Homework 2
CESG506
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2-1

1. Using Henky strain, a linear relation $\sigma = E\varepsilon$, $A = \text{const.}$ and equilibrium on the deformed system, derive the relationship between displacements and forces on the free node. Adjust it for path following as

$$\underline{R}(\gamma, \underline{u}) = \gamma \underline{\bar{P}} - \underline{F}(\underline{u}(s)) = \underline{0}$$

with reference load $\underline{\bar{P}} = -(0.99\ \mathrm{kN})\,\underline{j}$ load intensity factor γ and displacement $\underline{u}$

Find the tangents $\partial \underline{R} / \partial \gamma$ and $\partial \underline{R} / \partial \underline{u}$

$$\varepsilon = \frac{1}{2} \ln(\lambda^2) \qquad \lambda^2 = \frac{l^2}{L^2} = \frac{|\underline{l}|^2}{L^2} = \frac{|\underline{L} + \underline{u}|^2}{L^2}$$

$$\frac{\partial \underline{R}}{\partial \underline{u}} = [K_T] = \begin{bmatrix} \underline{k}^e & -\underline{k}^e \\ -\underline{k}^e & \underline{k}^e \end{bmatrix} \qquad \text{w/ } \underline{\underline{k}}^e = \frac{EA}{l^e} \underline{n}^e \otimes \underline{n}^e + \frac{f^e}{l^e}\left(\underline{\underline{1}} - \frac{1}{l^e} \underline{n}^e \otimes \underline{n}^e\right)$$

As we showed last week $\frac{\partial}{\partial \underline{u}} \gamma \underline{\bar{P}} = 0$

$$\frac{\partial \underline{R}}{\partial \gamma} = \underline{\bar{P}} - 0 = -(0.99\ \mathrm{kN})\,\underline{j}$$

2-1

2.

$$\begin{Bmatrix} \underline{R}(\gamma, \underline{u}) \\ g(\underline{u}) \end{Bmatrix} + \begin{Bmatrix} \underline{\bar{P}} \\ 0 \end{Bmatrix} \cdot \Delta\gamma + \begin{bmatrix} \underline{\underline{K}}_T \\ \underline{e}_k \end{bmatrix} \cdot \Delta\underline{u} = 0$$

$$\begin{bmatrix} \underline{\underline{K}}_T & -\underline{\bar{P}} \\ \underline{e}_k & 0 \end{bmatrix} \begin{Bmatrix} \Delta\underline{u} \\ \Delta\gamma \end{Bmatrix} = \begin{Bmatrix} \underline{R} \\ g \end{Bmatrix}$$

with $\underline{R} = \gamma\underline{\bar{P}} - F(u(s))$ & $g(\underline{u}) = \underline{e}_k \cdot \underline{u} - \bar{u}_k = 0$

1. Calculate disp. ($\underline{u}_0$) based on current residule forces ($\underline{R}$)

$$\underline{u}_0 = \underline{\underline{K}}^{-1} \cdot \underline{R}$$

and disp. ($\underline{u}_1$) based on reference force ($\underline{\bar{P}}$)

