SINGAPORE POLYTECHNIC

2019/2020 SEMESTER ONE EXAMINATION

ENGINEERING MATHEMATICS II Time allowed: 2 hrs

2nd Year Full-Time

School of Chemical and Life Sciences
DCHE

School of Electrical and Electronic Engineering
DASE, DCPE, DEB, DEEE, DES, DESM

School of Mechanical and Aeronautical Engineering
DARE, DBEN, DCEP, DME, DMRO

Instructions to Candidates:

1. The examination rules set out on the last page of the answer booklet are to be complied with.
2. **The questions are printed on Pages 2 – 4**.
3. This paper consists of THREE (3) sections.

   Section A: 5 multiple-choice questions (MCQ), 2 marks each.
   Answer ALL five (5) questions in this section.

   Section B: 7 short questions, 10 marks each.
   The total mark of the questions in this section is 70 marks. You may answer as many questions as you wish. The marks from all the questions you answered will be added, but the maximum mark you can obtain from this section is 50 marks.

   Section C: 3 questions, total of 40 marks.
   Answer ALL three (3) questions in this section.

3. All answers are to be written in the answer booklet provided.
4. Unless otherwise stated, leave your decimal answers correct to **two** decimal places.
5. A 'mathematical formulae and tables' card is provided for your reference.
   Please do not write anything on the card, and return it at the end of the examination.
6. Except for sketches, graphs and diagrams, no solutions are to be written in pencil. Failure to comply will result in loss of marks.

<u>Section A</u> (10 marks):

Answer <u>ALL</u> multiple choice questions on the MCQ answer sheet of the answer booklet.

A1. If partial differentiation is performed on a function, then this function must have

(a) only one independent variable.

(b) more than one dependent variable.

(c) two or more independent variables.

(d) equal number of dependent and independent variables.

A2. The rational expression $\dfrac{3x}{(x+1)(x^2-1)}$ can be expressed in partial fractions as:

(a) $\dfrac{A}{x+1}+\dfrac{B}{x^2-1}$ (b) $\dfrac{A}{x+1}+\dfrac{B}{x-1}+\dfrac{C}{(x+1)^2}$

(c) $\dfrac{A}{x+1}+\dfrac{B}{x-1}+\dfrac{C}{x+1}$ (d) $\dfrac{A}{(x+1)^2}+\dfrac{B}{x-1}$

A3. If the Fourier series of a periodic function $f(t)$ of period 2 is given by

$$f(t)=\frac{1}{4}-\frac{2}{\pi^2}\cos\pi t-\frac{2}{9\pi^2}\cos 3\pi t+\cdots+\frac{1}{\pi}\sin\pi t-\frac{1}{2\pi}\sin 2\pi t+\frac{1}{3\pi}\sin 3\pi t+\cdots,$$

then what is the value of the D.C. component?

(a) 0 (b) $-\dfrac{2}{\pi^2}$ (c) $\dfrac{1}{\pi}$ (d) $\dfrac{1}{4}$

A4. If $f(t)=\mathcal{L}^{-1}\{F(s-\pi)\}$, then what is $\mathcal{L}^{-1}\{F(s-2\pi)\}$?

(a) $e^{\pi t}f(t)$ (b) $f(t)-\pi$ (c) $e^{2\pi t}f(t)$ (d) $e^{t-2\pi}f(t)$

A5. Using Simpson's rule to estimate $\int_0^3 \sqrt{x^2+1}\,dx$, which of the following choices of h or n will give the most accurate estimate?

(a) $h=0.15$ (b) $h=1$ (c) $n=101$ (d) $n=40$

<u>Section B</u> (50 marks):

Each question carries 10 mark. The total marks of the questions in this section is 70. You may answer as many questions as you wish. The marks from all questions you answered will be added, but the maximum mark you can obtain from this section is 50 marks.

B1. (a) Find $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ when $f(x,y) = xy^2 + 3xy - x + 2$.

(b) If $g(x,y) = (y^2 + x)e^{-xy}$, evaluate $g_y(0,2)$.

