4.7 L'Hôpital's Rule

Theorem (★)

Assume that the limit $\lim_{x\to a}\frac{f(x)}{g(x)}$ *is of* $\frac{0}{0}$ *or* $\frac{\infty}{\infty}$ *type. Then*

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

if the limit on the right side exists.

- Examples

- Indeterminate Forms of Type $0 \cdot \infty$ or $\infty - \infty$

  - Convert the product or difference into a quotient

- Indeterminate Powers $\lim_{x \to a} [f(x)]^{g(x)}$

  - Set $y = [f(x)]^{g(x)}$ and take the logarithm of both sides

- Examples

4.8 Antiderivatives

Definition

A function F is called an antiderivative of f if $F'(x) = f(x)$.

- If F is an antiderivative of f, then all the antiderivatives of f have the form

$$F(x) + C$$

where C is an arbitrary constant.

- Examples

Definition

If F is an antiderivative of f, i.e $F'(x) = f(x)$, we write

$$\int f(x)dx = F(x) + C,$$

and $\int f(x)dx$ is called the indefinite integral of f. In other words,

Indefinite integral = Antiderivative

Proposition

1. $\int cf(x)dx = c\int f(x)dx$
2. $\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$

- Examples