5.3 Fundamental Theorem of Calculus (Continued)

Theorem (Fundamental Theorem of Calculus (★★))

1. *If F is an antiderivative of f, then*

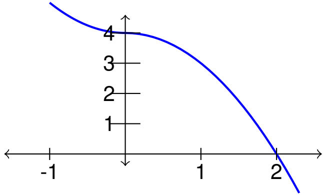
$$\int_a^b f(x)dx = F(b) - F(a).$$

2.

$$\frac{d}{dx}\int_a^x f(t)dt = f(x).$$

- Set
$$A(x) = \int_a^x f(t)dt.$$
Then the function $A(x)$ can be considered as the area function. It gives the net area between $t = a$ and $t = x$.
- We obtain from the FTC
$$A'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x).$$
- Examples

- Below is the graph of a function f.



Let $g(x) = \int_0^x f(t)\,dt$. Then

A. $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 0$.

B. $g(0) = 0$, $g'(0) = 4$ and $g'(2) = 0$.

C. $g(0) = 1$, $g'(0) = 0$ and $g'(2) = 1$.

D. $g(0) = 0$, $g'(0) = 0$ and $g'(2) = 1$.

5.4 Working with Integrals

Theorem

1. *If f is even, then*

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx.$$

2. *If f is odd, then*

$$\int_{-a}^{a} f(x)dx = 0.$$

- Examples

Definition

The *average value* *of an integrable function* f *on* $[a, b]$ *is defined to be*

$$\bar{f} = \frac{1}{b-a}\int_a^b f(x)dx.$$

- Example

Theorem (Mean Value Theorem for Integrals)

Suppose that f *is continuous on* $[a, b]$*. Then there exists* $c \in (a, b)$ *such that*

$$f(c) = \bar{f} = \frac{1}{b-a}\int_a^b f(x)dx.$$