B2. (a) Use integration by parts to find the integral $\int x^2 \cos x \, dx$.

(b) Evaluate the definite integral $\displaystyle\int_0^1 \frac{x^2}{(x^3+1)^2}\,dx$ by using the substitution $u = x^3 + 1$.

B3. Using Simpson's rule with $n = 4$, find the approximation for the integral $\int_0^2 \sin\left(x^2\right)dx$.

B4. (a) Solve the differential equation, $e^{-(2x-1)}\dfrac{dy}{dx} = e^{-y}$, by separating the variables.

(b) Find the RMS of the voltage $v(t) = 6\sin t$ over the interval $0 \le t \le 2\pi$. Show your detailed working clearly.

B5. (a) Find $\mathcal{L}\left\{3t^3 - 2e^{-5t} + 7\right\}$.

(b) Find $\mathcal{L}\{2t\sin 4t\}$. Hence, use first-shift theorem to find $\mathcal{L}\left\{2t\,e^{\pi t}\sin 4t\right\}$.

(c) By using the compound angle formula for $\cos(3t+\pi)$, find $\mathcal{L}\{\cos(3t+\pi)\}$.

B6. (a) Find $\mathcal{L}^{-1}\left\{\dfrac{3}{s^2} + \dfrac{2s}{s^2+9} + \dfrac{5}{s^2+4}\right\}$.

(b) By first shift theorem, find $\mathcal{L}^{-1}\left\{\dfrac{3}{(s-1)^2+9}\right\}$.

(c) By partial fraction, find $\mathcal{L}^{-1}\left\{\dfrac{5}{(s-2)(s+2)}\right\}$

B7. A second order differential equation is given as $y'' + 4y' + 4y = 0$.

(i) By using a suitable auxiliary equation, find its general solution.

(ii) If $y(0) = 1$ and $y'(0) = -2$, find the particular solution.

Section C (40 marks):

Answer all THREE questions below.

C1. When a current I passes through a resistor R, the voltage drop is given by $V = IR$. If V and R are measured with possible errors of 1% and 4% respectively, find the approximate percentage error in I.

(10 marks)

C2. A tank of hot oil at a temperature of 120°C is placed in a workshop at the start of the day to cool. The temperature T (°C) of the oil changes at a rate proportional to the difference in the temperature of the oil and the room temperature, R (°C).

(a) Given that k is a positive constant of proportionality, set up a differential equation that describes the rate of change of the temperature of the oil t hours from the start of the day in terms of T and R.

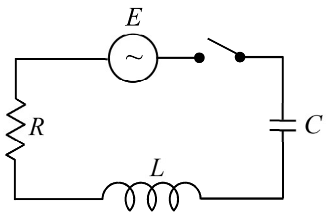
(b) Solve the differential equation set up in part (a), if k is 0.1 and the room temperature is maintained constant at 30°C.

(c) What is the temperature of the oil after two hours?

(12 marks)

C3. (a) Given $\dfrac{5000}{(s^2+100)(s^2+10s+100)} = \dfrac{As+B}{s^2+100} + \dfrac{Cs+D}{s^2+10s+100}$.

If $B = 0$ and $C = 5$, find the constants A and D.



(b) In the RLC circuit shown on the right, the electromotive force E is given as $E(t) = 500\sin 10t$ volts. It is known that initially, there is neither current nor charge in the circuit.

(i) Set up the differential equation with initial conditions to model the charge $q(t)$ flowing through the capacitor.

[Note: the voltage drops across the resistor, inductor, and capacitor are

respectively $v_R = iR,\ v_L = L\dfrac{di}{dt},\ v_C = \dfrac{q}{C}$]

(ii) If $L = 1$ henry, $R = 10$ ohms, and $C = 0.01$ farad, show that

$$\mathcal{L}\{q(t)\} = Q(s) = \frac{5000}{(s^2+100)(s^2+10s+100)}$$

(iii) Hence, by using the result obtained in part (a), solve the differential equation for the charge $q(t)$.

(18 marks)

- End of Paper -