Math 2110Q Practice Final Name: ______________________________

PART I. Five problems will be selected from the following. All these problems are from the previous practice exams.

1. Assume that a line ℓ passes through the origin and has the direction of $(3, 0, -4)$. Compute the distance between the line ℓ and the point $P(0, 2, 6)$.

2. Find the area of the triangle whose sides are $\mathbf{u} = (3, 3, 3)$, $\mathbf{v} = (6, 0, 6)$, and $\mathbf{u} - \mathbf{v}$.

3. Find an equation of the line passing through $(0, 4, 8)$ and $(10, -5, -4)$.

4. Find the unit tangent vector of the curve $\mathbf{r}(t) = (\sin t, \cos t, \cos t)$.

5. Find the length of the curve $\mathbf{r}(t) = (4\cos t, 4\sin t, 3t)$ for $0 \leq t \leq 4\pi$.

6. Find the arc length parameterization of the curve $\mathbf{r}(t) = (\cos t^2, \sin t^2)$ for $0 \leq t \leq \sqrt{\pi}$.

7. Consider the planes $Q : x + 2y - z = 1$ and $R : x + y + z = 1$. Find an equation of the line where the planes Q and R intersect.

8. Is the function $f(x, y) = \dfrac{x^2y^2}{x^4 + y^2}$ continuous at $(0, 0)$ if $f(0, 0) = 0$? Justify your answer.

9. Consider the vectors $\mathbf{u} = (-1, 2, 3)$ and $\mathbf{v} = (2, 1, 1)$. Express $\mathbf{u}$ as the sum $\mathbf{u} = \mathbf{p} + \mathbf{n}$, where $\mathbf{p}$ is parallel to $\mathbf{v}$ and $\mathbf{n}$ is orthogonal to $\mathbf{v}$.

10. Consider the plane $Q : x + y = 0$ and the curve $\mathbf{r}(t) = (\cos t, \sin t, t)$ for $0 \leq t \leq 4\pi$. Find the points at which the plane and curve intersect.

11. Consider the curve $\mathbf{r}(t) = (t - \sin t, 1 - \cos t)$. Find the length of the curve for $0 \leq t \leq 2\pi$.

12. Find the curvature of the curve $\mathbf{r}(t) = (7\cos t, \sqrt{3}\sin t, 2\cos t)$.

13. Consider the function $z = \sqrt{25 - x^2 - y^2}$. Graph the level curves corresponding to $z = 0$ and $z = 3$ on the xy-plane.

14. Evaluate the limit $\displaystyle\lim_{(x,y)\to(4,5)} \frac{\sqrt{x+y} - 3}{x + y - 9}$.

15. Let $f(x, y) = \sqrt{4 + x^2 + y^2}$. Find f_{xx}, f_{xy} and f_{yy}.

16. Let $w = \sqrt{x + y + z}$, where $x = \sin t$, $y = \cos t$ and $z = t$. Using the Chain Rule, compute $\dfrac{dw}{dt}$.

17. Let $f(x, y) = \sin \pi(2x - y)$ and $u = (\frac{5}{13}, -\frac{12}{13})$. Compute $D_u f(-1, -1)$.

18. Let $f(x, y) = x^4 - x^2y + y^2$. Find the unit vectors that give the direction of steepest ascent and steepest descent at $P(-1, 1)$.

19. Consider the surface given by $z^2 = x^2/16 + y^2/9 + 1$. Find an equation of the tangent plane to the surface at $(4, 3, -\sqrt{3})$.

20. Let $f(x, y) = xye^{-x-y}$. Find the critical points of f and determine whether each critical point is a local maximum, local minimum, or saddle point.

21. Let $R = \{(x, y) : x^2 + y^2 \leq 6\}$. Consider the function $f(x, y) = -x^2 - y^2 + \sqrt{3}x - y - 1$. Find the absolute maximum and minimum values of the function f on the set R.

22. Consider $f(x, y, z) = x^2 + y^2 + z^2$. Find the maximum and minimum value of f subject to the constraint $z = 1 + 2xy$.

23. Consider the region $R = \{(x, y) : 1 \leq x \leq 4, 1 \leq y \leq 2\}$. Evaluate the double integral

$$\iint_R \frac{x}{(1 + xy)^2}\, dA.$$

24. Evaluate the integral $\displaystyle\int_0^\pi \int_x^\pi \sin y^2\, dy\, dx$.

25. Find the volume of the solid above the region $R = \{(x, y) : 0 \leq x \leq 1,\ 0 \leq y \leq 2 - x\}$ and between the planes $-4x - 4y + z = 0$ and $-2x - y + z = 8$.

