SOLUTIONS/ MARKING SCHEME

SINGAPORE POLYTECHNIC
2018 / 2019 Semester 2 MST

Module Name: **Engineering Mathematics II**

Module Code: **MS2223/MS2216/MS4216/MS6216**

Course: **DCHE/DASE/DCPE/DEB/DEEE/DES/DESM/DARE/DCEP/DME/DMRO**

Year: 2 FT

No.	SOLUTION
1a	(i) $f(x,y)=\ln(x^2-y)$ $f_x=\dfrac{2x}{x^2-y}, \quad f_y=\dfrac{-1}{x^2-y}$ (ii) $f(x,y)=x\cos y+ye^x$ $f_x=\cos y+ye^x, \quad f_y=-x\sin y+e^x$
1b	$z=x^2+3xy^2, \ x=\cos t \ , \ y=e^{2t}$ $\dfrac{\partial z}{\partial x}=2x+3y^2, \ \dfrac{dx}{dt}=-\sin t$ $\dfrac{\partial z}{\partial y}=6xy, \ \dfrac{dy}{dt}=2e^{2t}$ $\therefore \dfrac{dz}{dt}=\dfrac{\partial z}{\partial x}\cdot\dfrac{dx}{dt}+\dfrac{\partial z}{\partial y}\cdot\dfrac{dy}{dt} \ =(2x+3y^2)(-\sin t)+6xy(2e^{2t})$ At $t=0$: $\dfrac{dx}{dt}=-\sin(0)=0, \ \dfrac{dy}{dt}=2e^{2(0)}=2, \ x=\cos(0)=1, \ y=e^{2(0)}=1$ $\therefore \dfrac{dz}{dt}=(2+3)(0)+6(2)=12$
1c	Let $V=\pi r^2 h$ = volume of cylinder, r = radius and h = height Let $c=2\pi r$ = circumference of the cylinder → $r=\dfrac{c}{2\pi}$ Thus $V=\pi\left(\dfrac{c}{2\pi}\right)^2 h=\dfrac{hc^2}{4\pi} \quad or \quad \dfrac{1}{4\pi}hc^2$ $\dfrac{\partial V}{\partial h}=\dfrac{1}{4\pi}c^2 \ , \ \dfrac{\partial V}{\partial c}=\dfrac{1}{2\pi}ch$ $\dfrac{dV}{dt}=\dfrac{\partial V}{\partial h}\cdot\dfrac{dh}{dt}+\dfrac{\partial V}{\partial c}\cdot\dfrac{dc}{dt}=\dfrac{1}{4\pi}c^2\cdot(0.5)+\dfrac{1}{2\pi}ch\cdot(0.2)$ $=\dfrac{1}{4\pi}\left(1.5^2(0.5)+2(1.5)(6)(0.2)\right)=0.38 \ \text{m}^3\text{/yr}$
2a (i)	$\int 4(5x-2)^3dx=\dfrac{1}{5}(5x-2)^4+C$

