2020-05-27

Homework 6

CESG506

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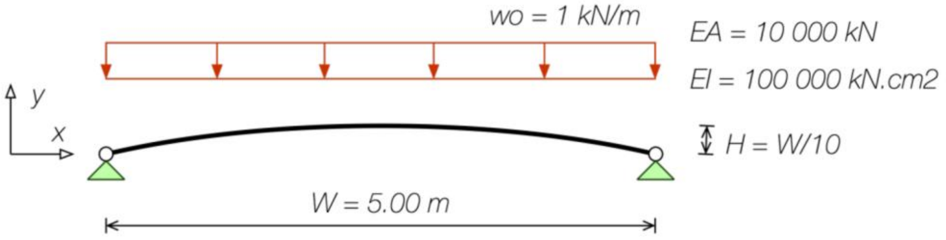
Problem 3

Problem 5-3: Study the snap-through behavior of a shallow arch

Model the snap-through behavior of the shown shallow arch using your arc-length implementation.



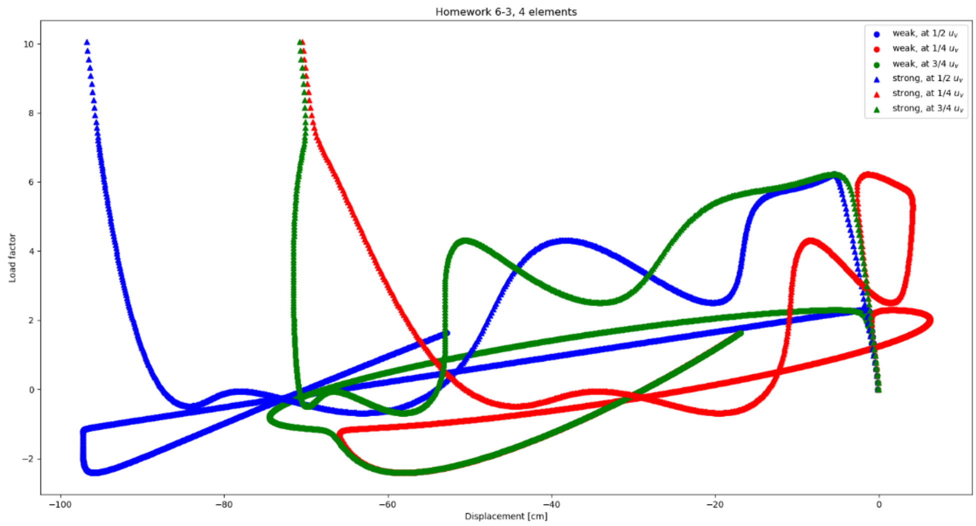
1. You will need a tiny imperfection to discover the full complex response. I suggest to vary the load pattern linearly to $0.99\,w_0$ on the left end and $1.01\,w_0$ on the right end of the arch. Note, that this doesn't change the load much over a single element and you may use the mean value over each element and assume the load constant for each element. That way, you can pre-integrate the load vector.

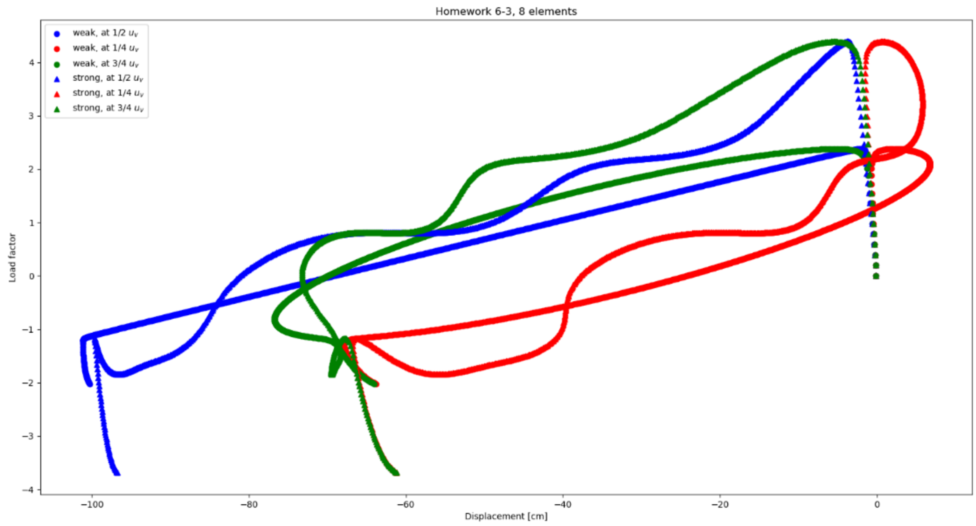
2. Perform a mesh refinement study: start with 4 elements, then double to 8, 16, 32, ... until the solution paths converge.[2] Check convergence for mid-point and quarter points.

