2020-04-29

Assignment 3

CESG 506

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3-1.

1. Find tangent stiffness

$$K_T = \frac{dF(u)}{du}$$

$$F(u) = EA \cdot \varepsilon \cdot \sin(\theta)$$

with

$$\varepsilon = \ln\left(\frac{l}{L}\right)$$

$$L = \sqrt{30.5} \qquad l = \sqrt{30.5 + u^2 + u}$$

$$\sin(\theta) = \frac{H+u}{l} = \frac{0.5+u}{\sqrt{30.5+u^2+u}}$$

$$\Rightarrow F(u) = EA \cdot \ln\left(\frac{\sqrt{30.5+u^2+u}}{\sqrt{30.5}}\right) \cdot \frac{0.5+u}{\sqrt{30.5+u^2+u}}$$

$$K_T = \frac{15.125 \cdot EA\left(\ln(l) + 0.0661157 \cdot (u^2+u-51.4437)\right)}{l^3}$$

or

$$K_T = \frac{EA\,(H+u)(2u+1)}{2\,l^3} - \frac{EA\left((2H-1)+H-2\cdot L^2\right)\ln\left(\frac{l}{L}\right)}{2\,l^3}$$

3-1

2.

$$\begin{Bmatrix} R(\gamma, \underline{u}) \\ g(\underline{u}) \end{Bmatrix} + \begin{Bmatrix} \bar{P} \\ -2\alpha(\gamma-\gamma_n)\bar{P}\cdot\bar{P} \end{Bmatrix} \Delta\gamma + \begin{bmatrix} \underline{\underline{K}}_T \\ -2(\underline{u}-\underline{u}_n) \end{bmatrix} \Delta\underline{u} = 0$$

$$\begin{bmatrix} \underline{\underline{K}}_T & -\bar{P} \\ -2(\underline{u}-\underline{u}_n) & -2\alpha(\gamma-\gamma_n)\bar{P}\cdot\bar{P} \end{bmatrix} \begin{Bmatrix} d\underline{u} \\ d\gamma \end{Bmatrix} = \begin{Bmatrix} \underline{R} \\ g \end{Bmatrix}$$

w/ $\underline{R} = \gamma\bar{P} - F^{int}(\underline{u}(s))$ & $g = (\underline{u}-\underline{u}_n)(\underline{u}-\underline{u}_n) + \alpha(\gamma-\gamma_n)^2\bar{P}\cdot\bar{P} - \Delta s^2$

1. Calculate disp. based on current residule forces ($\underline{u}_0$ due to $\underline{R}$)

$$\underline{u}_0 = \underline{\underline{K}}_T^{-1} \cdot \underline{R}$$

and disp. based on refereee force $\bar{P}$ ($\underline{u}_1$ due to $\bar{P}$)

$$\underline{u}_1 = \underline{\underline{K}}_T^{-1} \cdot \bar{P}$$

2 Calculate required change in reference force to match arc length controlled

$$\Delta\gamma = -\frac{g + 2(\underline{u}-\underline{u}_n)\cdot\underline{u}_0}{2(\underline{u}-\underline{u}_n)\underline{u}_1 + 2\alpha(\gamma-\gamma_n)\bar{P}\cdot\bar{P}}$$

3. Calculate new disp. incrament

$$\Delta\underline{u} = \underline{u}_0 + \Delta\gamma\cdot\underline{u}_1$$

4. Update Residule $\underline{R}(\gamma, \underline{u})$ based on new total $\underline{u}$ and γ

repeat 1-4 until $norm(\tilde{R}) = norm\begin{Bmatrix} \underline{R} \\ g \end{Bmatrix}$ is within desired tolerance.

To make sure that code converges to next solution (and not last) I calculate the 0-th step in next iteration as

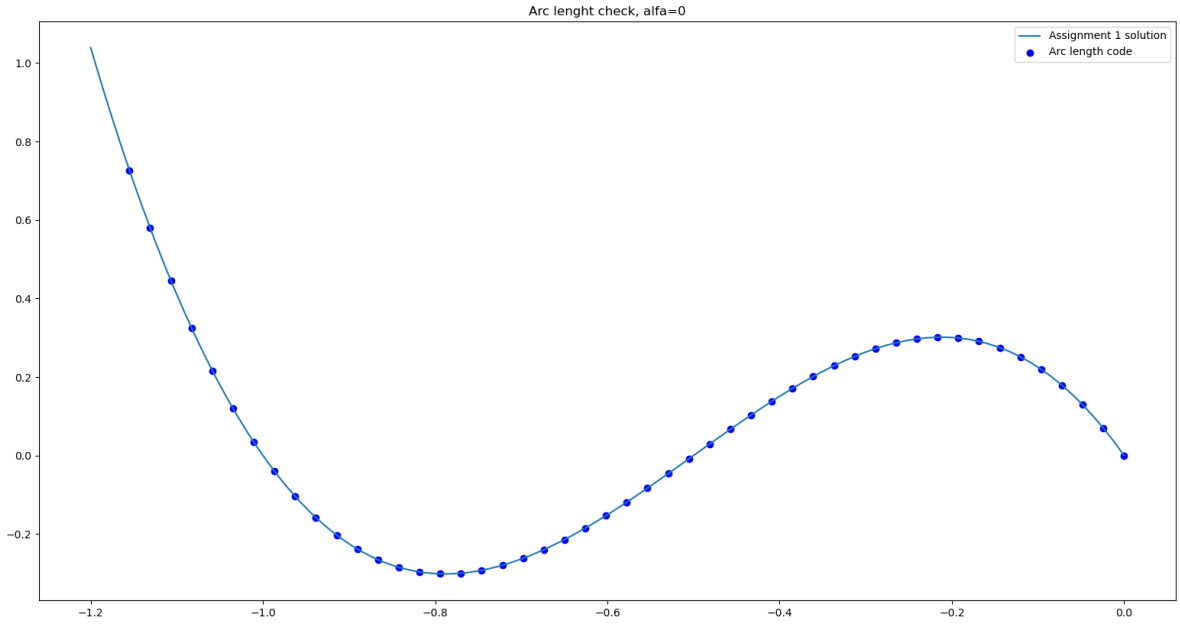
$$\underline{u}_{n+1}^0 = \underline{u}_n + \Delta\underline{u}_n = 2\underline{u}_n - \underline{u}_{n-1}$$

