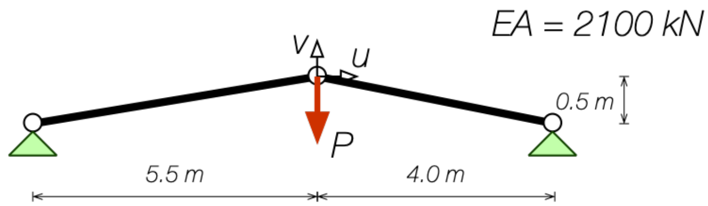
With the last assignment, you explored two simple nonlinear systems. You solved Problem 1-1 using displacement control (in its most primitive), and Problem 1-2 using incremental load control (prescribing the load level).

This week, you'll be exploring path following techniques for tracking equilibrium paths beyond a snap-through[1] or a snap-back[2] point.

Problem 2-1: Displacement control for a two-degrees-of-freedom (2 DOF) problem



This problem expands Problem 1-2 from Assignment #1 such that you shall track the entire equilibrium path from $(u = 0, v = 0)$ and $\lambda = 0$ at least through $\lambda \geq 2.0$.

1. Using Henky strain, a linear relation $\sigma = E\varepsilon$, $A = const.$, and equilibrium on the deformed system, derive the relationship between displacements and forces on the free node. Adjust it for path following as

$$\mathbf{R}(\gamma, \mathbf{u}) = \gamma\bar{\mathbf{P}} - \mathbf{F}(\mathbf{u}(s)) = \mathbf{0} \tag{1a}$$

   with reference load, $\bar{\mathbf{P}} = -(0.99\ \text{kN})\,\mathbf{j}$, load intensity factor γ and displacement $\mathbf{u}$.

   Find the tangents $\partial\mathbf{R}/\partial\gamma$ and $\partial\mathbf{R}/\partial\mathbf{u}$ as you will need them in what follows.

   You should have all the necessary parts for this question from Assignment #1.

2. Develop a, or adjust your existing Newton method to incrementally find γ and $\mathbf{u}$ using displacement control on the vertical displacement, $v = \bar{v}$. The respective constraint equation is

$$g(\mathbf{u}) := \mathbf{e}_v \cdot \mathbf{u} - \bar{v} = 0 \tag{1b}$$

   with

$$\Delta\mathbf{u} \Rightarrow \left\{ \begin{array}{c} u \\ v \end{array} \right\} \quad \text{and} \quad \Delta\mathbf{e}_v \Rightarrow \left\{ \begin{array}{cc} 0 & 1 \end{array} \right\} . \tag{1c}$$

3. Plot load level γ versus vertical displacement v, as well as γ versus horizontal displacement u for the converged points on the equilibrium path. Observe how only one of them actually looks like the curve you (might have) expected.

4. Add a plot u versus v to explore yet another view of the solution path. Try and overlay this curve on a contour plot of $F_x(u, v)$ and on a contour plot for $F_y(u, v)$[3].

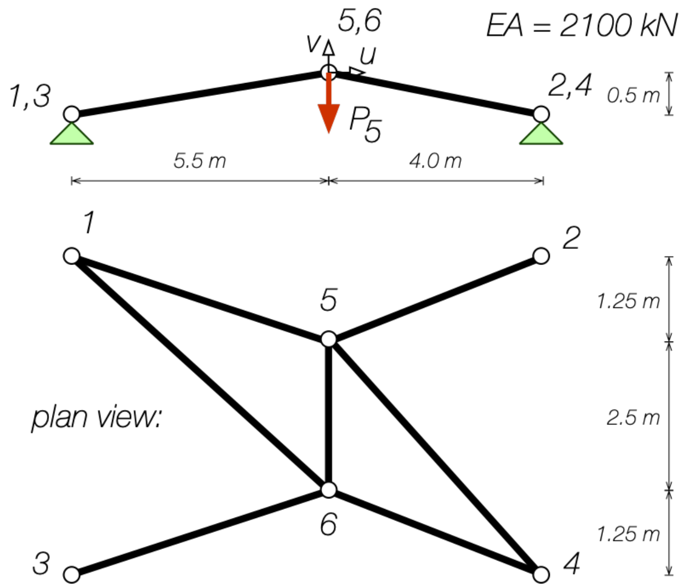
[1] This is where load control will fail.

[2] This is where displacement control will fail.

[3] The first should show that the solution path is hugging the $P_x(u, v) = 0$ contour line. The latter is like a road going through a hilly landscape with elevation along the road being $\lambda(s)\bar{P}$.

Problem 2-2: Going into higher dimensions

Now let's take the problem into the third dimension and look at a 3D-truss system with 6 nodes and 7 truss members as shown.



Only node 5 shall be loaded by a vertical load, $P_5 = \gamma \bar{P}$.

(a) Adjust your code to accommodate the higher dimensional system. At this point, it is highly advisable to drop the brute-force approach and use appropriate functions or my TrussElement class to represent each system component. Assembly may still be done beforehand, though transitioning to a more generic concept will be helpful for future assignments.

(b) Use the vertical displacement $v_5 = \bar{v}$ as control parameter and find the equilibrium path using displacement control. Trace the equilibrium path until members 3–6 and 4–6 are in tension.

(c) Present load-displacement diagrams by plotting the load factor γ against displacement components of nodes 5 and 6.

(d) Plot the planar view of the equilibrium path for nodes 5 and 6 (can be in one or two plots).