Homework 1

1. Show the followings.
   (a) (1pt) For all positive integer n,
   $$10n^3 + 1024n^2 \log_2^{10} n + 3n \log_{10} n - 144 = O(n^3).$$
   (b) (1 pt) For any fixed but arbitrarily small real number $c > 0$,
   $$n \log n = o(n^{1+c}).$$

2. (1 pt) Rank the following functions in increasing order of their asymptotic growth. That is, if $f_i(n) = O(f_j(n))$ then $f_i(n)$ comes before $f_j(n)$.
   $f_1(n) = \frac{n^2}{\log_2^7 n}$, $f_2(n) = 2^{\log_2 n}$, $f_3(n) = \frac{n!}{n^{1024}}$, $f_4(n) = 1024n \log_{10} n$
   $f_5(n) = n^{1.5} \log_2 n^{1024}$, $f_6(n) = 2^n$, $f_7(n) = 10^{1.9 \log_{10} n}$.

3. (2 pts) Observe that we can compute the n-th Fibonacci number F_n by the following matrix multiplication. Based on the observation, give an $O(\log n)$-time algorithm for computing F_n and analyze its running time.

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^n \begin{bmatrix} F_1 \\ F_0 \end{bmatrix} = \begin{bmatrix} F_{n-1} \\ F_n \end{bmatrix}$$

Due: 23:59, March 8 (Wednesday), 2023