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2a (ii)	$\int \frac{3}{6u+1}\,du = \frac{1}{2}\ln\lvert 6u+1\rvert + C$
2a (iii)	$\int 5\cos 3t\cos 2t\,dt = \frac{5}{2}\int (\cos 5t + \cos t)\,dt = \frac{1}{2}\sin 5t + \frac{5}{2}\sin t + C$
2b	$y_{rms} = \sqrt{\frac{1}{2-0}\int_0^2 \left(7e^{2t+3}\right)^2 dt} = \frac{7}{\sqrt{2}}\sqrt{\frac{1}{4}e^{4t+6}\Big\rvert_0^2} = \frac{7}{2\sqrt{2}}\sqrt{e^{14}-e^{6}} \approx 2713.57$
3a	$\frac{2}{(x+3)^2(x+2)} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x+2}$ Use 'cover-up" method to find B and C: $B = \frac{2}{x+2}\Big\rvert_{x=-3} = -2, \quad C = \frac{2}{(x+3)^2}\Big\rvert_{x=-2} = 2$ $2 = A(x+3)(x+2) + B(x+2) + C(x+3)^2$ Compare coefficient of x^2 : $A = -2$ $\therefore \frac{2}{(x+3)^2(x+2)} = \frac{-2}{x+3} + \frac{-2}{(x+3)^2} + \frac{2}{x+2}$ $\int \frac{2}{(x+3)^2(x+2)}\,dx = \int \left(\frac{-2}{x+3} + \frac{-2}{(x+3)^2} + \frac{2}{x+2}\right) dx$ $= -2\ln\lvert x+3\rvert + \frac{2}{x+3} + 2\ln\lvert x+2\rvert + C$
3b	$x^2+2x+17 = (x+1)^2 - 1^2 + 17 = (x+1)^2 + 16$ $\int \frac{1}{x^2+2x+17}\,dx = \int \frac{1}{(x+1)^2+4^2}\,dx = \frac{1}{4}\tan^{-1}\left(\frac{x+1}{4}\right) + C$ $\int \frac{2x+1}{x^2+2x+17}\,dx = \int \left(\frac{2x+2}{x^2+2x+17} - \frac{1}{x^2+2x+17}\right) dx$ $= \ln\lvert x^2+2x+17\rvert - \frac{1}{4}\tan^{-1}\left(\frac{x+1}{4}\right) + C$

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4a	Let $u = x^3$ $\frac{du}{dx} = 3x^2 \to du = 3x^2 dx \to \frac{1}{3}du = x^2 dx$ $\int x^2 \cos x^3 \, dx = \int \frac{1}{3}\cos u \, du = \frac{1}{3}\sin u + C = \frac{1}{3}\sin x^3 + C$
4b	$x = \frac{1}{\sqrt{2}}\sin\theta \to \sin\theta = \sqrt{2}x \to \theta = \sin^{-1}(\sqrt{2}x)$ $x = 0 \to \theta = \sin^{-1}(\sqrt{2}\times 0) = 0$ $x = \frac{1}{\sqrt{2}} \to \theta = \sin^{-1}\left(\sqrt{2}\times\frac{1}{\sqrt{2}}\right) = \frac{\pi}{2}$ $dx = \frac{1}{\sqrt{2}}\cos\theta \, d\theta$ $1-2x^2 = 1-2\left(\frac{1}{\sqrt{2}}\sin\theta\right)^2 = 1-\sin^2\theta = \cos^2\theta$ $\int_0^{\frac{1}{\sqrt{2}}} \sqrt{1-2x^2}\, dx = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2}}\cos^2\theta \, d\theta = \frac{1}{2\sqrt{2}}\int_0^{\frac{\pi}{2}}(1+\cos 2\theta)\, d\theta$ $= \frac{1}{2\sqrt{2}}\left(\theta + \frac{1}{2}\sin 2\theta\right)_0^{\frac{\pi}{2}} = \frac{1}{2\sqrt{2}}\left(\left[\frac{\pi}{2}-0\right] + \frac{1}{2}\left[\sin\pi - \sin 0\right]\right)$ $= \frac{\pi}{4\sqrt{2}}$ or $= \frac{\pi\sqrt{2}}{8}$
5a	$\begin{array}{ccc} u & & dv \\ 2x-1 & \xrightarrow{+} & e^x \\ 2 & \xrightarrow{-} & e^x \\ 0 & & e^x \end{array}$ $\therefore \int (2x-1)e^x \, dx = (2x-1)e^x - 2e^x + C$

