5-1

$$\varepsilon_o = u_o' + h'v' + \frac{1}{2}(v')^2 \qquad \phi = v''$$

$$\delta\pi(u_o, v) \equiv -G(u_o, v, \delta u_o, \delta v)$$

$$= \int_L \delta\bar{W}(\varepsilon_o, \phi)\, dx + \int_L w(x)\,\delta v(x)\, dx - \bar{P} u_o(L) + \bar{R} v(L) - \bar{M}\delta v_L'$$

$$= \int_L \left(F(\varepsilon_o, \phi)\,\delta\varepsilon_o + M(\varepsilon_o, \phi)\,\delta\phi\right) dx + \int_L w(x)\,\delta v(x)\, dx - \bar{P}\delta u_{o_L} + \bar{R}\delta v_L - \bar{M}\delta v_L'$$

$$\delta\varepsilon_o = \delta\left(u_o' + h'v' + \frac{1}{2}(v')^2\right) = \delta u_o' + h'\delta v' + \delta v' v'$$

$$d\delta\varepsilon_o = d\left(\delta u_o' + h'\delta v' + \delta v' v'\right) = dv'\delta v'$$

$$\delta\phi = \delta(v'') = \delta v''$$

$$d\delta\phi = d(\delta v'') = 0$$

$$d\delta\pi = d\left(\int_L \left(F(\varepsilon_o, \phi)\delta\varepsilon_o + M(\varepsilon_o, \phi)\delta\phi\right) dx + \int_L w(x)\delta v\, dx - \bar{P}\delta u_{o_L} + \bar{R}\delta v_L - M\delta v_L'\right)$$

$$= \int_L \Big(\frac{\partial F}{\partial \varepsilon_o} d\varepsilon_o \delta\varepsilon_o + \frac{\partial F}{\partial \phi} d\phi\,\delta\varepsilon_o + \frac{\partial M}{\partial \varepsilon_o} d\varepsilon_o \delta\phi + \frac{\partial M}{\partial \phi} d\phi\,\delta\phi + F(\varepsilon_o, \phi)\, d\delta\varepsilon_o \ldots$$

$$\ldots + M(\varepsilon_o, \phi)\underbrace{d\delta\phi}_{=0}\Big)\, dx$$

Linear elastic case with $F(\varepsilon_o, \phi) = EA\varepsilon_o$ & $M(\varepsilon_o, \phi) = EI\phi$

$$d\delta\pi = \int_L EA\left(du_o' + (h' + v')dv'\right)\left(\delta u_o' + h'\delta v' + \delta v' v'\right) dx \ldots$$

$$\ldots + \int_L EA\left(u_o' + h'v' + \frac{1}{2}(v')^2\right) dv'\delta v'\, dx$$

$$\ldots + \int_L EI\, dv''\delta v''\, dx$$

5-2

$$u_0^h(x) = \left(1-\frac{x}{L_x}\right)u_i + \left(\frac{x}{L_x}\right)u_j$$

$$v^h(x) = N_1(x)v_i + N_2(x)\theta_i + N_3(x)v_j + N_4(x)\theta_j$$

$$N_1(x) = 1-3\xi^2+2\xi^3, \quad N_2(x) = L(\xi-2\xi^2+3\xi^3), \quad N_3(x) = 3\xi^2-2\xi^3 \quad N_4(x) = L(\xi^3-\xi^2)$$

$$w/ \quad \xi = x/L$$

$$h(x) \approx \left(1-\frac{x}{L^e}\right)y_i + \left(\frac{x}{L^e}\right)y_j$$

1. $-\delta u \cdot \underline{R}(\underline{u}) := \delta\pi(u, \delta u)$

$$-\delta\underline{u}\cdot\underline{R}(u) = \int_L \left(F(\varepsilon_0,\phi)\delta\varepsilon_0 + M(\varepsilon_0,\phi)\delta\phi\right)dx$$

$$= \int_L \left(F(\varepsilon_0,\phi)\right)(\delta u_0' + h'\delta v' + v'\delta v')\,dx + \int_L M(\varepsilon_0,\phi)\,\delta v''\,dx$$