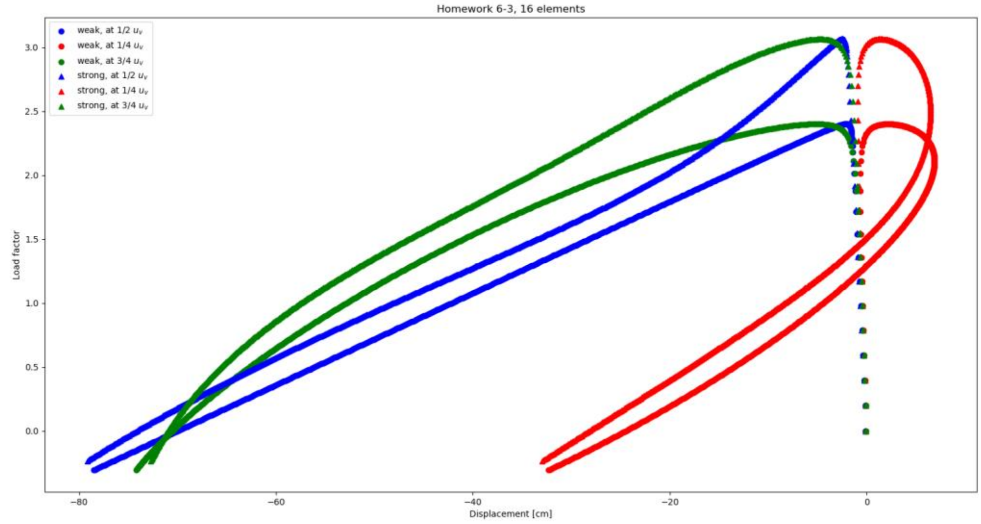
Solution

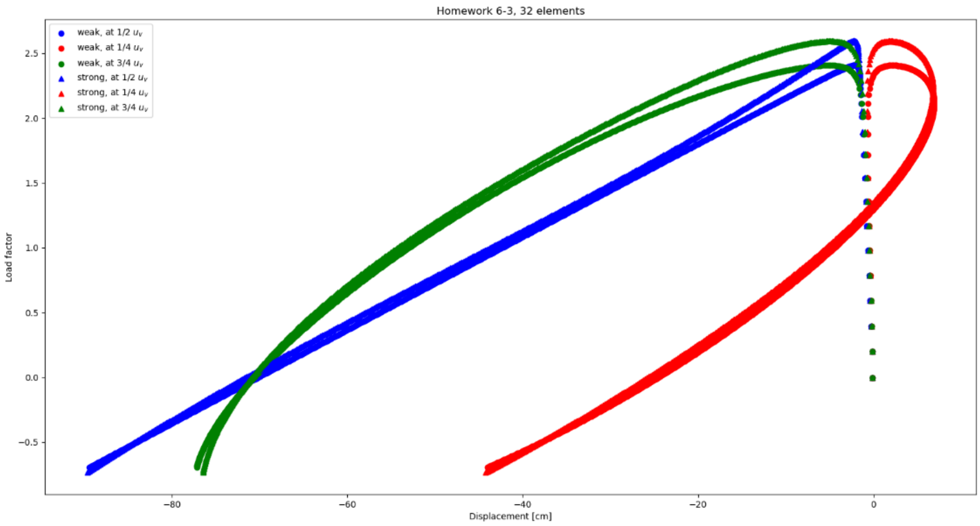
Code was run with increasingly finer mesh and vertical displacement of ¼ point, ½ point and ¾ point were plotted. The results can be seen in the images below.