$$\underline{u}_1 = \underline{\underline{K}}^{-1} \cdot \underline{\bar{P}}$$

2. Calculate required change in reference force to match disp. in controlled DoF

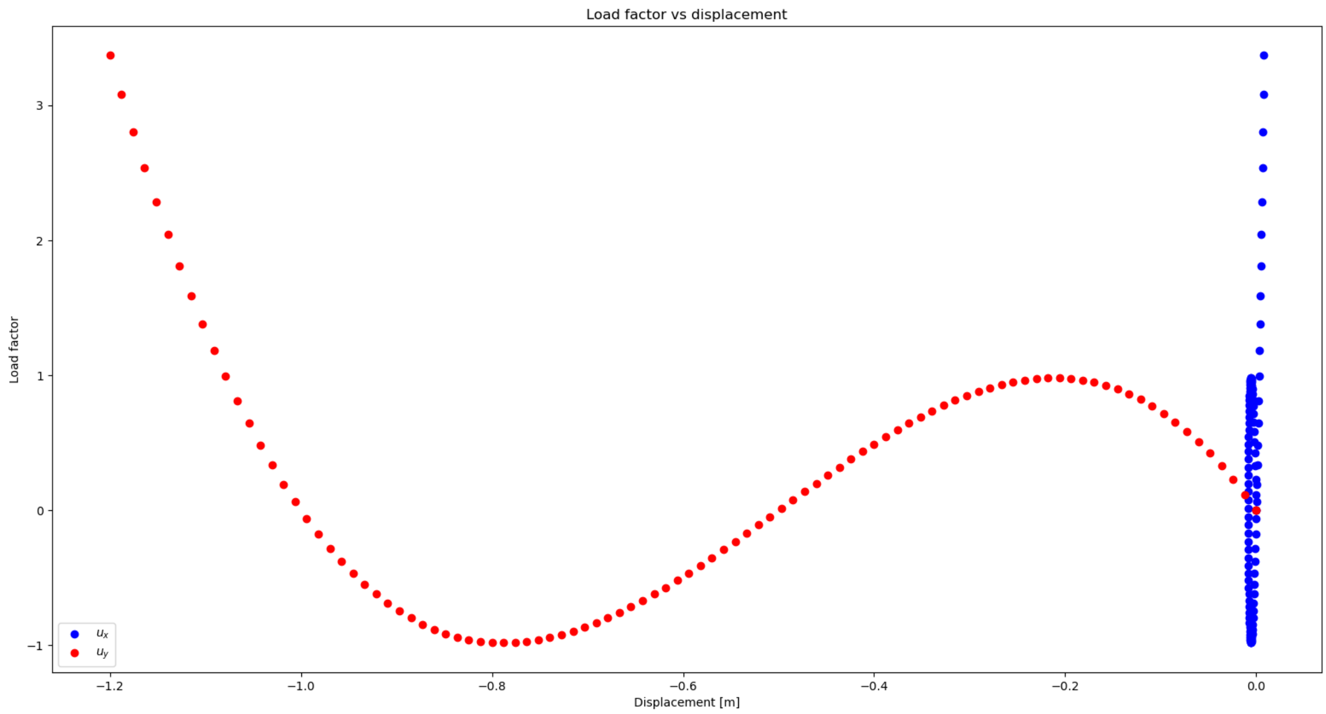
$$\Delta\gamma = -\frac{g + \underline{e}_k \cdot \underline{u}_0}{\underline{e}_k \cdot \underline{u}_1}$$

