3.8 Derivatives of Logarithmic Functions

Recall the following properties of e^x and $\ln x$:

- $e^{\ln x} = x, \quad \ln(e^x) = x$
- $a^x = \left(e^{\ln a}\right)^x = e^{(\ln a)\,x}$

Proposition

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \textit{for } x > 0, \qquad \frac{d}{dx}(\ln|x|) = \frac{1}{x} \quad \textit{for } x \neq 0.$$

- Proofs
- Examples

Corollary

$$[\ln g(x)]' = \frac{g'(x)}{g(x)},$$

$$[a^x]' = (\ln a)\, a^x, \quad \textit{and} \quad [\log_a x]' = \frac{1}{(\ln a)\, x}.$$

- The logarithmic function converts a product into a sum.
- Logarithmic Differentiation

  (when there are complicated products, quotients and powers)

  1. Take natural log of both sides and simplify.
  2. Differentiate implicitly.
  3. Solve the result for y'.

- The derivative $\dfrac{dy}{dx}$ of $y = x^{\pi} + \pi^{x}$ is

A. $y' = \pi x^{\pi - 1} + x\pi^{x-1}$

B. $y' = \pi x^{\pi - 1} + \pi^{x} \ln x$

C. $y' = \pi x^{\pi - 1} + \pi^{x} \ln \pi$

D. $y' = \pi x^{\pi - 1} + \pi^{x} / \ln x$

E. $y' = \pi x^{\pi - 1} + \pi^{x} / \ln \pi$

3.9 Derivatives of Inverse Trigonometric Functions

Proposition

$$[\sin^{-1} x]' = \frac{1}{\sqrt{1-x^2}}, \qquad [\tan^{-1} x]' = \frac{1}{1+x^2}.$$

- Proofs
- Examples
- We also have

$$[\cos^{-1} x]' = -\frac{1}{\sqrt{1-x^2}}, \qquad [\csc^{-1} x]' = -\frac{1}{x\sqrt{x^2-1}},$$

$$[\sec^{-1} x]' = \frac{1}{x\sqrt{x^2-1}}, \qquad [\cot^{-1} x]' = -\frac{1}{1+x^2}.$$