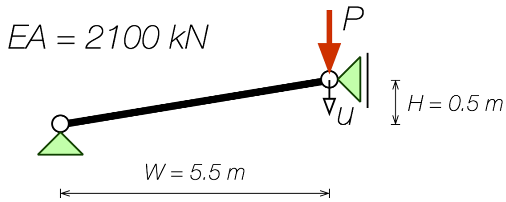
Problem 1-1: Study of different formulations for large deformation problems.



We looked at alternative formulations for strain, the constitutive relation, and the way to formulate equilibrium. For reference, here are the four strain measures:

$$\epsilon = \frac{\ell - L}{L} \tag{1a}$$

$$E = \frac{1}{2}\frac{\ell^2 - L^2}{L^2} \tag{1b}$$

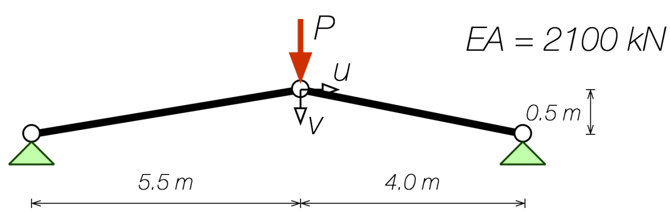
$$\varepsilon = \frac{1}{2}\frac{\ell^2 - L^2}{\ell^2} \tag{1c}$$

$$\tilde{\varepsilon} = \frac{1}{2}\ln\left(\frac{\ell^2}{L^2}\right) \tag{1d}$$

Assume the cross section area, A, does not change during the deformation process.

1. Derive the force-displacement relation for all possible combinations (8 options) of kinematic relations and equilibrium on the undeformed and the deformed configuration and compare them in a single plot for $0 \le u \le 2.5\,H$. Include the linear solution in the same plot.
2. Include a brief discussion on why some formulations may be better than others. Which one would you pick if you had to design this structure?

Problem 1-2: Two degree of freedom (2 DOF) problem – Load control.



This problem expands problem 1-1 to a small system with two degrees of freedom, u and v. Due to the non-symmetric nature of the problem, the vertical load P will activate both degrees of freedom.

1. Using Henkey strain, a linear relation $\sigma = E\varepsilon$, $A = const.$, and equilibrium on the deformed system, derive the relationship between displacements and forces on the free node. i.e.,

$$\left\{ \begin{array}{c} P_x \\ P_y \end{array} \right\} = \left\{ \begin{array}{c} F_x(u,v) \\ F_y(u,v) \end{array} \right\} \tag{2a}$$

2. Find the tangent stiffness matrix of the system. Keep it simple and do not substitute long expressions into your answer.

3. Formulate the condition $P_x = 0$ (no horizontal load) to the system of equations. Will this allow you to eliminate the horizontal displacement from the system of equations? Why? Why not?

4. Write a simple program that allows you to solve for the displacement vector using the generalized Newton method (also known as Newton-Raphson method) with

$$\left\{ \begin{matrix} P_x \\ P_y \end{matrix} \right\} = \lambda \left\{ \begin{matrix} 0 \\ P_{cr} \end{matrix} \right\} \tag{2b}$$

   with $\lambda = 0, 0.25, 0.5, 0.75, 0.99, 0.999$. List the residual for each iteration step at each of the load levels in a table.

   Hint: you will need to iterate to find P_{cr} even before solving for the given load levels. You may use your code to work up to that limit or use any other iterative or graphic technique to find that limit.

5. To what error limit could you compute the solution? Did that limit depend on the load level?

   Present the relative error for each load level in a table with one row per iteration step and one column per load level. Combine the error versus iteration step for all cases in a single semi-log plot (the relative error goes on the logarithmic axis).