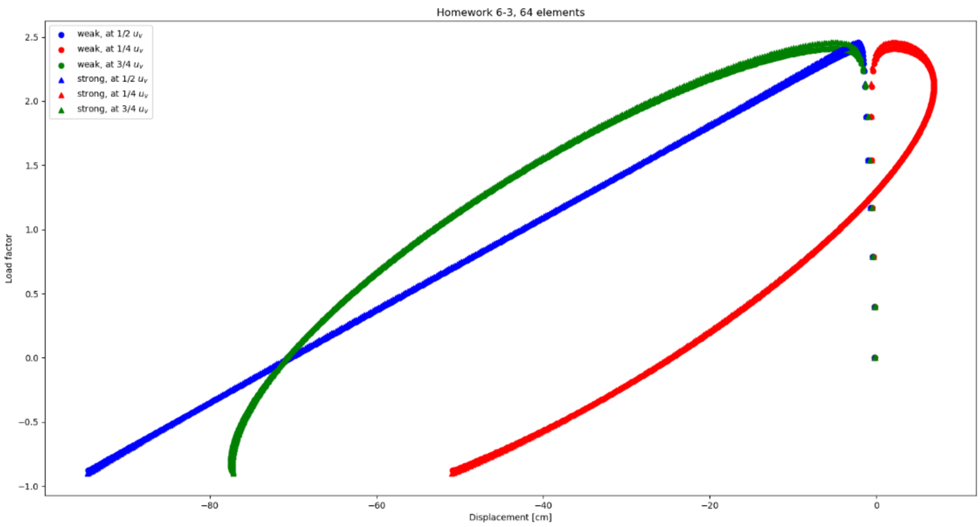
For meshes with 4 and 8 elements the strong formulation behaves in a wildly different manner compared to the weak form formulation. Once the mesh refinement hits 16 elements and higher, the strong form can be seen converging towards the weak form solution. While the weak form has converged to a solution much faster than the strong form.





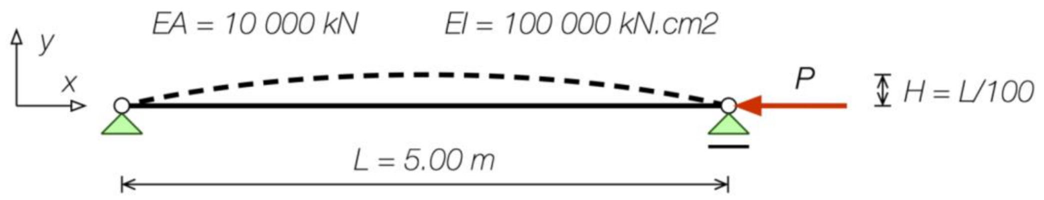

Homework 6-3, 16 elements
weak, at 1/2 u_y
weak, at 1/4 u_y
weak, at 3/4 u_y
strong, at 1/2 u_y
strong, at 1/4 u_y
strong, at 3/4 u_y
Load factor
Displacement [cm]


Homework 6-3, 32 elements
weak, at 1/2 u_y
weak, at 1/4 u_y
weak, at 3/4 u_y
strong, at 1/2 u_y
strong, at 1/4 u_y
strong, at 3/4 u_y
Load factor
Displacement [cm]


Homework 6-3, 64 elements
weak, at 1/2 u_y
weak, at 1/4 u_y
weak, at 3/4 u_y
strong, at 1/2 u_y
strong, at 1/4 u_y
strong, at 3/4 u_y
Load factor
Displacement [cm]

