Answers:

I. Partial Differentiation

MCQ: 1. (b) 2. (b)

Structured questions:

1. (a) $\frac{\partial f}{\partial x} = 5x^4 + 3x^2y^2 + 3y^4$; $\frac{\partial f}{\partial y} = 2x^3y + 12xy^3$

   (b) $\frac{\partial f}{\partial x} = 3x^2 + 10xy$; $\frac{\partial f}{\partial y} = 5x^2 + 6y^2$

   (c) $\frac{\partial f}{\partial x} = 3x^2y^2 - \frac{y}{x^2}$; $\frac{\partial f}{\partial y} = 2x^3y + \frac{1}{x}$

2. (a) $f_x(1,2) = 1$; (b) $h_y(1,0) = 4$

3. (a) $f_r(r,s) = \ln\left(r^2 + s^2\right) + \frac{2r^2}{r^2 + s^2}$; $f_s(r,s) = \frac{2rs}{r^2 + s^2}$

   (b) $h_u = \frac{u}{u^2 - v^2}$; $h_v = \frac{-v}{u^2 - v^2}$

   (c) $\frac{\partial z}{\partial x} = 2x\sin(xy) + x^2y\cos(xy)$; $\frac{\partial z}{\partial y} = x^3\cos(xy) - 9y^2$

4. 3.3π

5. $L \approx 8.096\times10^{-4} \pm 6.6\times10^{-6}$ microhenrys

6. 10%

7. 1%

II. Integration Techniques

MCQ

1. (b) 2. (a) 3. (d) 4. (d) 5. (b) 6. (d) 7. (c) 8. (a)

Structured questions

1.(a) (i) $-\frac{1}{8(2x-3)^4} + C$ (ii) $-\frac{2}{9}(4-3x)^{3/2} + C$ (iii) $\frac{1}{8}\ln\left|8x+3\right| + C$

1.(b) (i) $\frac{1}{10}\left(x^2+1\right)^5 + C$ (ii) $\frac{1}{3}\sin^3 x + C$ (iii) $\ln\left|\ln x\right| + C$ (iv) $-5\sqrt{1-e^{2x}} + C$

1.(c) (i) $-\ln\left|x+3\right| + \frac{2}{3}\ln\left|3x-1\right| + C$ (ii) $\ln\left|x+1\right| - \frac{3}{4}\ln\left|2x-1\right| + \frac{1}{4(2x-1)} + C$

(iii) $2\ln\left|s\right| + \frac{1}{2}\ln\left|s^2+4\right| - \frac{1}{2}\tan^{-1}\frac{s}{2} + C$

1.(d) (i) $2\tan^{-1}(x-1) + C$ (ii) $\frac{1}{5}\tan^{-1}\left(\frac{x-5}{5}\right) + C$

1.(e) (i) $\frac{1}{2}\left(\frac{\cos 2x}{2} - \frac{\cos 8x}{8}\right) + C$ (ii) $\frac{1}{2}\left(x - \frac{\sin 4x}{4}\right) + C$ (iii) $\frac{1}{2}\left(x + \frac{\sin 6x}{6}\right) + C$

1.(f) (i) $\frac{1}{2}x^2e^{2x} + C$ (ii) $-\frac{x^2}{3}\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C$ (iii) $\frac{1}{29}e^{5x}(2\sin 2x + 5\cos 2x) + C$

2. (a) (i) 4 (ii) 0.2882 (b) (i) 1/24 (ii) 1.07 (c) (i) 0.0384 (ii) 4.575

3. (a) 6.25 (b) 2.41 (c) 2

4. (a) $2\ln\left(1+\sqrt{\mathrm{x}}\right)+\mathrm{C}$ (b) $\frac{\sin 3\theta}{6}-\frac{\sin 5\theta}{20}-\frac{\sin\theta}{4}+C$

5. (a) 13.33 (b) $\frac{3\pi}{16}$

III. Simpson's Rule & Fourier Series

MCQ
1. b 2. b 3. d 4. b 5. c 6. a 7. d

Structured Questions
1. (a) 1.11; (b) 1.19; (c) 0.24 (d) 17.35

2. 0.390 mC

3. (a) $\frac{1}{2}$ (b) $\frac{1}{\pi}\sin(\pi t)$ (c) $\frac{2}{\pi}\cos\left(\frac{3\pi}{2}t\right)$

4. even

5. (a) odd (b) $f(t)=-\frac{4}{\pi^2}\sin\left(\frac{\pi}{2}t\right)+\frac{1}{\pi}\sin\left(\pi t\right)+\frac{4}{9\pi^2}\sin\left(\frac{3\pi}{2}t\right)+\cdots$

IV. 1st ODE & Applications

MCQ
1. d 2. a 3. b 4. b 5. d 6. b

Structured Questions

1. (a) $-\frac{1}{y}=\frac{1}{2}\tan^{-1}2x+C$ (b) $\frac{y^2}{2}+3y=\frac{1}{2}\left(-\frac{\cos 4x}{4}-\frac{\cos 2x}{2}\right)+C$

(c) $-\frac{1}{2}\ln\left|\csc(2y)+\cot(2y)\right|=\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$ (d) $\frac{1}{2}\ln\left|y^2-1\right|=\ln x+C$

(e) $\ell n\left|y\right|=\frac{1}{2}\ell n\left|1+x^2\right|+C$ or $y=A\sqrt{1+x^2}$ (f) $y=\frac{x^3}{3}-x+C$

