4.1 Maxima and Minima (Continued)

Theorem (Fermat's Theorem)

If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

Definition

A critical point of a function f is a point c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

- If f has a local max or min at c, then c is a critical point of f.

Procedure (Locating Absolute Max and Min (★★))

Assume that f is continuous on a closed interval $[a, b]$.

1. *Find all the critical points in (a, b).*
2. *Evaluate f at the critical points and at the end points of $[a, b]$.*
3. *The largest of the values from Step 2 is the absolute max; the smallest of these values is the absolute min.*

- Examples

Proposition (Increasing/Decreasing Test (★))

1. *If $f'(x) > 0$ on an interval, then f is increasing on that interval.*
2. *If $f'(x) < 0$ on an interval, then f is decreasing on that interval.*

- Example

Proposition (Concavity Test (★))

1. *If* $f''(x) > 0$ *on an interval, then* f *is concave up on that interval.*
2. *If* $f''(x) < 0$ *on an interval, then* f *is concave down on that interval.*

Definition

A point P *on a curve* $y = f(x)$ *is called an inflection point if* f *is continuous and changes its concavity at* P*.*

- Example

An article in the Wall Street Journal's "Heard on the Street" column (*Money and Investment* August 1, 2001) reported that investors often look at the "change in the rate of change" to help them "get into the market before any big rallies." Your stock broker alerts you that the rate of change in a stock's price is increasing. As a result you

A. can conclude the stock's price is decreasing;

B. can conclude the stock's price is increasing;

C. cannot determine whether the stock's price is increasing or decreasing.

Proposition (The First Derivative Test)

Suppose that c is a critical point of a continuous function f.

1. *If f' changes from $+$ to $-$ at c, then f has a local max at c.*
2. *If f' changes from $-$ to $+$ at c, then f has a local min at c.*

- Examples

Proposition (The Second Derivative Test)

1. *If $f'(c) = 0$ and $f''(c) > 0$, then f has a local min at c.*
2. *If $f'(c) = 0$ and $f''(c) < 0$, then f has a local max at c.*

- Examples