3.9 (Continued)

Proposition (Derivative of the Inverse Function)

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

In particular,

$$(f^{-1})'(b) = \frac{1}{f'(a)} \quad \text{where } b = f(a).$$

- Proof
- Example

4.1 Maxima and Minima

Definition

Let f be defined on I and $c \in I$.

1. *f has an absolute maximum at c if $f(c) \geq f(x)$ for all $x \in I$.*
2. *f has an absolute minimum at c if $f(c) \leq f(x)$ for all $x \in I$.*

- Examples

Definition

Let f be defined on I and $c \in I$.

1. *f has a local maximum at c*

   if $f(c) \geq f(x)$ for all x in some open interval containing c.

2. *f has a local minimum at c*

   if $f(c) \leq f(x)$ for all x in some open interval containing c.

- Examples

Question

Why is it important to know the locations of maxima and minima?

- When do an absolute max and an absolute min exist?

Theorem (The Extreme Value Theorem)

If f is continuous on a closed interval $[a, b]$, then there exist an absolute maximum $f(c)$ and an absolute minimum $f(d)$ at some points c and d in $[a, b]$.

Theorem (Fermat's Theorem)

If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

- Examples