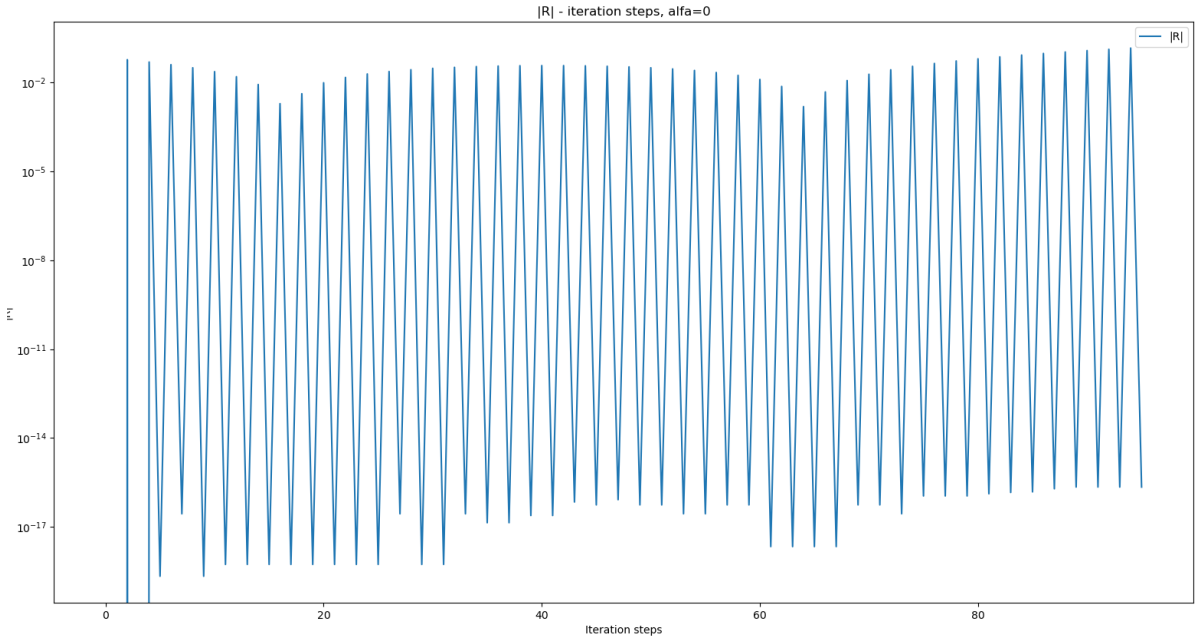
$$\gamma_{n+1}^0 = \gamma_n + \Delta\gamma_n = 2\gamma_n - \gamma_{n-1}$$

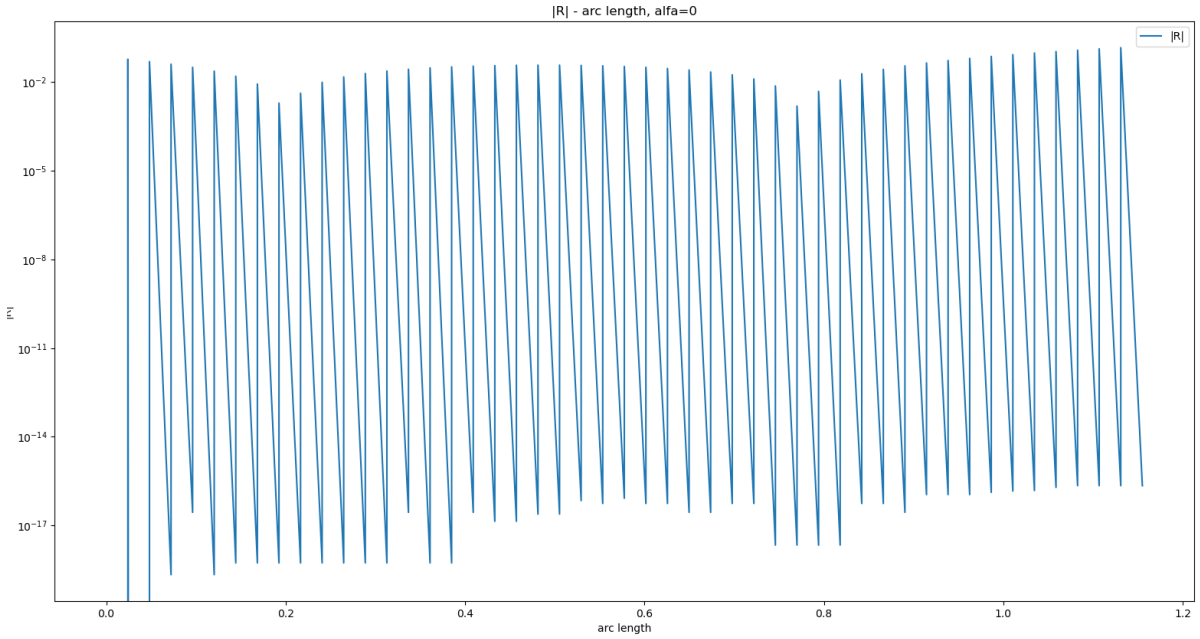
To get started on first iteration. I solve the linear problem for given γ and choose the resulting $\underline{u}$ as $\underline{u}_1^0$ and set $\Delta s^2 = \underline{u}\cdot\underline{u} + \alpha\gamma^2\bar{P}\cdot\bar{P}$