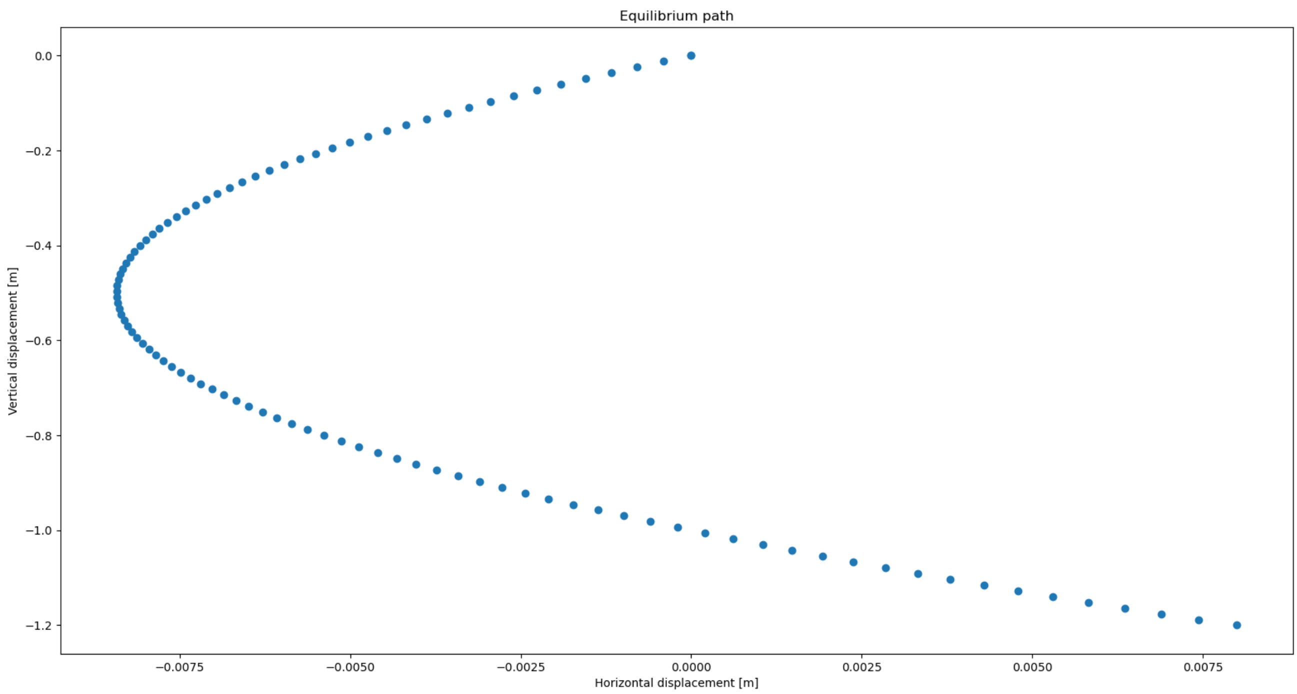
3. Calculate new displacement increment

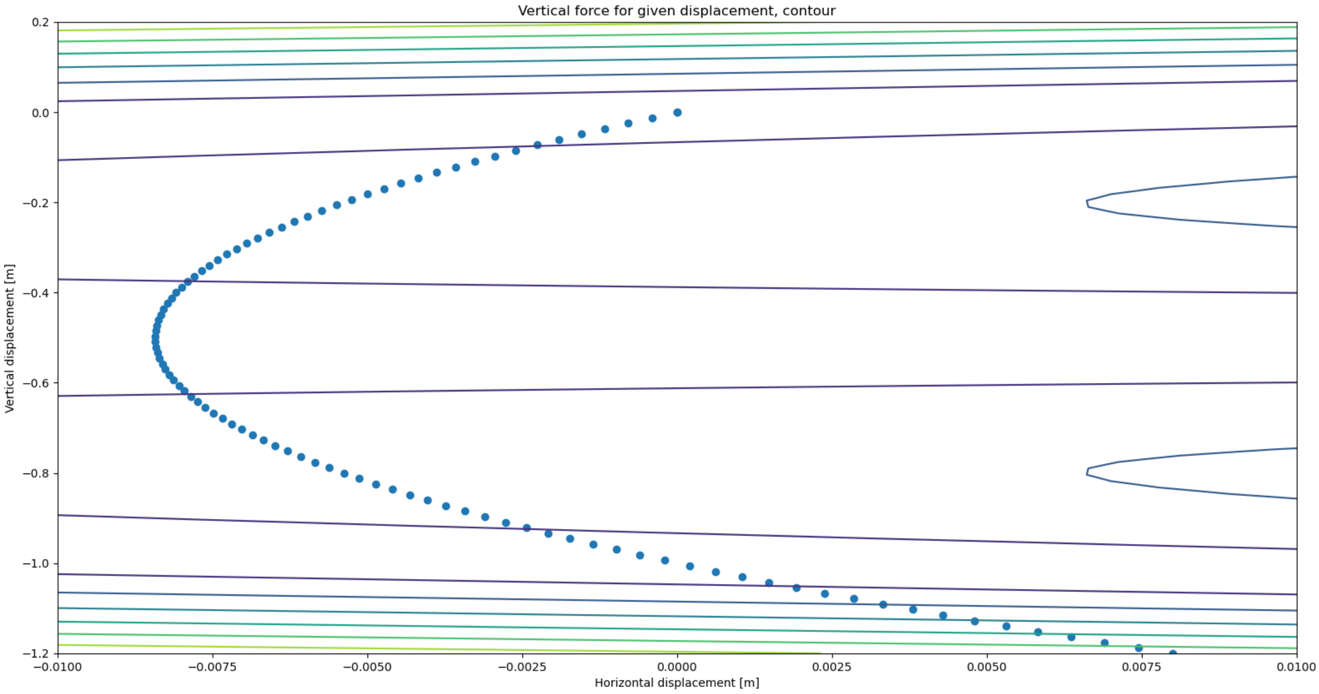
$$\Delta u = \underline{u}_0 + \Delta\gamma \cdot \underline{u}_1$$

