3.8 Problems

Q3-1 Find the *z*-transform of the following sequences:

(a) $x(n) = \{1, -2, 0.5, 0, -3\}$

(b) $x(n) = 2u(n) - 3(0.5)^n u(n)$

(c) $x(n) = 3(0.5)^n u(n-1)$

(d) $x(n) = ne^{2n}u(n)$

Q3-2 Find the inverse *z*-transform of:

(a) $X(z) = 1$

(b) $X(z) = 0.5 - 2z^{-1}$

(c) $X(z) = \dfrac{0.5z}{z-1}$

(d) $X(z) = \dfrac{1-z^{-1}}{1-ez^{-1}}$

(e) $X(z) = \dfrac{1}{1+0.81z^{-2}}$

(f) $X(z) = \dfrac{1-z^{-1}}{8-6z^{-1}+z^{-2}}$

Q3-3 Draw the digital networks of the systems where:

(a) $y(n) = x(n) + 0.4x(n-1) + 0.3x(n-2) + 0.2y(n-1) + 0.1y(n-2)$

(b) $H(z) = \dfrac{1-0.4z^{-1}-0.3z^{-2}}{1+0.2z^{-1}+0.1z^{-2}}$

Q3-4

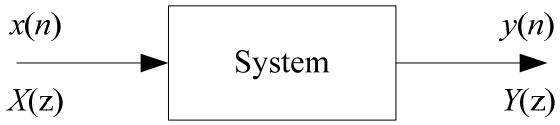

Figure 3.21 A discrete-time system

Which of the following equations about the system in Figure 3.21 is NOT valid?

(a) $Y(z) = H(z) \cdot X(z)$

(b) $Y(z) = X(z) \cdot H(z)$

(c) $y(n) = h(n) * x(n)$

(d) $y(n) = x(n) * h(n)$

(e) $y(n) = h(n) \cdot x(n)$

(f) $y(n) = x(n) \cdot h(n)$

Q3-5 Determine the difference equations and system functions of Figure 3.22(a), (b) and (c) below.