Problem 1

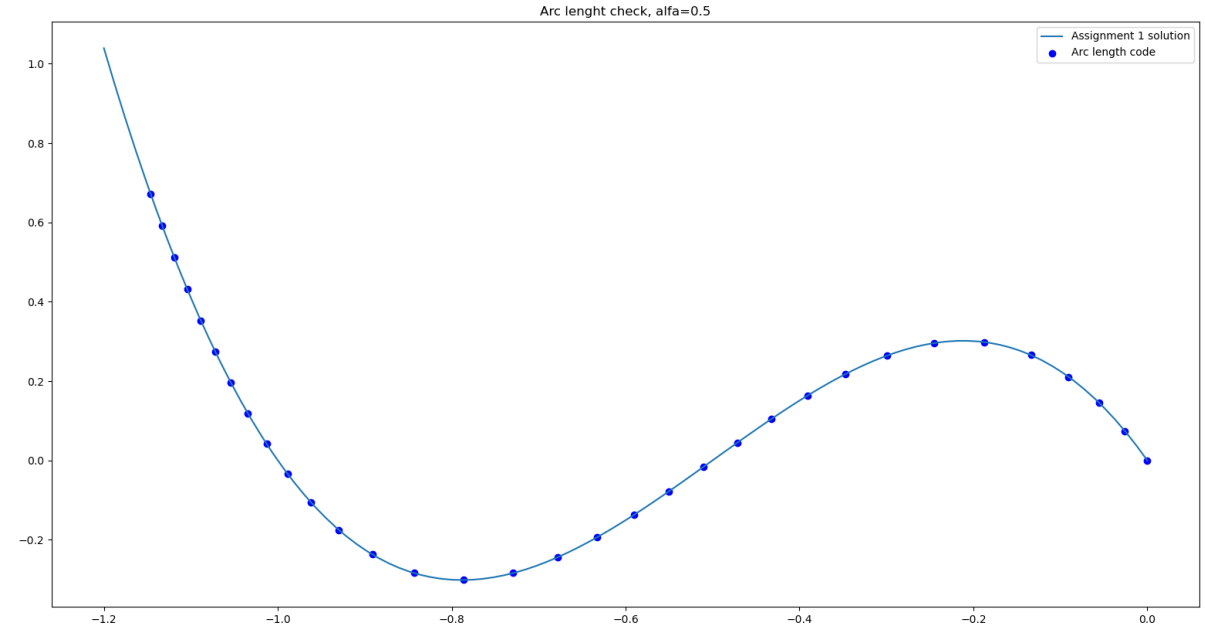
Part 3

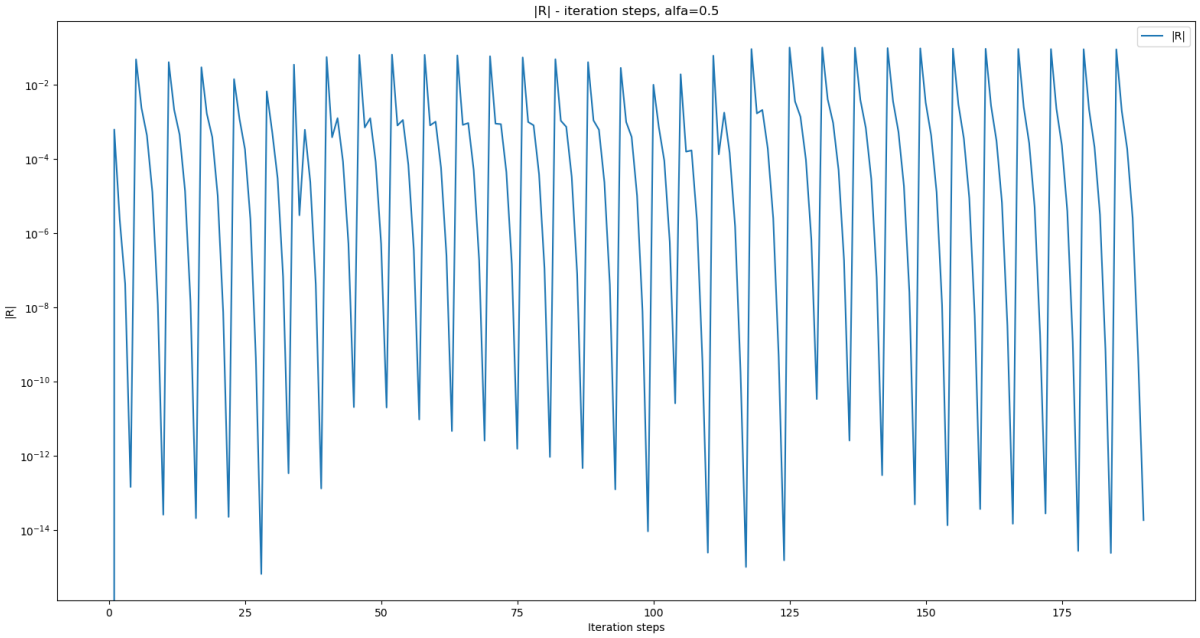


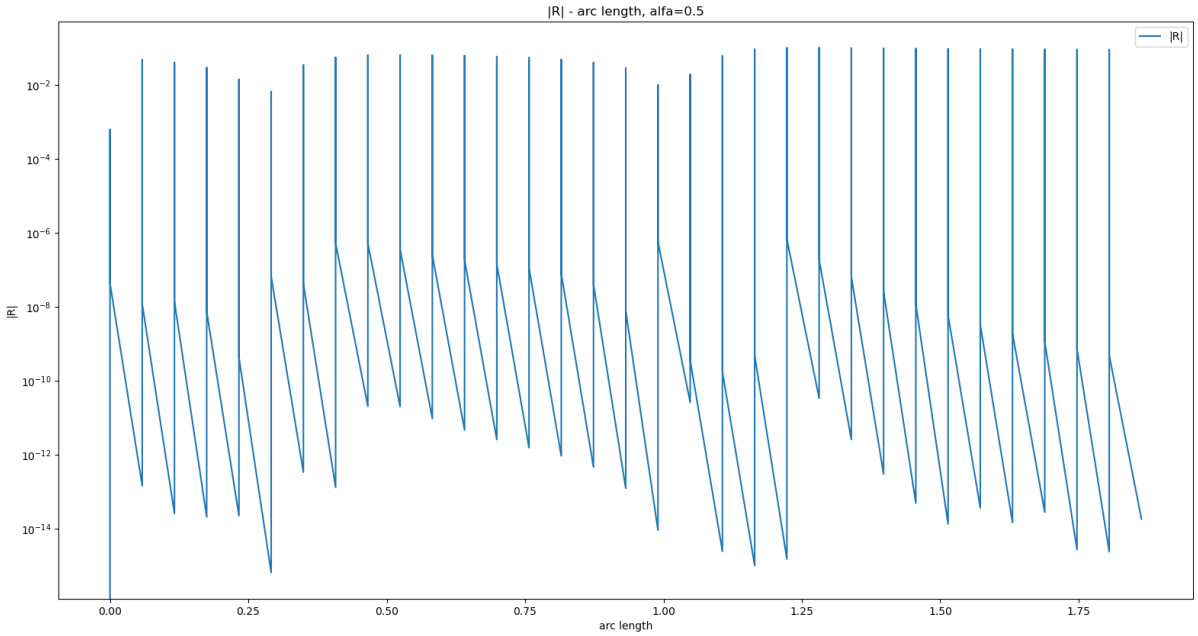



|R| - arc length, alfa=0
|R|
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-14}
10^{-17}
|R|
0.0
0.2
0.4
0.6
0.8
1.0
1.2
arc length