4. Update Residule force ($R(\gamma, \underline{u})$) based on new total $\underline{u}$ and γ

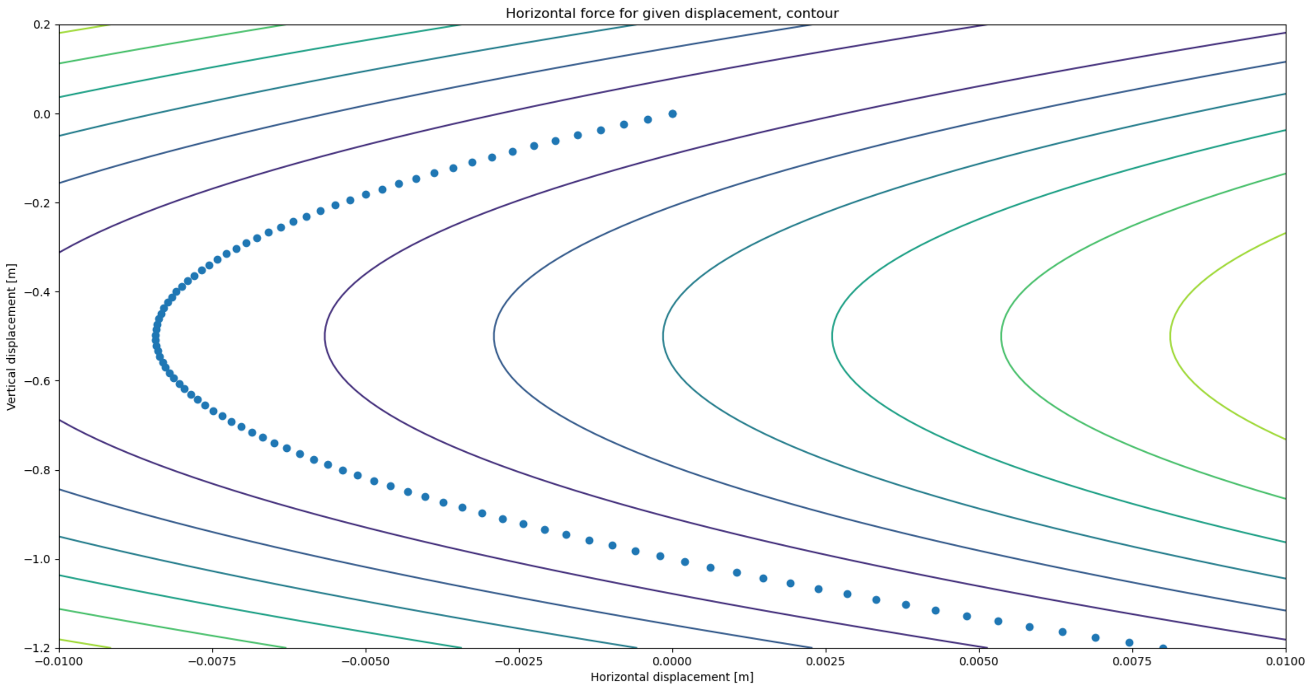
Repeat 1-4 until tolarance norm (R) is within desired limits.


Load factor vs displacement
Load factor
Displacement [m]
u_x
u_y
3
2
1
0
−1
−1.2
−1.0
−0.8
−0.6
−0.4
−0.2
0.0

