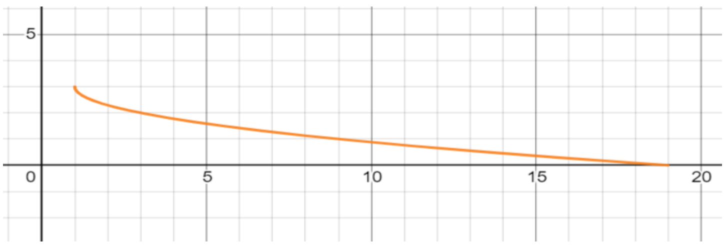
EP0604 MST Solutions (AY21/22 S1)

No.	SOLUTION
1a(i)	$S_\infty = \dfrac{256}{1-\frac{3}{4}} = 1024$
(ii)	$S_n = \dfrac{256\left(1-\left(\frac{3}{4}\right)^n\right)}{1-\frac{3}{4}} = 1000$ $\left(\frac{3}{4}\right)^n = 0.0234375$ $n = 13.04$ 13 pieces must be cut
1(b)	Midpoint at $\left(\frac{-4+2}{2}, \frac{3+1}{2}\right)$ Midpoint at $(-1{,}2)$ $r = \sqrt{(-1-2)^2+(2-1)^2} = \sqrt{10}$ Centre $(-1{,}2)$, radius = $\sqrt{10}$ Equ of circle: $(x+1)^2+(y-2)^2 = 10$

No.	SOLUTION
2a	$x = 2t^2+1$ y= $3-t$
(i)	t= 5 $x = 51, y = -2$ $(51, -2)$
(ii)	

No.	SOLUTION
(iii)	$9 = 2t^2 + 1$ $t = \pm 2$ When t=2, y is indeed 1, Hence the object passes through (9,1) at t =2. (can use the graph to explain too)
(iv)	t =1, $x = 3, y = 2$ $\frac{dy}{dx} = -\frac{1}{4t}$ Gradient of tangent is -1/4 Gradient of normal is 4 $y - 2 = 4(x - 3)$ $y = 4x - 10$
2(b)	$y = \frac{1}{2} acos^2(2t)$ $y = \frac{1}{2} a[cos(2t)]^2$ $y = \frac{1}{2} a[1 - 2sin^2(t)]^2$ $y = \frac{1}{2} a\left[1 - 2(\frac{x}{a})^2\right]^2$ since $x = asin(t)$, that is, $sin(t) = \frac{x}{a}$

No.	SOLUTION
3a	$f(x) = \ln(x - 3)$ $g(x) = 2 + \sqrt{t - 2}$
(i)	$D_f = (3, \infty)$ $R_f = (-\infty, \infty)$
(ii)	$D_g = [2, \infty)$ $R_f = [2, \infty)$
3(b)(i)	Let $y = \frac{1}{x-1}$ $x = \frac{1}{y} + 1$ $f^{-1}(x) = \frac{1}{x} + 1$

No.	SOLUTION
	$(f°f^{-1})(x) = f\left(\frac{1}{x}+1\right)$ $(f°f^{-1})(x) = \frac{1}{\frac{1}{x}+1-1}$ $(f°f^{-1})(x) = x$ (shown)
(ii)	$(f°f)(x) = f\left(\frac{1}{x-1}\right)$ $(f°f)(x) = \frac{1}{\frac{1}{x-1}-1}$ $(f°f)(x) = \frac{1}{\frac{1+1-x}{x-1}}$ $(f°f)(x) = \frac{x-1}{2-x}$
(iii)	$(f°f°f)(x) = f(\frac{x-1}{2-x})$ $(f°f°f)(x) = \frac{1}{\frac{x-1}{2-x}-1}$ $(f°f°f)(x) = \frac{2-x}{2x-3}$

No.	SOLUTION
4a.	$y = e^x cosx$ $\frac{dy}{dx} = e^x(-sinx) + cosxe^x$ $e^x(-sinx + cosx) = 0$ $e^x = 0(N.A.)\ or - sinx + cosx = 0$ $tanx = 1$ $x = \frac{\pi}{4}$ $y = e^{\frac{\pi}{4}}\cos(\frac{\pi}{4})$=1.55
(b)	$x = 0, \frac{dy}{dx} = 1$

No.	SOLUTION
	$y = 1$ Gradient of normal = -1 $y - 1 = -1(x - 0)$ Equ of normal: $y = -x + 1$ Cut x-axis, y=0, x =1

No.	SOLUTION
5a (i)	$y = a\sqrt{1 + x^2} + b(10 - x)$
(ii)	$\dfrac{dy}{dx} = \dfrac{ax}{\sqrt{1 + x^2}} - b$ $\dfrac{ax}{\sqrt{1 + x^2}} - b = 0$ $ax = b\sqrt{1 + x^2}$ $a^2x^2 = b^2(1 + x^2)$ $x^2 = \dfrac{b^2}{a^2 - b^2}$ $x = +\frac{b}{\sqrt{a^2-b^2}}$ or $-\frac{b}{\sqrt{a^2-b^2}}$ $(N.A.)$
(b)	$f(x) = x^n$ $f'(x) = \lim\limits_{\Delta x \to 0}\left(\dfrac{f(x + \Delta x) - f(x)}{\Delta x}\right)$ $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{(x + \Delta x)^n - x^n}{\Delta x}$ $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{x^n + \binom{n}{1}x^{n-1}\Delta x + \binom{n}{2}x^{n-2}\Delta x^2 + \cdots + \Delta x^n - x^n}{\Delta x}$ $f'(x) = \lim\limits_{\Delta x \to 0}\left(\binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2}\Delta x + \cdots + \Delta x^{n-1}\right)$ $f'(x) = \binom{n}{1}x^{n-1}$ $f'(x) = nx^{n-1}$