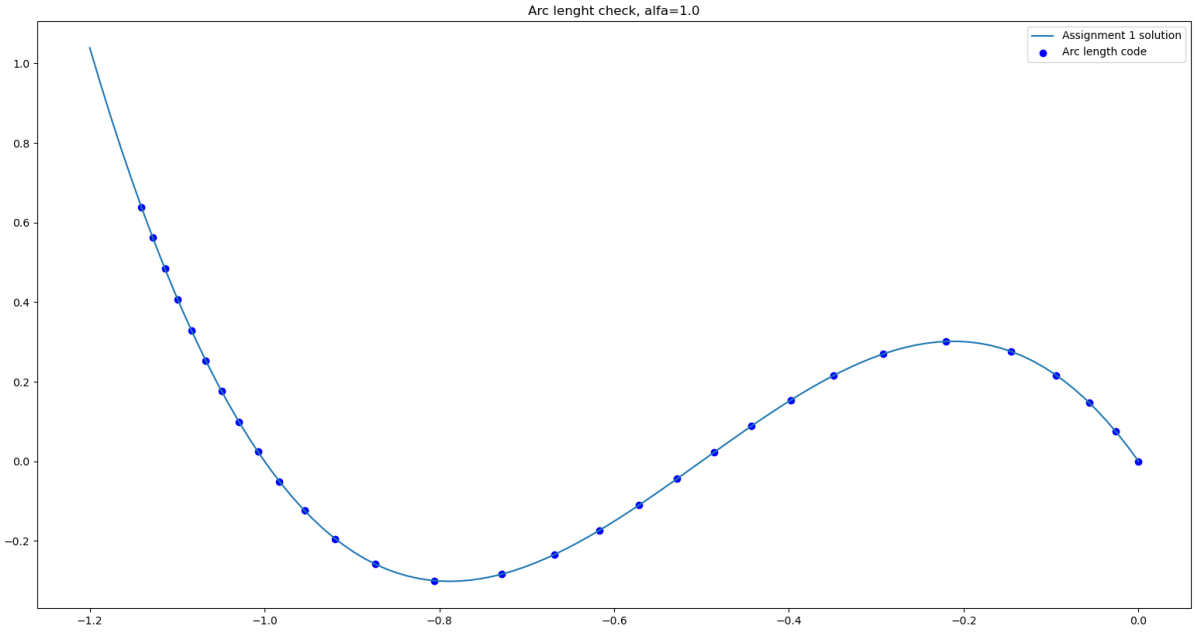
ALFA = 0.5

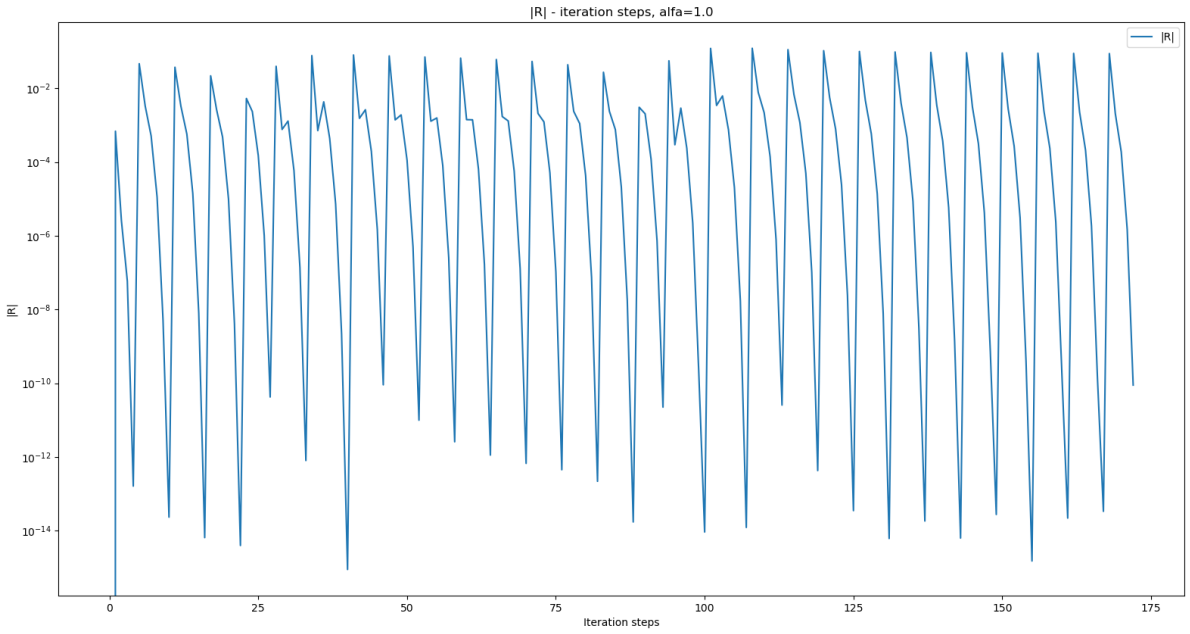

Arc lenght check, alfa=0.5
Assignment 1 solution
Arc length code
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−1.2
−1.0
−0.8
−0.6
−0.4
−0.2
0.0

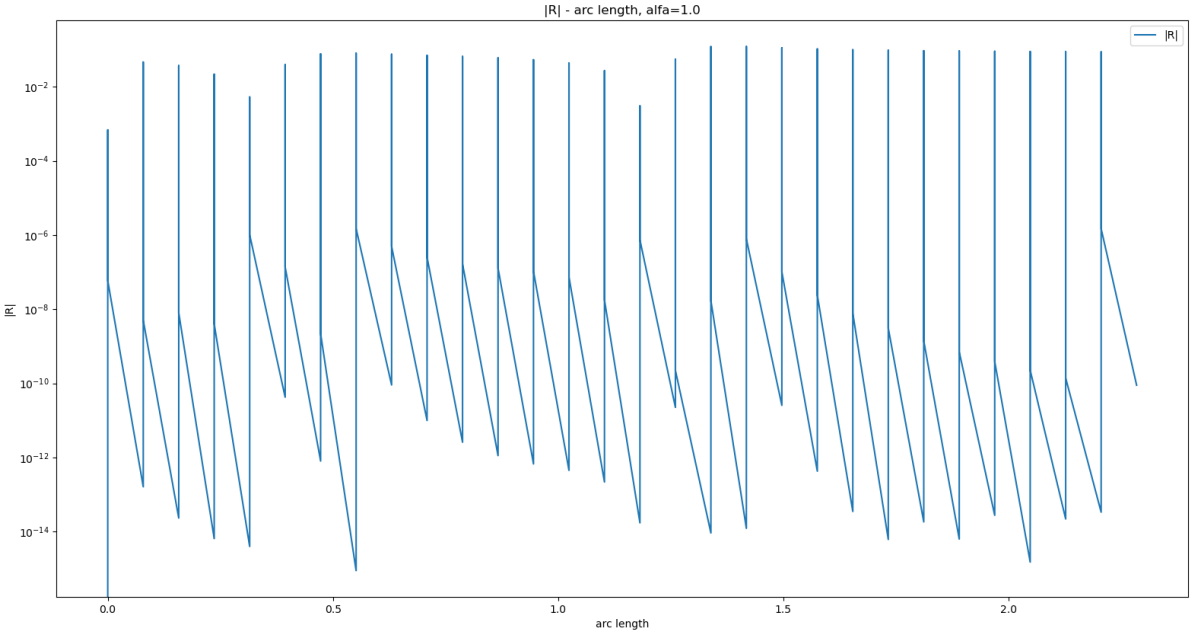

|R| - iteration steps, alfa=0.5
|R|
Iteration steps
0
25
50
75
100
125
150
175


|R| - arc length, alfa=0.5
|R|
arc length
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75

ALFA = 1.0


Arc lenght check, alfa=1.0
Assignment 1 solution
Arc length code
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−1.2
−1.0
−0.8
−0.6
−0.4
−0.2
0.0


|R| - iteration steps, alfa=1.0
|R|
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
0
25
50
75
100
125
150
175
Iteration steps


|R| - arc length, alfa=1.0
|R|
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
|R|
0.0
0.5
1.0
1.5
2.0
arc length

3-2

1. Find the tangents $\partial \underline{R}/\partial \gamma$ and $\partial \underline{R}/\partial \underline{u}$ for $\underline{R}(\gamma, \underline{u}) = \gamma(s)\,\bar{P} - \underline{F}(\underline{u}(s))$

using Henky strain, $\sigma = E\varepsilon$, $A = \text{const.}$ and equilibrium on deformed system

From assignment 1 we have $\partial \underline{R}/\partial \underline{u}$

$$\frac{\partial \underline{R}}{\partial \underline{u}} = \underline{\underline{K}}_{T,Fa} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

w/

$$K_{11} = EA\left(\frac{c_1^2(1-\varepsilon_1)+\varepsilon_1}{L_1} + \frac{c_2^2(1-\varepsilon_2)+\varepsilon_2}{L_2}\right)$$

$$K_{12} = K_{21} = EA\left(\frac{c_1 s_1(1-\varepsilon_1)}{L_1} - \frac{c_2 s_2(1-\varepsilon_2)}{L_2}\right)$$

$$K_{22} = EA\left(\frac{s_1^2(1-\varepsilon_1)+\varepsilon_1}{L_1} + \frac{s_2^2(1-\varepsilon_2)+\varepsilon_2}{L_2}\right)$$

