Final Exam **Math 115** Fall 2007

Name:______________________________ Section:__________

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IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. (20 pts) For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F.

(a) If the radius of a sphere is increasing at 3 ft/sec then is volume is increasing at 27 cubic feet per second. (Volume $= \frac{4}{3}\pi r^3$.) (a) T F

(b) The derivative of a product is the product of the derivatives. (b) T F

(c) If $f(x)$ is differentiable and $f(-1) = f(1)$ then there exists a number c such that $f'(c) = 0$. (c) T F

(d) According to L'Hopital's rule, $\lim_{x\to 0} \frac{\cos(2x)}{3x^2} = \lim_{x\to 0} \frac{-2\sin(2x)}{6x} = \lim_{x\to 0} \frac{4\cos(2x)}{6} = \frac{4}{6}$ (d) T F

(e) If $f''(x) > 0$ and $g''(x) < 0$ down then $f(x) - g(x)$ is concave up. (e) T F

2. (10 pts) Use the fundamental theorem to find the area between the curve $y = x - x^3$ and the x-axis.

3. (10 pts) Find the derivative of the following functions.

(a) $\displaystyle\int_{-3}^{x} \sqrt{1+t^2}\, dt.$

(b) $\displaystyle\int_{\cos x}^{3} \sqrt{1+t^2}\, dt.$

4. (10 pts) a) For the function $f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & \text{if } x < 2\,; \\ 0 & \text{if } x = 2\,. \\ k - 3x & \text{if } x > 2 \end{cases}$, find $\begin{cases} \lim\limits_{x\to 2^-} f(x) = \underline{\qquad} \\ \text{and} \\ \lim\limits_{x\to 2^+} f(x) = \underline{\qquad} \end{cases}$

b) Is it possible to choose k so that the function $f(x)$ is continuous at $x = 2$? Explain your reasoning in such a way that you demonstrate an understanding of the limit definition of continuity.

5. (15 pts) Given the function $f(x) = \frac{1}{3}x^3 + \frac{16}{x}$.

a) Find $f'(x)$.

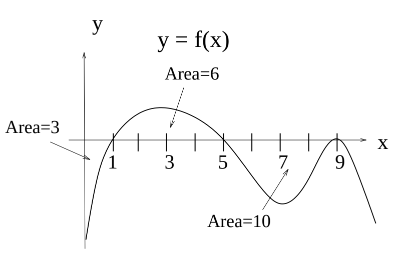
b) Determine where f is increasing and where it is decreasing.

c) Find where the local extrema (local maxima and local minima) of f occur.

d) Find where the graph of $f(x)$ is concave down.

e) Does $f(x)$ have an absolute maximum value on the interval $[-5, 5]$? Explain.

6. (10 pts) Let $f(x)$ be a continuous function given by the following graph.

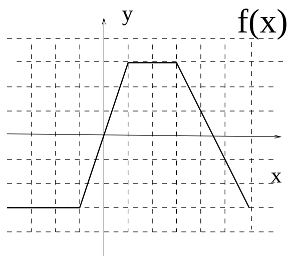


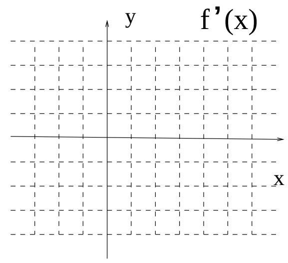
(a) Find $\int_1^9 f(x)\,dx$

(b) If $F'(x) = f(x)$ and $F(1) = -2$, determine the value of $F(9)$.

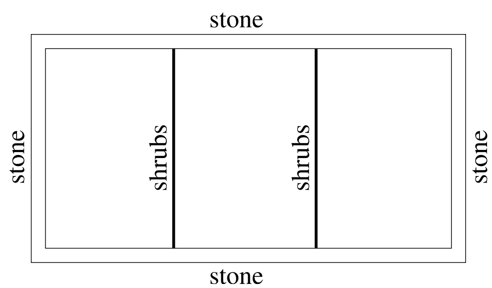
(c) If $F'(x) = f(x)$, for which x-values does $F(x)$ have a local maximum value?

7. (5 pts) Given the graph of the function $f(x)$ below, sketch the graph of its derivative $f'(x)$.

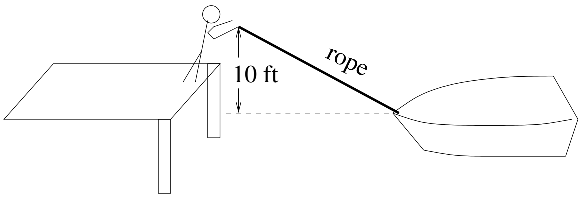




8. (10 pts) A landscape architect is planning borders for a 3000 square foot botanical garden. She will build a stone wall cosing \$50 per foot along all four sides of the outer border. The garden will then be divided into three equal sized rectangular plots (as shown) by two parallel rows of shrubs costing \$25 per foot. Find the dimensions that minimize cost.



9. (10 pts) A boy on a dock is pulling in a rope fastened to the bow of a small boat. If the boy's hands are 10 feet higher than the point where the rope is attached to the boat and if he is retrieving the rope at a rate of 2 feet per second, how fast is the boat approaching the dock when 25 feet of rope are still out? (Assume that the rope is taut and the lies along a straight line.)



10. (10 pts) Consider the function $f(x) = \dfrac{\ln(x) + 3}{x}$, $x > 0$.

(a) Find the equation of the tangent line at $x = 1$.

(b) Use the tangent line approximation of $f(x)$ to approximate $f(1.02)$.

11. (10 pts) An unknown amount of a radioactive substance is being studied. After two days, the mass is 15.231 grams. After eight days, the mass is 9.086 grams.

a) How much was there initially?

b) What is the half-life of this substance?

12. (10 pts) A worker is drawing cider from a storage vat at the rate of $R(t) = 25 - t^2$ liters per minute.

(a) Write a definite integral that represents the total amount of cider that flows from the tank during the first 4 minutes.

(b) Use a Riemann sum with 4 subintervals, taking the sample points to be left endpoints, to estimate the amount of cider that flows from the tank during the first 4 minutes. Show all work and illustrate the Riemann sum with a diagram.