0. Introduction

- What is calculus? Calculus = "Calculations"
- 1. Differentiation: tangent lines, velocity, ...
  2. Integration: area, volume, work, ...
- Making a linear approximation and taking the limit.
- We will have to deal with infinity.
- E.g. the area of a circle

- More precisely, we study differentiation of functions and integration of functions

Definition

- *A function f is a rule that assigns to each element x in a set A exactly one element $f(x)$ in a set B.*
- *The set A is called the domain of the function f.*
- *The range of f is the set of all values of $f(x)$ as x varies throughout the domain.*

- Examples

Definition

Given two functions f and g, the composite function $f \circ g$ is defined by

$$(f \circ g)(x) = f(g(x)).$$

- Diagram
- Examples
- In general, $f \circ g \neq g \circ f$

Definition

- *A function f is even* $\Leftrightarrow f(-x) = f(x) \Leftrightarrow$ *the graph of f is symmetric with respect to the y-axis.*
- *A function f is odd* $\Leftrightarrow f(-x) = -f(x) \Leftrightarrow$ *the graph of f is symmetric with respect to the origin.*

- Examples

1.2 Representing Functions

- A catalogue of essential functions
  1. Polynomial functions
  2. Rational functions
  3. Algebraic functions
  4. Exponential functions
  5. Logarithmic functions
  6. Trigonometric functions
  7. Inverse trigonometric functions
- The graphs of power and root functions

Suppose $c > 0$. To obtain the graph of

- $y = f(x) + c$, shift the graph of $y = f(x)$ c units upward
- $y = f(x) - c$, shift the graph of $y = f(x)$ c units downward
- $y = f(x - c)$, shift the graph of $y = f(x)$ c units to the right
- $y = f(x + c)$, shift the graph of $y = f(x)$ c units to the left

Suppose $c > 1$. To obtain the graph of

- $y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c
- $y = (1/c)f(x)$, compress the graph of $y = f(x)$ vertically by a factor of c
- $y = f(cx)$, compress the graph of $y = f(x)$ horizontally by a factor of c
- $y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c
- $y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis
- $y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis

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- If $a > 1$ then $f(x) = a^x$ is increasing.
- If $0 < a < 1$ then $f(x) = a^x$ is decreasing.
- The functions $f(x) = a^x$ and $g(x) = (1/a)^x$ are symmetric about the y-axis.

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- $(ab)^x = a^x b^x$

Question: What is the most natural choice of a for $f(x) = a^x$?

Answer: $a = e$