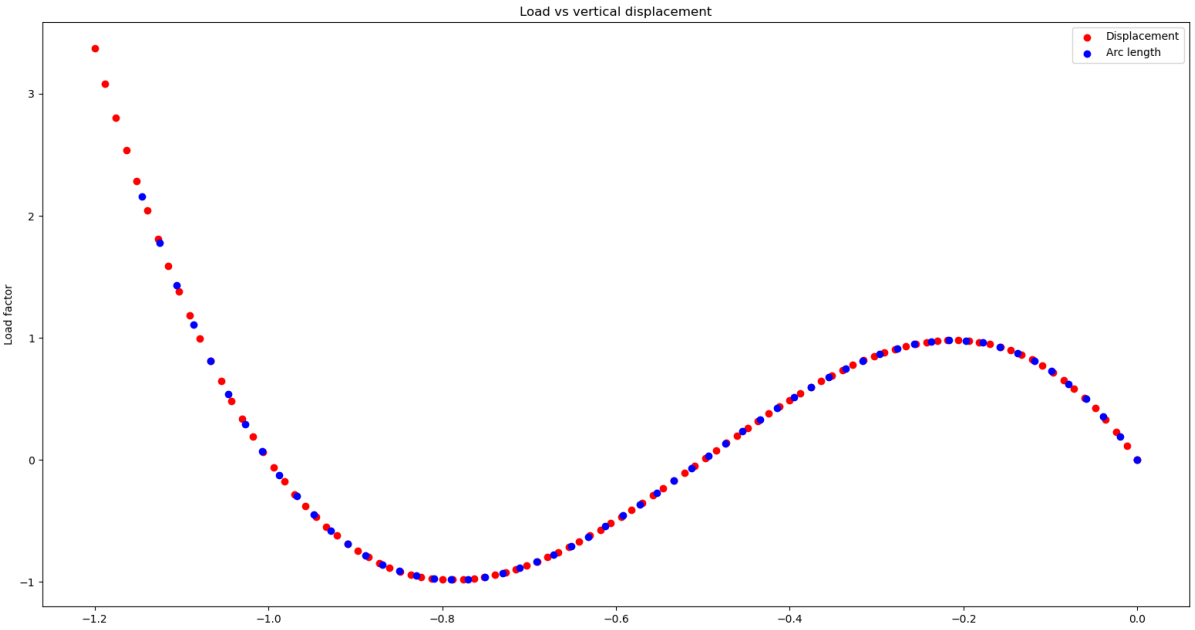
ε_1 & ε_2 Henky strain in elements 1 & 2

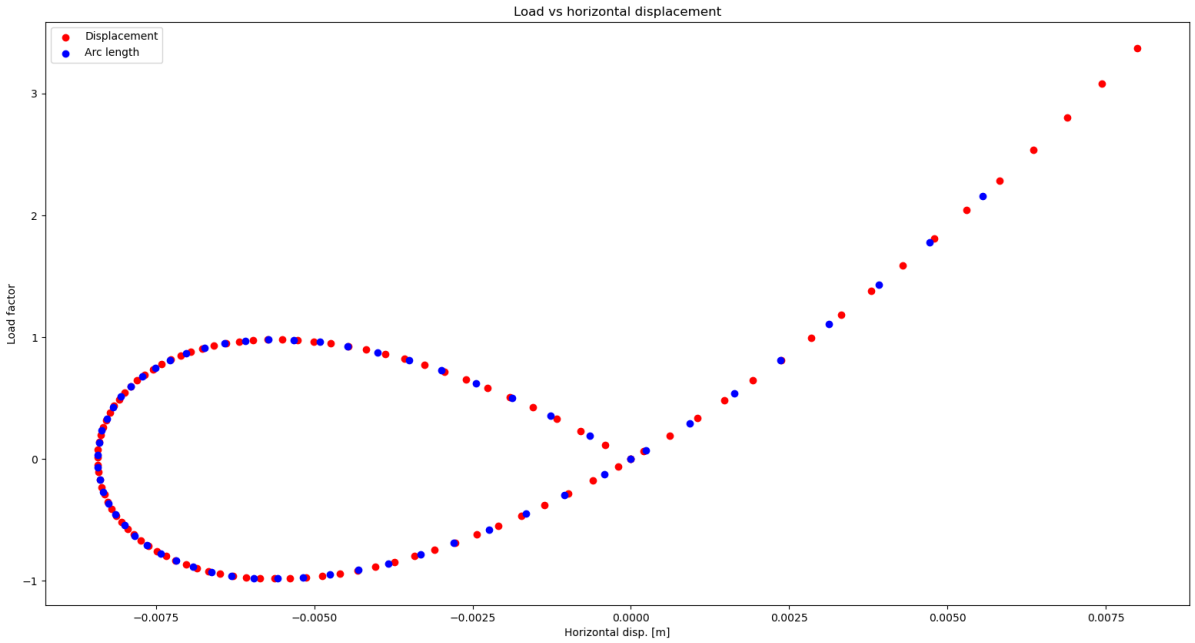
c_1 c_2 s_1 s_2 cos and sin of direction of elements 1 & 2

L_1 & L_2 deformed length of the elements

$$\frac{\partial \underline{R}}{\partial \gamma} = \bar{P} - 0 = \bar{P} = -0.99\,kN\ j$$

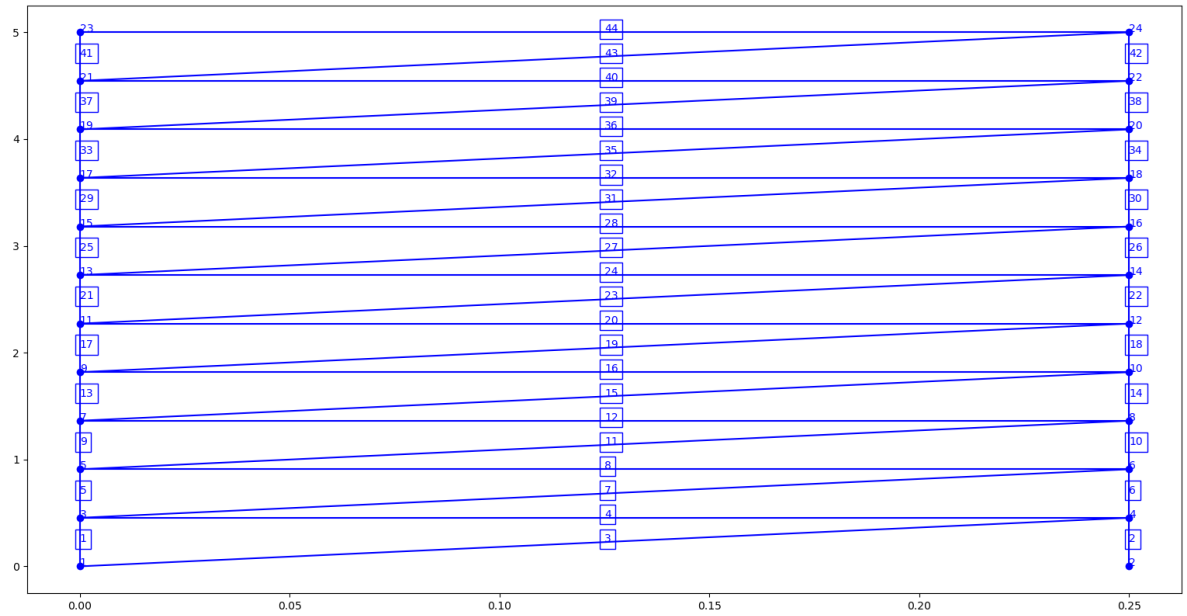
Problem 2


Load vs vertical displacement
Displacement
Arc length
Load factor
3
2
1
0
−1
−1.2
−1.0
−0.8
−0.6
−0.4
−0.2
0.0


