FM EP0604/MS837M

2021 S1 EST Solutions

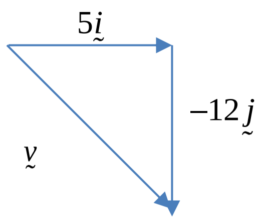
No.	**SOLUTION**
1(a)	$\dfrac{1}{x-2} \geq \dfrac{2x-3}{(x-2)(x-3)}$
	(zero marks for cross multiply)
	$\dfrac{1}{x-2} - \dfrac{2x-3}{(x-2)(x-3)} \geq 0$
	$\dfrac{x-3-(2x-3)}{(x-2)(x-3)} \geq 0$
	$\dfrac{x}{(x-2)(x-3)} \leq 0$
	From number line: $x \leq 0 \;\; or \;\; 2 < x < 3$
(b)	$\left\|\dfrac{x+1}{x-1}\right\| \leq 2 \;\to -2 \leq \dfrac{x+1}{x-1} \leq 2$
	$-2 \leq \dfrac{x+1}{x-1} \qquad and \qquad \dfrac{x+1}{x-1} \leq 2$
	$\dfrac{x+1}{x-1} + 2 \geq 0 \qquad \dfrac{x+1}{x-1} - 2 \leq 0$
	$\dfrac{x+1+2x-2}{x-1} \geq 0 \qquad \dfrac{x+1-2x+2}{x-1} - 2 \leq 0$
	$\dfrac{3x-1}{x-1} \geq 0 \qquad \dfrac{-x+3}{x-1} \leq 0$
	From number line: $x \leq \frac{1}{3} \;\; or \;\; x > 1 \qquad and \qquad x < 1 \; or \; x \geq 3$
	Hence, $x \leq \frac{1}{3} \; or \; x \geq 3$

No.	**SOLUTION**
2(a)	$u = \ln x \qquad dv = x dx$
	$du = \dfrac{1}{x} dx \qquad v = \dfrac{x^2}{2}$
	$y = \int x \ln x dx$
	$y = \ln x \dfrac{x^2}{2} - \int \dfrac{1}{x} \dfrac{x^2}{2} dx$
	$y = \ln x \dfrac{x^2}{2} - \dfrac{x^2}{4} + C$

No.	SOLUTION
2(b)	$(1,0),\ C=\frac{1}{4}$ $y=\ln x\frac{x^2}{2}-\frac{x^2}{4}+\frac{1}{4}$ Let $u=\cos x$ $du=-\sin x\,dx$ $-du=\sin x dx$ $\int \sin(x)\left[\cos(x)\right]^2\ dx$ $=-\int u^2 du$ $=-\frac{1}{3}u^3+C$ $=-\frac{\cos^3 x}{3}+C$

No.	SOLUTION
3(a)	$\sqrt{2x-1}=x-0\cdot 5$ $2x-1=x^2-x+0.25$ $x^2-3x+1.25=0$ $x=0.5, x=2.5$
(b)	$$A=\int_{0.5}^{2.5}\sqrt{2x-1}-x+0.5\ \ dx$$ $$=\left[\frac{(2x-1)^{\frac{3}{2}}}{2\left(\frac{3}{2}\right)}-\frac{x^2}{2}+0.5x\right]_{0.5}^{2.5}$$ = 2/3 units square

No.	SOLUTION
4	• Volume of revolution generated by $4x^2+4y=41$ $V=\pi\int_0^{5/2}\left(-x^2+\frac{41}{4}\right)^2dx$ $=\pi\int_0^{5/2}\left(x^4-\frac{41}{2}x^2+\frac{1681}{16}\right)dx$ $=\pi\left[\frac{x^5}{5}-\frac{41}{6}x^3+\frac{1681}{16}x\right]_0^{5/2}$ $=175\frac{5}{12}\pi\ unit^3$ • Volume of revolution generated by $y=\lvert 2x-3\rvert+2=\begin{cases}-2x+5\\ 2x-1\end{cases}$ $V=\pi\int_0^{3/2}(-2x+5)^2dx+\pi\int_{3/2}^{5/2}(2x-1)^2dx$ $=\pi\int_0^{3/2}\left(4x^2-20x+25\right)dx+\pi\int_{3/2}^{5/2}\left(4x^2-4x+1\right)dx$ $=\pi\left[\frac{4x^3}{3}-10x^2+25x\right]_0^{3/2}+\pi\left[\frac{4x^3}{3}-2x^2+x\right]_{3/2}^{5/2}$ $=\pi\left(19\frac{1}{2}-0\right)+\pi\left(10\frac{5}{6}-1\frac{1}{2}\right)$ $=28\frac{5}{6}\pi\ unit^3$ • Volume of revolution of region R $=\left(175\frac{5}{12}-28\frac{5}{6}\right)\pi\ unit^3=146\frac{7}{12}\pi\ unit^3$

