3-1.

1. Find tangent stiffness

$$K_T = \frac{dF(u)}{du}$$

$$F(u) = EA \cdot \varepsilon \cdot \sin(\theta)$$

with

$$\varepsilon = \ln(l/L)$$

$$L = \sqrt{30.5} \qquad l = \sqrt{30.5 + u^2 + u}$$

$$\sin(\theta) = \frac{H+u}{l} = \frac{0.5+u}{\sqrt{30.5+u^2+u}}$$

$$\Rightarrow F(u) = EA \cdot \ln\left(\frac{\sqrt{30.5+u^2+u}}{\sqrt{30.5}}\right) \cdot \frac{0.5+u}{\sqrt{30.5+u^2+u}}$$

$$K_T = \frac{15.125 \cdot EA\left(\ln(l) + 0.0661157 \cdot (u^2+u-51.4437)\right)}{l^3}$$

or

$$K_T = \frac{EA(H+u)(2u+1)}{2l^3} - \frac{EA\left((2H-1)+H-2 \cdot L^2\right)\ln\left(\frac{l}{L}\right)}{2l^3}$$

3-1

2.

$$\begin{Bmatrix} R(\gamma, \underline{u}) \\ g(\underline{u}) \end{Bmatrix} + \begin{Bmatrix} \bar{\underline{P}} \\ -2\alpha(\gamma-\gamma_n)\,\bar{\underline{P}}\cdot\bar{\underline{P}} \end{Bmatrix} \Delta\gamma + \begin{bmatrix} \underline{\underline{K}}_T \\ -2(\underline{u}-\underline{u}_n) \end{bmatrix} \Delta\underline{u} = 0$$

$$\begin{bmatrix} \underline{\underline{K}}_T & -\bar{\underline{P}} \\ -2(\underline{u}-\underline{u}_n) & -2\alpha(\gamma-\gamma_n)\,\bar{\underline{P}}\cdot\bar{\underline{P}} \end{bmatrix} \begin{Bmatrix} d\underline{u} \\ d\gamma \end{Bmatrix} = \begin{Bmatrix} \underline{R} \\ g \end{Bmatrix}$$

w/ $\underline{R} = \gamma\bar{\underline{P}} - F^{int}_{(\underline{u}(s))}$ & $g = (\underline{u}-\underline{u}_n)(\underline{u}-\underline{u}_n) + \alpha(\gamma-\gamma_n)^2\,\bar{\underline{P}}\cdot\bar{\underline{P}} - \Delta s^2$

1. Calculate disp. based on current residule forces ($(\underline{u}_0)$ due to $\underline{R}$)

$$\underline{u}_0 = \underline{\underline{K}}_T^{-1}\cdot\underline{R}$$

and disp. based on referece force $\underline{P}$ ($(\underline{u}_1)$ due to $\bar{\underline{P}}$)

$$\underline{u}_1 = \underline{\underline{K}}_T^{-1}\cdot\bar{\underline{P}}$$

2 Calculate required change in reference force to match arc length controlled

$$\Delta\gamma = -\frac{g + 2(\underline{u}-\underline{u}_n)\cdot\underline{u}_0}{2(\underline{u}-\underline{u}_n)\underline{u}_1 + 2\alpha(\gamma-\gamma_n)\,\bar{\underline{P}}\cdot\bar{\underline{P}}}$$

3. Calculate new disp. incrament

$$\Delta\underline{u} = \underline{u}_0 + \Delta\gamma\cdot\underline{u}_1$$

4. Update Residule $\underline{R}(\gamma,\underline{u})$ based on new total $\underline{u}$ and γ

repeat 1-4 until $\text{norm}(\tilde{\underline{R}}) = \text{norm}\begin{Bmatrix} \underline{R} \\ g \end{Bmatrix}$ is within desired tolerance.

To make sure that code converges to next solution (and not last)

I calculate the 0-th step in next iteration as

$$\underline{u}^0_{n+1} = \underline{u}_n + \Delta\underline{u}_n = 2\underline{u}_n - \underline{u}_{n-1}$$

$$\gamma^0_{n+1} = \gamma_n + \Delta\gamma_n = 2\gamma_n - \gamma_{n-1}$$

To get started on first iteration. I solve the linear problem for given γ and choose the resulting $\underline{u}$ as $\underline{u}^0_1$ and set $\Delta s^2 = \underline{u}\cdot\underline{u} + \alpha\gamma^2\,\bar{\underline{P}}\cdot\bar{\underline{P}}$

3-2

1. Find the tangents $\partial \underline{R}/\partial \gamma$ and $\partial \underline{R}/\partial \underline{u}$ for $\underline{R}(\gamma, \underline{u}) = \gamma(s)\,\bar{\underline{P}} - \underline{F}(\underline{u}(s))$

using Henky strain, $\sigma = E\varepsilon$, $A = \text{const.}$ and equilibrium on deformed system

From assignment 1 we have $\partial \underline{R}/\partial \underline{u}$

$$\frac{\partial R}{\partial \underline{u}} = \underline{\underline{K}}_{T.FA} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

w/ $$K_{11} = EA\left(\frac{c_1^2(1-\varepsilon_1)+\varepsilon_1}{l_1} + \frac{c_2^2(1-\varepsilon_2)+\varepsilon_2}{l_2}\right)$$

$$K_{12} = K_{21} = EA\left(\frac{c_1 \cdot s_1(1-\varepsilon_1)}{l_1} - \frac{c_2 s_2(1-\varepsilon_2)}{l_2}\right)$$

$$K_{22} = EA\left(\frac{s_1^2(1-\varepsilon_1)+\varepsilon_1}{l_1} + \frac{s_2^2(1-\varepsilon_2)+\varepsilon_2}{l_2}\right)$$

ε_1 & ε_2 Henky strain in elements 1 & 2

$c_1\ c_2\ s_1\ s_2$ cos and sin of direction of elements 1 & 2

l_1 & l_2 deformed length of the elements

$$\frac{\partial R}{\partial \gamma} = \bar{\underline{P}} - 0 = \bar{\underline{P}} = -0.99\ \text{kN}\ j$$