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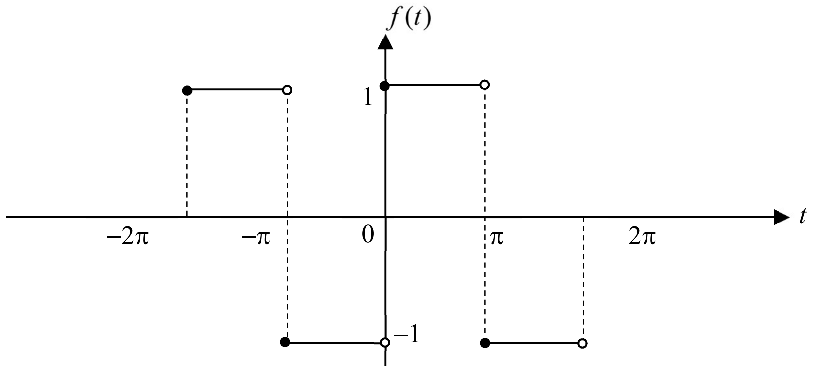
5b	u dv $\ln x \xrightarrow{+} x^3$ $\dfrac{1}{x} \xrightarrow{-\int} \dfrac{x^4}{4}$ $\therefore \int x^3 \ln x \, dx = \frac{1}{4}x^4 \ln x - \int \frac{1}{x}\cdot\frac{1}{4}x^4 \, dx = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$
6	$h = \dfrac{\frac{\pi}{3}-0}{6} = \dfrac{\pi}{18}$ Let $y = \ln(\sec x) = \ln\left(\dfrac{1}{\cos x}\right)$ x: 0, $\frac{\pi}{18}$, $\frac{2\pi}{18}$, $\frac{3\pi}{18}$, $\frac{4\pi}{18}$, $\frac{5\pi}{18}$, $\frac{6\pi}{18} = \frac{\pi}{3}$ y: 0, 0.015309, 0.062202, 0.143841, 0.266515, 0.441941, 0.693147 $\int_0^{\pi/3} \ln(\sec x)\, dx \approx \frac{1}{3}\left(\frac{\pi}{18}\right)\times[0+0.693147+4(0.015309+0.143841+0.441941)+2(0.062202+0.266515)]$ ≈ 0.218
7a	
7b	$T = 2\pi \rightarrow \omega = \dfrac{2\pi}{T} = 1$ $a_0 = \frac{1}{T}\int_k^{k+T} f(t)\, dt = \frac{1}{2\pi}\left[\int_0^{\pi} dt - \int_{\pi}^{2\pi} dt\right] = \frac{1}{2\pi}\left[t\Big\|_0^{\pi} - t\Big\|_{\pi}^{2\pi}\right] = 0$ $a_n = \frac{2}{T}\int_k^{k+T} f(t)\cos n\omega t\, dt = \frac{1}{\pi}\left(\int_0^{\pi}\cos nt\, dt - \int_{\pi}^{2\pi}\cos nt\, dt\right) = \frac{1}{\pi}\left(\frac{\sin nt}{n}\Big\|_0^{\pi} - \frac{\sin nt}{n}\Big\|_{\pi}^{2\pi}\right) = 0$

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	$b_n = \frac{2}{T}\int_k^{k+T} f(t)\sin n\omega t\,dt = \frac{1}{\pi}\left(\int_0^{\pi}\sin nt\,dt - \int_{\pi}^{2\pi}\sin nt\,dt\right)$ $= \frac{1}{\pi}\left(-\frac{\cos nt}{n}\Big\vert_0^{\pi} + \frac{\cos nt}{n}\Big\vert_{\pi}^{2\pi}\right) = \frac{1}{n\pi}\left(\cos 2n\pi - 2\cos n\pi + 1\right)$ $b_1 = \frac{1}{\pi}\left(\cos 2\pi - 2\cos\pi + 1\right) = \frac{4}{\pi}$ $b_2 = \frac{1}{2\pi}\left(\cos 4\pi - 2\cos 2\pi + 1\right) = 0$ $b_3 = \frac{1}{3\pi}\left(\cos 6\pi - 2\cos 3\pi + 1\right) = \frac{4}{3\pi}$ Or $b_n = \begin{cases} \frac{4}{n\pi} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$ $\therefore f(t) = \frac{4}{\pi}\sin t + \frac{4}{3\pi}\sin 3t + \cdots$