26. Find the volume of the solid below the paraboloid $z = 4 - x^2 - y^2$ and above the region

$$R = \{(r, \theta) : 0 \leq r \leq 1,\ 0 \leq \theta \leq 2\pi\}.$$

27. Find the average distance between points within the cardioid $r = 1 + \cos\theta$ and the origin.

28. Evaluate the integral $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xz\, dz\, dy\, dx$.

29. Find the average of the *squared* distance between the origin and points in the solid cylinder

$$D = \{(x, y, z) : x^2 + y^2 \leq 4,\ 0 \leq z \leq 2\}.$$

30. Evaluate the integral $\displaystyle\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_0^2 \frac{1}{1 + x^2 + y^2}\, dz\, dy\, dx$.

31. Evaluate the integral $\displaystyle\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{x^2+y^2}} \frac{1}{\sqrt{x^2 + y^2}}\, dz\, dy\, dx$.

32. Evaluate the integral $\int_0^{2\pi}\int_0^{\pi/3}\int_0^{4\sec\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$.

33. Find the volume of the region inside the sphere $\rho = 2\cos\phi$ and outside the sphere $\rho = 1$.

PART II. Five problems will be selected from the following.

1. Evaluate the integral $\iint_R x^2\sqrt{x+2y}\,dA$ using the change of variables $x = 2u$ and $y = v - u$, where

$$R = \{(x, y) : 0 \le x \le 2, -x/2 \le y \le 1 - x\}.$$

2. Evaluate the integral $\iiint_D z\,dV$ using the change of variables $x = 4u\cos v$, $y = 2u\sin v$, $z = w$, where D is bounded by the paraboloid $z = 16 - x^2 = y^2$ and the xy-plane.

3. Find the gradient vector field F for the potential function $\phi(x, y, z) = e^{-z}\sin(x + y)$.

4. Evaluate the line integral

$$\int_C (y - z)\,ds,$$

where C is the helix $r(t) = (3\cos t, 3\sin t, t)$ for $0 \le t \le 2\pi$.

5. Consider the force field $F = \dfrac{(x, y, z)}{(x^2 + y^2 + z^2)^{3/2}}$. Find the work required to move an object on the line segment from $(1, 1, 1)$ to $(10, 10, 10)$.

6. Consider the vector field $F = (y + z, x + z, x + y)$. Determine a potential function of F.

7. Let $\phi(x, y, z) = (x^2 + y^2 + z^2)/2$. Evaluate the integral $\int_C \nabla\phi \cdot dr$ where C is given by

$$r(t) = (\cos t, \sin t, t/\pi) \qquad \text{for } 0 \le t \le 2\pi.$$

8. Let C be the closed curve given by $y = \sin x$ and $y = 0$ for $0 \le x \le \pi$ and oriented counterclockwise, and consider the vector field $F = (2y, -2x)$. Compute $\int_C F \cdot dr$.

9. Let C be the triangle with vertices $(0, 0)$, $(2, 0)$, $(0, 4)$ and oriented counterclockwise, and consider the vector field $F = (0, xy)$. Compute the flux $\int_C F \cdot n\,ds$.

10. Find a nonzero vector field F in $\mathbb{R}^3$ such that $\nabla \cdot F = 0$ and $\nabla \times F = 0$. Justify your answer.

11. Find the surface integral $\iint_S (x^2 + y^2)\,dS$ where S is the hemisphere $x^2 + y^2 + z^2 = 36$ for $z \ge 0$.

12. For the vector field $F = (0, 0, -1)$, compute the flux across the slanted face of the tetrahedron $z = 4 - x - y$ in the first octant when normal vectors point in the positive z-direction.

13. Consider the vector field $F = (x^2 - z^2, y, 2xz)$. Evaluate the integral $\int_C F \cdot dr$ using Stokes' Theorem, where C is the boundary of the plane $z = 4 - x - y$ in the first octant.

14. Let S be the cap of the sphere $x^2 + y^2 + z^2 = 25$ for $3 \leq x \leq 5$, and consider the vector field $F = (2y, -z, x - y - z)$. Compute $\iint_S (\nabla \times F) \cdot n \, dS$.

15. Let S be the sphere $x^2 + y^2 + z^2 = 6$, and consider the vector field $F = (x, -2y, 3z)$. Compute the flux across the surface S.

16. Let $F = (z - x, x - y, 2y - z)$, and D be the region between the spheres of radius 2 and 4 centered at the origin. Compute the net outward flux of F across boundary of the region D.

17. Let $F = r/|r|^3 = \dfrac{(x, y, z)}{(x^2 + y^2 + z^2)^{3/2}}$. Prove Gauss' law: If S is a surface that encloses the origin we have $\iint_S F \cdot n \, dS = 4\pi$.