

1. (10 pts) Find the general solution to the system
$$\begin{cases} \frac{dx}{dt} = x + 2y + 1 \\ \frac{dy}{dt} = 3y \end{cases}.$$

2. (10 pts) Consider the system

$$\frac{dx}{dt} = x - y, \quad \frac{dy}{dt} = -x + y.$$

Note that this system has infinitely many equilibrium points.

(a) Draw the direction field of the system on the xy -plane.

(b) Sketch a number of solution curves on the phase plane in the part (a).

3. (10 pts) Consider the system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2 & 6 \\ 2 & -2 \end{pmatrix}.$$

(a) Find the general solution to the system.

(b) Sketch the phase portrait of the system on the xy -plane.

4. (10 pts) Consider the system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

(a) Find the general solution to the system.

(b) Determine the particular solution satisfying the initial condition $\mathbf{Y}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

5. (10 pts) Consider the system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2 & 1 \\ -1 & 4 \end{pmatrix}.$$

(a) Find the solution to the system with initial condition $\mathbf{Y}_0 = (1, 0)$.

(b) Sketch the solution curve obtained in the part (a) on the xy -plane.