6.2 Regions Between Curves

Proposition (Area Between Curves)

Assume that $f(x) \geq g(x)$ on $[a, b]$. The area A of the region bounded by $y = f(x)$, $y = g(x)$, and $x = a$, $x = b$, is given by

$$A = \int_a^b [f(x) - g(x)]dx.$$

Proof.

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} [f(\bar{x}_k) - g(\bar{x}_k)]\Delta x$$

Corollary

The area A of the region bounded by $y = f(x)$, $y = g(x)$, and $x = a$, $x = b$, is

$$A = \int_a^b |f(x) - g(x)|dx.$$

Corollary

The area A of the region bounded by $x = f(y)$, $x = g(y)$, and $y = c$, $y = d$, is

$$A = \int_c^d |f(y) - g(y)|dx.$$

- Given the fact $\int_0^{\pi/4} \cos\theta - \sin\theta d\theta = \sqrt{2} - 1$, what is the area between $y = \cos x$ and $y = \sin x$ from 0 to 2π?

A. 0 B. 1 C. $\sqrt{2} - 1$ D. $4\sqrt{2}$ E. $4\sqrt{2} - 4$