Homework 5 Due: 12:59 pm, May 30, 2023

Problem 1 (2pts) You are given the following points in the plane:
$(1, 3), (2, 5), (3, 7), (5, 11), (7, 14), (8, 15), (10, 19)$.

You want to find a line $ax + by = c$ that approximately passes through these points (no line is a perfect fit). Write a linear program (you don't need to solve it) to find the line that minimizes the *maximum absolute error*, $\max_{1 \leq i \leq 7} |ax_i + by_i - c|$.

Problem 2 There are two variations of the maximum flow problem of a directed network $G = (V, E)$ with $s, t \in V$. Give an efficient way that each variation can be reduced to the max-flow problem.

(a) (2 pts) Instead of s and t, there are source set $S = \{s_1, s_2, \ldots\}$ and sink set $T = \{t_1, t_2, \ldots\}$. We want to maximize the total flow from all sources in S to all sinks in T .

(b) (2 pts) Each vertex also has a capacity like edges on the maximum flow that can enter it.

Problem 3 (2 pts) Given a directed graph $G = (V, E)$ with $s, t \in V$, show how to compute the maximum number of vertex-disjoint paths from s to t efficiently. (Hint: Use an algorithm for the variant of the maximum flow problem in Problem 2 (b).)

Problem 4 Consider a special type of SAT where each clause is a disjunction of at most four literals. The goal is to find a satisfying assignment, if one exists.

(a) (2 pts) Prove that this problem belongs to NP.

(b) (3 pts) Prove that this problem belongs to NP-hard using a polynomial time reduction.

Problem 5 (4 pts) **Rectangle packing problem**: given n axis-parallel rectangles of arbitrary sizes, find a smallest-area axis-parallel rectangle that contains all input rectangles. You can translate the input rectangles but they must be disjoint in their interiors.
Show that the **Rectangle packing problem** is NP-hard. (Hint: Use a reduction from **Partition problem**: decide whether a given multiset S of positive integers can be partitioned into two subsets S_1 and S_2 such that the sum of the numbers in S_1 equals the sum of the numbers in S_2.)