Load vs horizontal displacement
Displacement
Arc length
Load factor
3
2
1
0
−1
−0.0075
−0.0050
−0.0025
0.0000
0.0025
0.0050
0.0075
Horizontal disp. [m]

Problem 3

Plot of structure



Nodes

[['no.' 'State' '[x, y, z]']
 [1 array([0, 0]) array([0., 0.])]
 [2 array([0, 0]) array([0.25, 0.])]
 [3 array([1, 1]) array([0.   , 0.45454545])]
 [4 array([1, 1]) array([0.25   , 0.45454545])]
 [5 array([1, 1]) array([0.   , 0.90909091])]
 [6 array([1, 1]) array([0.25   , 0.90909091])]
 [7 array([1, 1]) array([0.   , 1.36363636])]
 [8 array([1, 1]) array([0.25   , 1.36363636])]
 [9 array([1, 1]) array([0.   , 1.81818182])]
 [10 array([1, 1]) array([0.25   , 1.81818182])]
 [11 array([1, 1]) array([0.   , 2.27272727])]
 [12 array([1, 1]) array([0.25   , 2.27272727])]
 [13 array([1, 1]) array([0.   , 2.72727273])]
 [14 array([1, 1]) array([0.25   , 2.72727273])]
 [15 array([1, 1]) array([0.   , 3.18181818])]
 [16 array([1, 1]) array([0.25   , 3.18181818])]
 [17 array([1, 1]) array([0.   , 3.63636364])]
 [18 array([1, 1]) array([0.25   , 3.63636364])]
 [19 array([1, 1]) array([0.   , 4.09090909])]
 [20 array([1, 1]) array([0.25   , 4.09090909])]
 [21 array([1, 1]) array([0.   , 4.54545455])]
 [22 array([1, 1]) array([0.25   , 4.54545455])]
 [23 array([1, 1]) array([0., 5.])]
 [24 array([1, 1]) array([0.25, 5.])]]

Elements

[['no.' 'i-node' 'j-node' 'element']
 [1 1 3 <truss.TrussElement object at 0x000001A77DD1C240>]
 [2 2 4 <truss.TrussElement object at 0x000001A70D996EF0>]
 [3 1 4 <truss.TrussElement object at 0x000001A70DBCE470>]
 [4 3 4 <truss.TrussElement object at 0x000001A70DDB3D30>]
 [5 3 5 <truss.TrussElement object at 0x000001A70D9E58D0>]
 [6 4 6 <truss.TrussElement object at 0x000001A70D9E5128>]
 [7 3 6 <truss.TrussElement object at 0x000001A70CCA0358>]
 [8 5 6 <truss.TrussElement object at 0x000001A70CC9AEF0>]
 [9 5 7 <truss.TrussElement object at 0x000001A70DF13860>]
 [10 6 8 <truss.TrussElement object at 0x000001A70DF13898>]
 [11 5 8 <truss.TrussElement object at 0x000001A70DF138D0>]
 [12 7 8 <truss.TrussElement object at 0x000001A70DF13908>]
 [13 7 9 <truss.TrussElement object at 0x000001A70DF139B0>]
 [14 8 10 <truss.TrussElement object at 0x000001A70DF13C50>]
 [15 7 10 <truss.TrussElement object at 0x000001A70DF13C88>]
 [16 9 10 <truss.TrussElement object at 0x000001A70DF13CC0>]
 [17 9 11 <truss.TrussElement object at 0x000001A70DF13CF8>]
 [18 10 12 <truss.TrussElement object at 0x000001A70DF13D30>]
 [19 9 12 <truss.TrussElement object at 0x000001A70DF13D68>]
 [20 11 12 <truss.TrussElement object at 0x000001A70DF13DA0>]
 [21 11 13 <truss.TrussElement object at 0x000001A70DF13DD8>]
 [22 12 14 <truss.TrussElement object at 0x000001A70DF13E10>]
 [23 11 14 <truss.TrussElement object at 0x000001A70DF13E48>]
 [24 13 14 <truss.TrussElement object at 0x000001A70DF13E80>]
 [25 13 15 <truss.TrussElement object at 0x000001A70DF13EB8>]
 [26 14 16 <truss.TrussElement object at 0x000001A70DF13EF0>]
 [27 13 16 <truss.TrussElement object at 0x000001A70DF13F28>]
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 [29 15 17 <truss.TrussElement object at 0x000001A70DF13F98>]
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 [32 17 18 <truss.TrussElement object at 0x000001A70DF2D048>]
 [33 17 19 <truss.TrussElement object at 0x000001A70DF2D080>]
 [34 18 20 <truss.TrussElement object at 0x000001A70DF2D0B8>]
 [35 17 20 <truss.TrussElement object at 0x000001A70DF2D0F0>]
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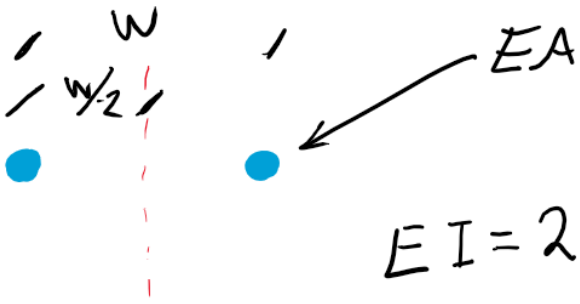
 [39 19 22 <truss.TrussElement object at 0x000001A70DF2D1D0>]
 [40 21 22 <truss.TrussElement object at 0x000001A70DF2D208>]
 [41 21 23 <truss.TrussElement object at 0x000001A70DF2D240>]
 [42 22 24 <truss.TrussElement object at 0x000001A70DF2D278>]
 [43 21 24 <truss.TrussElement object at 0x000001A70DF2D2B0>]
 [44 23 24 <truss.TrussElement object at 0x000001A70DF2D2E8>]]

