4.8 Antiderivatives (Continued)

Definition

If F is an antiderivative of f, i.e $F'(x) = f(x)$, we write

$$\int f(x)dx = F(x) + C,$$

and $\int f(x)dx$ is called the indefinite integral of f. In other words,

Indefinite integral = Antiderivative

Proposition

1. $\int cf(x)dx = c\int f(x)dx$
2. $\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$

5.1 Approximating Areas under Curves

- The area under a velocity curve represents the displacement.
- How can we calculate the area under a curve?
- Example: The area under $y = x^2$ from 0 to 1

- Sigma Notation: $\displaystyle\sum_{k=1}^{n} a_k = a_1 + a_2 + \cdots + a_n$

- 1. $\displaystyle\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
  2. $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
  3. $\displaystyle\sum_{k=1}^{n} k^3 = \left\{\frac{n(n+1)}{2}\right\}^2 = \frac{n^2(n+1)^2}{4}$

Definition

Suppose that f is defined on $[a, b]$. Let $\Delta x = \frac{b-a}{n}$. If $\bar{x}_k$ is any point in the kth subinterval $[x_{k-1}, x_k]$, for $k = 1, 2, \ldots, n$, then

$$\sum_{k=1}^{n} f(\bar{x}_k)\Delta x = f(\bar{x}_1)\Delta x + f(\bar{x}_2)\Delta x + \cdots + f(\bar{x}_k)\Delta x$$

is called a Riemann sum for f on $[a, b]$.

- This sum is called a left Riemann sum, a right Riemann sum or a midpoint Riemann sum if $\bar{x}_k$ is the left endpoint, right endpoint, or midpoint of $[x_{k-1}, x_k]$, respectively.