Math2410   Practice Midterm-I

1. (10 pts) Find the general solution to the following differential equations.

   (a) $\dfrac{dy}{dt} = \dfrac{t^2}{1+y^2}$

   (b) $\dfrac{dy}{dt} + y^2 \sin t = 0$

2. (10 pts) Find the general solution to the following differential equations.

   (a) $\dfrac{dy}{dt} - 2y = 3e^t$

   (b) $\dfrac{dy}{dt} + \left(\dfrac{2}{t}\right) y = \dfrac{\sin t}{t^2}$

3. (10 pts) Solve the following initial-value problems.

   (a) $\dfrac{dy}{dt} + 2y = te^{-2t}$, $y(1) = 0$.

   (b) $\dfrac{dy}{dt} = 2ty^2 + 3t^2 y^2$, $y(1) = -1$.

4. (10 pts) Consider the differential equation

$$\frac{dy}{dt} = y(y+2)(y-3).$$

   (a) Draw the phase line.

   (b) Identify the equilibrium points as sinks, sources, or nodes.

   (c) Sketch the graphs of the solutions satisfying the initial conditions $y(0) = -2$, $y(-1) = -1$, $y(1) = 1$, $y(0) = 5$, respectively, in the ty-plane.

5. (10 pts) Consider the one-parameter family $\dfrac{dy}{dt} = y(y-3) + \alpha$.

   (a) Locate the bifurcation value and draw the phase lines for values of the parameter smaller than, larger than, and at the bifurcation value.