Problem 4

Problem 5-4: Buckling of a beam



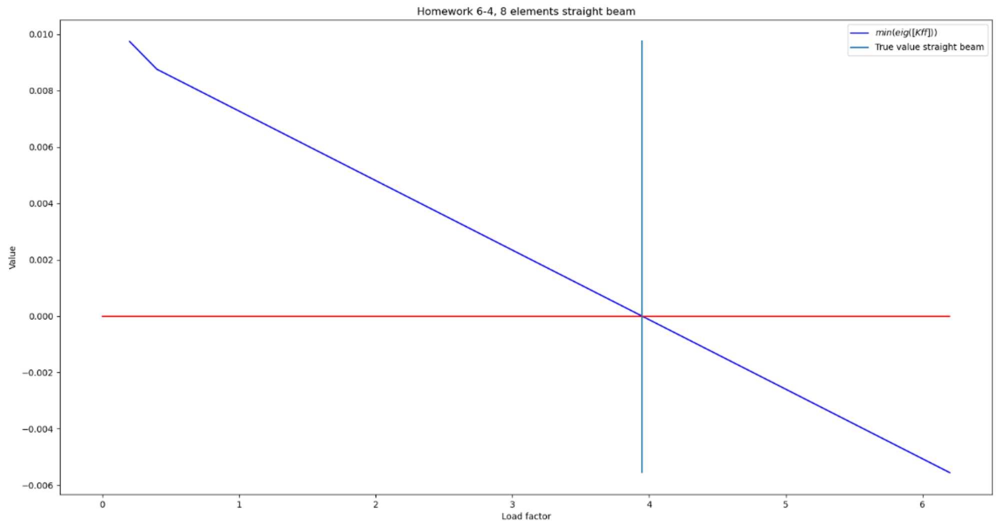
1. Test your element by modeling a straight beam subjected to an axial load. You may use either load stepping as in Assignment #1 or displacement control as in Assignment #2 or the arc-length method as in Assignment #3 for gradually increasing the load level. Plot $\det \mathbf{K}_T$ over load level λ. You have reached the critical load when $\det \mathbf{K}_T = 0$.

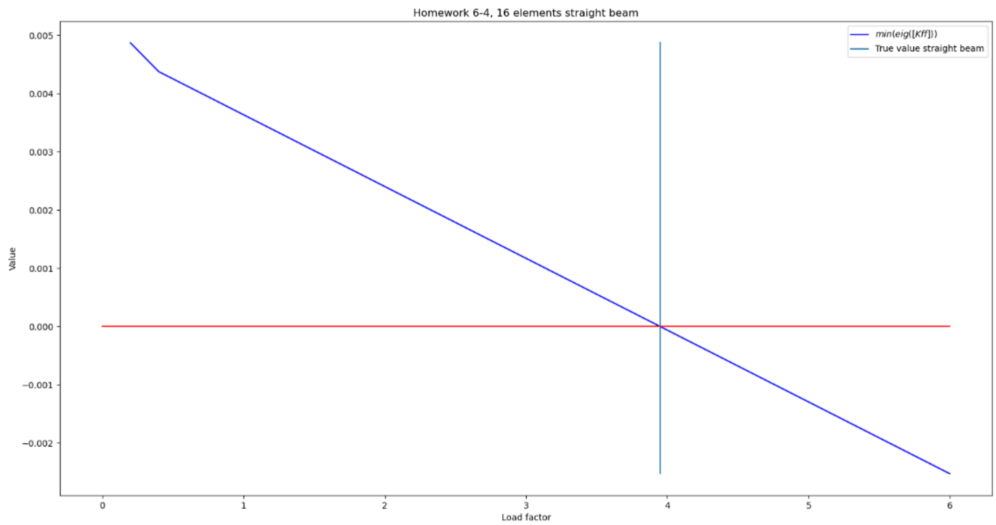
   You may need mesh refinement similar to Problem 5-3 for a good result.