(g) $2(y-\ell n\left|\,y+1\right|)=\frac{1}{x}-1$

2. (a) $ye^{2x}=\frac{1}{3}e^{3x}+C$ (b) $yx=x^2+1$ (c) $y\left(x+1\right)=x-2\ln\left|x+3\right|+C$ (d) $ye^{2x}=\frac{1}{6}e^{6x-1}-\frac{1}{6}$

(e) $y=x^3+Cx^{-3}$ (f) $y=(x^3+C)e^{-3x}$

3. (a) $(1+e^x)\sec y=2\sqrt{2}$ or $(1+e^x)=2\sqrt{2}\cos y$ (b) $y\,x=\frac{1}{4}(x^2-x\sin 2x-\frac{1}{2}\cos 2x)+C$

(c) $y = 5e^{-\frac{x^2}{2}} - 4$

4. (b) $T = 25 + 75e^{-0.155t}\ (°C)$ (c) $46.7°C$

5. (b) $T = 20 + 80e^{-0.029t}\ (°C)$ (c) 14.1 min

6. (i) $q = 0.00001\left(1 - e^{-1000t}\right)$ coulombs, $i = \frac{dq}{dt} = 0.01e^{-1000t}$ amperes (ii) 0.0674 volts

V. Laplace Transform & Inverse Laplace Transform

MCQ

1. c 2. d 3. d 4. c 5. c 6. c

Structured Questions

1. (a) $\frac{4}{s} - \frac{9}{s+4}$ (b) $\frac{30}{s^4} + \frac{6}{s^2+4}$ (c) $\frac{(s-2)^2 - 25}{\left[(s-2)^2 + 25\right]^2}$ (d) $\frac{2}{s^3} + \frac{3}{s^2} + \frac{2}{s}$ (e) $\frac{e^3}{s-2}$

   (f) $\frac{\sqrt{3}+s}{2\left(s^2+1\right)}$ or $\frac{0.866 + 0.5s}{s^2+1}$ (g) $\frac{1}{2}\left[\frac{1}{s^2} + \frac{s^2-36}{(s^2+36)^2}\right]$ (h) $\frac{1}{2}\left[\frac{s^2-9}{(s^2+9)^2} - \frac{s^2-49}{(s^2+49)^2}\right]$

2. (a) $2 - 4t^2 + \frac{2}{3}t^4$ (b) $e^{-6t} - 3\cos 5t + \frac{1}{7}\sin 7t$ (c) $t\cos 10t - \frac{2}{9}t\sin 9t$ (d) $\frac{1}{2}e^{\frac{3}{2}t}$

   (e) $\frac{1}{8}t^4 + \frac{1}{2}t^3$ (f) $3\cos 6t + \frac{1}{3}\sin 6t$ (g) $3t^2e^t$ (h) $e^{2t}\sin 3t$ (i) $e^{-2t}\cos 5t$

   (j) $\frac{3}{2} - 2e^t + \frac{3}{2}e^{2t}$ (k) $\frac{2}{3}e^t + \frac{1}{3}\left(\cos\sqrt{2}\,t + \frac{1}{\sqrt{2}}\sin\sqrt{2}\,t\right)$

3. (a) $(s+3)\mathcal{L}\{v\} - 6 - \frac{26}{s^2+4}$ (b) $(s^2+2s+5)\mathcal{L}\{y\} - s - \frac{s+2}{(s+2)^2+9}$

   (c) $e^{2t}\left(\cos 4t + \frac{1}{4}\sin 4t\right)$ (d) $e^{-3t}(1-t)$

VI. 2nd ODE & Applications

MCQ

1. (c) 2. (c) 3. (b)

Structured Questions

1. (a) $y = Ae^{3x} + Be^{-2x}$ (b) $y = (A + Bx)e^{2x}$ (c) $y = e^{-2x}\left(A\cos 3x + B\sin 3x\right)$

2. (a) $y = (5 - 14x)e^x$ (b) $y = \frac{1}{4}\left(3e^{-2x} + e^{2x}\right)$ (c) $y = e^{-\frac{1}{2}x}\left(\cos\frac{3}{2}x + \sin\frac{3}{2}x\right)$

   (d) $y = -3 - 2t + 2e^{2t}$

3. $q = \frac{2}{3}\sin 3t$

4. (a) $\frac{1}{2(s+2)} + \frac{1}{(s+2)^2} - \frac{s}{2(s^2+4)}$ (b) $v = \frac{3}{2}e^{-2t} + 3te^{-2t} - \frac{1}{2}\cos 2t$

5. (a) Simple Harmonic Motion , (b) 3 cm below equilibrium position (c) 2.9 Hz

6. (b) 10.4 above equilibrium position (c) 0.15m., 1.15s, 0.87 Hz

7. (b) $x = -\frac{1}{40}\cos 5t + \frac{1}{40}e^{-4t}[\cos 3t + \frac{4}{3}\sin 3t]$ (c) $\frac{1}{40}$ m (d) 4.22

8. (b) $x = e^{-3t}[0.2\cos(4t)+0.15\sin(4t)]$ $x' = -1.25\, e^{-3t}\sin(4t)$

9. $q(t) = 6e^{-20t} - 2e^{-60t}$ C

10. (b) $i(t) = 60e^{-3t}\sin 3t$ A

11. (b) $q(t) = \frac{3}{2} - \frac{1}{2}e^{-10t}(\cos 10t + \sin 10t)$ C