SOLUTIONS/ MARKING SCHEME

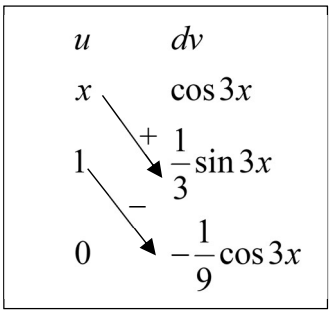
SINGAPORE POLYTECHNIC
2019 / 2020 Semester 1 MST

Module Name: Engineering Mathematics II

Module Code: MS2216/MS4216/MS6216

Course: DCHE/DASE/DCPE/DEB/DEEE/DES/DESM/DARE/DCEP/DME/DMRO

Year: 2 FT

No.	SOLUTION
1a(i)	$\int\left[(4u-1)^5+\frac{7}{2u+5}\right]du=\frac{(4u-1)^6}{6(4)}+\frac{7}{2}ln\lvert 2u+5\rvert+C=\frac{(4u-1)^6}{24}+\frac{7}{2}ln\lvert 2u+5\rvert+C$
a(ii)	$\int 2\sin^2 2\theta d\theta=\int(1-\cos 4\theta)\,d\theta=\theta-\frac{\sin 4\theta}{4}+C$
a(iii)	$\int\cot^2 4\theta\,d\theta=\int\left(\csc^2 4\theta-1\right)d\theta=-\frac{\cot 4\theta}{4}-\theta+C$
1b	$\int_{-1}^{1}\frac{5}{e^{3x-2}}dx=\int_{-1}^{1}5e^{-3x+2}dx=-\frac{5}{3}\left[e^{-3x+2}\right]_{-1}^{1}=-\frac{5}{3}\left(e^{-1}-e^{5}\right)=246.74$
1c	$y=\frac{1}{\sqrt{3t+1}}$ $y_{rms}=\sqrt{\frac{1}{3-1}\int_1^3\frac{1}{3t+1}dt}=\sqrt{\frac{1}{2\times 3}\left[\ln\lvert 3t+1\rvert\right]_1^3}=\sqrt{\frac{1}{6}(\ln 10-\ln 4)}=0.39$
2a	$\int x^2\left(4+x^3\right)^{20}dx\underset{u=4+x^3}{=}\frac{1}{3}\int u^{20}\,du=\frac{1}{3}\left(\frac{u^{21}}{21}\right)+C=\frac{1}{63}\left(4+x^3\right)^{21}+C$
2b	$\int_0^1\sqrt{e^{2x}\left(1+e^x\right)}dx=\int_0^1 e^x\sqrt{\left(1+e^x\right)}dx\underset{u=1+e^x}{=}\int_2^{1+e}\sqrt{u}\,du=\frac{2}{3}\left[u^{\frac{3}{2}}\right]_2^{1+e}$ $=\frac{2}{3}\left((1+e)^{\frac{3}{2}}-(2)^{\frac{3}{2}}\right)=2.89$
3a	$\int x\cos 3x\,dx=\frac{1}{3}x\sin 3x+\frac{1}{9}\cos 3x+C$ 

