Math 2110Q   Practice Midterm-II   Name: _____

1. Let $f(x,y) = \sqrt{4 + x^2 + y^2}$. Find f_{xx}, f_{xy} and f_{yy}.

2. Let $w = \sqrt{x + y + z}$, where $x = \sin t$, $y = \cos t$ and $z = t$. Using the Chain Rule, compute $\dfrac{dw}{dt}$.

3. Let $f(x,y) = \sin \pi(2x - y)$ and $u = (\frac{5}{13}, -\frac{12}{13})$. Compute $D_u f(-1, -1)$.

4. Let $f(x,y) = x^4 - x^2 y + y^2$. Find the unit vectors that give the direction of steepest ascent and steepest descent at $P(-1, 1)$.

5. Consider the surface given by $z^2 = x^2/16 + y^2/9 + 1$. Find an equation of the tangent plane to the surface at $(4, 3, -\sqrt{3})$.

6. Let $f(x,y) = \sqrt{x^2 + y^2}$. Estimate $f(3.06, -3.92)$ using the linear approximation.

7. Let $f(x,y) = xye^{-x-y}$. Find the critical points of f and determine whether each critical point is a local maximum, local minimum, or saddle point.

8. Let $R = \{(x,y) : x^2 + y^2 \le 6\}$. Consider the function $f(x,y) = -x^2 - y^2 + \sqrt{3}x - y - 1$. Find the absolute maximum and minimum values of the function f on the set R.

9. Consider $f(x,y,z) = x^2 + y^2 + z^2$. Find the maximum and minimum value of f subject to the constraint $z = 1 + 2xy$.

10. Consider the region $R = \{(x,y) : 1 \le x \le 4, 1 \le y \le 2\}$. Evaluate the double integral

$$\iint\limits_{R} \frac{x}{(1+xy)^2} \, dA.$$

11. Evaluate the integral $\displaystyle\int_0^\pi \int_x^\pi \sin y^2 \, dy \, dx$.

12. Find the volume of the solid above the region $R = \{(x,y) : 0 \le x \le 1, 0 \le y \le 2 - x\}$ and between the planes $-4x - 4y + z = 0$ and $-2x - y + z = 8$.

13. Find the volume of the solid below the paraboloid $z = 4 - x^2 - y^2$ and above the region

$$R = \{(r, \theta) : 0 \le r \le 1, 0 \le \theta \le 2\pi\}.$$

14. Find the average distance between points within the cardioid $r = 1 + \cos\theta$ and the origin.

15. Evaluate the integral $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xz \, dz \, dy \, dx$.

16. Find the average of the *squared* distance between the origin and points in the solid cylinder

$$D = \{(x,y,z) : x^2 + y^2 \le 4, 0 \le z \le 2\}.$$

17. Evaluate the integral $\displaystyle\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_0^2 \frac{1}{1 + x^2 + y^2} \, dz \, dy \, dx$.

18. Evaluate the integral $\displaystyle\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{x^2+y^2}} \frac{1}{\sqrt{x^2 + y^2}} \, dz \, dy \, dx$.

19. Evaluate the integral $\displaystyle\int_0^{2\pi} \int_0^{\pi/3} \int_0^{4\sec\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$.

20. Find the volume of the region inside the sphere $\rho = 2\cos\phi$ and outside the sphere $\rho = 1$.