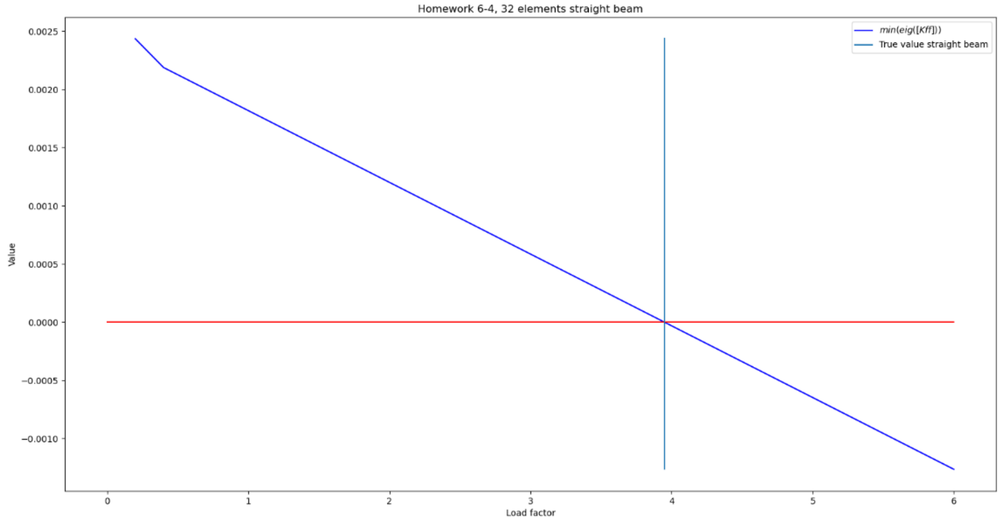
2. Add a small imperfection by curving the beam such that the center deflection $\max h = h(L/2) = L/100$. Now perform the same test. Did $\det \mathbf{K}_T$ go to zero? Why? or Why not?

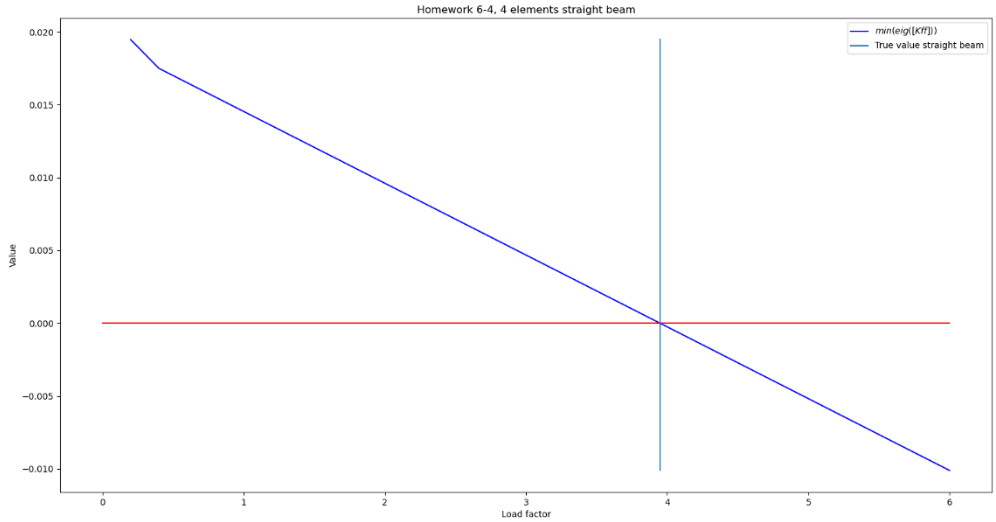
Solution – Straight beam

Plotting determinant of the stiffness matrix over load factor shows that the stiffness matrix becomes singular at the theoretical buckling capacity.



