1. For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. For each T/F question, write a careful and clear justification or describe a counterexample.

(a) If f and g are increasing on an interval (a,b) then fg is also increasing on (a,b).

(b) $\frac{d}{dx}(\ln(10) = \frac{1}{10}$

(c) If f has an absolute minimum value at $x = c$ then $f'(c) = 0$.

(d) If f, f', f'' are continuous then the inflection points of f are the local extrema of f'.

(e) A function defined on all points of a closed interval $[a, b]$ always has a global maximum and a global minimum on $[a, b]$.

2. Multiple choice questions.

(i) If $f(t) = \sqrt{4t+1}$, Find $f''(2)$

(a) $\frac{-4}{27}$ (b) 3 (c) $\frac{-2}{3}$ (d) $\frac{2}{3}$ (e) $\frac{4}{27}$

(ii) Find the points on the curve $y = 2x^3 + 3x^2 - 36x + 7$ where the tangent is horizontal

(a) (-4,71),(4,39) (b) (-3,88),(4,39) (c) (-3,88),(2,-37) (d) (-4,71),(2,-37) (e) None of these

(iii) If a tennis ball is thrown vertically upward with a velocity of 72 ft/s, then its height after t seconds is $h = 72t - 6t^2$. What is the maximum height reached by the ball?

(a) 6 ft (b) 216 ft (c) 36 ft (d) 225ft (e) 81 ft

(iv) Find all the critical numbers of the function $g(x) = 4x + \sin(4x)$.

(a) $\frac{\pi}{4}$ (b) $\frac{\pi(2n+1)}{8}$ (c) $\frac{\pi n}{2}$ (d) $\frac{\pi(2n+1)}{4}$ (e) None of these

3. Use the table to compute the following. The functions f and g are differentiable for all real numbers.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	8	0	7	-5
2	4	3	4	2
3	1	4	4.75	2.5
4	-3	7	5	3

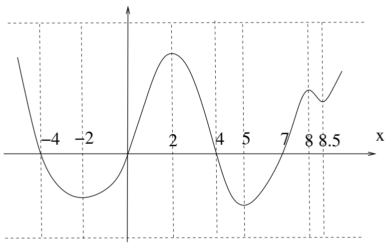
(a) Let $h(x) = f(x)g(x) + x$. Calculate $h'(2)$

(b) Let $k(x) = g(f(x))$. Calculate $k'(3)$

4. Find the derivative of $y = (\sin(x)^{\ln(x)})$

5. For the function $f(x) = x^4 - 2x^2 + 3$, $[-2, 3]$ find the absolute Maximum and Minimum values the given interval $[-2, 3]$.

6. For the function $f(x) = (\frac{1}{4})x^4 - 2x^3 + (\frac{9}{2})x^2 + 1$, use the techniques of calculus to answer the following. Show your work and proper justification for your answers.



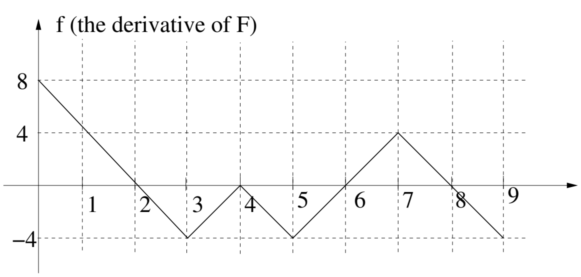
(a) Find $f'(x)$.
(b) Determine all critical points of f. Classify each as a local maximum, a local minimum, or neither.
(c) Find the intervals on which the function is increasing.
(d) Determine any global maxima or minima (if any).
(e) Find any inflection points and the intervals on which the graph of f is concave down.

7. The graph of the derivative f' of a function f is shown.
i) At what values of x does f have a local maximum.
ii) Using the graph, indicate on which intervals the graph of f is concave up and concave down.

8. (a) Find the tangent line approximation to $f(x) = x^{2/3}$ at $a = 8$.
(b) Use your answers to (a) to obtain a linear approximation estimating the value of $(7.98)^{2/3}$.
(c) Will this be an overestimate or an underestimate? Explain your reasoning.[Hint: Look at the shape of the graph.]

9. $f(x) = x^3 - 6x^2 - 15x + 4$
(a) Find the critical numbers and intervals where $f(x)$ is increasing and decreasing.
(b) Find the inflection points and intervals where $f(x)$ is concave up and concave down.

10. The graph of f' is shown below. (The graph of f is not shown.) Use the graph of f' to answer the following questions.



(a) On which intervals, if any, is f increasing?
(b) At which values of x, if any, does f have a local maximum? A local minimum?
(c) On which intervals, if any, is f concave up?
(d) Which values of x, if any, correspond to inflection points on the graph of f?
(e) Assume that $f(0) = 0$. Sketch a graph of f.

11. Sketch the curve for $f(x) = \dfrac{x}{x^2 - 9}$
Show all the necessary work and steps to receive full credit. Only graph of the curve without the work even if it is correct will not receive any credit

(a) Find vertical asymptotes
(b) Find the critical points
(c) Find the intervals of increase
(d) Find the intervals of decrease
(e) Find the intervals where the graph is concave up
(f) Find the intervals where the graph is concave down.
(g) Sketch the graph and indicate the important points.

12. A man 6 feet tall is walking away from a street light 12 feet high. It the man is walking at a rate of 1.8 ft/min, how fast is the length of his shadow increasing? At what rate is the top end of the shadow moving?

13. Two cars start moving from the same point. One travels south at 27 miles/hr and other travels west at 50 miles/hr. At what rate is the distance between the cars increasing 3 hours later?

14. A bacteria culture contains 300 cells initially and grows at a rate proportional to its size. After two hours the population has increased to 420 cells.
(a) Find the number of bacteria after t hours.
(b) Find the rate of growth after 4 hours.
(c) When will the population reach 15,000?

15. A rectangular building is to cover 20,000 square feet. Building lots are rectangular. Zoning regulations require 20 foot borders in front and back of the building and a 10 foot border on each side. Use the optimization techniques of calculus to find the dimensions of the smallest piece of property on which the building can be legally built.

16. A potato is launched with an initial upward velocity of 112 ft/sec from the roof of a dorm 128 ft above the ground. Throughout its flight, the acceleration of the potato is a constant -32 ft/sec^2.
(a) Find the formulas for **both the velocity** of the potato at time t sec and **the height** of the potato at time t sec.
(b) Find the maximum height of the potato.

17. A paper cup has a shape of a cone with height 10 cm and radius 3 cm (at the top). If the water is poured into the cup at a rate of 2 cm^3/s. How fast is the water level rising when the water is 5 cm deep? **Volume of the cone is given by** $V = \frac{1}{3}\pi r^2 h$