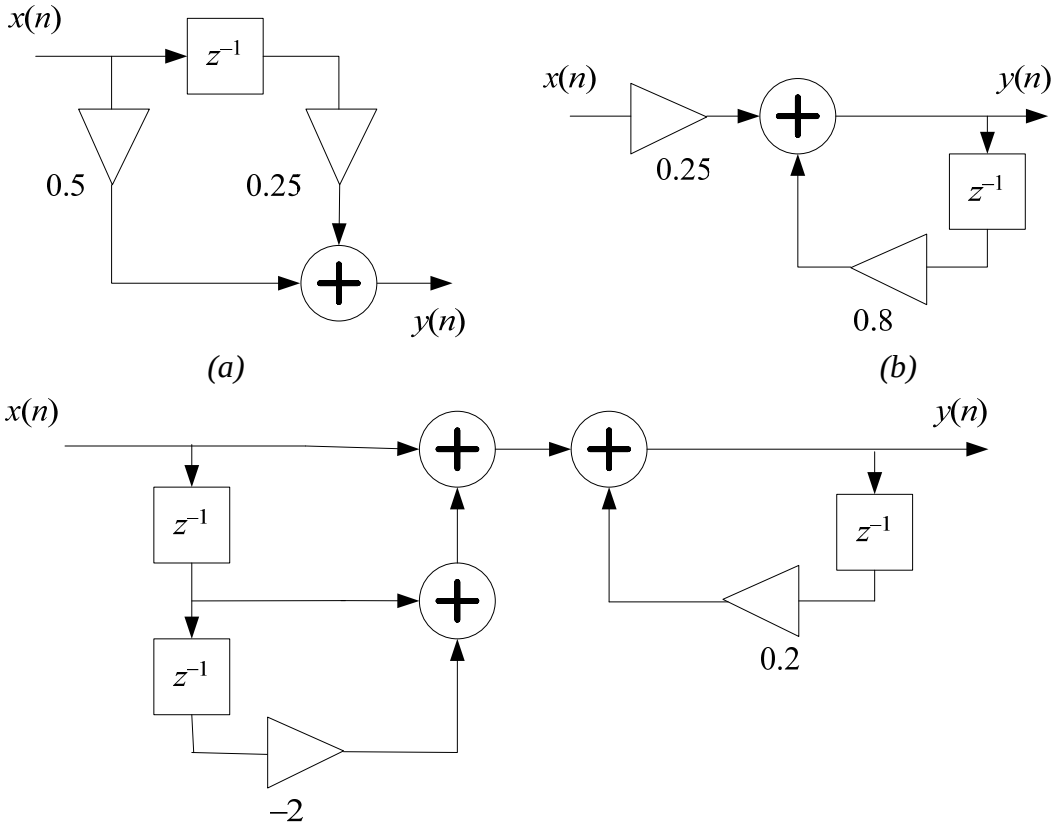

Figure 3.22 Discrete-time systems

Q3-6 Find $y(n)$ if $x(n) = u(n)$ for Figure 3.22(a).

Q3-7 For Figure 3.22(b),

(a) Find $y(n)$ if $x(n) = \delta(n)$.

(b) Find the impulse response of the system.

(c) Find $y(n)$ if $x(n) = 3.2(0.8)^{n-1}u(n-1)$.

Q3-8 Find the magnitude and phase response of the systems described by Figure 3.22(a) and (b).

Q3-9 A moving average filter is defined as $y(n) = \frac{1}{3}x(n) + \frac{1}{3}x(n-1) + \frac{1}{3}x(n-2)$.

(a) Calculate the system function, $H(z)$.

(b) Derive the magnitude response of the system $\left|H(e^{j\omega})\right|$ for $\omega = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$ and sketch the magnitude response.

(c) By observing the magnitude response, comment on the function of this filter.

Q3-10 A system operating with a sampling frequency of 10 kHz has magnitude response shown in Figure 3.23 below.

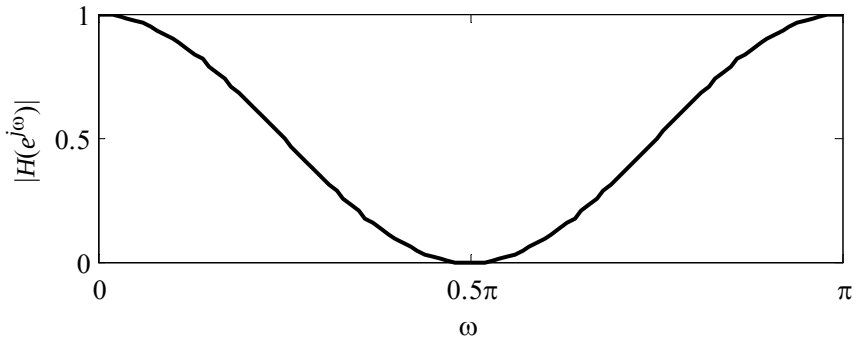

Figure 3.23 Magnitude response of a digital system

(a) Sketch the magnitude response of the system from $\omega = -2\pi$ to $\omega = 2\pi$.

(b) Sketch the magnitude response of the system from 0 Hz to 10 kHz.

(c) Determine the frequency in Hz corresponding to $\omega = \frac{\pi}{4}$.

(d) If the system is a digital filter, determine the filter type. Explain your answer.

Q3-11 Figure 3.24 shows a theoretical DSP system without quantization problem.

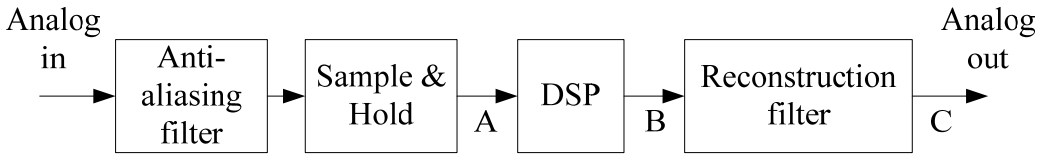

Figure 3.24 A DSP system

Figure 3.25 shows the magnitude spectrum of the signal at the output of the anti-aliasing filter.