No.	SOLUTION
5 (a)	

5(b)	$\left\lVert 2\underset{\sim}{u}+\underset{\sim}{v} \right\rVert = \left\lVert 11\underset{\sim}{i}-4\underset{\sim}{j} \right\rVert$ $= \sqrt{11^2+(-4)^2}$ $= \sqrt{137}$
(c)	$\lvert\underset{\sim}{u}\rvert = \sqrt{3^2+(4)^2} = 5 = 3$ $\hat{\underset{\sim}{u}} = \frac{1}{\lvert\underset{\sim}{u}\rvert}\underset{\sim}{u} = \frac{1}{5}\left(3\underset{\sim}{i}+4\underset{\sim}{j}\right)$
(d)	$\underset{\sim}{u}\cdot\underset{\sim}{v} = \lVert a\rVert\lVert b\rVert\cos\theta$ $15-48 = 5(13)\cos\theta$ $\cos\theta = -\frac{33}{65}$ $\theta = 120.51^\circ$

No.	**SOLUTION**
6(a) (i)	$\boldsymbol{r} = \begin{pmatrix}4\\-1\\2\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\1\end{pmatrix}$
(ii)	$(4+\lambda)-2(-1-2\lambda)+(2+\lambda) = 20$ $\Longrightarrow 4+2+2+\lambda+4\lambda+\lambda = 20$ $\Longrightarrow \lambda = 2$ The point of intersection is: (4+2, -1-4, 2+2)=(6, -5, 4)
(b)(i)	$\overrightarrow{BC} = \begin{pmatrix}-2\\0\\0\end{pmatrix}$ $\overrightarrow{BV} = \begin{pmatrix}-2\\-2\\4.5\end{pmatrix}$ $\underset{\sim}{n} = \begin{vmatrix}\underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k}\\-2 & 0 & 0\\-2 & -2 & 4.5\end{vmatrix}$ $= \underset{\sim}{i}(0)-\underset{\sim}{j}(-9-0)+\underset{\sim}{k}(4-0)$ $= 9\underset{\sim}{j}+4\underset{\sim}{k}$
b(ii)	B is on the plane, $\begin{pmatrix}x-2\\y-2\\z-0\end{pmatrix}\cdot\begin{pmatrix}0\\9\\4\end{pmatrix} = 0 \Rightarrow \boldsymbol{9y+4z=18}$

No.	SOLUTION
7(a)	$f(x) = 2x+1$ $f^2(x) = 2(2x+1)+1$ $= 4x+3$ (shown)
(b)	Let P_n be the statement $f^n(x) = 2^n x + 2^n - 1$ STEP 1: Prove that P_1 is true. When n = 1, LHS = $f^1(x) = 2x+1$ and RHS = $2^1(x) + 2^1 - 1 = 2x+1$ Hence LHS = RHS. Therefore P_1 is true. STEP 2: Assume that P_n is true for an arbitrary $n \in Z^+$. P_n: $f^n(x) = 2^n x + 2^n - 1$ STEP 3: Prove that P_{n+1} is true. P_{n+1}: $f^{n+1}(x) = 2^{n+1}x + 2^{n+1} - 1$ $L.H.S. = f^{n+1}(x)$ $= f^n f(x)$ $= f^n(2x+1)$ $= 2^n(2x+1) + 2^n - 1$ $= 2^{n+1}x + 2^n + 2^n - 1$ $= 2^{n+1}x + 2^{n+1} - 1 = R.H.S(shown)$ Hence P_n is true implies P_{n+1} is true. Since P_1 is true, it follows by the principle of mathematical induction that P_n is true for all $n \in Z^+$