Math2410 Practice Final Exam Name: ______________________________

1. (10 pts) Find the general solution to the following differential equations.

   (a) $\dfrac{dy}{dt} = \dfrac{e^t}{1+y^2}$

   (b) $\dfrac{dy}{dt} + 2y = \sin(5t)\, e^{-2t}$

2. (10 pts) Solve the following initial-value problems.

   (a) $\dfrac{dy}{dt} = \dfrac{1-y^2}{y}, \qquad y(0) = -2. \qquad$ (Write your solution in an explicit form.)

   (b) $y' + \left(\dfrac{2}{t}\right) y = \dfrac{\sin t}{t^2}, \qquad y(\pi/2) = 0.$

3. (10 pts) Consider the differential equation

$$\frac{dy}{dt} = 3y^3 - 12y^2.$$

   (a) Draw the phase line.

   (b) Identify the equilibrium points as sinks, sources, or nodes.

   (c) Sketch the graphs of the solutions satisfying the initial conditions $y(0) = -1$, $y(0) = 0$, $y(0) = 3$, $y(1) = 5$, respectively, in the ty-plane.

4. (10 pts) Consider the one-parameter family $\dfrac{dy}{dt} = (y^2 - \alpha)(y^2 - 1)$. Sketch the bifurcation diagram in the αy-plane and locate the bifurcation values. Your bifurcation diagram must include the phase lines for values of the parameter α smaller than, larger than, and equal to each bifurcation value.

5. (10 pts) Consider the system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} \mathbf{Y}.$$

   (a) Find the general solution.

   (b) Sketch the phase portrait.

   (c) Determine the type of the equilibrium point at the origin.

6. (10 pts) Suppose $\frac{d\mathbf{Y}}{dt} = e^{(-5+i)t}\begin{pmatrix} i \\ -3 \end{pmatrix}$ is a complex solution to the system $\frac{d\mathbf{Y}}{dt} = A\mathbf{Y}$, where the matrix A has real entries.

(a) Find the general solution to the system.

(b) Find the particular solution with $\mathbf{Y}(\mathbf{0}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

7. (a) (5 pts) Find the general solution to the following differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 2\cos 2t.$$

(b) (5 pts) Using the Method of Undetermined Coefficients, solve the initial-value problem

$$y'' - y = e^{2t}, \qquad y(0) = 0,\ y'(0) = 0.$$

8. (10 pts) Solve the following initial-value problems.

(a) $\frac{dy}{dt} + 5y = u_3(t)\ , \qquad y(0) = -2$

(b) $\frac{dy}{dt} + 4y = 3u_5(t)e^{-2(t-5)}\ , \qquad y(0) = 1$