3.3 The Product and Quotient Rules (Continued)

Proposition (★★)

1. $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$
2. $\left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$

- Proofs
- Examples

3.4 Derivatives of Trigonometric Functions

- We have

$$\sin x \approx x \approx \tan x \qquad \text{around } x = 0.$$

- Graphs

Proposition

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \qquad \textit{and} \qquad \lim_{x\to 0} \frac{\tan x}{x} = 1$$

★ $\frac{0}{0}$-type with $\sin mx$ or $\tan mx$: make replacement with mx

★ $\frac{0}{0}$-type with $\cos x - 1$: make multiplication by $\cos x + 1$

- Examples

Proposition (★★)

$$[\sin x]' = \cos x \qquad [\cos x]' = -\sin x$$

$$[\tan x]' = \sec^2 x \qquad [\cot x]' = -\csc^2 x$$

$$[\sec x]' = \sec x \tan x \qquad [\csc x]' = -\csc x \cot x$$

- Proofs: We need to use

$$\sin(x+y) = \sin x \cos y + \cos x \sin y,$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y.$$

- Examples