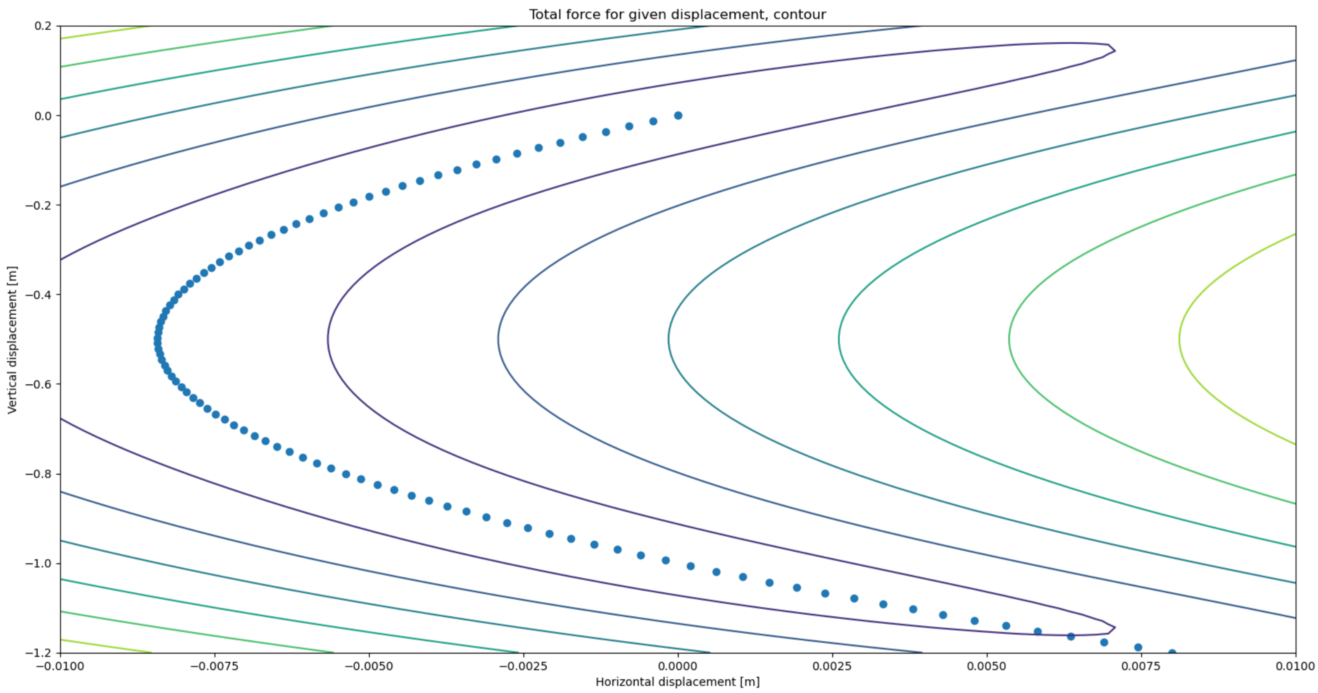

Equilibrium path
Vertical displacement [m]
Horizontal displacement [m]
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−0.0075
−0.0050
−0.0025
0.0000
0.0025
0.0050
0.0075



Vertical force for given displacement, contour
Vertical displacement [m]
Horizontal displacement [m]
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−0.0100
−0.0075
−0.0050
−0.0025
0.0000
0.0025
0.0050
0.0075
0.0100

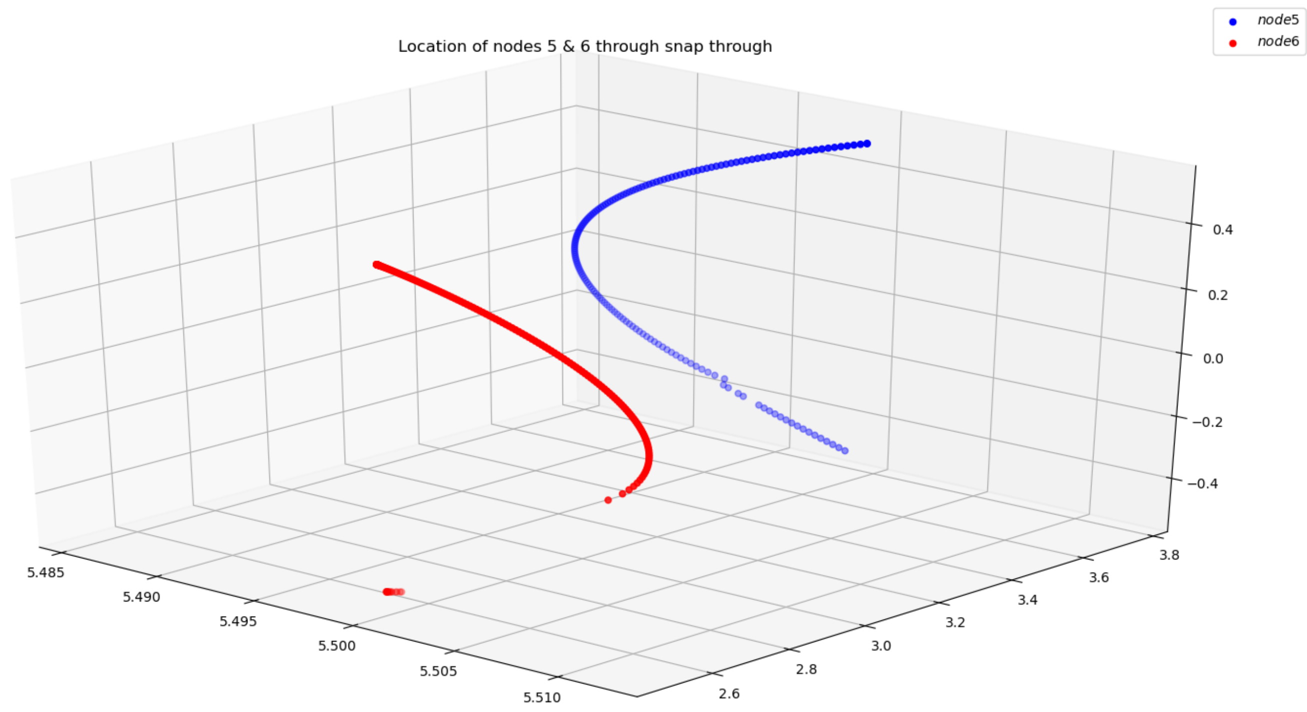


Horizontal force for given displacement, contour
Vertical displacement [m]
Horizontal displacement [m]
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−0.0100
−0.0075
−0.0050
−0.0025
0.0000
0.0025
0.0050
0.0075
0.0100