Part 2

Estimated P.cr = 6.1685 kN



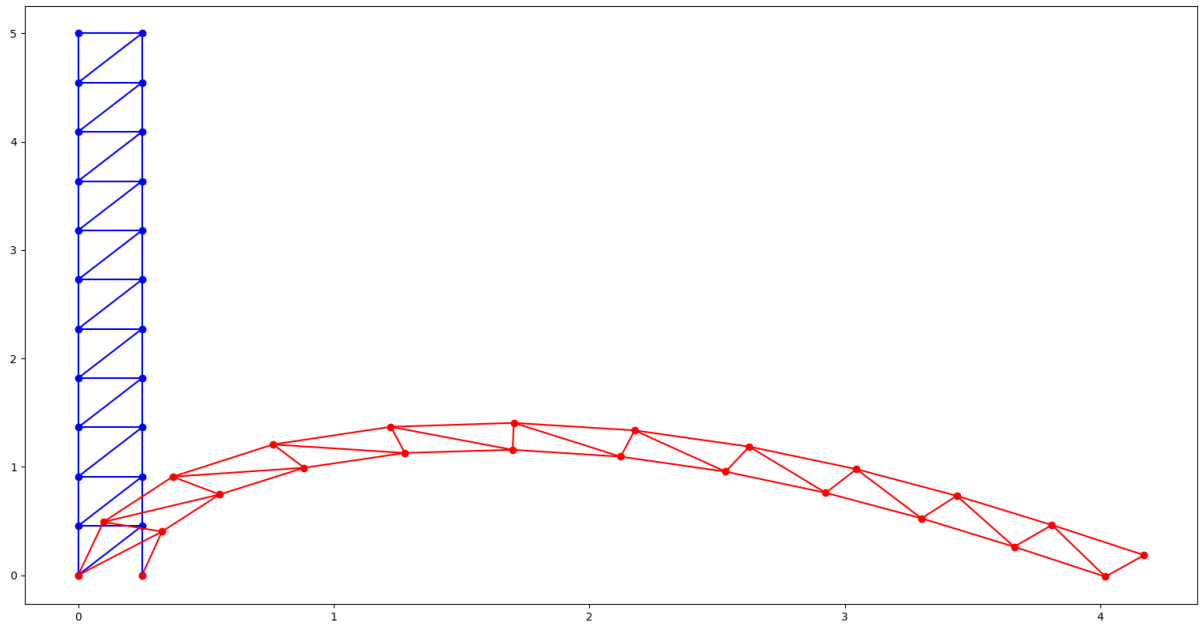
$$EI = 2 \cdot EA \left(\frac{W}{2}\right)^2$$

$$= \frac{EA}{2} W^2$$

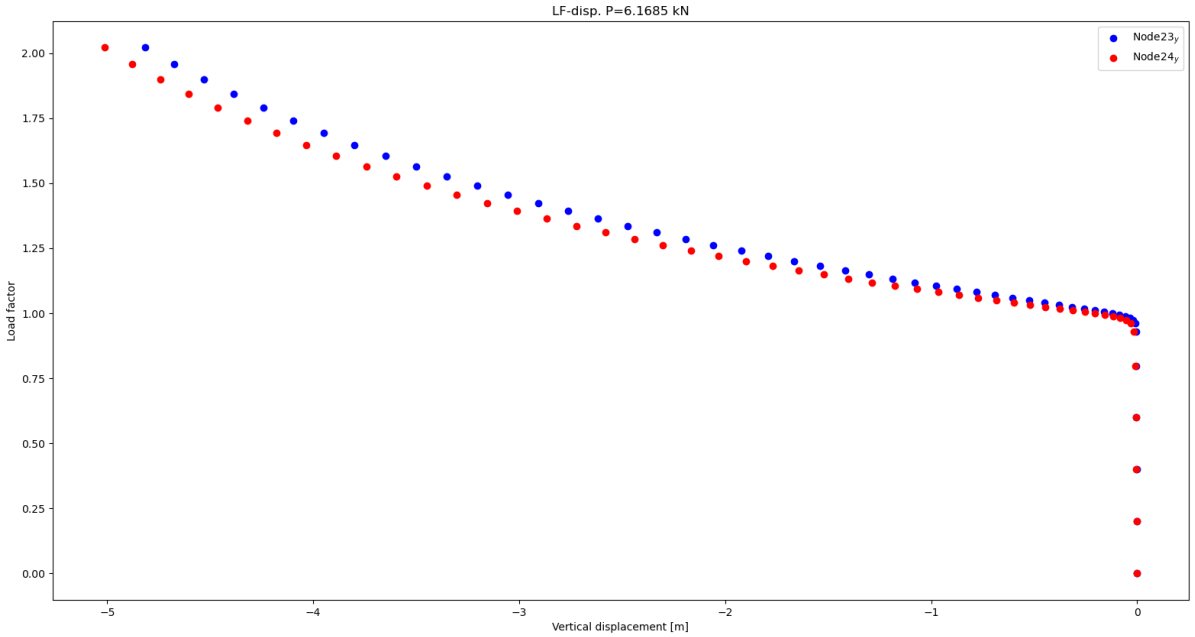
$$= 62{,}5\ kN\,m^2$$

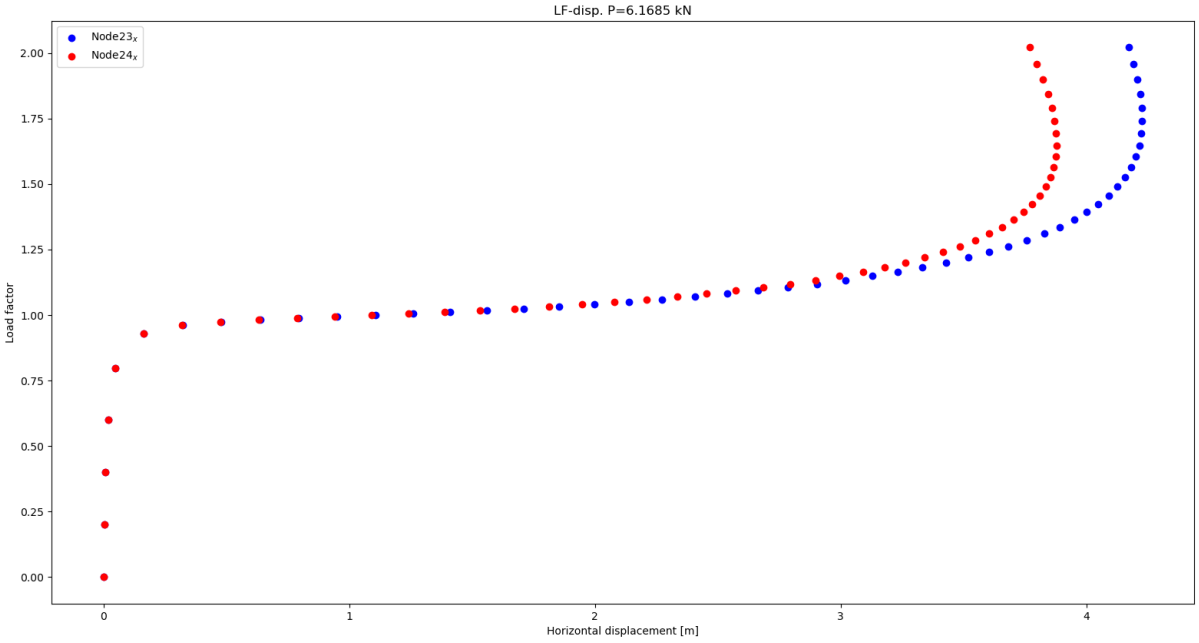
$$P_{cr} = \frac{EI \cdot \pi^2}{L^2} = \frac{62{,}5\ kN m^2 \cdot \pi^2}{(10\,m)^2} = 6.1685\ kN$$