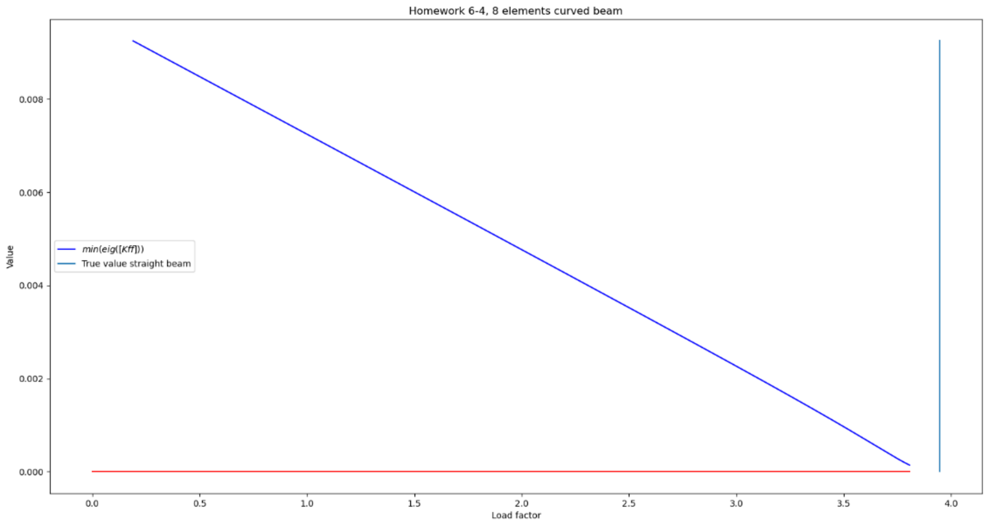


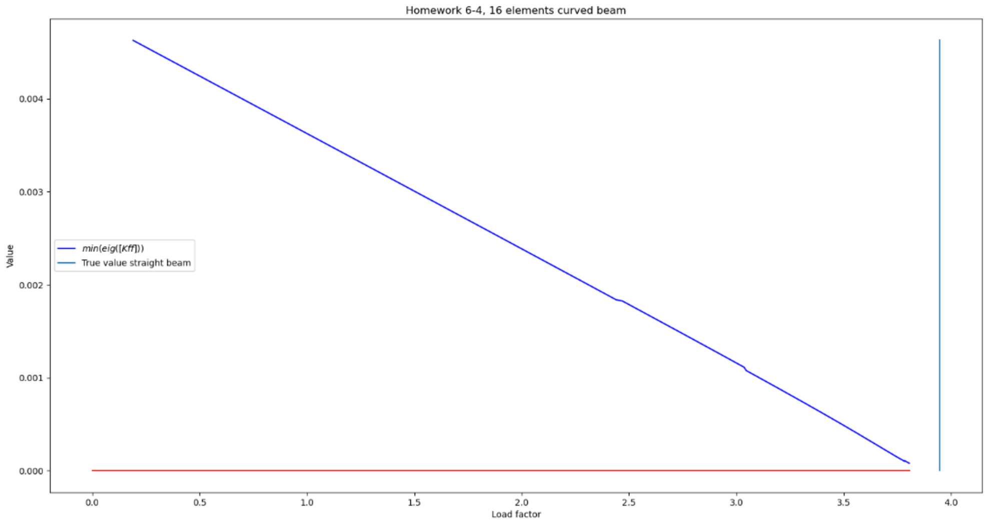

Homework 6-4, 32 elements straight beam
$min(eig([Kff]))$
True value straight beam
Value
Load factor
0.0025
0.0020
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
0
1
2
3
4
5
6

