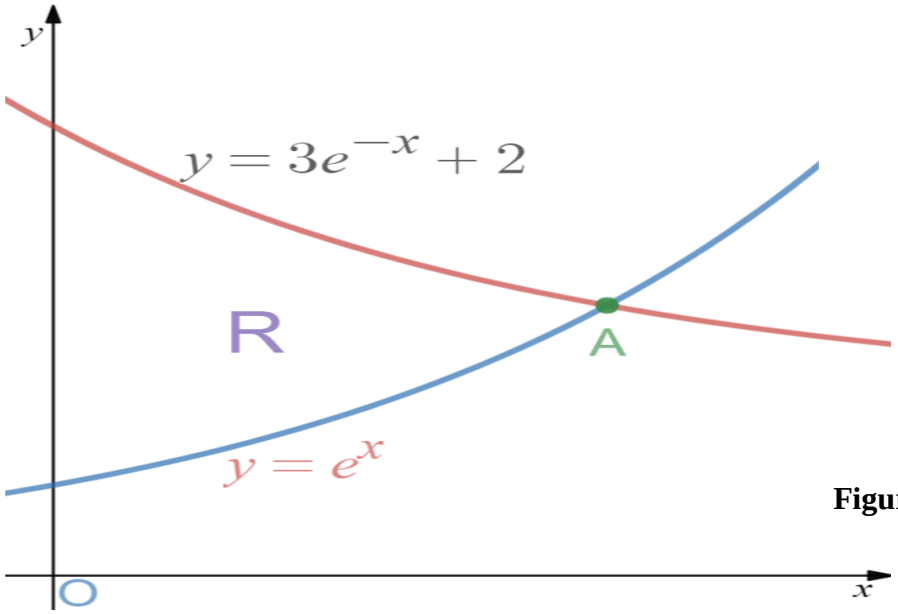



**Figure 2**

(15 marks)

5. (a) Given two points $A(2, 0, 4)$, $B(1, -1, 6)$ and a force $\vec{F}$ of 10N acting in the direction of $4\underset{\sim}{i} + 3\underset{\sim}{j}$, find the

(i) force vector $\vec{F}$, (2 marks)

(ii) work done by the force $\vec{F}$ in moving an object from B to A, distance being measured in metres. (5 marks)

(b) Explain why work done is maximized when a force is applied in the same direction as the displacement vector. (3 marks)

6. The line $L_1$ passes through the point $(3, -1, 6)$ and parallel to the line given by $x = 1 - \lambda,\ y = 2 + \lambda,\ z = 3$. Another line $L_2$ is given by $\langle 3,2,1\rangle + \mu\langle 1,1,-\frac{10}{3}\rangle$.

(a) Find the parametric equation of line $L_1$. (4 marks)

(b) Find the point of intersection of line $L_1$ and line $L_2$. (6 marks)

7. (a) Find the point of intersection of the plane $2x + y - z = 0$ and the line through $(0, -6, 0)$, given that the line is perpendicular to the plane. (8 marks)

(b) Determine, with workings, if the line and the plane are parallel, perpendicular or neither:

$$x = 2 + 2\lambda,\ y = 3 + 2\lambda,\ z = 2 + 4\lambda$$
$$-x + 5y - 2z = 3$$

(5 marks)

8. Find the range of values of $x$ for each of the following inequalities:

(a) $x^3 \geq x(4x + 12)$ (7 marks)

(b) $\dfrac{|3x-2|}{|x-3|} \geq 1$ (8 marks)

~END OF PAPER~

**Answers**

1 Step 3 need to prove: $1 + 2 \times 2^1 + 3 \times 2^2 + \cdots + (n+1) \times 2^n = n2^{n+1} + 1$

2 (a) $e^{x^2+x} + C$ (b) $y = x\frac{e^{2x}}{2} - \frac{e^{2x}}{4}$

3 $\frac{\pi}{2}$ units$^2$

4 (b) $4\pi(2+\ln 3)$ unit$^3$ or $38.9\,unit^3$

5 (a) (i) $\vec{F} = 10\frac{\langle 4,3,0\rangle}{\sqrt{4^2+3^2}} = \langle 8,6,0\rangle$ (ii) Work done = $\vec{F} \cdot \vec{S} = 14$ N

5 (b) When in same direction $\theta = 0$.

Work done $= \vec{F} \cdot \vec{S} = |\vec{F}||\vec{S}| \cos\theta$ and $cos\,\theta = 1$ when $\theta = 0$, work done is maximized.

6 (a) $x = 3 - t, y = -1 + t, z = 6$ (b) $\left(\frac{3}{2}, \frac{1}{2}, 6\right)$

7 (a) $(2, -5, -1)$

(b) $\begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 5 \\ -2 \end{pmatrix} = -2 + 10 - 8$

$= 0$

The line is perpendicular to the normal of the plane. Hence, the line and the plane is **parallel**.

8 (a) $x \geq 6$ Or $-2 \leq x \leq 0$ (b) $\frac{5}{4} \leq x < 3$ or x > 3 or $x \leq -1/2$