$$= \int_L \left(F(\varepsilon_0,\phi)\left([\underline{N}^u]'\{\delta\underline{u}\} + (h'+v')[\underline{N}^v]'\{\delta\underline{v}\}\right)dx \;\dots\right.$$

$$\dots + \int_L M(\varepsilon_0,\phi)\,[\underline{N}^v]''\{\delta v\}\,dx$$

$$= \begin{Bmatrix} \int_L F(\varepsilon_0,\phi)\,[\underline{N}^u]'\,dx \\ \int_L M(\varepsilon_0,\phi)\,[\underline{N}^v]''\,dx + \int_L \underbrace{}_{F(\varepsilon_0,\phi)}(h'+v')[\underline{N}^v]'\,dx \end{Bmatrix}^T \begin{Bmatrix} \delta\underline{u} \\ \delta\underline{v} \end{Bmatrix}$$

5-2

2.

From 5-1 we have

$$d\delta\pi = \int_L EA(du_0' + (h'+v')dv')(\delta u_0' + (h'+v')\delta v')\,dx \ldots$$

$$\ldots + \int_L EA(u_0' + h'v' + \tfrac{1}{2}(v')^2)\,dv'\delta v'\,dx + \int_L EI\,dv''\delta v''\,dx$$

Sub in our approximations

$$d\delta\pi^L = \int_L EA([\underline{N}^u]'\{d\underline{u}\} + (h' + [\underline{N}^v]'\{\underline{v}\})[\underline{N}^v]'\{d\underline{v}\})([\underline{N}^u]'\{\delta\underline{u}\} + (h' + [\underline{N}^v]'\{\underline{v}\})[\underline{N}^v]'\{\delta\underline{v}\})\,dx \ldots$$

$$\ldots + \int_L EA([\underline{N}^u]'\{\underline{u}\} + h'[\underline{N}^v]'\{\underline{v}\} + \tfrac{1}{2}([\underline{N}^v]'\{\underline{v}\})^2)[\underline{N}^v]'\{d\underline{v}\}[\underline{N}^v]'\{\delta\underline{v}\}\,dx \ldots$$

$$\ldots + \int_L EI[\underline{N}^v]''\{d\underline{v}\}[\underline{N}^v]''\{\delta\underline{v}\}\,dx$$

$$= \int_L EA([\underline{N}^u]'\{d\underline{u}\})([\underline{N}^u]'\{\delta\underline{u}\})\,dx + \int_L EA((h' + \overbrace{[\underline{N}^v]'\{\underline{v}\}}^{v'})[\underline{N}^v]'\{d\underline{v}\})([\underline{N}^u]'\{\delta\underline{u}\})\,dx \ldots$$

$$+ \int_L EA([\underline{N}^u]'\{d\underline{u}\})((h' + [\underline{N}^v]'\{\underline{v}\})[\underline{N}^v]'\{\delta\underline{v}\})\,dx \ldots$$

$$+ \int_L EA((h' + [\underline{N}^v]'\{\underline{v}\})[\underline{N}^v]'\{d\underline{v}\})((h' + [\underline{N}^v]'\{\underline{v}\})[\underline{N}^v]'\{\delta\underline{v}\})\,dx \ldots$$

$$+ \int_L F(\varepsilon_0, \phi)[\underline{N}^v]'\{d\underline{v}\}[\underline{N}^v]'\{\delta\underline{v}\}\,dx \ldots$$

$$+ \int_L EI[\underline{N}^v]''\{d\underline{v}\}[\underline{N}^v]''\{\delta\underline{v}\}\,dx$$

$$[d\underline{u}, d\underline{v}]\underbrace{\begin{bmatrix} \int_L EA[\underline{N}^u]'^T[\underline{N}^u]'\,dx & \int_L EA(h'+v')[\underline{N}^u]'^T[\underline{N}^v]'\,dx \\ \int_L EA(h'+v')[\underline{N}^v]'^T[\underline{N}^u]'\,dx & \int_L (EA(h'+v')^2 + F(\varepsilon_0,\phi))[\underline{N}^v]'^T[\underline{N}^v]'\,dx + \int_L EI[\underline{N}^v]''^T[\underline{N}^v]''\,dx \end{bmatrix}}_{[\underline{K}_T]}\begin{Bmatrix} \delta\underline{u} \\ \delta\underline{v} \end{Bmatrix}$$