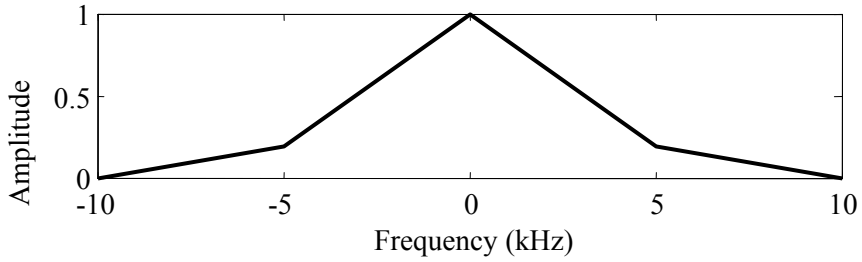

Figure 3.25 Magnitude spectrum of the signal at the output of the filter

Figure 3.26 shows the frequency response of the DSP module between points A and B.

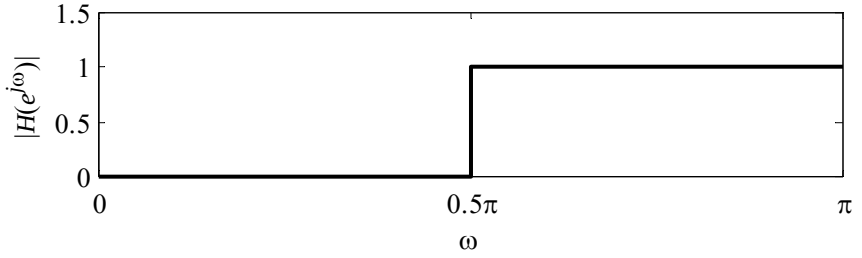

Figure 3.26 Frequency response of the DSP module

Assume that the reconstruction filter is an ideal low pass filter with a cut-off frequency at half sampling frequency.

(a) If the sampling frequency is equal to the Nyquist rate, sketch the spectra at points A, B and C.

(b) If the sampling rate is only 15 kHz,
   (i) Sketch the spectrum at point A.
   (ii) Explain why aliasing is a problem in the DSP system.

Q3-12* The unit step response of a digital system is found to be:

$$y(n) = \left(\frac{1}{3}\right)^n u(n) + \frac{1}{5}\left(\frac{1}{3}\right)^{n-1} u(n-1)$$

(a) Show that the system function is $H(z) = \dfrac{1 - \frac{4}{5}z^{-1} - \frac{1}{5}z^{-2}}{1 - \frac{1}{3}z^{-1}}$.

(b) What is the impulse response of this system? Calculate the value of $h(0)$.

Q3-13* Find the inverse *z*-transform of the following sequences:

(a) $X(z) = \dfrac{9z}{z^2 + 0.6z - 1}$

(b) $X(z) = 1 + \dfrac{3z^2 - 2z}{z^2 - 6z + 8}$

(c) $X(z) = \dfrac{1 - 0.4z^{-1}}{1 - 0.8z^{-1} + 0.16z^{-2}}$

Q3-14* A digital system is described with the difference equation:

$$y(n) = x(n) + 2x(n-1) + 0.5y(n-1)$$

where $x(n)$ is the input and $y(n)$ is the output of the system.

(a) Determine the system function, $H(z)$.

(b) Use inverse *z*-transform to determine the impulse response, $h(n)$, of the system.

(c) Determine the frequency response $H(e^{j\omega})$ of the system.

(d) Determine the magnitude and phase of the system at ω = 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$ and π, respectively.

(e) Plot the magnitude response and determine the type of this filter?

Q3-15* For the following systems:

(a) $H_1(z) = \dfrac{1}{1 + 1.4z^{-1}}$

(b) $H_2(z) = \dfrac{1}{1 + 1.3z^{-1} + 0.5z^{-2}}$

(c) $H_3(z) = \dfrac{1}{0.1 + 0.2z^{-1} - z^{-2}}$

(d) $H_4(z) = \dfrac{1}{1 - 0.3z^{-1}}$

(i) Draw their pole-zero diagrams.

(ii) Determine whether these systems are stable.