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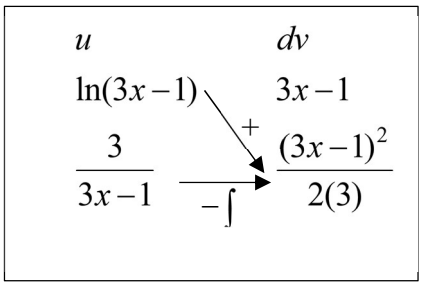
3b	$\int (3x-1)\ln(3x-1)\,dx$ $= \frac{(3x-1)^2}{2(3)}\ln(3x-1) - \int \frac{3}{3x-1}\frac{(3x-1)^2}{2(3)}\,dx$ $= \frac{(3x-1)^2}{6}\ln(3x-1) - \frac{1}{2}\int (3x-1)\,dx$ $= \frac{(3x-1)^2}{6}\ln(3x-1) - \frac{1}{2}\frac{(3x-1)^2}{2(3)} + C$ $= \frac{(3x-1)^2}{6}\ln(3x-1) - \frac{(3x-1)^2}{12} + C$ 
4a	$\frac{1}{x^2(x+1)(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} + \frac{Dx+E}{x^2+1}$ or $\frac{1}{x^2(x+1)(x^2+1)} = \frac{Ax+B}{x^2} + \frac{C}{x+1} + \frac{Dx+E}{x^2+1}$
4b	$\frac{1}{(x+2)(x+1)^2} = \frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ $1 = A(x+1)^2 + B(x+2)(x+1) + C(x+2)$ $x=-2$: $1 = A$ $x=-1$: $1 = C$ Coeff x^2: $0 = A + B \Rightarrow B = -A = -1$ $\int \frac{1}{(x+2)(x+1)^2}\,dx = \int \frac{1}{x+2} - \frac{1}{x+1} + \frac{1}{(x+1)^2}\,dx$ $= \ln\lvert x+2\rvert - \ln\lvert x+1\rvert - \frac{1}{x+1} + C$ or $= \ln\left\lvert\frac{x+2}{x+1}\right\rvert - \frac{1}{x+1} + C$ (optional)

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4c	$\int \frac{1}{x^2+2x+5}\,dx = \int \frac{1}{(x+1)^2+2^2}\,dx = \frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)+C_1$ $\int \frac{1}{10x^2+20x+50}\,dx = \frac{1}{10}\int \frac{1}{x^2+2x+5}\,dx$ $= \frac{1}{10}\left(\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right)+C_1\right) = \frac{1}{20}\tan^{-1}\left(\frac{x+1}{2}\right)+C_2$
5a	$A = f(x,y,z) = \frac{1}{2}z\sqrt{x^2-\left(\frac{z}{2}\right)^2}+yz = \frac{1}{2}z\sqrt{x^2-\frac{z^2}{4}}+yz$ $= \frac{1}{2}z\sqrt{\frac{4x^2-z^2}{4}}+yz = \frac{1}{4}z\sqrt{4x^2-z^2}+yz$
5b	$\frac{\partial A}{\partial x} = f_x(x,y,z) = \left(\frac{1}{4}z\right)\left(\frac{1}{2}\right)\frac{8x}{\sqrt{4x^2-z^2}}+0 = \frac{xz}{\sqrt{4x^2-z^2}}$ $\frac{\partial A}{\partial y} = f_y(x,y,z) = 0+z = z$ $\frac{\partial A}{\partial z} = f_z(x,y,z) = \left(\frac{1}{4}\right)\sqrt{4x^2-z^2}+\frac{1}{4}(z)\left(\frac{1}{2}\right)\left(\frac{-2z}{\sqrt{4x^2-z^2}}\right)+y = \frac{1}{2}\left(\frac{2x^2-z^2}{\sqrt{4x^2-z^2}}\right)+y$
5c	$\frac{\partial A}{\partial x} = f_x(2,5,3) = \frac{2(3)}{\sqrt{4(4)-3^2}} = 2.2678$ $\frac{\partial A}{\partial y} = f_y(2,5,3) = 3$ $\frac{\partial A}{\partial z} = f_z(2,5,3) = \frac{1}{2}\left(\frac{2(2)^2-3^2}{\sqrt{4(2)^2-3^2}}\right)+5 = 4.8110$

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	Given that $\frac{dx}{dt} = 0.015$ m/s, $\frac{dy}{dt} = 0.01$ m/s and $\frac{dz}{dt} = 0.01$ m/s $\frac{dA}{dt} = \frac{\partial A}{\partial x}\frac{dx}{dt} + \frac{\partial A}{\partial y}\frac{dy}{dt} + \frac{\partial A}{\partial z}\frac{dz}{dt}$ $\frac{dA}{dt}(2,5,3) = f_x(2,5,3)\frac{dx}{dt} + f_y(2,5,3)\frac{dy}{dt} + f_z(2,5,3)\frac{dz}{dt}$ $= (2.2678)(0.015) + (3)(0.01) + (4.8110)(0.01)$ $= 0.034017 + 0.03 + 0.04811$ $= 0.11\, m^2/s$