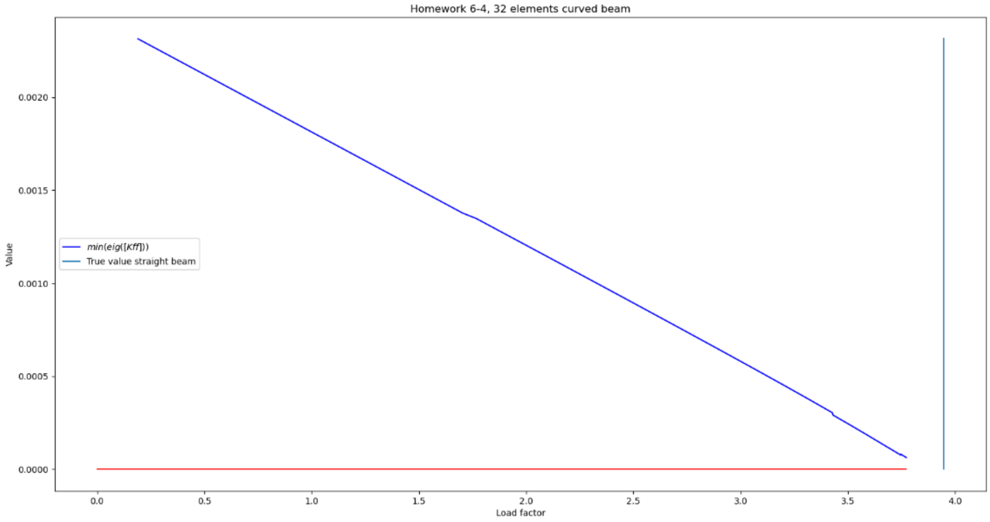

Homework 6-4, 4 elements straight beam
$min(eig([Kff]))$
True value straight beam
Value
Load factor
0.020
0.015
0.010
0.005
0.000
−0.005
−0.010
0
1
2
3
4
5
6

Solution – Curved beam

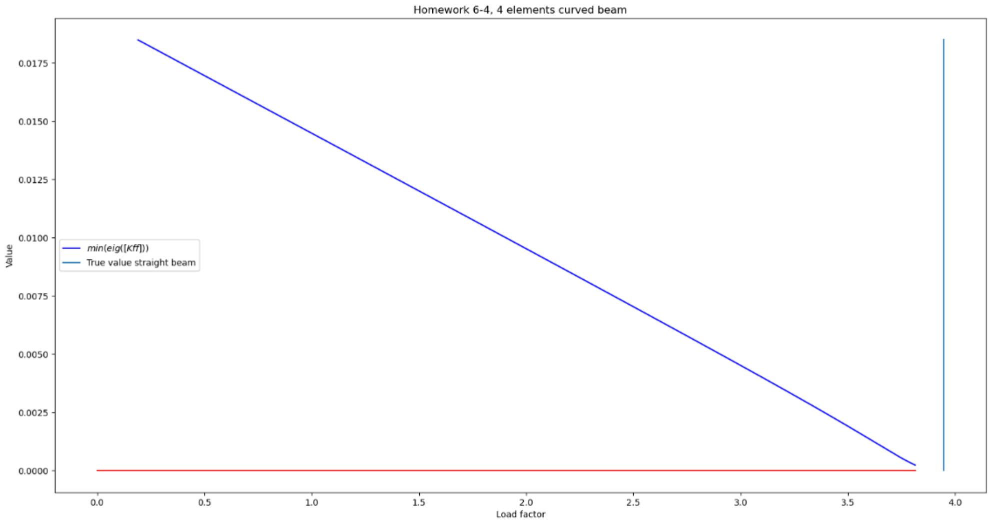
Adding a small curve to the beam changed the behaviour. The matrix no longer turned singular at any point. Furthermore, when the minimum eigenvalue of the stiffness matrix was plotted, it showed it also never quite became zero but clearly trended towards it.







Homework 6-4, 32 elements curved beam
Value
Load factor
min(eig([Kff]))
True value straight beam
0.0020
0.0015
0.0010
0.0005
0.0000
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0



Homework 6-4, 4 elements curved beam
Value
Load factor
min(eig([Kff]))
True value straight beam
0.0175
0.0150
0.0125
0.0100
0.0075
0.0050
0.0025
0.0000
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0