Part 3

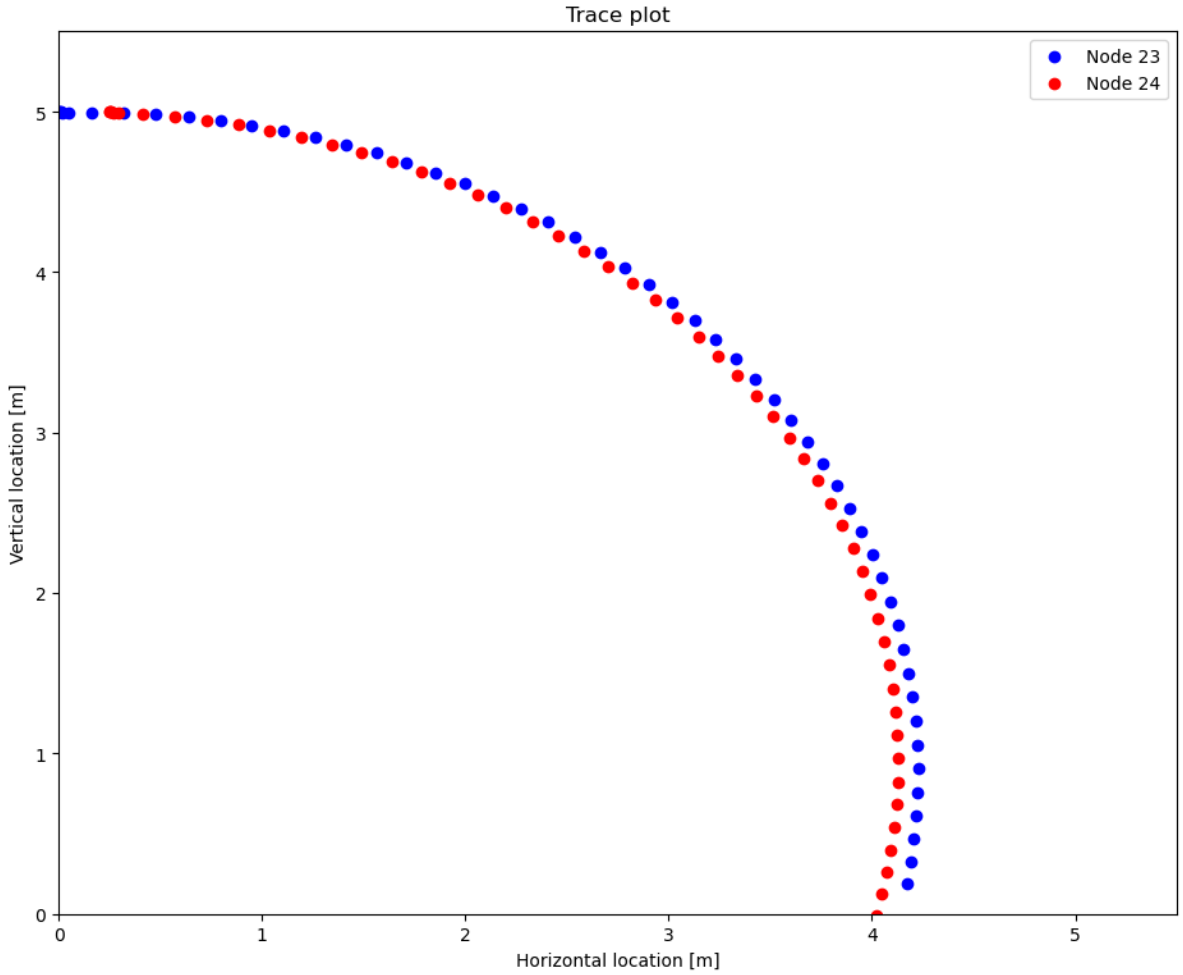


Part 4

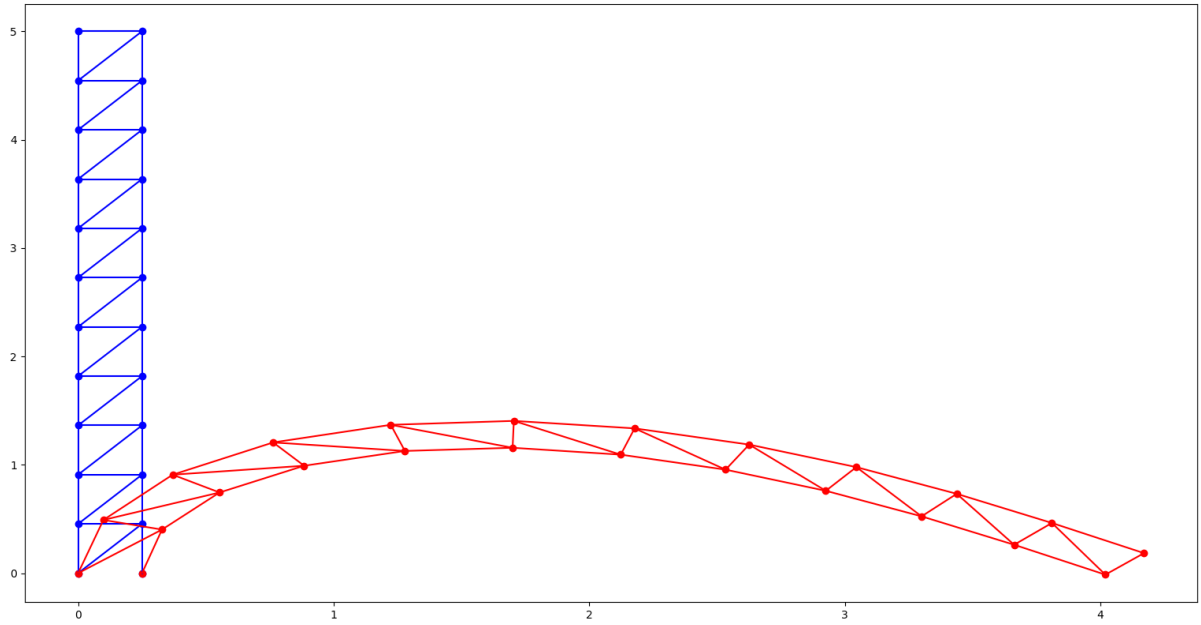

LF-disp. P=6.1685 kN
Node23$_y$
Node24$_y$
Load factor
Vertical displacement [m]


LF-disp. P=6.1685 kN
Node23$_x$
Node24$_x$
Load factor
Horizontal displacement [m]

Part 5


Trace plot
Node 23
Node 24
Vertical location [m]
Horizontal location [m]
0
1
2
3
4
5

Part 6 – Deflected structure



Code for all problems is accessible through github.
main.py runs all problems in secuence.