4.6 Mean Value Theorem

Theorem (Rolle's Theorem)

Let f be a function that satisfies the following:

1. *f is continuous on $[a, b]$*
2. *f is differentiable on (a, b)*
3. *$f(a) = f(b)$*

Then there exists $c \in (a, b)$ such that $f'(c) = 0$.

Theorem (Mean Value Theorem)

Let f be a function that satisfies the following:

1. *f is continuous on $[a, b]$*
2. *f is differentiable on (a, b)*

Then there exits $c \in (a, b)$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

or, equivalently,

$$f(b) - f(a) = f'(c)(b - a).$$

- There must be a point at which the instantaneous rate of change is equal to the average rate of change.

- Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all x. How large can $f(2)$ possibly be?

  A. -3 B. 0 C. 7 D. 10 E. 100

Theorem

If $f'(x) = 0$ for all $x \in (a, b)$, then f is constant on (a, b).

Proof. Let x_1 and x_2 be any two numbers in (a, b) with $x_1 < x_2$. Now use the MVT.

Corollary

If $f'(x) = g'(x)$ for all $x \in (a, b)$, then $f(x) = g(x) + c$ where c is a constant.