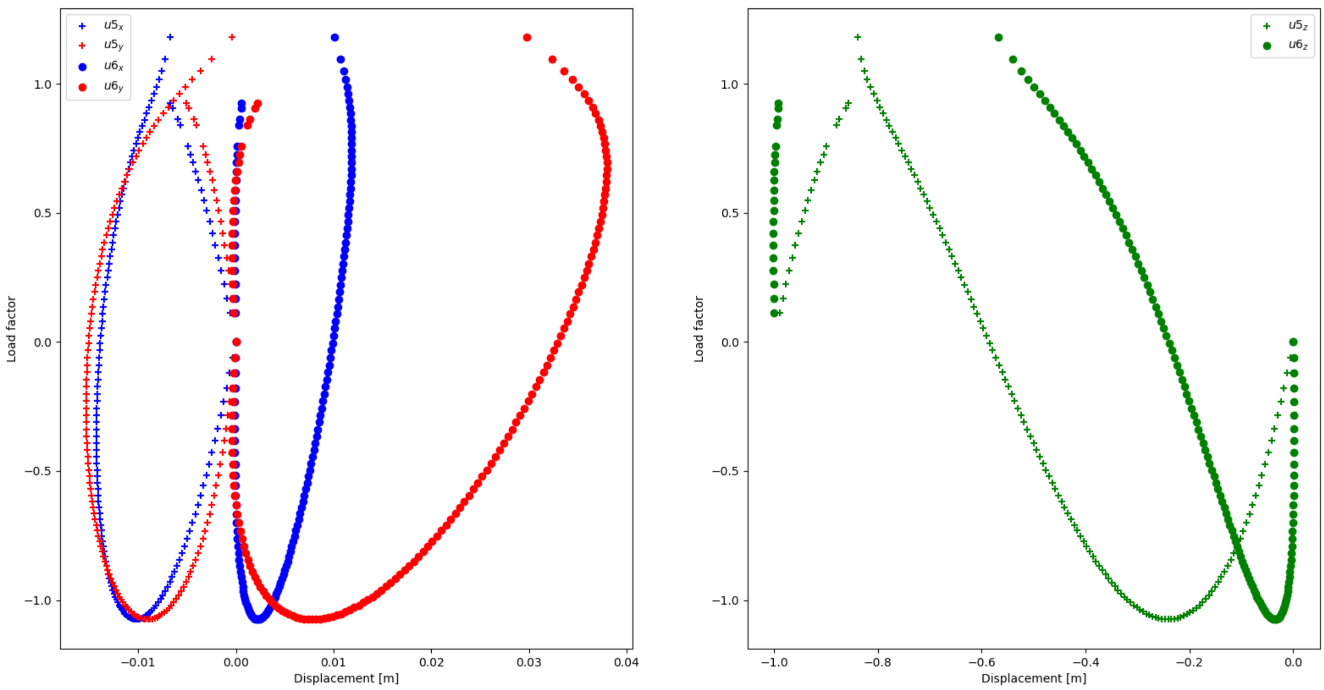


Total force for given displacement, contour
Vertical displacement [m]
Horizontal displacement [m]
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−0.0100
−0.0075
−0.0050
−0.0025
0.0000
0.0025
0.0050
0.0075
0.0100

Problem 2-2



Location of nodes 5 & 6 through snap through
node5
node6
5.485
5.490
5.495
5.500
5.505
5.510
2.6
2.8
3.0
3.2
3.4
3.6
3.8
0.4
0.2
0.0
−0.2
−0.4



$u5_x$
$u5_y$
$u6_x$
$u6_y$
Load factor
Displacement [m]
−0.01
0.00
0.01
0.02
0.03
0.04
1.0
0.5
0.0
−0.5
−1.0
$u5_z$
$u6_z$
Load factor
Displacement [m]
−1.0
−0.8
−0.6
−0.4
−0.2
0.0

All code for problem 2-1 and 2-2 can be found in github file structure. Running main.py will run through all calculations and produce all figures shown here.